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NVO-1163-TM-15

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THE CAVITY FORMED BY A CONTAINED UNDERGROUND NUCLEAR DETONATION

MASTER

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FEBRUARY 1970

P4804

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THE CAVITY FORMED BY A CONTAINED
UNDERGROUND NUCLEAR DETONATION

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February 16, 1970

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Prepared under
Contract AT(29-2)-1163
for the
Nevada Operations Office
U. S. Atomic Energy Commission

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ABSTRACT

An equation is derived that directly relates the radius of the cavity formed by an underground nuclear detonation to, among other parameters, several physical properties of the geologic environment of the detonation. These physical properties are the vaporization pressure, adiabatic expansion coefficient, density, Young's modulus, cohesive strength, and angle of internal friction. The values of Young's modulus, cohesive strength and angle of internal friction required are those values characteristic of the geologic environment as altered by the outgoing shock wave from the explosion. Assuming what would appear to be reasonable values for the required physical properties, statistical analyses suggest that the derived equation accurately describes cavity radii for underground nuclear events in five geologic media; alluvium, tuff, granite, salt and rhyolite. The derived equation may be used to predict the cavity radius for an underground nuclear detonation in a geologic environment in which no previous detonations have occurred if the required physical property data are obtained.

ACKNOWLEDGMENT

The author wishes to thank P. P. deCaprariis,
W. W. Hakala, R. A. Mueller, and J. R. Murphy for their
valuable advice and assistance.

CHAPTER 1

INTRODUCTION

1.1 General Background

When an underground nuclear detonation takes place, the energy released by the explosion is deposited in the surrounding earth material. The strong shock wave initially established in the earth material results in vaporization of the material out to a given radius forming a "vaporization cavity." The size and the internal pressure of the vaporization cavity depend on the yield of the explosion and the geologic medium in which the detonation takes place.

The shock wave propagating beyond the vaporization cavity is initially strong enough to cause the earth material to fail. Near the vaporization cavity, where the stress levels associated with the shock wave are high, failure of the medium probably occurs by plastic yielding. At greater distances, the stress levels associated with the outgoing shock are lower than required to produce plastic failure and the principal mode of failure for the geologic medium is probably brittle fracture. At still greater

distances, because of the dissipative character of the earth material and spherical divergence, the stress level associated with outgoing waves is degraded to such a degree that the medium responds elastically.

The vaporization cavity formed by the detonation contains vaporized rock material at a very high pressure and temperature. This high-pressure, high-temperature gas expands the initial cavity to its final size.

The accurate prediction of the final size of the cavity formed by an underground nuclear detonation is required for many purposes; one example is underground engineering with nuclear explosives where the size of the cavity, fracture and chimney regions is of primary importance in determining the technical and economic feasibility of a project.

1.2 Brief Review of Work by Other Investigators

1.2.1 Hydrodynamic - Plastic - Brittle Computer Code Calculations

A number of investigators have modeled the major early-time close-in effects of an underground nuclear detonation (i.e. cavity growth, plastic and brittle failure of the medium and shock wave propagation) using one-

dimensional and two-dimensional hydrodynamic-plastic-brittle computer codes (see for example Cherry, 1967; Beaudet, et al., 1969; and Cassity, et al., 1969). In addition to a determination of the final cavity size, a hydrodynamic-plastic-brittle code permits a study of cavity growth as a function of time. Hydrodynamic-plastic-brittle codes, assuming the necessary physical property data for the geologic medium are available, can lead to very useful and accurate results. The use of a hydrodynamic-plastic-brittle code, however, can be relatively time consuming and expensive.

1.2.2 Statistical Studies

For many applications, such as underground engineering using nuclear explosives, only the final magnitude of the cavity radius is required and not its time history. In these instances, it is highly desirable to have a method to predict the magnitude of the close-in effects that does not require extensive use of a computer. Therefore, several investigators have attempted to relate the size of the cavity formed by an underground nuclear detonation to the major event parameters such as yield, depth of burial and geologic medium using statistical analyses.

Rogich and Rich* in an early statistical study, found that cavity radii for over 100 underground nuclear events in tuff and alluvium could be predicted using an equation of the form

$$R = KW^n$$

where R is the cavity radius in feet, W is the event yield in kilotons and K and n are regression constants. For events in tuff or alluvium, Rogich and Rich found $K = 54.92$ and $n = 0.286$.

Boardman, et al., (1964) on the basis of dimensional analysis and the assumptions that: 1) the cavity gas expands adiabatically, 2) the adiabatic expansion coefficient of the gas is $4/3$, 3) the internal cavity pressure when cavity growth terminates is equal to the weight of the overlying earth material, derived the equation

$$R = C \frac{W^{1/3}}{(\rho h)^{1/4}}$$

* This work is contained in a classified report to the U. S. Atomic Energy Commission and is available to those with proper access clearance and the need to know.

to predict the cavity radius for events in four geologic media; alluvium, tuff, granite and salt. In this work, R is the cavity radius in meters, W is the event yield in kilotons, ρ is the average density of the overlying earth materials in grams per cubic centimeter, h is the depth of burial of the event in meters and C is a statistically determined regression constant. The value of the regression constant depends on the geologic medium in which the detonation takes place.

Dependence of the value of the regression constant on the medium in which the detonation takes place is a significant limitation of the equation reported by Boardman, et al. There is no reliable method to determine the value of the regression constant in order to predict the cavity radius for an event in a geologic medium for which there is no previous experience. This limitation becomes more serious as greater numbers of events, many in connection with the Plowshare program, are detonated in geologic environments not previously encountered.

This limitation of the equation reported by Boardman, et al. has been recognized and two investigators have

recently reported work statistically relating the cavity formed by an underground nuclear detonation to some of the physical properties of the geologic environment of the event, along with other event parameters.

Heard,* using dimensional analysis techniques, reports that the cavity radius can be described by the following statistically derived equation:

$$R = 16.3 \left[\frac{W^{0.29} E^{0.62}}{h^{0.11} \rho^{0.24} \mu^{0.67}} \right]$$

where R is the cavity radius in meters, W is the event yield in kilotons, E is Young's Modulus of the geologic environment in megabars, h is the depth of burial in meters, ρ is the density of the overburden in grams per cubic centimeter, and μ is the shear modulus of the earth material in megabars. The equation reported by Heard is based on a statistical analysis of data from 147 underground nuclear events.

*This work is contained in a classified report to the U. S. Atomic Energy Commission and is available to those with the proper access clearance and the need to know.

