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POTENTIAL WELL SHAPES FOR TENSOR TERMS INVOLVING SPIN

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ABSTRACT

Potential well shapes for spin-orbit interactions are predicted using heuristic arguments. The results are generalized to interactions of the second rank in spin. Comparison with well known potential interactions are in agreement with the conclusions of this report.

INTRODUCTION

A suggestion by Fermi that the spin-orbit contribution to a nucleon-nucleus interaction is a surface effect¹ can be generalized to determine the shape of a second-rank tensor term for any particle-spin-nucleus interaction. This generalization reduces the number of ad hoc form factors that might be chosen for such a potential by establishing a consistent set of guidelines. For example, the same logic that leads to the Thomas form for the spin-orbit term of a phenomenological potential describing deuteron-nucleus interactions also leads to a particular successive derivative form for the spin-alignment tensor term. That is, the shape of the $\vec{S} \cdot \vec{L}$ term is connected to the shape of the central term V_c by the relation $\left[\frac{1}{r} \frac{dV_c}{dr} \right]$, and the shape of the $\{(\vec{S} \cdot \hat{r})^2 - \frac{1}{3} s(s+1)\}$ term is connected to the central term by the relation $\left[r \frac{d}{dr} \left(\frac{1}{r} \frac{dV_c}{dr} \right) \right]$, where $\vec{r}$ is the radius vector between the centers of the two interacting particles. This shape for the spin-alignment tensor term agrees with the folding model for the deuteron-nucleus interaction² and with that for the classical electrostatic quadrupole interaction. Similarly, the same logic applied to spin-spin interactions contributes a tensor term $\sim \left[r \frac{d}{dr} \left(\frac{1}{r} \frac{dV_c}{dr} \right) \right] \left\{ (\vec{I}_1 \cdot \hat{r})(\vec{I}_2 \cdot \hat{r}) - \frac{1}{3} \vec{I}_1 \cdot \vec{I}_2 \right\}$ to the potential. This shape for the spin-spin term agrees with the one pion exchange potential (OPEP) for nucleon-nucleon interactions and with that for classical magnetic dipole-dipole interactions.

Therefore, whenever a local potential containing nuclear spins is being devised without a specific theory, at least these simplicity arguments that will lead to form factors for the various spin contributions can be invoked. The purpose of this report is to generalize Fermi's suggestion by using simple algebraic techniques and to propose an internally consistent criterion for choosing well shapes that might be used to describe spin interactions.

THE BASIC IDEA

Around 1950 introduction of the nucleon-nucleus spin-orbit potential was necessary to explain the splitting of the energy levels in the nuclear shell model³ and to explain nucleon polarization after elastically scattering from nuclei.⁴ Fermi concluded that an $\vec{S} \cdot \vec{L}$ term would be predominantly a surface effect.¹ He suggested that a nucleon passing through a uniform distribution of nucleons cannot locate the center of the distribution because the range of nucleon-nucleon forces is short (see Fig. 1). Therefore, any potential term containing angular momentum must vanish in the nuclear interior because the sign and magnitude of $\vec{L}$ cannot be determined. Near the surface, however, there is a preferred direction, namely, the gradient of the nucleon density $\rho(r)$. At the surface three vectors occur that might be used as "building blocks" to construct a potential term, which must be a scalar. These vectors are the particle spin $\vec{S}$, the particle momentum $\vec{p}$, and

Fig. 1. A projectile of spin $\vec{S}$ immersed in a uniform nuclear density cannot locate the center of the nucleus and therefore cannot determine angular momentum $\vec{L} = \vec{r} \times \vec{p}$. However, at the surface the gradient of the nucleon density does supply a preferred direction.

the gradient of the nucleon density $\vec{\nabla}\rho(r)$. The simplest combination is an inner product between $\vec{S}$ and $\vec{\nabla}\rho \times \vec{p}$, which, because $\vec{r} \times \vec{p} = \vec{L}$, results in

$$\vec{S} \cdot \vec{\nabla}\rho \times \vec{p} = \frac{1}{r} \frac{d\rho}{dr} \vec{S} \cdot \vec{L}. \quad (1)$$

For short-range forces the shape of the nuclear potential $f(r)$ can be identified with the nucleon density distribution, and Eq. (1) yields the Thomas form for the spin-orbit term. Clearly all three available vectors were required to construct a scalar, and other simple combinations can be reduced to the form of Eq. (1).

