APR 8 1964

UNCLASSIFIED

AD 416 721

MASTER

DEFENSE DOCUMENTATION CENTER

FOR

SCIENTIFIC AND TECHNICAL INFORMATION

CAMERON STATION, ALEXANDRIA, VIRGINIA

UNCLASSIFIED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

REPORT CODE SELECTION

Prime No. PWA-2170

Secondary Nos. AD-416721

RADC-TDR-62-464 (Vol. I)

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

64-1-1

RADC-TDR-62-464, Vol. I
DDC Document No.

MAY 1963

CATALOGED BY DDC 416721

AD No.

416721

(Unclassified)
FINAL REPORT

INVESTIGATION OF MAGNETOHYDRODYNAMIC POWER GENERATION
VOLUME I - SYSTEM ANALYSIS AND ENVIRONMENTAL TESTING

PRATT & WHITNEY AIRCRAFT
DIVISION OF UNITED AIRCRAFT CORPORATION
EAST HARTFORD, CONNECTICUT

REPORT PWA-2170

CONTRACT AF30 (602)-2487

PREPARED FOR
ROME AIR DEVELOPMENT CENTER
RESEARCH AND TECHNOLOGY DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
GRIFFISS AIR FORCE BASE
NEW YORK

PATENT NOTICE: When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

DDC NOTICE: Qualified requestors may obtain copies of this report from DDC, Cameron Station, Alexandria, Virginia. DDC services for Department of Defense contractors are available through the "Field of Interest Register" on a "need-to-know" certified by the cognizant military agency of their project or contract.

RADC-TDR-62-464, Vol. I
DDC Document No.

MAY 1963

(Unclassified)
FINAL REPORT
INVESTIGATION OF MAGNETOHYDRODYNAMIC POWER GENERATION
VOLUME I - SYSTEM ANALYSIS AND ENVIRONMENTAL TESTING

PRATT & WHITNEY AIRCRAFT
DIVISION OF UNITED AIRCRAFT CORPORATION
EAST HARTFORD, CONNECTICUT

REPORT PWA-2170

CONTRACT AF30(602)-2487
PROJECT NO. 5561
TASK NO. 556101

PREPARED FOR
ROME AIR DEVELOPMENT CENTER
RESEARCH AND TECHNOLOGY DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
GRIFFISS AIR FORCE BASE
NEW YORK

The investigation of magnetohydrodynamic power generation under Contract AF30(602)-2487 is reported in the following six volumes:

- Volume I - System Analysis and Environmental Testing
- Volume II - Integration with Cross-Field Microwave Devices
- Volume III - Integration with Cross-Field and Linear Beam Microwave Devices
- Volume IV - Materials
- Volume V - Theoretical Plasma Conductivity Studies
- Volume VI - Preliminary Design of 1000 and 250 Megawatt MHD Generators

Title of Report RADC-TDR-62-464, Vol I

PUBLICATION REVIEW

This report has been reviewed and is approved.

Approved: *Daniel R. Loreto*
DANIEL R. LORETO
Chief, Development Engineering Laboratory
Directorate of Engineering

Approved: *William P. Bethke*
WILLIAM P. BETHKE
Director of Engineering

FOR THE COMMANDER:

IRVING J. GABELMAN
Director of Advanced Studies

FOREWORD

The system analysis and the environmental testing of MHD generators was conducted by the Pratt & Whitney Aircraft Division of the United Aircraft Corporation during the period April 19, 1961 to December 31, 1962. This work was authorized under Contract AF30(602)-2487, Investigation of Magnetohydrodynamic Power Generation. Included in this report is information generated under this contract and information generated by the contractor in other investigations, some of which were conducted under other contracts with the Government. This report, "System Analysis and Environmental Testing", is Volume I of six volumes describing the investigation of MHD power generation. Technical direction of the project was administered by Rome Air Development Center, Griffiss Air Force Base, New York, under the supervision of Frank J. Mollura, RASSM.

The following personnel participated in this study:

S. J. Markowski, Program Manager

System Analysis

S. Hamilton, Project Engineer
E. O. Cheney, Principal Investigator
H. A. Jeremias, Senior Analytical Engineer
J. P. Tobin, Senior Analytical Engineer
J. Kniat, Analytical Engineer
J. H. Hirschenhofer, Analytical Engineer

Environmental Testing

S. J. Keating, Jr. Principal Investigator
W. K. Gilroy, Jr. Assistant Project Engineer
E. F. Cole, Assistant Project Engineer
H. A. Harmon, Senior Analytical Engineer
R. D. Sawyer, Senior Analytical Engineer
R. V. Conde, Senior Experimental Engineer
R. W. Higgins, Senior Experimental Engineer
J. M. King, Analytical Engineer
J. W. Lane, Analytical Engineer
D. R. McVay, Analytical Engineer
K. A. Potz, Analytical Engineer
A. H. Reinhardt, Analytical Engineer

Valuable technical assistance was given by many other engineering personnel at both Pratt & Whitney Aircraft and at United Aircraft Corporation Research Laboratories.

ABSTRACT

This report is Volume I of six volumes submitted as the final report to RADC on contract AF30(602)-2487, Investigation of Magnetohydrodynamic Power Generation. It presents summaries of all other volumes and overall conclusions and recommendations resulting from the project in addition to presenting the system analyses and environmental testing information. Volumes II through VI present the results of investigations of crossed field and linear beam rf converters, MHD materials work, theoretical plasma conductivity study, and preliminary designs of 1000 and 250-megawatt power generators.

In the system analysis of 1000 and 250-megawatt generators, Volume I, the necessary equations are derived, the assumptions discussed, loss mechanisms are considered, and design conditions determined and presented in parametric form. An environmental testing program which investigated the areas of bulk and boundary layer conductivity, structural concepts, and combustion phenomena is also presented in Volume I.

A study of crossed-field rf converters conducted by Raytheon Company (Volume II) and a parallel study of crossed-field and linear-beam tubes performed by Litton Industries (Volume III) are presented. A new concept in rf converters resulting from Raytheon's work, the electromagnetic amplifying lens (EAL), is discussed and cold tests conducted on a test anode are reported. A design of a large size (1000 megawatt) EAL tube is described. The design conditions of a 200-KW klystron tube are also presented by Litton Industries.

The results of an investigation of materials for use in MHD generators are presented in Volume IV. The results include the experimental determination of electrical resistivity, thermal expansion, tensile strength and tensile modulus of selected materials. The results of chemical compatibility tests and an environmental testing program are also included.

In Volume V the formulas for the electrical conductivity and diffusion coefficients of a magnetohydrodynamic generator plasma including ion flow are derived. The MHD electrode boundary layer is investigated to find the electrode voltage drop. The electron energy equation is developed, leading to an expression for electron temperature which is evaluated for segmented and nonsegmented MHD generators.

The results of preliminary engineering designs of 1000 and 250-megawatt MHD DC power generators are presented in Volume VI in the form of preliminary design layouts, schematic flow diagrams, performance graphs and tables. An evaluation of the design and design philosophy is provided.

TABLE OF CONTENTS

	<u>Page</u>
Title	i
Foreword	iv
Abstract	v
Table of Contents	vi
List of Figures	vii
I. Introduction to Final Report	1
II. System Analysis	6
A. One Dimensional MHD Performance Calculation	7
B. Two-Dimensional Problems	26
C. Conclusions	37
III. Environmental Testing	38
A. Plasma Conductivity Measurements	39
B. Structural Concepts	113
C. Combustion Phenomena	131
IV. Summaries of Volumes II - VI	159
V. Conclusions of the Final Report	173
VI. Recommendations of the Final Report.	175
Appendix 1 - Derivation of Basic Equations	176
Appendix 2 - Derivation of MHD Generator Performance Equations	194
Appendix 3 - Nomenclature	218
Appendix 4 - Tables	221
Appendix 5 - Experimental Determination of Conductivity	248
Appendix 6 - Test Equipment and Procedures	262
Appendix 7 - Theoretical Prediction of Conductivity Based on Experimental Conditions	282
Appendix 8 - Special Support Work	306
Appendix 9 - Figures	320

LIST OF FIGURES

Number	Title	Number	Title
1	Relative Power/Area vs Oxygen/Fuel for Various Fuels	38	Segmented Electrode Performance. Optimum $c/(c+g)$ vs $(c+g)/h$ for Various Values of α
2	Combustion Temperature vs Oxygen/Fuel for Various Fuels	39	Segmented Electrode Performance. Conductivity Ratio vs α for Various Values of $(c+g)/h$ at Optimum $c/(c+g)$
3	Conductivity vs Weight Per Cent Potassium for Various Seeding Compounds	40	Segmented Electrode Performance. α_c/α_o vs Conductor + Insulator-to-Height Ratio for Various Values of α at Optimum $c/(c+g)$
4	Relative Power/Area vs Weight Per Cent Potassium for Various Seeding Compounds	41	Segmented Electrode Performance. E/VB vs α_c/α_o
5	Theoretical Chamber Temperature. Gaseous O_2 and JP-4 vs Oxygen/Fuel	42	Segmented Electrode Performance. E/E_o vs α_c/α_o
6	Oxygen/Fuel Ratio Optimization. 1000 MW MHD Generator	43	Segmented Electrode Performance. $E_x/E_{yo} [\eta_o/(1-\eta_o)]$ vs $c/(c+g)$ for Various Values of $(c+g)/h$ and α
7	Seed Rate Optimization. 1000 MW MHD Generator	44	Segmented Electrode Performance. $E_x/E_{yo} [\eta_o/(1-\eta_o)]$ vs $c/(c+g)$ for Various Values of $(c+g)/h$ and α
8	Seed Rate Optimization. 250 MW MHD Generator	45	Segmented Electrode Performance. $E_x/E_{yo} [\eta_o/(1-\eta_o)]$ vs for Various Values of $(c+g)/h$ at Optimum $c/(c+g)$
9	Axial Voltage vs Relative Cost. 1000 MW MHD Generator	46	Current Flux for Unstaggered Segmented Electrodes
10	Cross-Stream Voltage vs Relative Cost. 1000 MW MHD Generator	47	Current Flux for Staggered Segmented Electrodes
11	Axial Voltage vs Relative Cost. 250 MW MHD Generator	48	Effect of Uniform Electrode Staggering
12	Cross-Stream Voltage vs Relative Cost. 250 MW MHD Generator	49	Effect of Stagger on Current Distribution in Conductor Portion of Electrode
13	Magnetic Flux Density Optimization. 1000 MW MHD Generator	50	Method of Electrode Interconnection
14	Inlet Aspect Ratio Optimization. 1000 MW MHD Generator	51	Axial Potential Distribution
15	Electrode Wall Angle Optimization. 1000 MW MHD Generator	52	Typical Variation of Critical Interconnection Voltages with Output Current
16	Current vs Axial Distance. 1000 MW MHD Generator	53	Schematic Representation of End Current Loops
17	Electrode Spacing and Segmentation Diagram for the 1000 MW Generator	54	Effectiveness of Constant Current Density Generator vs Aspect Ratio with $\omega T = 0$
18	Current vs Axial Distance. 250 MW MHD Generator	55	Effectiveness of Constant Voltage Generator vs Aspect Ratio for Various Electrical Efficiencies with $\omega T = 0$
19	Electrode Spacing and Segmentation Diagram for the 250 MW Generator	56	Efficiency of Constant Current Density Generator vs Aspect Ratio for Various Electrical Efficiencies with $\omega T = 0$
20	Mach number vs Distance for the 1000 MW MHD Generator	57	Efficiency of Constant Voltage Generator vs Aspect Ratio for Various Electrical Efficiencies with $\omega T = 0$
21	Temperature vs Distance for the 1000 MW MHD Generator	58	Pressure Drop Ratio vs Aspect Ratio for Various Electrical Efficiencies with $\omega T = 0$
22	Plasma Velocity vs Axial Distance for the 1000 MW MHD Generator	59	End Current Leakage Schematic
23	Pressure vs Distance for the 1000 MW MHD Generator	60	Hall Leakage Current Effect on MHD Generator Power Output
24	Density vs Distance for the 1000 MW MHD Generator	61	Conductivity vs O/F for JP4 + O_2 + 0.75% Hydrous Solution of KNO_2
25	Conductivity vs Distance for the 1000 MW MHD Generator	62	Conductivity vs O/F for JP4 + O_2 + 0.75% Hydrous Solution of KNO_2
26	Mach Number vs Distance for the 250 MW MHD Generator	63	Characteristic Velocity vs O/F
27	Temperature vs Distance for the 250 MW MHD Generator	64	Ratio of Experimental to Corrected Theoretical Conductivity vs O/F
28	Velocity vs Distance for the 250 MW MHD Generator	65	η_{c^*} vs O/F. 6-Element Injector
29	Pressure vs Distance for the 250 MW MHD Generator	66	η_{c^*} vs O/F. 37, 48, and 61 -Element Injectors
30	Density vs Distance for the 250 MW MHD Generator	67	Plasma Conductivity vs Nozzle Area Ratio
31	Conductivity vs Distance for the 250 MW MHD Generator	68	Total Mole Fraction of Free Oxygen vs O/F
32	Oxygen Plant Requirements	69	Button Electrodes after Run. Build 14301-L-1. Run Date 13 August 1962
33	Oxygen Plant Cost		
34	Segmented Electrodes		
35	Segmented Electrode Performance. α_c/α_o vs $c/(c+g)$ for Various Values of α . $(c+g)/h = 0.1$		
36	Segmented Electrode Performance. α_c/α_o vs $c/(c+g)$ for Various Values of α . $(c+g)/h = 1.0$		
37	Segmented Electrode Performance. Optimum $c/(c+g)$ vs α for Various Values of $(c+g)/h$		

List of Figures (Cont'd.)

PWA-2170

Number	Title	Number	Title
70	Keystone Electrode Rig (Build 14279-L-6). Bottom Electrodes and Right Sidewall with Voltage Probes. Date of Run June 25, 1962. Total Run Time on Build 78 Seconds	104	Chase-Around Rig (Build 14406-L-1) Equipped with ZTA Graphite Electrodes after Two Firings and an Accumulated Time of 690 Seconds. Electrode Cover and Left Sidewall. Date of Run October 29, 1962.
71	Keystone Electrode Rig (Build 14279-L-6). Downstream End. Date of Run June 25, 1962. Total Run Time on Rig 98 Seconds	105	Applied Voltage vs Current for Electrode 3
72	Keystone Electrode Rig (Build 14279-L-6). Bottom Electrodes and Right Sidewall with Voltage Probes, Showing Crack Surface. Date of Run June 25, 1962. Total Run Time on Rig 98 Seconds	106	Voltage vs Current for Electrode 3
73	Button Electrode Rig (Build 14301-L-1). Bottom Electrode Assembly. Date of Run August 13, 1962. Total Run Time on Build 150 Seconds	107	Voltage vs Current for Electrode 3
74	Chase-Around Rig (Build 14403-L-1). Top Electrode Housing Assembly. Date of Run October 11, 1962. Total Run Time on Build 15.9 Minutes	108	Applied Voltage vs Current for Electrode 3
75	Chase-Around Rig (Build 14403-L-1). Bottom Electrode Housing Assembly. Date of Run October 12, 1962. Total Run Time on Build 13.9 Minutes	109	Applied Voltage vs Current for Electrode 3
76	SV vs O/F	110	Voltage vs Current for Electrode 3
77	4 Injectors Used for MHD Tests	111	Chase-Around Rig (Build 14404-L-2-R). Electrode Contact Pins. Date of Run December 10, 1962. Total Run Time on Build 27.9 Minutes
78	Electrode Backup Spacer Configurations for Various Electrode Face Temperatures	112	Chase-Around Rig (Build 14404-L-2-R). Electrode Contact Pins. Date of Run December 10, 1962. Total Run Time on Build 27.9 Minutes
79	Voltage vs Current for Electrode 3. Run No. 3	113	Chase-Around Rig (Build 14404-L-2-R). Electrode and Sidewall. Date of Run December 10, 1962. Total Run Time on Build 27.9 Minutes
80	Voltage vs Current for Electrode 3. Run No. 4	114	Chase-Around Rig (Build 14404-L-2-R). Electrode and Sidewall. Date of Run December 10, 1962. Total Run Time on Build 27.9 Minutes
81	Voltage vs Current for Electrode 3.	115	Sidewall Block Temperature vs Time
82	Probe Voltage vs Distance from Cathode	116	Applied Voltage vs Current for Electrode 3
83	Probe Voltage vs Distance from Cathode	117	Terminal Voltage vs Current. Electrode 3
84	Applied Voltage vs Current Density for Electrode 3	118	Power vs Load Resistance. Electrode 3
85	Current vs Voltage for Middle Button Electrode	119	Chase-Around Rig (Build 14394-L-4). Date of Run December 12, 1962. Total Run Time on Build 15.7 Minutes
86	Current vs Voltage for Downstream Button Electrode	120	Chase-Around Rig (Build 14394-L-4). Date of Run December 12, 1962. Total Run Time on Rig 15.7 Minutes
87	Current vs Voltage for Electrode Plate	121	Applied Voltage vs Current for Electrode 3
88	Static Pressure vs Time	122	Temperature vs Time
89	Applied Voltage vs Current for Electrode 3	123	Voltage Probe Data
90	Electrode Temperature vs Time	124	Voltage Probe Data
91	Chase-Around Rig (Build 14403-L-1). Left Sidewall. Date of Run October 12, 1962. Total Run Time on Build 13.9 Minutes	125	Voltage vs Current for Electrode 3
92	Total Power vs Time	126	Power vs Resistance for Electrode 3
93	Static Pressure vs Time and Distance Down MHD Channel	127	Applied Voltage vs Current for Electrode 3
94	Voltage vs Current for Electrode 3	128	Applied Voltage vs Current for Electrode 3
95	Voltage vs Current for Electrode 3	129	Applied Voltage vs Current for Electrode 3
96	Voltage vs Current for Electrode 3	130	Terminal Voltage vs Load Current for Electrode 3
97	Voltage vs Current for Electrode 3	131	Power vs Load Resistance for Electrode 3
98	Static Pressure vs Time and Distance Down MHD Channel	132	Chase-Around Rig (Build 14403-L-3). Top Electrodes and Right Sidewall. Run Date 19 December 1962. Total Run Time on Build 32.6 Minutes
99	Total Power vs Time	133	Chase-Around Rig (Build 14403-L-3). Bottom Electrodes and Left Sidewall. Run Date 19 December 1962. Total Run Time on Build 32.6 Minutes
100	Voltage vs Current for Electrode 3		
101	Voltage vs Current for Electrode 3		
102	Voltage vs Current for Electrode 3		
103	Chase-Around Rig (Build 14404-L-1) Equipped with SiC + 25 Per Cent Graphite Electrodes after Two Firings and an Accumulated Time of 973 Seconds. Date of Run 24 October 1962		

List of Figures (Cont'd.)

PWA-2170

Number	Title	Number	Title
134	Electrode Temperature vs Time	165	Keystone Electrode Rig (Build 14221-L-6). Left Sidewall. Total Run Time 1.7 Minutes. Run Date 15 June 1962
135	Applied Voltage vs Current for Electrode 3	166	Broken Carburized Tantalum Pin Showing Large Crystal Size
136	Voltage Probe Data from Cycle No. 1	167	Toadstool Electrode Rig (Build 14307-L-2). Right Sidewall and Top Electrode Assembly. Total Run Time 29 Minutes. Run Date 20 July 1962
137	Voltage Probe Data from Cycle No. 2	168	Magnetohydrodynamic Test Rig
138	Chase-Around Rig (Build 14406-L-3). Electrodes and Sidewall. Date of Run December 20, 1962. Total Run Time on Build 8.3 Minutes	169	Chase-Around Rig (Build 14394-L-1). Inlet End. Total Run Time 4.4 Minutes. Run Date August 28, 1962
139	Chase-Around Rig (Build 14406-L-3). Electrodes and Sidewall. Date of Run December 20, 1962. Total Run Time on Build 8.3 Minutes	170	Chase-Around Rig (Build 14394-L-1). Right Sidewall and Bottom Electrodes. Total Run Time 4.4 Minutes. Run Date 28 August 1962
140	Applied Voltage vs Current for Electrode 3	171	Chase-Around Rig (Build 14422-L-1). Ramp Electrodes and Sidewall. Total Run Time - Unused
141	Temperature at #1 Electrode Back Face vs Time	172	Chase-Around Rig (Build 14422-L-1). Downstream End of Duration Rig. Total Run Time 88 Minutes. Run Date 15 November 1962
142	Probe Voltage vs Distance from Centerline	173	Chase-Around Rig (Build 14422-L-1). Left Sidewall of Duration Rig. Total Run Time 88 Minutes. Run Date 15 November 1962
143	Probe Voltage vs Distance from Centerline	174	Chase-Around Rig (Build 14422-L-1). Right Sidewall of Duration Rig. Total Run Time 88 Minutes. Run Date 15 November 1962
144	Applied Voltage vs Current for Electrode 3	175	Chase-Around Rig (Build 14422-L-1). Bottom Electrode Assembly of Duration Rig. Total Run Time 88 Minutes. Run Date 15 November 1962
145	Terminal Voltage vs Current for Electrode 3	176	Chase-Around Rig (Build 14422-L-1). Top Electrode Assembly of Duration Rig. Total Run Time 88 Minutes. Run Date November 15, 1962
146	Power vs Resistance. Electrode 3	177	Chase-Around Rig (Build 14422-L-2). Sidewall and Electrodes after Long Duration Run on Propane. Total Run Time 88 Minutes. Run Dates January 4, 7, 8, 1963
147	Chase-Around Rig (Build 14403-L-3). Top Electrodes and Right Sidewall. Run Date 21 December 1962. Total Run Time on Rig 15.4 Minutes	178	Chase-Around Rig (Build 14422-L-2). Sidewall and Electrodes after Long Duration Run on Propane. Total Run Time 88 Minutes. Run Dates January 4, 7, 8, 1963
148	Chase-Around Rig (Build 14403-L-3). Bottom Electrodes and Left Sidewall. Run Date 21 December 1962. Total Run Time on Rig 15.4 Minutes	179	Detail of Cooled Plate Electrode Contact
149	Applied Voltage vs Current. Electrode 3	180	MHD Gas Generator
150	Terminal Voltage vs Current to Load for Electrode 3	181	Calculated Efficiency Data. Configuration 42
151	Applied Voltage vs Current. Electrode 3	182	Calculated Efficiency Data. Configuration 54
152	Chase-Around Rig (Build 14394-L-5). Bottom Electrodes and Right Sidewall. Date of Run December 27, 1962. Total Run Time on Build 18.4 Minutes	183	Calculated Efficiency Data. Configuration 61
153	Chase-Around Rig (Build 14394-L-5). Top Electrodes and Left Sidewall. Date of Run December 27, 1962. Total Run Time on Build 18.4 Minutes	184	C* Efficiency Correlation Method
154	Chase-Around Rig (Build 14394-L-5). Rig Entrance. Date of Run December 27, 1962. Total Run Time on Build 18.4 Minutes	185	Gas Generator Performance Correlation. Pressure Coefficient vs Normalized Pressure
155	Downstream End View of 1.65 x 3.30 x 7.0-Inch Channel, Keystone Electrode MHD Generator Test Section	186	Gas Generator Performance Correlation. Mixture Ratio Coefficient vs Normalized Mixture Ratio
156	Electrode Wall and Sidewall Assembly for 1.65 x 3.30 x 7.0-Inch Channel, Keystone Electrode MHD Generator	187	Gas Generator Performance Correlation. Fuel = JP4, Oxidant = O ₂ (Gas)
157	Build 14221-L-1. Molybdenum Sidewall of MHD Test Rig. Run Date 23 November 1961. Run Duration 6 Seconds	188	Gas Generator Performance Correlation. C* Efficiency vs Effective Length
158	Build 14221-L-1. Back Surface of Molybdenum Sidewall of MHD Test Rig. Run Date 22 November 1961. Run Duration 6 Seconds	189	Atomization of Seeding Solution by Injection into Oxygen Supply Pipe Ahead of Main Injector
159	Alternating Sidewall MHD Rig	190	Injection of Nitrogen with Entrained NaHCO ₃ Powder Through 18-Hole MHD Injector
160	Keystone Electrode Rig (Build 1437-L-4). Electrode and Spacer. Total Run Time 0.8 Minute. Run Date 5 April 1962	191	Mixture of K ₂ CO ₃ and JP4 + Additives at 300 Hrs. Showing Trapped Pockets of JP4
161	Keystone Electrode Rig (Build 14279-L-4). Electrodes and Spacer. Total Run Time 0.8 Minute. Run Date 5 April 1962	192	Settling of K ₂ CO ₃ Suspensions in JP4
162	Toadstool Electrode Rig (Build 14307-L-1). Top Electrode Assembly. Total Run Time 127 Seconds. Test Date 5 June 1962	193	Settling of K ₂ CO ₃ Suspensions in JP4
163	Toadstool Electrode Rig (Build 14307-L-1). BeO Interelectrode Spacer. Total Run Time 127 Seconds. Test Date 5 June 1962.		
164	Keystone Electrode Rig (Build 14221-L-6). Top Electrode Assembly. Total Run Time 1.7 Minutes. Run Date 15 June 1962		

Number	Title	Number	Title
194	Settling of K_2CO_3 Suspensions in JP4	227	Electrical Circuit for MHD Test Stand X-197C
195	Power per Unit Area vs Weight Per Cent Potassium	228	Wiring and Instrumentation Diagram. X-197C Test Stand
196	Relative Power/Area vs Temp.	229	Water-Cooled Copper Nozzles Used in MHD Test Program
197	JP4 Unreacted and Heated	230	Basic 1.65 x 3.30 x 7.0-Inch Channel MHD Generator Rig Shown with Segmented, Keystone Type, Nickel Electrodes
198	Effect of Combustion Efficiency on MHD Performance for Several Assumed Mechanisms of Combustion Inefficiency	231	Basic 1.65 x 3.30 x 7.0-Inch MHD Generator Rig Shown with Segmented ZTA Graphite Electrodes. Keystone Electrode Configuration
199	Enthalpy vs Temperature of JP4 Vapor	232	Power Take-off Configuration. Keystone Electrode Rig
200	Effect of Combustion Efficiency on MHD Performance for the Droplet Effect Mechanism	233	Keystone Electrode Rig (Build 14279-L-6). Run Date June 25, 1962. Length of Run 98 Seconds
201	Concentric Testing. Local Weight Flow per Unit Area vs Probe Location	234	Basic 1.65 x 3.30 x 7.0-Inch Channel MHD Generator Rig with Toadstool Type Electrodes Installed
202	Concentric Testing at Five Inches from Injector Face	235	Schematic Diagram of Button Electrode Configuration
203	Maximum Normalized Concentration vs Gas-to-Liquid Injection Velocity Head Ratio	236	Major Components of Modified Button Electrode Rig
204	Concentric Testing. Normalized Spread vs Normalized Liquid Concentration	237	Temperature vs Pressure-Density Ratio for Various Pressures
205	Low Pressure Spray Nozzle Testing. Local Weight Flow per Unit Area vs Probe Location	238	Nozzle Thrust and Flow Performance. Nominal Area Ratio = 1.50
206	Comparison of Inverted and Normal Concentric Spray Geometry at Equal Liquid Flow Conditions	239	Nozzle Thrust and Flow Performance. Nominal Area Ratio = 2.0
207	Princeton Theoretical Light Scattering Profile	240	Nozzle Thrust and Flow Performance. Nominal Area Ratio = 3.75
208	Schematic of Optical Technique of Measuring Droplet Size	241	Velocity Ratio vs η_{LC}
209	Drop Size Measurement Rig	242	Rearward-Facing Step Vortex Generator
210	Typical Experimental Light Scattering Profile	243	Forward-Facing Step Vortex Generator
211	Electron Temperature Determination for Ion Probe	244	Forward-Facing Ramp Vortex Generator. Combined Forward-Facing Ramp, Rearward-Facing Step Vortex Generator
212	Voltage-Current Characteristics of the Double Probe Method	245	Velocity vs Area Ratio for Shifting Equilibrium and Frozen Expansions
213	Ionization Level Profiles in JP4/O ₂ Rocket Exhaust	246	Temperature and Conductivity vs Mach Number for Shifting Equilibrium and Frozen Expansions
214	Circuit Setup for Determining Plasma Conductivity in the Presence of a Magnetic Field	247	Location of Voltage Probes
215	Diagram of Current Flow Lines to Illustrate "Blooming"	248	Schematic Diagram of Voltage Probe Circuit
216	Determination of Apparent Plasma Conductivity from Voltage-Current Characteristic	249	Voltage Probe Readings
217	Average Plasma Conductivity Obtained from Plot of Applied Voltage vs Current	250	Voltage Probe Characteristic Curve
218	Electrical Circuit Diagram for the Probing Technique in an MHD Generator	251	Voltage Probe Characteristic Curve
219	Circuit Setup for Determining Plasma Conductivity. No Magnetic Field	252	High Frequency Filter Circuit for Voltage Probes
220	Average Conductivity Measurement by Means of a Variable Applied Voltage	253	Voltage Probe Installation for Chase-Around Rig
221	Measurement of Free Stream Conductivity by Means of Variable Electrode Spacing.	254	Grid-Dip Meter Circuit
222	Electrical Circuit Diagram for Probing Technique	255	Allowable Size of AC Conductivity Probe as Function of Generator Parameters
223	1.65 x 3.30 x 7.0-Inch Channel MHD Rig Mounted on Test Stand (Magnet Not in Place)	256	AC Plasma Conductivity Probe Configurations
224	1.65 x 3.30 x 7.0-Inch Channel MHD Rig and Facility at X-197 Test Stand	257	Major Components of AC Conductivity Probe
225	MHD Generator and Auxiliary Equipment	258	Predicted Gross/Allowable Gross vs Hot Wall Temperature for Various Chamber Pressures and Thicknesses
226	Piping Schematic of X-197C MHD Test Stand as of January 1, 1963	259	Assembly of AC Conductivity Probe
		260	Velocity Probe Installation
		261	Velocity Probe Circuit

I. INTRODUCTION TO FINAL REPORT

This volume is Volume I of six volumes which constitute the final report submitted by Pratt & Whitney Aircraft to Rome Air Development Center under Contract AF30(602)-2487, Investigation of Magnetohydrodynamic Power Generation.

Volume I has been written to include a comprehensive report on all aspects of this program and to, therefore, represent the final report submitted. Volume I contains an introduction to the final report, summaries of all other volumes, and conclusions of the program in addition to self-contained sections on system analysis and environmental testing. The remaining five volumes are autonomous and report on various subjects as indicated by title.

Objective of Program

The purpose of this program was to establish, by analytical and experimental studies, the design and performance characteristics of a 1000 megawatt MHD generator and compatible continuous wave radio frequency converter. The design and performance characteristics of an alternate size (250 megawatts) generator as determined by potential Air Force needs were also to be established.

Pursuant to these objectives the principal MHD problem areas of plasma conductivity and materials were investigated. The objective of the plasma conductivity studies was to develop improved analytical methods for determining plasma and boundary layer conductivity and to establish the validity of these analytical methods through experimental tests conducted on devices of practical configuration. The materials investigation was performed to evaluate various materials for use in the fabrication of MHD generator electrodes and insulating walls.

Principal types of radio frequency converters were to be studied to determine the type or types most compatible with MHD power generation. The design characteristics of those types most compatible with MHD power generation were to be determined and used in conjunction with an MHD generator system analysis to ascertain the optimum MHD - rf converter system. The required characteristics of the overall system are that it should be capable of:

- 1) Rapid starting,
- 2) Operating for periods between ten seconds and one-half hour,
and,
- 3) Cumulative operation for twenty hours.

The results of these studies were to form the basis for preliminary designs of both the 1000 megawatt and 250 megawatt MHD generators. The purpose of these designs was to serve as a step toward the ultimate demonstration of the applicability of MHD power generation or an integrated MHD - rf system to present or future military applications.

Scope of Program

The scope of the program as outlined below was sufficient to allow the establishment of the design and performance characteristics of the required 1000 megawatt - rf system and of the 250 megawatt generator.

The principal problem areas of MHD power generation and the problem of superpower radio frequency converters compatible with MHD generators were investigated. This investigation included a system analysis and economic evaluation of 1000 megawatt MHD generators operating in a manner compatible with superpower rf converters as well as of 250 megawatt generators to be utilized in other possible Air Force applications. Superpower rf converters of both the crossed-field and linear-beam type were considered, including a crossed-field device which was conceived as a result of this study. The compatibility of the rf converters with the MHD generator as well as an economic evaluation of the most feasible schemes was determined.

The basic physics of electron and ion motion was analyzed and applied to the calculation of the conductivity and diffusion coefficients of the bulk plasma. The boundary layer problem was studied by interrelating the fluid mechanics problem with the study of diffusion of the electrons and ions, of the electron energy transfer, and of the electron formation rates under the highly nonuniform condition existing in the boundary layer. These problems are solved simultaneously to find the cathode and anode voltage drops as a function of current density.

A program for the evaluation of materials for MHD generator applications was pursued. The program was divided into three phases, 1) a literature survey and preliminary analysis, 2) a laboratory investigation, and 3) an environmental testing program. The electrical resistance of insulating materials and the electrical conductivity of

ceramic conductors were measured at high temperatures. Those materials having promising electrical properties were tested for chemical compatibility and erosion resistance to plasma of the type found in MHD generators. The properties of linear thermal expansion and modulus of rupture were determined at high temperature for the most promising of these. The best materials were then tested in actual MHD generators under operating conditions.

An environmental testing program was carried out using MHD generators of practical configuration *. The purpose of the program was to yield information on conductivity of the plasma, boundary layer voltage drops, structural concepts, and combustion problems. The effects of variation in oxidant-to-fuel ratio, electrode temperature, boundary layer condition and the existence of a magnetic field on conductivity were studied. Various structural concepts were tested and the various causes and effects of combustion inefficiencies studied.

