MAGNETIC FIELDS FROM STRAIGHT
FILAMENTS, RECTANGULAR SHEETS
AND RECTANGULAR BARS

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Magnetic Fields from Straight Filaments,
Rectangular Sheets and Rectangular Bars

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ABSTRACT

This report describes the derivation and the use of three
fortran programs:

FIFL = Field from a straight filament of finite length.

FIRS = Field from a rectangular sheet of infinitesimal
thickness.

FIRB - Field from a rectangular bar of finite length.

The three geometries in question might suggest a logical
extension of the solution for the simplest geometry, i.e.,
the linear filament, into the successively more complicated
cases, i.e., the rectangular sheet and the rectangular bar. The
programs to be discussed were; however, developed independently.
Each program is, therefore, the result of a different mathematical
approach and will, accordingly, be discussed in separate sections
in the body of this report.

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FIFL - FIELD FROM A FINITE LINE

The derivation of FIFL is based on the geometry shown in figure 1. Figure 1 shows a straight, current-carrying wire, parallel to the x-axis. The end points of the line are the terminal points of the vectors $\vec{A}$ and $\vec{B}$ which originate at the origin of the coordinate system. (The origin is also the spatial point at which the magnetic field is to be evaluated.) The vector $\vec{R}$ is the perpendicular distance from the line to the point of interest, i.e. the origin. The vector $\vec{n}$ is a unit vector, normal to the plane defined by vectors $\vec{A}$ and $\vec{B}$, which points in the direction of the magnetic field at point O.

The magnetic induction at point O may be obtained from the general expression

$$\vec{B} = \frac{\mu_0 I}{4\pi |\vec{R}|} (\cos\theta_1 - \cos\theta_2) \vec{n} \quad (1)$$

where:

I = current (in amperes).

μ_0 = permeability constant = $4.0\pi \times 10^{-7}$ henry/meter.

$|\vec{R}|$ = absolute magnitude of the perpendicular distance from the line to point O.

θ_1 = angle between the x-axis and $\vec{B}$.

θ_2 = angle between the x-axis and $\vec{A}$.

$\vec{n}$ = direction of magnetic field at O.

From figure 1

$$\vec{n} = \frac{(\vec{A} \times \vec{B})}{|\vec{A} \times \vec{B}|} \quad (2)$$

$$\vec{A} + \vec{C} = \vec{B} \quad (3)$$

$$\vec{A} \cdot \vec{C} = |\vec{A}| |\vec{C}| \cos \theta_2 \quad (4)$$

$$\vec{B} \cdot \vec{C} = |\vec{B}| |\vec{C}| \cos \theta_1 \quad (5)$$

Substituting equations (2) through (5) into (1)

$$\vec{B} = \frac{\mu_0 I}{4\pi |\vec{R}|} \left[\frac{\vec{B} \cdot (\vec{B} - \vec{A})}{|\vec{B}| |\vec{B} - \vec{A}|} - \frac{\vec{A} \cdot (\vec{B} - \vec{A})}{|\vec{A}| |\vec{B} - \vec{A}|} \right] \frac{(\vec{A} \times \vec{B})}{|\vec{A} \times \vec{B}|} \quad (6)$$

The geometry of figure 1 shows that:

$$|\vec{R}| = \frac{|\vec{A} \times \vec{B}|}{|\vec{B} - \vec{A}|} \quad (7)$$

Substituting (7) into (6) and rearranging terms:

$$\vec{B} = \frac{\mu_0 I}{4\pi} (\vec{B} - \vec{A}) \cdot \left[\frac{\vec{B}}{|\vec{B}|} - \frac{\vec{A}}{|\vec{A}|} \right] \frac{(\vec{A} \times \vec{B})}{|\vec{A} \times \vec{B}|^2} \quad (8)$$

Equation (8) forms the basis of the Fortran program FIFL.

