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DERIVATION OF CORRECTIONS TO K_{∞}
IN THE TWO-GROUP APPROXIMATION

E. P. Lippincott

May 1971

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DERIVATION OF CORRECTIONS TO K_{∞} IN THE
TWO-GROUP APPROXIMATION

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ABSTRACT

An improved derivation of the correction terms necessary to evaluate k_{∞} from experimental data in the unpoisoned and poisoned techniques is presented. This **work** extends earlier derivations to include epithermal fissions and corrects minor errors in the earlier results. A new formula for k_{∞} is derived in which the buckling has been eliminated in order to allow a more accurate value of k_{∞} to be determined.

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I. INTRODUCTION

Recent experiments in the High Temperature Lattice Test Reactor (HTLTR) have required improved derivations of corrections necessary for the evaluation of the infinite medium neutron multiplication factor, k_{∞} . These corrections are derived for both the poisoned and unpoisoned techniques in the two energy group approximation. In these derivations, the multigroup definition of k_{∞} is used, which reduces to:

$$k_{\infty} = \frac{\text{neutrons produced}}{\text{neutrons absorbed}} = \frac{\nu_1 \Sigma_1^f \phi_1 + \nu_2 \Sigma_2^f \phi_2}{\Sigma_1^a \phi_1 + \Sigma_2^a \phi_2} \quad (1)$$

in the two group approximation. In this expression, 1 refers to the epithermal neutron group and 2 to the thermal group. The fluxes to be substituted in equation (1) are those for the bare critical reactor and thus the value for k_{∞} is taken to be that appropriate to such a system and not a true infinite system.

II. DERIVATION OF THE MISMATCH CORRECTION TO k_{∞} IN THE UNPOISONED TECHNIQUE

The formula for the mismatch correction to k_{∞} in the unpoisoned technique was originally presented by Heineman.⁽¹⁾ Errors in sign were suspected in this formulation and so the mismatch correction has been rederived using the same formalism that was used in a previous paper on the definition of k_{∞} and the derivation of the two group expression for the unpoisoned technique without mismatch.⁽²⁾

Using first order perturbation theory, it is shown⁽¹⁾ that

$$\chi \Delta \rho = - \sum_i \int_V \phi_i^+ D_i \nabla^2 \phi_i dV \quad (2)$$

where the integral extends over the central cell and the ϕ_i are the perturbed fluxes in the cell. Using suitable average fluxes, this reduces to:

$$\frac{\chi \Delta \rho_{\text{cell}}}{V} = - \phi_1^+ D_1 \nabla^2 \phi_1 - \phi_2^+ D_2 \nabla^2 \phi_2 \quad (3)$$

Using for the worth of poison (represented by superscript p),

$$\chi \Delta \rho^p = - \phi_1^+ \Sigma_1^p \phi_1^p V_p - \phi_2^+ \Sigma_2^p \phi_2^p V_p, \quad (4)$$

the relationship is obtained

$$\left(- \frac{\Delta \rho_{\text{cell}}}{\Delta \rho^p} \right) \frac{\left(\Sigma_2^p \phi_2^p V \right)_p}{\left(\Sigma_2^a \phi_2^a V \right)_{\text{cell}}} \left(1 + \frac{\Sigma_1^p \phi_1^p \phi_1^+}{\Sigma_2^p \phi_2^p \phi_2^+} \right) = - \frac{D_2 \nabla^2 \phi_2}{\Sigma_2^a \phi_2} - \frac{D_1 \nabla^2 \phi_1}{\Sigma_2^a \phi_2} \frac{\phi_1^+}{\phi_2^+} \quad (5)$$

where the right hand side is still the suitable volume average over the cell.

If ϕ_1 and ϕ_2 are the correct fluxes for the bare critical reactor, then the wave equation holds,

$$\nabla^2 \phi_i + B^2 \phi_i = 0. \quad (6)$$

Using this relation and the two-group equations,

$$D_1 \nabla^2 \phi_1 - \Sigma_1^a \phi_1 - \Sigma_1^R \phi_1 + \nu_1 \Sigma_1^f \phi_1 + \nu_2 \Sigma_2^f \phi_2 = 0 \quad (7a)$$