Preliminary design studies of a 250 MW and of a 1000 MW MHD generator were made, based on the conclusions of the above studies. The study consisted of a systematic survey, analysis and evaluation of the problems involved, an evaluation of available means and materials, formulation of a design philosophy, design of the generator and an evaluation of the resulting design. Combustion, aerodynamic, thermodynamic, electrical and magnetic, thermal, chemical and mechanical effects and their interdependence were studied and evaluated.

Program Background

The feasibility of generating large amounts of electrical power by MHD means was studied earlier at Pratt & Whitney Aircraft under Air Force Contract Number AF30(602)-2268. The results of that study are reported in RADC TR61-104. The principal problem areas noted in the previous feasibility study were investigated in this study. In particular the principal problem areas of the MHD generator were shown to be plasma conductivity and materials and structures. The problem of superpower rf converters and their compatibility with MHD generators was also shown to be a major consideration in the practicality of the overall system.

To demonstrate the relative success of the solution to the plasma conductivity, materials, and structures problems and to further advance the MHD generator concept toward a working status, an extensive

*The test generators used produced power outputs up to 0.81 megawatts/ft³ and have been run repeatedly during power generation for periods up to 1 1/2 hours

program of environmental testing was deemed necessary. This testing program in conjunction with a system analysis and economic evaluation of the MHD generator led to the ability to make a reasonable preliminary design of the MHD generator complex, thereby determining design philosophy, concepts and procedures consistent with the practical solution of the MHD problem.

The level of this investigation was augmented not only by the previous feasibility study but by an extensive in-house effort in all of the above areas except the rf converter problem. For the latter reason Pratt & Whitney Aircraft let subcontracts to two established electronics firms with backgrounds in superpower rf converters. Thus a high level of effort was extended for the solution of all the problem areas under investigation.

The possibility of other Air Force applications for MHD generators and a consideration of development cost variation with generator size, indicated the desirability of considering more than one size of generator. Therefore two sizes of MHD generators (1000 megawatts and 250 megawatts net output) were considered.

Breakdown of the Problem

In this section the overall problem will be broken down into those phases considered in each of the separate volumes. The individual volume can be consulted for a more detailed breakdown of that phase of the investigation reported.

The system analysis, which includes derivations of the necessary equations, optimization of the operating conditions and definition of the performance characteristics of the MHD generator, is included in Volume I. The environmental testing program consisting of plasma conductivity studies, testing of structural concepts and studies of combustion phenomena, is also reported in Volume I.

In Volume II the principal crossed-field types of radio frequency converters were studied to determine the types most compatible with MHD generation. This study, conducted by Raytheon Company on a subcontract from Pratt & Whitney Aircraft, led to the conception of a new class of cross-field rf converter named the electromagnetic amplifying lens (EAL). The design and performance characteristics of this tube are considered and an experimental model was constructed for cold testing.

Both crossed-field and linear-beam rf converters were considered by the Electron Tube Division of Litton Industries in Volume III. Consideration of possible tube types was limited to those which had at least progressed to the experimental model stage. The klystron was selected as the best compromise and the characteristics of a 200 kilowatt tube were outlined.

A study of materials for MHD generator applications including a literature survey, laboratory investigation and environmental testing is contained in Volume IV. Insulating materials and ceramic conductors were investigated. Properties such as electrical resistance (or conductivity), chemical compatibility, erosion resistance, coefficient of linear thermal expansion and modulus of rupture were determined at high temperature and are reported.

The relationships between the current, electric field, and other current driving forces are derived in Volume V. These relationships are used to develop improved methods for determining the plasma conductivity. The boundary layer problem is analyzed and the results in terms of anode and cathode voltage drops are presented. Derivations are presented for the equations of elevated electron temperature, ion conductivity, ambipolar diffusion and mobility coefficients.

The analytical and design efforts which made up a preliminary engineering design study of two sizes of MHD generators (1000 megawatt and 250 megawatt net output) are presented in Volume VI. The study includes design layouts, concepts, and supporting data which are presented and discussed with regard to performance capabilities, from both the structural and operational points of view.

II. SYSTEM ANALYSIS

The purpose of this portion of the program was to establish the design and performance characteristics of two high power (1000 megawatt and 250 megawatt net output) MHD generators. The system analysis objectives were to establish the conditions for minimum overall cost, consistent with the technical requirements of the system and reasonable design procedures. The 1000 megawatt generator was to be used in conjunction with compatible rf converters in an integrated system. The other generator size was determined by Air Force requirements to be 250 megawatts.

The required characteristics of the MHD generator to be used in conjunction with rf converter system were that it should be capable of:

- 1) Rapid starting,
- 2) Operating for periods between ten seconds and one-half hour, and
- 3) Cumulative operation for twenty hours.

Compatibility with rf converters was the governing factor in choosing the operating voltage of 5000 volts DC for this system. This voltage is suitable for driving klystron tubes as designed by Litton Co. (see Volume III). This tube was selected over the EAL tube (see Volume II) in consideration of the fact that it could be more readily developed from existing data.

Systems analysis procedures developed previously* were utilized, improvements in computer programs and in various other areas being incorporated. Derivations of the necessary equations are shown and the independent flow variables are optimized. Losses are accounted for and performance characteristics determined.

Highlights of Results

Two MHD generators, burning JP4 and oxygen seeded with a hydrous solution of potassium nitrite at an O/F ratio of 2.9 and combustion chamber total pressure of 350 psia, were analytically sized to produce 1000 and 250 megawatts of DC electrical power. The active section of the 1000-megawatt generator is 13.3 feet long and has a volume of 1035 cubic feet. This generator develops nominal voltages of 15,000 volts cross-stream and 40,000 volts Hall voltage. Interconnection of the segmented electrodes is utilized to provide 30,000 volts net to klystron converters. The active section of the 250-megawatt generator

* Report PWA-1940, RADC TR-61-104

is 7 feet long with a volume of 120.5 cubic feet and develops 8000 volts cross-stream and 17,000 volts Hall voltage. The products of combustion flow at supersonic velocities in both generators with flow rates of 1650 and 387 lbs/sec for the 1000 and 250-megawatt generators respectively.

A. One-Dimensional MHD Performance Calculation

The purpose of the one-dimensional performance calculation was to determine generator configurations to satisfy predetermined performance requirements. These configurations were then compared on a relative cost basis to determine optimum characteristics. The one-dimensional performance calculation was then supplemented by two-dimensional loss considerations to refine the performance calculation of the optimum generator.

The assumptions are listed and explained. The pertinent equations are presented in general form and in the specific form resulting from the application of the assumptions. Several derivations utilizing these equations are presented. The optimized flow variables are determined by economic considerations. The optimum flow variables are then combined to determine optimum generator configurations. The electrical and thermodynamic characteristics of the optimum generators are shown.

Assumptions

To make the solution of the MHD equations tractable it is necessary to make certain simplifying assumptions, as follows:

- 1) One-dimensional flow exists with the directions of gas velocity, applied magnetic field and current, mutually perpendicular at any point. One-dimensionality in this context implies that all variables (such as velocity or pressure) can be expressed completely in terms of a single parameter, in this case length (x). The assumption of one-dimensional flow allows the

spatial derivatives $\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial z}$ to be set equal to zero.

- 2) Two-dimensional losses such as heat transfer, segmentation, etc., can be considered separately from the one-dimensional considerations. It is assumed that the consideration of these losses will not affect the relative position of the various generators in the optimizations.

- 3) Electrical conductivity of the plasma is a tensor and is returned to the isotropic scalar value by the use of segmented electrodes. The losses inherent in segmentation are applied as an efficiency after one-dimensional performance has been determined.
- 4) Ion current and ion slip are assumed to be negligible. The assumption that ion current is negligible yields a calculation that is slightly pessimistic since the net result of ion conductivity is to increase overall plasma conductivity. Ion slip is actually Hall ion current. The Hall parameter for ions is much lower than one for the types of ions found in the combustion products and magnetic field of the strength used here. Hence the Hall ion current is much less than the normal ion current and is also negligible.
- 5) The plasma is neutral and homogeneous. The high gradients of the various chemical species that would exist in the thermal boundary layer and the attendant boundary layer voltage drops are taken to be small. Normal experimental boundary layer voltage drops are on the order of ten to one hundred volts, which is only about 1 per cent of the cross-stream potential of typical generators in the one thousand megawatt power range.
- 6) The induced magnetic field is small compared to the applied magnetic field. The magnetic Reynolds number, which is approximately equal to the ratio of the induced field to the applied field, can be shown to be small for the range of conductivity, velocities and scale factors typical in the generator sizes being considered.

1. One-Dimensional Performance Calculation Programs

Two basic classes of solutions to the system of MHD flow equations have been programmed for high speed digital computers. The first class of solutions involves the specification of a thermodynamic variable such as pressure, temperature (enthalpy), velocity, or Mach number. The second class assumes the specification of geometric and/or electrical parameters such as area, ratio of electrode wall separation-to-insulator wall separation, length, voltage, or magnetic field. Specification of the flow variables of an MHD generator involved the simultaneous solution of the

equations of momentum, energy, continuity, state, Maxwell's and Ohm's laws, and an analytic expression for conductivity. A summary of the equations used in the solution follows. For derivations of these equations, and to note the implicit assumptions see Appendices 1 and 2. For a list of pertinent nomenclature see Appendix 3.

Energy equation

$$\rho V \frac{dh}{dx} + \rho V^2 \frac{dV}{dx} = -J_y E_y \quad (1)$$

Momentum equation

$$\rho V \frac{dV}{dx} + \frac{dP}{dx} = -J_y B_z \quad (2)$$

Continuity

$$\dot{m} = \rho A V \quad (3)$$

State

$$P = \rho R T \quad (4)$$

Ohm's law

$$J = \sigma (V_x B_z - E_y) \quad (5)$$

Maxwell's equations

$$\begin{aligned} E_y Y &= \text{constant} \\ B_z Z &= \text{constant} \end{aligned} \quad (6)$$

Conductivity

$$\sigma = f(T, P, \text{composition})$$

Specified Thermodynamics - In this class of solutions one of the thermodynamic variables is kept constant. This permits a relatively simple solution as evidenced by a constant static enthalpy derivation (Appendix 2). Among the variables specified in this type of solution are the duct aspect ratio Y/Z (to minimize magnet cost), and E/VB . If the ratios E/VB and Y/Z are kept constant as well as $B_z Z$ it can be shown that $E_y Y$, the generator cross-stream voltage, will vary. The resultant generator wall shape will curve outward.

Alternatively, it is possible to allow a variation of duct aspect ratio consistent with a constant cross-stream voltage. Such an expansion process has the following effects:

- 1) The MHD exhaust dimension in the magnetic field direction (Z) is reduced relative to the case where Y/Z is constant, causing an increase in the magnetic field strength and a reduction in generator length.
- 2) The MHD exhaust dimension in the electric current direction (Y) is increased, resulting in an increase in MHD exit wall angle or gas flow angle entering the diffuser.
- 3) The voltage delivered to various loads is more nearly equal as a result of the constant cross-stream potential.

The disadvantages of the specified thermodynamics are:

- 1) The specification of the thermodynamic, aerodynamic and electric variables leads to a calculated wall shape. For optimized thermodynamic variables the wall angle increases during the expansion process in such a manner that large wall angles are obtained at the generator exit. This condition is disadvantageous due to difficulty of fabrication and length of diffuser required. A long transition section is required to turn the flow leaving the generator into the diffuser without causing boundary layer separation. Experience with nozzle tests indicates that approximately thirteen degrees is the maximum wedge angle permissible without producing separation of flow from the walls as the flow enters the diffuser.
- 2) Due to reduction of the Mach number toward the exhaust end of the generator, the conductivity is higher at this end than at the inlet end. This condition leads to problems in interconnection which require deviations from the specified optimum values of Mach number and electric field strength.

Specified Geometric and Electric Variables - In view of the disadvantages of the specified thermodynamics cases, the final optimization for this study was based on a derivation where the geometric and electrical variables are specified (Appendix 2). Two programs with these basic criteria have been written. The first utilizes essentially frozen equilibrium and has been derived considering infinite segmentation ($J_x = 0$). It considers the neutral species of the composition fixed, constant gas constant, and constant specific heat. The variation in electron concen-

tration is taken into account by use of Saha's equation considering the reaction $K \rightleftharpoons K^+ + e^-$. The initial composition is determined by use of the chemical thermodynamics program which allows for the existence of the neutral species CO, H₂, H₂O, N₂, CO₂, O₂, O, OH, H, NO, N, CH₄, C, CN, C₂N₂, HCN, HN, C₂H₂, CH₂, CH, C₃, C₂, COH, K, KOH, K₂, KH, K₂O. The ionic species considered are e⁻, K⁺, H⁺, C⁺, O⁺, N⁺, N⁺⁺, O⁺⁺, H⁻, O⁻, OH⁻, O⁺⁺⁺, CO⁺, N₂⁺, O₂⁺, NO⁺, C⁺⁺, H₃O⁺.

The only ionized species that exist in quantities greater than 10⁻⁶ are e⁻, K⁺, O⁻ and OH⁻. Experience has indicated that even considering shifting equilibrium, little change in the composition of the neutral species occurs. Therefore results for frozen equilibrium approximate those for shifting composition quite closely in many aspects. The concentration of K⁺ is greater than the negative ions, so use of the Saha equation where only the reaction $K \rightleftharpoons K^+ + e^-$ can occur is a reasonably good approximation.

The MHD equations may be solved by prescribing certain geometric properties instead of a thermodynamic variable. The basic purposes of considering such a derivation are:

- 1) A straight wall generator presents fewer construction problems,
- 2) The wall angle at the exit can be prescribed to alleviate the problem of transition to the diffuser, and
- 3) Specifying the area variation and length of the generator permits the computation of off-design performance.

Six process types have been considered in the program. In each process type, four variables are specified as a function of length, and maintained at these values for the noniterating cases. Thus the program can be used to calculate off-design performance by specifying exactly how the geometry and electrical properties vary as a function of the length. The power output and exit-stagnation conditions are determined at the prescribed length.

The six process types are determined by the specification of the following variables:

1) Area	Y/Z	E/VB	BZ
2) Area	Y/Z	$E_y Y$	BZ
3) Area	Y/Z	E/VB	B
4) Area	Y/Z	$E_y Y$	B
5) Area	E/VB	$E_y Y$	BZ
6) Area	E/VB	$E_y Y$	B

Given the initial values of the static thermodynamic variables and electric parameters, differential equations of flow, Appendix 2, Equations (2-34) and (2-36) are solved simultaneously using the Runge-Kutta method over an interval of the independent variables, X .

The following variables are predicted as a function of length, gas density, Mach number, cross-stream voltage, conductivity, axial voltage, current, temperature, pressure, velocity, electron concentration, and current. In addition to design performance, the program can also be used to compute off-design point performance. The initial area schedule will not, in general, yield the desired power level required for a particular length, exit stagnation pressure, or exit Mach number requirement. An iteration procedure adjusts the inlet flow area according to the power requirement, maintaining the same values of density, velocity, and Mach number at the inlet, therefore requiring an adjustment in the mass flow. Changing the inlet flow area in the iteration-type cases necessitates revision of the area and Y/Z schedules. The new schedules are defined by maintaining either the same wall angle or area ratio as the initial schedule. The usual criterion for design-point generator studies is exit total pressure, the selection of which is discussed below under Generator Optimization. Integrated into the program is an equation for calculating magnet weight and a method for determining magnet, fuel, oxidant, seed, generator, diffuser, injector, heat exchanger, pump, oxygen plant, combustion chamber, and nozzle costs.

This program has been supplemented by one which incorporates complete shifting chemical equilibrium of the plasma into the solution of the flow equations. The derivation is found in Appendix 2. The shifting equilibrium program was completed

too late in the contract period to utilize in optimization of the generator but has been used to validate test results. Improvements of the shifting equilibrium program over previous programs are:

- 1) It simulates the effects of shifting equilibrium. Although assuming that the composition is frozen and that all of the electrons result from the reaction $K \rightleftharpoons K^+ + e^-$ as was done in the frozen equilibrium program is a good approximation, it does result in some error which this shifting equilibrium program eliminates.
- 2) Changes in specific heat, gas constant, and the ratios of specific heat have been included.
- 3) A constant value of boundary layer voltage drop per increment of length may be included.
- 4) Performance for a nonsegmented electrode generator, where $E_x = 0$ has been included.
- 5) A more accurate method of computing conductivity has been included. In previous programs the calculation of conductivity assumed that some of the species had constant collision cross-section and that others had constant collision frequency*. In the more exact conductivity calculation method utilized in this program, the integrated collision frequency vs electron velocity for each species has been programmed in conjunction with the chemical thermodynamic program.

To incorporate shifting equilibrium into this method of solution, tables of thermodynamic data, conductivity, and compositions are generated from the chemical thermodynamic program. The method of solution is the same as the previously-mentioned specified geometric and electrical variable program. The derivation is more complex, however, due to the existence

of terms such as $\frac{dR}{dx}$ which result in three simultaneous equations instead of two, Equations (2-68), (2-69), and (2-70).

* Report PWA-1940, RADC TR-61-104

2. Generator Optimization

The objective of the optimization program is to arrive at an MHD generator configuration that will meet specifications with minimum expense and be compatible with reasonable design procedures. The problem requires the simultaneous optimization of the variables that affect the power output of the generator. The following is a list of variables:

- 1) Oxygen/fuel ratio,
- 2) Seeding rate,
- 3) Stagnation pressure ratio,
- 4) Inlet Mach number - exit Mach number,
- 5) Magnetic field strength,
- 6) E/VB (electrical efficiency), and
- 7) Duct geometry (wall angles, aspect ratios).

Simultaneous optimization of all these variables would result in a great number of case studies. A more feasible approach is to determine local optimization points, when possible, and to combine them to form an optimum generator.

It was necessary to determine an expansion process in order to obtain a solution of the flow variables in the MHD generator. Specified thermodynamic expansions lead to solutions with wall shape being determined. For optimized thermodynamic variables, the wall angle increases during the expansion process in such a manner that large wall angles are obtained at the generator exit. This condition is disadvantageous due to the difficulty of fabrication, and to diffusion of the supersonic exhaust gases.

A straight wall presents fewer construction problems. The assumption of one-dimensional flow dictates that the cross-stream voltage and the product of magnetic field strength and coil separation be maintained approximately constant during expansion (see Appendix 1). These conditions have been satisfied in determining the optimum parameters. A specified geometric and electrodynamic variable MHD computer program with the input items area, Y/Z , $E_y Y$, and $B_z Z$ was used in conjunction with a combustion chamber and nozzle thermodynamic program to arrive at the optimum parameters.

Choice of Reactants and Seeding Material - The primary considerations in selecting the reactants were cost, relative performance, and handling. Oxygen was chosen as the oxidizer. The use of air, the only practical alternative, would be suitable from a performance standpoint if it were highly preheated and/or enriched with oxygen. However neither of these methods would be economical for a system with only a 20-hour operating life. Oxygen can also double as the magnet coolant, thereby increasing the efficiency of the magnet by allowing operation at cryogenic temperatures. The manufacture of oxygen in an oxygen plant can be carried out at an operating cost of 0.006 dollar per pound of oxygen produced.

An evaluation was made to compare the relative performance of various fuels with oxygen at standardized combustion conditions. The fuels evaluated were hydrogen, several hydrocarbons, coal, coke, pure carbon, and cyanogen. The relative values of power per unit area ($\sigma V^{3/2} \rho^{-1/2}$)* are shown in Figure 1 as a function of oxygen-to-fuel mass flow ratios. The theoretical combustion temperatures of these fuels are shown in Figure 2.

Those fuels having a performance significantly higher than JP4 are cyanogen, acetylene, pure carbon and coke. Considerations of fuel cost, storability, availability in large quantity, handling safety, toxicity of exhaust products, and ease of flow measurements and control, favor the use of JP4 as the fuel.

For the performance study described above, the seed material was assumed to be solid potassium. Since potassium in this form would be extremely difficult to utilize in a practical MHD system, a study was made of the relative performance of a generator using various potassium compounds and solutions of potassium compounds that would allow easier introduction of the seed material into the plasma. Choice of the seeding material to increase the plasma conductivity was restricted to potassium compounds because of the high cost of more easily ionized compounds such as cesium, and the smaller effect of other compounds including sodium.

*The quantity ($\sigma V^{3/2} \rho^{-1/2}$) is an optimization parameter with units of power per unit area. It is derived and discussed in Report PWA-1940, RADC-TR-61-104.

Figures 3 and 4 present plasma conductivity and power per unit area versus percentage of elemental potassium in the plasma. It can be seen that of the solutions of potassium compounds considered, the 75 per cent KNO_2 - 25 per cent H_2O solution has the highest performance for conditions having above 2 per cent potassium (by weight) in the plasma. Below 2 per cent potassium concentration, 50 per cent potassium hydroxide solution and 75 per cent potassium cyanide solution are somewhat superior but they present marked problems in handling. The effectiveness of the other solvents is shown to be lower than that of water. At very low seeding rates, such as 0.1 per cent potassium, the performance differences are minor.

A large increase in power per unit area can be realized for a given concentration of potassium in the plasma by eliminating the water required in the solutions. Figures 3 and 4 also show the relative increase possible with several undiluted seed materials assumed to be in the solid state. Potassium cyanide gives high performance, but would require special handling as well as thorough scrubbing of the exhaust, because of the very high toxicity of the HCN that would be formed on contact with water. Potassium hydroxide would also require special handling. Potassium carbonate appears to be a very promising seed material because of its high performance and safe handling characteristics. Below a concentration of about 3 per cent potassium in the plasma, potassium bicarbonate also is attractive. Both of the carbonates are hygroscopic and must be protected from moisture, but would give a significant performance increase over any of the solutions investigated at seed rates greater than 2 per cent. Again, at low seed rates there is little difference in performance between the solid seeding materials and the solutions. The seeding solutions are probably easier to measure and control accurately. Hence the best of these, 75 per cent KNO_2 - 25 per cent H_2O was selected as the seed material.

Optimization of O/F - The oxygen/fuel ratio influences the chamber temperature as indicated in Figure 5. It is noted that chamber temperature does not vary greatly with O/F in the stoichiometric range. The O/F tends to optimize fuel-rich at 2.9 due to the higher obtainable velocities with the low molecular weight combustion products of CO and H_2 , and the low collision cross-section of H_2 relative to H_2O . The cost optimization of O/F is presented in Figure 6. The optimum is not expected to vary significantly with changes in other parameters.

Optimization of Seed Rate - The optimum seeding rate for the 1000 MW generator appears to result in a compromise between minimum cost and voltage requirements (Figure 7). A 0.5 per cent seed rate gives minimum cost, but the voltage obtained with this seed rate is considerably less than the desired value. The required voltage, 30,000 volts D.C. net, was obtained by a reduction in seeding rate to 0.1 per cent resulting in slightly higher costs. Figure 7 shows the variation of axial voltage with seeding rate. The lowering of seed rate yields a greater voltage increase per dollar than other possible methods such as variations in Y/Z , E/VB , P_T , etc.

The seeding rate choice for the 250-MW generator was not affected by voltage considerations. Unlike the 1000-MW generator which utilizes the Hall potential for the load, the 250-MW generator uses cross-stream potential which is not as sensitive to changes in seed rate. An optimum seeding rate value in the range of 0.8 per cent of the total flow was determined (see Figure 8).

Optimization of Inlet Total Pressure, Mach Number, and Inlet E/VB - The interdependence of inlet total pressure, inlet Mach number, and E/VB in determining the power per unit volume and other significant parameters for the 1000 MW generator necessitated simultaneous optimization.

Within the range of the parameters selected (based on previous analytical studies) there appear to be several cases in the voltage range required (above 30,000 volts). Figure 9 indicates a minimum cost at $P_{T_0} = 350$, $M_{N_0} = 3.0$, and $(E/VB)_0 = 0.6$ with an axial voltage of 32,000 volts. It is interesting to note the trend indicated on Figure 10 which presents cross-stream voltage vs relative cost at the design inlet total pressure. To obtain higher cross-stream voltages without deviating from the optimum inlet aspect ratio (see paragraph below) an increase in Mach number and E/VB would be required. However, this would effect considerable cost increases without yielding a substantial increase in cross-stream voltage.

A similar study was performed for the 250-MW generator, the primary difference being the voltage requirement (approximately 7500 volts). Figures 11 and 12 present axial voltage and cross-stream voltage versus cost. These figures show that it is within reason to accept the same inlet total pressure ,

Mach number, and $E/V\beta$ as the 1000-MW design case, since these values occur in the minimum cost range.

In both the 1000-MW and the 250-MW generators, an exit total pressure of 22 psia was selected. This value is expected to be sufficient for exhaust to the atmosphere (see Volume VI on the MHD diffuser). Figure 12 presents the extremes of the pressure range investigated (200 psia and 350 psia). The cost and cross-stream voltages are typical of pressures within this range.

Inlet Magnetic Field Strength - Choice of an inlet magnetic field strength (β_o) influences the magnet weight as follows

$$\text{magnet weight} = \frac{(\text{constant}) (\beta_o)^2}{\text{field power}} \quad (7)$$

and the power per unit volume of the MHD generator as follows

$$\text{power per unit volume} = \sigma V^2 \beta^2 (E/V\beta)(1-E/V\beta) \quad (8)$$

To increase power per unit volume by way of magnetic field strength increase, magnet weight must increase. Therefore, it becomes apparent that an optimum value of β_o will occur versus cost. (Note: The value of β_o determines the magnetic field strength at any point along the duct center line, since $\beta_z Z = \text{constant}$ and Z versus length is input in the performance calculation). An inlet magnetic field strength of $\beta_o = 3.0$ webers/meter² has been found to result in minimum cost (see Figure 13).

The magnetic field is assumed to be produced by a magnet of the type described in Volume VI, operating at cryogenic temperatures. The use of superconducting magnets was investigated for this application and eliminated from consideration due to prohibitive cost. Many practical problems would also have to be overcome before the superconducting magnet could be used in an application requiring the magnet size discussed here.

Duct Geometry Optimization - The inlet aspect ratio (Y/Z), the ratio of the distance between electrodes to the distance between insulating walls, has a significant effect on the size of

the magnet. Figure 14 indicates a range of optimum inlet aspect ratios between 1.4 and 1.7. The design generators are at $Y/Z = 1.7$.

The specified geometry computing procedure is based on straight insulating and electrode walls. An important factor determining the expansion process is the amount of divergence of these walls. The angle θ_y which is the angle that each electrode wall makes with the duct center line, has a considerable effect on the power density in a constant cross-stream voltage study. The equation

$$\text{cross-stream voltage} = (E/VB)(V)(B)(Y) = \text{constant}$$

shows that lower Y values (smaller θ_y) will necessitate an increase in E/VB , hence lowering the power per unit volume, (E/VB in all cases being greater than 0.5 initially). However, increasing θ_y will eventually increase cost due to constructional complications. Therefore an optimum exists (see Figure 15). The design value chosen was $\theta_y = 13$ degrees. Larger angles also tend to create problems in the diffuser with boundary layer separation, etc. The effect of θ_y for the 250-MW generator is the same as for the 1000-MW generator.

The insulating wall angle, θ_z , has little effect on the generator total cost in the range from $\theta_z = 0$ to $\theta_z = 3$ degrees, except that it influences exit Mach number with a corresponding influence on diffuser cost. The angles used in the 250-MW generator and the 1000-MW generator were chosen to allow the Mach number to remain above 1.0 during the optimization of inlet total pressure, inlet Mach number, and inlet E/VB . Consideration of reasonable diffuser design indicates that the MHD exit Mach number should not be too close to 1.0, since minor stream disturbances could move the shock upstream. The effects due to shocks in the generating portion of MHD generators are unknown. It is believed that the exit Mach numbers should be in the range of 1.3 to 1.6 to eliminate this problem. The insulating wall angle for the 1000-MW generator was selected at 2.5 degrees and 1 degree for the 250-MW generator.

Summary of Optimum Parameters - Selection of the operating parameters was based upon the discussion presented above, where the variables of the generators have been optimized without consideration for losses such as combustion inefficiencies,

combustion chamber and nozzle heat transfer, end effects, interconnection, and segmentation. The application of these losses is not expected to affect the relative position of the optimum parameters.

The parameters that appear optimum on a cost comparison for an MHD generator using JP4 and O_2 as the reactants are as follows:

	<u>1000 MW</u>	<u>250 MW</u>
oxygen/fuel ratio	2.9	2.9
seeding rate, weight % K	.5	.8
stagnation pressure ratio	16	16
inlet Mach number	3.0	3.2
magnetic field strength, inlet	3.0 $\frac{\text{webers}}{\text{meter}^2}$	3.0 $\frac{\text{webers}}{\text{meter}^2}$
electrical efficiency, inlet	.6	.6

These parameters were used to determine design generators, except where off-optimum conditions were required in order to meet generator specifications.

3. Losses in the Optimum Parameter Generator

In determining the electrical properties and the geometric configurations that exist in a generator utilizing the optimum parameters, the following losses have been considered:

- 1) Combustion losses,
- 2) Combustion chamber and nozzle heat loss,
- 3) End losses, and
- 4) Segmentation and interconnection losses.

Combustion Losses - The actual mechanisms causing inefficiencies in any particular combustion system are unknown, but a number of possible mechanisms can be imagined. It was postulated that if each of the possible mechanisms had the same quantitative effect on MHD power output, any one of them could

be used to predict the performance loss from the theoretical standpoint. Some of these possible mechanisms are:

- 1) Incomplete combustion of the reacting mixture with either fuel or oxidant unreacted,
- 2) Heat loss from the combustion chamber,
- 3) Nonuniform distribution of the fuel and the oxidant, and
- 4) Liquid droplets of unburned fuel in the exhaust gases.

Experimentation has shown that a total theoretical heat loss of one to five times the normal calculated heat rejection in the burner would result in a prediction that would approximate the actual thermodynamic conditions resulting from combustion inefficiencies (see Section III of this volume). Therefore this method was selected for the theoretical studies to effect a closer approximation of an actual MHD generator. This additional amount of heat was removed from the plasma in the performance calculation by reducing the initial total enthalpy of the reactants.

Combustion Chamber Heat Loss - For the 1000 MW generator a heat transfer of approximately 100 MW is rejected from the plasma to the coolant in the walls of the combustion chamber and nozzle. To account for combustion losses, a nominal value of 300 MW heat loss was chosen for the 1000 MW generator.

The same reasoning used in the 1000-MW generator was applied to the 250-MW generator to arrive at a 75 MW reduction in the enthalpy of the reactants. This includes the heat transfer loss and the adjustment used to approach actual conditions.

End Losses - As outlined in Section II. B.3 below, end losses may be reduced by the extension of the magnetic field and by utilization of splitter vanes. In each generator design the magnetic field was extended a distance equal to approximately one-half the spacing between splitter vanes at the downstream end. The 1000-MW generator with four separate burners has an upstream magnetic field extension of approximately one-half the spacing between the separate burner walls. The 250-MW generator has one burner, therefore the upstream magnetic field extension is approximately one-half the channel inlet height.

Approximately 5 per cent of the generated power has been allowed for end losses.

Application of Segmentation and Interconnection Losses to the Design Generators - The 1000-MW and the 250-MW generators require segmentation for the reasons discussed in Section II. B.1 below. The 1000-MW generator requires interconnection to operate at voltages in the vicinity of 30,000 volts. The 250-MW generator will not be interconnected because the cross-stream voltage (approximately 7,500 volts) meets the requirements.

The segmentation performance curves, Figures 35 to 49, permit determination of the voltage loss at each segment. The segment width has been determined for fabrication simplicity to consist of modules of two-inch increments. An attempt to arrive at maximum segmentation efficiency resulted in the choice of small conductor width, two inches, and insulator width, also two inches. An exception to these modular widths was made in the case of the magnet power electrodes where the segment width was determined by the amount of current required by the magnet (see Figure 16). The magnet power electrode was positioned at the inlet of the generator. Any other position will involve complexities when interconnecting, and magnetic field instabilities whenever upstream flow conditions fluctuate. The average cross-stream voltage loss is on the order of 5 per cent excluding the magnet electrode voltage loss which is approximately 40 per cent. These losses have been applied to the generator prior to interconnection (see Figure 17).

The 250-MW MHD generator segmentation was designed in a similar manner. Since interconnection is not required, the number of segments has been minimized to reduce the complexities involved in load connections at the slight expense of segmentation efficiency. The segmentation losses were on the order of 10 per cent (see Figures 18 and 19).

The interconnection procedure discussed in Section II. B.2 below was used to determine the interconnection range and the amperage to the load. Segmentation losses were applied to the cross-stream voltage prior to determining interconnection. Interconnection resulted in voltage variations in the interconnection region on the order of 20 per cent when the upstream load

segments were connected to the downstream load segments with equal current. The compound losses due to segmentation and interconnection amounted to 130 megawatts. Figure 17 shows representative voltages and currents at each segment. This study does not include other two-dimensional effects and assumes that current density does not change with the voltage adjustments.

Description of Generator Losses - The 1000-MW generator was designed for 1300 MW gross output. The following is a disposition of the gross power:

magnet load (including losses)	110	MW
segmentation and interconnection losses	130	
allowance for end losses, heat transfer in the generator, etc.	60	
	<hr/>	
net losses	300	MW
gross power	1300	
losses	300	
	<hr/>	
net power	1000	MW

The 250-MW generator was designed with 325 MW gross. The following is the disposition of the gross power:

magnet load (including losses)	29	MW
segmentation losses	36	
allowance for end losses, heat transfer in generator, etc.	10	
	<hr/>	
net losses	75	MW
gross power	325	
losses	75	
	<hr/>	
net power	250	MW

4. Design Generators Data Summary

The system analysis has been conducted to provide a 1000-MW MHD generator with a capacity for supplying 30,000 volts net and with 20 hours fuel supply at a minimum unit cost. A 250-MW MHD generator design is being presented as an alternative to the 1000-MW design. The primary interest in the smaller generator is its development cost. The 250-MW development cost appears to be significantly less than that of the 1000-MW generator. To provide 1000 megawatts with 30,000 volts, four 250-MW generators connected in series (7500 volts each) is recommended. It is expected that the development savings would offset the additional cost of the four smaller generators.

