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# A New Approach to Testing Dark Energy Models by Observations

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## Abstract

We propose a new approach to the consistency test of dark energy models with observations. To test a category of dark energy models, we suggest introducing a *characteristic* $Q(z)$ that in general varies with the redshift $z$ but in those models plays the role of a (constant) distinct parameter. Then, by reconstructing $dQ(z)/dz$ from observational data and comparing it with zero we can assess the consistency between data and the models under consideration. For a category of models that passes the test, we can further constrain the distinct parameter of those models by reconstructing $Q(z)$ from data. For demonstration, in this paper we concentrate on quintessence. In particular we examine the exponential potential and the power-law potential via a widely used parametrization of the dark energy equation of state, $w(z) = w_0 + w_a z/(1 + z)$, for data analysis. This method of the consistency test is particularly efficient because for all models we invoke the constraint of only a single parameter space that by choice can be easily accessed. The general principle of our approach is not limited to dark energy. It may also be applied to the testing of various cosmological models and even the models in other fields beyond the scope of cosmology.

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## I. INTRODUCTION

The accelerating expansion of the universe in the present epoch was discovered in 1998 [1, 2] via Type Ia supernova (SN Ia) distance measurement. This has been confirmed by more recent observations, including SN Ia [3, 4, 5], cosmic microwave background (CMB) [6] and large-scale structure (LSS) [7] observations. Models with a wide variety of strategies have been proposed to explain this salient phenomenon. One of the approaches invokes an energy source, generally referred to as "dark energy", that provides a significant negative pressure and therefore a repulsive gravity (anti-gravity). Examples of dark energy candidates include a positive cosmological constant [8, 9, 10] and a dynamical scalar field such as quintessence [11, 12, 13] and phantom [14].

So far many of these models remain consistent with observational results. In this situation the dark energy information obtained by comparing the individual theoretical model with the observational data is indecisive. Responding to this deficiency, recently cosmologists attempt to extract the generic features of dark energy, such as (the constraints on) the equation of state (EoS) or the energy density as a function of the redshift, from observational results by invoking model-independent parametrizations in data analysis [15, 16, 17, 18]. It is hoped that several generic questions can be addressed through this approach. A particularly important question is: Is dark energy a cosmological constant? If not, it is essential to explore how the dark energy density evolves with time. A specific manifestation of this would be the deviation of the dark energy EoS from $-1$.

Along a similar line with the utilization of a parametrization, in the present paper we propose a new approach to the testing of the consistency between observational results and dark energy models or, more generally, cosmological models. For each category of dark energy models we suggest introducing a distinct *characteristic* $Q(z)$ that in general varies with time and the redshift $z$ but is equivalent to an essential constant parameter within the scope of that category of models. In general the quantities, $Q(z)$ and $dQ(z)/dz$, by design can be reconstructed from data with no reliance on the other parameters of the models. The consistency between data and models can then be assessed by comparing the observational constraint on $dQ(z)/dz$ with the theoretical prediction, $dQ(z)/dz = 0$. This approach in principle provides a simple "litmus test" for each category of dark energy models. In addition, for a category of models that passes the test we can further constrain the distinct parameter of those models by reconstructing $Q(z)$ from data.

Close in spirit to our approach, Zunckel and Clarkson [19] investigated the consistency test of the flat $\Lambda$CDM model with a parametrization of the luminosity distance-redshift relation invoked. For the consistency test of $\Lambda$CDM, a natural choice of the distinct characteristic is the dark energy density $\rho_{\rm DE}(z)$ that in general is time-varying but is constant within $\Lambda$CDM. An equivalent alternative choice invoked by Zunckel and Clarkson in [19] is $Q_\Lambda(z) = 1 - \rho_{\rm DE}(z)/\rho_{\rm c}$ that within flat $\Lambda$CDM is equal to the matter density fraction $\Omega_{\rm m}$, an essential constant parameter. By comparing the observational constraint on $dQ_\Lambda(z)/dz$ with zero, one can assess the consistency between data and the $\Lambda$CDM model.

A conventional method of comparing models with observational results is the *model-based approach*. In this approach one directly invokes a specific category of models to fit data and obtains the constraint on the parameter space associated with those models (e.g., see [20, 21, 22]). One can then assess the goodness of fit (e.g., see Ref. [23]) for manifesting how well a model fits the data. In contrast, the consistency test is for examining whether the condition required for a category of models is excluded by observational results, which is different in spirit from the model-based approach. These two methods are complimentary to each other in the quest for revealing the nature of dark energy.

Our approach to the consistency test and to the constraining of dark energy models can be efficiently performed because for all models we invoke the constraint of only a single parameter space that by choice can be easily accessed. Our approach is particularly simple and fast when applied to quintessence. It is because in this approach one does not need to numerically solve the coupled field equations, the quintessence field equation and the Einstein equations. This benefit will be demonstrated in Sec. III and discussed in Sec. IV. In contrast to the benefit, on the other hand we note that the utilization of parametrization may be accompanied with a bias against certain models. For all the approaches invoking parametrization, the model-(in)dependence and the potential concomitant bias of the chosen parametrization are a separate and essential issue that requires further exploration. We are currently investigating this issue for our approach [24].

