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AUTOMATIC LENS DESIGN BY STATISTICAL ANALYSIS*

by

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LECTURE

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DISCLAIMER

The title of this lecture may have startled you. You may have asked yourselves, "Why look for new lens designing techniques when the classical methods produce such good results?" The answer is that classical methods are not producing enough new lenses to meet today's needs. I use myself as an example. At the Los Alamos Scientific Laboratory I am in charge of a small group of engineers and technicians whose primary job is to develop high-speed cameras. For some years I have been having difficulty obtaining the lenses I need. Lenses to meet our performance specifications can be obtained only after great loss of time and at great expense. We have not been able to hire a classical lens designer to solve our problems because of the unglamorous nature of our needs, the small number of new designs needed, and the limited application of the resulting lenses, no matter how well they perform in our cameras.

The obvious solution seemed to be for me to learn to design lenses. But a brief study of the classical methods convinced me that I would need a lifetime of study to master the elaborate approximations developed for designing by hand computation. I am a mechanical engineer, not a physicist. If I were to design lenses, I would have to use a different approach. The electronic computer seemed to offer the answer. Because of the computer's speed, it does not need to use the short cuts that only a trained designer knows. Instead of using simplifying approximations, the computer can trace skew rays rapidly and precisely. Bundles of rays can be used to evaluate lens performance statistically. As you know, the technique for determining the minima of complicated functions is not new. Effective methods date back to Newton, Euler, Gauss, Cauchy, and others. My mathematical problems were solved for me by a mathematician in our theoretical division, John C. Holladay. He quickly understood my needs and prepared the first edition of the program for the IBM 704. When he subsequently enlarged the program he was assisted by Charles A. Lehman, who recoded it for the IBM 7090. Although the designing section of this program has mathematical approximations which limit its accuracy, the code has produced amazingly good solutions to many lens problems. The need for a more precise mathematical analysis will be indicated briefly at the conclusion of this talk.

As I said earlier, the ability of the computer to calculate millions or even billions of times faster than hand calculation, makes it possible to use methods which previously were impractical. A wide variety of problems can now be solved by means of statistical analysis, lens designing being one of these problems, and one particularly suited to statistical analysis. I shall now go on to explain what I mean.

First there is the problem of evaluating lens performance. For this evaluation the statistical element chosen is the path of a ray, which is the probe for determining the behavior of the lens under investigation. Each ray is traced from an object point, through a specified region of the lens, and on to the image. The traced ray samples the focusing ability of the part of the lens through which it passes. In a perfect lens system, the ray will intersect its specified image point. All other rays passing through the lens from this object point will also intersect the image, with a perfect geometrical image as the result. Rays traced from other object points will also intersect their specified image points without any deviation whatsoever.

However, lenses of practical interest are far from perfect. In an imperfect lens each of the traced rays will usually deviate from its specified image point. The smaller the deviation, the better the lens performance. Conversely, large deviations of the rays indicate poor performance. Many lens designs have one or more defects large enough to make them objectionable for particular uses. These defects are especially noticeable when the lenses are used under conditions differing greatly from those for which they were designed. Whatever the causes of the image defects, their extent can be evaluated as accurately as is desired by tracing an appropriate number of rays from the object points to the focal plane of interest. As the rays intersect the plane in a spot-like pattern, the various classes of image defects can be evaluated merely by observing the ways in which the rays deviate from their specified image points. The ray deviation is measured from the ray intersection on the focal plane of interest, which is perpendicular to the optical axis.

Each spot-like pattern, or image spot, is made of the ray intersections, which may be treated collectively to form a useful assessment of lens performance. The main reference point in the image spot is the centroid, which is the mean position of the ray intersections. The size and position of the image spot are the most important considerations. The size of the image spot is defined as the rms radius of the ray intersections from the spot centroid. The position of the image spot is defined as the position of its centroid. By evaluating the size and position of the image spot we obtain the information necessary for assessing overall lens performance. The resolution is defined as the rms radius of the spot. And experience has shown that this estimate of resolution agrees well with visual resolution as measured on the optical bench. The effect of longitudinal chromatism is automatically included in the rms spot radius calculation. The effect of lateral chromatism is given by the lateral variation of the centroid positions of the colored spots in each image. Distortion is the deviation of the image centroid position from the position required by rectilinear geometry. This distortion deviation could also, if desired, be expressed in terms of the percent deviation of the lateral magnification from the central magnification.

