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FRACTIONAL PARENTAGE COEFFICIENTS
IN INTERMEDIATE COUPLING

by

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Abstract

Fractional parentage coefficients are obtained which connect certain low-lying states in the lp -shell without the necessity of referring to either an LS or jj representation. The possibility of doing this algebraically is based on the generator method of producing the wave functions of these states.

1. INTRODUCTION

In order to calculate various properties of nuclei, it is often useful to express the wave function for n nucleons in a shell in terms of the different ways of coupling the n th nucleon to particular states of the configuration with $(n-1)$ nucleons in the shell. The coefficients connecting particular states are known as fractional parentage coefficients (CFP) and many configurations have been treated to present these coefficients in tabular form. Generally these coefficients are calculated in a given representation, either the limit of negligible spin-orbit coupling (the LS-representation) or for very strong spin-orbit coupling (the jj-representation).

However, these limiting cases are seldom realized in actual nuclei and the advantage of using a particular representation disappears rapidly if a mixing of states of such a representation becomes necessary. In the 1p shell this mixing is especially great so that the intermediate-coupling wave functions are very cumbersome in either representation. The possibility⁽¹⁾ of generating intermediate-coupling functions from a Nilsson representation offers a simple way to obtain fractional parentage coefficients without referring to the LS or jj representations.

In the first part of the paper this procedure is carried through for those cases of the 1p shell in which there is no ambiguity about which generator state to use. Tables are given for these cases. A discussion of what is possible for more complicated cases is also included.

2. PROCEDURE

The cases wherein the procedure is straight-forward are the "simple cases" of reference 1 where two identical nucleons or one, three, or four nucleons are put into a single Nilsson level. These cases include He^5 , He^6 , Li^7 , and Be^8 and their mirror partners which come from filling level No. 3

1) D. Kurath and L. Picman, Nuclear Phys. 10, 313 (1959).

of the Nilsson scheme, Fig. 1, with prolate deformation ($\eta > 0$). In similar fashion one can treat C^{12} , C^{13} , C^{14} , and N^{15} and their mirror partners by putting holes into level No. 4 with oblate deformation ($\eta < 0$).

In these simple cases, the generator for each case is a single $n \times n$ determinant which has (T, T_3) as good quantum numbers for isobaric spin and K as a good quantum number for the projection of the total angular momentum on the nuclear symmetry axis. The generator can be expressed as a sum of components of different total angular momentum I :

$$\chi_{Kp}^T (1 \dots n) = \sum_{I, s} c_{Kp}^{IsT} \phi_{Kp}^{IsT} = \sum_I \left[N_{Kp}^{IT} \right]^{-1} \psi_{Kp}^{IT} (1 \dots n). \quad (1)$$

Here p is an added label that identifies the Nilsson configuration and the label s identifies states from the particular representation used to make the expansion. The N_{Kp}^{IT} are normalization coefficients given by

$$\left[N_{Kp}^{IT} \right]^{-2} = \sum_s \left| c_{Kp}^{IsT} \right|^2. \quad (2)$$

Although the expansion coefficients c_{Kp}^{IsT} depend on the representation used (such as LS or jj), the normalization coefficients (whose sign is positive by definition) do not depend on the representation and could be extracted from χ by repeated application of the J^+ and J^- operators. We shall see that the CFP for these simple cases require only the N_{Kp}^{IT} .

In order to obtain the CFP one makes the usual determinant expansion of χ into the sum of products of $(n-1) \times (n-1)$ determinants by single-nucleon functions. There are n such terms which can be coupled to n functions having good isobaric spin T , namely,

$$n^{\frac{1}{2}} \chi_{Kp}^T (1 \dots n) = \sum_r B_r \left\{ \chi_{K'r}^{T_0} (1 \dots n-1) \phi_{kr}^{(n)} \right\}_K^T. \quad (3)$$

Fig. 1. Energy levels of the $1p$ shell as a function of the parameter η (according to Nilsson). The energy scale is plotted in units of the spin-orbit strength a .

