DESTABILIZATION OF THE TRAPPED
ELECTRON MODE BY MAGNETIC
CURVATURE DRIFT RESONANCES

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ABSTRACT

Electron curvature drift resonances, ignored in earlier work on the trapped-electron modes, are found to exert a strong destabilizing influence in the lower collision frequency range of these instabilities. Effects arising from ion temperature gradients, shear, and finite ion gyroradius are included with these ∇B -drifts in the analysis, and the resultant eigenvalue equation is solved by numerical procedures rather than the commonly used perturbation techniques. For typical tokamak parameters the maximum growth rates are found to be increased over earlier estimates by roughly a factor of 4, and requirements on magnetic shear strength for stabilization are likewise more severe and very difficult to satisfy. For inverted density profiles, this new destabilizing effect is rendered ineffective, with the result that the modes can be stabilized for achievable values of shear provided the temperature gradients are not too severe. Estimates of the particle and thermal energy transport are given for both normal and inverted profiles.

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I. INTRODUCTION

The dissipative trapped-electron modes are potentially dangerous instabilities in tokamaks, and have been the subject of extensive recent studies. It has generally been concluded¹ that when the effective electron collision frequency becomes smaller than the characteristic mode frequency ($\omega \sim \omega_*$, the diamagnetic drift frequency) the mode becomes stable unless $\eta_e = d \ln T_e / d \ln n$ is negative. The purpose of this paper is to demonstrate that a driving mechanism suggested in Ref. 2, namely, electron curvature drift resonances, can destabilize the mode even in this collisionless regime provided the aspect ratio is not too large. The growth rates found under these conditions are much larger than those for the usual modes driven only by collisions and temperature gradients. The requirement on magnetic shear for stability is likewise more severe and very difficult to satisfy. We then demonstrate that for reversed density profiles, ($\eta_e < 0$) which have been shown to be very favorable from numerous points of view,³ the new destabilizing effect described above is effectively suppressed. The usual dissipative trapped-electron modes remain unstable in this lower collision frequency regime for inverted profiles. In this case, however, if the temperature gradients are not too severe, the modes can be stabilized for achievable values of shear. Finally, rough estimates of the transport coefficients for both modes of operations are given.

II. TRAPPED-ELECTRON MODES INCLUDING CURVATURE DRIFT RESONANCES

The dispersion relation for the dissipative trapped-electron instability is derived in the usual way by solving the Vlasov-Maxwell equations in the electrostatic limit. For simplicity, collisions are described by an energy dependent Krook model.¹ Recalling that the characteristic mode frequency lies between the thermal ion and electron bounce frequencies ($\omega_b^i < \omega < \omega_b^e$), approximate forms for the respective perturbed distribution functions are obtained in the limit of small ω/ω_b^e and ω_b^i/ω .⁴ Application of the quasineutrality condition then yields the basic mode equation. The essential difference with the usual dispersion relation is the inclusion of the curvature drift term of the electrons. In what follows the $\vec{v}_B$ -drift of the ions is not retained, since it does not appreciably alter the growth rate of these instabilities.

Taking into account finite ion gyroradius effects,⁵ the dispersion relation may be written as

$$Q(\omega, b) = \left[1 + \tau - \left(\tau + \frac{\omega_*}{\omega} \right) \Gamma_0 - \eta_i \frac{\omega_*}{\omega} b (\Gamma_1 - \Gamma_0) \right] \phi$$

$$= \left\langle \frac{\omega - \omega_* \left[1 + \eta_e \left(\frac{E}{T_e} - \frac{3}{2} \right) \right] \bar{\phi}}{\omega - \bar{\omega}_D(E, g) + i \hat{v}(E/T_e)^{-3/2}} \right\rangle . \quad (1)$$

Here, $E = mv^2/2$ is the particle energy, $\bar{\phi}$ denotes the bounce average of the potential along the field line given by

