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A SOLUTION TO THE NUCLEON-MESON CASCADE
EQUATIONS UNDER VERY SPECIAL CONDITIONS

R. G. Alsmiller, Jr.

OAK RIDGE NATIONAL LABORATORY

operated by

UNION CARBIDE CORPORATION

for the

U.S. ATOMIC ENERGY COMMISSION

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UNDER VERY SPECIAL CONDITIONS

R. G. Alsmiller, Jr.

MARCH 1964

OAK RIDGE NATIONAL LABORATORY
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Abstract

The equations which describe a one-dimensional nucleon-meson cascade are solved under the following assumptions:

1. the secondary-particle production kernels are taken to be of the special form

$$F_{ij}(E', E) = \alpha_i \gamma_j e^{\nu(E' - E)} \text{ for all } i, j$$

$$\alpha_i, \gamma_j, \nu = \text{const.};$$

2. pion decay is neglected;
3. pion and nucleon nonelastic cross sections are taken to be constant;
4. the stopping powers of all charged particles are equal to the same nonzero constant.

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I. Introduction

In a series of recent reports numerical solutions to the set of equations describing a one-dimensional nucleon-meson cascade have been presented.¹⁻⁴ An improved version of the IBM code used in these calculations is in preparation.⁵ To check the numerical accuracy of this code, particularly the truncation error after calculating over very large distances, it is desirable to have analytic solutions to the equations for particular cases.

A variety of such solutions have been obtained previously under the assumption that the stopping power of all charged particles is zero.^{6,7} In this report a solution is given for the case when the stopping power of all charged particles is the same nonzero constant and the secondary-particle production kernels are of a very special form.

In Section II the equations describing a one-dimensional nucleon-meson cascade are given and the physical assumptions used to simplify the equations are introduced. In Section III the solutions to the equations for arbitrary initial values are given. In Section IV the specific case of an incident monoenergetic proton flux is considered. In Section V the

1. R. G. Alsmiller, Jr., F. S. Alsmiller, and J. E. Murphy, Nucleon-Meson Cascade Calculations: Transverse Shielding for a 45-GeV Electron Accelerator, Part I, ORNL-3289 (1962), Part II, ORNL-3365 (1962), and Part III, ORNL-3412 (1963).
2. R. G. Alsmiller, Jr., and J. E. Murphy, Space Vehicle Shielding Studies Calculation of the Attenuation of a Model Solar Flare and Monoenergetic Proton Beams by Aluminum Shields, ORNL-3317 (1963).
3. R. G. Alsmiller, Jr., and J. E. Murphy, Nucleon-Meson Cascade Calculations. The Star Density Produced by a 24-GeV Proton Beam in Heavy Concrete, ORNL-3367 (1963).
4. R. G. Alsmiller, Jr., and J. E. Murphy, Nucleon-Meson Cascade Calculations: Shielding Against an 800-MeV Proton Beam, ORNL-3406 (1963).
5. R. G. Alsmiller, Jr., F. S. Alsmiller, and J. E. Murphy, Proceedings of the Symposium on the Protection Against Radiation Hazards in Space, TID-7652, p 698 (1962).
6. C. Passow, Deutsches Elektronen-Synchrotron, Hamburg, Germany, Phenomenologische Theorie zur Berechnung einer Kaskade aus schweren Teilchen (Nukleonkaskade) in der Materie, Desy-Notiz A.285 (Feb. 1962).
7. F. S. Alsmiller and R. G. Alsmiller, Jr., "Two Categories of Soluble Nucleon-Meson Cascade Equations," Neutron Phys. Space Radiation Shielding Research Annual Progress Report for Period Ending August 31, 1962, ORNL CF 62-10-29, p 138.

case of an incident proton flux with an exponential energy spectrum is considered. This case is of particular interest because the solutions to the equations can be given completely in terms of elementary functions. In the appendix the inverse Laplace transform of a specific function needed to obtain the solutions given in Section III is carried out.

II. Equations and Physical Assumptions

Neglecting charged-pion decay and making no distinction between positively and negatively charged pions, the equations describing a one-dimensional nucleon-meson cascade may be written*

$$\Phi_{ij}(E, r) = \Phi_j(E_j) \frac{S_j(E_j)}{S_j(E)} e^{-\int_E^{E_j} \frac{Q_j(E')}{S_j(E')} dE'}, \quad j = N, P, \pi, \quad (2.1)$$

$$\int_E^{E_j} \frac{dE'}{S_j(E')} = r, \quad (2.2)$$

$$\frac{\partial}{\partial r} \Phi_{sj}(E, r) + Q_j(E) \Phi_{sj}(E, r) - \frac{\partial}{\partial E} \left[S_j(E) \Phi_{sj}(E, r) \right] \quad (2.3)$$

$$= \sum_k \int_E^\infty F_{jk}(E', E) Q_k(E') \left[\Phi_{sk}(E', r) + \Phi_{ik}(E', r) \right] dE', \quad j, k = N, P, \pi,$$

where

N, P, π = neutron, proton, and charged pion, respectively,

$\Phi_j(E)$ = arbitrary initial values,

*The neutral pion decays very rapidly into two photons and is not included because photons are not included.

