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released Jan 1960

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WRITTEN BY J. S. McDonald & F. Perez	ATOMICS INTERNATIONAL A Division of North American Aviation, Inc.	TDR NO. 4578
SECTION Sodium Graphite Reactors		GO 7519
GROUP Sodium Components	TECHNICAL DATA RECORD	LEDGER ACCT. 3621
UNIT Coolant Systems Equipment		
APPROVED BY: (SUPERVISOR) J Leppard	PROGRAM Advanced Sodium Graphite Reactors	SUB-ACCT. 4451
OTHER		TWR 5253
	PROJECT Operating Components for Sodium Systems	DATE October 29, 1959
		PAGE 1 OF 28

TO: J. B. Williams*

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L. E. Glasgow	794-42 SS	A. C. Williams	785 VO

SUBJECT:

Heat and Mass Transfer Analysis of Proposed Experimental Cold Trap System

CONTENTS:

I STATEMENT OF PROBLEM

Investigate the possibility of removing oxide impurities from sodium-cooled reactor systems by cold trapping in conjunction with fill-drain tanks. Perform detailed analysis of concepts which are most advantageous from the standpoint of efficiency and economy. Prepare a plan of action for the experimental evaluation of the proposed unit (s).

II SUMMARY OF RESULTS AND CONCLUSIONS

1. The cold trapping rate and efficiency will be experimentally determined for a simple batch precipitation process which utilizes the fill/drain tank as the cold trap.

2. A promising circulation-type cold trap concept was developed during this study. The design of the unit (shown in Figure 2) incorporates both the cold trap and the economizer with the fill-drain tank.

a. Generalized transfer functions were obtained for the heat transfer elements of the system. Based on these relations, the variations of minimum cold trap temperature, T_{in}, with economizer input (flow and sodium temperature) for the proposed experimental cold trap were determined and are presented in Figure 3.

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b. The experimental unit will fulfill the cold trap design criteria (III, B) for a million pound sodium system inventory, except that total oxide storage capacity of the unit will be only 200 lbs (which corresponds to a smaller reactor sodium inventory).

3. A mass transfer anaylsis was made to estimate the transient re-entry of oxide into solution when sodium is dumped into the tank (during a scram). This expression will be used in conjunction with results of one experimental phase to estimate the coefficient of diffusivity of oxide in sodium.

4. An expression was developed for the average total sodium inventory oxide concentration as a function of cold trap by-pass flow rate and transport time.

5. A plan of action for the proposed experiment is included.

III DISCUSSION

A. Introduction

Cold traps for sodium cooled reactor systems are expensive. Because there are many unknown factors regarding the transfer and precipitation of oxides in sodium, the proper design and operation of cold traps is difficult. Therefore, there is strong incentive to obtain basic information on the cold trapping process and to develop a reliable and economical sodium cold trap concept that may be used for any reactor-size sodium cooling system.

Reference 1 suggested an approach that may lead to significant savings in sodium system capital cost. Since the cost of fill and drain tanks represents a significant investment in components which are used only a small percentage of the time during reactor operation, it proposed that such tanks be used for cold trapping. A study of this possibility was made and is discussed herein.

In judging the suitability of drain tank cold trap arrangements for reactor systems, it is well to compare them from the standpoints of effectiveness and economy with the HNPF cold traps. The two circulation-type cold trap models tested for HNPF demonstrated high efficiency, but the unit cost of the trap (probably a total of three for the reactor) is expected to be about $20,000.[2]

1. Diffusion Cold Trap

Considering the basic tank design shown in Figure 1, two cold trapping arrangements in this general category were considered. The first was a batch process in which a charge of sodium is admitted to the tank, and the tank is valved off from the system. Following this the tank is cooled and the oxide precipitates out of the sodium; then the cleaned sodium batch is returned to the reactor system and another sodium charge is transferred to the tank.

Secondly, a continuous diffusion-type cold trapping process was considered. In this operation a continuous stream of sodium is by-passed from the reactor system through the drain tank. The heel of the tank (see Figure 1) is cooled and acts as a large cold finger.

2. Circulating Cold Trap

In this process the temperature of practically all elements of the by-pass sodium stream passing through the drain tank are reduced to a desired (oxide saturation) temperature. The excess oxide comes out of solution and is retained in the cold trap. This operation is inherently more efficient than the diffusion process, whose action depends on oxide concentration gradients.

Additional hardware must be added to the tank for this type cold trap, and several simple arrangements were considered.

B. Objectives and Design Criteria

The objectives and design criteria for the tank-cold trap are listed below. The cold trap should be:

a. Of sufficient capacity to trap and store, over the reactor lifetime, 1200 ppm of oxygen based on the total system sodium inventory. The unit will be considered as filled to capacity when the oxide volume occupies 1/3 of the total volume of the cold trap.

b. Capable of cold trapping, within a three-day period (with the trap initially clean) a total of 500 ppm of oxygen based on the total sodium inventory.

c. Capable of cold trapping at the rate of 5 ppm/day after being filled to 75% of oxygen storage capacity.

d. Low in capital cost and capable of satisfactory operation with a minimum of maintenance and attention.

e. Free from flow-stoppage due to oxide plugging.

Furthermore, the unit is desired to operate over the entire reactor life, and no removable or disposable features are considered. This approach attempts to make maximum use of the inherent large capacity of fill-drain tanks.

Other design factors to be considered are: (1) the need to minimize the amount of oxides that go back into solution when the reactor sodium is dumped into the tank, and (2) for continuous cold trapping arrangements in which the elevation of the fill-drain tank is lower than that of the core tank, the need to prevent the drainage of sodium from the system as a result of the loss of fill-drain tank cover gas pressure.

C. Diffusion Cold Trap

Several structural configurations were considered for this type cold-trapping process. Any advantages offered by the arrangements studied were offset by the increased complexity of the structural design. Thus the simple design shown in Figure 1 is the basis for a feasibility study of this process.

A calculation was made to estimate diffusion cold trapping rates in a drain tank using the heel as the cold-finger oxide sink. In making the calculation, it was assumed that the sodium velocity through the drain tank (during continuous cold trapping) would be sufficiently low to be considered stagnant for purposes of calculation. Dimensions were assumed for the tank, heel, and depth of sodium. Using a curvilinear-square potential plot, values for the isothermals within the sodium were obtained for the cases of the heel being externally cooled by air and by tetralin. Since the sodium-sodium oxide temperature-concentration saturation curve is nearly linear below temperature of about 500°F, the temperature-potential plot was used as a concentration potential plot, temperature values on the potential lines being replaced by the saturation oxide concentrations. Then, assuming a coefficient of diffusivity, D, of $2 \times 10^{-5} \frac{cm^2}{sec}$, it was calculated that a 2- ft. diameter by 3-ft. long heel would trap at an approximate rate of $2 \times 10^{-6} \frac{lbs.}{hr}$ A rough comparison of this value with the results reported in reference 3 for cold trapping with a 3-inch diameter cold finger indicates that the assumed value of D may be off by a factor as great as 200.

If the batch process is used and the entire bottom of the tank is cooled, faster trapping can be expected. The size of oxide crystals or polycrystalline aggregates which would come out of solution is not known, and the length of time required for complete precipitation must be experimentally determined.