The factor $n^{\frac{1}{2}}$ on the left is the ratio, $[n!/(n-1)!]^{\frac{1}{2}}$, of normalization factors for χ_K and $\chi_{K'}$. Next the two functions within brackets are expanded into sums of angular momentum eigenfunctions. The products of these eigenfunctions can be combined via the usual angular momentum coupling procedure so that Eq. (3) becomes

$$n^{\frac{1}{2}} \chi_{Kp}^T(1 \dots n) = \sum_{r, I_0 s_0, j, J} B_r c_{K'r}^{I_0 s_0 T_0} d_{kr}^j (I_0 j K' k | JK) \psi_K^{JT}(I_0 s_0 T_0 j), \quad (3')$$

where $(I_0 j K' k | JK)$ is a vector-coupling coefficient, and ψ_K^{JT} is defined by

$$\psi_K^{JT}(I_0 s_0 T_0 j) = \sum_{\mu, \nu} (I_0 j \mu \nu | JK) \left\{ \phi_{\mu}^{I_0 s_0 T_0} \phi_{\nu}^j \right\}_K^T.$$

The n -nucleon function of total angular momentum I which one obtains by coupling the n th nucleon with angular momentum j to the $(n-1)$ -nucleon function with angular momentum I_0 , arising from the generator $\chi_{K_0 r}^{T_0}$, is given by

$$N_{K_0 r}^{I_0 T_0} \sum_{s_0} c_{K_0 r}^{I_0 s_0 T_0} \psi_K^{IT}(I_0 s_0 T_0 j) \equiv \Psi_K^{IT}(I_0 K_0 r j). \quad (4)$$

From the fundamental definition one sees that the CFP are

$$\langle ITKp(1 \dots n) || I_0 T_0 K_0 p_0(1 \dots n-1) j(n) \rangle = N_{Kp}^{IT} \int \chi_{Kp}^{T*} \Psi_K^{IT}(I_0 K_0 p_0 j). \quad (5)$$

With the help of Eqs. (3') and (4), this expression becomes

$$\begin{aligned} & \langle ITKp || I_0 T_0 K_0 p_0 j \rangle \\ &= n^{-\frac{1}{2}} N_{Kp}^{IT} N_{K_0 p_0}^{I_0 T_0} \sum_r B_r d_{kr}^j (I_0 j K' k | IK) \sum_{s_0} c_{K_0 p_0}^{I_0 s_0 T_0} c_{K'r}^{I_0 s_0 T_0}. \end{aligned} \quad (5')$$

So far the procedure has been general in that it just relates the states obtained from a single n -nucleon generator to those obtained from a single $(n-1)$ -nucleon

generator. This is a basic step for any of the cases, and the further applications to complicated cases will be treated later.

However, this completes the solution for the simple cases defined at the beginning of this section, because they are characterized by having a single determinant as generator. Furthermore, in the determinant expansion, indicated by the index r , the terms which have the $I_0 T_0$ of the simple states all have

$$c_{K'r}^{I_0 s_0 T_0} \propto c_{K_0 p_0}^{I_0 s_0 T_0}$$

with proportionality factors independent of s_0 so that Eq. (2) can be used to carry out the sum over s_0 . Then with the help of the relationships between the angular momentum expansion coefficients of χ_K and χ_{-K} , the sum over r can also be carried out, and in all the simple cases the CFP expression can be reduced to a single term.

