NDA 2147-4

**REVIEW OF INVESTIGATIONS PERTINENT TO THE
PREDICTION OF THE THERMAL BEHAVIOR OF FAST
REACTORS UNDER ABNORMAL OPERATING CONDITIONS**

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1. INTRODUCTION

An investigation of methods for predicting fuel element and coolant behavior in fast reactors under abnormal operating conditions constitutes part of a study of fast reactor safety problems for the United States Atomic Energy Commission under Contract AT(30-1)-2303 (XIII). The first phase of this investigation consists of a survey to determine the status of experimental and analytical work pertinent to the overall problem. This report summarizes the results of this survey and presents a compilation of pertinent material for reference.

Fast breeder power reactors are characterized by very high power densities. As a result, maintenance of adequate coolant flow is imperative. Factors which may lead to inadequate coolant flow are: abnormally high fuel element power generation, blockage of coolant passage, primary coolant system leakage, or pump failure. Under such conditions, the coolant is unable to remove all of the generated heat satisfactorily. For certain time sequences of events, fuel element overheating, failure, and meltdown may result. Fuel meltdown in a fast power reactor may lead to reassembly in a supercritical fuel mass. For this reason, the circumstances prior to, during, and after meltdown are of particular interest in fast reactor safety studies.

The analysis of this transient situation involves the mechanics and thermal behavior of the coolant as well as the energy transfer within the fuel element and cladding. Complications arise when the coolant undergoes a change of phase, is voided from part of the reactor, or fuel elements begin to melt. At present no convenient approaches are available to account simultaneously for the various modes of behavior of coolant and fuel when attempting to predict these transients. This is particularly the case when phase changes occur.

This report serves the purpose of consolidating present technology, which will aid in the preliminary analysis of the problem, and also, of highlighting certain areas where basic information is not available. Since liquid sodium is almost exclusively used as the coolant for fast breeder power reactors, the survey is directed toward specialized information related to this fluid.

The technical portion of the report is separated into several sections. Section 2, Results of Survey, provides a summary of investigation relating to fuel behavior during transients and of coolant behavior, and indicates areas where information is sparse. Section 3, Survey, enters into more detail regarding actual sodium-cooled fast reactor excursions, the theory of fuel element thermal transients, reactor coolant transients in single and two-phase flows, pertinent information and sources thereof, and specific topics such as two-phase flow patterns, two-phase pressure drop, and the calculation of transient voids in boiling-coolant reactors.

Section 4 presents certain of the most important mathematical treatments which apply to the report. These are presented in a separate section in order that they may not influence continuity, and for ready reference.

Section 5 is an annotated bibliography.

2. RESULTS OF SURVEY

2.1 FUEL BEHAVIOR DURING REACTOR TRANSIENTS

The prediction of local transient temperatures in fuel and cladding for specified power excursions and boundary conditions and no internal fuel melting can be handled by existing numerical or analytical methods. These methods give local temperatures vs distance from the center of the fuel. They are applicable to studies of fuel elements and cladding alone but do not lend themselves to the study of transient heat transfer over the length of fuel element and coolant passage configurations. Lumped parameter methods have been developed for unclad fuel rods and plates for use in the solution of such problems. The lumped parameter technique essentially replaces the partial differential equations in space and time for the fuel by an ordinary differential equation in terms of characteristic fuel temperatures. The treatment neglects effects of axial heat conduction in the fuel.

The treatment of transient heat transfer in fuel and cladding when internal melting occurs in the fuel is less certain. Some numerical methods have been developed for obtaining temperature histories in the presence of melting when there is no internal heat generation. Apparently there have been no analyses conducted to determine the effect of internal melting of the fuel on the surface heat transfer during an excursion. Such effects should be considered in order to obtain better estimates of surface heat flux and cladding surface temperatures when studying the transient behavior of the coolant up to the point where the fuel element is still dimensionally intact. Lumped parameter relations should be developed to account for the heat capacity of both the cladding and fuel and for internal fuel melting to obtain transient temperatures in fuel element and coolant passage configurations.

2.2 COOLANT BEHAVIOR

For studies of coolant behavior during reactor excursions which are not too severe, it appears that the available data and theoretical relations for the physical and thermodynamic properties of (liquid and vapor) sodium are sufficiently accurate. Steady state heat transfer coefficients for nonboiling forced convection of liquid sodium in tubes, though known with considerably less accuracy than those for the usual coolants such as air or water, are probably of sufficient accuracy for use with accident studies. However, there should be some study of methods for treating transient convection heat transfer within the coolant passage for the case of reactor excursions. For this situation, it may be warranted to assume that the heat is transferred entirely by thermal conduction with the possible use of an increased effective thermal conductivity to account for forced convection effects.

There are few experimental data on boiling heat transfer with liquid metals. Practically all experiments to date have been for pool boiling. There are no data on boiling burnout with liquid metals. There appears to be no justification for applying existing correlations to the prediction of boiling burnout with sodium. The effect of transient heating on the boiling process and on boiling

burnout has been investigated only slightly, and only for water. The results indicated that, for rapid transients and for bulk temperatures well below the boiling point, the surface temperature increased well above the boiling point before boiling commenced. The burnout heat flux was well below that for steady state.

The pressure drop for liquid sodium in tubes is correlated by the equations for ordinary fluids. Since sodium has properties similar to those of water with respect to pressure drop calculations, it might be expected that the available data on two-phase flow pressure drop for water could be extended to sodium. The major difference in properties is the ratio of liquid to vapor densities at reactor design operating conditions. Approximate correlations are available for predicting the two-phase flow pressure drop for water under steady state conditions. However, there appear to be no data available on two-phase flow pressure drop under transient conditions such as would be encountered in the case of sodium coolant expulsion during a reactor excursion or the failure of the primary coolant pump at constant reactor power.

Approximate correlations are available for predicting the steady state coolant vapor volume fraction for water in the quality range. As in the case of the two-phase flow pressure drop correlation, an extension of these data to sodium may be warranted. Prediction of the vapor volume fraction for water in the subcooled boiling range is less certain. In this case extension of existing data to sodium would not appear to be warranted, in view of the differences in surface tensions and the coolant temperature profiles.

Analyses to date indicate that the problem of expulsion of coolant is more severe for sodium than for water. This is a result of the larger ratio of coolant-to-vapor density under reactor operating conditions for the former. Estimates have been made of the time required for expulsion of the sodium coolant. For the Enrico Fermi Reactor, it is estimated that the coolant would be expelled in about 1/10 of a second. There is a need for more refined calculations of the coolant expulsion time.

The existing treatments of fuel meltdown and freezing ignore the effects of the sodium coolant on heat transfer to the fuel and on fuel movement. It might be expected that the coolant would have a significant effect on these processes.

3. SURVEY

3.1 REACTOR TRANSIENTS OF INTEREST

By way of orientation, it is beneficial to describe the results of some actual reactor transients. These transients, which are essentially power excursions, indicate an erratic behavior of fuel and coolant when the fuel becomes molten.