In the selection of design parameters for the 250-MW generator, several parameters such as total pressure and inlet Mach number optimized at values close to those values for the 1000-MW generator. In those cases where no significant cost penalty was incurred, the 250-MW parameters were selected to be the same as the 1000-MW variables. This allowed a simplification in the design effort without compromising the performance of the 250-MW generator.

Optimum Design Conditions - The application of heat loss considerations and optimum parameters resulted in an analytical MHD design generator for the 1000 MW and the 250 MW generators (see Tables 1 and 2). The magnetic field has been extended in each case as stated in Section II. A.3. Figures 20 to 31 present the thermodynamic conditions existing in each generator.

5. Cost Basis

The primary factor in obtaining an optimized generator has been cost (see Section II. A. 2). Prior to arriving at an optimum configuration, a near optimum generator was costed in detail. The component costs of this generator were used to obtain cost figures for each case presented in the optimization. It was assumed that although design detail changes could appreciably affect costs, they would not change the relative position of generators considered in the optimization. The following items were costed:

- 1) magnet,
- 2) components (related to mass flow), combustion chamber and nozzle, tanks, lines and valves, heat exchanger, and pumps,

- 3) MHD duct,
- 4) diffuser,
- 5) oxygen plant,
- 6) 20 hours (reactant and seed) mass flow, and
- 7) miscellaneous.

To utilize the costs of these items, the following scaling factors and assumptions were made:

- 1) Magnet - The magnet cost was divided into three major classes of field, housing, and capital equipment and tooling. The scaling relationship used was \$3.95 per pound of field weight, \$463 per square foot of housing surface area, and \$517,000 for capital equipment and tooling. The sum of the costs for each class is equal to the total magnet cost. For purposes of cost comparison it was assumed that these base costs would not change with total costs.
- 2) The items listed below have been assumed to vary in cost only due to changes in mass flow. It was judged that this variation with mass flow would not be a direct relationship but would vary as:

$$\text{Cost A} = \text{Cost B} \left[\frac{\text{mass flow A}}{\text{mass flow B}} \right]^{.7} \text{ or}$$

- a. Combustion chamber and nozzle cost =

$$[\$198/\text{lb mass flow}] [9.27 (\text{mass flow})^{.7}]$$
- b. Tanks, lines, and valves cost =

$$[\$348/\text{lb mass flow}] [9.27 (\text{mass flow})^{.7}]$$
- c. Heat exchanger costs =

$$[\$89/\text{lb mass flow}] [9.27 (\text{mass flow})^{.7}]$$
- d. Pump costs = $[\$753/\text{lb mass flow}] [9.27 (\text{mass flow})^{.7}]$
- e. Injector cost was assumed to be directly related to mass flow.

$$\text{Injector cost} = [\$109/\text{lb mass flow}] [\text{mass flow}]$$

The factor $9.27 (\text{mass flow})^{.7}$ was determined by the mass flow of the case costed. The exponent .7 was used to depress the direct mass flow relationship, since it was believed that

the costs of these items would not vary greatly from the base cost.

- 3) The MHD duct cost was divided into the cost of the electrode walls, and the insulator walls as follows:
MHD duct cost = $(\$894/\text{ft}^2)$ (surface area of the electrode walls) + $(\$414/\text{ft}^2)$ (surface area of the insulator walls)
- 4) The diffuser cost was assumed to vary directly with its surface area. The diffuser cost was assumed to be of the duct type described previously*.
diffuser cost = $(\$191/\text{ft}^2)$ (surface area of diffuser)
- 5) The cost of the oxygen plant is established directly by the oxygen mass flow as shown in Figures 32 and 33.
- 6) The cost of 20 hours of total mass flow is based upon the cost of each reactant and the seed

JP₄ cost = \$.015/lb

O₂ cost = \$.006/lb

seed cost = \$.2/lb

- 7) The miscellaneous items are

gas turbine drive = \$600,000

controls = \$360,000

installation = \$360,000

These items were considered constant for each case.

All costs (except the cost of propellants for 20 hours of mass flow) were increased by 20 per cent which includes the normal factors for General and Administrative and Fee.

B. Two-Dimensional Problems

All of the problems found in MHD generators are not tractable by one-dimensional or quasi one-dimensional analysis. This section treats some of these problems and presents the considerations they received. In particular, segmented electrodes, interconnection, and end effects are considered.

* Report PWA-1940, RADC TR-61-104

1. Segmented Electrodes

The one-dimensional calculations indicate the need for a strong magnetic field in a practical MHD generator. The magnetic field gives rise to the Hall effect resulting in an angle between the electric field and the current density vectors. It was shown (see Appendix 1) that the deleterious effects of Hall currents on MHD generator performance can be eliminated by precluding the flow of Hall currents. An obvious means of achieving this end is to establish an electric field in the proper direction and of the proper magnitude. In the case of the linear generator this field needs to be applied in the axial direction. The existence of an axial electric field indicates a potential gradient in the axial direction. This potential gradient cannot be supported by continuous electrodes. Therefore it is necessary to break up the electrode walls into conducting and nonconducting segments. In theory, there could be an infinite number of these segments and one-dimensional theory could properly predict the generator performance. Practically, the segments must be finite in number and reasonable in size. Since the axial electric field vanishes at the surface of the conducting part of the electrode wall, local Hall currents can flow. The effect of these local Hall currents on generator performance has been the subject of several papers.

A method for predicting segmentation effects has been formulated by the United Aircraft Research Laboratory*. The method consisted of analyzing the two-dimensional electrodynamic fields, holding the gasdynamic variables (velocity, pressure, temperature, etc) and the applied magnetic field constant. This procedure was utilized at Pratt & Whitney Aircraft and the performance was found for a range of segment geometries and for various values of the anisotropic conductivity parameter α . (α , the product of the cyclotron frequency and the collision time, is also commonly denoted by $\omega\tau$).

* Crown, J. C., Analysis of Magnetogasdynamic Generators Having Segmented Electrodes and Anisotropic Conductivity, United Aircraft Corporation Research Laboratories Report R-1852-2, February 1961

Segmentation Performance - A parametric study of segmented electrode performance has been made defining performance in terms of an actual electrical efficiency, η , and an actual required axial electric field, E_x . The actual electrical efficiency has also been related to the ratio of an effective conductivity to ideal scalar conductivity. The performance parameters are presented for various geometric variables, defined in Figure 34, and a range of the anisotropic conductivity parameter, α .

The relationship between effective conductivity and scalar conductivity can be arrived at by the following:

Consider Ohm's law for a scalar or effectively scalar (this is the case when Hall currents are precluded) conductivity plasma

$$J = \sigma VB(1 - E/VB) \quad (1)$$

Let the subscript $_o$ designate the ideal case and the subscript $_e$ the actual or effective case.

Then

$$J_o = \sigma_o VB [1 - (E/VB)_o]$$

and

$$J_e = \sigma_e VB [1 - (E/VB)_e]$$

If VB is constant and the current density is held constant then

$$\sigma_o [1 - (E/VB)_o] = \sigma_e [1 - (E/VB)_e]$$

Hence

$$\frac{\sigma_e}{\sigma_o} = \frac{1 - (E/VB)_o}{1 - (E/VB)_e} \equiv \frac{1 - \eta_o}{1 - \eta} \quad (9)$$

The significance of η_o and η in this expression will be covered later. It was found that the parameter σ_e / σ_o is a function of geometry and α , and is independent of the other independent variable η_o . Figures 35 and 36 show the values of σ_e / σ_o for a range of values of α and $c/(c+g)$ for two different values of $(c+g)/A$. From these curves two important effects can be seen. First the $\alpha = 0$ curve shows the losses incurred by the reduction in conductor area alone. Secondly, optimum values of $c/(c+g)$ can be found for each $(c+g)/A$ as a function of α . There also exists a fairly wide band of values of $c/(c+g)$ which yield performance within two per cent of the optimum. This allows quite a latitude in selecting conductor and insulator sizes, and allows the use of modular construction without further performance losses.

Figures 37 and 38 show the optimum values of $c/(c+g)$ for ranges of both $(c+g)/A$ and α . Figure 39 and 40 relate the conductivity ratio to $(c+g)/A$ and α for the optimum values of $c/(c+g)$.

From Equation (9) it can be seen that

$$\eta = \left(\frac{E}{VB} \right)_e = \left(1 - \frac{1 - \eta_o}{\sigma_e / \sigma_o} \right) \quad (10)$$

Equation (10) defines the relationship between actual electrical efficiency, η , ideal electrical efficiency, η_o , and the parameter σ_e / σ_o . The actual electrical efficiency defined by this equation represents the electrical efficiency that would exist in a segmented electrode generator designed to yield the assumed ideal electrical efficiency, and having geometric and α values to yield the appropriate σ_e / σ_o . The significance of this is that the actual generator will have the design current flow at a lowered voltage. To achieve this result the external resistance must be adjusted from the ideal value to a new value as determined by the relationship

$$R_{\text{LOAD actual}} = R_{\text{LOAD ideal}} \left(\frac{\eta}{\eta_o} \right) \quad (11)$$

Figures 41 and 42 show the resulting actual efficiencies and voltage ratios. Another important result of this study is that the axial electric field necessary to preclude Hall currents is somewhat less than predicted by one-dimensional means. Figures 43, 44, and 45 present this information in a nondimensional form. It should be noted that the required axial field decreases with both increasing $(c+g)/h$ and increasing $c/(c+g)$.

A byproduct of the computer program used to calculate the above performance data was the determination of the current flux lines. A plot of these flux lines is shown in Figure 46 for a typical geometry and value of α . Two effects can be noted in this figure, 1) the current bunches up at one end of the electrode on the bottom and at the other end of the top electrode, and 2) the current does not flow perpendicular to the fluid flow at the centerline.

The existence of an angle between the current and fluid flow at the centerline of the generator does not mean that Hall current is flowing. It is apparent from the current flux lines that there is no net current flow between adjacent electrode pairs, hence no net Hall current. This condition shows that a boundary condition assuming the current to be perpendicular to the fluid flow at the centerline would be in error if the electrodes were considered to be placed directly across the channel from each other.

A condition of perpendicular current and fluid flow can be achieved by staggering the electrodes as shown in Figure 47. The ratio of effective conductivity to ideal conductivity, σ_e / σ_o , and the

axial electric field parameter $\frac{E_x}{E_{y_o}} \left(\frac{\eta_o}{1 - \eta_o} \right)$ are presented

in Figure 48 for various electrode stagger angles. An unexpected result is that the zero stagger case is slightly more efficient than the case yielding straight across current at the centerline.

Although there may be an underlying reason why the unstaggered electrode case was more efficient, it appears that this may have been fortuitous, since two opposing effects were noted as the stagger angle changed. These effects were the increased bunching of current at the electrode-insulator junction as shown

in Figure 45, and the decreased current path in the region outside of the "current boundary layer" as the stagger angle is increased. Another effect, which could be expected, is the continuous increase in the axial electric field required to yield no net Hall current. Since the gain or loss in efficiency is so slight with stagger it was decided to use unstaggered electrodes for this study. The use of staggered electrodes would be advocated only to adjust the Hall voltage if this appeared to be necessary.

2. Interconnection

The possibility of obtaining high voltage output from MHD generators by utilizing the Hall potential has been discussed in a previous report*. The method used to accomplish this requires the series connection of a portion of the upstream end of the generator to a portion of the downstream end. This series connection is illustrated in Figure 50 where the connection yields a load voltage of 9 KV while the cross-stream voltage is only 6 KV. Note that it was not necessary to interconnect the same number of electrodes as there are output electrodes. The various conditions that must be satisfied in an interconnected generator can be illustrated by referring to Figure 50. It is obvious that the voltages on the various electrodes connected together must be equal, and that the electrode lengths must be adjusted to yield equal current flow through each.

It is convenient to construct a diagram such as the one shown in Figure 51 when considering the interconnection of a machine. Figure 51 shows the potential on the electrodes vs axial distance. The distance between the two curves represents the cross-stream potential difference between the two electrode walls. The slope of the curves represents the Hall potential gradient. It may be observed that the high potential ϕ_s on the downstream end of the top electrodes (relative to the potential at the upstream end of the bottom electrodes ϕ_o) results from the addition of cross-stream and Hall voltages. It should be noted that the ground point selected is arbitrary and that all voltages are relative.

The conditions that must be satisfied simultaneously for a machine with no interconnection losses can be written:

* Report PWA-1940, RADG TR 61-104

$$\phi_1 = \phi_3$$

$$\phi_2 = \phi_4$$

$$I_2 = I_3$$

These conditions imply that I_1 would be identical with I_4 .

The output voltages can be seen from Figure 51 to lie between the limits $\phi_5 - \phi_1$, and $\phi_4 - \phi_6$. Ideally, ($\phi_5 - \phi_1$) would equal ($\phi_4 - \phi_6$) and the voltage output would be constant for all the parallel loads. Voltage output variation could be caused by the variation with x of either the cross-stream potential difference ϕ_y or of the Hall electric field. Since these effects are somewhat independent of one another, it may in some cases be possible to partially counteract one with the other, thereby reducing the range of output voltages.

When a generator is designed to match a set of optimized thermodynamic and electrodynamic properties or parameters, it is not established a priori that the machine will interconnect satisfactorily. The following discussion will attempt to point up this problem, explain possible methods of solution, and explain the consequences of living with a part of the problem. When interconnection of the generator is determined by the conditions $\phi_1 = \phi_3$ and $\phi_2 = \phi_4$, I_2 is usually not equal to I_3 . If this fact were ignored and the generator were built with these regions interconnected as indicated by the equal potentials, the performance of the generator would be quite different than predicted. Another method for determining the zone of interconnection is illustrated in Figure 52. This method involves satisfying the condition $I_3 = I_2$ first and comparing ϕ_1, ϕ_2, ϕ_3 and ϕ_4 to see how closely the conditions $\phi_1 = \phi_3$ and $\phi_2 = \phi_4$ have been satisfied. Arbitrary amounts of current flowing through the interconnection or conversely to the load have been taken, and ϕ_1, ϕ_2, ϕ_3 , and ϕ_4 can be determined and plotted versus amount of current. As expected, $\phi_1 = \phi_3$ and $\phi_2 = \phi_4$ are not satisfied at the same current. From Figure 52 three conditions of interest can be found, these are:

$$(1) \quad \phi_1 = \phi_3, \phi_2 \neq \phi_4$$

$$(2) \quad \phi_2 = \phi_4, \phi_1 \neq \phi_3$$

$$(3) \quad |\phi_4 - \phi_2| = |\phi_1 - \phi_3|$$

One of these conditions probably represents the best interconnection condition that can be found without changing the thermodynamic or electrodynamic properties. It is an arbitrary choice as to which of these conditions is selected for any particular generator.

These above voltages should be the values after other two-dimensional losses such as segmentation, have been accounted for. Any of the above criteria would result in losses since the actual requirements outlined above must be met or two-dimensional flow will result. Prediction of these losses is very difficult and would require a complete two-dimensional solution. The approach to the problem has been to find the best condition for the interconnection of the generator and then to estimate the losses (see Section II.B.3.).

3. End Effects

The electrical loss mechanisms that may occur in the regions immediately upstream and downstream of the active MHD generators are commonly classified as end effects. Two of these mechanisms that could have large effects on generator performance will be discussed. These mechanisms, referred to here as end current loops and end current leakage, can occur independent of one another and can, therefore, be dealt with separately.

End Current Loop - The electric potentials established on the electrodes of the MHD generator result in an electric field within the generator. This field fringes beyond the limits of the active MHD generator. If the magnetic field is terminated at, or close to, the limits of the active generator, the fringing electric field may be greater than the induced electric field in the regions upstream and downstream of the active generator. If the fluid in these regions is conducting, the current in the generator would see a return circuit in parallel with the load (see Figure 53). Any current following this short circuit path would result in a degradation of generator performance. The case to be considered for a practical MHD generator using combustion gases is:

- 1) Low magnetic Reynolds number,
- 2) Tensor conductivity within the region encompassed by the magnetic field,
- 3) Finite segmented electrodes, and
- 4) Duct height of generator of the same order as the generator length.

The effectiveness of the various methods for reducing end current loops, magnetic field extension and insulating baffles placed parallel to the fluid flow but perpendicular to the current path, is also of interest. The prediction of performance loss under certain of the above conditions has been discussed in the literature ^{1, 2, 3, 4}. Reference 1 considers the case of scalar conductivity and evaluates the effectiveness of both baffles and various types of magnetic field extensions for reducing this loss. The assumption of scalar conductivity makes the application of these results to the present type generators questionable. In Reference 2, Fishman has also considered the case of scalar conductivity and the effectiveness of magnetic field extension. Again the assumption of scalar conductivity makes the conclusions only qualitatively applicable to the type of generator being considered. His main finding was that "the loss of efficiency of a generator channel due to end effects is roughly minimized if the magnetic field extends half a channel height or more beyond the electrodes".

- 1 Sutton, G. W., Electrical and Pressure Losses in a Magnetohydrodynamic Channel Due to End Current Loops, General Electric Missile and Space Vehicle Department Report, R59SD431, July 22, 1959
- 2 Fishman, F., End Effect in Magnetohydrodynamic Channel Flow, AVCO Research Note 135, June 1959

Sutton in Reference 3 analyzed end losses for the case of tensor conductivity and infinitely small electrode segments. Although the effects of finite segmentation were not determined by this study, the results will be approximately correct for a finely segmented generator. Some of the results of this study are presented, in modified form, in Figures 54 through 58. The effectiveness, ϵ , or the ratio of power actually delivered to the loads, to the ideal power to the loads is shown in Figures 54 and 55 for different types of generators. Figure 54 shows that for the constant current density generator, effectiveness is independent of ideal electric efficiency $(E/VB)_0$. It can also be seen that effectiveness increases with increase in aspect ratio (channel length)/(channel height). The latter is true for the constant potential generator. However, in this case the effectiveness is a function of ideal electric efficiency, the effectiveness increasing with decreasing ideal electric efficiency. The efficiency of these generators is shown in Figures 56 and 57. The constant current density generator is somewhat more efficient than the constant potential case. The efficiency of both increases with increasing aspect ratio while at any aspect ratio there exists an optimum ideal electrical efficiency. This optimum efficiency increases with increasing aspect ratio. In the constant current density case the loss is manifest as a reduction in cross-stream voltage and a pressure loss at each end of the generator. The ratio of the pressure loss to the ideal pressure drop is shown in Figure 58 as a function of aspect ratio for various electrical efficiencies. The end loss in the constant voltage generator results from a reduction in the amount of current flowing across the generator and into the load. The current flow within the generator is the same as under ideal conditions, however, a portion of this current is shunted back through the conducting fluid outside the generator. Sutton also shows that the efficiency of the constant current density case increases with increasing Hall effect or Hall parameter α

- 3 Sutton, G. W., End Losses in Magnetohydrodynamic Channels with Tensor Electrical Conductivity and Segmented Electrodes, R 62SD35, General Electric Missile and Space Vehicle Department, March 1962

The problem of end current loops has also been investigated by Dzung in Reference 4. Both tensor conductivity and finite length segmentation were considered. Finite segmentation is considered in an approximate manner by the use of a factor, γ , which is determined by the modified ratio of Hall electric field to cross-stream electric field with segmentation as seen in the Lagrangian coordinate system. This factor is seen to be determined by the relationship

$$\frac{\sigma_{EFF}}{\sigma_0} = \frac{1 + \gamma \beta^2}{1 + \beta^2} \quad (12)$$

or in our notation

$$\frac{\sigma_{EFF}}{\sigma_0} = \frac{1 + \gamma (\omega \tau)^2}{1 + (\omega \tau)^2} \quad (13)$$

The factor γ can be determined from the results of Section 1. above. Hence the end effect power loss can be determined. The effects of magnetic field extension and the use of insulating baffles was not investigated.

The calculation of end effects as reported in Section II.B.3. of this report is based on the conclusions and results of all these methods.

End Current Leakage - The existence of grounded electrically conducting surfaces upstream and downstream of the MHD generator, as shown in Figure 59, could result in the establishment of axial current. These currents would flow through the generator decreasing the effectiveness of the segmentation. The losses resulting from this current can be expressed as the ratio of actual power to ideal power. This factor is plotted in Figure 60 for a range of $\omega \tau$ versus the Hall current leakage factor Λ . For a derivation of the equations used to determine this plot see Volume VI of this final report. The main conclusions that can be reached by considering this curve are that power output is very strongly affected by Hall current leakage and that this detrimental effect increases with increasing Hall parameter

- 4 Dzung, L. S., Hall Effect and End Loop Losses in Magnetohydrodynamic Generators, Symposium on Magneto-Plasma-Dynamic Electrical Power Generation, Inst. of Elec. Engrs., Univ. Durham, Newcastle upon Tyne, Sept. 6-8, 1962

(α or $\omega\tau$). Care must be taken either to eliminate grounded structures within these regions or to quench the conductivity between the active MHD generator and the grounded points.

C. Conclusions

Based on the system analysis of the 1000 megawatt - rf converter system and the 250 megawatt generator the following conclusions have been reached.

1. The integration of MHD power generation with rf converters is possible with either crossed-field or linear-beam rf devices. No major areas of incompatibility exist between the MHD generator and the rf converter. The high voltage requirement of klystron rf tubes can be met either by an interconnected 1000 megawatt generator or by four series-connected 250 megawatt generators, to achieve a 1000 megawatt MHD generator - rf system.
2. The 250 megawatt power range yields an MHD generator that offers more advantages if development were to be undertaken. Some possible advantages are lower development cost and greater flexibility of operation in powerplants requiring more than one unit of this size.
3. Physical integration of the rf converter and the MHD generator to take advantage of the fringing magnetic field of the generator is not desirable. It is both more efficient and less costly to distribute electrical energy rather than rf energy. Therefore, placing the rf converters as close to the load as possible is indicated.
4. The correlation between one-dimensional prediction modified by two-dimensional losses and actual test performance must be improved. An extensive program of analysis and testing would be necessary to achieve the accuracy necessary in performance prediction. Experimental testing should evaluate whether the plasma is frozen or shifting in composition.

III. ENVIRONMENTAL TESTING

This section of Volume I presents the results of environmental tests conducted with magnetohydrodynamic generators and components of practical configuration to obtain information on plasma conductivity, structural concepts and combustion phenomena. Results from these tests were used in support of the conceptual design studies of large MHD generators reported in Volume VI "Preliminary Design of 1,000 and 250 Megawatt MHD Power Generators," of the final report on Contract AF 30(602)-2487.

Measured plasma conductivities agreed nearly exactly with the most optimistic theory for hydrogen-oxygen plasmas. For JP4 + oxygen plasmas, the measured conductivity was consistently lower than ideal theoretical by factors of 2 to 3 depending on O/F, but these values were bracketed by theory, corrected by two different methods for heat and/or combustion losses.

An aerodynamic method of reducing boundary-layer voltage drop was conceived and tested.

Problem areas involved in the construction and testing of MHD generators of practical configuration under realistic conditions were isolated, studied, and eliminated or else practical solutions were suggested for future development testing.

Methods of detecting, evaluating, and correcting for deviations from ideal conditions were developed or investigated and suggestions for future work made.

Test rigs, facilities, and procedures were developed to a high point of capability and flexibility. MHD generators utilizing nominal 5000°F plasmas and capable of operation and power generation for periods up to 1-1/2 hours* either continuously or with numerous starts were developed. Fuel, seed, and other operating condition changes including switchover from applied voltage conductivity testing to power generation could be achieved at will during a test run without shutdown.

Plasma conductivity and structural concept tests were conducted using hydrogen and hydrocarbons as fuel with oxygen to provide high temperature plasmas which were seeded with a hydrous solution of potassium nitrite, when required, to enhance the electrical conductivity. Tests with and without applied magnetic fields were run. Supersonic plasma

* limited by propellant supply of test facility

flow was provided in the test section by various area-ratio, converging-diverging nozzles so that Mach numbers could be varied from 1.7 to 2.4.

Combustion studies using hydrocarbon fuels with oxygen to provide high-temperature plasma were conducted with chamber pressures from 30 to 120 psia and oxygen-to-fuel mass ratios of 2.0 to 3.3. Supplementary work on fuel distribution, seed injection and plasma temperature measurements was accomplished. The effects on MHD generator performance of several mechanisms which contribute to combustion inefficiency were studied analytically.

The discussion of these studies is divided into three basic areas:

- 1) Plasma conductivity measurements,
- 2) Structural concepts, and
- 3) Combustion phenomena.

A. Plasma Conductivity Measurements

This section presents results of a test program conducted with devices of practical configuration to provide high-velocity plasma measurements over a broad area of variables in an effort to experimentally substantiate analytical methods of predicting plasma conductivity. Two principal areas were investigated, core flow conductivity, and boundary-layer effects. Local and overall measurements were made both with and without a magnetic field present. The detailed, theoretical discussion of plasma conductivity is included in Volume V and will not be repeated here.

Due to the extreme nature of the conditions encountered in a test MHD generator of practical configuration where velocities approach 6,500 ft/sec, plasma temperatures range from 4,500 to 5,700°R, heat flux is 100 to 300 BTU/ft² - sec*, the flow is highly reactive, and high voltages are present, very careful attention must be given to the mechanical and electrical integrity of the rig. Careful attention must also be given to the experimental techniques used to obtain information. The equations used, and experimental techniques employed based on these equations are presented in Appendix 5. Rigs, facilities, instrumentation and test procedures used are described in Appendix 6.

* This range of heat flux falls within that experienced for short periods by ICBM and satellite re-entry vehicle nose cones

Test conditions do not usually coincide with the assumptions made in the derivation of the equations used in the investigation. This includes both the equations used to theoretically predict plasma conductivity and those used as a basis for the experimental determination of plasma conductivity. If the departure from ideal conditions is not compensated for, errors may result. A discussion of the more important loss mechanisms which contribute to these errors is presented in Appendix 7. Methods of compensating for them are also presented.

Plasma conductivity may be affected by many variables. The variables experimentally studied in this investigation included:

- 1) Fuel composition,
 - a) JP4
 - b) Hydrogen
- 2) Oxidizer-to-fuel ratio,
- 3) Magnetic field strength,
- 4) Mach number (nozzle area ratio),
 - a) $A/A^* = 1.5$
 - b) $A/A^* = 2.0$
 - c) $A/A^* = 3.75$
- 5) Electrode temperature (4), and
- 6) Boundary layer thickness and temperature.

Factors to be held constant were:

- 1) MHD channel static pressure (8 psia nominally)
- 2) Part per cent seed (2%),
- 3) MHD generator channel size
(1.65 inches wide x 3.30 inches high x 7.0 inches long), and
- 4) Oxidizer (oxygen).

The test program based on these parameters is presented below in table form.

SCHEDULED TEST RUNS

Test	A/A*	Mach No. Appr.	B WEB/M ²	Fuel	O/F	PC PSIA	Electrode Boundary Layer	Electrode Temp.	Type of Measurement	
									Overall	Local
I-a-1	2.0	2.0	0	JP4	2.6	60	Thick-Cool	T ₁	Yes	Yes
I-a-2	"	"	"	"	"	"	"	T ₂	"	"
I-a-3	"	"	"	"	"	"	"	T ₃	"	"
I-a-4	"	"	"	"	"	"	"	T ₄	"	"
I-b-1	"	"	"	"	"	"	Thin-Hot	T ₁	"	"
I-b-2	"	"	"	"	"	"	"	T ₂	"	"
I-b-3	"	"	"	"	"	"	"	T ₃	"	"
I-b-4	"	"	"	"	"	"	"	T ₄	"	"
I-c-1	"	"	"	"	3.2	"	Thick-Cool	T ₂	"	"
I-c-2	"	"	"	"	"	"	"	T ₄	"	"
I-d-1	"	"	"	"	"	"	Thin-Hot	T ₂	"	"
I-d-2	"	"	"	"	"	"	"	T ₄	"	"
II-a-1	"	"	2.0	"	2.6	"	Thick-Cool	T ₁	"	-
II-a-2	"	"	"	"	"	"	"	T ₂	"	-
II-a-3	"	"	"	"	"	"	"	T ₃	"	-
II-a-4	"	"	"	"	"	"	"	T ₄	"	-
II-b-1	"	"	"	H ₂	6.0	"	"	T ₂	"	-
II-b-2	"	"	"	"	"	"	"	T ₄	"	-
III-a	"	"	0	JP4	3.2	"	"	T ₄	"	Yes
III-b	"	"	0	"	2.6	"	"	"	"	Yes
III-c	"	"	2.0	"	2.6	"	"	"	"	-
IV-a	1.5	1.7	0	"	3.2	37	"	"	"	Yes
IV-b	"	"	0	"	2.6	"	"	"	"	Yes
IV-c	"	"	2.0	"	2.6	"	"	"	"	-
V-a	3.75	2.4	0	"	3.2	145	"	"	"	Yes
V-b	"	"	0	"	2.6	"	"	"	"	Yes
V-c	"	"	2.0	"	2.6	"	"	"	"	-

1. Discussion

Tests using MHD generators of practical configuration were run to obtain data on the conductivity of plasma flowing at supersonic speeds. The plasma was obtained by the combustion of either JP4 or hydrogen with oxygen. The electrical conductivity of the plasma was increased to practical levels through the addition of an easily-ionizable alkali metal in the form of a hydrous solution of potassium nitrite (KNO_2). Some of the thermal energy of the plasma was converted to kinetic energy by expanding it through converging-diverging nozzles of various area ratios, providing flows whose nominal Mach number ranged from 1.7 to 2.4. The tests, whose duration ranged from seconds to 1 1/2 hours, were run both with and without a magnetic field imposed on the plasma. When a magnetic field was imposed, the EMF used to provide the electric fields and current flow necessary to analyze the plasma was generated within the plasma in the MHD generator channel and the power produced was measured and dissipated through suitable equipment. When no magnetic field was applied, the necessary EMF was supplied by a variable output DC generator. Overall and probing type measurements of plasma conductivity under these conditions were made using methods described in Appendix 5. Measurements of both core-flow (free-stream) and boundary-layer conductivity were included. Factors affecting the latter, such as boundary-layer thickness, boundary-layer temperature and electrode temperature were varied. Electrode composition and configuration were also varied.

Efforts to obtain various boundary-layer conditions (thick-cool and thin-hot) through the use of devices which sucked off the boundary layer ahead of the electrodes were abandoned, in favor of a much simpler method which relied, for its operation, on the interaction of flow curvature with the boundary-layer profile to provide high energy, streamwise boundary-layer vortices whose axes were parallel to the stream. The results of tests employing this method, which not only provided a convenient and simple way of altering boundary-layer conditions, but which also has direct application to the improved performance of large MHD generators, are reported and discussed.

Combustion pressures were varied, depending on nozzle area ratio, to provide static pressures in the MHD generator channel of approximately eight pounds per square inch.

The test equipment used in this project, including MHD test rigs, facilities and instrumentation is described, discussed and shown in Appendix 6. The run procedures are also outlined in Appendix 6.

A narrative of the work performed is presented. The data is summarized, discussed and compared with theory. Tables and plots showing the effect of the most powerful variables as revealed by the tests are shown. This includes two summary tables containing pertinent information on scheduled and support runs. Reports of individual test runs which include information on variables encountered during the course of the program and necessary for a proper evaluation of the data are presented. Tests run in support of the contract effort are also reported.

Negative findings, some of which led to or suggested improvements in technique and equipment are discussed. Areas revealed as needing additional study are outlined and discussed.

Special analytical and experimental work undertaken in support of the contract effort to provide a better understanding of the data, to increase the precision of the results and/or to provide new methods of obtaining the data not subject to the restrictions of other methods is described in Appendix 8.

The discussion of the results of the test program to experimentally determine plasma conductivity is presented before the section containing the record of work performed (summary tables and abbreviated run reports). The work performed section may be referred to for supplementary details on individual runs as desired.

2. Results

The most significant results are summarized first and then discussed in greater detail along with other results.

The most significant effect observed was that of O/F on boundary-layer voltage drop. It appeared to overshadow all other boundary-layer effects* except possibly those due to the applied magnetic field during power generation. When this effect was small or constant, the effect

* The only effect that approached this one in magnitude was that due to the use of ablating electrodes

of ramped electrodes used to energize the electrode boundary layer through the generation of high energy streamwise vortices, was observed. Combustor efficiency was observed to have a strong effect on the core-flow plasma conductivity as were the secondary effects of an applied magnetic field.

The test methods referred to in this section and the equations upon which they are based are presented in Appendix 5. The facilities, rigs, instrumentation, and test procedure are described in Appendix 6. The methods used to theoretically predict and analytically correct plasma conductivity are discussed in Appendix 7 and the nomenclature is defined.

Ideal conductivity refers to that which would be present at the entrance to the test section of the MHD generator, based on theoretical calculations, if no losses were present, the flow were in complete thermochemical equilibrium, were perfectly homogenous, one-dimensional and had been expanded isentropically from measured * combustion chamber conditions through a nozzle of measured ** area ratio.

Summary Tables I and II, which follow this section, provide information on test runs made to study plasma conductivity. Summary Table I contains both the proposed runs and conditions and data on the runs made to satisfy these conditions. All of the runs presented in Summary Table I were made with one type of rig ("Chase-Around" Rig) in an attempt to minimize unwanted variables. These runs are grouped according to test conditions. Summary Table II contains data on other runs presented in chronological order which illustrate the evolution of test rigs, technique, test stand flexibility, and rig performance,

* The total pressure used, P_t^* , is that calculated to be present in the nozzle based on the pressure measured at the injector face, P_c , corrected for momentum losses as described in Appendix 7

** The area ratio used is the measured geometrical area ratio divided by the discharge coefficient of the nozzle

Plasma conductivity as obtained by Method No. 3* and reported in Summary Table I is plotted in Figures 61 and 62 versus oxygen-to-fuel mass-flow ratio, O/F , to illustrate graphically the effect of this variable and to provide a basis for other plots.

