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ELASTIC STABILITY AND VIBRATION OF TOROIDAL

MAGNETS FOR FUSION REACTORS**

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ABSTRACT

The vibration and elastic stability of a set of discrete superconducting toroidal field magnets arranged to form a "bumpy" torus is examined. The mutual destabilizing magnetic forces between magnet pairs are calculated using a numerical differential inductance technique. It is shown that the mutual attractive magnetic forces can produce elastic buckling of the entire toroidal set. The vibration modes of the set are also found as functions of the coil current. The response of the set of magnets to an earthquake type motion of the toroidal base is calculated. The calculations have been incorporated in a computer code which accompanies the report.

Measurements are made of the lateral stiffness of a flexible, planar, superconducting coil between two rigid coils in series. These tests show a dramatic decrease in the natural bending frequency with subsequent elastic instability or "buckling" at a critical value of the current in the coils. These observations support a magnetoelastic analysis which shows that proposed designs, of toroidal field coils for Tokamak fusion reactors, have insufficient lateral support for mechanical stability of the magnets.

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TABLE OF CONTENTS

	page
Abstract	i
Table of Contents	ii
Summary	iii
I. Introduction	1
II. Theoretical Analysis and Vibration Results	3
III. Buckling Instability of Proposed Toroidal Magnets Designs	13
IV. Experimental Results	14
V. Conclusions	17
Appendix I - List of Symbols	19
Appendix II - Description and Users Guide for Computer Code "BUCKLE"	20
Appendix III - Listing of Computer Code Buckle	29
References	60
List of Tables	61
List of Figures	62

SUMMARY OF RESEARCH UNDER

CONTRACT At (11-1) 2493

The objectives of this program were

- I,II to investigate the magnetoelastic stability of a single toroidal field coil in its own magnetic field and in a toroidal field.
- III,IV,V to investigate the magnetoelastic stability and vibration of a set of elastic superconducting coils and to determine their response to transient excitation.
- VI to provide for experimental confirmation of the theory (with NSF support).

A progress report on this research was produced in Dec. 1974 [4].

The work summarized in that report and the progress to date include the following:

1. A computer code has been developed to calculate the mutual forces between toroidal field coils when subject to bending. This code called "BUCKLE" can calculate the critical magnetoelastic instability current, and the natural frequencies of the set of magnets as well as determine the transient response of the magnets to ground excitation as a function of current.
2. Both analytical and computer methods have been used to analyze the stability of a single toroidal field magnet under its own field. This analysis takes into account both the tension forces due to the magnetic field as well as the perturbed magnetic forces due to bending.
3. Comparison of self magnetic forces and intercoil magnetic forces

show that the intercoil perturbation forces under bending lead to much lower buckling currents than the self field analysis.

4. At present the computer code is being used to analyze various proposed fusion reactor designs using toroidal field magnets. The code examined the stability of the magnets under inter-coil magnetic forces under various lateral constraints. The results show that designs of superconducting toroidal field coils for proposed Tokamak fusion reactors have insufficient lateral support for mechanical stability of the magnets.
5. To test the theoretical predictions, experimental apparatus has been constructed at Cornell, using a National Science Foundation Grant, to determine the magnetoelastic stability of superconducting coils. These tests show a dramatic decrease in the lateral stiffness of a flexible coil between two identical rigid coils, with subsequent "buckling" of the coil at a critical value of the current in the coils.

I. INTRODUCTION

It has been shown by Demichele and Darby [1] that the limiting factor in the design of large toroidal field magnets for fusion reactors is the stress in the magnet structure. Recently however the principal investigator and coworkers [2]-[4] have shown that a magnetoelastic instability of the magnet structure may be another limiting factor on the amount of current in the superconducting magnet.

Increasingly as magnetic systems get larger and excess structure becomes costly, mechanical factors such as stress, deformation, and mechanical instabilities are becoming more important factors in the design of these systems. Thus more detailed analyses of the magnetic forces and stresses in these magnets become mandatory. While certain forces and effects in small magnets may be neglected with good reason, neglect of these effects in large systems may lead to either damage or significantly lower operating fields and currents than designed. In the case of magnetoelastic instability of magnets, the Principal Investigator has shown that the critical current density for mechanical instability decreases as the reciprocal of the square of the magnet size [2]. Thus buckling may arise in a large scale system which may not be observed in a conventional laboratory magnet system.

Reference designs for full scale fusion reactors have proposed using a set of discrete superconducting coils to form the toroidal magnetic field [5], [6] (Figure 1). Each magnet in the torus will be over 3 meters in radius and may have up to $7 \cdot 10^6$ ampere turns.

In previous studies [2], the question of the magnetoelastic stability of a single coil due to out of plane bending under perturbed magnetic forces has been examined.

These perturbation forces are similar to those in a kink instability in a plasma. However, besides the destabilizing self magnetic forces, the mutual attractive forces between each magnet have been found to be of greater importance. Thus, while the net magnetic force on a coil is zero when symmetrically placed between two similar magnets in the torus, slight out of plane bending of the coil will introduce a magnetic force which tends to increase the bending. For a large enough coil current, the elastic restraining forces will not be able to restore the coil to its unbent shape and the whole set will elastically bend to a new equilibrium position, (Figure 2) providing the coils don't fail structurally or go normal. An analysis of this problem is presented in this report.

Even if the structure is stiff enough to maintain the unbent equilibrium (i.e. the current I is below the critical buckling current I_c) the current can still couple the coils dynamically and affect the vibration modes of the system. In fact the vibration analysis also includes the buckling problem since the unbent configuration will become unstable when one of the natural frequencies goes to zero with increasing current. The position of the bent equilibrium configuration of the coils requires a nonlinear analysis of the coil bending (see e.g. [3])

II THEORETICAL ANALYSIS AND VIBRATION RESULTS

To analyze the vibrations of the set of magnets, each coil is treated as a planar elastic ring with a single current filament along the elastic axis. The coils may be circular, elliptical or "D" shaped. Their planes are normal to a common toroidal plane and intersect at a common line or toroidal axis. The rings are assumed to be constrained along part of their boundary by more rigid structures and interact with each other only through mutual magnetic forces produced by circulating currents in the rings.

Let z be a distance variable along the length of each coil or ring. Then at each point z there is assumed a bending displacement $v_n(z)$ normal to the plane of the n th magnet in the set and a twist $\phi_n(z)$ about the elastic axis. Once given v_n , ϕ_n as functions of z , and time, one can calculate the elastic energy V_n , and kinetic energy T_n of the n th magnet from which a Lagrangian is obtained for the system. In principal one can expand v_n , ϕ_n in a complete set of modal functions ($f_i(z)$) each of which satisfies the boundary constraints on the ring i.e.

$$v_n = \sum_{i=1}^{\infty} u_{ni}(t) f_i(z) \quad (1)$$

$$\phi_n = \sum_{i=1}^{\infty} \phi_{ni}(t) f_i(z)$$

Thus, for a "D" coil constrained along the straight edge, the $f_i(z)$ are taken as the vibration modes of a straight beam with built in ends at $z = 0, s$, i.e.

$$f_i(z) = \cosh\left(\frac{\lambda_i z}{s}\right) - \cos\left(\frac{\lambda_i z}{s}\right) - p_i \left(\sinh\left(\frac{\lambda_i z}{s}\right) - \sin\left(\frac{\lambda_i z}{s}\right) \right), \quad (2)$$

In practice one truncates the sum (1) and in this paper a one mode expansion is used. Thus for the lowest mode of vibration

$$\lambda_1 = 4.73, \quad p_1 = 0.9825$$

The mechanics of rings have been treated in many texts and papers, (e.g. Love [7]). The vibration of circular rings using this modal expansion has been treated in [2] based on the work of Culver [8].

While there may be elastic constraints between each coil, in this paper we assume that each coil in the torus is mechanically independent of the neighboring coils and is constrained only by the toroidal base. Then the elastic energy of each magnet depends only on its bending displacement. In the case of a circular ring or "D" with a circular arc of radius a , V_n is found by calculating the integral

$$V_n = \int_0^s \frac{1}{2} \left\{ A \left[\frac{\phi}{a} - \frac{\partial^2 v}{\partial z^2} \right]^2 + C \left[\frac{\partial \phi}{\partial z} + \frac{1}{a} \frac{\partial v}{\partial z} \right]^2 \right\} dz$$

where A is the bending stiffness and C is the torsional stiffness of the coil cross section. This neglects the elastic energy due to the self magnetic forces in each magnet which could be added in if desired.

Further if one neglects the twisting inertia, the twist variable ϕ_n in (1) can be eliminated. For a one mode expansion within each coil, the n th coil bending amplitude $u_n(t)$ satisfies a simple equation

for a harmonic oscillator (using u_n as independent variables in Lagrange's equations);

$$\ddot{u}_n + \omega_0^2 u_n = \frac{1}{m_0} \sum_{m=1}^{N-1} F_{nm} = Q_n/m_0 \quad (3)$$

where m_0 is the modal mass and the F_{nm} are the mutual magnetic forces between the N coils when the coils are bent out of their planes. The natural frequency ω_0 is a function of the elastic stiffnesses A, C , coil radius a , and modal function $f_i(z)$.

The calculation of the magnetic forces F_{nm} is best done by considering pairs of the m th nearest neighbors to the n th coil. The details are discussed in the next section. With this procedure the large magnetic force between the n th and $(n+m)$ th coils is cancelled by the force between the n th and $(n-m)$ th coils except for a force linearly proportional to the deflections of the coils, i.e. the right hand side of (3) becomes

$$Q_n = \sum_{m=1}^M K_m (2u_n - u_{n-m} - u_{n+m}), \quad M = (N-1)/2 \text{ (N odd)}, \quad M = N/2 \text{ (N even)} \quad (4)$$

where $\{K_m\}$ act as negative spring constants between the n th coil and its m th nearest neighbors.

Thus the magnetically coupled free vibrations of a set of toroidal magnets can be represented by a linear chain of masses coupled by negative springs and having periodic boundary conditions.

The motions of such a set of oscillators are well known and are treated in many texts, e.g. Brillouin [9].

The general motion of such a set of N oscillators can be represented by N modes each of which has a frequency ω_α , i. e.

$$u_n(t) = \sum_{\alpha=0}^{N-1} U_\alpha e^{i\omega_\alpha t} e^{i\frac{2\pi n\alpha}{N}} \quad (5)$$

The frequency of each mode is found by substituting a particular mode into the equations (4), from which one obtains

$$\omega_\alpha^2 = \omega_0^2 \left[1 - \sum_{m=1}^M 4 C_m \sin^2 \frac{\pi m \alpha}{N} \right] \quad (6)$$

where $C_m \equiv K_m / m\omega_0^2$ describes the differential magnetic force between a coil and its m th nearest neighbors and monotonically decreases with m . The relation (6) can be shown to be periodic by replacing α by $\alpha + N$.

One should recall that the C_m couple bending vibrations of one coil mode with the same bending mode in another coil. One can construct a more complicated model by coupling two or more bending modes of one coil with similar modes in another. Also one should not confuse the term "mode" as used here. The bending modes of each coil refer to the functions $f_i(z)$. We have assumed only one f_i is present. The vibratory modes $U_\alpha e^{i\left(\frac{2\pi n\alpha}{N}\right)}$ refer to the relative amplitudes of the different coils of the whole set vibrating together at one frequency ω_α .

To determine the critical buckling current we observe first that the C_m will be proportional to the square of the coil current so $C_m = I^2 \hat{C}_m$, and also that buckling occurs for a particular mode when $\omega_\alpha^2 \rightarrow 0$ as $I^2 \rightarrow I_\alpha^2$. Thus for a particular mode α ,

$$I_\alpha^{-2} = \sum_{m=1}^M \hat{C}_m \sin^2 \frac{\pi m \alpha}{N} \quad (7)$$

If the C_m decrease monotonically with α , then the smallest I_α^2 is obtained for $\alpha = \frac{N-1}{2}$ for N odd or $\alpha = N/2$ for N even. When N is even, the buckling mode is given by

$$u_n = U_{n/2} e^{i\pi n}$$

or each coil bends in an equal and opposite direction to its nearest neighbor.

A. Calculation of Magnetic Forces

Since each coil is assumed to be constrained by a relatively rigid base, there are no intercoil mechanical constraints. The only interaction between coils is magnetic. The large magnetic forces between pairs of magnets is cancelled out when the coils are symmetrically placed around the torus. However, when one coil bends toward one neighbor and away from the other neighbor a differential or perturbed force arises which is destabilizing - i.e. it tends to increase the bending. To calculate these differential magnetic forces the energy method is used (see e.g. [2], [3]), according to which these forces are proportional to the change of inductance L with bending.

In the energy or variational method one introduces the magnetic energy W into the Lagrangian for the system

$$\mathcal{L} = T(\dot{u}_{ni}) - V(u_{ni}) + W(u_{ni}, I) \quad (8)$$

where u_{ni} is the amplitude of the bending shape functions $u_{ni} f_i(z)$ of the n th coil. Since we will use only one bending mode we drop the subscript i . If the u_n and I are considered as generalized variables, then the generalized magnetic force is given by

$$Q_n = \frac{\partial W(u_n, I)}{\partial u_n} \quad (9)$$

In this paper we will not consider self forces on each coil. Thus, the generalized magnetic force in each coil depends on the mutual inductance for each pair of coils. In the previous section we used the perturbed force between a coil and its m th nearest neigh-

bors. The force constant K_m in (4) can then be determined from the relation

$$K_m 2u_n = I^2 \frac{\partial}{\partial u_n} (L_{n(n+m)} + L_{n(n-m)}) \quad (10)$$

with $u_{n-m} = u_{n+m} = 0$, and where L_{kl} is the mutual inductance between the k th and l th coils.

When the displacements $u_{n-m} = u_{n+m} = 0$ the sum of the mutual inductances in (10) is quadratic in u_n . Since the coils are symmetrically placed in a circle around the torus, the force constants K_m depend only on the m th neighbors and not the particular n th coil.

B. Numerical Solution and Results

To calculate the mutual magnetic force between coils when they are bent, we numerically calculated the mutual inductance between one bent coil and the two n^{th} nearest neighbors. Each of the coils are replaced by as many as 200 straight filaments which are described by the vectors $\Delta \vec{S}_i^{(n)}$ where (n) is the coil number and the subscript "i" is the number of the element in the n^{th} coil. The inductance is found by the sum

$$L_{nm} = \frac{\mu_0}{4\pi} \sum_i \sum_j \frac{\Delta \vec{S}_i^{(n)} \cdot \Delta \vec{S}_j^{(m)}}{R_{ij}}$$

where R_{ij} is the distance between the centers of the two current elements.

