MEASUREMENT OF NP (PP) TRIPLE SCATTERING PARAMETERS \( R'(R) \) AND \( R''(R') \) 510 TO 670 MeV

Kwok-Chu Leung
(Ph.D. Thesis)

March 23, 1970

AEC Contract No. W-7405-eng-48
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.
MEASUREMENT OF NP (PP) TRIPLE SCATTERING PARAMETERS \( R^t(R) \) AND \( R'^t(R') \) 
510 TO 670 MeV

Kwok-Chu Leung
(Ph.D. Thesis)

March 23, 1970
MEASUREMENT OF NP (PP) TRIPLE SCATTERING PARAMETERS $R^t(R)$ AND $R'^t(R')$
510 TO 670 MeV

Kwok-Chu Leung

Lawrence Radiation Laboratory
University of California
Berkeley, California

March 23, 1970

ABSTRACT

Wolfenstein triple scattering parameters $R$ and $R'$ for the proton-proton system and $R^t$ and $R'^t$ for the neutron-proton system have been measured. The p-p data were measured at three angles at 510, 600, and 670 MeV, and the n-p data were measured at four angles at 510 and 600 MeV. No strong energy dependence is observed in either case.

A spark chamber spectrometer system was used to analyze the second and third scattering. Polarized proton and neutron beams were produced by scattering the primary proton beam off liquid hydrogen and liquid deuteron targets respectively.

We find good agreement between our proton-proton data and the published data at 430 and 660 MeV. Phase shift predictions are compared to our results.
MEASUREMENT OF NP (PP) TRIPLE SCATTERING PARAMETERS $R_t(R)$ AND $R'_t(R')$
510 to 670 MeV

Abstract

I. Introduction ................................................................. 1

II. N-N Scattering Matrix .................................................. 3

III. Experimental Method and Apparatus
   A. Method of Measurement ............................................. 8
   B. Polarized Beams and Targets ..................................... 10
   C. Wire-Spark-Chamber-Spectrometer ............................... 11
   D. Electronics .......................................................... 12
   E. Data Acquisition ................................................... 13

IV. Data Analysis
   A. Chamber Alignment .................................................. 14
   B. Reconstruction of the Third Scatter Angle ..................... 14
   C. Momentum Reconstruction ......................................... 15
   D. Calculation of Asymmetry ......................................... 16
   E. Determination of Beam Polarization ............................. 17
   F. Determination of Effective Carbon Analyzing Power .......... 18
   G. Combination of Data ................................................. 19

V. Results and Discussion
   A. $R_{pp}$ and $R'_{pp}$ ............................................... 20
   B. $R^t_{np}$ and $R'^t_{np}$ ........................................... 21
   C. Contribution of Errors ............................................. 21
   D. Conclusion .......................................................... 21
I. INTRODUCTION

After thirty years of intensive experimental and theoretical research, our understanding of the two nucleon elastic interaction at medium energy has come to a cross road, not knowing where to continue. By medium energy, we mean the energy range from about 10 MeV to below 1 GeV. The interaction seems to be as complicated as the invariance principles allow and no basic understanding of it has emerged, in spite of all the effort.

On the experimental side, data on the pp system is rather complete, but data on the np system is still scanty and of poor quality. The lack of a high intensity polarized neutron beam and the lack of a stationary neutron target present great difficulties to experimentalists. In many of the neutron–proton scattering experiments, one measures the quasi–elastic scattering of the neutron inside a deuteron with an incoming proton and relates the result to the elastic scattering by using the Impulse Approximation. The approximation sometimes gives a correction as large as 20% to the measured value, and large uncertainties exist concerning means of calculating off mass shell matrix elements.

Many approaches have been taken to explain the experimental data. The earliest one is the use of potentials. Even though the data could be reproduced successfully by using a general potential with many parameters, the inherent non–relativistic character of a potential function limits the calculations to in–coming nucleon energies below about 350 MeV.

Another approach with some success is the One–Boson–Exchange calculation. This kind of calculation has relatively few parameters, and the calculated values of these parameters are consistent with other scattering processes. Its shortcomings are the facts that the correction due to unitarity condition is large and that a large contribution from the \( \sigma \)
meson (a particle yet to be observed) is required to fit the data.

Two phenomenological approaches to explain the experimental result are also taken up. One is the partial wave analysis. Three groups in particular, contribute enormously to this kind of calculation—- Yale, Livermore and Dubna. Their results agree at energies below 450 MeV but show disagreement at higher energies where inelastic scattering becomes important. The trouble is that one has no guidance for taking into account the inelastic absorption. Recently MacGregor presented a rather grim view of the situation.

The other phenomenological approach is the direct reconstruction of the scattering amplitudes as advocated by Puzikov et. al., and again by Schmacher and Bethe, and has been worked out by Golovin and Rozanova. The simplest way of reconstruction involves the measurement of quantities which are the interferences between these amplitudes, for example, the measurements of correlation parameters $C_{NN}$ and $C_{KP}$, and the triple scattering parameters $R$ and $R'$.

The measurement of triple scattering parameters was first proposed by Wolfenstein in 1954. The goal of the experiment described herein is to provide the needed np and pp triple scattering data. It is hoped that the information will help to better determine the scattering amplitudes and eventually may help further theoretical work in this field of physics.
II. N–N SCATTERING MATRIX

In this section we give a short summary on the form of the nucleon–
nucleon scattering matrix and some formulae relevant to our experiment.
Our notations follow that of Bilenkii et.al.\(^{10}\)

The strong interaction scattering matrix for the two–nucleon system
with requirements of spatial rotational invariance, parity conservation,
time reversal invariance, and charge independence can be written as:\(^{(10, 2)}\)

\[
M = (u+v) + c(a_1 + a_2) \cdot \hat{n} + (u-v) \cdot (a_1 \cdot \hat{n}) \cdot (a_2 \cdot \hat{n}) \\
+ (g+h) \cdot (a_1 \cdot \hat{l}) \cdot (a_2 \cdot \hat{l}) + (g-h) \cdot (a_1 \cdot \hat{m}) \cdot (a_2 \cdot \hat{m})
\]

where \(a_i (i=1,2)\) is the Pauli spinor of the nucleon in either the
initial or the final state. \(n, m\) and \(l\) are kinematic vectors defined
by

\[
\hat{n} = (\vec{p} \times \vec{p'})/|\vec{p} \times \vec{p'}|
\]
\[
\hat{m} = (\vec{p'} - \vec{p})/|\vec{p'} - \vec{p}|
\]
\[
\hat{l} = (\vec{p'} + \vec{p})/|\vec{p'} + \vec{p}|
\]

\(\vec{p}, \vec{p'}\) are c. m. s. momenta of the incident and scattered nucleons.