In the assessment of lens performance preliminary to designing by machine, the image-spot size and position characteristics just described are the most important factors. However, several useful accessory image-spot size and position characteristics can also be obtained by observing these evaluations on a series of equally spaced accessory focal planes adjacent to the focal plane of interest. The longitudinal chromatism can be obtained by noting the longitudinal spacing of the focal planes which give the smallest spot sizes for each of the colored images. This method of evaluating longitudinal chromatism is useful both for the axial and the lateral images. From these same evaluations the astigmatism can also be obtained because the rms-x and the rms-y values are tabulated along with the rms-spot-radius values. In this way the field-curvature characteristics for the astigmatic and the mean foci can be found for each color. The ray-intersection coordinates on the focal plane of interest are also available for making spot diagrams. Other optical characteristics of the lens can be evaluated by inventing the appropriate technique for obtaining the information by this general-purpose ray-tracing program.

To summarize what I have just said, I will repeat that before designing lenses by machine each image is examined for three main characteristic defects: the mean deviation of the rays in the spot, the distortion deviation, and the lateral chromatic deviation. When these defects have been evaluated, designing can begin. The aim of designing is to reduce the sizes of the defects. The larger the defect, the harder the program will work to reduce its size. In this program the apparent size of the defect is controlled by weight--a numerical coefficient assigned to the defect to express its relative importance in the problem. Particular aspects of lens performance can be emphasized by making the corresponding defects appear larger by using a large weight. Although a defect given a weight of zero will not be minimized by the program, even this defect may change in size as a result of lens changes which minimize other defects that are being worked on. Each of the three characteristic image defects just mentioned may be assigned any desired weight. The three characteristic defects of each image for the 7 images possible with this program give a maximum of 21 individual weights available for assignment. The sum of all the weights used can be considered as a unit for estimating the effect of the individual weights.

The next step, after assigning the weights in order to emphasize the desired performance characteristics, is to use the designing routine to reduce the image defects. These are reduced by changing the lens parameters so that the bundles of rays intersect the image surface closer to the desired positions than they did before and in patterns of smaller size. This sharpening of focus is achieved by means of an iterative least-squares system. (Least squares, as you doubtless know, is a method of fitting a curve to a set of points in such a way as to minimize the sum of the squares of the distances of the points from the curve.) In the system we use, the lens parameters represent the parameters of the equations which determine the coordinates of the ray intersections forming the image spots. The points to be fitted are the desired image points, which are specified by position on the focal plane of interest. The fit of each ray intersection is measured by comparing the position of the ray intersection with the specified position. As is usual with least squares, the problem is simplified by reducing it to a linear approximation, which is then solved by a linear matrix. Because the result is a linear solution of a nonlinear function, the parameter changes must be restricted to a region of unknown size in the vicinity of the initial prescription. The upper limit on the parameter change is determined by a number selected arbitrarily as a coefficient for the parameter increment which was used to generate the solution. Numbers in the range of 1 to 100 have been used successfully. However, as a general rule, if the parameter change is small, the design progress will be slow. If the change is large, there may be no progress because of violent oscillation. The result may even be that the progress will be reversed.

I have described how lens performance is evaluated statistically, how weighting is used to emphasize particular characteristics of lens performance, and how a least-squares system is used to change selected lens parameters and thus minimize image defects. I will now give the main details of the procedure we follow in order to make this iterative least-squares designing system work effectively. The principal lens parameters are the radii of curvature of the surfaces, the thicknesses of the optical media, the positions of the entrance and exit pupils, the eccentricities which determine the conic sections of the refracting surfaces, and the refractive indexes of the optical media. All of these parameters can be made into variables by incrementing them by a small amount. The designing program then calculates a new value and by this means improves lens performance. When the effects of incrementing two parameters are calculated simultaneously, the resulting interaction improves the precision of calculation and speeds design progress. Up to a certain point, the more parameters calculated simultaneously, the faster the design progress. It has been our experience, however, that ten is the maximum number of parameter changes which can be calculated simultaneously and still give useful answers. The parameter-increment storage therefore has been limited to ten. I emphasize this point because people who have tried using our program have had a tendency at first to make too many changes at once, with disappointing results. We usually start with single increments and then advance to doubles, triples, etc., after each type has ceased to improve the design. Our method is to use an iterative sequencing procedure to work with all possible parameter changes as often as possible. The sequencing procedure consists of a scheme for obtaining different sets of parameters which are used in succession when the total number of parameter changes exceeds the number used simultaneously. The sequencing procedure is described in detail in the published reports and will require some study to understand.