Small R Approximation

The condition of small $|\vec{R}|$, i. e. when $|\vec{R}| \rightarrow 0$, presents three cases. Figure 2 shows the projection of the vectors $\vec{R}$, $\vec{A}$, $\vec{B}$ and $\vec{C}$ on the XY-plane. The end points of the current-carrying line are designated as ℓ_1 and ℓ_2 . The three cases are: Case I, which includes all spatial positions for which the field point, O, lies to the left of ℓ_1 and ℓ_2 , thereby making both ℓ_1 and ℓ_2 positive; Case II, which includes all spatial positions for which the field point, O, lies to the right of ℓ_1 and ℓ_2 , thereby making both ℓ_1 and ℓ_2 negative; and Case III, which includes all spatial positions for which the field point, O, lies to the left of ℓ_1 and to the right of ℓ_2 , thereby making ℓ_1 positive and ℓ_2 negative.

From any one of the three cases shown in figure 2:

$$\cos \theta_1 = \frac{\ell_1}{|\vec{B}|} \quad (9)$$

$$\cos \theta_2 = \frac{\ell_2}{|\vec{A}|} \quad (10)$$

$$\ell_1^2 + |\vec{R}|^2 = |\vec{B}|^2 \quad (11)$$

$$\ell_2^2 + |\vec{R}|^2 = |\vec{A}|^2 \quad (12)$$

Considering, for the moment, only the magnitude of $\vec{B}$ as given by equation (1):

$$|\vec{B}| = \frac{\mu_0 I}{4\pi |\vec{R}|} (\cos \theta_1 - \cos \theta_2) \quad (13)$$

Substituting (9) through (12) into (13) and rearranging terms:

$$|\vec{B}| = \frac{\mu_0 I}{4\pi |\vec{R}|} \left[\text{sgn}(\ell_1) \sqrt{1 - \frac{|\vec{R}|^2}{|\vec{B}|^2}} - \text{sgn}(\ell_2) \sqrt{1 - \frac{|\vec{R}|^2}{|\vec{A}|^2}} \right] \quad (14)$$

Where

$$\text{sgn}(\ell) = \frac{\ell}{|\ell|} = \pm 1 \quad (14a)$$

Expanding the square root terms and retaining only the first two terms of each expansion:

$$|\vec{B}| = \frac{\mu_0 I}{4\pi} |\vec{R}| \left[\text{sgn}(\ell_1) \left(\frac{1}{|\vec{R}|^2} - \frac{1}{2|\vec{B}|^2} \right) - \text{sgn}(\ell_2) \left(\frac{1}{|\vec{R}|^2} - \frac{1}{2|\vec{A}|^2} \right) \right] \quad (15)$$

For Case I, $\ell_1 = \ell_2 = +1$, and (15) becomes

$$|\vec{B}| \cong \frac{\mu_0 I}{4\pi} |\vec{R}| \left[\frac{1}{2|\vec{A}|^2} - \frac{1}{2|\vec{B}|^2} \right] \quad (16)$$

Similarly, for Case II, $\ell_1 = \ell_2 = -1$ and (15) becomes

$$|\vec{B}| \cong \frac{\mu_0 I}{4\pi} |\vec{R}| \left[\frac{1}{2|\vec{B}|^2} - \frac{1}{2|\vec{A}|^2} \right] \quad (17)$$

For Case III, $\ell_1 = +1$ and $\ell_2 = -1$ and (15) becomes

$$|\vec{B}| \cong \frac{\mu_0 I}{4\pi} |\vec{R}| \left[\frac{2}{|\vec{R}|^2} - \left(\frac{1}{2|\vec{A}|^2} + \frac{1}{2|\vec{B}|^2} \right) \right] \quad (18)$$

Equations (16) and (17) show that for small $|\vec{R}|$, i.e. when $|\vec{R}| \rightarrow 0$, the magnetic induction vanishes at points beyond the end points of the line. Equation (18) shows that, for points on the line, as $|\vec{R}| \rightarrow 0$, the solution for $|\vec{B}|$ blows up. In other words, the point $|\vec{R}| = 0$ is a singular point for spatial positions which fall on the current-carrying line itself.

FIFL may be called by any main program, provided that the main program supplies the cartesian coordinates of the end points of each current-carrying line and the current in each line. Any number of lines may be considered. The contributions from all lines to each magnetic induction component are summed and returned by the program.

The Fortran "CALL" statement for FIFL is:

CALL FFL (PX, PY, PZ, SBX, SBY, SBZ)

where:

$\left. \begin{array}{l} \text{PX} \\ \text{PY} \\ \text{PZ} \end{array} \right\} = \text{Cartesian coordinates of the field point O.}$

SBX }
SBY } = Returned values of the magnetic induction components at point 0.
SBZ }

The variables in the common statiment have the following meaning.