Closmann (1969) reports a similar equation for predicting cavity radius based on a statistical analysis of unclassified data from 46 underground nuclear events:

$$R = 21.0 \left[\frac{W^{.306} E^{.514}}{h^{.161} \rho^{.244} \mu^{.576}} \right]$$

where the parameters are defined as in the work by Heard.

1.2.3 Cavity Formation by High Explosive Detonations

In addition to the work on the cavity formed by underground nuclear detonations outlined above, there has been considerable work reported on cavity formation in various materials by high explosive detonations. Hopkins (1960) reviews extensively the theoretical work that has been performed to describe the expansion of cavities in metals due to high explosive detonations. A theoretical description of cavity formation by a high explosive detonation in an "ideal soil" (i.e. an isotropic elastic-rigid plastic material that obeys Hooke's law within the elastic range and Coulomb's criterion at yield) is reported in papers by Chadwick (1959, 1961) and Chadwick, et al., (1964). Vesic and Barksdale (1963) also report work on cavity formation by an explosion in ideal soils.

1.3 Objectives and Description of Work Reported in this Technical Memorandum

The purpose of the work reported here is to apply the theoretical work described by Hopkins (1960), Chadwick (1959, 1961) and Chadwick, et al., (1964) to the problem of predicting the size of the cavity formed by an underground nuclear detonation.

Cavity formation by an underground nuclear detonation is an extremely complex process. In order to make the mathematics easier to handle and to derive a relatively simple equation relating the radius of the cavity to pertinent event parameters, some drastic compromises were made with physical realities. Some of the basic assumptions underlying the present work are:

- 1) the growth of the cavity can be treated as spherical and due to the quasi-static, adiabatic expansion of the cavity gas.
- 2) the geologic environment surrounding the detonation is an isotropic elastic-perfectly plastic material obeying Hooke's law within the elastic range and Coulomb's criterion at yield.

- 3) the plastic material surrounding the expanding cavity is incompressible.
- 4) the ratio of the radius of the elastic-plastic boundary to the cavity radius, b/a , instantaneously reaches a limiting value that depends on the properties of the geologic environment (for a more detailed discussion of this assumption see Section 2.3.3).
- 5) cavity growth terminates when the internal pressure inside the cavity is no longer sufficient to maintain the limiting ratio of the elastic-plastic radius to the cavity radius. That is, the internal pressure at termination of cavity growth is assumed to be that pressure required to establish the limiting value of b/a .

Using these assumptions, an equation is derived in Chapter 2 to calculate the final radius of the cavity formed by an underground nuclear detonation. The equation derived relates the cavity radius to the following event parameters:

1. the pressure of vaporization of the geologic medium
2. the radius of vaporization of the geologic medium for a one kiloton explosion
3. the adiabatic expansion coefficient of the cavity gas
4. the angle of internal friction of the geologic medium
5. the cohesive strength of the geologic medium
6. the Young's modulus of the geologic medium
7. the density of the geologic medium
8. the depth of burial of the detonation
9. the energy released by the detonation

In Chapter 3, the determination of appropriate physical property values is discussed. It is shown in Chapters 4 and 5 that reasonable choices of values for the physical parameters result in scaling relationships similar to those reported by other investigators and permits

accurate prediction of cavity radius for events in five geologic media (alluvium, tuff, granite, salt and rhyolite). The derived equation directly relates the cavity radius to some of the physical properties of the geologic environment. Therefore, the equation may be used to predict the cavity radius for an underground nuclear detonation in a geologic environment in which no previous detonations have occurred if the required physical property data are obtained.

CHAPTER 2

CAVITY FORMATION BY A CONTAINED UNDERGROUND NUCLEAR DETONATION

2.1 Formation of the "Vaporization Cavity"

When an underground nuclear detonation takes place, the strong shock wave initially established in the surrounding earth materials results in vaporization of the material out to some radius, R_v , forming a "vaporization cavity." The internal pressure in the vaporization cavity, P_v , is taken as the shock pressure sufficient to cause the geologic material surrounding the detonation to vaporize after decompression.

Butkovich (1967) has developed gas equations of state for several geologic materials and reports values of R_v and P_v for these materials.

2.2 Quasi-Static Adiabatic Expansion of the Vaporization Cavity

The formation of the final cavity is assumed to be the result of the spherical, quasi-static expansion of the gas contained in the vaporization cavity. The assumption of spherical symmetry is appropriate due to the nearly

point source nature of a nuclear detonation. The assumption of an adiabatic expansion is deemed reasonable considering the very short time (perhaps 100 milliseconds) in which the cavity is formed. The further assumption of a quasi-static expansion is justified on the basis of work reported by Hopkins (1960). Under the assumptions of no work-hardening or strain rate effects (i.e. the material surrounding the detonation is assumed to be perfectly plastic) and incompressibility of the medium, Hopkins demonstrates that the size of the final cavity formed by a high explosive detonation is dependent only on the total work done by the working pressure. That is, the final cavity radius achieved under dynamic conditions is also achieved under quasi-static conditions for the same energy expenditure.

The quasi-static adiabatic expansion of the cavity gas can be described by the equation

$$P_v V_v^\gamma = P_f V_f^\gamma \quad (1)$$

where P_v = the internal pressure in the vaporization cavity (i.e. the shock pressure sufficient

to cause the geologic environment
surrounding the detonation to vapor-
ize after decompression - see
Butkovich, 1967)

V_v = the volume of the "vaporization
cavity" (Butkovich, 1967)

P_f = the internal cavity pressure at
termination of cavity growth

V_f = the volume of the final cavity

γ = the adiabatic gas constant of the
cavity gas (see Higgins and
Butkovich, 1967)

Assuming spherical expansion, equation (1) becomes

$$R_f = \left[\frac{P_v}{P_f} \right]^{1/3\gamma} R_v \quad (2)$$

where R_v and R_f are the radii of the vaporization and final
cavity respectively.

Butkovich (1967) reports that

$$R_v = R_v (1 \text{ kt}) W^{1/3} \quad (3)$$

where $R_v (1 \text{ kt})$ = the radius of the vaporization cavity for a one kiloton underground nuclear detonation. The value of $R_v (1 \text{ kt})$ ($\text{ft/kt}^{1/3}$) depends on the geologic environment of the detonation

W = the event yield in kilotons (kt).