$\Phi_{ij}(E, r)$ = primary flux per unit energy range of particles of type j ,

$\Phi_{sj}(E, r)$ = secondary flux per unit energy range of particles of type j ,

E = kinetic energy,

r = dimensionless distance variable defined by the relation

$$r = \frac{\rho}{\lambda_0} R,$$

ρ = density of medium, in g/cm^3 ,

R = distance, in cm,

λ_0 = an arbitrary constant with dimensions g/cm^2 which determines the units in which r is measured,

$$Q_j(E) = \frac{\lambda_0 N_0}{A} \sigma_j(E),$$

N_0 = Avogadro's number,

$\sigma_j(E)$ = nonelastic cross section for particles of type j in the medium being considered,

A = atomic weight of the medium being considered,

$$S_j(E) = \frac{\lambda_0 N_0}{A} \epsilon_j(E),$$

$\epsilon_j(E)$ = stopping cross section for particles of type j ,

$F_{jk}(E', E) dE$ = the number of secondary particles of type j in the energy interval E to $E + dE$ produced by the nonelastic collision of a particle of type k with energy E' .

To reduce the equations to soluble form, we introduce the assumptions

$$Q_j(E) = Q = \text{const.}, \quad j = N, P, \pi, \quad (2.4)$$

$$S_j(E) = S = \text{const.}, \quad j = P, \pi, \quad (2.5)$$

$$S_N(E) = 0, \quad (2.6)$$

$$F_{ij}(E', E) = \alpha_i \gamma_i e^{\nu(E' - E)} \Theta(E' - E), \quad i, j = N, P, \pi, \quad (2.7)$$

$$\alpha_i, \gamma_i, \nu = \text{const.},$$

$$\begin{aligned} \Theta(E' - E) &= 1, \quad E' \geq E, \\ &= 0, \quad E' < E, \end{aligned}$$

and obtain

$$\Phi_{ij}(E, r) = \Phi_j(E + Sr) e^{-Qr}, \quad j = P, \pi, \quad (2.8)$$

$$\Phi_{iN}(E, r) = \Phi_N(E) e^{-Qr}, \quad (2.9)$$

$$\frac{\partial}{\partial r} \Phi_{sj}(E, r) + Q \Phi_{sj}(E, r) - S \frac{\partial}{\partial E} \Phi_{sj}(E, r) \quad (2.10)$$

$$= \alpha_j Q e^{-\nu E} \int_E^\infty e^{\nu E'} \sum_k \gamma_k \left[\Phi_{sk}(E', r) + \Phi_{ik}(E', r) \right] dE', \quad \begin{aligned} j &= P, \pi, \\ k &= N, P, \pi, \end{aligned}$$

$$\frac{\partial}{\partial r} \Phi_{sN}(E, r) + Q \Phi_{sN}(E, r) \quad (2.11)$$

$$= \alpha_n Q e^{-\nu E} \int_E^\infty e^{\nu E'} \sum_k \gamma_k \left[\Phi_{sk}(E', r) + \Phi_{ik}(E', r) \right] dE', \quad k = N, P, \pi.$$

Equations 2.8 to 2.11 are the equations for which solutions will be obtained in the next section. It is to be understood that the assumptions contained in Eqs. 2.4 to 2.7 are chosen for their simplicity and represent only approximately any real physical system.

III. Solution of the Equations

To solve Eqs. 2.8 to 2.11 it is convenient to eliminate the absorption terms and to reduce the number of equations by adding the proton and pion equations. To this end we introduce the definitions

$$\Psi_{iN}(E, r) = e^{Qr} \gamma_N \Phi_{iN}(E, r), \quad (3.1)$$

$$\Psi_{ic}(E, r) = e^{Qr} \left[\gamma_P \Phi_{iP}(E, r) + \gamma_\pi \Phi_{i\pi}(E, r) \right], \quad (3.2)$$

$$\Psi_{sN}(E, r) = e^{Qr} \gamma_N \Phi_{sN}(E, r), \quad (3.3)$$

$$\Psi_{sc}(E, r) = e^{Qr} \left[\gamma_P \Phi_{sP}(E, r) + \gamma_\pi \Phi_{s\pi}(E, r) \right], \quad (3.4)$$

$$\beta_N = \alpha_N \gamma_N Q, \quad (3.5)$$

$$\beta_c = (\alpha_P \gamma_P + \alpha_\pi \gamma_\pi) Q, \quad (3.6)$$

and obtain from Eqs. 2.10 and 2.11

$$\frac{\partial}{\partial r} \Psi_{sc}(E, r) - S \frac{\partial \Psi_{sc}(E, r)}{\partial E} \quad (3.7)$$

$$= \beta_c e^{-\nu E} \sum_{j=c, N} \int_E^\infty e^{\nu E'} \left[\Psi_{sj}(E', r) + \Psi_{ij}(E', r) \right] dE'$$

$$\frac{\partial}{\partial r} \Psi_{sN}(E, r) = \beta_N e^{-\nu E} \sum_{j=c, N} \int_E^\infty e^{\nu E'} \left[\Psi_{sj}(E', r) + \Psi_{ij}(E', r) \right] dE'. \quad (3.8)$$

From Eq. 2.10 it follows that

$$\frac{\Phi_{sP}(E, r)}{\alpha_P} = \frac{\Phi_{s\pi}(E, r)}{\alpha_\pi}, \quad (3.9)$$

so for future reference we note that

$$\Phi_{SP}(E, r) = \frac{\alpha_P}{\alpha_P \gamma_P + \alpha_\pi \gamma_\pi} e^{-Qr} \bar{\Psi}_{SC}(E, r), \quad (3.10)$$

$$\Phi_{S\pi}(E, r) = \frac{\alpha_\pi}{\alpha_P \gamma_P + \alpha_\pi \gamma_\pi} e^{-Qr} \bar{\Psi}_{SC}(E, r). \quad (3.11)$$