VX }
VY } = End coordinates of line elements
VZ }
VI = Current in the line elements.
NL = Number of consecutive lines in a group.
IG = Number non-connected groups.
MU = Unit system desired by user.

```

SUBROUTINE FIF1 ( PX, PY, PZ, SHX, SHY, SHZ )
COMMON /DAL1/ VX(200),VY(200),VZ(200),VI(200),NL(200),IG,MU
C      (MU=1) UNITS IN INCH - WEEER/M**2 (B)
C      (MU=2)          INCH - GAUSS (B)
C      (MU=3)          METER - WEEER/M**2 (B)
C      (MU=4)          METER - AMP/M (H)
C      (MU=5)          INCH - AMP/M (H)
1  FORMAT (' ECINT',3F6.3,' FALLS ON LINE NO.',I5)
   IF(IG.EQ.0) RETURN
   NU=IABS(MU)
   GC TO (11,12,13,14,15),NU
11  C=39.370079E-7
   GO TO 300
12  C=39.370079E-3
   GO TO 300
13  C=1.0E-7
   GO TO 300
14  C=0.079577472
   GO TO 300
15  C=3.1328213
300  NF=0
   IF(MU.LT.0) GO TO 301
   SHX=0.0
   SHY=0.0
   SHZ=0.0
301  DO 323 I=1,IG
   NB=NF+1
   NF=NL(I)
302  DO 323 L=NB,NF
   VFAC=VI(L)*C
304  SX2=VX(L)-PX
   SY2=VY(L)-PY
   SZ2=VZ(L)-PZ
   S22= SX2*SX2+SY2*SY2+SZ2*SZ2
307  AS2=SQRT(S22)
308  IF(L.EQ.NB) GO TO 320
C
309  CX=VX(L)-VX(L-1)
   CY=VY(L)-VY(L-1)
   CZ=VZ(L)-VZ(L-1)
   C2= CX*CX+CY*CY+CZ*CZ

```

C
 313 FX=SY1*SZ2-SZ1*SY2
 PY=SZ1*SX2-SX1*SZ2
 FZ=SX1*SY2-SY1*SX2
 F2=FX*FX+FY*FY+FZ*FZ F=S1XS2
 R2=F2/C2 R2=R**2
 IF(R2/C2 .LT.1.0E-6) GO TO 201

E=(SX2/AS2-SX1/AS1)*CX+(SY2/AS2-SY1/AS1)*CY+(SZ2/AS2-SZ1/AS1)*CZ
 316 VH=VFAC*E/F2
 GO TO 317

C
 C SMALL R APPROX.
 201 C=SQRT(C2)
 CL1=SX1*CX+SY1*CY+SZ1*CZ
 CL2=SX2*CX+SY2*CY+SZ2*CZ
 IF(CL1*CL2.GT.0.0) GO TO 202

C*L1

C
 IF(R2/C2.LT.1.0E-20) GO TO 205
 VH=VFAC*2.0*C/F2
 GO TO 317

C
 202 IF(R2/C2.LT.1.0E-20) GO TO 203
 VH=VFAC*0.5*ABS(1.0/S12-1.0/S22)/C
 GO TO 317

203 VH=0.0

C
 317 SHX=SHX+FX*VH
 SHY=SHY+FY*VH
 SHZ=SHZ+FZ*VH

C
 320 SX1=SX2
 SY1=SY2
 SZ1=SZ2
 S12=S22
 323 AS1=AS2
 325 RETURN
 205 PRINT 1, PX, PY, PZ, L
 SHX=1.0E16
 SHY=1.0E16
 SHZ=1.0E16
 RETURN
 END

FIRS - FIELD FROM A RECTANGULAR SHEET

The derivation of FIRS is based on the geometry shown in figure 3. Figure 3 shows a rectangular conducting sheet of infinitesimal thickness whose plane is parallel to, and above the xz-plane. The corners of the sheet are denoted by the numbers 1 through 4. The linear current density, K, is shown to flow in the positive x-direction and is assumed to be of uniform magnitude across the width of the sheet. The field point is taken to be at the origin of the coordinate system.

The derivation begins by calculating the magnetic field at the end of a linear filament carrying a current, Kdz , in the positive x-direction.