The program can store instructions for changing as many as 49 independent parameters. Sets of as many as 6 dependent parameters may also be changed as a group. For each set of dependent parameters the individual alterations must be simultaneous and identical in magnitude but may be in any arbitrary plus or minus combination. Dependent parameters are very useful when one is working with symmetrical lenses or with lens systems which have a fixed-length requirement.

The versatility of this program can be extended by using the substitution routine which makes it possible to design a lens simultaneously for as many as five conditions of use, a zoom lens, for example. For each of these additional conditions of use, the program has storage for 100 substitute parameters. The substitutions about which I am talking here include all the lens data--the alphabetic problem identification, the weights, the prescription, and all the associated control numbers required for the proper functioning of the program.

Next to be described are the ray patterns, which generate the ray bundles used to evaluate lens performance. Each ray of the pattern must be specified. By specifying the x and y coordinates in a unit-radius entrance pupil, the ray is generated. But before the ray is traced, the unit-pupil dimensions are scaled by the program to the actual pupil dimensions. Although the rays used to sample the entrance pupil are usually generated in a grid pattern, as many as 25 rays may be positioned individually in any pattern desired. A maximum of 100 rays can be traced from each object point for each color. Because rays traced through a centered optical system are symmetrical, it is necessary to trace only half the rays of symmetrical ray patterns. The ray intersections of the other half may be obtained by a sign change of the x coordinate. I have devised a series of ray patterns with from 2 to 100 rays which has proved convenient both for designing lenses and for testing the performance of already designed lenses. For the preliminary design stage, the 2-ray pattern is especially useful because it is economical of machine time and still evaluates the major lens defects for most design problems. For the intermediate designing sequences, the 6-ray pattern is usually adequate. During the final design stages it may be possible to obtain considerable improvement by using the 12-ray or 26-ray patterns. For testing the performance of an already designed lens, the many-ray patterns are useful--especially when there is a widely variable vignetting of the beams. In this case the aperture radius or the central obstruction radius may be specified for each optical surface.

Because the general procedure for using this program is versatile, the problem may be run by the lens designer or by the IBM-7090 machine operator. By means of data and instructions read from a series of punched cards, a complete sequence of calculations can be scheduled; and each item of data can be located as needed since it has been addressed to a memory cell specifically reserved for it by the 213-card code. One frequently effective sequence of parameter sets for designing starts with the curvatures; follows with the curvatures combined with some air spaces; continues with the curvatures and all the air spaces; and concludes with the curvatures, the air spaces, and the glass thicknesses. Any parameters which tend to create undesirable conditions are fixed at tolerable values, and designing is continued with the remaining parameters. Sometimes these fixed parameters may profitably be made variable again at a later design stage. When starting with a crude prescription, one usually designs with only one or two incremented parameters per cycle. As the design is stabilized, the number of parameters per cycle is increased until the design progress becomes erratic because of the program's inability to evaluate precisely the too-complex problem.

In this program, the control of boundary conditions has not been automated. A boundary is a data-field limit beyond which satisfactory calculation is not obtained. When the program tries to use any data outside its established boundary, a boundary violation occurs. Boundary violations are considered for both lens calculation and machine operation. There are several machine-discovered boundary violation indicators which help the designer to locate the error, which is then manually corrected. These indicators are printed statements giving the type and location of the first violation found. If the violation occurs during a design cycle, the program will try to restore the previous violation-free prescription and if successful will proceed to the next cycle. If the program is unable to restore a violation-free prescription, the machine stops automatically after printing the complete lens data. A lens parameter generally may be fixed at or near its boundary.