Equations 3.7 and 3.8 may be solved by introducing Laplace transforms. Letting

$$\psi_{SC}(E, \lambda) = \int_0^\infty e^{-\lambda r} \bar{\Psi}_{SC}(E, r) dr \quad (3.12)$$

etc., we have after taking the Laplace transform of the equations

$$\lambda \psi_{SC}(E, \lambda) = S \frac{\partial}{\partial E} \psi_{SC}(E, \lambda) \quad (3.13)$$

$$+ \beta_c e^{-\nu E} \sum_{j=c, N} \int_E^\infty e^{\nu E'} \left[\psi_{sj}(E', \lambda) + \psi_{ij}(E', \lambda) \right] dE',$$

$$\lambda \psi_{SN}(E, \lambda) = \beta_N e^{-\nu E} \sum_{j=c, N} \int_E^\infty e^{\nu E'} \left[\psi_{sj}(E', \lambda) + \psi_{ij}(E', \lambda) \right] dE'. \quad (3.14)$$

The integral terms may be eliminated from the equations by differentiating with respect to energy. After differentiating and carrying out the obvious algebra, one obtains

$$\begin{aligned} \left[SD^2 + (\nu S - \lambda) D - (\nu\lambda + \beta_c) \right] \psi_{sc}(E, \lambda) - \beta_c \psi_{sN}(E, \lambda) \\ = \beta_c \left[\psi_{ic}(E, \lambda) + \psi_{iN}(E, \lambda) \right], \end{aligned} \quad (3.15)$$

$$\begin{aligned} \left[\lambda D + \nu\lambda + \beta_N \right] \psi_{sN}(E, \lambda) + \beta_N \psi_{sc}(E, \lambda) \\ = -\beta_N \left[\psi_{ic}(E, \lambda) + \psi_{iN}(E, \lambda) \right], \end{aligned} \quad (3.16)$$

where

$$D = \frac{\partial}{\partial E}. \quad (3.17)$$

By operating on Eq. 3.15 with the operator

$$\lambda D + \nu\lambda + \beta_N$$

and multiplying Eq. 3.16 by β_c , the quantity ψ_{sN} may, by adding the two equations, be eliminated. In this manner one obtains

$$\begin{aligned} \left[(SD - \lambda)(D + \nu) - \beta_c + (SD - \lambda) \frac{\beta_N}{\lambda} \right] \psi_{sc}(E, \lambda) \\ = \beta_c \left[\psi_{ic}(E, \lambda) + \psi_{iN}(E, \lambda) \right], \end{aligned} \quad (3.18)$$

and in like manner eliminating ψ_{sc} from Eqs. 3.15 and 3.16 one obtains

$$\begin{aligned} \left[(SD - \lambda)(D + \nu) - \beta_c + (SD - \lambda) \frac{\beta_N}{\lambda} \right] \psi_{sN}(E, \lambda) \\ = -\frac{\beta_N}{\lambda} (SD - \lambda) \left[\psi_{ic}(E, \lambda) + \psi_{iN}(E, \lambda) \right]. \end{aligned} \quad (3.19)$$

Thus the problem is reduced to solving two uncoupled second-order differential equations with constant coefficients. It is, of course, clear that the kernels given by Eq. 2.7 were specifically chosen so this would be the case.

Using the standard methods for solving such equations, one obtains the solutions

$$\psi_{sc}(E, \lambda) = \beta_c \int_E^{\infty} g(E' - E, \lambda) \left[\psi_{ic}(E', \lambda) + \psi_{in}(E', \lambda) \right] dE', \quad (3.20)$$

$$\begin{aligned} \psi_{sN}(E, \lambda) \\ = \beta_N \int_E^{\infty} g(E' - E, \lambda) \left(-\frac{S}{\lambda} \frac{\partial}{\partial E'} + 1 \right) \left[\psi_{ic}(E', \lambda) + \psi_{in}(E', \lambda) \right] dE', \end{aligned} \quad (3.21)$$

where

$$g(E' - E, \lambda) = \frac{e^{-a_2(E' - E)} - e^{-a_1(E' - E)}}{S(a_1 - a_2)}, \quad (3.22)$$

$$a_1 = \frac{1}{2S} \left(\lambda - Sv - \frac{\beta_N S}{\lambda} \right) + \frac{1}{2S} \left[\left(\lambda + Sv + \frac{\beta_N S}{\lambda} \right)^2 + 4\beta_c S \right]^{\frac{1}{2}}, \quad (3.23)$$

$$a_2 = \frac{1}{2S} \left(\lambda - Sv - \frac{\beta_N S}{\lambda} \right) - \frac{1}{2S} \left[\left(\lambda + Sv + \frac{\beta_N S}{\lambda} \right)^2 + 4\beta_c S \right]^{\frac{1}{2}}.$$

Taking the inverse transform of Eqs. 3.20 and 3.21, we obtain

$$\begin{aligned} \Psi_{sc}(E, r) = \beta_c \int_0^r dr' \int_E^{\infty} dE' G(E' - E, r') \left[\Psi_{ic}(E', r - r') + \right. \\ \left. \Psi_{in}(E', r - r') \right], \end{aligned} \quad (3.24)$$

$$\begin{aligned} \Psi_{sN}(E, r') = \beta_N \int_0^r dr' \int_E^\infty dE' & \quad (3.25) \\ \left\{ G(E' - E, r') \left[\Psi_{ic}(E', r - r') + \Psi_{iN}(E', r - r') \right] \right. \\ + \int_0^{r'} dr'' G(E' - E, r'') \left(-S \frac{\partial}{\partial E'} \left[\Psi_{ic}(E', r - r') \right. \right. \\ \left. \left. + \Psi_{iN}(E', r - r') \right] \right) \left. \right\}, \end{aligned}$$

where

$$G(E' - E, r') = \mathcal{L}^{-1} g(E' - E, \lambda), \quad (3.26)$$

$\mathcal{L}^{-1}$ = inverse Laplace operator.

