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Informal Report

THE VELOCITY OF THE NEUTRINO

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CP and/or Lorenz invariance violations might be manifested in the physical velocity of propagation of neutrinos (and antineutrinos). Their velocities may differ from c (the velocity of light in vacuum), and possibly also from one another. The deviation of the velocity from c that might be expected is of the order of $10^{-3}c$.

Special Relativity for $m_0 \neq 0$ Particles (also applicable to tachyons of mass $\mu = im_0$). Einstein's postulate gives the relation between energy and mass as

$$E = mc^2 = \gamma m_0 c^2, \quad (1)$$

where m_0 is the "rest" mass ($\gamma = 1$, $\beta = 0$),

$$\text{where } \gamma = (1 - \beta^2)^{-1/2} \quad \text{and} \quad \beta = v/c. \quad (2)$$

Using the canonical equation $p = mv$ for the momentum we have

$$pc = mvc = \beta mc^2 = \beta \gamma m_0 c^2 = \beta E. \quad (3)$$

Thus

$$\frac{pc}{E} = \beta \quad (\text{or } \frac{pc}{E} = \frac{\beta \gamma m_0 c^2}{\gamma m_0 c^2} = \beta, m_0 \neq 0). \quad (4)$$

For $\gamma \rightarrow \infty$ (or $m_0 \rightarrow \epsilon$, $\epsilon \ll E$)

$$\lim_{\gamma \rightarrow \infty} \left(\frac{pc}{E} \right) = \lim_{\gamma \rightarrow \infty} \beta = 1. \quad (5)$$

Thus the limiting velocity of propagation is c . This has been tested with threshold Cerenkov ($\checkmark$) counters. They measure $(1 - \beta) \approx (2\gamma^2)^{-1}$ for $\gamma \lesssim 10^3$ to some accuracy. At BNL energies, $\gamma \sim 100$ for

~ 14 GeV/c pions; the pressure threshold is typically large and probably checked to $\sim 1\%$. At Fermilab, $\gamma \sim 10^3$ for ~ 140 GeV/c pions; the pressure threshold is typically small and checked to $\sim 10\%$. At SLAC, Guiragossian, et al.,¹ via very precise timing, measured the time of flight difference of 15 GeV electrons ($\gamma = 3 \times 10^4$) versus light to be $(1 - \beta) \leq 2 \times 10^{-7}$, i.e. the equivalent of $\gamma \sim 2 \times 10^3$. The tuning of the RF frequency at accelerators, e.g. SLAC and Fermilab, also checks this relation. For $\delta L/L$ or $\delta R/R \sim 10^{-6} - 10^{-7}$ ($L =$ SLAC length = 2 miles; $R =$ radius of 400 GeV synchrotron ≈ 1 km.), $(2\gamma^2)^{-1} = (1 - \beta)$ is $\sim 10^{-6} - 10^{-7}$, i.e. an equivalent of $\gamma \sim 3000$. For $\gamma = 400$ protons at Fermilab, this is a $\approx 10\%$ check. Thus $\gamma = (1 - \beta^2)^{-1/2}$ holds for $\gamma \lesssim 100$, holds reasonably to 10^3 , but has not been tested for $\gamma \geq 10^4$.

Massless Particles, $m \equiv 0$. For massless particles we have pc and E . The ratio $\frac{pc}{E}$ is experimentally equal to one[†] with high accuracy for photons and the different neutrinos. This is quantified in the mass limits of γ , ν_e , λ_e , ν_μ , λ_μ and their antiparticles (ν neutrinos from pion decay, λ neutrinos from kaon decay). The equality of ν and λ is yet to be tested experimentally.² The mass limits are given in the following table.^{3,4}

Table of Mass Limits

	γ	ν_e	λ_e	ν_μ	λ_μ
M (90% C.L.) limits	$< 2 \times 10^{-21}$ MeV	< 60 ev (tritium decay)	< 0.45 MeV (K^0_L decay)	< 1.15 MeV (π decay)	< 0.65 MeV (K^0_L decay)

Note: Cosmological m_{ν_1} limits ≤ 8 ev on average.⁵

[†] Conservation of energy and momentum.

These mass limits were deduced independent of the properties of the γ or the neutrinos, using the relations between pc and E for $m \neq 0$ particles, known to hold. Thus, although the neutrino masses may not be identically zero, $pc \approx E$.