\(u, v, c, g\) and \(h\) are complex functions of energy and angle.

If we add an index \(j (j=1,0)\) to \(M\) to denote the isospin state, the
scattering matrix for the \(pp\) system is given by

\[
M_{pp} = M_1
\]

and the corresponding matrix for \(np\) system is

\[
M_{np} = (M_1 + M_0)/2
\]

using Pauli principle, one obtains the following; \((j\) is isospin index).

\[
u_j (\pi - \theta) = (-1)^j u_j (\theta)
\]
\[
c_j (\pi - \theta) = (-1)^{j+1} c_j (\theta)
\]
\[
h_j (\pi - \theta) = (-1)^{j+1} h_j (\theta)
\]
\[
v_j (\pi - \theta) = (-1)^j g_j (\theta)
\]
Because they are all complex quantities, we have a total of 19 quantities (10 each for pp and np, minus one for arbitrary normalization) to determine in order to completely describe the NN system.

A convenient way to characterize the two-nucleon system in spin space is in terms of the density matrix. Denoting $\Psi_n$ as any spin state for the two nucleons and $\rho_n$ the relative probability of finding the system in state $\Psi_n$, the density matrix is given by:

$$\rho = \sum \rho_n \Psi_n \Psi_n^+$$

The density matrix after a scattering can be written in terms of the initial density matrix and the scattering matrix as:

$$\rho^{(f)} = M \rho^{(i)} M^+$$

The average value $< S >$ of any spin operator after the scattering is given by:

$$< S > = Tr \rho^{(f)} = Tr(S \rho^{(f)})$$

Any nucleon-nucleon scattering experiment can be thought of as a preparation of two nucleons in certain states prior to scattering, and a subsequent measurement of the states of the nucleons after scattering. We may measure expectation values of certain components of the 2 final nucleons, or ignore the spin conditions of one or both nucleons.

Thus we are measuring:

$$< \sigma_\mu^{(1)(f)} \sigma_\nu^{(2)(f)} > = Tr(\sigma_\mu^{(1)(f)} \sigma_\nu^{(2)(f)} \rho^{(f)})/(\rho^{(f)})$$

where the Greek subscripts on the $\sigma$'s go over 0, 1, 2, and 3, $\sigma=0$ implies we ignore that spin, the other three values refer to the 3 components of $\sigma$ in a coordinate system. Superscript (f) or (i) refers to the final or initial state, and the superscripts (1), and (2) refer to the two nucleons.
If the initial density matrix is equal to $\sigma_\tau^{(1)(1)}$, we will be measuring

$$<\sigma_\mu^{(1)(f)} \sigma_\nu^{(2)(f)} ; \sigma_\tau^{(1)(1)} \sigma_\omega^{(2)(1)}>$$

$$= \text{Tr}(\sigma_\mu^{(1)(f)} \sigma_\nu^{(2)(f)} M \sigma_\tau^{(1)(1)} \sigma_\omega^{(2)(1)} M^+) / \text{Tr} \rho(f)$$

For convenience, the abbreviation $<\mu, \nu, \tau, \omega>$ will be used for

$$<\sigma_\mu^{(1)(f)} \sigma_\nu^{(2)(f)} ; \sigma_\tau^{(1)(1)} \sigma_\omega^{(2)(1)}>$$

In this way, the polarization measurement can be written as

$$P = (n, 0; o, 0)$$

where $n$ is normal to the scattering plane.

Let us introduce in the laboratory frame the following orthonormal vector systems:

$$\hat{n}_k, \hat{k}_l, \hat{s}_k = \hat{n}_k \times \hat{k}_l$$

$$\hat{n}_k, \hat{k}_{1l}, \hat{s}_{k1} = \hat{n}_k \times \hat{k}_{1l}$$

$$\hat{n}_k, \hat{k}_{2l}, \hat{s}_{k2} = \hat{n}_k \times \hat{k}_{2l}$$

There $\hat{k}_l, \hat{k}_{1l}$ and $\hat{k}_{2l}$ are unit vectors in the directions of the momenta of the incident nucleon, scattered nucleon and recoil nucleon, respectively, and $\hat{n}_k$ is the normal to the scattering plane.

The triple scattering parameters as defined by Wolfenstein(9) are given by:

$$D = (0, n_k; 0, n_k)$$

$$R = (0, s_{k1}; 0, s_{k1})$$

$$R' = (0, k_{1l}; 0, s_{k1})$$

$$A = (0, s_{k1}; 0, k_k)$$

$$A' = (0, k_{1l}; 0, k_k)$$
Similar parameters can be defined for the recoil particle, and we add a superscript "t" to signify polarization transfer. They are:

\[ D^t = (n_q, 0; 0, n_q) \]
\[ R^t = (s_{q1}, 0; 0, s_{q1}) \]

and similarly for \( R'^t \), \( A^t \) and \( A'^t \). All the triple scattering parameters are a measure of how large a fraction of the initial polarization of the incident nucleon is transferred to the scattered particle after an elastic scattering. In terms of the amplitudes \( u, v, c, g, \) and \( h \), we obtain:

\[ R = D_+ \cos(\alpha + \theta/2) - D_m \sin(\alpha + \theta/2) - D_\cos(\alpha - \theta/2) \]
\[ R' = D_+ \cos(\alpha + \theta/2) + D_m \cos(\alpha + \theta/2) - D_\sin(\alpha - \theta/2) \]

where

\[ D_+ = 4 \text{ Re}(uv^*)/\sigma \]
\[ D_- = 4 \text{ Re}(gh^*)/\sigma \]
\[ D_m = 4 \text{ Im}(cv^*)/\sigma \]

\[ \sigma = 2(\mid u \mid^2 + \mid v \mid^2 + \mid c \mid^2 + \mid g \mid^2 + \mid h \mid^2) \]

is the differential cross-section.