To demonstrate our approach, in the rest of the present paper we will concentrate on quintessence. We attempt to extract physical information about quintessence from the current observational results via a widely used parametrization of the EoS of dark energy

(DE) [15, 25],

$$w_{\rm DE}(z) \text{ or } w_\phi(z) = w_0 + w_a(1-a) = w_0 + w_a z/(1+z)\,. \tag{1}$$

Here $z$ is the redshift and $a$ the scale factor of the universe; the present scale factor $a_0$ has been set to unity by assuming the (spatial) flatness of the universe for simplicity. The constraints on $w_0$ and $w_a$ from observational data have been obtained by Riess et al. [4]. We adopt the constraints of these two parameters with the "weak prior" [4], for which they invoked the SN Ia data [4] as well as the constraints from the LSS measurement [26, 27], the baryon acoustic oscillation (BAO) measurement [28], the Cepheid measurement [29, 30] and the 3-year WMAP results for CMB [31]. With the observational constraint on the chosen parameter space we proceed the testing and the constraining of quintessence in three directions: (A) We reconstruct the quintessence potential with data. (B) We assess the consistency between the quintessence models and data. (C) We constrain the distinct parameter of the quintessence models by data.

**(A) Reconstruction of potential**

To reconstruct the quintessence potential, we will convert the constraint on the chosen parameter space $(w_0, w_a)$ to that on the potential $V(\phi)$. The results will be presented in Sec. III A.

Quintessence can be reconstructed [32, 33, 34] from data via the parametrization of its potential [35] or other relevant physical quantities. In the present demonstration of our approach we invoke the parametrization of the dark energy EoS (e.g., see [36]) in Eq. (1). The reconstruction in this way is particularly simple when the observational constraints on the EoS parameters can be easily accessed or have already been obtained.

**(B) Assessment of consistency**

To assess the consistency between quintessence models and data, for each category of quintessence models we invoke a characteristic $Q(z)$ that possesses the following features, as our guidelines of constructing $Q(z)$.

1. In general $Q(z)$ is time-varying.
2. Within the scope of the models under consideration $Q(z)$ is constant.
3. For those models $Q(z)$ plays the role of a distinct parameter.
   (This feature is oriented to fulfilling Direction C.)

4. By our construction $Q(z)$ is a functional of the parametrized physical quantity $P(z)$, which in the present demonstration is the dark energy EoS, $w_{\rm DE}(z)$. This is so designed that $Q(z)$ and its derivative $dQ(z)/dz$ can be reconstructed from data via the constraint on the parameters involved in the $P(z)$ parametrization, for example, via the constraint on $(w_0, w_a)$ as considered in the present demonstration.

5. Accordingly, the (in)compatibility between the observational constraint on $dQ(z)/dz$ and the theoretical prediction, $dQ(z)/dz = 0$, tells the (in)consistency between data and models.

We will convert the constraint on the chosen parameter space $(w_0, w_a)$ to that on $dQ(z)/dz$, and compare it with zero, thereby assessing the consistency between the data and the models under consideration. The results will be presented in Sec. III B.

**(C) Constraining the distinct parameter**

For a category of quintessence models with a characteristic $Q(z)$ introduced in the above-mentioned manner, we will convert the constraint on the chosen parameter space $(w_0, w_a)$ to that on $Q(z)$, thereby giving the constraint on the distinct parameter of the potential. The results will be presented in Sec. III C.

As a demonstration of the effectiveness of our approach, for Direction B we examine the exponential potential (for a recent study see [22] and references therein) and the power-law potential (that with a negative power is invoked in the tracker quintessence [37]). We conclude: (Direction B) at the 68% confidence level both are ruled out; at the 95% confidence level the exponential potential remains disfavored, while the power-law potential is consistent with data. Since the exponential potential is inconsistent with data, for Direction C we deal with only the power-law potential for which the power-law index is the distinct parameter in our treatment. We conclude: (Direction C) for the power-law potential, at the 95% confidence level a negative power-law index between $-2$ and 0 can fit the data, whereas a positive power is ruled out.

