24-75
eg. 1715
DAS

DEPARTMENT of APPLIED SCIENCE

FAILURE PROBABILITIES OF STEAM GENERATOR TUBES

ANNUAL REPORT

August 1975

MASTER

Structural Analysis Group

for the
Materials Engineering Branch, Division of Technical Review,
Nuclear Regulatory Commission

**BROOKHAVEN NATIONAL LABORATORY
UPTON, NEW YORK 11973**

INFORMAL REPORT

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

FAILURE PROBABILITIES OF STEAM GENERATOR TUBES

ANNUAL REPORT

M. REICH

**Structural Analysis Group
Department of Applied Science
Brookhaven National Laboratory
Upton, New York 11973**

**for the
Materials Engineering Branch, Division of Technical Review,
Nuclear Regulatory Commission**

Notice: This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United Nuclear Regulatory Commission, or any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

rlb
REPRODUCTION OF THIS DOCUMENT IS UNLIMITED

TABLE OF CONTENTS

	<u>Page</u>
List of Figures	i
List of Tables	ii
1. Introduction	1
2. Statistical Evaluations	2
3. Finite Element Structural Analysis Failure Evaluation	7
4. Conclusions and Recommendations	17
5. References	20
APPENDIX I	
APPENDIX II	
APPENDIX III	
APPENDIX IV	
APPENDIX V	

List of Figures

<u>Figure</u>	<u>Title</u>	<u>Page</u>
1	Pressure Expansion Curves for Aluminum Tube $\frac{R_1}{R_0} = 2$	9
2	Stress-Strain Curve for 17SO Aluminum	10
3	NONSAP 4-Node Finite Element Model	13
4	Pressure Expansion Curve for 17SO Aluminum Finite Element Theory	14

List of Tables

<u>Table</u>	<u>Title</u>	<u>Page</u>
I	Extreme Value Failure Predictions	3
II	Weibul Distribution Failure Prediction	6
III	Pressure vs Strain in Element 20	12
IV	Burst Pressure vs Percent Wastage	16

1. Introduction

As was indicated when this task was undertaken, close liaison between Brookhaven National Laboratory and the Materials Engineering Branch, Division of Technical Review, Nuclear Regulatory Commission was maintained for all phases of this work carried out during FY 1975. Thus, it was mutually agreed upon to focus BNL's efforts on the following specific items;

a) The probabilities of failure for perfect steam generator tubes.

b) The probabilities of failure for steam generator tubes containing long axisymmetrically thinned sections, and

c) The probabilities of failure for steam generator tubes containing finite length (relatively short) axisymmetric wastages.

2. Statistical Evaluations

At the urging of members of the Regulatory Staff, an attempt was made to answer item (a), with a probability model that was based on very small sample data. This resulted in an article entitled "Statistical Study of Piping Failure" by J.M. Frankel issued in November, 1974. A copy of this paper is included at the end of this text under the heading Appendix I.

As more data became available for Inconel 600 at 600°F, further statistical studies were undertaken. As was to be expected, the additional investigations showed that the log-normal distributions assumed for the previous studies were not acceptable. In fact, as shown in Appendix II, entitled "Burst Pressure Statistics for Non-Degraded and Degraded Tubing" by J.M. Frankel issued in April, 1975, the W-test indicated that there is a less than 1% probability that the test data is from a log-normal distribution.

Instead of the log-normal distribution, Frankel concluded that the type I extreme value distribution was applicable on the basis of the available data. Frankel's model which includes work-hardening expressions for the stress and strain relationships, can best be summarized from the table shown below.

Table I

<u>Percentile</u>	<u>c=1</u>	<u>c=.5</u>	<u>c=.4</u>	<u>c=.3</u>	<u>c=.2</u>	<u>c=.1</u>
50	9.57 ksi	4.78	3.83	2.87	1.91	0.96
25	9.21 ksi	4.61	3.68	2.76	1.84	0.92
10	8.95 ksi	4.48	3.58	2.69	1.79	0.89
5	8.82 ksi	4.41	3.53	2.65	1.76	0.88
2.5	8.71 ksi	4.36	3.48	2.61	1.74	0.87
1.0	8.60 ksi	4.30	3.44	2.58	1.72	0.86
0.5	8.53 ksi	4.26	3.41	2.56	1.70	0.85
0.1	8.39 ksi	4.20	3.36	2.52	1.68	0.84

The above table taken from Appendix II (page 13) assumes a relationship

$$P = \frac{\sigma_u' t}{(1+\epsilon_u') r}$$

for the burst pressure of a perfect tube and

$$P = \frac{\sigma_u' t}{r} \left(\frac{100+x \epsilon_n'}{100 (1+\epsilon_n')} \right)$$

for the burst pressure of a defected tube,

where σ_u' = nominal stress at ultimate

t = wall thickness

r = inner radius

ϵ_u' = engineering strain at ultimate

x = amount of wall thickness reduced by corrosion.

As indicated from the table, the 50th percentile of perfect tubes ($c=1$) will fail below 9.57 ksi while only 0.1 of one percent of the tubes will fail below 8.39 ksi. Similarly, for 90% degraded tube ($c=.1$), the 50th percentile will fail below .96 ksi and only 0.1 of one percent of all tubes will fail below 0.84 ksi.

In order to varify some of the distributions assumed in the studies described in Appendix II, several additional statical tests such as the Kolmogorov-Smirnov, the chi-square tests, as well as Kendall's Rank correlation were employed for the data used by Frankel. Results of this study issued in May of 1975, and included as Appendix III of this paper, showed that the assumed distributions were indeed adequate.

As pointed out by Han and Shapiro⁽¹⁾ and others⁽²⁾ the test type I extreme value distribution can take either of two forms. The largest value type I, has a curve with a short tail section on the left and a long tail on the right, while the smallest type I extreme value distribution has a large tail to the right and a short tail to the left. Originally, Frankel interpreted the available data as only applying to the largest value type I extreme value distribution. However, further work to be presented shortly by Kao indicates that the Weibull distribution which is essentially an extreme value

curve with a long tail to the left and a short tail to the right appears to present the most acceptable distribution for the burst pressure from both the physical and statistical standpoint. This work (which is still in the development stage) is based on a graphical method called the density-gram instead of the usual histogram. It indicates the following probabilities for a perfect tube:

$$\text{Prob \{ B.P. (burst pressure) <3 kpsi\} } \approx 1.60 \times 10^{-9}$$

$$\text{Prob \{ B.P. <3.5 ksi\} } \approx 2.07 \times 10^{-8}$$

$$\text{Prob \{ B.P. <4 ksi\} } \approx 1.87 \times 10^{-7}$$

$$\text{Prob \{ B.P. <5 ksi\} } \approx 7.4 \times 10^{-6}$$

$$\text{Prob \{ B.P. <7 ksi\} } \approx 1.89 \times 10^{-3}$$

Table II gives the Weibull probabilities for failure in the same manner as the results shown previously for the extreme value distributions in Table I.

Table II

<u>Percent</u>	<u>c=1</u>	<u>c=.5</u>	<u>c=.4</u>	<u>c=.3</u>	<u>c=.2</u>	<u>c=.1</u>
50	10.0156 ksi	5.0078	4.0062	3.0047	2.0031	1.0013
10	8.9328	4.4663	3.5737	2.6803	1.7868	.8930
5	8.5507	4.2753	3.4210	2.5658	1.7104	.8548
2.5	8.1918	4.0958	3.2776	2.4582	1.6386	.8190
1	7.7448	3.8723	3.0990	2.3243	1.5493	.7743
0.5	7.4244	3.7121	2.9710	2.2282	1.4853	.7422
.1	6.7322	3.3660	2.6944	2.0208	1.3469	.6730
$\hat{\eta}$:	16.4654	16.4639	16.4897	16.4883	16.4788	16.4671
$\hat{\sigma}$:	10.2410	5.1205	4.0962	3.0722	2.0482	1.0238

While these results are more conservative than those previously given in Appendix II, and in Table I, it needs to be pointed out that these studies were only recently initiated and thus the results shown are at the moment only preliminary.

Further work in this area is indeed necessary in order to reach a firm conclusion with regards to the Weibull failure probabilities for the steam generator tubes.

3. Finite Element Structural Analysis Failure Evaluation

In addition to the probabilities studies mentioned above, structural analysis evaluations by means of finite element techniques were carried out to evaluate the stresses and strains for various finite length wastages. Initially, the work in this area was confined to studies involving the elastic analysis of axial and circumferential cracks and wastages. Later on the studies were extended to include large deformations that include tube failure predictions. Results of the elastic analysis are contained in Appendix IV, while those for the large deformation studies are detailed in Appendix V.

In reviewing the analysis presented in Appendix V, it needs to be pointed out that the finite element computer code developments involving Lagrangian large displacement analysis methods are relatively new. NONSAP, the code used for the results depicted and discussed in Appendix V, has been available to the public for only a little over a year. Although some check problems were attempted by the authors, no closed-form solution check problems for the Lagrangian portions of the code were carried out either by the code authors or BNL. After presenting the data shown in Appendix V to NRC personnel it was suggested by the Material Engineering

people of NRC that attempts to test the program vs. an appropriate analytical closed-form solution that could be found in the literature should be undertaken. The writer is indeed indebted to Dr. Hartzman of NRC Mechanical Engineering Branch, for steering us to a closed form solution of a thick cylinder undergoing large strains.

Figure 1, taken from C.W. MacGregor et al⁽³⁾ depicts results for an aluminum 17S0 alloy thick tube with a R_i/R_o ratio of 2 and a uniaxial stress-strain curve of the type shown in Figure 2. As can be seen from Figure 1, the infinitesimal strain theory indicates an indefinitely increasing bore pressure, while the large strain theory indicates a maximum pressure corresponding to an external tangential strain of about 0.04% after which the tube bulges and the pressure falls off. As far as the difference between the finite strain theory and the experimental data is concerned, the authors state;

"No great significance can be attached to the smaller disagreement between the predictions of the large strain theory and the actual behavior of the aluminum tube...study of the material of the tube showed it to be highly anisotropic with differences in the stress strain curves for axially and tangentially oriented specimens."

Figure 1

Figure 2

The significance of the results of MacGregor et al. is that ultimate bulging can only be predicted by large deformation theory and not by infinitesimal theory. Thus the method chosen for the analysis presented in Appendix V for the various axisymmetric finite length wastages with different corner configurations could indeed yield valuable information for steam generator tube ultimate failure prediction. Unfortunately, when NONSAP was applied for the MacGregor et al., thick walled aluminum tube using identical dimensions, loads, and the stress-strain curve assumed by the above mentioned authors, the results did not match those shown for the finite strain theory depicted in Figure 1.