Consideration of fuel behavior during and after an actual core meltdown in the EBR-I is presented in Reference 8. The reactor was uranium fueled, unmoderated, reflected with natural uranium, and cooled with NaK. During the accident described, the reactor was made critical at 11 watts, the reactivity increased to a 60 sec period at 50 watts power, then the power level was allowed to increase on the latter period until it reached 500 watts. At this point the period began to decrease, and at an increasing rate. The reactor was scrammed, but not in time to prevent a severe accident.

The integrated core power was 14 MW/sec. It is believed that some 50% of the core reached melting temperature, but did not exceed it due to the heat of fusion involved. Much of the heat would go toward vaporization of the NaK, so that in a short time the vaporization temperature would provide a new upper limit for the temperature in the central core region.

It is believed that molten eutectic first fell to the bottom of the core through the coolant (NaK). Then molten fuel froze upon contact with NaK and fell through the eutectic at the bottom of the core. The normal flow of NaK tended to deposit some material toward the top of the core. The question arose as to whether the core had been shut down by its own disassembly, or by the actual scram induced by the operators. The former would have been caused by the motion of the material away from the center of the core. The source of the impulse required for such action would presumably be the vapor pressure of the NaK bond between clad and fuel. The reduction of the ultimate strength of the stainless steel clad with temperature (8000 psi at 1000°C to 0 psi at 1430°C) coupled with the increase in NaK vapor pressure and the eutectic formation may be used to estimate the NaK pressure at rupture. This is stated to be about 100 psi, along some 10% of the core length. Although the rupture would disperse material with some force, the conclusion is reached that the effect, in this case, was not sufficiently violent to cause core shutdown.

Reference 93 describes the first results obtained from the Test Reactor Transient (TREAT) facility. Tests involve uncooled fuel elements of the EBR-II type. The experiments are simply to determine meltdown temperatures and behavior. A single fuel element is vertically suspended, within a capsule, in the center of the TREAT core. The core is brought up to critical at low power, and then the control rods are withdrawn at high speed. Power surge varied from 0.2 to 0.5 sec in duration, and is defined as:

$$t_d = \frac{\int_0^{\infty} P dt}{P_{\text{peak}}}$$

where P = reactor power level

t = time variable

t_d = power surge duration time.

The duration time is thus the time required to expend the total actual power if it had all been expended at a rate equal to P_{peak} , the maximum power attained during the excursion.

Experimental TREAT power surges have raised cladding surface temperatures to 970°C with little damage other than the formation of unbonded areas (probably caused by boiling and ejection of the sodium bond between fuel and cladding). From 970°C to 1015°C, failure occurs by penetration of the stainless steel cladding in the lower region of the element, and subsequent ejection of uranium fuel. Above 1015°C penetration of the cladding occurs over broad areas and uranium ejection is violent enough to deposit the fuel on the inside surface of the capsule. In an operating reactor, such violent behavior of the molten fuel could communicate damage rapidly to previously unaffected pins.

These reports indicate the necessity for the prediction of transient fuel element temperatures during abnormal reactor behavior.

3.2 PREDICTION OF FUEL ELEMENT THERMAL TRANSIENTS

Any heat added to a reactor coolant, excepting radiation heat, must first travel through the fuel material and the clad. This section reviews some of the techniques developed to predict those temperatures during transients. The work reports cases with and without fuel melting.

The results of Reference 26 are most useful, providing an approximate relationship between fuel element surface temperature, an average fuel element temperature and their time derivatives.

Reference 11 employs the standard conduction equations for a cylindrical fuel rod with cladding, and for the temperature distribution in the coolant. A solution is found where the initial conditions are assumed in order to facilitate the mechanics of the problem. It is stated that the initial temperature distribution has a strong effect in determining whether the coolant will boil first or the fuel will melt first, although this distribution is arbitrarily assumed. The primary purpose in Reference 11 is to estimate coolant void effectiveness in reducing fuel temperature excursions through reactivity effects.

In Reference 9 it was desired to estimate a rate of power increase, or period, beyond which heat transfer out of the source to the coolant might be neglected. To determine where thermal inertia becomes important, the time variation of stored heat in a sodium-cooled uranium rod was studied. The temperature distribution in a two-region cylindrical geometry was found assuming an exponentially rising heat source in the central region. From these results the heat content of the fuel may be found, and thus the actual fraction of total heat generation remaining in the rod. The temperature distribution corresponding to the exponentially rising heat source is a cumbersome expression involving Bessel functions of various types with time and radial position as arguments. Integral transform methods are employed. Predictions of the fuel rod heat content during various power excursions for the EBR-I configuration are shown in Fig. 1.

The discussion so far treats an intact fuel and cladding. Behavior of the fuel near and above its melting point is deemed more important in light of the EBR-I accident.

Reference 22 calculates a melting pattern for the Enrico Fermi Reactor assuming that all of the sodium coolant has voided the core. Longitudinal heat conduction is neglected, and heat is

Fig. 1 — Fraction of heat remaining in a fuel rod for exponentially rising heat source of period $1/\alpha$; EBR-I configuration; uranium rod diameter = 0.208 cm; sodium annulus OD = 0.338 cm (Reference 9).

produced in a subcritical assembly by the delayed neutrons and attendant radiation. The fuel is assumed to melt in situ; any possible molten fuel run-off and effects of such are neglected. The pattern of melting for the Fermi Reactor, as predicted by this analysis, is shown in Fig. 2. Five seconds have elapsed since the transient began. The total volume of molten fuel as a function of time for this transient is shown in Fig. 3. A brief of the mathematical development is given in Section 4.1.

A study of the behavior of a solid-liquid interface during melting, under the same conditions, is found in Reference 24. The mathematics are outlined in Section 4.2.

The analyses reviewed in this section have employed boundary conditions which are governed by the coolant. In the following portion of the report, the static and dynamic aspects of the coolant are discussed.

3.3 PREDICTION OF REACTOR COOLANT TRANSIENTS

Several studies have been made of the transient characteristics of the Enrico Fermi Reactor. These analyses of the transient coolant behavior serve as a useful foundation for future investigations of the safety of fast breeder reactors.

The coolant behavior of a fast reactor during transients, for a liquid-phase coolant, is discussed in Reference 76. It is stated that, in calculations for the APDA reactor, heat transfer to the coolant may be neglected for transients resulting from reactivity insertions of 5 dollars/sec or more. The governing equations are given in Section 4.10. Solution of these equations coupled with kinetics equations yields an estimate of coolant temperatures for the nonboiling region. A comparison with studies previously made for the APDA reactor, for the case of a ramp insertion of reactivity of 1 dollar/sec shows that 1.8 sec elapse before fuel melting is reached when the coolant behavior is included, and is 64% greater than if sodium cooling is neglected.

An analysis has been made for the behavior of a sodium coolant during an accident wherein a large leak develops in the lower portion of a sodium-cooled fast reactor. This is reported in Reference 23. The coolant will begin to flow in the reverse direction. This is assumed to occur immediately with the sodium starting from rest. Application to the Enrico Fermi Reactor indicates a velocity of 1/3 normal is attained in the reversed direction. The entrance temperature of the sodium will be equal to its previous exit temperature. The reactor core thus heats rapidly, and in a few seconds the sodium coolant would be expected to boil. The report presents a transient analysis of temperatures just prior to boiling, and thus a method to determine time lapse before boiling. The fluid is assumed incompressible, and there is no allowance for axial heat transfer within the fuel. Methods of solution are discussed, and a difference-equation technique is chosen. Results are shown in Figs. 4 and 5, for the Fermi Reactor. Fig. 4 shows the variation of the sodium coolant temperature distribution with time. Fig. 5 presents the coolant exit temperature variation with time. Both figures are for a central channel.