## II. THE BASICS

Consider a Friedmann-Lemaitre-Robertson-Walker (FLRW) universe described by the Robertson-Walker metric,

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \tag{2}$$

and assume that it is dominated by pressureless matter and quintessence in the present epoch. Quintessence is a dynamical scalar field described by the Lagrangian density,

$$\mathcal{L} = \sqrt{|g|}\left[\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)\right]. \tag{3}$$

For simplicity, we assume that the universe is spatially flat ($k = 0$) and that the spatial dependence of quintessence is weak so that the spatial curvature and the spatial derivative terms are ignored. The governing equations, i.e. the Einstein equations and the quintessence field equation, for the cosmic evolution are then as follows.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\left(\rho_{\rm m} + \rho_\phi\right), \tag{4}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right) = -\frac{4\pi G}{3}\left(\rho_{\rm m} + \rho_\phi + 3p_\phi\right), \tag{5}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \tag{6}$$

where the Hubble expansion rate is defined as $H \equiv \dot{a}/a$, and the energy density and pressure of quintessence are given by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) = K + V, \tag{7}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) = K - V. \tag{8}$$

The EoS of quintessence is therefore

$$w_\phi = p_\phi/\rho_\phi = (K - V)/(K + V). \tag{9}$$

Next we derive the expressions for the relevant quantities in terms of $w_\phi(z)$ and the essential cosmological parameters such as $\Omega_\phi$ and $\Omega_m$. From the Einstein equations, we have

$$\rho_\phi(z) = \rho_c\Omega_\phi \exp\left(3\int_0^z \left[1 + w_\phi(z')\right]\frac{dz'}{1+z'}\right), \tag{10}$$

where the critical density $\rho_c \equiv 3H_0^2/8\pi G_N$. Therefore,

$$H^2(z) = \frac{8\pi G_N}{3} (\rho_m + \rho_\phi)$$
$$= H_0^2 \left[ \Omega_m (1+z)^3 + \Omega_\phi \exp\left( 3 \int_0^z [1 + w_\phi(z')] \frac{dz'}{1+z'} \right) \right]. \quad (11)$$

In addition, from Eqs. (7) – (9), we have

$$K(z) = [1 + w_\phi(z)]\, \rho_\phi(z)/2\,, \quad (12)$$
$$V(z) = [1 - w_\phi(z)]\, \rho_\phi(z)/2\,, \quad (13)$$

from the former of which we obtain

$$\phi(z) - \phi_0 = \pm \int_0^z \frac{\sqrt{[1 + w_\phi(z')]\, \rho_\phi(z')}}{H(z')} \frac{dz'}{1+z'}\,, \quad (14)$$

where $\phi_0$ is the present value of the quintessence field.

For a given parametrization of $w_\phi(z)$, a parametric relationship between the potential $V$ and the quintessence field $\phi$ can be deduced from Eqs. (13) and (14) with the help of Eqs. (10) and (11). Based on this relation, from the observational constraint of $w_\phi(z)$ one can then in principle reconstruct the quintessence potential $V(\phi)$ as well as other quantities which can be expressed in terms of $V(z)$, $\phi(z)$ and their derivatives.

## III. TESTING AND CONSTRAINING QUINTESSENCE

In this section we test and constrain quintessence with the current observational results via the $w_\phi(z)$ parametrization in Eq. (1) in three directions: (A) reconstructing the quintessence potential $V(\phi)$, (B) assessing the consistency between a category of quintessence models and data via the derivative of a distinct characteristic, $dQ(z)/dz$, and (C) constraining the distinct parameter of a category of quintessence models played by the characteristic $Q(z)$. The observational constraints on the two parameters in $w_\phi(z)$ have been obtained by Riess et al. in Ref. 4, as shown in Fig. 10 therein. We invoke the 68% and the 95% confidence contours in the left panel, i.e. with a weak prior, of that figure and focus on the region satisfying $w(z) > -1\, \forall\, z$ for quintessence.[1] These contours were deduced from the SN Ia data [4] and the constraints from other measurements [26, 27, 28, 29, 30, 31]. The

[1] By definition, no quintessence can be reconstructed from the region where $w < -1$ for some $z$.

redshift range of these supernovae is $0 < z < 1.8$. In the reconstruction we set $\Omega_m = 0.28$ and $\Omega_\phi = 0.72$.

### A. Reconstruction of potential

The reconstructed potential is sketched in Fig. 1, where the dark gray and the light gray areas correspond to the 68% and the 95% confidence regions, respectively. From this figure the shape of the quintessence potential is already apparent.

Note that the 68% and the 95% confidence contours in the $(w_0, w_a)$ parameter space, from which the potential is reconstructed, enclose both the quintessence and the non-quintessence cases. It is therefore the probability space of $(w_0, w_a)$, but not that of the quintessence models, that the confidence of the constraints in Fig. 1 is measured against.

### B. Assessment of consistency

To assess the consistency between quintessence models and data, we deal with one category of potentials (in principle embracing infinitely many specific potentials) at once. We facilitate this consistency assessment by introducing a *characteristic* $Q(z)$ with several features listed in Introduction. By construction, $Q(z)$, although is in general time-varying, plays the role of a constant distinct parameter within the scope of the models under consideration.

The characteristic $Q(z)$ by design can be expressed in terms of $V$, $dV/d\phi$, $d^2V/d\phi^2$, ....[2] As shown in the previous section, $V(z)$ and $\phi(z)$ are the functionals of $w_\phi(z)$ and the cosmological parameters $\{H_0, \Omega_m, \Omega_\phi\}$. Accordingly, the characteristic $Q(z)$ so constructed and its derivative $dQ(z)/dz$ are also the functionals of $w_\phi(z)$ and those cosmological parameters, and can therefore be reconstructed from data via the constraint on $w_0$ and $w_a$ (in the same way as that for the potential reconstruction). Then, by comparing the reconstructed $dQ/dz$ with zero within the redshift range $0 < z < 1.8$, one can determine the consistency between the data and the quintessence potentials under consideration.

