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PROGRESS REPORT ON STUDIES IN
XPL-8 SOLID REACTORS

R. L. Stewart
et al.

OAK RIDGE NATIONAL LABORATORY
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Applied Mechanics Section

REACTOR DIVISION

PROGRESS REPORT ON STUDIES IN APPLIED SOLID MECHANICS

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AUGUST 1970

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PROGRESS REPORT ON STUDIES IN APPLIED SOLID MECHANICS

ABSTRACT

The accomplishments made during the calendar year 1969 under Oak Ridge National Laboratory programs for developing structural design technology for piping components and for nozzle-to-pressure vessel attachment regions are described. These programs, being conducted in cooperation with industry through the Pressure Vessel Research Committee (PVRC) of the Welding Research Council, have as their primary objective the development of improved design methods for assuring, on a practical level, the structural integrity of the pressure vessels and primary coolant systems of nuclear power plants. The piping and nozzle programs, discussed in Parts I and II of the report, respectively, are directed at obtaining experimental data on structural behavior and establishing experimentally proven methods of design and analysis. This information is used in the development of design rules and analytical procedures for incorporation into current industry-developed standards and USAEC Division of Reactor Development and Technology (RDT) supplemental standards. The technology is also applicable to non-nuclear systems.

INTRODUCTION AND SUMMARY

This report describes the progress during the calendar year 1969 on programs for developing structural design technology for piping components and for nozzle-to-pressure vessel attachment regions; the combination is termed studies in applied solid mechanics. These are on going programs which are reported in the Oak Ridge National Laboratory Nuclear Safety Program bimonthly and annual reports. However, the results are also given separately at this time because of the importance of the work, the developments during the period, and the desirability of emphasizing these portions of the overall Nuclear Safety Program.

Both the nozzle and piping programs are being conducted in cooperation with industry through the Pressure Vessel Research Committee (PVRC) of the Welding Research Council. The objectives of the programs are to improve, on a practical level, the assurances of integrity of the pressure vessels and primary coolant systems of nuclear power plants. Applications of the technology developed are not limited, however, to nuclear reactor systems. The nozzle and piping programs are directed at producing experimental data

on structural behavior and at establishing experimentally proven methods of design and analysis that will be incorporated into current industry-developed standards.

Design Criteria for Piping, Pumps, and Valves

The ORNL program on design criteria for piping, pumps, and valves is the AEC-supported portion of a cooperative effort with industry to develop background information and design techniques for use in safety codes and standards for present-generation nuclear power plant piping systems. Close cooperation with industry is maintained through the Pressure Vessel Research Committee, and with the various code-writing bodies such as the American National Standards Institute (ANSI), formerly the United States of America Standards Institute (USASI), and the American Society of Mechanical Engineers (ASME). During the past year, code rules and the associated stress indices for the design of small branch connections were developed under the ORNL program and were incorporated into the USASI Standard Code for Pressure Piping, Nuclear Power Piping - USA standard B31.7, published by the ASME in 1969.

A brief summary of work under the ORNL portions of the program is given below. A more detailed description is given in Part I of this report.

The experimental analysis of a thin-walled cylindrical shell with a branch connection was completed, and the results were compared with a finite-element elastic analysis. Comparisons between the experimental and analytical results for the internal pressure loading case are presented and show overall good agreement, especially for the maximum values. Results from the entire study, consisting of 13 different loadings - three mutually perpendicular force and moment components applied on the ends of the run and branch - as well as internal pressure, will be reported separately. The experimental data provide a much-needed basis for the further development of finite-element techniques for the stress analysis of shell structures.

Both the experimental and analytical studies of ASA B16.9 tees are well under way. Experimental work is being done at ORNL and, under subcontract, at Southwest Research Institute, Combustion Engineering, Inc., and Westinghouse Research Laboratories. Analytical development work is being done at the University of California. The interrelationship between

these tasks and the individual progress accomplished during the year are discussed in detail. During the coming year a great deal of data from these tasks will become available, and the remaining tests can be planned in detail.

Experimental and analytical studies for elbows and miters were also initiated during 1969. The experimental work includes plastic collapse tests of standard short- and long-radius elbows and elastic response tests of machined elbows with perfect geometry and with controlled distortions. Analytical studies include theoretical development work for miters, studies of the influence of straight pipe welded to the elbows, and analysis of the experimental models.

Overall progress under the ORNL piping program has been very good. Many different task efforts are fully under way, and the influence of this program is already being recognized in the new codes and standards.

Experimental and Analytical Investigations of Nozzles

The Oak Ridge National Laboratory has been assigned the responsibility for managing and directing as well as participating in experimental and analytical studies related to the structural behavior of nozzle attachments in spherical and cylindrical shells. These studies provide basic engineering data for design of nuclear reactor pressure vessels, as well as pressure vessels for other applications and are being carried out in close cooperation with industry through the Pressure Vessel Research Committee. Nozzles attached singly and in clusters are being studied. A summary statement of the work under this program is given below, with a more detailed report contained in Part II.

An analytical study using the method of finite elements on the effects of reinforcement of a radial nozzle in a spherical shell was initiated. In order to provide background information regarding the particular finite-element computer code used, two nozzle-to-spherical-shell models which had been tested previously were analyzed. The finite-element results were compared with the experimental results and with results from a thin-shell analysis. Close agreement between the finite-element results and the thin-shell theory results was obtained except in the immediate vicinity of the junction, where the finite-element method reflected local stress concentrations.

In another study on closed-form solution type shell analysis, analytical and experimental comparisons for a large radial nozzle attached to a spherical shell were made. The experimental model was loaded under

internal pressure and an axial force applied to the nozzle. The experimental and theoretical results agree satisfactorily in both distribution and magnitude, except for the stresses at the inner surface of the sphere. The highest stresses calculated for the model agreed quite well with the experimental data.

A single nonradial nozzle attached to a spherical shell at an oblique angle of 22.5° was tested to examine elastic behavior under internal pressure and force and moment loadings acting in the nozzle. In addition, a large radial nozzle attached to a spherical shell was tested. Results from the latter nozzle were used for the above analytical comparison.

A theoretical solution for a radial nozzle attached to a cylindrical shell loaded with either an internal pressure or an out-of-plane bending moment applied to the nozzle was completed, programmed for the computer, and checked. The analysis of a photoelastic model, Westinghouse WC-2AY, for the case of internal pressure gave reasonably good agreement with experimental results for the maximum stresses. However, near the nozzle-to-shell intersection the stress distributions predicted by the analysis differed considerably from the photoelastic results.

A finite-element study is being made to determine the stress concentrations occurring in stepped cylindrical shells with various fillet radii. Two cases have been analyzed for internal pressure loading, both of which had dimensionless parameter ratios of $t/T = 1.5$ and $a/t = 5.0$, where t and T are the wall thicknesses of the two cylinders and a is the radius of the smaller cylinder. In one case there was no fillet at the transition, while in the other case the fillet had a radius equal to $0.3 \times t$. The effect of the fillet was to reduce the maximum stress by 9%.

A machined 45° lateral was tested under internal pressure. The maximum stresses occurred at the acute inside corner in the circumferential direction. At the fillet opposite the acute inside corner, the circumferential stresses were compressive and much smaller in magnitude. The highest stress on the outside surface was on the transverse plane of the run.

Series of photoelastic tests were made for lateral and hillside nozzles in cylindrical shells, one hillside opening in a cylindrical shell, oblique nozzles in spherical shells, and one oblique opening in a spherical shell. In all, 15 nozzles and openings were studied. These nozzles and openings were contained in three cylindrical pressure vessel models with hemispherical end closures. The first two models each contained five nozzles and one opening, while the third model contained five nozzles

and two openings. However, only one nozzle and the two openings contained in the third model were analyzed.

Lower bound limit pressures have been calculated and have been experimentally verified for radial nozzles attached to cylindrical shells within the parameter range $d/D \leq 0.5$. Upper bound limit pressures have been determined analytically within the range $0 < d/D \leq 1.0$, and experimental data are being obtained for verification. In a separate study a limit load and pressure burst test was conducted on a thin-walled cylindrical shell with a single radial nozzle. Both the load vs deflection curves and the load vs strain curves were nonlinear over practically the entire pressure range. It thus appears that the limit load concept is invalid for thin shells of this type.

Analytical solutions for the stresses in square flat plates with clusters of holes have been calculated for plates with one, two, three, four, and five holes. Experimental results were obtained for plates with one hole and with two holes for both uniaxial and biaxial tension in the plates, and comparisons between the analytical and experimental results for the one-hole plate show excellent agreement. Comparisons for the two-hole plate are currently being prepared.

A computer program is being developed to analyze clusters of nozzles attached to square flat plates. At present the program is being used to analyze one nozzle attached to a circular plate for comparison with an existing program based on shell theory. Comparisons for the internal pressure load and for an axial force load on the nozzle were excellent. Discrepancies, however, exist for other loads, and further study is required to resolve the differences.

A model consisting of two closely spaced large radial nozzles attached to a spherical shell was tested for an internal pressure load and for an axial thrust load and a bending moment load on one of the nozzles. For the internal pressure case, the meridional stresses between the two nozzles were lower than those measured elsewhere, and the circumferential stresses between the two nozzles were higher than those measured elsewhere. Experimental results from the axial thrust case are being studied.

Finally, a set of alternate pressure vessel code rules was developed, which pertain to reinforcement of radial nozzles attached to spherical and cylindrical shells. These rules have been submitted to the ASME Design Committee's Subgroup on Openings and Attachments for use in pressure vessel codes.

PART I
DESIGN CRITERIA FOR PIPING, PUMPS, AND VALVES

W. L. Greenstreet	S. E. Moore
S. E. Bolt	J. M. Corum

Introduction

Basic structural information and design data are being developed for assuring the continued adequate and safe design of piping systems for nuclear service. The ORNL work is primarily concerned with piping components and is part of a larger AEC-industry cooperative program for developing design criteria for piping components, pumps, and valves. Through this program, factors urgently needed for use both in industry and in AEC-RDT codes and standards are being developed which delineate design practices commensurate with meeting reliability and safety requirements.

Primary tasks to be carried out are accurate, thorough experimental and analytical stress analyses of pipe fittings. At present, the experimental work is confined to tees, branch connections, and elbows, while analytical studies are planned for all standard pipe fittings. From the analyses and companion studies, data correlations and evaluations will be made; design charts, graphs, and tables will be prepared; and code rules will be drafted for use by the various code bodies. Data will be presented in terms of stress indices and flexibility factors for direct use. Overall interpretive reports will also be written on work sponsored under this program and on that done by others.

The ORNL program is based on a research proposal¹ consisting of 12 tasks for the development of stress indices and methods of analysis which was developed by the Ad-Hoc Committee of the Pressure Vessel Research Committee (PVRC) of the Welding Research Council. The AEC agreed² to sponsor a portion of the work on piping components in a cooperative effort between AEC and industry and asked that industry sponsor the remaining tasks on pumps and valves and the development of low-cycle fatigue criteria. Oak Ridge National Laboratory was charged with the responsibility for planning, managing, and directing the AEC portion of the cooperative program, with specific management functions to include the following:

1. detailed technical planning of the program and development of work priorities;
2. procurement of consultant services, materials, and placing of contracts for research and development;

3. a coordination of work with cooperating organizations;
4. periodic reporting of progress of work;
5. preparation of summary reports;
6. development of recommended requirements for specifications, codes, and standards necessary to provide for the assured continued safety of nuclear plant piping, pumps, and valves.

A permanent subcommittee of the PVRC, the Subcommittee to Develop Stress Indices for Piping, Pumps, and Valves, was formed to coordinate the non-AEC portion of the work and to advise the code-writing bodies. This subcommittee is composed of four working task groups, one of which was asked to review, consult with, and advise ORNL in its program. Periodically the subcommittee and the working task groups meet to discuss the various research efforts. Results from the ORNL program as well as the industry-supported work are used to form recommendations concerning the methods of analysis and design rules for the various code-writing bodies. Annual information meetings are also held, usually in conjunction with a subcommittee meeting. The 1969 information meeting was a two-day conference, Studies in Applied Solid Mechanics, held April 28 and 29 in conjunction with the Experimental and Analytical Investigation of Nozzles (ORNL nozzles program) information meeting. A tentative date for the next annual meeting has been set for April 1-3, 1970.

The scope of the ORNL piping program includes both analytical and experimental studies for the basic components that may be used in class I nuclear piping systems. The objectives of the work are to provide basic information on the structural behavior of piping components and to organize this information in a convenient manner for use in safety codes and standards such as the USA Standard Code for Pressure Piping, Nuclear Power Piping - Section B31.7 (ref. 3) and the AEC Division of Reactor Development and Technology (RDT) standards.^{4,5} Both these codes require a detailed stress analysis and stress report for class I piping system components similar to the requirements of

¹Program and Request for Proposals for Development of Stress Indices and Methods of Analysis for Piping Components, Pumps, and Valves, unnumbered PVRC report dated July 1, 1967.

²Letter from AEC Division of Reactor Development and Technology (DRDT), Director, Milton Shaw, to C. F. Larson, Executive Secretary, Pressure Vessel Research Committee, Jan. 9, 1968. Subject: PVRC Proposed Program for Development of Stress Indices for Piping, Pumps, and Valves.

³USA Standard, Code for Pressure Piping, Nuclear Power Piping, USAS B31.7, American Society of Mechanical Engineers, New York, 1969.

⁴Nuclear Piping Analysis, RDT E7-1, USAEC Division of Reactor Development and Technology (to be published).

⁵Design Specification for Nuclear Piping, RDT E7-2, USAEC Division of Reactor Development and Technology (to be published).

Section III of the ASME Boiler and Pressure Vessel Code⁶ for class A vessels.

Satisfactory analytical stress analysis methods, however, have not been developed for many of the components used in class I piping systems, thus making individual design analyses both difficult and costly relative to the cost of the hardware. On the other hand, many types of components are, to a certain extent, standardized, and once analytical methods or sufficient experimental data are available to predict the behavior of such standardized components, the future cost of the analyses will become relatively small. This economy is further enhanced by developing design graphs and tables using analytical methods or empirical correlations from experimental data and by introducing simplified methods for using the results in structural evaluations. The stress indices being developed under the ORNL piping program are intended for use with the simplified analytical procedures of USA standard B31.7.

Table 1 shows a listing of the various piping components and the specific studies planned for each of the components, indicated by an "X" in the appropriate column. The program is somewhat flexible in that, as the work progresses, additional studies will be added as the needs are identified. One important function of the PVRC Subcommittee to Develop Stress Indices for Piping, Pumps, and Valves is to provide a continuing assessment of the needs and objectives of the work and to assist ORNL in planning extensions or revisions to the program. A summary of the work under the ORNL piping program during the past year is given below.

Survey Reports

Literature Survey

As implied in the first column of Table 1, a great deal of applicable information that can be used in this program already exists in the literature. The first major objective was therefore to identify and evaluate existing test data and analytical methods. The preliminary draft of a comprehensive literature review⁷ has been compiled at Battelle Memorial Institute with the help of Southwest Research Institute, under subcontract to ORNL. Over 700 references on various aspects of stress analysis of piping system components were collected and analyzed for application in the development of stress indices and structural design methods suitable for code use. Copies of the draft were distributed to the AEC, to the RDT Standards Program Office at ORNL, to the PVRC Subcommittee, and to various individuals for comment and review and to elicit additional information. The review process was completed August 15, 1969, and the final draft is in preparation. Plans have been made to publish the report as a USAEC Division of Technical Information document and to prepare a condensed version for consideration as a *Welding Research Council Bulletin*. It is intended that

⁶ ASME Boiler and Pressure Vessel Code, Section III, Nuclear Vessels, American Society of Mechanical Engineers, New York, 1969.

⁷ E. C. Rodabaugh and A. G. Pickett, *Survey Report on Structural Design and Piping Components* (to be published).

Table 1. Studies for Class I Piping System Components

Component	Evaluation of Existing Data and Analytical Methods	Analytical Methods Development	Experimental Stress Analysis	Limit Load Test and Analysis	Thermal Stress Analysis and/or Tests	Fatigue Tests
Straight pipe (out of roundness, flat spots, peaks)	X					
Circumferential butt welds	X				X	
Local attachments to straight pipe (lugs, supports, anchors)	X				X	
Socket welding and threaded fittings	X				X	
Transition joints	X				X	
Concentric reducers	X				X	
Elbows, miter bends, and curved pipe	X	X	X	X		X ^a
Flanged joints	X				X	
Branch connections	X	X	X	X	X	
ASA B16.9 tees		X	X	X	X	X

^aOval and thinned elbows.

this report will be a major source of reference for existing information and for planning future additions to the ORNL program.

Class I Piping System Components

A survey report is being prepared on the number and type of piping components being used in class I piping systems in new water-cooled nuclear power plant construction. This information is being developed in order to assure that the scope of the ORNL piping program properly reflects the needs of present-generation nuclear power plant design. The types of components most often used, the range of sizes, the design conditions such as pressure and temperature, the material used, and the fabrication procedures should all be considered in order to obtain maximum benefit from the research efforts. In this survey we plan to document the piping systems of several typical plants of both the boiling-water (BWR) and the pressurized-water (PWR) type.