The field from a line current of finite length is given by:

$$dH = \frac{Kdz}{4\pi a} (\cos \alpha_2 - \cos \alpha_1) \quad (1)$$

From figure 1, the following relationships may be written:

$$H_y = |\vec{H}| \frac{a}{a} \quad (2)$$

$$H_z = -|\vec{H}| \frac{y}{a} \quad (3)$$

$$\cos \alpha = \frac{x}{r} \quad (4)$$

$$r^2 = x^2 + y^2 + z^2 \quad (5)$$

$$a^2 = y^2 + z^2 \quad (6)$$

$$dH_y = \frac{K}{4\pi} \frac{\cos \alpha}{a} \cdot \frac{z}{a} dz = \frac{K}{4\pi} \frac{xz}{ra^2} dz \quad (7)$$

$$dH_z = -\frac{K}{4\pi} \frac{\cos \alpha}{a} \cdot \frac{y}{a} dz = -(K/4\pi) \frac{xy}{ra^2} dz \quad (8)$$

Substituting (5) and (6) into (7) and (8)

$$dH_y = \frac{Kxzdz}{4\pi (y^2+z^2) (x^2+y^2+z^2)^{1/2}} \quad (9)$$

$$dH_z = -\frac{Kxydz}{4\pi (y^2+z^2) (x^2+y^2+z^2)^{1/2}} \quad (10)$$

The field components for a single end point of a rectangular sheet are then obtained by integrating equations (9) and (10) over the width of the sheet.

$$H_y = \frac{K}{4\pi} \int \frac{xzdz}{(y^2+z^2) (z^2+b^2)^{1/2}} \quad (11)$$

where $b^2 = x^2 + y^2$

$$H_y = \frac{K}{4\pi} \frac{x}{(b^2-y^2)^{1/2}} \ln \frac{(z^2+b^2)^{1/2} - (b^2-y^2)^{1/2}}{(z^2+y^2)^{1/2}} \quad (12)$$

(for $b^2 > y^2$)

Since

$$\left. \begin{aligned} a &= (y^2+z^2)^{1/2} \\ |x| &= (b^2-y^2)^{1/2} \end{aligned} \right\} \quad (13)$$

Equation (12) becomes:

$$H_y = \frac{K}{4\pi} \frac{x}{|x|} \ln \frac{r-|x|}{a} \quad (14)$$

Using the notation

$$\text{sgn}(x) = \frac{x}{|x|} = \pm 1 \quad (14a)$$

Equation (14) may be written as:

$$H_y = \frac{K}{4\pi} \text{sgn}(x) \ln \frac{r-|x|}{a} \quad (14b)$$

Equation (14) is a "corner value" that must be summed, with alternating signs for each successive corner, to obtain the total field component at the point in question. Thus, summing the contributions from each corner,

$$H_y = \frac{K}{4\pi} \sum_{i=1}^4 (-1)^i \text{sgn}(x_i) \ln \frac{r_i - |x_i|}{a_i} \quad (15)$$

For the H_z -component, the derivation proceeds in similar fashion:

$$H_z = - \frac{K}{4\pi} \int \frac{xydz}{(y^2+z^2)(z^2+b^2)^{1/2}} \quad (16)$$

where $b^2 = x^2 + y^2$

$$H_z = - \frac{K}{4\pi} \frac{xy}{y(b^2-y^2)^{1/2}} \tan^{-1} \frac{z(b^2-y^2)^{1/2}}{y(z^2+b^2)^{1/2}} \quad (17)$$

$$H_z = - \frac{K}{4\pi} \frac{x}{|x|} \tan^{-1} \frac{|x|z}{yr} \quad (18)$$

Since the arctangent is a monotonic function, the sign of the fraction $x/|x|$ in equation (18) may be transferred to the argument of the arctangent by eliminating the $|x|$ (in the argument) and using, instead, the algebraic value of x . Equation (18) then becomes:

$$H_z = - \frac{K}{4\pi} \tan^{-1} \frac{xz}{yr} \quad (18a)$$

Again summing the contributions from each corner,

$$H_z = - \frac{K}{4\pi} \sum_{i=1}^4 (-1)^i \tan^{-1} \frac{x_i z_i}{y r_i} \quad (19)$$

The total magnetic field is obtained by combining equations (15) and (19)

$$\vec{H} = \frac{K}{4\pi} \sum_{i=1}^4 (-1)^i \left(\hat{u}_y \operatorname{sgn}(x_i) \ln \frac{r_i - |x_i|}{a_i} - \hat{u}_z \tan^{-1} \frac{x_i z_i}{y r_i} \right) \quad (20)$$

The Fortran program for FIRS is a user-oriented program and may be called by any main program by the Fortran statement:

CALL FIRS (XD, ZD, CD, XP, YP, ZP, HY, HZ)

where:

XD = length of the rectangular sheet (in meters).