As a demonstration of the effectiveness of this approach, in the following we will examine

[2] For a category of potentials $V(\phi; q_i)$ involving N parameters, $\{q_i\,,\ i = 1, 2, \ldots, N\}$, one can treat $V, V', V'', \ldots, V^{[N]}$ as $N+1$ equalities for $N+1$ variables: $\phi$ and $q_i$. Then, in general, one can obtain $q_i$ as a function of $V, V', V'', \ldots, V^{[N]}$ by solving these equalities.

the exponential potential and the power-law potential,

$$V_{\rm exp}(\phi) \;=\; V_A \exp\left[-\phi/M_0\right], \tag{15}$$

$$V_{\rm power}(\phi) \;=\; m^{4-n_0}\phi^{n_0}. \tag{16}$$

To examine the exponential potential in Eq. (15), we introduce the following characteristic,

$$Q_{\rm exp}(z) = M^{-1}(z) \equiv -\frac{1}{V(z)}\frac{dV}{d\phi}(z)\,, \tag{17}$$

which for the exponential potential is equal to the essential parameter $1/M_0$. This characteristic, by construction, does not explicitly involve $\phi(z)$. Naively, to check whether a (reconstructed) potential follows the exponential behavior in Eq. (15) one can plot $\ln V$ versus $\phi - \phi_0$ and see whether there exists a straight line passing through the shaded region, for which the characteristic $Q_{\rm exp}$ in Eq. (17) is the slope. Note that in this exponential case the value of $\phi_0$ does not affect the existence of the straight line within the shaded region. Instead of exhausting all straight lines with the try-and-error method, we take a more efficient approach by comparing the derivative of the characteristic, $dQ_{\rm exp}(z)/dz$, with the zero-line (a single line), as mentioned earlier.

The derivative of the characteristic w.r.t. the redshift $z$, $dM^{-1}(z)/dz$, reconstructed from data for $0 < z < 1.8$ is shown in Fig. 2 (dark gray for 68% confidence and light gray for 95%). From this figure, one can see that the horizontal zero-line lies outside the 68% confidence constraint for all $z$ and is on the margin of the 95% confidence constraint for $z > 0.8$. Accordingly, at the 68% confidence level [with regard to the $(w_0, w_a)$ probability space] the quintessence model with the exponential potential is ruled out, and at the 95% confidence level the likelihood of this model to describe the cosmic evolution is marginal for $z > 0.8$.

To examine the power-law potential in Eq. (16), we introduce the following characteristic,

$$Q_{\rm power}(z) = n(z) \equiv \left[1 - V(z)\left(\frac{dV}{d\phi}(z)\right)^{-2}\frac{d^2V}{d\phi^2}(z)\right]^{-1}, \tag{18}$$

which for the power-law potential is equal to the power-law index $n_0$, an essential constant that characterizes the potential. This characteristic, by construction, does not explicitly involve $\phi(z)$. Naively, to check whether a (reconstructed) potential follows the power-law behavior in Eq. (16) one can plot $\ln V$ versus $\ln\phi$ and see within the constrained region whether there exists a straight line, of which the characteristic $Q_{\rm power}$ in Eq. (18) is the slope. However, the variable we reconstruct is $\phi(z) - \phi_0$, not $\phi(z)$ itself. Therefore what we can directly plot is $\ln V$ versus $\ln(\phi - \phi_0)$, but not $\ln V$ versus $\ln \phi$. We note that in this power-law case the value of $\phi_0$ plays an essential role in the existence of the straight line, and accordingly in this naive method one cannot assess the consistency without the information about $\phi_0$. This is the reason why we avoid invoking $\phi(z)$ in constructing the characteristics, such as those in Eqs. (17) and (18). Following the same procedure mentioned in the previous paragraphs, we proceed again with the derivative of the characteristic.

The quantity, $dn(z)/dz$, reconstructed from data for $0 < z < 1.8$ is shown in Fig. 3 (dark gray for 68% confidence and light gray for 95%). As shown in this figure, the horizontal zero-line is outside the 68% confidence constraint for most of the redshift $z$ but within the 95% confidence constraint for all $z$ under consideration. Accordingly, while the quintessence model with the power-law potential is ruled out at the 68% confidence level, it can fit the current data at the 95% confidence level [with regard to the $(w_0, w_a)$ probability space].

### C. Constraining the distinct parameter

In addition to the derivative, $dn(z)/dz$, the characteristic $n(z)$ itself is also reconstructed from data, as illustrated in Fig. 4 (dark gray for 68% confidence and light gray for 95%). According to the 95% confidence constraint, a negative power-law index between $-2$ and 0 is favored, whereas the model with a positive power is inconsistent with the current data.