\[ \alpha + \theta/2 - \theta_q \]

\( \theta \) is the scattering angle of the scattered nucleon in c.m. frame
\( \theta_q \) is the corresponding angle in lab. frame

and

\[ R^t_{np} = - K_+ \sin(\alpha' - \phi/2) + K_m \cos(\alpha' - \phi/2) + K_\sin(\alpha' - \phi/2) \]
\[ R'^t_{np} = - K_+ \cos(\alpha' - \phi/2) - K_m \sin(\alpha' - \phi/2) + K_\cos(\alpha' + \phi/2) \]

where

\[ K_+ = 4 \text{ Re}(ug^*)/\sigma \]
\[ K_- = 4 \text{ Re}(vh^*)/\sigma \]

(contd)
\[ K_{2m} = 4 \, \text{Im}(cg^*)/\sigma \]
\[ \alpha' = \phi/2 - \phi \]

\( \phi \) is the scattering angle of the recoil nucleon in c.m. frame

\( \phi \) is the corresponding angle in lab. frame. Similar relations exist for \( A, A' \) and \( D \).

Because of the symmetry properties of \( u, v, c, g \) and \( h \), the following relations are true:

\[
R_{pp}(\theta=0) = D_{pp}(\theta=0)
\]
\[
R'_{pp}(0) = 0
\]
\[
R'_{pp}(\pi/2) = - R_{pp} \cos(\alpha+\pi/4)/\cos(\alpha-\pi/4)
\]
\[
R_{np}(0) = D_{np}(0)
\]
\[
R'_{np}(0) = 0
\]
\[
R_{np}(\pi) = 0
\]
\[
R_{np}(0) = 0
\]
\[
R_{np}(\pi) = 0
\]

The differential cross-section \( I \), and the polarization \( \langle \vec{\sigma}_\perp \rangle \) of the scattered nucleon for elastic nucleon-nucleon scattering with the incident nucleon polarized along \( \hat{s}_\perp \) is given by:

\[
I \langle \vec{\sigma}_\perp \rangle = \sigma (P \hat{n}_\perp + R P_B \hat{s}_1 + R' P_B \hat{k}_1)
\]

where \( \sigma, P, R \) and \( R' \) have been defined earlier, and \( P_B \) is the magnitude of the incident nucleon polarization.
III. EXPERIMENTAL METHOD AND APPARATUS

A. Method of Measurement

The procedure of the experiment is first to produce a beam of polarized nucleons, and after this beam interacts with a nucleon target, to measure the final polarization of the scattered nucleon. Because both the production and analysis of polarization involve scattering, we call this kind of an experiment a triple scattering experiment.

A plan view of our experimental setup is shown in Fig. 1. In this experiment we concerned ourselves only with components of polarization which were in the plane of scattering. The plane of scattering was arranged to coincide with the horizontal plane in space. The extracted beam of the 184-inch Cyclotron, after leaving the shielding wall, passed through a quadrupole magnet Q1 and was deflected by two bending magnets B1 and B2 before hitting the first target. The magnets and their supports were so designed that the angle $\gamma$ between the real beam line and the undeflected beam line could be any value between $+28$ degrees and $-28$ degrees, producing at the first target a polarized secondary beam with either positive or negative polarization $P_B$ in the up-down direction. The solenoid could rotate this polarization $P_B$ by $\pm 90$ degrees perpendicular to the direction of motion. By changing $\gamma$ from positive to negative, and because for each sign of $\gamma$ we could have both polarities of the solenoid, we had four ways of preparing a polarization in the horizontal plane before reaching the second target. After the second scattering, there would be in the horizontal plane a component of polarization perpendicular to the direction of motion, which was by definition $R_P B$, and the polarization along the direction of motion was $R'P_B$. (In neutron-proton scattering, since we used a polarized neutron beam and then measured the recoil proton
we should add a superscript "t" to R and R' to signify "polarization transfer". For simplicity of argument we will neglect such a distinction unless otherwise stated.) The values of R and R' are measured in this experiment. The magnet B4 bent the momentum of the scattered nucleon and also precessed the polarization by 45 degrees with respect to the direction of motion. By reversing the polarity of B4 we could also precess the polarization by -45 degrees, giving us a total of eight possible ways of measurement.

Figure 2 shows the eight different combinations for the measurements of R and R', together with the direction of polarization. The actual polarization we analyzed using proton-carbon scattering was the component perpendicular to the direction of motion. Inspection of Fig. 2 shows that in cases 1, 2, 3 and 4 the effective component is

\[ \pm \frac{1}{\sqrt{2}}(R + R')P_B' \]

and in cases 5, 6, 7 and 8 the effective component is equal to

\[ \pm \frac{1}{\sqrt{2}}(R - R')P_B' \]

We could solve for R and R' once the effective polarizations were known.

Because we required a nucleon to undergo three scatters the rate was very low. To overcome this we used large targets, designed large solid angle for the second and third scattering. The amount of shielding that was required to stop the primary beam determined the minimum separation between the first and second target. The separation was set to be minimum to increase the number of beam particles impinging on the second target.
B. Polarized Beams and Targets

The Cyclotron beam is pulsed at 16 msec intervals and has a fine structure of about 18.5 megacycles. We tuned an internal quadrupole magnet, not shown in Fig. 1, and Q1 until the spot size of the primary beam was a minimum at the first target.