As mentioned in Appendix V, the elements used for the NONSAP failure analysis were eight node elements. In attempting to find out why NONSAP answers did not match those of the closed form solution, a review of the NONSAP formulations was initiated. After carrying out some initial investigations of the code formulations and algorithms, etc., it was decided to first concentrate the efforts on a simpler four node element. After some effort, it was possible to modify NONSAP and by use of a four node element model depicted in Figure 3, the results shown in Table III and Figure 4 were obtained. As can be seen, the plotted points

Table III

<u>Pressure (psi)</u>	<u>% Strain at Element 20</u>
4608	0.028
5632	0.036
6656	0.045
7680	0.058
8704	0.079
9728	0.123
10752	0.195
11776	0.273
12800	0.375
13824	0.504
14848	0.682
15872	0.949
16384	1.123
16896	1.340
17408	1.615
17920	2.025
18432	2.714
18688	3.348

PRESSURE EXPANSION CURVE
FOR ALUMINUM TUBE
(1750 ALUMINUM, $R_i/R_o = 20$)

Figure 4
-14-

are almost identical with those obtained by use of the closed-form solution.

One of the modifications carried out on NONSAP involves the ability to input actual uniaxial stress-strain curves instead of simulating the curve outline with a bilinear stress-strain relationship.

Results of a run using a typical bilinear material of mode for the 17 aluminum are also plotted in Figure 4. As can be seen, the internal failure pressures are somewhat higher and the strains at failure are less than those obtained for the actual non-linear stress-strain curve. With respect to the last remark, it needs to be pointed out that the slope of the bilinear curve will greatly effect the strains at failure. A bilinear curve with a small plastic slope, requires very small pressure increments once the material becomes fully plastic. Thus, when using bilinear input, the pressure at failure can readily be obtained from a run that uses fairly course pressure increments, while the strains at failure require very small pressure increments, once the tube becomes fully plastic.

Having established the validity of the four node elements and knowing that the eight node element results do not check

with the closed form solution, it is obvious that the runs carried out for tube failure prediction in Appendix V should be rerun with the modified NONSAP four node elements.

Table IV gives the bursting pressure results of modified NONSAP runs for uniformly degraded tubes having 50%, 60%, 70%, 80% and 90% wastages.

Table IV

<u>Percent Wastage</u>	<u>Burst Pressure</u>
50%	3790 psi
60%	3027 psi
70%	2300 psi
80%	1435 psi
90%	709 psi

Comparing these results, with those of Table II, for the Weibull distribution, it can be seen that the burst pressures nearly fit those of the 0.5 percentile. In view of the fact that the statistical formulations include only infinitesimal stress-strain theory and not finite strain theory, it seems the Weibull distribution needs to be adjusted to account for large deformation theory.

4. Conclusions and Recommendations

Substantial progress has occurred in both the statistical and the structural evaluation areas. The statistical work derived and described in Appendices II and III, regarding the application of the extreme value distribution has served as a stepping stone to the more advanced work currently being carried out with the Weibull distribution methods. As mentioned in Section II of the text, where some of the initial Weibull probabilities already arrived at are given, this work, which is still in the development stages, will be reviewed in a progress report. All indications at the present are that the Weibull distribution is the most acceptable from both the physical and statistical viewpoint.

As far as the analytical work is concerned, it has been demonstrated that ultimate bulging can be predicted by large deformation theory and that the BNL modified NONSAP large deformation computer code does indeed match a closed-form analytical solution for the burst pressure obtained from the literature. Furthermore, as discussed in Section III of the text, results of computer runs for uniformly degraded tubing with 50%, 60%, 70%, 80% and 90% wastages match the burst pressures predicted in Table II for the 0.5 percentile Weibull distribution. Obviously, since the calculations for

the Weibull distribution does not contain the large displacement theory, the results indicate that a greater number of tubes than the 0.5 percentile would fail at the calculated pressures. Since these results effect the probabilities of failure for various burst pressures, it seems that further investigations possibly leading to a modification of the Weibull formula is warranted. Additionally, since the computer code has now been corrected and modified, the cases with the various finite length and different corner configurations described in Appendix V should now be rerun to ascertain the correct burst pressures.

Finally, it needs to be pointed out that the modified computer code can only be applied to axisymmetric wastages. Since in many instances the wastages are arbitrary and thus require a three-dimensional evaluation for burst pressure, the large deformation code should be modified to include three-dimensional burst pressure calculation capabilities.

In conclusion, it needs to be emphasized that the scarcity of material property information for inconel 600 at 600^oF has placed a severe handicap on all aspects of this investigation. A broader data base will unquestionably place the statistical results on a much firmer foundation. Also, as pointed out in Appendix V (see Figure 2, Appendix V),

even the quality of the limited data could stand improvement. The only stress-strain data available for material input into the large deformation code for failure prediction bilinear in nature. As pointed out in the text, a complete stress-strain curve yields more realistic results. Unfortunately, we were unable to locate such a curve even after an extensive search. Furthermore, during the last few weeks, in anticipation of future fracture mechanics investigations, attempts have been made to locate fracture toughness and other pertinent data but have thus far been unsuccessful.

We strongly urge the initiation of a test program to yield the necessary backup and as mentioned in some cases input data for this work.

References

1. G.J. Han and S.S. Shapiro, Statistical Methods in Engineering, J. Wiley and Sons, (1967).
2. E.J. Gumbel, Statistic of Extremes, Columbia University Press, (1958).
3. C.W. MacGregor, L.F. Coffin, and J.C. Fisher, The Plastic Flow of Thick Walled Tubes with Large Strains Journal of Applied Physics, Vol 19, March 1948.

Appendix I

Statistical Study of Piping Failures

**Judah Frankel
Stress Analysis Group
Brookhaven National Laboratory**

DRAFT

SAG-2

Statistical Study of Piping Failures

Judah Frankel
Stress Analysis Group
Brookhaven National Laboratory

1. Introduction. The initial analysis of the burst pressure statistics for Inconel 600T 0.750" OD, 0.048" nominal wall tubing is concerned with "perfect tubing," no chemical corrosion, stress cracking, or other physical faults are assumed to be present. Variations of the materials stress and strain properties and of the tube diameter and wall thickness will be considered. Under these assumptions the minimal (internal) pressure which will cause the tube to burst can be theoretically determined from an equation of the form,

$$(1) \quad b = g(s, t, r)$$

where s is a vector of the material's properties

t = tube-wall thickness

$$r = \frac{1}{2} OD - \frac{1}{2} t$$

Probabilistic statements concerning burst pressures will therefore be based on the estimation and accuracy of the estimates of the distributions of s , t , and r . Throughout let B , S , T , and R denote the random variables whose specific values are b, s, t, r respectively.

2a. For any single tube we are interested in obtaining an estimate of $P(B \leq b)$ = probability the tube bursts at a pressure less than or equal to b . If we let $H(t, r, b)$ be determined by $g(S, T, R) \leq b$ when $S \in H(t, r, b)$ then

$$(2) \quad P(B \leq b) = P(g(S, T, R) \leq b) = \int P(S \in H(t, r, b) \mid T = t, R = r) dP(T \leq t, R \leq r) = \int P(S \in H(t, r, b)) dP(T \leq t, R \leq r).$$

When S is independent of T and R

$$(3) \quad P(B \leq b) = \iint P(S \in h(t, r, b)) dP(T \leq t, R \leq r),$$

and if S is one dimensional $g(h(t, r, b), t, r) = b$.

If $S_1 \dots S_n$ are the stress data and $t_1 \dots t_m$ are the wall thickness data then we can estimate $P(B = g(S, T, R) \leq b)$ without first estimating the distributions of S and T by

$$\sum_{j=1}^m \sum_{i=1}^n \chi[g(s_i, t_j, r) \leq b] / m \cdot n$$

where $\chi[g(s_i, t_j, r) \leq b] = 1(0)$ if $g(s_i, t_j, r) \leq (\geq) b$, S and T are independent, and R is taken to be fixed at $R=r$.

The difficulty now is that this gives us no estimate for the tails of the distribution unless m and n are quite large.

To get estimates of (3) when $P(B \leq b)$ is either close to zero or close to one we will estimate the tails of the distributions for S, T , and R and consequently the tails of the distribution for B .

The accuracy of these estimates depend on the availability of data and can be expressed in terms of confidence bounds for $P(B \leq b)$.

2b. Calculations with Available Data

The data currently available is very limited and only rough estimates can be made to illustrate some of the techniques which will be utilized. No data for the distribution of either R or T is available so we will assume that $R = .350 (= \frac{1}{2} OD - \frac{1}{2} t)$ since small percentage variations will not much effect the burst pressure and we will set T at its nominal value .048 which will give us conservative burst pressures.

Equation (1) will be approximated by

$$(4) \quad b = st/r$$

where s is now the ultimate stress; this is the burst pressure equation for thin wall tubing without allowance for deflection under load.

Only four data points for the ultimate stress of Inconel 600T are available at the moment and only a preliminary estimate of the distribution for S can be made. Since ultimate stress and burst pressure are determined by a product of many factors we might reasonably assume that S and B have lognormal distributions. Assuming $r = .350$ and $t = .048$ and the data for S given in Table I we calculate the burst pressures b, log b and

the sample mean and variance of $\log b$. Then if B has a lognormal distribution, its parameters μ and σ are estimated by $\hat{\mu} = 9.523$ and $\hat{\sigma} = .04$.

Note: Equation (4) is only an approximation for the true burst pressures; if we consider tube deflection the burst pressure would be 7-8% lower than those given in Table I. An even larger deviation in the pressure calculations results from the use of (4) because the tubes are not really "thin-walled," more accurate calculation procedures are in preparation by the Stress Analysis Group and will be utilized in conjunction with the more comprehensive data which has been requested.

Computation of $P(B \leq b)$ is now relatively simple, $P(B \leq b) = P(\log B \leq \log b) = P(X \leq \log b) = P\left(\frac{X - \mu}{\sigma} \leq \frac{\log b - \mu}{\sigma}\right) =$

$$(5) \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log b - \mu}{\sigma}} e^{-z^2/2} dz \approx \frac{\exp\left(-\left(\frac{\log b - \mu}{\sigma}\right)^2\right)}{\sqrt{2\pi} \left| \frac{\log b - \mu}{\sigma} \right|}$$

where B is lognormal with parameters μ , σ , and the approximation holds for large negative $(\log b - \mu)/\sigma$.

If $\hat{\mu} = 9.523$, $\hat{\sigma} = .04$, $b = 7000$, then $P(B \leq 7000) \approx \frac{1}{\sqrt{2\pi} (16) \exp(280)}$ which is negligible; $\hat{\mu}$, however, is

probably substantially smaller (see note above).

If we consider the burst test results of apparently perfect tubes (Table 2) and assume a lognormal distribution, we get

$\hat{\mu} = 9.21$, $\hat{\sigma} = .032$ (since no tubes are truly perfect we would expect μ for perfect tubes to be somewhat larger). For an MSLB burst test we then have

$$(6) \quad P(B \leq 2300) = P\left(\frac{\log B - 9.21}{.032} \leq \frac{\log 2300 - 9.21}{.032}\right) \approx (\sqrt{2\pi} \cdot 46 \cdot e^{46^2})^{-1}$$

which is negligible. Even for $b = 6000$ $P(B \leq 2300) \approx (\sqrt{2\pi} \cdot 16 \cdot e^{256})^{-1}$
 $\approx (\sqrt{2\pi} \cdot 16 \cdot 10^{110})^{-1}$

It seems that the chance of a perfect tube failure under MSLB conditions is virtually negligible.