On occasion the reactor system will be drained, thus bringing relatively hot sodium into contact with the oxide which has been trapped out. To estimate the amount of previously trapped oxide which goes back into solution, a calculation was made based on a residence time of four hours. The calculation appears in Appendix I, and the result indicates that for four hours and a sodium depth of 8 ft. (9 ft. diameter tank), the average increase in concentration is 224 ppm/ft^2 of exposed oxide surface. Therefore, if the oxide is contained in a 2-ft. heel, the average increase in oxygen concentration in the entire inventory of a 20-ft. long tank (approximately 50,000 lbs.) is 15 ppm, whereas for the same tank with oxide covering a 1-ft. wide band along the total length of the bottom, the increase would be 180 ppm. If the drain tank contained about a half-million pounds, the oxide increase would probably be less than 50 ppm. The above estimates are based on the assumption that the oxide-sodium contact area is a plane surface. Actually, it is probable that several faces of the oxide crystals are in contact with the sodium and that adjacent crystals (or aggregates) in the assumed contact area are separated from one another by sodium.

D. Circulation Cold Trap

A cold trap of this type requires the provision of internal equipment for the tank and perhaps minor tank modifications. Appendix II illustrates several possible configurations which are representative of those considered and lists advantages and disadvantages of each.

Of the configurations studied, case A in Appendix II is considered to hold the most promise. For cases B and C, heat transfer analyses and the oxide storage requirements dictate the need for very large heel dimensions and for an economizer between the cold trap and the system.

Case A is a relatively simple and economical arrangement. It makes effective use of the large storage capacity afforded by the dump tank, and the necessary heat transfer areas in the cold trap and economizer can be obtained without significantly changing the tank basic features and size. The derivation of heat transfer expressions relating various cold trap parameters appears in Appendix III. The applications of these expressions in designing the proposed experimental model is discussed in III, F.

E. General Analysis of Cold Trap Systems

The calculations discussed in this section are pertinent to either of the previously discussed cold trap-tank arrangements. Some of the results below further apply to cold trapping systems in general.

Appendix IV shows the development of an expression whichmay be used to determine the reactor system oxide concentration as a function of cold trap characteristics, sodium by-pass flow rate through the cold trap (see figure in Appendix IV), and transport time through the cold trap. The expression is

$$c_n = (1 - \frac{w\tau}{W})^n c_o + \frac{w\tau c}{W} \sum_{i=0}^{n-1} (1-\frac{w\tau}{W})^i. \qquad (1)$$

All terms are defined in the Nomenclature and in Appendix IV. This expression may be used to determine the time required for a given cleanup operation.

For the initial system oxide cleanup, it is assumed that the cold trap will be in series with the flowing system (see figure in Appendix IV), and that no sodium will by-pass the cold trap. Then the expression for average system oxide concentration is

$$c_t = \frac{w(c_s - c)_t}{W} \, . \qquad (2)$$

For this case, the inlet concentration will remain constant at the saturation level of the inlet temperature. The excess oxygen in the sodium system will initially be deposited throughout the system, and will for a considerable time serve as a constant oxide source for saturating the cold trap inlet sodium.

A generalized analysis of the cold trap-economizer system appears in Appendix III. The results may be used to examine the cold trap performance as affected by input changes (flow and temperature) and changes in heat transfer characteristics as oxide builds up. This is discussed further in the next section in connection with the proposed cold trap experimental model.

F. Proposed Experimental Cold Trap and Test Procedure

An experiment is proposed which will examine two promising concepts discussed above. A drain tank will be used to investigate the batch process discussed under III, A, 1. Then the tank, which will be fitted with the internals shown for case A in Appendix II, will be used in an investigation of a circulation-type cold trap. The design of the experimental unit is discussed below in 1, and the variables to be studied and experimental procedure are discussed in 2.

1. Design

The experimental unit will be designed to cold trap at the rates required for a reactor system of 10^6 pounds sodium inventory. However, a smaller oxide storage capacity will be used than would be required for this size system. The cold trapping performance of the model as the oxide stored nears the unit storage capacity should nevertheless be indicative of results to be expected from full-size units. The desired (oxide saturation) temperature for normal operation of the cold trap is taken as 260°F. The normal economizer inlet temperature is taken as 600°F.

Using equations (1) and (2), the flow rate necessary to fulfill the required cold trapping rate was found to be 8.5 gpm. The heat transfer area around the heel periphery was determined to provide the desired temperature difference between T_{in} and T_{out} (this is the same as the economizer primary-secondary temperature difference). For a T_{in}-T_{out} of 30°F a heel area of 18 ft^2 is required. The overall heat transfer coefficient (based on water external cooling) used for the heel area determination was 20 B/hr°F ft^2. This figure makes allowance for a possible heel wall region thermal resistance build up, since a small amount of oxide may carry over from the storage bin and deposit in the heel over a long period of time. When the unit is clean, the overall heat transfer coefficient will be about 40. The area required for heat transfer in the economizer is 25 ft^2. The resulting design and dimensions are shown in Figure 2. The downcomer in the heel is thickwalled to provide a relatively high thermal resistance, thereby minimizing the degenerative heat transfer to the sodium entering the heel region.

Figure 3 illustrates, for the proposed experimental circulating cold trap, the variation of T_{in} with flow rate for various inlet temperatures. The minimum temperature is seen to be sensitive to flow rate, and care must be taken to avoid freezing.

In an effort to obtain a design with less sensitivity of T_{in} to flow rate (see characteristic curves in Figure 3) the values of and will be optimized by methods of calculus before construction of the proposed experimental unit. This less sensitive design is desired in order to minimize the operation of the heel auxiliary coolant temperature control valve to be used to maintain the desired value of T_{in} and corresponding value of T_L.

2. Variables to be Studied and Experimental Procedure

The parameters of principal interest in this experiment are:

a. The optimum residence time for oxide precipitation for the batch-cooling process.
b. The efficiency* of the unit when clean for the circulation-type cold trap.
c. The storage capacity of the unit (the unit will be filled to 50% oxide design capacity with a sodium inlet temperature to the economizer of 350°F, and an additional 25% of oxide capacity will be filled with sodium inlet temperature of 600°F).
d. The efficiency of the unit after being filled (with oxide) to 75% of capacity.

The above characteristics will be investigated in the order listed.

An existing test loop will be modified to include the experimental cold trap as shown in Figure 5 for this experiment. The independent variables to be measured are: (1) sodium inlet temperature, (2) inlet oxide content, (3) sodium flow rate, (4) water coolant flow rate and inlet temperature. In addition, the following dependent variables will be measured: (1) minimum cold trap sodium temperature, (2) outlet sodium temperature, (3) outlet oxide concentration, (4) sodium pressure drop across the unit, (5) water outlet temperature.

*Cold trap efficiency as used here is defined as the ratio of the cold trap outlet oxide concentration to the saturation concentration of sodium at the minimum cold trap temperature.

Further study will be devoted to the problem of detecting the manner of oxide deposition (distribution) in the storage bin. It is hoped that such information may be deduced from thereadings over the longterm of thermocouples located within the oxide storage bin. Oxygen will be added to the system in gaseous form at the addition tank (Figure 4), and the quantity of oxide being trapped and stored will be determined from a mass balance based on steady state oxide plugging meter indications.

In the early stages of the circulation cold trap experiment, cold trap performance will be studied at various inputs of oxide content, flow rate, and temperature.