Substituting equation (3) into equation (2),

$$R_f = \left[\frac{P_v R_v^{3\gamma} (1 \text{ kt})}{P_f} \right]^{1/3\gamma} W^{1/3} \quad (4)$$

Equation (4) is taken as the basic equation for calculating the radius of the cavity formed by an underground nuclear detonation of yield W and is equivalent to an equation reported by Higgins and Butkovich (1967). Higgins and Butkovich assume that the final internal cavity

pressure, P_f , is equal to the hydrostatic overburden pressure, ρgh . In the work reported here, an attempt is made to relate the internal pressure at termination of cavity growth to the physical properties of the geologic environment surrounding the cavity.

2.3 Termination of Cavity Growth

In order to determine, P_f , the internal cavity pressure at termination of cavity growth, an analysis was made of the quasi-static expansion of a spherical cavity in an ideal soil.

2.3.1 Mechanical Behavior of Ideal Soils

For the purpose of this study, an ideal soil is defined as a solid that obeys Hooke's law within the elastic range and Coulomb's criterion at yield. When the ideal soil yields, it is assumed to behave as a perfectly plastic material and to be incompressible. Chadwick, et al., (1964) present a more detailed discussion of the above assumptions and compare the mechanical behavior of such an ideal soil to what is known of the mechanical behavior of real soils.

Coulomb's yield criterion states that the critical

shear stress required for yielding of the material depends on the cohesive strength and angle of internal friction of the material. If σ and τ are the normal and shear stresses respectively exerted across a plane surface through a point in the soil, Coulomb's law states that plastic flow will occur at that point in the soil when (taking σ as positive for tensile stress)

$$|\tau| = c - \sigma \tan \phi \quad (5)$$

where c = the cohesive strength of the soil

ϕ = the angle of internal friction of the soil

Shield (1955) has shown that equation (5) may be written in terms of the principal stress components ($\sigma_1, \sigma_2, \sigma_3$) as

$$\sigma_3 - \sigma_1 = 2 c \cos \phi - (\sigma_3 + \sigma_1) \sin \phi \quad (6)$$

for $\sigma_1 \leq \sigma_2 \leq \sigma_3$

$$\text{Letting} \quad \alpha = \frac{2 \sin \phi}{1 - \sin \phi} \quad (7)$$

$$\text{and} \quad Y = \frac{2 c \cos \phi}{1 - \sin \phi} \quad (8)$$

equation (6) may be written

$$(1 + \alpha) \sigma_3 - \sigma_1 = Y \quad (9)$$

Equation (9) is the form of Coulomb's yield criterion used in this report. The quantity Y will be referred to as the "yield strength" (Chadwick, et al., 1964) of the material. It should also be noted that for $\phi = 0$ (i.e., $\alpha = 0$) equation (9) reduces to the Tresca yield criterion.

2.3.2 Quasi-Static Expansion of A Spherical Cavity in an Ideal Soil: Elastic Solution

Assume a spherical cavity with radius " a " and internal pressure " p " in an ideal soil. If the internal pressure p is such that the soil responds elastically and the displacements and stresses are everywhere small, the stresses and strains are related by Hooke's law, i.e., assuming spherical symmetry

$$\frac{\partial u}{\partial r} = \frac{1}{E} \left[\sigma_r - 2\nu \sigma_\theta \right] = \epsilon_r \quad (10)$$

and

$$\frac{u}{r} = \frac{1}{E} \left[(1-\nu)\sigma_\theta - \nu \sigma_r \right] = \epsilon_\theta \quad (11)$$

where u = the displacement

r = radial distance from the center
of the cavity

E = Young's modulus

σ_r = stress in radial direction

σ_θ = stress in tangential direction

ν = Poisson's ratio

ϵ_r = strain in radial direction

ϵ_θ = strain in tangential direction

Equations (10) and (11) may also be written

$$\sigma_r = \frac{E}{(1+\nu)(1-2\nu)} \left[(1-\nu) \frac{\partial u}{\partial r} + 2\nu \frac{u}{r} \right] \quad (12)$$

$$\sigma_\theta = \frac{E}{(1+\nu)(1-2\nu)} \left[\frac{u}{r} + \nu \frac{\partial u}{\partial r} \right] \quad (13)$$

Under the assumption of elastic equilibrium (i.e. quasi-static motion) the equation of motion may be written

$$\frac{\partial \sigma_r}{\partial r} + \frac{2}{r} (\sigma_r - \sigma_\theta) = 0 \quad (14)$$

The boundary conditions are:

$$\sigma_r (a) = -p \quad (15)$$

$$\sigma_r (\infty) = -\Pi \quad (16)$$

where $\sigma_r(a) = \sigma_r$ at $r = a$, the cavity radius

$$\sigma_r(\infty) = \sigma_r \text{ at } r = \infty$$

p = internal cavity pressure

Π = hydrostatic (overburden) pressure

The solution of equations (12), (13) and (14), subject to the boundary conditions given by equations (15) and (16), is

$$\sigma_r = -\Pi - p \left(\frac{a}{r} \right)^3 \quad r \geq a \quad (17)$$

$$\sigma_\theta = -\Pi + \frac{p}{2} \left(\frac{a}{r} \right)^3 \quad r \geq a \quad (18)$$

$$u = \frac{1}{2} p \left(\frac{1+\nu}{E} \right) \frac{a^3}{r^2} \quad r \geq a \quad (19)$$

From equations (17) and (18) we see that $\sigma_r < \sigma_\theta$ and the Coulomb yield condition, equation (9), may be written

$$(1+\alpha)\sigma_\theta - \sigma_r = Y \quad (20)$$

Substituting equations (17) and (18) into equation (20) we find

$$(1+\alpha) \left[-\Pi + \frac{p}{2} \left(\frac{a}{r} \right)^3 \right] + \Pi + p \left(\frac{a}{r} \right)^3 = \gamma \quad (21)$$

It is clear that the left hand side of equation (21) assumes its greatest value at $r = a$. Yielding of the soil therefore begins at the cavity surface when the internal cavity pressure is

$$p_1 = \frac{2(\gamma + \alpha \Pi)}{(3 + \alpha)} \quad (22)$$

2.3.3 Quasi-Static Expansion of a Spherical Cavity in an Ideal Soil: Elastic - Plastic Solution

For $p > p_1$ (see equation (22)), a plastic zone is formed around the cavity. Symmetry considerations dictate that the outer boundary of the plastic zone be spherical. The outer radius of the plastic zone is denoted as "b".

It is assumed that the plastic region around the cavity only experiences quasi-static motion and, as in Section 2.3.2,

the equation of motion is

$$\frac{\partial \sigma_r}{\partial r} + \frac{2}{r} (\sigma_r - \sigma_\theta) = 0 \quad (23)$$

Also the yield condition

$$(1+\alpha)\sigma_\theta - \sigma_r = Y \quad (24)$$

is satisfied for all points in the plastic region.