### D. Distinguishing between models

Here we demonstrate how well the two models considered above can be distinguished from other models via our approach with the future supernova observations. We invoke the SNAP-quality [38] simulated data with 2023 SNe [39] distributed in the redshift range between 0 and 1.7, as well as the current-quality simulated CMB [6] and BAO [28] data. We take the other models one by one as the fiducial model to generate 1000 realizations of the simulated data, with which we obtain the observational constraints on $dQ_{\rm exp}(z)/dz$ and $dQ_{\rm power}(z)/dz$. If the observational constraint is inconsistent with the theoretical prediction, $dQ(z)/dz = 0$, the fiducial model can be distinguished from the model with which the characteristic $Q(z)$ is associated.

We consider eight fiducial models:

M1: $w_{\rm DE} = w_\Lambda = -1$,

M2: $w_{\rm DE} = -0.8$,

M3: $w_{\rm DE} = -1 + 0.5z/(1+z)$,

M4: $w_{\rm DE} = -1 + 1.5z/(1+z)$,

M5: $w_{\rm DE} = -0.8 - 0.2z/(1+z)$,

M6: $w_{\rm DE} = -1.05 + 0.2z/(1+z)$,

M7: $w_{\rm DE} = -0.6 - 0.5z/(1+z)$,

M8: $w_{\rm DE} = -1.05 + 1.0z/(1+z)$.

(The first four models are considered in [19].) The observational constraints obtained w.r.t. these fiducial models on $dQ_{\rm exp}(z)/dz$ and $dQ_{\rm power}(z)/dz$ are presented in Fig. 5 (dark gray for 68% confidence and light gray for 95%). As shown by this figure, with the future observations considered above and via our approach, the exponential-potential quintessence model is distinguishable from all the eight dark energy models, while the power-law potential can be distinguished from the models with faster evolving $w$: M3 (68% confidence), M4, M7 and M8 (95% confidence), but not from those with more slowly evolving $w$: M1, M2, M5 and M6.

## IV. SUMMARY

In this paper we propose a new approach to the testing and the constraining of dark energy models based on observational results. To demonstrate our approach, we concentrate on quintessence and proceed in three directions: (A) We reconstruct the quintessence potential. (B) We assess the consistency between quintessence models and data. (C) We obtain the constraint on the distinct parameter of one category of quintessence models.

For Direction B, to assess the consistency between quintessence models and data, we introduce a characteristic $Q(z)$ for each category of theoretical potentials. This characteristic $Q(z)$ is in general time-varying, but within the scope of those potentials it is constant and equivalent to a distinct parameter therein. By comparing the reconstructed $dQ(z)/dz$ with zero we can assess the consistency between data and that category of potentials. This approach provides a simple "litmus test" for each category of quintessence models. For Direction C, since the characteristic $Q(z)$ plays the role of the distinct parameter within the scope of a category of models, via reconstructing $Q(z)$ from data we obtain the constraint on that distinct parameter.

Our approach to the consistency test and the constraining of dark energy models is simple and efficient to perform. This is because in this approach (i) for all models the characteristics $Q(z)$ and their derivatives $dQ(z)/dz$ are reconstructed from data via the observational constraints of a single parameter space that by choice can be easily accessed, and (ii) with our design of the characteristic $Q(z)$ we can test the models and constrain their distinct parameter without the knowledge of the other parameters of the models. The simplicity and the efficiency of our approach is particularly manifest when it is applied to quintessence. In this case, in addition to the above-mentioned two general features, a specific technical reason for this benefit is that in our approach one does not need to numerically solve the quintessence field equation and the Einstein equations coupled together.

To demonstrate the effectiveness of our approach, we invoked a widely used parametrization of the dark energy EoS, and investigated the exponential potential and the power-law potential of quintessence models. For each of them a characteristic was introduced. Via these characteristics, the two theoretical potentials were compared with the current data. We found that at the 68% confidence level both the exponential and the power-law potentials were ruled out. When relaxed to the 95% confidence constraint, the power-law potential with a negative power-law index between $-2$ and 0 can fit the current data. In contrast, the exponential potential remains disfavored.

To perform our approach, one may choose other parametrizations. Generally speaking, an approach invoking parametrization may be accompanied with a bias against certain models. This is an essential issue that requires further study. The potential bias of performing our approach via the parametrization of the dark energy EoS in Eq. (1) is currently under our investigation [24].

To sum up, our approach provides a useful tool for testing and constraining dark energy models based on observational results. This approach can be applied to a variety of quintessence models and other dark energy models, and probably to other explanations of the cosmic acceleration. The general principle of our approach may also be applied to different cosmological models and even the models in other fields beyond the scope of cosmology.