3. Other Investigations

The proposed experiment has been set up keeping in mind the objective of obtaining, where and if possible, fundamental information on the cold trapping process. At present, extremely little is known of the mechanism of oxide crystallization in sodium or of the kinetics of oxygen in solution in sodium. The information obtained in the batch process experiment will permit the deduction of gross values for the coefficient of diffusivity (using the expression in Appendix I). Also, based on Stokes' Law precipitation considerations, a very rough idea may be obtained of the effective average oxide crystal size precipitating in static, large volume sodium vessels. Consideration is being given the performance of a small carefully controlled experiment to obtain reasonably accurate values for the coefficient of diffusivity. Also, a study is being made of possible methods to obtain estimates of the oxygen content of the deposits in the storage bin as a function of location.

4. Estimated Cost of Experiment

A breakdown of the estimated cost of approximately $21,000 for the proposed experiment is presented below:

Material & Equipment*	Cost
Steel tank modification (an existing tank will be used)	$1000
Piping and other hardware	800
Valves	500
Plugging meter	400
Heaters and wiring	1500
Thermocouples	200
Miscellaneous	500

Labor	
Mechanics	5 man-months
Engineering	3 man-months

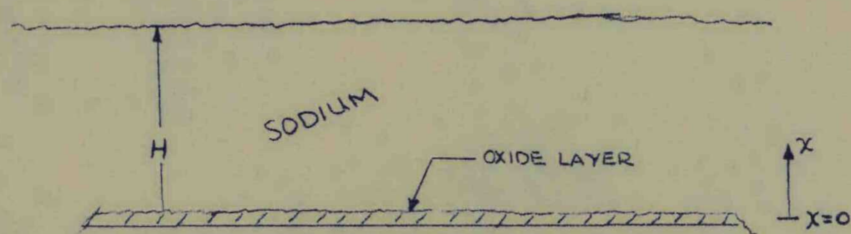
*Needed in addition to existing equipment.

IV REFERENCES

1. R. W. Dickinson IOL of September 17, 1958 to R. L. Olson.
2. Personal communication with R. Cygan.
3. Mausteller, J. W. and McGoff, M. J., MSA TR-22 (1953).

APPENDIX I

Estimate of Oxide Re-entry into Hot Sodium



At time t = 0, sodium at 400°F is dumped into the drain tank thus contacting the layer of oxide. The assumed sodium depth H is 8 ft, and the coefficient of diffusivity is taken to be 2×10^{-5} cm^2/sec oxide diffuses into the sodium so that the sodium returns to reactor system with a higher oxide content. Assuming a constant oxide source at x = 0 and that $x \to \infty$ (since H is very large for this process for the times involved), we have

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial t^2} \; ; \quad x > 0 \;,\; t > 0 \;, \qquad \text{(I-1)}$$

and

$$c = c_o \; ; \quad x = 0, \quad t > 0, \qquad \text{(I-2)}$$

$$c = 0 \; ; \quad x > 0 \;, \quad t = 0, \qquad \text{(I-3)}$$

where c = concentration, lb mols/ft^3

t = time, hrs

D = coefficient of diffusivity, ft^2/hr.

Employing Laplace transforms we define:

$$\bar{c} = \int_0^\infty e^{-pt} c \, dt \;,$$

where p is the transform variable in frequency space.
(I-1) is transformed as

$$D \frac{d^2 \bar{c}}{dt^2} = p \bar{c} \;, \quad x > 0 \qquad \text{(I-4)}$$

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and (I-2) is transformed to be

$$\frac{c_0}{p} \quad , \quad x = 0 \; , \; t > 0 \, . \qquad \text{(I-5)}$$

The solution os (I-4), using determinative conditions (I-3) and (I-5), is

$$\bar{c} = \frac{c_0}{p} \, e^{-\sqrt{p/D}\, x} \, . \qquad \text{(I-6)}$$

From an inversion table for Laplace Transforms:

$$c = c_0 \, \text{erfc} \left\{ x / 2\sqrt{Dt} \right\}$$

where erfc is the complementary error function. As a basis for estimating the oxide contamination of sodium, a sodium residence time of 4 hrs. is assumed. A plot of c <u>vs</u> x is made, and numerical integration yields a value of 0.084 c_o lb mols/ft^2 as the total oxygen diffusion into the sodium.

Assuming the density of sodium oxide in the oxide-sodium matrix is 50 lb/ft^3 (oxide layer):

$$c_0 = \frac{50}{\left(\frac{62}{16}\right) \text{Mol Wt. } Na_2O} = 0.206 \; lbmol/ft^3$$

$$0.084 c_0 = 0.067 \; lbmol/ft^2 = 1.07 \; lb/ft^2 \, .$$

For H = 8 ft, the average increase in concentration/ft^2 would be

$$\frac{1.07}{8 \times 55} = 0.00244 \; \frac{lb\,O_2}{lb\,Na/ft^2} = 244 \; ppm/ft^2$$

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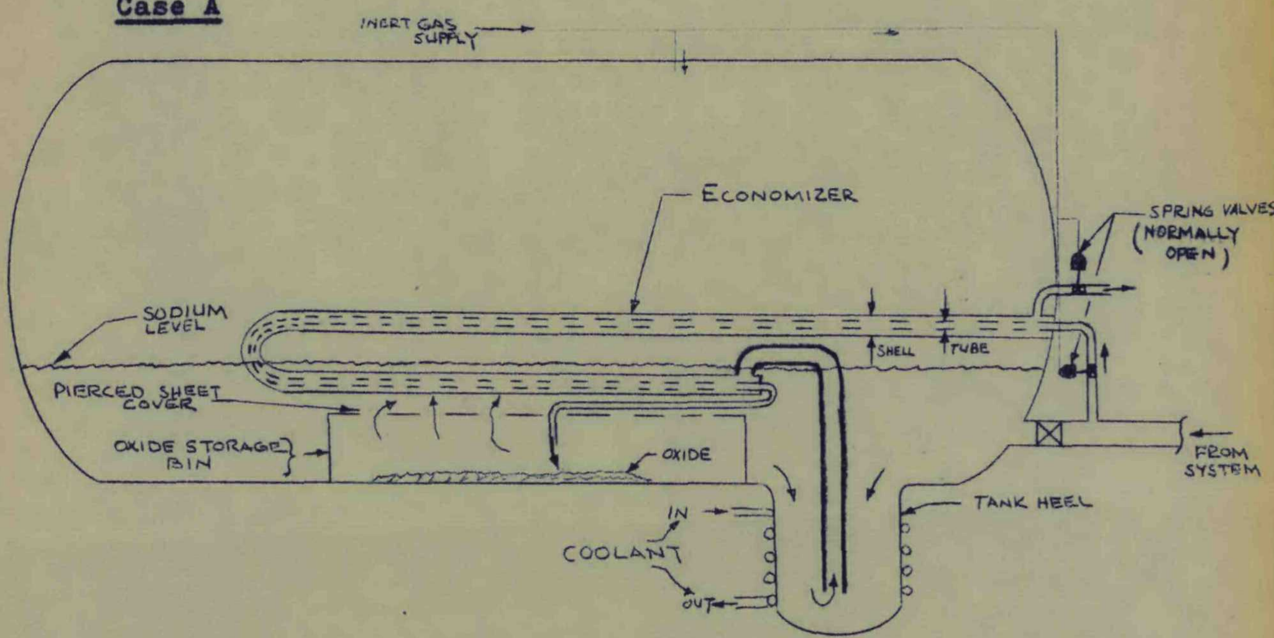
APPENDIX II

Forced Circulation Cold Trap Integrated with the Drain Tank

Case A



Advantages

1. **Provides high rates of oxide removal.**

The sodium stream is cooled to desired oxide saturation temperature and then the direction of flow is changed to promote separation of the oxides (which will be deposited in the oxide storage bin) from the sodium.