Solving equations (23) and (24) simultaneously we find

$$\frac{\partial \sigma_r}{\partial r} + \frac{2}{r} \left[\frac{\alpha \sigma_r - Y}{1+\alpha} \right] = 0 \quad (25)$$

The solution to equation (25) is (Chadwick, 1959)

$$\sigma_r = \frac{Y}{\alpha} + Br^{\frac{-2\alpha}{1+\alpha}} \quad (26)$$

$$\sigma_{\theta} = \frac{Y}{a} + \left(\frac{B}{1+\alpha} \right) r^{\frac{-2\alpha}{1+\alpha}} \quad (27)$$

In order to evaluate the constant B, we apply the boundary condition given by equation (15) and find

$$B = -a^{\frac{2\alpha}{1+\alpha}} \left[p + \frac{Y}{a} \right] \quad (28)$$

and equations (26) and (27) become

$$\sigma_r = \frac{Y}{a} - \left(\frac{a}{r} \right)^{\frac{2\alpha}{1+\alpha}} \left[p + \frac{Y}{a} \right], \quad a \leq r \leq b \quad (29)$$

$$\sigma_{\theta} = \frac{Y}{a} - \left(\frac{a}{r} \right)^{\frac{2\alpha}{1+\alpha}} \left[\frac{p + \frac{Y}{a}}{1 + \alpha} \right], \quad a \leq r \leq b \quad (30)$$

The material on the elastic side of the plastic - elastic boundary, $r = b$, is at incipient failure. Therefore the following boundary conditions apply for the elastic region, $r \geq b$.

$$\sigma_r (b) = -p_1 = \frac{-2(Y+a\Pi)}{3+a} \quad (31)$$

$$\sigma_r (\infty) = -\Pi \quad (32)$$

where $\sigma_r (b)$ is the radial stress at $r = b$.

Applying the above boundary conditions, the stresses and the displacement in the elastic region can be shown to be (see Section 2.3.2)

$$\sigma_r = - \frac{2 (Y+a\Pi)}{(3+a)} \left(\frac{b}{r} \right)^3 - \Pi, \quad r \geq b \quad (33)$$

$$\sigma_\theta = \left(\frac{Y+a\Pi}{3+a} \right) \left(\frac{b}{r} \right)^3 - \Pi, \quad r \geq b \quad (34)$$

$$u = \frac{(Y+a\Pi) (1+\nu) b^3}{(3+a) (E) r^2}, \quad r \geq b \quad (35)$$

A further condition must be satisfied. It is required that the stresses σ_r and σ_θ be continuous across the elastic-plastic boundary, $r = b$. Setting $r = b$ in equation (29), equating equation (29) and equation (33) and solving for p we find

$$p = \left[\frac{3(1+\alpha)(Y+\alpha\Pi)}{\alpha(3+\alpha)} \right] \left[\frac{b}{a} \right]^{\frac{2\alpha}{1+\alpha}} - \frac{Y}{\alpha} \quad (36)$$

The pressure, p , given by equation (36) is the internal cavity pressure required to produce yielding of the ideal soil out to a radius $r = b$.

An expression for the ratio b/a may be derived assuming incompressibility in the plastic region. Under this assumption, conservation of mass requires

$$r^3 - r_0^3 = a^3 - a_0^3 \quad (37)$$

for

$$r > r_0, \quad a \leq r \leq b$$

and

$$a > a_0$$

if $r = b$ and $r_o = b - (u)_{r=b}$, where $(u)_{r=b}$ is the displacement at the elastic - plastic boundary,

$$b^3 - \left(b - (u)_{r=b} \right)^3 = a^3 - a_o^3 \quad (38)$$

Assuming $(u)_{r=b}$ is small, equation (38) can be approximated as

$$3(u)_{r=b} b^2 \approx a^3 - a_o^3 \quad (39)$$

We assume $a \gg a_o$ and equation (39) reduces to

$$3(u)_{r=b} b^2 \approx a^3 \quad (40)$$

Substituting equation (35) into equation (40),

$$\frac{b}{a} = \left[\frac{(3+\alpha) E}{3(Y+\alpha\Pi) (1+\nu)} \right]^{\frac{1}{3}} \quad (41)$$

Chadwick, et al., (1964) show that for a point source model of an underground explosion, under the assumptions that the plastic region is incompressible and that there are no inertial effects in the elastic region, the ratio b/a is a constant. The value of the constant depends on the material properties of the soil in which the explosion takes place. For a spherical charge model in which the initial cavity radius is not zero, with the assumption of incompressibility, they report that the ratio b/a is not constant but is a function of time, increasing very rapidly from unity, when plastic deformation first occurs, to a value closely approximating that predicted by the point source model. Hopkins (1960) reports essentially identical results for cavity formation in metals. On the basis of this work, it is assumed that the ratio b/a may be approximated as constant. In addition, the values of the ratio b/a calculated from equation (41) and using the soil properties assumed by Chadwick, et al., (1964) very closely approximate the maximum values of b/a obtained by these workers through numerical solutions of two non-linear second-order differential equations for $a(t)$ and $b(t)$. It is therefore assumed that equation (41) adequately represents the maximum or limiting

value of the ratio b/a .

Chadwick (1959) has shown that the effect of elastic compressibility is small when the cavity is assumed to expand quasi-statically. Inspection of equation (41) also shows that b/a is little affected by the value of ν . Therefore, for convenience, it is assumed that $\nu = 0.5$ in equation (41).

Assuming $\nu = 0.5$ and substituting equation (41) into equation (36),

$$p = \left[\frac{3(1+\alpha)(Y+\alpha\Pi)}{\alpha(3+\alpha)} \right] \left[\frac{(3+\alpha)E}{3(Y+\alpha\Pi)(1.5)} \right] \frac{2\alpha}{3(1+\alpha)} - \frac{Y}{\alpha} \quad (42)$$

Equation (42) gives the internal cavity pressure required to establish the maximum ratio of b/a .

Hill (1950) and Hopkins (1960) show that the internal cavity pressure required to establish the maximum ratio b/a is that pressure associated with the work required to increase the cavity volume by unity at an infinitely large plastic strain. In addition, Chadwick, et al., (1964) show that cavity formation in a soil having a realistic angle

of internal friction, ϕ , (Chadwick, et al., suggest $\alpha \approx 10^{-4}$ as a critical value) is essentially deadbeat (i.e. there is no significant contraction phase).