Starting from a reference planar configuration of current filaments, the heads and tails of the vectors $\Delta \vec{S}_i^{(n)}$ are moved out of the plane according to the bending function (2) (i.e. 1st mode, 2nd mode ect.). The resulting change of inductance was found to be quadratic in the modal amplitude, thus giving a linear modal magnetic force according to (9), (10).

Values of the intercoil modal magnetic force between a coil and its various pairs of neighbors in a torus are shown in Figure 4 (for 1st bending mode). The major radius of the torus is 5.0m and the minor radius of each coil is 3.0m. For a fixed number of coils, the force becomes very small past the 4th neighbors compared with the force due to nearest pair. The number of coils in the torus was varied and the currents scaled to produce the same average magnetic field at the major radius. For a constant center field, the forces do not seem to vary much with number of coils in the torus.

Knowing the intercoil magnetic force constants K_m (eqn 10) the dispersion relation for vibration frequency versus current can be found using (6). A typical set of normalized curves is shown in Figure 6. The ordinate gives the multi-

coil vibration mode number. At a certain value of the normalized current, one of these modes has a zero frequency or becomes elastically unstable. This means that the coils will deflect to a new, bent equilibrium position. The amount of deflection depends on nonlinear bending a brief discussion of which is found in [3]. The effect of different lateral supports on the critical buckling current for the toroidal field coil design [10], illustrated in Figure 6, is shown in Table 1. The modal stiffnesses used in (3) were calculated according to the theory presented in Reference 3. For this "D" coil the straight segment is clamped and pinned at various points along the curved arc of the coil, as shown in Fig. 6, according to the positions of the zeros of the bending function given in (2). One can see that if the design current is to be $7 \cdot 10^6$ A then the coil must be pinned at three points to raise the critical current above this value to $11 \cdot 10^6$ A.

As discussed in the previous sections, once the intercoil magnetic force constants are determined the coupled dynamics of the entire set can be found. One source of excitation could be through the Ohmic heating coils. Another possible source is ground motion due to a horizontal motion in an earthquake [11]. This example is shown in Figure 7. Horizontal motion of the Toroidal base will produce a vibration according to

$$U_n = U_1 e^{i\omega t} \cos(2\pi n/N)$$

i.e. only the mode $x = 1$ in (5) is excited. From Figure 5 one can see that the frequency of this mode is not very sensitive to the current and is close to the zero current natural bending frequency of the coil. The maximum response is found in the coil 90° to the direction of disturbance.

If one coil is disturbed, however, many modes are excited and two disturbance waves propagate around the set in clockwise and anti clockwise directions. An example of this is shown in Figure 8. In Figures 7,8 a fast Fourier computer algorithm was used to solve the equations (3). To reduce the computer running time artificially high structural damping was used in the testing of the computer programs.

III. BUCKLING INSTABILITY IN PROPOSED TOROIDAL MAGNET DESIGNS

Designs for full scale or test reactors for fusion power have been presented by various national laboratories [5],[6],[12],[13], Table II. These designs have been analyzed using the code to determine the critical buckling current under various support conditions. The coils were assumed to be rigidly clamped along part of the inner arc of each coil and to have various number of pinned supports around the rest of the circumference of each magnet. Each support supplies resistance to lateral bending of the coils, (see figure in Table III. It is found that in the full scale designs the design currents in the magnets exceed the critical buckling current when the magnet is just clamped on the inner arc. For example, in the Argonne magnets [10] the design currents in each magnet is around 7.10^6 Ampere turns. From Table III one can see that three lateral supports are required, in addition to the clamped arc, to attain a buckling current greater than the design value. The structures of superconducting magnets for large scale applications should be carefully examined for magnetoelastic instabilities early in the design and feasibility stages of proposed devices.

IV EXPERIMENTAL RESULTS

A. Introduction

While the mutual magnetic forces between toroidal magnets, for proposed fusion reactors, cancel for perfect symmetry, small out of plane bending of the coils lead to destabilizing lateral forces. These forces can be characterized by negative springs between the coils. The ratio of force to bending displacement is the negative magnetic stiffness and is proportional to the square of the current in the magnets. For mechanical stability the negative magnetic stiffness must be less than the elastic out of plane bending stiffness. Thus there exists a magnet current at which the combined elastic and magnetic lateral stiffness of the coils goes to zero and the planar configuration of the coils becomes unstable. This phenomenon is called "buckling" and is similar to the instability of a column under axial load.

Experimental observation of the magnetoelastic buckling of a small flexible superconducting coil, centered between identical coils have recently been made at Cornell University by the principal investigator and coworkers and is described briefly below. Intensive experimental work on magnetoelastic buckling of superconducting coils is continuing and will be reported in more detail in subsequent papers.

B. Description of Experiment

Three identical pancake coils were wound on a 20.3 cm diameter form, with 12 turns of 0.406 mm diameter Nb Ti multifilament superconducting wire (180 filaments, copper matrix/superconductor = 1.8, twist pitch = 2.5 cm). The turns were wound radially and potted in epoxy to form a very flexible elastic structure, (Figure 9). The side coils were secured to a very stiff plastic base and the center coil was clamped over a 90° arc, leaving the rest of the coil free to move laterally. The coils were arranged to form a three coil segment of a 24 coil torus (15° between the planes of the coils) with a major toroidal radius of ~~77.5~~ cm (Figure 10).

Strain gages were attached to the lateral sides of the flexible center coil near the clamped portion, where the strain would be large. The lateral stiffness of the coil, which is proportional to the square of the natural bending frequency, was measured using resonance and free vibration tests of the center coil. To excite these vibrations two copper wound exciter coils were positioned near the center coil as shown in Figure 9.

The assembly was placed in a 31.8 cm diameter cylindrical dewar with liquid helium at atmospheric pressure. With the coils in series, the current was increased and the bending deflection and natural frequency of the lowest lateral vibration mode recorded. Up to 50 amperes per turn were applied to the coils.

C. Experimental Results

The effective lateral stiffness of the flexible coil, as measured by the square of the natural frequency, decreased linearly with the square of the current (Figure 11). At 50 Amps/turn, the center coil buckled against one of the two side coils. The initial frequency of the coil was 7.2 Hz and decreased to 4.4 Hz at 540 Amp-turns in the coil before buckling at a current of 600 Amp-turns. The lateral deflection of the center coil, as measured by the strain gages, at first increased slowly with current then becoming greater near the critical current, and finally taking a jump in value at 600 Amp-turns.

Throughout the test the coils remained superconducting. Upon removal of the current, after buckling, the coil showed a permanent set as indicated by the strain gages and from post test inspection. Two more buckling tests were performed on the bent coil with a snap instability occurring at about 504 Amp-turns per coil.

The use of a three coil segment will give an upper bound on the buckling current for a full set of toroidal coils. In Ref. 2 it was shown that all neighboring pairs of coils lead to a destabilizing force on the center coil for out of plane bending. For a 24 coil torus, calculations of magnetic forces have shown that nearest neighbors exert 3-4 times the destabilizing force of the 2nd nearest neighbors, [2].

There is now increasing evidence that elastic structures carrying high currents in magnetic fields may exhibit instabilities analogous to those found in plasmas. The structures of superconducting magnet systems for large scale applications should be carefully examined for magnetoelastic instabilities early in the design phases of proposed devices.

V. CONCLUSIONS

On the basis of this study of toroidal field coils for Tokamak fusion reactors, we can conclude that out of plane magnetic forces between magnets can dramatically lower the lateral bending stiffness of the coils. The lateral or out of plane bending stiffness decreases with the square of the current and can in some cases lead to buckling of the coils if sufficient lateral support is not provided. These conclusions are supported by both analysis and experimental observation.

Application of the computer code to proposed reference designs for Tokamak reactors shows that all superconducting toroidal magnet systems will require more lateral support or increased elastic stiffness than proposed if the buckling currents are to greater than the design values.

Continuing Program

While a number of magnet design groups at national laboratories are carrying out design studies on magnet systems for fusion reactors, including the determination of stresses due to magnetic forces, the continuing program at Cornell University will complement these efforts in the areas of

i) magnetoelastic stability

ii) dynamic interaction between coils under transient conditions

The problems investigated thus far include the elastic rod with high current, out of plane and in-plane buckling of planar superconducting coils under their own magnetic fields and the mutual interaction of a set of toroidal field coils. The continuing program will extend this research to investigate the following problems;

1. A theoretical analysis of combined out of plane and in plane magnetoelastic buckling will be undertaken. This is like a spiral deformation along the arc of the magnet.

2. Study of the magnetoelastic stability of other magnetic systems for fusion reactors including ohmic heating coils and equilibrium field coils.

Under an NSF grant, cryogenic equipment has been purchased to look at the mechanics of superconducting materials. Using this equipment an extensive experimental program on the stability of a scale model of a superconducting magnet coil system will be carried out. This program includes;

1. study of the elastic stability of small superconducting coils with up to 10^4 amp turns with variable center of current and center of tension.
2. investigation of the post buckling behavior of these coils. i.e., the buckling deflection versus current.
3. construction of a model torus with four, six, and eight coils, each coil 5 inch (12.7 cm) in diameter, to determine the inter-coil buckling current or instability and vibration properties as functions of current.
4. to measure the dynamic motion of a small model of an ohmic heating coil and a toroidal field coil under transient current conditions.

These small scale experiments will be used to check the assumptions of the theory and computer program developed under the Princeton program.

APPENDIX I

LIST OF SYMBOLS

a	radius of coil
f_i	nodal bending functions
m_o	modal mass of coil
s	length of unsupported arc of coil
t	time variable
u_n, u_{n_i}	modal bending amplitude of n^{th} coil
v_n	lateral displacement of n^{th} coil
z	distance along arc of coil
A	bending stiffness of coil
C	torsional stiffness of coil
$C_m, \hat{C}_m$	normalized magnetic force constants
F_{nm}	magnetic force between coils
I, I_α	electric current
$\mathcal{L}$	Lagrangian
K_m	magnetic force constant between coils
L_{nm}	mutual inductance between n^{th}, m^{th} coil
Q_n	total magnetic force on n^{th} coil
R_{ij}	distance between points on a coil
T	kinetic energy
U_n	<i>DISPLACEMENT</i>
V	potential energy
W	magnetic energy
α	vibration mode number
μ_o	magnetic permeability of vacuum
ϕ_n	angular twist of arc of coil
ω_o, ω_α	natural vibration frequency

APPENDIX II

Description and Users Guide for Computer Code "BUCKLE"

A computer code has been written which analyzes the elastic stability of a set of toroidal magnets under offcentering magnetic forces between the coils. Critical currents for lateral buckling of the magnets under different support conditions are presented for seven fusion reactor designs. In addition to the buckling analysis, the computer code can calculate the natural frequencies for lateral vibration of the magnets as a function of magnet current. The response of the magnets to ground motion can also be calculated.

It is well known that the lateral magnetic force on a planar magnet in a toroidal set is zero when symmetrically placed between two similar magnets. However slight out of plane bending of the coil will introduce a magnetic force which tends to increase the bending. For a large enough current the elastic restraining force will not be able to restore the coil to its unbent shape and the whole set of magnets will bend to a new equilibrium position, providing the coils don't fail structurally or go normal if superconducting. The magnet current at which the toroidal set becomes elastically unstable is called the critical buckling current. The corresponding magnetic field at the torus major radius is the critical buckling field.

The destabilizing magnetic forces between magnet pairs are calculated using a numerical differential inductance technique. The simultaneous mutual interactions of all the coils are included self consistently. The deformation of all the coils is analyzed using a modal expansion and treating the set of magnet coils as a periodic structure. The details of this analysis are presented in Section II.

Description of Code

The computer code BUCKLE uses the techniques of analysis derived above to calculate the vibration and elastic stability limits of a set of discrete super conducting magnets arranged to form a "bumpy" toroidal field. It also calculates such limits for individual torus coils. The mutual destabilizing forces between magnet pairs and the destabilizing self-forces of each magnet are calculated using a numerical differential inductance technique. Using methods from the theory of periodic structures, the vibration modes of the set are found as functions of the coil current. The response of the set of magnets to an earthquake type motion of the base, and to a disturbance of one of the coils (such as a hammer blow) is also computed.

The Flow Chart of Fig.II-1 displays the functional arrangement of the program. The main program contains the input data such as the dimensions and specifications of the magnet system, and branching commands that determine which output quantities are to be computed. Based on these, BUCKLE proceeds to call a series of subprograms which perform all the actual computations.

There are four major subroutines which are called by the main program. They are:

SELFIDL Which calculates the total self inductance and differential modal self inductance of a single toroidal field coil, and the critical buckling current for each mode due to the coil's self field.

It computes the modal self inductances and corresponding critical currents for in-plane and out of plane deformations as well as twisting.

TORUS Which calculates the interpair mutual inductance of the torus

Figure II-1

coils as well as differential modal inductance between pairs, for deformations in the coil planes or perpendicular to them.

WOVALU Which uses as input the self and mutual differential modal inductances from SELF DL and TORUS, and computes the frequencies of the collective vibration modes of the TORUS, and the critical buckling current and critical toroidal magnetic field for each buckling mode.

FFTRAN Which calculates the dynamic response of the magnets as a function of time to several types of mechanical disturbances, including transverse or rotational motion of the base and the excitation of a single magnet such as from a hammer blow.

Each of these programs perform a distinct independent function and will operate independently of the other three if supplied with the proper input data.

In addition, there are seven other subroutines which are called by the four above. They are:

COIL Which sets up coordinates of a single filament coil with adjustable modal deformations in the radial and Z directions, and a variety of possible constraints which control the stretching of the filament length during deformation. COIL is called by SELF DL and TORUS.

OFFSET Is used by SELF DL in computing self inductance of a magnet coil. It divides the coil cross section into (NDIVZ*NDIVR) rectangular subsections, produces coordinates for filaments at the center of each subsection, then computes the mutual inductance of all such filaments.

FILIA Which computes the self inductance of a single filament coil,

omitting the self inductance of each filament segment upon itself.

FFTP Which is called from the FORTRAN IMSL library. It performs the fast fourier transform on an input array ω_i , producing an output array x_i of the same dimension, where

$$x_i = \sum_{j=1}^N \omega_j e^{ij2\pi/N}.$$

It is called by FFTRAN to generate the dynamic responses of the magnet coils to a disturbance.