The reason for using wave functions of holes in the upper half of the $1p$ shell is that in this way it is very simple to obtain the CFP between states arising from the ground-state bands in the adjacent nuclei. Actually there are also states which do not belong to the ground-state band in the parentage of these nuclei, but such CFP are usually not of interest since most experimental observations concern only the low-lying states. In order to keep phases straight for the hole-particle relationships, a conventional way⁽²⁾ was selected for choosing the hole-partners of the generating functions for particles. The relationship between hole CFP and particle CFP is

$$\frac{\left\langle I T K p (1 \dots n) \parallel I_0 T_0 K_0 p_0 (1 \dots n-1) j(n) \right\rangle}{\left\langle I_0 T_0 K_0 p_0 (n \dots 12) \parallel j(n) I T K p (n+1 \dots 12) \right\rangle} \equiv H_n \quad (6)$$

$$= (-1)^{n+1+I_0+T_0-I-T} \left[\frac{(13-n)(2I_0+1)(2T_0+1)}{n(2I+1)(2T+1)} \right]^{\frac{1}{2}},$$

2) Such procedure is discussed by G. Satchler, Nuovo Cim. 4, 549 (1956).

where n is the number of nucleons in the $1p$ shell. Therefore for the latter half of the shell the hole CFP and factors H_n are listed separately.

3. SIMPLE CASES

Here expression (5') for the CFP reduces to the following two forms. The first is applicable for the first half of the shell, $n < 6$, while the second is the hole CFP, used for $n > 6$. They are

$$\begin{aligned} \langle I T K p \mid \mid I_0 T_0 K_0 p_0 j \rangle &= \left[N_{Kp}^{IT} / N_{K_0 p_0}^{I_0 T_0} \right] d_{\frac{1}{2}}^j(3) (I_0 j K_0 K - K_0 \mid IK) f_n(j) \\ \langle I_0 T_0 K_0 p_0 \mid \mid j I T K p \rangle &= \left[N_{K_0 p_0}^{I_0 T_0} / N_{Kp}^{IT} \right] d_{\frac{1}{2}}^j(4) (j I K - K_0 - K \mid I_0 - K_0) f_n(j). \end{aligned} \quad (7)$$

The generators appropriate to a given number of nucleons are given in the second column of Table I and indicate the meaning of the labels p and p_0 . The normalization coefficients N_{Kp}^{IT} appropriate to these generators are listed in Table II for the various values of I . The coefficients d_{kp}^j refer to particular Nilsson functions, and phases were chosen as in reference 1 such that

$$d_{-kp}^j = (-1)^{j+\frac{1}{2}} d_{kp}^j. \quad (8)$$

Aside from the vector-coupling coefficient, the remaining factor is $f_n(j)$ which contains the effects of determinant normalization and isobaric-spin couplings; it is listed in Table I. In order to convert the hole CFP to particle CFP the value of H_n from Eq. (6), as tabulated in Table I, is needed for the latter half of the shell.

The factors N_{Kp}^{IT} and d_{kp}^j depend on the importance of deformation relative to spin-orbit coupling, expressed by the Nilsson parameter η . One can go to the LS limit and check these CFP against those compiled by Jahn and van Wieringen⁽³⁾. In order to do this, one must first convert

3) H. A. Jahn and H. van Wieringen, Proc. Roy. Soc. (London) A209 502 (1951).

the CFP of Jahn and van Wieringen, who used wave functions in an $M_L M_S$ representation. These CFP can be converted with the aid of the Wigner 9j-coefficients

$$\frac{\langle p^n [f] \text{ TSLI} \parallel p^{n-1} [f_0] T_0 S_0 L_0 I_0 p_n j \rangle}{\langle p^n [f] \text{ TSL} \parallel p^{n-1} [f_0] T_0 S_0 L_0 \rangle} = [(2L+1)(2S+1)(2I_0+1)(2j+1)]^{\frac{1}{2}} \times \begin{pmatrix} L & L_0 & 1 \\ S & S_0 & \frac{1}{2} \\ I & I_0 & j \end{pmatrix}. \quad (9)$$

Comparison of the generated CFP with those in the numerator on the left of Eq. (9) shows that there is a sign difference which is listed in the last column of Table I. This is no doubt due to the choice of over-all sign of the generated angular momentum eigenfunctions. One could of course choose the phases such that in the LS-limit the CFP would go over into the CFP of Jahn and van Wieringen.

