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Inclusive Momentum and Angular Distributions
from Electron Positron Annihilation
at $\sqrt{s} = 3.0, 3.8, \text{ and } 4.8 \text{ GeV}$

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ABSTRACT

Inclusive features of multi-hadron final states produced in the annihilations of electrons and positrons are presented. Data were taken at the colliding beam machine, SPEAR, at center-of-mass energies 3.0 GeV, 3.8 GeV, and 4.8 GeV. Reaction products were detected in a $\sim 20 \text{ m}^3$ collection of spark chambers and counters, cylindrically arranged, in an axial magnetic field of $\sim 4 \text{ KG}$, around the e^+e^- intersection region. Distributions of single-particle momenta and production angle and two-particle correlations are presented and compared with dynamical models. The results are in disagreement with expectations based on the successful parton-quark model of hadron structure. No generally satisfactory interpretation is available.

INTRODUCTION

Since the original discovery by workers at Frascati, Italy, that the cross section for the annihilation of an electron and a positron into a pair of hadrons, or a state with many hadrons, was higher than could be expected from simple models, a great deal of theoretical and experimental effort has been expended in this field. The Frascati measurements were limited to a center of mass energy ($\sqrt{s}$) of less than 3.0 GeV and had large uncertainties due to the small number of events collected, the limited solid angle, and the very small efficiency (~3%) of their detectors. The experimental effort of the past few years has been to try for higher center of mass energies and larger solid angle detectors. This thesis describes the apparatus known as the SPEAR Magnetic Detector which was designed to investigate the annihilation process at the SPEAR Storage Ring Facility. In addition it describes some of the problems which were encountered with backgrounds at the storage ring, the methods used for analyzing the data from the detector, and the results of that analysis on the high statistics data taken at $\sqrt{s} = 3.0, 3.8$, and 4.8 GeV.

To understand why all this effort should be expended and why the results are exciting, one must look at some of the theoretical predictions for the annihilation. The excitement stems from the fact that while the electron positron initial state is believed to be well understood in terms of the elegant theory of Quantum

Electrodynamics (QED), the hadron final state does not seem to fit into any simple theoretical scheme. The QED theory has been remarkably successful in predicting the detailed structure of atomic levels (Lamb Shift), the magnetic moments of the electron and muon, and the processes of electron positron annihilation into electrons, muons, or photons up to presently available energies. For the hadron final state, this theory suggests that if all of the hadrons are made up from some set of elementary constituents, then the hadron production cross section measures the sum of the squared charges of these constituents. The desire for a theory which uses such elementary constituents is generated by the fact that there are now more than 13 more or less stable "elementary" particles, many with several charge states. In addition to this, there is an ever-expanding list of resonances, and no theory explains the mass spectrum of either the stable particles or the resonances. Some successes have been achieved in explaining the observed types of particles and their interactions with theories involving SU(3) symmetries. The elementary constituents of these theories are called quarks. Each of these theories predicts an asymptotic annihilation cross section related to the sum of the squared charges of the quarks.

The results of this experiment indicate that the hadron production cross section does not seem to have reached the asymptotic region for $\sqrt{s}$ below 4.8 GeV but yet is approximately six times higher than one could expect from the simplest quark theory.

(The ratio is an increasing function of $\sqrt{s}$.) The fact that the cross section is enhanced rather than suppressed is also interesting since if there were something wrong with the assumption that the interactions take place through point-like constituents, this would eventually lead to a suppression of the cross section. In addition to the fact that the total cross section does not behave as expected, irregularities have been found in the angular and momentum distributions of the produced hadrons. These problems arise when one tries to compare what is learned about electron hadron interactions from the scattering of electrons from a proton target with what one learns in the annihilation process. These processes should be related, in which case the electron proton results predict that the momentum distribution in the annihilation channel should be a universal function of dimensionless quantities (scaling) and that the angular distribution should be non-isotropic. These predictions appear to be valid only for particles with energy functions $\frac{2E}{\sqrt{s}} \geq .5$.

As can be seen, most of the interest in electron positron annihilation is generated by the disagreement of the cross section and distribution results with the predictions of otherwise successful theories. But since the process being investigated probes the basic underlying structure of the hadrons and since part of the interaction is believed to be understood (i.e. the simple initial state), one can hope that all of this disagreement will eventually lead to modified theories which enhance our knowledge of the basic laws of nature.

I. The SPEAR Magnetic Detector

A. Introduction

The magnetic detector apparatus is of generally cylindrical design, the components being concentric and centered on the beam axis. To describe the apparatus, it is best to start at the beam axis (point of interaction) and move radially outward following the path of the detected particle. Closest to the beam of course is the vacuum chamber. The vacuum chamber must be very thin so as to provide as little interference as possible to the particles traveling through it, but it must of course be able to support the atmospheric pressure and its own weight. The present vacuum chamber is made of 6 mil stainless steel and is corrugated to provide additional structural strength. Two semi-cylinders of plastic scintillator form a cylindrical counter around the vacuum pipe which is used in the detector trigger. Eight cylindrical wire spark chambers provide position information for the charged particles. These chambers are paired, and the wires have different angles to the axis to aid in ambiguity resolution. Thus there are four sets of chambers, each having two high voltage cylinders and two ground cylinders located at average radii of .66, .91, 1.1, and 1.35 meters. Surrounding the aluminum can, which provides support for the spark chambers, are 48 trigger counters consisting of 1-inch-thick scintillators with a 56DVP phototube at each end. The signals from these counters are used in the trigger logic and are fed to TDC's which provide time-of-flight information. The

trigger counters are just interior to the solenoid windings. Just exterior to the windings are 24 shower counters, again arranged to form a cylinder. These shower counters are also included in the trigger logic and in addition provide pulse height information which is used to distinguish electrons from pions and muons. The shower counters are surrounded by the 20-cm-thick iron flux return of the solenoid. Exterior to the flux return are two spark chambers which, since the flux return provides some hadron absorption, are capable of rough muon identification. The general layout of the apparatus is shown in Figs. 1-3.

B. Pipe Counter

The pipe counter was constructed from two semi-cylindrical sheets of 1/8" scintillator with a length of 90 cm. It was mounted just exterior to the vacuum chamber. Each end of each semi-cylinder was connected to a lucite light pipe which extended outside the magnetic field region. Each of the four light pipes was fitted with a 56DVP phototube, a discriminator, and a latch. The status of the latches was recorded with each event.

The primary purpose of such a pipe counter is to reduce the cosmic ray background. This is accomplished by requiring a pipe counter latch or combination of them for the main trigger. Since the magnetic detector has a horizontal projected area of 7.5 m^2 , the cosmic ray rate is approximately 1.3 KHz. By adding the pipe counter to the trigger requirement, the effective area is reduced

XBL 753-404

Fig. 1

XBL 753-405

Fig. 2

Fig. 3

to $1.8 \times 10^2 \text{ cm}^3$. Further, the apparatus is sensitive only during 30 nsec of the 780 nsec revolution time, so the cosmic ray trigger rate is reduced to 1 per second. In addition, the pipe counter requirement discriminates against those backgrounds which come from beam losses upstream from the detector and which do not produce particles which pass through the pipe counter.

The typical occupancy rate in the pipe counter due to backgrounds which did cause particles to pass through the counter was 6% with a variation of about $\pm 4\%$ depending on the machine configuration and the quality of the particular fill. The pipe counter has a negligible effect on the detection efficiency for hadron events.

C. Vacuum Chamber

The dimensions of the vacuum chamber are shown below.

The chambers were built at CERN.

$$\text{OD} = 6.713'' \text{ to } 6.73''$$

$$\text{ID} = 5.914'' \text{ to } 5.898''$$

$$\text{WALL} = .006''$$

$$\text{OD} = .1705 \text{ to } .1709 \text{ m}$$

$$\text{ID} = .1502 \text{ to } .1498 \text{ m} \quad R_0 = .085$$

$$\text{WALL} = 1.5 \times 10^{-4} \text{ m} \quad R_I = .075$$

$$\frac{L}{L_{\text{rad}}} = \frac{1.5 \times 10^{-4}}{1.77 \times 10^{-2}} = 8.5 \times 10^{-3}$$

$$\sqrt{\langle \theta^2 \rangle}_{\text{rms}} \bigg|_{\beta=1}^{p=400} = \frac{(\sqrt{2})(.015)}{.4} \sqrt{\frac{L}{L_{\text{rad}}}} \sim 0.5 \times 10^{-2}$$

As one can see, the rms scattering angle of .4 GeV/c particles is on the order of 5 m radians. However, due to the corrugated shape of the vacuum chamber, the amount of material traversed can be much larger than the wall thickness, in fact as great as 1 mm.

D. Spark Chambers

There are four spark chambers in the magnetic detector, each consisting of four wire cylinders. All spark chambers share a common gas volume but are optically isolated by sheets of mylar.

The nominal radii and lengths of the chambers are given in Table 1.

Chamber #	R	L	Wire Spacing	Approx. # of Wires	# of Wands
1	53"	106"	1/24"	31500	16
2	44"	95.6"	1/24"	26500	24
3	36"	86.4"	1/24"	21700	28
4	26"	86.7"	1/24"	15700	32

Table 1

The two inner wire planes of each chamber form a spark gap and have wires oriented at approximately $\pm 4^\circ$ with respect to the axis of the chamber. An additional spark gap is formed by the outer two wire planes, the wires being oriented at $\pm 2^\circ$ with

respect to the axis so that ambiguities can be resolved by comparing the information in the two gaps. The positions of the cylinders are shown in Table 2 and Fig. 4.

GAP	PLANE	R _{plane} -Mean Chamber R	Sign of dφ/dz of Wires	Readout end
4°	Ground	-.58"	+	-Z
	HV	-.19"	-	+Z
2°	HV	.19"	-	-Z
	Ground	.58"	+	+Z

Table 2

A magnetostrictive readout is employed, consisting of a total of 100 wands distributed as shown in Table 1. Both the HV and the ground planes are read out, and in order to keep the electrical path length independent of the z position of the spark, the two planes are read out at opposite ends of the detector. The bias angle of the wands is 30° (angle between the wand normal and the magnetic field), providing the small longitudinal magnetic field necessary for the operation of the magnetostrictive wands.

Using the stereo angle, radius of the wire plane at the readout end, length of the chamber, and some hypothesis about the spark formation, the wand information can be converted to spark locations relative to the chamber center. Additional

TYPICAL CROSS SECTION
WIRE SPARK CHAMBER
SPEAR DETECTOR

TABULATION

CHAMBER NUMBER	FINAL L" (IN.)	ACCEPT ANGLE	RADIUS (IN.)
4	86.7	36°	26
3	86.4	39.8°	36
2	95.6	42.6°	44
1	106.0	45°	53

Fig 4

"optical" constants are needed to relate the Chamber Centered Coordinate System (CCCS) to the Beam Centered Coordinate System (BCCS). These constants consist of chamber shifts, rolls, pitches, and yaws relative to the beam axis and are determined by an optical constant fitting procedure (see Appendix I).

The major sources of particle scattering and/or absorption within the spark chamber volume are the six support posts situated between chambers 3 and 4. These posts are used to provide additional structural strength to the "spider" structure at the ends of the chamber which supports the tension in the spark chamber wires. They are made of tubular aluminum. Any track passing within .5 cm of any of the posts was eliminated from further consideration in the event analysis. Corrections were later applied for this missing solid angle.

The spark chamber wires are Aluminum (5056) with a diameter of .0075". Thus the probability that a track will hit a wire is

$$\frac{\text{wire diameter}}{\text{wire spacing}} = \frac{.0075}{1/24} \sim .18 \text{ per plane}$$

and the amount of material traversed

$$L = 2D\sqrt{x(1-x)}$$

where x is the distance from the center of the wire to the particle path in units of the wire radius. If a track has $p = .4$ GeV, $\beta = 1$, and traverses the full diameter of the wire, the rms scattering angle is

$$\sqrt{\theta^2} = \frac{\sqrt{2}(.015)}{.4} \sqrt{\frac{L}{L_{\text{rad}}}} = \frac{\sqrt{2}(.015)}{.4} \sqrt{.0021} = 2.4 \text{ m rad.}$$

The 70 cm of air in the gaps between the chambers, $L/L_{\text{rad}} \approx 22.5 \times 10^{-4}$, produces a scattering angle of 2.4 m rad at this energy. Adding the air and one wire scatter on average for each particle passage gives at the outer chamber

$$\sigma \approx 1/\sqrt{2} \times r \times \Delta\theta = 1.8 \text{ mm.}$$

The only other significant material associated with the spark chambers is the support can consisting of a 1/2-inch Al cylinder surrounding the spark chambers. It does not affect the spatial resolution of the chambers, but does affect the triggering of the detector, since it absorbs some of the hadrons coming from the interaction region.

The spark chamber efficiencies were determined by taking the tracks from hadron events and counting a success for each chamber when all four spark chambers contributed a space point (see analysis section) to the track, and a failure for the chamber which missed when only three chambers contributed. (Three chambers are required in the definition of a track.) The efficiency is then the ratio of the successes to the sum of successes and failures. Each efficiency was then examined as a function of the azimuthal angle ϕ and the distance z along the beam axis and found to be consistent with a uniform distribution. The average efficiency for each chamber in a particular run was typically 97-98% and was always greater than 96% for the data at $\sqrt{s} = 3.0, 3.8, \text{ and } 4.8 \text{ GeV}$. The measured values of the efficiency were used in the Monte Carlo calculation of the experimental efficiencies. As a function

of event multiplicity, the multi-track efficiency was found to vary by less than 2% for multiplicities between 2 and 8. As a function of transverse momentum, the efficiency varied by 1.5% for transverse momenta greater than .2 GeV/c. For the outermost chamber, the efficiency for $.1 \leq p \leq .2$ GeV/c dropped by 8% due to the greater inclination of these tracks relative to the spark chamber surface.

E. Trigger Counters

The trigger counters are made of 1" x 7.9" x 102" Pilot y scintillator viewed from both ends by 56 DVP photomultipliers. The attenuation length is $\sim 124"$, and the propagation velocity is constant over the counter length, $|V|^{-1} \sim 1.95$ nsec/ft. The average efficiency of the trigger counters over the time in which the data were accumulated was found by using the tracks in the data sample and determining the number of times the latch (logic level used in the trigger) fired. The efficiency determined in this way is 100% for $0 \leq |\cos\theta| < .5$, 97% for $.5 \leq |\cos\theta| < .6$, and 71% for $.6 \leq |\cos\theta| < .7$.

F. Shower Counters

The shower counters are constructed with 5 sheets of lead $1/4" \times 18" \times 122"$ interleaved with 5 sheets of $1/4" \times 18" \times 122"$ Pilot F scintillator. The measured l/e attenuation length of the plastic is approximately 58" before fabrication. After fabri-

cation, however, the $1/e$ attenuation length was found to be about 29". This degradation was caused by minute scratches on the surface of the sheets due to wiping of the surface during fabrication. The later attenuation length produces a factor of 20 variation in signal from the center of the counter to the phototube mounting. Because of this, it is necessary to correct the pulse height information off-line for the position of the track. Each end of the shower module is viewed by a 5" RCA 4522 phototube. The signal from each of these tubes is fed to an LRS Model 2248 ADC which is run in the bilinear mode with the breakpoints set at 32 pc. In this mode, the scale is .25 pc/count up to channel 32 and 1 pc/count from 32 to 255. The minimum ionizing peak occurs at channel 13 with a FWHM of 10 channels. The electrons from e^+e^- elastic scattering give a pulse height with a mean of 165 counts with a FWHM of 120 channels.

G. Trigger

The highest non-background rate within the detector comes from the Bhabha process $e^+e^- \rightarrow e^+e^-$. It has a rate of approximately .3 events/second within the detection region for a luminosity of $10^{31} \text{ cm}^{-2} \text{ sec}^{-1}$. The rate for hadron events at the same luminosity is about .1/second, and the rate for muon pairs is about .03/sec. The spark chambers have a dead time of .1 sec. These facts indicate that, to avoid having a large contamination in the recorded events, and to avoid being limited

by the dead time, it is necessary to reduce the trigger rate to a few events per second. The rate of coincidences of 2 or more trigger counters with the solenoid off and 25 ma of stored current was ~ 15 KHz. Turning on the solenoid (and thus eliminating charged particles with $p \gtrsim .1$ GeV/c) reduces this rate by a factor of 5. This indicates that the backgrounds are dominated by processes which produce soft particles. An examination of the events collected with solenoid on and the requirement of 2 or more trigger counters indicated that ~ 2 KHz of the remaining rate had at least one shower counter fired. Though the background is dominated by soft particles, the majority of the events are accompanied by at least one hard particle. The extra material between the trigger counters and the shower counters provides additional absorption of the soft background. The remaining 1 KHz rate containing 2 or more shower counters was found to be dominated by cosmic rays. Two or more of the four pipe counter latches were required in the trigger to eliminate the cosmic ray background, and a requirement of 2 or more shower counters was used to eliminate the "one hard, multi-soft" machine background. Shower counters which fired without a coincidence in any of the adjacent four trigger counters could not have been fired by a charged particle, and thus were not counted as a part of the required 2 or more. All coincidences on the trigger counters were made with a 15 ns gate derived from the beam pickup electrodes. The final trigger was

beam pickup gate,

2 of 4 pipe counter latches,

and 2 or more trigger-associated showers (TASH).

The typical event rate was 2 or 3 events per second.

H. Magnetic Field

The magnetic field is generated by a main solenoid winding in series with two compensation coils. The iron flux return is octagonal, and thus the entire system has an 8-fold symmetry. This symmetry is marred only by the 24 holes in each end plate for shower and trigger counter phototubes. The field was mapped with a Hall-effect probe at a current producing a field $B_z = 3891 \pm 1$ gauss at the beam axis. Measurements were also taken of the hysteresis of the magnet, and indicated that this effect is approximately .05%. The field was then fit to a polynomial of the form

$$B_z = 4000 \left\{ \sum_{j=1}^7 c_j P_{zj}(r, z') - c_{10} \frac{(3r^2 - 2\rho_1^2)}{\rho_1^5} - c_{12} \frac{(3z'^2 - 2\rho_2^2)}{\rho_2^5} \right\}$$

$$B_r = 4000 \left\{ \sum_{j=1}^7 c_j P_{rj}(r, z') + 3c_{10}r \frac{(c_9 + a')}{\rho_1^5} - 3c_{12}r \frac{(c_{11} - z')}{\rho_2^5} \right\}$$

$$B_\phi = 0$$

where

$$z' = z - c_8$$

$$\rho_1^2 = (z + c_9)^2 + r^2$$

$$\rho_2^2 = (z - c_{11})^2 + r^2$$

$$P_{z_1} = 1$$

$$P_{z_3} = 3(z')^2 - (3/2)r^2$$

$$P_{z_5} = 5(z')^4 - 15r^2(z')^2 + (15/8)r^4$$

$$P_{z_7} = 7(z')^6 - (105/2)r^2(z')^4 + (315/8)r^4(z')^2 - (35/16)r^6$$

$$P_{r_1} = 0$$

$$P_{r_3} = -3rz'$$

$$P_{r_5} = (-10(z')^2 + (15/2)r^2)rz'$$

$$P_{r_7} = (-21(z')^4 + (105/2)r^2(z')^2 - (105/8)r^4)z'r$$

The terms containing P_z and P_r are the solutions for Laplace's equation. The coefficients of the even order terms are set to zero due to the symmetry about the plane $z' = 0$. The constant c_8 adjusts the position of the symmetry in the beam coordinate system. Included also are source terms of variable strength and position for the compensating solenoids. The resultant coefficients are shown in Table 3.