ZD = width of the rectangular sheet (in meters).

CD = linear current density in amperes/meter.

$\left. \begin{matrix} XP \\ YP \\ ZP \end{matrix} \right\} = \text{coordinates of the field point.}$

$\left. \begin{matrix} HY \\ HZ \end{matrix} \right\} = \text{components of the magnetic field at the point XP, YP, ZP, which are returned by FIRS.}$

Note that the FIRS program differs from the above-discussed derivation of field components in the relative locations of field point and rectangular sheet. The derivation assumed the sheet to be located parallel to, and above the xz-plane with the field point located at the origin of the coordinate system. The less restrictive and more general arrangement of the sheet located in the xz-plane and the field point located at any point in space was used in the program.

```

C  SUBROUTINE FIRS (XD,ZD,CD,XP,ZP,YP,HY,HZ)
    FIELD FROM RECTANGULAR SHEET
    IMPLICIT REAL*8 (A-H,O-Z)
    REAL*4 XD,ZD,CD,XP,ZP,YP,HY,HZ
    DIMENSION X(2),Z(2)
    X(1)=-0.5*XD-XP
    X(2)= 0.5*XD-XP
    Z(1)=-0.5*ZD-ZP
    Z(2)= 0.5*ZD-ZP
    Y=-YP
    ONEM=-1.0
    HYD=0.0
    HZD=0.0
    DO 12 J=1,2
    A=1.0D0
    IF(X(1)*X(2).LT.0.0) A=DSQRT(Y**2+Z(J)**2)+1.0D-20
    M=0
    DO 12 I=1,2
    R=DSQRT(X(I)**2+Y**2+Z(J)**2)
    DRX=R-DABS(X(I))
    IF(DRX.LE.1.0D-08) M=1
    IF(M.EQ.1) DRX=1.0/(DABS(X(I))+1.0D-20)
    DY=(ONEM**(I+J))*DSIGN(1.0D0,X(I))*DLOG(DRX/A)
    PRINT 1, R,X(I),DRX,DY
1  FORMAT (4D25.16)
10  HYD=HYD+DY
    YR=Y*R
    DZ=0.0
    IF(DABS(YR).LE.1.0D-10) GO TO 11
    DZ=(ONEM**(I+J))*DATAN(Z(J)*X(I)/YR)
11  HZD=HZD-DZ
12  CONTINUE
    HY=0.0795774715*CD*HYD
    HZ=0.0795774715*CD*HZD
    RETURN
    END

```

FIRB - FIELD FROM A RECTANGULAR BAR

The derivation of FIRB is based on the geometry shown in figure 4. Figure 4 shows a conductive, rectangular bar of dimensions $2l$, $2h$, $2b$, positioned with its center coincident with the origin of the coordinate system. A current, I , which flows in the positive x -direction, is assumed to be uniformly distributed over the cross-sectional area of the bar, i.e. over the area $2b \times 2h$. The field point P (R_x , R_y , R_z) is spatially located with respect to the origin and a volume element within the bar by the vector relationship:

$$\vec{R} = \vec{r} + \vec{\rho} \quad (1)$$

The magnetic field at P may be found by using the Biot-Savart Law

$$\vec{H} = \frac{1}{4\pi} \iiint \frac{\vec{J} \times \vec{r}}{r^3} dx dy dz \quad (2)$$

and performing the necessary integration over dx , dy and dz . Equation (2) may be separated into the two field components, H_y and H_z (the H_x component is zero because the current flows uniformly in the x -direction) as follows:

Let:

$$\left. \begin{aligned} X &= R_x - x; dX = -dx \\ Y &= R_y - y; dY = -dy \\ Z &= R_z - z; dZ = -dz \end{aligned} \right\} \quad (3)$$