As one can see from Table II, the normalization coefficients can be conveniently expressed in terms of the $[d_{\frac{1}{2}}^j]^2$ which are functions of the Nilsson deformation parameter η . The coefficient for $n = 5$, $I = \frac{1}{2}$ has been included because this particular state also arises from only a single generator since the competing generator has $K = \frac{3}{2}$. Therefore the states with $I = \frac{1}{2} = T$ for Be^9 and B^{11} have the simple CFP expressions

$$\begin{aligned} \langle \text{Be}^9 (\tfrac{1}{2} \tfrac{1}{2}) \parallel \text{Be}^8 (I_0 0) j \rangle &= \left[N_0^{I_0} / N_{\frac{1}{2}}^{\frac{1}{2}} \right] d_{\frac{1}{2}}^j(4) (I_0 j 0 \tfrac{1}{2} \mid \tfrac{1}{2} \tfrac{1}{2}) \left(\frac{2I_0 + 1}{\sqrt{5}} \right) \\ \langle \text{C}^{12} (I_0) \parallel \text{B}^{11} (\tfrac{1}{2} \tfrac{1}{2}) j \rangle &= \left[N_0^I / N_{\frac{1}{2}}^{\frac{1}{2}} \right] d_{\frac{1}{2}}^j(3) (\tfrac{1}{2} j \tfrac{1}{2} - \tfrac{1}{2} \mid I_0) (-1)^{j+\frac{1}{2}}. \end{aligned} \quad (10)$$

Numerical values for the factors in the CFP expressions are contained in Table III (for the lower half of the shell) and in Table IV (for the

Table I. Generators used in obtaining the CFP are listed in column 2 for different numbers of lp nucleons. The numbers refer to the Nilsson levels of Fig. 1; overlined numbers mean $k < 0$; ν, π refer to neutrons and protons; [] means a full level. The factor $f_n(j)$ in the CFP expression of Eq. (7) is in column 3. The factor H_n of Eq. (6), which relates hole CFP to particle CFP, is given in column 4. Column 5 lists the phase factor relating generated CFP at the LS limit to the CFP of Jahn and van Wieringen.

n	Generators (TKp, T ₀ K ₀ p ₀)	$f_n(j)$	H_n	Phase factor at LS limit
2	$ 3\nu \overline{3\nu} , 3\nu$	$(-1)^{j+1/2}\sqrt{2}$	-	$(i)^{L+L_0-2I_0}$
3	$ 3\pi 3\nu \overline{3\nu} , 3\nu \overline{3\nu} $	$(1/\sqrt{2})$	-	$(i)^{L+L_0-2I}$
4	$ 3\pi 3\nu \overline{3\pi} \overline{3\nu} , 3\pi 3\nu \overline{3\nu} $	$(-1)^{j+3/2}\sqrt{2}$	-	$(i)^{L+L_0+2I_0}$
9	$ [2] [3] 4\nu , [2] [3] $	$-\sqrt{2}$	$(-1)^{I+1/2} \sqrt{\frac{2(2I_0+1)}{9(2I+1)}}$	$(i)^{L+L_0-2I}$
10	$ [2] [3] 4\nu \overline{4\nu} , [2] [3] 4\nu $	$(-1)^{j+3/2}(1/\sqrt{2})$	$(-1)^{I_0+1/2} \sqrt{\frac{2I_0+1}{5(2I+1)}}$	$(i)^{L+L_0+2I_0}$
11	$ [2] [3] 4\nu 4\pi \overline{4\nu} , [2] [3] 4\nu \overline{4\nu} $	$-\sqrt{2}$	$(-1)^{I-1/2} \sqrt{\frac{3(2I_0+1)}{11(2I+1)}}$	$(i)^{L+L_0+2I}$