<u>j</u>	<u>C_j</u>
1	.98396
3	-.82239 x 10 ⁻²
5	-.15860 x 10 ⁻³
7	.58682 x 10 ⁻⁴
8	.91698 x 10 ⁻¹
9	1.7121
10	-.12952 x 10 ⁻¹
11	1.5220
12	-.12527 x 10 ⁻¹

Table 3

This expansion fits the measured, ϕ averaged values of the field to about .05% in B_z and about 3 gauss in B_r . The resultant error in momentum due to the uncertainty in the field is much smaller than the minimum error due to wire spacing in the spark chambers. Plots of the field are shown in Figs. 5-8.

XBL 753-406

Fig. 5

XBL 753-361

Fig. 6

XBL 753-362

Fig. 7

XBL 753-363

Fig. 8

II. Data Analysis

A. PASS 1 Filter

All events satisfying the hardware trigger were recorded on tape, and these tapes were passed through a filter program to eliminate the most obvious sources of background events. The criteria of this PASS 1 filter were

1. Rejection of "illegal" hardware triggers

$$NTASH \geq 2$$

2. "Early Time" (counters 25, 26 excluded for run numbers less than 601)

$$\geq 2 \text{ trigger counters with } TOF > 2 \text{ ns}$$

3. "Cosmic" filter (counters 25, 26 excluded for run numbers less than 601)

for 2-trigger counter events where the trigger counters have a separation of more than 12 counters require $-2 \text{ ns} < (\Delta T - \text{calculated flight time}) < 1 \text{ ns}$

4. Minimum number of points in the wire spark chambers

$$\# \text{ points} \geq 4$$

5. At least one "road" containing 2 points

a road is the football-shaped area formed by two tracks of opposite charge and .12 GeV transverse momentum hitting a latched trigger counter.

Table 4 shows the number of events eliminated by each of these filters. Only events satisfying all of these criteria were output to the filtered tapes. These criteria were also included in the Monte Carlo simulation of the detection efficiency. The tapes from this PASS 1 filter were analyzed by the PASS 2 program discussed in the next section to separate events into the various event types.

Filter

Data Sample	NTASH		EARLY TIME		COSMIC		NPTS		NROAD		Total Number of Events	% Survival
3.0 GeV B = 4kg	5	0%	18246	12.3%	75893	51.1%	7741	5.2%	5180	3.5%	148575	27.9%
3.8 GeV B = 4kg	9	0%	42801	15.5%	116556	42.3%	14186	5.2%	10719	3.9%	275357	33.1%
4.8 GeV B = 4kg	149	0%	51712	13.8%	143688	38.3%	11842	3.2%	37440	10.0%	375073	34.7%
4.8 GeV B = 2kg	32	0%	41380	19.8%	66099	31.7%	6368	3.1%	21680	10.4%	208630	35.0%

Table 4. Number of Events Eliminated by Filters

B. PASS 2 Analysis

1) Construction of Space Points from Spark Chamber Readouts

The raw data from the magnetostrictive wands of the spark chambers consist of azimuthal readouts of the positions of the sparks projected along the skewed wires to the proper readout end of the chamber. It is necessary to form three-dimensional space points from these wire azimuths (WAZM's) for use in the track recognition and track fitting programs. The equations of Appendix I which relate the position of points on the surface of any wire plane to the resultant WAZM at the readout end can be inverted and used to reconstruct the space points from the WAZM's. Each WAZM defines a line (or wire) in 3 space and one would like to search for intersections of the lines found on the ground surface with those on the high voltage surface. (These surfaces have opposite skew angles.) Of course, these lines do not intersect at all in three dimensions since the surfaces are always separated radially. Thus, to complete the reconstruction, one needs to make a hypothesis about the way in which the spark passes from one surface to the other. Visual observations of the sparks indicated that even in the cylindrical geometry the sparks tended to form normal to the cylinder surfaces roughly independent of the track inclination relative to this normal. When the inclination of the track relative to the normal is greater than 45° there is a tendency for the spark

to begin to follow the track, and, at very large angles, multiple sparking occurs. Because tracks coming from the interaction region are restricted to angles of less than 45° relative to the chamber normal, the hypothesis used was that the spark always formed normal to the wire surface and that the position of the spark and the track coincided at a point a fraction $DR = .5$ of the distance between the two surfaces. This hypothesis translates to the assumption that the azimuthal angles ϕ and the positions along the beam axis z are the same on the two wire surfaces. The hypothesis was checked by looking for correlations between the chi-square of track fits with the inclination of the track and between the chi-square and changes in the value assumed for DR . No such correlations were found.

Using the spark hypothesis discussed above, all "intersections" of the wires in each of the eight gaps were found for each event. Each such intersection on the inner gap of a chamber was used to define a search volume with $\Delta\phi = .1$ radian and $\Delta z = .15$ meters on the outer gap. Wires or intersections on the outer gap within this search volume were combined with the inner intersection to form a 3-wire or 4-wire space point respectively. All intersections on either gap which remained unpaired were used to define 2-wire space points. A 4-wire space point differs slightly from the others in that it defines two coordinates in 3 space (i.e. one on each gap). The innermost coordinate of such a pair was used for track recognition purposes. In this way, the 4-wire, 3-wire,

and 2-wire points could be used identically for the track recognition programs, the only difference being that a 4-wire point contributed two coordinates to the track fit. This type of classification scheme has the advantage of making the track recognition efficiency independent of variations in the spark efficiency on a single gap. In addition, it increases the spark chamber efficiency for the purposes of track recognition since a gap efficiency of 85% to 90% gives a space point efficiency of 98% to 99%.

2) Track Reconstruction

For the purpose of reconstructing tracks, the points are separated into 4 groups, each group consisting of the points from a single chamber. Points which are constructed from readouts on 4, 3, or 2 wire surfaces are treated identically. Several tracking programs have been written with increasing levels of sophistication. The basic philosophy in each is to choose a pair of points on separate chambers and to use these points along with the origin to define a rough searching volume for other points. The tolerances in all cases have been adjusted (using simulation) to be large enough that a point on a real track will not be eliminated due to spark chamber resolution. The tolerances must be enlarged beyond this, however, so that whether a track is found or not is not dependent on the accuracy of the initial track parameter estimates based on two

points and the origin. This accuracy is determined in part by the spark chamber resolution but depends much more critically upon whether the track does indeed come from the origin. Observations of the performance of these programs indicates that all tracks whose projected radial distance from the origin of the detector was <70 cm were found in this way. Higher levels of sophistication might involve using combinations of 3 points to define the original track parameters, but we did not find it necessary to use these types of reconstruction procedures.

There were two levels of tracking used in the PASS 2 analysis. The algorithms are described below.

3) Level 2 Tracking Program: Subroutine ELSIE

Beginning with each point on chamber 1, this level looks for any point on chamber 2, which satisfies the following relations:

$$|Z_0| = |Z_1 - R_1 \frac{(Z_1 - Z_2)}{(R_1 - R_2)}| \leq .45$$

where Z_0 is the predicted Z at $R = 0$ based on the two points (Z_1, R_1) and (Z_2, R_2) . If there are no points on chamber 2, no tracks are found by this level.

Because of the inability of this algorithm to reliably distinguish tracks which are closely spaced in the XY plane, this algorithm is not applied to events with more than 35 points

The parameterization of a circle of radius ρ , passing through the origin, can be written as (see diagram below)

$$\sin \frac{\phi}{2} = \frac{R/2}{\rho} \quad \text{where } R \text{ is the radius of point } P$$

so that

$$\phi = 2 \sin^{-1} \frac{R}{2\rho} + \phi_0$$

$$\phi = \phi_0 + 2\left(\frac{R}{2\rho}\right) + \frac{1}{3!}\left(\frac{R}{2\rho}\right)^3 + \frac{3}{5!}\left(\frac{R}{2\rho}\right)^5 + \dots$$

In cases where $\frac{1}{3!}\left(\frac{R}{2\rho}\right)^2 \ll 1$, the radius of curvature and initial angle of the track can be estimated by using

$$\phi_1 = \phi_0 + \frac{R_1}{\rho}$$

$$\phi_2 = \phi_0 + \frac{R_2}{\rho}$$

$$\left(\phi_1 - \frac{R_1}{\rho}\right) = \left(\phi_2 - \frac{R_2}{\rho}\right)$$

$$\rho = \frac{R_1 - R_2}{\phi_1 - \phi_2}, \quad \phi_0 = \phi_1 - \frac{R_1}{\rho}$$

R_1 and R_2 are approximately 1.4 and 1.1 respectively so that

$$\frac{1}{3!} \left(\frac{R}{2\rho} \right)^2 \leq .1 \quad \text{gives}$$

$$\rho \geq \sqrt{\frac{(1.4)^2}{6 \times .1}} = 1.7 \text{ meters}$$

and therefore

$$P_1 = .03B\rho \sim .100 \text{ GeV/c at } B = 2$$

$$\sim .2 \text{ GeV/c at } B = 4.$$

The track can thus be well represented by a straight line in R vs ϕ where the slope of the line can be determined by the two points. The predicted ϕ at chambers 3 and 4 can now be calculated using

$$\phi_i = \phi_1 - (R_1 - R_i) \frac{(\phi_1 - \phi_2)}{(R_1 - R_2)}.$$

(One must of course always normalize angular differences between $-\pi$ and π .) Further, Z_i can be estimated as

$$Z_i \approx Z_0 + \frac{R_i(Z_1 - Z_2)}{R_1 - R_2}.$$

For this level to find a track, there must be 1 and only 1 point on each of chambers 3 and 4 satisfying

$$|\phi - \phi_i| \leq .1 \text{ Rad} \quad \text{and}$$

$$|Z - Z_i| \leq .2 \text{ m}.$$

The algorithm is applied for all pairs of points on chambers 1 and 2. Following this, it is also applied to pairs of points on chambers 2 and 3, provided the predicted Z_i for chamber 1 lies outside the spark chamber volume.

4) Level 4 Tracking Program: Subroutine PEARL

The level 4 track recognition program is designed to take

care of all the ambiguous situations ignored in level 2, namely

1. Tracks with $P < .200 \text{ GeV/c}$
2. Tracks with missing chambers
3. Tracks with more than 1 candidate point on a chamber
4. Tracks which turn around within the chamber volume
5. Events with >35 points.

Every point on chamber 1 which has not so far been used in a "track" is paired with the points on chamber 2 and a test is again made to see if the two points together with the origin make a good track. The procedure is:

1. calculate approximate 2ρ

$$2\rho \approx \frac{-\Delta R}{\Delta\phi} \text{ as in Level 2;}$$

2. if 2ρ is small enough that the estimate would be more than 10% off, calculate ρ exactly as illustrated below.

Using two points R_1, ϕ_1 and R_2, ϕ_2 , we can solve for ϕ_0 and ρ . From law of cosines applied to two points:

$$R_1^2 + R_2^2 - 2R_1R_2 \cos(\phi_1 - \phi_2) = [2\rho \sin(\phi_1 - \phi_2)]^2$$

$$4\rho^2 = \frac{R_1^2 + R_2^2 - 2R_1R_2 \cos(\phi_1 - \phi_2)}{\sin^2(\phi_1 - \phi_2)}.$$

Having solved for ρ , we can find ϕ_0 by using

$$\phi_0 = \cos^{-1} \left(\frac{R}{2\rho} \right) + \phi.$$

(If we can approximate $2\rho \approx \frac{\Delta R}{\Delta \phi}$

$$\text{then } \phi_0 \approx \frac{\pi}{2} + \phi + \frac{R}{2\rho}.)$$

A search is made to find all pairs of points which define circles such that

1. The two points lie on the same side of the center of curvature so that they form a contiguous track segment to the origin.

2. The Z at the origin

$$Z_0 = Z_1 - S_{xy} \frac{dZ}{dS_{xy}}$$

satisfies

$$|Z_0| \leq .85 \text{ m}$$

where S_{xy} is the transverse arc length.

For each pair of points determining a valid helix, the predicted points on chambers 3 and 4 can be calculated as

$$Z_i = S_i \frac{dZ}{dS} + Z_0$$

where S_i = arc length in XY from origin to point at radius R_i

$$S_i = 2\rho \sin^{-1}(R_i/2\rho).$$

A search is made for points on chamber 3 which satisfy

$$|Z - Z_i| \leq .12 \text{ m}$$

$$|\phi - \phi_i| \leq .11 \text{ rad.}$$

If points are found, only that point with the smallest $|\phi - \phi_i|$ is selected. A search is then made among all the points on chamber 4 with the same Z tolerance as above but with the ϕ tolerance increased to .14. Each point within tolerance on chamber 4 is used to make a new track with the points accumulated thus far. In cases where the track does not go through all four chambers, due to low transverse momentum or $P_z/P_\perp$ large, the program checks for points only on the allowed chambers. To define a track it requires that there be at most one missing chamber.

Having completed the algorithm with all possible pairs of points on chambers 1 and 2, the program now takes unused points on chamber 2, pairs them with all possible points on chamber 3 to find acceptable helicities. For each acceptable pair, a track is made with each acceptable point on chamber 4, and an additional search is made in chamber 1 for the best point within tolerance. Continuing in the same way, unused points on chamber 3 are paired with all points on chamber 4 and a search is made on chambers 1 and 2.

The algorithm is capable of handling all cases where there is a missing point and handles low momentum tracks by using more accurate calculations than Level 2. In cases where multiple points occur within tolerance on a single chamber, all

possible tracks are found, but the algorithm does not find all possible tracks when multiple points lie on more than one chamber. Instead, it finds a subsample of the possible point collections by choosing only the best points while searching chambers one to three. For example, if there are two sparks on both chambers 2 and 3, only 2 or 3 of the 4 possible combinations will be used. Though this feature does not decrease the efficiencies for finding tracks coming from the origin, it does tend to bias against tracks not coming from the origin since the selection of the "best" point is based on the initial approximation of two points with the origin. The major advantage in this procedure is merely to limit the possible combinatorial growth of track possibilities in cases where many points lie within the searching volume. The observed relative frequency of N points appearing within the search volume is shown in Table 5 for the data sample used in this experiment.

5) Initial Track Fits

Each combination of space points assembled by the track recognition programs is passed to a fitting package which estimates the track parameters using the approximation of a uniform field in the $\hat{z}$ direction. In this approximation the track is a helix so that the track can be completely parameterized by fitting the points in the X,Y plane to a circle and by fitting the arclength (as measured from the point of closest approach

N	P(N)
0	.007
1	.900
2	.076
3	.010
4	.005
≥5	.002

$P(\text{missing a track}) = 6 \times (.007)^2 \sim 3 \times 10^{-4}$
 $P(>1 \text{ point on some chamber}) \sim 4 \times (.095) \sim .38$
 $P(\text{extra track from doubling on 3 chambers}) \sim 4 \times 10^{-3}$

Table 5. Probability of Finding N Points Within the Tracking Tolerances

in X,Y to the origin) vs Z to a straight line. The resultant parameters are

(X_0, Y_0) point of closest approach to the origin in the XY plane

ρ radius of curvature of the circle

Z_0 Z at the point (X_0, Y_0)

$\tan\lambda$ tangent of the angle from the XY plane.

a) Circle Fit

The equations for a least-square fit to a circle are

$$X^2 = \sum_i (x_i - \rho)^2 \quad r_i = [(X_1 - X_0)^2 + (Y_1 - Y_0)^2]^{1/2}$$

$$\frac{\partial X^2}{\partial \rho} = 0, \quad \frac{\partial X^2}{\partial X_0} = 0, \quad \frac{\partial X^2}{\partial Y_0} = 0$$

$$\rho = \frac{\sum x_i}{N}, \quad X_0 = \frac{\sum x_i}{N} - \frac{\rho}{N} \sum \frac{x_i - X_0}{r_i}, \quad Y_0 = \frac{\sum y_i}{N} - \frac{\rho}{N} \sum \frac{y_i - Y_0}{r_i}$$

The equations are highly coupled, and there is no convenient way to iterate them. The problem of fitting points to a circle has been solved by Ascoli. The method consists of minimizing

$$\begin{aligned} X_A^2 &= \sum_i \left[\frac{r_i^2 - \rho^2}{2\rho} \right]^2 \\ &= \sum_i (x_i - \rho)^2 \left[\frac{r_i + \rho}{2\rho} \right]^2 \\ &= \sum_i (x_i - \rho)^2 \left[1 + \frac{r_i - \rho}{2\rho} \right]^2 \\ &= \sum_i (x_i - \rho)^2 \left[1 + \frac{(x_i - \rho)^2}{4\rho^2} \right] \end{aligned}$$

Note that the altered χ_A^2 differs from the true χ^2 by a term of order $\chi^2/4\rho^2$

$$\chi_A^2 = \chi^2 \left[1 + O\left(\frac{\chi^2}{4\rho^2}\right) \right].$$

The equations for minimizing χ_A^2 require

$$\frac{\partial \chi_A^2}{\partial \rho^2} = 0, \quad \frac{\partial \chi_A^2}{\partial \bar{X}_0} = 0, \quad \frac{\partial \chi_A^2}{\partial \bar{Y}_0} = 0.$$

These equations can be considerably simplified if one makes a transformation of coordinates, with $\bar{X}$, $\bar{Y}$ the average values of the

$$\begin{aligned} X_L, Y_L: \quad X' &= X - \bar{X} \\ Y' &= Y - \bar{Y}. \end{aligned}$$

One can also rotate the coordinate system such that

$$\overline{X'Y'} = 0$$

the angle of rotation being determined by

$$\cot 2\theta = \frac{\overline{X'^2} - \overline{Y'^2}}{2\overline{X'Y'}}.$$

The equations are now

$$(1) \quad \frac{\partial \chi_A^2}{\partial \rho^2} = \sum_i \left(\frac{r_i^2 - \rho^2}{4\rho^2} \right) (X - X_0)(-4) = 0$$

$$(2) \quad \frac{\partial \chi_A^2}{\partial Y_0} = \sum_i \left(\frac{r_i^2 - \rho^2}{4\rho^2} \right) (Y - Y_0)(-4) = 0$$

$$(3) \quad \frac{\partial \chi_A^2}{\partial \rho^2} = \frac{-1}{4\rho^2} \left[\sum_i \frac{(r_i^2 - \rho^2)^2}{\rho^2} + 2 \sum_i (r_i^2 - \rho^2) \right] = 0$$

from equation (3), neglecting terms of order $\chi^2/4\rho^2$ with respect to 1

$$\sum (x_i^2 - \rho^2) = 0, \text{ so } \rho^2 = \frac{\sum x_i^2}{N}$$

and since $\bar{X} = \bar{Y} = \overline{XY} = 0$

$$\begin{aligned} \rho^2 &= \frac{1}{N} \sum (X - X_0)^2 + (Y - Y_0)^2 \\ (4) \quad &= \bar{X}^2 + X_0^2 + \bar{Y}^2 + Y_0^2 . \end{aligned}$$

Equations (1) and (2) yield

$$(5) \quad \bar{X}^3 + \overline{XY}^2 - 2X_0\bar{X}^2 = 0$$

$$(6) \quad \bar{Y}^3 + \overline{XY}^2 - 2Y_0\bar{Y}^2 = 0 .$$

Equations (4), (5), (6) can easily be solved for X_0 , Y_0 , and ρ .