Reference 75 reports a study, done for the Enrico Fermi Reactor, of failure of effective cooling. The three coolant pumps were assumed to fail simultaneously and the eight safety rods became inoperable. Results of the study predict sodium coolant boiling in 8 sec, and fuel melting in 4 sec.

Reference 52 discusses the results of sodium vapor formation within the same reactor. Sodium vapor might be formed as a result of pump failure and coolant slowdown, or a power excursion over 300% of operating power. Two-phase pressure drop calculations were made, and will be discussed in a later section. The results indicate that a stable boiling condition could not occur at any power level above 25% of operating power.

Reference 28 discusses the first stage of sodium coolant boilout in application to the APDA reactor. If the reactor power should suddenly attain a level high enough to cause boiling at the core exit, the increased volume would lead to increased flow resistance. This would tend to de-

Fig. 2 — Predicted melting configuration in the Enrico Fermi reactor core with 5 sec time elapsed from sodium coolant voiding (Reference 22)

Fig. 3 — The predicted volume of uranium melted following sodium coolant voiding of the Enrico Fermi reactor (Reference 22)

Fig. 4 — The temperature of the Fermi reactor sodium coolant during a period of flow reversal (Reference 10)

Fig. 5 — The exit temperature of the Fermi reactor sodium coolant during a period of flow reversal (Reference 10)

crease coolant flow thus increasing the extent of the boiling region. Thus, eventually, the hottest subassembly might become void of sodium.

Sodium coolant pressure drop through the APDA reactor during two-phase flow is treated in Reference 81. Flow friction is taken proportional to the square of the liquid-phase velocity and independent of volume fraction. The slip ratio is assumed constant over the core length. Liquid and vapor are assumed to be in equilibrium. The resulting differential equation is shown in Section 4.5. A sample problem is presented, wherein exit pressure is 1.5 atm, flow is 1/3 normal, and the slip ratio is 5. A heat input of 110% of that required to cause boiling at the core exit is assumed. Assuming stable boiling, a plenum pressure of 3 atm is calculated necessary to sustain flow.

Reference 11 also treats coolant boiling in the APDA reactor, but arbitrarily assumes vapor to be formed and to remain in the central core region. The stability question is not mentioned. The coolant boils, and thus raises the system pressure and the boiling point of the sodium. An expression is used for the vapor pressure of sodium as a function of temperature. This is:

$$p = 1.043 \times 10^{-18} T^{6.67}$$

where T is in $^{\circ}\text{F}$ and p is in lb/ft^2 . Since the vapor remains in the core, two-phase flow considerations are avoided. For the reactor in question, the analysis indicates that a period of 0.108 sec would bring the sodium to boiling (at 2000°F) simultaneously with central fuel melting (at 2200°F). A longer period would presumably lead to sodium boiling and possible burnout.

Reference 75 describes an analysis of fast reactor behavior for the case where there is initially a constant sodium-coolant flow and the power level is abruptly raised to the point where boiling begins in one of the hotter subassemblies. It is predicted that if flow was the normal 32 ft/sec of the Fermi Reactor, the flow would be quickly choked in the subassembly, and the liquid level would drop to the core entrance level. Thus the fuel would melt into a high-velocity sodium-vapor stream. The time required to change from normal conditions to a condition of dry vapor was estimated to be of the order of a tenth of a second.

3.4 GENERAL INFORMATION

3.4.1 Coolant Heat Transfer

Steady State Nonboiling

The analyses and experimental data found in the literature for nonboiling forced convection heat transfer with liquid metals can be separated, roughly, into three categories: (1) Nusselt number for fully developed velocity and temperature profiles, (2) Nusselt number variation in the entrance region of the test section, and (3) average Nusselt number over the test section. Boundary conditions for the analyses consider uniform wall temperature or uniform surface heat flux in the direction of flow. Most of the tests and analyses have been concerned with tubular or annular flow passages.

For liquid metals, which have low values of the Prandtl number, $Pr = C_p \mu / K$, the radial transport of heat by thermal conduction in the turbulent core of the fluid can be significant compared to that due to eddy diffusion. This characteristic of liquid metals results in heat transfer coefficients that are nearly independent of the Reynolds number, Re , up to values of 10^4 to 10^5 . As an example, for sodium at reactor operating temperatures ($Pr \approx 4 \times 10^{-3}$), the eddy diffusion in the core is estimated to increase the heat transfer coefficient by only 30% for a Reynolds number of 100,000. The semiempirical correlations developed for ordinary fluids, such as air or water,

do not apply to liquid metals since eddy diffusion with these ordinary fluids is controlling in the core.

The analogy between heat and momentum transfer was first extended to the case of low values of the Prandtl number by Martinelli^{73*} who treated fully developed heat transfer to a fluid flowing through a tube having uniform wall temperature. Lyon⁶⁸ considered the case of uniform surface heat flux. The analysis was based on the assumptions of steady state conditions, axial symmetry, uniform fluid properties, no end effects, and a constant ratio of the eddy diffusivities of heat (ϵ_H) and momentum (ϵ_m). The Lyon relation for the Nusselt number ($Nu = 2hr_w/K$) is

$$\frac{1}{Nu} = 2 \int_0^1 \frac{\left[\int_0^{r/r_w} \left(\frac{u}{u_m} \right) \left(\frac{r}{r_w} \right) d\left(\frac{r}{r_w} \right) \right]^2}{\left(\frac{r}{r_w} \right) \left(1 + \frac{\epsilon_H}{\epsilon_m} Pr \frac{\epsilon_m}{\nu} \right)} d\left(\frac{r}{r_w} \right)$$

where r and r_w are the local radius and tube inner radius, u is the local axial velocity, u_m is the average axial velocity and ν is the kinematic viscosity. The above equation gives a Nusselt number of 8 for a flat velocity profile and 4.36 for a parabolic velocity profile. Using Nikuradse's experimental data on velocity profiles and numerical integration procedures, Lyon found that values of Nu from the above equation for Prandtl numbers from 0 to 0.1 and Reynolds numbers from about 4×10^3 to 3×10^6 could be expressed approximately by the relation

$$Nu = 7 + 0.025 (Pe)^{0.8}$$

where Pe is the Peclet number, given by $Re \cdot Pr$. In this expression ϵ_H/ϵ_m was taken equal to unity.

Deissler¹⁹ and Jenkins⁴⁸ attempted to account for the effect on turbulent heat transfer of heat conduction to the eddies during their movement. This effect tends to reduce ϵ_H/ϵ_m and, consequently, to lower the predicted heat transfer coefficient, particularly at lower values of the Peclet number.