TABLE II. Normalization coefficients for the generated functions. The quantity $\left[N_{Kp}^{IT} \right]^{-2}$ is tabulated as a function of a^2 and β^2 , and N_{Kp}^{IT} is defined to be positive. For prolate deformation $a^2 = \left[d_{\frac{1}{2}}^{\frac{1}{2}} \right]^2 = 1 - \beta^2$. For oblate deformation $a^2 = \left[d_{\frac{1}{2}}^{\frac{1}{2}}(4) \right]^2 = 1 - \beta^2$.

n	T	K	I	$\left[N_{Kp}^{IT} \right]^{-2}$
1	1/2	1/2	1/2	a^2
			3/2	β^2
2	1	0	0	$\frac{1}{2} [\beta^4 + 2a^4]$
			2	$\frac{1}{2} [\beta^4 + 4\beta^2 a^2]$
3	1/2	1/2	1/2	$\frac{1}{70} [7\beta^6 + 63\beta^4 a^2 + 70a^6]$
			3/2	$\frac{1}{70} [42\beta^6 + 42\beta^4 a^2 + 126\beta^2 a^4]$
			5/2	$\frac{1}{70} [3\beta^6 + 33\beta^4 a^2 + 84\beta^2 a^4]$
			7/2	$\frac{1}{70} [18\beta^6 + 72\beta^4 a^2]$
4	0	0	0	$\frac{1}{70} [21\beta^8 + 28\beta^6 a^2 + 126\beta^4 a^4 + 70a^8]$
			2	$\frac{1}{70} [40\beta^8 + 180\beta^6 a^2 + 150\beta^4 a^4 + 280\beta^2 a^6]$
			4	$\frac{1}{70} [9\beta^8 + 72\beta^6 a^2 + 144\beta^4 a^4]$
5	1/2	1/2	1/2	$\frac{1}{10} [3\beta^6 + 3\beta^4 a^2 + 9\beta^2 a^4]$

TABLE III. The reciprocals of the normalization coefficients, $\left[N_{Kp}^{IT}\right]^{-1}$, as functions of the Nilsson parameter η for prolate deformation appropriate to the first half of the 1p shell. The quantities $d_{\frac{1}{2}(3)}^j$ are also listed.

n	T	K	I	η						
				+4	+6	+8	+10	+12	+20	$+\infty$
1	1/2	1/2	1/2	.350	.411	.447	.470	.487	.521	.577
			3/2	.937	.912	.895	.882	.874	.853	.816
2	1	0	0	.632	.612	.600	.594	.589	.582	.577
			2	.775	.791	.800	.805	.808	.813	.816
3	1/2	1/2	1/2	.393	.409	.418	.423	.426	.434	.447
			3/2	.697	.676	.665	.658	.653	.644	.632
			5/2	.299	.329	.347	.360	.368	.386	.414
			7/2	.520	.517	.513	.509	.506	.497	.478
4	0	0	0	.482	.467	.460	.456	.454	.450	.447
			2	.763	.762	.760	.759	.758	.757	.756
			4	.431	.449	.459	.464	.468	.474	.478
5	1/2	1/2	1/2	.493	.478	.470	.465	.462	.455	.447
$d_{1/2(3)}^{3/2}$				.937	.912	.895	.882	.874	.853	.816
$d_{1/2(3)}^{1/2}$				-.350	-.411	-.447	-.470	-.487	-.521	-.577

TABLE IV. The reciprocals of the normalization coefficients, $\left[N_{Kp}^{IT} \right]^{-1}$, as functions of the Nilsson parameter η for oblate deformation appropriate to the upper half of the 1p shell. The quantities $d_{\frac{1}{2}(4)}^j$ are also listed.