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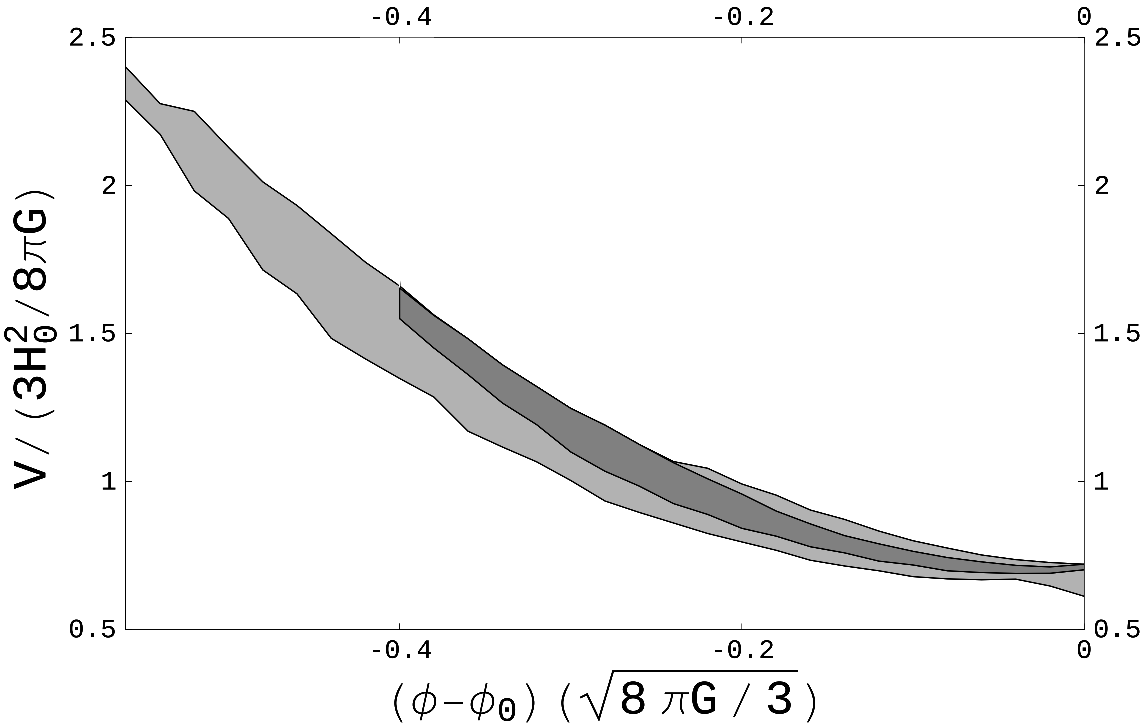


FIG. 1: The potential reconstructed from data via the $w_\phi(z)$ parametrization in Eq. (1).

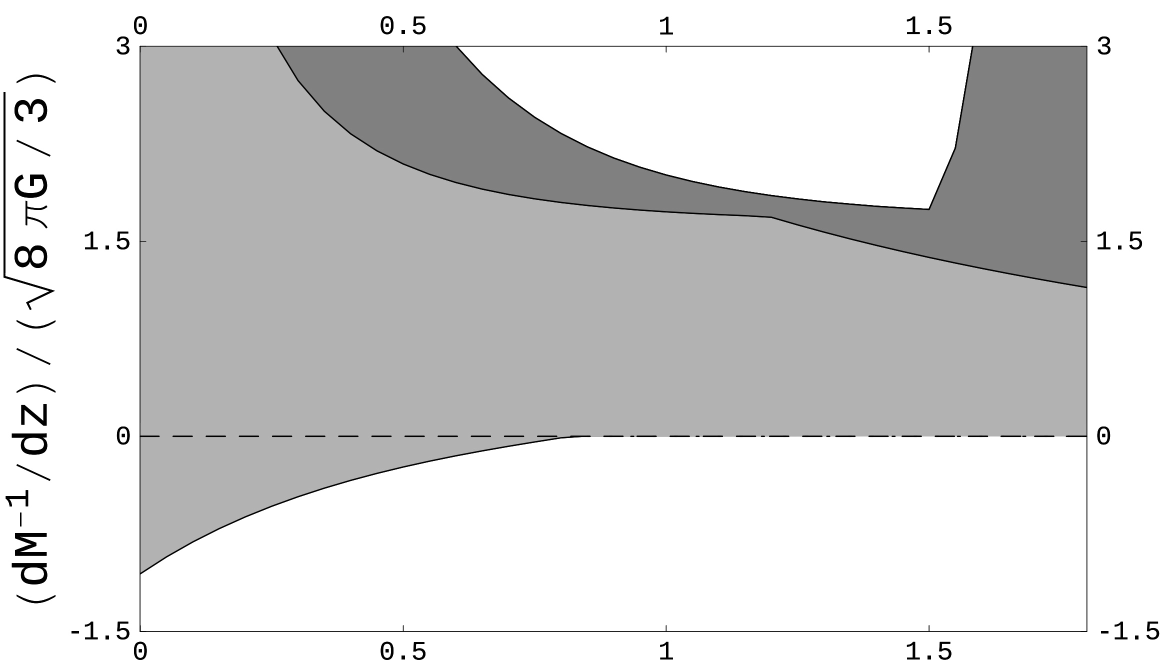


FIG. 2: The derivative of the characteristic of the exponential potential reconstructed from data.