2c. Tube Failure in System

In a system containing N tubes assumed to be perfect with single tube failure f , the probability of at least one tube failing is

(7) $1 - (1-f)^N \approx 1 - e^{-Nf}$ for small f . If f is of the order indicated in section 2b, then even for large N , ($N \leq 10^4$ in typical reactor systems), $1 - e^{-Nf}$ is negligible.

3. Defective Tubing Failures

From the discussion above it is obvious that tube failures will almost surely occur in defective tubing which will be the subject of a later report; some observations, however, may be made at this time. A failure in a defective tube should occur at the position of the most serious defect and a tube-wall degradation of $x\%$ will at worst behave as if the entire tube-wall were $(100-x)\%$ of its specified thickness. We will consider

360° milled defects varying from 68% to 90% degradation of wall thickness. For defects of this magnitude the resulting tube is "thinwalled" and if we assume that the axial length of the defect is sufficiently large we may disregard any wall stiffening and treat the tube as if it were defective in its entirety. Under these conditions a good approximation to the burst pressure is $b = st/r$ where s is the ultimate stress, $t = .048(1-x/100)$ is the wall thickness for a milled defect of $x\%$ and $r = .325" + t/2$. We will compute b for the four stresses in Table I; the results for various size defects are listed in columns 2-6 of Table 3a. We note that the nominal wall thickness of .048" was used in the computations so that actual burst pressures might be somewhat higher for large defects; these values then compare favorably with the results of the burst tests performed on tubing with 1 ½" long 360° milled defects of varying sizes as listed in Table 3b.

The Analysis of more complex defects is being pursued by the Stress Analysis Group at this time and a more comprehensive discussion of these problems will appear in the future.

1. D. L. Starr, "Predicting Burst Strength," Machine Design,
August 1968, pp. 173-175.
2. Morris Reich, Personal Communication.
3. Don Gardner, Personal Communication.
4. G. J. Hahn and S. S. Shapiro, Statistical Models in Engineering,
J. Wiley & Sons, 1967.

Table 1

Sample	Ultimate Stress	$b = \frac{sxt}{t}$	$\chi = \log b$
5086 W	103.0×10^3	14,420	9.576
5014 W	96.5×10^3	13,510	9.511
5006 W	93.5×10^3	13,090	9.480
4986 W	97.5×10^3	13,650	9.521

$$\hat{\mu} = \frac{\sum_{i=1}^4 \chi_i}{4} = 9.523$$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^4 (\chi_i - \hat{\mu})^2}{3} = .0016 \quad \hat{\sigma} = .04$$

Table 2

Burst Tests

Stress	b	$\chi = \log b$
	9,550	9.16
	9,600	9.17
	9,850	9.20
	10,150	9.23
	10,150	9.23
	10,200	9.23
	10,350	9.24

$$\hat{\mu} = \frac{\sum_{i=1}^7 \chi_i}{7} = 9.21$$

$$\hat{\sigma}^2 = .0010$$

$$\hat{\sigma} = .032$$

Table 3a

$$b = st/r$$

% Degradation

<u>Ultimate Stress</u>	<u>72%</u>	<u>80%</u>	<u>90%</u>
103.0x10 ³	4,120	3,000	1,510
96.5x10 ³	3,910	2,800	1,420
93.5x10 ³	3,790	2,720	1,370
97.5x10 ³	3,950	2,840	1,430

Table 3b

Burst Test Results at 600°F

1-1/2" 360° Milled Defect

<u>Sample #</u>	<u>% Wall Degradation</u>	<u>Burst Pressure</u>
59	70	3750 psi
54	72	3550 "
57	72	3300 "
58	72	3100 "
64	80	2500 "
65	80	2250 "
66	80	2700 "
67	80	3150 "
75	90	1750 "
76	90	1750 "
77	88	1850 "

**Burst Pressure Statistics for
Non-degraded and Degraded Tubing**

Judah Frankel

**Structural Analysis Group
Brookhaven National Laboratory**

April, 1975

1. Introduction

The relationship between burst pressure and stress strain properties for cylinders of varying dimensions is investigated and a burst pressure equation is proposed for marginally thick-walled cylinders. The distribution of burst pressure statistics is discussed; results are compared with experimental data for perfect and degraded tubing.

The tubing under consideration is Inconel 600T with outer diameter 0.750" and tube-wall thickness .048". Throughout let

r = inner radius

t = wall thickness

σ_u' = nominal stress at ultimate

ϵ_u' = engineering strain at ultimate

n = $\log (1 + \epsilon_u')$

For simplicity stresses and burst pressures will be given in kpsi throughout.

2a. Non Degraded Tubing

Assuming that the tubing is not flawed and that it behaves as if it were of infinite length several equations for determining the maximum internal or burst pressure for work-hardening materials possessing a stress-strain curve of the form $\sigma = \sigma_0 \epsilon^n$ have been proposed.^(1,2) The simplest procedure is to treat the cylinder as thin-walled, then the approximate burst pressure is given by

$$P_1 = \frac{2}{(\sqrt{3})^{n+1}} \frac{t}{r} \sigma_u' \quad . \quad (1)$$

Since the tube is not really thin-walled the use of the inner radius will result in a somewhat larger burst pressure than might be expected. A more conservative estimate is obtained by using $R = r + t/2$; then

$$P_2 = \frac{2}{(\sqrt{3})^{n+1}} \frac{t}{R} \sigma_u' \quad . \quad (2)$$

Alternatively since $t/r = .15$ we may treat it as thick-walled and a relatively simple approximation to the burst pressure is given by¹

$$p_3 = \left(\frac{.25}{n+.227} \right) \left(\frac{e}{n} \right)^n \sigma'_u \log (1 + t/r) \quad (3)$$

Table 1 lists the nominal stress and strain values for Inconel 600T based on tension tests performed on plates with dimensions .047" x .866" and .042" x .5" at 600°-660°F. If we examine the sample correlation coefficient, we have

$$\frac{\sum (\sigma_i - \bar{\sigma}) (\epsilon_i - \bar{\epsilon})}{\sqrt{\sum (\sigma_i - \bar{\sigma})^2 \sum (\epsilon_i - \bar{\epsilon})^2}} = -.61$$

indicating a somewhat negative correlation. We have calculated p_1 and p_2 for each (σ', ϵ') in columns 1 and 2 of Table 1 and arranged them in increasing order in columns 3 and 4. We then have $\bar{p}_1 = 13,240$, $\bar{p}_2 = 12,330$, and $\bar{p}_3 = 11,530$ psi as the mean values where $\bar{p}_3$ was obtained by using the sample means of σ' and $n = \log (1 + \epsilon')$ in equation (3). These values, indeed the smallest values of p_1 and p_2 in Table 1 are somewhat larger than the burst test results in Table 2a. The difficulty of course is that the tube is not truly thin-walled and that the thickwall approximation (3) has not been shown to be accurate for $n \neq 0$ unless $(r+t)/r$ is substantially larger than .375/.327.

We might, however, consider the following procedure; assume that the cylinder is thick-walled initially and it deflects uniformly until it becomes plastic whereupon we can apply the thinwall equilibrium equation $p = \sigma t/r$. If we assume that the maximum deflection occurs at ultimate then the radius at ultimate yield is $r_u = (1 + \epsilon')r$ and t_u is approximately t ; so we get

$$p_4 = \frac{\sigma'_u t}{(1 + \epsilon'_u) r} \quad (4)$$

The values calculated from the data are listed in increasing order in Table 1 and comparison with the burst test results in Table 2a shows that they are in excellent agreement. We will therefore assume that the burst pressure of cylinders with $t/r \approx .15$ is approximated by equation (4).

2b. Degraded Tubing

If the thickness of the tube wall has been reduced over some area by corrosion or other means an immediate (and usually conservative) simplification is to

assume that the entire tube wall or a large section of it has been uniformly degraded to the extent of the deepest corrosion. Then if the corrosion has reduced the wall thickness a maximum of $x\%$ we simply view the tube as if it has thickness $t' = t(1 - x/100)$. Now as x increases the tube becomes thin-walled and equation (4) is no longer appropriate; the burst pressure would now be given by (1) or (2) which are approximately equal for large x . The mean burst pressures for various levels of degradation as predicted by equations (1), (2) and (4) for the data in Table 1 are listed in columns 3, 4, and 5 of Table 2b. It seems reasonable to assume that the burst pressure is given by some combination of (1) and (4); the simplest linear function being

$$p_5 = \frac{x}{100} p_1 + (1 - \frac{x}{100}) p_4 \approx \frac{\sigma_u' t}{r} \left(\frac{100 + x \epsilon_n'}{100 (1 + \epsilon_n')} \right) \quad (5)$$

In column 2 of Table 2b the burst test results for $1\frac{1}{2}$ " 360° milled defects of varying size are listed. Although the precise nature of the end effects are not yet known the burst pressure of a tube degraded over a moderate area should not differ significantly from that for a tube which is degraded to the same depth in its entirety. Indeed the values in column 2 compare favorably with those in columns 3, 4, and 5 of Table 2b. The precise effect that various shape and size defects have on the burst pressure is being investigated by the stress analysis group using finite element methods; some of these results will be available shortly.

3. Distribution of Burst Pressure

In order to estimate the probability that a randomly selected tube will burst at a specific pressure we will test several proposed distribution functions for adequacy of fit to the data. To test the hypotheses that the log-normal distribution is an adequate fit³ to the burst pressures calculated by equation (4) we compute the W statistic⁴. We have,

$$s^2 = \sum_1^n (\log x_i)^2 - \left(\sum_1^n \log x_i \right)^2 / 17 = .057 \quad (6)$$

$$b^2 = \sum_1^k A_{k-i+1} (\log x_{n-i+1} - \log x_i) = .048 \quad (7)$$

where x_i is the i -th smallest burst pressure in column 5 of Table 1, $n = 17$, $k = 8$, and the coefficients A_j are given in Tables for the W-test. Then

$$W = b^2 / s^2 \approx .84 \quad (8)$$

which is below even the one percentage point for the W-test indicating that there is a less than 1% probability that the data is from a log-normal distribution. So we reject the log-normal hypothesis.³

3b. The type I extreme value distribution is frequently utilized to describe component failure and it has the form,

$$P(P_4 \leq t) = F_4(t) = \exp(-e^{-y}) \quad , \quad (9)$$

where p_4 is the burst pressure calculated by (4) and $y = \frac{t - \mu_4}{\alpha_4}$ and μ_4 and α_4 are location and scale parameters; their usual estimates are:

$$\hat{\alpha}_4 = .779s_4 = .51 \quad \hat{\mu}_4 = \bar{p}_4 - .577 \hat{\alpha}_4 = 9.62 \quad (10)$$

using the values for p_4 in Table 1. To apply the chi-square goodness of fit test we divide the data into four cells and compute the test statistic X (the calculations appear in Table 3a). Since $X^2 = 1.18$ is well below $\chi^2_{.05,1} = 3.85$ we have no reason to reject the hypothesis. So we will assume that P_4 has the distribution given by (9).