Using X_0 , Y_0 , and ρ , χ^2 can be estimated, and the solution can be corrected for the neglect of the terms of order $\chi^2/4\rho^2$. After the final X_0 , Y_0 , ρ have been determined, each crossing is required to be less than 3 standard deviations from the fit. If any crossings are found which do not fall within 3σ , the worst one is deleted from the track and the fit is redone and rechecked. The purpose of the point deletion is to eliminate crossings which, though they fall within the large tolerances of the recognition programs, do not properly belong to the rest of the track segment.

b) Z Fit

Since the resolution of the spark chambers is 10-20 times worse in Z than in XY, it is sufficient to ignore the errors in the radius of curvature from the XY fit and use it to calculate arclength and fit arclength vs Z to a straight line. This eliminates the need for a simultaneous non-linear fit of the points

to a helix. Thus, the fit for Z_0 (the origin in Z) and $\tan\lambda$ (the tangent of the dip angle) uses

$$Z_i = Z_0 + S_i \tan\lambda$$

where

$$S_i = 2\rho \sin^{-1} \left(\frac{L_i}{2\rho} \right)$$

$$L_i = [(X_i - X_0)^2 + (Y_i - Y_0)^2]^{1/2}$$

with X_0, Y_0, ρ determined by the XY fit. The method is to minimize

$$\chi_Z^2 = \sum \frac{[Z_i - (Z_0 + S_i \tan\lambda)]^2}{\sigma_i^2}$$

where σ_i is used to account for the different resolution on the 4° and 2° spark gaps. The solutions of the minimization equations are

$$\tan\lambda = \frac{\left(\sum \frac{Z_i S_i}{\sigma_i^2} \right) \left(\sum \frac{1}{\sigma_i^2} \right) - \left(\sum \frac{Z_i}{\sigma_i^2} \right)}{D}$$

$$Z_0 = \frac{\left(\sum \frac{S_i^2}{\sigma_i^2} \right) \left(\sum \frac{Z_i}{\sigma_i^2} \right) - \left(\sum \frac{S_i}{\sigma_i^2} \right) \left(\sum \frac{Z_i S_i}{\sigma_i^2} \right)}{D}$$

where

$$D = \left(\sum \frac{S_i^2}{\sigma_i^2} \right) \left(\sum \frac{1}{\sigma_i^2} \right) - \left(\sum \frac{S_i}{\sigma_i^2} \right)^2$$

Again if there are crossings on the track which are more than 3 standard deviations from the fit, the worst one is removed from the track and the Z fit redone. This procedure is repeated until all points are satisfactory. Finally because of the coupling

between the XY fit and the Z fit, if crossings are eliminated due to the cut in Z, the track is refitted in XY and Z without deletions. After the completion of both the XY and the Z fit, each track is required to have points on a minimum of 3 of the 4 chambers. All other tracks are eliminated from further consideration.

6) Elimination of Duplicate Tracks

The track recognition programs and fitting programs produce a sample of tracks containing most of the point combinations within 3 standard deviations of the true track. Thus multiple sparking or spurious sparks in the vicinity of the track can lead to several tracks being found in that region. These tracks share the same points on all chambers where there was no multiple choice of space points within 3σ . The probability of a spurious spark is small since the volume is only $(3\sigma_x)(3\sigma_y)(3\sigma_z) = (.15 \text{ cm})^2(1.7 \text{ cm}) = .04 \text{ cm}^3$. Multiple sparking, on the other hand, can be a serious problem, depending upon the condition of the spark chambers at the time the data were taken. The probability that more than one point will be reconstructed in the vicinity of the true spark can be estimated by the number of points found in recognized elastic events and is found to be $\sim .15/\text{chamber}$. (See also Table 5.)

Selection of the best track from the duplicate sample is handled by the program DUMPDUP. The basic philosophy is that

no two tracks are allowed to share a point unless

1. The tracks are oppositely charged
2. The shared point is on chamber 4.

The purpose of the exceptions are to allow the program to recognize pair conversion in the material before the spark chambers. The justification of this philosophy is that the probability that any two real tracks will share a point is the product of the probability that they cross within $(.04)\text{cm}^3$ and that they together produce only one space point. Since the total active volume of the chambers is approximately

$$4 \times 100 \text{ cm} \times (2\pi \times 100 \text{ cm}) \times 2 \text{ cm} \approx 5 \times 10^5 \text{cm}^3,$$

the probability that tracks which are not correlated in angle would cross is of order 10^{-7} .

The criteria for selecting tracks from duplicate sets is:

1. find the set of tracks with the largest number of degrees of freedom;
2. select the track with the best χ^2_{xy} ;
3. for 3-point tracks ($\chi^2_{xy} = 0$) select the track with the best χ^2_z .

For approximately 1% of the events, there are multiple sparks on 3 or more chambers. In this situation it is possible for the reconstruction programs to find two tracks which do not share any points. These situations are easily recognized by the fact that the separation in the xy plane of the two tracks reconstructed is very small on each of the 4 chambers. Thus

the pair-wise separation in space is calculated. The average,

$$S_{12} = \frac{\sum_{i=1}^2 R_i \Delta\phi_i}{4}$$

where $\Delta\phi_i$ is the angle difference between the two tracks on the i-th chamber normalized to $-\pi \leq \Delta\phi_L \leq \pi$, is required to satisfy

$$S_{12} > .04 \text{ m}$$

For a pair of tracks which fails this test, the track with the least number of degrees of freedom is eliminated. This criterion is found to affect 2% of the $e^+e^- \rightarrow e^+e^-$ events and about 4% of the hadron events. It is easy to Monte Carlo the effect of this correction. Though the resolution of the spark chambers is of the order of half a wire spacing, determined by the timing resolution of pulses on the magnetostrictive wands, the width of the pulses can cause two pulses separated by less than a centimeter to merge into a single pulse. Thus the spark chambers do not always resolve two real pulses with small separations, and this cut eliminates the necessity of a detailed investigation of the spark chamber performance in the presence of two tracks with very small separations.

7) Vertex Determination

In order to determine the event type, Bhabha, multi-hadron, cosmic ray, etc., it is necessary to examine the vertex topology of the event. To do this, the tracks which remain after duplication elimination are separated into primary and secondary tracks. A

primary track must satisfy

1. $R_{\min} \leq .15 \text{ m}$
2. $Z \leq .6 \text{ m at } R_{\min}$

where $R_{\min}$ is the radius of closest approach of the track to the beam axis. Tracks not satisfying the above criteria are classed as secondaries. The primary group includes tracks from beam-beam events, beam-gas events, beam-wall events, and clean two-prong cosmic rays passing within the .15 m radius about the beam. The secondary group consists mainly of background tracks from outside the interaction region including cosmic ray knock-on electrons and tracks from extensive cosmic ray showers. It may, however, include some decay products from long-lived particles produced in beam-induced events such as K_S^0 , or back-scatters from interactions of produced particles in the material surrounding the interaction region ($\bar{p}$ annihilations in the coil, etc.).

There may also be background tracks within the sample of primary tracks as for instance would be the case for an event which contains both a beam-induced event and a cosmic ray, or a beam-induced event with a machine-associated background track. Old tracks can be left over from a previous event due to the finite clearing time of the chambers, but these tracks are eliminated by the requirement of a 3σ fit of the points to the track since the EXB drift of the ions in the chamber is much larger for these tracks than for the event-associated tracks. The probabilities for these contaminations can be estimated

by looking at events with a recognized $e^+e^- \rightarrow e^+e^-$ and searching for additional tracks not associated with interactions of the final state particles with the chamber material. This occurs in fewer than 2% of the events.

a) First Estimate of Vertex Position

The following methods are used to estimate the location of the vertex formed by the tracks from the primary group:

1. If there is one primary track the point of closest approach is used as the vertex.
2. If there are 2 primary tracks which have $\Delta\phi \leq 20^\circ$ or $\Delta\phi \geq 160^\circ$, the average of their points of closest approach is taken as the vertex.
3. If there are 2 primary tracks with $20^\circ < \Delta\phi < 160^\circ$, then the vertex is found by finding the point which minimizes the sum of the perpendicular distances to each track in three dimensions, using a χ^2 minimization procedure. Weights were assigned based on individual track fits, taking into account position resolution and multiple scattering.
4. If there are 3 primary tracks and 2 within .02 m of the origin, method 3 is used. The initial estimate for the vertex location in this minimization is the average of the 3 points of closest approach (unweighted centroid of the three tracks).

5. If there are more than 2 tracks within .15 m of the origin and all of these also fall within .02 m of the beam axis, the method is the same as method 3.
6. All other cases use method 3 except that the initial estimate is the average of the pairwise intersections of the tracks.

The radial distribution of the event vertices is not Gaussian but indicates that the resolution of the vertex location is 1.5 cm.

Possible secondary vertices are found by locating all projected pair-wise intersections of the secondary tracks. Each of these intersections is recorded, though they were not used in the classification of hadron events or hadron multiplicities.

8) Constrained Vertex and Track Parameter Fit

Each vertex which did not consist of a pair of tracks with $\Delta\phi \leq 20^\circ$ or $\Delta\phi \leq 160^\circ$ and whose estimated radial position lay within .15 m of the beam axis was refitted using a procedure which allows for the simultaneous minimization of the χ^2 of all the tracks from the vertex using the constraint that the tracks come from a common vertex. By parameterizing each track in terms of its initial momentum components at the vertex and by using a third order Runge-Kutta integration routine along with the magnetic detector's fitted field, all corrections due to field non-uniformity can be made. This procedure is limited only by the

finite step size of the integration and the precision of the field representation. The additional constraint of a common vertex for each of the tracks increases the effective sagitta for the tracks, and since the momentum resolution depends on the square of the sagitta, the improvement in resolution can be substantial. Further, the circle fitting programs minimize a χ^2 which is

$$\chi^2 = \sum_{i=1}^{\text{\#points}} \frac{(x_{\text{point}} - x_{\text{track}})^2}{\sigma_x^2} + \frac{(y_{\text{point}} - y_{\text{track}})^2}{\sigma_y^2}$$

and thus do not properly account for the fact that the spark chamber points are constrained to lie on the cylindrical surface of the chambers.

In the constrained vertex fit, the χ^2 is formed from

$$\sum_{i=1}^{\text{\#points}} \frac{R_{\text{chamber}}^2 (\phi_{\text{point}} - \phi_{\text{track}})^2}{\sigma_{R\phi}^2}$$

where $\sigma_{R\phi}^2$ is determined by the wire spacing of the chambers.

By adjusting the relative weights of the tracks in inverse proportion to the tracks RMS multiple scattering at the various chamber radii, it is also possible to account for multiple coulomb scattering effects. The entire correlation matrix of the fit is available for possible kinematic fitting.

The constrained vertex-fitting procedure is based on an algorithm developed by D. E. Fries⁽¹⁾ for analysis of streamer chamber data at the Stanford Linear Accelerator Center. The major modifications are the inclusion of the multiple scattering

error matrix for the magnetic detector's distribution of material, the alteration of the definition of the χ^2 as noted above, and the rewriting of the Runge-Kutta integration routine to account for the specific geometry of the magnetic detector. The resultant resolution in the vertex location is now ± 0.6 cm.

C. Event Classification

Using the vertex information, track parameters, and counter information, the events were sorted into event classifications to select the events of interest. Each event was given a classification number (ECODE) and within each classification various sub-classifications (SUBTYP) were made.

The first step in this procedure was to compare all pairs of tracks and test them for collinearity. To specify collinearity, a cut was made on the angle between the two tracks and on the difference of the radii of closest approach. The angular cut was 10° , and for tracks with $R_{ij} < .15$ m, the cut on the difference of the radii was .04 m where R_{ij} is the root mean radius of closest approach

$$R_{ij} = \frac{1}{2} (R_i^2 \min + R_j^2 \min)^{1/2}$$

If $R_{ij} > .15$ m, the radius cut was

$$\frac{.04}{.15} R_{ij}.$$

If neither prong was identified as an electron by its pulse height (pulse height > 50), and if the timing in the TOF

counters was consistent with cosmic ray timing, the event was classed as cosmic ray induced. With no times recorded, both particle minimum ionizing, the classification was also cosmic. If either track was identified as an electron, the event is a Bhabha candidate.

Following the search for collinear pairs, all events not classed as cosmics, containing more than 4 tracks outside of the .15 m fiducial volume, more than 10 shower counters without any recognized charged tracks, and more than 100 space points were classed as "Beam Burps." These events may be due to sudden losses of large numbers of electrons from the stored beams. These losses are infrequent and would not be a serious problem for stability of the beams since the fraction of the beam which is lost is very small. There is also a possibility that some of these events may be associated with the operation of experiments in the SLAC end station A at SLAC which is adjacent to the SPEAR site.

For the final classification of events not classed as cosmic rays, the fiducial cuts for the interacting region were $R \leq .04$ m, $|Z| \leq .6$ m. Events originating from $.04$ m $< R \leq .06$ m come from a region with no sources and were classified as unknowns. The region $.06 < R \leq .10$ m contains the vacuum pipe, and events from this region were placed in background classes along with events from $R > .10$ or $|Z| > .6$.

1) 1-Prong Events

In order to recognize Bhabha or mu pair events in which only

one track was found, due to inefficiencies in the spark chambers or tracking programs, all events which had a single prong within the interaction region with momentum

$$.75 E_{\text{beam}} \leq P \leq 1.5 E_{\text{beam}}$$

were analyzed in terms of trigger counter and shower counter information alone. The presence of a shower counter and trigger counter with beam timing and opposite the found track was sufficient to classify the event as ECODE 3, SUBTYPE 5 (see Table 6 for event classes) or ECODE 4, SUBTYPE 2 depending on the pulse height in the shower counter. Approximately 2% of the total Bhabha or mu pair events fell into these classifications. All other 1-prong events were classified as illegal triggers ECODE 0, SUBTYPE 5.

2) 2-Prong Events

The two-prong events were sorted into collinear and non-collinear classes with the collinearity cuts defined as in the original search for collinear cosmic ray pairs. Collinear tracks with electron-like pulse heights were classed as Bhabhas independent of the timing information, while timing information had to be used to separate mu-pairs and cosmic rays. Non-collinear events with vertices in the vacuum pipe region, large R region, large Z region and unknown region were sorted into the respective event classes. The remaining non-collinear events originating from the interaction region were further examined to

Table 6. Event Classes

<u>Event Code</u>	<u>Subtype</u>
0 Garbage	0 No vertex found 1 <2 tracks not hitting posts 2 illegal trigger
1 Cosmic	0 2-prong, cosmic timing 1 2p, no TDC information 2 >2p with a cosmic pair
2 Wall	0 2p, vertex at vacuum chamber 1 >2p, vertex at vacuum chamber 2 1p, vertex at vacuum chamber 3 vertex outside Z cut 4 vertex outside radial cut
3 QED, EE	0 2p, large showers, good TDC 1 2p, large showers, bad TDC 2 >2p, with collinear EE pair 3 2p, one large shower
4 Mu Pair	0 collinear 2p, low pulse height, good timing 1 $\mu\mu$ or non-collinear $\mu\mu$ candidates
5 Hadron	0 >2p 1 2p non-coplanar 2 2p charge ± 2
6 Unknown	0 vertex fit failure 1 1p from $4 < R < 6$ cm 2 2p, non-coplanar, $4 < R < 6$ cm 3 vertex lies at $R > 10$ cm 4 $\geq 3p$, $4 < R < 6$ cm 5 2p, coplanar, $4 < R < 6$ cm 6 collinear pair with no TDC in a multiprong 7 collinear or coplanar π pair with $0 < R < 2$ cm

determine whether they were radiative QED events, hadron events, or unknowns. Events of the type $e^+e^- \rightarrow \gamma\gamma$ can be detected in the magnetic detector if one of the γ 's converts in the vacuum pipe. These events can be found by looking for the converting pair, and requiring a shower counter opposite the pair. Such events were classed as ECODE 3, SUBTYPE 4. All non-coplanar events ($\Delta\phi > 20^\circ$) where both particles were not electrons were classified as 2-prong hadron events. Coplanar events with recognized electrons or muons were placed in the electron and muon classes, and all other coplanar 2 prongs were classified in the unknown ECODE 6, SUBTYPE 7.

3) 3 or More Prongs in the Interaction Region

Events with two of the tracks satisfying the requirement for identification as $e^+e^- \rightarrow e^+e^-$ or a cosmic ray were classified as ≥ 3 -prong ee or ≥ 3 -prong cosmic events. The remaining events were all classified as hadron events with the exception of those events for which both the initial vertex minimizations and the final constrained vertex fit failed. A visual scan of events with these vertex failures was undertaken to verify that they were all background.

4) Verification of Event Identification

An event display program was written which produced pictures of the reconstructed events. The pictures produced by this program showed the status of all the trigger and shower counter latches,

the positions of the vacuum chamber and spark chamber support posts, the reconstructed space points, the trajectories of the tracks found, the location of the event vertex, and the final event identification. The events could be viewed in the x,y projection, the y,z projection, or the x,z projection, or in all three projections simultaneously. These pictures could be made on a variety of media including on IBM 2250 display terminal, Tektronix 4013 terminal, microfilm cassette, or microfiche. In addition, the IBM 2250 version allowed interactive control of the analysis program and could supply the viewer with any of the track or vertex parameters assembled by the analysis program. Under control of the console, tracks and/or vertices could be removed, event identifications changed, or parameters of the analysis varied and the resultant modifications in identification viewed.

Using this display console, a sample of data containing approximately 1000 Bhabha events was scanned at each energy and a comparison made of the program's identification and the scanner's identification. Misidentification of Bhabha events used for normalization was found to be less than .5%. The corresponding percentage of events found by the scanners for 2-prong hadrons and multiprong hadrons are shown in Table 7.

Event Type	C.M. Energy (GeV)		
	3.0	3.8	4.8
2-prong hadron	-12 $\pm$ 6	-7 $\pm$ 4	-6 $\pm$ 4
$\geq$ 3-prong hadron	1.5 $\pm$ 1.0	2.9 $\pm$ 1.5	0.7 $\pm$ 0.7

Table 7. Percentage Correction to Number of Events Found by
Hand Scan

III. Monte Carlo Simulation of the Detector

In order to calculate cross-sections from the observed number of hadronic events and their momentum and angular distributions, it is necessary to understand the response of the detector to the produced particles. Since the detector does not cover the entire 4π solid angle and since the types of particles in the final state are not well known, this procedure is necessarily model dependent. Models which predict large numbers of particles entering the missing solid angle will of course yield a lower estimate of the efficiency than invariant phase space models. Similarly, production of large numbers of particles with masses greater than that of the pion would reduce the available kinetic energy in the final state, thus changing the shape of the momentum spectrum. The parameters of the models are not arbitrary, however, since they must all reproduce the same observed momentum and angular distributions within the detection region.

A. Detector Response

To estimate detection efficiencies, the Monte Carlo simulation must combine the appropriate model for the production of hadronic states with an estimate of the response of the detector to those states. Given that a produced particle lies within the detector solid angle, the response of the detector is model dependent only because the particle type is unknown. But since the raw data contains the correct spectrum of particle types to first order, by

using the data to estimate those efficiencies which depend upon particle type, some of this dependence can be removed. This method was used to find the efficiency for forming trigger and shower counter combinations. The same argument can be applied to the other efficiencies such as that for the pipe counter and spark chambers although in these cases there is very little dependence on particle type.

1) Determination of the TASH Efficiency

The major factor determining the response of the detector is the efficiency for the formation of trigger-associated shower (TASH) combinations, two of which are required in the hardware trigger. The software analysis of the events requires in addition that there be at least two reconstructed tracks responsible for the valid TASHes. This requirement eliminates the dependence of the hardware trigger on the formation of TASHes by background tracks or neutral particle conversions. The TASH efficiency, $\epsilon_{\text{TASH}}(p, z)$, is determined by many complicated effects which are difficult to estimate since it depends strongly on the nuclear absorption cross-section of the produced particles in the material of the detector and on the response of the trigger and shower counters to the secondaries produced in the nuclear interactions. The problems inherent in calculating this efficiency can be avoided however since the efficiency can be determined directly from the available experimental data.