Figures 61 and 62 also include data on theoretical predictions of conductivity. Theoretical adiabatic conductivity, based on measured, corrected** combustion-chamber conditions and measured nozzle area ratio is plotted for each run along with a curve of theoretical adiabatic conductivity for a nozzle-exit static pressure of 8 psia and a weight per cent seed of 2.0. Values of conductivity, based on measured, corrected combustion-chamber conditions, obtained theoretically from two methods, described in Appendix 5, which attempt to compensate for nonideal test conditions are also included. The first method adjusts plasma conductivity for measured heat-out in BTU/lb of plasma by analytically reducing the initial enthalpy of the plasma before its isentropic expansion through the nozzle. The other method includes the preceding correction plus an analytical removal of fuel to match the measured C^* through the effect of an increase in O/F on C^* ideal (see Figure 63). The predicted conductivity is then based on the lowered temperature, flow rate, and adjusted O/F . All three of the above methods assume complete thermochemical equilibrium. The dashed lines in Figures 61 and 62 are fitted to the experimental and corrected theoretical points.

The experimental data in Figure 61 exhibits some scatter, as do the points representing the corrected theoretical values. However, the lines fitted to the data show that the experimental values fall between the theoretical values determined by two different methods. There appears to be little agreement in trend since both of the theoretical corrections yield a decrease in conductivity with increasing O/F (above $O/F = 2.9$) while the experimental conductivity value increases continuously for all O/F values. In general, the experimental data is about a factor of two lower than the measured heat-out prediction but is approximately 4 to 10 times greater than the measured "heat-out-plus-unburned-fuel-correction" values (except for $O/F < 2.8$). Since

* See Appendix 5

** See Appendix 7

most of the experimental conditions noted in Appendix 5* as contributing to errors in measured values of conductivity would give values lower than those which actually exist in the plasma, the line fitted to the experimental data can be taken as a lower limit. Data from electrode three, where blooming is at a minimum, as shown by Teledeltos paper analog studies, is used.

The determination of the magnitude of these errors would take extensive additional study. The data from which Figure 62 was plotted is too sparse to provide as strong a basis for comparison as that in Figure 61. However, the same general trends can be noted.

In an effort to reduce the apparent scatter in data, the ratio of experimentally measured conductivity to theoretical conductivity (corrected for measured heat out) was calculated and is plotted in Figure 64. This should remove apparent scatter due to variations in chamber pressure, per cent seed, nozzle throat area changes, fuel and oxidizer temperature, combustor efficiency, and viscosity, etc. The results do show less scatter and indicate increasingly better agreement with the theoretical measured heat out correction as O/F increases. This increasingly better agreement may be due to one or more factors. The reduction in corrected theoretical conductivity for measured heat out increases as O/F increases. At the same time, the actual conductivity may be improving with O/F due to better performance of the injectors at higher O/F in spite of the fact that the values of η_C^* determined for the injectors (Figures 65 and 66) used did not change appreciably with O/F. The change may have been due to better seed distribution or to some other undetermined cause.

* Except for the blooming effect and possibly the effect of the boundary layer in region 3 (see Figures 216 and 217). If, as assumed, the boundary layer voltage drop is a constant in this region, then the resistance of the boundary layer, R_{bL} , is proportional to I^{-1} where I is the current through the boundary layer and the conductivity determined by Methods 1 or 3 should be accurate. If however, $R_{bL} \sim I^{-n}$ where $n > 1$, then the conductivity determined from the slope of the line in region 3 will be lower than that which exists in the core flow of the plasma

A plot of conductivity versus nozzle area ratio, A/A^* for two values of O/F (2.6 and 3.2) is presented in Figure 67. Both the curves representing experimental data and those representing the measured heat out approximation were obtained by crossplotting the curves in Figures 61 and 62. Again, the agreement with the theoretical value is within a factor of two to three but the trends do not agree over the entire range. The experimental values do, however, approach the theoretical values, as A/A^* increases. An explanation for this may be that the combustion efficiency, η_{c^*} , for most injector-combustion chamber combinations*, increases as the ratio of combustion chamber area-to-nozzle throat area, $(\frac{A_c}{A^*})$, increases. This effect could manifest itself through the effect that η_{c^*} has on plasma conductivity, that is, the effect of η_{c^*} may be superposed on the A/A^* variation. Since several different injectors were used in the course of the investigation, each having different characteristics as shown in Section III.C. and in Figures 65 and 66, some scatter and uncertainty in the comparability of the data is introduced.

It should be noted that changes apparently due to O/F may, in fact, be due to other effects induced by changes in O/F , that is, the measured conductivity variation with O/F may be, in part, a secondary effect. For instance, in nearly all cases present, the electrodes were made of silicon carbide + 25 per cent graphite. Since the free oxygen present in the very hot plasma increases with O/F , as shown in Figure 68, oxidation of the electrodes may alter the character of the boundary layer and/or the coatings commonly found on this type of electrode, effectively altering the assumptions upon which the method (No. 3) for determining plasma conductivity is based. Experimental evidence of this effect is available. For instance Figures 69, 70, 71, 72, 73, 74 and 75 show that oxidation of carbide-type electrodes does take place even during nominally fuel-rich conditions. The report on the run of 25 June, 1962 discusses this effect in connection with zirconium carbide electrodes in which case the oxidation appears to be more severe.

* An exception to this trend is the 37-element injector used exclusively for the $A/A^* = 1.5$ tests (see Section III.C. below)

These secondary O/F effects may also account for the behavior of the boundary layer electrical characteristics illustrated by the data in Figure 76 , which shows electrode boundary-layer voltage drop vs O/F. The effect of O/F on boundary-layer voltage drop appears to override all other effects except possibly those encountered during power generation runs. Neither electrode temperature nor boundary-layer thickness appears to have nearly as powerful an effect. Proof of this thesis is demonstrated in the results of the modifications made to the 61 fuel element injector (Figure 77). The modifications, involving intentional blocking of approximately half (13) of the fuel orifices in the peripheral ring of orifices and five of the oxygen orifices near the center of the injector (called 48-element injector in Figure 77), almost certainly resulted in a much leaner (high O/F) region of plasma near the walls. It would also result in a higher concentration of seed near the walls since the seed is carried into the combustion chamber by the oxygen, though this effect may have been secondary. Prior to this modification, the boundary layer voltage drop obtained with this injector (for the run of 11 Oct. 1962, Test Point I-c-2) was 160 volts. After modification, the boundary-layer voltage drop was 7.0 volts (run of 27 Nov. 1962, Test Point I-a-2). The slight variations in test conditions can be seen in Summary Table I. Note that the measured core-flow conductivity was somewhat higher for the 61-element case, indicating some change in core-flow conditions which may be attributed to the higher η_{c*} . Unfortunately, damage to these injectors precluded any further testing within the contract period.

The data presented in Figure 76 , for $A/A^* = 2.0$, suggests that phenomena associated with an applied magnetic field and power generation result in a much higher boundary-layer voltage drop than observed during applied voltage runs. This is not in disagreement with theory since it has been shown (Eq. 12 in the Appendix 5) that the effective conductivity of the plasma is a very strong function of the degree to which the Hall (axial) field (E_x) is shorted.

That is

$$\sigma_{\Lambda} = \left(\frac{\sigma'}{1 + \alpha' \Lambda} \right) \quad (14)$$

where

$$\Lambda = \frac{J_x}{J_y} = \frac{[\alpha' (uB - E_y) - E_x]}{[uB - E_y] + \alpha' E_x} \quad (14a)$$

At the surface of the electrode, the Hall field is completely shorted, so that other effects* aside, the effective plasma conductivity would be reduced by a factor of about 5 since the limit of Λ for $E_x = 0$ is α which, for our case, is approximately 2. The surface effect decreases** as the distance from the electrode increases. Naturally, the effect may be alleviated by compensating effects such as ohmic heating of the boundary layer and/or nonequilibrium ionization due to the high electric field and severe gradients of temperature and density present. Intensive additional experimental study, including development of suitable instrumentation, would be necessary to reveal exactly what is occurring. For instance, oxidation of the electrode may be accelerated by the greater ohmic heating of the boundary layer and subsequent increase in electrode temperature during power runs due to the Hall-induced increase in boundary-layer resistivity, with the result that the boundary layer is more contaminated with lower-conductivity gases. Another effect possibly contributing to the higher observed electrode boundary-layer voltage drops during power runs may be that due to the $J \times B$ body force on the plasma. This retarding force would tend to thicken the electrode boundary layer thus increasing its resistance.

Neither the effect of electrode temperature nor boundary-layer thickness was clearly defined, since in the first case measurements of electrode temperature indicate no appreciable change in spite of the changes in backup spacer configuration shown in Figure 78. In the second case, equipment was not available to quantitatively determine the effect of the ramps used on the boundary layer and no clearly defined change in electrical behavior was noted with these ramps during the scheduled contract runs. It should be noted, however, that in previous runs with higher ramps, significant changes were observed. However, the height of the ramps used in the scheduled contract tests was reduced by a factor of two in order to avoid unpredictable shock wave-induced variables in the stream and boundary layer. Much more work is necessary in both these areas, especially in the instrumentation used.

* Temperature, nonequilibrium, etc.

** UAC Report R-1852-2, Analysis of Magnetogasdynamic Generators having Segmented Electrodes and Anisotropic Conductivity, by J. C. Crown

The effect of magnetic field on plasma conductivity due to variations in collision time τ will not be discussed as this effect would be expected to be smaller than the Hall effect illustrated by Equation 14, and hence not separable in the data. In spite of good MHD generator electrical integrity, as evidenced by the interelectrode Hall voltages presented in some of the abbreviated run reports, the uncertain electrical integrity of the exhaust flame shield apparently allowed indeterminate amounts of Hall-induced current (I_x) to flow. As previously pointed out, this effect alone can cause the apparent plasma conductivity to vary by as much as a factor of five or more for the conditions under which these tests were made. Unfortunately, Hall electrodes, designed to short out the axial field and thus cause Λ to go to its limit α' , were not completed in time to use on the contract tests*. The values of measured and predicted conductivities are, however, presented in Summary Tables I and II.

The runs made using hydrogen as fuel gave conductivities that were surprisingly high (60-64 per cent of ideal-adiabatic)** in view of the low values of η_c * obtained (0.69 to 0.78)***. These runs also yielded the highest output power densities (0.82 megawatts/ft³) obtained although some of the runs made on JP4 closely approached this value. Part of the explanation for this high performance is the very low boundary-layer voltage drop (0-5 volts) observed. The equation derived from equations in Appendix 5 for generated power density at maximum power transfer, given below

$$P = \left(\frac{uBH - \delta V}{2H} \right)^2 \left(\frac{\sigma'}{1 + \alpha'(J_x/J_y)} \right) \quad (14-b)$$

illustrates the effect of boundary-layer voltage drop. It is also apparent that not only the rig but the exhaust flame shield electrical integrity must have been very good to minimize the effect of the quantity J_x/J_y in Equation (14-b).

- * α' can be checked analytically, and by measuring I_x and obtaining the ratio $J_x/J_y = \alpha'$. It can then be used in Equation 14 to correct the value of apparent conductivity measured during a power run
- ** If the allowance for the effect of segmented electrodes is included (0.6) as reported in UAC Report R-1852-2, the values agree with theoretical, ideal-adiabatic conductivity
- *** These values are doubtful due to uncertainties in the accuracy of the fuel flowmeter used in the hydrogen fueled runs. The true value was likely higher

Effect of Electrode Ramps - A series of runs was made to evaluate the effect of the electrode ramps. The rigs used in this run had a nominal channel size of 1.65 x 1.65 x 4.2 inches and a nozzle area ratio of 2.0. Each rig had 3 pairs of 1.0-inch long electrodes made of zirconium carbide. The BeO interelectrode spacers were 0.4 inch long. The insulating sidewalls were of BeO. In some runs, the center pair of electrodes for the runs had straight ramps. The ramped electrodes were 0.050 to 0.060 inch higher at the downstream end than at the upstream end.

There was considerable electrical leakage in these runs. The exit of the rig was a metal flange and this provided a shorting path. A Tele-deltos paper analog of the rig was made and tested. This test showed that, with the bare metal end plate, the ratio of apparent conductances for the three electrodes would be 1.1 : 1 : 1.65. In addition to the leakage to the end plate, leakage through the rig housing may have occurred. The resistance of the path through the rig housing was found by the method discussed in Appendix 6 under Electrical Integrity. The necessary corrections were then made to the raw data.

The run of March 7, 1962 did not have the ramps. Conductance ratios for the analog run, the raw data from this run, and the corrected data from the run are given below

	<u>G₁/G₂</u>	<u>G₂/G₂</u>	<u>G₃/G₂</u>
Analog run	1.1	1	1.65
Raw data (7 March, 1962)	1	1	1.9
Corrected data (7 March, 1962)	1	1	1.7

Note that the corrected data agrees very well with the analog data.

Two more runs were made at similar conditions except that these rigs had ramps on the center electrodes. The run conditions, leakage resistances, and voltage-current slope are tabulated below. Using the March 7th run as a base case, the change in plasma resistance for each electrode is shown in the lower table.

	P_c	O/F	%K	R_{Λ_1}	R_{Λ_2}	R_{Λ_3}	dV_t/dI_m	dV_c/dI_m	dV_c/dI_{m_3}
March 7	57.8 psia	3.13	1.87	∞	∞	32 Ω	7.7	7.7	4
* March 16	63.5	3.26	1.99	300 Ω	600 Ω	120	7.15	4.22	3.2
* March 28	74.3	3.23	2.00	337	19.7	23.6	10	3.48	2.68

* Ramps on Electrode 2

dV_t/dI_m is the slope of the averaged raw data.

R_{Λ} is leakage resistance measured after seed has been shut off (see Appendix 7)

	$\% \Delta R_{g1}$	$\% \Delta R_{g2}$	$\% \Delta R_{g3}$
	--	BASE CASE	--
March 7			
* March 16	3-16-I	-6.5%	-46%
* March 28	3-28-I	+30%	-44%

* $R_g = R_{core} + R_{boundary layer}$

The great reduction in resistance for electrode 2 and 3 for the ramped runs can easily be seen from the second table. It is significant that the effect extends downstream to the next electrode, from the ramped electrode (No. 3).

As can be seen from the above discussion, much work still remains to be done in evaluating the effect of ramps. The ramp effect seems to be greater for the rigs run in this test series. This could be because the boundary-layer thickness is less with this rig. Another possibility is that the chamfered blocks (See Appendix 6) at the entrance to the chase-around rigs could be mixing the boundary layer thoroughly before it reaches the electrodes so that the vortices are already present whether or not the electrode ramps are used.

3. Work Performed

The work performed is presented in two supplementary forms, summary tables and abbreviated run reports.

Summary Table I contains a list of the scheduled runs and conditions grouped according to variables. Each run, designated by a test point number, is followed, in the table, by the test runs that were made to satisfy it. Run data and results are included.

* See Appendix 6 for description of chase-around rig

Summary Table II contains a list of all runs made during the contract period, except those listed in Summary Table I. In this case, the runs are listed in chronological order.

The abbreviated run reports are presented in chronological order so that the evolution of techniques, equipment procedures, and refinement of data may be seen. These include the test point designation where applicable so that they may be readily referred to Summary Table I.

N.A. = Not Attempted
 P.S. = Not Successful
 N.C. = Not Calculated

Contract Test Designation	Date of Run	A/A*	B. W/M ²	Fuel	O/F	P _c Meas. (PSIA)	P _c Calc. (PSIA)	P _g Meas. (PSIA)	% K	q ₁ lbm/sec.	Electrode Boundary Layer T-C (thick-cool); T-H (thin-hot)	Electrode Face Temperature, °F	Rig Type (Chase-Around, Rotation, T-rod, etc.)	Build Number	Total Run Time This Build (Min.)	Electrode Surface Shape (Flat or Ramp)	Contact Method	Contact Pin Material	Contact Pin Plating
I-a-1	14 Dec. 62	2.0 0	0	JP 4	2.6	60.0	57.1	8.0	2.0	1.07	T-C	T ₁ 2350-2950	CAR	14406-L-2	11.3	Flat	Positive Press.	C	Rh
I-a-2	27 Nov. 62	2.0 0	0	"	2.6	60.0	55.1	2.0	2.0	0.98	"	T ₂ N.A.	"	14404-L-2	8.1	Flat	"	Ta	None
	10 Dec. 62	2.0 0	0	"	2.72	59.5	57.5	N.A.	2.14	1.00	"	N.A.	"	14404-L-2-R	27.9	Flat	"	C	Rh
I-a-3	27 Dec. 62	2.0 0	0	"	2.6	60.0	56.8	2.0	2.0	1.1-1.11	"	T ₃ N.S.	"	14394-L-5	10.4	Flat	"	Ta	None
	27 Dec. 62	2.0 0	0	"	2.72	60.8	58.8	N.A.	1.9-2.2	1.1-1.11	"	N.S.	"	14394-L-4	10.4	Flat	"	Ta	None
I-a-4	11 Oct. 62	2.0 0	0	"	2.6	60.0	56.0	2.0	2.0	1.03	"	T ₄	"	14403-L-1	15.9	Flat	"	C	None
	12 Dec. 62	2.0 0	0	Run was unsuccessful, electrode contacts failed	2.56	57.5	56.0	N.A.	2.01	1.03	"	2400-3000	"	14394-L-4	15.0	Flat	"	C	None
I-b-1	20 Dec. 62	2.0 0	0	"	2.6	60.0	61.0	N.A.	1.61	1.17	T-H	T ₁ 2400-2900	"	14406-L-3	6.3	Ramp	"	C	Rh
I-b-2	20 Dec. 62	2.0 0	0	"	2.6	60.0	57.4	2.0	2.0	1.06	"	T ₂ 2100-2700	"	14404-L-3	15.4	Ramp	"	Ta	None
I-b-3		2.0 0	0	"	2.6	60.0	8.0	2.0	2.0		"	T ₃	"						
				Not attempted - Substituted propane run for this															
I-b-4	18 Dec. 62	2.0 0	0	"	2.6	60.0	58.4	N.S.	1.99	1.07	"	T ₄ N.S.	"	14403-L-3	15.4	Ramp	"	Ta	None
	18 Dec. 62	2.0 0	0	"	2.68	60.5	58.5	N.S.	1.97	1.07	"	N.S.	"	14403-L-3	15.4	Ramp	"	Ta	None
I-c-1		2.0 0	0	JP 4	3.2	60.0	6.0	2.0	2.0		T-C	T ₂							
				Scheduled for 10 Dec. 62 missed conditions, run conditions those of I-a-2															
I-c-2	11 Oct. 62	2.0 0	0	"	3.2	60.0	58.1	10.0	1.87	1.02	"	T ₄ 2400-3000	CAR	14403-L-1	15.9	Flat	Positive Press.	C	None
	12 Dec. 62	2.0 0	0	"	3.07	57.0	54.5	N.A.	1.96	1.05	"	N.A.	"	14394-L-4	13.0	Flat	"	C	Rh
I-d-1	20 Dec. 62	2.0 0	0	"	3.2	60.0	55.5	10.5	2.04	1.07	T-H	T ₂ 2100-2700	"	14404-L-3	15.4	Ramp	"	Ta	None
	20 Dec. 62	2.0 0	0	"	3.21	57.5	60.5	N.A.	1.77	1.15	"	(Elect. Config. was for T ₂) 2400-2900	"	14406-L-3	6.3	Ramp	"	C	Rh
I-d-2	18 Dec. 62	2.0 0	0	"	3.2	60.0	56.8	N.S.	1.99	1.07	"	T ₄ N.S. - T.C. - Failed	"	14403-L-3	32.6	Ramp	"	Ta	None
II-a-1	14 Dec. 62	2.0 2.0	2.17	"	2.6	60.0	57.1	N.A.	2.17	1.07	T-C	T ₁ 2350-2550	"	14406-L-2	11.3	Flat	"	C	Rh
II-a-2	10 Dec. 62	2.0 2.0	2.17	"	2.6	60.0	55.8	N.A.	1.99	0.99	"	T ₂ N.A.	"	14404-L-2-R	27.9	Flat	"	C	Rh
II-a-3	27 Dec. 62	2.0 2.0	2.17	"	2.6	60.0	58.7	N.A.	1.9-2.2	1.1-1.11	T-C	T ₃ N.S.	"	14394-L-5	18.4	Flat	"	Ta	None
II-a-4	11 Oct. 62	2.0 2.0	2.17	"	2.6	60.0	56.6	N.A.	2.06	1.02	"	T ₄	"	14403-L-1	15.9	Flat	"	C	None
	12 Oct. 62	2.0 2.0	2.17	Tests unsuccessful, electrode contacts failed									"	14403-L-1	15.9	Flat	"	C	None
	12 Dec. 62	2.0 2.17		"	2.53	59.1	56.6	N.A.	2.06	1.02	"	2400-3000	"	14394-L-4	15.0	Flat	"	C	Rh
II-b-1	21 Dec. 62	2.0 2.0	2.17	H2	6.0	60.0	46.2	N.S.	1.5-2.9	0.6	T-H	T ₂ N.A.	"	14404-L-3	15.4	Ramp	"	Ta	None
II-b-2	17 Dec. 62	2.0 2.0	2.17	"	6.0	60.0	52.2	N.S.	1.99	0.79	T-C	T ₄ N.S.	"	14403-L-3	32.6	Ramp	"	Ta	None
	19 Dec. 62	2.0 2.0	2.17	"	6.1	54.0	52.2	N.S.	1.99	0.79	T-H	N.S.	"	14403-L-3	32.6	Ramp	"	Ta	None

SUMMARY TABLE I

Electrode Surface Shape (Flat or Ramp)	Contact Method	Contact Pin Material	Contact Pin Plating	Electrode Plating	Electrode Material	Inter-electrode Spacer Material	Sidewall Type	Sidewall Material	Injector	D_{out}/A Combustion Chamber (Btu/in ² sec)	D_{out}/A Nozzle (Btu/in ² sec)	C^* measured (ft/sec)	C^* ideal adiabatic (ft/sec)	η_{c^*} ideal adiabatic	σ meas. (Btu/dly) Elect (MHC/M)	σ meas. (Btu/dy) Elect (MHO/M)	σ ideal (MHO/M)	σ meas. Heat Removed Approx. (MHO/M)	σ Heat Removed to Match C^* (MHO/M)	σ meas. Heat Removed & Unburned Fuel to Match C^* (MHO/M)	$V_{c.c.}$ (Volts) Ideal Adiabatic	$V_{c.c.}$ (Volts) Measured	δV (Volts) (Voltage Axis Int.)	V_{Hall} - o.c. (Volts)	V_{Hall} - Max. Power (Volts)	Power (Watts)	Remarks					
Flat	Positive Press.	C	Rh	Rh	SiC 36% Graphite	BeO	BeO Bars	BeO	6	0.92	2.32	4727	5560	0.85	7.0	31	25.7	19.8	<1.0	5.77		26						Note 1				
Flat	"	Ta	None	Rh	"	"	"	"	48	N.S.	N.S.	4690	5570	0.84	5.1	N.S.	26.5	N.C.	<1.0	4.72		7						Note 2				
Flat	"	C	Rh	Rh	"	"	"	"	6	0.80	1.95	4771	5570	0.86	7.5	N.A.	75.8	19.4	<1.0	6.3		27.87										
Flat	"	Ta	None	Rh	"	"	"	"	6	0.97	2.45	4546	5562	0.82	6.9	N.S.	25.7	20.8	<1.0	1.7		47						Note 3				
Flat	"	Ta	None	Rh	"	"	"	"	6	0.7	2.45	4546	5562	0.82	10.9	N.S.	75.7	20.5	<1.0	1.7		96						Note 3				
Flat	"	C	None	Pt	"	"	"	"	61	Run failed because contact pins did not work												32								Note 4		
Flat	"	C	None	Pt	"	"	"	"	6	0.53	1.65	4704	5590	0.84	6.5	N.S.	25.7	21.8	<1.9	5.4												
Ramp	"	C	Rh	Rh	"	"	"	"	6	1.0	3.9	4542	5491	0.83	8.5	18.5	26.0	19.0	<1.0	1.5		10										
Ramp	"	Ta	None	Rh	"	"	"	"	6	0.68	2.52	4630	5580	0.83	6.9	8.5	26.8	20.7	<1.0	2.7		27						Note 5				
Not attempted																																
Ramp	"	Ta	None	Rh	"	"	"	"	6	0.73	2.0	4572	5500	0.82	7.2	N.S.	25.4	18.3	<1.0	3.5		7						Note 6				
Ramp	"	Ta	None	Rh	"	"	"	"	6	0.66	2.6	4690	5590	0.84	8.3	N.S.	25.6	19.6	<1.0	4.0		31						Note 7				
Scheduled for 10 Dec. 1962 missed conditions																																
Flat	Positive Press.	C	None	Pt	"	"	"	"	61	N.C.	N.C.	4710	5460	0.87	8.4	N.S.	26.0	N.C.	N.C.	N.C.		160										
Flat	Positive Press.	C	Rh	Rh	"	"	"	"	6	0.58	2.12	4517	5450	0.83	10.1	N.S.	26.3	22.4	<1.0	1.2		9										
Ramp	"	Ta	None	Rh	"	"	"	"	6	0.79	2.52	4440	5410	0.82	11.2	N.S.	27.3	21.0	<1.0	<1.0		10										
Ramp	"	C	Rh	Rh	"	"	"	"	9	1.24	4.14	4373	5315	0.72	11.1	19.8	22.0	19.4	<1.0	<1.0		2										
Ramp	"	Ta	None	Rh	"	"	"	"	6	0.81	2.7	4570	5360	0.85	10.8	21	23.8	18.8	1.46	1.7		3.3						Note 8				
Flat	"	C	Rh	Rh	"	"	"	"	6	0.92	2.32	4727	5560	0.85	5.6-7.0	N.A.	25.0	19.4	<1.0	5.67	372	266	71	0	70	5320						
Flat	"	C	Rh	Rh	"	"	"	"	6	0.88	1.92	4676	5570	0.84	4.6	N.A.	25.6	18.3	<1.0	3.6	363	266	68	N.A.	N.A.	4620						
Flat	"	Ta	None	Rh	"	"	BeO Bars	"	6	0.87	2.45	4546	5562	0.82	3.2	N.A.	25.6	20.3	<1.0	1.3	368	236	31	0	12+	2800						
Flat	"	C	None	Pt	"	"	"	"	61	Runs not successful																						
Flat	"	C	None	Pt	"	"	"	"	61																							
Flat	"	C	Rh	Rh	"	"	"	"	6	0.39	2.01	4829	5630	0.86	8.7	N.A.	25.5	N.C.	<1.0	8.2	360	263	131	N.C.	N.C.	3156						
Ramp	"	Ta	None	Rh	"	"	"	"	6mcdif.	1.6	5.0	4935	7196	0.69	6.85	N.A.	11.3	N.C.	N.C.	N.C.	N.C.	317	0	0	85	13,700			Note 9			
Ramp	"	Ta	None	Rh	"	"	"	"	6mcdif.	4.2	5.6	5678	7277	0.75	7.1	N.A.	11.1	N.C.	N.C.	N.C.	N.C.	435	337	9	9	170	10,000					
Ramp	"	Ta	None	Rh	"	"	"	"	6mcdif.	4.2	5.6	5678	7277	0.75	7.1	N.A.	11.1	N.C.	N.C.	N.C.	N.C.	455	337	5	5	170	10,000					

2

Contract Test Designation	Date of Run	A/A*	B W/M ²	Fuel	O/F	P _c meas. (PSIA)	P _c (Calc.) (PSIA)	P _s meas. (PSIA)	% X	W _t lbm/sec	Electrode Boundary Layer T-C:thick-cool; T-H:thin-hot	Electrode Face Temperature, °F	Rig Type (Chase-A ₁ round, Button Toolsteel, Doubly Segmented, etc.)	Build Number	Total Run Time This Build (Min.)	Electrode Surface Shape (Flat or Rump)	Contact Method	Contact Pin Material	Contact Pin Plating
III-a	11 Oct. 62 12 Dec. 62	2.0 2.0	0 0	JP 4 "	3.2 3.07	60.0 60.0	58.1 54.5	10 N.A.	2.0 1.87 1.96	1.02 1.05	T ₄ 2400-3000 N.A.	"	CAR	14403-L-1 14394-L-4	15.9 15.0	Flat Flat	Positive Press. C	None Rh	F F
III-b	11 Oct. 62 12 Dec. 62	2.0 2.0	0 0	" "	2.6 2.56	60.0 58.5	Run unsuccessful - contact pins failed 56.0	N.A.	2.0 2.01 1.03	"	T ₄ 2400-3000	"	"	14403-L-1 14394-L-4	15.5 15.0	Flat Flat	" C	None Rh	F F
III-c	11 Oct. 62 12 Dec. 62	2.0 2.0	2.0 2.17	" "	2.60 2.53	60.0 59.1	Run unsuccessful - contact pins failed 56.6	N.A.	2.0 2.06 1.02	"	T ₄ 2400-3000	"	"	14403-L-1 14394-L-4	15.9 15.0	Flat Flat	" C	None Rh	F F
IV-a	24 Oct. 62 29 Oct. 62	1.5 1.585	0 0	" "	3.2 3.23	37.0 41.3	N.A.	2.0 2.11	0.84	"	T ₄ N.A.	"	"	14404-L-1 14406-L-1	16.2 11.4	Flat Flat	" TabC	None None	F F
IV-b	24 Oct. 62 29 Oct. 62	1.5 1.585	0 0	" "	2.6 2.61	37.0 41.0	39.2 N.C.	N.A.	2.0 2.13 2.16	0.82	" N.A.	"	"	14404-L-1 14406-L-1	16.2 11.4	Flat Flat	" TabC	None None	F F
IV-c	24 Oct. 62 24 Oct. 62 26 Oct. 62	1.5 1.585 1.585	2.0 2.17 2.17	" " "	2.6 2.65 2.65 2.49	37.0 41.0 41.3 41.1	39.1 39.4 39.2	11.0 11.0 N.S.	2.0 2.06 2.09 2.04	0.82 0.82 0.85	" N.A. N.A. N.A.	"	"	14404-L-1 14404-L-1 14406-L-1	16.2 16.2 11.4	Flat Flat Flat	" TabC " TabC " TabC	None None None	F F F
V-a	7 Dec. 62	3.75 3.75	0 0	" "	3.2 3.35	145 148	146	N.A.	2.0 2.05	1.31	" N.A.	"	"	14404-L-2-R	27.9	Flat	" C	Rh	F
V-b	28 Nov. 62	3.75 3.75	0 0	JP 4 "	2.6 2.73	145 149	147	N.A.	2.0 2.11	1.26	" N.S.	"	"	14403-L-2	8.9	Flat	Positive Press. C	Rh	Rh
V-c	6, 7, 8 Nov. 62 7 Dec. 62	3.75 3.75 3.75	2.0 Unsuccessful 2.17	" JP 4 "	2.6 2.95	145 132	130	N.A.	2.0 1.99	1.16	" N.A.	"	"	14394-L-3 14404-L-2-R	14.2 27.9	Flat	" C	Rh	Rh

Note 1 Hysteresis (used desc. data only)

Note 2 Injector mod. apparently responsible for Hysteresis in V-I data

Note 3 Same firing change in slope of V-I data

Note 4 Sidewall temperature measured and equal to about 2900°F (B=O) voltage probe test made during the run

Note 5 Hysteresis in V-I data

Note 6 First of 3 cycles - ramps probably in good condition

Note 7 Third of 3 cycles - ramps probably in worn down condition

Note 8 Second of 3 cycles - ramps getting worn down

See abbreviated run reports which follow. for more detailed information

Note 9 Conditions changing

Note 10 Sidewall temperature measured and equal to about 2900°F (ReO); voltage probe test made during the run

Note 11 Break

Note 12 Chamber pressure reading failed

Note 13 Hysteresis scatter in data

Note 14 Chamber pressure reading failed

Note 15 Scatter in data

SUMMARY TABLE I (Cont'd.)

