TMD measurements at CLAS

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Outline

Transverse structure of the nucleon and partonic correlations

- Introduction
- Hard scattering processes and correlations between spin and transverse degrees of freedom
- $k_T$-effects with polarized SIDIS
- Higher twist effects in SIDIS
- Summary
PDFs, $q(x)$: Probabilities to find a quark with a fraction $x$ of proton momentum $P$.

Semi-Inclusive processes and transverse momentum distributions

Hard exclusive processes and spatial distributions of partons

Detection of final state particles in semi-inclusive and hard exclusive processes allows access also to transverse distribution of quarks.

Collinear analysis of observables in semi-inclusive and hard exclusive processes will be sensitive to integration region over transverse degrees of freedom due to correlations of $x$, spin and transverse degrees of freedom.
SIDIS: partonic cross sections

\[ \nu = \frac{(qP)}{M} \]
\[ Q^2 = (k - k')^2 \]
\[ y = \frac{(qP)}{(kP)} \]
\[ x = \frac{Q^2}{2(qP)} \]
\[ z = \frac{(qP_h)}{(qP)} \]

\[ \sigma = F_{UU} + P_t F_{UL}^{\sin \phi} \sin 2\phi + P_b F_{LU}^{\sin \phi} \sin \phi \ldots \]

Transverse momentum of hadrons in SIDIS provides access to orbital motion of quarks

\[ P_T = p_\perp + z k_T \]

\[ d\sigma^{\gamma^* H \rightarrow hX} \propto \sum e_q^2 \int d^2 \vec{k}_T d^2 \vec{p}_\perp f^{H \rightarrow q}(x, \vec{k}_T) D^{q \rightarrow h}(z, \vec{p}_\perp) \delta^{(2)}(z \vec{k}_T + \vec{p}_\perp - \vec{P}_T) \]

\[ d\sigma^h \propto \sum f^{H \rightarrow q}(x) d\sigma_q(y) D^{q \rightarrow h}(z) \]

H. Avakian, QCD-N'12, Oct 23
Azimuthal moments in SIDIS

\[ \frac{d\sigma}{dx \, dy \, d\psi \, dz \, d\phi_h \, dP_{h\perp}^2} = \]

\[ \frac{\alpha^2}{x y Q^2} \frac{y^2}{2 (1 - \varepsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \left\{ F_{UU, T} + \varepsilon F_{UU, L} + \sqrt{2 \varepsilon (1 + \varepsilon)} \cos \phi_h F_{UU}^{\cos \phi_h} \right. \]

\[ + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2 \varepsilon (1 - \varepsilon)} \sin \phi_h F_{LU}^{\sin \phi_h} \]

\[ + S_{||} \left[ \sqrt{2 \varepsilon (1 + \varepsilon)} \sin \phi_h F_{UL}^{\sin \phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \]

\[ + S_{\perp} \lambda_e \left[ \sqrt{1 - \varepsilon^2} \cos(\phi_h - \phi_S) F_{LL}^{\cos(\phi_h - \phi_S)} + \varepsilon \sin(\phi_h + \phi_S) F_{UL}^{\sin(\phi_h + \phi_S)} \right. \]

\[ + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \]

\[ + \sqrt{2 \varepsilon (1 + \varepsilon)} \sin \phi_S F_{UT}^{\sin \phi_S} + \sqrt{2 \varepsilon (1 + \varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \]

\[ + S_{\perp} \lambda_e \left[ \sqrt{1 - \varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2 \varepsilon (1 - \varepsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} \right. \]

\[ + \sqrt{2 \varepsilon (1 - \varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \} \]
Quark distributions at large $k_T$: lattice

Higher probability to find a quark anti-aligned with proton spin at large $k_T$

Higher probability to find a $d$-quark with at large $k_T$

$k_T$-distributions of TMDs may depend on flavor and spin

$$f_1^q(x, k_T) = f_1(x) \frac{1}{\pi \mu_0^2} \exp \left(-\frac{k_T^2}{\mu_0^2}\right)$$

$$g_1^q(x, k_T) = g_1(x) \frac{1}{\pi \mu^2} \exp \left(-\frac{k_T^2}{\mu^2}\right)$$

$g_1^q = \Delta q = (q^+ - q^-)/2$

B. Musch et al arXiv:1011.1213

$u^+(x, k_T^2) \propto \frac{(xM + m)^2}{(k_T^2 + \lambda_R^2)2\alpha}$

$u^-(x, k_T^2) \propto \frac{k_T^2}{(k_T^2 + \lambda_R^2)2\alpha}$

JMR model

$M_R, R = s, a$
CLAS configurations

$ep \to e' \pi X$

Inner Calorimeter (DVCS experiments)

Unpolarized, longitudinally and transversely polarized targets
- Polarized NH$_3$/ND$_3$ (no IC, ~5 days)
- Unpolarized H (with IC ~ 60 days)
- Polarized NH$_3$/ND$_3$ with IC 60 days 10% of data on carbon
- Polarized HD-Ice (no IC, 25 days)