The piping system of Millstone Point I, a 549-MW_e BWR nuclear power plant located at Niantic, Connecticut, has been audited. We also plan to examine Millstone Point II, an 828-MW_e PWR, and perhaps one or two other plants that are under construction. Piping systems in the Millstone I plant identified as class I are the nuclear steam supply systems, all the reactor emergency systems, and the piping in the condensate feedwater system. The nuclear steam supply systems include the recirculating piping system and all piping connections from the reactor vessel up to and including the first isolation valve external to the dry well.

In order to develop a documented "as-built" listing, it was necessary to examine site construction drawings, called spool piece drawings, and lists of purchased materials at the plant location. This proved to be the only way that a complete list of components could be obtained, including those purchased to ASA standards and those which are specialty items, because the detailed design of the piping system is the responsibility of the architectural engineer and is done after the plant is actually under construction. It is interesting to note that in the class I piping systems in this plant, there are 422 individual components, excluding straight pipe sections, and many of these are one of a kind. If this piping were constructed according to the rules of USAS B31.7, a complete stress analysis of each component would be required and a stress report covering the entire class I piping written.

Dimensional Survey of Pipe Fittings

A survey and dimensional analysis of standard pipe fittings was conducted⁸ in order to investigate the dimensional variations that might be obtained in commercially available butt welding and screwed fittings. The objectives of this study were to assemble, on a limited basis, a detailed dimensional description of commercial fittings in order to determine, if possible, a "nominal shape," and to statistically evaluate the dimensional variations from this nominal shape. Since stress indices and simplified design methods are being developed for standard fittings based on their nominal shape, an indication of the dimensional variations to be expected is needed to assure that proper consideration is given in the development of conservative and realistic design factors.

A small number of piping components were purchased "off the shelf" from four different manufacturers, and the dimensions and shape of these components were studied in detail. Sixty-four carbon steel (WPB) and stainless steel (type 304) fittings consisting of elbows, tees, reducers, and caps were examined. Most of these were 4-in. sched 40 fittings; however, a few 6-in. sched 80 fittings were included. We also obtained complete dimensional data from 18 ASA standard B16.9 tees, with nominal diameters ranging from 6 to 24 in. and wall thicknesses from sched 40 to sched 160.

Dimensional information on this small number of fittings cannot be expected to represent, even in a reasonable fashion, all the fittings which might be used in a nuclear power plant. For example, there were fewer fittings used in this study than in the Millstone I plant discussed above. The study does, however, give some perspective to the kind of dimensional variations that can be expected for the different components, and, through the use of statistical methods in reducing the data, it was possible to discuss the results in a quantitative manner. In general, the results indicate that rather large dimensional variations can be expected for most piping components, especially when the product of several different manufacturers is considered. It is therefore prudent, in the development of generally applicable design factors, to stress the use of conservative numbers.

⁸J. P. Callahan, J. N. Robinson, and S. E. Moore, *Dimensional Study of ASA Standard Pipe Fittings* (to be published).

Theoretical and Experimental Analysis of Thin-Shell Cylinder-to-Cylinder Models

ORNL-DWG 68-94098

The first in a series of four thin-shell cylinder-to-cylinder models has been tested at ORNL, and the experimentally determined stresses have been compared with theoretical predictions obtained from a finite-element analysis. The models in this series, listed in Table 2, are idealized thin-shell structures consisting of two circular cylindrical shells intersecting at right angles. There are no transition regions, reinforcements, or fillets at the junction. Model 1 has been analyzed, and models 2 and 3 are being fabricated; they will be analyzed during the coming year. These model studies serve two basic purposes: (1) the experimental data will directly provide design information applicable to nozzles in cylindrical vessels and (2) the idealized models serve a basic need for test results for use in developing theoretical analyses applicable to piping tees as well as nozzles in cylindrical pressure vessels.

The first model, together with the loadings and boundary conditions to which it was subjected, is depicted schematically in Fig. 1. The mean diameter of the cylinder was 9.9 in., and its thickness was 0.10 in. The nozzle mean diameter was 4.95 in., and the thickness was 0.05 in. The mean diameter-to-thickness ratio for both cylinder and nozzle was thus 99. A total of 13 separate loading cases were analyzed. In addition to an internal pressure of 50 psi, these consisted of three mutually perpendicular force components and three mutually perpendicular moment components applied at the free end of the cylinder and at the free end of the nozzle. The left end of the cylinder, as seen in Fig. 1, was rigidly fixed in the plane $X = 0$, while the free end of the cylinder and the end of the nozzle were

DIMENSIONS ARE IN INCHES

Fig. 1. Schematic Drawing of Thin-Shell Cylinder-to-Cylinder Model.

closed off by thick plates. These plates restrained the boundary circles in such a way that they remained plane circles, and at the edges the shells were largely restrained against rotation with respect to the planes of the edge circles.

In the following sections, the construction and experimental test of this first cylinder-to-cylinder model are briefly described, the finite-element analysis of the model is described, and comparisons of the experimental results with the finite-element predictions are presented for the internal pressure loading case. A topical report covering the entire study is being prepared.

Model Construction and Experimental Analysis

The material for the first cylinder-to-cylinder model was a low-carbon steel. The model was made by welding together two thick-walled cylinders and then carefully machining the thin-walled model from the weldment. After finishing the inner surface of the cylinder and nozzle by boring, the outer surfaces were machined down to the junction region. Finally, the junction region was finished by hand. The resulting model was of relatively high quality. Despite the care taken in machining the model, however, there were inevitably slight geometric imperfections. These imperfections caused some discrepancies in experimental data from regions that should have, theoretically, behaved exactly the same.

Table 2. Thin-Shell Cylinder-to-Cylinder Tee Models

Model No.	Major Dimensions ^a (in.)				Dimensionless Parameters			
	D_o	d_o	T	t	d_o/D_o	D_o/T	d_o/t	s/S^b
1	10.0	5.0	0.1	0.05	0.5	100	100.0	1.00
2	10.0	10.0	0.1	0.1	1.0	100	100.0	1.00
3	10.0	1.29	0.2	0.168	0.129	50	7.7	0.154
4 ^c	10.0	1.29	0.2	0.064	0.129	50	20.2	0.40

^a D_o is the outside diameter of the cylindrical shell, d_o is the outside diameter of the nozzle, T is the wall thickness of the cylindrical shell, and t is the wall thickness of the nozzle.

^bThe dimensionless parameter s/S is the ratio of membrane stress in the nozzle to the membrane stress in the cylindrical shell due to internal pressure loading; in terms of other parameters, $s/S = d/D \times T/t$.

^cModel 4 will be made by boring out the nozzle of model 3.

The strain-gage layout for the model is given in Fig. 2, which shows developed views of the cylinder and nozzle. A total of 322 three-gage strain rosettes were used. These rosettes were arranged primarily in two opposite quadrants of the model, as shown in the figure. Each quadrant had four lines, or strings, of gages. On the nozzle, the gage lines were spaced at 30° intervals and were oriented in the axial direction along the nozzle. On the cylinder, the gage lines were helical curves perpendicular to the junction at the points where the nozzle gage lines intersected the junction line. Rosettes were placed on both the inside and outside surfaces at the points indicated in Fig. 2. The rosettes were Micro-Measurements type EA-06-30YB-120, which is a very compact foil gage with an individual active gage length of 0.030 in.

The instrumented model is shown in Fig. 3. In this photograph, an out-of-plane moment is being applied to the nozzle.

Theoretical Analysis

The purpose of the theoretical analysis of the first model was to evaluate the accuracy and general capability of the finite-element method in the analysis of thin-shell cylinder-to-cylinder configurations. This is a difficult task because, unlike the well-developed and accepted finite-element formulations for solid bodies, there are no completely satisfactory formulations for general curved shell structures. Both flat-plate and curved-shell elements have been used. The representation of curved surfaces by flat elements has obvious

Fig. 2. Strain Gage Layout for Thin-Shell Cylinder-to-Cylinder Model (Inside and Outside Surfaces).

Fig. 3. Instrumented Thin-Shell Cylinder-to-Cylinder Model in Test Frame.

Fig. 4. Finite-Element Mesh Layout.

shortcomings; on the other hand, the formulation of satisfactory curved elements faces many difficulties, so that they are at a relatively early stage of development.⁹

The finite-element shell program chosen for this comparison is believed to be representative of the present state of the art. It was originally developed by C. P. Johnson as a Ph.D. dissertation¹⁰ under Professor R. W. Clough at the University of California, Berkeley. The program was subsequently modified to adapt it to the analysis of intersecting shell surfaces, such as those found in cylinder-to-cylinder configurations,¹¹ and it was this modified program, called JOINT, which was used in the present study. The specific analysis that has been compared with experimental results was performed at the University of California under the direction of Professor R. W. Clough.

⁹R. H. Gallagher, "Analysis of Plate and Shell Structures," pp. 155-205 in *Proceedings of the Symposium on Application of Finite Element Methods in Civil Engineering*, American Society of Civil Engineers, November 1969.

¹⁰C. P. Johnson, *The Analysis of Thin Shells by a Finite Element Procedure*, Structures and Materials Research, Department of Civil Engineering, University of California, Berkeley, Report No. 67-22 (September 1967).

The basic elements used in this program are flat-plate elements of general quadrilateral shape and are built up as an assemblage of four triangular elements. Each triangular element has a membrane stiffness property based on assumed quadratic displacement patterns in the plane of the element and a bending stiffness based on cubic displacements normal to the element. The resulting quadrilateral element has five degrees of freedom at each corner, the rotation about the normal to the plate being neglected.

The finite-element representation that was used is shown in Fig. 4. Because of symmetry considerations it was only necessary to consider one half of the structure. The mesh had 649 nodes, resulting in about 3000 equations.

The principal boundary conditions used in the analysis included full fixity at all nodes attached to the left-hand support. The heavy plates that closed the free ends of the cylinder and nozzle were included in the finite-element idealization. The boundary conditions used for the nodes along the symmetry plane provided

¹¹Ojars Greste and Ray W. Clough, *Finite Analysis of Tubular Joints: A Report on a Feasibility Study*, Structures and Materials Research, Department of Civil Engineering, University of California, Berkeley, Report No. 67-7 (April 1967).

for either symmetrical or antisymmetrical deformations, depending on the loading.

Comparison of Theory and Experiment for Internal Pressure Loading

The experimentally determined stresses are compared with the finite-element predictions in Figs. 5-12 for all of the gage lines. These figures are arranged in pairs that should, theoretically, match. The results at 0 and 180° (refer to Fig. 2) are shown in Figs. 5 and 6, respectively; the 330 and 150° results in Figs. 7 and 8; the 300 and 120° results in Figs. 9 and 10; and the 270 and 90° results in Figs. 11 and 12. In each of these figures, "longitudinal stress" refers to the normal stress in the direction of the line of gages, and "transverse stress" refers to the normal stress perpendicular to the line of gages.

An examination of Figs. 5 and 6, which show the results in the longitudinal plane of symmetry, reveals very good agreement between theory and experiment. Furthermore, the experimental points seem consistent, and the agreement between the 0 and the 180° experimental results is good.

An examination of the remaining figures reveals two trends. First, as the transverse plane of symmetry (270 and 90°) is approached, the agreement between theory and experiment becomes less satisfactory, although it is still reasonably good except for the stresses in the nozzle at 270 and 90°. This decrease in the agreement between theory and experiment is not surprising, because the shortcomings of the flat-plate-element representation would be expected to have a greater influence in the transverse plane than in the longitudinal plane. These shortcomings are described by Gallagher.⁹

The second trend is that the experimental results from lines which should exhibit identical results often disagree. It is felt that this disagreement is a consequence of the cumulative effect of the very small geometric imperfections in the model. The two halves of the experimental model, defined by the longitudinal plane of symmetry, were not identical.

The comparison of predictions with experimental results thus shows reasonably good qualitative agreement in all cases except for the nozzle at the transverse plane. In addition, the quantitative agreement is excellent at many points. In particular, the maximum stresses, which occur in the longitudinal plane for this model, are closely predicted by the finite-element analysis.

Structural Analysis of Branch Connections and ASA B16.9 Tees

In addition to the thin-shell models discussed in the previous section, two types of tee-joint piping components are being studied. These are branch connections and ASA B16.9 tees, both of which consist of a straight section or run pipe and a branch pipe which intersects the run at a right angle. The term "branch connection" is used to designate tee joints constructed in accordance with the rules of USAS B31.7 for nuclear piping. For application of these rules the maximum diameter ratio of the branch pipe to the run pipe is limited to a value of $d/D \leq 1/2$, where d is the diameter of the branch pipe and D is the diameter of the run pipe.

ASA B16.9 tees are commercially available butt-welding tees fabricated in accordance with the USA B16.9 or the MSS SP-48 standards.^{12,13} The diameter ratio d/D is normally greater than $1/2$ but may be as small as $1/3$. Other commercially available tee-joint fittings sold under registered trade names are not being studied in this program.

Branch Connections

Branch connections are a class of tee joints in which the diameter of the branch pipe is normally much smaller than the diameter of the run pipe, and they may or may not have reinforcing material at the junction. The nuclear piping code, USAS B31.7, requires reinforcing which is equal to 100% of the cut-out area, called the area replacement rule, unless the mean diameter of the branch satisfies the inequality

$$d \leq 0.2 \sqrt{R_m T},$$

where R_m is the mean radius of the run and T is the nominal wall thickness of the run pipe.

A considerable amount of information has been developed for branch connections, through the efforts of the PVRC Subcommittee on Reinforced Openings, because of their use in pressure vessels. In this case, they are simply radial nozzles attached to cylindrical shells. For pressure vessel application the primary loading of interest is internal pressure, and consequently relatively little information has been developed for applied bending loads and direct thrust, especially when applied on the run. Therefore, additional studies

¹² B16.9 - 1964, *Wrought Steel Butt Welding Fittings (ISO R-285)*, American National Standards Institute, New York.

¹³ MSS SP-48 - 1969, *Steel Butt-Welding Fittings (26-in. and Larger)*, Manufacturers Standardization Society of the Valve and Fittings Industry, Arlington, Va.

Fig. 5. Measured and Predicted Stresses on 0° Gage Lines for Cylinder-to-Cylinder Model Subjected to an Internal Pressure of 50 psi.

Fig. 6. Measured and Predicted Stresses on 180° Gage Lines for Cylinder-to-Cylinder Model Subjected to an Internal Pressure of 50 psi.

Fig. 7. Measured and Predicted Stresses on 330° Gage Lines for Cylinder-to-Cylinder Model Subjected to an Internal Pressure of 50 psi.

Fig. 8. Measured and Predicted Stresses on 150° Gage Lines for Cylinder-to-Cylinder Model Subjected to an Internal Pressure of 50 psi.

Fig. 9. Measured and Predicted Stresses on 300° Gage Lines for Cylinder-to-Cylinder Model Subjected to an Internal Pressure of 50 psi.

Fig. 10. Measured and Predicted Stresses on 120° Gage Lines for Cylinder-to-Cylinder Model Subjected to an Internal Pressure of 50 psi.

Fig. 11. Measured and Predicted Stresses on 270° Gage Lines for Cylinder-to-Cylinder Model Subjected to an Internal Pressure of 50 psi.

Fig. 12. Measured and Predicted Stresses on 90° Gage Lines for Cylinder-to-Cylinder Model Subjected to an Internal Pressure of 50 psi.

are to be undertaken as a part of this program. Two such studies, undertaken during the past year, are described below.

Stress Indices for Branch Connections. — The USA Standard Code for Pressure Piping, Section B31.7, was first issued in draft form for review and comment in February 1968. One of the comments¹⁴ pointed out that the stress indices for branch connections as presented in the draft could be unconservative for configurations in which either part or all of the required reinforcement is supplied by the run pipe. Consequently, a study of the available data for branch connections with external loadings was conducted. Empirical correlations were then established and used to formulate stress indices for small branch connections¹⁵ — $d/D \leq 0.5$ — which overcome these earlier problems. Modified rules were also written and proposed to the code committee for using these stress indices in connection with the simplified design procedures of USAS B31.7. The new rules were adopted and are included in the latest issue of the code.

Analytical Parameter Study. — An analytical solution for the internal pressure loading case developed previously by Eringen¹⁶ is available for relatively small branch connections; but, at the present time, there are no theoretical solutions available for externally applied force and moment loadings. The solutions developed by Bijlaard¹⁷⁻¹⁹ for distributed loads over a small rectangular area on a cylindrical shell give some guidance for the design of branch connections, however. In addition, analytical solutions being developed for externally applied loads on nozzles in cylindrical shells under the ORNL nozzles program, discussed in another part of this report, will be available in the near future.

In order to develop more immediate analytical information which can be used to extend the coverage of stress indices to include combined loadings, a series of eight models, listed in Table 3, are being analyzed using the finite-element computer program JOINT discussed earlier. The last four models in this series are reinforced at the branch pipe junction, as indicated in the last column, and they have been experimentally analyzed photoelastically²⁰⁻²² for the case of internal pressure loading using the frozen stress technique. Thirteen different loading conditions will be studied for each model. These include internal pressure and three mutually perpendicular force and bending moment components applied to the branch pipe and to the run pipe.

The study will be conducted in three stages. First, the unreinforced models will be analyzed using the computer program JOINT. Model 1 has also been photoelastically analyzed²² for internal pressure loading, and a comparison of the analytical results with these data will provide further confidence in the computer program. The program will then be modified, as necessary,

²⁰M. M. Leven, "Photoelastic Determination of the Stresses in Reinforced Openings in Pressure Vessels," *Welding Research Council Bulletin No. 113*, April 1966.