On the basis of the above results reported by Hill, Hopkins, and Chadwick, et al., it is assumed that the pressure, p , given by equation (42) is equal to P_f , the internal cavity pressure at termination of cavity growth (see Section 2.2). Therefore,

$$P_f = \left[\frac{3(1+\alpha)(Y+\alpha\Pi)}{\alpha(3+\alpha)} \right] \left[\frac{(3+\alpha)E}{3(Y+\alpha\Pi)(1.5)} \right]^{\frac{2\alpha}{3(1+\alpha)}} - \frac{Y}{\alpha} \quad (43)$$

2.4 The Radius of the Cavity Formed by a Contained Underground Nuclear Detonation

Substituting equation (43) into equation (4),

$$R_f = \left\{ \frac{P_v R_v^{3\gamma}(1 \text{ kt})}{\left[\frac{3(1+\alpha)(Y+\alpha\Pi)}{\alpha(3+\alpha)} \right] \left[\frac{(3+\alpha)E}{3(Y+\alpha\Pi)(1.5)} \right]^{\frac{2\alpha}{3(1+\alpha)}} - \frac{Y}{\alpha}} \right\}^{\frac{1}{3\gamma}} w^{\frac{1}{3}} \quad (44)$$

Equation (44) gives the radius of the cavity formed by a contained underground nuclear detonation as a function of the yield, W , of the detonation and the following properties of the geologic environment of the detonation:

1. the pressure of vaporization, P_v
2. the radius of vaporization for a one kiloton explosion, $R_v(1 \text{ kt})$
3. the adiabatic expansion coefficient of the cavity gas, γ
4. the angle of internal friction, ϕ

$$\left(\alpha = \frac{2 \sin \phi}{1 - \sin \phi}, \gamma = \frac{2 c \cos \phi}{1 - \sin \phi} \right)$$

5. the cohesive strength, c

$$\left(\gamma = \frac{2 c \cos \phi}{1 - \sin \phi} \right)$$

6. Young's Modulus, E
7. the hydrostatic (overburden) pressure, Π . $\Pi = \rho g h$ where ρ is the density of the overburden material, g is the

acceleration of gravity and
h is the depth of burial of
the event.

In order to compare cavity radii calculated by equation (44) with those observed from underground nuclear detonations, it is necessary to determine numerical values of the above physical properties for the appropriate geologic media.

CHAPTER 3

DETERMINATION OF PHYSICAL PROPERTY CONSTANTS

For the purpose of the initial study presented here, no attempt was made to obtain measured physical property values for the geologic environment of each nuclear event. Instead, representative physical property values were chosen for the gross geological categories (i.e. alluvium, tuff, granite, salt and rhyolite) generally used to describe the geologic media in which underground nuclear events have taken place. The choice of physical property values was based on previously published data (for example see Boardman, et al., 1964; Butkovich, 1967; Higgins and Butkovich, 1967).

3.1 Determination of Densities

The densities chosen as representative for alluvium, tuff, granite, salt and rhyolite are based on previously published values and are given in Table 3-1.

TABLE 3-1

DENSITY, ρ , FOR SEVERAL GEOLOGIC MEDIA

	<u>Alluvium</u>	<u>Tuff</u>	<u>Granite</u>	<u>Salt</u>	<u>Rhyolite</u>
ρ (g/cc)	1.7	1.9	2.7	2.3	2.3

3.2 Determination of P_v

Butkovich (1967) reports calculations of the shock pressure, P_v , required for vaporizing alluvium, tuff, granite and salt. In this work, it is assumed that silicate rocks may be treated as mixtures of SiO_2 and H_2O and that when any of the above geologic materials are shocked, unloading takes place along the experimentally determined shock Hugoniot. When the material thus unloads to a pressure near zero, there is a net gain in the internal energy of the material. Butkovich assumes that a net increase in internal energy of 2,800 calories per gram is sufficient to totally vaporize SiO_2 - H_2O mixtures (silicate rocks). Net increases in internal energy of 1,185 cal./g, and 620 cal./g. are taken as sufficient to vaporize salt and water respectively.

Using the above mentioned assumptions, a range of values for P_v are reported, depending on such parameters as the values chosen for the unshocked density and the water content of the material (Butkovich, 1967 and Higgins and Butkovich, 1967).

Values of P_v for the geologic media considered in this study are based on the work cited above. These values of P_v are given in Table 3-2.

TABLE 3-2
PRESSURE OF VAPORIZATION, P_v , FOR
SEVERAL GEOLOGIC MEDIA

	<u>Alluvium</u>	<u>Tuff</u>	<u>Granite</u>	<u>Salt</u>	<u>Rhyolite</u>
P_v (bars)	7.5×10^5	9.9×10^5	1.8×10^6	9.6×10^5	1.3×10^6

3.3 Determination of $R_v(1 \text{ kt})$

The extent of vaporization is reported by Butkovich (1967) and Higgins and Butkovich (1967) for several geologic materials. Based on this work, values for the radius of vaporization for a one kiloton explosion, $R_v(1 \text{ kt})$, have been chosen for the geologic media considered in this study. These values are given in Table 3-3.

TABLE 3-3

RADIUS OF VAPORIZATION FOR A 1 KILOTON
EXPLOSION, $R_v(1 \text{ KT})$, FOR SEVERAL
GEOLOGIC MEDIA

	<u>Alluvium</u>	<u>Tuff</u>	<u>Granite</u>	<u>Salt</u>	<u>Rhyolite</u>
$R_v(1 \text{ kt})(\text{ft}/\text{kt}^{1/3})$	7.4	7.1	6.0	7.4	6.5

3.4 Determination of γ

The values of γ used in the present study are based on the work of Higgins and Butkovich (1967) and are given in Table 3-4.

TABLE 3-4

ADIABATIC EXPANSION COEFFICIENT, γ ,
FOR SEVERAL GEOLOGIC MEDIA

	<u>Alluvium</u>	<u>Tuff</u>	<u>Granite</u>	<u>Salt</u>	<u>Rhyolite</u>
γ	1.125	1.14	1.03	1.10	1.08

3.5 Determination of the Cohesive Strength, c , Angle of Internal Friction, ϕ , and Young's Modulus, E

The task of choosing numerical values for c , ϕ and E for use in equation (44) is not an easy one. Physical

property values reported from field or laboratory measurements of the preshot geologic environment and used in equation (44) result in cavity radius predictions that are much too low (with the exception of events conducted in salt). This difficulty is believed due to the fact that the values of c , ϕ and E which influence the size of the cavity are associated with the geologic medium surrounding the detonation that has been significantly altered by the outgoing shock wave from the detonation.