The inversion of the transform is quite tedious and will be carried out in the appendix. From the appendix we have

$$\begin{aligned} G(E' - E, r) = I_0 \left[2 \sqrt{\beta_c(E' - E - Sr)} r \right] e^{v(E' - E - Sr)} \odot(E' - E - Sr) & \quad (3.27) \\ + \int_0^r dr' I_0 \left[2 \sqrt{\beta_c(E' - E - Sr')} r' \right] \\ \cdot \left[\frac{\beta_N(E' - E - Sr')}{r - r'} \right]^{\frac{1}{2}} I_1 \left[2 \sqrt{\beta_N(E' - E - Sr')(r - r')} \right] \\ \cdot e^{v(E' - E - Sr')} \odot(E' - E - Sr'), \end{aligned}$$

where I_0 and I_1 are the usual hyperbolic Bessel functions.

The expression for Ψ_{sN} may be simplified slightly by carrying out the integration over r'' . Noting that

$$\begin{aligned} \frac{\partial}{\partial r'} I_0 \left[2 \sqrt{\beta_N (E' - E - S r'') (r' - r'')} \right] \\ = \left[\frac{\beta_N (E' - E - S r'')}{r' - r''} \right]^{\frac{1}{2}} I_1 \left[2 \sqrt{\beta_N (E' - E - S r'') (r' - r'')} \right], \end{aligned} \quad (3.28)$$

we have after interchanging integrations and integrating by parts

$$\begin{aligned} H(E' - E, r') &= \int_0^{r'} dr'' G(E' - E, r'') \\ &= \int_0^{r'} dr'' I_0 \left[2 \sqrt{\beta_c (E' - E - S r'') r''} \right] \\ &\quad \cdot I_0 \left[2 \sqrt{\beta_N (E' - E - S r'') (r' - r'')} \right] \\ &\quad \cdot e^{\nu(E' - E - S r'')} \Theta(E' - E - S r''), \end{aligned} \quad (3.29)$$

and thus

$$\begin{aligned} \Psi_{3N}(E, r) &= \beta_N \int_0^r dr' \int_E^\infty dE' \\ &\quad \cdot \left\{ H(E' - E, r') \left(-S \frac{\partial}{\partial E'} \left[\Psi_{ic}(E', r - r') + \Psi_{iN}(E', r - r') \right] \right) \right. \\ &\quad \left. + G(E' - E, r') \left[\Psi_{ic}(E', r - r') + \Psi_{iN}(E', r - r') \right] \right\}. \end{aligned} \quad (3.30)$$

Utilizing the definitions given in Eqs. 3.1 to 3.6 together with the Eqs. 3.10 and 3.11, one has finally

$$\Phi_{SP}(E, r) = \alpha_{PQ} e^{-Qr} \int_0^r dr' \int_E^\infty dE' G(E' - E, r') \Phi_i(E', r - r'), \quad (3.31)$$

$$\Phi_{s\pi}(E, r) = \alpha_p Q e^{-Qr} \int_0^r dr' \int_E^\infty dE' G(E' - E, r') \Phi_i(E', r - r'), \quad (3.32)$$

$$\Phi_{sN}(E, r) = \alpha_N Q e^{-Qr} \int_0^r dr' \int_E^\infty dE' \quad (3.33)$$

$$\left\{ H(E' - E, r') \left[-S \frac{\partial}{\partial E'} \Phi_i(E', r - r') \right] + G(E' - E, r') \Phi_i(E', r - r') \right\},$$

where

$$\Phi_i(E', r - r') = \gamma_P \Phi_P(E' + Sr - Sr') + \gamma_\pi \Phi_\pi(E' + Sr - Sr') + \gamma_N \Phi_N(E'). \quad (3.34)$$

Equations 3.31 to 3.34 together with Eqs. 2.8 and 2.9 are the desired solutions. Since the solutions contain triple integrals they are somewhat formidable for arbitrary initial values. In the next two sections specific cases of the initial values for which the solutions simplify considerably will be considered.

IV. The Case of an Incident Flux of Monoenergetic Protons

In this section the solutions will be given for the case of an incident flux of monoenergetic protons. That is, the solutions for the case

$$\Phi_P(E) = P_0 \delta(E_0 - E), \quad (4.1)$$

$$\Phi_j(E) = 0, \quad j = N, \pi, \quad (4.2)$$

where

$$P_0 = \text{const.},$$

$$E_0 = \text{energy of incident protons},$$

will be given.