2. Cost is reduced when using a built-in oxide storage bin, as shown. Also, the nuisance of oxide disposable units is eliminated by using the inherent large capacity of the drain tank as an oxide storage tank capable of retaining all of the oxide trapped during the reactor's life.

2. cont.

A small volume for storage has been included at the bottom of the cooling section (tank boot) to retain possible oxides being carried over by the Na after leaving the main oxide storage tank.

3. Cost is reduced by using a built-in economizer as shown.

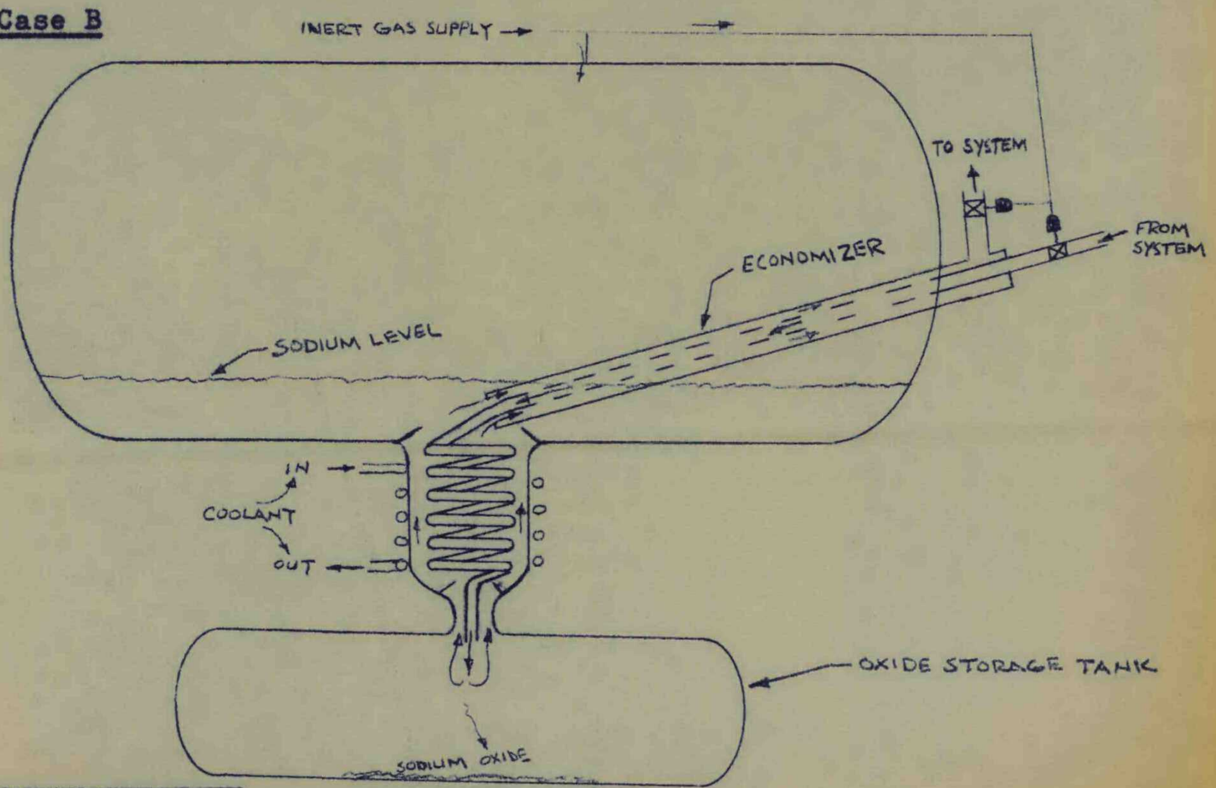
4. Free from flow stoppage due to oxide plugs. There are no small flow passages that are conducive to plugging.

Disadvantages

1. The very improbable but possible double failure of P_t and the *spring valves will drain the reactor system thus causing a reactor scram.

2. Some redissolving of the oxides into the Na is expected when hot Na is dumped since the oxide storage bin is within the drain tank.

Case B



*These spring actuated valves are held open by P_t; if cover gas system develops a leak, the spring will close them.

Case B cont.

Advantages

1. Little internal modification of the drain tank is required.

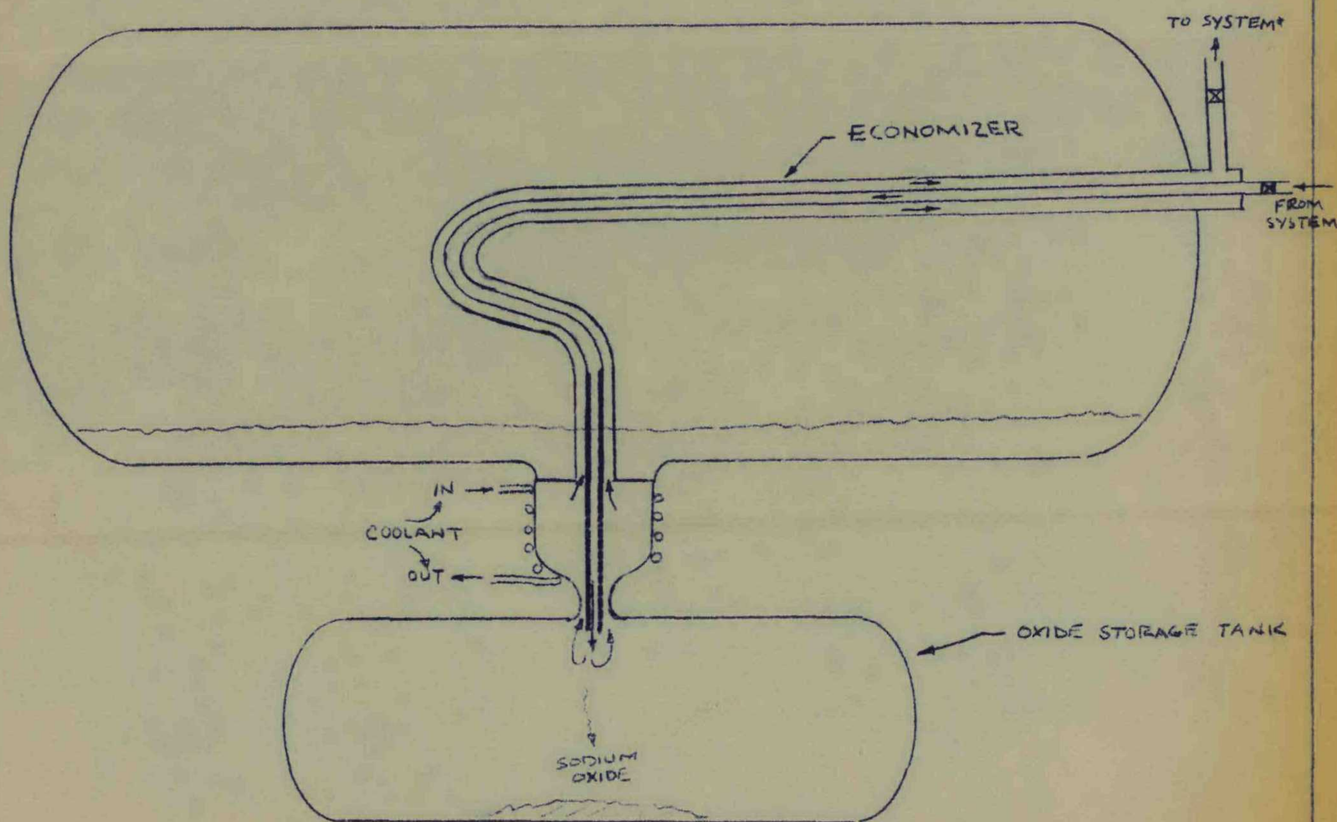
2. The trapped oxides are less likely to redissolve in Na in the drain tank.