$\bar{\phi} = (\oint \phi dl / v_{||}) / \oint dl / v_{||}$, $\langle A \rangle$ denotes a velocity-space average of A over trapped particles, $\Gamma_n = I_n(b) \exp(-b)$ with I_n a modified Bessel function, $b = k_{\perp}^2 \rho_i^2 / 2$, $\rho_i = v_i / \Omega_i$, $v_i = (2T_i / m_i)^{1/2}$, $\eta_j = d \ln T_j / d \ln n_j$, $\omega_* = k_{\perp} T_e / e B r_n$, $r_n = -(d \ln n / dr)^{-1}$, $\tau = T_e / T_i$, $\varepsilon = r / R$, and $\hat{v} = v_e / \varepsilon$. The bounce average of the electron curvature drift frequency is given by

$$\bar{\omega}_D(E, g) = \omega_* \frac{r_n}{R} \frac{E}{T} G(\xi, g)$$

where $\xi = r d \ln q / dr$, and

$$G(\xi, g) = \left(\frac{2F(g)}{K(g)} - 1 \right) + 4\xi \left(\frac{F(g)}{K(g)} - (1 - g^2) \right),$$

with F and K designating complete elliptic integrals of the first and second kind respectively, $g = \sin(\theta_0/2)$, and θ_0 is the azimuthal coordinate of the turning point of a given trapped particle.

To arrive at the preceding dispersion relation, the local approximation is invoked, which involves assuming that the modes are sufficiently radially localized near rational surfaces that stabilization by Landau damping is negligible. It can be further simplified by making the usual approximation $\phi = \bar{\phi}$, i.e., by neglecting ballooning effects along the field lines. This approximation still gives the growth rate to within a factor 2 of the exact solution.⁶ In Ref. 7 it is shown that $G(\xi, g)$ depends

very weakly on g especially in the presence of shear ($\xi \sim 1$), and it is also shown that for $\xi \sim 1$ the curvature is bad almost everywhere. This fact allows us to further simplify the dispersion relation by replacing G with a constant. The energy average on the right hand side of Eq. (1) then reduces approximately to

$$\langle A \rangle = \frac{2(2\varepsilon)^{1/2}}{\pi^{1/2} T^{3/2}} \int_0^\infty dE E^{1/2} \exp(-E/T_e) A .$$

In order to estimate the effect of the curvature drift resonance in Eq. (1), we first consider the collisionless limit $\nu = 0$. The energy integral can be readily performed with the imaginary part reducing to

$$\frac{\gamma}{\omega_*} = 2\pi^{1/2} \eta_e (2\varepsilon)^{1/2} \int_0^\infty dx x_0 x^{1/2} e^{-x} (x - \frac{3}{2}) \delta(x - x_0)$$

where $x = E/T_e$ and $x_0 = R/r_n G$. Following the usual perturbative approach, we have taken $\omega \approx \omega_*$ to lowest order. This leads to a growth rate that scales as

$$\frac{\gamma}{\omega_*} \approx 2 \left(\frac{2\pi r}{R} \right)^{1/2} \eta_e \left(\frac{R}{r_n G} \right)^{3/2} \left(\frac{R}{r_n G} - \frac{3}{2} \right) \exp \left(- \frac{R}{r_n G} \right) . \quad (2)$$

Taking $r/R = r_n/R = 1/4$ and $G = 1.2$ (a rough average value) gives $\gamma \approx \omega_*$. For such a large growth rate, the perturbation

theory obviously breaks down. Hence, for an accurate estimate of the destabilizing influence of the drift resonances, it becomes necessary to numerically solve the dispersion relation given in Eq. (1). As noted earlier, the curvature function $G(\xi, g)$ can be taken as a constant with respect to g ; i.e., $G \approx 1.2$ for $\xi \sim 1$. Checking this approximation for a few individual cases, we found that the results obtained with the approximate treatment were in agreement to within 10% of the exact values.