The work of Allen and Duff (1968), Beaudet, et al., (1969) and Cassity, et al., (1969), among others, suggests that the strength of geologic materials is reduced after the material experiences brittle failure due to shock loading.

Giardini, et al., (1968) report triaxial compression data on normal, nuclear explosion shocked and mechanically shocked granodiorite. This work shows a decrease in Young's modulus, E , for the nuclear and mechanically shocked granodiorite up to a confining pressure of about 4.5 kilobars. Above a confining pressure of approximately 4.5 kilobars, the shocked and unshocked granodiorite exhibit the same values for E .

The use of pre-shot laboratory measurements of c , ϕ and E (Azery and Redmond, 1962) for the salt environment of the Salmon underground nuclear explosion results in a calculated cavity radius in good agreement with post-shot field measurements. Salt is a highly plastic material and probably experienced relatively little gross brittle fracture as a result of the Salmon detonation (Rawson, et al., 1966).

It is tentatively concluded on the basis of the information presented above that the brittle failure of the geologic environment due to shock loading from an underground nuclear explosion results in a decrease in the yield strength and Young's modulus of the geologic material. Further work is required in order to define quantitatively the effects of shock loading on the yield strength and elastic properties of geologic materials.

The final choice of values of c , ϕ and E for use in calculating cavity radii for events in alluvium, tuff, granite, salt and rhyolite was based on the following considerations: 1) values published in the literature for these and similar geologic materials (for example see Hardin, 1966; Boardman, et al., 1964) 2) the work cited

above on nuclear and mechanically shocked geologic materials and 3) field measurements of cavity radii formed by underground nuclear detonations. Specifically, sets of values were chosen for the parameters c , ϕ and E for each of the geologic media under consideration. The sets of values for these parameters were not chosen randomly but rather were chosen within the limits suggested by the published data for the natural state of the geologic materials considered and the work on shocked granodiorite reported by Giardini, et al., (1968) suggesting a reduced value of E for shocked earth materials. For each set of parameters, cavity radii were calculated, using equation (44), for events conducted in the appropriate geologic environment. The set of physical property values resulting in cavity radius predictions in good agreement with field measurements were assumed to be approximately representative of the shock altered geologic environment of an underground nuclear explosion in that medium. In this manner, values for c , ϕ and E were chosen for the five geologic media under consideration. These values of c , ϕ and E are shown in Table 3-5.

TABLE 3-5

COHESIVE STRENGTH, c , ANGLE OF INTERNAL FRICTION,
 ϕ AND YOUNG'S MODULUS, E , ASSUMED REPRESENTATIVE
 OF SEVERAL NUCLEAR EXPLOSION SHOCKED GEOLOGIC MEDIA

	<u>Alluvium</u>	<u>Tuff</u>	<u>Granite</u>	<u>Salt</u>	<u>Rhyolite</u>
c (bars)	1	50	200	50	100
ϕ (degrees)	40	35	55	30	50
E (bars)	2×10^4	2×10^4	$1 \times 10^{5*}$	2×10^5	8×10^4

*It is of interest to note that the value of E chosen for granite on the empirical basis described above agrees reasonably well with the range of values reported by Giardini, et al., (1968) for nuclear and mechanically shocked granodiorite. For granodiorite shocked to estimated peak pressures of 20 kb. and 35-40 kb., these workers report values of $E \approx 1.8 \times 10^5$ bars and 1×10^4 bars respectively at a confining pressure of 1 bar.

3.6 Summary of Physical Property Constants

A summary of the physical property constants used for calculating cavity radii from equation (44) is given in Table 3-6.

TABLE 3-6

SUMMARY OF PHYSICAL PROPERTIES
FOR SEVERAL GEOLOGIC MEDIA

	<u>Alluvium</u>	<u>Tuff</u>	<u>Granite</u>	<u>Salt</u>	<u>Rhyolite</u>
ρ (g/cc)	1.7	1.9	2.7	2.3	2.3
P_v (bars)	7.5×10^5	9.9×10^5	1.8×10^6	9.6×10^5	1.3×10^6
$R_v(1 \text{ kt})(\text{ft/kt}^{1/3})$	7.4	7.1	6.0	7.4	6.5
γ	1.125	1.14	1.03	1.10	1.08
c (bars)	1	50	200	50	100
ϕ (degrees)	40	35	55	30	50
E (bars)	2×10^4	2×10^4	1×10^5	2×10^5	8×10^4

3.7 Calculated "Yield Strength" for Alluvium, Tuff, Granite, Salt and Rhyolite

The "yield strength" is given by the equation (8)

(see Section 2.3.1)

$$Y = \frac{2 c \cos \phi}{1 - \sin \phi} \quad (45)$$

The yield strengths for alluvium, tuff, granite, salt and rhyolite were calculated using this equation and the physical properties given in Table 3-6. The resulting yield strengths are given in Table 3-7.

TABLE 3-7

CALCULATED "YIELD STRENGTH" FOR
ALLUVIUM, TUFF, GRANITE, SALT AND RHYOLITE

	<u>Alluvium</u>	<u>Tuff</u>	<u>Granite</u>	<u>Salt</u>	<u>Rhyolite</u>
Y (bars)	4	192	1270	173	549

CHAPTER 4

SCALING RELATIONSHIPS

An inspection of equation (44) reveals that the dependence of the cavity radius on the depth of burial of the event, h , and the relationship between the cavity radius and the Young's modulus, E , of the geologic environment surrounding the detonation are both functions of the parameters ϕ and γ . Therefore the scaling relationships between the cavity radius and the parameters h and E are dependent on the geologic environment in which the detonation takes place. This result is in contrast with the work reported by Heard* and Closmann (1969) where the scaling of the cavity radius with the parameters h and E is assumed to be independent of the geologic environment of the detonation.

The scaling relationships between the cavity radius and both the depth of burial of the event and the Young's modulus of the geologic environment of the detonation were

*This work is contained in a classified report to the U. S. Atomic Energy Commission and is available to those with the proper access clearance and the need to know.

investigated in order to examine quantitatively the variation of these scaling relationships for the five geologic media under consideration.

4.1 Dependence of the Cavity Radius on the Depth of Burial, h

For a given geologic medium the variables P_v , $R_v(1 \text{ kt})$, γ , ϕ , c , ρ and E were assumed to be fixed. Under this assumption, calculations were performed using equation (44) and the physical properties given in Chapter 3 to determine the quantity $R/W^{1/3}$ as a function of h for each of the geologic media under consideration. A log-log regression was then performed on the values of $R/W^{1/3}$ as a function of h for each geologic medium to determine the scaling relationships. The results of these calculations are given in Table 4-1.