$$\Sigma_1^R \phi_1 + D_2 \nabla^2 \phi_2 - \Sigma_2^a \phi_2 = 0 \quad (7b)$$

$$D_1 \nabla^2 \phi_1^+ - \Sigma_1^a \phi_1^+ - \Sigma_1^R \phi_1^+ + \nu_1 \Sigma_1^f \phi_1^+ + \Sigma_1^R \phi_2^+ = 0 \quad (8a)$$

$$\nu_2 \Sigma_2^f \phi_1^+ + D_2 \nabla^2 \phi_2^+ - \Sigma_2^a \phi_2^+ = 0 \quad (8b)$$

where Σ_1^R is the cross section for transfer from group 1 to group 2, the right hand side of Equation 5 can be related to k_∞ and the following relationship is obtained:⁽²⁾

$$k_\infty = 1 - \left(\frac{\Delta \rho_{\text{cell}}}{\Delta \rho_p} \right) \left(\frac{\Sigma_{2p}^{pV} \phi_2^p}{\Sigma_{2p}^{aV} \phi_2} \right)_{\text{cell}} \left\{ 1 + \frac{\phi_1^+}{\phi_2^+} \frac{\Sigma_{11}^{p\phi} \phi_1^p}{\Sigma_{22}^{p\phi} \phi_2^p} \right\} \quad (9)$$

$$- \frac{(1+\tau B^2)(1+L^2 B^2)(1-p) L^2 B^2}{1 + (1-p) L^2 B^2}$$

$$+ \frac{\tau B^2(1+L^2 B^2) [\tau B^2 - \eta_1 f_1 (1-p)]}{1 + \tau B^2 - \eta_1 f_1 (1-p)}$$

However, if the fluxes and adjoints in the reactor, which are used to measure the left side of Equation 5 differ from the fundamental mode values, i.e., are not matched, then Equation 9 is not correct. In this case, we denote the actual reactor fluxes and adjoints by primed quantities, and Equations 7 and 8 still hold for these primed quantities within the two-group theory.

If we denote the right hand side of Equation 5 by A, then in the mismatch case:

$$A' = \frac{D_2 \nabla^2 \phi_2'}{\Sigma_2^a \phi_2'} + \frac{D_1 \nabla^2 \phi_1'}{\Sigma_2^a \phi_2'} \frac{\phi_1^+}{\phi_2^+} \quad (10)$$

and substituting from Equations 7 and 8 (using primed fluxes)

$$A' = \frac{\Sigma_1^R}{\Sigma_2^a} \frac{\phi_1'}{\phi_2'} - 1 - \frac{(\Sigma_1^R + \Sigma_1^a) \phi_1'}{\Sigma_2^a \phi_2'} \frac{\phi_1^+}{\phi_2^+} + \frac{v \Sigma_1^f}{\Sigma_2^a} \frac{\phi_1^+ \phi_1'}{\phi_2^+ \phi_2'} + \frac{v \Sigma_2^f}{\Sigma_2^a} \frac{\phi_1^+}{\phi_2^+} \quad (11)$$

We also define*

$$\phi = \frac{\phi_1'}{\phi_2'}, \quad \phi^+ = \frac{\phi_1^+}{\phi_2^+} \quad (12)$$

$$\Delta\phi = \left(\frac{\phi_1'}{\phi_2'} - \frac{\phi_1}{\phi_2} \right) \text{ and } \Delta\phi^+ = \left(\frac{\phi_1^+}{\phi_2^+} - \frac{\phi_1}{\phi_2} \right) \quad (13)$$