n	T	K	I	η						
				-2	-4	-6	-8	-10	-12	$-\infty$
11	1/2	1/2	1/2	.943	.838	.766	.723	.694	.675	.577
			3/2	.333	.546	.643	.691	.720	.738	.816
10	1	0	0	.892	.733	.656	.622	.605	.597	.577
			2	.451	.680	.755	.783	.796	.803	.816
9	1/2	1/2	1/2	.844	.636	.547	.510	.492	.482	.447
			3/2	.407	.563	.599	.610	.615	.617	.632
			5/2	.333	.455	.470	.466	.460	.454	.414
			7/2	.108	.266	.348	.388	.411	.425	.478
8	0	0	0	.801	.576	.500	.474	.463	.458	.447
			2	.580	.747	.764	.763	.761	.760	.756
			4	.146	.332	.409	.440	.454	.462	.478
7	1/2	1/2	1/2	.288	.398	.423	.431	.434	.437	.447
$d_{1/2(4)}^{3/2}$				.333	.546	.643	.691	.720	.738	.816
$d_{1/2(4)}^{1/2}$				-.943	-.838	-.766	-.723	-.694	-.675	-.577

upper half). In order to obtain an idea of the values of η which best describe the physical situation, we can employ the relationship obtained in reference 1 between η and γ , the usual intermediate-coupling parameter which indicates the relative strength of spin-orbit coupling to central interaction. Rough values of γ have been obtained by comparing intermediate-coupling calculations with experiment. Translating these values into corresponding values of η indicates that for the first half of the shell η varies from about +8 or +10 for $A = 6$, and about +10 for $A = 7$,[†] to about +10 to +12 for $A = 8$ and 9. Similarly, for the upper half-shell η starts at about -8 to -6 for $A = 11$, is about -6 for $A = 12$, -6 to -4 for $A = 13$, and finally becomes about -4 to -2 for $A = 14$.

In evaluating the CFP, the same value of η was chosen both for the n -nucleon and the $(n-1)$ -nucleon functions. Since the physical situation indicates that this is not the case, an approximation has already been made in carrying out the overlap integrals to evaluate the CFP. If we had kept different values of η for the two nuclei, the CFP expressions would be multiplied by factors

$$N_K^I(\eta) N_K^I(\eta') \sum_s c_K^{Is}(\eta) c_K^{Is}(\eta'). \quad (11)$$

However the wave functions are rather slowly varying functions of η in general, so that expression (11) gives factors which lie between 0.99 and 1.00 for the values of η pertinent to the physical region. Therefore the CFP are good to about 1% insofar as this factor is concerned, which is the same sort of accuracy one would assume considering the imperfect overlaps of radial wave functions in adjacent nuclei of different size.

4. COMPLEX CASES

In these cases more than one generator comes into consideration for producing the wave functions of the low-lying states. While the

[†]Note that the $I = \frac{5}{2}$ state of Li^7 is not well represented by the generated function farther away from the LS limit, but it is still quite good at $\eta = 10$. (See ref. 1.)

procedure for obtaining CFP can still be employed, individual cases must be considered carefully to see how well the wave-functions represent particular states.

$$\underline{\text{Li}^6 (T = 0)}$$

Here the generator $|3\pi \ 3\nu|$ with $K = 1$ gives accurate wave functions for $I = 2$ and $I = 3$ but only a rough approximation for the lowest $I = 1$ state. To obtain a better $I = 1$ state, the $K = 0$ generator must be considered¹ also.

$$\underline{\text{Be}^9 (T = \frac{1}{2})}$$

Examination of the wave functions indicates that except very close to the LS limit there is very little band-mixing between the $K = \frac{1}{2}$ and $K = \frac{3}{2}$ generators in Be^9 . In terms of a rotational approach, this is because the band heads are quite widely separated and the Coriolis term is not strong. For $\eta < +14$, the generator $|[3] \ 4\nu|$ with $K = \frac{1}{2}$, which gives an accurate $I = \frac{1}{2}$ function for Table III, also provides adequate excited $I = \frac{3}{2}, \frac{5}{2}, \text{ and } \frac{7}{2}$ functions. Similarly the generator $|[3] \ 2\nu|$ with $K = \frac{3}{2}$ provides wave functions for the lowest lying states with $I = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \text{ and } \frac{9}{2}$. There is a lack of orthogonality between states of the same I from the two generators, but it is small except for $I = \frac{9}{2}$. The resultant CFP are certainly good to about 5%, which is adequate for most purposes.

