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Abstract

A comparison of a previous gas-embedded Z-pinch experiment (by E. Smårs) is made using a nonlinear theory of kink instability (by W. Manheimer, et al.). Good agreement is found for some experimental

cases while for others the theory predicts too fast a growth of the kink. To account for this discrepancy the evolution of topologically different secondary currents is postulated.

Introduction

The kink mode appears to be the most important magnetohydrodynamic (MHD) instability mode of the gas-embedded Z-pinch. Experimental observations¹ show that under some conditions a tightly pinched column distorts into a helix, launching shock waves into the surrounding gas. Manheimer, et al.² have developed a nonlinear theory of the kink mode which predicts secular growth of the helix at late times. In this paper that theory is modified to account for time variation of the pinch current

and the predictions thereby obtained are compared with experiment. If all the discharge current is assumed to flow in the pinched column, the predicted growth considerably exceeds the observed value. If, on the other hand, the pinch current is assumed to vary freely in response to changes in the pinch inductance the predicted growth is less than the observed value. In this latter case, secondary currents are assumed to arise to account for the full discharge current.

Experimental Observations

The experimental study of the gas embedded Z-Pinch of Ref. 1 was conducted by discharging a capacitor bank ($C = 9.6 \mu F$, $di/dt = 1.5 \times 10^{11}$ A/sec at $t = 0$, and $V \leq 30$ kV) through a preionized channel formed by a vortex-stabilized, dc arc. Two initial channel conditions were studied in hydrogen gas at 1 atm, using a high-gas-flow arc and a low-gas-flow arc.

In Table 1, we list the experimental

parameters for the high- and low-gas-flow arc conditions studied.

Kerr-cell pictures and spectroscopic studies give a general view of pinch evolution in hydrogen under these conditions. A tightly pinched column is formed from the initial preionized mass formed by the arc. The column then kinks into a helix. In some cases the helix grows to a fixed radius and rotates, and in others expands

Table 1. Experimental parameters for high- and low-gas-flow arcs.

Parameter	Description	High-gas-flow arc	Low-gas-flow arc
r_0	Arc channel radius (cm)	0.05	0.18
n_0	Ion density in the arc (cm^{-3})	6.5×10^{18}	6.5×10^{18}
τ	Time of maximum compression (μsec)	0.07	0.33
I_0	Current at maximum compression (A)	10^4	5×10^4
r_p	Pinch radius (cm)	10^{-2}	5×10^{-2}
λ_z	$\lambda_z = 2\pi/k$. λ_z is the axial wavelength of the kink (cm)	0.45	0.3
kr_p	k is the axial wave number	0.14	1.0
ρ_p/ρ_g	Ratio of pinch density to surrounding gas density	0.58	0.30
T^*	Bennett temperature at maximum compression (eV)	15.8	30.7

to launch shocks into the surrounding gas. In still other cases, the helical deformation propagates in from the ends. Although light is emitted mainly from the luminous shock fronts, the pinched column is occasionally visible in photographs and appears to retain its pinched aspect during the kinking. This observation is further enforced by spectroscopic observations of the plasma temperature and by the obvious helical structure retained by the shock

fronts during expansion. Since the kink motion takes place over many transits of sound across the plasma column, radial-momentum balance given by the Bennett relation ($i^2 = 2 \times 10^9 n a^2 T$) holds, implying that either T remains high or that na^2 increases. It is concluded in Ref. 1 that na^2 is constant, suggesting that radial confinement of the column is nearly stable and relatively unaffected by the kinking.

Theory

In the nonlinear kink-instability theory,² the pinched-plasma column is assumed to remain essentially unchanged during the helical deformation and is approximated by a current filament. This assumption appears to approximate experimental results well since the column itself shows no strong filamentation or plasma loss.

The theory assumes that the pinch current is constant while in the experiment only the total discharge current from the capacitor bank is known, and this increases nearly linearly with time during most of the period of interest. Since alternate current paths may form during kinking, two assumptions are made about the pinch current in modifying the theory. These are:

(a) No alternate current paths form and the pinch current increases linearly with time in response to the total discharge current and, (b) The applied voltage remains nearly constant and the pinch current decreases in accordance with the increase in inductance as the helix forms. Here, secondary currents spring up to account for the full bank current. Because the external inductance is large, no reliable distinction between (a) and (b) can be made using the observed shape of the current traces.