Substituting in Equation 10 and grouping terms,

$$\begin{aligned} A' = & \frac{\Sigma_1^R}{\Sigma_2^a} \frac{\phi_1'}{\phi_2'} - 1 - \frac{\Sigma_1^R + \Sigma_1^a}{\Sigma_2^a} \frac{\phi_1'}{\phi_2'} \frac{\phi_1^+}{\phi_2^+} + \frac{v \Sigma_1^f}{\Sigma_2^a} \frac{\phi_1^+ \phi_1'}{\phi_2^+ \phi_2'} + \frac{v \Sigma_2^f}{\Sigma_2^a} \frac{\phi_1^+}{\phi_2^+} \\ & + \frac{\Delta\phi^+}{\phi^+} \left[- \frac{\Sigma_1^R + \Sigma_1^a}{\Sigma_2^a} \frac{\phi_1'}{\phi_2'} \frac{\phi_1^+}{\phi_2^+} + \frac{v \Sigma_1^f}{\Sigma_2^a} \frac{\phi_1^+ \phi_1'}{\phi_2^+ \phi_2'} + \frac{v \Sigma_2^f}{\Sigma_2^a} \frac{\phi_1^+}{\phi_2^+} \right] \\ & + \frac{\Delta\phi}{\phi} \left[\frac{\Sigma_1^R}{\Sigma_2^a} \frac{\phi_1'}{\phi_2'} - \frac{\Sigma_1^R + \Sigma_1^a}{\Sigma_2^a} \frac{\phi_1'}{\phi_2'} \frac{\phi_1^+}{\phi_2^+} + \frac{v \Sigma_1^f}{\Sigma_2^a} \frac{\phi_1^+ \phi_1'}{\phi_2^+ \phi_2'} \right] \\ & + \frac{\Delta\phi^+}{\phi^+} \frac{\Delta\phi}{\phi} \left[- \frac{\Sigma_1^R + \Sigma_1^a}{\Sigma_2^a} \frac{\phi_1'}{\phi_2'} \frac{\phi_1^+}{\phi_2^+} + \frac{v \Sigma_1^f}{\Sigma_2^a} \frac{\phi_1^+ \phi_1'}{\phi_2^+ \phi_2'} \right] \quad (14) \end{aligned}$$

*When evaluating $\Delta\phi$ and $\Delta\phi^+$, recall that ϕ is the average over the cell and ϕ^+ is evaluated at the surface of the void.

If $\Delta\Phi$ and $\Delta\Phi^+$ are zero, then $A' = A$, the expression with no mismatch. Since the terms in brackets in Equation 14 involve unprimed quantities, we can use Equations 3 and 4 to reduce these expressions to the following, (calling the mismatch correction $g = A - A'$)

$$g = - \frac{D_1 B^2}{\Sigma_2} \frac{\phi_1^+}{\phi_2} \frac{\phi_1}{\phi_2} \left[\frac{\Delta\Phi}{\Phi} + \frac{\Delta\Phi^+}{\Phi^+} + \frac{\Delta\Phi}{\Phi} \frac{\Delta\Phi^+}{\Phi^+} \right] + (1+L^2 B^2) \frac{\Delta\Phi}{\Phi} \frac{\Delta\Phi^+}{\Phi^+} \quad (15)$$

or, again using Equations 3 and 4, this may be written

$$g = - \frac{\tau B^2 (1+L^2 B^2)}{1+\tau B^2 - \eta_1 f_1 (1-p)} \left(\frac{\Delta\Phi}{\Phi} + \frac{\Delta\Phi^+}{\Phi^+} \right) + \frac{(1+L^2 B^2)(1-\eta_1 f_1 (1-p))}{1+\tau B^2 - \eta_1 f_1 (1-p)} \frac{\Delta\Phi}{\Phi} \frac{\Delta\Phi^+}{\Phi^+} \quad (16)$$

This quantity is seen to reduce to the poisoned technique expression^(3,4) when $B^2 = 0$.

The expression for k_∞ may now be written as Equation 9 with primed fluxes and adjoints and the correction g added on, i.e.

$$k_\infty = 1 - \left(\frac{\Delta\rho^{\text{cell}}}{\Delta\rho^p} \right) \frac{(\Sigma_2^p V \phi_2^p)}{(\Sigma_2^a V \phi_2^a)_{\text{cell}}} \left(1 + \frac{\phi_1^+}{\phi_2^+} \frac{\Sigma_1^p \phi_1^p}{\Sigma_2^p \phi_2^p} \right) - \frac{(1+\tau B^2)(1+L^2 B^2)(1-p)L^2 B^2}{1+(1-p)L^2 B^2} + \frac{\tau B^2 (1+L^2 B^2) [\tau B^2 - \eta_1 f_1 (1-p)]}{1+\tau B^2 - \eta_1 f_1 (1-p)} - \frac{\tau B^2 (1+L^2 B^2)}{1+\tau B^2 - \eta_1 f_1 (1-p)} \left(\frac{\Delta\Phi}{\Phi} + \frac{\Delta\Phi^+}{\Phi^+} \right) + \frac{(1+L^2 B^2)(1-\eta_1 f_1 (1-p))}{1+\tau B^2 - \eta_1 f_1 (1-p)} \frac{\Delta\Phi}{\Phi} \frac{\Delta\Phi^+}{\Phi^+} \quad (17)$$

where now $\Delta\rho^{\text{cell}}$ and $\Delta\rho^{\text{p}}$ are measured in the mismatch spectrum. If epithermal fissions are neglected, this expression agrees with Heineman's⁽¹⁾ except for the sign of the $\Delta\phi^+/\phi^+$ term.