Rather than use the general results given by Eqs. 3.31 to 3.34, it is convenient to return to Eqs. 3.20 and 3.21. We have

$$\begin{aligned}\psi_{iP}(E, \lambda) &= P_0 \gamma_P \int_0^{\infty} e^{-\lambda r} \delta(E_0 - E - Sr) dr \\ &= \frac{P_0 \gamma_P}{S} e^{-\frac{\lambda}{S} (E_0 - E)} \Theta(E_0 - E)\end{aligned}\quad (4.3)$$

and

$$\left[-\frac{S}{\lambda} \frac{\partial}{\partial E} + 1 \right] \frac{P_0 \gamma_P}{S} e^{-\frac{\lambda}{S} (E_0 - E)} \Theta(E_0 - E) = P_0 \frac{\delta(E_0 - E)}{\lambda} . \quad (4.4)$$

Utilizing these results and taking the inverse transform of Eqs. 3.20 and 3.21, one obtains

$$\Psi_{sc}(E, r) = \beta_c \gamma_P \int_0^r dr' \int_E^{\infty} dE' G(E' - E, r') P_0 \delta(E_0 - E' - Sr + Sr') \quad (4.5)$$

$$\Psi_{on}(E, r) = \beta_N \gamma_P \int_0^r dr' G(E_0 - E, r') P_0 \Theta(E_0 - E), \quad (4.6)$$

where $G(E_0 - E, r)$ is given by Eq. 3.27.

Carrying out the integration as before, we have

$$\Psi_{sc}(E, r) = P_0 \beta_c \gamma_P \int_0^r dr' G(E_0 - E - Sr + Sr', r') \Theta(E_0 - E - Sr + Sr') \quad (4.7)$$

$$\Psi_{sn}(E, r) = P_0 \beta_N \gamma_P H(E_0 - E, r) \Theta(E_0 - E), \quad (4.8)$$

where

$$G(E_0 - E - Sr + Sr', r') \quad (4.9)$$

$$\begin{aligned} &= I_0 \left[2 \sqrt{\beta_c (E_0 - E - Sr) r'} \right] e^{v(E_0 - E - Sr)} \Theta(E_0 - E - Sr) \\ &\quad + \int_0^{r'} dr'' I_0 \left[2 \sqrt{\beta_c (E_0 - E - Sr + Sr' - Sr'') r''} \right] \\ &\quad \quad \quad \left[\frac{\beta_N (E_0 - E - Sr + Sr' - Sr'')}{r' - r''} \right]^{\frac{1}{2}} \\ &\quad \bullet I_1 \left[2 \sqrt{\beta_N (E_0 - E - Sr + Sr' - Sr'') (r' - r'')} \right] \\ &\quad e^{v(E_0 - E - Sr + Sr' - Sr'')} \Theta(E_0 - E - Sr + Sr' - Sr''), \end{aligned}$$

and $H(E_0 - E, r)$ is given by Eq. 3.29. Considering the first term in the equation for ψ_{sc} , we note that

$$\int_0^r dr' I_0 \left[2 \sqrt{\beta_c (E_0 - E - Sr) r'} \right] e^{v(E_0 - E - Sr)} \quad (4.10)$$

$$\begin{aligned} &\Theta(E_0 - E - Sr) \Theta(E_0 - E - Sr + Sr') \\ &= \left[\frac{r}{\beta_c (E_0 - E - Sr)} \right]^{\frac{1}{2}} I_1 \left[2 \sqrt{\beta_c (E_0 - E - Sr) r} \right] e^{v(E_0 - E - Sr)} \Theta(E_0 - E - Sr), \end{aligned}$$

so

$$\Psi_{sc}(E, r) = P_0 \gamma_P \beta_c K(E_0 - E, r), \quad (4.11)$$

where

$$\begin{aligned}
 K(E_0 - E, r) = & \left[\frac{r}{\beta_c(E_0 - E - Sr)} \right]^{\frac{1}{2}} I_1 \left[2 \sqrt{\beta_c(E_0 - E - Sr)r} \right] \\
 & e^{\nu(E_0 - E - Sr)} \Theta(E_0 - E - Sr) \\
 + \int_0^r dr' \Theta(E_0 - E - Sr + Sr') & \left\{ \int_0^{r'} dr'' \left[I_0 \left[2 \sqrt{\beta_c(E_0 - E - Sr + Sr' - Sr'')r''} \right] \right. \right. \\
 & \cdot \left(\frac{\beta_N(E_0 - E - Sr + Sr' - Sr'')}{r' - r''} \right)^{\frac{1}{2}} I_1 \left[2 \sqrt{\beta_N(E_0 - E - Sr + Sr' - Sr'')(r' - r'')} \right] \\
 & \left. \left. e^{\nu(E_0 - E - Sr + Sr' - Sr'')} \Theta(E_0 - E - Sr + Sr' - Sr'') \right] \right\}.
 \end{aligned}
 \tag{4.12}$$

Thus using the previous definitions the solutions for the case being considered may be written

$$\Phi_{iP}(E, r) = P_0 \delta(E_0 - E - Sr) e^{-Qr}, \tag{4.13}$$

$$\Phi_{sP}(E, r) = P_0 \alpha_P \gamma_P Q e^{-Qr} K(E_0 - E, r), \tag{4.14}$$

$$\Phi_{s\pi}(E, r) = P_0 \alpha_\pi \gamma_P Q e^{-Qr} K(E_0 - E, r), \tag{4.15}$$

$$\Phi_{sN}(E, r) = P_0 \alpha_N \gamma_P Q e^{-Qr} H(E_0 - E, r) \Theta(E_0 - E). \tag{4.16}$$

V. The Case of an Incident Proton Flux with an Exponential Energy Spectrum

Let us next consider the solutions to the equations when

$$\Phi_P(E) = P_0 e^{-bE}, \tag{5.1}$$

$$\Phi_j(E) = 0, \quad j = N, \pi, \tag{5.2}$$

$$P_0 = \text{const.},$$

$$b = \text{const. } (> \nu)^*.$$

This case is of particular interest because the solutions can be given completely in terms of elementary functions.