Contact Method	Contact Pin Material	Contact Pin Plating	Electrode Plating	Electrode Material	Inter-electrode Spacer Material	Sidewall Type	Sidewall Material	Injector	Quat-A Combustion Chamber (Blu/in ² /sec)	Quat-A Nozzle (Blu/in ² -sec)	Ce measured (lb/sec)	Ce Ideal adiabatic (lb/sec)	T _{ce} Ideal adiabatic	Q' meas. (dW/dly) Elec. 3 (MHO/M)	Q' meas. (dW/dly) Elec. 3 (MHO/M)	Q' Ideal (MHO/M)	Q' meas. Heat Removed Approx. (MHO/M)	Q' Heat Removed to Match Ce (MHO/M)	Q' meas. Heat Removed & Unburned Fuel to Match Ce (MHO/M)	V _{o.c.} (Volts) Ideal Adiabatic	V _{o.c.} (Volts) Measured	(Volts) (Voltage Adv Int.)	V _{Hall} - o.c. (Volts)	V _{Hall} - Max. Power (Volts)	Power (Watts)	Remarks
Active res. C	C	None Rh	Pt Rh	S ₁ C+25% Graphite	BeO	BeO Bars	BeO	61	N.C.	N.C.	4730	5460	0.87	8.4	N.S.	26.0	N.C.	N.C.	N.C.			160.0				Note 11
"	C	None Rh	Pt Rh	"	"	"	"	6	0.58	2.12	4517	5450	0.83	10.1	N.S.	26.3	22.4	1.0	1.22			9.0				Note 10
"	C	None Rh	Pt Rh	"	"	"	"	61			Run unsuccessful	- contact pins failed														
"	C	None Rh	Pt Rh	"	"	"	"	6	0.53	1.68	4704	5590	0.84	6.5	N.S.	25.7	21.8	1.9	4.4			37.0				
"	C	None Rh	Pt Rh	"	"	"	"	61			Run unsuccessful	- contact pins failed														
"	C	None Rh	Pt Rh	"	"	"	"	6	0.39	2.01	4849	5630	0.86	8.7	N.A.	25.5	N.C.	1.0	8.2	160	263	151.0	N.C.	N.C.	3156	
"	Ta&C	None None	None	S ₁ C+25% Graphite	"	"	"	37	1.2	1.4	5059	5392	0.91	8.9	N.S.	27.9	23.0	N.C.	18.8			105.0				Note 11
"	Ta&C	None None	None	ZTA Graphite	"	"	"	37	1.6	1.8	N.C.	N.C.	N.C.	11.2	N.S.	27.9	21.0	N.C.	17.9			120.0				Note 12
"	Ta&C	None None	None	S ₁ C+25% Graphite	"	"	"	37			5185	5570	0.931	4.5	N.S.	29.9	23.4	N.C.	22.4			6-50				Note 13
"	Ta&C	None None	None	ZTA Graphite	"	"	"	37	1.1	1.4	N.C.	N.C.	N.C.	6.1	N.S.	N.C.	23.4	N.C.	22.4			78-110				Note 14
"	Ta&C	None None	None	S ₁ C+25% Graphite	"	"	"	37	0.97	1.1	5200	5560	0.94	8.4		30.6	23.3	12.9	23.2	N.C.	215	75.0			3.6KW	
"	Ta&C	None None	None	Graphite	"	"	"	37	0.97	1.1	5190	5560	0.94	6.0		30.6	23.4	12.6	23.1	N.C.	231	68.0	0	N.C.	3.7KW	
"	Ta&C	None None	None	ZTA Graphite	"	"	"	37	1.00	1.2	4990	5590	0.89	11.8		30.5	21.6	3.06	19.7	N.C.	236	180.0	0	N.C.	1.2KW	Note 15
"	C	Rh	Rh	S ₁ C+25% Graphite	"	"	"	6	N.S.	N.S.	5158	5440	0.95	6.5	N.A.	16.8	12.1	7.9	9.59			24.0				
Active res. C	C	Rh	Rh	S ₁ C+25% Graphite	"	"	"	48	N.S.	N.S.	5333	5614	0.95	7.1	N.A.	17.1	11.7	6.9	12.39			11				
"	C	None Rh	Pt Rh	"	"	"	"	6	Unsuccessful	N.P.	Unsuccessful	5104	5550	0.94	4.4	N.A.	18.1	11.5	6.7	10.6	414	N.S.	N.C.	0	195	7030

2

N.A. = Not Attempted
N.C. = Not Calculated
N.S. = Not Successful

Date of Run	A/A*	B/W/MZ	Y/M	O/F	P _c Meas. (PSIA)	P _c (Calc.) (PSIA)	P _s Meas. (PSIA)	% K	W _g lbm/sec.	Ele. Inlet Boundary Layer T-C = Thin + Cool; T-H = Thin + Hot	Electrode Face Temperature, °F	Rig Type (Chase Around Bottom Toadstool, Doubly Segmented, etc)	Bolt Number	Total Run Time This Build (Min.)	Electrode Surface Shape (Flat or Ramp)	Contact Method	Contact Pin Material	Contact Pin Plating	Electrode Plating
22 Nov. 61	2.0	0	JP 4	2.43	40.0	49.9	9-13	2.0	1.03	T-C	NA	Keystone	14221-L-1	0.1	Flat	Threaded	-	None	None
19 Dec. 61	2.0	0	JP 4	2.63	75.0	NC	NA	0	NC	T-C	NA	Keystone	14221-L-2	0.5	Flat	Threaded	-	"	"
20 Dec. 61	2.0	0	JP 4	4.9	NC	NC	Run Unsuccessful				NA	Keystone	14221-L-2	0.5	Flat	Threaded	-	"	"
30 Jan. 62	2.0		JP 4	3.46	61.2	59.5	10.4-14.5	1.2	1.11	T-C	NA	Alternating Sidewalls	14279-L-1	0.1	Flat	Threaded	ZnC	"	"
16 Feb. 62	2.0	2.4	JP 4	3.49	62.3	60.2	NA	NS	1.05	T-C	NA	Keystone	14221-L-4	2.5	Flat	Threaded	Steel	"	"
	2.0	2.4		3.59	65.4	63.1	NA	NS	1.11	T-C	NA	Keystone	14221-L-4	2.5	Flat	Spring Loaded	Steel	"	"
21 Feb. 62	2.0	2.4	JP 4	3.305	61.0	61.0	NA	1.42	1.08	T-C	NA	Keystone	14221-L-4	2.9	Flat	Threaded	Steel	"	"
	2.0	2.4		2.454	60.0	58.0	NA	1.72	0.99	T-C	NA	Keystone	14221-L-4	2.8	Flat	Spring Loaded	Steel	"	"
19 Mar. 62	2.0	0	JP 4	3.41	54.6	52.9	NA	0.92	1.07	T-C	NA	Keystone	14221-L-5	1.0	Flat	Threaded	T ₈ Carburized	"	"
	2.0	0		3.5	58.0	56.0	NA	1.09	1.13	T-C	NA	Keystone	14221-L-5	1.0	Flat	Spring Loaded	"	"	"
22 Mar. 62	2.06	0	JP 4	2.72	51.0	49.5	NA	No Seeding	Not Given	T-C	1550 0.25"	Keystone	14279-L-3	1.0	Flat	Threaded	Steel	"	"
	2.06	0		2.57	61.5	59.7	NA	"	"	T-C	2100 From Face	Keystone	14279-L-3	1.0	Flat	Threaded	"	"	"
	2.06	0		3.65	59.4	57.4	NA	"	"	T-C	2200 Not Steady	Keystone	14279-L-3	1.0	Flat	Threaded	"	"	"
5 Apr. 62	2.0	2.28	JP 4	3.2	75.0			2.0	1.1	T-C	NA	Keystone	14279-L-4	0.9	Flat	Threaded	-	None	None
				2.51	55.0	53.8	14.8	2.78											
2 May 62	1.967	2.5	JP 4	3.2	75.0			2.0	1.34	T-C	NA	Keystone	14279-L-5	1.0	Flat	Threaded	T ₈	"	"
				3.53	78.0	75.5	13.0	2.06											
5 June 62	1.966		JP 4	3.2	60.0			2.0	1.11	T-C	2940-3140	Toadstool	14307-L-1	12.2	Flat	Threaded	T ₈ Carburized	"	"
				2.94	67.2	65.0	11.5	2.14											
15 June 62	1.97		JP 4	3.07	57.3	55.5	NA	0		T-C	1920-2100	Keystone	14221-L-6	1.7	Flat	Threaded	-	"	"
18 June 62	2.016	0	JP 4	3.2	60.0			2.0	1.03	T-C	NS	Keystone	14221-L-6	1.7	Flat	Threaded	-	"	"
				3.09	59.9	58.0	NA	2.19											
25 June 62	2.054		JP 4	3.2	60.0			2.0	1.05	T-H	NA	Keystone	14279-L-6	1.6	See Note 12	Pos. Press.	T ₈	"	"
				3.03	62.0	60.1	N.S.	2.32											
20 July 62		2.8	JP 4	3.2	75.0			2.0	1.2	T-H	NA	Toadstool	14307-L-2	2.9	Ramp	Integral Pin	T ₈ Carburized	"	"
				3.35	67.0	65.0	11-15	1.5-3.0											
27 July 62	2.0		JP 4	NC	67.0	NC	NA	NC	NC	T-C	NA	Toadstool	14221-L-7	0.5	Flat	Threaded	-	"	"
6 Aug. 62			Run Unsuccessful							T-H	NA	Keystone	14279-L-7	-	Ramp	-	-	"	"
13 Aug. 62	2.0		JP 4	2.4	60.0	NC	NA	2.0											
	1.99			2.65	65.2	NC	NA	2.13	1.04	T-C	N.S.	Button	14301-L-1	2.5	Flat	Threaded	T ₈ Carburized	"	"
22 Aug. 62	2.0		JP 4	NC	55.0	NC		Run Unsuccessful				Toadstool	14307-L-1					"	"

1

[illegible]

Date of Run	A/A*	B W/M ²	Fuel	O/F	P _c Meas. (PSIA)	P _c * (Calc.) (PSIA)	P _a Meas. (PSIA)	% K	W _T lbm/sec.	Electrode Boundary Layer T-C = Thick-Cool; T-H = Thin-Hot	Electrode Face Temperature, °F	Rig Type (Chamber, Nozzle, Bulb, Tapered, Doubly Nozzled, etc.)	Build Number	Total Run Time This Build (Min.)	Electrode Surface Shape (Flat or Ramp)	Contact Method	Contact Pin Material	Contact Pin Plating	Electrode Plating
25 Aug. 62	2.0		JP-4	2.6 2.77	60.0 56.0	54.2	NA	2.0 1.74	1.01	T-C	NA	CAR	14394-L-1	3.0	Flat	Pos. Press.	T ₂ Carburized	None	None
26 Sept. 62	2.03	2.13	JP-4	2.6 2.74	60.0 61.6	59.7	NA	2.0 1.99	1.01	T-C	NA	CAR	14394-L-2	8.0	Flat	"	C	Rh	Rh
15 Nov. 62	2.0	2.33	JP-4	2.6 2.68	60.0 58.6	56.8	NA	2.0 1.74-2.02	1.0	T-H	NA	CAR	14422-L-1	86.0	Ramp	"	T ₂	Rh	Rh
29 Nov. 62	3.75		JP-4		Run Unsuccessful							CAR	14403-L-2	35.0	Flat	"	C	Rh	Rh
30 Nov. 62	3.75		JP-4		Run Unsuccessful							CAR	14403-L-2	35.0	Flat	"	C	Rh	Rh
12 Dec. 62	2.0	2.0 2.17	JP-4	2.6 3.02	60.0 56.3	54.5	NA	2.0 1.81	1.04	T-C	T ₂ (2400-3000)	CAR	14394-L-4	15.0	Flat	"	C	Rh	Rh
4,7,8 Jan. 63	2.0	2.3	Propane	2.8 NS	60.0 37-42	NC	NA	2.0	NS	T-C	NA	CAR	14422-L-2	88.0	Flat	"	T ₂	None	Rh

Note 1 Fire occurred and normal shutdown proceeded.

Note 2 Supersonic Inconel nozzle used - thrust chamber liner collapsed, Nozzle collapsed

Note 3 Electrodes failed at ignition

Note 4 Seed flow rate and current not recorded, electrical integrity poor, ammeters failed, $\dot{W}_T = \dot{W}_F + \dot{W}_{O_2}$

Note 5 Rig electrical integrity poor

Note 6 No data recorded - strip charts went off scale

Note 7 Purpose of runs - electrode temperature measurement

Note 8 Rig electrical integrity poor, rig shorted out internally progressively with time, carbon deposit 0.030" thick - formed on interelectrode insulators

Note 9 Several attempts were made to fire before successful start - spark plug wire failed - rig saturated with JP-4, 4th electrode contact broken

Note 10 Electrode 1 shorted, con

Note 11 No. 4 electrode shorted

Note 12 Conductivity from deaer electrode pairs 24+-flat

Note 13 Rig electrical integrity 1

Note 14 No useful heat transfer

SUMMARY TABLE II (Cont'd.)

Conduct Pin Plating	Electrode Housing	Electrode Material	Interleaved Spacer Material	Sidewall Type	Sidewall Material	Injector	Quartz Combustion Chamber (Btu/hr-sq-c)	Quartz Nozzle (Btu/hr-sq-c)	C ₁ Measured (ft/sec)	C ₁ Ideal Adiabatic (ft/sec)	η_{CC} Ideal Adiabatic	C ₁ Meas. (W/W) Effect 3 (MHO/M)	C ₁ Meas. (W/W) Effect 3 (MHO/M)	C ₁ Ideal (MHO/M)	C ₁ Meas. Heat R. moved Approx. (MHO/M)	C ₁ Heat Removes in Match (MHO/M)	C ₁ Meas. Heat Removed & Temperature Match (MHO/M)	V _{cc} (Volts) Ideal Adiabatic	V _{cc} (Volts) Measured	V _g (Volts) (Voltage Axis Int.)	V _{gall} = 0.6 (Volts)	V _{hall} = Mod. Power (Volts)	Power (Watts)	Remarks
None	None	SIC-25%NC	BeO	BeO Bars	BeO	61	0.57	0.64	4620	5560	0.83	5.9	NA	26.3	NC	NC	NC	-	-	40.0	-	-	-	Note 15
"	Ph	SIC-25%NC	"	"	"	61	NC	NC	5070	5550	0.92	5.1	NA	25.0	NC	6.4	NC	365	280	0	190	10,100	Note 16	
Rh	Ph	SIC-25%NC	"	"	"	Modif. 48	1.3	1.9	4800	5575	0.86	9.4	NA	24.8	15.9	0.7	7.3	400	280	61.0	0	150	13,000	Note 17
Rh	Rh	"	"	"	"	48	5 Starts on this build, the 4 starts on these dates were unsuccessful																	
Rh	Rh	"	"	"	"	48																		
Rh	Rh	"	"	"	"	6	0.55	2.45	4500	5500	0.82	7.1	NA	26.0	NC	1.0	2.9x10 ⁻⁶	366	250	38.0	NC	NC	7,360	
None	Ph	"	"	"	"	Modif. 6	NS	NS	NC	NC	NC	6.7	NA	NC	NC	NC	NC	NC	245	-	0	200	14,300	Note 18

Electrode 1 shorted, conductivity data questionable

No. 4 electrode shorted out and arced to the sidewall

Conductivity from descending data. Electrode pairs 1, 2 at 1000 mg.
Electrode pairs 2&4-flat, carbon pillow blocks.

Alg electrical integrity poor

No useful heat transfer or voltage probe data obtained

Note 15 First run on chase-around configuration

Note 16 First power run on chase-around configuration, 3 cycles, combustion instabilities

Note 17 Four electrodes only; curved ramps, combustion oscillations

Note 19 Propane long duration run - propane flow meter failed, did not reach desired flow rate of 12; 1 specimen unusable

Abbreviated Run Reports *Keystone Electrode Rig (Build 14221-L-4)Run Dates - 16 Feb. 1962 and 21 Feb. 1962

Four runs were made with applied magnetic field for a total of 164 seconds firing time. Data obtained during the first two runs is not valid because of instrument failures.

During runs 3 and 4 measured conductivities were 5.7 and 3.0 mhos/m respectively.

Hall voltage could not be maintained. 3.5 KW was generated during run 3 and 1.9 KW during run 4.

Four runs were made on this rig to obtain conductivity data by Method 1 at 1.5 per cent K, and O/F's of 3.2 and 2.6. Firing duration was 36.8, 33.5, 47.8, 50.9 seconds for runs 1-4 respectively. The Keystone Electrode Rig was used with zirconium carbide electrodes for runs 1 and 2, and with PO₃ graphite electrodes for runs 3 and 4. The H₂ cooled nickel sidewalls were coated with Rokide A (Al₂O₃ base coating). The 37-element injector was used. Nozzle area ratio was 2.0. No voltage probes, thermocouples or pressure taps were installed in the rig.

After coolant flows and magnet current were adjusted, the rig was lit on JP4 and O₂. As soon as these flows were set, KNO₂ was initiated and the load sequence was started. Automatic load switching was used.

Conditions for the four runs were

<u>Run</u>	<u>Pc</u>	<u>O/F</u>	<u>%K</u>	<u>η_{c*} ideal</u>	<u>B</u>
1	62.3 psia	3.49	----	----	2.4 W/M ²
2	65.4	3.59	----	----	2.4
3	63.0	3.31	1.42	.92	2.4
4	60.0	2.45	1.62	.91	2.4

The KNO₂ flow rate was not obtained during the first two runs because of an instrument failure. The ammeters failed to function

* Descriptions of test procedures, apparatus, instrumentation and facilities are given in Appendices 5, 6 and 7

properly during the first two runs, so no conductivity data was obtained. However, voltmeter readings indicated about 8 KW was generated during the first run.

Runs three and four were more successful. Plots of voltage vs current for electrode 3 are shown in Figures 79 and 80. Conductivities calculated from these data by method 1 are: 5.9 and 3.0 mhos/meter. Boundary layer voltage drop was 60 and 93 volts for runs 3 and 4 respectively.

Theoretical conductivities for these runs were

	<u>Run 3</u>	<u>Run 4</u>	
σ'	22.5	23.0	mhos/meter
σ_t'	3.5	3.7	" "

for ideal adiabatic combustion and expansion. Hall voltages were low and erratic indicating poor electrical insulation between adjacent electrodes, so measured conductivity should be compared to σ_t .

Measured conductivity was higher than σ_t for run 3. This may be because the Hall field was not completely shorted out.

Maximum power generated was 3.5 and 1.9 KW for runs 3 and 4 respectively.

Runs 1 and 2 were unsuccessful because of instrumentation failures.

Runs 3 and 4 were successful, but electrical insulation in the rig was not good enough to sustain the Hall voltage. Measured conductivities were representative of σ_t . Considerable losses were incurred due to the boundary layer voltage drop which was 60 volts for run 3 and 93 volts for run 4.

Keystone Electrode Rig (Build 14279-L-4)

Run Date 5 Apr. 1962

A run of 46 seconds duration was made to obtain conductivity data by Method 1.

The rig shorted out so that only about 3 KW of power was generated, and the data are not valid for determination of conductivity.

The purpose of this test was to demonstrate power generation with ZTA graphite electrodes, and obtain conductivity data with an applied magnetic field. The keystone electrode rig was used, with ZTA graphite electrodes and BeO interelectrode insulators. A more complete description is given in the section on structural concepts. The 37-element injector and nozzle with an area ratio of 2.0 were used. No voltage probes or electrode thermocouples were installed.

The rig was fired on JP4 and O₂ without warmup, after coolant flows and magnet current had been set. Seed flow was initiated as soon as JP4 and O₂ flows were stabilized. Automatic load sequence was then started. As soon as the load cycle was completed the rig was shut down. Run duration was 46 seconds.

Target and actual test conditions were

	<u>Pc</u> <u>psia</u>	<u>O/F</u>	<u>%K</u>	<u>C*</u> <u>ft/sec</u>	<u>C* ideal-</u> <u>adiabatic</u> <u>ft/sec</u>	<u>η_c*</u>	<u>B</u> <u>w/m²</u>
Target	75	3.2	2.0				2.0
Actual	55.5	2.51	2.78	4360	5540	0.79	2.28

The large discrepancy was due to a faulty oxygen regulator.

Voltage - current data for electrode 3 is shown in Figure 81. Conductivity calculated from this data by Method 1 is 11 mhos/meter. Total power output was 3 KW. Open circuit voltage was 324 volts initially, and decreased to 200 volts before the first load was applied. Hall voltage was not zero at open circuit, and decreased as the run progressed. These two factors indicate that the rig was shorting internally in both the transverse and axial directions, as the run progressed.

If this is the case, conductivity and boundary-layer voltage drop as determined by Method 1 have very little meaning. For this reason no further analysis of this data was made. The mechanical performance of the rig is discussed in the section on structural concepts. However, it is worth mention here that a carbon deposit about 0.030-inch thick formed on the interelectrode insulators which was probably responsible for the axial short circuit.

The rig shorted out progressively with time, causing power output to be low, and voltage-current data to be useless for determination of conductivity.

Keystone-Electrode Rig (Build 14279-L-6)Run Date - 25 June 1962

A run of 98 seconds duration made on the keystone electrode rig is described and discussed. The scalar conductivity, σ_o , was obtained by Methods 3 and 5. Method 5 indicates a conductivity approximately twice that of Method 3 (14.13 mhos/meter vs 7.84 mhos/meter). The difference may be explained from the fact a ZrO_2 coating built up on the ZrC electrodes during the run and suggests that Method 5 is the better of the two techniques available, at least for the case when coating and contact resistance of indeterminate value are present.

The object of this test was to measure plasma conductivity by two different methods in the absence of a magnetic field, and compare the results.

The keystone-electrode rig with 75 per cent ZrC - 25 per cent graphite electrodes was used. Electrode pairs 1, 3, and 5 were the ramp-step-type, while pairs 2 and 4 were the flat type. The spacers between the electrodes were of boron nitride. The sidewalls were of stainless steel coated with Rokide A which was stoned smooth. The right sidewall (looking upstream) had four SiC voltage probes installed in it. The 61 fuel-element injector was used. The nozzle area ratio was 2.05. The exit was an uncooled steel flange.

The standard procedure for battery-applied voltage was followed.

The conditions for a stabilized point were

	P_c , psia		O/F	$\%K$	$\dot{W}_t$, lbs/sec	C^* , Meas		C^* , ideal	γ_{c^*}
	measured					ft/sec	ft/sec	adiabatic	ideal
Nominal	60.0	3.2	2.0	---	---	---	---	---	---
Measured	62.0	3.03	2.32	1.05	4857	5452	0.89		

No combustion instabilities were noted during the run.

Figures 82 and 83 show probe voltage versus distance from the cathode for various applied voltages. The probe readings were corrected by algebraically adding the zero applied voltage reading to the potential measured by the probes. The figures indicate a variation in the scalar conductivity across the stream. The core conductivity is higher as expected. The values, however, are subject to uncertainty because the data is obtained by subtracting two relatively large numbers (i.e. the

potential at each point) and small discrepancies in measuring the potential can lead to unreliable values of conductivity. The values of σ_0 obtained by using the slope of the line obtained by a least squares fit to the data are

Volts									
Applied	23	17	11.5	6	0	6	12	17.2	23.2
Conductivity									
mhos/m	10.67	14.13	16.05	10.83	0	11.70	11.47	10.88	13.01
	descending voltage					ascending voltage			

Figure 84 shows voltage versus current density for electrode 3. The other four electrodes exhibit similar trends.

The values of σ_0 obtained by Method 3 are given below

	1	2	3	4	5
<u>Electrode</u>	<u>mhos/m</u>	<u>mhos/m</u>	<u>mhos/m</u>	<u>mhos/m</u>	<u>mhos/m</u>
For descend-					
ing voltage	7.84	6.30	7.41	7.47	10.90
For ascend-					
ing voltage	5.89	4.42	5.32	4.47	6.45

Predicted adiabatic conductivity is 26.8 mhos/meter. Note that in every case the measured conductivity is higher for descending voltage than for ascending voltage. The coolant temperature data appeared to be invalid, hence there was no combustion chamber and nozzle heat flux data for this run.

An examination of the rig after teardown showed the ZrO_2 buildup on the ZrC electrodes was heavier on the downstream end (see Figure 70). An explanation may be that the boundary layer is getting hotter and more reactive as it is warmed up due to the turbulence and consequent mixing with the hot core gas and to ohmic heating when current is passed through the boundary layer. There has been some question as to when this coating forms on the electrodes. Several of the electrodes were found cracked and pushed into the duct after the run, (See Figure 71). There was no rounding of the edges on the breaks and the split electrodes were pushed so far into the duct that the pieces would have been swept out if gas had been flowing. Thus the cracking occurred at or near shutdown and not during the run. If the coating had occurred because air rushed in and oxidized the hot carbide electrodes, the interfaces of the cracks would also have been coated. An inspection of Figure 72 reveals no coating on the interfaces, hence the coating occurs during the run.

Button Electrode Rig* (Build 14301-L-1)Run Date: 13 Aug. 1962

A conductivity run using Methods 3 and 5 is described. The button electrode rig was used. The measured conductivity between the buttons was significantly higher than the conductivity between the electrode plate, indicating high resistance in the corner and sidewall boundary layers and/or a marked conductivity profile across the core flow.

An applied-voltage conductivity run was made with a button electrode configuration. The purpose of this configuration was to obtain conductivity data by Methods 3 and 5, minimizing the effects of fringing and leakage to the sidewalls. For this build, the 61-element injector was used. The measured nozzle area ratio was 1.99. The rig was equipped with a thermocouple behind one of the electrode plates.

The standard run procedure was used. The total firing duration was approximately 150 seconds. One applied voltage cycle was executed, lasting for less than 15 seconds. During the time when voltage and current data was collected the combustion conditions were constant, and were

	<u>Actual</u>	<u>Nominal</u>
P_c (measured chamber pressure - psia)	65.2	60
W_t (total weight flow of propellants - lbs/sec)	1.04	---
O/F (oxygen - fuel ratio)	2.65	2.6
%K (per cent potassium)	2.13	2.0
C^* (measured - ft/sec)	5223	---
C^* (ideal adiabatic - ft/sec)	5500	---
η_{c^*} (ideal-adiabatic)	0.932	---
B W/m^2	0	0

Plots of applied voltage versus current for the middle and downstream buttons and for the electrode plate are shown in Figures 85, 86, and 87 respectively. No useful data was collected from the upstream button because of a short circuit. The conductivities calculated by Method 3 are given

middle button	18.5 mhos/m
downstream button	18.0
electrode plate	11.0

* See Appendix 6 for description of this rig

The theoretical conductivity, corrected for ∇C^* by the heat removal method to match C^* , was 8.1 mhos/m; the ideal-adiabatic theoretical conductivity was 25 mhos/m. The voltage axis intercepts indicating boundary-layer voltage drop were

middle button	47.7 volts
downstream button	42.6
electrode plate	37.1

No useful heat transfer or voltage probe data was obtained. Figure 73 is a photograph of the top electrode after the run. The buttons were not cracked at all, but both electrode plates were cracked in several places (Figure 69). It is believed that the cracking occurred late in the run or while the rig was cooling, for the reasons discussed in the June 25, 1962 run report. The electrodes were coated with zirconium oxide. The coating appeared to be heavier on the buttons than on the plate. A temperature difference between button and plate may have been the cause but no temperature data was available. Another possible explanation for the heavier coating on the buttons is that the buttons may have been slightly recessed during the run, thus subjecting the coating to less erosion.

The measured conductivities were 18.5 mhos/m and 18.0 mhos/m for the middle and downstream buttons respectively, and 11.0 mhos/m for the plate. This indicates that the conductivity in the center of the stream may be much higher than the conductivity along the edge where the plasma is affected by both temperature and velocity boundary layers..

Chase-Around Rig (Build Number 14403-L-1)
Test Points IIIa, IIIb, IIIc - Run Date 11 Oct. 1962

One firing of approximately 13.9 minutes duration was made. Two applied-voltage cycles with no magnetic field were made, and one load cycle with magnetic field was made to obtain data for Test Points IIIa, b, and c. Data obtained during the first cycle gave a conductivity of 8.4 mhos/meter. Loss of electrical continuity during the second cycle resulted in unusable data for the remainder of the run.

One firing was made to measure conductivity by Methods 1, 3 and 5, Test Points IIIa, b, and c in 3 consecutive cycles. The basic rig was used with the following modifications or conditions:

- 1) T₄ electrode configuration,
- 2) Platinum plating on the backs of the electrodes,
- 3) Carbon contact pins,
- 4) Nominal nozzle area ratio was 2.0,
- 5) 61-element injector, and
- 6) Flat electrodes.

Instrumentation was standard except:

- 1) Voltage probe leads were tungsten wire brazed to nickel plating on the probes, and
- 2) Static pressure taps were connected at the 2nd and 14th sidewall blocks and 1 each in the upper and lower corner spring chambers

Nominal and actual conditions for Test Point IIIa were

	<u>O/F</u>	<u>%K</u>	<u>W_t</u> lb/sec	<u>P_c</u> psia	<u>C*</u> ft/sec	<u>C* Ideal</u> ft/sec	<u>η_{C*}</u>	<u>B</u> W/m ²
Nominal	3.20	2.00		60.0			-	0
Actual	3.07	1.87	1.02	60.0	4730	5460	0.866	0

The static pressure tap in the upper spring chamber failed to give data. The data for the other 3 taps is plotted on Figure 88. A pressure rise of about 2.3 psi is apparent. This is an indication that there may be oblique shocks present in the channels since the theoretical pressure rise is an order of magnitude less.

There were significant deposits of silica on the BeO electrode wall, insulators, (Figure 74). The electrode contact pins seized in their insulating tubes during the early part of the second cycle of this run, resulting in unusable electrical data for the remainder of the run. Voltage probe data was not obtained because of failure of the brazed joint between the probes and their respective leads.

A plot of applied voltage vs current for the third electrode pair, Figure 89, shows δV to be 160 volts. This was constant within 20 volts for all electrodes, and the data shown is representative of that for the other electrodes. Conductivity determined by Method 3 for all 5 electrode pairs is shown below

Electrode	1	2	3	4	5
Conductivity, mhos/m	8.0	8.0	8.4	8.1	11.2

The relatively high conductivity indicated for the 5th electrode pairs may be due to the scatter in the data above the knee of the voltage-current curve or to excessive blooming effects. Figure 90 is a plot of temperature measured at the back surfaces of 4 electrodes during the warmup phase of the run. Temperature increase with time is seen to be quite unsteady, and still increasing when the thermocouples were disconnected.

The calculated ideal adiabatic conductivity is 26.0 mhos/meter.

The second and third cycles, which were to have supplied data for points IIIb and c were unsuccessful because of binding between the contact pins and insulating tubes which resulted in loss of electrical continuity. The warmup procedure imposes quite high initial rates of heating in the electrodes, and probably also in the sidewalls.

Brazed joints are not satisfactory for the voltage probe leads.

Chase-Around Rig (Rig No. 14403-L-1)
Test Point IIIc - Run Date 12 Oct. 1962

A run of 154 seconds duration was made to obtain data for test point IIIc, but was unsuccessful because some of the electrode contact pins seized in their insulators, breaking electrical continuity in three load circuits.

This run was a second attempt to obtain test point IIIc. The first, on 11 Oct. 1962, was unsuccessful. All apparatus and instrumentation was identical to that of the run of 11 Oct. 1962 except that the voltage probes and thermocouples were inoperative. Repairs to the electrode contacts were made.

Standard warmup and test procedure for a run with magnetic field was used, except that the run was terminated at 154 seconds because of poor performance. No useful conductivity data was obtained from this run because the electrode contact pins again seized in the insulator, breaking circuit continuity on 3 electrode pairs. The photographs, Figure 75 and 91, show that the condition of each of the electrode walls and sidewalls is very good after a total run time of 13.9 minutes with two starts. Note the SiO_2 deposits downstream of the SiC electrodes. Apparently the electrodes oxidize during a run because of the free, unreacted, oxygen present at the high temperatures, in spite of the fuel-rich mixture.

Mechanical performance of the rig was good except for the electrode contact pins. The electrodes oxidized during the run to form SiO_2 .

Chase-Around Rig (Build 14404-L-1)
Test Point IV-c - Run Date 24 Oct. 1962

A conductivity run of 6.9 minutes duration was made. Two load cycles were made giving measured conductivities of 8.35 and 5.95 mhos/meter. Power generation was 3.56 and 3.67 KW respectively.

The purpose of this run was to obtain data for Test Point IV-c by Method 1. The basic rig configuration was used with flat electrodes. Electrode walls were built in the T_4 configuration. The contact pins for the first and second electrode pairs were tantalum while the other pins were carbon. There were no electrode thermocouples. The area ratio of the nozzle was 1.58. The 37-element injector was used.

Standard test procedure was followed. The actual and nominal conditions were

	<u>1st Load Cycle</u>	<u>2nd Load Cycle</u>	<u>Target</u>
	Actual	Actual	
P_c (psia)	41.0	41.3	40.0
$\dot{W}_T^c$ (lbs/sec)	0.815	0.821	---
O/F	2.65	2.65	2.6
%K	2.06	2.09	2.0
C^* (measured - ft/sec)	5200	5190	---
C^* (ideal-adiabatic -ft/sec)	5560	5560	---
ηC^* (ideal - adiabatic	0.936	0.934	---
B (webers/m ²)	2.17	2.17	2.0
Maximum total power generated			
(kilowatts)	3.56	3.67	---

During this run the propellant flows and seed flows remained fairly constant. However, some combustion surges were observed. These combustion surges caused the power generated to be erratic. Figure 92 shows a plot of total power versus time for the two load cycles. At open circuit the Hall voltmeters read zero, indicating no leakage in the transverse direction.

In Figure 93 the static pressure is plotted as a function of time and distance down the MHD channel. The measured value of the pressure gradient is much larger than the predicted gradient. Figures 94 and

95 are plots of terminal voltage versus output current for the third electrode pair at various points during the load sequence. The apparent conductivities calculated by Method 1 are 8.35 mhos/m and 5.95 mhos/m for the first and second load cycles respectively.

The calculated theoretical conductivities were

	<u>1st Load Sequence</u>	<u>2nd Load Sequence</u>
Ideal	30.6 mhos/m	30.6 mhos/m
Measured heat removed approx.	23.3	23.4
Heat removal to match C* approximation	12.9	12.6
Unburned fuel plus measured heat-out to match C* approximation	23.2	23.1

The measured conductivities were 28 to 30 per cent lower during the second load sequence. It is possible that the contact resistance changed after or during the first load sequence. It was also noted that open circuit voltage increased 15 volts. This is an indication that flow conditions may have changed, causing the drop in conductivity.

Boundary-layer voltage drops for electrodes 2, 3 and 4 were

<u>Electrode</u>	<u>1st Load Cycle</u>	<u>2nd Load Cycle</u>
2	79.6 volts	75.2 volts
3	76.2	68.9
4	70.1	70.3

The heat flux from the nozzle was 1.1 Btu/in²-sec while the heat flux from the combustion chamber was about 0.97 Btu/in²-sec during the run.

This run lasted for a total of 413 seconds. The rig was in excellent condition after the run and another run was made shortly thereafter without the applied magnetic field.

Chase-Around Rig (Build No. 14404-L-1)

Test Points IV-a and IV-b - Run Date 24 Oct. 1962

A run of 9.3 minutes duration was made to measure the plasma conductivity by Method 3. The measured conductivity was 8.9 mhos/meter

for Test Point IV-a. Test Point IV-b was unsuccessful because of excessive scatter in the data.