²¹M. M. Leven, *Photoelastic Determination of Stresses at an Opening in a Thin-Wall Cylindrical Pressure Vessel*, Report No. 67-9D7-PHOTO-R1, Westinghouse Research Laboratories (August 1967).

²²M. M. Leven, *Photoelastic Determination of Stresses in a Reinforced Steam Charging Connection with a Thermal Sleeve*, Report No. 69-9D7-PHOTO-R1, Westinghouse Research Laboratories (Jan. 14, 1969).

¹⁴R. N. Zogran to E. C. Rodabaugh, personal communication, Dec. 27, 1968.

¹⁵E. C. Rodabaugh, *Stress Indices for Small Branch Connections with External Loadings*, BMI Phase Report 115-1, Battelle Memorial Institute (April 1969).

¹⁶A. C. Eringen *et al.*, *Analysis of Stress and Deformation in Two Normally Intersecting Cylindrical Shells Subjected to Internal Pressure*, Technical Report No. 3-9, General Technology Corp. (January 1967).

¹⁷P. P. Bijlaard, "Stresses from Local Loadings in Cylindrical Pressure Vessels," *Trans. ASME* (August 1955).

¹⁸P. P. Bijlaard, "Stresses from Radial Loads and External Moments in Cylindrical Shells Under Local Loads," *Welding Research Supplement*, December 1955.

¹⁹P. P. Bijlaard, "Additional Data on Stresses in Cylindrical Shells Under Local Loadings," *Welding Research Council Bulletin No. 50*, May 1959.

Table 3. Branch Connection Models for Analytical Parameter Study

Model No.	Dimensionless Parameters ^a				
	D/T	d/D	t/T	s/S	r/T
1	59.4	0.115	0.235	0.49	0
2	20.0	0.02	0.02	1.00	0
3	20.0	0.08	0.473	0.169	0
4	20.0	0.32	1.00	0.322	0
1F	59.4	0.115	0.235	0.49	0.60
2F	20.0	0.02	0.02	1.00	1.00
3F	20.0	0.08	0.08	1.00	1.46
4F	20.0	0.32	0.32	1.00	3.34

^aThe dimensionless parameters listed here are the same as those discussed earlier in Table 2 with the exception of the ratio r/T , which is the ratio of the reinforcement fillet radius to the wall thickness of the run pipe.

to properly account for the reinforcement in the remaining models, 1F through 4F, using the photoelastic data for the pressure loading case as a reference. The third stage of the study will be a complete analysis of the reinforced models and the computation of stress indices for combined loadings suitable for inclusion in the codes and standards.

ASA B16.9 Tees

ASA B16.9 tees are commercially available butt-welding fittings fabricated in accordance with the USA B16.9 or MSS SP-48 standards.^{12,13} The diameter ratio of the branch pipe to the run pipe is normally greater than $\frac{1}{2}$ but may be as small as $\frac{1}{3}$. These fittings are characterized by a smooth transition region between the branch and run connections and are normally forged as a single unit. The wall thickness is large in comparison with the diameter, as shown in Fig. 13, so that thin-shell theory is not expected to apply.

Essentially no information is available in the literature which can be applied directly in the development of stress indices and flexibility factors for these fittings. Consequently, an extensive program consisting of both

analytical and experimental investigations has been undertaken.

Various experimental analyses will be conducted on the series of 15 tees listed in Table 4 ranging in nominal size from 6 to 24 in. These tees represent the "standard" product of three different manufacturers and include both full outlet and reducing tees; nominal wall thickness ratings of sched 40, 80, and 160; and two different materials, A106 grade B carbon steel and type 304L stainless steel.

Planned experimental work includes elastic response tests for internal pressure and applied bending moment and thrust loads on the branch and run, low-cycle fatigue-to-failure tests, thermal stress tests, and photoelastic model studies. The test assemblies for all the experimental work will be fabricated by welding pipe extensions of the same nominal size and material to all three of the tee outlets according to the welding requirements of USAS B31.7. The indicated photoelastic studies will be made using models which are scaled models of the respective tee assemblies, including the pipe extensions and weld design.

Fig. 13. A 3-in. Sched 160 Stainless Steel Tee with One Quarter Removed.

Table 4. Experimental and Analytical Studies for ASME B16.9 Tees

Tee No.	Manufacturer	Nominal Size (in.)	Nominal Wall (sched)	Material	Type of Strain Gage and Fatigue Test ^a	Type of Photoelastic Test ^b	Thermal Stress Test ^c	Detailed Finite-Element Analysis ^d
T-1	I	6 X 6 X 6	40	SS ^e	1B-M			
T-2	I	6 X 6 X 6	160	C ^f	1B-P			
T-3	I	6 X 6 X 6	40	SS	1B-M			
T-4	I	12 X 12 X 12	80	C	1A-M			
T-5	II	12 X 12 X 12	80	C	1B-P			
T-6	III	12 X 12 X 12	80	C	1A-M	8B-M	8A-T	4A-F
T-7	II	12 X 12 X 12	160	SS	1B-M	8B-P		4A-F
T-8	II	12 X 12 X 6	40	SS	1A-M	8B-M	8A-T	4A-F
T-9	II	12 X 12 X 6	160	SS	1B-P	8B-P		
T-10	III	24 X 24 X 24	40	C	1B-M			
T-11	III	24 X 24 X 24	160	C	1B-P			
T-12	III	24 X 24 X 10	40	C	1B-M			
T-13	III	24 X 24 X 10	160	C	1B-P			
T-14	III	12 X 12 X 6	40	SS	1B-U			
T-15	I	12 X 12 X 6	40	SS	1A-M			

^aTest 1A is a detailed elastic response test using approximately 225 three-gage strain rosettes; test 1B will use fewer gages to measure the maximum stress intensity; the loading for the fatigue test is designated as M for moment loading, P for internal pressure, and U as yet undecided.

^bPhotoelastic analysis as outlined under PVRC Task 8 - Phase B, with either internal pressure (P) or moment loading (M); see *Program and Request for Proposals for Development of Stress Indices and Methods of Analysis for Piping Components, Pumps, and Valves*, unnumbered PVRC report dated July 1, 1967.

^cThese tests will be run in place of PVRC Task 8 - Phase A.

^dPVRC Task 4 - Phase A, Analytical Development.

^eType 304 or 304L stainless steel.

^fA-106 grade B.

Elastic Response Tests. - The elastic response tests have been divided into two groups, phase A and phase B, with the amount of instrumentation required being the major difference. The objective of the phase A tests is to determine both the general stress distributions over the body of the tee and the location and magnitude of the maximum stresses caused by the different loadings. The detailed stress distribution data will be used to compare with analytical results obtained from using different finite-element formulations. The tees will be strain gaged on both the inside and outside surfaces with three-gage strain rosettes at approximately 225 locations. Brittle lacquer tests will be used to determine whether additional gages will be needed to measure the maximum stresses on the outside surface. Mechanical dial indicators will also be used to measure angular rotations and deflections of the pipe extensions for calculating flexibility factors.

The objective of the phase B tests is primarily to obtain maximum stress data, once the expected locations have been identified from the phase A tests. Data from these tests, together with the data from the phase

A tests, will be used to develop empirical correlations which can be used for piping system design purposes. Since the majority of the tests are designated as phase B, as indicated in Table 4, and since the phase A test results are needed to identify the number and location of strain gages required to measure the maximum stresses, it follows that the phase A test series should be completed before the phase B tests are started. However, in order to complete the entire test series in a reasonable amount of time, specifications were written for two of the four 24-in. tees (T-10 and T-11) and for the 12-in. sched 160 tee (T-7) for the same number of strain-gage locations as for the phase A tests, that is, about 225 three-gage strain rosettes on the body of the tee. Tests have now been scheduled for the four phase A tees (T-4, T-6, T-8, and T-15), the 12-in. sched 160 tee (T-7), and the four 24-in. tees (T-10, T-11, T-12, and T-13).

The 12-in. tees are being tested at Southwest Research Institute (SwRI) in San Antonio, Texas, under subcontract to ORNL. Elastic strain-gage data from the first of the 12-in. tees to be tested (T-4) should be

available for analysis by the end of January. Figure 14 shows this tee mounted in the test frame during recent brittle lacquer tests. Although not shown in this figure, all the internal gages have been installed. Elastic strain-gage tests for the other four 12-in. tees also being tested at SwRI are scheduled for completion by the end of the fiscal year, June 30, 1970. After the data from these tests have been studied, detailed plans can be made for the experimental analysis of the remaining tees listed in Table 4.

In the meantime, the four 24-in. tees will be experimentally stress analyzed using the three-gage strain rosettes and fatigue tested at Combustion Engineering, Inc. (C.E.) in Chattanooga, Tennessee. The present plan calls for these tests to be completed during fiscal year 1971, that is, by the end of June 1971. The work at C.E. was started in July 1969, and fabrication of the first test assembly, for tee T-10, will be

completed in February. Fabrication of the remaining test assemblies will follow at two-month intervals.

The first two tees to be tested at C.E. (T-10 and T-11) will be instrumented in the same manner as the phase A tests on the 12-in. tees, with approximately 225 three-gage strain rosettes on the body of the tee and additional gages along the pipe extensions. Detailed strain-gage layouts have been made for T-10 and are being prepared for T-11. The elastic response tests for T-10 should be completed by the end of June, and instrumentation of T-11 will be in progress. Instrumentation drawings for tees T-12 and T-13 will be made after studying the results from T-10, in order to establish the optimum number and locations for the strain gages.

Low-Cycle Fatigue Tests. — Low-cycle fatigue-to-failure tests are also scheduled for the 15 tees listed in Table 4. The fatigue tests will be run with either an

Fig. 14. A 12 X 12 X 12 in. Sched 80 ASA B16.9 Tee, No. T-4, Mounted in the Loading Frame During Brittle Lacquer Tests at Southwest Research Institute.

internal pressure or an applied moment, as indicated. The particular moment loading, and whether it is applied on the branch pipe or the run pipe, will depend on the results obtained from the elastic response strain-gage tests and will be selected to give the maximum stresses in the tee. The magnitude of the load will be based on an expected fatigue failure within the range of 500 to 100,000 cycles and will be fully reversed displacement controlled. That is, the pipe will be deflected a given amount in one direction, unloaded to zero displacement, and then deflected in the opposite direction the same amount during each cycle. The necessary loading fixtures and controls for these tests have been specified, and orders for their purchase have been issued. Fatigue tests for the five 12-in. tees being tested at SwRI should be completed during the coming year, and fatigue tests for the 24-in. tees being tested at C.E. will be conducted following the strain-gage tests.

Photoelastic Tests. — Photoelastic stress analysis investigations are being conducted at Westinghouse Research Laboratories on the four 12-in. tees indicated by 8B in Table 4 (column 7); 8B indicates that these tests were specified under task 8, phase B, of the PVRC research proposal.²³ Frozen stress techniques are being used, and the letter M or P in the table indicates the type of loading. Two of the tees (T-7 and T-9) are to be analyzed for internal pressure loading, and the other two (T-6 and T-8) will be analyzed for an applied moment loading. The particular moment loading to be used will depend on which one of the possible six loadings, three on the branch and three on the run, will produce the maximum stresses in the tee. The elastic response strain-gage tests discussed above will provide this information.

The objective of these investigations is to provide stress distribution data through the wall thickness at locations of high surface stress. These data are needed in order to identify nonlinearities in the stress distribution and thus separate secondary and peak stress components as required by the B31.7 piping code. Surface stresses will also be measured along lines which correspond to a line of strain-gage positions on the prototype steel tees.

The photoelastic tests will be conducted on $\frac{1}{2}$ -scale epoxy models of the prototype tees using three-dimensional frozen stress techniques. Photoelastic models have been fabricated for two of the tees (T-6

and T-9). Each model was made by first constructing an accurate $\frac{1}{2}$ -scale wooden pattern and using this wooden pattern as a template for machining the model from a solid epoxy casting. The upper half of the wooden pattern for the 12 X 12 X 12 in. sched 80 tee (T-6) is shown in Fig. 15. An exploded view of the model for T-9 (the 12 X 12 X 6 in. sched 160 tee) is shown in Fig. 16. Both the pipe extensions and the weld details are accurate scale replicas of the prototype test assemblies. All presently scheduled photoelastic test work will be completed by the end of June 1970.

Thermal Stress Tests. — Thermal stress tests have been scheduled for two of the tees (T-6 and T-8) listed in Table 4. Tests on these tees were substituted for the plastic model tests originally proposed by the PVRC as task 8, phase A, after careful consideration of the difficulties expected in conducting those tests. Approximately 80 locations on the body of each tee will be instrumented with thermocouples and three-gage strain rosettes. Both inside and outside surfaces will be instrumented. Short pipe extensions, about 2 ft long, welded to each of the three tee outlets will be wrapped with electrical heaters and cooling coils, and the entire assembly will be insulated on both the inside and outside surfaces. The objective of the tests is to produce thermal gradients in the tee by holding two of the pipe extensions at one temperature and raising the temperature of the third extension; ΔT 's of 400°F are planned. For example, the branch pipe extension will be heated to 500°F while holding the temperature of both run-pipe extensions at 100°F. Alternately, one of the run-pipe extensions will be held at 100°F while heating the branch pipe and the other run-pipe extension to 500°F. Temperatures and strains will then be measured and used to compute the thermal stresses. Procurement of material and fabrication of the test assemblies will begin in January. The tests will be conducted at Oak Ridge.

Finite-Element Elastic Analyses. — Concurrently with the experimental analyses of the tees, analytical development is under way at the University of California under Professor R. W. Clough. The objective of this work is, in broad terms, to develop suitable analytical techniques which may be used for generating parameter studies over a wide range of sizes and wall thicknesses and, with these results, to provide stress indices which may be used in piping system design. Three 12-in. tees, listed in Table 4 as T-6, T-7, and T-8, which have nominal wall thicknesses of sched 80, 160, and 40, respectively, will be analyzed in detail. Four different types of finite-element formulations are being considered in order to develop the most accurate and

²³ *Program and Request for Proposals for Development of Stress Indices and Methods of Analysis for Piping Components, Pumps, and Valves*, unnumbered PVRC report dated July 1, 1967.

Fig. 15. One-Half-Scale Wooden Pattern of Tee No. T-6, 12 X 12 X 12 in. Sched 80 ASA B16.9 Tee, Used as a Template for Machining the Photoelastic Model.

Fig. 16. An Exploded View of the Photoelastic Test Assembly for the 12 X 12 X 6 in. Sched 160 ASA B16.9 Tee, No. T-9, to Be Analyzed for an Internal Pressure Loading at Westinghouse Research Laboratory.

efficient computer code: (1) a flat-plate quadrilateral element, (2) a flat-plate quadrilateral with added transverse shear, (3) an 8-node rhombohedral element, and (4) a 20-node curved rhombohedral. The analysis of the thin-shell tee discussed earlier was accomplished using the first type of element. The three-dimensional elements, types 3 and 4, are being considered because of the relatively thick walls of the ASA B16.9 tees.

During the past year computer code development work was accomplished in two areas. First, a general thin-shell analysis program which originally was based on using the first type of element was modified to include the second type of element; that is, transverse shearing distortion of the element was added. Second, large-capacity programs for the analysis of three-dimensional solids using either the type 3 eight-node element or the type 4 twenty-node element were developed. The capacity of existing programs was found to be insufficient for analyzing the tees. Therefore new programs, with greater capacity and improved efficiency, were developed. Each of the three tees will eventually be analyzed for all 13 loading conditions using the "best" methods. This year the 12 X 12 X 6 in. sched 40 tee (T-8) was analyzed for two loading conditions, internal pressure and an in-plane bending moment on the branch, using each of the four different types of elements. It was noted that the different elements gave similar results; however, there were significant differences at certain locations, particularly between the thin-shell-element results and the three-dimensional-element results. Further analysis will be done when the experimental results are available for comparison.

The 12 X 12 X 12 in. sched 160 tee (T-7) was analyzed for internal pressure and an in-plane bending moment on the branch using the three-dimensional elements, types 3 and 4; however, the results have not been studied in detail. Thin-shell analyses were not made because the tee is so thick that it did not appear feasible to use the thin-shell idealization. Data preparation has been completed for analyzing T-6, the 12 X 12 X 12 in. sched 80 tee, for the same two loading conditions. Work was also started on the thermal stress problem, that is, the addition of thermal strain terms in the finite-element description.

During the coming year we plan to compare the preliminary analytical results with the experimental results, select the most promising element or elements for use in a "production" computer program, and complete the analyses of the three tees for all 13 loading conditions. Automatic mesh generation capability, particularly designed for ASA B16.9 tees, will be

developed, and thermal stress analyses of the experiments described above will be in progress.

Structural Analysis of Elbows, Curved Pipe, and Miters

Currently four task efforts are in progress on the stress analysis of elbows, curved pipe, and miters: (1) analytical determinations of stress indices and flexibility factors for elbows and curved pipe, (2) experimental stress analyses of machined elbows, (3) experimental limit load studies for elbows, and (4) theoretical stress analyses of miters. Each of these areas is discussed below.