Using all tracks in the sample of recognized hadronic events with at least 3 prongs, a trigger-associated shower is defined to have fired if both the trigger counter pointed to by the track and either of that trigger counter's two adjacent shower counters have fired. For each event, a count is kept of how many charged tracks form TASHes and how many do not. If the event has only two TASHes associated with tracks, these are not counted as successful firings since they were required for the event to trigger. Failures are counted for all other tracks in such events. Events with fewer than two TASHes are illegal triggers since the trigger was not formed from charged tracks in the event, but rather was due to background tracks or the conversion of neutral particles. These events are not used. In events with more than two TASHes associated with charged tracks, successes are counted for each track with a TASH and failures for each track without a TASH.

Now suppose one had an event with N charged prongs, and suppose that the TASH efficiency of the i -th prong is $\epsilon(p_i, z_i)$ where p_i is the momentum and z_i is the point at which the track hits the trigger counter. Then the probability that we could count a success for prong i using the above method is

$$\begin{aligned} P_S &= \epsilon(p_i, z_i) \times P(\text{at least 2 other TASHes}) \\ &= \epsilon(p_i, z_i) \times (1 - P_{N-1,1} - P_{N-1,0}) \end{aligned}$$

where $P_{N-1,m}$ is the probability of m TASHes from the other $N-1$ prongs.

$$P_S = \prod_{k=1}^m \epsilon_k(p_k, z_k) \prod_{j=1}^{N-1-m} [1 - \epsilon(p_j, z_j)].$$

Similarly, the probability of counting a failure is

$$\begin{aligned} P_F &= [1 - \epsilon(p_i, z_i)] \times P(\text{at least 2 other TASHes}) \\ &= [1 - \epsilon(p_i, z_i)] (1 - P_{N-1,1} - P_{N-1,0}). \end{aligned}$$

For each ensemble of events in the data with N prongs and parameters (p_i, z_i) the estimate of the efficiency is given by

$$\begin{aligned} \epsilon_{\text{est}} &= \frac{\# \text{successes}}{\# \text{successes} + \# \text{failures}} \\ &= \frac{\epsilon(p_i, z_i) (1 - P_{N-1,0} - P_{N-1,1})}{\epsilon(p_i, z_i) (1 - P_{N-1,0} - P_{N-1,1}) + [1 - \epsilon(p_i, z_i)] (1 - P_{N-1,0} - P_{N-1,1})} \\ \epsilon_{\text{est}} &= \epsilon(p_i, z_i). \end{aligned}$$

The above derivation assumes that no two tracks have the same parameters (p_i, z_i) , but it is easy to show that the result does not depend on this assumption. Thus, for each ensemble in the data, the method provides the correct estimate of the efficiency. Events in which a shower counter was hit by more than 1 track were eliminated from the sample since this would increase the efficiency of that TASH. The remaining sample contained 6503 events and 15714 total prongs for full field runs and 5743 events with 13512 total prongs for half field runs. The resultant histogram together with the smoothed functions used in the Monte Carlo are shown in Figs. 9-13. The calculation of the TASH efficiency from particle absorption is discussed in Appendix II

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Fig. 9

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Fig. 10

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Fig. 11

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Fig. 12

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Fig. 13

and the result of such a calculation is compared to the measured values in Fig. 14.

2) Hardware Trigger, PASS 1 Filter, and PASS 2 Simulation

If the total number of charged particles in the final hadronic state is known, the hardware trigger efficiency can be easily simulated using the measured efficiency for the pipe counter and the TASH efficiency. Though it is possible for a neutral particle to assist a charged particle in the formation of a TASH, neutrals in the simulation were not allowed to affect the latch status of unlatched shower counters associated with charged tracks. This effect is already in the measured TASH efficiency. Thus neutral particles which do not decay or convert have no effect on the triggering efficiency. The charged decay products of neutrals are considered along with the produced charged particles in the simulation of TASH formation. Photons were allowed to convert to e^+e^- pairs in the beam pipe and pipe counter with spectrum given by electromagnetic theory.⁽²⁾ Dalitz decay of produced π^0 's with a branching ratio of 1.17%⁽³⁾ was considered in addition to the usual two photon decay mode. Produced η 's were allowed to decay with the following ratios

$\gamma\gamma$	38.0%
$\pi^0\pi^0\pi^0$	30.0%
$\pi^+\pi^-\pi^0$	23.9%
$\pi^+\pi^-\gamma$	5.0%
$\pi^0\gamma\gamma$	3.1%

CALCULATED TASH EFFICIENCY

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Fig. 14

Half of the produced K^0 's were treated as K_L the decay modes of which were not simulated. The branching ratio for $K_S^0 \rightarrow$ charged particles used was 68.81%.

The additional criteria of the PASS 1 filter were implemented by applying the same cuts to the simulated events. Measured spark chamber efficiencies were used to predict the number of space points, and these points were used to test for a minimum number of sparks and a minimum number of "roads." Since the TDC's on each end of the trigger counters were stopped in the experiment by the first pulse after the event start pulse, it was possible for an apparently early time to be created when several particles strike the same trigger counter. In simulating the TDC's, the arrival time of a pulse at each tube was used in cases where there were more than one. Following the TDC simulation, tests were made to determine if events would have been eliminated by the early time or cosmic ray criteria.

To implement the criteria of the PASS 2 analysis, errors were applied to the produced momenta and angles of each track using the experimentally determined resolutions. The minimum requirements of space points and track separation were applied to determine which of the charged tracks would be found by the track recognition programs. Coplanarity and collinearity cuts were applied to all resultant two-prong events, and each event was required to have at least two prongs not hitting spark chamber support posts.

B. Models

1) Simple Model of the Detector Response

By taking a simplistic physics model and combining it with an approximation of the detector's performance, it is possible to calculate the detection efficiency. Monte Carlo simulations with more complex models give estimates of the magnitude of the model dependence of the efficiency. The simplest model of the produced states is that in which the particles are produced according to invariant phase space. By combining this model of the production of the final state particles with various assumptions about the detector's performance, one can come to a qualitative understanding of the degree to which the efficiency depends on the detector parameters. Combining these measures of the sensitivity of the efficiency to the model and the detector with limits on the variability of these parameters yields the uncertainty in the efficiency.

The probability that a particle will fall within the solid angle of the detector and form a trigger-associated shower combination (TASH) is

$$\epsilon = \frac{\Omega}{4\pi} \epsilon_{\text{TASH}}(p)$$

$$\Omega = 4\pi(.65)(.92)$$

where the calculation of the solid angle includes the exclusion (.92) of regions shadowed by the spark chamber support posts.

For an event with N prongs produced, we can approximate $\epsilon_{\text{TASH}}(p)$

by $\epsilon_{\text{TASH}}(\langle p \rangle_N)$ where $\langle p \rangle_N$ is the average momentum of particles in an N-prong event and is approximately

$$\langle p \rangle_N \approx \frac{E_{cm}}{N}.$$

We can now calculate the probability of forming two TASHes.

$$\begin{aligned} P(N,m) &= \text{detection efficiency for } N \text{ prongs,} \\ &\quad m \text{ of which are charged} \\ &\approx 1 - (1 - \epsilon_N)^m - m\epsilon_N(1 - \epsilon_N)^{m-1} \end{aligned}$$

where

$$\epsilon_N = (.65)(.92)\epsilon_{\text{TASH}}\left(\frac{E_{cm}}{N}\right).$$

Results of this simple model are shown in Tables 8-11. States with all neutral pions are $C = +1$ and thus do not contribute. The state with two charged pions only would be eliminated by the coplanarity cut so that its efficiency is zero. The effect of the coplanarity cut on higher multiplicity states has not been considered.

The efficiency for detecting events, $\bar{\epsilon}$ and the efficiency for detecting particles of momentum p , $\epsilon(p)$ can now be calculated if we assume a multiplicity distribution. As an example, let the multiplicity have a Poisson distribution with mean 6, 7.4, 8.7 at $\sqrt{s} = 3.0, 3.8, 4.8$. Further, let the various allowed charge states within each multiplicity be equally populated. Then

Table 8. Simple Model Detection Efficiency for N-Prong Events
with m-Charged Prongs $P(N,m) \sqrt{s} = 3.0 \text{ GeV}$

$P(N,m)$ N/M	2	4	6	8	10	12	14	16	18	20
2	0.0									
3	0.32									
4	0.30	0.75								
5	0.26	0.71								
6	0.24	0.67	0.88							
7	0.22	0.63	0.85							
8	0.20	0.60	0.83	0.93						
9	0.18	0.57	0.81	0.92						
10	0.17	0.54	0.78	0.90	0.96					
11	0.15	0.51	0.75	0.88	0.95					
12	0.14	0.48	0.72	0.87	0.94	0.97				
13	0.12	0.43	0.67	0.83	0.91	0.96				
14	0.10	0.39	0.62	0.78	0.88	0.94	0.97			
15	0.09	0.35	0.58	0.74	0.85	0.91	0.95			
16	0.08	0.32	0.54	0.71	0.82	0.89	0.94	0.96		
17	0.07	0.29	0.50	0.67	0.79	0.87	0.92	0.95		
18	0.06	0.26	0.47	0.64	0.76	0.84	0.90	0.94	0.96	
19	0.06	0.24	0.44	0.60	0.73	0.82	0.88	0.92	0.95	
20	0.05	0.22	0.41	0.57	0.70	0.79	0.86	0.91	0.94	0.96

Table 9. $P(N,m)$ for $\sqrt{s} = 3.8 \text{ GeV}$

$P(N,m)$ N/M	2	4	6	8	10	12	14	16	18	20
2	0.0									
3	0.32									
4	0.32	0.77								
5	0.30	0.75								
6	0.27	0.71	0.91							
7	0.25	0.69	0.89							
8	0.23	0.66	0.87	0.96						
9	0.21	0.63	0.85	0.95						
10	0.20	0.60	0.83	0.93	0.98					
11	0.19	0.58	0.82	0.92	0.97					
12	0.17	0.55	0.79	0.91	0.96	0.99				
13	0.16	0.53	0.77	0.90	0.95	0.98				
14	0.15	0.51	0.75	0.88	0.95	0.98	0.99			
15	0.14	0.49	0.73	0.87	0.94	0.97	0.99			
16	0.13	0.45	0.69	0.84	0.92	0.96	0.98	0.99		
17	0.11	0.41	0.65	0.81	0.90	0.95	0.97	0.99		
18	0.10	0.38	0.61	0.78	0.87	0.93	0.96	0.98	0.99	
19	0.09	0.35	0.58	0.74	0.85	0.91	0.95	0.97	0.99	
20	0.08	0.32	0.55	0.71	0.83	0.90	0.94	0.97	0.98	0.99

Table 10. $P(N,m)$ for $\sqrt{s} = 4.8$ GeV

$P(N,m)$ N-m	2	4	6	8	10	12	14	16	18	20
2	0.0									
3	0.32									
4	0.32	0.77								
5	0.32	0.77								
6	0.30	0.76	0.93							
7	0.28	0.73	0.92							
8	0.26	0.71	0.90	0.97						
9	0.25	0.68	0.89	0.96						
10	0.23	0.66	0.87	0.96	0.99					
11	0.22	0.64	0.86	0.95	0.98					
12	0.21	0.62	0.84	0.94	0.98	0.99				
13	0.20	0.60	0.83	0.93	0.97	0.99				
14	0.19	0.58	0.81	0.92	0.97	0.99	1.00			
15	0.18	0.56	0.79	0.91	0.96	0.99	0.99			
16	0.17	0.54	0.78	0.90	0.96	0.98	0.99	1.00		
17	0.16	0.52	0.76	0.89	0.95	0.98	0.99	1.00		
18	0.15	0.50	0.75	0.88	0.95	0.98	0.99	1.00	1.00	
19	0.14	0.49	0.73	0.87	0.94	0.97	0.99	0.99	1.00	
20	0.13	0.46	0.70	0.85	0.92	0.96	0.98	0.99	1.00	1.00

Table 11. Average Defection Efficiency $\bar{\epsilon}$

$\sqrt{s}$ (GeV)	$\bar{\epsilon}$	
	Simple Model	All Pion Monte Carlo
3.0	.515	.454
3.8	.610	.536
4.8	.679	.615

$$\bar{\epsilon} = \sum_{n=2}^{10} \frac{e^{-\bar{n}} (\bar{n})^n}{n!} \left\{ \sum_m P(n, m) \right\} / \left\{ \sum_m 1 \right\}$$

$$\epsilon(p) \sim \frac{\sum_m P(n, m)}{\sum_m \frac{\left\{ \sum_{i=2}^m i \Omega^i (1 - \Omega)^{m-i} \frac{m!}{i! (m-i)!} \right\}}{1 - (1 - \Omega)^m - m (1 - \Omega)^{m-1}}}$$

$$m = 2, 4, 6$$

$$n = \frac{E_{cm}}{p}$$

$$\Omega = (.65)(.92)$$

The estimate of $\bar{\epsilon}$ using this simple model is about 6% lower than that of the full Monte Carlo calculation with the all pion model and a uniform angular distribution. The inclusive efficiency $\epsilon(p)$ differs by less than 8% (Fig. 15) everywhere between .3 and 1.0 GeV (the limits of the model due to the assumption that all prongs have the average momentum E_{cm}/N). The calculation using this simple model shows that these efficiencies are not very sensitive to the details of the particular model used, but rather are in the main determined by the effective solid angle $\left[\int d\Omega f(\Omega) \right]$ where $f(\Omega)$ is the angular distribution], the mean charged multiplicity and the energy available to charged particles (or equivalently the total multiplicity).

2) All Pion and Heavy Particle Models

In addition to modelling the response of the detector, the Monte Carlo calculation must provide a model for the physics. The simplest model is of course the invariant phase space model. Using

SIMPLE MODEL EFFICIENCY

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Fig. 15

this, one need only specify the charge, strangeness and baryon numbers of each particle (taking care to constrain the sum of each to zero) to complete the description of the final state. Since the efficiencies depend most heavily on the mean charged multiplicity and mean charged momentum, each model must be adjusted to match the multiplicity and momentum yields from the experiment. The following model parameters were chosen for the general case of invariant phase space production where the multiplicity is Poisson in all particles and binomial in the particle types.

MP1 - mean total multiplicity

MP2 - kaon fraction

MP3 - nucleon and anti-nucleon fraction

1-MP2-MP3 - pion and eta fraction

MP4 - fraction of π^0 's in pion-eta class

MP5 - fraction of η 's in pion-eta class

1-MP4-MP5 - $\pi^+\pi^-$ fraction in pion-eta class

MP6 - fraction of K^0 's in kaon class

MP7 - fraction of N's in nucleon class

The restriction $MP2=MP3=MP5=0$ reduces this model to an all pion model with two parameters MP1 and MP4. The detailed procedure for generating events in all models was as follows:

- 1) Determine center of mass energy and momentum using radiative corrections to order α^3 assuming no radiation from the final hadronic state. (4)

- 2) Pick a multiplicity between 3 and 20 from a Poisson distribution.
- 3) Determine the multiplicity from each of three classes, pion-eta, kaon, nucleon, as follows
 - a) if the multiplicity is odd, pick one from π -n class
 - b) continue picking pairs from each of the three classes in the ratio 1-MP2-MP3:MP2:MP3 until the full multiplicity is reached.
- 4) Within each class pick a charged or neutral particle depending on MP4, MP5, MP6, MP7, the charge being chosen so that the sum of the charges is zero
- 5) If the state contains all π^0 's (charge conjugation +1) discard the state and repeat step 3.
- 6) Events with a phase space weight of less than 1 part in 100,000 were eliminated since such events would not be observed in the experiment.

3) Jet Models

The all pion and heavy particle models generate states whose angular distributions are uniform in $\cos\theta$. Since a non-uniform angular distribution would change the effective solid angle of the detector, thus changing the triggering efficiencies, a jet model was devised with the same parameters as the all-pion model except that a jet axis is chosen with a distribution $1 + a\cos^2\theta$. The

transverse momentum of the particles relative to the jet axis is limited by using a matrix element squared of the form

$$|M|^2 = e^{-\Sigma} p^2/4R^2.$$

R is in principle adjustable but has been chosen to be .3 GeV/c in accordance with the usual limited transverse momentum behavior observed in hadronic interactions.

C. Inclusive Efficiencies

The inclusive efficiency is defined as the ratio between the number of charged hadrons seen in the detector and the total number produced. In the analysis of the inclusive cross sections, this efficiency must be approximated by the ratio of detected to produced hadrons based on one of the models. Further, the efficiency will generally have some dependence on each of the parameters of the track (momentum, angle, vertex) and on the global properties of the event (produced charged multiplicity, particle types, and angular distribution of produced prongs). The global properties of the events cannot be deduced on an event-by-event basis due to the limited solid angle and resolution of the detector. But since the raw data was found to be closely reproduced by the models, the global properties of the events could be accounted for by a simple average over all events. The use of this simplification is also justified by the simple model calculation, which indicates that the global properties of the model

(excepting angular distribution) affect the efficiency only at the 10% level. An additional simplification can be achieved by recognizing that since the bunch length is of the order of .1 meter, the effect of the vertex position is small and can be accounted for by putting the observed bunch length in the model calculation. We are thus left with an efficiency which in the single particle inclusive case can be expressed as a function of p and θ . For the two-particle case, the efficiency becomes a function of $p_1, p_2, \theta_1, \theta_2$.

1) Cuts for the Inclusive Spectra

a) Pulse Height Cut

The measured pulse height spectra for prongs from hadron events are shown in Fig. 16. The spectra for the regions $.45 \leq x < .9$ and $x \geq .9$ show an enhancement for prongs with pulse height greater than 75. (See reference 5 for typical electron and muon pulse height spectra.) Since possible contamination of the higher x regions with electrons from the reaction $e^+e^- \rightarrow e^+e^-$ would bias the angular distributions, a cut was made at a pulse height of 50. A correction factor is then made to the normalization of the distributions which is

$$\frac{\text{number of prongs } x < .45}{\text{number of prongs } x \leq .45 \text{ and pulse height } \leq 50.}$$

This correction represents a decrease in the total number of prongs relative to accepting all pulse heights of 1%.

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Fig. 16

b) Momentum Cut

Prongs with transverse momenta less than .090 GeV in the full-field data and .045 GeV in the half-field data will not reach the trigger counters and thus not participate in the trigger. Further, the requirement that all tracks hit at least 3 spark chambers in order to be found by the tracking programs dictates that the transverse momentum be at least .066 GeV with $B = 4$ Kg and .033 GeV with $B = 2$ Kg. Within the angular region $|\cos\theta| \leq .6$, this means that, for (full, half) field, prongs with total momenta greater than (.113, .056) GeV participate in the trigger, while tracking extends to (.083, .041) GeV. To the extent that the Monte Carlo model matches the lower end of the momentum spectrum, it can account for the fact that some of these prongs do not participate in the trigger. The behavior of the track recognition programs at these low momenta is, however, much less certain. Thus a cut was made requiring each prong to have a total momentum greater than .025 B.

c) Polar Angle Cut

The detector solid angle extends to approximately $|\cos\theta| = .65$. Measurements of the trigger counter and shower counter efficiencies show no change in the efficiency within $0 \leq |\cos\theta| \leq .6$ and a 30% drop between .6 and .65. In order to eliminate the necessity of parameterizing the inclusive efficiency as a function of both p and θ and to eliminate the need for detailed knowledge of the

positions of the counter edges in the detector coordinate system, a cut was made at $|\cos\theta| = .6$ on all prongs in the inclusive distributions.