Since we are assuming $m \approx 0$, we have no a priori relationship between $\frac{pc}{E} = 1$ and the physical velocity of propagation. The photon is clearly an example of the limiting case $\lim_{m \rightarrow 0} (\frac{pc}{E} = \beta) = 1$. Whether this holds also for neutrinos must be experimentally tested. Since CP violation exists, T and/or CPT violations and therefore possibly Lorentz invariance violations may exist. Neutrinos may be connected with these violations due to their relationship with CP violating K_L^0 decays.

Current Knowledge of the Velocity of the Neutrinos. The electronic experiments with neutrinos (and antineutrinos) at BNL have observed the RF structure of the AGS in their events.⁶ Twelve (or 13) 25 ns FWHM peaks separated by 220 ns, with neutron background in between, are observed for the one-turn extraction used in the U-line. The width corresponds to the (expected) RF bunch length. No accurate absolute timing has been done to my knowledge. The neutrino flight path is (at velocity c) $\sim 500 \pm 100$ ns (decay space is 200 feet long). If the velocity of the neutrino β_ν differed from c by 10%, the flight time would differ from 500 ns by 50 ns with a spread (due to decay space length) of ± 10 ns, i.e. ~ 20 ns FWHM, to be folded with the poorly known RF bunch length of the AGS. Thus the BNL neutrino experiments require β_ν to differ by less than 10% from c .

The Fermilab neutrino experiments have not previously been instru-

observe the mented to/RF structure = 53 MHz (or 18.83 ns spacing), with width ~ 1 ns.⁷ With a timing resolution of ~ 6 ns FWHM, one should be able to observe the RF structure of 18.83 ns. A time spread ~ 6 ns versus ± 550 ns dispersion due to the 1100 foot decay space would imply that β_v differs from c by less than 1%, if the RF structure were in fact observed in the neutrino data.

On the theoretical side, such a violation might be estimated to be equivalent to a small admixture of V+A to the usual V-A interaction. This changes nothing in π^+ , $K^+ \rightarrow e^+ \nu$, $\mu^+ \nu$ decay/ rates. However, the ξ parameter³ in μ decay would be changed some from $\xi = +1$; the experimental value of $\xi = 0.972 \pm 0.013$ allows for a few percent admixture of V+A.

We will assume that β_v ($\beta_{\bar{v}}$) differs from c by at most $\sim 1\%$.

Possible Magnitude of Deviation of β_v from c. Such a deviation would find its root in CPT violation, CP violation or Lorenz noninvariance. The very small mass difference of K^0 , $\bar{K}^0$ implies CPT in strong interactions is good to extremely high accuracy.⁸ For the weak interaction, CPT has been tested to much lower accuracy by measuring the difference in the lifetimes τ of particle-antiparticle pairs. Define

$$\delta_{AA} = \left| \frac{\tau_A - \tau_{\bar{A}}}{\tau_A} \right|. \quad (6)$$

Then, the 95% confidence level limits are³

$$\left. \begin{aligned} \delta_{\Lambda\bar{\Lambda}} &< 2.2 \times 10^{-3} \\ \delta_{K^+K^-} &< 2.4 \times 10^{-3} \\ \delta_{\pi^+\pi^-} &< 1.9 \times 10^{-3} \\ \delta_{\mu^+\mu^-} &< 2 \times 10^{-3} \end{aligned} \right\} \text{CPT tests.} \quad (7)$$

CP violation is known to exist in K_L^0 decay. No other manifestation has been observed. The "superweak" model describes all the data well;⁹ all effects are related to one parameter. We take this parameter to be the decay asymmetry in the K_L^0 semileptonic decay. Positive leptons (e^+ and μ^+) are in slightly greater abundance than negative ones by $(3.4 \pm 0.2) \times 10^{-3}$ (3.5×10^{-3} for electrons and 3.2×10^{-3} for muons)³ which implies a slight excess of K^0 over $\bar{K}^0$ in the K_L^0 particle mixture (assuming validity of $\Delta S = \Delta Q$ rule). One might speculate that this difference comes about due to a slight velocity difference between neutrino and antineutrino states, such that the final states $\pi^- \ell^+ \nu$ and $\pi^+ \ell^- \bar{\nu}$ are not quite symmetric.