B2 was mounted on a pivot which in turn could travel along a pair of tracks perpendicular to the undeflected beam line. To line up the primary beam at the target, we used a pair of split ion chambers mounted on an arm pivoted at the center of the target table. With the ion chambers set at the desired angle, the magnets B1 and B2 were tuned until the readings from both sides of the ion chambers were balanced, assuring us that the beam was lined up correctly, see Fig. 3.

The first target consisted of a LH2 flask and a LD2 flask, each 11-inches long and 5-inches in diameter, in a horizontal position along the undeflected beam line with .0055 inch wall of aluminum. The whole target assembly was mounted on a platform which could be moved up and down by remote control, allowing us to change from the hydrogen flask to the deuterium flask in a very short time.

When we ran with the deuterium target, we had seven pairs of small telescope counters to detect the slow recoil protons corresponding to the quasi-elastic scattering of the neutron in the deuteron with the incoming proton. These telescope counters will be described later. The effect of the telescope counters on the energy distribution of the neutron and the proton beams are shown in Fig. 4.

The polarized neutrons or protons from the first scattering went through a brass collimator and entered a superconducting solenoid. The collimator was 24 inches long and 3 inches in diameter.
The superconducting solenoid magnet had a 4-1/4 in. diameter warm bore and an overall length of 3-1/2 feet. It had a winding of 39,000 turns of 52% Nb and 48% Ti wire. The winding was tapered to minimize the variation of the line integral $\int B \cdot dz$. The detailed construction has been reported elsewhere.\(^{(11)}\) It worked extremely well throughout the entire experiment. A conventional magnet with the same capacity as this superconducting magnet would have to be 11 feet long and would consume about half a megawatt of power.

B3 was a vertical sweeping magnet for getting rid of charged particles in the neutral beam. Counters M1 (=L1+R1) and M2 defined a charged particle down the channel and these counters were used to eliminate charged particles when we ran on polarized neutrons. M3 was a large counter with a hole of five inches diameter serving as an anticoincidence. Fig. 5 shows the apparatus for the detection of the second scattering.

The second target was a conventional one with a liquid hydrogen flask 6-inches in diameter and 12-inches in length along the beam direction. The scattered protons from the second target were detected by a wire-spark-chamber spectrometer described below. The recoil protons were detected by a pair of spark chambers on the opposite side of the beam. A counter called Rec was placed behind these chambers and served as part of the trigger logic. These chambers were not used when the expected recoil particle was a neutron.

C. Wire-Spark-Chamber-Spectrometer

The scattering angles and momenta of scattered charged particles from the LH$_2$ target were determined by the magnetic spectrometer, shown
in Fig. 6. The spectrometer consisted of four parts: (1) An array of three small wire spark chambers placed between the magnet and the second target, called the front package. (2) An 8 in. gap 16 in. by 36 in. bending magnet with iron shields at the entrance and exit to shield the spark chambers from residual magnetic field. (3) An array of seven spark chambers four inches apart. Carbon slabs were placed between the chambers with the exception of the first two and the last two chambers. This whole array will be referred to as the back package. (4) A set of counters S1, S2, S3 and S4 were used to detect the proton. S4 was placed behind all chambers to insure that the proton went through all the chambers. S2 served as an anti-coincidence counter.

Each of our chambers consisted of a pair of orthogonal wire planes separated by 3/8 inch. The wires were 7 mil copper wires spaced 1 mm apart. For the last three chambers in the back package which had active areas of 28 in. by 32 in., we had sheets of aluminized mylar on the outsides of the wire planes which were connected electrically to them. The aluminized mylar sheets reduced the inductance of the spark chambers, and the high voltage pulses were able to travel down the chambers in a shorter time. This improved the performance of the large chambers. Table 2 gives the relevant information on the chambers.

A clearing field of 50 volts was applied to all chambers and 10% of the gas used (about 90% neon and 10% helium) was saturated with ethanol gas. The ethanol acted like a quenching gas and prevented the formation of spurious sparks.

D. Electronics

We had seven pairs of two-fold coincidence telescope counters near the first target. They were added together to form the T coincidence.
Each of the T counters was viewed at by a LP21 photomultiplier. The discriminators and coincidence circuits associated with the T counters were of the Chronetic 151 series. M₁ (=L₁+R₁), M₂ with M₃ in anticoincidence formed the M coincidence. The M counters were viewed by 56AVP photomultipliers. Counters S₁, S₃, S₄ with S₂ in anticoincidence formed the S coincidence. These counters were viewed by RCA 6810A photomultipliers. The proton-proton trigger consisted of a coincidence between M₁S and counter Rec. The neutron-proton trigger consisted of a coincidence in T₁S and no count in any of the M counters. If the trigger requirement was satisfied, the spark chambers were triggered and all the electronics were gated off for 30 msec to allow the data to be collected by the computer and the spark chambers to recover.

E. Data Acquisition

The system for recording outputs from magnetostrictive wands was similar to that described in Ref. 12. Normally each wand would have three signals. The first and the last were signals produced by currents flowing down the fiducial wires, the middle one was produced by the spark current in the chamber. Associated with each wand were two scalers. The first signal turned both scalers on and they would start to count a MHz clock. The second signal from the same wand stopped the first scaler and the third signal stopped the second scaler.

After a suitable time delay, all of the scalers were read into a PDP-5 computer. These data were blocked into records of 23 events each and written on magnetic tape. The computer was used mainly for data collection and monitoring. No on-line data reduction or selection was performed.
IV. DATA ANALYSIS

The analysis is essentially divided into two parts. The first part is the determination of the scattering angle between the incoming proton and carbon nucleus in the back package, and the elimination of unscattered events. The second part is the momentum reconstruction and the selection of elastic events.

A. Chamber Alignment

Because in the trigger logic we accepted all protons that underwent scatterings off the second target and entered the back package, only about 5% of the total protons achieved another scattering in the carbon plates of the back package. Our first task is to pick out all these events. The rest are straight through events or events which had very small angles of scattering. These events are used to determine the alignment of the chambers, as is described below.