Taking $G = \text{constant}$, and considering the limit $\nu = 0$, the energy integral in Eq. (1) can be performed analytically,⁸ and the results expressed in terms of the plasma dispersion function. As a further check on our numerical procedures, the answers obtained from solving this equation were found to be in excellent agreement with our numerical results obtained from Eq. (1) in the collisionless limit ($\nu \rightarrow 0$). At high collision frequencies, where the $\vec{v}B$ drift resonances become ineffective, our results from Eq. (1) correspond very well with previously calculated results.¹

In Fig. 1 the effect of the $\vec{v}B$ resonances on the growth rate of the trapped-electron mode is illustrated. Here, it is seen that there is an increase of γ as ν decreases, with the mode eventually reaching its maximum growth rate in the region where previous calculations, ignoring such resonances, would have predicted stability. Since finite ion gyroradius effects are included in the analysis, the usual normalization of γ in

terms of ω_* is rather misleading because of the dependence of ω_* on k_1 . A more appropriate choice is to plot γ/ω_{be}^t versus the banana-regime parameter $\nu_* = \hat{\nu}/\omega_{be}^t$ with $\omega_{be}^t = \epsilon^{1/2} v_e/qR$. For each value of ν_* , we determined the value of b that gives rise to the maximum growth rate, and then plotted this growth rate as a function of ν_* . The corresponding values of b are indicated on Fig. 1 for the case $r_n/R = \epsilon = 1/4$, $\tau = 1$, $q = 2.5$, and $\eta_e = 1$ for a deuterium plasma. In Fig. 1, the dotted curve shows the stabilizing influence of ion temperature gradients. This effect can be explained in terms of the shift of the real part of the frequency. Assuming that the perturbation techniques at least indicate the basic tendency of a particular effect to exert either a stabilizing or destabilizing influence, one can readily see by examining the lowest order dispersion relation, i.e.,

$$1 + \tau - \left(\tau + \frac{\omega_*}{\omega} \right) \Gamma_0 = - \eta_i \frac{\omega_*}{\omega} b(\Gamma_0 - \Gamma_1) ,$$

that the inclusion of η_i tends to decrease the real part of the frequency;

$$\frac{\omega}{\omega_*} = \frac{\Gamma_0 - \eta_i b(\Gamma_0 - \Gamma_1)}{1 + \tau(1 - \Gamma_0)} .$$

It would at first appear from Eq. (1) that by reducing ω , the growth rate would also increase because an increased number of lower energy particles can now resonate with the waves. However,

the dominant effect is that this downward shift of the resonant energy also increases the weight of the negative (stabilizing) contribution of the integrand.

In Fig. 2, we have shown the range of unstable perpendicular wave numbers for two typical values of v_* . The fact that these are reasonably broad indicates that a quasilinear type theory should be applicable in estimating transport from these instabilities.

We have also considered the dependence of the resonant growth rate on the density-gradient scale length. Results are shown in Fig. 3 for two representative values of the aspect ratio. Here, the growth rates were evaluated in the $v = 0$ limit of Eq. (1). It can be seen that they are weak functions of r_n , especially when the aspect ratio is not too small. The sharp drop of γ when R/r_n becomes smaller than 3 (with $R/r = 4$) is due to the fact that the resonant energy becomes smaller than $3/2$ so that the term proportional to ∇^2 becomes stabilizing. The weak dependence of the growth rate on R/r_n is primarily due to the variation of the real part of the frequency. When r_n is decreased, the real part of the frequency also decreases, thus yielding a nearly constant resonant energy over a wide range of r_n .

III. SHEAR CRITERIA

In the preceding section, we have shown that the inclusion of curvature drift resonances gives rise to growth rates that are

much larger than those predicted in earlier work. Hence, the shear criteria for marginal stability must also be modified. In treating this problem, we adopt the usual approach of neglecting the ballooning effects along the field lines,¹ and follow the procedure of Pearlstein and Berk⁹ in treating the radial mode equation. We obtain the following equation

$$\begin{aligned} \frac{\rho_i^2}{2} \frac{\partial^2 \phi}{\partial r^2} + \left\{ -\xi Z \left\{ \left[1 - \frac{\omega_{*i}}{\omega} \left(1 - \frac{\eta_i}{2} \right) \right] \frac{d\Gamma_o}{db} - \eta_i \frac{\omega_{*i}}{\omega} \frac{d}{db} b (\Gamma_1 - \Gamma_o) \right\} \right. \\ \left. + \eta_i \frac{\omega \omega_{*i}}{k_{||}^2 v_i^2} \frac{d\Gamma_o}{db} (1 + \xi Z) \right\}^{-1} \left\{ \left[1 + \xi Z \left\{ \left[1 - \frac{\omega_{*i}}{\omega} \left(1 - \frac{\eta_i}{2} \right) \right] \Gamma_o \right. \right. \right. \\ \left. \left. - \eta_i \frac{\omega_{*i}}{\omega} b (\Gamma_1 - \Gamma_o) \right\} - \eta_i \frac{\omega_{*i} \omega}{k_{||}^2 v_i^2} \Gamma_o (1 + \xi Z) + \frac{1}{\tau} (1 - \langle E \rangle) \right] \right\} \phi = 0. \quad (3) \end{aligned}$$