III. POISONED TECHNIQUE K_{∞}

In the poisoned technique, the cell is poisoned to a zero net reactivity by addition of copper poison. In this case, if the flux is matched, the two group equations are:

$$-\Sigma_1^a \phi_1' - \Sigma_1^R \phi_1' - \Sigma_1^p \phi_1' + v\Sigma_1^f \phi_1' + v\Sigma_2 \phi_2' = 0 \quad (18a)$$

$$-\Sigma_2^a \phi_2' - \Sigma_2^p \phi_2' + \Sigma_1^R \phi_1' = 0 \quad (18b)$$

where the prime indicates poisoned fluxes and Σ^p represents the absorption in the copper added to each cell. Eliminating ϕ_1 and ϕ_2 , these equations reduce to:

$$-\frac{(\Sigma_2^a + \Sigma_2^p)}{\Sigma_1^R} (\Sigma_1^a + \Sigma_1^R) - \frac{\Sigma_1^p}{\Sigma_1^R} (\Sigma_2^a + \Sigma_2^p) + \frac{v\Sigma_1^f}{\Sigma_1^R} (\Sigma_2^a + \Sigma_2^p) + v\Sigma_2^f = 0 \quad (19)$$

Using the definition of k_{∞} (Equation 1) and the unpoisoned two-group equations (Equation 7) to eliminate ϕ_1 and ϕ_2 ,

$$k_{\infty} = \frac{v\Sigma_1^f \Sigma_2^a (1+L^2 B^2) + v\Sigma_2^f \Sigma_1^R}{\Sigma_2^a \Sigma_1^R + \Sigma_1^a \Sigma_2^a (1+L^2 B^2)} \quad (20)$$

where B^2 is the unpoisoned buckling. Combining (19) and (20) to eliminate Σ_2^f gives:

$$k_{\infty} \Sigma_2^a \left[\Sigma_1^R + \Sigma_1^a (1+L^2 B^2) \right] = (\Sigma_2^a + \Sigma_2^p) (\Sigma_1^R + \Sigma_1^a) + \Sigma_1^p (\Sigma_2^a + \Sigma_2^p) + L^2 B^2 \nu \Sigma_1^f \Sigma_2^a - \Sigma_2^p \nu \Sigma_1^f \quad (21)$$

Using

$$p = \frac{\Sigma_1^R}{\Sigma_1^R + \Sigma_1^a} \quad (22)$$

this may be written

$$k_{\infty} \left[1 + L^2 B^2 (1-p) \right] = 1 + \frac{\Sigma_2^p}{\Sigma_2^a} + \frac{\Sigma_1^p}{\Sigma_1^R} \left(1 + \frac{\Sigma_2^p}{\Sigma_2^a} \right) + \eta_1 f_1 (1-p) \left(L^2 B^2 - \frac{\Sigma_2^p}{\Sigma_2^a} \right) \quad (23)$$

or

$$k_{\infty} \left[1 + (1-p) L^2 B^2 \right] = 1 + \frac{(\Sigma_{2\phi}^p \nu)^p}{(\Sigma_{2\phi}^a \nu)^{\text{cell}}} \left\{ 1 + p \frac{(\Sigma_{1\phi}^p \nu)^p}{(\Sigma_{2\phi}^p \nu)^p} \right\} + \eta_1 f_1 (1-p) \left(L^2 B^2 - \frac{\Sigma_2^p}{\Sigma_2^a} \right) \quad (24)$$

This formula is adequate to solve for k_{∞} , but to compare with the unpoisoned technique this form must be modified slightly. Using (cf. Ref. 2)

$$k_{\infty} = \frac{(1+\tau B^2)(1+L^2 B^2)}{1+(1-p)L^2 B^2} \quad (25)$$

and solving Equation 24 for Σ_2^p/Σ_2^a

$$\frac{\Sigma_2^p}{\Sigma_2^a} = \left\{ (1 + \tau B^2)(1 + L^2 B^2) - 1 - \eta_1 f_1 (1-p) L^2 B^2 \right\} \left\{ 1 + p \frac{(\Sigma_{1\phi}^p \gamma)^p}{(\Sigma_{2\phi}^p \gamma)^p} - \eta_1 f_1 (1-p) \right\}^{-1} \quad (26)$$