For each run, or a few adjacent runs, we use either the surveyed chamber positions or the values from preceding runs to reconstruct spark positions in space. An event is considered a straight-through if the sparks in the middle chambers all lie within a narrow corridor defined by the first spark and last spark in the chamber package, for both X and Y views. A straight line is fitted to these sparks and the deviation from the best fit is recorded. The mean values of these deviations are used to correct for the positions of our chambers for these sets of runs. This procedure is applied to both the front and back package.

B. Reconstruction of the Third Scatter Angle

This phase of reconstruction is concerned exclusively with the seven chambers in the back package. If the sparks in both the X and Y views in the back package can be fitted to a straight line, the event is eliminated as a
straight-through. Besides this, any event having more than three sparks missing out of a total of 14 sparks, and any event having more than three extra sparks is eliminated. A possible candidate is fitted to two straight lines in both views with a vertex constrained to fall in a carbon plate. A combined $\chi^2$ value is compared to a fixed limit. If it is smaller than that limit, it will be accepted as a good event and the azimuthal angle and the scattering angle are recorded. Because of the fact that carbon caused a lot of multiple scattering, dependent strongly on energy, we have to determine experimentally the $\chi^2$ limit to be used. Details of the determination are presented in the Appendix.

For each of the accepted events we also ask whether the event will still be within our fiducial volume in the package if the scatter angle and the azimuthal angle in carbon are reversed in sign. If this is not true, the event is eliminated.

C. Momentum Reconstruction

After the event is accepted as a good scatter event in carbon, we reconstruct the momentum using the entrance angle to the back spark chambers and the angle in the front package. The former angle is already calculated when we fit the third scattering angle and the latter is obtained by fitting a straight line to the three sparks in the front chambers. The relative positions between the two packages were obtained by turning off $B_4$ and triggering on events that went straight through the two packages. The second scattering angle $\theta_3$ is determined completely by the front package. The momentum of the charged particle is estimated by using a square field approximation to $B_4$ with a correction for the non-uniformity of the line integral across the magnetic field. A Monte-Carlo program was written to compare our simple estimation versus the
step-by-step integration across the magnet field. More than 97% of the random events show an agreement within 1%. Thus our simple formula is used to obtain the momentum since this error in momentum does not affect the kinematic constraints. The reason is that the only competing inelastic channel is that of one pion production. This will cause a drastic change in the proton momentum and is easily detected.

Knowing the momentum and the scattering angle $\theta_3$ is enough to determine the incoming momentum, assuming elastic scattering. For proton-proton scattering we also measured the recoil angle as an additional constraint on the event. This additional constraint, together with the momentum selection, allow us to collect almost all elastic events for pp scattering and the target empty runs have almost zero events and are neglected. For neutron-proton scattering, we run into some problems. The motion of the neutron inside deuterium caused a widening of the polarized neutron beam width and the momentum cut is not as clean as in the proton-proton case. The other problem is that we did not detect the recoil neutron and could not reconstruct the interaction point at the target. The best that can be done is to extend the proton trajectory back to the target position and eliminate events which intercept the central beam line outside of the target volume. This selection eliminates about 20% of our data.

D. Calculation of Asymmetry

To get the distribution in the azimuthal angle for each run we bin all events which have scatterings in the carbon of greater than six degrees and less than 22 degrees. The lower limit is to get rid of coulomb scattering and the upper limit is the designed limit of the solid angle acceptance of the back package. The azimuthal angular distribution is
fitted by a two parameters \((e_1, e_2)\) formula:

\[
f(\phi) = 1 + A(E)e_1 \cos(\phi) + A(E)e_2 \sin(\phi)
\]

where \(e_1\) is the polarization normal to the plane of scattering and
\(e_2\) is the polarization in the plane of scattering and is related to
\(R\) and \(R'\), as shown in Fig. 2. \(A(E)\) is the effective analyzing power
of carbon and is determined experimentally, see Section IV F.

The expression (1) is fitted to the events of each run, and runs
with a chi-square probability of less than 5% are eliminated. These
discarded runs are mainly those having very low spark-chamber efficiency,
and they amount to about 4% of the total sample. The data from the re-
main ing runs are combined to give composite value for the two parameters.
Once the \(e_2\)'s are known for each set of running conditions, we can cal-
culate \(R\) and \(R'\), see Section IV G. For np scattering, unlike the pp case,
the data are corrected by the target empty data in the usual way.

**E. Determination of Beam Polarization**

It is well known that the angular distribution of a polarized nucleon
scattered from an unpolarized nucleon has the distribution at angle \(\theta_3(2a)\)

\[
\sigma = \sigma_0(\theta_3)(1 + P_1 P_2(\theta_3))
\]

where \(P_1\) is the polarization of the incident beam normal to the
scattering plane, and \(P_2\) is the N-N polarization at the scattering
angle \(\theta_3^0\). By setting \(\theta_3\) equal to \(\gamma\), Fig. 1, and comparing the
counting rates of accepted events with solenoid on, \(R_1\) and off, \(R_0\),
we measure a quantity which was related to the square of beam polari-
zation, i.e.,

\[
\frac{R_0 - R_1}{R_0} = P_1(E, \gamma) P_2(E_2, \gamma).
\]
Using the shape of the polarization as a function of energy from other data, we can calculate the actual beam polarization. For pp runs, we normalize the counting rate by the M coincidence rate, and for the np runs we normalize the counting rate by the counts in M2, which is proportional to the intensity of the neutron beam. The results of the beam polarization measurements are shown in Table 1.

F. Determination of Effective Carbon Analyzing Power

As was mentioned in Section IV D, the experimental quantity we measured is $P_1A(E)$, we can get $A(E)$ if we know $P_1$. This was done by putting the spectrometer in the polarized proton beam. The beam polarization is measured by the method described in Section IV E. $A(E)$ was measured at seven energies from 670 down to 200 MeV. The low energy points were measured by degrading the polarized beam at 510 MeV with copper degrader situated just behind the first target. The results are shown in Fig. 8.