Unpolarized and longitudinally polarized targets

Polarizations:
- Beam: ~80%
- NH$_3$ proton 80%, ND$_3$ ~30%
- HD (H-75%, D-25%)
\( A_1(x, z, P_T) = A_1(x) \frac{\langle P^2_{T, unp} \rangle}{\langle P^2_{T, pol} \rangle} \exp \left( -\frac{P^2_T}{\langle P^2_{T, pol} \rangle} - \frac{P^2_T}{\langle P^2_{T, unp} \rangle} \right) \)

\[ f_1^q(x, k_T) = f_1(x) \frac{1}{\pi \mu_0^2} \exp \left( -\frac{k_T^2}{\mu_0^2} \right) \]

\[ g_1^q(x, k_T) = g_1(x) \frac{1}{\pi \mu_0^2} \exp \left( -\frac{k_T^2}{\mu_0^2} \right) \]

\(< P^2_T(z) > = z^2 \mu_{0/2}^2 + \mu_D^2 \)

\( \mu_2^2/\mu_0^2 = 0.692 \pm 0.039 \pm 0.045 \)

\( \pi^+ A_1 \) suggests broader \( k_T \) distributions for \( f_1 \) than for \( g_1 \)

The new data is consistent with old measurements, now available in several bins in \( x \)

0.4 < \( z < 0.7 \)

H. Avakian, QCD-N'12, Oct 23
Worm gear TMDs are unique (no analog in GPDs)
The $\sin \phi$ moments of $\pi^0$ and $\pi^+$ of target SSA comparable, while $\pi^-$ SSA seem to have an opposite to $\pi^0/+$ sign at CLAS (also HERMES).
Longitudinally Polarized Beam SSA

\[ A_{LU}(\phi) = \frac{N^+ - N^-}{P_b N^+ + N^-} \]

HT function related to force on the quark. Burkardt (2008), Qiu (2011)

\[ A_{LU}^{Collins} \sim e H_1^\perp \]


Collins type contribution may be dominant for \( \pi^- \)

Sivers type contribution may be dominant for \( \pi^0 \)

W. Gohn

M. Aghasyan

W. Mao, Z. Lu (arXiv:1210.4790)

0.4 < \( P_T \) < 0.6

x vs \( k_T \) correlations matter

CLAS e1-dvcs

CLAS 5.5 GeV
Model predictions: unpolarized target

Lattice provides important cross check with data and models for all HT TMDs (Musch et al, arXiv:1011.1213)

\[ f_1^{[1]}(k_\perp^2) \equiv \int_{-1}^{1} dx \, f_1(x, k_\perp^2) \]

\[ \frac{e^{[1]}}{f_1^{[1]}} = \frac{\tilde{A}_1}{\tilde{A}_2} \]

- Models agree on a large beam SSA for \( \pi\pi \) pair production
- Lattice results for u-d can be directly compared to models and data.
Dihadron production kinematics

$z_1, z_2$ - fractions of energy carried by a hadrons

$D_{1\rightarrow h_1, h_2}(z_1, z_2, R_T^2)$

- Factorization proven
- Evolution known
- Extracted at BELLE for $\pi\pi$ pairs, planned for $\pi K$ pairs

Dihadron productions offers exciting possibility to access HT pdfs as we deal with the product of functions instead of convolution

H. Avakian, QCD-N'12, Oct 23
Dihadron productions offers exciting possibility to access HT parton distribution surviving $k_T$-integration.
Spin-azimuthal asymmetries in hard exclusive photon (DVCS) and hadron (DVMP) production give access to underlying GPDs and PDFs.
Summary

• Measurements of azimuthal dependences of double and single spin asymmetries in hard scattering (SIDIS, DVMP) indicate that there are significant correlations between spin and transverse distribution of quarks.

• Current JLab data are consistent with a partonic picture and measurements performed at higher energies.

• Sizable higher twist asymmetries measured both in SIDIS and exclusive production indicate the quark-gluon correlations may be significant at moderate $Q^2$.

JLab measurements at 6 GeV provide important input for model independent flavor decomposition of TMDs and GPDs. Tools are required to extract the 3D PDFs in multidimensional space.
Support slides....
The complete mapping of the multi-dimensional SIDIS phase space will allow a comprehensive study of the TMDs and the transition to the perturbative regime.

Flavor separation will be possible by the use of different target nucleons and the detection of final state hadrons.

Measurements with pions and kaons in the final state will also provide important information on the hadronization mechanism in general and on the role of spin-orbit correlations in the fragmentation in particular.

Higher-twist effects will be present in both TMDs and fragmentation processes due to the still relatively low $Q^2$ range accessible at JLab, and can apart from contributing to leading-twist observables also lead to observable asymmetries vanishing at leading twist. These are worth studying in themselves and provide important information on quark-gluon correlations.
Lattice calculations and $b_T$-space

(PDFs in terms of Lorenz invariant amplitudes Musch et al, arXiv:1011.1213)

\[ f_1^{[1]}(k_{\perp}^2) = \frac{c_2 \sigma_2^2}{4\pi} e^{-\frac{k_{\perp}^2}{(2/\sigma_2)^2}} \]

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**CLAS12** $A_{UT}$ with transverse proton target

Stat. error for a 4D analysis of the $\pi^+$ Sivers asymmetry on proton ($x1.5$ on D) target