For the case of constant wall temperature, the analysis of Seban and Shimazaki⁹² indicated that the Nusselt number relations for fully developed flow and temperature profiles in a tube could be approximated by

$$Nu = 5 + 0.025 (Pe)^{0.8}$$

Deissler²⁰ made an analysis of the variation in the local Nusselt number along the entrance region of a smooth tube for turbulent heat transfer to a liquid with a Prandtl number of 0.01. The results were used in Reference 91, to obtain a predicted average Nusselt number vs Peclet number for various values of the test section length-diameter ratio, L/D .

Reactor application is usually concerned with Peclet numbers greater than 500 and length-diameter ratios greater than 50. The predictions of References 91 and 20 indicate that the average Nusselt number is about 10% higher than the Nusselt number for fully developed flow and heat transfer at an L/D of 50, the difference being less at higher values of L/D .

Some analytical work has been done for the case of turbulent heat transfer to parallel flat plates with heat addition from one side (see Reference 66). For fast reactor application, it would appear that the case of heat transfer to a round tube would be more pertinent.

A survey of experimental data on forced convection heat transfer to liquid metals is given

*Super numbers refer to references (see Section 5).

in Reference 91. There is a relatively small amount of data for the forced convection heat transfer coefficient with sodium.

In Fig. 6, experimental data on the average sodium heat transfer coefficient in round tubes are compared with the Lyon-Martinelli relation. Curves one, two, and three, and the striated region (4) for experimental data were taken from Fig. 5.64 of Reference 7. The experimental points were obtained for small, parallel-flow, sodium heat exchangers (Reference 69). Curve 5 is the Martinelli-Lyon relation. Curve 6 is the semiempirical relation presented by Lubarsky and Kaufmann in their survey⁶⁵ of forced convection heat transfer for a number of liquid metals. This relation, which is given by

$$Nu = 0.625 (Pe)^{0.4},$$

fitted the majority of the experimental points.

There is a rather large scatter in the experimental results, particularly at low values of the Peclet number. The Lyon-Martinelli relation generally overestimates the Nusselt number by perhaps 50%.

Steady State Boiling Heat Transfer

Although there has been a considerable amount of investigation in the general field of boiling heat transfer, experimental and analytical work on boiling liquid metals is limited. All of the experimental investigations, which were with pool boiling, were made with mercury, sodium or sodium-potassium alloy.^{7, 66, 69}

Lyon⁶⁶ investigated the boiling of sodium and NaK from a horizontal rod heater at surface heat fluxes up to about 130,000 Btu/hr-ft² as limited by apparatus. The heat transfer coefficients in Btu/hr-ft²-°F ranged from approximately 3000 to 15,000 for Na and from about 5000 to 15,000 for NaK. There were no indications of having approached maximum heat flux.

Madsen and Bonilla⁶⁹ report data on the boiling of NaK from a flat plate. These results show much lower heat transfer coefficients than those obtained by Lyon⁶⁶. It was suggested that this difference was due to the geometry of the heating surface.

It is noted that a recent correlation by Forster and Greif for nucleate boiling³² has been applied with reasonable success to predict the heat transfer coefficients for mercury and sodium. The authors suggest use of the equation to predict the heat transfer coefficient for nucleate boiling with and without forced convection.

The only reported experiments on boiling of liquid metal inside tubes are presented in Reference 54. The data were for mercury.

Since there have been few experiments on boiling burnout with liquid metals, the predictions of this phenomenon are questionable. Calculations using the correlations of Kutateladze,⁵⁴ Zuber and Tribus,¹⁰⁴ and Rohsenow and Griffith,⁸⁹ give predicted values of the critical heat flux for sodium with pool boiling of about 500,000 Btu/hr-ft²-°F. Apparently, the most extensive correlations of boiling burnout studies to date are presented in the Westinghouse Atomic Power Department report, WAPD-188. These correlations are for forced convection water systems at 2000 psia, and extension to physically different liquid metal systems is not intended.

Transient Heat Transfer

Little has been found in the literature relating to the prediction of nonboiling or boiling convection heat transfer during transients.

Fig. 6 — Comparison of experimental and predicted values of average sodium heat transfer coefficient in tubes. 1 — Deriugin, Mikheev, and Fedynskii; 2 — Novikov and Khabakhpasheva; 3 — Borishanskii, Kutateladze, and Shneideman; 4 — MacDonald and Quittenton; 5 — Martinelli-Lyon;⁷³ 6 — Lubarsky and Kaufman.⁶⁵ ● Reference 44, 0.130 in. diam tube; + Reference 44, 0.125 in. diam tube.

With nonboiling systems the assumption is sometimes made that an appropriate steady state value of a heat transfer coefficient can be used as an approximation. There is some basis for this assumption for fluids of moderate Prandtl number in turbulent flow when transients are not too rapid. For liquid metals and with the kind of transients of interest to fast reactor safety studies, this assumption is probably not sufficiently accurate.

Rosenthal,⁸⁰ describes transient boiling experiments, conducted in water, using platinum and aluminum ribbons. The ribbons are treated electrically to yield an exponentially increasing heat generation rate, in simulation of a nuclear reactor excursion. Ribbon temperatures are measured during the transient and associated with the type of boiling through the use of high-speed motion pictures. The conclusions of interest are: for rapid heating of a metal surface, in contact with water below the boiling point, a finite rise in surface temperature above the boiling point occurs before boiling begins; prior to boiling, the water-coolant acts as a semi-infinite solid during an excursion, and its temperature rise may be computed from transient conduction theory; once quasi-steady state boiling has begun during an excursion, for periods greater than 15 msec with large subcooling, behavior of the system is essentially that of steady boiling at any instant.

3.4.2 Boiling Coolant Hydraulics

Two-Phase Flow Patterns

In order to be able to predict the behavior of boiling coolant flow and heat transfer from physical models it is important to know the way liquid and vapor distribute themselves in a channel. Although there are several visual studies of two-phase flow systems without boiling, mostly for air-water and air-oil mixtures, there do not seem to be any such studies for boiling systems. As a result it is necessary to borrow from these nonboiling studies with respect to expectation of liquid-vapor distributions or flow patterns for boiling systems.

Nonboiling system two-phase flow regimes with vertical upflow have been divided into six categories, more or less, depending upon the observer. Reference³³ describes the five types commonly employed. Bubble flow, which occurs at low velocities, is characterized by gas bubbles within the liquid mass. Slug flow, which occurs at slightly larger velocities, is composed of alternate liquid and gas pistons. Further increase of gas velocity results in dispersed flow, where the two phases are in a violently agitated mixture. A broad transition region is termed semiannular flow. The flow at high gas velocities is termed annular flow, with liquid adjacent to the containing wall and a central flow of gas. Further increase in gas velocity results in mist flow, where gas and liquid have the appearance of being uniformly dispersed.

Steady State Pressure Drop and Void Fraction

The classical method for calculation of pressure drop during forced-circulation boiling of water is presented in Reference 74. This was published in 1948, by R.C. Martinelli and D.B. Nelson. The pressure drop of a boiling mixture is made up of two parts: the pressure loss due to frictional forces and the loss due to the rate of increase of momentum of the mixture as it vaporizes. It is assumed that both liquid and vapor flow turbulently and that a linear relationship exists between flow quality and axial position. A summary of the calculation procedure is given in Section 4.9.