2) All Pion Model Efficiency Results

The single particle inclusive efficiencies are shown in Fig. 17 as predicted by the all pion model. The corresponding average trigger efficiencies are given in Table 11. Since the angular distribution in the all pion model is uniform, the maximum possible value of the inclusive efficiency will be .6 due to the polar angle cut. Thus the remaining features of the model contribute an additional factor of .83 in the region where the momentum spectrum peaks. The drop in the efficiency at lower momenta is a reflection of the steep drop in the TASH efficiency for such prongs. It is not as severe, however, because the low momentum prongs tend to come from events with higher multiplicities so that the efficiency is not strongly dependent on the TASH efficiency of any one prong. At the upper end of the momentum spectrum one has two effects tending to decrease the efficiency. First, the average multiplicity for events with a high momentum prong will be lower than the average multiplicity for all events. Second, with a high momentum prong, the constraint of momentum conservation will increasingly constrain the other particles in the event to be approximately collinear with the high momentum prong. The probability that there will be two particles within

EFFICIENCY (ENERGY DEPENDENCE)

XBL 753-372

Fig. 17

the solid angle increases from $(\Omega/4\pi)^2$ to $\Omega/4\pi$ due to this effect, but all such events are eliminated by the coplanarity cut.

3) Model Dependence of the Inclusive Efficiency

The observed yields of particles versus the cosine of the production angle vary by less than 6% from $\cos\theta = 0$ to $|\cos\theta| = .6$ at all energies. Restricting the variation in the yield from the jet models within this region to 6% requires that $-.62 \leq \alpha \leq 1.0$ in the $1 + \alpha \cos^2$ distribution of jet axes. This is for a transverse momentum cutoff of .3 GeV. Changing the value of the transverse momentum parameter merely changes the degree of non-uniformity for a given value of α , and since it is the non-uniformity that results in a different estimate of the efficiency, the transverse momentum parameter was not changed. Further, the reason that the two limits on α are not symmetric is that what is important is the fractional change in the probability that a particle lies within the solid angle $|\cos\theta| \leq .6$. For a uniform distribution this probability is simply .6, but for a $1 + \alpha \cos^2\theta$ particle angular distribution it is given by

$$P = \frac{.6 + \frac{\alpha}{3} (.6)^3}{1 + \frac{\alpha}{3}}$$

The fractional change in the yield will be approximately

$$\frac{|P - .6|}{.6}$$

which is the same for $\alpha = -.62$ and $\alpha = 1.0$.

To investigate the model dependence of the efficiency, the inclusive efficiency was calculated using the all pion model, the heavy particle model, and the two jet models $\alpha = -.62$ and $\alpha = 1.0$. The resultant efficiencies at 2.4 GeV are shown in Fig. 18, with the model parameters given in Table 12. Similar results were obtained at 1.9 and 1.5 GeV. The efficiency from the all pion model was used to calculate the cross-sections and distributions, and the variations in the efficiency due to model dependence was added in quadrature with the statistical errors.

EFFICIENCY (MODEL DEPENDENCE)

XBL 753-971

Fig. 18

Model	All Pion			Heavy Particle			Jet $\alpha = 1.0$	Jet $\alpha = -.6$
	$\sqrt{s}$ (GeV)	3.0	3.8	4.8	3.0	3.8	4.8	4.8
MP1		6.05	7.40	8.65	6.05	7.40	8.65	8.45
MP2		0	0	0	.33	.33	.33	0
MP3		0	0	0	.33	.33	.33	0
MP4		.427	.462	.480	.427	.462	.480	.525
MP5		0	0	0	.427	.462	.480	0
MP6		0	0	0	.5	.5	.5	0
MP7		0	0	0	.5	.5	.5	0

Table 12. Monte Carlo Model Parameters

IV. RESULTS

A. Background Subtraction

During the collection of data, several processes can produce events which look like multihadron events but are not the result of electron-positron collisions. For this reason, data were taken at each of the three energies with only a single beam circulating in the machine, or with two beams which were separated in the interaction regions so that no collisions could occur. The events found in these runs are primarily the result of electro-production of hadrons off the residual gas in the interaction region. The raw multiplicities are approximately 2 to 3 and the event vertices are distributed uniformly in z within statistics. The fraction of background events within each run of a given set of data can vary by factors of order 2 or 3 due to dissimilarities of machine configurations, pressure in the interaction region, and detector conditions. Raw trigger rates increase by a factor of 3 in going from 10 ma stored current to 25 ma, and since the machine stores more current at the higher energies, background fractions increase with increasing energy. Also, the trigger rate is found to increase by a factor of 5 over the full field value when the solenoid is turned off indicating that a large fraction of the background is coming from low energy ($P < 100$ MeV) charged particles. Thus, the background fractions are higher in runs taken with the solenoid at half field, and in addition, the momentum spectrum of the background is different for half field runs, being much more strongly peaked at

low momenta. The z distribution of all of the background events found is shown in Fig. 19. It is consistent with a flat distribution in the range $-.4$ meters to $+.4$ meters.

It is possible to account for all of the variations in the background discussed above by taking advantage of the fact that the distribution of true annihilation events is gaussian with a standard deviation of approximately $.02\sqrt{s}$ meters. The fraction of background events can thus be found by fitting the z distribution of the hadron events to a constant plus a gaussian. These distributions together with the fitted curves are shown in Fig. 20 for the full field runs. The value of the constant can then be used to give the number of background events in the data sample between $-.4$ meters and $+.4$ meters. Momentum and angular distributions are taken from the background runs and multiplied by C_B , the ratio of the number of background events found in the real data (from the fit) to the number found in the background runs. These distributions are used to make background subtractions. The results of the fits together with the fraction of background events and C_B are shown in Table 13. A large fraction of the data at 1.5 GeV was taken in the "high eta" mode which allowed greater current to be stored in the machine, with corresponding increases in the available luminosity. For this reason, the background fraction is higher in the 1.5 data than it would have been in the normal operating mode.

Z DISTRIBUTION FROM BACKGROUND RUNS

Fig. 19

XBL 753-373

Z DISTRIBUTION FOR HADRON EVENTS

XBL 753-374

Fig. 20

Beam Energy (GeV)	Field (kg)	$\bar{z}$ (meters)	σ_z (meters)	# background events/total	C_B
1.5	2	-.02	.06	.065	16.0
1.5	4	-.02	.06	.036	5.82
1.9	2	-.02	.07	.048	2.33
1.9	4	-.02	.07	.024	6.74
2.4	2	-.02	.09	.107	33.3
2.4	4	-.02	.10	.081	9.41

Table 13. Background Determination

B. Momentum and Cos θ Yields

Figures 21-23 show the observed momentum distributions of all tracks. These show background-subtracted yields, raw background yields multiplied by C_B and the Monte Carlo predicted yield at full field, and each of the three energies. It can be seen that the phase space momentum spectra match the data qualitatively although at each of the energies the yield is rather less than the phase space prediction for the lowest momentum bin, and is higher for several bins past the peak. The simple model efficiency calculation indicates however that the efficiency calculation will not be affected by this difference since it is roughly independent of the detailed shape of the momentum spectrum. The requirement that the Monte Carlo give the same mean observed multiplicity and mean observed momentum as the data fixes the area under the curve and the first moment of the distribution. Adjustment of these parameters was done to $\pm 4\%$. The results are shown in Table 14.

The unsubtracted yield of particles versus cos θ and the background yield multiplied by C_B are shown in Figs. 24-26. The fractional background contamination is that given in Table 13 and thus is between 2% and 10%. The cos θ distribution in the background runs is not necessarily the same as that of the background in the data runs due to the presence of single beam background runs which must, of course, be asymmetric and rate dependent effects (such as pipe counter occupancy). As an example, there is a 1%

Energy (GeV)	$\langle n \rangle_{ch}$ observed within cuts		$\langle p \rangle_{ch}$ observed within cuts	
	Monte Carlo	Data	Monte Carlo	Data
1.5	2.68	2.74	.44	.45
1.9	2.83	2.92	.47	.48
2.4	2.88	3.11	.51	.52

Table 14

XBL 759-375

Fig. 21

XBL 753-376

Fig. 22

XBL 753-377

Fig. 23

XBL 753-378

Fig. 24

XBL 753-379

Fig. 25

XBL 753-380

Fig. 26

forward backward asymmetry in the yield at $\sqrt{s} = 4.8$ GeV and this asymmetry is not seen in the total background sample. Due to asymmetries in the occupancy of the pipe counter, the single beam positron background rates were always higher than the electron rates at the same current. Since there are too few background events to determine any momentum dependence for the $\cos \theta$ distributions, no such dependence was included in the subtraction of background in these distributions.

C. Inclusive Phenomenology

The detailed dynamics of hadron interactions at these energies are characterized by strong couplings and a large number of contributing resonant states. One approach to unravelling such a complicated situation would be to study each exclusive final state (i.e. a state where all of the final particles are explicitly specified) in turn and try to determine as many as possible of the details of each particular reaction. This is practical if all of the particles emanating from the interactions are observed and unambiguously identified. As one goes to higher and higher energies, however, the number of states which need to be investigated increases dramatically both because the maximum possible number of produced particles increases and because more and more types of particles become available. For such an analysis, the experimentalist must restrict himself to a bubble chamber, build an apparatus which subtends the entire 4π solid angle, or content himself

with the fraction of events in which all of the particles enter his solid angle. The latter fraction is likely to be quite small unless the state of interest has few particles or the solid angle is large. Also, as the energies of the particles increase, it becomes increasingly difficult to design an apparatus which will unambiguously determine the type of particle observed. Further, in cases where there are a large number of possible final states, the contribution of any particular one in the scheme of things (as evidenced by the fraction of the total number of interactions proceeding through that channel) is likely to be small and will probably decrease with energy as the total number of possibilities grows.

Even if one is able to measure all the details of a particular final state, the final result will be difficult to present (if not to understand) if it involves many particles. This is because for each of the N particles involved, there are 4 parameters necessary to describe it, i.e. 3 components of momentum, and its energy or mass. Since energy and momentum conservation only provide 4 constraints, the total number of variables left in the problem is

$$4(N-1) ,$$

In the present experiment N is approximately 6 to 8 so there are approximately 20 variables to deal with. Obviously, one must ignore the dependence of the dynamics upon some of these variables in order to present the results of such a measurement in an

understandable and useful manner.

The inclusive approach to the situation described above is to ignore the details of the particular final states and concentrate on the dynamical dependence of the interaction on a specific set of observed particles. This is equivalent to integrating out the dependence on all variables except the selected ones, which can be a definite advantage for the theorist who is trying to compute theoretical predictions from any production model. A single particle inclusive approach would be to look for final states of the form

$$a + b \rightarrow \text{particle 1} + \text{anything else}$$

where a and b are the initial particles. This reduces the problem to a 4-variable problem (energy and momentum conservation no longer supply any constraint). Similarly one can look for two particle inclusive reactions

$$a + b \rightarrow c_1 + c_2 + \text{anything},$$

for n body inclusive reactions,

$$a + b \rightarrow c_1 + c_2 + \dots c_n + \text{anything}.$$

The complexity of the situation can be gradually increased until one is again studying states which are completely specified. The major advantage of this approach to the experimentalist is that one can now treat final states where some of the particles are not observed. For the theorist, the inclusive approach reduces

the complexity of the problem to a tractable level. The major disadvantage for both is that the many summations performed could simplify the situation so much that only trivial results common to all models would remain. That is, after doing all these summations, one may not be able to find any dynamics at all! The hope is, however, that one will be left with significant regularities which can be used in the formulation of new models.

1) Sum Rules and Integrals of Inclusive Distributions

By making use of the various conservation laws available, it is possible to derive sum rules which place restrictions on the inclusive distributions. For example, in the reaction

$$e^+ + e^- \rightarrow c + X$$

one can integrate the single particle distribution function over all momenta to obtain

$$\int d^3p \frac{d^3\sigma}{d^3p} = \langle n \rangle \sigma_T$$

since each event in the inclusive distribution is weighted by its multiplicity. Using energy conservation,

$$\int d^3p \left(E \frac{d^3\sigma}{d^3p} \right) = \sqrt{s} \sigma_T$$

where s is the total center of mass energy. Similarly conservation of momentum yields

$$\int d^3 p_i p_i \frac{d^3 \sigma}{d^3 p} = 0 \quad i = 1, 2, 3$$

Most often it is the case that only charged particles are observed. In this case the momentum conservation sum rule does not apply and the other sum rules are modified to read

$$\int d^3 p \frac{d^3 \sigma}{d^3 p} = \langle n \text{ charged} \rangle \sigma_T$$

$$\int d^3 p E \frac{d^3 \sigma}{d^3 p} = \langle E \text{ in charged particles} \rangle \sigma_T$$

If it were possible to separate the particle types, the multiplicity and fractional energy for each particle type would be related to the integral over the respective inclusive distribution. Similar restrictions will occur for each quantum number which is conserved in the reaction, i.e. charge, isospin, parity, strangeness, lepton number, etc. In the case of the two particle inclusive distributions, the integral relation yields

$$\int d^3 p_1 d^3 p_2 \frac{d^2 \sigma}{d^3 p_1 d^3 p_2} = \langle n(n-1) \rangle \sigma_T$$

The weight of each event being $n(n-1)$. It is often more useful to subtract from the two particle distribution the square of the single particle distribution to remove the factorizable part of the distribution. Thus if

$$f_2(\vec{p}_1, \vec{p}_2) \cong \frac{1}{\sigma_T} \frac{d\sigma}{d^3p_1 d^3p_2} - \frac{1}{(\sigma_T)^2} \left(\frac{d\sigma}{d^3p_1} \right) \left(\frac{d\sigma}{d^3p_2} \right)$$

we have

$$\begin{aligned} f_2 &\equiv \int d^3p_1 d^3p_2 f_2(\vec{p}_1, \vec{p}_2) = \langle n(n-1) \rangle - \langle n \rangle^2 \\ &= \{ \langle n^2 \rangle - \langle n \rangle^2 \} - \langle n \rangle \end{aligned}$$

Note that this is simply the difference between the dispersion and the mean of the multiplicity distribution so that, if the particles are produced in a completely uncorrelated manner (i.e. according to a Poisson distribution), f_2 will be zero. However, there must be correlations in the distributions because of the various conservation laws. As an example, consider the consequences of four-momentum conservation in a two particle inclusive reaction.⁽⁷⁾

$$(p^\mu - q_1^\mu) \frac{d\sigma}{d^3p_1} = \int d^3p_2 q_2^\mu \frac{d\sigma}{d^3p_1 d^3p_2}$$

where p^μ is the four-momentum of the initial state. Taking $\mu = 0$ and multiplying both sides by E_1/σ_T , we obtain

$$(\sqrt{s} - E_1) \frac{E_1}{\sigma_T} \frac{d\sigma}{d^3p_1} = \int d^3p_2 \frac{E_1 E_2}{\sigma_T} \frac{d\sigma}{d^3p_1 d^3p_2}$$

Now integrating over d^3p_1 we have

$$s - \langle E^2 \rangle = \int d^3p_1 d^3p_2 E_1 E_2 \frac{d\sigma}{d^3p_1 d^3p_2}$$

Thus when one integrates $f_2(p_1, p_2)$, one obtains

$$\begin{aligned} \int d^3 p_1 d^3 p_2 E_1 E_2 f_2(\vec{p}_1, \vec{p}_2) &= \int d^3 p_1 d^3 p_2 \frac{E_1 E_2}{\sigma_T} \frac{d\sigma}{d^3 p_1 d^3 p_2} \\ &= \frac{1}{\sigma_T} \left(\int d^3 p_1 E_1 \frac{d\sigma}{d^3 p_1} \right) \left(\int d^3 p_2 E_2 \frac{d\sigma}{d^3 p_2} \right) \\ &= s - \langle E^2 \rangle - s \\ &= - \langle E^2 \rangle \end{aligned}$$

From this relationship it can be seen that f_2 cannot be zero everywhere; that is energy conservation requires the particle production to be correlated. Also since $\langle E^2 \rangle$ is positive, we should expect $f_2(\vec{p}_1, \vec{p}_2)$ to be negative when the conservation constraint puts severe restrictions on the single particle momenta.

Some additional insight into the shape of $f_2(\vec{p}_1, \vec{p}_2)$ can be gotten by viewing it as the momentum space correlation coefficient. If two particles are far apart in momentum space, we expect the correlations to vanish. Further, the rate with which these correlations vanish will be related to the range of the interactions in momentum space. Thus we have a good deal of intuition about what the shape of the two particle distribution should be.

1. f_2 will be approximately zero and probably negative.
2. $f_2(\vec{p}_1, \vec{p}_2)$ will be negative in regions where four-momentum conservation is important, i.e.

$$p_1^\mu + p_2^\mu \approx p^\mu \quad \mu = \begin{pmatrix} \sqrt{s} \\ 0 \end{pmatrix}$$

3. $f_2(p_1 - p_2) \equiv \int d\Omega_1 d\Omega_2 d(p_1 + p_2) f_2(\vec{p}_1, \vec{p}_2)$ will approach zero as $p_1 - p_2$ becomes large.

The results of integrating the observed distributions are shown in Table 15. Errors quoted include statistical errors, model dependence, background subtraction statistical errors, and in the case of $\langle E_{ch} \rangle$ and $\langle E_{ch} \rangle / \sqrt{s}$ the effect of introducing a 10% charged K fraction.

D. Parton Model

The advantage of studying hadron reactions with

$$e^+ + e^- \rightarrow \text{hadrons}$$

consists primarily of the fact that we believe that we understand at least part of the interaction well. The successes of quantum electrodynamics in explaining the interactions of electrons and muons are well known and limits on possible modifications to this theory parameterized by changes to the theoretical propagator of the form

$$\frac{1}{q^2} + \frac{1}{q^2} \left(1 + \frac{\Lambda^2}{q^2} \right)$$

have now been increased to $\Lambda \sim 20\text{-}30 \text{ GeV}$.⁽⁵⁾ In terms of this model, the reaction $e^+ + e^- \rightarrow \text{hadrons}$ is understood in terms of the diagram shown below.

$\sqrt{s}$ (GeV)	B (kg)	σ_T (nb)	$\langle n_{ch} \rangle$	$\langle p_{ch} \rangle$ (GeV/c)	$\langle E_{ch} \rangle$ GeV	$\frac{\langle E_{ch} \rangle}{\sqrt{s}}$	f_2
3.0	2	26.3±2.0	3.50±.20	1.62±.20	1.73±.10+.10	.58±.03+.03	-.93±.25
3.0	4	26.8±1.5	3.43±.17	1.62±.09	1.70±.08+.10	.57±.03+.03	-1.19±.24
3.8	2	24.9±1.5	3.90±.22	1.91±.15	2.02±.15+.12	.53±.04+.03	-1.06±.20
3.8	4	24.5±1.3	4.00±.20	2.00±.11	2.10±.10+.12	.55±.03+.03	-1.06±.21
4.8	2	19.3±1.3	4.38±.25	2.21±.20	2.33±.20+.14	.49±.04+.03	
4.8	4	18.04±.98	4.46±.22	2.40±.13	2.51±.13+.14	.52±.03+.03	-1.07±.21

Table 15. Integrals of the Inclusive Distributions

The interaction of the electron and positron with the photon is believed to be well understood, and since the hadrons come from a single photon, the quantum numbers of the hadronic system are known. What is not understood is the way in which the photon converts itself into the hadron final state. The simplest model consists of considering the interaction to proceed via point-like objects called partons such that the photon always couples to a parton anti-parton pair with the partons converting later into hadrons as in the diagram below. (8)

We now treat the coupling of the parton anti-parton pair to the photon in analogy to the way the electron positron pair couple to a photon and hope that the conversion of the partons into hadrons will not obscure the situation too much.

What does this model predict? To answer this question, it is best to begin by calculating some analogous processes using quantum electrodynamics. For example, in Appendix III the reaction $e^+ + e^- + \mu^+ + \mu^-$ is calculated using the standard theory.