The boundary condition most frequently violated is the minimum thickness between optical surfaces, which is automatically zero until the designer assigns another value. Another violation, the loss of rays during a design cycle, is intolerable because the matrix solution loses direction. (The ray loss may be a result of total reflection or because of missing the surface.) The program requires a performance improvement at the end of each design cycle, when the merit number is calculated and compared with the previous value. The linear system solver, to function properly, must have determinants within specified ranges. An error in the prescription or designing data is an obvious boundary violation. Missing punches or extra punches in the cards being read are immediately recognized as violations of the machine's card-reading program. Finally, it should be mentioned that there is always the possibility of a problem too large for the machine to solve. The first calculation made by the program is always to estimate the problem size. If the problem should be too large, the program would immediately report, "PROBLEM TOO BIG FOR MACHINE." But so far this situation has not occured at Los Alamos.

To illustrate the way our IBM-7090 program is used to solve lens design problems, I will briefly describe the steps taken to design a 600-mm, f/2.0 lens which was to cover a 24° field of view with the highest resolution and the lowest distortion possible. At the start it was clear that a lens of this size, achromatized for the C-F range, would have its white-light resolution limited by the secondary spectrum. Because the lens was intended for a ballistic camera, however, the spectrum could be limited by a red filter to the C-d range. I decided to let the machine find the most suitable design.

The first step in making a fresh design was to select the glass types to be used. Since I wanted to limit the choice to high-quality objective-grade blanks which would be readily available in the large sizes needed, I chose borosilicate crown, 517645, and dense flint, 617366. Next, in a series of monochromatic design runs, I assigned the estimated average index of refraction, 1.55, to all elements because I did not know which elements would be made of crown glass and which of flint. Because previous work had indicated that eight lens elements were enough, I confined myself to this number. Similarly I kept the lens length the same as that of an existing design so that the new lens could be adapted without difficulty to the ballistic camera.

The design was started with eight sheets of 35-mm thick, 1.55-index glass, spaced 70-mm apart, with the entrance pupil 150 mm to the right of the first surface. Two-ray-trace monochromatic designing was started on the 16 surfaces only, with the focal plane of interest held 70-mm behind the last surface. After the initial merit number, 21,450, had been reduced to 2.2, the ray pattern was enlarged successively to 6, 12, and 26 rays, to yield a final merit number of 8.0. The average rms spot-size radius was 0.037 mm, a very satisfactory value for the initial design, which is shown in the top illustration.

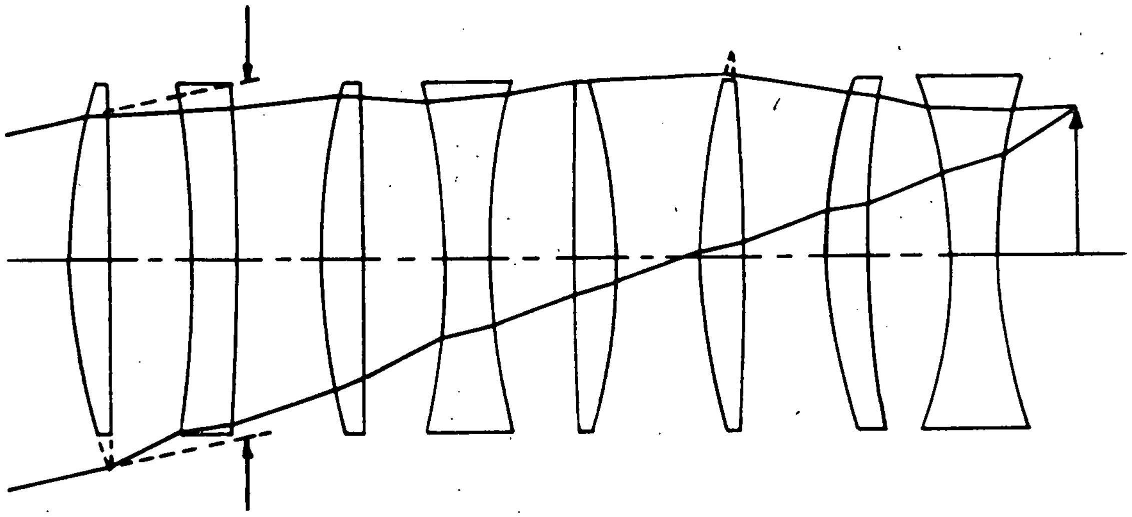
The next step was to achromatize the monochromatic design by substituting crown and flint glasses, respectively, for the glasses in the positive and negative elements. Continued designing with a 6-ray pattern, using the C- and d-light indexes of refraction, adjusted the prescription to the substitute glasses. The thicknesses were adjusted manually to make the elements practical for construction. Since the resulting design had large lateral chromatism at the field edge, the glass in the eighth element was changed from flint to crown. Because the resulting reduction in lateral chromatism was still not enough, the glass in the seventh element was changed from 517645 crown to 541599 crown, an arrangement which made the lateral chromatism very small. Now designing with 12-ray and later with 26-ray patterns was continued on the 16 surfaces and on all air spaces. The final runs reduced the merit number to 12. This design is shown in the lower illustration.