The preceding argument is general in the sense that specifying the magnitude of $\vec{S}$ is not necessary and also in the sense that spin tensors of higher rank than one may be included. It therefore provides a guide for choosing the form factors of second-rank tensor terms and the only rationale for using a Thomas form for the $\vec{S} \cdot \vec{L}$ portion of a phenomenological deuteron optical potential.

To summarize, when a particle of momentum $\vec{p}$ and spin $\vec{S}$ encounters a central potential $V_c = \text{constant} \times f(r)$, potential terms of rank 1, 2, etc. in spin can be constructed from $\vec{p}$, $\vec{S}$, and $\vec{\nabla}f(r)$. This construction consists of an invariant contraction of tensors that, in the case of rank 1, is the scalar product of the two pseudovectors $\vec{S}$ and $\vec{\nabla}f(r) \times \vec{p}$. Each term of the potential is considered independently and is therefore required to be even in time and parity.

INVARIANT CONTRACTIONS OF CARTESIAN TENSORS

The Cartesian elements S_{ij} of a second-rank tensor can be constructed from vectors $\vec{A}$ and $\vec{B}$ in Cartesian space by

$$S_{ij}(\vec{A}, \vec{B}) = \frac{1}{2} (A_i B_j + A_j B_i) - \frac{1}{3} \vec{A} \cdot \vec{B} \delta_{ij}, \quad (2)$$

where δ_{ij} is the Kronecker delta and A_i, B_i are components of the vectors $\vec{A}$ and $\vec{B}$, respectively. The invariant contraction of Cartesian tensors (or tensor scalar product) is designated by

$$[\vec{A} \vec{B}] : [\vec{C} \vec{D}] = \sum_{i,j=x,y,z} S_{ij}(\vec{A}, \vec{B}) S_{ij}(\vec{C}, \vec{D}) \quad (3)$$

and expanded to

$$\begin{aligned} [\vec{A} \vec{B}] : [\vec{C} \vec{D}] &= \left(A_x B_x - \frac{1}{3} \vec{A} \cdot \vec{B} \right) \left(C_x D_x - \frac{1}{3} \vec{C} \cdot \vec{D} \right) \\ &+ \left(A_y B_y - \frac{1}{3} \vec{A} \cdot \vec{B} \right) \left(C_y D_y - \frac{1}{3} \vec{C} \cdot \vec{D} \right) \\ &+ \left(A_z B_z - \frac{1}{3} \vec{A} \cdot \vec{B} \right) \left(C_z D_z - \frac{1}{3} \vec{C} \cdot \vec{D} \right) \\ &+ \frac{1}{2} (A_x B_y + A_y B_x) (C_x D_y + C_y D_x) \\ &+ \frac{1}{2} (A_y B_z + A_z B_y) (C_y D_z + C_z D_y) \\ &+ \frac{1}{2} (A_z B_x + A_x B_z) (C_z D_x + C_x D_z). \end{aligned} \quad (4)$$

By a simple matter of expansion, requiring only that vectors $\vec{B}$ and $\vec{C}$ commute,

$$\begin{aligned} (\vec{A} \cdot \vec{C}) (\vec{B} \cdot \vec{D}) &= \frac{1}{3} (\vec{A} \cdot \vec{B}) (\vec{C} \cdot \vec{D}) \\ &+ \frac{1}{2} (\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D}) + [\vec{A} \vec{B}] : [\vec{C} \vec{D}]. \end{aligned} \quad (5)$$