TABLE 4-1

DEPENDENCE OF THE QUANTITY $R/W^{1/3}$
ON THE DEPTH OF BURIAL, h^*

Alluvium	$R/W^{1/3}$	$\propto$	$h^{-0.14}$
Tuff	$R/W^{1/3}$	$\propto$	$h^{-0.11}$
Granite	$R/W^{1/3}$	$\propto$	$h^{-0.08}$
Salt	$R/W^{1/3}$	$\propto$	$h^{-0.12}$
Rhyolite	$R/W^{1/3}$	$\propto$	$h^{-0.09}$

*It must be noted that the values of h for which the scaling relationships given in Table 4-1 are theoretically valid are limited by the implicit assumption in the derivation of equation (44) that $b < h$, i.e. the plastic region around the cavity does not intersect the surface.

The scaling relationships given in Table 4-1 show a significant dependence on the geologic environment of the detonation. The above scaling relationships for events in alluvium and tuff agree well with the empirically derived dependence of the cavity radius on the depth of burial as reported by Closmann (1969) and Heard (see Chapter 1).

Closmann and Heard report that

$$R \propto h^{-0.161}$$

and

$$R \propto h^{-0.11}$$

respectively for data samples dominated by events in alluvium and tuff.

4.2 Dependence of the Cavity Radius on Young's Modulus, E

The dependence of the cavity radius on Young's modulus was determined in the manner described in Section 4.1, using equation (44), the physical properties given in Chapter 3, and assuming all variables constant except E for a given geologic medium. The resulting scaling relationships are shown in Table 4-2.

TABLE 4-2

DEPENDENCE OF THE CAVITY RADIUS
ON YOUNG'S MODULUS, E

Alluvium	R	$\propto$	$E^{-0.16}$
Tuff	R	$\propto$	$E^{-0.14}$
Granite	R	$\propto$	$E^{-0.19}$
Salt	R	$\propto$	$E^{-0.14}$
Rhyolite	R	$\propto$	$E^{-0.18}$

The scaling relationships between the cavity radius and Young's modulus given in Table 4-2 show a significant dependence on the geologic environment of the detonation. The scaling relationships between the cavity radius and E given above do not agree well with those reported by Closmann (1969) and Heard (see Chapter 1). Closmann and Heard report

$$R \propto (E^{.514} / \mu^{.576}) \propto E^{-.062}$$

and

$$R \propto (E^{.62} / \mu^{.67}) \propto E^{-.05}$$

respectively.

The scaling of cavity radius with depth of burial and Young's modulus is dependent, in both cases, on the parameters ϕ and γ . It is not clear at this time why the scaling of cavity radius with depth agrees so well with the work of Closmann and Heard and the scaling of cavity radius with Young's modulus does not.

CHAPTER 5

STATISTICAL ANALYSIS OF OBSERVED VERSUS CALCULATED CAVITY RADII

Preliminary statistical analyses were performed on the observed (measured) cavity radii versus the cavity radii calculated using equation (44) and the physical properties given in Chapter 3. These analyses were performed in order to determine the usefulness of equation (44) for predicting cavity radii.

5.1 The Data Sample

The data sample used in the statistical analyses consisted of cavity radii measured for 172 underground nuclear events. Separating the data sample by geologic environment, the data sample contained 112 events in alluvium, 52 events in tuff, 2 events in granite, 2 events in salt and 4 events in rhyolite. The events in each of the above geologic media encompassed the approximate ranges in yield, depth of burial and scaled depth of burial shown in Table 5-1.

TABLE 5-1

APPROXIMATE RANGES OF YIELD, DEPTH OF BURIAL AND SCALED DEPTH OF BURIAL FOR CAVITY RADIUS DATA USED IN STATISTICAL ANALYSIS

	<u>Approximate Yield Range, kt</u>	<u>Approximate Depth of Burial Range, ft</u>	<u>Approximate Scaled Depth of Burial Range, ft/kt^{1/3}</u>
Alluvium	<0.1 - ≈20	<200 - >1500	292 - 1238
Tuff	≈1 - ≈200	≈300 - ≈4000	325 - 1419
Granite	4.8 - 12.2	950 - 1200	522 - 562
Salt	3 - 5.3	1184 - 2700	822 - 1547
Rhyolite	65 - 1200	≈1800 - ≈3800	354 - 542
Total Sample	<0.1 - 1200	<200 - ≈4000	292 - 1547

5.2 Statistical Analysis of Observed versus Calculated Cavity Radii for Events in the Same Geologic Environment

5.2.1 Cavities Formed in Granite, Salt and Rhyolite

No regression analysis was performed on the cavity radius data for events in these geologic media because of the extremely small data sample. The eight events in granite, salt and rhyolite were, however, included in the regression analysis of the total cavity radius data sample (see Section 5.3).

5.2.2 Cavities Formed in Alluvium

It was concluded after plotting the observed versus calculated radii for events in alluvium in linear space, that the data could be fit with a linear regression.

A linear regression analysis resulted in the following statistical relationship between the observed cavity radius, R_O , and the calculated cavity radius, R_C :

$$R_O = 3. + 0.96 R_C \quad (46)$$

where R_O and R_C are in feet. The correlation coefficient and standard error on the slope (Crow, et al., 1960) were calculated to be 0.94 and 0.03 respectively.

Equation (46) is close to the desired relationship between R_O and R_C ; namely

$$R_O = 0. + 1.0 R_C$$

A significance test (Crow, et al., 1960) was performed on the slope of equation (46) to determine whether the slope of 0.96 was statistically different from the desired slope of 1.00.

On the basis of this calculation, it was concluded that at the 95% confidence level the slope of 0.96 resulting from the regression analysis was not statistically different from a slope of 1.00.

On the basis of the above analysis, it was concluded that the cavity radii formed by underground nuclear events in alluvium are well represented by equation (44).

Based on the form of equation (44), it was assumed that errors in predicting cavity radius using equation (44) would be percentage type errors. Therefore, a regression analysis of the observed versus calculated cavity radii for events in alluvium was also performed in log-space in order to determine a standard error of estimate, σ , for equation (44) when used to predict cavity radius for an event in alluvium. The resulting standard error of estimate is $\sigma = 1.13$.