Here, $Z \equiv Z(\xi)$ is the plasma dispersion function, $\xi = \omega/k_{||} v_i$, and $\langle E \rangle$ represents the energy integrals of Eq. (1).

In the vicinity of a rational surface, $k_{||} \sim 0$ implies $|\xi| \gg 1$ and allows one to expand the Z function. After some tedious but straightforward calculations, one can cast the result into the form of a Weber equation,¹

$$A \frac{\partial^2 \phi}{\partial x^2} + (B - Cx^2) \phi = 0 \quad (4)$$

where x is the radial distance from the rational surface, and where A , B , C are given by

$$A = - \frac{\rho_i^2}{2} \left[\left(1 - \frac{\omega_{*i}}{\omega} \right) (\Gamma_0 - \Gamma_1) - \eta_i \frac{\omega_{*i}}{\omega} (\Gamma_0 + 2b(\Gamma_1 - \Gamma_0)) \right] \quad (5)$$

$$B = 1 - \left(1 - \frac{\omega_{*i}}{\omega} \right) \Gamma_0 + \frac{1}{\tau} (1 - \langle E \rangle) + \eta_i \frac{\omega_{*i}}{\omega} b(\Gamma_1 - \Gamma_0) \quad (6)$$

$$C = \frac{1}{2} \left(\frac{k_y v_i}{\omega L_s} \right)^2 \left[\left(1 - \frac{\omega_{*i}}{\omega} \right) \Gamma_0 - \eta_i \left(\frac{\omega_{*i}}{\omega} \right) (\Gamma_0 + b(\Gamma_1 - \Gamma_0)) \right] \quad (7)$$

where $L_s = |(\epsilon/q) d \ln q / dr|^{-1}$. The general solution of Eq. (4) is

$$\phi = H_n(\sigma^{1/2} x) \exp(-\sigma x^2/2) , \quad (8)$$

where

$$\sigma = \pm (C/A)^{1/2} , \quad (9)$$

$$A\sigma(2n + 1) = B , \quad (10)$$

and the appropriate sign for σ is determined by invoking outgoing wave boundary conditions.⁹ Comparing the solution to the eikonal form gives

$$\hat{\phi} = \exp(-\sigma x^2/2 - i\omega t) = \exp(i \int k_x dx - i\omega t) ,$$

which implies

$$ik_x = -\sigma x = \pm (C/A)^{1/2} x ,$$

or from Eqs. (5) and (7),

$$k_x = \mp \frac{k_{\theta} v_{ix}}{\omega L_s \rho_i} \left(\frac{\left(1 - \frac{\omega_{*i}}{\omega}\right)(\Gamma_o - \Gamma_1) - \eta_i \frac{\omega_{*i}}{\omega} [\Gamma_o + 2b(\Gamma_1 - \Gamma_o)]}{\left(1 - \frac{\omega_{*i}}{\omega}\right)\Gamma_o - \eta_i \frac{\omega_{*i}}{\omega} [\Gamma_o + b(\Gamma_1 - \Gamma_o)]} \right)^{-1/2} . \quad (11)$$