Stress Indices and Flexibility Factors for Elbows

Stress indices and flexibility factors for combined moment loading which can be used directly with the simplified design procedures of the B31.7 piping code have been developed and a report written.²⁴ An extensive review of the published literature, including both experimental and analytical work, was conducted in order to assure the use of the most complete and up-to-date theory. Although more recent work has been published, the analytical solution based on minimum potential energy by Rodabaugh and George²⁵ was selected as being the most appropriate, after modifying the circumferential membrane force terms as suggested by Gross.²⁶ The resulting solution for the longitudinal and circumferential stresses is expressed in terms of three independent nondimensional parameters: a bend parameter, a radius-ratio parameter, and an internal pressure parameter. Stresses resulting from an arbitrary bending moment — expressed in terms of three mutually perpendicular components, in-plane, out-of-plane, and torsional components — can be calculated as a function of angular position on the elbow.

A computer program written for the ORNL IBM 360 computer was used to conduct a parameter study and to compute stress indices and flexibility factors. The stress index for moment loading is defined as twice the maximum shear stress at any point on the elbow resulting from a unit bending moment applied in an

²⁴W. G. Dodge and S. E. Moore, *Stress Indices and Flexibility Factors for Elbows and Curved Pipe* (to be published).

²⁵E. C. Rodabaugh and H. H. George, "Effect of Internal Pressure on Flexibility and Stress-Intensification Factors of Curved Pipe or Welding Elbows," *Pressure Vessel and Piping Design, Trans. ASME*, 467-76 (1957).

²⁶N. Gross, "Experiments on Short-Radius Pipe Bends," *Inst. Mech. Engrs. Proc.* 1B, 465-79 (1952-1953).

arbitrary direction, divided by the bending stress which would be produced in straight pipe with the same section modulus. The flexibility factor is defined as the relative rotation of two transverse planes in the elbow divided by the relative rotation of corresponding planes in a similar straight pipe. Both stress indices and flexibility factors are given in graphical and tabular form in the report.

Unfortunately, the solution used to obtain stress indices and flexibility factors does not consider the influence of straight pipe or other components welded to the elbows as would occur in a real piping system. Experimental evidence by Vissat and del Buono²⁷ and others suggests that these components would influence the deformation of an elbow and thereby affect both the maximum stresses and the flexibility. It is expected that both of these could be reduced, depending on the relative stiffness and arc length of the elbow or bend. The maximum stresses calculated using the above stress indices would still be conservative. However, if the flexibility of an elbow in a real piping system were less than calculated using the above factors, higher forces and moments would result, and the piping system analysis could be unconservative.

This problem has been under intensive investigation during the past year, following the work of Kalnins.^{28,29} A computer program based on numerical integration of the system of eight linear partial differential equations was modified for the ORNL IBM 360 computer, and a number of cases have been studied. To date, however, the analytical results have not been entirely satisfactory. A number of possible sources for the errors have been suggested and are being systematically investigated.

Experimental Analysis of Elbows

Both of the analytical studies discussed above are based on the assumption of idealized geometry, whereas in practice the shape of an elbow or pipe bend may deviate from a true torus because of fabrication practices. Rather than dictate allowable fabrication tolerances, as done by other codes and standards (e.g., the 8% maximum ovality permitted by the nonnuclear piping code USAS B31.1.0, 1967), the nuclear piping

code, USAS B31.7, requires that geometric deviations be accounted for in the stress analysis. The ORNL piping program, therefore, includes a task directed toward the analytical and experimental study of elbows with carefully controlled geometric deviations.

The experimental portion of this study will be an extensive strain-gage analysis of three or possibly four 10-in. standard long-radius elbows with a nominal sched 40 wall thickness. A long-radius elbow is one for which the bend radius is three times the radius of the tube. The bend radius for a short-radius elbow is twice the tube radius. Two types of geometric deviations will be studied: nonuniform wall thickness and ovality. For these studies, machined elbows welded to short pipe extensions will be used. The first elbow will be machined as a true torus with a uniform wall 0.365 in. thick in order to obtain base-line data for comparison with the results from the other models and for comparison with the idealized theoretical solutions discussed above. The second model will be machined as a true torus but will be eccentrically bored so that the minimum wall thickness, along the back of the elbow, will be 0.272 in. The third model will be machined like the first, that is, as a true torus with a uniform wall thickness; then it will be flattened out of round to form an elliptical cross section with a 5 to 8% ovality. The fourth model will be both thinned and ovoided; that is, an elbow similar to the second will be machined and then flattened out of round like the third model. All four elbows have been ordered and are being machined; delivery of the first model is expected in January, and the remaining three are expected shortly afterward.

Each of the models will be strain gaged on both the inside and outside surfaces and will be tested with internal pressure and externally applied moments on the pipe. Moment loadings will first be run with zero internal pressure. Subsequent moment loadings will then be run combined with internal pressures corresponding to one-third, two-thirds, and full rated design pressure. The first three models will be tested during the coming year, and, if funds permit, the fourth will also be tested.

Plans have also been made to analyze all the experimental models using a modified version of the finite-element computer program JOINT, which was used to analyze the thin-shell tees. The computer code modifications are presently being programmed.

Limit Load Studies for Elbows

Although not contained in the original PVRC research proposal, plastic limit load studies were added to the program at the request of the AEC after consideration

²⁷P. L. Vissat and A. J. del Buono, "In-Plane Bending Properties of Welding Elbows," *Trans. ASME* 77, 161-75 (1955).

²⁸A. Kalnins, "Analysis of Curved Thin-Walled Shells of Revolution," *AIAA J.* 6, 584-88 (1968).

²⁹A. Kalnins, "Stress Analysis of Curved Tubes," pp. 223-36 in *First International Conference on Pressure Vessel Technology. Part I. Design and Analysis*, ASME, 1969.

of the simplified analytical procedures of USAS B31.7 for primary stresses. This design procedure is based on satisfaction of the inequality

$$B_1 \frac{PD_0}{2t} + B_2 \frac{D_0}{2I} M_i \leq 1.5S_m,$$

where S_m is the allowable stress intensity, P and M_i are internal pressure and applied moment loading, respectively, $D_0/2t$ and $D_0/2I$ are normalizing factors based on stresses in straight pipe, and B_1 and B_2 are stress indices based on plastic limit loads.

Values for B_1 and B_2 are included in USAS B31.7 for piping components manufactured according to the appropriate ASA standard, and the B_1 stress index values are supported by the pressure burst test requirement of the ASA standard. However, the code values for B_2 were based on engineering judgment because of the lack of direct experimental evidence. Although the present code values are believed to be conservative, experimental confirmation is required. A series of limit load tests on elbows has therefore been initiated.

Plastic limit load tests on elbows will be conducted on the 6-in. A-106 grade B carbon steel elbows listed in Table 5. Both short- and long-radius elbows and two wall thicknesses, sched 40 and 80, are included. Pipe extensions will be welded to the elbows, and the assemblies will be loaded as indicated in Table 5. For

those cases where an internal pressure is indicated, for example, No. PE-4, the assembly will be pressurized to the rated design pressure, and then the moment loading will be superposed until the plastic collapse load is reached. Additional tests may be added for different internal pressure levels after evaluation of the results from the present series.

The elbow assembly for the first test is shown mounted in the load frame in Fig. 17. Since the object of the test is simply to determine the load at which plastic collapse occurs, a minimum amount of instrumentation was used. This consisted of four strain gages mounted on the outside surface of the elbow, mechanical dial indicators to measure deflections, and a load cell to measure the applied thrust. The plastic collapse load was determined from the force-deflection curve. The present series of tests will be completed by midyear.

Theoretical Stress Analysis of Miter

Typically, miter bends, which consist of segments of straight pipe cut at an angle and welded together, are used in a piping system to provide a change in direction when, for some reason or another, standard elbows or field-bent pipe cannot be used. Even though their use is expected to be a minimum in nuclear power piping, the class of problems is important in the overall scope of a

Table 5. Plastic Limit Load Tests on 6-in. Elbows

Number ^a	Elbow	Bend Radius (in.)	Loading Conditions			Pressure, <i>P</i>
	Wall Thickness (sched)		Moment			
			In Plane ^b		Out of Plane <i>M_y</i>	
			+ <i>M_x</i>	- <i>M_x</i>		
PE-1	40	9	X			
PE-2	40	9		X		
PE-3	40	9			X	
PE-4	40	9	X			X
PE-5	40	9		X		X
PE-6	40	9			X	X
PE-7	80	9	X			
PE-8	80	9		X		
PE-9	80	9			X	
PE-10	40	6	X			
PE-11	40	6		X		
PE-12	40	6			X	
PE-13	40	6	X			X
PE-14	40	6			X	X

^aThe letters PE stand for plastic collapse, elbow.

^bA positive in-plane moment ($+M_x$) causes the elbow to close, whereas a negative in-plane moment ($-M_x$) causes the elbow to open.

Fig. 17. A 6-in. Sched 40 Standard Long-Radius Elbow Plastic Collapse Test Assembly Mounted in the Loading Frame for an In-Plane Bending Test.

basic research program designed to provide fundamental information for the stress analysis of piping system components.

The long-range objective in the ORNL program is to develop stress indices for combined loadings and flexibility factors similar to those developed for elbows and pipe bends. In the case of miters, however, theoretical development has not progressed to the point where directly applicable solutions are available. With this in mind, an intensive review³⁰ of theoretical work published to date³¹ was conducted during the summer.

A number of papers have been published on the stress analysis of miters or on cylindrical shells with an oblique edge. Most of the work has been done for the symmetrical loading cases, such as internal pressure and in-plane bending. Very little has been published for the out-of-plane bending case. The best work has been done using the general thin-shell theory, as presented by Goldenveiser³² in 1953. Because of the complex nature of the general equations and the approximations used in obtaining a solution to a particular problem, it appears unlikely that a general solution will be forthcoming in the near future. The work that has been done, however, suggests a number of promising approaches which may be used in the development of stress indices for design use. We plan to resume theoretical work later, when personnel and funding become available.

Structural Analysis of Other Piping Components

Studies on a number of other piping components besides tees and elbows are being actively pursued. During the past year analytical parameter studies have been initiated on concentric reducers; girth-welded joints, including the effects of piping mismatch; tapered-wall transitions, lug supports, bolted flanges, and out-of-round pipe. These studies are based on the use of analytical solutions in the literature or on "strength of materials" type stress analyses for various loadings on the piping component. The principal objective is to compute stresses, stress intensities (which are defined as twice the maximum shear stress at a point), and normalized stress indices over a sufficiently wide range of dimensional variables and to present these results in graphical and tabular form for direct use in piping system design. In some cases the results from the

parameter studies will be used to draft design rules which can be incorporated directly into the codes and standards.

Parameter studies have been completed for concentric reducers with internal pressure loading and moment loadings. The results were used to prepare design rule modifications which will be presented to the code-writing bodies for consideration. A report on these studies is being prepared.³³ Parameter studies have also been partially completed for transition joints, including (1) a balanced taper where the wall thickness increases uniformly on either side of the midsurface, (2) an outside taper where the inside diameter is constant, and (3) an inside taper where the outside diameter is constant. The maximum stress index was calculated as a function of nondimensional length and thickness ratio parameters. The report on this work will be prepared early in 1970.³⁴

Conclusions

Extensive effort has been expended under the ORNL piping program during the past year. Several studies were completed, and work on a number of important tasks was initiated. The literature review study and report started at the beginning of the program were completed after a thorough review by experts from AEC, industry, and members of the PVRC. While still in draft form, the report has been a valuable reference document, providing source material for several analytical studies. One such study, an empirical correlation of experimental data for small branch connections, was used to develop a new set of design rules and stress indices, which were incorporated into the 1969 edition of USAS B31.7, the nuclear piping code. Parameter studies were completed for concentric reducers using published analytical solutions; and parameter studies were initiated for girth-welded transition joints, tapered-wall transitions, lug supports, bolted flanges, and out-of-round pipe.

The experimental stress analysis of a thin-shell tee (cylinder-to-cylinder model) was completed, and the experimental results were compared with numerical

³⁰E. D. Gurley (North Carolina State University), "On the Mitered Bend," unpublished, 1969.

³¹E. C. Rodabaugh and A. G. Pickett, *Survey Report on Structural Design and Piping Components* (to be published).

³²A. L. Goldenveiser, *Theory of Elastic Thin Shells*, Pergamon, New York, 1961.

³³E. C. Rodabaugh, *Stress Indices and Flexibility Factors for Reducers*, BMI Phase Report 115-4, Battelle Memorial Institute, to be published.

³⁴E. C. Rodabaugh and W. G. Dodge, *Stress Indices and Flexibility Factors at Girth-Welded Joints, Including Mis-Match and Tapered-Wall Transitions*, to be published.

results obtained from the finite-element analyses reported last year. The comparisons show good overall agreement, especially for the maximum stresses. Even better agreement should be possible with further development of finite-element techniques.

The experimental stress analysis and fatigue tests of ASA B.6.9 tees being conducted at Southwest Research Institute and at Combustion Engineering, Inc., are well under way, with the first results to be forthcoming early in 1970. Results from the photoelastic studies at Westinghouse Research Laboratories will be available before midyear 1970, and thermal stress tests being conducted in Oak Ridge should be

completed during the coming year. Corresponding finite-element analyses will be completed for several of these tests.

Other experimental studies initiated during the past year include elastic response tests on machined elbows and plastic collapse tests on standard long- and short-radius elbows. The machined models are being fabricated, and results from the experiments are expected in 1970.

The overall progress during 1969 has been excellent. Next year should be even more fruitful, with completed results from many of the tasks becoming available.

PART II
EXPERIMENTAL AND ANALYTICAL INVESTIGATIONS OF NOZZLES

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Introduction

The Oak Ridge National Laboratory is conducting a combined experimental and analytical investigation of stresses and strains in the region of nozzle attachments in both spherical and cylindrical shells. The basic objective of this investigation is to establish analytical design procedures for nuclear pressure vessel penetrations and nozzle connections that have been thoroughly substantiated by experimental data.

The investigations cover single nozzles radially and nonradially attached to spherical and cylindrical shells and clusters of nozzles attached to spherical shells and flat plates. The loadings of principal interest are internal pressure, axial thrust forces acting on the nozzles, and bending and torsional moments applied to the nozzles. Parameters being studied are diameter-to-thickness ratios for the nozzle (d/t), the ratio of the diameter of the nozzle to the diameter of the shell (d/D), the diameter-to-thickness ratio of the shell (D/T), and the ratio of the nominal membrane stress in the nozzle to the nominal membrane stress in the shell (σ/σ_s). Other parameters of interest are the effects of the length internal to the vessel for internally protruding outside nozzles and the angle of attachment in the spherical-shell-nozzle configurations. In the clustered nozzle configurations, the number and pattern of nozzle attachments and the spacing between nozzles are being studied.

The program is divided into two portions: (1) an AEC (ORNL) sponsored program and (2) an AEC-PVRC (Pressure Vessel Research Committee of the Welding Research Council) cooperative program. The latter program is coordinated through the PVRC Subcommittee on Reinforced Openings and External Loadings. The AEC (ORNL) effort is directed primarily toward multiple nozzle connection studies, whereas the AEC-PVRC cooperative effort is concerned with single nozzle connections. Much of the research is being done under subcontract, either to ORNL or to the PVRC, depending upon the source of funds. The current research projects for the two portions of the program are listed in Table 6.

ORNL has the task of coordination, review, and evaluation of the research program under the PVRC

Subcommittee on Reinforced Openings and External Loadings. This management function includes (1) participating in and directing AEC-sponsored work, (2) generating various analytical parameter studies in support of correlation and evaluation studies being done at Battelle Memorial Institute, (3) making reports and recommendations to the Subcommittee on non-AEC-sponsored projects, and (4) soliciting recommendations from the Subcommittee on AEC-sponsored projects.

As a part of this coordination task, ORNL staff members attend the PVRC Design Division meetings and meetings of the Subcommittee. Visits are also made to the facilities of all subcontractors who are working on the program in order to evaluate the work and to obtain additional information for use in reports to the Subcommittee. Summary descriptions of each task and status reports on the work being done by the participating organizations are contained in a project report.³⁵ The project report will be updated on a yearly basis and will contain, in addition to the status reports, detailed descriptions of any changes in the program. In addition, an annual information meeting — Studies in Applied Solid Mechanics — is sponsored by ORNL. The 1969 symposium was held April 28–29, and the next meeting will be held April 1–3, 1970, in Oak Ridge. Summaries of the current status of the work at ORNL are given below. These are followed by summaries on the work being done by other organizations participating in the program. For a more complete description of each project, the reader should refer to the project report.³⁵

Oak Ridge National Laboratory Projects

In addition to carrying out the program management responsibilities, ORNL is directly engaged in some phases of the work on the structural behavior of shells and of nozzle-to-shell attachments. Descriptions of current studies, along with indications of the status in each case, are given below.

³⁵W. L. Greenstreet and R. C. Gwaltney, *Experimental and Analytical Investigations of the Structural Behavior of Nozzle-to-Shell Attachments*, ORNL-TM-2526 (April 1969).