Substituting in Equation 24:

$$k_\infty = 1 + \frac{(\Sigma_{2\phi}^p \gamma)^p}{(\Sigma_{2\phi}^a \gamma)^{\text{cell}}} \left\{ 1 + p \frac{(\Sigma_{1\phi}^p \gamma)^p}{(\Sigma_{2\phi}^p \gamma)^p} \right\} - \frac{(1+\tau B^2)(1+L^2 B^2)(1-p)L^2 B^2}{1 + (1-p)L^2 B^2} + \frac{\eta_1 f_1 (1-p)}{1+p \frac{(\Sigma_{1\phi}^p \gamma)^p}{(\Sigma_{2\phi}^p \gamma)^p} - \eta_1 f_1 (1-p)} \left\{ p L^2 B^2 \frac{(\Sigma_{1\phi}^p \gamma)^p}{(\Sigma_{2\phi}^p \gamma)^p} - \tau B^2 (1+L^2 B^2) \right\} \quad (27)$$

After identifying the amount of poison to give zero reactivity for the poisoned cell with the predicted amount in the unpoisoned technique, $\left(\frac{\Delta \rho^{\text{cell}}}{-\Delta \rho^p} \right)$, this equation is seen to be very similar to Equation 17 with no mismatch. The difference in the correction terms arises only in the last term and in the epithermal copper term. Calling this difference δ ,

$$\delta = \frac{(\tau B^2)^2 (1+L^2 B^2)}{1+\tau B^2 - \eta_1 f_1 (1-p)} \left[1 - \eta_1 f_1 (1-p) \right]^{-1} + \frac{(\Sigma_{1\phi}^p \gamma)^p}{(\Sigma_{2\phi}^p \gamma)^p} \left\{ \left(\frac{\phi_1^+}{\phi_2^+} - p \right) \frac{(\Sigma_{1\phi}^p \gamma)^p}{(\Sigma_{2\phi}^p \gamma)^{\text{cell}}} - \frac{\eta_1 f_1 (1-p) p \left[L^2 B^2 + \frac{\tau B^2 (1+L^2 B^2)}{1 - \eta_1 f_1 (1-p)} \right]}{1 + p \frac{(\Sigma_{1\phi}^p \gamma)^p}{(\Sigma_{2\phi}^p \gamma)^p} - \eta_1 f_1 (1-p)} \right\} \quad (28)$$

In general, the second term in Equation 28 may be neglected, and then,

$$\delta = \frac{(\tau_B^2)^2(1+L_B^2)}{1+\tau_B^2 - \eta_1 f_1 (1-p)} \left[1 - \eta_1 f_1 (1-p) \right]^{-1} \quad (29)$$

If the epithermal fissions are neglected, this term reduces to the correction term as derived by Heineman.⁽¹⁾

IV. MISMATCH CORRECTION IN THE POISONED TECHNIQUE

If the poisoned case is not matched, the criteria for null reactivity is (using primes for mismatched quantities)

$$\begin{aligned} \chi \Delta \rho^{\text{cell}} = 0 = & -\phi_1^+ (\Sigma_1^R + \Sigma_1^a + \Sigma_1^{p'}) \phi_1' + \phi_1^+ \nu \Sigma_1^f \phi_1' \\ & + \phi_1^+ \nu \Sigma_2^f \phi_2' - \phi_2^+ (\Sigma_2^a + \Sigma_2^{p'}) \phi_2' + \phi_2^+ \Sigma_1^R \phi_1' \end{aligned} \quad (30)$$

where $\Sigma_2^{p'}$ will differ from Σ_2^p . Using ϕ and $\Delta\phi$ as defined in Equation 12 and 13, and Equation 18 for matched fluxes,*

$$\begin{aligned} 0 = & -(\Sigma_1^R + \Sigma_1^a + \Sigma_1^{p'} - \nu \Sigma_1^f) \frac{\Sigma_2^p + \Sigma_2^a}{\Sigma_1^R} \frac{\Sigma_2^a + \Sigma_2^p}{\nu \Sigma_2^f} \left(1 + \frac{\Delta\phi}{\phi}\right) \left(1 + \frac{\Delta\phi}{\phi^+}\right) \\ & + (\Sigma_2^a + \Sigma_2^p) \left(1 + \frac{\Delta\phi}{\phi^+}\right) - (\Sigma_2^a + \Sigma_2^{p'}) + \Sigma_1^R \frac{\Sigma_2^p + \Sigma_2^a}{\Sigma_1^R} \left(1 + \frac{\Delta\phi}{\phi}\right). \end{aligned} \quad (31)$$