To modify the theory of Ref. 2 we begin with the equation of motion of the filament with helix radius ξ [see Eq. (23) of Ref. (2)]:

$$M \ddot{\xi} = F_B - \frac{5}{2} r_p p_g \times \left[\frac{\xi^2}{C_{sg}^2} - 1 \right] \theta(\xi - C_{sg}), \quad (1)$$

where F_B is the kinking force per unit length, M is the mass per unit length, and the term following F_B is the force due to launching a cylindrical bow shock of width r_p in surrounding gas with pressure p_g and sound speed C_{sg} . The function $\theta(\xi - C_{sg}) = 0$ if $\xi < C_{sg}$ and $\theta(\xi - C_{sg}) = 1$ if $\xi > C_{sg}$. The magnetic kinking force is given by

$$F_B = 2\pi \frac{B_0^2}{2\mu_0} \frac{r_p^2}{R_c} = 2\pi \frac{B_0^2}{2\mu_0} \frac{r_p^2}{\left[\xi + (k^2 \xi)^{-1} \right]}, \quad (2)$$

where $R_c = \xi + (k^2 \xi)^{-1}$ is the radius of curvature of the helical column and k is the axial wavenumber of the helix. The

term $B_0^2 r_p^2 = \mu_0 I_p^2 / 4\pi^2$ is taken to be a constant in Ref. 2.

To modify the theory of Ref. 2 we consider early times in the discharge. For assumption (a) above, we take

$$I_p = I_0(t/\tau) \quad (3)$$

in F_B . For assumption (b), constant voltage is assumed, giving

$$V = d/dt (L_p I_p) = \text{constant}, \quad (4)$$

which implies that secondary currents have a small inductive effect. As a simple model of the pinch inductance, which, for a straight pinch, is $L_{p0} = (\mu_0/2\pi) \ln(b/a)$, variations in the logarithmic term are neglected and it is assumed that because of the helical deformation, only the increase in length l increases L_p . Then, for a pinch of initial length l ,

$$l = l_0 / (1 + k^2 \xi^2)^{1/2}$$

so that the pinch inductance is approximately

$$L_p = L_{p0} / (1 + k^2 \xi^2)^{1/2}.$$

Using Eq. (4), the pinch current for assumption (b) is then

$$I_p = I_0 (t/\tau) / (1 + k^2 \xi^2)^{1/2}. \quad (5)$$

Now, to examine Eq. (1) we consider only the secular limit $\xi > C_{sg}$, and $M \ddot{\xi} = 0$, since the time for exponential growth is typically short in the experiment. Instead of r_p in the bow shock force we use R_c , the corona radius.

Equation (1) can be written

$$\xi^2 = \frac{\eta v_{ti}^2 (t/\tau)^2 k^2 \xi r_c}{1 + k^2 \xi^2} \quad (6)$$

for assumption (a), and

$$\xi^2 = \frac{\eta v_{ti}^2 (t/\tau)^2 k^2 \xi r_c}{(1 + k^2 \xi^2)^2} \quad (7)$$

for assumption (b). In Eqs. (6) and (7) the quantity $\eta = (4\gamma/5) (\rho_p r_p^2 / \rho_g r_c^2)$ is a mass-loading factor where ρ_p and ρ_g are the pinch and gas density respectively, and $v_{ti}^2 = 2eT^*/m_i$ is the square of the ion thermal velocity at the Bennett temperature T^* , where $I_0^2 = 16\pi^2 e^2 / \mu_0 \eta_p r_p^2 T^*$.

The secular Eqs. (6) and (7) can be solved in the limit $k\xi < 1$ and at late times when $k\xi > 1$. The limit $k\xi < 1$ implies that $1 + k^2 \xi^2 \approx 1$. In this case, Eqs. (6) and (7) reduce to the same equation,

so that the early secular growth of the kink is then given by

$$\xi = \left[\frac{\eta v_{ti}^2 k^2 r_c}{16\tau^2} \right] t^4. \quad (8)$$

At late times, when $k\xi > 1$, the solutions are,

$$\xi_{(a)} = \left[\frac{3}{4} \frac{\eta^{1/2} v_{ti}^* r_c^{1/2}}{\tau} \right]^{2/3} t^{4/3} \quad (9)$$

and

$$\xi_{(b)} = \left[\frac{5}{4} \frac{\eta^{1/2} v_{ti}^* r_c^{1/2}}{k\tau} \right]^{2/5} t^{4/5} \quad (10)$$

for assumptions (a) and (b) respectively. Note in Eqs. (9) and (10) that ξ is relatively insensitive to the quantities in the brackets.

Comparison with Experiment

To evaluate Eqs. (9) and (10) using the experimental values given in Table 1, the corona radius r_c is needed and will be taken to be $r_c = r_0$, the initial arc radius. This assumption is consistent with the experimental picture of the initial arc channel being pinched while leaving the surrounding gas relatively unaffected. Using the experimental quantities, it is found that the initial secular growth phase $k\xi < 1$ given by Eq. (7) takes place in typically $t \leq 0.1$ μ sec, which is short compared with the observed helix-growth time.

The subsequent secular growth of ξ , given by Eqs. (9) and (10), and evaluated using Table 1, is plotted in Fig. 1 along

with the observed radius of the helical luminous fronts for the two initial arc conditions studied. The luminous-front radius constitutes an upper limit on the actual helix radius.