Similarly,

$$\begin{aligned} & (\vec{A} \cdot \vec{B})(\vec{C} \cdot \vec{D}) + (\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D}) \\ &= (\vec{A} \cdot \vec{C})(\vec{B} \cdot \vec{D}) + (\vec{A} \times \vec{C}) \cdot (\vec{B} \times \vec{D}), \end{aligned} \quad (6)$$

which is sometimes useful in manipulating expressions. Equation (5) can be used to express invariant contractions of tensors in terms of simple vector relations. The results are, of course, independent of the fact that Cartesian tensors were used. The same results could have been obtained with spherical tensors by defining

$$[\vec{A} \vec{B}]:[\vec{C} \vec{D}] = \sum_q (-1)^q T_{kq}(\vec{A}, \vec{B}) T_{kq}(\vec{C}, \vec{D}) \quad (3A)$$

for $k = 2$, as was done by Brink and Satchler.⁵

For the deuteron-nuclear interaction, Satchler has shown⁶ that of the possible second-rank tensor terms only three need to be considered, namely,

$$\begin{aligned} T_r &= \left\{ (\vec{S} \cdot \hat{r})^2 - \frac{1}{3} s(s+1) \right\}, \\ T_p &= \left\{ (\vec{S} \cdot \vec{p})^2 - \frac{1}{3} s(s+1) \right\}, \text{ and} \\ T_L &= \left\{ (\vec{S} \cdot \vec{L})^2 + \frac{1}{2} \vec{S} \cdot \vec{L} - \frac{1}{3} s(s+1) l(l+1) \right\}. \end{aligned} \quad (7)$$

These scalars are constructed from Eq. (5) with the four vectors $\vec{r}$, $\vec{p}$, $\vec{L}$, and $\vec{S}$ by forming $T_r = \{\hat{r} \hat{r} : [\vec{S} \vec{S}]\}$, etc., where $\hat{r} = \vec{r}/|\vec{r}|$, etc. These results are consistent with this generalization as will be shown.

APPLICATION TO THE DEUTERON-NUCLEAR TENSOR POTENTIAL

The three vectors $\vec{V}_f$, $\vec{S}$, and $\vec{p}$ are available to construct scalars for terms in the deuteron-nuclear potential as described above. The simplest cases are selected for this report by requiring that each element of the second-rank tensors to be contracted be even in parity and time and contain an even number of $\vec{S}$ vectors, and that each term contain at least one $f(r)$, which serves to connect the spin to the potential (thus ruling out, for example, $[\vec{p} \vec{p}]:[\vec{S} \vec{S}]$). In addition, the concept is illustrated by assuming spherical symmetry, $f(\vec{r}) = f(r)$.

The results obtained by applying the above conditions on Eq. (5) are shown in Table I. For example,

$$\begin{aligned} [\vec{V}_f \vec{V}_f]:[\vec{S} \vec{S}] &= \left(\frac{1}{r} \frac{df}{dr} \right)^2 [\vec{r} \vec{r}]:[\vec{S} \vec{S}] \\ &= \left(\frac{1}{r} \frac{df}{dr} \right)^2 \left[(\vec{r} \cdot \vec{S})^2 - \frac{1}{3} r^2 s(s+1) \right] \\ &= \left(\frac{df}{dr} \right)^2 T_r, \end{aligned} \quad (8)$$

where T_r is defined in Eq. (7). Also,

$$\begin{aligned} [\vec{V} \vec{V} f]:[\vec{S} \vec{S}] &= (\vec{V} \cdot \vec{S})(\vec{V}_f \cdot \vec{S}) - \frac{1}{3} s(s+1)(\nabla^2 f) \\ &= \frac{1}{r} \frac{d}{dr} \left(\frac{1}{r} \frac{df}{dr} \right) (\vec{S} \cdot \vec{r})^2 + s(s+1) \left(\frac{1}{r} \frac{df}{dr} \right) \\ &\quad - \frac{1}{3} s(s+1) \nabla^2 f \\ &= r \frac{d}{dr} \left(\frac{1}{r} \frac{df}{dr} \right) T_r. \end{aligned} \quad (9)$$