If we let

$$x = \frac{\Sigma_2^{p'} - \Sigma_2^p}{\Sigma_2^p} \quad (32)$$

*Since primes now represent mismatched fluxes, the primes in Equation 18 should be eliminated.

we will also have (since only the amount of poison changes),

$$\Sigma_1^p = \Sigma_1^p(1+x) \quad (33)$$

and substituting in 31 and simplifying with Equation 18,

$$\begin{aligned} 0 = & -(\Sigma_2^a + \Sigma_2^p)(1 + \frac{\Delta\Phi}{\Phi})(1 + \frac{\Delta\Phi^+}{\Phi^+}) - x\Sigma_1^p \frac{\Sigma_2^p + \Sigma_2^a}{\Sigma_1^R} \frac{\Sigma_2^a + \Sigma_2^p}{v\Sigma_2^f} (1 + \frac{\Delta\Phi}{\Phi})(1 + \frac{\Delta\Phi^+}{\Phi^+}) \\ & + (\Sigma_2^a + \Sigma_2^p)(\frac{\Delta\Phi^+}{\Phi^+} + \frac{\Delta\Phi}{\Phi}) + \Sigma_2^p(1-x) + \Sigma_2^a \end{aligned} \quad (34)$$

$$= -(\Sigma_2^a + \Sigma_2^p) \frac{\Delta\Phi}{\Phi} \frac{\Delta\Phi^+}{\Phi^+} - x \left\{ \Sigma_2^p + \Sigma_1^p \frac{\Sigma_2^p + \Sigma_2^a}{\Sigma_1^R} \frac{\Sigma_2^a + \Sigma_2^p}{v\Sigma_2^f} (1 + \frac{\Delta\Phi}{\Phi})(1 + \frac{\Delta\Phi^+}{\Phi^+}) \right\}$$

Solving for x, therefore,

$$x = - \frac{\Delta\Phi}{\Phi} \frac{\Delta\Phi^+}{\Phi^+} \left[\frac{\Sigma_2^p}{\Sigma_2^a + \Sigma_2^p} + \frac{\Sigma_1^p (\Sigma_2^p + \Sigma_2^a)}{\Sigma_1^R v\Sigma_2^f} (1 + \frac{\Delta\Phi}{\Phi})(1 + \frac{\Delta\Phi^+}{\Phi^+}) \right]^{-1} \quad (35)$$

Substituting measured quantities in this expression

$$x = - \frac{\frac{\Delta\Phi}{\Phi} \frac{\Delta\Phi^+}{\Phi^+} \left((1+x)(\Sigma_2^a \phi_2^v)^{cell} + (\Sigma_2^p \phi_2^v)^p \right)}{(\Sigma_2^p \phi_2^v)^p \left(1 + \frac{\phi_1^+ (\Sigma_1^p \phi_1^v)^p}{\phi_2^+ (\Sigma_2^p \phi_2^v)^p} \right)} \quad (36)$$

In terms of the mismatched quantities, we may write from Equation 27

$$\begin{aligned} k_\infty = & 1 + \frac{(\Sigma_2^p \phi_2^v)^p}{(\Sigma_2^p \phi_2^v)^{cell}} \frac{1}{1+x} \left\{ 1 + p \frac{(\Sigma_1^p \phi_1^v)^p}{(\Sigma_2^p \phi_2^v)^p (1 + \frac{\Delta\Phi}{\Phi})} \right\} \\ & - \frac{(1+\tau B^2)(1+L^2 B^2)(1-p)L^2 B^2}{1 + (1-p)L^2 B^2} + \frac{\eta_1 f_1 (1-p)}{1+p \frac{(\Sigma_1^p \phi_1^v)^p}{(\Sigma_2^p \phi_2^v)^p (1 + \frac{\Delta\Phi}{\Phi})} - \eta_1 f_1 (1-p)} \\ & \left\{ p L^2 B^2 \frac{(\Sigma_1^p \phi_1^v)^p}{(\Sigma_2^p \phi_2^v)^p} \frac{1}{1 + \frac{\Delta\Phi}{\Phi}} - \tau B^2 (1+L^2 B^2) \right\} \end{aligned} \quad (37)$$