WPLOT1 Is also called from the IMSL library. It plots graphs and scales the coordinate axes, when given two arrays X and Y of equal length. The points of the graph correspond to $(X_{(i)}, Y_{(i)})$, where X is the horizontal axis, Y the verticle axis. It is called by WOVALU in plotting the dispersion curve and by FFTRAN in displaying coil displacements vs time.

COFIC Computes the dimensionless elasticity parameter which scales the bending stiffness according to the deformation mode. It is used to calculate the self-frequency and the critical current.

YESNO Is a subroutine used to format the output. It converts the values of certain integer variables to format statements of "NO" or "YES" when the value of the variable is 0 or 1.

The actual operation of BUCKLE is governed by the array OPTION, which contains six entries, each having the value -1, 0, or +1. As indicated in Fig. II-1, the option statements have the following effect:

OPTION (1) governs the computation of the mutual inductances and differential modal inductances in the torus. OPTION (1) = 1 causes subroutine TORUS to be called, and the values of differential inductance for the coil pairs to be returned in the array BETA. If OPTION (1) = -1

the values of BETA will be read from the data cards. If $\text{OPTION (1)} = 0$ no action is taken on BETA.

OPTION (2) controls the self-inductance and differential modal self-inductance calculation. When $\text{OPTION (2)} = 1$, SELF DL computes the differential self-inductance and returns it as DLSELF, a real variable. When $\text{OPTION (2)} = 0$ or -1 , no action is taken, though DLSELF may be specified among the input parameters if the self-inductance is to be included as a correction to mutual effects later in the program.

OPTION (3) controls the computation of the frequencies of the torus normal modes. When $\text{OPTION (3)} = 1$, these frequencies are calculated by calling WOVALU. The differential inductance from the array BETA is needed for input, and the modal frequencies are returned in the array WO2. If $\text{OPTION (3)} = -1$, the values of WO2 are read from data cards, and if $\text{OPTION (3)} = 0$, the program takes no action and skips to the next step.

OPTION (4) determines whether the torus modal frequencies computed by WOVALU should be corrected by including the effect of self-inductance. $\text{OPTION (4)} = 1$ causes WOVALU to include this effect before generating the modal frequencies WO2. $\text{OPTION (4)} = 0, -1$ implies that this correction is to be skipped.

OPTION (5) governs the subroutine FFTRAN. If it equals 1, FFTRAN is called and the dynamic responses of the torus coils to a disturbance is calculated. If $\text{OPTION (5)} = 0$ or -1 , FFTRAN is not run.

OPTION (6) selects which of three approximations are to be used when the coil self-inductance program SELF DL is run. $\text{OPTION (6)} = -1$ selects the fastest but least accurate method, in which the coil is approximated by a single filament. If $\text{OPTION (6)} = 0$ the

self-inductance is approximated by the mutual inductance of an array of filaments which approximate the current distribution in the coil. If OPTION (6) = 1, the coil is quite accurately approximated by the mutual plus the self-inductance of an array of filaments.

In addition to the option statements, there are several input variables which determine the type of coil modal deformation for which the differential inductances and critical currents are calculated.

TYPEMD specifies in which direction the coil is to be deformed. TYPEMD = 1 specifies a deformation normal to the plane of the coil, the z direction, 0 specifies an in-plane distortion, the radial mode, and TYPEMD = -1 specifies a twisting deformation or Phi mode.

SHAPEM determines whether the mode shape will be cantilever or sinusoidal (pinned). SHAPEM = 1 causes a cantilever deformation, SHAPEM = 0, -1 produces a sinusoidal deformation.

MODE specifies the mode number for the bending or twisting mode. MODE = 1 is the lowest cantilever mode or lowest pinned mode.

When the coil is deformed in-plane or out-of-plane, its length changes, and this accounts for most of its modal differential self-inductance. If the stretching is constrained by fixing the total coil length at some radius (not necessarily the center of current, which is RAD) the differential self-inductance will be changed, and stable configurations can be found. The following variables control this freedom:

RJUST determines whether coil constraints are to be applied. RJUST = 0 means that the coil length is allowed freely to stretch in accordance with modal perturbations in and out-of-plane. RJUST = 1 means that the total coil length will be constrained during deformation according to the next two variables.

RFIX specifies the radius at which the length constraint is applied. If the center of current is at 2.0m, but the center of tension at 2.5 m, then RAD = 2.0 m, and RFIX = 2.5m.

STRECH is the ratio by which the total coil length at RFIX is allowed to change. STRECH = 1.0 means the total length at RFIX is to be held constant. RFIX = 1.01 allows for a small stretching with the modal distortion.

The labelling of the dimensions and mechanical constants of the torus, as defined in the code print out, should be self-explanatory. All units are MKS. The x-direction is defined as the direction parallel to the central axis of the torus. The y-direction is perpendicular to x, and also in the plane of each coil. The axes cross at the geometrical center of each coil. This defines the x and y directions, and eliminates any ambiguity in quantity XOVERY, the ellipticity of each coil, the ratio of the coil radii in the x and y direction.

AMP is needed in WOVALU and FFTRAN in plotting the dispersion curve for the torus, and determining the coupling between coils for their dynamic response characteristics.

MODMAS the modal mass may be omitted if either SELF DL or TORUS are called, for they calculate it.

The subroutine FFTRAN has a number of degrees of freedom which should be mentioned. First, in scaling the output of the fast fourier transform, one must compromise between how many coil oscillations one wishes to see, and the detail of each cycle.

NCYCLE specifies the number of oscillations in the fast fourier time frame-10 cycles has been found to be a useful figure.

EFOLD determines the dynamic coil damping of the coils. The time

dependent damping then is $\exp(-EFOLD \cdot t)$ where t is time. EFOLD should be large enough so the coil oscillation is negligible at the end of the time frame.

The program is provided with two time dependent forcing functions for the coil disturbance. One is a triangular pulse, the other an exponentially damped sine wave. QKSHAP determines this selection.

ALPH determines the width of the triangular pulse, while QKDAMP, QKFREQ, and QKNCYC delineate the characteristics of the damped sinusoid.

QSTART specifies at what time T_0 the disturbance begins, where $QSTART = T_0/T_{max}$ and T_{max} is the elapsed time at the end of the time frame.

NFORCE determines which coils are driven by the disturbance. When NFORCE = 1, only one coil is driven. This corresponds to a hammer blow or other mechanical disturbance of a single coil. When NFORCE = 2, all the coils are driven by a translational force, such as would be produced by translational disturbances of the base by an earth quake. NFORCE = 3 drives all coils at the source phase and amplitude, such as a torsional disturbance of the base would cause.

Q is the magnitude of the translational disturbance in meters

A few additional comments on information transfer between sub-routines is also useful:

SELF DL and TORUS require no input data beyond that specified in the initialization of the main program, and can therefore be run independently. However, WOVALU requires the values in the array BETA the differential inductances, as input. These must be supplied by data cards (OPTION (1) = -1) or by the program TORUS (OPTION (1) = 1) if WOVALU is called. WOVALU also requires the quantities MODMAS, the

modal mass, and WSELF, the coil self-frequency. These are supplied by TORUS and by SELFDL. If neither of these are called, the values must be inserted in the initialization section of BUCKLE.

And finally, FFTRAN requires as input the array W02 of torus modal frequencies squared. This must be supplied by either calling WOVALU or from data cards.

APPENDIX III

LISTING OF COMPUTER CODE BUCKLE

COMPUTER CODE "BUCKLE"

THIS PROGRAM CALCULATES THE VIBRATION AND ELASTIC STABILITY LIMITS OF A SET OF DISCRETE SUPERCONDUCTING MAGNETS ARRANGED TO FORM A "BUMPY" TOROIDAL FIELD. IT ALSO CALCULATES SUCH LIMITS FOR INDIVIDUAL TORUS COILS. THE MUTUAL DESTABILIZING FORCES BETWEEN MAGNET PAIRS AND THE DESTABILIZING SELF FORCES OF EACH MAGNET ARE CALCULATED USING A NUMERICAL DIFFERENTIAL INDUCTANCE TECHNIQUE. USING METHODS FROM THE THEORY OF PERIODIC STRUCTURES, THE VIBRATION MODES OF THE SET ARE FOUND AS FUNCTIONS OF THE COIL CURRENT. THE RESPONSE OF THE SET OF MAGNETS TO AN EARTHQUAKE TYPE MOTION OF THE BASE, AND TO A MECHANICAL VIBRATION OF ONE OF ITS COILS IS ALSO COMPUTED.

THERE ARE FOUR MAJOR SUBROUTINES WHICH ARE CALLED BY THE MAIN PROGRAM. THEY ARE:

SELFDL WHICH CALCULATES THE TOTAL SELF INDUCTANCE AND DIFFERENTIAL MODAL SELF INDUCTANCE OF A SINGLE TOROIDAL FIELD COIL, AND THE CRITICAL BUCKLING CURRENT FOR EACH MODE DUE TO THE COIL'S SELF FIELD. IT COMPUTES THE MODAL SELF INDUCTANCES AND CORRESPONDING CRITICAL CURRENTS FOR IN-PLANE AND OUT-OF PLANE DEFORMATIONS AS WELL AS TWISTING.

TORUS WHICH CALCULATES THE INTERPAIR MUTUAL INDUCTANCE OF THE TORUS COILS AS WELL AS DIFFERENTIAL MODAL INDUCTANCE BETWEEN PAIRS, FOR DEFORMATIONS IN THE COIL PLANES OR PERPENDICULAR TO THEM.

WOVALU WHICH USES AS INPUT THE SELF AND MUTUAL DIFFERENTIAL MODAL INDUCTANCES FROM SELFDL AND TORUS, AND COMPUTES THE FREQUENCIES OF THE COLLECTIVE VIBRATION MODES OF THE TORUS, AND THE CRITICAL BUCKLING CURRENT AND CRITICAL TOROIDAL MAGNETIC FIELD FOR EACH BUCKLING MODE.

FFTRAN WHICH CALCULATES THE DYNAMIC RESPONSE OF THE MAGNETS AS A FUNCTION OF TIME TO SEVERAL TYPES OF MECHANICAL DISTURBANCES, INCLUDING TRANSVERSE OR ROTATIONAL MOTION OF THE BASE AND THE EXCITATION OF A SINGLE MAGNET SUCH AS FROM A HAMMER BLOW.

IN ADDITION, THERE ARE THREE OTHER SUBROUTINES WHICH ARE CALLED BY THE FOUR ABOVE. THEY ARE:

COIL WHICH SETS UP COORDINATES OF A SINGLE FILAMENT COIL WITH ADJUSTABLE MODAL DEFORMATIONS IN THE RADIAL AND Z DIRECTIONS, AND A VARIETY OF POSSIBLE CONSTRAINTS WHICH CONTROL THE STRETCHING OF THE FILAMENT LENGTH DURING DEFORMATION. COIL IS CALLED BY SELFDL AND TORUS.

OFFSET IS USED IN COMPUTING SELF INDUCTANCE OF A MAGNET COIL.

IT DIVIDES THE COIL CROSS SECTION INTO NDIVZ*NDIVR RECTANGULAR SUBSECTIONS, PRODUCES COORDINATES FOR FILAMENTS AT THE CENTER OF EACH SUBSECTION, THEN COMPUTES THE MUTUAL INDUCTANCE OF ALL SUCH FILAMENTS.

FILIA WHICH COMPUTES THE SELF INDUCTANCE OF A SINGLE FILAMENT, OMITTING THE SELF INDUCTANCE OF EACH FILAMENT SEGMENT UPON ITSELF.

INITIALIZE

DIMENSION W02(40)

DIMENSION BETA(20)

DIMENSION OPTION(10)

REAL Q

INTEGER OPTION

INTEGER QKSHAP

COMPLEX IIMAG

REAL QSTART

INTEGER OPT

REAL QKFREQ

REAL QKDAMP

INTEGER QKNCYC

REAL MODMAS

PI=3.1415926

PI=3.14159

SELECT OPTIONS WHICH DETERMINE PROGRAM BRANCHING

OPTION 1: GENERATE THE DLDA FOR THE TORUS. -1=GET THE DATA FROM CARDS; 0=DO NOT GET DLDA; 1=GET DLDA FROM SUBROUTINE TORUS PROG.

OPTION(1)= 1

OPTION 2: IS SELFINDUCTANCE TO BE CALCULATED? -1=DLSELF IS SPECIFIED IN A CARD IN MAIN PROG; 0=DO NOT GET DLSELF; 1=DLSELF IS TO BE COMPUTED BY CALLING SUBROUTINE SELFDL.

OPTION(2)= 1

OPTION 3: GENERATION OF TORUS MODAL FREQUENCIES. -1=GET THE VALUES OF W02 FROM DATA CARDS; 0=DO NOT GET THE MODAL FREQUENCIES; 1=GET THE VALUES OF W02 BY CALLING SUBROUTINE WOVALU.

OPTION(3)= 1

OPTION 4: ARE THE MODAL FREQUENCIES CALCULATED BY WOVALU TO BE CORRECTED BY ADDING THE SELF INDUCTANCE DLSELF?