The data in Column 3 of Table 1 for p_1 is similar to that for p_4 and the goodness of fit statistic $X^2 = .97$ is even smaller than that for p_4 so we will assume that the distribution of p_1 has the form,

$$P(P_1 \leq t) = F_1(t) = \exp(-e^{-y}) \quad (11)$$

where $y = \frac{t - \mu_1}{\alpha_1}$ and μ_1 and α_1 are estimated by $\hat{\alpha}_1 = .56$ $\hat{\mu}_1 = 12.92$.

Although the number of observations is small the test statistic is sufficiently small so that we would not expect it to increase to a value near 3.85 if more observations were available or if the cells were chosen differently.

If we use the Kolmogorov-Smirnov test to test the hypothesis that P_4 has the distribution in (9) we get

$$\sqrt{17} \sup | \hat{F}(x) - F_4(x) | = .387 \quad (12)$$

which is less than the upper 5% percentage point, 1.36; so we do not reject the hypothesis; the same would hold for P_1 .

For degraded tubing the situation is essentially the same. If the tubing is degraded by $100(1-c)\%$ the burst pressure calculated by (1) or (4) multiplied by c will be the burst pressure for degraded tubing, and it is easy to see that the test statistics are unchanged and we do not reject the Extreme value distribution. So for $100(1-c)\%$ degraded tubing we assume the burst pressures by method (1) or (4) are distributed according to

$$P(P_1^c \leq t) = F_1^c(t) = \exp(-e^{-y}) \quad (13)$$

where $y = \frac{t - \hat{\mu}_1}{c \hat{\alpha}_1}$, $\hat{\alpha}_1 = .56$, $\hat{\mu}_1 = 12.92$ and

$$P(P_4^c \leq t) = F_4^c(t) = \exp(-e^{-y}) \quad (14)$$

where $y = \frac{t - \hat{\mu}_4}{c \hat{\alpha}_4}$, $\hat{\alpha}_4 = .51$ and $\hat{\mu}_4 = 9.62$ respectively.

4. Estimation of Probabilities

Assuming that the distributions (9), (11), (13), and (14) are correct we could calculate the probability of a tube bursting under an internal pressure p for models (1) and (4). For a $100(1-c)\%$ degraded tube with $c=.1, .2, .3, .4, .5, 1$ several percentiles of interest are tabulated in Tables 4a and 5a. In general the k -th percentile, K_k^i , of the burst pressure distribution for $100(1-c)\%$ degraded tubing is given (for model i , $i = 1, 4$) by

$$K_k^i = c \hat{\mu}_i - \hat{\alpha}_i c \log \log \left(\frac{100}{k} \right) \quad i = 1, 4. \quad (15)$$

The difficulty with this is that these percentiles are based on the assumed distributions and on the estimates of μ and α so that we have very little information about the true form of the distribution below the smallest observed value.

The assumed extreme value distributions (like the data) have very short lower end tails which result in negligible probabilities for burst pressures even 10% below the smallest observation, for example $F_4(8.50) = 5 \times 10^{-6}$ and the values are not very meaningful in view of the assumptions involved. To obtain a more reliable and conservative estimate we may account for the variation in the mean by using the t-test⁵.

$$P(E(P_i) > \bar{p}_i - t_{n-1, \beta} \frac{s_i}{\sqrt{n}}) \sim \beta \quad (i = 1, 4) \quad (16)$$

From the values of $\bar{p}_i, s_i$ from Table 1 and $\beta = .95$ we get,

$$P(E(P_1) > 12.98) = P(E(P_4) > 9.67) = .95 \quad (17)$$

Now we have $\alpha'_1 = .52$ $\mu'_1 = 12.98 - .577 \alpha'_1 = 12.68$ and $\alpha'_4 = .46$ $\mu'_4 = 9.67 - .577 \alpha'_4 = 9.40$. Some percentiles based on these estimates are tabulated in Tables 4b and 5b and assuming the distributions and the estimate of α are correct then we could say that the true percentiles are at least as large as the tabulated ones with probability .95. Then

$$P(P_i \leq K_k^i) \leq \frac{k}{100} \quad i = 1, 4 \quad (18)$$

where P_i is the burst pressure for model i and K_k^i is taken from Table 4b or 5b.

To avoid choosing a distribution to fit the data and then estimating probabilities on that basis it might be preferable to apply nonparametric methods.⁶ The paucity of data, however, prohibits any meaningful use of distribution free methods. The probability that the smallest observation falls below the k -th percentile of the underlying distribution is $1 - (1 - k/100)^n$; for $n = 17$ and $k = 10$ we get $1 - (.9)^n = .833$. To obtain a probability of .95 that the smallest observation will fall below the 5-th percentile we would need at least 58 observations. Heuristically speaking, the probability that less than five percent of the bursts pressures lie below the smallest observed pressure as calculated by equations (1), (2), or (4) is $1 - (.95)^{17} = .52$ which indicates the unreliability of any estimates about the tail of the distribution. If the burst pressures do follow a Type I extreme value distribution then the estimates in Section 3 will be far more reliable.

Conclusions

Several methods of predicting burst pressures in both non-degraded and degraded tubing are discussed and general agreement is found with test results. A Type I extreme value distribution for burst pressure is proposed on the basis of the data available and probability estimates for various pressures are given.

The reliability of the assumptions and estimates are limited by the current scarcity of data; requests for more extensive data have produced no response as yet. A more comprehensive set of data should provide more definitive information as to the basic distribution of the burst pressure statistics and would lend a much higher degree of accuracy to the probability estimates obtained.

References

1. N. L. Svensson, "The bursting pressure of Cylindrical and Spherical Vessels," *Journal of Applied Mechanics*, 1958.
2. F. B. Seely and J. O. Smith, Advanced Mechanics of Materials, J. Wiley and Sons, 1957.
3. J. M. Frankel, "Statistical Study of Piping Failures."
4. G. J. Hahn and S. S. Shapiro, Statistical Methods in Engineering, J. Wiley and Sons, 1967.
5. B. W. Lindgren, Statistical Theory, Macmillan Co., 1968.
6. J. D. Gibbons, Nonparametric Statistical Inference, McGraw-Hill, 1971.

<u>Sample</u>	<u>σ_u - kpsi</u>	<u>ϵ_u</u>	<u>p_1 - kpsi</u>	<u>p_2 - kpsi</u>	<u>p_4 - kpsi</u>
6578W	88.0	.37	12.55	11.69	9.35
6581W	89.0	.36	12.57	11.71	9.36
6590W	90.0	.42	12.65	11.79	9.37
NX	90.2	.40	12.68	11.81	9.43
6586W	90.5	.42	12.71	11.84	9.30
6579W	90.5	.37	12.71	11.84	9.46
6562W	90.6	.42	12.76	11.89	9.58
6585W	91.7	.44	12.92	12.04	9.60
6593W	92.7	.42	12.96	12.08	9.70
6578W	93.0	.40	13.11	12.21	9.75
5086W	93.4	.39	13.21	12.31	9.86
5046W	94.3	.37	13.46	12.54	10.10
6376W	96.2	.36	13.67	12.76	10.20
5045W	96.6	.39	13.78	12.81	10.37
4986W	97.5	.38	13.86	12.91	10.38
4973W	97.5	.30	14.31	13.33	11.00
5026W	103.0	.30	15.12	14.09	11.63
<hr/>					
$\bar{x}$	93.2	.383	13.24	12.33	9.91
$\Sigma(x-\bar{x})^2$	242.6	.02	8.17	7.08	6.69
$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$	3.89	.034	.71	.67	.65

TABLE 1

Burst Test Results, Non-Degraded Tube

pressure - kpsi

9.55
9.60
9.85
10.15
10.15
10.20
10.35

TABLE 2a

Burst Tests - Degraded Tube - kpsi				
<u>% Degradation</u>	<u>Burst Pressure</u>	<u>P₁</u>	<u>P₂</u>	<u>P₄</u>
70	3.75	3.97	3.89	2.98
72	3.55	3.71	3.63	2.78
72	3.30	3.71	3.63	2.78
77	3.10	3.71	3.63	2.78
80	2.50	2.65	2.61	2.0
80	2.25	2.65	2.61	2.0
80	2.70	2.65	2.61	2.0
80	3.15	2.65	2.61	2.0
88	1.85	1.59	1.57	1.20
90	1.75	1.32	1.31	.99
90	1.75	1.32	1.31	.99

TABLE 2b

Goodness of Fit Test For P_4

<u>Cell</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
C_i	(0, 9.40]	(9.40, 9.65]	(9.65, 10.30]	(10.30, ∞)
M_i	4	4	5	4
f_i	.20	.17	.41	.22
E_i	3.39	2.87	7.04	3.67

$$\chi^2 = \sum_{i=1}^4 \frac{(M_i - E_i)^2}{E_i} = 1.18$$

TABLE 3a

Goodness of Fit For P_1

<u>Cell</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
C_i	(0, 12.70]	(12.70, 13.05]	(13.05, 13.70]	(13.70, ∞)
M_i	4	5	4	4
f_i	.20	.24	.35	.21
E_i	3.48	4.08	5.91	3.52

$$\chi^2 = \sum_{i=1}^4 \frac{(M_i - E_i)^2}{E_i} = .97$$

M_i is the number of observations in cell C_i and f_i and E_i are the expected frequency and expected number of observations in cell C_i for the assumed distribution.

TABLE 3b

Percentiles of F_1^c , $\hat{\mu}_1 = 12.92$, $\hat{\alpha}_1 = .56$

<u>Percentile</u>	<u>c=1</u>	<u>c=.5</u>	<u>c=.4</u>	<u>c=.3</u>	<u>c=.2</u>	<u>c=.1</u>
50	13.12	6.56	5.25	3.94	2.62	1.31
25	12.74	6.37	5.09	3.82	2.55	1.27
10	12.45	6.23	4.98	3.73	2.49	1.24
5	12.30	6.15	4.92	3.69	2.46	1.23
2.5	12.19	6.09	4.87	3.66	2.44	1.22
1	12.06	6.03	4.82	3.62	2.41	1.21
0.5	11.99	5.99	4.79	3.59	2.40	1.20
0.1	11.84	5.92	4.73	3.55	2.37	1.18

TABLE 4a

Percentiles of F_1^c , $\mu_1' = 12.66$, $\alpha_1' = .56$

<u>Percentile</u>	<u>c=1</u>	<u>c=.5</u>	<u>c=.4</u>	<u>c=.3</u>	<u>c=.2</u>	<u>c=.1</u>
50	12.86	6.43	5.15	3.86	2.57	1.29
25	12.48	6.24	4.99	3.74	2.49	1.25
10	12.19	6.10	4.88	3.66	2.44	1.22
5	12.04	6.02	4.82	3.61	2.41	1.20
2.5	11.93	5.96	4.77	3.58	2.38	1.19
1	11.80	5.90	4.72	3.54	2.36	1.18
0.5	11.73	5.86	4.69	3.52	2.34	1.17
0.1	11.58	5.79	4.63	3.47	2.31	1.16