If we call g the correction to k_∞ , then

$$\begin{aligned}
g = & - \frac{x}{1+x} \frac{(\Sigma_2^{\phi} \hat{\nu})^p}{(\Sigma_2^{\phi} \hat{\nu})^{\text{cell}}} \left\{ 1 + \frac{p (\Sigma_1^{\phi} \hat{\nu})^p}{(\Sigma_2^{\phi} \hat{\nu})^p} \frac{1}{1 + \frac{\Delta\phi}{\phi}} \right\} \\
& - \frac{\frac{\Delta\phi}{\phi}}{1 + \frac{\Delta\phi}{\phi}} p \frac{(\Sigma_1^{\phi} \hat{\nu})^p}{(\Sigma_2^{\phi} \hat{\nu})^p} \left\{ \frac{(\Sigma_2^{\phi} \hat{\nu})^p}{(\Sigma_2^{\phi} \hat{\nu})^{\text{cell}}} + \frac{\eta_1 f_1 (1-p)}{1 - \eta_1 f_1 (1-p) + p \frac{(\Sigma_1^{\phi} \hat{\nu})^p}{(\Sigma_2^{\phi} \hat{\nu})^p}} \right. \\
& \left. \left[L^2 B^2 - \frac{p L^2 B^2 \frac{(\Sigma_1^{\phi} \hat{\nu})^p}{(\Sigma_2^{\phi} \hat{\nu})^p} \frac{1}{1 + \frac{\Delta\phi}{\phi}} - \tau B^2 (1 + L^2 B^2)}{1 - \eta_1 f_1 (1-p) + p \frac{(\Sigma_1^{\phi} \hat{\nu})^p}{(\Sigma_2^{\phi} \hat{\nu})^p} \frac{1}{1 + \frac{\Delta\phi}{\phi}}} \right] \right\} \quad (38)
\end{aligned}$$

If the corrections to the epithermal copper absorptions are neglected, and x is substituted from Equation 36, then

$$g = \left[1 + \frac{(\Sigma_2^p \hat{\nu})}{(\Sigma_2^a \hat{\nu})^{\text{cell}}} \right] \frac{\frac{\Delta\phi}{\phi}}{\frac{\Delta\phi}{\phi} +} \quad (39)$$

to first order in $\frac{\Delta\phi}{\phi} \frac{\Delta\phi}{\phi} +$.

It is again seen that the earlier derivation^(3,4) is obtained

if epithermal fissions and epithermal copper absorptions are neglected.

V. SOLUTION FOR K_{∞} WITHOUT THE BUCKLING

In the above equations for k_{∞} , there are several correction terms involving the buckling to first order. Since $(k_{\infty}-1)$ is proportional to the buckling in first order these correction terms do not get smaller relative to $k_{\infty}-1$ as the buckling is reduced, and, in general, contribute significantly to the value of $k_{\infty}-1$ derived from these equations. Since the calculated value of the buckling is quite sensitive to the input to the calculation and the calculational technique (which are intended to be checked by the experiment) the best experimental value of k_{∞} can be obtained by eliminating the buckling (using Equation 25) and using the calculation only for the other (less sensitive) parameters.

To carry out this substitution, we first solve Equation 25 for B^2 to second order,

$$B^2 = \frac{1}{\tau + pL^2} (k_{\infty}-1) - \frac{pL^2 [\tau - L^2(1-p)]}{(\tau + pL^2)^3} (k_{\infty}-1)^2 \quad (40)$$