5.2.3 Cavities Formed in Tuff

The regression analysis performed on the observed versus calculated cavity radii for events in tuff was identical to that described in Section 5.2.2. The resulting

statistical relationship between the observed cavity radius, R_O , and the calculated cavity radius, R_C , was found to be

$$R_O = 13. + 0.93 R_C \quad (47)$$

where R_O and R_C are in feet. The correlation coefficient and standard error on the slope were calculated as 0.97 and 0.03 respectively.

Once again, equation (47) is close to the desired relationship between R_O and R_C .

A significance test (Crow, et al., 1960) was performed on the slope of equation (47) to determine whether this slope was statistically different from a slope of 1.00. The results of the calculation showed that the slope of 0.93 was statistically different from 1.00 at the 95% confidence interval but not at the 98% confidence level.

A further significance test was performed to determine whether the slopes of equation (46) for the alluvium data and equation (47) for the tuff data were statistically distinct. At the 95% confidence level, these two slopes are not statistically different.

As discussed in Section 5.2.2, a regression analysis in log-space was performed to determine the standard error of estimate for equation (44) when used to predict cavity radius for an event in tuff. The resulting standard error of estimate was $\sigma = 1.12$.

On the basis of the above results, it is tentatively concluded that equation (44) adequately represents cavities formed by underground nuclear explosions in tuff. It is also concluded that the tuff and alluvium data populations are not statistically distinct when analyzed in the manner described.

5.3 Statistical Analysis of Observed versus Calculated Cavity Radii for Events in Alluvium, Tuff, Granite, Salt and Rhyolite

The regression analysis performed on the observed versus calculated cavity radii for the total data sample as described in Section 5.1 was identical to that discussed above for events in tuff and alluvium. The resulting statistical relationship between R_O and R_C was found to be

$$R_O = 4. + 0.97 R_C \quad (48)$$

where R_0 and R_c are again in feet. The correlation coefficient and standard error on the slope were calculated to be 0.97 and 0.02 respectively.

As discussed in section 5.2.2, a regression was performed in log-space to determine a standard error of estimate for equation (44) when used to predict cavity radius for events in alluvium, tuff, granite, salt or rhyolite. The standard error of estimate was found to be $\sigma = 1.13$.

A significance test (Crow, et al., 1960) was performed to determine whether the slope of equation (48) was statistically different from 1.00. The slope 0.97 was found not to be statistically different from a slope of 1.00 at the 95% confidence level.

Additional significance tests showed that the slopes of equations (46) and (47), for the separate alluvium and tuff data samples, were not statistically different from the slope of equation (48) at the 95% confidence interval.

Figure 5-1 is a plot of observed versus calculated cavity radii for 49 underground nuclear events in alluvium, tuff, granite, salt and rhyolite for which the necessary

FIGURE 5-1 OBSERVED VERSUS CALCULATED CAVITY RADIUS FOR UNDERGROUND NUCLEAR EVENTS IN ALLUVIUM, TUFF, GRANITE, SALT AND RHYOLITE

data are unclassified. Much of these data were previously reported by Higgins and Butkovich (1967). The data have been plotted in log-space in order to conveniently show the standard error of estimate, $\sigma = 1.13$, found applicable to equation (44) when used to predict cavity radii for events in these geologic media. Also shown in Figure 5-1 is the $R_o = R_c$ line.

It will be noted that in Figure 5-1, the tuff data are not evenly distributed about the $R_o = R_c$ line. Examination of Figure 5-1 shows that for eight of eleven events detonated in tuff, the observed cavity radius is larger than that predicted by equation (44). It's important to note, however, that the uneven distribution of the tuff data shown in the figure is not typical of the distribution of the total tuff data sample used in the analysis.

CHAPTER 6

SUMMARY AND RECOMMENDATIONS

6.1 Summary of Results

The results of this study may be summarized as follows:

1. An equation was derived that can be used to predict cavity radii for underground nuclear events in five geologic media; alluvium, tuff, granite, salt and rhyolite.
2. The derived equation directly relates the radius of the cavity formed by an underground nuclear detonation to, among other parameters, several physical properties of the geologic environment of the detonation. These physical properties are the:
 - vaporization pressure
 - adiabatic expansion coefficient of the cavity gas

- density
- Young's modulus
- cohesive strength and
- angle of internal friction.

3. The values of Young's modulus, cohesive strength and angle of internal friction which influence the size of the cavity are associated with the geologic environment surrounding the detonation that has been altered by the outgoing shock wave from the explosion.

4. The scaling relationships between the cavity radius and the depth of burial of the event and the Young's modulus of the geologic environment of the detonation are dependent upon the geologic environment in which the detonation takes place.

The principal advantage of the equation reported here over other methods of predicting the cavity radius is that equation (44) directly relates the cavity radius to measurable physical properties of the geologic environment of the detonation. Therefore, equation (44) may readily be used to predict the cavity radius for an underground nuclear explosion in a new geologic environment if the required physical property data are obtained. It is envisioned that the equation derived here will find extensive application in the Plowshare program for predicting the size of the cavity for underground nuclear detonations in new geologic media.

6.2 Recommended Additional Work

The work presented in this Technical Memorandum has indicated several areas where additional work is required. The additional work required may be summarized as follows:

1. Much more information is required concerning the physical properties of the shock altered geologic material outside the cavity. Lacking such information, a complete verification

of the usefulness of equation (44) for calculating cavity radius was not possible and values for the required physical properties were chosen on a somewhat empirical basis. The required data can probably most easily be obtained by shocking samples of geologic media, taken at depth, in the laboratory and then measuring the physical properties of the shocked geologic material. Giardini, et al., (1968) have reported such measurements on the granodiorite of the Hardhat event at the Nevada Test Site.

2. Further examination of equation (44) is required to investigate more completely the influence of the cohesive strength and the angle

of internal friction on the
cavity radius.

3. It is envisioned that the equation derived in this report for predicting cavity radius will find extensive application in the Plowshare program where underground nuclear detonations are planned in new geologic media. The geologic environments for many of these planned detonations are saturated with hydrocarbons, either in liquid or gaseous form. Therefore, a study needs to be performed to determine the effect on the cavity radius of the porosity and liquid content of the geologic environment of an underground nuclear detonation.
4. Finally, it has been noted that values of the maximum ratio of the plastic-elastic radius and the cavity radius

calculated from equation (41) are in some cases very similar to values reported for the ratio of the cracking radius and cavity radius (Boardman, et al., 1964). An investigation is planned, in conjunction with hydrodynamic computer code calculations (Beaudet, et al., 1969 and Cassity, et al., 1969), to determine the relationship, if any, between the maximum plastic-elastic radius as given by equation (41) and the outer limit of fracturing produced by an underground nuclear detonation.

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