For positive values of η_i the term under the square root is always positive, so that the sign of the group velocity is determined by the factor before the parenthesis. This in turn determines the sign of σ . The shear criteria is then obtained by combining Eqs. (9) and (10) and setting $n = 0$ (corresponding to the lowest eigenvalue); i.e.,

$$(-AC)^{1/2} = B . \quad (12)$$

To derive the shear criterion, one has to solve this equation for marginal stability. It then reduces to a system of two uncoupled equations that may be written in the following schematic way:

$$\frac{\omega}{\omega_*} [1 + (1 - \Gamma_o)\tau] - \Gamma_o - \eta_i b(\Gamma_1 - \Gamma_o) = \frac{\omega}{\omega_*} \langle \text{Re}(\omega, b) \rangle \quad (13)$$

$$\Gamma_o (AC)^{1/2} = \frac{\omega_*}{\omega} \left(\frac{L_n}{L_s} \right) \langle \text{Im}(\omega, b) \rangle \quad (14)$$

where $\langle \text{Re}(\omega, b) \rangle$ and $\langle \text{Im}(\omega, b) \rangle$ refer to the real and imaginary part of $\langle E \rangle$ respectively in Eq. (6).

In the usual perturbation theory, Eq. (13) is automatically satisfied once ω is set equal to the value obtained by solving the dispersion relation to the lowest order (i.e., $\omega \approx \omega_*$ if one neglects ion gyroradius effects and sets $\eta_i = 0$). The shear criteria is then calculated by inserting the perturbatively computed growth rate into Eq. (14). Because of the large shift in the real part of the frequency, which occurs when γ is large, this procedure is inapplicable and Eqs. (13) and (14) must be solved numerically.

Typical results are presented in Fig. 4. Here again, we recover the usual values of shear necessary for stabilization in the high collision frequency limit, while in the low collision range the shear length, required for marginal stability, is smaller by nearly one order of magnitude. These values of shear are clearly not achievable in tokamak experiments. In the following sections we address the problems of: (i) Suppressing the dangerous resonant effects; and (ii) estimating the particle and energy transport which would result if they cannot be suppressed:

IV. INVERTED PROFILES

To avoid the resonances considered, it is necessary that $\bar{\omega}_D \omega < 0$. Since ω has the sign of ω_* , this can be effected by creating a situation where the density profile is inverted. This

reverses the sign of ω_* but leaves ω_D unchanged. As noted earlier, the average curvature seen by trapped particles (for $\xi \sim 1$) is "bad" for almost all such particles so that the destabilizing resonance is now effectively eliminated.

For the reversed gradient situation, maximum growth rates, as defined in Sec. II, are plotted in Fig. 5. Now the contribution of η_i ($\eta_i < 0$) is destabilizing. The corresponding values of critical shear are plotted in a similar way in Fig. 6. Both figures show that as long as $|\eta_e|$ remains of the order of unity the growth rate is reduced compared with normal profiles, and shear stabilization may be obtained. When $|\eta_e|$ becomes of the order of 2 or larger, the required amount of shear again becomes difficult to obtain. This situation is likely to occur because some inverted profile configurations may involve flatter density profiles than the normal case.³

Finally, we would also like to emphasize that for negative values of η_i , the shear stabilization can be inhibited in some range of b if $|\eta_i|$ becomes larger than 2. This is because the product of AC, defined by Eqs. (5) and (7), can become positive thus yielding an imaginary value for k_x . The wave is then evanescent around the rational surface, with the corresponding wave energy remaining confined in a very narrow radial range. However, numerical computations have shown that for $|\eta_i| < 2$ this never occurs as long as v_* remains larger than 5×10^{-3} , i.e., in the relevant regimes of interest.

V. PARTICLE AND ENERGY FLUX DUE TO TRAPPED-ELECTRON MODES

In this section we compute the mean particle flux and the mean energy flux across a magnetic surface ψ . This is done, as usual, by averaging over time the components of the equation of motion of the electrons in the magnetic surface, i.e., by computing (for the particle flux),

$$(nv)_{\psi} = \left[\left(\frac{\vec{E} \times \vec{B}}{B^2} \right)_{\psi} \delta n_j^* \right],$$

where δn_j is the perturbed density response of the trapped electrons and the square brackets denote a spatial average over a magnetic surface.