Then, using the vector relationship shown in (1)

$$\begin{aligned} r^3 &= (r_x^2 + r_y^2 + r_z^2)^{3/2} = \left[(R_x - x)^2 + (R_y - y)^2 + (R_z - z)^2 \right]^{3/2} \\ &= \left[X^2 + Y^2 + Z^2 \right]^{3/2} \end{aligned} \quad (4)$$

Substituting equations (3) and (4) into (2) and taking the cross product of $\vec{J} \times \vec{r}$, the field components, H_y and H_z are:

$$H_y = \frac{J_x}{4\pi} \int_{x_1}^{x_2} dx \int_{y_1}^{y_2} dy \int_{z_1}^{z_2} \frac{z dz}{(x^2 + y^2 + z^2)^{3/2}} \quad (5)$$

$$H_z = - \frac{J_x}{4\pi} \int_{x_1}^{x_2} dx \int_{y_1}^{y_2} dy \int_{z_1}^{z_2} \frac{y dz}{(x^2 + y^2 + z^2)^{3/2}} \quad (6)$$

where:

$$\left. \begin{aligned} x_i &= R_x - (-1)^i a \\ y_i &= R_y - (-1)^j b \\ z_i &= R_z - (-1)^k c \end{aligned} \right\} \quad i, j, k = 1, 2 \quad (7)$$

and J_x = current density in the x-direction.

The final expressions resulting from the integrations shown in equations (5)

and (6) are:

$$H_y = - \frac{J_x}{4\pi} \sum_{i,j,k=1}^2 (-1)^{i+j+k} \left\{ x_i \sinh^{-1} \left[\frac{y_j}{(x_i^2 + z_k^2)^{1/2}} \right] + y_j \sinh^{-1} \left[\frac{x_i}{(y_j^2 + z_k^2)^{1/2}} \right] \right. \\ \left. - |z_k| \sin^{-1} \left[\frac{x_i y_j}{\sqrt{(x_i^2 + z_k^2)(y_j^2 + z_k^2)}} \right] \right\} \quad (8)$$

$$H_z = \frac{J_x}{4\pi} \sum_{i,j,k=1}^2 (-1)^{i+j+k} \left\{ x_i \sinh^{-1} \left[\frac{z_k}{(x_i^2 + y_j^2)^{1/2}} \right] + z_k \sinh^{-1} \left[\frac{x_i}{(y_j^2 + z_k^2)^{1/2}} \right] \right. \\ \left. - |y_j| \sin^{-1} \left[\frac{x_i z_k}{\sqrt{(x_i^2 + y_j^2)(y_j^2 + z_k^2)}} \right] \right\} \quad (9)$$

Since the Fortran programming language does not contain functions for the inverse hyperbolic sine, the conversion of the first two terms in equations (8) and (9) to their algebraically equivalent forms in terms of natural logarithms is desirable. For example,

$$\sinh^{-1} \left[\frac{Y_j}{(x_i^2 + z_k^2)^{1/2}} \right] = \ln \left[Y_j + (x_i^2 + y_j^2 + z_k^2)^{1/2} \right] \quad (10)$$

Thus, equations (8) and (9) become, after substitution of relations similar to the one shown in (10)

$$H_Y = - \frac{J_x}{4\pi} \sum_{i,j,k=1}^2 (-1)^{i+j+k} \left\{ x_i \ln \left[Y_j + (x_i^2 + y_j^2 + z_k^2)^{1/2} \right] + y_j \ln \left[x_i + (x_i^2 + y_j^2 + z_k^2)^{1/2} \right] - |z_k| \sin^{-1} \left[\frac{x_i y_j}{\sqrt{(x_i^2 + z_k^2)(y_j^2 + z_k^2)}} \right] \right\} \quad (11)$$

$$H_z = \frac{J_x}{4\pi} \sum_{i,j,k=1}^2 (-1)^{i+j+k} \left\{ x_i \ln \left[z_k + (x_i^2 + y_j^2 + z_k^2)^{1/2} \right] + z_k \ln \left[x_i + (x_i^2 + y_j^2 + z_k^2)^{1/2} \right] - |y_i| \sin^{-1} \left[\frac{x_i z_k}{\sqrt{(x_i^2 + y_j^2)(y_j^2 + z_k^2)}} \right] \right\} \quad (12)$$

Equations (11) and (12) form the basis of the Fortran program FIRB. FIRB returns the field components at any point in space due to a conductive bar whose center may be arbitrarily located, but whose sides must be parallel, respectively, to the three orthogonal axis of a cartesian

coordinate system.