The invariant contractions of Table I always result in a form factor times T_r , T_p , or T_L , or a linear combination thereof as discussed in the previous section. Not shown in this table are contributions to the rank 1 and rank 0 terms derived from these calculations. The conclusion from this exercise is that possible form factors for T_r are $\left(\frac{df}{dr} \right)^2$, $\left(k \frac{df}{dr} \right)^2$, and $r \frac{d}{dr} \left(\frac{1}{r} \frac{df}{dr} \right)$, with no special preference among them. This problem has been studied using a folding model for the deuteron-nucleus interaction.² Results show that the last term is in good agreement with the actual calculation and that there is negligible contribution to the potential from the T_p and T_L terms. This conclusion could not have been foreseen using the present approach.

Extracting the essence of the folding model by a Taylor expansion is worthwhile because it reinforces the choice of $r \frac{d}{dr} \left(\frac{1}{r} \frac{dV_c}{dr} \right)$ as a form factor and because the approach has more general applications. If $\vec{\alpha}$ is the radius vector between the neutron and proton, the effective potential between the deuteron and the nucleus is assumed to be

$$v(\vec{r}, \vec{\alpha}) = v_p \left(\vec{r} + \frac{1}{2} \vec{\alpha} \right) + v_n \left(\vec{r} - \frac{1}{2} \vec{\alpha} \right), \quad (10)$$

where the subscripts p and n refer to the proton and neutron, respectively, and v_p , v_n are the

TABLE I
DEUTERON-NUCLEAR POTENTIAL FORM FACTORS^a

Invariant Contractions	T_r	T_p	T_L
$(\vec{\nabla}f \cdot \vec{\nabla}f) : (\vec{S} \cdot \vec{S})$	$\left(\frac{df}{dr}\right)^2$	0	0
$(\vec{\nabla} \cdot \vec{\nabla} f) : (\vec{S} \cdot \vec{S})$	$r \frac{d}{dr} \left(\frac{1}{r} \frac{df}{dr}\right)$	0	0
$(\vec{\nabla}f \cdot \vec{\nabla}f) : (\vec{p} \times \vec{S} \cdot \vec{p} \times \vec{S})$	0	$\left(k \frac{df}{dr}\right)^2$	$\left(\frac{1}{r} \frac{df}{dr}\right)^2$
$(\vec{\nabla} \cdot \vec{\nabla} f) : (\vec{p} \times \vec{S} \cdot \vec{p} \times \vec{S})$	0	$k^2 r \frac{d}{dr} \left(\frac{1}{r} \frac{df}{dr}\right)$	$\frac{1}{r} \frac{d}{dr} \left(\frac{1}{r} \frac{df}{dr}\right)$
$(\vec{\nabla}f \times \vec{p} \cdot \vec{\nabla}f \times \vec{p}) : (\vec{S} \cdot \vec{S})$	0	0	$\left(\frac{1}{r} \frac{df}{dr}\right)^2$
$(\vec{\nabla}f \times \vec{p} \cdot \vec{\nabla}f \times \vec{p}) : (\vec{p} \times \vec{S} \cdot \vec{p} \times \vec{S})$	0	$\ell(\ell+1) \left(\frac{k}{r} \frac{df}{dr}\right)^2$	$\left(\frac{k}{r} \frac{df}{dr}\right)^2$
$(\vec{p} \cdot \vec{p}) : (\vec{S} \times \vec{\nabla}f \cdot \vec{S} \times \vec{\nabla}f)$	$\left(k \frac{df}{dr}\right)^2$	0	$\left(\frac{1}{r} \frac{df}{dr}\right)^2$