For a perturbation of the form $\tilde{\phi} = \phi \exp i(S - \omega t)$ where $S = \ell(\zeta - q\theta) + \int k_{\psi} d\psi$, the electric field is given by $\vec{E} = -\vec{\nabla}\tilde{\phi} = -i\phi\vec{\nabla}S$ with

$$\vec{\nabla}S = \frac{\ell}{R} \vec{e}_{\zeta} - \frac{\ell q}{r} \vec{e}_{\theta} + k_{\psi} |\vec{\nabla}\psi| \vec{e}_{\psi}.$$

Writing $\vec{B}$ as $\vec{B} = B_{\zeta} \vec{e}_{\zeta} + B_{\theta} \vec{e}_{\theta}$ gives

$$\left(\frac{\vec{E} \times \vec{B}}{B^2} \right)_{\psi} = i \frac{\ell \phi}{RB_{\theta}} = i \frac{\ell q \phi}{r B_{\zeta}}.$$

We know from the linear theory that the perturbed density of the electrons is given by

$$\tilde{\delta n}_e = \frac{en_o}{T} \tilde{\phi} + (2\varepsilon)^{1/2} \left\langle \frac{\omega - \omega_* \left[1 + \eta_e \left(\frac{E}{T} - \frac{3}{2} \right) \right] \bar{\phi}}{\omega - \bar{\omega}_D(E, g) + i\hat{v}(E/T)^{3/2}} \right\rangle$$

where notations are the same as in Sec. I. The particle flux is then given by

$$\sum_{\ell} \frac{i\ell\phi}{RB_{\zeta}} \tilde{\delta n}_e^*$$

In the summation over ℓ only the terms quadratic in ℓ contribute, i.e., assuming ω has the sign of ω_* , only the imaginary part of $\tilde{\delta n}_e^*$ contributes. The particle flux becomes

$$(nv)_{\psi} = (2\varepsilon)^{1/2} \frac{\ell\phi}{RB_{\zeta}} \text{Im} \left\langle \frac{\omega - \omega_* \left[1 + \eta_e \left(\frac{E}{T_e} - \frac{3}{2} \right) \right] \bar{\phi}}{\omega - \bar{\omega}_D(E, g) + i\hat{v}(E/T)^{-3/2}} \right\rangle \quad (15)$$

The approximations made in the linear analysis, i.e., $\phi = \bar{\phi} = \text{constant}$ and $G = \text{constant}$, allow us to ignore the θ average within the ψ surface. Equation (15) now takes the form

$$\Gamma = (nv)_{\psi} =$$

$$\frac{n_o e |\phi|^2}{T_e B \rho_i} (2b)^{1/2} (2\varepsilon)^{1/2} \text{Im} \int \frac{\left\{ \omega - \omega_* \left[1 + \eta_e \left(\frac{E}{T_e} - \frac{3}{2} \right) \right] \right\} F\left(\frac{E}{T_e}\right) \left(\frac{E}{T_e}\right)^{1/2} d\left(\frac{E}{T_e}\right)}{\omega - \bar{\omega}_D(E/T_e) + i\hat{v}(E/T_e)^{-3/2}}$$

(16)

The expression for the energy flux is obtained in the same way after multiplying the fluctuating density by $\frac{1}{2} m v^2$. This gives

$$Q = \frac{n_o e |\phi|^2}{B \rho_i} (2b)^{1/2} (2\epsilon)^{1/2} \text{Im} \int \frac{\left\{ \omega - \omega_* \left[1 + \eta_e \left(\frac{E}{T_e} - \frac{3}{2} \right) \right] \right\} \left(\frac{E}{T_e} \right)^{3/2} F \left(\frac{E}{T_e} \right) d \left(\frac{E}{T_e} \right)}{\omega - \bar{\omega}_D(E/T_e) + i \hat{v}(E/T_e)^{3/2}} \quad (17)$$

Hence, the ratio of the particle to energy flux is simply the ratio of the integrals in Eqs. (16) and (17), and does not require an estimate of the saturation amplitude of $|\phi|$.

