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ANALYSIS OF DISTRIBUTIONS PRODUCED BY CHARGED PARTICLES

(and Other Compound-Poisson Processes)

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ANALYSIS OF DISTRIBUTIONS PRODUCED BY CHARGED PARTICLES*
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This paper is about how to transform certain charged-particle distributions into what they would have been under somewhat different circumstances. It provides a technique useful in analyzing certain experimental data. The distributions that can be transformed are those known as compound-Poisson distributions.

COMPOUND-POISSON DISTRIBUTIONS

Since the interactions between charged particles and an irradiated medium occur at individual molecules of the medium, some of the quantities produced by the particles are sums of increments due to individual interactions, and the number of individual interactions whose increments are summed has a Poisson distribution. Such quantities are said to be the result of a compound-Poisson process. Since the increments in individual interactions may have a distribution and since the number of increments has a (Poisson) distribution, the quantity itself has a distribution of values.

A good example of a charged-particle, compound-Poisson process is the energy lost by a particle in passing through a thin sheet of material. The energy lost in the sheet is the sum of the energy losses in interactions with individual molecules in the sheet. The number of those interactions is Poisson distributed.

In this example it is assumed that the total energy loss of the particle is so small that its properties do not change significantly while passing through the sheet. In particular, this means that the distribution of energy losses in the individual interactions is the same from one interaction to the next. The same assumption about the total energy of the particle is made in the remainder of this paper. However, it is possible and desirable to consider quantities for which the distribution of the increments does change -- but for reasons other than changes in the properties of the particles. For example, the energy absorbed within the sheet considered above differs from the energy lost there by the particle because of the transport of energy into and out of the sheet by secondaries. The energy transport by the secondaries depends on where the interaction was along the path of the particle. Roesch and Glass (1) showed that this change along the track, since it is not a stochastic variation, can be absorbed within the theory and the absorbed energy treated as a compound-Poisson process also. The only difference from a process such as the energy loss is that the energy absorbed is treated as though it were due to increments of absorbed energy from a common distribution that is the average of

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the actual distributions along the track of the particle. The same sort of averaging permits dealing with quantities such as ionization, number of chemical species produced, Z-distributions of microdosimetry, etc.

TRANSFORMATIONS OF DISTRIBUTIONS

Distributions resulting from a compound-Poisson process can be transformed into what they would have been had the distribution (or its track-average) of increments in individual interactions remained the same but the average number of events changed. The Laplace transform of a distribution resulting from a compound-Poisson process is an exponential function of an argument that contains the average number of interactions as a factor (cf. Eq. (8) in reference (1)). Consequently, if the transform is raised to a power, the inverse of the result is the distribution that would be obtained for the same increment-distribution but for an average number of interactions equal to the power times the original number.

CALCULATIONS

The Fourier transform can be used as well as the Laplace transform, because it is an exponential function also. The numerical calculations reported here were made with discrete Fourier transforms (2).

Ordinary computer software for raising complex numbers to powers assumes that the phase of the number is between -180° and $+180^\circ$; it may assume a value that is incorrect by an integral multiple of 360° . Ordinarily this makes no difference

Raising to a power multiplies the phase of the complex number by the power. If the phase is not between -180° and $+180^\circ$ and the power is an integer, the resulting phase will be in error by an integral multiple of 360° and the inverse transformation will be carried through correctly. However, the interesting applications are those where the power is non-integral; then the resulting phase is in error by non-integral multiples of 360° and the inverse transform is done incorrectly.

To find the correct phase angle for each complex number in the Fourier transform array, it was found sufficient to assume that the phase angle varied regularly from one member of the array to the next. The first member of the array, the one for zero "frequency", has a phase angle of zero, which serves as the initial value. Figure 1 illustrates how the phases were determined. First, the angle whose tangent is the ratio of the imaginary and the real parts of the complex number (the arctangent referred to below) is calculated in the usual way for successive members of the array. It starts at zero and decreases rapidly. After a few frequency channels, it suddenly changes from a large negative value to a large positive value, i.e. it jumps up to the curve labeled "incorrect" in the figure. This change is not a "regular" one; it is too large. Therefore, it is assumed that the phase angle has passed -180° , and 360° is subtracted from the arctangent. 360° is also subtracted from each succeeding arctangent till the latter exhibits another sudden jump. For example, there is another jump near 300 channels from a large positive to a large negative value. Here it is assumed that the phase angle has increased to above -180° again and the 360° is no longer subtracted from the arctangents. Large multiples of 360° may

have to be added or subtracted from the arctangent; in some experimental data, values larger than $75 \times 360^\circ$ were subtracted.