The dimensionality of the Fermi constant G_w (cm^2) is usually taken as a unitarity limit regarding the breakdown of the simple Fermi local four point interaction. That is

$$G_w m_p^2 = 1.05 \times 10^{-5} \text{ (dimensionless; } \hbar = c = 1) \quad (8)$$

gives

$$G_w^{1/2} = 3.2 \times 10^{-3} \frac{\hbar c}{m_p} \text{ cm.} \quad (9)$$

This corresponds to a "range" of the weak interaction of $(m_p / (3.2 \times 10^{-3})) \sim 300$ GeV. The mass of the W boson is expected to be < 100 GeV and thus the 300 GeV value (in center of mass) must be a more drastic unitarity breakdown point. However, an additional interpretation can be made. Using the K^0 mass as the basis one finds

$$G_w^{1/2} = 1.8 \times 10^{-3} \frac{\hbar c}{m_K} = \left(\frac{\hbar}{m_K} \right) (1.8 \times 10^{-3} c) = (\text{Time}) (\text{Velocity}). \quad (10)$$

This velocity, $1.8 \times 10^{-3}c$, might represent the deviation of the velocity of the neutrino (in K_L^0 decay) from c . Note that $2(1.8 \times 10^{-3}) = 3.6 \times 10^{-3}$ equals the K_L^0 semileptonic asymmetry $(3.4 \pm 0.2) \times 10^{-3}$.

If the velocity of the neutrino differs from c , these considerations lead one to expect a deviation $\sim 10^{-3}c$, i.e. $\sim 0.1\%$.

Speculations. Assuming $\beta_v \neq c$, a spectrum of β_{v_i} values might exist. Assuming $\lambda_i \neq v_i$ (neutrinos in K and π decay) we have eight velocities $\beta_{v_e}, \beta_{\lambda_e}, \beta_{v_e}^-, \beta_{\lambda_e}^-, \beta_{v_\mu}, \beta_{\lambda_\mu}, \beta_{v_\mu}^-$ and $\beta_{\lambda_\mu}^-$ clustered about c , the velocity of light in vacuum. Some or all of these velocities might be equal. If $\beta_v \neq \beta_{v_i}$, then the separation of right and left handed states would be explained in a new way. If $\beta_{v_e} \neq \beta_{v_\mu}$, the difference between e and μ would have a new basis, and universality would be (slightly at least) broken. The opposite helicity neutrinos (and antineutrinos) might form an additional spectrum and be connected with neutral currents (i.e. $V+A$ instead of $V-A$). The velocity of the neutrino would presumably change under Lorentz transformations. This might cause a dependence of the velocity upon the energy of the neutrino in a given frame of reference (e.g. the laboratory).

Conclusions. The velocity of neutrinos might differ from c by $\sim 0.1\%$ due to violation of Lorentz invariance and/or CP invariance by neutrinos. Fermilab experiments E21/E254 (Cal. Tech., Fermilab, BNL, Purdue) will attempt an absolute measurement of the velocity of their neutrinos and antineutrinos using the RF structure of the machine. A timing accuracy of ~ 2 ns FWHM will result in an event by event measurement to 10^{-3} (2 ns

over 2000 ns flight path) and an average over the sample measured to $\sim 0.3 \times 10^{-3}$. This can be calibrated against a leakage flux of high energy muons (of velocity $(1 - \beta_\mu) < 10^{-4}$) with the beam tuned to a high momentum. Any result will be modulo 18.83 ns, i.e. $\Delta t = t(\text{mod } 18.83) = t_0 + 18.83 n$. However, for $n \neq 0$, the FWHM of the timing curve will be greater than ~ 10 ns (due to decay space dispersion, see above), and the machines' RF structure will not be seen in the neutrino data.

References and Footnotes

1. Z. G. T. Guiragossian, et al., Phys. Rev. Lett. 34, 335 (1975).
2. Fermilab Experiment E254, BNL-Purdue-Cal. Tech.-Fermilab.
3. Particle Data Group, Rev. Mod. Phys. 45, S1 (1973).
4. A. R. Clark, et al., Phys. Rev. D9, 533 (1974).
5. R. Cowsik and J. McClelland, Phys. Rev. Lett. 29, 669 (1972).
6. Private Communication (H. H. Williams, U. Penna.; L. Sulak, Harvard).
7. J. A. Appel, et al., Phys. Rev. Lett. 32, 428 (1974).
8. We must remember to distinguish $(CPT)_{\text{Strong}}$, $(CPT)_{\text{EM}}$, $(CPT)_{\text{Weak}}$.
 $(K^0, \bar{K}^0)$ Mass equality tests $C_S P_S T_S$ whereas lifetime tests verify $C_W P_W T_W$. See e.g., N. Christ and T. D. Lee, Phys. Rev. 143, 1310 (1966).
9. See e.g., W. Carithers, et al., Phys. Rev. Lett. 34, 1240 and 1244 (1975); L. Wolfenstein, in Theory and Phenomenology of Particle Physics, edited by A. Zichichi (Academic, New York, 1969), p. 218.