The average spot sizes obtained with this design suggest that a resolution of about 40 lines per mm would be maintained to 6°, dropping to 25 lines per mm at the 12° field edge. Distortion is held to ± 10 microns. The maximum lateral chromatism is 8 microns.

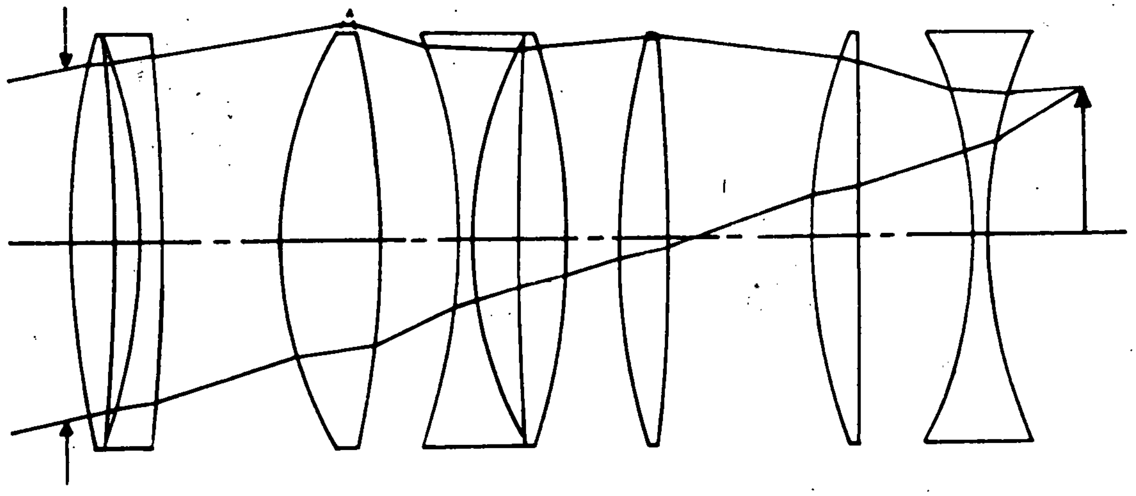
Later this design was achromatized for the C-e-F range to make a prototype lens suitable for aerial and general photographic problems requiring a large-aperture, long-focal-length, medium-field camera lens. The average spot sizes obtained suggest that a resolution of at least 20 lines per mm would be maintained over the entire field of view. Distortion in this design is held to ± 8 microns. The maximum lateral chromatism is 17 microns.

The prescriptions and the performance evaluations for the three designs are given on the three machine prints which are posted at the side. My experience with this program indicates that lenses constructed according to these specifications will make exceptionally brilliant images which will give the predicted resolution.

In conclusion I will mention three areas in which the code needs a more precise mathematical treatment than it receives in its present form. T. C. Doyle is now preparing a new program for the IBM Stretch machine which it is hoped will correct these deficiencies. First, the numerical differentiation obtained by incrementing the parameters needs sharpening. This calculation will either be automated or, preferably, the differentiation will be obtained analytically, as assumed by Wachendorf in 1953. Second, the least-squares minimization procedure needs attention because of a tendency to oscillate. It fails to give a continuously decreasing sequence of merit function values. The new method will correct this deficiency by employing the method proposed by Cauchy in 1847. Third, there is the present limitation on the number of parameters which can be calculated simultaneously. It is expected that this last limitation will disappear as a result of greater precision in the first two areas.



Monochromatic design for 600-mm *f*/2.0 lens.



C-d achromatic design for 600-mm *f*/2.0 lens.