OPTION(4)= 1

OPTION 5: ARE THE DYNAMIC RESPONSES OF THE COILS TO A DISTURBANCE TO BE CALCULATED? -1=NO, 0=NO; 1=YES, BY CALLING SUBROUTINE FFTRAN

OPTION(5)= 0

OPTION 6: WHICH APPROXIMATION IS TO BE USED FOR THE COIL SELF INDUCTANCE CALCULATION? -1=COIL IS TO APPROXIMATED AS A SINGLE FILAMENT; 0=COIL IS APPROXIMATED BY THE MUTUAL INDUCTANCES

OF AN ARRAY OF FILAMENTS WHICH APPROXIMATE THE COIL CROSS SECTION; 1=COIL SELF INDUCTANCE IS APPROXIMATED BY THE MUTUAL PLUS THE SELF INDUCTANCE OF AN ARRAY OF FILAMENTS WHICH APPROXIMATE THE COIL

LISTING OF COMPUTER CODE BUCKLE

```

C   CROSS SECTION.
C   OPTION(6)= 1
C   *****
C   SPECIFY DIMENSIONS AND PARAMETERS OF TORUS
C
C   NRING IS THE NUMBER OF COILS IN THE TORUS
C   NRING=24
C
C   RR IS THE TOROIDAL MAJOR RADIUS
C   RR=4.5
C
C   WEDGE IS THE TOTAL ANGLE SUBTENDED BY THE FIXED PART OF THE COIL.
C   WEDGE=PI/2.0
C
C   RAD IS THE RADIUS, OR MEAN RADIUS, FOR EACH MAGNET COIL.
C   RAD=2.35
C
C   XOVERY IS THE RATIO OF THE MAXIMUM COIL RADIUS IN THE X DIRECTION
C   TO THAT IN THE Y DIRECTION FOR ELLIPTICAL COILS.
C   XOVERY=1.0
C
C   EYOUNG: YOUNGS MODULUS,NT/METER**2.
C   EYOUNG=15.0E10
C
C   WIDER: COIL THICKNESS IN THE RADIAL DIRECTION.
C   WIDER=0.198
C
C   WIDEZ: COIL THICKNESS IN THE Z DIRECTION.
C   WIDEZ=0.384
C
C   DENS:COIL AVERAGE DENSITY,KG/METER**3
C   DENS=8.510E3
C
C   MODAL MASS OF THE MAGNET COIL
C   MODMAS=9.4E3
C
C   TOTAL CURRENT IN AMPERE TURNS FOR EACH COIL
C   AMP=1.0E6
C
C   *****
C
C   IF OPTION(2)=0 OR 1, MUST CHOOSE NUMBER OF SUBDIVISIONS IN
C   *SELF INDUCTANCE CALCULATION
C   NDIVZ IS THE NUMBER OF SUBDIVISIONS IN THE COIL CROSS SECTION IN
C   THE Z DIRECTION.
C   NDIVZ=2
C
C   NDIVR IS THE NUMBER OF SUBDIVISIONS IN THE COIL CROSS SECTION IN
C   THE R DIRECTION.
C   NDIVR=2
C   *****
C
C   IS BENDING MODE DESIRED? CHOOSE PARAMETERS FOR BENDING MODE.
C
C   IS THE MODAL DEFORMATION TO BE IN THE Z DIRECTION, I.E. PERPENDICULAR

```

LISTING OF COMPUTER CODE BUCKLE

TO THE COIL PLANE? 1=YES, 0=NO.
ZYES=1.0

IS THE MODAL DEFORMATION TO BE IN THE R DIRECTION, I.E. IN THE
PLANE OF THE COIL? 1=YES, 0=NO.
RYES=0.0

CYCLE THE ENTIRE PROGRAM OVER THE MODE NUMBERS DESIRED.
DO 700 MODE=1,1

APIN: IS THE BENDING MODE SINUSOIDAL, I.E. PINNED? 1=YES, 0=NO.
APIN=0.0

CANT: IS THE BENDING MODE A CANTELEVER SHAPE? 1=YES, 0=NO.
CANT=1.0

IS TWISTING MODE DESIRED? CHOOSE PARAMETERS FOR TWISTING MODE.
PMODE IS THE MODE NUMBER FOR THE PHI DEFORMATION
PMODE=1

IS THE PHI (TWIST) MODE A CANTELEVER SHAPE? 1=YES, 0=NO.
PCANT=1.0

IS THE PHI MODE A SINUSOIDAL SHAPE? 1=YES, 0=NO.
PAPIN=0.0

PHIMAG IS THE INITIAL MAGNITUDE OF TWIST.
PHIMAG=0.0

SPECIFY CONSTRAINTS ON COIL STRETCHING: IF RJUST=0, THE REST ARE NOT
NECESSARY.

RJUST DETERMINES COIL STRETCHING DURING DEFORMATION. IF ANY
CONSTRAINT IS TO BE APPLIED, RJUST=1. IF NOT, RJUST=0.
RJUST=1.0

RFIX IS THE RADIUS AT WHICH THE TOTAL COIL LENGTH IS TO BE CONSTRAINED.
RFIX=RAD

STRECH: RATIO BY WHICH THE COIL LENGTH AT RFIX IS TO BE CHANGED.
STRECH=1.0 CORRESPONDS TO THE COIL LENGTH AT RFIX HELD CONSTANT.
STRECH=1.0

IF OPTION(5)=1, THE FOLLOWING PARAMETERS MUST BE SELECTED FOR THE
COIL FORCING FUNCTION AND DYNAMIC RESPONSE

SCALE THE GRAPHICAL OUTPUT FOR THE COIL DYNAMIC RESPONSE

LISTING OF COMPUTER CODE BUCKLE

NPROVE=1

BATA=7.8E3

NCYCLE=TIME SCALING FACTOR-NUMBER OF COIL OSCILLATIONS IN THE
TIME FRAME OF THE FAST FOURIER.

NCYCLE=10

EFOLD:DAMPING FACTOR.

EFOLD=1.5

PARAMETERS OF THE COIL DISTURBANCE FUNCTION

CHOOSE THE FORM OF THE COIL OSCILLATION

HAMMER BLOW

NFORCE=1

TORSIONAL QUAKE

NFORCE =2

TRANSVERSE OSCILLATION QUAKE

NFORCE=3

Q: MAGNITUDE OF QUAKE MOTION (METERS)

Q=0.1

PHIO:DIRECTION OF QUAKE OSCILLATION RELATIVE TO COIL #1.

PHIO=0.0

QKSHAP: SHAPE OF FORCING FUNCTION. QKSHAP=1 PRODUCES A TRIANGULAR
PULSE, CENTERED AT T0, OF WIDTH 1/ALPH. QKSHAP=2 PRODUCES A DAMPED
SINUSOID, OF FREQUENCY QKFREQ, DAMPING QKDAMP, AND QKNCYC HALF CYCLES.
QKSHAP=1

QSTART: THE FRACTION OF TMAX AT WHICH THE EARTHQUAKE BEGINS.T0=TMAX*QSTART
QSTART=0.1

DAMPING OF THE QUAKE WAVEFORM.

QKDAMP=0.1

QKFREQ:FREQUENCY OF THE QUAKE

QKFREQ=4.0

QKNCYC:NUMBER OF HALF CYCLES IN THE QUAKE QSCILLATION

QKNCYC=3.0

PROGRAM RUNS

DERIVED PARAMETERS

NRING2=IF1X(NRING/2.0+0.001)

GET THE VALUES OF THE MUTUAL INDUCTANCES(BETA(I), BY CALLING

LISTING OF COMPUTER CODE BUCKLE

```

C      TORUS OR READING DATA CARDS
      IF(OPTION(1)) 500,501,502
500    CONTINUE
C
C      READ DATA CARDS
      WSELF=SQRT(39.4)
      DO 15 I=1,NRING2
      READ(5,103) BETA(I)
103    FORMAT(F16.0)
      WRITE(6,104) BETA(I)
104    FORMAT('0','DLDA',F16.3)
      CHKLST=BETA(I)
C
C      MAKE SURE THERE ARE THE PROPER NUMBER OF DATA CARDS
      IF(CHKLST-10000000.0) 20,19,19
19     CONTINUE
      WRITE(6,200)
200    FORMAT('0','BETA HAS TOO FEW DATA CARDS')
      GO TO 31
20     CONTINUE
15     CONTINUE
23     CONTINUE
      READ(5,103) CHKLST
      IF(CHKLST-10000000.0) 21,22,22
21     GO TO 23
22     CONTINUE
      GO TO 503
501    CONTINUE
      GO TO 503
502    CONTINUE
      CALL      TORUS(RR,RJUST,WEDGE,RAD,RFIX,XOVERY,ZYES,RYES,
*MODE,APIN,CANT,STRECH,EYOUNG,WIDER,WIDEZ,DENS,NRING,BETA,WSELF,
*MODMAS)
503    CONTINUE
      OPT=OPTION(6)
C
C      DECIDE WHETHER TO CALL SELF DL.
      IF(OPTION(2)) 510,511,512
510    CONTINUE
      GO TO 513
511    CONTINUE
      GO TO 513
512    CONTINUE
      CALL SELF DL(RR,RJUST,WEDGE,RAD,RFIX,
*XOVERY,ZYES,RYES,PCANT,PAPIN,PMODE,PHIMAG,NDIVR,NDIVZ,MODE,
*APIN,CANT,STRECH ,EYOUNG,WIDER,WIDEZ,DENS,DLSELF,OPT,MODMAS)
513    CONTINUE
C
C      GET THE VALUES OF THE MODAL FREQUENCIES SQUARED, W02(I), BY EITHER
C      CALLING WOVALU OR READING DATA CARDS.
      IF(OPTION(3)) 520,521,522
C
C      READ DATA CARDS.
520    CONTINUE
      DO 14 I=1,NRING

```

LISTING OF COMPUTER CODE BUCKLE

```

    READ(5,100) W02(1)
    .00  FORMAT(F10.4)
        WRITE(6,101) W02(1)
    101  FORMAT('0','W02',F10.6)
        CHKLST=W02(1)
C
C    CHECK FOR PROPER NUMBER OF DATA CARDS
    IF(CHKLST-10000000.0) 24,25,25
    25  CONTINUE
        WRITE(6,201)
    201  FORMAT('0','W02 HAS TOO FEW DATA CARDS')
        GO TO 31
    24  CONTINUE
    14  CONTINUE
    521  CONTINUE
        GO TO 523
    522  CONTINUE
C
C    ARE THE MODAL FREQUENCIES TO BE CORRECTED FOR THE EFFECTS OF THE
C    SELF INDUCTANCE, BY INCLUDING DLSELF?
    IF(OPTION(4)) 530,531,532
    530  CONTINUE
    531  CONTINUE
        DLSELF=0.0
        GO TO 533
    532  CONTINUE
    533  CONTINUE
        CALL WOVALU(BETA,W02 ,NRING2,NRING,MODMAS,RR,WSELF,DLSELF,AMP)
    523  CONTINUE
C
C    CALCULATE THE RESULTANT MOTION OF THE TOROIDAL COILS DUE TO A DISTURBANCE
    IF(OPTION(5)) 540,541,542
    540  CONTINUE
    541  CONTINUE
        GO TO 543
    542  CONTINUE
        CALL      FFTRAN(QSTART,NTEST      ,NRING,NFORCE,Q,BATA,PHI 0,
*NCYCLE,EFOLD,QKSHAP,QKDAMP,QKFREQ,QKNCYC,W02,MODMAS)
    543  CONTINUE
    31  CONTINUE
    700  CONTINUE
        STOP
        END
        SUBROUTINE SELFDL(RR,RJUST,WEDGE,RAD,RFIX,
*XOVERY,ZYES,RYES,PCANT,PAPIN,PMODE,PHIMAG,NDIVR,NDIVZ,MODE,
*APIN,CANT,STRECH ,EYOUNG,WIDER,WIDEZ,DENS,DLSELF,OPT,MODMAS)
        INTEGER OPT
        INTEGER KORSEL
        DIMENSION SUMFIL(16),ZEROFL(16),DLDAFL(16),DLFL(16),SUMLFL(16)
        REAL ICRIT
        REAL MODMAS
        PI=3.141592
        DELTA=0.5E-3
        DELTA=1.0E-3
        DELTA=0.0

```

LISTING OF COMPUTER CODE BUCKLE

```

DDEL=0.05*RAD
MOKEY=NDIVR*NDIVZ
VAB=FLOAT(MOKEY)
AMOKEY=VAB
AMOK2=(AMOKEY**2)*1.0E7
MKEY=1
KORSEL=0
WRITE(6,301)
301  FORMAT('1')
WRITE(6,996)
996  FORMAT('0','INPUT VARIABLES FOR SELF INDUCTANCE SELF DL')
WRITE(6,313) RAD
313  FORMAT(' ','MINOR RADIUS (RAD)=' ,F16.6)
WRITE(6,315) XOVERY
315  FORMAT(' ','ELLIPTICITY (XOVERY)=' ,F16.6)
WRITE(6,316) WEDGE
316  FORMAT(' ','ANGLE OF STRAIGHT BACK (WEDGE)=' ,F16.6)
WRITE(6,317) WIDER
317  FORMAT(' ','COIL THICKNESS IN RADIAL DIRECTION (WIDER)=' ,F16.6)
WRITE(6,318) WIDEZ
318  FORMAT(' ','COIL THICKNESS IN Z DIRECTION (WIDEZ)=' ,F16.6)
WRITE(6,320)
320  FORMAT(' ','IS LENGTH CONSTRAINED?' )
CALL YESNO(RJUST)
WRITE(6,321) RFIX
321  FORMAT(' ','RADIUS AT WHICH CONSTRAINT IS APPLIED (RFIX)=' ,F16.6)
WRITE(6,322) STRECH
322  FORMAT(' ','RATIO OF STRETCHING AT RFIX (STRECH)=' ,F16.6)
WRITE(6,323) EYOUNG
323  FORMAT(' ','AVERAGE YOUNGS MODULUS (EYOUNG)=' ,E16.6)
WRITE(6,324) DENS
324  FORMAT(' ','AVERAGE COIL DENSITY (DENS)=' ,F16.6)
WRITE(6,325)
325  FORMAT(' ','INPUT DATA ON THE BENDING MODE')
WRITE(6,326) MODE
326  FORMAT(' ','MODE NUMBER (MODE)=' ,I4)
WRITE(6,327)
327  FORMAT(' ','IS THIS A PINNED MODE?')
CALL YESNO(APIN)
WRITE(6,328)
328  FORMAT(' ','IS THIS A CANTELEVER MODE?')
CALL YESNO(CANT)
WRITE(6,329)
329  FORMAT(' ','IS DEFORMATION IN Z DIRECTION?')
CALL YESNO(ZYES)
WRITE(6,330)
330  FORMAT(' ','IS DEFORMATION IN R DIRECTION?')
CALL YESNO(RYES)
IF(PAPIN+PCANT) 940,940,941
941  CONTINUE
WRITE(6,331)
331  FORMAT(' ','INPUT DATA ON TWISTING MODE')
WRITE(6,332) PMODE
332  FORMAT(' ','TWISTING MODE NUMBER (PMODE)=' ,I4)
WRITE(6,333)