The purpose of this test was to measure conductivity by Method 3 for test points IV-a and IV-b.

The 37-element injector was used. The nozzle area ratio was 1.585. The rig did not have electrode thermocouples. The rig had been run for 413 seconds prior to this run.

Standard test procedure was used. Two applied voltage cycles were made: one at an O/F of 2.6 and another at an O/F of 3.2. The fuel and oxygen flows were quite steady, but the KNO_2 flow drifted slightly and had to be reset several times.

The nominal and actual conditions were

	<u>Cycle 1 (IV-b)</u>		<u>Cycle 2 (IV-a)</u>	
	<u>Actual</u>	<u>Nom.</u>	<u>Actual</u>	<u>Nom.</u>
P_c , psia	41.0	40.0	41.3	40
$\dot{W}_T$ lbs/sec	0.818	--	0.844	--
O/F	2.60	2.6	3.23	3.2
%K	2.13	2.0	2.11	2.0
C* Measured-ft/sec	5185	--	5059	--
C* ideal - adiabatic - ft/sec	5570	--	5392	--
γC^* ideal - adiabatic	0.931	--	0.908	--
B (W/m^2)	0	0	0	0

The plots of applied voltage versus current for the middle electrode pair are shown in Figures 96 and 97 for both ascending voltage and descending voltage. The least-squares fits to the data points are also shown. There is considerable scatter in the data, especially for the cycle at an O/F of 2.6. It may be that there was arcing or poor electrical contact at the terminal pins.

The apparent conductivities for electrode 3 calculated by using the slopes of the least squares straight lines are shown below

	<u>Cycle 1</u>	<u>Cycle 2</u>
Ascending applied voltage	4.70 mhos/m	8.9 mhos/m
Descending applied voltage	4.29 "	8.9 "

The boundary-layer voltage drop ranged between 6.0 volts and 48 volts for the first cycle and was 105 volts for the second cycle. Because of excessive scatter the data from Cycle 1 is unreliable.

The heat fluxes from the combustion chamber and nozzle were

	<u>Combustion Chamber</u>	<u>Nozzle</u>
Cycle 1 (O/F = 2.6)	0.86 Btu/in ² sec	1.3 Btu/in ² sec
Cycle 2 (O/F = 3.2)	1.2	1.4

Figure 98 shows plots of static pressure versus time and pressure versus distance down the MHD channel. The chamber pressure was unstable during this run and this instability is reflected by the static pressures. The insets of Figure 98 show the pressure gradient down the rig for oxygen-fuel ratios of 2.6 and 3.2 when the flows and chamber pressure were fairly constant. The drop in pressure near the rig entrance may be due to incomplete expansion of the plasma as it expands from the nozzle throat. No voltage probe data was obtained because of high frequency hash.

Cycle 1 was unsuccessful. Cycle 2 resulted in data for Test Point IV-a. This rig ran for a total of 973 seconds during two separate runs. The sidewalls showed some cracking and erosion but the electrodes and spacers were in excellent condition.

Chase-Around Rig (Build 14406-L-1)
Test Point IV-C - Run Date 26 Oct. 1962

A run of 250 seconds was made. Conductivity measured by Method 1 was 11.8 mhos/meter. Some comparisons are made with the performance of an identical rig equipped with SiC - 25 per cent graphite electrodes which indicate that the latter is a superior electrode material. 1.2 KW was generated.

The purpose of this run was to obtain data for Test Point IV-C in a rig equipped with ZTA graphite electrodes and compare its performance to a rig equipped with SiC-25 per cent graphite electrodes. The rig used for this run was the same as for the runs of 24 October 1962 except that flat ZTA graphite electrodes were used. The 37-element injector was used.

The test procedure for this run was standard. The target and actual

conditions for this run were

<u>Conditions</u>	<u>Actual</u>	<u>Target</u>
P_c - psia	41.1	40
$\dot{W}_t$ - lbs/sec	0.852	--
O/F	2.49	2.6
%K	2.04	2.0
c^* measured - ft/sec	4990	--
c^* ideal-adiabatic - ft/sec	5590	--
η_{c^*} ideal-adiabatic	0.893	--
B - webers/m ²	2.17	2.0
Maximum total power generated (kilowatts)	1.19	--

Figure 99 is a plot of total power generated versus time. Power output was rather erratic which is probably due to combustion surges. Figure 100 is a plot of terminal voltage versus output current for the middle electrode pair. The apparent plasma conductivity calculated by Method 1 was 11.8 mhos/m. This is significantly higher than the 8.35 and 5.95 mhos/m calculated for the run of 24 October 1962 with SiC - 25 per cent graphite electrodes.

The large amount of scatter in the data makes the 11.8 mhos/m measurement somewhat doubtful.

The calculated theoretical conductivities are

Ideal	30.5 mhos/m
Measured heat removed approximation	21.6
Heat removal to match c^* approximation	3.06
Unburned fuel plus measured heat-out to match c^* approximation	19.7

Boundary-layer voltage drops for electrodes 2, 3, and 4 were 178, 180 and 180 volts respectively. The boundary-layer voltage drop for the ZTA electrodes is more than twice as high as for the SiC electrodes. A possible reason for the high boundary-layer voltage drop with graphite electrodes is ablation of the graphite. Ablation keeps the electrode boundary layer cool and makes the resistivity of the boundary layer quite high. The heat flux from the nozzle and combustion chamber was 1.2 Btu/in² -sec and 1.0 Btu/in² -sec, respectively.

No useful data was obtained from the four static pressure taps.

The results of this run indicate that SiC-25 per cent graphite is superior to ZTA graphite as electrode material. The rig equipped with ZTA graphite electrodes had a much higher boundary-layer voltage drop, and lower output. The apparent conductivity was higher for ZTA graphite electrodes, but the data showed considerable scatter. The run lasted for 250 seconds, including hydrogen-oxygen warmup and cool-down. The electrodes showed some signs of erosion. The data from this run does not indicate any appreciable difference between the carbon and tantalum contact pins.

Chase-Around Rig (Build No. 14406-L-1) ZTA Graphite Electrodes
Test Points IV-a and IV-b - Run Date 29 Oct. 1962

A conductivity run of 7.3 minutes duration without an applied magnetic field was run in a rig equipped with ZTA graphite electrodes to obtain data for Test Points IV-a and IV-b and to compare the results of this run with a similar run made on 24 October 1962 with a rig equipped with SiC - 25 per cent graphite electrodes. There was considerable scatter in the voltage-current data and the conductivity data is not very reliable. The conductivity measured for the graphite electrode appeared higher, which may have been due to the absence of the high resistivity oxide coating commonly noted on the SiC electrodes. The electrode boundary-layer voltage drop was considerably higher for the rig equipped with the graphite electrodes, which is probably due to the ablation products in the boundary layer, and the cooler electrode temperature. Silicon carbide was far superior to graphite in its ability to withstand the high temperature and erosive action of the MHD plasma. The graphite electrodes were almost completely eroded away after 11.5 minutes and two startups, while the silicon carbide electrodes showed no appreciable wear after 16 minutes and two firings.

The purpose was to obtain data for Test Points IV-a and IV-b by Method 3 and to compare the performance of ZTA graphite with SiC - 25 per cent graphite as electrode material. The rig for this run was the same one run on 26 October for 250 seconds. The 37-element injector was used. The standard run procedure was followed. Two applied voltage cycles were executed. The oxygen and fuel flow rates were fairly steady but the KNO_2 flow had to be reset several times because it had a tendency to drift lower as the pressure in the tank decreased. The chamber pressure transducer failed, consequently no reliable chamber pressure data was available and accurate calculations of C^* , η_{C^*} , and theoretical conductivity cannot be made. The chamber pressure read from a gage in the control room was about 40 psia. The data indepen-

ent of chamber pressure is shown in the following table for two applied voltage cycles.

	<u>Cycle 1 IV-b</u>		<u>Cycle 2 IV-a</u>	
	Actual	Target	Actual	Target
$\dot{W}_T$ - lbs/sec	.812-.818	---	.838-.847	---
O/F	2.56-2.67	2.6	3.11-3.26	3.2
%K	2.15-2.16	2.0	2.09-2.15	2.0
B - W/m ²	0	0	0	0

Figures 101 and 102 are plots of applied voltage versus current for the middle electrode pair.

The apparent conductivities and boundary-layer voltage drops calculated by Method 3 from the high current data of Figures 101 and 102 are shown below

	<u>Cycle 1</u>	<u>Cycle 2</u>
Ascending applied voltage	6.1 mhos/m	11.2 mhos/m
Descending applied voltage	5.0	---
Boundary-layer voltage drop	75 - 100	120 volts

The heat flux from the combustion chamber and nozzle was

	<u>Combustion Chamber</u>	<u>Nozzle</u>
Cycle 1 (O/F = 2.6)	1.1 Btu/sec-in ²	1.4 Btu/sec-in ²
Cycle 2 (O/F = 3.2)	1.6	1.8

Total time for this run was 7.3 minutes, for a total run time on the build of 11.6 minutes.

The SiC - 25 per cent graphite electrodes are superior to the ZTA graphite in durability as shown by Figures 103 and 104. The values of the measured apparent conductivity are somewhat higher for the graphite. Electrode boundary-layer voltage drop is significantly higher for the graphite electrodes. The fact that the electrode boundary-layer voltage drop is higher for graphite electrodes may be due to cooling and contamination of the boundary layer by ablation and erosion. The higher measured conductivity for the graphite electrodes may be due to better contact resistance and the lack of a high resistance oxide coating commonly noted on the SiO₂. In both rigs the sidewalls held

up very well. There was some cracking and a slight amount of erosion of the BeO near the exit. The BeO spacers were in excellent condition.

Chase-Around Rig (Build No. 14404-L-2)

Test I-a-2 - Run Date 27 Nov. 1962

A test of 8.3 minutes duration was made to determine conductivity for test point I-a-2. A conductivity of 5.1 mhos/m was measured at the center electrode pair. The voltage-current characteristic for the center electrode pair was essentially a straight line with a voltage intercept of 7 volts. An injector modification apparently was responsible for this unusually low boundary-layer voltage drop.

This test was made to determine the plasma conductivity using Methods 3 and 5 for Test Point I-a-2. The basic rig configuration was used with flat electrodes. The nozzle area-ratio was 2.04. The BN backup pieces were arranged to give electrode temperature T_2 . Standard instrumentation was provided, except that measurement of electrode temperature was not attempted. Electrical contact was made with tantalum pins. The back faces of the electrodes were coated with rhodium to improve electrical contact between pins and electrodes. The 48-element injector was used in this test, in an attempt to provide more even distribution of propellant and oxygen.

The test procedure was standard. Actual test conditions are compared to target conditions for Test Point I-a-2.

	P_c	O/F	%K	$\dot{W}_t$	$C^*(\text{meas.})$	$C^*(\text{ideal-adiabatic})$	η_{C^*}	B
				lb/sec	ft/sec	ft/sec		
Nominal	60 psia	2.6	2.0					0
Actual	57 psia	2.64	2.19	0.975	4690	5570	0.84	0

Combustion was stable during the first cycle and no abnormalities were noted. A second cycle attempted during the same firing was discontinued due to loss of electrical contact to electrode pairs 4 and 5. Total run time was 8.3 minutes. A plot of applied voltage vs current for the third electrode pair is shown in Figure 105. The static pressure in the test section was essentially constant at 9.7 psia during the applied voltage portion of the test.

The measured conductivity determined by Method 3 for the third electrode pair was 5.1 mhos/m. Theoretical conductivities were:

<u>Ideal</u>	<u>Measured Heat Removed</u>	<u>Heat Removal to Match C*</u>	<u>Unburned* Fuel Approximation to Match C*</u>
26.5 mhos/m	Not calculated	< 1.0 mho/m	4.72 mhos/m

The voltage probes used in this test failed to operate properly and conductivity could not be calculated using Method 5. The boundary-layer voltage drop for the third electrode pair was 7 volts. The voltage-current characteristic was essentially a straight line except for the usual data scatter. The thermocouple used to measure combustion chamber coolant temperature rise failed to operate properly. This prevented calculation of heat flux and theoretical conductivity for the measured heat removed condition.

The voltage-current characteristic for the I-a-2 test shows no break and a voltage axis intercept of only seven volts. This is believed due in part to the injector modification that was made. The original 61-element injector was designed to give an excess of fuel near the walls of the combustion chamber. This design was used to partially cool the chamber walls and it is believed that this cool, fuel-rich boundary layer carried through the nozzle into the MHD test section, adversely affecting the boundary-layer conductivity. The injector was modified by plugging a number of fuel holes in the outside ring of injector elements and oxygen holes in the center portion of the injector, thus presumably producing a more uniform O/F distribution across the test section.

Chase-Around Rig (Build 14403-L-2)
Test Point V-b - Run Date 28 Nov. 1962

A test of 8.9 minutes duration was made to satisfy Test Point V-b. The measured conductivity was 7.1 mhos/m and the indicated boundary-layer voltage drop was 11 volts. The low boundary-layer drop is believed due to the use of the 48-element injector which presumably provided a more uniform O/F distribution and hotter boundary layer.

This test was made to determine conductivity at the conditions specified for Test Point V-b. Methods 3 and 5 were to be used to determine conductivity. The basic rig was used. The flat type electrodes plated on the back face with rhodium were used. The electrode contact pins were carbon with rhodium plated tips. The 48-element injector and

* Without measured heat removed

area-ratio 3.75 nozzle were used. No pressure taps were installed in the rig. Thermocouples were installed in the back faces of both upstream electrodes.

Nominal and actual conditions for the test were

	P_c	O/F	%K	$\dot{W}_t$	$C^*(\text{measured})$	$C^*(\text{ideal-adiabatic})$	η_{c*}	$\frac{B}{W/m^2}$
				lb/sec	ft/sec	ft/sec		
Nominal	145 psia	2.6	2.0					0
Actual	149 psia	2.73	2.11	1.26	5333	5614	0.95	0

No combustion instabilities were noted during the test. The normal test sequence was followed throughout the run. A plot of applied voltage vs current for electrode three is presented in Figure 106. The thermocouples that were installed to record electrode temperature failed. The voltage probes also failed to operate properly during the test.

The conductivity determined by Method 3 was 7.1 mhos/m. Predicted conductivities were

	σ (mhos/m)
Ideal	17.1
Measured heat removed	Not calculated
Heat removal correction to match C^*	6.9
Measured heat removed plus unburned	
Fuel correction to match C^*	16.7

The boundary-layer voltage drop was 11 volts. Instrumentation failure prevented measurement of coolant temperature rise, thus making heat flux calculations impossible.

The voltage-current characteristic obtained in this test was very nearly a straight line, showing a boundary-layer voltage drop of 11 volts. This is the same type characteristic as was noted in Test I-a-2 made on 27 November, 1962. It appears that the 48-element injector used in both tests produces a more uniform distribution of fuel and oxygen within the test section, thus giving lower boundary-layer voltage drop.

Chase-Around Rig (Build 14404-L-2-R)
Test Points V-a and V-c - Run Date 7 Dec. 1962

A run of 13.9 minutes duration was made to obtain data test points V-a and V-c. The measured conductivity for Test Point V-a was 6.5 mhos/m and the boundary-layer voltage drop was 24 volts. Measured conductivity for Test Point V-c was 4.4 mhos/m. Open circuit voltage was not recorded for Test V-c, thus boundary-layer voltage drop could not be determined. 7.03 KW was generated.

This test was made to obtain conductivity data for Test Points V-a and V-c. Conductivity was to be determined by Methods 3 and 1 respectively. The basic rig was used. Flat electrodes with rhodium plating on the back faces were used. Except for two, all the electrode contact pins used were rhodium-tipped carbon pins. One of the two remaining pins was of tantalum and the other was a carbon pin without rhodium plated tips. The 6-element injector and area ratio 3.75 nozzle were used. No thermocouples, voltage probes, or pressure taps were installed.

Standard test procedure was used. Nominal and actual conditions for Test Point V-c were

	P_c	O/F	%K	W_t	$C^*(\text{Measured})$	$C^*(\text{ideal})$	η_{c^*}	B
				lb/sec	ft/sec	ft/sec		W/m ²
Nominal	145 psia	2.6	2.0					2.0
Actual	132 psia	2.95	1.99	1.16	5194	5550	0.94	2.17

Combustion was smooth. The plot of voltage vs current for Test Point V-c which is presented in Figure 107 shows some scatter at the lower load resistances. These data in general, however, appear to be valid.

The conductivity calculated by Method 1 for electrode 3 was 4.4 mhos/m. Predicted conductivities were

	$\sigma(\text{mhos/m})$
Ideal	18.1
Measured heat removed - not calculated	----
Heat removal correction to match C^* -	6.7
Measured heat removed + unburned fuel correction to match C^* -	13.5

Maximum power generated was 7.03 KW. The Hall voltage at maximum power was 195 volts. Open circuit voltage was not recorded for this

test so boundary-layer voltage drop was not obtained. The voltage-axis intercept was 274 volts which was only 63 per cent of the ideal open circuit voltage, 434 volts. The heat transfer data recorded during this test appeared to be in error and thus were not recorded.

Nominal and actual conditions for Test V-a were

	P_c	O/F	$\%K$	$\dot{W}_t$ lb/sec	$C^*(\text{Measured})$ ft/sec	$C^*(\text{Ideal})$ ft/sec	$\eta_{c*} \frac{B}{W/m^2}$
Nominal	145 psia	3.2	2.0				0
Actual	148 psia	3.35	2.05	1.31	5158	5440	0.95 0

The normal test procedure was used. No abnormalities were noted. Combustion appeared to be stable. A plot of applied voltage vs current is presented in Figure 108. The fifth electrode circuit opened during this test so the voltage-current data recorded beyond this time are not presented in Figure 108.

The conductivity calculated by Method 3 was 6.5 mhos/m. Predicted conductivities were

	σ (mhos/m)
Ideal	16.8
Measured heat removed	not calculated
Heat removal correction to match C^*	7.9
Measured heat removed + unburned fuel correction to match C^*	10.8

The boundary-layer voltage drop for electrode 3 was 24 volts. The heat transfer data recorded during this test was inconsistent and is not reported. The rhodium - tipped carbon pins were in good condition after the test. The uncoated carbon pin was badly damaged on both ends while the tantalum pin was coated with oxide on the electrode end. The total run time for this test was 13.5 minutes.

The rhodium-tipped carbon pins gave better performance than uncoated carbon pins or tantalum pins. The C^* efficiencies recorded with the 3.75 area-ratio nozzle were higher than those recorded in tests with area ratio 2.0 nozzles. It would thus appear that C^* efficiency for the injector-combustion chamber system used is a function of combustion chamber pressure and/or chamber-to-throat area ratio, since all

tests were run to obtain a channel static pressure of 8 psia.

Chase-Around Rig (Build 14404-L-2-R)

Test Points II-a-2 and I-a-2-R - Run Date 10 Dec. 1962

A test of 14.5 minutes was made to satisfy Test Points I-a-2-R and II-a-2. The conductivity measured in Test I-a-2-R was 7.5 mhos/m and the conductivity for II-a-2 was 4.6 mhos/m. The test showed that carbon pins were not suitable for use as electrode contact pins.

The purpose of this test was to obtain conductivity data by Methods 3 and 1 to satisfy Test Points I-c-1 and II-a-2. Test conditions for Test I-c-1 were not achieved and actual test conditions were those specified for test point I-a-2. This test was designated as I-a-2-R since it was actually a repeat of Test Point I-a-2 which was run on 27 November 1962. Test conditions for Test Point II-a-2 were achieved. The rig used was a rebuild of build 14404-L-2. The electrodes used for this test were of the standard flat-face configuration, with back faces rhodium plated in the area of the electrode contact pins. The electrode contact pins were carbon with rhodium plated tips. The 6-element injector and area ratio 2.0 nozzle were used.

No voltage probes or static pressure taps were used in this test and no electrode temperature measurements were attempted.

The conditions achieved in Test I-a-2-R as compared to nominal conditions were

	P_c	O/F	$\%K$	$\dot{W}_t$	$C^*(\text{measured})$	$C^*(\text{ideal})$	ηC^*	B
				lb/sec	ft/sec	ft/sec		
Nominal	60 psia	2.6	2.0					0
Measured	59.5 psia	2.72	2.14	1.00	4771	5570	0.86	0

The test was carried out according to standard procedure with no apparent abnormalities. No combustion instabilities were noted. Chamber pressure and total weight flow remained essentially constant throughout the test. A plot of applied voltage vs current for the center electrode pair is presented in Figure 109. Inspection of Figure 109 shows that as voltage was increased the apparent conductivity also increased, giving a break in the curve at about 150 volts. It is noted that as the voltage was decreased, the characteristic showed no break and did not follow the same curve as the increasing voltage characteristic.

The measured conductivity calculated by Method 3 from the data for $I > 12$ amperes of the voltage-current characteristic was 7.5 mhos/m. Predicted conductivities were

	σ (mhos/m)
Ideal	25.8
Measured heat removed	19.4
Heat removal correction to match C^*	3.8
Measured heat removed plus unburned fuel correction to match C^*	6.3

The indicated boundary-layer voltage drop for $I > 12$ amperes from the voltage-current characteristic was 82 volts. For the descending voltage portion ($3 < I < 12$ amperes) the indicated boundary-layer drop was 27 volts. Test conditions achieved for this test are essentially those achieved for Test I-a-2 made on 27 November, 1962. The conductivities for Tests I-a-2 and I-a-2-R are similar while the boundary-layer voltage drop is 20 to 75 volts higher for the I-a-2-R test. The 48-element injector was used for Test I-a-2 while the standard 6-element injector was used for Test I-a-2-R. Since the basic difference between the tests was the different injectors used, it appears that the 48-element injector gave a better propellant and oxygen distribution, thus giving a lower boundary-layer voltage drop. The core-flow composition temperature and pressure appear to be similar in view of the identical conductivities observed, unless compensating variations are present. The average heat flux for the chamber was 0.8 Btu/in² sec. For the nozzle the average heat flux was 1.95 Btu/in² sec.

Test II-a-2 was made to determine gas conductivity by Method 1. Actual test conditions are compared to nominal conditions in the following table.

	P_c	O/F	$\%K$	$\dot{W}_t$	$C^*(\text{meas.})$	$C^*(\text{ideal})$	C^*	B
				lb/sec	ft/sec	ft/sec		W/m ²
Nominal	60 psia	2.6	2.0					2.0
Measured	57.6	2.68	1.99	0.99	4676	5570	0.84	2.17

Combustion appeared to be stable during this test. The voltage-current characteristic for the third electrode pair is presented in Figure 110. Inspection of Figure 110 shows a considerable amount of scatter in the data, which is believed due primarily to poor electrical contact. Pictures of the contact pins taken after the test are presented in Figures 111 and 112. These pictures show that the contact pins suffered

considerable damage on the ends that were in contact with the electrodes. The damage appears to be due to overheating probably caused by electrical arcs between pins and electrodes. Pictures of the test rig after this test series are presented in Figures 113 and 114. The rig appears to be in good condition in view of the fact that a large number of the liner parts had been used previously.

The conductivity calculated by Method 1 for the third electrode pair was 4.3 mhos/m and the boundary-layer voltage drop was 68 volts. The maximum power output of the rig was 4.6 KW at a nominal load of 13.5 ohms. Predicted conductivities are presented in the following table.

	σ (mhos/m)
Ideal	25.6
Measured heat removed	18.3
Heat removal correction to match C*	0.3
Measured heat removed plus unburned fuel correction to match C*	3.6

Measured open-circuit voltage for this test was 266 volts. Ideal open-circuit voltage for complete combustion is 363 volts, indicating a velocity deficiency of $(363-266)/363 = 27$ per cent. Average heat flux for the combustion chamber was 0.88 Btu/in² sec while average heat flux for the nozzle was 1.92 Btu/in² sec.

The carbon contact pins did not perform satisfactorily even when coated with rhodium. The hysteresis effect noted in the voltage-current characteristic of the I-a-2-R test appears as a function of applied voltage but it should be noted that the effect is also a function of time. It is thus possible that factors other than voltage or current density could be responsible for the hysteresis effect.

Chase-Around Rig (Build Number 14394-L-4)

Test Points III-a, and III-b - Run Date 12 December 1962

A test was made to satisfy Test Points III-a and III-b. The conductivity calculated by Method 3 for Test III-a was 10.1 mhos/m. The voltage-current characteristic was a straight line. The conductivity calculated by Method 3 for Test III-b was 6.5 mhos/m. The voltage-current characteristic exhibited marked hysteresis. The cause for the hysteresis is attributed to boundary-layer phenomena and electrode temperature changes. The sidewall face temperature was about 2860°F.

This test was made to satisfy Test Points III-a, III-b, and III-c and to obtain information on voltage probe. Conductivities were to be obtained by Methods 1, 3, and 5. Test Point III-c is discussed under a separate heading. A variable resistance was connected in parallel with the galvanometer reading data from one of the voltage probes. By varying the resistance, the variation of indicated probe voltage with probe current can be obtained. The results of this test are reported under voltage probes in the section on Support Testing. The basic rig configuration was used with the following exceptions. The backs of the flat electrodes were plated with rhodium in the contact area. The electrode contact pins were carbon with rhodium-plated tips. An Inconel sheathed platinum-platinum-10 per cent rhodium ungrounded thermocouple was installed in the left sidewall of the rig. The thermocouple was located in a pressure tap hole in the No. 2 sidewall block as shown in Figure 115.

The electrode face temperature and rig static pressures were not measured. The 6-element injector and area ratio 2.0 nozzle were used.

The nominal and actual conditions for Cycle I, Test Point III-a were

	P_c	O/F	$\%K$	$\dot{W}_t$	$C^*(\text{meas.})$	$C^*(\text{ideal-adiabatic})$	$\eta_{c^*} \frac{B}{W/m^2}$
				lbm/sec	ft/sec	lbs/sec	
Nominal	60.0 psia	3.2	2.0	---	---	---	0
Actual	57.0	3.07	1.96	1.05	4517	5450	0.830

The rig ran very smoothly with no combustion surges noted. Standard test procedure was followed. The heat transfer data is given below

	$\dot{Q} \frac{BTU}{sec}$	$\dot{Q}/A \frac{BTU}{in^2 sec}$
Combustion chamber	101.0	0.58
Nozzle	113.0	2.12

Temperature vs time for the sidewall thermocouple is shown in Figure 115. The thermocouple was disconnected before voltage was applied to the rig. When the thermocouple was plugged back in at the end of the run, the temperature indicated was about 300°F higher. However, the temperature vs time curve had not leveled off when the thermo-

couple was disconnected.

This suggests that warmup times of one minute are not sufficient to reach steady state on the sidewalls. The maximum temperature recorded on the thermocouple was 2340°F. Based on one-dimensional slab heat transfer and the measured Q/A , which was 0.4 BTU/in² sec, the temperature of the front face of the sidewall was calculated to be 2860°F. The plot of voltage applied vs current for electrode 3 is shown in Figure 116. The characteristic was a straight line.

The conductivities calculated by Method 3 were

Electrode	1	2	3	4	5
σ (mhos/m)	8.4	7.9	10.1	9.1	12.0

The voltage probes did not function.

The predicted conductivities are given below

	σ (mhos/m)
Ideal	26.3
Measured heat removed	22.4
Heat removal approximation to match C*	<1.0
Unburned fuel plus measured heat removed to match C*	1.2

The boundary-layer voltage drop for electrode 3 was 8.7 volts.

The nominal and actual conditions for Cycle II, Test Point III-b are shown below

P_c	O/F	%K	$\dot{W}_t$	C*(meas.)	C*(ideal-adiabatic)	η_{c*}	B
			lbm/sec	ft/sec	ft/sec		W/m ²
Nominal 60.0 psia	2.6	2.0	---	---	---		0
Actual 58.5	2.56	2.01	1.03	4704	5590	0.84	0

The combustion was smooth as in Cycle I. The heat transfer data is given below

	$\dot{Q}$ BTU/sec	$\dot{Q}/A$ BTU/in ² sec
Combustion Chamber	92.4	0.53
Nozzle	87.0	1.68

The plot of voltage vs current for electrode 3 is shown in Figure 116. The characteristic for this point exhibits marked hysteresis with a sharp break in the curve at 205 volts. The reason for this hysteresis has not been determined. Among the possible causes is ohmic heating of the boundary layer. The phenomenon seems to be a function of O/F, electrode temperature, or electrode erosion since these are the major differences between Cycles I and II.

The apparent conductivities calculated by Method 3 for decreasing voltages were

<u>Electrode</u>	1	2	3	4	5
σ mhos/m	4.2	5.0	6.5	7.0	8.9

Again the voltage probes did not function. The predicted conductivities are

	<u>σ (mhos/m)</u>
Ideal	25.7
Measure heat removed	21.8
Heat removal approximation to match C*	1.9
Unburned fuel plus measured heat removed to match C*	5.4

The boundary-layer voltage drop for electrode 3 was 32 volts.

The measured voltage axis intercepts for Test Points III-a and III-b are 8.7 volts and 32.1 volts respectively. The thermocouple data indicates that total warmup times in excess of 2 minutes are necessary in order to achieve steady-state sidewall face temperature.

Chase-Around Rig (Build 14394-L-4)

Test Point III-c (also II-a-4)-Run Date 12 Dec. 1962

Two cycles were made, one at an O/F of 2.53 to satisfy Test Point III-c and the other at an O/F of 3.02. The maximum power generated was 3156 watts, and 7355 watts, respectively. The difference in power is due to the fact that the boundary-layer voltage drop for the O/F of 2.53 is more than twice that for 3.02. The measured conductivity at the third electrode for an O/F of 2.53 was 8.7 mhos/m. The conductivity for O/F = 3.02 was 7.1 mhos/m. Four separate tests were made without shutting the gas generator down, demonstrating rig durability and stand flexibility. Total run time for the four tests was 15 minutes.

The purpose of this test was to satisfy Test Point III-c which required measurement of plasma conductivity by Method 1. The run was to be continued for a test at an O/F of 3.2. A description of the rig can be found in the preceding report. The standard test procedures were employed during these tests. The voltage probes were disconnected during power generation to prevent burning out the recording galvanometers. The conditions for each cycle were

	Pc, psia	B			W _t	C*, ft/sec	C*, ft/sec	η C*,
	measured	W/m ²	O/F	%K	lbs/sec	measured	adiabatic	ideal
Nominal	60	2.0	2.6	2.0	----	----	----	----
Actual	59.1	2.17	2.53	2.06	1.02	4829	5630	0.86
(Cycle I)								
Nominal	60	2.0	3.2	2.0	----	----	----	----
Actual	56.3	2.17	3.02	1.81	1.04	4496	5500	0.82
(Cycle II)								

No combustion instabilities were noted during the tests. Figures 117 and 118 are plots of voltage versus current and power versus calculated load resistance at each condition for electrode 3. The other four electrodes are comparable. No Hall voltage measurements are available.

The conductivities obtained by Method 1 were

Electrode	Test Point III-c O/F 2.53	O/F - 3.02
1	$\sigma'_{app} = 7.8$ mhos/m	$\sigma'_{app} = 7.9$ mhos/m
2	7.6	7.1
3	8.7	7.1
4	9.1	8.3
5	10.4	9.5

The predicted conductivities were

	Test Point III-c O/F = 2.53	O/F 3.02
Ideal adiabatic conductivity (mhos/meter)	$\sigma' = 25.5$ $\sigma_t = 6.0$	$\sigma' = 26.0$ 6.1
Conductivity for heat removal approximation (mho/meter), to match C*	$\sigma' = 1.0$ $\sigma_t = 1.0$	<1.0 <1.0

	Test Point III-c O/F = 2.53	O/F 3.02
Conductivity for unburned fuel plus	$\sigma' = 10.8$	1.1
measured heat out approximation	$\sigma_t = 1.9$	<1.0
(mhos/meter) to match C*		

Two least squares fits were made to the data shown in Figure 117. The apparent conductivity for the low current leg is 2.95 mhos/meter and that for the high current leg is 8.72 mhos/meter. The break in the curve is at approximately 7 amperes. There was a similar break noted in the applied voltage vs current curve for Test Point III-a, when it occurred at approximately 10 amperes. It has been suggested that the reason for the break in the voltage-current curve is a change in the character of the boundary layer from one that acts as a constant resistance to one that acts as a constant voltage drop. More data is necessary before any conclusions can be drawn. The maximum power generated for each O/F was: Cycle I (Test Point III-c) 3156 watts, Cycle II - 7355 watts. Since the gas conductivity in each case is approximately the same, the apparent reason for the large difference in power is the fact that the boundary-layer voltage drop at an O/F of 2.53 was more than twice that for an O/F of 3.02 (see Figure 117).

The boundary-layer drop for electrode 3 was 131 volts for Point III-c and 38 volts for the O/F = 3.02 cycle. Heat transfer data for a stabilized point in each cycle were

	Cycle I(III-c)		Cycle II	
	$\dot{Q}$ BTU/sec	$\dot{Q}/A$ $\frac{\text{BTU}}{\text{in}^2 \text{ sec}}$	$\dot{Q}$ BTU/sec	$\dot{Q}/A$ $\frac{\text{BTU}}{\text{in}^2 \text{ sec}}$
Combustion chamber	67.0	0.39	95.0	0.55
Nozzle	104.7	2.01	127.0	2.45

Pictures of the rig taken during teardown are shown in Figures 119 and 120. The erosion on the sidewalls was quite marked. As noted in other runs, the erosion appears to start from the cracks between the sidewall blocks and becomes progressively worse downstream. Very little erosion was evidenced on the electrodes.