Rather than use the general results given by Eqs. 3.31 and 3.34 it is convenient to return to Eqs. 3.20 and 3.21 because in this case the inversion of the transforms are particularly simple. For the case being considered we have

$$\Psi_{ic}(E, r) = \gamma_P P_O e^{-b(E+Sr)}, \quad (5.3)$$

$$\psi_{ic}(E, \lambda) = \frac{\gamma_P P_O e^{-bE}}{\lambda + Sb}, \quad (5.4)$$

$$\left[-\frac{S}{\lambda} \frac{\partial}{\partial E} + 1 \right] \psi_{ic}(E, \lambda) = \frac{\gamma_P P_O e^{-bE}}{\lambda}. \quad (5.5)$$

Introducing these results into Eqs. 3.20 and 3.21 and carrying out the integration over energy, one finds

$$\psi_{sc}(E, \lambda) = \frac{\beta_c \gamma_P P_O e^{-bE}}{S} \frac{1}{(\lambda + Sb)(a_2 + b)(a_1 + b)}, \quad (5.6)$$

$$\psi_{sN}(E, \lambda) = \frac{\beta_N \gamma_P P_O e^{-bE}}{S} \frac{1}{\lambda(a_2 + b)(a_1 + b)}, \quad (5.7)$$

where a_1 and a_2 are given by Eq. 3.23. Using the form of a_1 and a_2 one finds

$$S(a_2 + b)(a_1 + b) = \frac{(b - \nu)}{\lambda} (\lambda - b_1)(\lambda - b_2) \quad (5.8)$$

where

*It is necessary to assume $b > \nu$ because otherwise the integration over the primary flux in Eqs. 3.7 and 3.8 diverges.

$$b_1 = \frac{1}{2} \left[\frac{\beta_N + \beta_c}{b - v} - Sb \right] + \frac{1}{2} \sqrt{\left[\frac{\beta_N + \beta_c}{b - v} - Sb \right]^2 + 4 \frac{Sb \beta_N}{b - v}}, \quad (5.9)$$

$$b_2 = \frac{1}{2} \left[\frac{\beta_N + \beta_c}{b - v} - Sb \right] - \frac{1}{2} \sqrt{\left[\frac{\beta_N + \beta_c}{b - v} - Sb \right]^2 + 4 \frac{Sb \beta_N}{b - v}},$$

so

$$\psi_{sc}(E, \lambda) = \frac{\beta_c \gamma_P P_O e^{-bE}}{b - v} \frac{\lambda}{(\lambda + Sb)(\lambda - b)(\lambda - b_2)}, \quad (5.10)$$

$$\psi_{sN}(E, \lambda) = \frac{\beta_N \gamma_P P_O e^{-bE}}{b - v} \frac{1}{(\lambda - b_1)(\lambda - b_2)}. \quad (5.11)$$

The inversion of the transforms is straightforward so we have

$$\Psi_{sc}(E, r) = \frac{\beta_c \gamma_P P_O e^{-bE}}{b - v} \left[- \frac{Sb}{(b_1 + Sb)(b_2 + Sb)} e^{-Sbr} \right. \\ \left. + \frac{b_1}{(b_1 + Sb)(b_1 - b_2)} e^{b_1 r} - \frac{b_2}{(b_2 + Sb)(b_1 - b_2)} e^{b_2 r} \right], \quad (5.12)$$

$$\Psi_{sN}(E, r) = \frac{\beta_N \gamma_P P_O e^{-bE}}{b - v} \frac{1}{b_1 - b_2} \left[e^{b_1 r} - e^{b_2 r} \right], \quad (5.13)$$

and from Eqs. 3.1 to 3.6

$$\Phi_{iP}(E, r) = P_O e^{-Qr} e^{-b(E+Sr)}, \quad (5.14)$$

$$\Phi_{sP}(E, r) = P_O \alpha_P \gamma_P Q e^{-Qr} e^{-bE} \left[- \frac{1}{(\alpha_P \gamma_P + \alpha_\pi \gamma_\pi) Q} e^{-Sbr} \right. \\ \left. + \frac{b_1}{(b - v)(b_1 - b_2)(b_1 + Sb)} e^{-b_1 r} - \frac{b_2}{(b - v)(b_1 - b_2)(b_2 + Sb)} e^{-b_2 r} \right], \quad (5.15)$$

$$\Phi_{s\pi}(E, r) = P_O \alpha_{\pi} \gamma_P Q e^{-Qr} e^{-bE} \left[- \frac{1}{(\alpha_P \gamma_P + \alpha_{\pi} \gamma_{\pi}) Q} e^{-Sbr} \right. \\ \left. + \frac{b_1}{(b - \nu)(b_1 - b_2)(b_1 + Sb)} e^{b_1 r} - \frac{b_2}{(b - \nu)(b_1 - b_2)(b_2 + Sb)} e^{b_2 r} \right], \quad (5.16)$$

$$\Phi_{sN}(E, r) = P_O \alpha_N \gamma_P Q e^{-Qr} e^{-bE} \left\{ \frac{1}{(b_1 - \nu)(b_1 - b_2)} \left[e^{b_1 r} - e^{b_2 r} \right] \right\}, \quad (5.17)$$

where

$$b_1 = \frac{1}{2} \left[\frac{\gamma Q}{b - \nu} - Sb \right] + \frac{1}{2} \sqrt{\left[\frac{\gamma Q}{b - \nu} - Sb \right]^2 + 4 \frac{Sb \alpha_N \gamma_N Q}{b - \nu}}, \quad (5.18)$$

$$b_2 = \frac{1}{2} \left[\frac{\gamma Q}{b - \nu} - Sb \right] - \frac{1}{2} \sqrt{\left[\frac{\gamma Q}{b - \nu} - Sb \right]^2 + 4 \frac{Sb \alpha_N \gamma_N Q}{b - \nu}},$$

$$\gamma = \alpha_P \gamma_P + \alpha_{\pi} \gamma_{\pi} + \alpha_N \gamma_N.$$

As the equations indicate the solutions in this case have a particularly simple form. Note that at all values of r the protons, neutrons, and pions have the same energy spectrum ($\sim e^{-bE}$) and this energy spectrum is the same as that of the incident protons.