G. Combination of Data

After doing all the calculations described in Section IV D, we end up, for each pair of R and R', with eight values for the asymmetry $e_2(I)$, $I$ goes from 1 to 8 corresponding to the 8 different measurements in Fig. 2. We first compute the average values of $e_2(1)$, $e_2(2)$, $-e_2(3)$, and $-e_2(4)$ weighted by the number of events in each sample. The average value $e_2^+$ is related to $R$ and $R'$ by

$$e_2^+ = P_B (R+R')/\sqrt{2}.$$
A similar average value for $e_2$ is computed for $e_2 (5), e_2 (6), e_2 (7)$ and $-e_2 (8)$, giving

$$e_2 = P_B (R - R')/\sqrt{2}$$

In the above formulae, we have assumed that the angle of precession of the polarization of the proton in $B_4$ with respect to the momentum is 45 degrees. Corrections must be made when the angle is not exactly 45°. These corrections, however are very small, typically about 2% of the 45° value. Solving for $R$ and $R'$, we obtained for proton-proton scattering

$$R_{pp} = (e_2^+ + e_2^-)/(|\sqrt{2} P_B|)$$
$$R'_{pp} = (e_2^+ - e_2^-)/(|\sqrt{2} P_B|)$$

For neutron-proton scattering, because the $g$ - factor of the neutron is negative, the solenoid precesses the spin in opposite direction to that for proton, we have

$$R^+_{np} = - (e_2^+ + e_2^-)/(|\sqrt{2} P_B|)$$
$$R'^+_{np} = - (e_2^+ - e_2^-)/(|\sqrt{2} P_B|)$$

The $e_1$ values for all eight measurements are averaged to give the polarization $P$ at the second scattering angle $\theta_3$. Since polarization values are quite well established, we have a very good way of checking our equipmental biases. The result of our measurements on np system are shown in Figs. 9 and 10, together with other published data. The agreement is good.
V. RESULTS AND DISCUSSION

A. $R_{pp}$ and $R'_{pp}$

Our results for $R_{pp}$ and $R'_{pp}$ at 520, 600 and 670 MeV are given in Table 3. The same results are presented graphically in Figs. 11 and 12.

In contrast to the rich amount of data for differential cross-section and polarization and depolarization (14) there is no duplication in the measurement of $R$, and there is only one experiment done to measure $R'$ above 400 MeV.

Figure 13 shows our 520 MeV result together with data at 425 by the Princeton group (15) and the Chicago group (16). The curves are phase shift predictions of Kazarinov et. al. (17) and Stapp (18) at 400 MeV. The Princeton point at 30 degree seems to be lower than our value and the phase shift prediction.

Our 600 and 670 MeV data are plotted together with the 600 MeV data of Golovin et. al. (19) and the 635 MeV data by Kumekin et. al. (20) in Fig. 14. The curve is from Golovin. Our result seems to lie higher than predicted; though the general shape looks the same.

Only one experiment has been done for $R'_{pp}$ at an energy higher than 400 MeV, and that is the Chicago experiment at 425 MeV. Their results are plotted with ours at 520 MeV in Fig. 15. The data points by Roth et. al., are not true data points. They are calculated from the knowledge of $R, A, A', \text{ and } \theta_{\text{lab. scattering angle}}$ by the formula

$$\tan \theta_{\text{lab}} = (A+R')/(A'-R)$$

and have large uncertainties. Phase shift curves are from Stapp and Kazarinov, both at 400 MeV.

Figure 16 shows our $R'_{pp}$ data at 600 and 670 MeV together with the
phase shift predictions by Kazarinov at 635 MeV. Except for small angles, the agreement between phase shifts and our experimental results is fair for both $R'_{pp}$ and $R''_{pp}$.

B. $R^t_{np}$ and $R'^t_{np}$

This is the first experiment to measure $R^t_{np}$ and $R'^t_{np}$ for the np system, above 400 MeV. Our result is shown in Table 4. Figure 17 and 18 show the prediction by Livermore group at 500 MeV, with our data at 520 and 600 MeV. for $R^t_{np}$ and $R'^t_{np}$ respectively. Because most np data are rather inaccurate and incomplete, the prediction is subject to large errors and only qualitative agreement is expected. Their prediction is not strongly supported by our data.

C. Contribution of Errors

The errors of our results come mainly from the following sources: The statistical uncertainty in our number of events, the uncertainty in the determination of our carbon analyzing power and the uncertainty in our beam polarization.

At small angles, when we used thick carbon plates, the statistical uncertainty is about 1%. At large angles, we used thin carbon plates and collected fewer events, the uncertainty goes up to about 1.5%. The uncertainty in our carbon analyzing power is typically 3%.

D. Conclusion

In this experiment, we have successfully made use of a polarized neutron beam produced from a deuterium target by np quasi-elastic scattering. We have also tried to make selection on this neutron beam energy
by detecting the recoil proton angles with counters. This worked well at low beam level, but at maximum beam, the counters were flooded with singles. This problem can be solved in the future if one also makes a selection on the recoil proton energy by using a degrader or a bending magnet. Another possible improvement to the experiment will be the use of a solenoid capable of precessing the polarization a full 180 degrees instead of just 90 degrees. This eliminates the problem of having to align the magnet B2 frequently to change the sign of the beam polarization.

We have made no attempt to incorporate our result into phase shift analysis. In principle, one can express the quantities \( u, v, c, g \) and \( h \), as defined in Section II, in terms of partial waves, and the values for the partial wave amplitudes can be obtained by a \( \chi^2 \) fit to previous experiments and our new data on \( \text{D}, \text{R} \) and \( \text{R}' \).

In view of the difficulties in incorporating the inelastic absorption to the elastic partial wave amplitudes, a better understanding of one-pion production process is important. In particular, to find out which amplitudes are more responsible for the production process, we have to do polarization measurements on the initial and final two nucleon states similar to what one has been doing for the elastic scattering.