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Table 6. Current Experimental and Analytical Investigations of Nozzle Attachments

Program	Subcontractor ^a	Funding Agency	Problem
Joint AEC (ORNL)-PVRC; single nozzle studies	University of Tennessee (I)	AEC	Experimental stress analysis of single radial and non-radial nozzles in spherical shells; determination of the internal pressure required for plastic collapse of a thin cylindrical shell with a single radially attached nozzle
	Westinghouse Research Laboratories	AEC	Photoelastic studies of radial and nonradial nozzles attached to spherical and cylindrical shells; photoelastic studies of holes without nozzles
	University of Waterloo (I)	PVRC	Experimental stress analysis of a 45° lateral connection subjected to external loadings on the nozzle and the vessel; development of simplified analytical methods for predicting elastic stress concentrations at nozzle-to-shell junctions due to internal pressure; experimental determination of plastic limit pressures for commercial tees; analytical and experimental investigations in connection with plastic limit design for the general problem of a spherical shell with a radial nozzle in which the composite structure is subjected to combinations of internal pressure and force and moment loadings on the nozzle
	University of Waterloo (II)	PVRC	Derivations of upper- and lower-bound limit analysis equations for cylindrical shells with radially and non-radially attached nozzles where the composite structure is subjected to internal pressure and external loadings on the nozzle; experimental verification of selected limit loads
	University of Sherbrooke	PVRC	Theoretical and experimental investigations of flat plates with single skewed holes; experimental determination of plastic limit pressures for conical-spherical profile shells with single nozzles; theoretical and experimental limit analysis studies for spherical shells with single radial nozzles subjected to internal pressure; derivations of upper- and lower-bound limit analysis equations for cylindrical shells with radially attached nozzles where the composite structure is subjected to internal pressure or combinations of internal pressure and external force and moment loadings on the nozzle
	Auburn University (II)	AEC	Theoretical solutions for a single nonradial nozzle attached to a spherical shell
	Oak Ridge National Laboratory	AEC	Computer programming and completion of solutions for the radial-nozzle-to-cylindrical-shell attachment problems developed by General Technology Corporation; finite-element analysis studies of shell attachment regions; comparisons between analytical and experimental results; miscellaneous testing; parametric studies for program guidance and code use
	Oak Ridge National Laboratory	AEC	Construction of thin-shell hard models for experimental stress analysis
AEC (ORNL); multiple nozzle studies	University of Tennessee (II)	AEC	Experimental stress analysis of flat plates with clusters of holes and with clusters of nozzles (applicable to small nozzles on large vessels)
	Battelle Memorial Institute (II)	AEC	Theoretical solutions for flat plates with clusters of holes and for clusters of nozzles attached to flat plates
	Auburn University (I)	AEC	Experimental stress analysis of spherical shells with clusters of nozzles and holes
Joint AEC (ORNL)-PVRC; general	Battelle Memorial Institute (I)	AEC	Engineering evaluation and correlation of results for nozzle-to-shell attachments and preparation of material based on these results for use in support of ASME Code Committee work

^aThe Roman numeral after a subcontractor is used to indicate studies where a subcontractor has more than one subcontracted study.

Development of Analytical Methods for Nozzle-to-Cylindrical-Shell Attachments

The stress analysis equations for a radial nozzle attached to a cylindrical shell have been derived, checked, and programmed for the ORNL IBM 360 computer. The equations are for an out-of-plane bending moment applied to the nozzle and for an internal pressure loading. The analysis is applicable to thin-walled shells and is limited to those cases for which the nozzle-to-shell diameter ratio is less than $\frac{1}{3}$ ($d/D \leq \frac{1}{3}$). In addition, equations for calculating thermal stresses have been derived; the computer program will be extended to calculate the stresses associated with specific temperature distributions.

Comparisons have been made between analytical and experimental photoelastic results³⁶ obtained for Westinghouse Research Laboratories' model WC-2AY. These experimental results are discussed in a later section of this report. In general, reasonable agreement was obtained for the maximum stress values; however, significant differences exist for the stress distributions near the nozzle-to-shell intersection. Agreement between the analytical and experimental results was good for the membrane regions, away from the nozzle; and, although close agreement is to be expected, this is a significant step insofar as theory development is concerned. Future efforts will be directed toward resolving the discrepancies for this particular case study and toward further validation of the analyses for both internal pressure and out-of-plane bending loads.

Finite-Element Studies of Reinforcing for Nozzle-to-Spherical-Shell Configurations

A limited parametric study, using the finite-element method, is being made to investigate area reinforcement on the inside shell surface of nozzle-to-spherical-shell configurations. The purpose of the study is to determine whether or not special code rules or limitations are required for inside reinforcement. The study will cover both thin shells and relatively thick shells with various reinforcing arrangements. Outside reinforcing, in the form of circular fillets at the nozzle-to-shell junction, will also be examined for comparison purposes.

The first series of calculations will be performed for the nozzle-to-shell structures shown in Fig. 18. The

model parameters are $D_i/T = 24$, $d_i/D_i = 0.2$, and $d_i/t = 11.43$, where D_i and d_i are the inside diameters of the shell and nozzle and T and t are the corresponding wall thicknesses. In addition to the unreinforced case, two configurations of outside reinforcement and two configurations of inside reinforcement are being considered. Input data for these five cases have been prepared for internal pressure loading, and the calculations are being performed.

The finite-element analysis being used in the study is based on a two-dimensional, axisymmetric solid body formulation. Triangular ring elements are used, with linearly varying displacement fields within each element. It should be emphasized that this type of analysis has no connection with shell theory, and hence it has none of the inherent limitations of thin-shell theory or of finite-element analyses using thin-shell elements.

The specific computer program being used is SAFE-2D,³⁷ which was developed at Gulf General Atomic, Inc., as a part of the ORNL Prestressed Concrete Reactor Vessel Program. The computer program was modified at Oak Ridge to double precision and expanded to handle a maximum of 3600 nodal points and 7200 element

In order to verify the applicability of this computer code and the finite-element formulation for the analysis of relatively thin shell-type structures, two experimental nozzle-to-shell models were analyzed for comparison with the experimental results. The models were steel hemispherical shells with a single radial nozzle which were strain gaged and tested at the University of Tennessee³⁸ under subcontract to ORNL. The models identified as 1 and 5 in ref. 39 were chosen for the present study, partly because analyses using thin-shell theory had failed to adequately predict the stress distributions in the spherical shell for the internal pressure loading case. The radius-to-thickness ratio for the spherical shell was 40.7 for both models, while the nozzle radius-to-thickness ratio was 4.75 for model 1

³⁷D. C. Cornell, *SAFE-2D, a Computer Program for the Stress Analysis of Plane and Axisymmetric Composite Structures, a User's Manual*, GA-9076, Gulf General Atomic (February 1969).

³⁸R. L. Maxwell, R. W. Holland, and J. A. Cofer, *Experimental Stress Analysis of the Attachment Region of Hemispherical Shells with Radially Attached Nozzles*, ME7-65-1, University of Tennessee (June 1965).

³⁹F. J. Witt, R. C. Gwaltney, and B. L. Greenstreet, *Comparison of Theoretical and Experimental Stresses on a Spherical Shell with a Single Radially Attached Nozzle*, ORNL-TM-1634 (November 1966).

³⁶M. M. Leven, *Photoelastic Determination of Stresses in a Reinforced Steam Charging Connection with a Thermal Sleeve*, 69-9D7-PHOTO-R1, Westinghouse Research Laboratories (Jan. 14, 1969).

Fig. 18. Details of the Five Initial Configurations Being Examined in the Finite-Element Studies of Area Reinforcing.

and 10.0 for model 5. Model 1 had a protruding nozzle, while model 5 did not.

The finite-element idealization of the nozzle region is shown in Fig. 19 for model 1 and in Fig. 20 for model 5. In both cases the full hemispherical shell was considered in the analysis. A total of 1200 nodes and 1967 elements was used to represent model 1, while 1199 nodes and 1968 elements were used for model 5.

The results of the analyses for the two models are shown in Figs. 21-24. The predicted deflections are shown, to an exaggerated scale, superimposed on the original cross sections in Figs. 21 and 22 for models 1 and 5 respectively. The meridional and circumferential surface stress distributions are compared with the experimental stresses in Figs. 23 and 24 for models 1 and 5 respectively. Predictions based on thin-shell theory, taken from ref. 39, are also shown in Figs. 23 and 24.

In general, the finite-element predictions agree closely with shell theory predictions, except in the immediate vicinity of the junction, where the axial and meridional stresses predicted by the finite-element method reflect local stress concentrations. Since the theoretical basis for thin-shell theory is different from the solid body of revolution finite-element formulation, the agreement between the two sets of predictions seems to give credence to the more approximate shell theory predictions. The results also indicate that the finite-element method is capable of examining local stresses in the reinforced regions of the nozzle-to-shell configurations being examined in the parametric study. Left unanswered, however, is why the theoretical predictions do not agree with the experimental stresses in the spherical shell portions of models 1 and 5 for the internal pressure loading case.

ONR-DMG 70-822

Fig. 19. Finite-Element Idealization of the Junction Region of Model No. 1.

OSBL-8716 70-823

Fig. 20. Finite-Element Idealization of the Junction Region of Model No. 5.

ORNL-DWG 77-824

Fig. 21. Displaced Configuration of Model No. 1. Note that the displacements are shown to an exaggerated scale.

NACA-OR-70-825

Fig. 22. Displaced Configuration of Model No. 5. Note that the displacements are shown to an exaggerated scale.

Fig. 23. Comparison of Finite-Element Predictions with Shell Theory Predictions and Experimental Results for Model No. 1 Subjected to an Internal Pressure of 400 psi.

ORNL-DWG 70-827

Fig. 24. Comparison of Finite-Element Predictions with Shell Theory Predictions and Experimental Results for Model No. 5 Subjected to an Internal Pressure of 300 psi.

Comparison of Analytical and Experimental Results for a Large Radial Nozzle Attached to a Spherical Shell

As a part of a continued assessment of analytical methods, theoretical analyses for one of the radial-nozzle-to-spherical-shell models being tested at the University of Tennessee⁴⁰ were made using the CERL-II computer code.⁴¹ This model has the following dimensions and dimensional ratios: $D_i = 30.132$ in., $T = 0.375$ in., $d_i = 7.50$ in., $t = 0.188$ in., $D_i/T = 80.35$, $d_i/D_i = 0.249$, and $t/T = 0.5$. Here D is the shell diameter, T is the shell thickness, d is the nozzle diameter, t is the nozzle thickness, and the subscript i denotes inside dimension. The theoretical analysis was based on thin-shell theory, and the nozzle and spherical shell were assumed to be joined at the intersection of the midsurfaces. The loadings considered were internal pressure and an axial force applied to the nozzle.

Comparisons of the theoretical and experimental results are given in Figs. 25 and 26. The two sets of results give satisfactory agreement in both distribution and magnitude, except for the stresses at the inner surface of the sphere. The spherical portion of this model has a flat spot that measures about $1\frac{1}{4}$ in. along the meridian located at about 25° from the center line of the nozzle. Hence, there is probably some influence from this geometrical imperfection. However, the overall agreement is very good, particularly in the regions of highest stress.

Internal Pressure Test of a 45° Lateral

A machined 45° lateral, which is to be subjected to an internal pressure collapse test at the University of Waterloo, was instrumented with strain gages at ORNL so that strain measurements could be taken in critical regions during the collapse and a subsequent burst test. The regions of interest are at the branch (nozzle) to run (shell) intersection along the longitudinal and transverse axes. Of particular interest is the behavior at the acute inside corner along the longitudinal axis.

The lateral was machined from 30% carbon steel stock, and the branch and run have the same dimen-

sions. This specimen has been designated as Y_2 by the University of Waterloo (see writeup on University of Waterloo (II), which follows later). The dimensions, as measured from the lateral, are shown in Fig. 27, which also shows the strain-gage locations. The dimensional parameters are $D_i/T = 26.2$, $d_i/D_i = 1.0$, and $a/S = 1.0$. The specimen was instrumented as shown with 16 three-gage rosettes mounted on the inside and outside surfaces. The specifications for the gages used are given on the figure in the tabulations entitled "Strain Gage Schedule."

Prior to returning the lateral to the University of Waterloo, an internal pressure test was conducted to determine the elastic response. Strains were recorded at pressures of 0, 100, 200, and 0 psi using a Dextir data-logging system. The principal stresses and the maximum shear stresses recorded in Table 7 were computed from the strains measured at 200 psi. The direction of the maximum principal stress at each location is given with respect to the major axis of the rosette, which is defined as the center line of the horizontal gage shown in view DD of Fig. 27.

The maximum stresses occurred at the acute inside corner in the circumferential direction. The maximum stresses obtained were essentially identical for the two rosette locations, that is, 30,226 and 30,831 psi. At the fillet opposite the acute inside corner, the circumferential stress was compressive and much smaller in magnitude (-2909 psi). The highest stress on the outside surface (17,214 psi) was on the run on the acute corner side of the transverse plane through the nozzle. It can be concluded from these data that failure can occur at the acute inside corner before yielding is experienced on the outer surface on the longitudinal plane.

Finite-Element Studies of Cylindrical Shells with Step Changes in Wall Thickness

A finite-element parameter study is being made to determine the stress concentrations occurring in stepped cylindrical shells with various fillet radii (see Fig. 28). The change in thickness occurs at the outside surface only. This work is being performed at the request of the PVRC Subcommittee on Stresses in Ligaments.

A total of 72 cases have been identified as covering the range of interest, although it will probably not be necessary to analyze every one of these. The combinations of parameters of interest are:

$$T/t = 1.05, 1.1, 1.2, 1.5, 2.0, 4.0,$$

$$r/t = 0, 0.15, 0.3, 0.45,$$

$$a/t = 5.0, 10.0, 20.0,$$

⁴⁰R. L. Maxwell, R. W. Holland, and G. R. Stengl, *Experimental Stress Analysis of the Attachment Region of Hemispherical Shells with Attached Nozzles, Part 2b, Radial Nozzle 7.875 O.D.-7.500 I.D., 10.00 in. Penetration*, ME 7-69-4, University of Tennessee, to be published.

⁴¹S. E. Moore and F. J. Witt, *CERL-II - a Computer Program for Analyzing Hemisphere-Nozzle Shells of Revolution with Axisymmetric and Unsymmetric Loadings*, ORNL-3817 (October 1965).

Fig. 25. Comparison of Theoretical and Experimental Stresses on the Model with a $7\frac{1}{8}$ -in.-OD Nozzle with a Wall Thickness of 0.183 in. for an Internal Pressure of 200 psi.

Fig. 26. Comparison of Theoretical and Experimental Stresses on the Model with a $7\frac{7}{8}$ -in.-OD Nozzle with a Wall Thickness of 0.183 in. for an Axial Thrust Load on the Nozzle of 3000 lb.

STRAIN GAGE SCHEDULE			
DEXTR BLOCK NO. 1			
COLOR CODE	GAGE NO.	GAGE TYPE	FACTOR
WHITE-BLUE	1	MM-EA-06-025RA-120	2.045
WHITE-ORANGE	2		
WHITE-GREEN	3		
WHITE-BROWN	4		
WHITE-GRAY	5		
RED-BLUE	6		
RED-ORANGE	7		
RED-GREEN	8		
RED-BROWN	9		
RED-GRAY	10		
BLACK-BLUE	11		
BLACK-ORANGE	12		
BLACK-GREEN	13		
BLACK-BROWN	14		
BLACK-GRAY	15		
YELLOW-BLUE	16	MM-EA-06-062AK-120	2.045
YELLOW-ORANGE	17		
YELLOW-GREEN	18		
YELLOW-BROWN	19		
YELLOW-GRAY	20		
PURPLE-BLUE	21		
PURPLE-ORANGE	22		
PURPLE-GREEN	23		
PURPLE-BROWN	24		
DEXTR BLOCK NO. 2			
PURPLE-GRAY	*1	MM-EA-06-062AK-120	2.045
WHITE-BLUE	*2		
WHITE-ORANGE	*3		
WHITE-GREEN	4	MM-EA-06-025RA-120	2.045
WHITE-BROWN	5		
WHITE-GRAY	6		
RED-BLUE	7		
RED-ORANGE	8		
RED-GREEN	9		
RED-BROWN	10		
RED-GRAY	11		
BLACK-BLUE	12		
BLACK-ORANGE	13		
BLACK-GREEN	14		
BLACK-BROWN	15		
BLACK-GRAY	16		
YELLOW-BLUE	17		
YELLOW-ORANGE	18		
YELLOW-GREEN	19		
YELLOW-BROWN	20		
YELLOW-GRAY	21		
PURPLE-BLUE	22	MM-EA-06-062AK-120	2.045
PURPLE-ORANGE	23		
PURPLE-GREEN	24

- LEGEND:
- STRAIN GAGES LOCATED INSIDE LATERAL
 - STRAIN GAGES LOCATED OUTSIDE LATERAL
 - *○ STRAIN GAGES IN DEXTR BLOCK NO. 2 LOCATED INSIDE LATERAL

Fig. 27. Strain-Gage Locations on, and Dimensions of,

BLANK PAGE

NOTE:

1. PLACE SINGLE STRAIN GAGES (120 ohm), SHOWN IN VIEWS A-A AND B-B, AS CLOSE TOGETHER AS POSSIBLE AND AS CLOSE TO KNIFE EDGE AT HOLE INTERSECTION AS POSSIBLE.
2. PLACE SINGLE STRAIN GAGES (120 ohm), SHOWN IN VIEW C-C, AS CLOSE TOGETHER AS POSSIBLE AND CENTER ON CENTERLINE AS SHOWN.

DIMENSIONS ARE IN INCHES.

Dimensions of, the 45° Lateral.