TABLE 4b

Percentiles of F_4^c , $\hat{\mu}_4=9.62$, $\hat{\alpha}_4=.51$

<u>Percentile</u>	<u>c=1</u>	<u>c=.5</u>	<u>c=.4</u>	<u>c=.3</u>	<u>c=.2</u>	<u>c=.1</u>
50	9.81	4.90	3.92	2.94	1.96	0.98
25	9.45	4.73	3.78	2.84	1.89	0.94
10	9.19	4.60	3.68	2.76	1.84	0.92
5	9.06	4.53	3.62	2.72	1.81	0.91
2.5	8.95	4.48	3.58	2.69	1.79	0.89
1	8.84	4.42	3.54	2.65	1.77	0.88
0.5	8.77	4.38	3.51	2.63	1.75	0.88
0.1	8.63	4.32	3.45	2.59	1.73	0.86

TABLE 5a

Percentiles of F_4^c , $\hat{\mu}_4=9.38$, $\hat{\alpha}_4=.51$

<u>Percentile</u>	<u>c=1</u>	<u>c=.5</u>	<u>c=.4</u>	<u>c=.3</u>	<u>c=.2</u>	<u>c=.1</u>
50	9.57	4.78	3.83	2.87	1.91	0.96
25	9.21	4.61	3.68	2.76	1.84	0.92
10	8.95	4.48	3.58	2.69	1.79	0.89
5	8.82	4.41	3.53	2.65	1.76	0.88
2.5	8.71	4.36	3.48	2.61	1.74	0.87
1	8.60	4.30	3.44	2.58	1.72	0.86
0.5	8.53	4.26	3.41	2.56	1.70	0.85
0.1	8.39	4.20	3.36	2.52	1.68	0.84

TABLE 5b

Appendix III

**TESTING ON THE DISTRIBUTION OF BURST PRESSURE
FOR TUBES, WITH RELATED DISCUSSIONS**

**Samuel Kao
Applied Mathematics Department
Brookhaven National Laboratory
Upton, N. Y. 11973**

DRAFT

**TESTING ON THE DISTRIBUTION OF BURST PRESSURE
FOR TUBES, WITH RELATED DISCUSSIONS**

**Samuel Kao
Applied Mathematics Department
Brookhaven National Laboratory
Upton, N. Y. 11973**

1. Introduction and Summary

A statistical investigation on the distribution of the burst pressure of tubes has been made by Frankel (1) under assumptions for the simplest cases. The idea of assuming log-normality for the burst pressure in the earliest results on this subject was rejected after a hypothesis testing procedure and the Type I Extreme Value Distribution was proved as an adequate assumed underlying distribution for the burst pressure which is calculated by equations relating it to the stress and strain of the tubes.

In the present extensive study on the problem of burst pressure for tubing, it is found that, based on the limited data available to us there are several different distributions acceptable as the assumed underlying distribution for burst pressure when the standard hypothesis testing procedure is undertaken. Nevertheless, the Type I Extreme Value Distribution or its kind appears to be about the most reasonable and promising distribution. In addition, a modified form similar to the Type I Extreme Value Distribution is believed a technically better distribution to be assumed, in order to overcome certain significant defects.

The different sample sequences of calculated burst pressure obtained in (1) by the different formulations that relate it to the stress and strain based on the findings of (4) can be proved to be significantly different when their corresponding empirical distributions are concerned. Therefore it is believed that a

unified and exact calculation for the burst pressure is necessary in obtaining the best fitting distribution. For this aim a discussion on the general basis to apply the Type I Extreme Value Distribution is also made.

2. Results of Some Associated Statistical Tests

In this report several statistical tests are carried out in order to gain a better understanding about subjects related to the burst pressure. The results of these tests are the basis for the conclusions given in Section 4.

(1) The sample values of Table 2 (or see Table 2a in (1) are directly utilized for testing on the fit of the Type I Extreme Value Distribution. The Kolmogorov-Smirnov test and the Pearson's chi-square test are applied, with results as follows.

(i) Kolmogorov-Smirnov Test

Burst pressure	9.55	9.60	9.85	10.15	10.20	10.35
Empirical distribution	.1425	.2857	.4286	.5714	.7143	.8571
Expected distribution	.0441	.0776	.3908	.7538	.7934	.8808

So, $\sqrt{n} \sup |F_n(x) - F(x)| \leq \sqrt{n} \times .3510 = .9287$. Since .9287 is less than the critical value of 1.36 at 5% level, the fitting is accepted.

(ii) Chi-Square Test

Burst pressure	(0, 10.10]	(10.10, 1]
Empirical frequency	3	4
Expected frequency	4	3

$$\chi^2 = \frac{(3-4)^2}{4} + \frac{(4-3)^2}{3} = .58$$

Since .58 is less than the critical value of 3.84 at 5% level for chi-square distribution with one degree of freedom, the fitting is accepted.

(2) In contrast to the test on fitting with the Type I Extreme Value Distribution in [1], a test on fitting with a normal distribution is done by using the calculated p_4 values. The results are as follows: mean = 9.92, standard deviation = .58.

Burst pressure	(0, 9.43]	(9.43, 9.65]	(9.65, 10.30)	(10.30, ∞)
Observed frequency	4	4	5	4
Expected frequency	3.41	2.02	7.28	4.33

$$\chi^2 = \sum \frac{(\text{obs.} - \text{Expt.})^2}{\text{Expt.}} = 2.78$$

Since 2.78 is far less than the critical value of 7.82 at 5% level for chi-square with 3 degrees of freedom, the fitting with normal distribution $N(\mu=9.92, \sigma=.58)$ is accepted.

(3) To see if the calculated sequences of values of p_1 , p_2 and p_4 are statistically significantly different, a test is made for p_1 vs p_2 . The results are as follows:

	(0,12.2]	(12.2,12.7]	(12.7,13.1]	(13.1,13.6]	(13.6,14.1]	(14.1,∞)
P_1	0/17	3/17	10/17	12/17	15/17	16/17
P_2	9/17	12/17	15/17	16/17	17/17	17/17

$$D = \sup |F_1(x) - F_2(x)| = .5294,$$

$$\text{so } \chi^2 = 4D^2 \frac{n_1 n_2}{n_1 + n_2} = 9.53.$$

9.53 is greater than the critical value of 5.99 at level 5% for chi square with 2 degrees of freedom. Therefore the two distributions for p_1 and p_2 are significantly different.

It is also true that distributions of any two of P_1, P_2 and P_4 are significantly different at 5% level of significance.

(4) Some study is directed to the properties of the ultimate stress σ_u and the ultimate strain ϵ_u and the relation between them. It was indicated in [1] that these two variables have between them "a somewhat negative correlation". As it is found that the skewness of ϵ_u is estimated around -2.703 which is significantly departed from 0 of normally distributed random variable, non-parametric method of Kendall's Rank Correlation is employed. The results are as follows:

Rank Comparison

$\sigma_u = 1$	2.5	4	5	6	7	8	9	10	11	12.5	12.5	14
$\epsilon_u = 16.5$	11	9	15	13	9	3	1	3	3	13	3	7

S = relative number of pairs of ranks in right order

$$= -54$$

$T_\sigma = 3$, $T_\epsilon = 13$ (T corresponds to ties and equals

to $\frac{1}{2} \sum t(t-1)$) (cf. [3]).

$$\text{Then Rank correlation } \gamma = \frac{S}{\sqrt{\frac{1}{2}N(N-1)-T_\sigma} \sqrt{\frac{1}{2}N(N-1)-T_\epsilon}} = -0.42.$$

When transformed to corresponding standard normal statistic

$$r \rightarrow z = \frac{r}{\sqrt{\frac{2(2N+5)}{9N(N-1)}}} = -2.353.$$

Since $|-2.353| > 1.96$ where 1.96 is the 95% value for the two-sided tail of the standard normal distribution, it is thus clear that the existence of association between σ_u and ϵ_u is significant.

Note: σ'_u has a distribution close to the normal, with skewness about .76.

3. Rationale with the Type I Extreme Value Distribution

It has often been assumed that the probability of survival may be analyzed by the logarithmic normal probability function. In this case the probability of survival approaches unity in the same way as the logarithm of the stress approaches zero. This prediction is contradicted by the experimental fact that for large stress a small increase results in a large decrease of the probability of survival (the straw that broke the camel's back), while for small stress a considerable decrease is necessary in order to increase the probability. It is not surprising that with the present data of P_1 , P_2 , and P_4 the lognormal probability function is again rejected in the statistical test. Instead of the logarithmic normal theory, we use the theory of extreme values, which may be justified as follows: The difference between the calculated and the observed strength resides in the existence of weakening flaws. Therefore, a different amount of force will be needed to fracture the body at one or another point, and the strength of a given specimen is determined by its weakest point.

In the application of the extreme value theory a number of assumptions are made:

- (i) The flaws of different sizes are distributed independently at random in the body.
- (ii) The probability density of the sizes of flaws is of the exponential type.
- (iii) The number of flaws is very large (although by actual size they are very minute and undetectable by present means).
- (iv) The breaking strength (or burst pressure) is a linear function of the size of the largest flaw alone, whatever its size may be, and diminishes with increasing size.

Under these conditions, the distribution of the breaking strength (or burst pressure) is the asymptotic distribution of the smallest values. Such distribution is the Type I Extreme Value Distribution.

Type I Extreme Value Distribution: Possible Modifications

Because of the possible difference between the real conditions for the bursting of tubes and that of the previously mentioned four hypothetical conditions, it may be necessary to modify the Type I Extreme Value Distribution. There is an obvious defect that one should be aware of when applying Type I Extreme Value Method. If p indicates the burst pressure, the formula

$$\text{Prob } (p \geq t) = \exp (-e^{-\mu}) \text{ where } y = \frac{t - \mu}{\alpha}$$

is not true for $t=0$, since $\exp(-e^{-\mu})$ at $t=0 = \exp(-e^{\mu/\alpha}) > 0$ is contrary to the fact that $\text{Prob } (p \leq 0) = 0$. Therefore, the statistical test as done in (1) upon the fitting of the Type I Extreme Value Distribution will be invalid if $\exp(-e^{\mu/\alpha})$ is non-negligible. For the case $\hat{\mu}_1 = 12.94$, $\hat{\alpha}_1 = .52$, $\hat{\mu}_4 = 9.65$ and $\hat{\alpha}_4 = .46$ that gave rise to a value almost equal to zero for $\exp(-e^{\mu/\alpha})$, the conclusion in (1) is valid.

4. Some Conclusive Remarks

From the result of (1) in section 2 it seems reasonable that the burst pressure does have the Type I Extreme Value Distribution. Yet, in view of the result of (2) the justification for assuming Type I Extreme Value Distribution is not absolute. Nevertheless, the discussion in section 3 clearly provides us with more than sufficient basis to look into the Type I Extreme Value Distribution as the best candidate for the underlying distribution of the burst pressure of tubes. In light of the test results in (3) it seems that it would be better to use the thick vessel equations for cases where $R/t \geq 20$ and the thin vessel equations

for cases where $R/t \leq 20$. Moreover, it should be realized that the assumptions regarding linearity made in reference (1) probably will require some refinement and thus should be further investigated.

References

- [1] Frankel, J. (1975). Burst pressure statistics for non-degraded and degraded tubing. Technical report, Brookhaven National Laboratory.
- [2] Gumbel, E. J. (1958). Statistics of extremes. Columbia University Press, New York.
- [3] Siegel, S. (1956). Nonparametric statistics. McGraw-Hill.
- [4] Svensson, N. L. (1958). The bursting pressure of cylindrical and spherical vessels. Journals of Applied Mechanics.