^aTake $\hbar = 1$, so that $|\vec{p}| = k$, and $\vec{L} \times \vec{L} = i\vec{L}$.

proton-, neutron-nuclear potentials. The nucleon-nuclear potentials can be considered equal and comprised of a central part V_c , and a spin part $V_S(x)\vec{\sigma} \cdot \vec{L}$, where $|V_S(x)\vec{\sigma} \cdot \vec{L}| \ll |V_c|$. With these assumptions, Eq. (10) becomes

$$V(\vec{r}, \hat{\alpha}) = V_c(|\vec{r} + \frac{1}{2} \hat{\alpha}|) + V_c(|\vec{r} - \frac{1}{2} \hat{\alpha}|) + \frac{1}{2} V_S(x) (\vec{\sigma}_p + \vec{\sigma}_n) \cdot \vec{L}, \quad (11)$$

where $\vec{L}$ is the deuteron angular momentum and each nucleon contributes $\frac{1}{2} \vec{L}$ on the average. The central part of the equation can be expanded in a Taylor series:

$$V_c(|\vec{r} + \frac{1}{2} \hat{\alpha}|) + V_c(|\vec{r} - \frac{1}{2} \hat{\alpha}|) = \left\{ 2 V_c(x) + \frac{\alpha^2}{12} \left[\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dV_c}{dr} \right) \right] \right\} + \frac{\alpha^2}{6} \left[r \frac{d}{dr} \left(\frac{1}{r} \frac{dV_c}{dr} \right) \right] P_2(\hat{r} \cdot \hat{\alpha}). \quad (12)$$

The Legendre polynomial $P(\hat{r} \cdot \hat{\alpha})$ for $\ell = 2$ has been used in Eq. (12). The interaction energy W of the deuteron can be considered classically as⁷

$$W = \int |\chi(\alpha)|^2 V(\vec{r}, \hat{\alpha}) d\hat{\alpha}, \quad (13)$$

where $\chi(\hat{\alpha})$ is the internal deuteron wave function and $|\chi(\hat{\alpha})|^2$ represents the nucleon density distribution. Taking⁸

$$\begin{aligned} \chi(\hat{\alpha}) &= \frac{1}{\sqrt{4\pi}} \left[\frac{u(\alpha)}{\alpha} + \frac{w(\alpha)}{\alpha\sqrt{8}} S_{pn}(\hat{\alpha}) \right] \\ S_{pn}(\hat{\alpha}) &= \left\{ 3(\vec{\sigma}_p \cdot \hat{\alpha})(\vec{\sigma}_n \cdot \hat{\alpha}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right\} \\ &= 6 \left\{ (\hat{S} \cdot \hat{\alpha})^2 - \frac{1}{3} S(S+1) \right\}, \end{aligned} \quad (14)$$

where $\vec{S} = \frac{1}{2} (\vec{\sigma}_p + \vec{\sigma}_n)$, Eq. (13) can be calculated.

Remember that $S_{pn}(\hat{\alpha}) \sim P_2(\hat{S} \cdot \hat{\alpha})$ if $\hat{S}$ is considered simply as an axis of symmetry. Also note that $S_{pn}^2 = 8 - 2S_{pn}$, and that⁸

$$\frac{1}{2\pi} \int_0^{2\pi} S_{pn}(\hat{\alpha}) d\phi = S_{pn}(\hat{r}) P_2(\hat{r} \cdot \hat{\alpha}), \quad (15)$$

where ϕ is the azimuthal angle of $\hat{\alpha}$ relative to a vector $\hat{r}$.