```

LISTING OF COMPUTER CODE BUCKLE

```

333  FORMAT(' ', 'IS THIS A PINNED MODE?')
      CALL YESNO(PAPIN)
      WRITE(6,334)
334  FORMAT(' ', 'IS THIS A CANTELEVER MODE?')
      CALL YESNO(PCANT)
940  CONTINUE
      IF(OPT) 771,772,773
771  CONTINUE
      WRITE(6,302)
302  FORMAT('0', 'SELF INDUCTANCE IS APPROXIMATED BY A SINGLE FILAMENT')
      GO TO 774
772  CONTINUE
      WRITE(6,303)
303  FORMAT('0', 'SELF INDUCTANCE IS APPROXIMATED BY MUTUAL INDUCTANCE
*OF MULTIPLE FILAMENTS')
      WRITE(6,305) VAB
      GO TO 774
773  CONTINUE
      WRITE(6,304)
304  FORMAT('0', 'SELF INDUCTANCE APPROXIMATED BY MUTUAL PLUS SELF
*INDUCTANCE OF ARRAY OF FILAMENTS')
      WRITE(6,305) VAB
774  CONTINUE
      WRITE(6,300)
300  FORMAT('0', 'RESULTS ON SELF INDUCTANCE FROM SELF DL')
      DO 500 JJ=1,3
C
C   FIRST SET UP CENTRAL FILAMENT OF THE COIL
      CALL COIL(RFIX,RR,DELTA,WEDGE,RAD,XOVERY,MODE,CANT,
*APIN,WIDER,WIDEZ,EYOUNG,DENS,WSELF,AL,F1,ZYES,RYES,RJUST,ALAM1,
*STRECH,ALPHA,COEFF,JJ)
C
C   DECIDE WHICH OF THREE POSSIBLE APPROXIMATIONS TO USE.
      IF(OPT) 550,551,552
C
C   CALCULATE SELF INDUCTANCE OF THIS FILAMENT
550  CONTINUE
      CALL FILIA(SUMRR,MKEY,KORSEL)
      SUMRR=SUMRR*1.0E7
      GO TO 553
C
C   SET UP ADDITIONAL FILAMENTS TO APPROXIMATE ACTUAL COIL, AND
C   CALCULATE MUTUAL INDUCTANCES OF THESE FILAMENTS
551  CONTINUE
      CALL OFFSET(WIDER,WIDEZ,NDIVZ,NDIVR,PCANT,PAPIN,PHIMAG,PMODE,
*WEDGE,ALAM1,SUMRR)
      SUMRR=SUMRR/AMOK2
      GO TO 553
C
C   CALCULATE MUTUAL INDUCTANCES OF THESE FILAMENTS AND PREPARE TO INCLUDE
C   SELF INDUCTANCE OF THESE FILAMENTS AS A CORRECTION.
552  CONTINUE
305  FORMAT('0', 'NUMBER OF FILAMENTS USED TO APPROXIMATE COIL=',F3.0)
      KORSEL=1
      CALL OFFSET(WIDER,WIDEZ,NDIVZ,NDIVR,PCANT,PAPIN,PHIMAG,PMODE,

```

LISTING OF COMPUTER CODE BUCKLE

```

      *WEDGE,ALAM1,SUMRR)
      SUMRR=SUMRR/AMOK2
553  CONTINUE
      SUMAIN=SUMRR
C
C      IS THE HIGHER CORRECTION DESIRED?
      IF(KORSEL) 610,610,611
611  CONTINUE
C
C      CALCULATE THE CORRECTION DUE TO THE SELF INDUCTANCES OF THE FILAMENTS
C      COMPRISING THE COIL.
      SFIL=0.0
      DO 10 MKEY=1,MOKEY
      CALL FILIA(SUMRR,MKEY,KORSEL)
      SUMRR=SUMRR/AMOK2
      SUMFIL(MKEY)=SUMRR
      SFIL=SFIL+SUMFIL(MKEY)
10   CONTINUE
610  CONTINUE
C      INCREMENT THE MODAL PERTURBATION AND CALCULATE DIFFERENTIAL INDUCTANCES.
      IF(JJ-1) 150,150,151
150  CONTINUE
      ZEROL=SUMAIN
      DL=0.0
      DLDA=0.0
      IF(KORSEL) 700,700,701
701  CONTINUE
      ZEROSF=SFIL
      DLSF=0.0
      DLDAF=0.0
      DO 11 I=1,MOKEY
      ZEROFL(I)=SUMFIL(I)
      DLFL(I)=0.0
      DLDAFL(I)=0.0
11   CONTINUE
700  CONTINUE
      GO TO 160
151  CONTINUE
      DL=SUMAIN-SUML
      DLDA=DL/((DELTA**2-DELT1**2)
      IF(KORSEL) 710,710,711
711  CONTINUE
      DLSF=SFIL-SUMLSF
      DLDAF=DLSF/((DELTA**2-DELT1**2)
      DO 12 I=1,MOKEY
      DLFL(I)=SUMFIL(I)-SUMLFL(I)
      DLDAFL(I)=DLFL(I)/((DELTA**2-DELT1**2)
12   CONTINUE
710  CONTINUE
160  CONTINUE
      WRITE(6,306) JJ,DELTA,SUMAIN,DL,DLDA
306  FORMAT('0','JJ=',I2,3X,',DELTA=',E7.1,3X,',L(1ST APPROX)=' ,E10.5,
      *3X,',DL(1ST APPROX)=' ,E10.5,3X,',DLDA(1ST APPROX)=' ,E10.5)
      IF(KORSEL) 720,720,721
721  CONTINUE

```


LISTING OF COMPUTER CODE BUCKLE

```

    SUMLSF=SFIL
    WRITE(6,307) JJ,DELTA,SFIL,DLSF,DLDA SF
307  FORMAT('0','JJ=',I2,3X,'DELTA=',E7.1,3X,'L(CORRECTION)=' ,E10.5,
    *3X,'DL(CORRECTION)=' ,E10.5,3X,'DLDA(CORRECTION)=' ,E10.5)
    DO 13 I=1,MOKEY
    SUMFL(I)=SUMFIL(I)
    WRITE(6,308) JJ,DELTA,SUMFIL(I),DLFL(I),DLDAFL(I)
308  FORMAT('0','JJ=',I2,3X,'DELTA=',E7.1,3X,'L(INDIVIDUAL)=' ,E10.5,
    *3X,'DL(INDIVIDUAL)=' ,E10.5,3X,'DLDA(INDIVIDUAL)=' ,E10.5)
13  CONTINUE
720  CONTINUE
    DELT1=DELTA
    DELTA=DELTA+DDEL
    SUML=SUMAIN
500  CONTINUE
    REAL LTOT
    LTOT=SFIL+SUMAIN
    DLTOT=DLSF+DL
    DLSELF=DLDA SF+DLDA
    WRITE(6,309) JJ,DELTA,LTOT ,DLTOT,DLSELF
309  FORMAT('0','JJ=',I2,3X,'DELTA=',E7.1,3X,'L(2ND APPROX)=' ,E10.5,
    *3X,'DL(2ND APPROX)=' ,E10.5,3X,'DLDA(2ND APPROX)=' ,E10.5)
C
    WRITE(6,310) LTOT
310  FORMAT('0','FINAL SELF INDUCTANCE (LTOT )=' ,E16.8,'HENRYS')
    WRITE(6,312) DLSELF
312  FORMAT('0','FINAL MODAL 2ND DERIVATIVE DLSELF=' ,E10.6,
    *'HENRY/METER**2')
C
    USE THESE VALUES OF SELF INDUCTANCE TO CALCULATE THE CRITICAL BUCKLING
C
    CURRENT DUE TO SELF FORCES.
    TURNS=20.0
    DLDA=DLSELF
    DLDA=(TURNS/FLOAT(NDIVR*NDIVZ))*2*DLDA
    DLDA=1.0E-7*ABS(DLDA)
    GAM=1.0
    ALAM=ALAM1
    RAD1=RAD
C
C
    CALCULATE THE MODAL MASS
    MODMAS=WIDER*WIDEZ*AL*DENS*F1
    WIDZ=WIDEZ*ZYES+WIDER*RYES
    WIDR=WIDEZ*(1.0-ZYES)+WIDER*(1.0-RYES)
C
C
    CALCULATE THE BENDING MOMENT
    BENDMO=WIDR*(WIDZ**3)/12.0
    WRITE(6,108) BENDMO
108  FORMAT('0','BENDING MOMENT BENDMO=' ,E16.6)
C
C
    CALCULATE THE CRITICAL CURRENT ICRIT.
    CALL COFIC(MODE,CANT,APIN,KO,COEFF,RAD,AL,F1)
    WSELF=SQRT((EYOUNG*BENDMO*COEFF)/(MODMAS*RAD**4))
    ICRIT=WSELF*SQRT(MODMAS /DLSELF)
    WRITE(6,312) ICRIT
312  FORMAT('0','CRITICAL BUCKLING CURRENT (ICRIT)=' ,E16.8,'AMPERES')
    WRITE(6,311) WSELF

```

LISTING OF COMPUTER CODE BUCKLE

```

311  FORMAT('0','FUNDAMENTAL FREQUENCY OF COIL (WSELF)=' ,F16.8,
      *'RADIANS/SEC, FOR ZERO CURRENT')
      RETURN
      END
      SUBROUTINE FILIA(SUMRR,MKEY,KORSEL)
C
C  FILIA CALCULATES THE SELF INDUCTANCE OF A SINGLE FILAMENT
C
      DIMENSION S(201)
      DIMENSION XC(201),YC(201),ZC(201)
      DIMENSION XCV(201),YCV(201),ZCV(201)
      DIMENSION X1(201,4),Y1(201,4),Z1(201,4)
      DIMENSION X1V(201,4),Y1V(201,4),Z1V(201,4)
      COMMON XC,YC,ZC,XCV,YCV,ZCV,S,X1,Y1,Z1,X1V,Y1V,Z1V
      INTEGER KORSEL
      NM=200
      NY=201
      IF(KORSEL) 70,70,71
71  CONTINUE
C
C  IF THE SELF INDUCTANCE OF THE OFFSET FILAMENTS IS BEING CALCULATED, THEIR
C  VALUES ARE READ IN HERE.
      DO 15 I=1,NM
      XC(I+1)=X1(I,MKEY)
      YC(I+1)=Y1(I,MKEY)
      ZC(I+1)=Z1(I,MKEY)
      XCV(I+1)=X1V(I,MKEY)
      YCV(I+1)=Y1V(I,MKEY)
      ZCV(I+1)=Z1V(I,MKEY)
15  CONTINUE
70  CONTINUE
      SUMRR = 0.0
C
C  INTEGRATE THE INDUCTANCE OVER THE FILAMENTS
      DO 201 L=2,NY
      SUMR = 0.0
      DO 200 K=2,NY
      IF (K-L) 61,62,61
62  CONTINUE
      RX=0.0
      GO TO 200.
61  CONTINUE
      D1=XC(K)-XC(L)
      D2=YC(K)-YC(L)
      D3=ZC(K)-ZC(L)
      D = SQRT((D1*D1) + (D2*D2) + (D3*D3))
      CO=(XCV(K)*XCV(L)+YCV(K)*YCV(L)+ZCV(K)*ZCV(L))
      RX=CO/D
200  SUMR = SUMR + RX
201  SUMRR = SUMR + SUMRR
203  CONTINUE
202  CONTINUE
C
C  SELF INDUCTANCE OF THE FILAMENT IS SUMRR
      RETURN

```

```

END
SUBROUTINE OFFSET(WIDTH,THICK,NDIVZ,NDIVR,CANT,APIN,
*PHIMAG,MODE,WEDGE,ALAM1,SUMRR)

```

```
INITIALIZE THE MODAL PARAMETERS
GO TO (1,2,3,4), MODE
```

CONTINUE
ALAM=7.853
ALPHA=1.0008
GO TO 5

```
CONTINUE  
ALAM=11.0  
ALPHA=1.0  
GO TO 5
```

```
CONTINUE  
ALAM=14.0  
ALPHA=1.0  
CONTINUE
```

```
ADJUST SCALES THE ARGUMENTS FOR THE MODE SHAPE
ADJUST=ALAM*CANT+2.0*PI*FLOAT(MODE)*APIN
ADALAM=ADJUST/ALAM1
NM=200
NY=NM+1
```

THESE VARIABLES ARE USED IN DETERMINING THE OFFSET FILAMENTS POSITIONS
ANABR=FLOAT(NDIVR+1)
ANABZ=FLOAT(NDIVZ+1)

```

LOOP WHICH SETS UP ARRAYS OF COORDINATES FOR THE OFFSET FILAMENTS
DO 51 I=1,NY

```

```

LENGTH OF CENTRAL FILAMENT SEGMENT
SIP=SQRT(ZCV(I)**2+YCV(I)**2+XCV(I)**2)
CDP=XCV(I)/SIP
SDP=YCV(I)/SIP

```

CALCULATE UNIT OFFSET VECTOR, PERPENDICULAR TO CENTRAL FILAMENT SEGMENT AND IN THE PLANE OF THE UNDEFORMED COIL.