There is strong evidence pointing to the existence of a large boundary-layer resistance at the O/F of 2.53. The maximum power point for an MHD generator occurs when load resistance and internal rig resistance are equal. For the case in which rig resistance decreases with increas-

ing current, this point changes as the loads are changed. This would tend to flatten the power vs load resistance characteristic. In particular, for this run, the applied voltage vs current characteristic was approximated by 2 straight lines as shown in Figure 117. When points taken from these lines are plotted in Figure 118, two peaks are seen. Figure 117 shows that for Test Point III-c the plot of voltage versus current is not a straight line. If the boundary layer has a pure, constant resistance, the slope of the line can be expressed by the following equation

$$\frac{dV_T}{dI_L} = -[R_{BL} + R_g(1 + \alpha' \Lambda)]$$

The slope of the line in Figure 117 decreases with increasing current. It is possible that this occurs because ohmic heating decreases the resistance of the boundary layer. The current increased by almost 10 amperes from electrode 1 to electrode 5. This phenomenon may be the result of ohmic heating and turbulent mixing of the relatively cool boundary layer from the water-cooled nozzle. It may also be due to a temperature increase caused by weak oblique shocks in the generator.

The boundary-layer voltage drop and measured conductivity for Test III-c (O/F = 2.53) were both higher than those for the O/F 3.02 test.

Chase-Around Rig (Build 14406-L-2)

Test Points I-a-1 and II-a-1 - Run Date 14 Dec. 1962

Test Points I-a-1 and II-a-1 were run during a continuous firing of 11 minutes and 21 seconds duration. For Test I-a-1 the conductivity determined by Method 3 for the descending voltage data was 7.0 mhos/m and the boundary-layer voltage drop was 26 volts. The conductivity for Test II-a-1 determined by Method 1 was 5.9 mhos/m. Boundary-layer voltage drop was 71 volts. Electrode temperature was calculated to be about 2950°F.

This run was made to obtain data for Test Points I-a-1 and II-a-1 by Methods 1, 3, and 5. The basic rig was used with electrodes in the T₁ configuration. Modifications were

- 1) Electrode contact pins were rhodium tipped carbon,
- 2) Electrodes were rhodium plated on the back surface to provide good electrical contact,
- 3) Nozzle throat area was 2.72 in², giving A/A* = 2.0, and
- 4) The 6-element injector was used.

Instrumentation was standard except that two thermocouples were installed, one each in the top and bottom upstream electrodes. The standard $H_2 - O_2$ warmup cycle was run. After a switch to JP4 was made, conditions were held steady until electrode temperatures leveled off and thermocouples were disconnected. Seeding was then turned on and two sets of conductivity data were run, the first with applied voltage, and the second with magnetic field. Seed flow was then cut off, and electrode thermocouples reconnected. Conditions were held steady for 55 seconds at which time normal shutdown procedure was started. Run time was 681 seconds.

Target and actual test conditions are summarized in the following table for Test Points I-a-1 and II-a-1.

	<u>O/F</u>	<u>%K</u>	<u>P_c</u>	<u>A/A*</u>	<u>P_t*</u>	<u>W_t lbm/sec</u>	<u>C* ft/sec measure</u>	<u>C* ft/sec ideal adiabatic</u>	<u>η_{c*}</u>
Target	2.6	2.0	60.0	2.0	---	---	---	---	---
Actual	2.59	2.17	58.9	2.0	57.1	1.07	4727	5580	.85

Recorded data for Test Point I-a-1 was as follows. Figure 121 is a plot of voltage versus current for electrode 3. A hysteresis effect was observed that has not been satisfactorily explained at this time but may be due to ohmic heating of electrodes and/or boundary layers. Figure 122 shows curves of electrode back surface temperature versus time. Temperature was still increasing at 170 seconds. When the thermocouples were reconnected one was burned out. The other one indicated a temperature of 2690°F, and dropped to 2470°F in 55 seconds. The initial increase may have been due to ohmic heating in the electrode at the contact point, or in the boundary layer. Propellant flows and chamber pressure were essentially constant over the interval during which conductivity data were taken. Voltage probes were used for this section of the run. Data for 6 values of current density selected for their consistent readings are shown on Figure 123 along with values of conductivity calculated from the data. Figure 124 shows data taken over a 0.5-second interval at 266 seconds. The range of scatter is apparent.

Conductivity calculated from electrode 3 data is compared with theoretical conductivity below

	σ
1) Measured Method 3	5.6 - 7.0 mhos/m
2) Measured Method 5	23-35, average 31
3) Ideal adiabatic	25.7
4) Meas. heat rem. approx.	19.8
5) Heat rem. to match C*	0.34
6) Meas. heat rem. + unburned fuel approx. to match C*	5.77

Boundary-layer voltage drop was 23 volts for increasing voltages and 26 volts for decreasing voltages as determined from a least squares straight line fit to the voltage-current data for electrode 3. Measured heat flux was

	Average $\dot{Q}/A$
Chamber	0.92 BTU/in ² sec
Nozzle	2.32

The $\dot{Q}/A$'s for the electrode walls were used with electrode temperature measurements from Figure 122 to calculate electrode surface temperatures. These are 2950°F for both top and bottom electrodes at 170 seconds.

Recorded data for Test Point II-a-1 was as follows. Figures 125 and 126 show plots of voltage versus current and power versus load resistance for electrode 3. A least squares fit to the data is shown on Figure 125 and a line derived from this is shown on Figure 126. Hall voltages at short circuit are shown below

	Hall Voltage at Short Circuit				
Electrodes	1-2	2-3	3-4	4-5	
Top	42	44	40	44	volts
Bottom	42	46	38	42	volts

Voltage probes were disconnected before this test.

Conductivity as measured by Method 1 is compared with theoretical conductivity below

	σ'
Meas.	5.9 mhos/meter
Ideal adiabatic	25.0

	σ'
Meas. heat rem.	19.4
Heat rem. to match C*	0.34
Meas. heat rem. + unburned fuel to match C*	5.67

Power output for all electrodes totalled 5.3 KW. Boundary-layer voltage drop, along with open circuit voltage is shown below

Electrodes	1	2	3	4	5	
Voc	266	266	266	266	266	volts
V _{oint}	198	204	195	201	197	volts
δV	68	62	71	65	69	volts

Rig condition was excellent after this run. Many of the parts were reused in build 14406-L-3 which was run on 20 December 1962.

Boundary-layer voltage drop was appreciably higher during the cycle with magnetic field than the previous cycle with applied voltage. High conductivity calculated from voltage probe data along with the scatter shown on Figure 124, indicates that the probes are still not reliable. The difficulty is probably poor connections. Hysteresis was observed in the applied voltage test data. This may be due to heating of the electrodes or boundary layer at high currents. More study is required to be certain of this. Conductivity measured with no magnetic field was 5.7 mhos/meter for increasing voltages, 7.0 mhos/meter for decreasing voltage. Conductivity with magnetic field was 5.9 mhos/meter.

Chase-Around Rig (Build 14403-L-3)

Test Point I-b-4 and I-d-2 - Date of Run 18 Dec. 1962

A run of 630 seconds was made to obtain data for Test Points I-b-4 and I-d-2. The plasma conductivities calculated by Method 3 were 7.2 and 8.25 mhos/m for Cycles 1 and 3 (Test Point I-b-4), and 10.8 mhos/meter for Test Point I-d-2. Conductivity calculated by Method 5 was 18.8 to 22.8 mhos/meter for the top-to-center and bottom-to-center probes respectively, from data taken during Cycle 2.

The purpose of this run was to obtain data for Test Points I-b-4 and I-d-2 with ramped electrodes and to compare the results with Test Points III-b and III-a which were run with flat electrodes at the same conditions. Methods 3 and 5 were to be used to obtain plasma conduc-

tivity. The basic rig was equipped with ramped electrodes, rhodium-plated in the contact area. Tantalum contact pins were used. The upstream electrode on the bottom was equipped with a thermocouple. The electrode walls were built to yield temperature T_4 . The 6-element injector and area ratio 2.0 nozzle were used.

The test procedure for this run was standard. Three applied voltage cycles were executed, one at a nominal O/F of 2.6, one at a nominal O/F of 3.2, and the final cycle at nominal O/F = 2.6. These cycles are designated as Cycle 1, Cycle 2 and Cycle 3 respectively. The nominal and actual firing conditions were

Nominal Conditions

	Oxygen-Fuel Ratio	%K	Chamber Pressure
Cycle 1	2.6	2.0	60 PSIA
Cycle 2	3.2	2.0	60 PSIA
Cycle 3	2.6	2.0	60 PSIA

Measured Conditions

	I-b-4 Cycle 1	I-d-2 Cycle 2	I-b-4 Cycle 3
P_c (Measured chamber pressure - psia)	60.2	58.8	60.5
$\dot{W}_t$ (Total weight flow of propellants lbs/sec)	1.07	1.07	1.07
O/F (oxygen-fuel ratio)	2.65	3.38	2.68
%K (per cent potassium)	1.99	1.99	1.97
C^* (measured ft/sec)	4572	4570	4690
C^* (ideal-adiabatic - ft/sec)	5590	5380	5590
ηC^* (ideal-adiabatic)	0.837	0.850	0.840

During the applied-voltage cycles, the combustion was steady and the propellant and seed flows were essentially constant. Data was obtained only during the decreasing voltage portion of Cycle 1, so after Cycle 2 the test was repeated.

Figures 127 and 128 and 129 show the plots of applied voltage versus current for the third electrode pair. The apparent conductivities calculated using Method 3 are 7.2, 10.8, and 8.25 mhos/m for Cycles 1, 2, and 3, respectively. During the second applied-voltage cycle some useful data was obtained from the three voltage probes between

the third electrode pair. The conductivity from the data is 22.8 ± 2.4 mhos/m between the bottom probe and the center probe and 18.8 ± 2.7 mhos/m between the center probe and the top probe, averaged over 12 points where probe voltages were over 35 volts.

The calculated theoretical conductivities were

	<u>Cycle 1</u>	<u>Cycle 2</u>	<u>Cycle 3</u>
	mhos/m	mhos/m	mhos/m
Ideal	25.4	23.8	25.6
Measured heat out approximation	18.3	18.8	19.6
Heat removal approx. (to match C*)	0.21	1.46	0.34
Unburned fuel plus measured heat out approximation (to match C*)	3.5	1.7	4.0

Boundary-layer voltage drops were 7.1, 3.31, and 30.6 volts for Cycles 1, 2, and 3, respectively. Runs made on 12 December 1962 for Test Points III-b and III-a with flat electrodes showed boundary-layer voltage drops of 32 and 8.7 volts, which are to be compared with Test Points I-b-4 and I-c-2, respectively. It is not certain on the basis of these tests whether the ramps had a significant effect on the boundary layer. The ramps may have been eroded enough during the run to cause the increase in boundary-layer voltage drop observed during the third cycle. The heat flux from the combustion chamber and nozzle were

	<u>Combustion Chamber</u>	<u>Nozzle</u>
Cycle 1	0.73	2.0 BTU/in ² sec
Cycle 2	0.81	2.7 BTU/in ² sec
Cycle 3	0.66	2.8 BTU/in ² sec

The thermocouple imbedded in the bottom electrode of the first pair failed. No useful static pressure data was recorded.

There was not good agreement between the theoretical conductivity predictions and the measured conductivities.

Chase-Around Rig (Build 14403-L-3,
Test II-b-2 - Run Date 19 Dec. 1962

A run of 22.1 minutes was made using H₂ fuel to satisfy Test Point II-b-2. A comparison run was made with JP4/O₂. The maximum power for Point II-b-2 was 18,000 watts while the maximum power for the JP4 cycle was 2245 watts. Boundary-layer voltage drops of

100 volts for the JP4/O₂ cycle and 10 volts for the H₂/O₂ cycle were observed. The large difference in power between JP4 and H₂ is caused by the lower velocity and higher boundary-layer voltage drop noticed with JP4.

Chase-around rig 14403-L-3 was run to satisfy Test Point II-b-2. The purpose of the run was to measure the plasma's conductivity with a magnetic field present, by Method 1. This was the same rig run for 11.5 minutes on 18 December 1962. The injector was the 6-element configuration with the fuel orifices enlarged to 0.070 - 0.075 inch diameter. The rig was warmed up to 165 seconds. Conditions were set and seeding was turned on. With power being generated, the loads were cycled twice. For comparison a switch from H₂/O₂ to JP4/O₂ was made, seeding was turned on and the loads were cycled once. After the JP4 cycle a transfer back to hydrogen was made, another load was applied, and then the rig was cooled down in the usual manner. A comparison of nominal conditions and actual conditions at a typical point are given below

	P _c , psia				W _t	C* _{meas}	C* _{ideal}	η _c , %
	measured	O/F	B	W/m ²	%K	lbs/sec	ft/sec	adiabatic
Nominal (Test Point II-b-2)	60	6.0	2.0	2.0	----	----	----	----
Cycle I and II	54	6.09	2.17	1.99		5678	7277	0.78
Nominal (Comparison Run)	60	3.2	2.0	2.0				
JP ⁴ /O ₂ Cycle	55	3.43	2.17	1.88		4040	5120	0.79

The gas generator ran well. No instabilities were noted during the power cycles. The electrode thermocouple failed. It was checked after the run and found to have a broken lead. No useful static pressure data was recorded. Figures 130 and 131 are plots of voltage versus current and power versus calculated load resistance for electrode 3. The readings on the Hall voltmeters indicate that the electrical integrity of the rig was good. Typical values for the H₂/O₂ cycles were

	V No.3	I _y No.3	Flameshield to Ground (volts)	Interelectrode Voltage							
				TOP				BOTTOM			
				1-2	2-3	3-4	4-5	1-2	2-3	3-4	4-5
Open Circuit	337	0	120**	0	0	0	0	0	0	8	16
Maximum Power	116	21.6	324**	40	36	36	32	40	44	36	38
Maximum Current*	102	27.9	396**	50	46	46	48	54	48	40	46

The readings for both H₂ cycles were almost identical, indicating little or no deterioration during the interval. The JP4/O₂ Hall voltages were lost when the recording instrument failed.

The conductivities obtained by Method 1 from data in Figure 130 were

			Point II-b-2	JP4 Cycle
Apparent conductivity (mhos/meter)				
	Electrode	1	7.5	-
	"	2	7.7	-
	"	3	7.1	8.2
	"	4	8.0	-
	"	5	8.4	-
Ideal-adiabatic conductivity (mhos/meter)				
		σ	11.1	22.8
		α	2.95	4.4

During the JP4/O₂ cycle there was an instrument failure and no voltage-current data was recorded. The voltage and current at each load in the cycle were recorded by hand for electrode 3. The average open-circuit voltage for the JP4 cycle was 177 volts.

The boundary-layer drop for the JP4 cycle was 100 volts and that for the H₂ cycle was 5 volts. No predicted conductivities which use the standard correction approximations are available. The maximum

* Ammeters pegged, unable to reach short circuit

** Data from flameshield-to-ground voltmeter is questionable

power generated for the test was: Cycles I and II (Test Point II-b-2) 18,000 watts, JP4 comparison cycle - 2245* watts. The total ideal power for the actual conditions of Test Point II-b-2 based on predicted conductivity and predicted velocity is 36,250 watts. The measured power output of the rig is then about 50 per cent of ideal. The power density is 472 watts/in³. The JP4 power was 18 per cent of ideal with a power density of 59 watts/in³. The main reasons for the large difference in power between JP4 and H₂ are:

- 1) The boundary-layer drop for JP4 is 100 volts as compared with 5 volts for H₂.
- 2) The velocity is much less with JP4. This is apparent when one considers that, ideally, the open circuit voltage of JP4/O₂ should be about 78 per cent of H₂/O₂ and actually it is approximately 58 per cent of the H₂/O₂ open circuit voltage.

Heat transfer data for a point representing conditions in both cycles are given below

	<u>Cycle I and II (Test Point II-b-2)</u>		<u>JP4 Cycle</u>	
	$\dot{Q}$, BTU/sec	$\dot{Q}/A$, BTU/in ² sec	$\dot{Q}$, BTU/sec	$\dot{Q}/A$, BTU/in ² sec
Combustion chamber	707	4.2	142	0.84
Nozzle	295	5.6	157	3.0

The reason for the higher heat transfer rate of hydrogen is not obvious since the theoretical total temperatures are about the same (5700°R at O/F = 6.0 for H₂ versus 5830°R at O/F = 3.4 for JP4) and the theoretical heat transfer coefficients are approximately the same. One possibility may be that a film of cool, unreacted JP4 flows near the walls. Since the fuel orifices in the injector were enlarged to handle the high volume rate of hydrogen flow, the JP4 leaving the orifice would have been at too low a velocity, resulting in poor atomization. Also, because of a time shortage, the fuel elements were enlarged with a hand drill. This might result in poor impingement of the fuel streams with a further decrease in atomization and increased possibility of unreacted fuel near the combustion chamber walls. The comparison of the boundary-layer voltage drop of the JP4 cycle with that of the H₂/O₂ cycles gives further evidence of a cool film associated with the JP4 run.

* Five times the third electrode value

The total run time on this rig was 32.6 minutes. Figures 132 and 133 are photographs of the rig after teardown. As can be seen, damage to the sidewalls was extensive. There are two possible reasons for the heavy damage: 1) the high temperatures melted the Al_2O_3 pins which combined with the BeO to form a lower melting point complex, 2) the literature* suggests that, at high temperature, beryllia combines with water to form the hydroxide $\text{Be}(\text{OH})_2$. The reaction is as follows:

Since the product of combustion of hydrogen and oxygen is superheated steam, the reaction is possible and may be one of the causes of the deterioration of the sidewalls.

The stability of the H_2/O_2 flame was excellent, as evidenced by the almost identical data obtained during Cycles 1 and 2. The objective of the run to measure plasma conductivity with magnetic field present for Test Point II-b-2 was achieved.

Chase-Around Rig (Build 14406-L-3)

Test Point I-b-1 - Run Date 20 Dec. 1962

A run of 6.3 minutes duration was made to satisfy Test Point I-b-1 and an additional point at $\text{O/F} = 3.2$. The conductivity calculated by Method 3 was 8.5 mhos/m for Test Point I-b-1 and 11.1 for the nominal $\text{O/F} = 3.2$ run. The boundary-layer voltage drop for electrodes was 10 volts for Test Point I-b-1 and 2 volts for the $\text{O/F} = 3.2$ point. The removal of the Al_2O_3 dowel pins appears to improve the sidewall performance markedly.

This firing included two applied-voltage cycles in the absence of an applied magnetic field. The target conditions for the first cycle were those of Test Point I-b-1. The second cycle was made at identical conditions except that the target O/F was 3.2. The basic rig was used with the area-ratio 2.0 nozzle but with the following exceptions or modifications:

- 1) The electrodes were ramped. The straight ramps were 0.050 inch high. The backs of the electrodes were rhodium-plated in the contact area.

* Inorganic Chemistry by E. deBarry Barnett and C. L. Wilson, Longmans Green and Company, P. 148

- 2) The electrode contact pins were carbon and the tips were rhodium plated.
- 3) The 6-element injector was used.
- 4) The aluminum oxide pins were omitted from between the sidewall blocks.
- 5) The boron nitride backup pieces were ungrooved with the direction of maximum thermal conductivity parallel to the direction of heat flux. This orientation gives the T_1 test condition.
- 6) Teflon gaskets were used between the sidewalls and electrodes.
- 7) There were no static pressure taps on this rig. A thermocouple was located behind the first electrode on the top electrode assembly.

This rig had been run previously on 14 December, 1962. The sidewall pieces, BN backups and the electrode wall BeO pieces used in this run were all used in the previous run except for the sidewall piece in which the voltage probes were located. Thus most of the pieces had experienced 11 minutes of running prior to this run.

The run followed normal sequence. The nominal and actual run conditions for Cycle 1, Test Point I-b-1 were

	P_c (psia)	O/F	%K	$\dot{W}_t$ (lbm/sec)	C^*_{meas} (ft/sec)	$C^*_{\text{ideal, adiabatic}}$ (ft / sec)	η_{c^*}	B
Nominal	60.0	2.6	2.0	--	--	--	--	0
Actual	63.9	3.02	1.81	1.17	4542	5491	0.83	0

The discrepancy between nominal and actual conditions was caused by an improperly set O_2 flow rate.

The combustion was quite steady throughout the run. The heat transfer data for the combustion chamber and nozzle is shown below

	$\dot{Q}$ ($\frac{\text{BTU}}{\text{sec}}$)	$\dot{Q}/A$ ($\frac{\text{BTU}}{\text{in}^2 \text{ sec}}$)
Combustion chamber	170	1.0
Nozzle	203	3.9

The thermocouple located behind the first electrode gave fairly good readings. A plot of thermocouple reading versus time is shown in Figure 134. No reason is apparent for the sudden drop in temperature at 110 seconds. The temperature appeared to level off at 2150°F before the thermocouple was disconnected at the beginning of the run, however, when it was plugged in again at the end of the run, the temperature was

2700°F. The O/F at the beginning of the run was 3.02 while at the end of the run it was 3.65. This O/F difference would account for the differences in reading, especially in view of the increased heat flux noted at the higher O/F. The temperature recorded before the thermocouple was disconnected can be used along with the measured Q/A to calculate an electrode face temperature of 2400°F. After the run the calculated electrode face temperature was 2900°F. The applied voltage versus current plot for electrode 3 is shown in Figure 135. The voltage-current characteristic is a straight line with no hysteresis or change in slope apparent.

The conductivities calculated from a least squares fit to the voltage-current data for Cycle 1 were

Electrode	1	2	3	4	5
σ (mhos/m)	7.9	7.2	8.5	10.2	13.1

The voltage axis intercept for electrode 3 was 10 volts. The voltage probe data is presented below

	<u>Range of Values</u>	<u>Average</u>
σ 1-2 (mhos/m)	12.3 - 15.3	13.8
σ 2-3	10 - 47	----
σ 1-3	17 - 20.4	18.5

As can be seen the data is not reliable. The data is shown graphically in Figure 136. The theoretically predicted conductivities for the various correction methods are given below

	<u>σ (mhos/m)</u>
Ideal	26
Measured heat removed	19.0
Heat removal approximation to match C*	< 1
Unburned fuel plus measured heat removed to match C*	1.5

The conductivity obtained from the slope of the voltage-current curve is bracketed by the measured heat removed prediction and the heat removal prediction but does not fall close to either one.

The nominal and actual run conditions for Cycle 2 are shown below

	<u>P_c (psia)</u>	<u>O/F</u>	<u>%K</u>	<u>$\dot{W}_t$ (lbm/sec)</u>	<u>C* meas (ft/sec)</u>	<u>C*ideal, adiabatic (ft/sec)</u>	<u>η_{c*}</u>	<u>B</u>
Nominal	60.0	3.2	2.0	---	---	---	---	0
Actual	62.6	3.65	1.77	1.19	4373	5315	0.82	0

As in the first cycle, the discrepancies were caused by an improperly set oxygen flow rate. Again, the combustion was quite steady throughout the run. The heat transfer for chamber and nozzle is presented below

	$\dot{Q} \left(\frac{\text{BTU}}{\text{sec}} \right)$	$\dot{Q}/A \left(\frac{\text{BTU}}{\text{in}^2 \text{ sec}} \right)$
Chamber	211	1.24
Nozzle	215	4.14

The applied voltage versus current curve for electrode 3 is shown in Figure 135. The voltage-current characteristic is a straight line with no hysteresis or change in slope apparent.

The conductivities calculated from a least squares fit to the voltage-current data for Cycle 2 were

Electrode	1	2	3	4	5
σ (mhos/m)	12.1	10.2	11.1	13.1	15.9

The voltage axis intercept for electrode 3 was 2 volts. The voltage probe data is presented below:

	Range of Values	Average
σ 1-2 (mhos/m)	11.2 - 15.8	12.9
σ 2-3	15 - 59	----
σ 1-3	13.3 - 18.8	15.8

The data is shown graphically in Figure 137. There is considerable scatter. The theoretical conductivity is shown below for the various predictions

	σ (mhos/m)
Ideal	22.0
Measured heat removed	15.6
Heat removal approximation to match C*	<1.0
Unburned fuel plus measured heat removed to match C*	<1.0

Figures 138 and 139 show photographs taken of the rig after teardown. The voltage probes can be seen in Figure 138. The erosion on the sidewalls was much less severe than on previous runs. The decrease

in erosion is believed to result from the removal of the Al_2O_3 pins. The problem of materials compatibility at high temperature must be considered carefully when designing MHD rigs and other high temperature devices.

Another noteworthy point brought to light in the pictures is that the coloration of the beryllia pieces varies across the rig and the pattern is not symmetrical with respect to the center line. This suggests that the flow from the nozzle is not perfectly axial. This is probably because of poor alignment between the rig and the nozzle. This phenomenon could explain some of the variation in voltage probe data.

The characteristics of the voltage-current plot are much different than previous runs which had flat electrodes. The fact that the probe data is erratic is in all probability due to the poor electrical contact made with the probes and to the high erosion rates of the probe material.

Chase-Around Rig (Build 14404-L-3)

Test Points I-b-2 and I-d-1 - Run Date 20 Dec. 1962

A test of 7.8 minutes was made to obtain data for Test Points I-b-2 and I-d-1. Measured conductivities for Tests I-b-2 and I-d-1 were 6.9 and 11.2 mhos/m respectively and boundary-layer drops for the respective tests were 27 and 10 volts. Ramped electrodes were used.

This test was made to obtain conductivity data for Test Points I-b-2 and I-d-1. Methods 3 and 5 were used to determine conductivity. The basic rig was used, with electrodes in the T_2 configuration. The electrodes were rhodium plated. The electrode contact pins were of tantalum. The 6-element injector and the area ratio 2.0 nozzle were used. Only the three cross-stream voltage probes were used. One thermocouple was installed in the back face of electrode 1.

Nominal and actual conditions for Cycle 1, Test Point I-b-2 are compared in the following table

	P_c	psia	O/F	%K	$\dot{W}_t$ (lb/sec)	$C^*(\text{meas})$ (ft / sec)	$C^*(\text{ideal})$ (ft / sec)	η_{c^*}	B W/m ²
Nominal	60		2.6	2.0	----	-----	-----	----	0
Actual	59.3		2.64	2.04	1.06	4630	5580	0.83	0

Combustion appeared to be stable during the test and no abnormalities were noted. A plot of applied voltage vs current for the third electrode pair is presented in Figure 140. Inspection of Figure 140 shows that the voltage-current characteristic exhibits the previously noted hysteresis. A plot of the temperature at the back face of electrode 1 is

plotted vs time in Figure 141. The thermocouple recording temperature was disconnected when voltage was applied in Test I-b-2. The thermocouple was reconnected after the applied voltage cycle for Test I-d-1. An increase in electrode temperature of about 650 degrees during the test was indicated. Static pressures in the test section were essentially constant during the test. The downstream pressure tap failed to give readings during the test. The static pressures recorded by the upstream taps 1 and 2 are presented in the following table

<u>Pressure Tap</u>	<u>Static Pressure</u>
1	10.6 psia
2	11.2 psia

Chamber pressure and total weight flow were essentially constant during the test.

The conductivity determined from the slope of the descending voltage portion of the voltage-current characteristic for the third electrode was 6.9 mhos/m. Predicted conductivities are shown in the following table

	<u>σ (mhos/m)</u>
Ideal	26.8
Measured heat removed	20.7
Heat removal correction to match C*	0.2
Measured heat removed + unburned fuel correction to match C*	2.7

Cross-stream voltage profiles as recorded with the voltage probes are presented in Figures 142 and 143. The conductivities calculated from the voltage gradients between various probe pairs are shown in tables on Figures 142 and 143. The calculated conductivities show considerable variation during the test and in general look questionable. The boundary-layer voltage drop for the third electrode pair was 27 volts. Average heat flux for the combustion chamber was 0.68 BTU/in²sec while the average heat flux for the nozzle was 2.52 BTU/in²sec.

Actual and nominal test conditions for Cycle 2, Test Point I-d-1 are presented in the following table

	<u>P_c psia</u>	<u>O/F</u>	<u>%K</u>	<u>$\dot{W}_t$ (lb/sec)</u>	<u>C*(meas) (ft / sec)</u>	<u>C*(ideal) (ft / sec)</u>	<u>B η_{c*}</u>	<u>W/m²</u>
Nominal	60	3.2	2.0	----	----	----	---	0
Actual	57.5	3.21	2.04	1.07	4440	5410	0.82	0

Combustion was stable during this test and no abnormalities were noted. A plot of applied voltage vs current for the third electrode pair is presented in Figure 144. The plot is essentially a straight line showing none of the hysteresis effect noted in Test I-b-2. The static pressures recorded on pressure taps 1 and 2 are presented in the following table

<u>Pressure Tap</u>	<u>Static Pressure</u>
1	10.4 psia
2	10.6 psia

The conductivity calculated from the slope of the voltage-current characteristic for electrode 3 was 11.2 mhos/m. Predicted conductivities are shown in the following table

	<u>σ (mhos/m)</u>
Ideal	27.3
Measured heat removed	21.0
Heat removal correction to match C*	0.6
Measured heat removed + unburned fuel correction to match C*	<1.0

The voltage probes failed to operate properly during this test. The boundary-layer voltage drop for electrode 3 was 10 volts. The average heat flux for the combustion chamber was 0.79 BTU/in²sec. For the nozzle the average heat flux was 2.52 BTU/in²sec.

The hysteresis effect noted in the voltage-current characteristic obtained in Test I-b-2 appears to be a result of an increasing boundary-layer temperature. This conclusion is supported by the measured electrode temperature data which shows that electrode temperature had not reached steady state when test I-b-2 was begun. The voltage probe data obtained during Test I-b-2 is questionable at best. It appears, however, that with some refinements in the probe design and installation techniques the voltage probe will become a useful tool for conductivity measurements.

Chase-Around Rig (Build 14404-L-3)
Test II-b-1 - Run Date 21 Dec. 1962

An H₂/O₂ run of 7.6 minutes duration was made to satisfy Test Point II-b-1. Because of a low hydrogen supply pressure the flow could not be maintained constant during the run and the O/F increased from 6.49 to 7.84. The seeding decreased from 2.86 to 1.48 per cent K. The conductivities calculated by Method 1 for electrode 3 are 7.1 mhos/meter at 2.86 per cent K and 6.1 mhos/meter at 1.48 per cent K.

The maximum power generated was 13,700 watts.

Chase-Around Rig 14404-L-3 was run to obtain data for Test Point II-b-1. The purpose of the run was to measure the plasma conductivity with a magnetic field present. The rig was the same as that run on 20 December 1962 for 7.8 minutes. One hundred and thirty two seconds were allowed for warmup and the setting of conditions. The loads were cycled and the rig shut down without a cooldown because of lack of hydrogen. A comparison of nominal conditions and actual conditions at a typical point are given below

	Pc, psia	B measured	W/m^2	O/F	%K	W _t , lbs/sec	C* meas ft/sec	C* ideal adiabatic	η c* adiabatic
Nominal	60.0	2.0	6.0	2.0	---	---	---	---	---
Actual	47.7	2.17	6.49	1.48	.802	4935	7196	0.69	

Because of low hydrogen supply pressure, the weight flow of hydrogen could not be held constant. Consequently the O/F increased from 6.49 to 7.84. An attempt was made to keep the per cent K constant during this time. During the time the 19.5 load was connected, per cent K changed from 2.86 to 1.48 resulting in a reduction in conductivity, and an apparent increase in boundary-layer voltage drop from 8 volts to 30 volts. This can be seen on Figure 145. Later on in the run, when the 3.5-ohm load was connected, performance became very erratic, resulting in loss of data above 24 amperes. A plot of terminal voltage versus current and power versus calculated load resistance for electrode 3 is shown in Figure 145 and 146. The other four electrodes show similar trends. The readings on the Hall voltmeters indicate the electrical integrity of the rig was good. Typical values during the run are given below

	Flameshield			Interelectrode Voltage							
	V _t No.3	I _y No.3	to Ground	TOP				BOTTOM			
	(Volts)	(Amps)	(Volts)	1-2	2-3	3-4	4-5	1-2	2-3	3-4	4-5
Open Circuit	332	0	120*	0	0	0	0	0	0	6	8
Maximum Power	235	11.7	264*	24	20	20	20	12	14	12	13

* Data from flameshield to ground is questionable since it does not agree with the interelectrode Hall voltages

The Hall meter readings at short circuit conditions are not given because the ammeters were pegged at 30 amperes at a load of 1.5 ohms. The values measured for the interelectrode voltage indicate that the electrical integrity of the rig was good. No detailed analysis of the Hall field will be made. Static pressure instrumentation failed to give data.

No predicted conductivities have been made because of changing test conditions. The maximum power generated for the test was 13,651 watts. The ideal total power out for the conditions of this run based on a reasonable predicted conductivity and predicted velocity is 34,200 watts. The ideal total power was calculated from formula

$$P_{\text{MAX IDEAL}} = \frac{(C_v u B H)^2}{4 R_{\text{opt}}} \quad \text{where} \quad R_{\text{opt}} = \frac{H}{\sigma A_f}$$

where C_v = nozzle velocity coefficient

The measured power output is then 39.5 per cent of ideal and yields a power density of 358 watts/cubic inch.

A plot of power versus calculated load resistance for electrode 3 is shown in Figure 146. The other four electrodes show similar trends. The indicated boundary-layer voltage drop was small for all electrodes. The data for a point representing conditions during the load cycle are given below

	$\dot{Q}$, BTU/sec	$\dot{Q}/A$, BTU/in ² sec
Combustion chamber	614	3.6
Nozzle	258	5.0

The heat transfer rates are comparable with the H_2/O_2 power run made on 19 December, 1962.

The total run time on this rig was 15.4 minutes. Figures 147 and 148 are photographs of the rig after teardown. As can be seen, damage to the sidewalls was extensive. A discussion of the probable causes is given in the writeup on the run of 19 December, 1962.

This run was an attempt to obtain data for Test Point II-b-1. However, the actual run conditions during this test were appreciably different from the nominal conditions required and they varied considerably during the run. The run data can not be used for Test Point II-b-1.