The form of the solutions at large r is of particular interest. From Eq. 5.18 it follows that

$$b_1 > 0$$

$$b_2 < 0$$

so as r becomes large

$$\Phi_{iP}(E, r) + \Phi_{sP}(E, r) \rightarrow \frac{P_O \alpha_P \gamma_P Q b_1 e^{-bE}}{(b - \nu)(b_1 - b_2)(b_1 + Sb)} e^{-(Q-b_1)r}, \quad (5.19)$$

$$\Phi_{s\pi}(E, r) \rightarrow \frac{P_O \alpha_\pi \gamma_P Q b_1 e^{-bE}}{(b - \nu)(b_1 - b_2)(b_1 + Sb)} e^{-(Q-b_1)r}, \quad (5.20)$$

$$\Phi_{sN}(E, r) \rightarrow \frac{P_O \alpha_N \gamma_P Q e^{-bE}}{(b - \nu)(b_1 - b_2)} e^{-(Q-b_1)r}, \quad (5.21)$$

and thus at larger r all components are attenuating* with the same attenuation length. As a consequence one has finally as r becomes large

$$\frac{\Phi_{iP}(E, r) + \Phi_{sP}(E, r)}{\Phi_{sN}(E, r)} \rightarrow \frac{\alpha_P}{\alpha_N} \frac{b_1}{b_1 + Sb}, \quad (5.22)$$

$$\frac{\Phi_{s\pi}(E, r)}{\Phi_{sN}(E, r)} \rightarrow \frac{\alpha_\pi}{\alpha_N} \frac{b_1}{b_1 + Sb}; \quad (5.23)$$

i.e., at large r the ratio of the proton flux to the neutron flux and the ratio of the pion flux to the neutron flux is independent of both energy and distance.

*We are assuming that $Q - b_1 > 0$; otherwise one obtains divergent solutions which are of little physical interest.

Appendix. Inversion of the Transform

In this appendix the inverse Laplace transform of the function

$$g(E' - E, \lambda) = \frac{e^{-a_2(E' - E)} - e^{-a_1(E' - E)}}{S(a_1 - a_2)}, \quad (A.1)$$

$$a_1 = \frac{1}{2S} \left(\lambda - Sv - \frac{\beta_N S}{\lambda} \right) + \frac{1}{2S} \left[\left(\lambda + Sv + \frac{\beta_N S}{\lambda} \right)^2 + 4\beta_c S \right]^{\frac{1}{2}}, \quad (A.2)$$

$$a_2 = \frac{1}{2S} \left(\lambda - Sv - \frac{\beta_N S}{\lambda} \right) - \frac{1}{2S} \left[\left(\lambda + Sv + \frac{\beta_N S}{\lambda} \right)^2 + 4\beta_c S \right]^{\frac{1}{2}}, \quad (A.3)$$

will be obtained.

The function $g(E' - E, \lambda)$ may be written

$$g(E' - E, \lambda) = e^{\frac{\beta_N}{\lambda}(E' - E)} \left\{ \frac{e^{-\left(a_2 + \frac{\beta_N}{\lambda}\right)(E' - E)} - e^{-\left(a_1 + \frac{\beta_N}{\lambda}\right)(E' - E)}}{S(a_1 + a_2)} \right\}, \quad (A.4)$$

so using the theorem⁸

$$\mathcal{L}^{-1} e^{\frac{\beta_N}{\lambda}(E' - E)} = \left[\frac{\beta_N(E' - E)}{r} \right]^{\frac{1}{2}} I_1 \left[2\sqrt{\beta_N(E' - E)r} \right] + \delta(r) \quad (A.5)$$

($\mathcal{L}^{-1}$ = inverse Laplace operator) together with the convolution theorem we obtain

8. A. Erdelyi (Editor), Tables of Integral Transforms, McGraw-Hill Book Co., New York, 1954, Vol. 1, p 244.

$$G(E' - E, r) = \int_0^r dr' \quad (A.6)$$

$$\bullet \left\{ \left[\frac{\beta_N(E' - E)}{r - r'} \right]^{\frac{1}{2}} I_1 \left[2 \sqrt{\beta_N(E' - E)(r - r')} \right] + \delta(r - r') \right\} \\ \bullet \left\{ \mathcal{L}^{-1} \left[\frac{e^{-\left(a_2 + \frac{\beta_N}{\lambda}\right)(E' - E)} - e^{-\left(a_1 + \frac{\beta_N}{\lambda}\right)(E' - E)}}{S(a_1 - a_2)} \right] \right\}_{r'}$$