Procedures for calculating pressure drop and density of a turbulent steam-water mixture are found in Reference 29. Early two-phase work is summarized, such as found in Reference 74 and 64. It is stated that "The procedures employed are presumably adaptable to substances other than H₂O." The calculation procedure is given in Section 4.4.

Reference 95 discusses pressure drops for a flow model with annular flow. The method is presented in Section 4.11.

Reference 70 discusses results in the prediction of slip ratios in low-quality water flow. A gamma counter is used to obtain average density readings. Either a phase velocity ratio or a phase velocity difference is recommended for correlation of data.

Equations governing the effects of slip in the forced circulation of boiling water are derived in Reference 60.

A momentum model is postulated which gives equal friction and head losses for two phases. It is stated that good experimental agreement was obtained for cases with and without heat addition for pressures from 12 to 2000 psi.

Reference 52 is a simplified study of pressure drops for two-phase flow conditions in the Enrico Fermi Reactor. It is stated, "A precise prediction of two-phase flow characteristics would involve laborious and time-consuming iterative processes." The analysis is believed to be quite good for predicting major trends involved. It is assumed that the coolant is well-mixed, that is, there is no vapor slip. The pressure drop due to the flow of liquid sodium is neglected. Specific volume for two-phase flow is taken as the arithmetic average of that at inlet and outlet of the top blanket section: the assumption is made that boiling takes place only in the top blanket section.

The analysis is presented in Section 4.7. The large value of specific volume of sodium vapor, coupled with top blanket resistance of the Fermi Reactor, resulted in large pressure drops for even small values of vapor volume fraction. Pressure loss resulting from vapor formation was always far greater than the slight compensation due to flow reduction. The paper concludes that the formation of any vapor in the Enrico Fermi Reactor would trigger an unstable condition in the coolant and would result in fuel burnout.

Reference 64 is concerned with the prediction of coolant densities in boiling reactor channels under normal operating conditions. This is equivalent to describing the distribution of vapor in a channel. In the analysis, the velocity of vapor relative to that of liquid, i.e., the slip ratio, is considered constant. The statement is made that little or no data relating to two-phase friction drop are available for vertical-channel boiling. The analysis is presented in Section 4.8.

Transients

Reference 12 presents techniques for the solution of several transient hydrodynamic problems. Methods cover single-phase systems, but are presumably adaptable to two-phase cases.

Reference 25 develops relations for the prediction of transient void formation in a boiling-water reactor. The important assumptions made are: inlet flow properties are constant; properties of the two phases are constant except for liquid enthalpy in the nonboiling region; at a given pressure there is a unique relation between quality and void fraction; the principal mode of heat generation persists during a transient; the two phases are in thermodynamic equilibrium. The development is given in Section 4.3.

A model for calculating boiling reactor transients is presented in Reference 31. Water enters the bottom of a reactor core and increases in temperature until saturation, T_s , is reached. A hydrodynamic description of the system is furnished by three equations expressing conservation of mass, energy, and momentum. The conservation laws are used in integral form to derive a set of ordinary differential equations.

A mathematical description is given in Section 4.6.

3.4.3 Physical and Thermodynamic Properties of Sodium

Recommended values of the physical properties of liquid sodium, based on experimental data obtained prior to 1955, are given in Reference 67. A fairly recent survey of published and unpublished data on the properties of inorganic coolants available as of 1959, has been made in con-

nection with the possible use of these coolants for space application.⁹⁸ This reference contains recommended values of physical and thermodynamic properties of liquid sodium and of sodium vapor. The tabulation and working charts of the properties include density, viscosity, thermal conductivity, latent heat of vaporization, specific heat, enthalpy, entropy, vapor pressure, critical properties and surface tension. Data obtained in the USSR on the properties of liquid metals, including liquid sodium, are contained in a recent Russian survey article on liquid metal heat transfer.⁵⁴

4. APPENDIX – MATHEMATICAL TREATMENTS FROM SELECTED REFERENCES

4.1 TEMPERATURE DISTRIBUTION IN REACTOR DURING FUEL MELTING²²

The initial fuel temperature is given by:

$$\bar{T}_f(r,z,0) = Cq(r,z) + T_0 + Bq(r,z)dz$$

where $\bar{T}_f(r,z,0)$ = mean fuel temperature initially

C, B = constants

T_0 = normal operation inlet coolant temperature

$q(r,z)$ = normal operation volumetric heat generation

r = dimension from the axis of core symmetry

z = axial dimension from core center-plane.

The initial heat generation is taken to have a cosine distribution:

$$q(r,z) \approx A \cos(\alpha r) \cos(\beta z)$$

where A, α, β = constants. For times greater than zero, the local heat generation rate is:

$$q(r,z,t) = F(t)q(r,z)$$

where $F(t)$ = decay heat function of time. A final expression for the fuel temperature is written as:

$$\bar{T}_f(r,z,t) = q(r,z) \left[E \int_0^t F(t) dt + JC + G + (J+K)T_0 + (J+K)B \int_{-L}^z q(r,z) dz \right]$$

where $E, \dots, K$ = constants

L = half-height of core

The above equation is used as long as the core maintains integrity. Time to melting may be found. Once melting commences, however, temperature is constant. A fictitious temperature, T'_m is introduced, which corresponds to the temperature which would be achieved if the system were unmelted. Thus:

$$\text{Fraction melted} = \frac{\bar{T}_f(r,z,t) - T_m}{T'_m - T_m}$$

where T_m = melting temperature of fuel

T'_m = fictitious temperature, above T_m

4.2 BEHAVIOR OF SOLID-LIQUID INTERFACE DURING MELTING²⁴

The solid-liquid interface is of variable position and remains at a constant temperature. A flat, infinite surface is assumed, and is indicated in Fig. 7. The pertinent equation and its boundary conditions are as follows:

$$\frac{\partial^2 T}{\partial x^2} - q(x) = \frac{1}{K} \frac{\partial T}{\partial t} \quad (x \geq 0)$$

$$q(x) = 0 \quad (a < x < b)$$

$$T(x, 0) = -T_0 \quad (x < b)$$

$$T(x, 0) = 0 \quad (x > b)$$

$$\frac{\partial T(0, t)}{\partial x} = 0$$

$$T[f(t), t] = 0$$

$$K \frac{\partial T}{\partial x} [f(t), t] = L \rho \frac{df(t)}{dt}$$

where T = temperature in configuration

x = dimension as shown in Fig. 7

q = volumetric heat generation

K = thermal conductivity

a, b = cladding bounds as in Fig. 7

T_0 = original depression of fuel-clad temperature below melting temperature

$f(t)$ = liquid-solid interface position, as shown in Fig. 7

L = heat of fusion per unit mass.

If $G(x, x', t, t')$ is the temperature response at point x and time t , due to unit release of heat per unit area in the plane x' at time t' in an infinite medium, then by application of the method of sources and sinks:

$$G(x, x', t, t') = [2\rho c \sqrt{\pi k(t-t')}]^{-1} \exp - \frac{(x-x')^2}{4k(t-t')}$$

where k = thermal diffusivity

c = specific heat.