This reaction is represented diagrammatically as

The important characteristics of this process are that the cross section is proportional to $1/s$ and has a differential cross section which is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha}{2s} \frac{1}{2} (1 + \cos^2\theta) Q_\mu^2$$

$$\sigma = \frac{4}{3} \pi \frac{\alpha^2}{s} Q_\mu^2$$

where Q_μ is the muon charge in units of the electron charge. The $1/2(1 + \cos^2\theta)$ angular distribution is characteristic of the production of two fermions. The $1/s$ behavior can be broken up as follows:

- ($1/s^2$ from photon propagator squared)
- x ($1/s$ from initial state normalization)
- x (s^2 from Lorentz invariant matrix element squared).

If we recalculate the pair production calculation for the case of pion pairs so that the final state consists of bosons instead of fermions, the differential cross section becomes

$$\frac{d\sigma}{d\Omega} = \frac{\alpha}{8s} \sin^2\theta Q_\pi^2, \quad \sigma = \frac{1}{3} \pi \frac{\alpha^2}{s} Q_\pi^2$$

In this case the angular distribution is $\sin^2\theta$ due to the fact

that the quantum numbers of the photon $J^{PC} = 1^{--}$ require that the spin zero pions be in a pure $L = 1$ state, and by gauge invariance, only the transverse parts of the off mass shell photon ($J_z = \pm 1$) couple to the pions.

In the parton model, the assumption is made that at high enough energies, the partons will not interact with each other and will behave as quasi-free particles. The cross section for hadron production will then have the form⁽⁹⁾

$$\sigma = \frac{4}{3} \pi \frac{\alpha^2}{s} \left[\sum_{\text{fermion partons}} Q_i^2 + \frac{1}{4} \sum_{\text{boson partons}} Q_i'^2 \right]$$

where the Q_i are the charges of the partons. The angular distribution will be

$$\frac{1}{2} (1 + \cos^2\theta) \sum_{\text{fermions}} Q_i^2 + \frac{1}{4} \sin^2\theta \sum_{\text{bosons}} Q_i'^2.$$

If we express the angular distribution as

$$\frac{1}{2} (1 + a \cos^2\theta)$$

then

$$a = \frac{\sum Q^2 - \frac{1}{2} \sum Q'^2}{\sum Q^2 + \frac{1}{2} \sum Q'^2}$$

and if we define

$$R = \frac{\sigma}{\sigma_{\mu\mu}}$$

then

$$R = \sum Q^2 + \frac{1}{4} \sum Q'^2.$$

E. Problems with the Parton Model

The first problem encountered by the parton model when confronted with the present experimental data is the s dependence of the total cross-section. This can be seen from Table 15. If the cross-section fell as $1/s$ from its value at $\sqrt{s} = 3.0$ GeV of 26.8 ± 1.5 nb, it would be 16.7 nb at $\sqrt{s} = 3.8$ GeV, and 10.5 nb at $\sqrt{s} = 4.8$ GeV. While the cross section is falling, it is not falling fast enough. Generally if the parton were not exactly pointlike, we would find that the photon would have more difficulty coupling to a diffuse object (a form factor less than one) and the cross section would be less than expected and therefore fall faster than $1/s$. What is seen is that the coupling of the photon appears stronger than expected and thus there must be some mechanism which is enhancing the hadron production mechanism. Some possible explanations will be discussed later.

Since the muon pair cross-section is falling as $1/s$ and the hadron production is not falling that rapidly, the ratio R must be rising with s . If we take the parton prediction for R seriously,

$$R = \sum Q^2 + \frac{1}{4} \sum Q_i^2$$

we would conclude that perhaps more types of partons are becoming available as the energy increases. An alternative viewpoint is that the current energies are simply not high enough for the partons to be quasi-free and yield these simple properties. However, the magnitude of R at the present energies is as difficult

to understand as its energy dependence. From the values given in Table 15, it can be seen that R increases from approximately 2.8 to approximately 4.8. If we were to identify the partons with quarks we would expect a value of R which is approximately $2/3 \rightarrow 2$. The simplest (and most successful) quark model has three fractionally charged particles which, though they have never been seen, have been quite successful in predicting the symmetry properties of the observed particles along with many of the properties of their interactions. This model gives $R = 2/3$. More complicated quark models can yield higher values of R by changing the charges to integral charges or by increasing the number of quarks. Thus unless the value of R begins to decrease at higher energies, there is a severe problem in the simplest quark model.

A similar problem exists in the observed angular distributions. As can be seen from the angular yields in the main region of the detector (Figs. 24-26) and the angular yield and angular yield corrected for efficiency in runs with special chambers placed close to the beam axis (Figs. 27-28), the majority of the hadron production is essentially isotropic, i.e. $\alpha \sim 0$. In terms of the squares of the parton charges, this would mean that

$$\sum Q^2 = \frac{1}{2} \sum Q'^2.$$

This would seem to indicate that a large number of boson partons are involved in the interaction if the charges of the two groups are comparable. In contrast to this, the indications from the

Fig. 27

ANGULAR DISTRIBUTION FROM ENDCAP RUNS

XBL 753-381

Fig. 28

reaction $e^- + p \rightarrow e^- + \text{hadrons}$, shown diagrammatically below, are that the contribution of boson partons is small, of the order of .18.

F. Scaling

The primary motivation for the parton model came from the results of the SLAC-MIT collaboration studying the deep inelastic reaction $e^- + p \rightarrow e^- + \text{hadrons}$.⁽¹⁰⁾ Because of the Lorentz invariance properties of the interaction, it is possible to write the most general interaction in terms of a tensor.⁽⁸⁾

$$\begin{aligned}
 W_{\mu\nu} &= 4\pi^2 \frac{E_p}{M} \sum_n \langle p | J_\mu(0) | n \rangle \langle n | J_\nu(0) | p \rangle \times (2\pi)^4 \delta^4(q + P - P_\nu) \\
 &= - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) W_1(q^2, \nu) + \frac{1}{M^2} \left(P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \\
 &\quad \times \left(P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) W_2(q^2, \nu)
 \end{aligned}$$

where P is the four-momentum of the initial proton, q is the four-momentum of the virtual photon, and

$$\nu \equiv \frac{P \cdot q}{M}.$$

The reaction is thus described by two unknown functions, each of which are functions of q^2 and ν . The SLAC-MIT results indicate

that these functions can both be represented as functions of a single dimensionless variable, ω ,

$$\frac{1}{\omega} \equiv \frac{-q^2}{2Mv}$$

and further that MW_1 and vW_2 (which are both dimensionless) can be represented as functions of ω alone, independent of the values of q^2 and v . One way of interpreting this type of behavior is that the scattering of the electron occurs off point-like light constituents in the proton, and the point-like-ness of the "partons" and the smallness of their masses prohibit the reaction from showing any intrinsic scale of distance. In this case, the reaction must be describable in terms of dimensionless objects.

A similar tensor can be found for the reaction $e^+ + e^- \rightarrow p + \text{anything}$.⁽⁹⁾

$$\bar{W}_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \bar{W}_1(q^2, v) + \frac{1}{M^2} \left(P_\mu - \frac{P \cdot q}{q^2} q_\mu\right) \left(P_\nu - \frac{P \cdot q}{q^2} q_\nu\right) \bar{W}_2(q^2, v).$$

If the partons are important in this interaction, we would again expect $M\bar{W}_1$ and $v\bar{W}_2$ to be well represented by dimensionless functions of

$$x \equiv \frac{2Mv}{q^2} = \frac{2E}{\sqrt{s}}.$$

The differential cross-section in terms of $\bar{W}_1$ and $\bar{W}_2$ is given by

$$\begin{aligned} \frac{d^2\sigma}{dEd\cos\theta} &= \frac{4\pi\alpha^2}{(q^2)^2} \frac{M^2\nu}{\sqrt{q^2}} \left(1 - \frac{q^2}{\nu^2}\right)^{1/2} \\ &\times [2\bar{W}_1(q^2, \nu) + \frac{2M\nu}{q^2} \left(1 - \frac{q^2}{\nu^2}\right) \nu \frac{\bar{W}_2}{2M}(q^2, \nu) \sin^2\theta] \\ &= \frac{4\pi\alpha^2}{s^2} (MP) [2\bar{W}_1(q^2, \nu) + \frac{2}{M^2} \sin^2\theta \bar{W}_2(q^2, \nu)]. \end{aligned}$$

If both $\bar{W}_1$ and $\bar{W}_2$ are functions of x alone, then at high energies

$$S \frac{d\sigma}{dx} \sim F(x).$$

If parton model accounts for pion production, then scaling should occur in pion inclusive distributions also.

These distributions are shown in Figs. 29-30. To obtain these distributions from the yields shown in Figs. 21-23 the inclusion efficiency as determined by the all pion model was used with normalization taken from the elastic events.⁽⁵⁾ For the error estimates the model dependence, Fig. 17, was added in quadrature with the statistical error. The deviations from a universal curve are striking in view of the success of the model in the deep inelastic case. Deviations from scaling in the small x region could be explained if the partons with small x (wee partons) decayed into hadrons in a way which depended on their energy rather than solely on x . This might happen if there were some mass scale which was important for the partons. In this case we would expect scaling to be violated until

$$x > \frac{2M^*}{\sqrt{s}}$$

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Fig. 29

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Fig. 30

and since the distributions do seem to be exhibiting some scaling for $x > .5$, we have

$$M^* \sim .75 \text{ GeV} + 1.2 \text{ GeV}$$

which is suspiciously close to the meson resonance and nucleon mass region. Further, if the wee partons are the source of the deviations from scaling, one would expect that scaling would be valid to smaller values of x as the energy increases. Some suggestions of this type of behavior are seen in the data (Fig. 30), but higher energies are clearly required to make the statement quantitative. This does not, however, solve the problems with the large value of R discussed earlier.

The separation of $d^2\sigma/dE d\cos\theta$ into $\hat{W}_1$ and $\hat{W}_2$ is difficult because of the limited statistics available when the data are separated into both energy and angular bins. It is also possible that unknown systematic effects may distort the angular distribution in a way which would depend on the momenta. The problem of backgrounds is negligible above .5 GeV since there are no background prongs observed with such a high energy and the shower pulse height cut will eliminate possible contamination from Bhabha scattering. The problems in this region arise chiefly because of the increased model dependence of the efficiency at higher energies, larger statistical uncertainties, and a possible angular dependence of the triggering system. In the region below .5 GeV, the model efficiency is not as severe a problem, but the background contamination is greater, and the angular dependence of the background

as a function of momentum is unknown. An additional problem arises in both regions due to the presence of kinematical factors involving the particle's mass in the expression for the cross-section in terms of $\bar{W}_1$ and $\bar{W}_2$. Since the particle masses are not determined over the entire x region, and since the major fraction (90%) of the production appears to be pions in the region where it can be measured, all particles were treated as pions. The results of fitting the observed cross section for $\bar{W}_1$ and $\bar{W}_2$ are shown in Figs. 31-34 for $x > .5$. Note that the scale of Fig. 32 differs from that of Figs. 33 and 34. The data points do not seem to lie on a universal curve; however the differences do seem to diminish for both $\bar{W}_1$ and $\bar{W}_2$ as x increases. The errors in the determination of $\bar{W}_2$ grow rapidly as x approaches zero because the coefficient of $\bar{W}_2$ is

$$\frac{p^2}{m_\pi^2} \sin^2 \theta$$

but $M_\pi v \bar{W}_2$ is less than $.5 \times 10^{-6}$ GeV over the range $1.5 \leq M_\pi v < 12$, so that in the non-scaling region it is clear that $\bar{W}_1$ dominates. The values of $\bar{W}_1$ for the entire range of $M_\pi v$ are shown in Figs. 35 and 36.

G. x Dependence of the Angular Distribution

An equivalent angular dependence parameterization is

$$\frac{d\sigma}{dE d\cos\theta} = f(x) (1 + \alpha(x) \cos^2 \theta),$$

XBL 753-385

Fig. 31

XBL 753-386

Fig. 32

XBL 753-387

Fig. 33

XBL 753-388

Fig. 34

XBL 753-389

Fig. 35

XBL 759-990

Fig. 36

with α in terms of $\bar{W}_1$ and $\bar{W}_2$

$$\alpha(x) = \frac{\frac{p^2}{m^2} \bar{W}_2(x)}{2\bar{W}_1(x) + \frac{p^2}{m^2} \bar{W}_2(x)} .$$

The x dependence of α determined by fits to the observed angular distributions is shown in Figs. 37-39. Note that in the region where the majority of the hadron production occurs, $x \sim .25$, α is consistent with zero. For this reason, curves of the angular yields which are dominated by prongs with $x \approx .25$ appear to be isotropic. Yet, there does seem to be an upward trend in α as x increases. The small number of boson partons predicted by the results from deep inelastic scattering would suggest that α approach unity in the scaling limit since this is the angular distribution characteristic of the production of spin 1/2 pairs. An important constraint occurs at $x = 1$, however, since, if the fraction of pions and kaons is large compared to the nucleon fraction there, we must have the $\sin^2\theta(\alpha = -1)$ distribution characteristic of the production of boson pairs. The data at $\sqrt{s} = 3.0$ GeV and $\sqrt{s} = 3.8$ GeV show this type of trend, but unfortunately there are too few data for $x > .85$ and $\sqrt{s} = 4.8$ GeV to determine whether the same is true there. Spin 1/2 partons could not turn into spin zero pions and kaons at $x = 1$ without interacting, and this would limit the validity of the simple parton model with spin 1/2 partons near the end point of the inclusive distribution. One would have to restrict the tests of

ANGULAR DISTRIBUTION $1 + \alpha \cos^2 \theta$

XBL 753-391

Fig. 37

ANGULAR DISTRIBUTION $1 + \alpha \cos^2 \theta$

XBL 753-592

Fig. 38

XBL 753-393

Fig. 39

scaling near $x = 1$ to inclusive production of nucleons and anti-nucleons. From Fig. 30 it can be seen that the deviations from scaling behavior of $s d\sigma/dx$ appear to be larger in the region $.8 \leq x \leq 1$ than in the region $.5 \leq x \leq .8$ in support of such qualitative arguments about the region where scaling should be valid.

H. Form Factors and the Region Near $x = 1$

Feynman⁽¹¹⁾ has suggested that there may be a connection between the behavior of the inclusive reaction near $x = 1$ and the exclusive process of pair production at $x = 1$. For example, due to energy momentum conservation, we know that if we could measure the inclusive pion spectrum at the point $x = 1$, what we would actually be measuring would be the pair production cross-section for pions. The question remains, however, whether the exclusive cross-section connects smoothly to the inclusive cross-section near $x = 1$. The argument for such a connection is based primarily on a duality argument, namely that at any value of x it should be possible to represent the differential cross-section, $d\sigma/dx$, in terms of the resonances in the recoil system which contribute. The smooth part of the cross-section should be made up of the tails of resonances and the superposition of resonances which are too broad to identify clearly. If one then looks at the inclusive cross section for electron positron annihilation as a function of momentum transfer and missing mass, each resonance contributes at

a specific value of M_x^2 where

$$M_x^2 \equiv (P_1 + P_2 - P)^2 = s + m_\pi^2 - 2\sqrt{s} E$$

P_1 = four-momentum of initial electron

P_2 = four-momentum of initial positron

P = four-momentum of final pion.

Since

$$x = \frac{2E}{\sqrt{s}}$$

then

$$x = \frac{s - M_x^2 + m_\pi^2}{s}$$

and if

$$sx \frac{d\sigma}{dx} = G(x) \sim (1-x)^\gamma \text{ for } x \text{ near } 1,$$

then at fixed M_x

$$d\sigma \sim \frac{1}{s} \left(\frac{1}{s}\right)^\gamma \left(\frac{dx}{dM_x^2}\right) dM_x^2 \sim \left(\frac{1}{s}\right)^{\gamma+2} dM_x^2.$$

The cross section for pion pair production is given by

$$\frac{d\sigma}{d\Omega} \sim \frac{1}{s} \sin^2\theta |F_\pi|^2$$

and if there is a connection between the two processes, they must vary in the same way with $1/s$. This yields

$$|F_\pi|^2 \sim \left(\frac{1}{s}\right)^{\gamma+1}.$$

Therefore, if the inclusive and exclusive channels are smoothly connected, the s dependence of the pion form factor can be measured from the behavior of the inclusive cross section near

the endpoint. The existence of an inclusive-exclusive connection within the parton model for protons has been established by Drell, Yan, and West.⁽¹³⁾ General theorems relating the threshold behavior of the annihilation channel and the deep inelastic channel have been developed by Gatto and Vendramin⁽¹⁴⁾ indicating that such connections exist even in theories where these amplitudes are not analytic continuations of each other. A connection of this type has implications for the particle ratios near the endpoint as a function of s since a $(1/s)^4$ behavior for the proton form factor and a $(1/s)^2$ behavior for the pion form factor would produce a decreasing nucleon fraction at $x = 1$.

In Fig. 40, $s \times d\sigma/dx$ is plotted versus $1 - x$. The best slope appears to be $\gamma \approx 1/2$ although, as can be seen from the log-log plot in Fig. 41, the estimate of the slope could be between .1 and .6 depending upon where one chooses to measure it. If the production at the end point consists of pions, this yields

$$|F_{\pi}|^2 \sim \left(\frac{1}{s}\right)^{1.5}$$

Recent fits for the pion form factor indicate values for the $1/s$ dependence of the form

$$|F_{\pi}|^2 \sim \left(\frac{1}{s}\right)^P$$

give values⁽¹²⁾ of P between, roughly, 2 and 3 in the region $1.25 \text{ GeV}^2 < s < 4.25 \text{ GeV}^2$. It appears from the qualitative agreement that there is some validity to this connection between the form factor and $d\sigma/dx$ near $x = 1$.

XBL 753-396

Fig. 40

XBL 753-394

Fig. 41

I. New Models for $e^+e^- \rightarrow$ Hadrons

1) Modifications to the Parton Model

The simplest modification to the parton model would be to give up the idea of point-likeness by giving the partons or quarks some structure. Such structure could be supplied by the presence of strong binding forces between quark pairs (needed to keep quarks bound tightly enough so as not to have been produced in the unbound state at present energies). If the particle supplying this binding (gluon) were within the present energy range, some enhancement of the cross section would be seen. The gluon with mass m_g generates a form factor of the type⁽¹⁵⁾

$$F = \left(1 - \frac{s}{m_g^2}\right)^{-1}$$

which makes

$$R = |F|^2 \sum q_i^2 = \left(1 + \frac{2s}{m_g^2}\right) \sum q_i^2$$

the latter form being an expansion for $s \ll m_g^2$. However, since this model supplies a mass scale for parton interactions, the scaling results in the deep inelastic channel are disturbed. To rectify this situation, West and Zerwas⁽¹⁶⁾ have suggested the addition of magnetic moments for the quarks or partons. The ratio of hadron production to mu pair production in e^+e^- annihilation then becomes

$$R = \left\{1 + \left(\frac{1}{2}\mu^2 + \frac{2}{m_g^2}\right)s\right\} \sum q_i^2$$

but vW_2 for the deep inelastic case is

$$vW_2 = [1 - \left(\frac{1}{2}\nu^2 - \frac{2}{m_g^2}\right)q^2] \sum q_i^2.$$

Thus by adjusting the relative values of ν^2 and m_g^2 , the deep inelastic scaling results can be preserved. However, since this modification changes only the magnitude of the total cross section and its s dependence, the structure functions in e^+e^- annihilation should scale if one divides out this dependence. Thus when looking at

$$S \frac{d\sigma}{dx}$$

one should find non-scaling behavior because σ is not proportional to $1/s$, but by looking at

$$\frac{1}{\sigma} \frac{d\sigma}{dx}$$

this non-scaling behavior due to the anomalous s dependence of σ is removed. The data are shown in Figs. 42 and 43. Scaling of this type seems to be worse than the original type of scaling since there is no region where the cross-section appears to scale in this sense. At this point one cannot say whether it is possible to modify the structure functions in a non-scaling way in the reaction $e^+e^- \rightarrow$ hadrons and preserve scaling in deep inelastic processes without additional ad hoc assumptions about the parton's "structure." In any case, structure for the partons would destroy many of the simple properties of the model such as

XBL 753-395

Fig. 42

XBL 753-397

Fig. 43

the obvious dynamical basis of scaling (absence of dimensions), the simple high energy behavior of cross sections (through parton pair production) and the identification of partons with the bare particles in a field theoretic view. If there is something with structure mediating these reactions, we should probably call it something else instead of a parton, since it will exhibit few of the simple properties of partons.