<u>Sample</u>	<u>σ'_u - kpsi</u>	<u>c_u</u>	<u>p_1 - kpsi</u>	<u>p_2 - kpsi</u>	<u>p_4 - kpsi</u>
6578W	88.0	.37	12.55	11.69	9.35
6581W	89.0	.36	12.57	11.71	9.36
6590W	90.0	.42	12.65	11.79	9.37
MX	90.2	.40	12.68	11.81	9.43
6586W	90.5	.42	12.71	11.84	9.30
6579W	90.5	.37	12.71	11.84	9.46
6562W	90.6	.42	12.76	11.89	9.58
6585W	91.7	.44	12.92	12.04	9.60
6593W	92.7	.42	12.96	12.08	9.70
6578W	93.0	.40	13.11	12.21	9.75
5086W	93.4	.39	13.21	12.31	9.86
5046W	94.3	.37	13.46	12.54	10.10
6376W	96.2	.36	13.67	12.76	10.20
5045W	96.6	.39	13.78	12.81	10.37
4986W	97.5	.38	13.86	12.91	10.38
4973W	97.5	.30	14.31	13.33	11.00
5026W	103.0	.30	15.12	14.09	11.63
<hr/>					
$\bar{x}$	93.2	.383	13.24	12.33	9.91
$\Sigma(x-\bar{x})^2$	242.6	.02	8.17	7.08	6.69
$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$	3.89	.034	.71	.67	.65

TABLE 1

Burst Test Results, Non-Degraded Tube

Pressure - kpsi

9.55
9.60
9.85
10.15
10.15
10.20
10.35

TABLE 2.

PROGRESS REPORT

August 8, 1974

ELASTIC ANALYSIS OF DEFECTIVE
STEAM GENERATOR TUBING

Contract # AT(30-1)-16

FAILURE PROBABILITIES OF STEAM
GENERATOR TUBING

D. W. Gardner

Stress Analysis Group
Department of Applied Science
Brookhaven National Laboratory
Upton, New York 11973

This report details work accomplished to date on project titled, "Failure Probabilities of Steam Generator Tubes," contract number AT(30-1)-16.

Work has progressed in two directions. The initial thrust was a literature survey to determine input parameters. Unfortunately, a great deal of this data comes from propriety publications and private communications and cannot be referenced here. It was determined, however, that many steam generators were constructed with tubing that was .750 inches (1.90 MM) outside diameter with 0.48 inches (.12 MM) wall thickness.

Some of the older generators were built with 300 series stainless steel tubes, but many of these and most of the new units are now using Inconel 600 alloy tubes. This material is a nickel-chrome-iron alloy designated SB163 (Inconel), chosen because of its superior corrosion resistance, particularly to chlorides, at elevated temperatures. (3) (4) This material has the following properties at 600°F (3):

σ_{uts}	75000
σ_Y	35000 Psi
σ_m	26000 Psi (2/3 σ_Y)
E	29.2×10^6 Psi
μ	0.30
σ_s	70000 Psi (2 σ_Y)

where σ_{mp} is the allowable primary membrane stress, and σ_{ms} is the allowable primary plus secondary bending stress as defined by the ASME Nuclear Pressure Vessel Code.

The loading on the tubes is created by either of two conditions. An internal pressure, which may cause bursting failure in a tube, is created by a main steam line break (MSLB). A standard

test for this is outlined in reference (5). This results in an internal pressure of 2300 PSIG ($15.9 \times 10^6 \text{ N/M}^2$). This also results in an axial, tensile load of 774# (3440 N).

An external, collapsing pressure is caused by a loss of coolant accident (LOCA). This is also outlined in reference (5). This results in an external pressure of 1000 PSIG ($6.9 \times 10^6 \text{ N/M}^2$) and an axial compressive load of 442 pounds (1960 N).

The analysis was accomplished using two, two-dimensional, finite element computer programs: AXISYM and MORIS. These programs will handle isotropic, elastic, statically loaded problems. AXISYM is used for structures with an axis of symmetry, and MORIS is used for structures under plane stress, with the third dimension considered to be of infinite length.

Reports of corrosion damage to generator tubes indicated that defects consisted of either cracks or general wastage. Computer runs were made combining the types of load detailed above with circumferential and axial cracks and wastage. Figure 1 & 2 show these cases. Each figure will be explained in the paragraphs that follow.

Figure 3 depicts results of a case with a single axial crack, loaded with internal pressure by a MSLB test, analyzed with the MORIS code. As shown in the figure, this analysis indicates that the material yield stress is exceeded at about 23% penetration. This stress is a combination of a basic hoop stress plus a bending stress due to thickness plus a concentration factor.

Figure 3A shows the local nature of the stress in the extreme fibers across the crack area under a typical load. At locations greater than 10° from the crack, stresses are "normal," showing an average stress of about 14,000 PSI plus a differential effect due to thickness (poisson ratio effects). Because of the shift

in the neutral axis through the crack, a bending stress is created, causing compression in the inner fiber. The maximum stress location moves from the inner fiber to the crack tip, and its theoretical value approaches infinity. The computer code used will pick up this concentration in inverse proportion to the grid size used. In this case, with a .005" x .007" grid, the maximum value observed was 93200 PSI for this particular example. These values are plotted in Figure 3 and in subsequent plots. Note that using standard pressure vessel calculations, where $\sigma = P \cdot R / T$, the stress in a 53% cracked .048 wall vessel would be 38000 PSI. This would indicate that a stress concentration factor should be about 2.5 to produce the 93,200 PSI stress the computer predicted. Standard design practices would call for a factor of 3 minimum, which would yield as expected, a conservative design.

The question to be answered when considering steam generator tube failure (under non-corrosive environment) would entail knowing when the entire cross-section in the region of the crack tip becomes plastic. (μ - a code which takes into account the plastic behavior is currently being prepared for these cases and should show instructive results when applied to this problem.) The probability of a failure due to a crack (under static conditions) will of course depend on the above mentioned analysis results.

Figure 4 shows an axial wastage under a MSLB condition, which indicates a yield at about 22% penetration. Figure 4A shows, as did figure 3A, the maximum fiber stress under the preceding conditions. Because of limitations in the computer code, this wastage was modeled as a succession of steps as shown. Although a comparison with Figure 3A shows that at locations of about 10^0

or more from the defect centerline, the average of the two extreme fiber stresses is almost the same. It is to be noted, however, that bending forces cause greater differences between the extreme fiber stresses in the wastage example. For this case, in the wasted area, the average stress is about 29000 PSI, a bending stress of ± 22000 PSI is superimposed, and a concentration factor of 1.3 is added to produce a maximum stress of 67000 PSI.

The above stresses, while smaller in magnitude than those for the maximum stress of the tip of an equivalent depth crack, is potentially more likely to cause tube failure. The reason for this is the larger area over which stresses above the material yield act. In essence the stress at a crack tip is relieved by an extremely local plastic yield zone, while for wastage conditions the stress relief is substantially larger yield zones. In view of the above conditions tube failure for axial wastage under static loads occur at lower burst pressures than those with equivalent depth cracks.

Figures 5 and 6 represent the stress resulting from a MSLB test on a tube with a circumferential crack. This has been analyzed with the computer code AXISYM, which allows the axial forces to be added. Both hoop and principal maximum stresses have been plotted. These curves indicate yield stresses occurring at about 50% penetration. Figure 6A shows a typical longitudinal stress plot across the crack. The maximum stress is a principal stress, arising from a combination of a bending stress caused by the jog in the neutral axis of the cross sections at the crack, and a stress concentration factor. At distances remote from the crack, the inner and outer fiber principal stresses are almost identical, the remaining difference being caused by poisson

ratio loads from the hoop stresses. Using the usual formula for axial stresses in a cylinder, axial stress is:

$$\sigma = P \cdot R / 2 \cdot T$$

for a .048 wall, 50% cracked

$$\sigma = 2300 \cdot .375 / 2 \cdot .0269$$

$$\sigma = 16000 \text{ PSI.}$$

A commonly used stress concentration factor of 3 yields 48000 PSI which is conservative.

Figures 7 & 8 show the analysis of a circumferential wastage .080 inches in length under a MSLB loading condition. As shown in the figure, yield stresses would occur at penetrations of 65%. Comparing these results with those shown in Figure 4, it is seen that circumferential defects allow greater penetration before yielding (and eventually failure) occurs. The reason for this is that stress concentration factors are applied to the longitudinal stress, which is usually half the hoop stresses. Additionally wastage defects allow greater penetration than cracks because the change in the neutral axis across the defect is gradual. Figure 8A shows a longitudinal plot across a typical defect. This is quite similar to Figure 4A. Note that over most of the length of a tube, the inner fiber hoop stress is critical, but that near the defect, the outer fiber principal stress, due to the stress concentration becomes critical.

The defect lengths were chosen in an arbitrary manner, since at the time that this analysis was carried out no specific test data was available for comparative purposes. Essentially, the case shown in Figure 8A represents a relatively narrow circumferential groove where the hoop stress at the center of the defect is not fully developed because of the constraints present at the

thicker sections at the ends of the groove. If the length of the wastage were larger, (i.e., say one inch or more) than the stress situation will be somewhat different. These longer defects are currently being analyzed for stresses and strains. These results will then be compared to some of the burst test data made available to us recently.

The remaining figures show the responses of similar configurations to LOCA simulated loads. From the results it is rather obvious that for these cases the mode of failure would involve stability, or buckling, so no conclusions can be drawn from these figures.

It must be re-emphasized that these results are for static, completely elastic conditions. In addition, it should be noted that all wastages modeled in these studies assumed smooth bottoms and as thus only represent machine simulated wastages. Actual defects would most probably be "pitted" throughout and possibly also contain cracks. Thus, in reality, the corroded section will most probably be completely plastic at a lower pressure than the smooth bottomed model. Studies with pitted wastage are planned for the near future. Other future analyses will include the results of plasticity, strain hardening, fracture mechanics, instability, dynamic loading, and three-dimensional effects. Computer codes are being perfected to accomplish these analyses.

REFERENCES

1. ASME Boiler & Pressure Vessel Code Section III, Nuclear Power Plant Components, 1971 edition.
2. ASME Code Case 1484.
3. Westinghouse Report #WCAP 7832 "Evaluation of Steam Generator Tube, Tube Sheet and Divider Plate Under Combined LOCA plus SSE Conditions."
4. "Westinghouse Steam Generator Symposium" April 1973.
5. Private Communication "The Effect of Wall Degradation on Burst and Collapse Pressure of Inconel 600T Steam Generator Tubing."

CASE I, ANALYSED BY MORIS CODE

CASE II, ANALYSED BY AXISYM CODE

NOT TO SCALE

FIG 3

BROOKHAVEN NATIONAL LABORATORY

BY *Gardner* DATE *5/15/74*

SUBJECT

SHEET NO. OF

CHKD. BY

DEPT. OR PROJECT

JOB NO.