By combining sources, the response in a semi-infinite medium to the heat injection when face $x=0$ is insulated, may be written:

$$H(x, x', t, t') = [2\rho c \sqrt{\pi k(t-t')}] \left[\exp - \frac{(x-x')^2}{4k(t-t')} + \exp - \frac{(x+x')^2}{4k(t-t')} \right]$$

The solution of the partial differential equation given previously is obtained by superposing responses of the type $H(x, x', t, t')$ for three cases: at zero time there is an instantaneous subtraction of heat such that $T(0 \leq x \leq b, 0) = -T_0$. The response to this is:

Fig. 7 — Configuration involved in reactor fuel meltdown study (Reference 24)

$$T_1(x,t) = -\rho c T_0 \int_0^b H(x,x',t,t') dx'.$$

Beginning at time zero and continuing at a uniform rate, heat is added in the region $x > 0$. The response at time t is:

$$T_2(x,t) = \int_{x'=0}^{\infty} \int_{t'=0}^t q H(x,x',t,t') dt' dx'.$$

From the beginning of the process, the solid-liquid interface moves, and maintains itself at melting temperature. The temperature response to the interface motion is:

$$T_3(x,t) = \int_{t'=0}^t H[x,f(t'),t,t'] \cdot \left[\frac{df(t')}{dt} \rho L \right] dt'$$

The solution may be represented by the sum of T_1 , T_2 , and T_3 . If the freezing point is taken as the datum, then a condition is:

$$0 = \sum_{i=1}^3 T_i[f(t),t].$$

Results obtained through approximation are shown for the case of heat generation for $x > f(t)$ in Fig. 8.

4.3 TRANSIENT VOID FORMATION IN BOILING REACTOR²⁵

Relations for conservation of mass and energy within a coolant channel yield:

$$\frac{Q-Q_b}{x h_{fg}} = W_{in} + \left(\rho_w - \rho_s - \frac{\rho_s}{x} \right) \int_0^z A \frac{\partial \epsilon}{\partial t} dz$$

where Q = heat to coolant between $z = 0$ and z

z = axial dimension along flow

x = quality based upon mass flow

W_{in} = local mass flow rate

ρ_w, ρ_s = density of liquid, steam

A = flow area

ϵ = steam volume fraction at z .

Perturbation of this equation provides an expression for the increase of total voids with small nuclear transients:

$$\frac{\delta(Q-Q_b)}{Q-Q_b} - \frac{W_{in}}{\rho_w - \rho_s + \frac{\rho_s}{x}} = \frac{\partial Y}{\partial t} + \frac{\partial Y}{\partial \zeta}$$

Fig. 8 — Freezing on a slab surface where $(cT_0)(L)^{-1} = 3.5$,
 $(qb^2)(Lpk)^{-1} = 4.5$. (Reference 24)

where

$$Y = \int_0^z A \delta \epsilon dz$$

$$\xi = \int_{z_b}^z [\rho_S + x(\rho_W - \rho_S)] A dz / x'(\epsilon) W_{in}$$

The heat generation is separable:

$$q(z,t) = f(z)F(t)$$

Mean fuel temperature is then:

$$T'_m(t) - C_0 T_m = C_0 f(z) F(t)$$

It follows,

$$T_m(z,t) = f(z)G(t).$$

It may be concluded that the heat transfer rate from fuel to coolant is:

$$Q = H(t) \int_0^z f(z) dz$$

Perturbation, further analysis, and series approximation provide:

$$\frac{\partial Y}{\partial t} - \frac{\partial Y}{\partial \xi} = A(\xi) \delta H - B(\xi) \frac{d(\delta H)}{dt}$$

where A,B = steady state channel functions.

4.4 PRESSURE DROP IN STEAM-WATER MIXTURES²⁹

The wall shear stress may be found from Fig. 9. The associated nomenclature is:

W_l, W_s = mass flow rate of liquid, steam

τ = two-phase flow wall shear stress

τ_l = shear stress if liquid at rate W_l

τ_v = shear stress if vapor at rate W_s

P = system pressure.

Single phase stresses may each be computed from the formula, where n refers to l or v:

$$\tau_n = f \rho_n \frac{(W_n / \rho_n A)^2}{2g}$$

where f = friction factor.

Fig. 9 — Martinelli-Nelson chart for steam-water mixtures

The fraction, ϵ , of duct volume occupied by vapor is found from each of the two formulas, and the lesser value chosen:

$$\epsilon = 1 - (\tau/\tau_L)^{-5/9}$$

$$1/\epsilon = 1 + (\rho_v/\rho_L)(W_1/W_S)$$

Mixture density is then:

$$\rho = \rho_v \epsilon + \rho_L (1 - \epsilon)$$

The velocities of the phases are then:

$$v_1 = W_1/\rho_1 A (1 - \epsilon); \quad v_v = W_S/\rho_v A \epsilon$$

The static pressure gradient along the line of flow is given by:

$$\frac{dp}{dL} = \frac{4\tau}{D_h} + \frac{dz}{dL} \rho + \frac{W_1}{g} \frac{dv_1}{dL} + \frac{W_S}{g} \frac{dv_v}{dL}$$

where L = dimension along flow

D_h = hydraulic diameter.

A method is presented for the calculation of pressure drop across an enlargement or contraction for two-phase flow. First compute the drop that would occur if the actual liquid flow rate, W_1 , occupied the entire flow section. Then the result obtained is multiplied by the following factor:

$$1/(1 - \epsilon) \left[1 - (\rho_L/\rho_v) \left(\frac{1 - \epsilon}{\epsilon} \right) (W_S/W_1)^2 \right]$$

If a flow model with vapor centrally located in a liquid annulus is taken, then the two-phase wall shear stress is given by:

$$\tau = f \rho_1 (V_1^2/2g)$$

where an inherent Reynold's number of $\rho_1 v_1 D_h / \mu_1$ is used. (See also Reference 86.)

4.5 TWO-PHASE FLOW EQUILIBRIUM³⁰

Neglecting kinetic energy, an energy balance provides:

$$x = \frac{Q(L) - W C_p T_b(F)}{W h}$$

where $Q(L)$ = integrated heat input, inlet to L

L = dimension along fuel axis

x = vapor weight fraction of flow

W = mass flow

C_p = specific heat of coolant

$T_b(P)$ = temperature change from inlet to point at pressure, P

h = heat of vaporization.

Mass flow continuity provides:

$$\alpha = (x/s)(\rho_f/\rho_g)/[1-x-(x/s)(\rho_f/\rho_g)]$$

where α = vapor volume fraction

s = vapor slip ratio

ρ_f, ρ_g = density of liquid, vapor.

The pressure gradient is found from a force balance:

$$\frac{dP}{dL} = \frac{-W}{144g} \frac{dxv_g}{dL} - \frac{W}{144g} \frac{d(1-x)v_f}{dL} - fv_f^2$$

where v_g = vapor velocity

v_f = liquid velocity

g = gravitational constant

f = friction factor.