This simple way of determining the phase should work all right as long as there are enough frequency channels to give good resolution. In a few cases of very simple distributions that were used as illustrations and test cases, the arctangent became indeterminate, because the magnitude of the complex number became zero. In these cases it was found necessary to set the multiple of 360° being subtracted from the arctangent to zero again and the proceed with the remaining members of the array.

Once the correct phase angles are determined, powers are taken by raising the magnitude of the complex number to the power and multiplying the phase angle by the power.

EXAMPLE: EXPERIMENTAL DATA -- NOISE

Figure 2 shows an experimental distribution (the triangles) that was transformed by a power of 0.5 (to give the circles) and by a power of 2.0 (to give the crosses).

The theory given above does not apply to individual experimental distributions; it applies to the probability distributions which the experimental distributions are assumed to approach in the limit of an infinite number of samples. Nevertheless, one can go ahead and apply the transformation and consider that what he is doing is transforming a distribution that is the sum of the probability distribution of interest and a distribution of statistical fluctuations or "noise".

Figure 2 shows quite clearly that the noise gets worse when the power is less than 1.0, and it gets better, i.e. the data become smoother, when the power is greater than 1.0. Raising to the second power is equivalent to the well-known process of folding or convoluting the data with itself. Folding is known to be a smoothing process. "Unfolding" is an "unsmoothing" process.

The line drawn through the original data was an eyeball fit to the data in an attempt to minimize the noise. When values were measured from this curve with a ruler and transformed to the 0.5 power, the jagged curve through the circles was obtained. The jaggedness is the result of noise due to uncertainties in measurement of the original smooth curve.

LIMITATIONS

Evidently there is no way of transforming experimental data by a power that is too much less than 1.0 without introducing unacceptable noise.

On the other hand, one should also take care in transforming by a power much larger than 1.0. Raising to a large power is equivalent to increasing the number of interactions whose increments are summed. According to the Central Limit Theorem of probability theory, the resulting distribution approaches a Gaussian distribution regardless of the shape of the increment distribution. Therefore, raising to a large power makes insensitive any comparisons one might like to make.

EXAMPLES: IONIZATION DISTRIBUTIONS

To illustrate why one might want to transform distributions, Figs. 3 and 4 are shown, taken from a recent paper on the measurement of ionization distributions due to protons (3.0 MeV shown) in a site of tissue-equivalent gas $25 \mu\text{g}/\text{cm}^2$ in height and diameter (3). The protons were in a beam whose dimensions were much smaller than those of the site. The different distributions resulted for different distances, r , of the beam from the center of the site. Before transformation, the distributions were considerably different; the mean values covered a range of more than a factor of eight.

For reasons detailed in the original paper, it was believed that the distributions of increments might be quite similar for all protons that passed through the site regardless of how far from the center. Since the mean ionization is proportional to the mean number of interactions (1), the distributions were transformed to the same mean ionization by raising each transform to the power equal to the ratio of the mean for tracks at $4 \mu\text{g}/\text{cm}^2$ from the center to its own mean ionization. Figure 3 shows the results. The distributions for protons that passed through the site transformed into a common distribution. The distribution for $r = 12.8 \mu\text{g}/\text{cm}^2$, which is $0.3 \mu\text{g}/\text{cm}^2$ outside the site, is noticeably different, and the one for $r = 13.6 \mu\text{g}/\text{cm}^2$ is distinctly different. The agreement confirmed that the increment distributions were quite similar for tracks that crossed the site, but quite different for tracks that did not.

Another hypothesis that was tested was that the number of interactions that contributed to the ionization was proportional to the pathlength, t , of the protons in the site. (Obviously, this cannot be true for tracks which produced ionization but which passed outside the site. The idea was to see where the hypothesis breaks down.) To test the hypothesis, the distributions used in Fig. 3 that were for tracks crossing the site were transformed by powers equal to $25/t$, which should transform them to a common pathlength of $25 \mu\text{g}/\text{cm}^2$. The results are shown in Fig. 4. None of the transformed distributions are as near alike as those in Fig. 3; hence, the hypothesis must be rejected -- although it is approximately true for tracks near the center.

Other uses to which the transformation technique has been put include comparing distributions due to protons in the same track position but of different energies (using the ratio of the stopping powers for the transformation power) and comparing distributions due to protons of a given energy and track position in sites of different size (using the ratio of the site sizes for the transformation power). Also, when the appropriate power to transform one set of data to fit another is not known, a good value can be found by requiring the sum of the squares of the deviations between the transformed curve and the curve being matched to be a minimum.

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Fig. 1

Fig. 2

Fig. 3

Fig. 4