MSLB TEST
AXIAL CRACK
(NO AXIAL FORCE)

BROOKHAVEN NATIONAL LABORATORY
BY Jardine DATE 4/25/74 SUBJECT
CHKD. BY DATE
DEPT. OF PROJECT

FIG 3H
SHEET NO. . OF
JOB NO.
DEPT. OF PROJECT

MSLB TEST
AXIAL CRACK
EXTREME FIBER STRESS
56% PENETRATION
(TUBE 21-REF)

BY *Gardner* DATE *5/1/74* SUBJECT
CHKD. BY DATE
DEPT. OR PROJECT
SHEET NO. OF
JOB NO.

MSLB TEST
AXIAL WASTAGE
(NO AXIAL FORCE)

MSLB TEST
AXIAL WASTAGE
EXTREME FIBER STRESS
42% PENETRATION
(TUBE 45-REF)

MSLB TEST
CIRCUMFERENTIAL CRACK
(NO AXIAL FORCE)

FIG 10

MSLB TEST
CIRCUMFERENTIAL CRACK
AXIAL FORCES

MSLB TEST
CIRCUMFERENTIAL CRACK
WITH AXIAL FORCE
EXTREME FIBER STRESS
56% PENETRATION
(TUBE 74-REF)

——— PRINCIPAL STRESS
 - - - - - HOOP STRESS

MSLB TEST
CIRCUMFERENTIAL WASTAGE
(NO AXIAL FORCE)

MSLB TEST
CIRCUMFERENTIAL WASTAGE
AXIAL FORCE

MSLB TEST
CIRCUMFERENTIAL WASTAGE
WITH AXIAL FORCE
EXTREME FIBER STRESS
56% PENETRATION
(TUBE 94-PLF)

LOCA TEST
AXIAL CRACK
(NO AXIAL FORCE)

BROOKHAVEN NATIONAL LABORATORY

BY *J. J. J.* DATE *5/15/74* SUBJECT

CHKD. BY D. TE.

DEPT. OR PROJECT

FIG 10

SHEET NO. ... OF ...

JOB NO.

LOCA TEST

AXIAL WASTAGE

(NO AXIAL FORCE)

FIG 11

LOCA TEST
CIRCUMFERENTIAL CRACK
(NO AXIAL FORCE)

LOCA TEST

CIRCUMFERENTIAL CRACK

AXIAL FORCES

LOCA TEST
CIRCUMFERENTIAL WASTAGE
(NO AXIAL FORCE)

BROOKHAVEN NATIONAL LABORATORY

DATE 6/11/74 SUBJECT
CHKD. BY DATE
DEPT. OR PROJECT

FIG 14
SHEET NO. 01
JOB NO

LOCA TEST
CIRCUMFERENTIAL WASTAGE
(AXIAL FORCE)

PROGRESS REPORT

July 1, 1975

**ELASTIC ANALYSIS OF DEFECTIVE
STEAM GENERATOR TUBING**

Contract # AT(30-1)-16

**FAILURE PROBABILITIES OF STEAM
GENERATOR TUBING**

D. W. Gardner

**Stress Analysis Group
Department of Applied Science
Brookhaven National Laboratory
Upton, New York 11973**

Introduction

This report details work done on project titled "Failure Probabilities of Steam Generator Tube", contract number AT(30-1)-16, during the fiscal year from July 1974 to July 1975.

In a previous report (DAS Report 8.6.4) work done on the tube analysis using elastic computer codes was described. Work during this segment was to utilize existing codes such as ELASPLA and NONSAP which have the ability to carry the analysis accurately through the plastic range to failure. Work was concentrated on infinite length tubes with internal pressure, and variable length completely symmetrical circumferential thinned zones to simulate wastage defects.

Code ELASPLA

ELASPLA is a finite element computer code developed at BNL. The program can be utilized for the elastic-plastic-creep analysis of two-dimensional plane and axisymmetric structures subjected to time dependent mechanical and thermal loads. The program uses the initial strain method to incrementally solve the non-linear plasticity and creep problem. Plasticity is handled using either isotropic hardening, kinematic hardening, or combinations of both of the above rules.

Creep strains are currently not analyzed, but can be evaluated using mechanical equations of state in conjunction with either strain hardening or linear rules.

This code is based on infinitesimal theory and cannot be used for problems involving very large displacements or rotations requiring a finite strain formulation.

Code NONSAP

The computer code NONSAP is equipped to handle large deformations by making use of a total Lagrangian formulation of geometrical non-linearities. The element model used is a two dimensional, axisymmetric quadrilateral, geometrically defined by eight points or nodes. This allows the addition of certain higher-order terms, and is called

an "isoparametric" element. It has been shown that the accuracy of this type of element is higher than that of simple triangular elements. (1) The material model chosen was an elastic-plastic stress strain curve defined by an elastic modulus, a yield point, and a non-zero plastic modulus. There is no stress limit to this latter parameter.

Material Model

In order to set up the parameters for the material model, various industrial firms were approached for data. From the International Nickle Company, Huntington Alloy Products Division, we obtained a large quantity of engineering stress-strain plots of Incolloy 600 T at several temperatures. This was sufficient to establish a "first shot" model. Figure 1 shows a typical curve, with the computer model superimposed. The Huntington curves supplied shifted scales after the yield point to conserve paper and that portion of the curve from there to the UTS is not available. It is shown as a straight line for lack of better data.

1. Zeinkiewicz, Finite Element Method in Energy Science, pp 155.

The actual values used for these runs where:

Elastic Modulus 2.9×10^7 psi

Yield Point 4.2×10^4 psi

Plastic Modulus 1.3×10^5 psi

Poisson's Ratio 3.0×10^{-1}

This data is not enough for statistical work, however. Tubing manufacturers were queried to obtain data on the temper of the finished tubes, and on three dimensional variations, including wall thickness, eccentricity and local defects or blemishes which are apparently allowed but polished smooth. This type of data has proved impossible to obtain. One reply stated simply that "due to existing business conditions, we are unable to supply you with the data requested." The others indicated that all records kept are just minimum limits, or go-no-go statistics, and that actual values were not retained.

Physical Model

The computer runs were made modelling a perfectly concentric tube with .750 outside diameter and .048 wall thickness. The model started at the center of the reduced area and extended beyond it far enough to eliminate end effects of the axial loads.

The wasted area varied in axial length from .040 to .200 inches, and was reduced in thickness from the outside from 50% to 80% of the wall thickness. A straight sided transition zone about .050 inches long was inserted between the wasted zone and the perfect tube. Internal pressure was simulated by applying an appropriate outwards radial load on all interior nodes and an axial load, to simulate a capped end, was applied to all nodes in the upper row. The lower row, representing the center line of the wasted zone, was fixed against axial motion only, so that a plane cross section would remain plane. (Note that because of the axial loading, the upper plane cross section did not remain plane but was instead modeled far enough from the zone of interest that any interference was negligible.)

Computer Runs

The models used for the NONSAP runs are shown in figures 2, 15 and 25. The results of the computer runs are depicted on two types of plots. One plot is the effective stress at two significant locations vs. applied pressure, and the other is effective stress of an outside fiber vs. distance from the center line of the defect.

The effective stress is a three-dimensional stress that accurately represents the state of the material:

$$\sigma^* = \frac{1}{\sqrt{2}} \left[(\sigma_R - \sigma_A)^2 + (\sigma_R - \sigma_H)^2 + (\sigma_H - \sigma_A)^2 + 6(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2) \right]^{1/2}$$

where

σ_R = Radial stress

σ_A = Axial stress

σ_H = Hoop stress

$\sigma_{xy}, \sigma_{yz}, \sigma_{zx}$ = Shear stresses

.200 Long Defect

Figure 2 shows the element grid for a NONSAP run of a .200 long defect, 50% reduced in thickness. The axial dimensions for further reduction remain the same, effectively changing the slope of the transition zone outer surface.

Figure 3 shows the results of elastic-plastic runs of the tube, plotting maximum stresses which are at the center of the defect. The horizontal line of 42000 psi at the ends of each line represent the pressure addition necessary to create a fully plastic cross section.

Figures 4, 5 and 6 show lengthwise variations in stress in the outer fiber due to increasing pressure. In figure 6, the curve at 1300 psi is completely elastic. As pressure increases, the center of the defect reaches yield, and holds there. Increasing pressure moves the yielded portion axially without raising the maximum stress until the "plateau" extends well beyond the transition zone. When enough of the

elements are plastic, the stress at the center starts to rise again. A spike starts to develop at the start of the transition zone, however, and this spike increases at a faster rate than the stress at the center, quickly exceeding it and eventually leading to failure.

The stresses at these two important points are plotted vs. pressure in figures 7, 8 and 9. At low pressures, the still-elastic value of the stress at the center can be seen to be greater, but the rapid rise of the stress in the transition zone can be clearly seen.

In order to more clearly see the above effect, hoop and axial stresses were plotted vs. length for three pressures -- fully plastic, partly plastic and fully plastic.

In figure 10, which is all stressed below the yield point, it can be seen there is little variation in the inner or outer fiber hoop stresses. A significant difference between the inner and outer fiber axial stress can be seen at lengths of about .07 and .11 inches. This represents an axial bending moment caused by the change in the location of the neutral axis.

In figure 11, the whole of the wasted area is stressed beyond the yield point, but most of the transition zone is still elastic. Hence, the axial moment, as shown by the differences in the axial fiber stress is quite large, but it is such as to lessen axial tension on the outer fiber.

In figure 12, the plasticity has extended beyond the transition zone, but the outer fiber axial stress has now spiked to a high tensile value. It is this stress, when calculated into the formula on page 4 that creates the high effective stress.

Figure 13 shows a representation of the axial bending moment vs. length for increasing pressures. The positive moment developing between 3330 psi and 3920 psi can be clearly seen.

Figure 14 shows a physical interpretation of the preceeding data. In the partly plastic sketch, the outward deflection at the center of the defect is permitted by stretching at the inner fiber at the start of the transition zone. Angle θ is decreasing, and the bending moment here is such that a compressive force is superimposed on the basic tensile loading caused by the axial forces.

In the third sketch, as shown by the hatching, the plastic area extends well beyond the transition zone, and here the center of defect is really bulging. Stretching now occurs at both inner and outer fibers at the start of transition: angle θ is now increasing. This means a tensile load added to the basic load, resulting in the spike seen on the preceeding plots.

Figure 15 and 25 show the grids for reduced areas .100 and .040 inches long. Figures 16 and 26 show the equivalent stress vs. pressure plots in the elastic range for these shapes. Note that there is little difference in either the slope or the pressure at which the first element becomes plastic, but that the pressure difference between all elastic and all plastic increases as the defect shortens.

.100 and .040 Long Defects

Figures 20 and 30 show the critical elements in the plastic range for .100 long and .040 long defects 50% reduced in wall thickness. The previous observation can be seen in the respecting lengths of the horizontal section of the curves also by comparing them with figure 7. Note that for .040 long, the center-of-defect stress is never larger in the plastic range.

Figures 21, 22, 31 and 32 can likewise be compared.

Figures 17 and 27 show equivalent lengthwise plots of the above data. Comparing these with figure 4, we note that the figure shows the center of defect stress rising slightly above all others before the start-of-transition stress starts to spike. This effect is not shown in figure 17 and 27.