3. Only one oxide storage tank is required.

Disadvantages

1. An external oxide storage tank is required. This will occupy additional big space below the drain tank and will increase the overall cost.

Case C



Case C cont.

Advantages

1. Since the drain tank and the reactor system are not interconnected the disadvantages listed for Case A have been eliminated. Therefore, the spring actuated valves can be substituted for less expensive ones.

Disadvantages

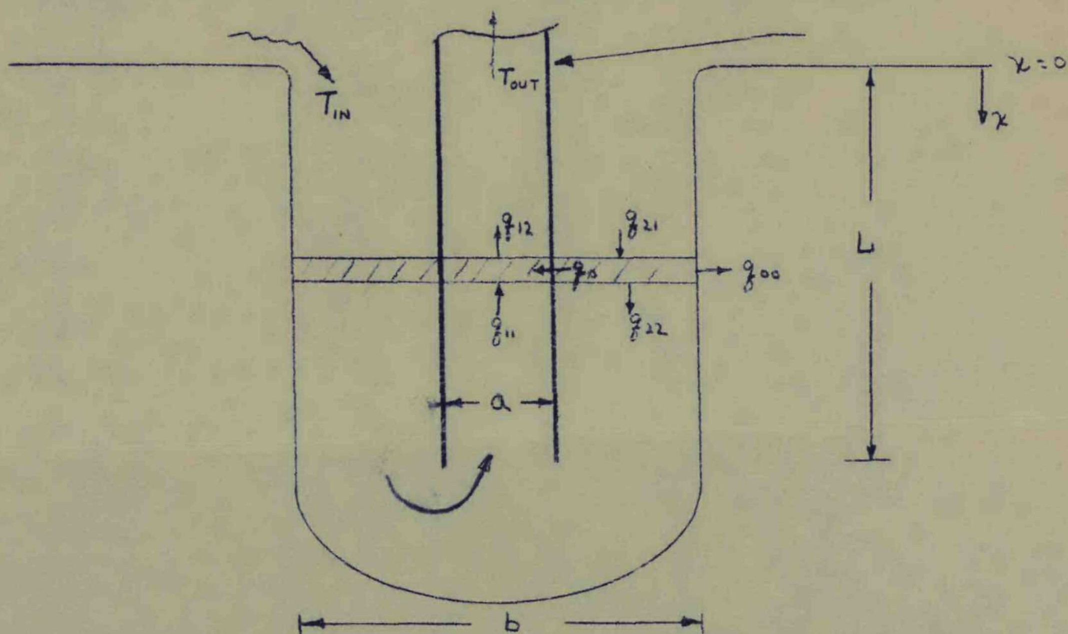
1. An external oxide storage tank is required. It will occupy additional big space below the drain tank and will increase the overall cost.

APPENDIX III

Heat Transfer Relations for Cold Trap and Economizer

Cold Trap

The simplified cold trap arrangement is shown together with the indicated nomenclature in the figure below.



The following energy balances may be written on the differential element shown in the figure (assuming no axial heat conduction)

$$q_{12} = q_{11} + q_0 \quad \text{(III-1)}$$

$$q_{21} = q_0 + q_{22} + q_{00} \quad \text{(III-2)}$$

Using the following nomenclature

c_p = specific heat, B/lb°F

U_{1-2} = overall heat transfer coefficient, tube-annulus, B/hr°F ft^2

U_{2-3} = overall heat transfer coefficient, annulus-external coolant, B/hr°F ft^2, we may write

$$q_{11} = wc_pT_1 \qquad \text{(III-3)}$$

$$q_{12} = wc_pT_1 + wc_p dT_1 \qquad \text{(III-4)}$$

$$q_0 = U_{12}\pi a\,(T_2 - T_1)dx \qquad \text{(III-5)}$$

$$q_{21} = wc_pT_2 \qquad \text{(III-6)}$$

$$q_{22} = wc_pT_2 + wc_p dT_2 \qquad \text{(III-7)}$$

$$q_{00} = U_{2-3}\pi b\,(T_2 - T_0)dx. \qquad \text{(III-8)}$$

Substituting the above relations into (1) and (2) results in

$$\frac{dT_1}{dx} = \alpha\left(T_2 - T_1\right) \qquad \text{(III-9)}$$

and

$$\frac{-dT_2}{dx} = T_2\left(\beta + \alpha\right) - \beta T_0 - \alpha T_1 \qquad \text{(III-10)}$$

where

$$\alpha = \frac{U_{1-2}\; a\pi}{wc_p}$$

and

$$\beta = \frac{U_{2-3}\; b\pi}{wc_p}$$

From (III-9)

$$T_2 = T_1 + \frac{1}{\alpha}\frac{dT}{dx} \qquad \text{(III-11)}$$

and

$$\frac{dT_2}{dx} = \frac{dT_1}{dx} + \frac{1}{\alpha}\frac{d^2T}{dx^2} \qquad \text{(III-12)}$$

Substituting in III-10 gives

$$\frac{d^2T_1}{dx^2} + \left(\beta + 2\alpha\right)\frac{dT_1}{dx} + \alpha\beta\left(T_1 - T_0\right) = 0 \qquad \text{(III-13)}$$

The solution of (III-13) is

$$T_1 = B_1 \exp\left\{-\left[\frac{(\beta+2\alpha)}{2} - \frac{1}{2}\sqrt{(\beta+2\alpha)^2+4\alpha\beta}\right]x\right\} + B_2 \exp\left\{-\left[\frac{(\beta+2\alpha)}{2} + \frac{1}{2}\sqrt{(\beta+2\alpha)^2+4\alpha\beta}\right]x\right\} + T_o \quad \text{(III-14)}$$

where B_1 and B_2 are constants of integration. Substituting as follows:

$$m = -\left(\frac{\beta+2\alpha}{2} - \frac{1}{2}\sqrt{(\beta+2\alpha)^2+4\alpha\beta}\right)$$

$$n = -\left(\frac{\beta+2\alpha}{2} + \frac{1}{2}\sqrt{(\beta+2\alpha)^2+4\alpha\beta}\right)$$

(III-14) may be written as

$$T_1 = B_1 e^{mx} + B_2 e^{nx} + T_o \,. \quad \text{(III-15)}$$

The boundary conditions are:

$$\frac{dT_1}{dx} = 0, \qquad x = L; \quad \text{(III-16a)}$$

$$T_1 = T_{out}, \quad x = 0 \,. \quad \text{(III-16b)}$$

Using equations (III-16) to evaluate B_1 and B_2 gives

$$T_1 = (T_{out} - T_o)\left(1 + \frac{m e^{mL}}{n e^{nL} - m e^{mL}}\right)e^{mx} - \left(\frac{(T_{out} - T_o)\, m e^{mL}}{n e^{nL} - m e^{mL}}\right)e^{nx} + T_o \quad \text{(III-17)}$$