SSAs in exclusive pion production

Transverse photon matters

\[ M^{\text{twist}-3}_{0-,++} \approx e_0 \sqrt{1 - \xi^2} f_{-1}^{+1} d\bar{x} \mathcal{H}_{0-,++} [H_T^{(3)} + \ldots] \]

<table>
<thead>
<tr>
<th>observable</th>
<th>dominant interf. term</th>
<th>amplitudes</th>
<th>low ( t' ) behavior</th>
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<tr>
<td>( A_{UT}^{\sin(\phi-\phi_0)} )</td>
<td>LL</td>
<td>Im[( M^*<em>{0-,0+} - M</em>{0+,0+} )]</td>
<td>( \propto \sqrt{-t} )</td>
</tr>
<tr>
<td>( A_{UT}^{\sin(\phi+\phi_0)} )</td>
<td>TT</td>
<td>Im[( M^*<em>{0-,++} - M</em>{0+,++} )]</td>
<td>( \propto \sqrt{-t} )</td>
</tr>
<tr>
<td>( A_{UT}^{\sin(3\phi-\phi_0)} )</td>
<td>TT</td>
<td>Im[( M^*<em>{0-,--} - M</em>{0-,++} )]</td>
<td>( \propto (-t')^{3/2} )</td>
</tr>
<tr>
<td>( A_{UT}^{\sin \phi_0} )</td>
<td>LT</td>
<td>Im[( M^*<em>{0-,++} - M</em>{0+,0+} )]</td>
<td>const.</td>
</tr>
<tr>
<td>( A_{UL}^{\sin \phi} )</td>
<td>LT</td>
<td>Im[( M^*<em>{0-,++} - M</em>{0-,0+} )]</td>
<td>( \propto \sqrt{-t} )</td>
</tr>
<tr>
<td>( A_{LU}^{\sin \phi} )</td>
<td>LT</td>
<td>Im[( M^*<em>{0-,++} - M</em>{0-,0+} )]</td>
<td>( \propto \sqrt{-t} )</td>
</tr>
<tr>
<td>( A_{LL}^{\cos \phi} )</td>
<td>LT</td>
<td>Re[( M^*<em>{0-,++} - M</em>{0-,0+} )]</td>
<td>( \propto \sqrt{-t} )</td>
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</table>

\[ A_{UL}^{\sin \phi} / A_{UL}^{\cos \phi} \approx \sqrt{(1 - \epsilon) / (1 + \epsilon)} \]

- HT SSAs are expected to be very significant
- Wider coverage (CLAS12,EIC) would allow measurements of \( Q^2 \) dependence of HT SSAs
Recent progress with GPD-based description

- Goloskokov&Kroll, Goldstein&Liuti. Include transversity GPDs $H_T$ and $\bar{E}_T = 2\bar{H}_T + E_T$ Dominate in CLAS kinematics. Successfully described data.
Sivers TMD evolution

Comparison of JLab12 data with HERMES and COMPASS will pin down the $Q^2$ evolution of Sivers asymmetry.

TMD Evolution may explain existing differences between HERMES and COMPASS.
Aybat, Prokudin & Rogers: arXiv:1112.4423

\[ f_{1T}^{q}(SIDIS) = -f_{1T}^{q}(DY) \]
$A_1(\pi) \propto \frac{\sum_{q} e_{q}^{2} g_{1}^{q}(x) D_{1}^{q \rightarrow \pi}(z)}{\sum_{q} e_{q}^{2} f_{1}^{q}(x) D_{1}^{q \rightarrow \pi}(z)} \quad A_{1}(x, z, P_{T}) = A_{1}(x, z) \left( \frac{\langle P_{T}^{2, unp} \rangle}{\langle P_{T}^{2, pol} \rangle} \right) \exp \left( -\frac{P_{T}^{2}}{\langle P_{T}^{2, pol} \rangle} - \frac{P_{T}^{2}}{\langle P_{T}^{2, unp} \rangle} \right)$

CLAS data consistent with model predictions and lattice predicting that width of $g_1$ is less than the width of $f_1$.

$\mu_2^2/\mu_0^2 = 0.692 \pm 0.039 \pm 0.045$
From JLab12 to EIC

- Study of high $x$ domain requires high luminosity, low $x$ higher energies
- Wide range in $Q^2$ is crucial to study the evolution
- Overlap of EIC and JLab12 in the valence region will be crucial for the TMD program

**JLab@12 GeV (25/50/75)**

$0.1 < x_B < 0.7$: valence quarks

**EIC** $\sqrt{s} = 140, 50, 15$ GeV

$10^{-4} < x_B < 0.3$: gluons and quarks, higher $P_T$ and $Q^2$.  