FIRB may be called by any main program by using the Fortran statement:

CALL FIRB (L, H, B, CD, XB, YB, ZB, XP, YP, ZP, HY, HZ) where:

L = half-length of bar in x-direction in meters.

H = half-height of bar in y-direction in meters.

B = half-width of bar in z-direction in meters.

$\left. \begin{array}{l} \text{XB} \\ \text{YB} \\ \text{ZB} \end{array} \right\} = \text{coordinates of the center of the bar.}$

$\left. \begin{array}{l} \text{XP} \\ \text{YP} \\ \text{ZP} \end{array} \right\} = \text{coordinates of the field point.}$

CD = current density in each bar (in amperes/square meter).

$\left. \begin{array}{l} \text{HY} \\ \text{HZ} \end{array} \right\} = \text{returned values of magnetic field components at the point (XP, YP, ZP).}$

SUBROUTINE FIRB(L, H, B, CD, XB, YB, ZB, XP, YP, ZP, HY, HZ)

FIRB CALCULATES THE MAGNETIC FIELD COMPONENTS (HY,HZ) AT A POINT (XP,YP,ZP) WHICH RESULTS FROM A RECTANGULAR BAR CENTERED AT (XB,YB,ZB) WITH DIMENSIONS (2L,2H,2B) AND CURRENT DENSITY (CD) IN THE X DIRECTION ONLY. UNITS ARE MKS.

IMPLICIT REAL*8 (A-H,O-Z)

REAL*8 L

DIMENSION X(2),Y(2),Z(2)

TINY=1.0D-8

BIG=DMAX1(L,H,B)

RX=XP-XB

RY=YP-YB

RZ=ZP-ZB

X(1)=RX+L

X(2)=RX-L

Y(1)=RY+H

Y(2)=RY-H

Z(1)=RZ+B

Z(2)=RZ-B

HY=0.0D0

HZ=0.0D0

DO 100 K=1,2

DO 100 J=1,2

DO 100 I=1,2

```

NUMB=1
A=DSQRT ((X(I)**2)+(Y(J)**2)+(Z(K)**2))
DT1Y=DSQRT (X(I)**2+Z(K)**2)
DT1Z=DSQRT (X(I)**2+Y(J)**2)
AA=Y(J)**2+Z(K)**2
AAA=DSQRT (AA)
IF (DABS (X(I)/BIG).LT.TINY) GO TO 10
T1Y=X(I)*DLOG ((Y(J)+A))
T1Z=X(I)*DLOG ((Z(K)+A))
GO TO 20
10 T1Y=0.0D0
T1Z=0.0D0
20 IF (DABS (Y(J)/BIG).LT.TINY) GO TO 22
T2Y=Y(J)*DLOG (X(I)+A)
21 IF (DABS (Z(K)/BIG).LT.TINY) GO TO 23
T2Z=Z(K)*DLOG (X(I)+A)
IF (NUMB.EQ.2) GO TO 24
T3Y=DABS (Z(K))*DARSIN ((X(I)*Y(J))/(DT1Y*AAA))
T3Z=DABS (Y(J))*DARSIN ((X(I)*Z(K))/(DT1Z*AAA))
GO TO 25
22 T2Y=0.0D0
NUMB=2
GO TO 21
23 T2Z=0.0D0
24 T3Y=0.0D0
T3Z=0.0D0
25 TERM1=T1Y+T2Y-T3Y
TERM2=T1Z+T2Z-T3Z
SGN=(-1.0)**(I+J+K)
HY=HY+(SGN*TERM1)
HZ=HZ+(SGN*TERM2)
100 CONTINUE
FOURPI=4.0D0*DARCOS (-1.0D0)
HY=-HY*CL/FOURPI
HZ=HZ*CD/FOURPI
RETURN
END

```

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FIGURE 1

Case I

Case II

Case III

FIGURE 2

FIGURE 3

FIGURE 4

NOTICE

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