$$\underline{\text{B}^{10} (T = 0, I = 3)}$$

There are so many possible generators for the B^{10} states that in general this approach is not useful. However, for the ground state which has $I = 3$, the generator $|[3] \ 2\pi \ 2\nu|$ with $K = 3$ provides a good wave function for prolate deformation, $\eta = +12$.

$$\underline{\text{B}^{11} (T = \frac{1}{2})}$$

It has been shown¹ that the $K = \frac{1}{2}$ and $K = \frac{3}{2}$ bands are quite strongly mixed in this nucleus. This arises because the band heads are much closer here than in Be^9 and the Coriolis term is stronger. The unmixed wave functions are therefore not useful here except for the $I = \frac{1}{2}$ case which is in Table III.

$$\underline{N^{14} (T = 0)}$$

The $I = 2$ and 3 wave functions are accurately given by the $|4\pi\ 4\nu\rangle$ hole function with $K = 1$. However, as in Li^6 , the $I = 1$ states require strong band-mixing.

The survey of wave functions indicates that, aside from the trivial two-nucleon cases, the method for generating CFP can be employed for relating the Be^9 functions to functions in the neighboring Be^8 and to the ground state of B^{10} . Any CFP between Be^9 and the $I = 0, 2, 4$ band of Be^8 is most easily obtained by expanding the configuration of 8 holes in levels Nos. 2 and 4. After extracting the hole CFP with a formula analogous to Eq. (5'), the hole relationship, Eq. (6), gives the desired CFP. In both this case and the ($B^{10} \leftrightarrow Be^9$) case, there are several terms in the sum over r in Eq. (5'). This necessitates considerably more work than for the simple cases. The CFP relating Be^9 to Be^8 are given by

$$\langle I_{\frac{1}{2}} K_p (Be^9) || I_0\ 00 (Be^8) j \rangle = \sqrt{4/5} \left[(2I_0+1)/(2I+1) \right] N_{Kp}^I N_0^{I_0} \sum_r d_{K'r}^j (I_0 j 0 K' | I K') \sum_s c_{Kp}^{I_s} c_{K'r}^{I_s}, \quad (12)$$

where the sum is over the two Be^9 wave function generators having $K' = \frac{1}{2}$ and $\frac{3}{2}$. Similarly the CFP relating Be^9 to the ground state of B^{10} involves an r sum over generators with $K' = \frac{3}{2}, \frac{5}{2},$ and $\frac{7}{2}$.

The resultant CFP are quite slowly varying functions of η in the region of physical interest. Since relatively few are likely to be needed for comparison with experiments, the CFP are listed in Tables V and VI only for $\eta = +12$, which should give a rough idea of the relative values. One point of interest for experimental test lies in the B^{10} parentage. By squaring the CFP of Table VI and finding the ratio of this number to the square of the CFP for the ground state of Be^9 ($I = \frac{3}{2} = K$), one gets an idea of the relative probability for reaching this state in a pickup experiment.

TABLE V. Coefficients of fractional parentage for $\langle \text{Be}^9 | \text{Be}^8 \rangle$, calculated with the approximate wave functions for Be^9 obtained from the $K = 1/2$ and $K = 3/2$ generators. The value $\eta = +12$, used for the Nilsson parameter, should be reasonable for the region of physical interest.