Next Equation 17 is written as a power series in B^2 to second order:

$$\begin{aligned}
k_{\infty-1} = A - \left\{ L^2(1-p) + \frac{\tau \eta_1 f_1(1-p)}{1-\eta_1 f_1(1-p)} + \frac{\tau}{1-\eta_1 f_1(1-p)} \left(\frac{\Delta\phi}{\phi} + \frac{\Delta\phi^+}{\phi^+} \right) \right. \\
\left. - \frac{L^2 \left[1-\eta_1 f_1(1-p) \right] - \tau \frac{\Delta\phi}{\phi} \frac{\Delta\phi^+}{\phi^+}}{1-\eta_1 f_1(1-p)} \right\} B^2 \\
- \left\{ (1-p)L^2 (\tau + pL^2) - \frac{\tau}{1-\eta_1 f_1(1-p)} \left[\frac{\tau}{1-\eta_1 f_1(1-p)} - L^2 \eta_1 f_1(1-p) \right] \right. \\
\left. + \frac{\tau L^2 \left[1-\eta_1 f_1(1-p) \right] - \tau^2 \left(\frac{\Delta\phi}{\phi} + \frac{\Delta\phi^+}{\phi^+} + \frac{\Delta\phi}{\phi} \frac{\Delta\phi^+}{\phi^+} \right)}{\left[1-\eta_1 f_1(1-p) \right]^2} \right\} B^4 \quad (41)
\end{aligned}$$

where

$$A = \left(- \frac{\Delta\rho^{\text{cell}}}{\Delta\rho^p} \right) \frac{(\Sigma_{2\phi_2}^p V)^p}{(\Sigma_{2\phi_2}^p V)^{\text{cell}}} \left(1 + \frac{\phi_1^+}{\phi_2^+} \frac{\Sigma_{1\phi_1}^p p}{\Sigma_{2\phi_2}^p p} \right) + \frac{\Delta\phi}{\phi} \frac{\Delta\phi^+}{\phi^+} \quad (42)$$

$$\text{Equation 41 may be written as } k_{\infty-1} = A - a_1 B^2 - a_2 B^4. \quad (43)$$

Substituting for B^2 from Equation 40, we can write Equation 43 as a power series for A in $k_{\infty-1}$ (keeping terms to second order),

$$A = (k_{\infty}-1)\left(1 + \frac{a_1}{\tau+pL^2}\right) + (k_{\infty}-1)^2 \left(\frac{a_2}{(\tau+pL^2)^2} - \frac{a_1 p L^2 [\tau-L^2(1-p)]}{(\tau+pL^2)^3} \right) \quad (44)$$

we can now solve for $k_{\infty}-1$ as a power series in A , keeping terms to second order,

$$k_{\infty}-1 = \frac{A}{1 + \frac{a_1}{\tau+pL^2}} - A^2 \frac{\left\{ a_2(\tau+pL^2) - a_1 p L^2 [\tau-L^2(1-p)] \right\}}{(\tau+pL^2+a_1)^3} \quad (45)$$

For small $k_{\infty}-1$, this is

$$k_{\infty}-1 = \frac{A}{1 + \frac{1}{\tau+pL^2} \left\{ (1-p)L^2 + \frac{\tau \eta_1 f_1 (1-p)}{1-\eta_1 f_1 (1-p)} + \frac{\tau}{1-\eta_1 f_1 (1-p)} \left(\frac{\Delta\Phi}{\Phi} + \frac{\Delta\Phi^+}{\Phi^+} \right) - \frac{L^2 [1-\eta_1 f_1 (1-p)]}{1-\eta_1 f_1 (1-p)} - \tau \frac{\Delta\Phi}{\Phi} \frac{\Delta\Phi^+}{\Phi^+} \right\}} \quad (46)$$

and for small flux mismatch this reduces to

$$k_{\infty}-1 = A \left(\frac{\tau+pL^2}{L^2 + \frac{\tau}{1-\eta_1 f_1 (1-p)}} \right) \quad (47)$$

It is important to note that all the parameters in this expression can be calculated quite accurately since none of them are directly dependent on $k_{\infty}-1$. This expression represents the first order value of $k_{\infty}-1$ that is least dependent on any theoretical models or cross sections used to analyze the experimental data.

VI. CONCLUSIONS

The theory for the evaluation of k_{∞} in the unpoisoned and poisoned technique has been rederived to use the fundamental mode k_{∞} and to include epithermal fissions in the two-group model. Minor errors in earlier derivations have been corrected and the former equations shown to be substantially correct within the approximations used in their derivations.

A new formula for the evaluation of k_{∞} from the measured reactivity and absorption ratios has been derived. In this expression (Equation 45) correction terms dependent on the buckling have been eliminated and first order correction terms are only dependent on the more accurately known parameters τ , L^2 , p , and $\eta_1 f_1$. The value of k_{∞} obtained in this way represents the best experimental value.

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