2) Diffractive Models

The suggestion was made by B. Richter, Greenberg, Yodh, and others,⁽¹⁷⁾ that the electron could have a strongly interacting core which could give rise to diffractive processes with constant cross-sections. These models must necessarily show strong peaking of the cross-section in the forward angles which appears to be absent. The extent to which the angular distribution will show peaking in the forward angles, of course, depends on the width of the diffraction peak. Greenberg and Yodh proposed a form

$$E \frac{d\sigma}{d^3p} = A \exp [-a p \sin\theta + 4b \frac{p^2}{s} \cos^2\theta]$$

with $a \approx 6 \text{ GeV}^{-1}$ and $b = 6$ and showed that, if one integrates over all momenta, the angular distribution is uniform to $\cos\theta = .4$ and rises by a factor of 1.36 at $\sqrt{s} = 4.8 \text{ GeV}$ between .4 and .6 and a factor of 4.55 between .4 and .9. This is clearly outside the experimental errors.

In addition, if one considers the problem of scattering from a black disc (strongly interacting core) in the optical model, one

finds that diffraction processes with a cross section of 20 nb cannot occur at these energies!

With a completely absorbing disc, partial waves with $\ell < kR$ are totally absorbed and partial waves with $\ell > kR$ are undisturbed and not absorbed. The partial wave expansion is

$$\sigma_{\text{elastic}} = \frac{\pi}{k^2} \sum_{\ell=0}^{kR} (2\ell + 1) |1 - a_{\ell}|^2 = \frac{\pi}{k^2} \sum_{\ell=0}^{kR} (2\ell + 1)$$

$$\sigma_{\text{inelastic}} = \frac{\pi}{k^2} \sum_{\ell=0}^{kR} (2\ell + 1) |1 - a_{\ell}|^2 = \sigma_{\text{elastic}}$$

when

$$a_{\ell} = n e^{2i\alpha_{\ell}} = 0 \quad \text{for } \ell < kR$$

and

$$a_{\ell} = e^{2i\alpha_{\ell}} = 1 \quad \text{for } \ell > kR.$$

Finally then,

$$\sigma_{\text{in}} = \sigma_{\text{el}} = \frac{\pi}{k^2} \sum_{\ell=0}^{(kR-1)} (2\ell + 1) = \pi R^2$$

$$\sigma_{\text{T}} = 2\pi R^2.$$

If we take

$$\sigma_{\text{inelastic}} = \pi R^2 \sim 20 \text{ nb}$$

then

$$R \sim 8 \times 10^{-18} \text{ cm}$$

$$|k| = \frac{|p|}{\hbar} \sim \frac{E}{\hbar c} \sim \frac{2.4 \text{ GeV}}{1.97 \times 10^{-14} \text{ GeV cm}} \sim 1.2 \times 10^{14} \text{ cm}^{-1}$$

so that

$$kR \sim 10^{-3}$$

and since $kR \ll 1$, no diffractive phenomena will be observed at these low energies with R as predicted by the inelastic cross-section.

3) Pati-Salam Model

Pati and Salam⁽¹⁸⁾ have proposed a model motivated by gauge theories of quark interactions which contains heavy spin 1 mesons with non-zero baryon and lepton numbers coupling to electrons. The new interaction is shown diagrammatically below.

The estimated mass of the new object is $15 \rightarrow 50$ GeV. The lower limit is necessary so as not to conflict with the present accuracy of QED tests. The upper limit is fixed by the requirement that the presence of the x particle modifies the $1/s$ behavior of the total cross-section in the experimentally observed manner. The total cross-section in this model is given by

$$\sigma_h(s) = \frac{4\pi s}{3} \left[\frac{a^2 \rho_{\gamma\gamma}(s)}{s^2} + \frac{2aa_x \rho_{\gamma\gamma}(s)}{s} + a_x^2 \rho_{xx}(s) \right]$$

The first term in this expression is the familiar $1/s$ cross-section, the second is the interference between the normal one photon mechanism and the new mechanism whose coupling constant is a_x , and the last term is the term coming solely from the new interaction. The spectacular claim of this model is that due to the last term the cross-section will fall slowly to a minimum and then begin to rise. The authors note that at high energies ($s \sim m_x^2$) this rise may be damped by variations in the s behavior of $\rho_{xx}(s)$. In addition, the presence of the new x meson in the t channel should cause significant deviations from uniformity in the forward angles when $s \sim m_x^2$. Tests of this model will have to await studies of the total cross-section at higher energies.

J. Phase Space Models and Thermodynamic Models

Phase space models cannot predict the behavior of the total cross section as a function of s , but they are useful for predicting the distribution functions which would be observed in the absence

of all dynamics. The distribution is generally found by Monte Carlo methods, but it is possible to do the integrations required analytically in certain limits. The earliest suggestions of the usefulness of such a model are due to Fermi,⁽¹⁹⁾ and the analytical evaluations are due to Lepore and Stuart (see Appendix IV).⁽²⁵⁾ The resultant expressions for the invariant distributions are

$$E \frac{d\sigma_n}{d^3p} \sim \frac{2^{n-2}}{(n-2)!(n-3)!} (M_X^2)^{\frac{2n-6}{2}} \frac{(\sum m_i)^2}{s} \ll 1$$

$$E \frac{d\sigma_n}{d^3p} \sim \frac{(2\pi)^{\frac{3n-6}{2}} m^{n-5/2}}{\Gamma(\frac{3n}{2} - 3)(n-1)^{3/2}} [M_X^2 - (n-1)m]^{\frac{3n-8}{2}} s - (\sum m_i)^2 \ll s$$

the first being the one applicable when all the particles are relativistic and the second when all the particles are non-relativistic. These must be summed over the multiplicity distribution at each value of s . This is rather complicated, but there are some things which can be learned from the general properties of the n body distributions. Note that in the non-relativistic case, $s = (nm)^2$,

$$M_X^2 = s + m^2 - 2E\sqrt{s}$$

$$\approx (nm)^2 \left[1 - \frac{2E}{nm} \right] \text{ for large } n$$

$$M_X \approx nm \left[1 - \frac{2E}{nm} \right]$$

and taking the limit

$$\lim_{n \rightarrow \infty} E \frac{d\sigma_n}{d^3p} \sim \left[\frac{M_x - (n-1)}{(n-1)m} \right]^{\frac{3}{2}(n-1)}$$

$$\sim \left[1 - \frac{E}{nm} \right]^{\frac{3}{2}n} \sim e^{-\frac{3}{2} \frac{E}{m}}$$

Thus at the lowest energy in the inclusive distribution where we expect the multiplicity to be highest, we should find a distribution which is exponential in E. At the highest energy, we should find a distribution more like the relativistic case so that the function will fall as a power of M_x^2 and therefore also as a power of E. Thus one expects an exponential distribution in E joined smoothly to a power behavior.

Since the absolute yields in our experiment seem to be fairly closely represented by phase space models (Figs. 21-23) it is of interest to compare the above predictions with the observed distributions. The experimental distributions are shown in Figs. 44-45. Note that the general prediction of an exponential shape is quite good, and that the power behavior seems to set in at $P \approx 1$ GeV/c. The absolute normalizations of the three curves are of course determined by the total cross section and charged energy fraction so a universal curve would be found if $\sigma \langle E_{ch} \rangle$ were constant. Experimentally this product is found to be falling slightly as a function of s.

XBL 753-398

Fig. 44

XBL 753-999

Fig. 45

K. The Soft Pion Region

Although the data are fairly well represented by the phase space models, a detailed comparison of the Monte Carlo predictions with the experimental yields does show that there are some deviations. As can be seen from Figs. 21-23, the yield in the region $.1 \leq p < .2$ GeV/c is suppressed at all there energies relative to the phase space prediction. The suppression factors are .73, .77, .93 at $\sqrt{s} = 3.0, 3.8, \text{ and } 4.8$ GeV respectively. Pais and Trieman⁽²⁰⁾ have discussed such suppressions within the context of current algebra and the soft pion theorems for the reaction

$$e^+ + e^- \rightarrow \text{soft pion} + x$$

Essentially what happens is that the coupling of the soft pion must vanish at an unphysical point ($E = 0$). If the amplitude is smooth enough (a requirement which Pais and Treiman point out becomes increasingly more stringent at higher s), this should suppress the coupling for low energy pions. They predict

$$\lim_{E_\pi \rightarrow 0} [(E^2 - m^2)]^{-1/2} \frac{d\sigma}{dE} = 0$$

and since

$$\frac{E d\sigma}{d^3p} = \frac{1}{p} \frac{d\sigma}{dE d\Omega}$$

we should have

$$\lim_{E_\pi \rightarrow 0} \frac{E d\sigma}{d^3p} = 0.$$

The experimental distributions (Fig. 44) seem to indicate that $E d\sigma/d^3P$ does not approach a constant from below as predicted by a phase space model. There is a definite suppression particularly in the $\sqrt{s} = 3.0$ and 3.8 GeV data for $P < .1$ GeV. (These data come from runs with $B = 2$ kg.) The situation at $\sqrt{s} = 4.8$ GeV is not as clear however. To make quantitative statements about the functional dependence of this suppression is impossible with the present data due to statistical limitation in the background subtractions at half field. Future detailed studies of this region with a low field spectrometer and careful measurements of the background spectrum would, however, seem to be justified.

L. Two-Particle Inclusive Distributions

The values of f_2 (Table 15) from the two-particle inclusive distributions are narrower than a Poisson distribution at each of the three energies with

$$[\langle n_{ch}^2 \rangle - \langle n_{ch} \rangle^2] - \langle n_{ch} \rangle = -1.$$

As was discussed previously, the value of f_2 is expected to be negative in reactions where energy and momentum conservation provide a significant constraint on the dynamics. The general predictions discussed for the shape of $f_2(P_2-P_1)$ in the region of large P_2-P_1 also seem to be verified in the data (Fig. 46). Simulations of this distribution using the all-pion model give the same general shape for $f_2(P_2-P_1)$ although the value of f_2 obtained is very sensitive to systematic errors in the average

XBL 753-400

Fig. 46

detection efficiencies for the single and two particle distributions. The peak at small values of $P_2 - P_1$ is a reflection of the exponential shape of the distribution functions.

Because of the possible systematic uncertainties in f_2 , it is more convenient to define

$$g_2 \equiv \frac{1}{n_{ch}(n_{ch}-1)} \frac{d\sigma}{dP_1 dP_2} - \frac{1}{n_{ch}^2} \frac{d\sigma}{dP_1} \frac{d\sigma}{dP_2}$$

so that

$$g_2 \equiv \int dP_1 dP_2 g_2(P_1, P_2) = 0.$$

The distribution $g_2(P_2 - P_1)$ is shown in Figs. 47-49 for all charged pairs, like charged pairs and unlike charged pairs. There is no significant difference between these three distributions.

As a function of P_1 and P_2 , $g_2(P_1, P_2)$ shows a strong maximum at $P_1 \approx P_2$ and $P_1 + P_2 < .4$ GeV/c and a strong minimum at $P_1 \approx P_2$ and $P_1 + P_2 \sim 1$ GeV/c. The soft pion arguments of Pais and Trieman⁽²⁰⁾ also apply to the two particle inclusive distributions in the case of $P_1 + P_2 \rightarrow 0$ and $P_1 - P_2 \rightarrow 0$. Comparison to the predictions of the phase space model reflect the same suppression seen in the single particle distributions for this case, although statistics and the uncertainties in the two particle efficiency for soft pions preclude any quantitative statement about the shape or s dependence of the suppression.

XBL 753-401

Fig. 47

XBL 753-402

Fig. 48

Fig. 49

XBL 753-403

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APPENDIX I

Spark Chamber Geometry

I. Equations of the Wires

The wires of the spark chambers are attached to two rings of radius R separated by a distance L . The wire at ϕ on the first ring is attached to the point at $(R, \phi + \Delta\phi)$ on the second ring. Let us for the moment ignore the complications introduced by the misalignments of the chambers and look at the chamber geometry. To find the mathematical form of the surface of rotation produced by the spark chamber wires, take a single wire and use it to generate the surface of revolution. For simplicity we choose the wire which is horizontal.

End View of Spark Chamber

ϕ_0 is the change in azimuth of the wire in going from one end of the chamber to the other (here greatly exaggerated).

R_0 is the radius of the cylinder at the ends.

The equation of the wire is

$$\Delta y = 0$$

$$\Delta x = 2R_0 \sin \frac{\phi_0}{2}$$

$$R_0^2 = x_0^2 + y_0^2$$

so

$$x = x_0 + \frac{2R_0 z \sin \left(\frac{\phi_0}{2} \right)}{L}$$

and

$$R^2(z) = x^2 + y^2$$

$$= x^2 + y_0^2$$

$$= R_0^2 + \frac{4R_0^2 z^2 \sin^2 \left(\frac{\phi_0}{2} \right)}{L^2} + \frac{4R_0 x_0 z \sin \left(\frac{\phi_0}{2} \right)}{L}$$

where of course $x_0 = -R_0 \sin \frac{\phi_0}{2}$. Thus, we have the surface

$$R^2(z) = R_0^2 \left[1 + 4 \frac{z}{L} - \frac{z}{L} \sin^2 \frac{\phi_0}{2} \right].$$

Notice that the center is at

$$\frac{\partial}{\partial z} R^2(z) = 0 \quad \text{i.e.} \quad z = \frac{L}{2}.$$

Now transform to center the surface at $z' = 0$

$$\begin{aligned} R^2(z) &= R_0^2 \left[1 + 4 \sin^2 \frac{\phi_0}{2} \left(\frac{z}{L} + \frac{1}{2} \right) \left(\frac{z}{L} - \frac{1}{2} \right) \right] \\ &= R_0^2 \left[1 + 4 \sin^2 \frac{\phi_0}{2} \left(\frac{z^2}{L^2} - \frac{1}{4} \right) \right]. \end{aligned}$$

We now rearrange the terms somewhat

$$\frac{R^2}{R_0^2} - 4 \sin^2 \frac{\phi_0}{2} \cdot \frac{z^2}{L^2} = 1 - \sin^2 \frac{\phi_0}{2} = \cos^2 \frac{\phi_0}{2}$$

so that

$$\frac{x^2 + y^2}{R_0^2 \cos^2 \frac{\phi_0}{2}} - \frac{4z^2}{L^2} \tan^2 \frac{\phi_0}{2} = 1 \quad \text{eqn. 1.}$$

Note that this is a confocal quadratic surface (see Morse and Feshbach p. 511). (26) The standard form is thus

$$\frac{x^2}{\xi^2 - a^2} + \frac{y^2}{\xi^2 - b^2} + \frac{z^2}{\xi^2 - c^2} = 1.$$

We have $a^2 = b^2$

$$\xi^2 - a^2 = R_0^2 \cos^2 \frac{\phi_0}{2}$$

$$\xi^2 - c^2 = -\frac{L^2}{4} \tan^2 \frac{\phi_0}{2}.$$

Note that $c > \xi > a = b$ so that the symmetry axis is the z axis, and this is a hyperboloid of 1 sheet. (See also Magnus and Oberhettinger p. 150, (27) who use a representation

$$\frac{x^2 + y^2}{c^2(1 - n_0^2)} - \frac{z^2}{c^2 n_0^2} = 1.$$

Thus

$$c^2(1 - n_0^2) = R_0^2 \cos^2 \frac{\phi_0}{2}$$

$$c^2 n_0^2 = \frac{L^2}{4} \operatorname{ctn}^2 \frac{\phi_0}{2} .)$$

So for this coordinate set

$$c^2 = R_0^2 \cos^2 \frac{\phi_0}{2} + \frac{L^2}{4} \operatorname{ctn}^2 \frac{\phi_0}{2}$$

$$n_0^2 = [1 + \frac{4R_0^2}{L^2} \sin^2 \frac{\phi_0}{2}]^{-1} .$$

The semi-axes of the surface are

$$a = c\sqrt{1-n_0^2}$$

$$b = cn_0$$

$$a = R_0 \cos \frac{\phi_0}{2}$$

$$b = \frac{L}{2} \operatorname{ctn} \frac{\phi_0}{2}$$

$$x = c\sqrt{1-n_0^2} \cos \phi \sqrt{1+\xi^2}$$

are the equations to constrain

$$y = c\sqrt{1-n_0^2} \sin \phi \sqrt{1+\xi^2}$$

x, y, z to the hyperbolic surface.

$$z = cn_0 \xi$$

II. Projection along the Wire to the Readout End

A. Consider the "wire" which is vertically oriented.

$$\Delta x = 0$$

$$y = y_0 + \Delta y \left(\frac{z}{L} + \frac{1}{2} \right)$$

where

$$\Delta y = -2R_0 \sin \frac{\phi_0}{2} \quad \text{and}$$

$$y_0 = R_0 \sin \frac{\phi_0}{2}$$

here we have taken the case for which ϕ is greatest at negative z . Other cases are listed in Section II-B.

The equation for the wire is

$$y(z) = \left[1 - 2 \left(\frac{z}{L} + \frac{1}{2} \right) \right] R \sin \frac{\phi_0}{2} = \frac{2z}{L} R_0 \sin \frac{\phi_0}{2}$$

using the results of section I,

$$\frac{R^2(z)}{c^2(1 - n_0^2)} - \frac{z^2}{c^2 n_0^2} = 1$$

or equivalently

$$\frac{R^2(z)}{R_0^2 \cos^2 \frac{\phi_0}{2}} - \frac{z^2}{\frac{L^2}{4} \tan^2 \frac{\phi_0}{2}} = 1$$

$$R(z) = \left[1 + (4z^2/L^2) \tan^2 \frac{\phi_0}{2} \right]^{1/2} R_0 \cos \frac{\phi_0}{2}$$

$$\sin \phi(z) = \frac{y(z)}{R(z)}$$

where

Suppose we know ϕ , z at the point of interest; we wish to predict ϕ at the end of the chamber. Thus,

$\Delta\phi$ to add to the present ϕ to project to the greatest ϕ end is

$$\Delta\phi = \frac{\phi_0}{2} - \sin^{-1} \frac{Y(z)}{R(z)}$$

$$= \frac{\phi_0}{2} - \sin^{-1} \left\{ \frac{\frac{2zR_0}{L} \sin \frac{\phi_0}{2}}{\left[1 + \frac{4z^2}{L^2} \tan^2 \frac{\phi_0}{2} \right]^{1/2} R_0 \cos \frac{\phi_0}{2}} \right\}$$

B. Correct Signs, Readout Ends, Etc.

If the greatest ϕ is at $+z$ and/or the readout is at the least ϕ end, then we must alter the signs appropriately.