Collection of equations provides:

$$dL = \frac{-\rho}{fw^2} f^2 \frac{(1-\alpha)^2}{(1-x)^2} dp + \frac{xs-1-x}{144fg} d(\rho_g s\alpha - \rho_f \alpha) - \frac{(s-1)\rho_f(1-\alpha)dx}{144fg(1-x)}$$

4.6 MODEL FOR CALCULATING BOILING REACTOR TRANSIENTS³¹

The nomenclature to be used in the development is as follows:

$f(x)$ = steam void fraction as a function of position

$q_s(x)$ = steam velocity as a function of position

$q(x)$ = water velocity as a function of position

γ = $q_s(x)/q(x)$ = slip ratio

ρ_0 = water density

ρ_s = steam density

p = pressure

q_s = steam velocity at the core outlet

h_w = enthalpy per unit mass of water at saturation

h_s = enthalpy of steam per unit mass

h_v = $h_s - h_w$ = heat of vaporization of steam per mass

e_s = internal energy of steam per unit mass

e_w = internal energy of water per unit mass

Q = total power in boiling volume.

The principle of mass conservation applied to the boiling region ($x_1 \leq x \leq x_2$) gives:

$$\frac{d}{dt} \int_{x_1}^{x_2} \{\rho_s f(x) + \rho_0 [1-f(x)]\} dx = \rho_0 [\dot{q}_0(x) - \dot{x}_1] - \rho_s q_s f - \rho_0 q(1-f)$$

The principle of energy conservation gives:

$$\frac{d}{dt} \int_{x_1}^{x_2} \{\rho_S e_S f(x) + \rho_0 [1-f(x)] e_W\} dx = \frac{Q}{A} + \rho_0 h_W (q_0 - \dot{x}_1) - f \rho_S q_S h_S + (1-f) \rho_0 h_W q$$

It should be noted that kinetic energies have been neglected on the right-hand side of the above equation. The principle of momentum conservation gives:

$$\frac{d}{dt} \int_{x_1}^{x_2} \{\rho_S q_S(x) f(x) + [1-f(x)] \rho_0 q(x)\} dx = F + [p_1 + \rho_0 q_0 (q_0 - x_1)] - [p_2 + f \rho_S q_S^2 + (1-f) \rho_0 q^2]$$

where F is the sum of all pressures resulting from forces such as gravity and friction, and p_1 and p_2 are pressures acting at x_1 and x_2 . The total force, F is represented by:

$$F = -g \int_{x_1}^{x_2} \{[1-f(x)] \rho_0 + \rho_S f(x)\} dx - \frac{\rho_0 f_0}{2} \int_{x_1}^{x_2} q^2(x) dx - \frac{f_0 \rho_0}{2} q_0^2 x_1$$

where the first term on the right is gravity force, and the next two are friction forces in the boiling and nonboiling sections. f_0 is the friction factor. It is assumed that the two-phase friction force is due to the increased velocity of the water phase alone.

The pressure difference, in the absence of an accelerating pressure developed within the core, is:

$$p_1 - p_2 = \rho_0 g \Delta - k_0 q_0^2 - \frac{d}{dt} q_0 \left(\frac{A}{A_p} \rho_0 x_2 + \rho_0 x \right)$$

where the terms on the right represent hydrostatic head, external pressure losses due to friction, constrictions, etc., and the acceleration of all mass not in the boiling volume, and A_p is the flow area in the downcomer.

The following assumptions are made: $q(x)$, $q_S(x)$, and $f(x)$ vary linearly with position; slip ratio, γ , is constant with position, but may depend upon q_0 ; e_W , e_S , h_W , and h_S are constant in time and position. The last assumption fixes the system pressure. The linear approximations are written:

$$f(x) = f \cdot \{x - x_1\} / \Delta$$

and

$$q(x) = q \cdot \{x - x_1\} / \Delta + q_0 \cdot (x_2 - x) / \Delta$$

The previous equations then become, along with the usual heat-transfer relationships:

$$\frac{df}{dt} = \frac{2}{\Delta} \left[\frac{Q - p A \dot{x}_1}{A \rho_S h_r} - f \cdot \left\{ \gamma q - \frac{1}{2} \dot{x}_1 \right\} \right]$$

$$\frac{dm}{dt} = \frac{1}{2} \rho_0 g f \cdot \Delta - k_0 q_0^2 - \frac{1}{2} f_0 \rho_0 q_0^2 \dot{x}_1 - \frac{1}{6} f_0 \rho_0 (q^2 + q q_0 + q_0^2) + \rho_0 q_0^2 - f \rho_S \gamma^2 q^2 - (1-f) \rho_0 q^2 - \rho_0 q_0 \dot{x}_1$$

$$\frac{dx_1}{dt} = - \frac{2}{T_S - T_0} \left[h_W \frac{A_0}{H_0} x_1 \left(\bar{T}_m - \frac{T_S + T_0}{2} \right) + \delta^* \frac{\varphi x_1}{H_0} - q_0 (T_S - T_0) \right]$$

$$Q = [A_S n_b (T^* - T_S) - \delta^* \varphi] \Delta$$

$$\frac{dT^*}{dt} = \frac{(1-\delta)}{H_m} \varphi - \frac{A_S h_B}{H_m} (T^* - T_S)$$

$$\frac{d\bar{T}_m}{dt} = \frac{(1-\delta)\varphi}{H_m} - \frac{h_w A_S}{H_m} \left(\bar{T}_m - \frac{T_0 + T_S}{2} \right)$$

The $\bar{T}_m$ is the average metal temperature in the nonboiling region, T_S is the saturation temperature, T_0 is the inlet temperature, δ^* is the fraction of fission energy deposited promptly in the water, T^* is the mean fuel temperature in the boiling region, φ is the power per unit length per fuel channel, Δ is the length of the boiling channel, A_S is the surface area of the fuel element per unit length, h_B is the boiling heat transfer coefficient, $1-\delta$ is the fraction of the fission energy deposited in the fuel, and H_m is the heat capacity per unit length of fuel element.

The above equations may be coupled with the reactor kinetic equations,

$$\frac{d\varphi}{dt} = \frac{k_{ex} - \beta}{\ell} \varphi + \sum_i \lambda_i C_i$$

and

$$\frac{dC_i}{dt} = \beta_i \frac{\varphi}{\ell} - \lambda_i C_i.$$

It is expressed that the excess reactivity is $k_{ex} = \epsilon - \alpha(1/2)\Delta(f/x_2)$ where ϵ is externally-induced reactivity and α is the void coefficient of reactivity.

4.7 TWO-PHASE PRESSURE DROP⁵²

A relationship between viscosity and specific volume is assumed:

$$\mu = 0.07776/\sqrt{v}$$

Total pressure drop is expressed as

$$\Delta P_t = \Delta P_m + \Delta P_f$$

where ΔP_m = the momentum pressure drop

ΔP_f = the frictional pressure drop.

These pressure drops are expressed, for the two-phase section, as:

$$P_m = \frac{W^2}{A^2 g} (\bar{V}_0 - \bar{V}_i)$$

$$P_f = \frac{2K}{g A^{1.8} D_e^{1.2}} W^{1.8} \bar{v} \Delta h \mu^{0.2}$$

where W = mass flow rate

A = flow area

g = gravitational constant

D_e = equivalent diameter

V_i, V_0 = inlet specific volume, outlet.