2

Table 7. Stresses for 200 psi Internal Pressure, Lateral Y_2

Location	Maximum Principal Stress (psi)	Minimum Principal Stress (psi)	Maximum Shearing Stress (psi)	Angle from Major Axis to Maximum Principal Stress (deg)
Inside gages 1, 2, 3	-354	-7060	3,352	20
Inside gages 4, 5, 6	-897	-5746	2,424	25
Inside gages 7, 8, 9	401	-5415	2,908	-21
Inside gages 10, 11, 12	-2,768	-4347	789	26
Inside gages 13, 14, 15	1,731	-5631	3,681	-6
Inside gages (view BB) 16, 17, 18	30,226	-440	15,333	-2
Inside gages (view BB) 19, 20, 21	30,831	196	15,317	-9
Inside gages (view AA) 22, 23, 24	8,407	1964	3,221	-2
Inside gages (view AA) 1, 2, 3	8,687	1126	3,780	4
Outside gages 4, 5, 6	9,997	1789	4,104	15
Outside gages 7, 8, 9	17,214	6400	5,407	29
Outside gages 10, 11, 12	10,831	6655	2,388	-34
Outside gages 13, 14, 15	12,084	3430	4,327	16
Outside gages 16, 17, 18	10,152	5834	2,159	-6
Outside gages (view DD) 19, 20, 21	3,376	3053	162	0
Outside gages (view CC) 22, 23, 24	-2,909	-7334	2,213	2

Fig. 28. Typical Stepped Cylinder Configuration.

where T is the thickness of the thick cylinder, t is the thickness of the thin cylinder, r is the fillet radius, and a is the inside radius. The stepped cylinder is loaded by an internal pressure, and both ends are considered to be capped at sufficient distances from the step region for end effects to have no influence on the region of interest. At present, only extreme combinations of the parameters are being examined to establish bounds on the stress concentration effect.

Two separate finite-element computer programs have been used. Both are based on a two-dimensional, axisymmetric solid body formulation. All the analyses are being performed using a version of Wilson's program.⁴² However, the SAFE-2D program, previously described,⁴³ was used to obtain duplicate analyses for two of the cases. This was done for two reasons. First, agreement between the two sets of results served to give confidence in the validity of the Wilson program results, and second, more extensive computer plotting capabilities were available at ORNL.

⁴²Edward L. Wilson, "Structural Analysis of Axisymmetric Solids," *AIAA J.* 3(12), 2269-74 (1965).

⁴³D. C. Cornell, *SAFE-2D, a Computer Program for the Stress Analysis of Plane and Axisymmetric Composite Structures, a User's Manual*, GA-9076, Gulf General Atomic (February 1969).

for the SAFE-2D program than for Wilson's program. Detailed computer plots of the two cases analyzed by SAFE-2D provided an insight into the basic behavior of stepped cylinders.

The results presented here are for the two cases that were analyzed by both programs. Some of the computer plots of the SAFE-2D results are presented, and comparisons of the Wilson program results with SAFE-2D results are shown.

The two cases chosen for the duplicate analyses both had a T/t ratio of 1.5 and an a/t ratio of 5.0. In one case, however, the r/t ratio was zero, while in the other, the r/t ratio was 0.3. The finite-element mesh layouts

for the SAFE-2D analyses are shown in Fig. 29. The entire mesh for the case with the fillet radius is shown on the left, and the stepped region is shown enlarged on the right. For the case of the zero fillet radius, the mesh was modified as shown in the cutout. A total of 1198 nodes and 2173 triangular elements were used for the case with the fillet radius.

In comparison, the mesh layouts for the Wilson program analyses used 723 nodes and 618 elements for the case with the fillet radius and 681 nodes and 568 elements for the case without the fillet radius. Most of the elements were quadrilaterals, which are internally divided by the program into four triangular elements

Fig. 29. Finite-Element Mesh Layouts Used for SAFE-2D Analyses of Stepped Cylinder.

with linearly varying displacement fields. The analyses using both Wilson's program and the SAFE-2D program thus used roughly the same number of triangular elements. The element layouts for use with the two programs were, however, prepared independently by different individuals and were quite different.

The deflections predicted by SAFE-2D for the two cases are shown, to an exaggerated scale, superimposed on the original cross sections in Fig. 30. As would be expected, the presence of the small fillet radius had no observable effect on the overall deflection behavior.

The principal stress trajectories, which are lines everywhere tangent to the directions of principal stress,

are shown in Fig. 31 for the step region of both configurations. These are again results from the SAFE-2D program. Principal stress trajectory curves have several useful properties, one of which is particularly important here. In the two configurations examined, the maximum stress in the plane of the cross sections shown in Fig. 31 is the tangential stress on the free boundary of the thin cylinder at a point just below the step. One property of stress trajectories is that the distances from a free boundary to a nearby stress trajectory curve are inversely proportional to the tangential stresses on the boundary. This allows one to estimate the peak stress from the stress trajectory plots.

Fig. 30. Displaced Configurations, Determined by SAFE-2D, for Stopped Cylinder With and Without Fillet Radius.

Fig. 31. Stress Trajectories, Determined by SAFE-2D, for Stepped Cylinder With and Without Fillet Radius.

Using the axial membrane stress in the lower part of the thin cylinder as a reference tangential stress, it can be seen that the peak tangential stress just below the step is about twice the reference value. The factor is somewhat less for the configuration with the fillet radius.

These observations are verified in Fig. 32, which shows the maximum stress contour levels in the plane of the figure for both cases. The third principal stress is always the circumferential stress, which is perpendicular

to the plane of the figure; these contours are shown in Fig. 33. Contours determined by both SAFE-2D and Wilson's program agreed very well, as shown in these figures.

The pressure level for the analyses was chosen to produce a nominal hoop stress of 1000 psi and a nominal axial stress of 500 psi in the thin cylinder. For reference purposes the calculated Lamé stresses at the surfaces of the cylindrical portions away from the discontinuity region are listed in Table 8. Figures 32

Table 8. Pressure Stresses in a Thick-Walled Stepped Cylinder

Portion	Inside-Radius-to-Thickness Ratio	Inside Surface Stresses (psi)			Outside Surface Stresses (psi)		
		Circumferential	Radial	Axial	Circumferential	Radial	Axial
Thin	5.00	1109	-200	455	909	0	455
Thick	3.33	780	-200	290	580	0	290

Fig. 32. Maximum Stress Contours in Plane of Cross Section for Stepped Cylinder With and Without Fillet Radius.

and 33 indicate that the largest stress in the configuration without the fillet radius occurs in the plane of the figure at a point just below the step with a value slightly above 1100 psi. Thus, if the nominal hoop stress is taken as a reference, the stress concentration factor is about 1.1. For the case with the fillet radius the nominal hoop stress is the largest stress. Thus the stress concentration factor is about 1.0. In summary then, the fillet radius reduces the maximum stress by only about 9% for this particular case.

Joint AEC (ORNL)-PVRC Cooperative Program - Single-Nozzle Studies

University of Tennessee (II) - Experimental Stress Analysis of Single Nozzles in Spherical and Cylindrical Shells

Experimental studies on two spherical shells with attached nozzles are being continued. In both cases the

dimensions for the shell are $D_i = 30.132$ in. and $T = 0.375$ in., giving a D_i/T ratio of 80.35. One model has a 7.875-in. outside diameter (d_o) nozzle radially attached at the apex, and the other has two 2.625-in.-OD oblique nozzles attached at angles of 22.5° and 45° respectively.

Tests on the 22.5° oblique nozzle of the second model were completed for a nozzle wall thickness t of 0.25 in., and the results have been partially reported.⁴⁴ The summary portion of the report, part 3d, is in preparation. It will contain plots of the data for all loading cases; a typical data plot is shown in Fig. 34.

⁴⁴R. I. Maxwell, R. W. Holland, and G. R. Stengl, *Experimental Stress Analysis of the Attachment Region of Monospherical Shells with Attached Nozzles, Part 3a, Part 3b, Part 3c, Nonradial Nozzle at $22\frac{1}{2}^\circ$ Degrees, 2.625 O.D.-2.125 I.D., 4.00-in. Penetration*, ME 7-69-1, The University of Tennessee (April 1969).

Fig. 33. Circumferential Stress Contour for Stepped Cylinder With and Without Fillet Radius.

The results in the figure correspond to an internal pressure loading and are from the downhill side of the nozzle along the plane which passes through the apex and contains the center line of the nozzle. Subsequent to completion of the tests, the nozzle was bored out to a wall thickness of 0.125 in. in preparation for the next series of tests.

The plots shown in Fig. 34 were done entirely by computer. The development of the associated computer program represents a significant advancement in providing means for handling the large amounts of data obtained from each test series and in the preparation of figures for use in final topical reports.

Tests were also completed on the second configuration for the shell with the 7.875-in.-OD nozzle. This configuration differs from the first in that the nozzle thickness was 0.375 in. for the first case, while it was 0.1875 in. for the second. A topical report on the results is being prepared.

The 7.875-in.-OD nozzle has an internal protrusion that is 10 in. long. For the next series of tests the length will be reduced to 2 in., but the inside diameter of the nozzle will not be changed.

In addition to the experimental investigations of spherical shells with attached nozzles, a cylindrical shell with a single radially attached nozzle was pressurized internally to determine first the plastic collapse load and finally the burst pressure. The plastic collapse load is that which corresponds to the onset of large deformation through plastic action. The model was originally tested at the IIT Research Institute under elastic conditions.⁴⁵ The dimensional parameters are $D_o/T = 230$, $d/D = 0.5$, and $s/S = 0.51$, where s/S is the

⁴⁵W. F. Riley, "Experimental Determination of Stress Distributions in Thin-Walled Cylindrical and Spherical Pressure Vessels with Circular Nozzles," *Welding Research Council Bulletin 108*, September 1965.

Fig. 34. Plot of the Total Stress vs the Shell Profile at the 0° Plane on the 22.5° Nozzle Model.

ratio of the membrane stress in the nozzle to that in the shell.

Plots of strains and deflections as functions of pressure did not give distinguishable indications of transitions from elastic to elastic-plastic material behavior. These indications were obscured, in part, by the geometrical changes in the shell as the pressure increased, and the responses measured gave almost entirely nonlinear curves. Some of the curves have linear portions between 0 and 30 psi (a crack developed at 170 psi), while the remainder are nonlinear even in this range. The shell was subjected to pressures up to 30 psi in the elastic tests performed by IIT Research Institute, and, although in retrospect it would have been advisable, cyclic loadings — that is, loadings and unloadings — were not imposed above this pressure level during the collapse and burst test. Since the limit load concept is based on a marked change in load response due to

changes from elastic to elastic-plastic behavior of the material, it appears there is little meaning to be associated with the concept in this case.

The maximum pressure reached during the test was 230 psi, but a leak due to a crack in the vessel actually occurred at 170 psi, as noted above. This crack was in the off-the-axis portion of the shell near the junction. An examination of the specimen after the test revealed that cracking occurred essentially at random around the junction. The cracks were in the circumferential direction along the nozzle-to-shell attachment weld.

Westinghouse Research Laboratories — Photoelastic Studies

A series of internal pressure tests was made on lateral and hillside nozzles in cylindrical shells, two hillside and one lateral opening in cylindrical shells, oblique nozzles

in spherical shells, and one oblique opening in a spherical shell. All conventional nozzles were nonprotruding. Table 9 lists the 15 nozzles and openings studied, along with the principal stress ratios. Nozzle UW-2 is included for comparison purposes. These nozzles and openings were contained in three cylindrical pressure vessel models with hemispherical end closures. The first and second models each contained five nozzles and one opening without a nozzle, and the third model contained one of the nozzles and two openings listed in Table 9.

Nozzle WC-2AY1 was one with a thermal sleeve attachment and balanced reinforcing, while nozzle WC-2AY was a conventional unreinforced nozzle attached to a cylindrical shell. A comparison of the maximum principal stress ratios given in Table 9 for these two models reveals that the unreinforced conventional nozzle experiences higher stresses than its thermal-sleeved counterpart. Additional information may be obtained from the topical report.⁴⁶

The end caps of model 1 contained a small hole (WS-63B), which was gasketed at the ridsurface, and nozzles WS-63-B1 and WS-63-B2. Nozzle WS-63-B1 was thin walled (unreinforced), while WS-63-B2 was thick walled (reinforced). The maximum stresses in these nozzles were in the longitudinal direction on the inside surface, with the unreinforced nozzle (WS-63-B1) being the most highly stressed.^{47,48} The last nozzle in the model, designated as WC-59-B1, was a lateral connection very similar to WC-2AY. The maximum stress at the acute inside corner of nozzle WC-59-B1 was 1.7 times the maximum stress measured in WC-2AY.⁴⁹

Model 2 contained four nozzles and one opening in the cylindrical portion of the shell. Nozzles WC-12B1, WC-12B2, and WC-12B are a set of lateral-nozzle-to-cylindrical-shell attachments. The first was unreinforced, while the remaining two were reinforced by

thickening the walls. These reinforced nozzles were identical, except that the acute inside corner was removed in the case of WC-12B3. The stress was somewhat higher in the unreinforced nozzle as compared with the reinforced nozzle. The reinforced nozzle that had the acute inside corner removed had the lowest maximum stress value.⁵⁰ The results for the opening WC-12C and the unreinforced hillside nozzle WC-12C1 are reported in ref. 50.

One hemispherical end cap of model 2 contained the nozzle WUW-2, which corresponds to UW-2, a nozzle that was tested at the University of Waterloo.⁵⁰ The essential difference is that WUW-2 had sharp inner corners, while those for UW-2 are generously rounded. The nozzle having the sharp inner corners developed a peak stress at the acute inside corner which was approximately 12% greater than that of the nozzle having the rounded corners.^{47,48,50}

A third model has been fabricated which is a relatively thick-walled cylindrical vessel with five nozzle attachments and two nonradial openings without nozzles [see Table 10 (WC-12C3, WC-12C2 were listed in Table 9)]. There are two closely spaced reinforced nozzles (WC-12DD) mounted along the longitudinal axis. There is also a single reinforced nozzle (WC-12D) with dimensions identical to those for the two closely spaced nozzles. This nozzle will be used for comparison with the other two to study the effect of closely spaced nozzles. The fourth nozzle (WC-12E) has the same dimensions as the above three, but it is reinforced internally. This is the only inwardly protruding nozzle. The fifth (WC-12C3) is an unreinforced hillside nozzle in the cylindrical portion of the model. One of the openings is a lateral hole (WC-12B), while the second (WC-12C2) is a hillside opening. The bore of the latter is tangential to the inside surface of the shell.

The data from this model are now being analyzed. As indicated by Table 9, the data analyses for nozzle WC-12C3 and the two openings, WC-12B and WC-12C2, have been completed.

A fourth model is to be fabricated which will be a thin-walled cylindrical shell ($D/T \approx 100$) with two closely spaced radial nozzle attachments and two single nozzle attachments for comparison purposes. These nozzles will be analogous to the first four nozzles of model 3.

All of the data for oblique openings (with and without nozzles) in cylindrical and spherical vessels

⁴⁶M. M. Leven, *Photoelastic Determination of Stresses in a Reinforced Steam Charging Connection with a Thermal Sleeve*, 69-9D7-PHOTO-R1, Westinghouse Research Laboratories (Jan. 14, 1969).

⁴⁷M. M. Leven, *Summary of Oblique Openings in Spherical Shells*, 69-9D7-PHOTO-R2, Westinghouse Research Laboratories (Mar. 20, 1969).

⁴⁸M. M. Leven, *Photoelastic Determination of the Stresses at Oblique Openings in Spherical Pressure Vessels*, 69-9E7-PHOTO-R2, Westinghouse Research Laboratories (Oct. 29, 1969).

⁴⁹M. M. Leven, *Photoelastic Determination of the Stresses at Oblique Openings in Cylindrical Pressure Vessels*, 69-9E7-PHOTO-R1, Westinghouse Research Laboratories (Sept. 10, 1969).

⁵⁰C. E. Taylor and N. C. Lind, "Photoelastic Determination of the Stresses near Openings in Pressure Vessels," *Welding Research Council Bulletin 113*, April 1966, p. 14.