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$A_{UT}^{sin(\phi - \phi_S)} = \frac{\sum_q e_q^2 f_1^{qT} D_1^q}{\sum_q e_q^2 f_1^{q} D_1^q}$

Aybat, Prokudin & Rogers

arXiv:1112.4423
From JLab12 to JLab50

For a given lumi (30min of runtime) and given bin in hadron z and \( P_T \), higher energy provides higher counts and wider coverage in \( Q^2 \), allowing studies of \( Q^2 \) evolution of 3D partonic distributions in a wide \( Q^2 \) range.
**k_Т and FSI**

- Factorization proven for small k_Т (Ji, Ma, Yuan 2005)
- Medium modifications of k_Т PDFs (Tang, Wang, Zhou 2008)
- Complete definition of TMDs (Collins 2011 “Foundation of Perturbative QCD”)
- Evolution of TMDs, (Collins, Aybat, Rogers 2011)
- TMDs on Lattice, (Musch, Haegler et al. 2011)
- Color Lorentz Force acting on ejected quark, torque along trajectory (Burkardt 2008, 2012)
- k_Т-dependent flavor decomposition (BGMP procedure, 2011)

- Experiments consistent with evolution on <k_Т^2> increasing with Q^2.
- What is the source of the k_Т (dynamical vs static)?
- What is the role of FSI and how they modify in medium

\[ f_q^N(x, \vec{k}_T) \]

BHS 2002
Collins 2002
Ji, Yuan 2002

**soft gluon exchanges included in the distribution function (gauge link)**
Forces and binding effects in the partonic medium

\[ xe = x\tilde{e} + \frac{m}{M} f_1 \]
\[ xh_L = x\tilde{h}_L + \frac{p_T^2}{M^2} h_{1L}^1 + \frac{m}{M} g_{1L} \]

“Wandzura-Wilczek approximation” is equivalent to setting functions with a tilde to zero.

\[ e_2 \equiv \int_0^1 dx x^2 \tilde{e}(x) \]

Quark polarized in the x-direction with \( k_T \) in the y-direction

Boer-Mulders Force on the active quark right after scattering (t=0)

Interpreting HT (quark-gluon-quark correlations) as force on the quarks (Burkardt hep-ph:0810.3589)

<table>
<thead>
<tr>
<th>N/q</th>
<th>U</th>
<th>L</th>
<th>T</th>
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<tr>
<td>U</td>
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<tr>
<td>L</td>
<td></td>
<td>h_L</td>
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<tr>
<td>T</td>
<td>g_T</td>
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</table>

current quark masses
**$k_T$-dependence of TMDs**

- Transition from low $p_T$ to high $p_T$

Directly obtained ETQS functions are opposite in sign to those from $k_T$ moments “sign mismatch”

Sivers function extracted assuming $k_T$ distribution is gaussian

With orbital angular momentum TMD can’t be gaussian

How to measure $k_T$-dependences of TMDs

(Z. Kang et al, 2011)
Q^2-dependence of beam SSA

\[ A_{LU} \propto g^\perp(x)D_1(z) \]

Study for Q^2 dependence of beam SSA allows to check the higher twist nature and access quark-gluon correlations.
$P_T$-dependence studies at Hall-C

H. Mkrtchyan (DIS2011)

**Experiment E00-108**

Beam energy 5.5 GeV

4 cm LH2 and LD2 targets

\[
\sigma_{d}^{\pi^+} \propto (4D^+ + D^-)(u + d)
\]

\[
\sigma_{d}^{\pi^-} \propto (4D^- + D^+)(u + d)
\]

\[
\frac{\sigma_{d}^{\pi^+}}{\sigma_{d}^{\pi^-}} = \frac{4D^+ + D^-}{4D^- + D^+}
\]

\[
D^-/D^+ = (4 - r) / (4r - 1)
\]

\[
r = \sigma_{d}(\pi^+)/\sigma_{d}(\pi^-)
\]

$x$-dependence of $\pi^+/\pi^-$ ratio is good agreement with the quark parton model predictions (lines CTEQ5M+BKK).
$A_1(P_T)$-dependence in SIDIS

\[ A_1(\pi) \propto \frac{\sum_q e_q^2 g_1^q(x) D_1^{q \to \pi}(z)}{\sum_q e_q^2 f_1^q(x) D_1^{q \to \pi}(z)} e^{-z^2 \frac{P_T^2}{\mu_D^2 + z^2 \mu_0^2} \frac{(\mu_0^2 - \mu_2^2)}{\mu_D^2 + z^2 \mu_2^2}} \]

M. Anselmino et al
hep-ph/0608048

$A_{LL}(\pi)$ sensitive to difference in $k_T$ distributions for $f_1$ and $g_1$

Wide range in $P_T$ allows studies of transition from TMD to perturbative approach

\[ f_1^q(x, k_T) = f_1(x) \frac{1}{\pi \mu_0^2} \exp \left( -\frac{k_T^2}{\mu_0^2} \right) \]
\[ g_1^q(x, k_T) = g_1(x) \frac{1}{\pi \mu_2^2} \exp \left( -\frac{k_T^2}{\mu_2^2} \right) \]
\[ D_1^q(z, p_T) = D_1(z) \frac{1}{\pi \mu_D^2} \exp \left( -\frac{p_T^2}{\mu_D^2} \right) \]

Perturbative limit calculations available for $g_1^q(x, k_T)$, $f_1(x, k_T)$:


\textbullet $A_{LL}(\pi)$ sensitive to difference in $k_T$ distributions for $f_1$ and $g_1$
\textbullet Wide range in $P_T$ allows studies of transition from TMD to perturbative approach