For the Fermi Reactor, the total pressure drop predicted is then:

$$[\Delta P_t]_{\text{fermi}} \approx 0.475W^2(\bar{V}_0 - \bar{V}_i) + 0.483W^{1.8}(\bar{V}_0 - \bar{V}_i)^{0.9}$$

and for qualities greater than 5% reduces to:

$$[\Delta P_t]_{\text{fermi}} \approx 0.475W^2\bar{V}_0 + 0.483W^{1.8}\bar{V}_0^{0.9}$$

4.8 COOLANT DENSITIES IN BOILING REACTOR CHANNEL⁶⁴

A pressure balance is made within the coolant loop:

$$\Sigma \Delta P = 0.$$

For the core channels, a heat balance gives:

$$Q_t = W_t[(h_f - h_{in}) + xh_{fg}] = W_g[(h_f - h_0) - h_{fg}]$$

where Q_t = total heat input to coolant
 W_t = total flow rate
 h_f = enthalpy of liquid
 h_{in} = enthalpy of coolant at inlet
 x = vapor flow weight-fraction
 h_{fg} = latent heat of vaporization for coolant
 W_g = vapor flow rate
 h_0 = enthalpy of condensate.

A mass balance provides:

$$\frac{W_g}{W_f} = \frac{xW_t}{(1-x)W_t} = \frac{\rho_g A_t V_g \alpha}{(1-\alpha)A_t V_f}$$

where W_f = liquid flow rate
 α = vapor volume fraction
 A_t = flow area
 V_g, V_f = vapor velocity, liquid.

In order to write the pressure drop equation above, pressure losses are expressed as frictional, acceleration, and hydrostatic losses. If frictional effect is primarily due to water-phase drag, then:

$$\frac{\bar{R}f_0\rho_f V_{in}^2}{2g} \frac{L}{D} = \frac{f_0\rho_f}{2gD} \int_0^{L_2} V_f dL = \frac{f_0\rho_f V_{in}^2}{2gD} \int_0^L (1-\alpha)^{-2} dL$$

where f_0 = single-phase friction factor
 V_{in} = inlet velocity of coolant
 $\bar{R}$ = ratio of two-phase to one-phase friction.

For low qualities,

$$\bar{R} = 1/3[1 + 1/(1-\alpha_e) - 1/(1-\alpha_e)^2]$$

where α_e = exit steam volume fraction.

An expression for acceleration pressure drop is:

$$\Delta P_a = 1/\rho_f[(1-x)^2/(1-\alpha) + (x^2/\alpha)(\rho_f/\rho_g) - 1]G^2/g$$

Hydrostatic head in the boiling section requires fluid density relations, and is given by:

$$\Delta P_h = (\bar{\rho}/\rho_f)\rho_f L_b$$

where $\bar{\rho}$ = mean density in two-phase region

L_b = two-phase length of channel.

Relations are developed for the average-to-liquid-density ratio, for conditions of constant heat flux along core length, and of cosine heat flux along core length:

$$\bar{\rho}/\rho_f = 1 - (1/x_e) \left[1 - \frac{\rho_g}{\rho_f} \int_0^{\lambda_e} \frac{x dx}{(V_g/V_f)(\rho_g/\rho_f)[1 - (\rho_g/\rho_f)(V_g/V_f)]x} \right]$$

and,

$$\bar{\rho}/\rho_f = 1 - \left(1 - \frac{\rho_g}{\rho_f} \right) \left[\frac{b'}{b} - \frac{a'b - ab'}{ba^2 - b^2} \frac{L_t}{L_b} \left(1 - \frac{2}{\pi} \tan^{-1} \frac{\sqrt{a^2 - b^2} \tan L_0/2L_t}{a+b} \right) \right]$$

where $a^2 > b^2$

$$a = 2/\alpha_e + b$$

$$b = [1 - V_g \rho_g / V_f \rho_f] b'$$

$$a' = 2 + b'$$

$$b' = -Q_t/Q_b = -[2/(1 - \cos L_b/L_t)]$$

4.9 CLASSICAL TWO-PHASE PRESSURE DROP CALCULATIONS⁷⁴

Two-phase pressure drop is arbitrarily expressed as:

$$(\Delta P/\Delta L)_{tpf} = (\Delta P/\Delta L)_1 \phi_{itt}^2$$

where ϕ_{itt}^2 = parameter defined by the above equation, to be obtained experimentally
 $(\Delta P/\Delta L)_1$ = pressure gradient assuming only liquid phase flowing.

At the critical pressure, 3206 psia, it is shown that:

$$\phi_{itt} = \left(\frac{x_{tt} + 1}{x_{tt}} \right)^{0.875}$$

where $x_{tt} = (\bar{V}_1/\bar{V}_g)^{0.571} (\mu_g/\mu_l)^{0.143} [(1/x) - 1]$

$\bar{V}_1$ = specific volume of saturated liquid

$\bar{V}_g$ = specific volume of saturated vapor

x = weight flow-quality of steam at any point.

Curves for intermediate pressures are established by trial and error fitting to experimental data, and are shown in Fig. 9. The expression developed in Reference 54 for acceleration or momentum pressure drop, ΔP_a , applies to the two-phase flow case wherein liquid and vapor are completely separated. Employing the nomenclature of the previous expression, the equivalent expression for the flow case of complete mixture or dispersion would be written:

$$\Delta P_a = 1/\rho_f[(1-x)+x(\rho_1/\rho_g)-1]G^2/g$$

where it is understood that G commonly represents weight flow of coolant per unit area.

4.10 BASIC EQUATIONS FOR TRANSIENT COOLANT TEMPERATURE ANALYSIS¹⁶

The equations developed are:

$$T_f - \bar{T}_f(0) = (m_f c_f)^{-1} \int_0^t \pi(\tau) d\tau$$

$$\pi = p - UA(T_w - T_c)$$

$$\bar{T}_f - T_w \approx \gamma(p - \pi)$$

$$m_c C_c \frac{dT_c}{dt} = (p - \pi) - 2W_c C_c (T_c - T_i),$$

where T_f, T_c = temperatures of fuel, coolant

m_f, m_c = mass of fuel, coolant

π = power generated retained by fuel

U = overall heat transfer coefficient, fuel to sodium

p = power level of reactor

A = heat transfer area of fuel

T_w = fuel surface temperature

γ = constant

C_c = specific heat of coolant

T_i = coolant inlet temperature

4.11 PRESSURE DROP FOR ANNULAR TWO-PHASE FLOW⁸⁶

Reference 95 calculates the frictional resistance of a two-phase mixture from the drag of the liquid phase along the channel wall.

$$\Delta P_{tpf} = \frac{f\rho}{2gD} \int_0^L v_f^2 dL = \frac{f\rho v_{in}^2}{2gD} \int_0^L (1-x)^2/(1-\alpha)^2 dL$$

where the nomenclature agrees with that previously defined, and tpf refers to two-phase flow. A friction-factor multiplier is then defined:

$$\bar{R} = \Delta P_{tpf}/\Delta P_0$$

and is actually the ratio of friction drop over the entire boiling channel to the friction drop at the same flow rate without boiling. Combining and integrating the above equation yields:

$$\bar{R} = 1/3 \left[1 + \left(\frac{1-x}{1-\alpha} \right)_{\text{exit}} + \left(\frac{1-x}{1-\alpha} \right)_{\text{exit}}^2 \right]$$

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