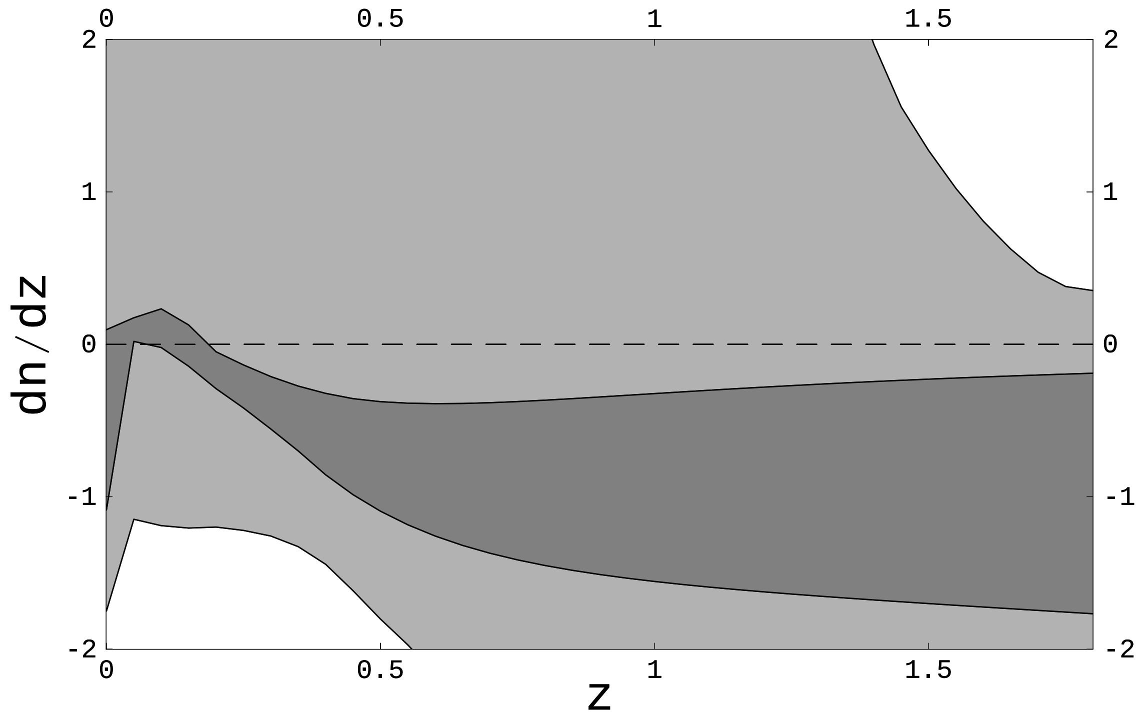


FIG. 3: The derivative of the characteristic of the power-law potential reconstructed from data.