H. Avakian, QCD-N'12, Oct 23
Quark distributions at large $k_T$: models

$\langle x \rangle \approx 0.3$

$u^+(k_T)/u^-(k_T)$

**B. Musch et al arXiv:1011.1213**

$\Delta u/u$ consistent between lattice and diquark model

JMR model

$$M_R, R=\frac{8\alpha}{a_2}$$

$$u^+(x, k_T) \propto \frac{(xM+\bar{m})^2}{(k_T^2 + \lambda_R^2)^{2\alpha}}$$

$$u^-(x, k_T) \propto \frac{k_T^2}{(k_T^2 + \lambda_R^2)^{2\alpha}}$$
E12-07-107: Studies of Spin-Orbit Correlations with Longitudinally Polarized Target

\[
\frac{d\sigma}{dx dy d\phi_S d\phi_h dP_{h\perp}^2} \propto S_L \left[ \sqrt{2\epsilon(1+\epsilon)} \sin\phi_h F_{UL}^\sin\phi_h + \epsilon \sin(2\phi_h) F_{UL}^\sin(2\phi_h) \right] \\
+ S_L \lambda_e \left[ \sqrt{1-\epsilon^2} F_{LL}^\cos(\phi_h) + \epsilon \cos(\phi_h) F_{LL}^\cos(\phi_h) \right]
\]

\[ h_1L \otimes H_i \]

\[ h_{1L} \otimes H_i \]

\[ g_{1L} \otimes D_1 \]

**A_1 \, P_T\text{-dependence provides access to helicity dependence of } k_T\text{-distributions of quarks**}

**p \& d data required for } P_T\text{-dependence flavor decomposition**

Jefferson Lab

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\[ \sigma_{LU(UL)}^{\sin \phi} \sim F_{LU(UL)} \sim \frac{1}{Q} \text{ (Twist-3)} \]

\[ A_{LU} \propto g^\perp(x) D_1(z) \]

Study for SSA transition from non-perturbative to perturbative regime. EIC will significantly increase the \( P_T \) range.
FAST-MC for CLAS12

SIDIS MC in 8D \((x,y,z,\phi,\phi_S,p_T,\lambda,\pi)\)

Simple model with 10% difference between \(f_1\) (0.2\(\text{GeV}^2\)) and \(g_1\) widths with a fixed width for \(D_1\) (0.14\(\text{GeV}^2\))

\[
f_q(x, k_\perp) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2/\langle k_\perp^2 \rangle}
\]

CLAS12 acceptance & resolutions

Events in CLAS12

Reasonable agreement of kinematic distributions with realistic LUND simulation

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For a given lumi (30min of runtime with L=10^{35} cm^{-2}s^{-1}) and given bin in hadron z and P_T, higher energy provides higher counts and wider coverage in x and Q^2.
Boer-Mulders Asymmetry with CLAS12 & EIC

Nonperturbative TMD

Perturbative region

Transversely polarized quarks in the unpolarized nucleon

\[ \sin(\phi_C) = \cos(2\phi_h) \]

\[ A_{UU}^{\cos 2\phi} \propto h_1^{(1)} H_1^{(1)} \]

\[ \langle \cos 2\phi \rangle |_{P_{h\perp} \gg \Lambda_{QCD}} \propto \frac{1}{P_{h\perp}^2} \]

Perturbative limit calculations available for \( f_1(x, k_T), h_1^{(1)}(x, k_T) \)


CLAS12 and ELIC studies of transition from non-perturbative to perturbative regime will provide complementary info on spin-orbit correlations and test unified theory (Ji et al)
At small $x$ of EIC Kaon relative rates higher, making it ideal place to study the Sivers asymmetry in Kaon production (in particular $K^-$).

Combination with CLAS12 data will provide almost complete $x$-range.
TMD Correlation Functions in other experiments

\[ \nu \approx h_{1q} \times h_{1\bar{q}} \]

BOER-MULDER
Spin Orbit effect

Fragmentation Functions (FF)

<table>
<thead>
<tr>
<th>Z</th>
<th>q/h</th>
<th>U</th>
<th>L</th>
<th>T</th>
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<td>U</td>
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Collins
Quark spin probe

In di-hadron case \( H_1 \)

Interference Fragmentation Function (IFF)

\[ A_{12} \approx H_{1q} \times H_{1\bar{q}} \]
Pretzelosity @ EIC

\[ A_{UT}^{\sin(3\phi-\phi_S)} \frac{1 - y}{1 - y + y^2/2} \frac{\sum_q e_q^2 h_{1T}^{\perp}(1) q H_{1}^{\perp} q}{\sum_q e_q^2 f_1^q D_1^q} \]

- EIC measurement combined with CLAS12 will provide a complete kinematic range for pretzelosity measurements
For a given lumi (30min of runtime with $10^{35}$) and given bin in hadron $z$ and $P_T$, higher energy provides higher counts and wider coverage in $x$ and $P_T$ to allow studies of correlations between longitudinal and transverse degrees of freedom.
For a given lumi (30min of runtime) and given bin in hadron z and $P_T$, higher energy provides higher counts and wider coverage in $Q^2$, allowing studies of $Q^2$ evolution of 3D partonic distributions in a wide $Q^2$ range.
Evolving TMD PDFs

\[ p = xP + k \]

\[ b_{T,\text{max}} = 0.5 \text{ GeV}^{-1} \]