Substituting (III-17) into (III-10) yields

$$\frac{dT_2}{dx} + (\beta+\alpha)(T_2 - T_o) = (T_{out} - T_o)\alpha\left[(1+K)e^{mx} - Ke^{nx}\right] \quad \text{(III-18)}$$

where

$$K = \frac{m e^{mL}}{n e^{nL} - m e^{mL}} \quad \text{(III-18a)}$$

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The solution to (III-18) is

$$T_2 = T_0 + (T_{out} - T_0)\,\alpha\left[\frac{(1+K)}{m+\beta+\alpha}e^{mx} - \frac{K}{n+\beta+\alpha}e^{nx}\right] + B_3 e^{-(\alpha+\beta)x} \qquad \text{(III-19)}$$

where B_3 is the constant of integration. Using

$$T_2 = T_{in} \; ; \quad x = 0 \, , \qquad \text{(III-20)}$$

B_3 may be evaluated. After substituting for B_3 in (III-19) and making algebraic adjustments, the result is

$$T_2 = (T_{out} - T_0)\,\alpha\left[\frac{(1+K)e^{mx}}{m+\beta+\alpha} - \frac{Ke^{nx}}{n+\beta+\alpha} - \left(\frac{1+K}{m+\beta+\alpha} - \frac{K}{n+\beta+\alpha}\right)e^{-(\alpha+\beta)x}\right] + T_0\left(1 - e^{-(\alpha+\beta)x}\right) + T_{in}\,e^{-(\alpha+\beta)x} \qquad \text{(III-21)}$$

At $x = L$, $T_1 = T_2$; therefore, setting $x = L$ in equations (III-17) and (III-21), equating them, and solving for $\frac{T_{out}}{T_{in}}$ yields

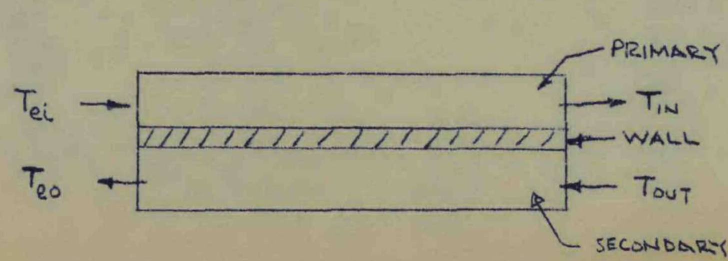
$$\frac{T_{out}}{T_{in}} = \frac{T_0}{T_{in}}\left[\frac{G - e^{-(\alpha+\beta)L}}{G}\right] + \frac{e^{-(\alpha+\beta)L}}{G} \qquad \text{(III-22)}$$

where

$$G = e^{mL}\left(1+K - \frac{\alpha(1+K)}{m+\beta+\alpha}\right) + e^{nL}\left(\frac{\alpha K}{n+\beta+\alpha} - K\right) + e^{-(\alpha+\beta)L}\left[\frac{\alpha(K+1)}{m+\beta+\alpha} - \frac{\alpha K}{n+\beta+\alpha}\right] \qquad \text{(III-22a)}$$

Economizer

The economizer is represented schematically in the figure below:



The symbol U_e represents the economizer overall heat transfer coefficient, and A_e the economizer heat transfer area. The following expressions for the heat transferred may be written:

$$wc_p (T_{ei} - T_{in}) = U_e A_e (T_{in} - T_{out}) \qquad \text{(III-23)}$$

Dividing through by T_{in} and inverting yields

$$\frac{T_{in}}{T_{ein}} = \frac{wc_p}{U_e A_e} \left(1 + \frac{wc_p}{U_e A_e} - \frac{T_{out}}{T_{in}} \right)^{-1} \qquad \text{(III-24)}$$

Substituting (III-22) for T_{out}/T_{in} and performing algebraic manipulations gives the economizer primary outlet temperature (cold trap inlet temperature) in terms of the economizer input and the economizer and cold trap physical properties:

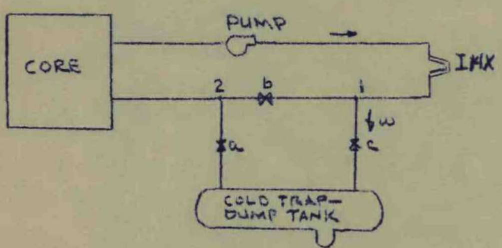
$$T_{in} = \frac{\frac{wc_p}{U_e A_e} T_{ein} + T_o \left[1 - \frac{e^{-(\alpha+\beta)L}}{G} \right]}{1 + \frac{wc_p}{U_e A_e} - \frac{e^{-(\alpha+\beta)L}}{G}} \qquad \text{(III-25)}$$

where G is given by (III-22a).

APPENDIX IV

System Oxide Reduction by By-Pass Cold Trapping

The figure below indicates schematically the system arrangement considered.



W = system sodium inventory, lbs.

w = cold trap flow rate, lbs/hr.

τ = transport time for travel of sodium element from point 1 to point 2, hrs.

c_n = average system oxygen concentration after n periods of length τ, ppm.

c = oxygen concentration in sodium at cold trap outlet, ppm.

c_0 = system oxygen concentration at the beginning of the cold-trapping period, ppm.

For initial oxide clean-up, valves (a) and (c) are open, and valve (b) is closed. During normal operational clean-up, all three valves are open.

At the end of first period of length τ,

$$c_1 = \left(1 - \frac{w\tau}{W}\right) c_0 + \frac{w\tau}{W} c \;;$$

after 2 τ,

$$c_2 = \left(1 - \frac{w\tau}{W}\right)\left[\left(1 - \frac{w\tau}{W}\right) c_0 + \frac{w\tau}{W} c\right] + \frac{w\tau}{W} c \;;$$

after 3 ,

$$c_3 = \left(1 - \frac{w\tau}{W}\right)\left\{\left(1 - \frac{w\tau}{W}\right)\left[\left(1 - \frac{w\tau}{W}\right) c_0 + \frac{w\tau}{W} c\right] + \frac{w\tau}{W} c\right\} + \frac{w\tau}{W} c$$

It is seen by induction that after $n\tau$,

$$c_n = \left(1 - \frac{w\tau}{W}\right)^n c_0 + \frac{w\tau}{W} c \sum_{i=0}^{n-1} \left(1 - \frac{w\tau}{W}\right)^i .$$

NOMENCLATURE
(Associated with Appendix III)

A_e	economizer heat transfer area, ft^2
a	cold trap tube diameter, ft
b	cold trap heel diameter, ft
B_1, B_2, B_3	constants of integration
c	oxide saturation content in sodium at cold trap exit temperature, ppm
c_n	oxide concentration in system after n periods of τ length, ppm
c_o	initial oxide content in sodium, ppm
c_s	oxide saturation content in sodium at cold trap inlet temperature, ppm.
c_p	specific heat, B/lb°F
c_t	oxide concentration in system at time t, ppm
D	coefficient of mass diffusivity, ft^2/hr
L	heel length, ft
p	transform variable in frequency space
q	elemental heat flux, B/hr
t	time, hr
T_{ei}	economizer inlet sodium temperature °F
T_{in}	cold trap inlet temperature = economizer primary outlet temperature °F
T_{out}	cold trap outlet temperature = economizer secondary inlet temperature °F
T_o	heel external coolant temperature °F
τ	transport time through cold trap, hr
u	overall heat transfer coefficient, B/hr°F ft^2
w	sodium weight flow rate, lb/hr
W	sodium inventory of system, lb