From Fig. 1 it can be seen that reasonably good agreement between theory and experiment is obtained for $t \approx 0.2$ μ sec while at later times the theoretical curves for assumption (a) and (b) tend to bracket the experimental quantities. This suggests the importance of secondary currents late in time. Figure 1 also illustrates the predicted insensitivity of ξ to pinch current (or bank voltage) with about the same helix growth observed for a bank voltage of 8 to 15 kV.

Fig. 1. Luminous-front diameter vs time for hydrogen discharges (in Ref. 1); and helix diameter vs time predicted by Eq. (9) for high- and low-gas-flow conditions.

Secondary Current Effects

With no secondary currents, the discrepancy between the observed velocity and the theory suggests the importance of slowing mechanisms other than the "mass pushing" effect. Because the secondary currents can introduce pinch forces which act on the primary helix, slowing of the helix growth can occur until eventually a reversed-field stabilized Z-pinch is formed.

A possible way secondary currents can form with different topology than the main helical pinch-current is shown sequentially in Fig. 2. After the initial exponential growth, the secular growth-phase evolves until the conducting-corona regions touch, in effect creating a conduction path parallel to the main pinch. If the applied voltage is still in the initial direction,

a Z-directed secondary current I_{sz} can form. Since this current is in the same direction as the pinch current I_p , I_{sz} will interact with I_p so as to give an inward force on the pinch helix. Roughly speaking, the outward kinking force per unit length

$$F_B = \frac{B_0^2 \pi r^2}{2\mu_0 R_c} \quad (11)$$

exerts an average outward pressure

$$\langle p \rangle = \frac{F_B dl}{2\pi \xi dz} = \frac{1}{2\mu_0} \left(\frac{\nu_0 I_p}{2\pi r_p} \right)^2 \times \frac{r_p^2 (1 + k^2 \epsilon^2)^{1/2}}{\epsilon R_c} \quad (12)$$

and the secondary sheet current ($i_{sz}/2\pi\zeta$) exerts an inward pressure

$$\langle p_s \rangle \approx \frac{B_s^2}{2\mu_0} \approx \frac{1}{2\mu_0} \left(\frac{\mu_0 I_{sz}}{2\pi\zeta} \right)^2. \quad (13)$$

To restrain the helix from growth requires $\langle p_s \rangle \geq \langle p \rangle$ or

$$\frac{I_{sz}}{I_p} \geq \frac{k\zeta}{\left[1 + \frac{k^2 \zeta^2}{1/4} \right]^{1/4}}. \quad (14)$$

Since the secular growth phase is entered when

$$k\zeta \approx \frac{1}{k r_p} \frac{p_g}{p_p} \approx 10^{-3} \text{ (typically).}$$

small secondary-currents may significantly alter the helix growth.

The secondary current may also have a θ -directed component so as to conserve the axial magnetic-flux generated by the helical pinch (Fig. 2 part c). Depending upon the effective area through which the return flux must pass, a net inward or outward force appears. This effect, however, only modifies somewhat the simple "mass pushing" model.

A tendency for conduction paths to appear in the shock-corona region may occur for the same reasons snowplow pinch insulators have often failed.³ If the Poynting flux ($\vec{p} = \vec{E} \times \vec{H}$) is directed radially inward through the insulator (or high-pressure gas), plasma created in the corona region is swept inward. On the other hand, if the Poynting flux is out-

Fig. 2 Nonlinear development of kink instability.

ward, plasma is carried radially outward to bombard the insulator (or gas). In this latter case a particle heat flow

$$q = 2\pi e l \frac{E}{B} \quad (15)$$

accompanies the electromagnetic energy flow $\vec{P}$.

In addition to the above "magnetic-flux

rectification" effect, substantial electric fields may appear because of the kinking motion itself, which could induce local breakdown in the adjacent gas.

Fusion Feasibility Constraints

As an alternative to the previous discussion, let us assume that the pinch current increases rapidly to a constant value so that the theory of Ref. 2 is approximately correct. Further, assume that the Z-pinch filament remains tightly pinched, up to $k_0^2 = 1$. It is shown in Ref. 2 that, when $k_0^2 = 1$, the energy-deposited by the shock in the gas is about the same as the plasma energy. This would appear to be an acceptable energy loss for a fusion-reactor pinch since, typically, the plasma energy is 10% of the magnetic energy. Using the helix growth-rate of Ref. 2, the time to reach $k_0^2 = 1$, measured in radial sound transits, is for

$$N = \frac{6}{\pi} \left[\frac{\rho_g r_c / r_p}{\rho_p k^2 r_p} \right]^{1/2}.$$

Evaluating N for $kr_p = 0.2$ (in the high-gas-flow case of Ref. 1, $kr_p = 0.14$), and for $r_c/r_p = 20$ (as calculated in a one-dimensional code⁴) gives

$$N = 70 (\rho_g/\rho_p)^{1/2}.$$

This result suggests that only modest values of ρ_g/ρ_p are required to achieve the values of $N \approx 100$ needed for feasible Z-pinch fusion.

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