Aybat & Rogers arXiv:1110.6099
Collins effect

Simple string fragmentation (Artru model)

\[ h_1 H_{1} \perp u \rightarrow \pi^+ \]

Leading pion out of page (\( \bar{d} \) - direction)

\[ h_1 H_{1} \perp u \rightarrow \pi^- \]

\[ H_{1} \perp u \rightarrow \pi^- > H_{1} \perp u \rightarrow \pi^+ \]

\( d \) kicked in the opposite to the leading pion (into the page)

Sub-leading pion opposite to leading (double kick into the page)

If unfavored Collins fragmentation dominates measured \( \pi^- \) vs \( \pi^+ \), why K- vs K+ is different?
SIDIS ($\gamma^* p \rightarrow \pi X$): Transversely polarized target

- Azimuthal moments in pion production in SIDIS
  - $\sin(\phi - \phi_S)$ (Sivers function $f_{1T}^\perp$) and relation with GPDs
  - $\sin(\phi + \phi_S)$ (Collins function $H_{1\perp}$ and transversity $h_1$)
  - $\sin(3\phi - \phi_S)$ (Collins function $H_{1\perp}$ and pretzelosity $h_{1T}^\perp$)

Pasquini and Yuan, Phys.Rev.D81:114013,2010
\[ A_1(x, z, P_T) = A_1(x, z) \frac{\langle P_{T}^{2, \text{unp}} \rangle}{\langle P_{T}^{2, \text{pol}} \rangle} \exp \left( -\frac{P_{T}^{2}}{\langle P_{T}^{2, \text{pol}} \rangle} - \frac{P_{T}^{2}}{\langle P_{T}^{2, \text{unp}} \rangle} \right) \]

\[ f_1(x, k_T) = f_1(x) \frac{1}{\pi \mu_0^2} \exp \left( -\frac{k_T^2}{\mu_0^2} \right) \]

\[ g_1(x, k_T) = g_1(x) \frac{1}{\pi \mu_2^2} \exp \left( -\frac{k_T^2}{\mu_2^2} \right) \]

\[ \langle P_{T}^{2}(z) \rangle = z^2 \mu_{0/2}^2 + \mu_D^2 \]

\[ \pi^+ A_1 \text{ suggests broader } k_T \text{ distributions for } f_1 \text{ than for } g_1 \]

\[ \pi^- A_1 \text{ may require non-Gaussian } k_T \text{ dependence for different helicities and/or flavors} \]
HT-distributions and dihadron SIDIS

Compare single hadron and dihadron SSAs

\[
\frac{M}{M_h} x e(x) H_1^L(z, \zeta, M_h^2) + \frac{1}{z} f_1(x) \tilde{G}^L(z, \zeta, M_h^2)
\]

\[
\frac{M}{M_h} x h_L(x) H_1^L(z, \zeta, M_h^2) + \frac{1}{z} g_1(x) \tilde{G}^L(z, \zeta, M_h^2)
\]

Only 2 terms with common unknown HT $G^{-}$ term!

Aurore Courtoy/Anselm Voosen - Spin session

• Higher twists in dihadron SIDIS collinear (no problem with factorization)
• Bell can measure $K^+\pi^-$ dihadron fragmentation functions

Projections for $(\pi^+K^-)(K^+\pi^-)$ for 580 fb$^{-1}$

arXiv:1104.2425

H. Avakian, QCD-N'12, Oct 23
Transverse momentum distributions of partons

\[ \langle P_T^2 \rangle \approx z^2 \langle k_T^2 \rangle + \langle p^2 \rangle \]

Transverse momentum distributions in hadronization may be flavor dependent => measurements of different final state hadrons required.

H. Avakian, QCD-N'12, Oct 23
Combined analysis of Collins fragmentation asymmetries from proton and deuteron and for $\pi$ and $K$ may provide independent to $e^+e^-$ (BELLE/BABAR) information on the underlying Collins function.
Chiral odd HT-distribution

How can we separate the HT contributions?

\[
\begin{align*}
F_{LU} \sin \phi & \quad F_{UL} \sin \phi \\
\vec{p}_T & \quad \phi_h \\
\phi_s=\pi & \quad y \\
x & \quad X
\end{align*}
\]

\[e H_\perp \sin \phi_h, \quad h_L H_\perp \sin \phi_h\]

HT function related to force on the quark. M.Burkardt (2008)

Compare single hadron and dihadron SSAs

\[
\frac{M}{M_h} x e(x) H_1(z, \zeta, M_h^2) + \frac{1}{z} f_1(x) \tilde{G}^\perp(z, \zeta, M_h^2)
\]

\[
\frac{M}{M_h} x h_L(x) H_1(z, \zeta, M_h^2) + \frac{1}{z} g_1(x) \tilde{G}^\perp(z, \zeta, M_h^2)
\]

Only 2 terms with common unknown HT $G^*$ term!