Table 9. Nozzle and Opening Tests Completed This Year at Westinghouse Research Laboratories

Model No.	Opening Designation	Type of Opening	Shell	D_f/T	d_f/D_f	s/S	α , Hillside	γ , Lateral	Section	Maximum Principal Stress Ratio ^a	
										Inner Surface	Outer Surface
1	WC-2AY1	Radial nozzle	Cylinder	59.4	0.098	0.467	0°	0°		1.60	1.02
		Thermal sieve	Cylinder	59.4	0.115	0.187					
1	WC-2AY	Conventional nozzle for comparison with WC-2AY1	Cylinder	59.4	0.115	0.488	0°	0°		2.96	2.65
1	WC-59B1	Lateral nozzle	Cylinder	59.5	0.115	0.50		45°	Acute Obtuse	5.13 3.71	2.31 ^b 3.90 ^b
1	WS-63B	Small opening with thin plate seal at midsurface	Sphere	63.4	0.008	0	45°		Acute Obtuse	2.72 1.25	1.35 2.68
1	WS-63B1	Flush nozzle	Sphere	63.4	0.04	0.92	45°		Acute Obtuse	3.70 4.18	1.67 3.50
1	WS-63B2	Flush nozzle	Sphere	63.4	0.04	0.10	45°		Acute Obtuse	3.50 2.40	1.55 ^c 1.80 ^c
3	WC-12B	Lateral opening	Cylinder	12.2	0.021	0		45°	Acute Obtuse	3.12 2.52	1.06 2.52
2	WC-12B1	Lateral nozzle	Cylinder	12.2	0.129	1.02		45°	Acute Obtuse	4.65 3.96	1.46 ^b 2.36 ^b
2	WC-12B2	Lateral nozzle	Cylinder	12.2	0.129	0.21		45°	Acute Obtuse	4.50 2.78	1.06 ^b 1.04 ^b
2	WC-12B3	Lateral nozzle with the acute inside corner removed	Cylinder	12.2	0.129	0.21		45°	Acute Obtuse	3.34 3.36	1.00 ^b 1.10 ^b
2	WC-12C	Hillside opening with thin plate seal at midsurface	Cylinder	12.2	0.02	0	45°		Acute Obtuse Longitudinal	1.17 1.01 ^c 2.33	0.94 1.04 1.86
2	WC-12C1	Hillside nozzle	Cylinder	12.2	0.129	1.01	60°35'		Acute Obtuse Longitudinal	1.25 2.23 1.87	1.45 ^{b,c} 2.35 ^{b,c} 1.77
3	WC-12C2	Hillside opening with thin plate seal at midsurface	Cylinder	12.2	0.021	0	78°18'		Acute Obtuse Longitudinal	2.20 1.43 1.67	0.87 ^c 1.23 1.67
3	WC-12C3	Hillside nozzle	Cylinder	12.2	0.067	0.99	50°		Acute Obtuse Longitudinal	1.50 2.21 1.87	1.29 ^c 2.31 ^c 1.70
2	WUW-2	Flush nozzle	Sphere	18.3	0.130	0.63	60°35'		Acute Obtuse	4.95 4.40	1.40 ^c 1.43
	UW-2 ^d	Flush nozzle - inner acute corner generously rounded	Sphere	18.3	0.129	0.63	60°38'		Acute Obtuse	4.0 4.4	1.30 ^c 1.50

^a σ_p/S unless otherwise specified, where $S = \sigma_{nom} = p(D_f + T)/2T$ and p = pressure (psi).

^bExtrapolated values for cement filler.

^c σ_p/S .

^dData from "Photoelastic Study of the Stresses near Openings in Pressure Vessels," by C. E. Taylor and N. C. Lind, *Welding Research Council Bulletin 113*, April 1966.

Table 10. Nozzle and Openings Planned for Model 3

Designation	Shell	D_j/T	d_j/D_i	s/S	α Hillside (deg)	γ Lateral (deg)	ϕ/T	Remarks
WC-12DD	Cylinder	12.0	0.129	1.0	0	0	0.50	Two closely spaced reinforced nozzles
WC-12D	Cylinder	12.0	0.129	1.0	0	0	0.50	
WC-12E	Cylinder	12.0	0.129	1.0	0	0	0.50	Inside reinforcing
WC-12C3	Cylinder	12.0	0.0667	1.0	60.0			
WC-12B	Cylinder	12.0	0.0207	0		45		Lateral opening
WC-12C2	Cylinder	12.0	0.0207	0	78.3			Hillside opening

subjected to internal pressure are contained in refs. 48 and 49. The material in these reports is to be included in a Welding Research Council Bulletin that is in preparation.

University of Waterloo (I) – Elastic and Plastic Analyses of Nozzle-to-Shell Attachments

Investigations are being made to provide theoretical and experimental data of elastic stresses at nozzle-to-vessel intersections. In addition, methods are being developed and experimental data obtained for plastic limit analysis and design of nozzle-to-vessel attachment configurations.

Upper and lower bound limit analysis methods and associated computer programs were developed to predict plastic limit loads for spherical shells with single rigid radial nozzle attachments. The loadings considered were internal pressure and external forces and moments applied to the nozzles. These loads may act singly or in combination. In support of the theoretical developments, six steel model tests were conducted using various combinations of internal pressure and external loadings. The results from both the theoretical and experimental studies are contained in a recent report.⁵¹

An experimental stress analysis of a 45° steel lateral ($D_j/T = 12$, $d_j/D_i = 0.50$, $s/S = 1.0$) which was subjected to external force and moment loadings was completed. One end of the intersected cylinder (run) was fixed, and force loads were applied to the specimen to give the equivalent of three forces and three moments acting separately on the end of the branch and on the free end of the run. Two parts⁵² of a three-part report on this test have been written, and the third part is being prepared. Part I describes the model, the loadings, and the methods used. It also gives the results in graphical form. The test data and computed results are given in tables in Part II. Part III presents a theory and the

results for calculation of the structural stiffness matrices for the lateral, and the principal modifications in classical piping stress analysis necessary for the analysis of systems which assume flexible joints are outlined.

University of Waterloo (II) – Plastic Limit Analysis Studies for Tees and Laterals

The research at the University of Waterloo is directed toward deriving upper bound limit loads for tees and laterals. The loadings to be considered are internal pressure and external forces and moments applied to the nozzles.

Analytical methods for determining upper bound plastic collapse loads (upper bound limit loads) for tees subjected to internal pressure were developed, and a computer program was written. A paper describing the analysis was published.⁵³ In this paper, configurations for which $0.4 \leq d_m/D_m \leq 1.0$, $D_m/T > 20$, $d_m/t > 20$, and $0.7 \leq s/S \leq 1.3$, where the subscript m is used to denote mean diameter, were considered.

Methods for calculating upper bound limit pressures for laterals with nozzle-to-shell (run) angles ranging from 30 to 90° (a 90° lateral is a tee) were derived, and analytical techniques for determining upper bound limits for in-plane bending moments applied to nozzles of tees were developed. In addition, the upper bound

⁵¹N. C. Lind and S. Palusamy, "Limit Analysis of Rigid Radial Nozzles in Spherical Pressure Vessels Under Combined Loading," draft report, University of Waterloo, December 1969.

⁵²P. K. Fung and N. C. Lind, *Experimental Stress Analysis of a 45° Lateral Branch Pipe Connection Subjected to External Loadings*, Parts I and II, University of Waterloo (September 1969).

⁵³J. Schroeder and P. Rangarajan, "Upper Bounds to Limit Pressures of Branch-Pipe Tee Connections," *Pressure Vessel Technology, Part I, Design and Analysis*, First International Conference on Pressure Vessel Technology, Delft, Netherlands, September 1969, pp. 277-91.

limit pressure analysis for tees was extended to include s/S ratios ranging from 0.25 to 3.0 and d_m/D_m ratios from 0 to 1.0.

Twelve models are to be tested to provide data for examining the theory. These models are listed in Table 11, where the status — that is, ordered, acquired, or tested — and type of loading to be used in each case are also given. Two of the laterals, Y_5 and Y_6 , were made by welding the nozzle to the shell, while the remainder were machined from one piece of material. To date, three tees (T_1 , T_2 , and T_3) and four laterals (Y_1 , Y_4 , Y_5 , and Y_6) have been tested under internal pressure loading. Good comparisons were obtained between analytical and experimental results.

The welded laterals (Y_5 and Y_6) were added to investigate the possibility of cracks developing in the welds during internal pressure limit load tests. From the results obtained, cracks did not develop in the welds until about twice the limit pressure was reached in each case.

University of Sherbrooke — Elastic Analysis of Skewed Holes in Flat Plates and Plastic Collapse Loads of Single Nozzles in Spherical and Cylindrical Shells

The investigations at the University of Sherbrooke serve two purposes. Elastic stresses around single skewed holes in flat plates are being examined in order to increase the understanding of skewed holes in pressure vessels, and information is being developed that is applicable to determining the plastic collapse loads of cylindrical and spherical shells with single cylindrical nozzles attached.

At present a three-dimensional theory is being developed for estimating the stress concentration factors for skewed holes in flat plates. A summary paper on skewed holes is being prepared for publication in a Welding Research Council Bulletin. This paper will contain some of the three-dimensional work as well as earlier work on comparisons between two-dimensional theory and experimental work.

Table 11. Models for Limit Load Tests

Model	d_m/D_m	D_m/T	s/S	Nozzle Angle (deg)	Status	Type	Loading, etc.
Y_1	1.0	34.5	1.0	45	Tested	Machined from one piece	Internal pressure
T_1	1.0	34.5	1.0	90	Tested	Machined from one piece	Internal pressure
Y_2	1.0	26.0	1.0	45	Acquired	Machined from one piece	Internal pressure with special strain gages on crotch
T_2	0.6	34.5	0.9	90	Tested	Machined from one piece	Internal pressure
T_3	0.85	25.0	1.0	90	Tested	Machined from one piece	Internal pressure
T_4	0.5	25.0	1.0	90	Acquired	Machined from one piece	External in-plane couple on branch
T_5	0.75	25.0	1.0	90	Ordered	Machined from one piece	External in-plane couple on branch
T_6	1.0	25.0	1.0	90	Ordered	Machined from one piece	External in-plane couple on branch
Y_3	0.75	25.0	1.0	45	Ordered	Machined from one piece	External in-plane couple on branch
Y_4	0.6	26.0	1.0	45	Tested	Machined from one piece	Internal pressure
Y_5	1.0	16.0	1.0	45	Tested	Welding lateral from Tube Turns	Internal pressure to fracture
Y_6	1.0	20.0	1.0	45	Tested	Welding lateral from Tube Turns	Internal pressure to fracture

Limit analysis methods for determining plastic collapse pressures for spherical shells with single radial nozzle attachments have been developed, and reports on this work were written.⁵⁴⁻⁵⁷

Equations were derived for calculating lower bound limit pressures for cylindrical shells with single radial nozzles.^{58,59} Although this analysis is presently limited to diameter ratios less than $\frac{1}{2}$ ($d/D \leq 0.5$), it can be extended to cover the full diameter ratio range. A preliminary formulation for upper bound limit pressures was obtained; however, since it appears that an exact formulation for the limit pressure can be obtained from the lower bound analysis, no further work is being done on the upper bound formulation at present.

Auburn University (II) – Theoretical Solution of a Single Nonradial Nozzle in a Spherical Shell

The investigations at Auburn University are to provide analytical methods for obtaining the stresses in the junction region of spherical shells with single nonradial nozzle attachments. These methods are to be based first on shallow-shell theory and later extended to use nonshallow-shell, or complete, theory.

The first analysis to be developed is based on an extension of an existing analytical method, which, in turn, is based on shallow-shell theory.⁶⁰ The existing method was modified by using an exact description for the nozzle-to-shell intersection instead of an approximate description such as was used in ref. 60. On this basis, the modified analytical method can be used to

analyze nozzles with larger angles of obliqueness than the previous method.

Formulations were developed for stress analysis of nonprotruding nozzles and for inwardly protruding nozzles, where the shell and nozzle are loaded by internal pressure. A report on the analysis for nonprotruding nozzles was written.⁶¹

Computer programs for the analysis of nonprotruding and protruding nozzles have been written. At present the analysis for the nonprotruding nozzle is being evaluated by comparing calculated and experimental results. University of Tennessee data for radial nozzles as well as the results for the 22.5° oblique nozzle are being used. Since the latter has an internal protrusion, a good assessment of the analytical results is not to be expected in this case.

As the angle of obliquity increases, the number of terms to be used in the equations and the number of boundary points to be matched increase. Studies have shown that the present computer program must be extended to meet the increased requirements before 45° nozzles, such as those tested by Westinghouse Research Laboratories, can be examined.

AEC (ORNL) Program – Multiple-Nozzle Studies

University of Tennessee (II) – Experimental Stress Analysis of Clusters of Holes and Clusters of Nozzles in Flat Plates

For relatively small nozzles attached to cylindrical or spherical vessels, stresses from local nozzle loadings such as axial thrust, moment, torsion, and shear are virtually independent of vessel curvature. In this study, nozzles attached to flat plates are used for experimental stress analysis of this configuration. Combinations of biaxial tension on the plates and pressure in the nozzles are used to simulate stresses resulting from vessel internal pressure loadings. A series of ten models consisting of the holes and nozzles listed in Table 12 are being tested. The hole and nozzle location patterns are to be the same. The 36 X 36 X 0.375 in. plate, along with the nozzle attachment configurations to be used in the tests, is shown in Fig. 35.

Initially, two unpierced flat plates were extensively strain gaged and tested in uniaxial tension and biaxial tension. One of these plates is shown in Fig. 36 positioned in the biaxial testing machine. The plate was

⁵⁴F. Ellyin, "Flush Nozzles in Conical-Spherical Pressure Vessels," *Pressure Vessel Technology, Part I, Design and Analysis*, First International Conference on Pressure Vessel Technology, Delft, Netherlands, September 1969.

⁵⁵F. Ellyin, "The Effect of Yield Surfaces on the Limit Pressure of Intersecting Shells," *Intern. J. Solids Structures* 5(7), 713-25 (July 1969).

⁵⁶F. Ellyin, "Elastic-Plastic Behavior of Intersecting Shells," *J. Eng. Mech. Div. ASCE* 95(EM1), 69-94 (February 1969).

⁵⁷F. Ellyin, *Experimental Investigation of Flush Nozzles in Conical-Spherical Pressure Vessels (the Tangent Generator)*, FE-7-69, University of Sherbrooke (March 1969).

⁵⁸F. Ellyin and S. Mouadeb, *Limite Inferieure De La Pression D'Effondrement A L'Intersection Perpendiculaire De Deux Cylindres Pressurises*, FE-8-69, University of Sherbrooke (June 1969).

⁵⁹F. Ellyin and S. Mouadeb, *Limit Pressure of Nozzles in Cylindrical Pressure Vessels*, University of Sherbrooke (August 1969).

⁶⁰D. E. Johnson, "Stresses Due to Forces on a Nonradial Nozzle in a Spherical Shell," Ph.D. dissertation, Cornell University, June 1963.

⁶¹J. C. Yu and C. H. Chen, *Stresses in the Vicinity of a Nozzle Nonradially Attached to a Spherical Pressure Vessel*, ME-UC2889-2, Auburn University (April 1969).

Table 12. Models for Flat-Plate Studies

Test Specimens	Model No.	Hole Diameter (in.)	Nozzle Dimensions (in.)		Hole or Nozzle Pattern
			d_o	t	
Two unperforated plates					
Plate for initial test of loading fixture	1				
Plate of material to be used in remaining studies	2				
Two plates with holes ^a					
One hole	3	2.625			
Two holes	3	2.625			
Three holes	4	2.625			Triangular
Four holes	3	2.625			Square
Five holes	5	2.625			Modified square; outer holes on 6.0-in. circle
Five plates with nozzles					
One nozzle	6		2.625	0.250, 0.125	
Two nozzles	7		2.625	0.250, 0.125	
Three nozzles	8		2.625	0.250, 0.125	Triangular
Four nozzles	9		2.625	0.250, 0.125	Square
Five nozzles	10		2.625	0.250, 0.125	Modified square; outer holes on 6.0-in. circle

^a3.0-in. hole spacing.

Fig. 35. Nozzle Design and Cluster Pattern Details for Flat-Plate Models.

Fig. 36. Biaxial Tensile Testing Machine.

loaded through 32 hydraulic rams positioned eight to a side along the edges of the specimen. The results of these tests indicated that the desired state of true biaxial stress was developed within the central 14-in.-diam region of the plate.

Plates with clusters of one, two, three, four, and five holes without nozzles are being tested in uniaxial and biaxial tension. Tests on the one- and two-hole configurations have been completed, and tests on the plate containing three holes are in progress.

A report containing the results of the uniaxial and biaxial tension testing of the plate containing a single hole has been completed.⁶² This plate was instrumented on one side with electrical resistance strain gages in the locations shown in Fig. 37. All tests were

conducted with the gaged side up in the loading machine and then repeated with the gaged side down. The averaging of the two readings for each gage compensated for possible bending or eccentricities in the loading system. Materials properties were determined from test coupons taken from the edges of the plate.

Figures 38 and 39 are examples of radial and tangential stress data along the 0 and 90° planes shown in Fig. 37 for the uniaxial tension test. Two experi-

⁶² A. Mathews, *Experimental Stress Analysis of Clusters of Holes of Equal Diameters in Flat Plates - Part 1, a Single Hole*, EM 69-2, Department of Engineering Mechanics, The University of Tennessee (April 1969).

POSITION	LOCATION	GAGE TYPE
1	1.00 R	SINGLE ELEMENT
2	1.07 R	DELTA ROSETTE
3	1.5 R	DELTA ROSETTE
4	2.0 R	DELTA ROSETTE
5	3.0 R	DELTA ROSETTE
6	4.0 R	DELTA ROSETTE

Fig. 37. Location of Strain Gages.

mental data points were shown for each gage location. One set of points was obtained by pulling the plate in one direction, and another set was obtained by subsequently pulling the plate in the transverse direction. The curves indicate theoretical results. The most scatter in experimental data occurred for the 45° plane. Stress values greater than theoretical values were measured at the edge of the hole, as shown in Fig. 39. In general, agreement between theoretical and experimental values was favorable.

Typical results of the biaxial tension test employing a 2:1 ratio of applied stresses are shown in Figs. 40 and 41. Experimental data are shown by triangles in Fig. 40 and by dots in Fig. 41. The solid curves indicate the theoretical solution obtained by superposition of two orthogonal uniaxial stress fields. The dashed curves in Fig. 40 were obtained by superposition of uniaxial test data. The biaxial test data fitted close to the dashed curves. This confirmed the validity of using superposition of test results. Good agreement between theory and experiment was observed.

Fig. 38. Radial and Tangential Stresses for the Uniaxial Tension Test. 0° plane.

A report is being written for the two-hole test. Once the testing of the five plates with holes is completed, plates to which the five clusters of nozzles shown in Fig. 35 and listed in Table 12 have been attached will be tested. A device for pressure loading a nozzle has been developed, and a loading frame for applying external force and moment loads to the nozzles is being fabricated.