LISTING OF COMPUTER CODE BUCKLE

```

XOFFR(I)=SOP
YOFFR(I)=-CDP
ZOFFR(I)=0.0

C
DIST=S(I)*ADALAM
DEFL=CANT*((COSH(DIST)-COS(DIST))-ALPHA*(SINH(DIST)-SIN(DIST)))
DEFL=DEFL+APIN*SIN(DIST)

C
C
C
ESTABLISH MAGNITUDE OF TWIST
PHI=PHIMAG*DEFL
C
TWIST THIS UNIT OFFSET VECTOR OUT OF THE PLANE ACCORDING TO THE
C
TWISTING MODE.
ZOFFR(I)=SIN(PHI)
XOFFR(I)=COS(PHI)*XOFFR(I)
YOFFR(I)=COS(PHI)*YOFFR(I)
71 CONTINUE

C
C
C
SET UP AN OFFSET VECTOR WHICH IS THE CROSS PRODUCT OF THE FIRST OFFSET
C
VECTOR WITH THE CENTRAL FILAMENT SEGMENT. THIS DETERMINES THE
C
SECOND OFFSET VECTOR IN THE PLANE OF THE COIL CROSS SECTION.
XOFFZ(I)=(YCV(I)*ZOFFR(I)-ZCV(I)*YOFFR(I))/SIP
YOFFZ(I)=(ZCV(I)*XOFFR(I)-XCV(I)*ZOFFR(I))/SIP
ZOFFZ(I)=(XCV(I)*YOFFR(I)-YCV(I)*XOFFR(I))/SIP

C
C
SCALE THE OFFSET VECTORS ACCORDING TO THE DIMENSIONS OF THE COIL.
ZSCALE=THICK/FLOAT(NDIVZ)
RSCALE=WIDTH/FLOAT(NDIVR)
XOFFZ(I)=XOFFZ(I)*ZSCALE
YOFFZ(I)=YOFFZ(I)*ZSCALE
ZOFFZ(I)=ZOFFZ(I)*ZSCALE
XOFFR(I)=XOFFR(I)*RSCALE
YOFFR(I)=YOFFR(I)*RSCALE
ZOFFR(I)=ZOFFR(I)*RSCALE

C
C
USING THESE OFFSET VECTORS, CALCULATE THE OFFSET FILAMENT COORDINATES.
DO 53 M=1,NDIVR
DO 54 K=1,NDIVZ
J=NDIVZ*(M-1)+K
XCOR=XOFFR(I)*ANABR/2.0+XOFFZ(I)*ANABZ/2.0
YCOR=YOFFR(I)*ANABR/2.0+YOFFZ(I)*ANABZ/2.0
ZCOR=ZOFFR(I)*ANABR/2.0+ZOFFZ(I)*ANABZ/2.0
X1(I,J)=XC(I)+M*XOFFR(I)+K*XOFFZ(I)-XCOR
Y1(I,J)=YC(I)+M*YOFFR(I)+K*YOFFZ(I)-YCOR
Z1(I,J)=ZC(I)+M*ZOFFR(I)+K*ZOFFZ(I)-ZCOR
54 CONTINUE
53 CONTINUE
51 CONTINUE

C
C
SET UP THE CORRESPONDING TANGENT VECTORS OF THE FILAMENTS
NCCOUNT=NDIVZ*NDIVR
DO 1001 J=1,NCCOUNT
X1V(1,J)=(X1(2,J)-X1(NM,J))/2.0
Y1V(1,J)=(Y1(2,J)-Y1(NM,J))/2.0
Z1V(1,J)=(Z1(2,J)-Z1(NM,J))/2.0

```

LISTING OF COMPUTER CODE BUCKLE

```

DO 110 I=2,NM
X1V(I,J)=(X1(I+1,J)-X1(I-1,J))/2.0
Y1V(I,J)=(Y1(I+1,J)-Y1(I-1,J))/2.0
Z1V(I,J)=(Z1(I+1,J)-Z1(I-1,J))/2.0
110 CONTINUE
1001 CONTINUE
C
C INTEGRATE THE MUTUAL INDUCTANCES
SUMRR = 0.0
NCM=NCOUNT-1
DO 202 J=1,NCM
MNUM=J+1
DO 203 M=MNUM,NCOUNT
DO 201 L=1,NM
SUMR = 0.0
DO 200 K=1,NM
D1=X1(K,J)-X1(L,M)
D2=Y1(K,J)-Y1(L,M)
D3=Z1(K,J)-Z1(L,M)
D = SQRT((D1*D1) + (D2*D2) + (D3*D3))
CO=(X1V(K,J)*X1V(L,M)+Y1V(K,J)*Y1V(L,M)+Z1V(K,J)*Z1V(L,M))
RX=CO/D
200 SUMR = SUMR + RX
201 SUMRR = SUMR + SUMRR
203 CONTINUE
202 CONTINUE
C
C SUMRR IS THE TOTAL MUTUAL INDUCTANCE
SUMRR=SUMRR*2.0
RETURN
END
SUBROUTINE COIL(RFIX,RR,DELTA,WEDGE,RADI,XOVERY,MODE,CANT,
*APIN,WIDER,WIDEX,EYOUNG,DENS,WSELF,AL,F1,ZYES,RYES,RJUST,ALAM,
*STRECH,ALPHA,COEFF,JJ)
C
C COIL SETS UP AN ELLIPTICAL OR D SHAPED FILAMENT WITH MODAL DEFORMATIONS IN
C R AND Z AND A VARIETY OF POSSIBLE STRETCHING CONSTRAINTS.
C
C COORDINATES OF FILAMENT
DIMENSION X(201),Y(201),Z(201)
DIMENSION S(201),THETA(201)
C
C COORDINATES AT WHICH LENGTH S1 IS TO BE CONSTRAINED.
DIMENSION X1(201),Y1(201),S1(201)
C
C TANGENT VECTORS OF X,Y,Z
DIMENSION XV(201),YV(201),ZV(201)
C
COMMON X,Y,Z,XV,YV,ZV,S
REAL MODMAS
PI=3.1415926
C DATA FOR FULL SIZE COIL
RAD=RADI
STARR=RFIX/RADI
YOVERX=1.0/XOVERY

```

LISTING OF COMPUTER CODE BUCKLE

```

INITIALIZE THE MODAL PARAMETERS
GO TO (1,2,3,4),MODE
1  CONTINUE
   ALAM=4.73
   ALPHA=0.9825
   COEFF=0.778
   GO TO 5
2  CONTINUE
   ALAM=7.853
   ALPHA=1.0008
   COEFF=4.19
   GO TO 5
3  CONTINUE
   ALAM=11.0
   ALPHA=1.0
   COEFF=19.5
   GO TO 5
4  CONTINUE
   ALAM=14.0
   ALPHA=1.0
   COEFF=58.71
5  CONTINUE
C
C  SET UP INTEGRATION INDICES
NM=200
M=IFIX(WEDGE*FLOAT(NM)/(2.0*PI))
N=NM-M
NX=N+1
NY=NM+1
BANGLE=0.5*FLOAT(NM)/FLOAT(NX)
DO 117 I=1,NY
  S(I)=0.0
117 CONTINUE
RADFIX=0.0
C
C  THIS LOOP RECURSIVELY CORRECTS THE LENGTH S1 OF THE FILAMENT BY
C  CHANGING RADFIX ACCORDING TO RFIX, STRECH, ANF RJUST.
DO 102 IND=1,3
  F1=0.0
  FSAVE=0.0
  DO 100 I=1,NX
    THETA(I)=(FLOAT(I-1)*2.0*PI)/FLOAT(NM)
C
C  Z DEFORMATION
Z(I) = CANT * ((COSH(S(I))-COS(S(I)))-ALPHA*(SINH(S(I))-SIN(S(I))))
Z(I)=Z(I)+APIN*SIN(S(I))
C
C  RADFIX ADJUSTS THE RADIUS TO ALLOW LENGTH CONSTRAINTS ON THE FILAMENT
RAD=RADI*(1.0-RADFIX*SIN(BANGLE*THETA(I)))
C
C  INCLUDE THE RADIAL MODAL DEFORMATION.
RAD=RAD+DELTA*RYES*Z(I)
C
C  SET UP COORDINATES

```

LISTING OF COMPUTER CODE BUCKLE

```

X(I)=RAD*COS(THETA(I)-(PI/4.0))*XOVERY
X1(I)=STARR*X(I)
Y(I)=RAD*SIN(THETA(I)-(PI/4.0))*YOVERX
Y1(I)=STARR*Y(I)

C
C FSAVE IS USED TO CALCULATE THE FIRST DEFORMATION MOMENT, THE
C MODAL MASS.
C FSAVE=ABS(Z(I))+FSAVE

C
C SCALE THE Z DEFORMATION
C Z(I)=DELTA*ZYES*Z(I)
100 CONTINUE
IF(IND-1) 112,112,115

C
C SET UP INITIAL LENGTHS S, S1, BEFORE DEFORMATION.
112 CONTINUE
S(I)=0.0
S1(I)=0.0
DO 114 I=2,NX
S(I)=S(I-1)+SQRT((X(I)-X(I-1))**2+(Y(I)-Y(I-1))**2)
S1(I)=S1(I-1)+SQRT((X1(I)-X1(I-1))**2+(Y1(I)-Y1(I-1))**2+
*(Z(I)-Z(I-1))**2)
114 CONTINUE
IF(JJ-1) 412,412,415
412 CONTINUE
AL=S(NX)
AL2=S1(NX)
415 CONTINUE

C
ADJUST=(ALAM/AL)*CANT+(2.0*PI*FLOAT(MODE))*APIN/AL

C
C SCALE THE COIL LENGTH FOR USE AS ARG OF SIN AND SINH.
DO 113 I=1,NX
S(I)=S(I)*ADJUST
113 CONTINUE
115 CONTINUE
S1(I)=0.0

C
C CALCULATE THE TOTAL CONSTRAINED LENGTH S1
DO 907 I=2,NX
S1(I)=S1(I-1)+SQRT((X1(I)-X1(I-1))**2+(Y1(I)-Y1(I-1))**2+
*(Z(I)-Z(I-1))**2)
907 CONTINUE

C
C CORRECTION TO RADFIX
RADFIX=RADFIX+RJUST*(S1(NX)-AL2*STRECH)/AL2
WRITE(6,909) RADFIX,S1(NX),AL2
909 FORMAT('0', 'ADJUSTMENT VBLS', 3E16.6)
102 CONTINUE

C
C SET UP STRAIGHT BACK OF THE "D" COIL
DX=(X(NX)-X(1))/FLOAT(M)
DO 111 I=NX,NM
X(I)=X(NX)-(FLOAT(I-NX)*DX)
Y(I)=Y(1)

```

LISTING OF COMPUTER CODE BUCKLE

```

      Z(1) = 0.0
111  CONTINUE
C
C   CONNECT THE ENDS OF THE COIL.
      X(NY) = X(1)
      Y(NY) = Y(1)
      Z(NY) = Z(1)
      DO 300 I=1,NY
300  CONTINUE
C
C   CALCULATE TANGENT VECTORS
      XV(1)=(X(2)-X(NM))/2.0
      YV(1)=(Y(2)-Y(NM))/2.0
      ZV(1)=(Z(2)-Z(NM))/2.0
      DO 303 I=2,NM
      XV(I)=(X(I+1)-X(I-1))/2.0
      YV(I)=(Y(I+1)-Y(I-1))/2.0
      ZV(I)=(Z(I+1)-Z(I-1))/2.0
303  CONTINUE
      XV(NY)=XV(1)
      YV(NY)=YV(1)
      ZV(NY)=ZV(1)
C
C   NORMALIZE THE MODAL MOMENT FSAVE.
      F1=FSAVE/FLOAT(NX)
      RETURN
      END
      SUBROUTINE TORUS(RR,RJUST,WEDGE,RAD,RFIX,XOVERY,ZYES,RYES,
*MODE,APIN,CANT,STRECH,EYOUNG,WIDER,WIDEZ,DENS,NRING,BETA,WSELF,
*MODMAS)
C
C   TORUS SETS UP COORDINATES OF ALL THE COILS IN A "BUMPY" TORUS AND
C   CALCULATES THEIR MUTUAL INDUCTANCE AND THE FIRST AND SECOND MODAL
C   DERIVATIVES OF INDUCTANCE PAIRWISE.
      DIMENSION S(201)
C
C   COORDINATES OF MIDDLE COIL
      DIMENSION X(201),Y(201),Z(201)
C
C   THE COORDINATE OF THE COIL ON THE LEFT
      DIMENSION XL(201),YL(201),ZL(201)
C
C   THE COORDINATE OF THE COIL ON THE RIGHT
      DIMENSION XR(201),YR(201),ZR(201)
C
C   THE CORRESPONDING TANGENT VECTORS
      DIMENSION XV(201),YV(201),ZV(201)
      DIMENSION XLV(201),YLV(201),ZLV(201)
      DIMENSION XRV(201),YRV(201),ZRV(201)
C
C   THE DIFFERENTIAL INDUCTANCE BETWEEN THE MIDDLE COIL AND THE
      NORDER-TH PAIR OF COILS
      DIMENSION BETA(20)
C
      COMMON X,Y,Z,XV,YV,ZV,S,XLV,YLV,ZLV,XRV,YRV,ZRV,XR,YR,ZR,XL,YL,ZL

```


LISTING OF COMPUTER CODE BUCKLE

```

REAL MODMAS
PI=3.1415926

C
C THE INCREMENT IN THE MODAL AMPLITUDE
DDEL=0.05*RAD

C
C RADIUS OF EACH COIL
RADI=RAD

C
C RATIO OF CONSTRAINED RADIUS TO COIL RADIUS
STARR=RFIX/RADI

C
C INITIALIZATION
YOVERX=1.0/XOVERY
NM=200
M=IFIX(WEDGE*FLOAT(NM)/(2.0*PI))
N=NM-M
NX=N+1
NY=NM+1
BANGLE=0.5*FLOAT(NM)/FLOAT(NX)
NRING2=IFIX(NRING/2.0+0.001)

C
WRITE(6,301)
301 FORMAT('1')
WRITE(6,996)
996 FORMAT('0','INPUT VARIABLES FOR DLDA FROM TORUS')
WRITE(6,313) RAD
313 FORMAT(' ','MINOR RADIUS (RAD)=' ,F16.6)
WRITE(6,314) RR
314 FORMAT(' ','MAJOR RADIUS (RR)=' ,F16.6)
WRITE(6,315) XOVERY
315 FORMAT(' ','ELLIPTICITY (XOVERY)=' ,F16.6)
WRITE(6,316) WEDGE
316 FORMAT(' ','ANGLE OF STRAIGHT BACK (WEDGE)=' ,F16.6)
WRITE(6,317) WIDER
317 FORMAT(' ','COIL THICKNESS IN RADIAL DIRECTION (WIDER)=' ,F16.6)
WRITE(6,318) WIDEZ
318 FORMAT(' ','COIL THICKNESS IN Z DIRECTION (WIDEZ)=' ,F16.6)
WRITE(6,319) NRING
319 FORMAT(' ','NUMBER OF COILS (NRING)=' ,I4)
WRITE(6,320)
320 FORMAT(' ','IS LENGTH CONSTRAINED?' )
CALL YESNO(RJUST)
WRITE(6,321) RFIX
321 FORMAT(' ','RADIUS AT WHICH CONSTRAINT IS APPLIED (RFIX)=' ,F16.6)
WRITE(6,322) STRECH
322 FORMAT(' ','RATIO OF STRETCHING AT RFIX (STRECH)=' ,F16.6)
WRITE(6,323) EYOUNG
323 FORMAT(' ','AVERAGE YOUNGS MODULUS (EYOUNG)=' ,E16.6)
WRITE(6,324) DENS
324 FORMAT(' ','AVERAGE COIL DENSITY (DENS)=' ,F16.6)
WRITE(6,325)
325 FORMAT(' ','INPUT DATA ON THE BENDING MODE')
WRITE(6,326) MODE
326 FORMAT(' ','MODE NUMBER (MODE)=' ,I4)