M.Radici

H. Avakian, QCD-N'12, Oct 23
Large PT may have significant nuclear contribution
Azimuthal moments with unpolarized target

### Quark Polarization

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### Azimuthal Anisotropy

$$A_{UU}^{\cos \phi} \propto \frac{M_h}{M} f_1 \frac{D_{1 \perp}}{z} - \frac{M}{M_h} x f_{1 \perp} D_1$$

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### Polarization Responses

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### Additional Table

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Azimuthal moments with unpolarized target

\[ A_{UU}^{\cos \phi} \sim -h_1^\perp \frac{H}{z} + xhH_1^\perp \]

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SSA with unpolarized target

\[ A_{L,U}^{\sin \phi} \propto \frac{M_h}{M} f^1 \frac{G^\perp}{z} - \frac{M}{M_h} x g^\perp D_1 \]

\[ q/h \quad U \quad L \quad T \]

| U | D^\perp | D_L^\perp | D_T, D_T^\perp |
| L | G^\perp | G_L^\perp | G_T, G_T^\perp |
| T | H, E | H_L, E_L | H_T, E_T, H_T^\perp, E_T^\perp |

\[ q/h \quad U \quad L \quad T \]

| U | D_1 | D_{1T} |
| L | G_{1L} | G_{1T} |
| T | H_1^\perp | H_{1L}^\perp | H_1^\perp, H_{1T}^\perp |
### SSA with unpolarized target

**Quark polarization**

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\[
A_{LU}^{\sin \phi} \sim h_{1L}^\perp \frac{E}{z} + x e H_1^\perp
\]

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### SSA with long. polarized target

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$$A_{UL}^{\sin \phi} \propto \frac{M_h}{M} \frac{G^\perp}{z} + \frac{M}{M_h} x f_L^\perp D_1$$

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### SSA with long. polarized target

#### Quark polarization

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#### Matrix Elements

$$A_{UL}^{\sin \phi} \sim h_1^L H_{UL} \frac{H}{z} + x h_L H_{1L}$$

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### References

H. Avakian, QCD-N’12, Oct 23
SSA with unpolarized target

quark polarization

\[
A_{LL}^{\cos \phi} \sim \frac{M_h}{M} g_{1L} \frac{D_{L}^{\perp}}{z} + x e_L H_{1L}^{\perp}
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\[
\begin{array}{c|c|c|c|c|}
q/h & U & L & T \\
\hline
U & D_1 & D_{L}^{\perp} & D_T, D_{T}^{\perp} \\
L & G_1 & G_{L} & G_T, G_{T}^{\perp} \\
T & H, E & H_{L}, E_{L} & H_T, E_T, H_{1T}, E_{1T} \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|}
q/h & U & L & T \\
\hline
U & D_{1T} & D_{1T} \\
L & G_{1L} & G_{1T} \\
T & H_{1T} & H_{1T} \\
\end{array}
\]

H. Avakian, QCD-N'12, Oct 23
SSA with unpolarized target

quark polarization

\[ A_{LL}^{\cos \phi} \sim \frac{M_h}{M} h_{1L}^\perp \frac{E}{z} + xg_L^\perp D_1 \]

\[ q/h \quad U \quad L \quad T \]
\[ U \quad D_1 \quad D_L^\perp \quad D_T, D_T^\perp \]
\[ L \quad G_L^\perp \quad G_L \quad G_T, G_T^\perp \]
\[ T \quad H, E \quad H_L, E_L \quad H_T, E_T, H_T^\perp, E_T^\perp \]

\[ q/h \quad U \quad L \quad T \]
\[ U \quad D_1 \quad \] \]
\[ L \quad G_{1L} \quad G_{1T} \]
\[ T \quad H_1^\perp \quad H_{1L}^\perp \quad H_1, H_{1T}^\perp \]
**Twist-3 PDFs**: “new testament”

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\[
\frac{1}{2M_x} \text{Tr} \left[ \Phi_{A\alpha} \sigma^{\alpha+} \right] = \tilde{h} + i \tilde{\epsilon} + \frac{\epsilon_T^\alpha p_T \rho S_T}{M} (\tilde{h}_T - i \tilde{\epsilon}_T),
\]
\[
\frac{1}{2M_x} \text{Tr} \left[ \Phi_{A\alpha} i \sigma^{\alpha+} \gamma_5 \right] = S_L (\tilde{h}_L + i \tilde{\epsilon}_L) - \frac{p_T \cdot S_T}{M} (\tilde{h}_T + i \tilde{\epsilon}_T),
\]
\[
\frac{1}{2M_x} \text{Tr} \left[ \Phi_{A\rho} (g_T^{\alpha\rho} + i e_T^{\alpha\rho} \gamma_5) \gamma^+ \right] = \frac{p_T^\alpha}{M} (\tilde{f}_L + i \tilde{g}_L) - e_T^{\alpha\rho} S_T^\rho (\tilde{f}_T + i \tilde{g}_T)
\]
\[- S_L \frac{\epsilon_T^\rho p_T^\rho}{M} (\tilde{f}_L + i \tilde{g}_L) - \frac{p_T^\rho}{M^2} \frac{1}{2} \frac{p_T^2 g_T^{\alpha\rho}}{M^2} \epsilon_T^\rho S_T (\tilde{f}_T + i \tilde{g}_T),\]
SIDIS ($\gamma^* p \rightarrow \pi X$) $x$-section at leading twist

$$\frac{d\sigma}{dxdydzd^2P_h} = \frac{4\pi\alpha^2_s}{Q^4} [x(1 - y + y^2/2) F_{UU} - x(1 - y) \cos(2\phi) F_{UU}^{\cos 2\phi}]$$