Utilizing the theorem*

$$\mathcal{L} \int_0^t du \left[\delta(t - u) - \left(\frac{au}{t - u} \right)^{\frac{1}{2}} J_1 \left[2 \sqrt{au(t - u)} \right] \right] f(u) \quad (A.7) \\ = h \left(\lambda + \frac{a}{\lambda} \right),$$

where

$$\mathcal{L} f(t) = h(\lambda), \quad (A.8)$$

Eq. A.6 becomes

*This theorem may easily be proven using the definition of the Laplace transform and the result**

$$\mathcal{L} \left\{ \delta(t) - \left(\frac{au}{t} \right)^{\frac{1}{2}} J_1 \left[2 \sqrt{auy} \right] \right\} = e^{-\frac{au}{\lambda}}.$$

**Reference 8, p 245.

$$G(E' - E, r) \quad (A.9)$$

$$= \int_0^r dr' \left\{ \left[\left(\frac{\beta_N (E' - E)}{r - r'} \right)^{\frac{1}{2}} I_1 \left(2 \sqrt{\beta_N (E' - E)(r - r')} \right) + \delta(r - r') \right] \right. \\ \cdot \int_0^{r'} du \left[\delta(r' - u) - \left(\frac{\beta_N Su}{r' - u} \right)^{\frac{1}{2}} J_1 \left(2 \sqrt{\beta_N Su(r' - u)} \right) \right] \\ \cdot \left[\mathcal{L}^{-1} \left(\frac{e^{\left[-\frac{1}{2S} (\lambda + Sv) + \frac{1}{2S} \sqrt{(\lambda + Sv)^2 + 4\beta_c S} \right] (E' - E)}}{\sqrt{(\lambda + Sv)^2 + 4\beta_c S}} \right. \right. \\ \left. \left. - \frac{e^{\left[-\frac{1}{2S} (\lambda - Sv) - \frac{1}{2S} \sqrt{(\lambda + Sv)^2 + 4\beta_c S} \right] (E' - E)}}{\sqrt{(\lambda + Sv)^2 + 4\beta_c S}} \right) \right]_u \right\}.$$

Furthermore⁹

$$\mathcal{L}^{-1} \left(\frac{e^{\left[-\frac{1}{2S} (\lambda - Sv) + \frac{1}{2S} \sqrt{(\lambda + Sv)^2 + 4\beta_c S} \right] (E' - E)}}{\sqrt{(\lambda + Sv)^2 + 4\beta_c S}} \right. \quad (A.10) \\ \left. - \frac{e^{\left[-\frac{1}{2S} (\lambda + Sv) - \frac{1}{2S} \sqrt{(\lambda + Sv)^2 + 4\beta_c S} \right] (E' - E)}}{\sqrt{(\lambda + Sv)^2 + 4\beta_c S}} \right) \\ = I_0 \left[2 \sqrt{\beta_c (E' - E - Su) u} \right] e^{\nu(E' - E - Su)} \mathcal{O}(E' - E - Su),$$

9. G. A. Campbell and R. M. Foster, Fourier Integrals for Practical Application, D. Van Nostrand Co., Inc., New York, 1948, p 113, No. 812.1.

so Eq. A.9 may be written

$$G(E' - E, r) \quad (A.11)$$

$$= \int_0^r dr' \left\{ \left[\left(\frac{\beta_N(E' - E)}{r - r'} \right)^{\frac{1}{2}} I_1 \left(2\sqrt{\beta_N(E' - E)(r - r')} \right) + \delta(r - r') \right] \right. \\ \cdot \int_0^{r'} du \left[\delta(r' - u) - \left(\frac{\beta_N Su}{r' - u} \right)^{\frac{1}{2}} J_1 \left(2\sqrt{\beta_N Su(r' - u)} \right) \right] \\ \left. \cdot \left[I_0 \left(2\sqrt{\beta_c(E' - E - Su)u} \right) e^{\nu(E' - E - Su)} \odot(E' - E - Su) \right] \right\}.$$

Interchanging the order of integration in Eq. A.11 and making the change of variable $y = r' - u$, we have

$$G(E' - E, r) \quad (A.12)$$

$$= \int_0^r du \left\{ \left[I_0 \left(2\sqrt{\beta_c(E' - E - Su)u} \right) e^{\nu(E' - E - Su)} \odot(E' - E - Su) \right] \right. \\ \cdot \int_0^{r-u} dy \left[\left(\frac{\beta_N(E' - E)}{r - u - y} \right)^{\frac{1}{2}} I_1 \left(2\sqrt{\beta_N(E' - E)(r - u - y)} \right) + \delta(r - u - y) \right] \\ \left. \cdot \left[\delta(y) - \left(\frac{\beta_N Su}{y} \right)^{\frac{1}{2}} J_1 \left(2\sqrt{\beta_N Suy} \right) \right] \right\}.$$

The inner integration in Eq. A.12 may now be carried out and we obtain the desired inversion*

*The method of integration used here is that explained on page 46 of Ref. 10.

10. A. Erdelyi (Editor), Higher Transcendental Functions, McGraw-Hill Book Co., New York, 1953, Vol. 2, p 46.

$$G(E' - E, r) \quad (A.13)$$

$$= I_0 \left[2 \sqrt{\beta_c (E' - E - Sr)} r \right] e^{\nu(E' - E - Sr)} \Theta(E' - E - Sr) \\ + \int_0^r dr' \left[I_0 \left(2 \sqrt{\beta_c (E' - E - Sr')} r' \right) e^{\nu(E' - E - Sr')} \Theta(E' - E - Sr') \right] \\ \cdot \left[\left(\frac{\beta_N (E' - E - Sr')}{r - r'} \right)^{\frac{1}{2}} I_1 \left(2 \sqrt{\beta_N (E' - E - Sr') (r - r')} \right) \right].$$

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