```

LISTING OF COMPUTER CODE BUCKLE

```

WRITE(6,327)
327 FORMAT(' ','IS THIS A PINNED MODE?')
CALL YESNO(APIN)
WRITE(6,328)
328 FORMAT(' ','IS THIS A CANTELEVER MODE?')
CALL YESNO(CANT)
WRITE(6,329)
329 FORMAT(' ','IS DEFORMATION IN Z DIRECTION?')
CALL YESNO(ZYES)
WRITE(6,330)
330 FORMAT(' ','IS DEFORMATION IN R DIRECTION?')
CALL YESNO(RYES)
WRITE(6,340)
340 FORMAT('0','RESULTS FROM SUBROUTINE TORUS')
AMD=1.0E-7
DO 908 I=1,NRING2
BETA(I)=0.0
908 CONTINUE
DO 117 I=1,NX
S(I)=0.0
117 CONTINUE
C
C CALCULATE INDUCTANCE OF MIDDLE COIL WITH NORDER-TH PAIR
DO 600 NORDER=1,1
C
C ANGLE BETWEEN NORDER-TH COIL AND MIDDLE COIL
OMEGA=(FLOAT(NORDER)/FLOAT(NRING))*2.0*PI
SOMEGA=SIN(OMEGA)
COMEGA=COS(OMEGA)
C
C INITIALIZE INTEGRATION VARIABLES
SUMO=0.0
SUMOL=0.0
SUMOR=0.0
DELT1=0.0
DELTA=0.0
RADFIX=0.0
C
C LOOP IN WHICH MODAL DEFORMATION IS INCREMENTED, CALCULATING DERIVATIVES
DO 500 JJ=1,3
C
C SET UP MIDDLE COIL, COIL 0.
CALL COIL(RFIX,RR,DELTA,WEDGE,RAD,XOVERY,MODE,CANT,
*APIN,WIDER,WIDEZ,EYOUNG,DENS,WSELF,AL,F1,ZYES,RYES,RJUST,ALAM1,
*STRECH,ALPHA,COEFF,JJ)
IF(JJ-1) 140,140,141
140 CONTINUE
C
C SET UP SIDE COILS
DO 142 I=1,NY
XL(I)=X(I)
YL(I)=(RR+Y(I))*COMEGA - RR
ZL(I)=(RR+Y(I))*SOMEGA
XR(I)=X(I)
YR(I)=YL(I)

```

LISTING OF COMPUTER CODE BUCKLE

```

      ZR(I)=(-1.0)*ZL(I)
142  CONTINUE
C
C    SET UP TANGENT VECTORS
      XLV(1)=(XL(2)-XL(NM))/2.0
      YLV(1)=(YL(2)-YL(NM))/2.0
      ZLV(1)=(ZL(2)-ZL(NM))/2.0
      XRV(1)=(XR(2)-XR(NM))/2.0
      YRV(1)=(YR(2)-YR(NM))/2.0
      ZRV(1)=(ZR(2)-ZR(NM))/2.0
      DO 110 I=2,NM
      XLV(I)=(XL(I+1)-XL(I-1))/2.0
      YLV(I)=(YL(I+1)-YL(I-1))/2.0
      ZLV(I)=(ZL(I+1)-ZL(I-1))/2.0
      XRV(I)=(XR(I+1)-XR(I-1))/2.0
      YRV(I)=(YR(I+1)-YR(I-1))/2.0
      ZRV(I)=(ZR(I+1)-ZR(I-1))/2.0
110  CONTINUE
141  CONTINUE
      SUMLL=0.0
      SUMRR = 0.0
C
C    INTEGRATE MUTUAL INDUCTANCE
      DO 201 L=1,NM
      SUMR = 0.0
      SUML=0.0
      DO 200 K=1,NM
      DL1=X(K)-XL(L)
      DR1=X(K)-XR(L)
      DL2=Y(K)-YL(L)
      DR2=Y(K)-YR(L)
      DL3=Z(K)-ZL(L)
      DR3=Z(K)-ZR(K)
      DL=SQRT((DL1*DL1) + (DL2*DL2) + (DL3*DL3))
      DR=SQRT((DR1*DR1) + (DR2*DR2) + (DR3*DR3))
      CL = (XV(K)*XLV(L)) + (YV(K)*YLV(L)) + (ZV(K)*ZLV(L))
      CR = (XV(K)*XRV(L)) + (YV(K)*YRV(L)) + (ZV(K)*ZRV(L))
      SL=CL/DL
      SR=CR/DR
      SUML=SUML+SL
      SUMR=SUMR+SR
200  CONTINUE
C
C    INDUCTANCE BETWEEN MIDDLE AND LEFT COIL
      SUMLL=SUMLL+SUML
C
C    INDUCTANCE BETWEEN MIDDLE AND RIGHT COIL
      SUMRR=SUMRR+SUMR
201  CONTINUE
C
C    DIMENSION PROPERLY
      SUMLL=SUMLL*AMO
      SUMRR=SUMRR*AMO
C
C    SUM THE PAIR

```

LISTING OF COMPUTER CODE BUCKLE

```

      SUMLR=SUMLL+SUMRR
C
C   CALCULATE DIFFERENTIALS
      IF (JJ-1) 150,150,151
150  CONTINUE
C
C   ON THE FIRST PASS SET DIFFERENTIALS =0, INITIALIZING.
      DL=0.0
      DLDA=0.0
      DLDAL=0.0
      DLDAR=0.0
      SUMO=SUMLR
      SUMOL=SUMLL
      SUMOR=SUMRR
      GO TO 160
C
C   CALCULATE DIFFERENTIALS AND DERIVATIVES
151  CONTINUE
      DL=SUMLR-SUMO
      DLL=SUMLL-SUMOL
      DLR=SUMRR-SUMOR
      DLDA=DL/((DELTA**2)-(DELT1**2))
      DLDAL=DLL/(DELTA-DELT1)
      DLDAR=DLR/(DELTA-DELT1)
160  CONTINUE
C
C   PRINT THE RESULTS
      WRITE(6,998) JJ,NORDER,DELTA,SUMLR,DLDA,DLDAL
998  FORMAT('0','JJ=',I2,3X,'NORDER=',I2,3X,'DELTA=',E7.1,
      *3X,'L=',E10.5,3X,'D2L/DA2=',E10.5,3X,'DL/DA=',E10.5)
C
C   INCREMENT MODAL AMPLITUDES
      DELTA=DELTA+DDEL
500  CONTINUE
C
C   STORE SECOND DERIVATIVE OF INDUCTANCE IN BETA
      BETA(NORDER)=DLDA
600  CONTINUE
      WRITE(6,107) F1
107  FORMAT('0','F1 VALUE IS',F12.6)
C
C   MODAL MASS
      MODMAS=WIDER*WIDEZ*AL*DENS*F1
C
      WIDZ=WIDEZ*ZYES+WIDER*RYES
      WIDR=WIDEZ*(1.0-ZYES)+WIDER*(1.0-RYES)
C
C   CALCULATE THE BENDING MOMENT
      BENDMO=WIDR*(WIDZ**3)/12.0
      CALL COFIC(MODE,CANT,APIN,KO,COEFF,RAD,AL,F1)
C   CALCULATE FUNDAMENTAL (NO MAGNETIC FIELD) OSCILLATING FREQUENCY OF COIL
      WSELF=EYOUNG*BENDMO*COEFF/(MODMAS*(RAD**4))
      WSELF=SQRT(WSELF)
      WRITE(6,341) BENDMO
341  FORMAT('0','BENDING MOMENT BENDMO=',E16.8)

```

LISTING OF COMPUTER CODE BUCKLE

```

WRITE(6,342) WSELF
42  FORMAT('0','SELF FREQUENCY WSELF=',E16.8)
WRITE(6,343) MODMAS
343  FORMAT('0','MODAL MASS MODMAS=',E16.8)
RETURN
END
SUBROUTINE WOVALU(BETA,Z,NN,N,MODMAS,RR,WSELF,DLSELF,AMP)
C
C  * THIS IS A SUBROUTINE TO CALCULATE *
C  * THE BUCKLING CURRENT FOR THE *
C  * SUPERCONDUCTING TORIOD. *
C
C  THE VALUES OF DLDA
DIMENSION BETA(N)
C
C  THE OUTPUT VALUES OF MODAL FREQUENCIES
DIMENSION Z(40)
C
C  MODE NUMBER FOR COLLECTIVE TOROIDAL OSCILLATION
DIMENSION X(43)
REAL MODMAS
DIMENSION SUMS(43)
REAL ICRIT
PI=3.141
WRITE(6,340)
340  FORMAT('1')
WRITE(6,345)
345  FORMAT('0','INPUT VALUES FOR WOVALU')
WRITE(6,342) WSELF
342  FORMAT('0','SELF FREQUENCY WSELF=',F16.8)
WRITE(6,343) MODMAS
343  FORMAT('0','MODAL MASS MODMAS=',F16.8)
WRITE(6,346) AMP
346  FORMAT('0','TOTAL AMPERE TURNS PER COIL AMP=',E16.8)
WRITE(6,347) N
347  FORMAT('0','NUMBER OF COILS NRING=',I4)
WRITE(6,348) RR
348  FORMAT('0','TORUS MAJOR RADIUS RR=',F16.8)
WRITE(6,349)
349  FORMAT('0','RESULTS FROM WOVALU FOR TORUS COLLECTIVE MODES')
C
AM01=(AMP**2)*DLSELF/MODMAS
VALI=AMP*AMP*4.0/MODMAS
C
C  SUM THE CONTRIBUTIONS TO EACH MODAL FREQUENCY FROM EACH COIL PAIR
C  FLOAT(J)=LITTLE K
C  X(M)=LITTLE N=0,1,...N-1
C
DO 7 M=1,40
Z(M)=0.0
X(M)=0.0
7  CONTINUE
WRITE(6,350)
50  FORMAT('VALUES OF MODAL FREQUENCY SQUARED')
WRITE(6,363)
363  FORMAT('0','FREQUENCY**2', 'MODE')

```

LISTING OF COMPUTER CODE BUCKLE

```

DO 20 M=1,N
X(M)=M-1
SUM=0
DO 30 J=1,NN
SUM=SUM+BETA(J)*(SIN(PI*X(M)*FLOAT(J)/FLOAT(N)))*2
30 CONTINUE
SUMS(M)=SUM
C
C CALCULATE THE SQUARED MODAL FREQUENCY
Z(M)=WSELF**2-VALI*SUM
C
C CORRECT FOR SELF INDUCTANCE EFFECTS
Z(M)=Z(M)-AMOI
WRITE(6,351) Z(M),M
351 FORMAT(F16.6,I4)
20 CONTINUE
ASUM1=SUMS(NN)
C
C PLOT THE VALUES OF FREQUENCY SQUARED VS MODE
WRITE(6,340)
CALL WPLOT1(X,Z,40,17,'FREQUENCY SQUARED')
C
WRITE(6,364)
364 FORMAT(40X,'MODE NUMBER')
C CALCULATE CRITICAL BUCKLING CURRENT ICRIT AND FIELD BCRIT.
ICRIT=SQRT(MODMAS*(WSELF**2)/(4.0*ASUM1 +DLSELF))
BCRIT=2.0*1.0E-7*ICRIT*FLOAT(N)/RR
WRITE(6,32)ICRIT,BCRIT
32 FORMAT('0','ICRIT=',E12.6,' BCRIT=',E12.6)
RETURN
END
SUBROUTINE FFTRAN(QSTART,NTEST, NRING,NFORCE,Q,BATA,PHIO,
*NCYCLE,EFOLD,QKSHAP,QKDAMP,QKFREQ,QKNCYC,W02,MODMAS)
C
C FFTRAN CALCULATES DYNAMIC RESPONSE OF TOROIDAL FUSION COILS TO A
C MECHANICAL DISTURBANCE.
C
DIMENSION IWK(1586),WK(1586),LL(1586)
DIMENSION U(40,256)
DIMENSION DATA(256)
DIMENSION BETA(20)
DIMENSION X(257),Y(257)
DIMENSION W02(40)
COMMON IWK,WK,LL,DATA,X,Y
COMPLEX DENOM
REAL QSTART
REAL KEEP1
REAL KEEP2
COMPLEX ETHIS
COMPLEX ETHAT
COMPLEX FAC
COMPLEX U
REAL Q
COMPLEX EQK
REAL QK1,QK2,QK3,QK4,QK5,QK6

```

LISTING OF COMPUTER CODE BUCKLE

COMPLEX DATA
COMPLEX ETHET
COMPLEX FACT
COMPLEX W3
COMPLEX FKEEP
COMPLEX FACTOR
COMPLEX NUMER
INTEGER QKSHAP
COMPLEX DENOM1
COMPLEX IIMAG
REAL QKFREQ
REAL QKDAMP
INTEGER QKNCYC
REAL MODMAS
PI=3.1415926

```
DEFINE IMAGINARY I  
IIMAG=CMPLX(0.0,1.0)
```

N=NUMBER OF SUBDIVISIONS USED BY THE FAST FOURIER TRANSFORM
N=256

SCALING OF WIDTH OF TRIANGLE PULSE IN QKSHAP=1.
ALPH=0.3

```
INITIALIZE VARIABLES DEPENDING ON TYPE OF EXCITATION
GO TO(60,61,62),NFORCE
CONTINUE
```

```

HAMMER BLOW
NPROVE=0
NDUMMY=1
PHIO=PI/2.0
GO TO 65
CONTINUE

```

```
TORSIONAL SHAKE
NPROVE=0
NDUMMY=NRING
PHI=PI/2.0
GO TO 65
CONTINUE
```

TRANSVERSE OSCILLATION SHAKE
NPROVE=1
NDUMMY=NRING
CONTINUE

```
INITIALIZE CONSTANTS  
NTEST=0  
NRING2=IFIX(NRING/2.0+0.001)  
N1=N/2.0  
N5=N1+1
```