For the direct reconstruction of the scattering matrix, the measurement of \( C_{nn} \), i.e., the quantity \( \langle n_f, n_f; 0, 0 \rangle \) in the notation of Section II, is required for the np system.
ACKNOWLEDGMENTS

I am grateful to Professor L. T. Kerth for continued guidance and encouragement as director of my thesis research. I would like to thank Professor Kerth and Dr. Pamela Surko, who was the other graduate student working on this project at the time and for their help in all phases of the experiment. I would like to thank Drs. David Cheng and Burns MacDonald for helping in the design and execution phases, and Dr. Charles Ankenbrandt for helping with the analysis.

I also would like to thank the technicians of the Lofgren Physics group, the Cyclotron crew, the Systems group, the Accelerator Design group and the Math and Computing group for their help. Mr. Mike Barnes helped considerably in the data handling.

I wish to thank Dr. Edward Lofgren for the hospitality of his research group.

This work was done under the auspices of the U. S. Atomic Energy Commission.
APPENDIX

For each scatter candidate in the back package, we fit two straight lines and one vertex to all the sparks and the vertex is constrained to the carbon plate positions. A chi-square value is calculated for the fit. We have to decide on the accepted chi-square value. If the calculated value is larger than the limit, the event is eliminated.

Two things contribute to the chi-square. The first is the resolution of our spark chambers and the second is the multiple scattering caused by carbon. As is well known, the rms projected angle $\theta_{\text{proj}}$ due to multiple Coulomb scattering of a charged particle is inversely proportional to $PV$, where $P$ is the momentum and $V$ is its velocity.

For protons at 600 MeV, $\theta_{\text{proj}}$ for 6 inches of carbon is about 0.6 degrees, and at 300 MeV the value goes up to about 1.2 degrees. If one plots the number of events with chi-square less than some $\chi^2_0$ as a function of $\chi^2_0$ we get a curve which shows a plateau. The knee position of the curve for low energy protons is at a larger $\chi^2_0$ than that for higher energy protons, as expected. Similar curves are obtained for the asymmetry $e$ as a function of $\chi^2_0$. Typical curves are shown in Fig. 7. The chi-square limit is selected to be just above the knee of our lowest energy curve.
REFERENCES

4. Many articles on this subject have been published. A very informative one is by M. H. MacGregor, R. A. Arndt and R. M. Wright, Phys. Rev. 182, 1714 (1969).
18. See discussion in Ref. 15.
\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Beam Type & \( \gamma \) & Kinetic Energy & Polarization & Intensity \\
\hline
p & \( \pm 15^\circ \) & 670 \( \pm \) 20 MeV & .51 \( \pm \) .01 & \( 2 \times 10^6 \) protons/sec \\
p & \( \pm 22^\circ \) & 600 \( \pm \) 20 MeV & .53 \( \pm \) .01 & \( 1 \times 10^6 \) \\
p & \( \pm 28^\circ \) & 520 \( \pm \) 20 MeV & .43 \( \pm \) .02 & \( 1 \times 10^6 \) \\
n & \( \pm 22^\circ \) & 600 \( \pm \) 30 MeV & .26 \( \pm \) .03 & \( 4 \times 10^5 \) neutrons/sec \\
n & \( \pm 28^\circ \) & 520 \( \pm \) 30 MeV & .32 \( \pm \) .04 & \( 3 \times 10^5 \) \\
\hline
\end{tabular}
\end{table}

Beam Cross Sectional Area: 28 in\(^2\)
**TABLE 2**

Spark Chamber Dimension

<table>
<thead>
<tr>
<th>Chamber Number(*)</th>
<th>Fiducial volume</th>
<th>height (inches)</th>
<th>width (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2</td>
<td></td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>3, 4, 5</td>
<td></td>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>6, 7</td>
<td></td>
<td>14</td>
<td>18</td>
</tr>
<tr>
<td>8, 9</td>
<td></td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>10, 11, 12</td>
<td></td>
<td>28</td>
<td>32</td>
</tr>
</tbody>
</table>

(*) For the numbering of the spark chambers, see Fig. 1.
### TABLE 3

<table>
<thead>
<tr>
<th>T(MeV)</th>
<th>$\theta_{cm}$</th>
<th>$R_{pp}$</th>
<th>$R'_{pp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>520</td>
<td>$35^\circ \pm 10^\circ$</td>
<td>.55 ± .10</td>
<td>-.19 ± .10</td>
</tr>
<tr>
<td></td>
<td>$99^\circ \pm 10^\circ$</td>
<td>.32 ± .13</td>
<td>-.45 ± .13</td>
</tr>
<tr>
<td></td>
<td>$118^\circ \pm 10^\circ$</td>
<td>.44 ± .10</td>
<td>-.22 ± .10</td>
</tr>
<tr>
<td>600</td>
<td>$35^\circ \pm 10^\circ$</td>
<td>.78 ± .09</td>
<td>-.30 ± .09</td>
</tr>
<tr>
<td></td>
<td>$99^\circ \pm 10^\circ$</td>
<td>.27 ± .15</td>
<td>-.32 ± .15</td>
</tr>
<tr>
<td></td>
<td>$118^\circ \pm 10^\circ$</td>
<td>.32 ± .10</td>
<td>-.27 ± .10</td>
</tr>
<tr>
<td>670</td>
<td>$35^\circ \pm 10^\circ$</td>
<td>.96 ± .15</td>
<td>-.46 ± .15</td>
</tr>
<tr>
<td></td>
<td>$79^\circ \pm 10^\circ$</td>
<td>.50 ± .15</td>
<td>-.26 ± .15</td>
</tr>
<tr>
<td></td>
<td>$99^\circ \pm 10^\circ$</td>
<td>.24 ± .15</td>
<td>-.39 ± .15</td>
</tr>
<tr>
<td></td>
<td>$118^\circ \pm 10^\circ$</td>
<td>.46 ± .13</td>
<td>-.23 ± .13</td>
</tr>
</tbody>
</table>
TABLE 4

Values of Measured $R_{np}^t$ and $R'_{np}^t$

The angle $\theta_{cm}$ refers to the center of mass scattering angle of the neutron for np scattering.