Subscripts

1	cold trap tube
2	cold trap annulus between tube and heel shell
3	heel external surface

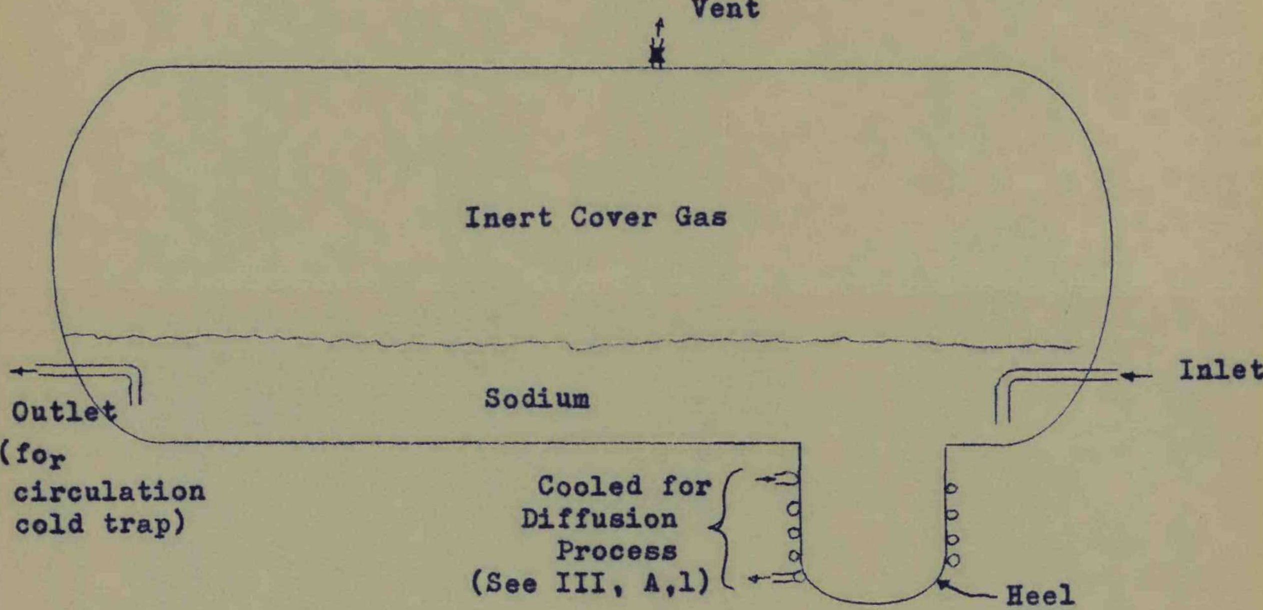
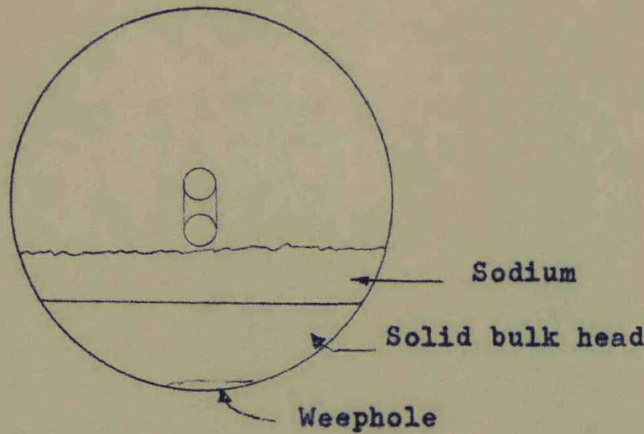
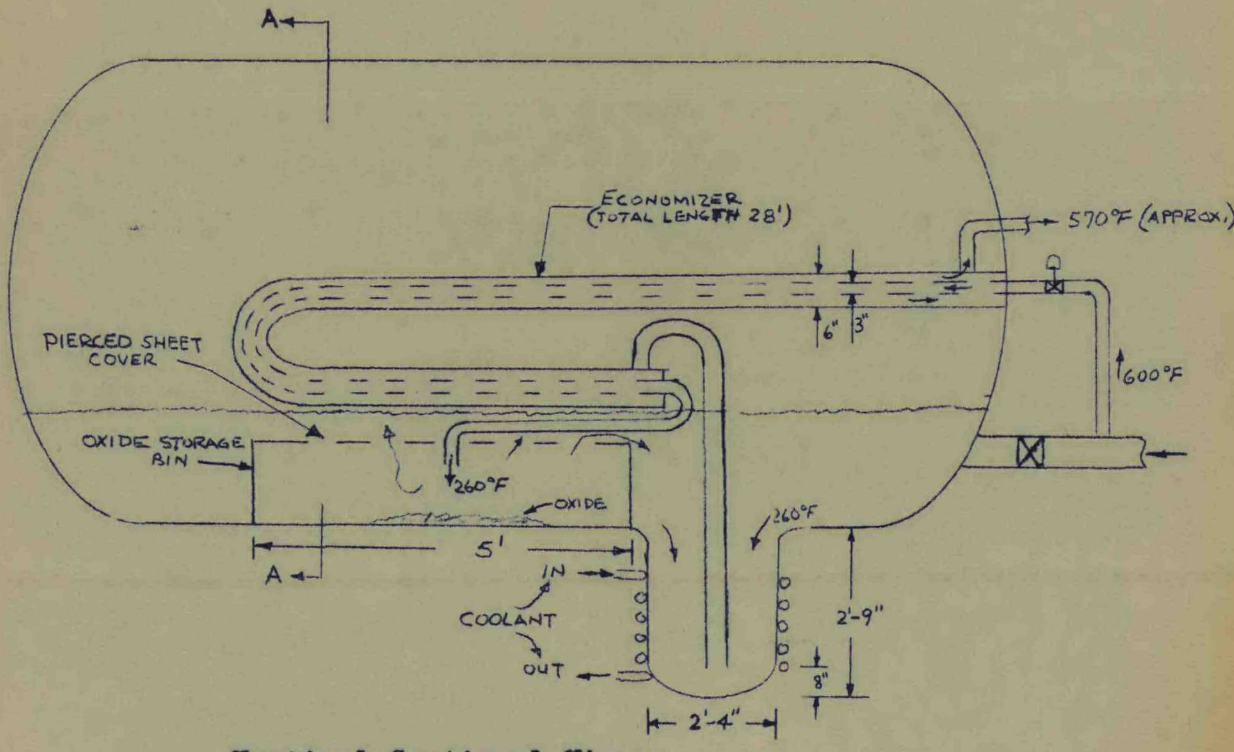


FIGURE I

Basic Fill-Drain Tank Configuration



Section A-A



Vertical Sectional View

FIGURE 2

Proposed Experimental Cold Trap

NOTES: Downcomer wall assumed to be perfect insulator, i.e. $U_{1-2} = 0$.

U_e = 800 B/hr°F ft^2,

A_e = 25 ft^2,

C_p = 0.31 B/lb°F.