In calculating the absolute value of the particle and energy flux, we invoke the usual condition¹⁰ that the mode saturates when the gradient of the density fluctuation ($\delta n_e \sim e n_o \phi / T_e$) reaches the level of the equilibrium gradient; i.e., $k_{\perp} e |\phi| n_o / T_e \sim dn_o / dr$ or $e |\phi| / T_e \sim 1 / k_{\perp} r_n$ where $k_{\perp}^2 = k_x^2 + k_{\theta}^2$. From the radial analysis, described earlier in this paper, we have $k_x = -i \sigma x$ with $\phi \sim \exp(-\sigma x^2 / 2)$. Since σ is imaginary, the mode varies approximately as $\sin(-i \sigma x^2 / 2)$ and passes between its largest maximum and minimum values over a distance determined by $-i \sigma x^2 = \pi$. This in turn implies $x = \pi / k_x$, so that Eq. (11) can be expressed as

$$k_{x \rho_i}^2 = \frac{2\pi}{\tau} \frac{r_n}{L_s} \frac{\omega_*}{\omega} H^{-1/2} \quad (20)$$

with H defined as the argument of the square root in Eq. (11).

The particle and energy flux are now given by

$$\begin{pmatrix} \Gamma \\ Q \end{pmatrix} = \begin{pmatrix} \hat{\Gamma}(b) \\ T_e \hat{Q}(b) \end{pmatrix} \frac{\rho_i}{r_n} D_B \frac{\partial n}{\partial r} \quad (21)$$

where $D_B = T_e / eB$, and

$$\begin{pmatrix} \hat{\Gamma}(b) \\ \hat{Q}(b) \end{pmatrix} = \frac{(2b)^{1/2} (2\epsilon)^{1/2}}{k_x^2 \rho_i^2 + 2b} \begin{pmatrix} I_\Gamma(b) \\ I_Q(b) \end{pmatrix}, \quad (22)$$

and $I_\Gamma(b)$ and $I_Q(b)$ are the integrals in Eqs. (16) and (17) respectively. We have numerically evaluated these integrals for normal and inverted profiles, and have taken $r_n / L_s \approx 0.1$ in specifying k_x in Eq. (20). The values for $\hat{\Gamma}$ and $\hat{Q}$, displayed on Figs. 7 and 8, were obtained after maximizing the expressions as a function of b over a relevant range of the collisionality parameter ν_* .

In the case of normal profiles, we recover the usual result that the thermal flux is larger than the particle flux. As shown on Fig. 7, the ratio of these fluxes is between 3 and 3.5, and the transport level is found to be rather insensitive to changes in collisionality. For inverted profiles, the situation is somewhat more complex. First of all, the energy flux is smaller than the particle flux. This is due to the fact that, for the

inverted profile, the mode is unstable when the negative part of the integrand in the bracket in Eq. (1) is dominant. Multiplying it by E , when computing the energy flux, increases the weight of the positive part, and thus gives a smaller net result. The sign of the computed quantities is such that both the particle and thermal fluxes are inwardly directed. As shown on Fig. 8, in the range of higher collisionality, $\nu_* \gtrsim 0.5$, the transport is effectively suppressed since both b and $k_x^2 \rho_i^2$ are quite large. Comparison with the magnitude of the flux given in Fig. 7 indicates that for $\nu_* < 0.5$ the particle flux here is close to that for the normal gradient case, while the thermal flux is reduced by about a factor of 6. Using these results in Eq. (21) and recalling that the neoclassical diffusion estimate in the plateau regime is approximately $q \rho_e D_B / R$, we find that for the parameters considered, the energy diffusion caused by trapped-electron modes is greater than the neoclassical plateau by about $(m_i/m_e)^{1/2}$ for a normal profile ($\eta_e = \eta_i = 1$) and by roughly $(m_i/m_e)^{1/2}/6$ for an inverted profile ($\eta_e = \eta_i = -2$).

VI. CONCLUSIONS

We have shown in this paper that, for normal profiles, curvature drift resonances can give rise to much larger growth rates than those obtained without this effect. More specifically, the mode is unstable in the lower collision frequency range, where it was previously thought to be stable. As a consequence, our analysis predicts enhanced anomalous transport in the lower collision frequency range of the trapped-electron modes. This effect is found to be quite important for relevant tokamak operating conditions. Inverted density profiles effectively suppress the destabilizing $\vec{v}_B$ drift resonances, and lead to a considerable reduction in the energy transport. They also result in an inward flux of both energy and particles in the region of density-gradient inversion, which may prove to be beneficial for energy confinement.

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Figure 1

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