Battelle Memorial Institute (II) – The Stress Analysis of Perforated Plates and Plates with Single and Clustered Nozzle Attachments

The basic purpose of this project is to provide stress analysis methods and associated computer programs applicable to the perforated flat plates and flat plates with attached nozzles being examined experimentally at the University of Tennessee (II). The loadings are in-plane forces applied at the edges of the plates containing perforations, and, in the case of plates with attached nozzles, the loadings will be simulated internal pressure loadings for cylindrical and spherical shells and external loads applied to the nozzles. The nozzle loads

Fig. 39. Radial and Tangential Stresses for the Uniaxial Tension Test. 90° plane.

Fig. 40. Radial and Tangential Stresses for the Biaxial Tension Test Employing a 2:1 Ratio. 0° plane.

Fig. 41. Radial and Tangential Stresses for the Biaxial Tension Test Employing a 2:1 Ratio. 90° plane.

will include an axial force, a shear force, and moment loadings. When the programs are developed and verified through comparisons between calculated and experimental studies, these programs will be used by ORNL for general studies of the stresses and displacements in the vicinities of nozzle clusters. The results will correspond to and be applicable for clusters of small nozzles attached to large-radius thin cylindrical and spherical shells.

The work is divided into five categories:

1. Membrane stress analyses of square plates with clusters of one, two, three, four, and five holes under uniaxial and biaxial (1:1 and 2:1) in-plane forces applied along the outside edges of the plates.
2. Modification of the computer program to analyze perforated plates under in-plane (membrane) loads for calculating bending stresses associated with edge loadings around the holes in perforated square plates.

3. Modification of an existing computer program for shell analyses to calculate sets of influence coefficients for nozzles.
4. Development of a matrix manipulation program to combine the matrix equations generated by the plate membrane, plate bending, and shell analysis programs and to calculate stresses, strains, and displacements for nozzle-plate assemblies.
5. Stress analyses of clusters of one, two, three, four, and five closely spaced nozzles attached to flat plates for simulated internal pressure loadings and external force and moment loadings on the nozzles. Combinations of these loadings are to be considered.

The analyses for plates having nozzle attachments are divided into two classes, *a* and *b*. For class *a* analyses, the loads are such that only one nozzle or a part of a nozzle is included in a symmetry element. In class *b* analyses, portions of more than one nozzle will be included in a symmetry element. Examples of problems belonging to class *a* are those for all nozzle clusters, except the five-nozzle cluster, when subjected to in-plane plate-edge forces which are the same in both directions, and those for the one-, two-, and four-nozzle clusters under uniaxial plate-edge loadings. Examples of problems belonging to class *b* are those of the five-nozzle cluster under all loading conditions and of the three-nozzle cluster with uniaxial plate-edge loads.

The approach to be used in solving problems of class *a* is to satisfy exactly the free-end conditions for the nozzles and the continuity conditions at the intersections of the nozzles and plate. The boundary conditions for the edges of the plate and the symmetry conditions along one line of symmetry are to be satisfied in the least-squares sense. Problems of class *b* probably will be solved by satisfying exactly the symmetry conditions in the plate and the free-end conditions for the nozzles, while the plate-edge boundary conditions and the continuity conditions between the plate and the nozzles are satisfied in the least-squares sense.

Stress analyses were completed for the square flat plates with the hole patterns, as shown in Fig. 35. Both uniaxial and biaxial (1:1 and 2:1) in-plane edge loadings on the plates were considered. A computer program previously developed by BMI was used for this purpose. This program is for generalized plane-stress conditions; the Airy stress function approach is adopted, and the solution to the governing differential equation is taken in series form. The boundary conditions are satisfied in a least-squares sense.

The stresses were calculated for locations consistent with strain-gage positions in the paralleling experimental tests at the University of Tennessee. The results of the analyses have been presented⁶³ as plots of tangential stress ratios around the edges of the holes. Figure 42 shows a plot of the stress distribution around one hole of the four-hole cluster for biaxial loadings. In this figure, σ_x is the stress in the *x* direction, σ_y is the stress in the *y* direction, $\bar{\sigma}_y$ is the nominal stress at the edge of the plate, and σ_t ($=\sigma_\theta$) is the tangential stress. The maximum tangential stress calculated for all loadings occurred at the edge of an outer hole of the five-hole cluster, as shown in Fig. 43.

To expand the in-plane (or membrane) boundary-point least-squares computer program to include plate bending, the primary modification was the addition of a subroutine to calculate the Kirchhoff shear terms. A method of shell analysis which was developed by Kalnins⁶⁴ and utilizes a multisegment approach for improved accuracy was adopted for relating the fundamental variables at one end of a cylindrical nozzle to those at the other end. A single computer program was then developed by combining the plate membrane, the plate bending, and the nozzle analyses to solve nozzle-plate problems.

Thus far, the following class *a* analyses have been made:

⁶³L. E. Hulbert, E. F. Rybicki, and A. T. Hopper, *The Stress Analysis of Clusters of Cylindrical Nozzles Attached to Large Spherical Shells*, BMI-X-576 (Sept. 29, 1969).

⁶⁴A. Kalnins, "Analysis of Shells of Revolution Subjected to Symmetrical and Nonsymmetrical Loads," *J. Appl. Mech.* 467-76 (September 1964).

Fig. 42. Dimensionless Tangential Stress for the Four-Hole Array.

Fig. 43. Dimensionless Tangential Stress for the Five-Hole Array.

1. a single nozzle attached to a circular plate with an axial force, a shear force, and a moment load on the nozzle and with internal pressure in the nozzle,
2. a single nozzle attached to a square plate with an axial force applied to the nozzle and with internal pressure in the nozzle,

3. two nozzles attached to a square plate with axial forces applied to the nozzles and with internal pressure in the nozzle.

The analysis of a single nozzle attached to a circular plate was undertaken so that the results obtained using the newly developed nozzle-plate program could be compared with results obtained using an existing shell analysis program. The latter program is based on a well established and verified computer analysis procedure. The mean radius of the nozzle was 1.1875 in., and the wall thickness was 0.25 in. The outside radius of the plate was 8 in., and its thickness was 0.375 in. The assumed elastic modulus and Poisson's ratio were 30×10^6 psi and 0.285 respectively.

Tables 13 and 14 give comparative values for the membrane stresses and the radial and circumferential moments in the plate as functions of the radius for the axial force and internal pressure cases. In these tables, h is the thickness of the plate, σ_r is the radial stress, σ_θ is the circumferential stress, M_r is the radial moment, and M_θ is the circumferential moment. The distributions are axisymmetrical. These tables show that there is excellent agreement between the two sets of calculated results. Discrepancies were found between the results

Table 13. Comparative Values of Radial and Circumferential Membrane and Bending Resultants

Axial thrust on top of nozzle: $N = 1$

Radius	Shell Analysis				Nozzle-Plate Analysis			
	$h\sigma_r$	$h\sigma_\theta$	M_r	M_θ	$h\sigma_r$	$h\sigma_\theta$	M_r	M_θ
1.1875	-3.60	3.77	-1.74	-1.58	-3.53	3.70	-1.70	-1.55
1.50	-2.23	2.39	-1.45	-1.51	-2.18	2.34	-1.42	-1.48
2.00	-1.22	1.38	-1.15	-1.37	-1.19	1.35	-1.12	-1.34
3.00	-0.50	0.66	-0.79	-1.12	-0.49	0.64	-0.75	-1.09
5.00	-0.13	0.29	-0.37	-0.76	-0.12	0.28	-0.34	-0.73
8.00	-0.00	0.16	0.00	-0.41	0.00	0.16	0.00	-0.39

Table 14. Comparative Values of the Radial and Circumferential Membrane and Bending Resultants

Unit pressure in the nozzle

Radius	Shell Analysis				Nozzle-Plate Analysis			
	$h\sigma_r$	$h\sigma_\theta$	M_r	M_θ	$h\sigma_r$	$h\sigma_\theta$	M_r	M_θ
1.1875	-0.227	0.237	-0.0217	0.0228	-0.211	0.220	-0.023	0.0210
1.5	-0.140	0.151	-0.0135	0.0144	-0.130	0.140	-0.0126	0.0133
2.0	-0.077	0.087	-0.0074	0.0083	-0.071	0.081	-0.0069	0.0077
3.0	-0.031	0.041	-0.0030	0.0040	-0.029	0.038	-0.0029	0.0036
5.0	-0.008	0.018	-0.0008	0.0017	-0.007	0.017	-0.0008	0.0016
8.0	-0.000	0.010	0.0000	0.0010	0.000	0.009	0.0000	0.0009

for the remaining two loading cases, however, and further study of the equations used is required to resolve the differences.

Reference 63 describes the results obtained thus far under this project. Future work will include completing the analyses of the remaining class *a* problems and making major modifications to the computer programs to permit the analysis of class *b* problems.

Auburn University (I) – Experimental Stress Analysis of Spherical Shells with Clusters of Nozzles

Two spherical shell models were fabricated, one with five separate clusters of small holes and the second with two large radial nozzles.⁶⁵ The first, model 1, was strain gaged and is being prepared for testing under internal pressure. Following the pressure test, model 1 will be fitted with nozzles and tested under separate loading conditions of internal pressure, axial nozzle load, and nozzle bending load. After the initial tests, the nozzles will be bored out to a larger diameter and retested. The nozzles will then be replaced by a second set, and the model will be tested again. These nozzles will also be bored out, and the model will be subjected to a final series of tests. A summary of the dimensions of model 1 and the nozzles is shown in Table 15.

The model containing the two large radial nozzles will eventually be modified to include four- and five-nozzle clusters, as described in Table 15. The present two-nozzle cluster model is shown in Fig. 44, and the strain-gage locations for this model are shown in Figs. 45 and 46. Figure 45 also includes an explanation of the numbering system employed for the gages, and the gage orientation at each location is indicated in Fig. 47.

Model 2 was tested under internal pressure and with an axial load and a bending moment applied to a nozzle. Some results for the pressure case are given in Figs. 48–51. From these figures it may be seen that the meridional stresses σ_x in the shell are generally lower and the circumferential stresses σ_y in the shell are generally higher in the region between the nozzles than elsewhere. To emphasize the differences in distributions as functions of angle around the nozzle, calculated results, obtained through the use of CERL-II,⁶⁶ for the shell region in the vicinity of a

Fig. 44. Details of Vessel with Attached Nozzles.

single nozzle attachment are plotted on the figures. Although the interaction between nozzles is apparent, the influence is not large in this case.

Joint AEC (ORNL)–PVRC Cooperative Program – General

Battelle Memorial Institute (I) – Evaluation and Correlation of Experimental and Analytical Results

The evaluation and correlation of experimental and analytical data on radial nozzles attached to pressure vessels by BMI has led to the development of proposed alternate pressure vessel code rules for reinforcement of radial nozzles attached to spherical and cylindrical shells. A report⁶⁷ describing these rules was submitted to the ASME Design Committee's Subgroup on Openings and Attachments.

⁶⁵W. L. Greenstreet and R. C. Gwaltney, *Experimental and Analytical Investigations of the Structural Behavior of Nozzle-to-Shell Attachments*, ORNL-TM-2526 (April 1969).

⁶⁶S. E. Moore and F. J. Witi, *CERL-II – a Computer Program for Analyzing Hemisphere-Nozzle Shells of Revolution with Axisymmetric and Unsymmetric Loadings*, ORNL-3817 (October 1965).

⁶⁷E. C. Rodabaugh, *Phase Report 117-3 on Proposed Alternate Rules for Use in ASME Codes*, Battelle Memorial Institute (August 1969).

Table 15. Dimensions of Models with Closely Spaced Nozzles in Spherical Shells

	D_o (in.)	T (in.)	D_o/T	L_o (in.)	L_i (in.)	a (in.)	h or t (in.)	d_n (in.)	a/h	T/h or T/t	t/T	d_i/D_i	a/S (Pressure)	Number of Holes or Nozzles	Minimum Spacing ^f (in.)	Maximum Spacing ^f (in.)	Hole or Nozzle Pattern
Model 1 ^a	40.90	0.503	81.40														
Quadrant 1 (2 holes)																	
Holes								1.572						2	0.570		
First nozzle				4		0.786	0.166	1.738	4.75	3.02	0.331	0.035	0.214		0.500		
Bored first nozzle				4		0.828	0.083	1.738	10.00	6.04	0.166	0.040	0.478		0.500		
Second nozzle				6		1.573	0.331	3.476	4.75	1.51	0.662	0.071	0.212		0.500		
Bored second nozzle				6		1.655	0.166	3.476	10.00	3.02	0.331	0.079	0.478		0.500		
Quadrant 2 ^b (3 holes)																	
Holes														3	0.825		Triangular
First nozzle															0.588		
Bored first nozzle															0.588		
Second nozzle															0.500		
Bored second nozzle															0.500		
Quadrant 3 ^b (4 holes)																	
Holes														4	1.125	2.750	Square
First nozzle															0.889	2.147	
Bored first nozzle															0.889	2.147	
Second nozzle															0.500	2.147	
Bored second nozzle															0.500	2.147	
Quadrant 4 ^b (5 holes)																	
Holes														5	1.540	2.820	Modified square
First nozzle															1.360	2.645	
Bored first nozzle															1.360	2.645	
Second nozzle															0.500	2.144	
Bored second nozzle															0.500	2.144	
Model 2 ^c	41.19	0.500	82.38														
Test 1				10		3.335	0.333	7.002	10.00	1.50		0.158	0.238	2	3.600	3.600	
Test 2				10		3.335	0.333	7.002	10.00	1.50		0.158	0.238	4	3.600	8.120	Square
Test 3				10	10 ^d	3.335	0.333	7.002	10.00	1.50		0.158	0.238	5	0.500	3.600	Modified square
Test 4 ^e				10	0	3.418	0.167	7.002	20.53	3.00		0.166	0.498	5	0.500	3.600	Modified square

^aNozzle dimensions based on 2.625-in.-OD, 0.25-in.-wall University of Tennessee nozzle.^bDimensions, dimensional ratios, and membrane stresses same as for quadrant 1.^cNozzle dimensions based on 5.25-in.-OD, 0.25-in.-wall University of Tennessee nozzle (not used).^dCenter nozzle only.^eAll five nozzles bored to reduced thickness.^fMeasurements taken on outside surfaces.

Fig. 46. Gage Location, Inside and Outside Hemispherical Vessel.

forces) on the outer edges of the plate. Also, a spherical shell with two closely spaced radial nozzle attachments ($D_o/T = 80.38$, $d/D_i = 0.158$, and $s/S = 0.158$) was tested under internal pressure and axial force and moment loadings on one nozzle.

In support of limit analysis development studies, an internal pressure collapse and burst test was done on a cylindrical vessel with a single radial nozzle ($D_o/T = 230$, $d/D = 0.5$, and $s/S = 0.51$). Internal pressure collapse tests were also done on three tees and four laterals; finally, six spherical shells with single rigid radial nozzles were tested using various combinations of internal pressure and external loads.

In the case of analytical methods development and verification, finite-element analysis techniques were applied successfully for the analysis of single-nozzle-to-spherical-shell and single-nozzle-to-cylindrical-shell attachments. Although the latter studies were reported under the program to develop design criteria for piping, pumps, and valves, they also apply to analyses of pressure vessels. The finite-element technique was also used in the analysis of cylindrical pressure vessels with

Fig. 47. Vessel 2, Coordinate Axes Alignment.

step changes in wall thickness loaded with internal pressure.

Additional verifications of the validity of analytical methods based on closed-form-type solutions for the governing differential equations for single radial-nozzle-to-spherical-shell attachments were obtained, and significant progress was made in the development of similar analytical methods which are applicable to cylindrical shells. Methods were developed and computer programs written for analyzing nonradial-nozzle-to-spherical-shell attachments. Analyses for square flat plates with one to five perforations and subjected to

membrane loads were completed, and flat plates with one and two nozzles were analyzed to determine the stresses and displacements corresponding to internal pressure and axial force loads on the nozzles.

Analytical methods for determining plastic limit loads for single nozzle-to-shell attachments have been established. These methods include those for limit pressures associated with spherical shells with single radial nozzles, and those for determining upper and lower bounds for plastic limit loads for spherical shells with single rigid radial nozzles under internal pressure, external loads on the nozzle, or combinations of these loadings. Equations were derived to calculate lower bound limit pressures for tees with nozzle-diameter-to-

shell-diameter ratios less than $\frac{1}{2}$ ($d/D < 0.5$). Upper bound analyses were also derived to calculate limit pressures for tees and laterals for the full range of diameter ratios. Finally, analytical techniques for determining upper bound limits for in-plane bending moments applied to nozzles of tees were developed.

Rules for use in design codes for pressure vessels were written and submitted to the ASME code writing group. In addition, 24 topical reports, drafts of topical reports, and papers were prepared.

Many difficult problems remain. However, solutions are to be expected through continued developments under this program.

Fig. 48. Distribution of Meridional Stresses Outside of the Interaction Region.

OSMA - DWS 70-050

Fig. 49. Distribution of Mesiodistal Strains in the Interaction Region.

CONFIDENTIAL - DANG 70-854

Fig. 50. Distribution of Circumferential Stresses Outside of the Interaction Region.

ORNL - DWG 70-852

Fig. 51. Distribution of Circumferential Stress in the Interaction Region.