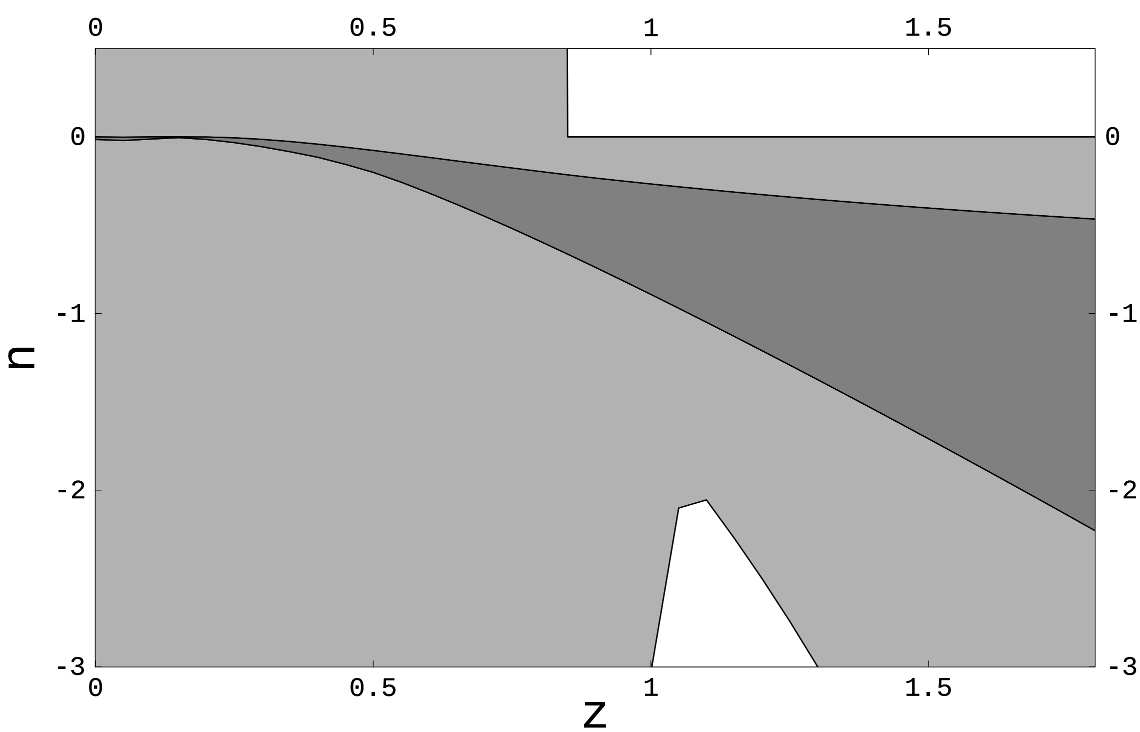


FIG. 4: The characteristic of the power-law potential reconstructed from data.

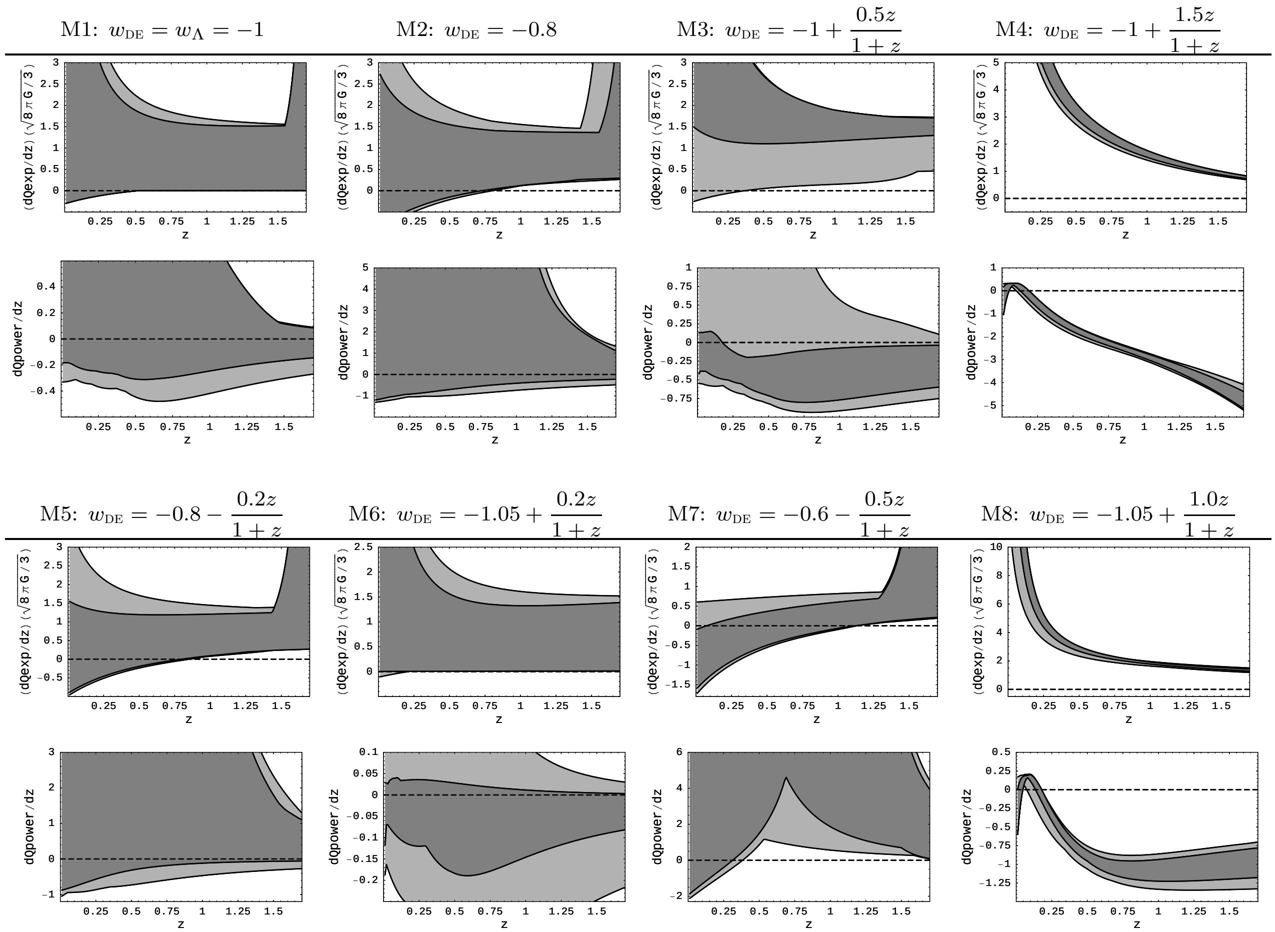


FIG. 5: Observational constraints on $dQ_{\rm exp}(z)/dz$ and $dQ_{\rm power}(z)/dz$ from the simulated data generated w.r.t. eight fiducial modes.