- Measure Boer-Mulders distribution functions and probe the polarized fragmentation function
- Measurements from different experiments consistent
SIDIS: partonic cross sections

\[ d\sigma^h \propto \sum f^{H\to q}(x, k_T) \otimes d\sigma_q(y) \otimes D^{q\to h}(z, p_\perp) \]

\[ d\sigma^h \propto \sum f^{H\to q}(x) d\sigma_q(y) D^{q\to h}(z) \]
Collins effect

Simple string fragmentation for pions (Artru model)

$\rho$ production may produce an opposite sign $A_{UT}$

<table>
<thead>
<tr>
<th>Fraction of $\rho$ in $e\pi X$</th>
<th>% left from $e\pi X$ asm</th>
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<td>~75%</td>
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<tr>
<td>40%</td>
<td>~50%</td>
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Fraction of direct kaons may be significantly higher than the fraction of direct pions.

Leading $\rho$ opposite to leading $\pi$ (into page)

$h_1 H_1 \frac{1}{u} \rightarrow \pi$

$LUND-MC$

hep-ph/9606390
Sivers effect in the target fragmentation

High statistics of CLAS12 will allow studies of kinematic dependences of the Sivers effect in target fragmentation region

A.Kotzinian
\[ q(x, k_\perp)_{k_\perp \gg \Lambda_{QCD}} = \frac{1}{(k_\perp^2)^n} \int \frac{dx'}{x'} f_i(x') \times H_{q/i}(x; x') , \] (23)

where \( q(x, k_\perp) \) represents the TMD quark distribution we are interested, \( f_i \) represents the integrated quark distribution for the \( k_\perp \)-even TMDs, and higher twist quark-gluon correlation function for the \( k_\perp \)-odd TMDs. For the latter case, \( x' \) should be understood as two variable for the twist-three quark-gluon correlation functions as we discussed in the last section. The overall power behavior \( 1/(k_\perp^2)^n \) can be analyzed by the power counting rule [48]. The hard coefficient \( H_{q/i}(x; x') \) is calculated from perturbative QCD. In this paper, we will show the one-gluon radiation contribution to this hard coefficient.

The \( k_\perp \)-even TMD quark distribution functions, \( f_1(x, k_\perp) \), \( g_{1L}(x, k_\perp) \), and \( h_1(x, k_\perp) \) be calculated from the associated integrated quark distributions [23]. For the non-s contributions, they are expressed as [23],

\[

t_1(x_B, k_\perp) = \frac{\alpha_s}{2\pi^2} \frac{1}{k_\perp^2} C_F \int \frac{dx}{x} f_1(x) \left[ \frac{1 + \xi^2}{(1 - \xi)_+} + \delta(1 - \xi) \left( \ln \frac{x_B^2 \xi^2}{k_\perp^2} - 1 \right) \right],
\]

\[
g_{1L}(x_B, k_\perp) = \frac{\alpha_s}{2\pi^2} \frac{1}{k_\perp^2} C_F \int \frac{dx}{x} g_{1L}(x) \left[ \frac{1 + \xi^2}{(1 - \xi)_+} + \delta(1 - \xi) \left( \ln \frac{x_B^2 \xi^2}{k_\perp^2} - 1 \right) \right],
\]

\[
h_1(x_B, k_\perp) = \frac{\alpha_s}{2\pi^2} \frac{1}{k_\perp^2} C_F \int \frac{dx}{x} f_1(x) \left[ \frac{2\xi}{(1 - \xi)_+} + \delta(1 - \xi) \left( \ln \frac{x_B^2 \xi^2}{k_\perp^2} - 1 \right) \right],
\]

where the color factor \( C_F = (N_c^2 - 1)/2N_c \) with \( N_c = 3 \), \( \xi = x_B/x \) and \( \xi^2 = (2v \cdot P)^2/v^2 \).
TMDs: QCD based predictions

<table>
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<tr>
<th>N</th>
<th>q</th>
<th>U</th>
<th>L</th>
<th>T</th>
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<td>g_{1T}</td>
<td>h_1</td>
<td>h_{1T}</td>
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</table>

Large-x limit

Brodsky & Yuan (2006)

\[ f_{1T}^\perp \sim (1 - x)^4 \quad g_{1T}^\perp \sim (1 - x)^4 \quad h_1 \sim (1 - x)^3 \]

Burkardt (2007)

\[ h_{1T}^\perp \sim (1 - x)^5 \]

Large-Nc limit (Pobilitsa)

\[ f_1^\perp u > 0, \quad f_1^\perp d > 0 \quad h_1^\perp u < 0, \quad h_1^\perp d < 0 \]

Do not change sign (isoscalar)

\[ f_{1T}^\perp u < 0, \quad f_{1T}^\perp d > 0 \]

All others change sign

u→d (isovector)