SCALING FACTOR FOR COIL DEFLECTION

LISTING OF COMPUTER CODE BUCKLE

```

C      BATA=BATA/MODMAS
C      INITIALIZE COIL DISPLACEMENTS U.
      DO 37 L=1,NRING
      DO 39 I=1,N
      U(L,I)=0.0
39     CONTINUE
37     CONTINUE
C
C      THE ZERO FIELD COIL FREQUENCY IS IDENTICAL TO THE FIRST MODAL FREQUENCY
      W022=W02(1)
      W022=SQRT(W022)
C
C      TMAX IS THE MAXIMUM TIME CUTOFF FOR THE FAST FOURIER TRANSFORM (FFT)
      TMAX=(NCYCLE*2.0*PI)/W022
C
C      TMIN IS THE TIME INCREMENT USED BY THE FFT
      TMIN=TMAX/N
C
C      WMIN IS THE SMALLEST FREQUENCY INTERVAL USED
      WMIN=(1.0/TMAX)*2.0*PI
C
C      WMAX IS THE HIGHEST FREQUENCY CUTOFF OF THE FFT
      WMAX=N*WMIN
C
C      DAMPING OF THE COIL RESPONSE
      DAMP=(2.0*PI/TMAX)*EFOLD
C
C      SCALE THE OSCILLATION AMPLITUDE FOR USE WITH THE FFT
      Q=Q/TMAX
C
C      TO IS THE TIME AT WHICH THE DISTURBANCE (QUAKE) OCCURS
      TO=-TMAX*QSTART
C
C      ITERATE OVER ALL THE TORUS MODES
      DO 21 NORDER=1,NRING
      IF (NTEST) 50,50,51
50     NTEST1=0
51     CONTINUE
C
C      THIS LOOP SETS UP THE FOURIER TRANSFORM IN FREQUENCY SPACE OF THE FORCING
C      FUNCTION AND THE COIL RESPONSE, IN THE ARRAY DATA(I).
      DO 10 I=1,N
      IF (I-N5) 13,14,14
14     W=WMIN*(FLOAT(I)-FLOAT(N)+0.0001)
      GO TO 15
13     W=WMIN*FLOAT(I)
15     CONTINUE
C
C      SET UP A PHASE SHIFT
      THAT=W*TO
      CTHAT=COS(THAT)
      STHAT=SIN(THAT)
      ETHAT=CMPLX(CTHAT,STHAT)
C
      GO TO (1,2),QKSHAP

```


LISTING OF COMPUTER CODE BUCKLE

QUAKE SHAPE 1: TRIANGLE PULSE

CONTINUE

AKEEP=(SIN(ALPH*W)/(ALPH*W))**2

AKEEP=ALPH*AKEEP

DATA(I)=AKEEP

DATA(I)=Q*DATA(I)

GO TO 6

QUAKE SHAPE 2: EXPONENTIALLY DAMPED SINUSOIDAL FUNCTION

CONTINUE

W=-W

QKL=QKNCYC*(PI/QKFREQ)

DENOM=CMPLX(W+QKFREQ,QKDAMP)

QK1=COS((W+QKFREQ)*QKL)

QK2=SIN((W+QKFREQ)*QKL)

EQK=CMPLX(QK1,QK2)

NUMBER=EQK*EXP(-QKDAMP*QKL)

NUMBER=NUMBER-1.0

DATA(I)=(-0.5)*(NUMBER/DENOM)

QKFREQ=-QKFREQ

DENOM=CMPLX(W+QKFREQ,QKDAMP)

QK1=COS((W+QKFREQ)*QKL)

QK2=SIN((W+QKFREQ)*QKL)

EQK=CMPLX(QK1,QK2)

NUMBER=EQK*EXP(-QKDAMP*QKL)

NUMBER=NUMBER-1.0

DATA(I)=DATA(I)+0.5*(NUMBER/DENOM)

QKFREQ=-QKFREQ

W=-W

DATA(I)=Q*DATA(I)

GO TO 6

CONTINUE

ON THE FIRST PASS OMIT THE COIL RESPONSE AND CALCULATE ONLY
THE FORCING FUNCTION

IF (NTEST1) 52,52,53

FOLD IN THE RESPONSE FUNCTION OF THE COILS

CONTINUE

KEEP1=W**2-W02(NORDER)

KEEP2=W*DAMP*2.0

KEEP2=-KEEP2

DENOM=CMPLX(KEEP1,KEEP2)

DATA(I)=(W**2)*(1.0/DENOM)*DATA(I)

CONTINUE

APPLY PHASE SHIFT

DATA(I)=DATA(I)*ETHAT

CONTINUE

PERFORM THE FOURIER TRANSFORM FROM FREQUENCY TO TIME

CALL FFTP(DATA,N,IWK,WK,LL)

CORRECT FOR COEFFICIENT

IF (NTEST1) 54,54,55

LISTING OF COMPUTER CODE BUCKLE

```

54 FACTOR=CMPLX(1.0,0.0)
   WRITE(6,57)
57  FORMAT('0','THIS IS EARTHQUAKE SHAPE')
   GO TO 56
55  CONTINUE
   FKEEP=CMPLX(0.0,0.0)

C
C   SUM THE FORCING FUNCTION OVER ALL THE COILS ON WHICH IT ACTS
DO 46 I=1,NDUMMY
   THAT=FLOAT(I-1)*FLOAT(NORDER-1)*(PI/NRING)
   THAT=THAT*2.0
   CTHAT=COS(THAT)
   STHAT=SIN(THAT)
   STHAT=-STHAT
   ETHAT=CMPLX(CTHAT,STHAT)
   ABB=2.0*NPROVE
   FKEEP=ETHAT*SIN(ABB*PI*(FLOAT(I-1)/FLOAT(NRING))+PHIO) +FKEEP
46  CONTINUE

C
C   SCALE THE FORCE PROPERLY
   FACTOR=BATA*FKEEP
56  CONTINUE

C
C   SHIFT BY PHASE FACTOR
DO 20 I=1,N
   THET=PI*(I)/N1
   CTHET = COS(THET)
   STHET = SIN(THET)
   ETHET = CMPLX(CTHET,STHET)
   DATA(I) = DATA(I)*ETHET
   DATA(I)=DATA(I)*FACTOR

C
C   X IS THE TIME COORDINATE
   X(I)=TMIN*FLOAT(I)

C
C   Y IS THE AMPLITUDE OF THE NORDER-TH MODE
   Y(I)=REAL(DATA(I))
20  CONTINUE

C
   WRITE(6,48) NORDER
48  FORMAT('0','NORDER=',I4)

C
C   GRAPH TH MODAL RESPONSE
   CALL WPLOT1(X,Y,256,15,'MODAL AMPLITUDE')
   WRITE(6,301)
301  FORMAT('0',40X,'TIME IN SECONDS')
   WRITE(6,300)
300  FORMAT('1')
   IF (NTEST1) 58,58,59
58  NTEST1=1
   NTEST=1
   GO TO 51
59  CONTINUE

C   TRANSFORM FROM MODAL RESPONSE TO RESPONSE OF INDIVIDUAL COILS

```

LISTING OF COMPUTER CODE BUCKLE

```

DO 23 L=1,NRING
DO 24 I=1,N
C
C   RELATIVE PHASE SHIFT OF COIL DISPLACEMENTS
THIS=FLOAT(L-1)*FLOAT(NORDER-1)*(PI/NRING)
THIS=THIS*2.0
STHIS=SIN(THIS)
CTHIS=COS(THIS)
ETHIS=CMPLX(CTHIS,STHIS)
ETHIS=ETHIS/NRING
C
C   DEFLECTION OF THE L-TH COIL
U(L,I)=DATA(I)*ETHIS*U(L,I)
C
24  CONTINUE
23  CONTINUE
21  CONTINUE
DO 41 L=1,NRING
DO 42 I=1,N
C
C   PREPARE FOR GRAPHING COIL DEFLECTION
Y(I)=REAL(U(L,I))
42  CONTINUE
WRITE(6,49) L
49  FORMAT('0',' COIL NUMBER L=',I4)
C
C   PLOT DEFLECTION OF L-TH COIL VS TIME
CALL WPLOT1(X,Y,256,17,'COIL DISPLACEMENT')
WRITE(6,301)
WRITE(6,300)
41  CONTINUE
RETURN
END
SUBROUTINE COFIC(MODE,CANT,APIN,K0,COEFF,RAD,AL,F1)
REAL K0,K1,K2,K3
PI=3.14159
GAM=1.0
RADI=RAD
IF (APIN) 10,10,11
11  CONTINUE
K0=AL/2.0
K1=(2.0*MODE)**4
K2=(2.0*MODE)**2
K3=K2
10  CONTINUE
IF (CANT) 12,12,13
13  CONTINUE
GO TO (14,15,16,17),MODE
14  CONTINUE
ALAM=4.73
ALPHA=0.9825
GO TO 20
15  CONTINUE
ALAM=7.853
ALPHA=1.0

```

LISTING OF COMPUTER CODE BUCKLE

```

16  GO TO 20
    CONTINUE
    ALAM=11.0
    ALPHA=1.0
    GO TO 20
17  CONTINUE
    ALAM=14.0
    ALPHA=1.0
20  CONTINUE
    K1=(ALAM/PI)**4
    K2=ALAM*ALPHA
    K2=K2*(K2-1.0)/(PI**2)
    K3=K2
    K0=AL
12  CONTINUE
    BLAM=(PI*AL)/AL
    COEFF=BLAM
    COEFF=(BLAM**2)*(K1*BLAM**2+K2/GAM)
    COEFF=COEFF-(BLAM**4)*(K3+K2/GAM)**2/(1.0+K2*BLAM**2/GAM)
    RETURN
    END
    SUBROUTINE YESNO(KAP)
    IF (KAP) 960,960,961
960  WRITE(6,963)
    GO TO 962
961  WRITE(6,964)
962  CONTINUE
963  FORMAT('+',40X,'NO')
964  FORMAT('+',40X,'YES')
    RETURN
    END
//GO.SYSIN DD *
4.27246
1.17797
0.53100
0.28000
0.16784
0.11520
0.08600
0.07705
0.00000
0.000000
0.00000
0.00000
0.00000
20000000.0
55.7
50.0
46.0
42.0
39.0
36.0
33.0
32.0
32.0
32.0

```

LISTING OF COMPUTER CODE BUCKLE

32.0
32.0
32.0
32.0
33.0
36.0
39.0
42.0
46.0
50.0
20000000.0

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LIST OF TABLES

- Table I: Critical Currents and Fields for Magnetoelastic Instability
Due to Intercoil Magnetic Forces.
- Table II: Fusion Reactor Design Specifications.
- Table III: Critical Buckling Currents and Fields for Seven Fusion
Reactor Designs

LIST OF FIGURES

- Figure 1: Toroidal field coils for fusion reactor.
- Figure 2: 1st and 2nd buckling mode shapes.
- Figure 3: Internal coordinates, stresses and moments for fusion toroidal field coil.
- Figure 4: Interpair 1st modal force for 10^8 amps total toroid current.
(4×10^4 gauss center toroidal field).
Newtons vs pair number
major radius - 5 m
minor radius - 3.03 m.
- Figure 5: Frequency dispersion vs mode and I.
- Figure 6: "D" Argonne coil figure and table.
Variable supports.
- Figure 7: Earthquake response
a) horizontal motion of base
b) 7th coil response with damping = 2 sec^{-1} .
- Figure 8: Wave propagation around torus due to disturbance of coil
#1; major radius 5 m, minor radius 3.03 m.
- Figure 9: Diagram of Experimental apparatus for observation of
buckling of a three coil superconducting toroidal segment.
- Figure 10: Photograph of experimental apparatus.
- Figure 11: Relative lateral stiffness (frequency squared) versus current.
squared for a flexible coil between two similar coils.

TABLE I

CRITICAL CURRENTS AND FIELDS FOR
MAGNETOELASTIC INSTABILITY DUE TO
INTERCOIL MAGNETIC FORCES

EXAMPLE:

ELLIPTICAL "D" TOROIDAL FIELD COIL

16 COIL TORUS

$$B_c = 8.2 \text{ kG}$$

$$I_c = 1.5 \text{ MA}$$

$$16.7 \text{ kG}$$

$$3.2 \text{ MA}$$

$$34.0 \text{ kG}$$

$$6.4 \text{ MA}$$

$$57 \text{ kG}$$

$$10.8 \text{ MA}$$

MAGNET CROSS SECTION

$$0.57 \times 0.90 \text{ m}^2$$

Young's Modulus

$$15 \times 10^{10} \text{ N/m}^2$$

TABLE II

FUSION REACTOR DESIGN SPECIFICATIONS

	Argonne	TFTR Westing- house	PLT	UWMAK-1	ORMAK	File/Mills D-T	File/Mills Cat-D
Major Radius	6.06	2.8	1.4	13.0	4.5	5.0	10.0
Minor Radius	1.01	1.72	1.01	8.75	2.35	3.03	5.85
No. of Coils	16	20	18	12	24	40	40
Coil Thick- ness	0.90	0.41	.3	2.0	.384	.20	.40
Width (Radial)	0.568	0.54	.45	1.0	.198	0.33	0.40
Ellip- ticity	1.15	1.0	1.0	1.58	1.0	1.26	1.36
Magnet Current 10^6 Amps	7.0	3.0	1.7	20.8	3.8	5.6	11.0

Effective Young's modulus in the calculations; $15 \cdot 10^{10}$ N/m²

Angle subtended by the straight clamped back of the coil = 90°.

All lengths are in meters.

The values in the table are those used in the computer code; in a few cases the values are approximations to the design specifications.

TABLE III

CRITICAL BUCKLING CURRENTS AND FIELDS
FOR SEVEN FUSION REACTOR DESIGNS

Magnet Design	1st mode	2nd mode	3rd mode	4th mode
Argonne I_c B_c	1.55 0.816	3.17 1.67	6.46 3.41	10.83 5.71
TFTR I_c B_c	2.77 3.96	5.68 8.12	11.82 16.89	19.9 28.5
PLT I_c B_c	3.54 9.1	7.1 18.2	14.4 37.06	24.1 61.8
UWMAK-1 I_c B_c	4.7 .87	10.8 2.0	23.6 4.37	40.7 7.5
ORMAK I_c B_c	.91 .966	1.9 2.0	4.0 4.2	6.7 7.1
File/Mills I_c (D-T) B_c	0.157 0.25	0.332 0.532	0.699 1.11	1.17 1.88
File/Mills I_c (Cat-D) B_c	0.183 0.146	0.392 0.314	0.832 0.666	1.41 1.12

Side view of coil and supports

I_c -- critical buckling current in Ampere-turns per coil
(multiply by 10^6)

B_c --- critical magnetic field at major radius of torus in Tesla

TOROIDAL FIELD COILS FOR FUSION REACTOR

FIGURE 1

FIRST AND SECOND BUCKLING MODE SHAPES
FIGURE 2

TOROIDAL FIELD FUSION
REACTOR COIL

MAGNET CROSS SECTION WITH INTERNAL FORCES AND MOMENTS
FIGURE 3

INTERPAIR MODAL FORCE : $I^2 \partial^2 L / \partial \alpha^2 (Nt/m^2)$

MODAL FREQUENCY vs MODE NUMBER

FIGURE 5

FIGURE 6

a)

b)

EARTHQUAKE RESPONSE

FIGURE 7

FIGURE 8

FIGURE 9

FIG. 10

FIGURE 11.