<table>
<thead>
<tr>
<th>T(MeV)</th>
<th>$\theta_{cm}$</th>
<th>$R_{np}^t$</th>
<th>$R'_{np}^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>520</td>
<td>180° ± 10°</td>
<td>-.22 ± .18</td>
<td>0(*)</td>
</tr>
<tr>
<td></td>
<td>124° ± 10°</td>
<td>-.19 ± .16</td>
<td>-.04 ± .16</td>
</tr>
<tr>
<td></td>
<td>102° ± 10°</td>
<td>.05 ± .15</td>
<td>-.10 ± .15</td>
</tr>
<tr>
<td></td>
<td>90° ± 10°</td>
<td>-.26 ± .18</td>
<td>-.18 ± .18</td>
</tr>
<tr>
<td>600</td>
<td>180° ± 10°</td>
<td>-.06 ± .15</td>
<td>0(*)</td>
</tr>
<tr>
<td></td>
<td>102° ± 10°</td>
<td>-.24 ± .16</td>
<td>.02 ± .16</td>
</tr>
<tr>
<td></td>
<td>90° ± 10°</td>
<td>-.01 ± .20</td>
<td>.09 ± .20</td>
</tr>
<tr>
<td></td>
<td>65° ± 10°</td>
<td>-.64 ± .16</td>
<td>-.08 ± .16</td>
</tr>
</tbody>
</table>

*Because of symmetry properties of the scattering matrix, $R'_{np}^t$ is expected to be zero, we use this fact to obtain the $R_{np}^t$ value at $\theta_{cm} = 180°$. 
FIGURE CAPTIONS

Figure 1. Schematic layout of the experiment. In the drawing $B_i$ represent bending magnets, $T$ is the telescope counters, $M_i$ are beam monitor scintillation counters, and $S_i$ are scintillation counters monitoring scattered particles.

Spark Chambers are numbered from 1 to 12.

Figure 2 a,b. The eight different ways of measuring $R$ and $R'$. The three signs associated with each measurement refer to the sign of the first scattering angle, the direction of the magnetic field of the solenoid along its axis, and the direction of bend of the momentum in $B_4$. Both the first and third signs are in left handed coordinate system and the sign of the magnetic field is defined to be positive pointing downstream.

Figure 3. Apparatus for the detection of the first scattering.

Figure 4. Effects of the telescope counters on the energy spectra of the polarized protons for both $LH_2$ and $LD_2$ targets. $MS$ means a coincidence between $M$ and $S$, and $TMS$ means a coincidence between $MS$ and the telescope counters $T$. The events at low energies are due to pion production. We can see clearly that the inclusion of $T$ cuts down the low energy contamination to the polarized beam.

Figure 5. Apparatus for the detection of the second scattering.

Figure 6. Wire-spark-chamber spectrometer.

Figure 7. Measured asymmetry as a function of $x^2$ cut, together with the number of accepted events for each case. The
curve on the right-hand side shows the increase of the number of accepted events as we relax the $X^2$ cut. The curve on the left-hand side shows the corresponding asymmetry from a fit to azimuthal scattering angles in carbon of that sample.

Figure 8. Effective analyzing power as a function of energy.

Figure 9. Measured np polarization at 520 MeV compared to that of D. Cheng at 500 MeV.

Figure 10. Measured np polarization at 600 MeV compared to that of D. Cheng at 600 MeV.

Figure 11. Results of $R_{pp}$ from this experiment.

Figure 12. Results of $R'_{pp}$ from this experiment.

Figure 13. Comparison of $R_{pp}$ around 500 MeV to phase shift predictions of Stapp (dotted curve) and of Kazarinov (17), (solid curve) both at 400 MeV.

Figure 14. Comparison of $R_{pp}$ at about 600 MeV with phase shift prediction of Golovin (19) at 600 MeV.

Figure 15. Comparison of $R'_{pp}$ at about 500 MeV with phase shift predictions of Stapp (dotted curve) and Kazarinov (solid curve) at 400 MeV.

Figure 16. Comparison of $R'_{pp}$ at about 600 MeV with phase shift prediction of Kazarinov.

Figure 17a, b. Comparison of the $R^t_{np}$ data at 520 and 600 MeV from this experiment with phase shift prediction at 500 MeV by MacGregor et al. (21) The two solid curves give the error corridor of the predicted value.
Figure 18a, b. Comparison of the $R_n^t$ data at 520 and 600 MeV with phase shift prediction at 500 MeV by MacGregor et al. The two solid curves give the error corridor of the predicted value.
Figure 1
(1) \( P_\perp = (R + R') P / \sqrt{2} \)

(2) \( P_\perp = (R + R') / \sqrt{2} \)

(3) \( P_\perp = -(R + R') P / \sqrt{2} \)

(4) \( P_\perp = -(R + R') P / \sqrt{2} \)

Figure 2a
(5) \[ P_{\perp} = (R - R') P / \sqrt{2} \]

(6) \[ P_{\perp} = (R - R') P / \sqrt{2} \]

(7) \[ P_{\perp} = -(R - R') P / \sqrt{2} \]

(8) \[ P_{\perp} = -(R - R') P / \sqrt{2} \]

Figure 2b
Figure 3

Split ion chambers

hodoscope

B2

H2/D2

\( \gamma \)
H₂ Target

D₂ Target

Arbitrary units

Trigger: MS ~
TMS ~
Figure 6

Beam path

Front package

TGT 2

Carbon scatterer

Back package

XBL703-2493
Asymmetry vs. Number of Accepted Events

Figure 7
Figure 8
Figure 9

- This work 520 MeV
- Cheng (1967) 500 MeV
○ This work 600 MeV
• Cheng (1967) 600 MeV

Figure 10
Figure 11
Figure 12

- 520 MeV
- 600 MeV
- 670 MeV
Figure 13

- 47 -

Figure 13

- 47 -
Figure 14
Figure 15

- this work 520 MeV
- Roth (1965) 430 MeV
- Handler (1965) 425 MeV
Figure 16

- this work  670 MeV
- this work  600 MeV
Figure 17a
Figure 18a
Figure 18b
This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.