$I (\text{Be}^9)$	$I_0 (\text{Be}^8) j$	CFP for $K = 3/2$	CFP for $K = 1/2$
1/2	0 1/2	- - -	-0.398
	2 3/2	- - -	-0.296
3/2	0 3/2	0.323	-0.161
	2 1/2	-0.044	0.442
	2 3/2	0.397	0.133
5/2	2 1/2	0.126	-0.355
	2 3/2	-0.439	0.221
	4 3/2	-0.225	-0.268
7/2	2 3/2	0.187	-0.126
	4 1/2	-0.084	0.463
	4 3/2	0.468	0.065
9/2	4 1/2	0.299 ^a	-0.299 ^a
	4 3/2	-0.391	0.391

^a The two χ_K generate the same 9/2 function with opposite sign.

TABLE VI. Coefficients of fractional parentage for $\langle B^{10} (I=3) | Be^9 \rangle$ calculated with the approximate wave functions obtained from the $K = 1/2$ and $K = 3/2$ generators for Be^9 and the $K = 3$ generator for the ground state of B^{10} . The value +12 is chosen for the Nilsson parameter η as suitable for the region of physical interest.

$I_0(Be^9) j$		CFP for $K = 3/2$	CFP for $K = 1/2$
3/2	3/2	-0.448	-0.044
5/2	1/2	-0.124	0.037
	3/2	0.430	-0.142
7/2	1/2	0.019	-0.007
	3/2	0.286	-0.032
9/2	3/2	0.094 ^a	-0.094 ^a

^a The two χ_K generate the same 9/2 function with opposite sign.

As French and Macfarlane found,⁽⁴⁾ the relative probabilities for reaching the ground state and the lowest $I = \frac{5}{2}$ are about equal, in agreement with observation. From Table VI one finds that the only other states one might see are the lowest $I = \frac{7}{2}$ and the next to lowest $I = \frac{5}{2}$ state with intensities of about 0.4 and 0.1, respectively, compared to the ground-state transition. It would be interesting to see if these can be found.

The property of being able to generate a state can still be useful even if the neighboring nuclei do not have simple generators. For example, the expansion of the ground state of B^{10} can be used to find the CFP for B^{11} states. However the B^{11} wave functions needed for the overlap integrals must be obtained from a usual intermediate-coupling calculation or an equally complicated band-mixing calculation for B^{11} with oblate deformation. The results have already been obtained by Bilaniuk and Hensel⁽⁵⁾ who used intermediate-coupling functions for both B^{10} and B^{11} . The Be^9 generators could be similarly expanded if one desired to find parents in Be^8 other than the $I_0 = 0, 2, 4$ states of the lowest band, but the wave functions of such parents again are difficult to obtain.

Finally the expansion in the form of Eq. (3') can be used to find the total amount of (I_0, j) component present in the parentage of any of the states with angular momentum I contained in the X_K generator. With the help of Eq. (1), Eq. (3') can be rewritten and functions with the same total angular momentum on the two sides of the equation can be equated separately to obtain

$$\psi_{Kp}^{IT} = \sum_{I_0 s_0 T_0 j} g(I_0 s_0 T_0, j) \psi_K^{IT}(I_0 s_0 T_0 j) . \quad (13)$$

4) M. H. Macfarlane and J. B. French, Revs. Modern Phys. 32, 567 (1960).

5) O. M. Bilaniuk and J. C. Hensel, Phys. Rev. 120, 211 (1960).

Then the total probability that the function on the left has $(I_0 T_0) (j t = \frac{1}{2})$ as components is

$$\sum_{s_0} |g(I_0 s_0 T_0 j)|^2 . \quad (14)$$

However, this is just the sum of all the CFP having the same $I_0 T_0$ as parents. Only if one has extracted the CFP for the low-lying parent state can one tell what fraction of the total $I_0 T_0$ parentage this supplies. In the simple cases for the first half of the shell this fraction is 100%, but in the other cases we have discussed it is often only a small part of the total.

In general, then, the generator method of representing intermediate-coupling functions provides a simplified method of extracting CFP directly in many cases. Even in the more complex cases for which it is not exact, it supplies a reasonable estimate with much less calculation than in the usual procedure.