T_o = 100°F

T_{ei} = Economizer inlet temperature

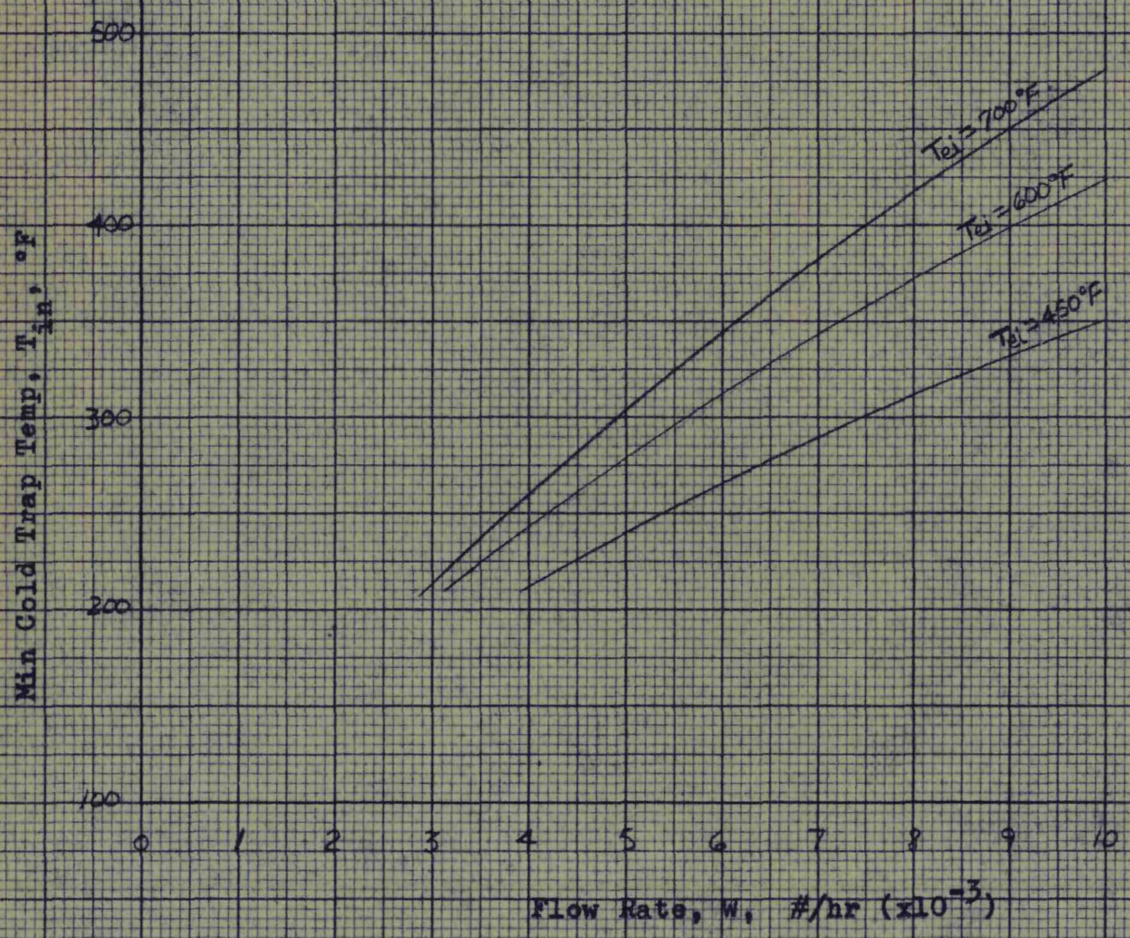


FIGURE 3

Minimum Cold Trap Temperature vs Economizer Input for Proposed Experimental Cold Trap

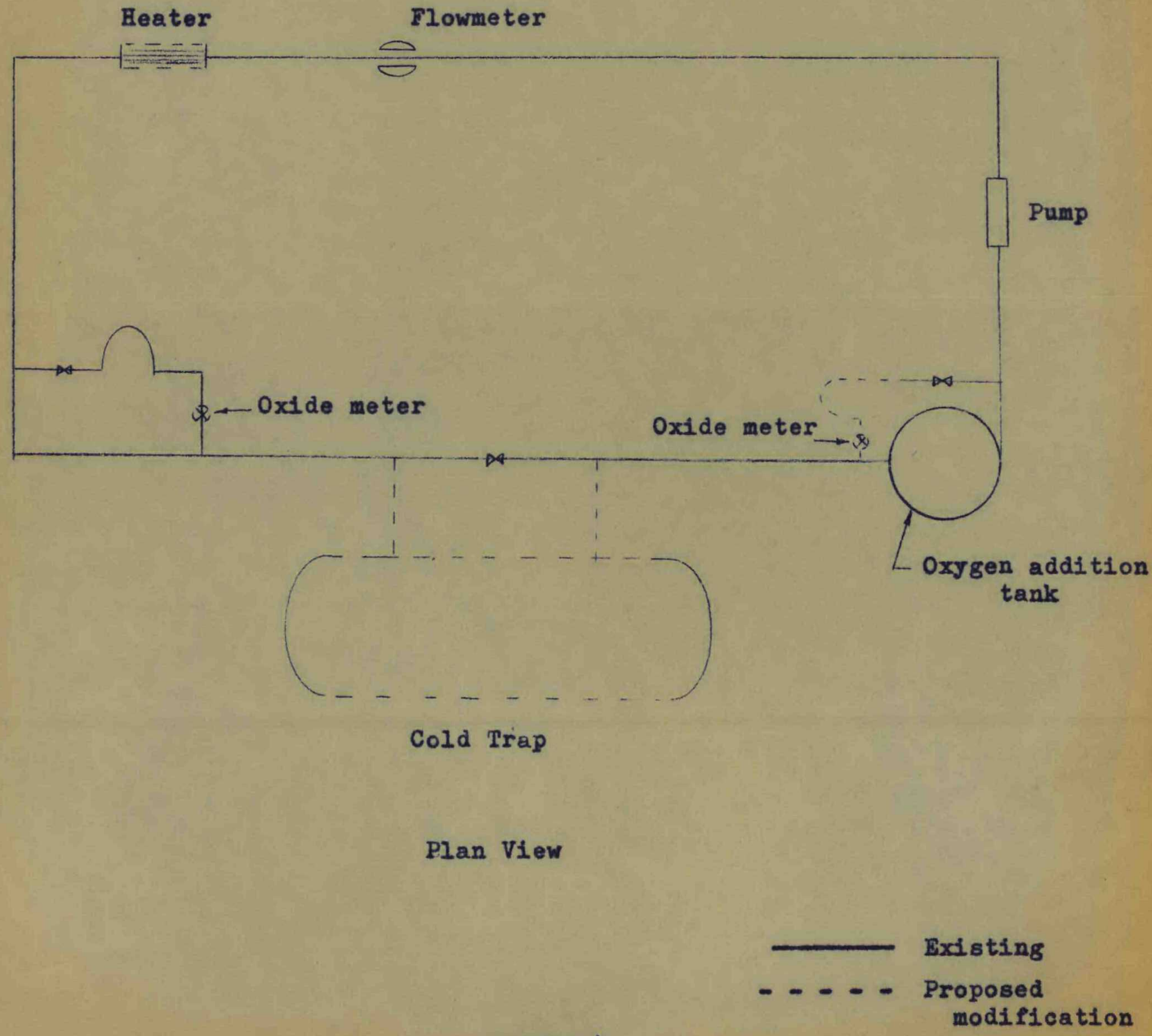


FIGURE 4
Experimental System