Figures 18, 19, 28 and 29 can be explained similarly.

Square Shoulder Defect

An additional run was made to test the effect of the shape of the transition zone. This modeled a defective zone .100 long, 65% reduced in wall thickness, similar to the previous run but with a square shoulder, i.e., no transition zone. The results are shown in figure 23 and 24. Comparing figure 24 with figure 21 it can be seen that for the square shoulder, the stresses are initially (4000 psi) somewhat lower, and that the slopes are shallower, the square shoulder σ^* reaching 88000 psi at about 6000 psi vs. 5600 psi for the tapered zone. This is because the additional material in the transition zone -- the additional triangular-section ring necessary to make the square shoulder -- stiffens this section, and keeps the angle θ (see figure 14) from starting to increase until a proportionally higher pressure has been reached. At that point, the element at the start of the square shoulder starts to spike.

Comments

From the preceeding discussion, it should be realized that simple elastic theory or even plastic infinitesimal theory cannot accurately predict the performance of these shapes. NONSAP provides a flexible tool to conduct these analyses. At the time of this report, work is in progress to check the accuracy of NONSAP against a known closed-form analytical solution, and to modify it to provide a more flexible, more accurate material model.

The results of this report, being two-dimensional, do not cover many actual defects which are really three-dimensional. At the present time, NONSAP, while it has some three-dimensional capability, cannot perform the three-dimensional, large deformation analyses required for these cases. In light of the preceeding discussion regarding prediction of failure, it would be highly desirable to modify NONSAP to allow it to perform these analyses.

Fig 1

TYPICAL STRESS-STRAIN CURVE

6578W NX2692 .636034IN. .005 4153 PSI. 37.
 SIZE PRODUCT CODE TEMP. °F SH. TO T.S. 0.02% YIELD STR. HEAT TREATMENT
 86X047 80AN 662 .050 50 PSI.

TCI 9944

GRAPHIC CONTROLS CORPORATION BUFFALO, NEW YORK

ELONGATION - COMPRESSION

PRINTED IN U.S.A.

BY Gardner DATE 4/2/75 BROOKHAVEN NATIONAL LABORATORY
 SUBJECT..... SHEET NO..... OF.....
 CHKD. BY..... DATE..... JOB NO.....
 DEPT. OR PROJECT.....

FIG 2

BY Jordan DATE 4/5/75
CHKD. BY _____ DATE _____

BROOKHAVEN NATIONAL LABORATORY
SUBJECT _____
DEPT. OR PROJECT _____

FIG 3
SHEET NO. _____ OF _____
JOB NO. _____

.200 LONG DEFECT
ELASTIC RUNS

FIG 4

BROOKHAVEN NATIONAL LABORATORY

BY Gardner DATE 4/2/75 SUBJECT
 CHKD. BY DATE
 SHEET NO. OF
 JOB NO.
 DEPT. OR PROJECT

.200 LONG DEFECT

50% REDUCED

BROOKHAVEN NATIONAL LABORATORY
BY Gardner DATE 4/2/75 SUBJECT _____
CHKD. BY _____ DATE _____
DEPT. OR PROJECT _____

FIG 5
SHEET NO. _____ OF _____
JOB NO. _____

.200 LONG DEFECT
65% REDUCED

BY J. J. J. DATE 4/14/75
CHKD. BY _____ DATE _____

BROOKHAVEN NATIONAL LABORATORY

SUBJECT _____

DEPT. OR PROJECT _____

SHEET NO. _____ OF _____

JOB NO. _____

FIG 6

200 LONG DEFECT

80% REDUCED

FIG 7

200 LONG DEFECT
50% REDUCED

BY Jordan DATE 4/1/75 BROOKHAVEN NATIONAL LABORATORY
 CHKD. BY _____ DEPT. OR PROJECT _____ SHEET NO. _____ OF _____
 JOB NO. _____

FIG 8

.200 LONG DEFECT
65% REDUCED

BY *Gen. L...* DATE *4/1/75*
CHKD. BY _____ DATE _____

BROOKHAVEN NATIONAL LABORATORY

SUBJECT _____

SHEET NO. _____

OF _____

DEPT. OR PROJECT _____

JOB NO. _____

FIG 9

.200 LONG DEFECT
80% REDUCED

BY Gardner DATE 4/2/75
CHD. BY _____ DATE _____

BROOKHAVEN NATIONAL LABORATORY

SUBJECT _____

DEPT. OR PROJECT _____

FIG 10

SHEET NO. _____ OF _____

JOB NO. _____

.200 LONG DEFECT

80% REDUCED

1300 PSI, ALL ELASTIC

BROOKHAVEN NATIONAL LABORATORY

BY J. J. J. DATE 9/2/75 SUBJECT _____
CHKD. BY _____ DATE _____
DEPT. OR PROJECT _____

FIG 11

SHEET NO. _____ OF _____
JOB NO. _____

200 LONG DEFECT

80% REDUCED

2460 PSI - PARTLY PLASTIC

J. J. J. DATE 9/2/75

SUBJECT

SHEET NO. OF

HKO. BY DATE

JOB NO.

DEPT. OR PROJECT

.200 LONG DEFECT80% REDUCED
3920 PSI - FULLY PLASTIC

BROOKHAVEN NATIONAL LABORATORY

FIG 13

BY _____ DATE _____ SUBJECT _____ SHEET NO. _____ OF _____
 CHKD. BY _____ DATE _____ JOB NO. _____
 DEPT. OR PROJECT _____

.200 LONG DEFECT
80% REDUCED

BY Andra DATE 5/15/75 SUBJECT _____ SHEET NO. _____ OF _____
 CHKD. BY _____ DATE _____ JOB NO. _____
 DEPT. OR PROJECT _____

ELASTIC TRANSITION POINTMAX γ^* $-\theta_{P1} < \theta_E$ PARTLY PLASTIC MAX γ^* $\theta_{P2} > \theta_{P1}$ FULLY PLASTIC

DISPLACEMENT of
REDUCED TUBE

BROOKHAVEN NATIONAL LABORATORY

FIG 13

BY *Gardner* DATE *9/2/75*
CHKO. BY _____ DATE _____

SUBJECT _____
DEPT. OR PROJECT _____

SHEET NO. _____ OF _____
JOB NO. _____

.100 LONG
DEFECT

Gardner

BROOKHAVEN NATIONAL LABORATORY

FIG 16

BY _____ DATE 9/5/55 SUBJECT _____
CHKD. BY _____ DATE _____
DEPT. OR PROJECT _____

SHEET NO. _____ OF _____
JOB NO. _____

.100 LONG DEFECT
ELASTIC RUNS

FIG 16

BY *g. J. Smev* DATE *4/2/75*
CHKD. BY _____ DATE _____

BROOKHAVEN NATIONAL LABORATORY

SUBJECT _____
DEPT. OR PROJECT _____

FIG 17
SHEET NO. _____ OF _____
JOB NO. _____

.100 LONG DEFECT
50% REDUCED

BROOKHAVEN NATIONAL LABORATORY

FIG 18

BY *Jardner* DATE *4/2/75* SUBJECT
 CKD. BY DATE
 DEPT. OR PROJECT

SHEET NO. OF
 JOB NO.

.100 LONG DEFECT
65% REDUCED

BY *Jandron* DATE *4/6/75*
CHKD. BY _____ DATE _____

BROOKHAVEN NATIONAL LABORATORY

SUBJECT _____
DEPT. OR PROJECT _____

FIG 19

SHEET NO. _____ OF _____
JOB NO. _____

.100 LONG DEFECT
80% REDUCED

BY *Jardner* DATE *4/2/75*
CHKD. BY _____ DATE _____

BROOKHAVEN NATIONAL LABORATORY

SUBJECT _____

DEPT. OR PROJECT _____

FIG 20
SHEET NO. _____ OF _____
JOB NO. _____

.100 LONG DEFECT
50% REDUCED

BY *John W.* DATE *4/2/75*

SUBJECT.....

SHEET NO. OF

CHKD. BY..... DATE.....

JOB NO.

DEPT. OR PROJECT.....

.100 LONG DEFECT65% REDUCED

BY garden DATE 4/6/75 BROOKHAVEN NATIONAL LABORATORY
CHKD. BY _____ DATE _____ SUBJECT _____
DEPT. OR PROJECT _____

FIG 22
SHEET NO. _____ OF _____
JOB NO. _____

.100 LONG DEFECT
80% REDUCED

BROOKHAVEN NATIONAL LABORATORY

BY Galina DATE 5/18/75 SUBJECT _____
CHKD. BY _____ DATE _____
DEPT. OR PROJECT _____

FIG 23

SHEET NO. _____ OF _____
JOB NO. _____

.100 LONG DEFECT
SQUARE SHOULDER
65% REDUCED

BROOKHAVEN NATIONAL LABORATORY
BY Jordan DATE 5/13/55 SUBJECT 100 LONG DEFECT
CHKD. BY DATE DEPT. OR PROJECT SQUARE SHOULDER
65% REDUCED

FIG 24

SHEET NO. OF
JOB NO.

FIG 25

.040 LONG DEFECT

BY _____ DATE _____ SUBJECT _____
 CHKD. BY _____ DATE _____
 DEPT. OR PROJECT _____

SHEET NO. _____ OF _____
 JOB NO. _____

.040 LONG DEFECT
ELASTIC RUNS

BY *Jarman* DATE *4/5/75*
CHKD. BY _____ DATE _____

BROOKHAVEN NATIONAL LABORATORY

SUBJECT _____
DEPT. OR PROJECT _____

FIG 27

SHEET NO. _____ OF _____
JOB NO. _____

.040 LONG DEFECT

50% REDUCED

BY Jardner DATE 4/5/75 BROOKHAVEN NATIONAL LABORATORY
CHKD. BY _____ DATE _____ SUBJECT _____
DEPT. OR PROJECT _____

FIG 28
SHEET NO. _____ OF _____
JOB NO. _____

.040 LONG DEFECT
65% REDUCED

BY gudmar DATE 2/5/75 BROOKHAVEN NATIONAL LABORATORY SHEET NO. F16 29 OF
 CHKD. BY DATE SUBJECT JOB NO.
 DEPT. OR PROJECT

.040 LONG DEFECT

80% REDUCED

BY Gardner DATE 4/5/75 SUBJECT
 CHKD. BY DATE
 DEPT. OR PROJECT

FIG 30
 SHEET NO. OF
 JOB NO.

.040 LONG DEFECT
50% REDUCED

BY *Grubbs* DATE *4/5/75*
CHRD. BY..... DATE

BROOKHAVEN NATIONAL LABORATORY

SUBJECT

DEPT. OR PROJECT

FIG 31

SHEET NO. OF

JOB NO.

.040 LONG DEFECT

65% REDUCED

BROOKHAVEN NATIONAL LABORATORY
BY *Jordan* DATE *4/5/75* SUBJECT.....
CHKD. BY..... DATE.....
DEPT. OR PROJECT.....

FIG 32
SHEET NO. OF
JOB NO.

.040 LONG DEFECT
80% REDUCED