Substituting Eqs. (11), (12), (14), and (15) into Eq. (13),

$$W = \left\{ 2 V_c(r) + \frac{\langle \alpha^2 \rangle}{12} \left[\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dV_c}{dr} \right) \right] \right\} \quad (16)$$

$$+ V_s(r) \vec{S} \cdot \vec{L}$$

$$+ \left[r \frac{d}{dr} \left(\frac{1}{r} \frac{dV_c}{dr} \right) \right] \left(\frac{1}{5\sqrt{8}} \int_0^\infty u(\alpha) w(\alpha) \alpha^2 d\alpha \right) T_r,$$

where $\langle \alpha^2 \rangle$ is the mean square radius of the deuteron. Thus W is effectively the deuteron-nucleus interaction potential containing a central term in braces, a spin-orbit term, and a $[(\vec{S} \cdot \hat{r})^2 - \frac{1}{3} S(S+1)]$ term of the shape $\left[r \frac{d}{dr} \left(\frac{1}{r} \frac{dV_c}{dr} \right) \right]$, which appears in Table I.

The form of Eq. (16) is correct for any density distribution containing a quadrupole term such as

$$\rho(\vec{\alpha}) \sim a(\alpha) + b(\alpha) P_2(\hat{S} \cdot \hat{\alpha}), \quad (17)$$

which is located at position $\vec{r}$, where $|\vec{r}| \gg |\vec{\alpha}|$, and is located in a potential $V_c(r)$. In particular, an electric quadrupole distribution in a Coulomb field caused by a point charge has form factors $V_c \sim 1/r$ and $r \frac{d}{dr} \left(\frac{1}{r} \frac{dV_c}{dr} \right) \sim 1/r^3$ as expected for T_r .

APPLICATION TO THE OPEP

The present generalization can be applied to the interaction of two spins, $\vec{I}_1$ and $\vec{I}_2$. As an example, consider

$$[\vec{\nabla} \cdot \vec{\nabla} f]: [\vec{I}_1 \vec{I}_2] = (\vec{\nabla} \cdot \vec{I}_1) (\vec{\nabla} f \cdot \vec{I}_2) - \frac{1}{3} (\nabla^2 f) (\vec{I}_1 \cdot \vec{I}_2), \quad (18)$$

$$= r \frac{d}{dr} \left(\frac{1}{r} \frac{df}{dr} \right) (\vec{I}_1 \cdot \hat{r}) (\vec{I}_2 \cdot \hat{r})$$

$$+ \left(\frac{1}{r} \frac{df}{dr} - \frac{1}{3} \nabla^2 f \right) \vec{I}_1 \cdot \vec{I}_2$$

$$= r \frac{d}{dr} \left(\frac{1}{r} \frac{df}{dr} \right) \left[(\vec{I}_1 \cdot \hat{r}) (\vec{I}_2 \cdot \hat{r}) - \frac{1}{3} \vec{I}_1 \cdot \vec{I}_2 \right],$$

where again Eq. (5) has been used. If the central force is derived from a Yukawa potential,

$$f(r) \sim \frac{e^{-\mu r}}{\mu r},$$

then the spin-spin interaction should be of the form

$$r \frac{d}{dr} \left(\frac{1}{r} \frac{df}{dr} \right) = \mu^2 \frac{e^{-\mu r}}{\mu r} \left[1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2} \right],$$

which is correct for the OPEP of the nucleon-nucleon interaction. On the other hand, if the central force is simply of the $1/r$ type, the spin-spin interaction is of the form $1/r^3$, which is correct for magnetic dipole-dipole interactions.

The calculation procedure for the additional scalar terms for spin-spin interactions using $\vec{I}_1$, $\vec{I}_2$, $\vec{\nabla} f$, and $\vec{p}$ is clear. However, a methodical application of this approach has not yet been applied to the nucleon-nucleon problem.

CONCLUSION

Spin interactions have been treated as though they were dominated by regions of high potential gradients, and applications to several known problems have shown that this approach gives the correct form factors. Whenever a phenomenological potential containing nuclear spins is being devised, the simplicity arguments presented in this report can be used to suggest form factors for the various spin contributions.

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