Now the present chambers have the following characteristics

Plane of a chamber group counting from outside	Readout end	Greatest ϕ end
1	$+z$	$+z$
2	$-z$	$-z$
3	$+z$	$-z$
4	$-z$	$+z$

So that for plane 1

$$\Delta\phi = \frac{\phi_0}{2} + \sin^{-1} \{ \} \quad \text{eqn. 2}$$

for plane 2

$$\Delta\phi = \frac{\phi_0}{2} - \sin^{-1} \{ \} \quad \text{eqn. 3}$$

for plane 3

$$\Delta\phi = -\frac{\phi_0}{2} \sin^{-1} \{ \} \quad \text{eqn. 4}$$

and for plane 4

$$\Delta\phi = -\frac{\phi_0}{2} + \sin^{-1} \{ \} \quad \text{eqn. 5.}$$

III. Accounting for Chamber Optical Constants

We now need to consider the effect of small shifts of the hyperboloids due to incorrect alignment. All rotations are treated only to 1st order so they may be treated exactly as if they were small shifts of the centers of the coordinate systems.

$$\begin{aligned} \text{Consider } \frac{R^2}{a^2} - \frac{z^2}{b^2} &= 1 \\ \text{shifted } \frac{(x - \Delta x)^2}{a^2} - \frac{(y - \Delta y)^2}{b^2} - \frac{(z - \Delta z)^2}{b^2} & \quad \text{eqn. 6} \end{aligned}$$

$$R^2 = a^2 \left\{ 1 + \frac{z^2}{b^2} + \frac{2}{a^2} (x\Delta x + y\Delta y) - \frac{2}{b^2} z\Delta z \right\} \quad \text{to 1st order} \quad \text{eqn. 7}$$

By Δx etc. we mean

$$\Delta x = \delta x_{\text{shift}} + \delta x(x, y, z)_{\text{rotations}} = x_{\text{BCCS}} - x_{\text{CCCS}} \quad \text{eqn. 8.}$$

The rotations are defined as follows:

$$\text{TP (pitch)} \equiv \theta x$$

$$\text{TY (yaw)} \equiv \theta y$$

$$\text{TR (roll)} \equiv \theta z.$$

For rotation of the chambers in the + θ direction, the rotations etc. take the form

$$x' = \begin{pmatrix} 1 & \Delta\theta \\ -\Delta\theta & 1 \end{pmatrix} x + \Delta x_{\text{shift}}$$

for the rolls

$$x' = x + \delta x + y\theta z$$

$$y' = y + \delta y - x\theta z$$

$$z' = z.$$

Making cyclic permutations of x, y, and z we get

$$x' = x + \delta x + y\theta z - z\theta y$$

$$y' = y + \delta y - x\theta z$$

$$z' = z + x\theta y$$

and finally for all three rotations

$$x' = x + \delta x + (y\theta z - z\theta y)$$

$$y' = y + \delta y + (z\theta x - x\theta z)$$

$$z' = z + \delta z + (x\theta y - y\theta x)$$

or

$$x_{\text{CCS}} = \begin{pmatrix} 1 & -\theta z & +\theta y \\ +\theta z & 1 & -\theta x \\ -\theta y & +\theta x & 1 \end{pmatrix} x_{\text{BCCS}} - \Delta x_{\text{shift}} \quad \text{eqn. 9}$$

$$x_{BCCS} = \begin{pmatrix} 1 & +\theta z & -\theta y \\ -\theta z & 1 & \theta x \\ \theta y & -\theta x & 1 \end{pmatrix} x_{CCCS} + \Delta x_{\text{shift}} \quad \text{eqn. 10.}$$

The convention is that θz is the rotation of the chamber, so in transforming from the BCCS system to the CCCS system one must rotate the points by $-\theta z$. (See figure below.)

where CCCS $\equiv$ chamber center coordinate system

BCCS $\equiv$ beam center coordinate system

so

$$x_{CCCS} = x_{BCCS} - x_0 - y\theta z$$

$$y_{CCCS} = y_{BCCS} - y_0 + x\theta z \quad \text{etc.}$$

To handle the shifts properly means that one must rewrite the equation of the hyperbola as

$$\frac{(x - \Delta x)^2}{a^2} + \frac{(y - \Delta y)^2}{b^2} - \frac{(z - \Delta z)^2}{b^2} = 1$$

with Δx , Δy , Δz given as above.

APPENDIX II

TASH Efficiency Calculation

Let the absorption cross section σ_{abs} be defined to be the cross section for an incident particle to have any inelastic interaction independent of the number or type of secondaries emitted. Let the corresponding interaction length be λ_{abs} . Then the probability that a secondary will be produced while traversing the material between x and $x + dx$ with minimum energy E_m is

$$P(x, \theta) dx = e^{-x/\lambda} \frac{dx}{\lambda} p(\theta, E_m, E)$$

where

$e^{-x/\lambda}$ = the fraction of particles remaining after traversing a distance x

$\frac{dx}{\lambda}$ = the probability per particle of an interaction within dx

and

$p(\theta, E_m, E)$ = the probability of emitting a charged secondary at angle θ , with energy greater than E_m from an incident particle of energy E .

The probability that an emitted secondary will not be reabsorbed in the remaining material $T-x$ is

$$P^*(x) = e^{-(T-x)/\lambda}$$

By integrating through the absorber of thickness T , the number of secondary particles emerging from the material with N_0 particles

incident is

$$N_{\text{sec}} = N_0 e^{-T/\lambda} \frac{T}{\lambda} p(\theta, E_m)$$

and the total number emerging is

$$N_{\text{TOT}} = N_{\text{pri}} + N_{\text{sec}} = N_0 e^{-T/\lambda} \left(1 + \frac{T}{\lambda} p(\theta, E_m, E) \right)$$

N_{pri} = the number of non-absorbed incident particles.

Neglecting the probability that in addition to charged particle emission, neutrals such as π_0 's could be emitted and subsequently convert to charged particles in the remaining material, the fraction of charged particles emerging is

$$\epsilon = \frac{N_{\text{TOT}}}{N_0} = e^{-T/\lambda} \left(1 + \frac{T}{\lambda} p(\theta, E_m, E) \right)$$

If we define

$$\lambda_0 \equiv \lambda_{\text{geometric}} = R(E) \lambda_{\text{abs}}$$

where

$$R(E) \equiv \frac{\sigma_{\text{abs}}(E)}{\sigma_{\text{geometric}}}$$

The above fraction can be rewritten as

$$\epsilon = e^{\frac{-TR(E)}{\lambda_0}} \left(1 + \frac{TR(E)}{\lambda_0} p(\theta, E_m, E) \right)$$

The TASH efficiency will be determined by the absorption of incident particles and the resultant secondary emission in the material between the spark chambers and the scintillator sheets of the shower counters. This material consists of the 1/2" Al support can for the spark chambers, the 1" trigger counter scintillator, the coil, the 1/2" Al face plate on the shower counters, and the first 1/4" Pb sheet in the shower counters. The coil consists of two sheets of 1/8" Al, two sheets of 5/8" hardwood, approximately 2.5" of Al and .7" of water in the windings, and 8 layers each of .007" Glass tape and .002" Mylar tape. The coil materials are equivalent to 10.32 cm of Al. In addition to determining the TASH efficiency, these materials reduce the soft photon and soft electron background at the storage ring (which has a trigger rate on the order of 100 KHz at the trigger counters) so that by using the shower counters in the trigger, a reasonable trigger rate (< 10 Hz) can be achieved.

To calculate the TASH efficiency, it is necessary to have measurements of the functions $R(E)$ and $p(\theta, E_{\text{min}}, E)$ for each of the materials. The measurements available indicate that to first order, $R(E)$ is independent of the type of material.⁽²¹⁾ In the calculation, the values of $R(E)$ used were those of Aluminum. There is much less information on $p(\theta, E_{\text{min}}, E)$ except for measurements in the range $140 \text{ Mev} < E < 400 \text{ Mev}$ ⁽²²⁾ The angular distribution of the secondaries is unknown and was assumed to be isotropic in the

incident particle and nucleon center of mass frame so that the θ dependence of $p(\theta, E_m, E)$ was approximated by the fraction of the secondaries emitted into the solid angle subtended by the shower counters. This fraction increases with energy due to the increasing velocity of the center of mass frame relative to the shower counters and the resultant peaking of the angular distributions toward the shower counters in the lab frame. The value for the minimum energy E_m for a detectable secondary was chosen to be 50 MeV. In addition to the above, it was necessary to account for the fact that incident particles with energy less than .22 GeV will range in the material. The efficiency below .22 GeV is thus determined by the portion of the absorption cross section which produces π^0 's and by their subsequent conversion probability. The calculation assumes that there is a negligible probability of emitting more than one charged secondary. Although there is no data on secondary multiplicities off complex nuclei, data on πN interactions⁽²³⁾ would indicate that multiple pion production cross sections are unimportant below 1 GeV.

Figure 14 shows the results of the calculation and for comparison the experimentally determined efficiency. The calculated efficiency without secondary emission is shown to illustrate the magnitude of the secondary contribution. The effect of the range cutoff can be seen at .2 GeV. Below this the calculation is not expected to be valid as mentioned previously. The small change in slope occurring between .35 and .2 GeV is due to an increase in $R(E)$

in this region. As can be seen the agreement between the calculated efficiency and the experimental efficiency is excellent. The additional efficiency for $p > 1$ GeV is probably due to the emission of more than one secondary and the enhancement for $p < .2$ GeV to π^0 production processes.

APPENDIX III

QED Calculation of Mu Pair Production

$$\begin{aligned}
 d\sigma &= \frac{1}{|v_{rel}|} |M|^2 \frac{m_e^2}{(E_e)^2} \frac{d^3 p_3}{(2\pi)^3} \frac{d^3 p_4}{(2\pi)^3} \frac{m_\mu^2}{E_\mu^2} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \\
 M &= e_0 e_\mu \bar{v}(p_2, s_2) (-i\gamma_\mu) u(p_1, s_1) \frac{(-i g_{\mu\nu})}{s} \bar{u}(p_3, s_3) (-i\gamma_\nu) v(p_4, s_4) \\
 &= \frac{ie_0 e_\mu}{s} \bar{v}(p_2, s_2) \gamma_\mu u(p_1, s_1) \bar{u}(p_3, s_3) \gamma_\mu v(p_4, s_4) \\
 M^* &= \frac{e_0^2 e_\mu^2}{s^2} \bar{v}(p_2, s_2) \gamma_\mu u(p_1, s_1) \bar{u}(p_1, s_1) \gamma_\nu v(p_2, s_2) \\
 &\quad \times \bar{u}(p_3, s_3) \gamma_\mu v(p_4, s_4) \bar{v}(p_4, s_4) \gamma_\nu u(p_3, s_3)
 \end{aligned}$$

Summing over the spins in the final state and averaging over the spins in the initial state:

$$\begin{aligned}
 d\sigma &= \frac{1}{2c} \frac{m_e^2}{E_e^2} m_\mu^2 \frac{d^3 p_3}{(2\pi)^3 E_\mu} \frac{d^3 p_4}{(2\pi)^3 E_\mu} \frac{1}{2} \cdot \frac{1}{2} (2\pi)^4 \frac{e_0^2 e_\mu^2}{s^2} \\
 &\quad \times \text{Tr} \left[\frac{\not{p}_2 - m_e}{2m_e} \gamma_\mu \frac{\not{p}_1 + m_e}{2m_e} \gamma_\nu \right] \\
 &\quad \times \text{Tr} \left[\frac{\not{p}_3 + m_\mu}{2m_\mu} \gamma_\mu \frac{\not{p}_4 - m_\mu}{2m_\mu} \gamma_\nu \right] \delta^4(p_1 + p_2 - p_3 - p_4)
 \end{aligned}$$

We now take advantage of the energy and momentum conserving delta function to do four of the integrals and evaluate the traces (a factor 1/2 comes from $\delta(2E_1 - 2E_2) dE$)

$$= \frac{1}{2c} \frac{1}{E_e^2} \frac{1}{E_\mu^2} \frac{e_0^2 e_\mu^2}{s^2} \frac{p_\mu^2}{(2\pi)^2} d\Omega \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$

$$\times [p_{2\mu} p_{1\nu} + p_{2\nu} p_{1\mu} - p_1 \cdot p_2 g_{\mu\nu} - m_e^2 g_{\mu\nu}]$$

$$\times [p_{3\mu} p_{4\nu} + p_{3\nu} p_{4\mu} - p_3 \cdot p_4 g_{\mu\nu} - m_\mu^2 g_{\mu\nu}]$$

dropping terms of order m_e^2 , m_μ^2 , using $E_e^2 = \frac{s}{4}$

$$= \frac{e_0^2 e_\mu^2}{s^2} \frac{1}{s} \frac{1}{2} \frac{\beta^2}{2} \frac{d\Omega}{(2\pi)^2} [2(p_2 \cdot p_3)(p_1 \cdot p_4)$$

$$+ 2(p_2 \cdot p_4)(p_1 \cdot p_3) - (p_3 \cdot p_4)(2p_1 \cdot p_2)$$

$$- (p_1 \cdot p_2)(2p_3 \cdot p_4) + 4(p_1 \cdot p_2)(p_3 \cdot p_4)].$$

Note that there is a $1/s^2$ from the square of the photon propagator, a $1/s$ from the initial state normalization, and no powers of s from the Lorentz invariant phase space and integrations. The Lorentz invariant matrix element squared has dimensions $(p^2)^2 \sim s^2$ so that $d\Omega$ is proportional to $1/s$. The above expression when evaluated in the center of mass frame yields

$$\frac{e_0^2 e_\mu^2}{s^3} \frac{\beta^2}{4} \frac{d\Omega}{(2\pi)^2} [2(p_2 \cdot p_3)(p_1 \cdot p_4) + 2(p_2 \cdot p_4)(p_1 \cdot p_3)]$$

$$= \frac{e_0^2 e_\mu^2}{s^3} \frac{1}{4} \frac{d\Omega}{(2\pi)^2} [2(E_i E_f + p_i p_f \cos\theta)^2 + 2(E_i E_f - p_i p_f \cos\theta)^2].$$

Using $E_i = E_f = p_i = p_f$

$$\begin{aligned} d\sigma &= \frac{e_o^2 e_\mu^2}{s^3} \frac{1}{4} \frac{d\Omega}{(2\pi)^2} 2E_i^4 [(1 + \cos\theta)^2 + (1 - \cos\theta)^2] \\ &= \frac{(4\pi\alpha)^2}{16s} (1 + \cos^2\theta) \frac{d\Omega}{(2\pi)^2} \left(\frac{e_\mu}{e_o}\right)^2. \end{aligned}$$

Integrating over the solid angle

$$\sigma = \frac{4}{3} \frac{\pi\alpha^2}{s} \left(\frac{e_\mu}{e_o}\right)^2.$$

Note that the cross section is proportional to the square of the muon's charge in units of the electron charge, and that before integration the angular distribution is $1 + \cos^2\theta$.

APPENDIX IV

Evaluation of Phase Space Integrals

The factorizability property of n body phase space⁽²⁴⁾ can be used to express the invariant momentum distribution from an n body reaction in terms of the (n-1) body phase space integral

$$\frac{E \, dR_n(\sqrt{s}, m)}{d^3p} = \frac{1}{2} R_{n-1}(M_x, m)$$

$$\text{where } M_x = \sqrt{s + m^2 - 2\sqrt{s} E}$$

Thus, if the function R_{n-1} can be calculated, it will yield the distribution function

$$\frac{E \, d\sigma_n}{d^3p} \quad (\sigma_n = \text{total cross section for } n \text{ body production})$$

which occurs when the Lorentz invariant matrix element is a simple constant. The early work on the analytic evaluation of the phase space integrals is due to Lepore and Stuart.⁽²⁵⁾

The phase space integral is

$$1 = \int \prod_{i=1}^n \frac{d^3p_i}{W_n} \delta^3(\Sigma p_i) \delta(\sqrt{s} - \Sigma w_i) (2\pi)^4.$$

Now following the method of Lepore and Stuart, we fourier transform the delta functions to obtain

$$I = \int_{i=1}^n \frac{d^3 p}{w_i} \int_{-\infty}^{\infty} d^3 \lambda e^{i \Sigma p_i \cdot \lambda} \int_{-\infty}^{\infty} d\alpha e^{i(\sqrt{s} - \Sigma w_i) \alpha}$$

$$= \int_{-\infty}^{\infty} d^3 \lambda \int_{-\infty}^{\infty} d\alpha e^{i \sqrt{s} \alpha} \prod_{i=1}^n I_i$$

where

$$I_i = \int \frac{d^3 p}{w} e^{i p \cdot \lambda - i \alpha w}$$

Doing the angular integral we have

$$I_i = \frac{4\pi}{\lambda} \frac{d}{d\lambda} \int_{-\infty}^{\infty} \frac{dp}{w} e^{-i \alpha w + i p \lambda}$$

Introduce the variable $p = M \sinh \theta$

then $w = (p^2 + M^2)^{1/2} = M(1 + \sinh^2 \theta)^{1/2} = M \cosh \theta$

and

$$I_i = \frac{4\pi}{\lambda} \frac{d}{d\lambda} \int_{-\infty}^{\infty} d\theta \exp\{iM(\lambda \sinh \theta - \alpha \cosh \theta)\}.$$

Note that the Hankel functions are

$$H_0^{(1)} = J_0 + iY_0 = \frac{2}{\pi} \int_0^{\infty} (-i) \exp(i x \cosh t) dt \quad x > 0$$

$$H_0^{(2)} = J_0 - iY_0 = \frac{2}{\pi} i \int_0^{\infty} \exp(-i x \cosh t) dt \quad x > 0$$

so that

$$I_i = \frac{e\pi}{\lambda} \frac{d}{d\lambda} \left(\frac{\pi}{2i} \right) H_0^{(2)} (M(\alpha^2 - \lambda^2)^{1/2}).$$

Using the recursion relation for hankel functions

$$I_i = \frac{2\pi^2}{i} \frac{M}{(\alpha^2 - \lambda^2)^{1/2}} H_1^{(2)}(M(\alpha^2 - \lambda^2)^{1/2})$$

so that the entire phase space integral can be written as

$$I = 4\pi \int_0^\infty \lambda^2 d\lambda \int_{-\infty}^\infty d\alpha e^{i\sqrt{2}\alpha} \prod_{i=1}^n \left(\frac{2\pi^2}{i} \right) \frac{M}{(\alpha^2 - \lambda^2)^{1/2}} H_1^{(2)}(M_i(\alpha^2 - \lambda^2)^{1/2}).$$

Lepore and Stuart were able to evaluate this function in the non-relativistic limit

$$s - (\Sigma m_i)^2 \ll s$$

and in the relativistic limit

$$\frac{(\Sigma m_i)^2}{s} \ll 1.$$

The expressions for the phase space integrals in the equal mass case are

$$(\text{relativistic}) I_n(E) = \frac{2^{n-1}}{(n-1)!(n-2)!} E^{2n-4}$$

$$(\text{non-relativistic}) I_n(E) = \frac{(2\pi)^{\frac{3n-3}{2}} m^{n-3/2}}{n^{3/2}} \frac{(E - nm)^{\frac{3n-5}{2}}}{\Gamma((3n-3)/2)}$$

Using the recursion relation for the invariant distribution we have

$$\begin{aligned} (\text{relativistic}) \frac{E dR_n}{d^3p} &= R_{n-1}(M_x) \\ &= \frac{2^{n-2}}{(n-2)!(n-3)!} (M_x)^{\frac{2n-6}{2}} \end{aligned}$$

$$(\text{non-relativistic}) \frac{E dR_n}{d^3p} = \frac{(2\pi)^{\frac{3n-6}{2}} m^{n-5/2}}{\Gamma(3n/2 - 3)(n-1)^{3/2}} [M_x - (n-1)m]^{\frac{3n-8}{2}}.$$