Chase-Around Rig (Build 14394-L-5)
Tests I-a-3 and II-a-3 - Run Date 27 Dec. 1962

A run of 18.4 minutes duration was made to satisfy Test Points I-a-3 and II-a-3. The measured conductivity for the first cycle on Test I-a-3 was 6.9 mhos/m for electrode 3. When this point was repeated later in the run the measured conductivity was 10.9 mhos/m. The voltage-current characteristic seems to be a strong function of electrode temperature. For Test II-a-3 the conductivity with magnetic field was 3.2 mhos/m. The maximum power generated was 2800 watts. Low power output was attributed to low flow velocity and to shorting of the Hall potential due to failure of the electrical isolation of the exhaust flame shield. The sidewall durability was very good. The improved performance is attributed to the removal of the Al_2O_3 dowel pins.

The firing included three cycles. The first cycle was made to obtain conductivity data for Test Point I-a-3. The second cycle was made to obtain conductivity data for Test Point II-a-3. The third cycle was made to obtain conductivity data for Test Point I-a-3, in an attempt to show the dependence of the voltage-current characteristic on time transients such as electrode temperature. The rig was Chase-Around Rig 14394-L-5. It was the standard rig with the following exceptions:

- 1) The flat electrodes were rhodium-plated in the contact area. These electrodes had been used previously. The electrode contact pins were tantalum.
- 2) The Al_2O_3 dowel pins between the individual sidewall blocks were omitted.
- 3) The gasket on the spring cover housing was Silastic. All other gaskets were Teflon.
- 4) A thermocouple was located behind the first electrode on the top.
- 5) There were no pressure taps on this rig.
- 6) The nozzle nominal area ratio was 2.0.
- 7) The 6-element injector was used.

Standard procedure was followed throughout the run except that in Cycle 3 the voltage was varied up and down successively several times. The nominal and actual run conditions are shown below. They were the same for all three cycles.

	P_c (psia)	O/F	%K	$\dot{W}_t$ lbs/sec	$C^*_{meas.}$ (ft / sec)	$C^*_{ideal-adiabatic}$ (ft/sec)	η_{c^*}
Nominal	60	2.6	2.0				
Actual	60.8	2.72	1.9-2.2	1.10-1.11	4564-4528	5572-5552	0.82

The KNO_2 flow varied during the run. The period of the variations was about 20 seconds. The variation is attributed to an improperly bled KNO_2 tank. The O/F and chamber pressure remained steady throughout the run. The thermocouple in the electrode failed. The heat transfer data is given below. It was taken at 500 seconds after firing. The data remained constant throughout the run.

	<u>BTU/sec</u>	<u>BTU/in²sec(average)</u>
Chamber	148	0.87
Nozzle	127	2.45

The voltage probe data exhibited considerable scatter and is considered to be unreliable.

The plot of applied voltage versus current to the plasma for Cycle 1 is shown in Figure 149. The slope of the curve changes quite markedly until currents greater than 9.5 amperes are reached.

The conductivities as determined by Method 3 are

Electrode	1	2	3	4	5
σ (mhos/m)	6.0	6.6	6.9	7.9	10.0

The conductivities above were determined for currents above 9.5 amperes. The boundary-layer voltage drop was 47 volts for electrode 3.

The magnetic field for Cycle 2 was 2.17 webers/m². The open-circuit voltage averaged 236 volts for electrodes 1 to 4 corresponding to a velocity of 4410 ft/sec. This velocity is much lower than theoretical prediction based on perfect one-dimensional expansion which is, $V_{oc\text{ ideal}} = 368$ volts. The plot of terminal voltage versus current to the load for electrode 3 is shown in Figure 150. The average boundary-layer voltage drop was 31 volts. The average of the interelectrode Hall voltages is given below along with the flameshield-to-combustion chamber Hall voltage.

	<u>Average of Interelectrode Hall Voltages (volts)</u>	<u>Flameshield-to- Combustion Chamber (volts)</u>
Open circuit	0	0
Maximum power	5	12
Short circuit	8	48

The conductivities as calculated by Method 1 are

Electrode	1	2	3	4	5
σ mhos/m	2.3	2.5	3.2	3.8	5.1

The maximum power generated was 2800 watts. The maximum power point occurred at a load resistance of 14.3 ohms. This indicates an average plasma conductivity of 3.9 mhos/m.

The plot of voltage applied versus current for Cycle 3 (a repeat of Cycle 1) is shown in Figure 151. The shape of the curve is the same as that observed in Cycle 1 and the region of constant slope once again occurs when the current is greater than 9.5 amperes. The conductivities as determined by Method 3 for points with current above 9.5 amperes are given below

Electrode	1	2	3	4	5
σ mhos/m	11.1	10.7	10.9	11.8	12.3

The boundary-layer voltage drop for electrode 3 was 96 volts. Shorting on electrodes 4 and 5 was observed at the end of this run. Figures 152, 153, and 154 are pictures of the rig after the run. Severe arcing damage can be seen on bottom electrodes 4 and 5. Arcing damage was also noted at the rig entrance. Current apparently flowed from the electrodes via the arc to the electrode backup structure and then back to the plasma through the arc at the rig entrance. The arc may have been started by breakdown of the BN and BeO resistivity because of water spraying into the rig from the flameshield. Aside from the arcing mentioned, which must have occurred very near the end of the run, the sidewalls performed very well.

The predicted conductivities are listed below. The predictions are the average of predictions for the two seed flow rates.

	σ_o	σ'	σ_t
Ideal	25.7	25.6	5.9
Measured heat removed	20.8	20.3	4.6
Heat removal approximation to match C*	<1.0	<1.0	<1.0
Unburned fuel plus measured heat removed to match C*	1.3	1.3	<1.0

As seen, the predictions bracket the measured conductivities but do not come close to hitting the measured conductivity.

The voltage-current characteristics for the applied voltage conductivity run (Cycles 1 and 3) demonstrate that the effect of boundary layer is important. It appears that there is a change in boundary-layer resistance with current and time. This change could be due to ohmic heating of the boundary layer or nonequilibrium ionization effects in the boundary layer. Since the maximum current of the power run data falls below electrode currents of 9.5 amperes, the main effect seen in the data is probably that of the boundary layer. The power generated was quite low. Reasons for the low Hall potential and the low flow velocity: The low flow velocity could be a result of poor alignment between rig and nozzle. There is often a slight step as the plasma enters the MHD section. This will cause at least weak oblique shock waves at the rig entrance resulting in a reduction in velocity and increase in static temperature and pressure.

4. Conclusions

Summary Tables I and II give pertinent data on the runs, rigs and results. The major effort was spent in developing a dependable, durable MHD test rig of practical configuration which had the necessary mechanical and electrical integrity. An MHD generator of novel design (chase-around rig) using very high temperature refractory ceramic, insulating sidewalls and electrodes was conceived and built. Its successful operation provided information on the performance, as it affected MHD generators, of the injector, combustion chamber, nozzle and other components. The rig also revealed the need for improvements in instrumentation and recording apparatus because of the extreme conditions present.

Efforts to remedy the various deficiencies in equipment were begun immediately and analytical procedures and corrections in some cases were accomplished in time. In general, however, the needed corrections could not be made within the time remaining in the contract period.

In spite of the knowledge that improvements were needed in various areas before truly refined data of high reliability was obtained, the tests that had been proposed were run as scheduled, since some of the more important improvements would have involved extensive developmental effort. In some cases, as can be seen in Summary Table I, more than one test was made in an attempt to satisfy the condition. The data is presented and compared with theory. However, the lack of a proven method of compensating for combustor inefficiency, one of the most powerful factors affecting MHD generator plasma conductivity, precluded a complete experimental verification of the plasma conductivity theory. The lack of knowledge of the state of the thermochemical

equilibrium also contributed to the uncertainties of the corrected theoretical values. In spite of these deficiencies, values of conductivity were measured during JP4 fueled runs. The measured values of conductivity were bracketed by theoretical predictions which included corrections for heat loss and combustion inefficiency made by two different methods. Measured values of conductivity made during hydrogen-fueled runs agreed almost exactly with theory uncorrected for heat loss or combustion inefficiency.

The lack of repeatable, reliable voltage probe data precluded experimentally obtaining plasma conductivity by two semi-independent methods. Thus the assumptions regarding the electrical characteristics of the boundary layer could not be verified. Two powerful mechanisms by which apparent boundary-layer voltage drop could be appreciably reduced were observed, however. One was the result of a deliberate attempt to decrease boundary-layer voltage drop by increasing boundary-layer energy and temperature through the use of ramped electrodes. The other and more powerful effect noted was the influence of high O/F on boundary-layer voltage drop, at least with ceramic carbide electrodes. The free oxygen present at the higher values of O/F apparently reacted with the carbides, oxidizing them and, in the process, adding thermal energy to the boundary layer, decreasing its resistance.

The experience gained during this effort provided the basis for future project activity.

5. Recommendations

Analysis of the test runs discussed previously has shown that many improvements in hardware, instrumentation, test procedures, and theoretical prediction methods must be made in order to achieve better agreement between theory and experiment. These improvements will also aid in the development of practical, larger-sized generators. The most important of the areas needing further study are noted here.

a. Hardware

Hardware improvements to components other than the MHD channel needed to refine experimental rigs are noted below. Improvements needed in the MHD channel structure are discussed below under Structural Concepts.

b. Nozzles

One problem encountered with the MHD rig has been that of aligning the MHD channel with the nozzles so that flow in the MHD channel is perfectly axial. Another problem associated with the nozzles is that they expand and warp as their temperature increases. Nozzle design should be improved so that nozzle dimensions change very

little or at least so that they do not warp because of the high heat flux encountered during testing. Design of the rig and/or nozzles should also allow for adjustment so that the rig and nozzle can be aligned easily and accurately. Nozzle design should include consideration of the variation of γ with axial distance as the plasma expands through the nozzle.

c. Combustion Chamber

To help reduce heat loss in the chamber it is suggested that a thermally insulating liner be used in the combustion chamber. Increasing the chamber-to-throat area ratio will reduce the heat flux per unit area and the chamber Mach number. Thus by increasing the chamber area it may be possible to decrease total heat loss and increase residence time with a resulting increase in combustion efficiency. Optimum sizing of the chamber area should be investigated analytically and experimentally.

d. Injector

A more efficient injector design must be found. The C^* efficiencies obtained with the current designs are so low that conductivity predictions cannot be made with any degree of assurance. Injectors must also produce a constant O/F and per cent K over a cross-section of flow so that these variables are known over the region in which the conductivity measurement is being made. It is also highly desirable to have an injector-combustion chamber combination which will provide a constant C^* efficiency regardless of chamber pressure, O/F, per cent K and nozzle area ratio.

e. Procedure

The time required to reach steady-state wall temperatures and the optimum warmup procedure should be investigated thoroughly by experimental and analytical techniques. It appears that addition of a significant percentage of nitrogen flow to the hydrogen/oxygen warmup combustants would decrease the total temperature and decrease the rate of increase of hot wall temperature. By experimentation, a warmup procedure could be found that would minimize the time required to reach steady-state wall temperature without damaging the rig through thermal shock.

f. Instrumentation

There is, of course, room for improvement in the reliability, resistance to high common mode voltages, and in the accuracy of much of the instru-

mentation. In addition to this obvious recommendation, more instrumentation, particularly with regard to electrode temperature measurement, static and total pressure measurement, and voltage probe measurements would be helpful in interpreting the test results.

Total pressure should be measured at the downstream end of the combustion chamber in order to determine experimentally the pressure losses associated with burning finite sized fuel droplets at finite velocities, and in order to correlate the theoretical predictions of total pressure loss.

The unreasonable static pressure gradients (decreasing pressure in the initial portion of the supersonic duct) noted, at times, in the rig give rise to suspicion of the static pressure data. This problem may be solely a result of faulty sealing of the pressure tap connections, but the problem should be investigated.

Experience with the chase-around rig has demonstrated that electrode temperature may increase by as much as 600°F during an applied-voltage cycle. The electrode temperature seems to have a large effect on the voltage current characteristic and the conductivity determined from the slope of this characteristic. It is suggested that instrumentation be developed that will allow accurate determination of electrode temperature when high common mode voltages are present.

Recommendations regarding the use of voltage probes, voltage probe instrumentation, and the AC conductivity probe are discussed in the section on Special Support Work.

g. Theoretical Predictions

As noted previously some of the important influences on MHD performance are:

- 1) Atomization, vaporization and acceleration of the liquid fuel droplets,
- 2) Nonuniform composition across the injector and MHD channel, and,
- 3) Finite reaction rates.

The first of these influences is discussed in the references* and in the combustion section of this volume. The second is also discussed in the combustion section. The third influence is discussed in Section III. A. above. It is quite possible that more thorough consideration of these effects may help to explain some of the discrepancies between theory and experiment. It is recommended that they be investigated thoroughly.

It is suggested that experimental methods of determining local plasma temperature and composition be investigated. Catalytic and spectrographic methods of determining plasma composition should be investigated. By using a sampling probe it would be possible to experimentally determine the O/F distribution. This area should be investigated. In general, then, improvements in MIID performance and prediction can be realized by refining the hardware and instrumentation, developing more realistic prediction methods, using new measurement techniques (voltage probes, AC conductivity probe) and by measuring more parameters (plasma temperature and composition). Another area that should receive further analytical and experimental attention is the boundary layer. This area should be studied extensively with regard to heat transfer and voltage drop.

* NASA Technical Report R-67, Propellant Vaporization as a Design Criterion for Rocket Engine Combustion Chambers, Richard J. Preim and Marcus F. Heidman

Stable Combustion Processes in Liquid Propellant Rocket Engines, 5th Colloquium of the Combustion and Propulsion Panel NATO, April 1962, S. Lambiris, L. P. Combs, and R. S. Levine

B. Structural Concepts

1. Introduction

The purpose of this section is to present the results of tests by which information on structural concepts for use in MHD generators was obtained. The results contributed to the design of the 1000 and 250-megawatt generators reported in Volume VI of the final report. Tests were run in MHD generators designed to generate high power densities using hydrocarbons or hydrogen as fuel with oxygen. The reactants produced plasma temperatures up to 6000°R. The flow Mach numbers ranged from 1.7 to 2.4 which represents flow velocities up to approximately 6,700 ft/sec. The flows were seeded with a hydrous solution of potassium nitrite to enhance electrical conductivity. Voltage gradients due either to the self-induced EMF through the application of magnetic fields to values of over 2 w/m² or to externally applied voltages approached 1500 volts/ft. Several different structural configurations including the use of numerous materials were investigated in an effort to develop a structural concept capable of meeting the requirements of the mission and environment.

MHD generators using alkali metal-seeded plasmas produced by the combustion of a hydrocarbon fuel with oxygen must operate with plasma temperatures in the range of 4500°R to 6000°R to obtain practical values of electrical conductivity. Plasma velocities must be very high since power output is proportional to velocity squared. Optimization studies reported in the System Analysis section of this volume show that the flow Mach numbers should be supersonic and in the range of 2 to 3. The same studies also show that plasma static pressures in the MHD generator channel should be about 8 psia. This combination of static pressure, Mach number and composition dictates a combustion chamber pressure ranging from 60 to 400 psia depending on the application. The heat fluxes resulting from this combination of conditions approximate those experienced, for short times, by the nose

cones of re-entering intercontinental ballistic missiles and earth satellites (50-300 Btu/ft² sec). The MHD generators must withstand these conditions for long periods of time and maintain not only mechanical but electrical integrity as well. The strong adverse effect which small amounts of electrical leakage can have on performance is discussed in Volume VI of the final report and will not be repeated here.

Because the generators must have mechanical as well as electrical integrity, both areas must be considered in the design of all components*. The electrode walls are required to operate at high temperatures ($T > 3000^{\circ}\text{R}$) to minimize boundary layer voltage drop, and must withstand intense erosive and corrosive conditions as well as severe thermal stress and thermal shock, with no loss in interelectrode electrical insulation or insulation from ground. The development of electrode walls also requires detailed examination of electrical leakage through insulators, leakage through debris (seed and/or combustion products) along insulator surfaces due to conducting deposits and arcing to sidewalls and conducting and/or grounded support structures. Gas-cooled metal electrodes require consideration of surface temperature measurement, surface temperature control, sealing of high pressure coolant, protection of seals from the hot gas stream, thermal stresses between unlike metals, warpage and embrittlement of welds, brazes, or electrode metal due to interaction with the coolant at high temperatures. Ceramic electrodes call for the investigation of erosion by the supersonic corrosive plasma, reaction with the plasma and other components, oxidation, resistive and/or conductive coating formation, thermal stress and shock, investigation of surface temperature control with the use of electrode backups with differing thermal conductivity and/or radiation gaps and fabrication problems.

Sidewalls must meet even more stringent requirements in order to perform as desired. Besides mechanical stress, thermal gradients and high velocity, reactive atmosphere, the sidewalls are subjected to large voltage gradients. The development of

* See Volume VI of the Final Report for an extensive discussion of the problems encountered due to the operating conditions in MHD generators

practical metal sidewalls requires the investigation of types of coolants and rate of cooling, prevention of warping, arcing from electrode to sidewall and magnetic permeability. Metal sidewalls must use an insulating coating to prevent shorting. The technology of refractory coating must be mastered. Methods of application, materials to be used, resistivity at high temperatures, porosity, adherence and scale-up problems are areas to be investigated in the field of coatings. Ceramic sidewalls have problems similar to ceramic electrodes. Warpage, contact with cooling surfaces, high dielectric strength coolants, allowance for thermal expansion, toxicity, plus the majority of the items mentioned for ceramic electrodes, all must be considered in the development of a useful sidewall.

The joining of the electrode wall to the insulating sidewall has unique problems which must be overcome. The allowance for thermal expansion in both walls, while maintaining minimum space between components to prevent collection of conductive debris are requirements which must be met. The housings for the electrode and sidewalls must be insulated from each other and from ground, must be properly cooled and must be fabricated in such a way as to minimize distortion due to thermal and mechanical stress.

2. Discussion

The results of tests which yielded information on structural concepts are discussed in the order in which they were run. Various problem areas are discussed along with solutions attempted to overcome weaknesses revealed in the tests. Some of the tests mentioned here are also discussed in the section on Plasma Conductivity Measurements. The evolution of rig design, construction and operation as experience was gained is illustrated. Unless otherwise noted, the tests were conducted with JP4 and O₂ at O/F's ranging from 2.0 to 4.0, chamber pressures from 40 to 90 psia and seed percentages from 1.0 to 4.0.

22 November 1961
Build 14221-L-1

The first Mho* rig run was built as follows:

*Refers to the 1.65 x 3.30 x 7.0 inch channel MHD test generator
(15 KW nominal output)

- 1) Hydrogen-cooled nickel electrodes (keystone type, Figures 155, 156)
- 2) Hydrogen-cooled molybdenum sidewalls, 0.005 inch Rokide A coating
- 3) Water-cooled, A/A*=2, Inconel nozzle

There was a fire 3 seconds after ignition. The rig was shut down and pressure checked. Large cracks were found in the molybdenum sidewalls (Figures 157 and 158). Failure was believed due to the preloading of the sidewalls at assembly and to thermal shock which cracked the extremely brittle molybdenum. Because of the difficulties of fabricating molybdenum (brittle, hard, difficult to braze, pieces crack during braze cycle), nickel and stainless steel sidewalls were used for all subsequent runs of this type of rig.

20 December 1961
(Build 14221-L-2)

Build 2 of the keystone electrode rig was assembled as follows:

- 1) Hydrogen-cooled nickel electrodes
- 2) Hydrogen-cooled nickel sidewalls. Al_2O_3 coating put on with plasma gun rather than Rokide process
- 3) Water-cooled Inconel nozzle

The test was to be an applied-voltage conductivity test but prior to the intended shutdown there was a fire and the run was terminated. Inspection showed the interelectrode gaskets were burned through, allowing hydrogen to leak into the test section. The Al_2O_3 coating cracked in spots. The Inconel nozzle collapsed when the combustion chamber and nozzle coolant (H_2O) was overpressurized. Subsequent runs used copper-lined nozzles because of the Inconel nozzle's prohibitive cost and marginal thermal characteristics.

17 January 1962

Samples of silicone resin varnish, Al_2O_3 , nickel-chrome spinel, BeO and insulating paint were submitted for dielectric strength tests to help evaluate the applicability of these materials as insulators.

30 January 1962
(Build 14279-L-1)

The alternating-sidewall rig was tested. The 1.65-inch wide wall was made up of BeO blocks. The 3.30-inch wide wall contained

five pairs of 0.7-spaced graphite-bonded ZrC electrodes insulated from the housing by Al_2O_3 blocks (see Figure 159).

Four of the ten electrodes blew out on startup and those remaining were found to be cracked. The Al_2O_3 backup block was badly cracked from thermal stress and mechanical shock. Subsequent runs were not attempted since the design would allow complete failure of a sidewall or electrode assembly if any one component cracked and fell into the stream.

16 February 1962
(Build 14221-L-4)

A modified version of the keystone electrode rig was run. The rig was built as follows:

- 1) Electrodes - Runs 1 and 2 spring-loaded zirconium carbide, Runs 3 and 4 spring-loaded PO_3 graphite, power takeoff studs - mild steel
- 2) Coolant bypass - stainless steel - coated with 0.020-inch Rokide A, interelectrode spacers - BeO
- 3) Sidewalls - hydrogen cooled - plasma sprayed with 0.010-inch Al_2O_3 0.040 inch had to be removed from Armco iron sideblock inserts to allow for warping of the sidewalls on previous runs

For Runs 1 and 2, total run time was 70 seconds and maximum power generated was 8 kilowatts. Runs 3 and 4 lasted a total of 99 seconds and a maximum of 3 to 4 kilowatts was generated. The difference in power was probably due to deteriorated rig electrical integrity and to the cool boundary layer formed because of ablating graphite electrodes.

A post-test parts inspection revealed that plasma-sprayed Al_2O_3 on dummy end electrodes and sidewalls was partially removed. The Al_2O_3 spacer between the electrode wall and sidewall (11 inches long) was shattered. It was noted that these spacers broke almost every time the rig was run. This was probably due to distortion and thermal stress while running. The piece was segmented on later runs. After both sets of runs, the majority of the electrodes were cracked and several BeO interelectrode spacers had cracked and fallen out.

5 April 1962
(Build 14279-L-4)

A power run was made using the modified keystone electrode rig.
The parts used were:

- 1) Electrodes - ZTA graphite
- 2) Interelectrode spacers - BeO
- 3) Sidewalls - hydrogen-cooled stainless steel coated with
0.020-inch plasma-sprayed Al_2O_3

A maximum of 3000 watts was generated during the 49-second run. A shorted Hall field and a cool electrode boundary layer due to the ablating carbon electrode is the probable reason for the low power.

A post-test parts inspection found all ZTA electrodes badly eroded. One electrode was broken and most of the BeO inter-electrode spacers were broken. A conductive coating found on the BeO spacers was analyzed and found to be pure carbon (see Figures 160 and 161). It was not possible to determine whether the carbon was from electrode erosion or from unburned carbon in the gas flow. This conductive coating, in effect, made the electrodes a continuous conducting slab with no segmentation and was the cause of the Hall field short circuit. The coating indicated the electrode wall spacers were running too cold. The stainless steel sidewalls were badly warped. A hydrogen leak developed between the electrode coolant bypass and the dummy end electrode causing a small external fire.

9 May 1962

The initial test results of dielectric strength tests of various coatings were:

	<u>Test Weight</u> <u>Corner Radius</u>	<u>Breakdown</u> <u>Voltage</u>
aluminum oxide, 0.010-inch coating at 70°F	sharp corner 0.060 inch	3800-4000 3800-4600
beryllium oxide, 0.010-inch coating at 70°F	sharp corner 0.060 inch	1500-1600 2500-3000

	Test Weight Corner Radius	Breakdown Voltage
nickel chrome spinel, 0.003-inch coating at 70°F	Sample broke down at very low voltages	
silicone resin varnish (Dow-Corning 806A) 0.002-inch coating at 70°F	sharp corner 0.060 inch	1000 1000
silicone resin varnish (Pratt & Lambert 480)	sharp corner	2000
0.004 inch coating at 70°F	0.060 inch	greater than 4600
0.004 inch coating at 500°F	sharp corner 0.060 inch	800-1300 700-1000

5 June 1962

(Build 14307-L-1)

This first run of a toadstool electrode rig was made to measure scalar plasma conductivity and to test the structural integrity of the rig. The electrodes were ZTA graphite. The electrode-retaining stems were tantalum coated with TaC. An attempt was made to decrease the cracking of parts noted in past runs by using a slow hydrogen-oxygen warmup prior to the run on JP4 and O₂. The total run time from ignition to shutdown was 127 seconds.

A post-test examination showed the sidewall insulating coating was undamaged but heavily coated with carbon. This allows detrimental leakage currents and indicates the sidewalls were running too cold. The estimated surface temperature was 1350°F.

The tantalum carbide on the plasma-side surface of the electrode retention pins was missing and the pins were eroded (Figure 162). The BeO interelectrode spacers were badly cracked (Figures 162 and 163). An inferior shipment of BeO was believed to be the cause of the spacer failures. The electrode coolant passage was badly warped.

The use of N_2 as a coolant for the electrode wall was an improvement since the fire problem was eliminated but the higher flow rates necessary required higher pressures. The O-ring seal used between the electrode cover plate and electrode coolant passage made it very difficult to seal the 500 psi nitrogen coolant. Later rigs were modified by welding tubes to the coolant passage.

18 June 1962
(Build 14221-L-6)

The keystone electrode rig used in this test was built as follows:

- 1) Hydrogen-cooled segmented nickel electrodes
- 2) Hydrogen-cooled stainless steel, coated with 0.010-inch Rokide A
- 3) Thermocouples were brazed to the face of electrodes No. 3-top and bottom

Total run time on this conductivity rig was 102 seconds. A post-test parts inspection revealed the Rokide coating was missing from the dummy end electrodes (see Figure 164). The aluminum oxide interelectrode spacers had cracked and fallen out. There was arcing between electrodes and sidewalls (Figure 165). During later runs all sharp corners in the joint between the electrode and sidewall were rounded to decrease arcing.

25 June 1962
(Build 14279-L-6)

This run, discussed in another section of this Volume, was the third configuration of the keystone electrode rig. Figure 71 illustrates one of the basic problems with this rig. Contact with the electrode was made by spring loading a tantalum pin on the electrode backface.

This method of contact superposed loading stresses on the induced thermal stresses in the electrodes and caused them to fail. Another problem with this method was that the contact resistance between the electrode and contact pin was much higher compared to other methods.

Figure 70 illustrates other problem areas. A heavy coating of ZrO_2 builds up on the ZrC electrodes. The zirconia coating acts as a resistance of undetermined value. The sprayed coating on the sidewalls obviously lacks mechanical integrity without which electrical integrity is impossible.

20 July 1962

(Build 14307-L-2)

This second run of the toadstool electrode rig was built as follows:

- 1) Graded ZrC , integral-pin electrodes with ramps
- 2) Sidewalls were hydrogen-cooled stainless steel with a 0.020-inch stabilized zirconia coating, Astroceramed and stoned smooth

The total firing time for this power run was 172 seconds. Maximum power generated was about 1500 watts. The electrodes were made in graded layers (pure ZrC in the front face to 20 per cent graphite in the back face) in an attempt to reduce cracking due to thermal stresses. Carburized tantalum pins were imbedded in the electrode to eliminate contact resistance. The carburized pins were very brittle and easily broken. Apparently, in the manufacture of the integral-stem electrode, the tantalum was brought above the recrystallization temperature during the hot-pressing cycle, allowing the crystal size in the tantalum to increase. In some cases, crystal sizes were almost half the diameter of the pin (see Figure 166).

The sidewalls were coated with lime-stabilized zirconia to improve the adherence of the coating. It was hoped the sidewalls would run cold enough to keep the resistivity of the zirconia at an acceptable level without causing undesirable condensation of carbon and seed products. The purpose of covering the coating with an Astroceram paste and stoning it smooth was to fill the pores of the zirconia coating. All sharp corners between the sidewall and electrode were rounded to reduce the tendency to arcing. A post-test examination of the parts showed that the zirconium carbide electrodes were heavily coated with zirconium oxide (see Figure 167). The tantalum pins were badly oxidized. The stabilized coating had chipped but exhibited better mechanical integrity than previously-used coatings. There were no arcing marks.

13 August 1962
Build 14301-L-1

This first run of the button-electrode configuration is also discussed in another section of this volume. The purpose of this electrode configuration was to allow measurement of the plasma core conductivity without errors due to blooming effects and leakage to the sidewall. The electrode and electrode plate were made of zirconium carbide. The sidewalls were hydrogen-cooled stainless coated with 0.010-inch Rokide A. After a successful conductivity run of 150 seconds, a post-test inspection of parts was made (see Figures 69 and 73). The main electrode plate was cracked in several places and there was a ZrO_2 coating on the buttons and plate.

The effect of cracking of the plate could be minimized by using a plate made of several pieces with a provision for maintaining electrical continuity among the segments.

The heavier coating on the buttons probably resulted from the fact that they were slightly recessed and out of the sweeping action of the plasma. The formation of the resistive ZrO_2 coating on ZrC appears to be a basic fault of the material. However, further investigation should be made into its use in MHD generators because of its good erosion resistance.

28 August 1962
Build 14394-L-1

Because of the problems mentioned above, a new rig configuration, the "Chase-Around Rig", was built and run. An extensive description of the rig is given in Appendix 6 (see Figure 168). The total run time for this first run was 278 seconds including warmup and cooldown. The rig behaved very well. The rig endured an applied voltage of 200 volts and currents up to 25 amps per electrode with no evidence of arcing.

A post-test parts inspection showed portions of the leading edge of one of the sidewall entrance pieces was broken away and had fallen out. (See Figure 169). All BeO sidewall blocks were cracked or broken. None of the cracked blocks fell out, thus proving the idea of retaining brittle ceramics by compressive forces and of using redundant supports (i. e., the Al_2O_3 pins) to maintain the mechanical integrity of the ceramic structure*. All the electrode spacers re-

*During some runs these Al_2O_3 dowel pins melted, indicating that the BeO sidewall blocks were operating at temperatures above 3720°F

mained intact although some were cracked (see Figure 170). The difference in the amount of cracking between the sidewalls and inter-electrode spacers is noteworthy. It indicates that the size of the BeO has an influence on its ability to resist cracking (i. e. the smaller pieces sustain less cracking). A clear siliceous coating covered the interelectrode spacers and the spacers downstream of the electrodes (see Figures 75 and 170). On this and some of the subsequent runs, when the coating was present, it showed definite evidence of having flowed downstream from the electrodes and over the spacers.

An analysis by x-ray diffraction showed the coating to be silicon dioxide whose melting point is 3056°F, thus indicating that the BeO spacers and electrodes were operating at or above this temperature. The electrodes themselves were in excellent condition with no electrodes cracked or broken and only the slightest erosion being noted. The backs of the electrodes were pitted in a circular area where the carburized tantalum pin contacted the electrode. The pitting may be a result of the relatively high contact resistance between the pin and the back surface of the electrode which would result in very severe local temperatures, gradients, and induced thermal stresses. Later runs used rhodium-plated electrodes in an attempt to reduce the contact resistance. All other parts were in excellent condition with no arcing evidence noted.

24 and 29 October, 1962

Builds 14404-L-1 and 14406-L-1

Two chase-around rigs approximately identical except that one was built with silicon carbide electrodes and the other was built with ZTA graphite electrodes were run in an attempt to compare effects of electrode materials. The electrical results of these runs are described in detail in the section on experimental plasma conductivity measurements.

While there was relatively little difference in the apparent plasma conductivity, there was a marked difference in the power generated and the boundary-layer voltage drop. The ZTA graphite-equipped rig generated 1.2 KW and had a boundary layer drop of over 175 volts while the rig equipped with silicon carbide electrodes generated 3.6 KW and had a voltage drop of about 80 volts. The high boundary layer voltage drop is probably due to cooling and contamination of the boundary layers by ablation of the graphite.

There is also a marked difference between the erosion-resistant properties of the silicon carbide electrodes and the ZTA graphite electrodes. Figures 103 and 104 are photographs of the rigs after running. Both rigs had gone through two separate runs, one for power generation and one with applied voltage for conductivity measurements. The graphite electrodes were almost completely eroded away after 11.5 minutes of running while the silicon-carbide electrodes showed no appreciable wear after 16 minutes.

15 November, 1962

Build 14422-L-1

This endurance run of the chase-around rig was built as follows:

- 1) The electrodes were ramped SiC - 25 per cent graphite. The ramps were curved to eliminate sharp corners in the flow and so minimize shock wave strength (Figure 171)
- 2) The electrode backups were ungrooved boron nitride arranged to give electrode temperature T_2 (see Figure 78)
- 3) Contact with the electrodes was made with tantalum pins
- 4) The back surface of the electrodes were coated with 0.0005 inch rhodium

Excluding the above modifications the rig was standard. The measured run conditions are given below

P_c (measured) psia	- 58.6
O/F	2.6
% K	1.74 - 2.02
B w/m ²	2.33

The length of the run was 86 minutes and power was produced for 78 minutes. The maximum power generated was 13,000 watts. This was generated with only four electrodes operating since electrical continuity was lost on Electrode 5. Had the faulty electrode operated, the maximum power would have been 16,000 watts.

Figures 172, 173, 174, 175, and 176, are photographs of the rig after the run. As can be seen, the erosion, in general, was quite bad. However in Figures 173, and 174 note that the upstream sidewall pieces were not eroded at all while the downstream pieces are badly eroded.

It is believed that the aluminum oxide pins (between the sidewall block), which have a relatively low melting temperature (3720°F) compared to the melting point of BeO (4658°F), melted and flowed downstream over the BeO, combining with it to form a lower melting point complex. In Figure 174 the fact that the Al_2O_3 melted and flowed downstream over the BeO blocks is evident. An American Ceramic Society paper* contains an Al_2O_3 -BeO liquid-solid phase diagram which shows the marked effect of a small percentage of dissolved Al_2O_3 on the melting point of BeO. The Al_2O_3 pins were to be replaced with BeO pins to prevent recurrence of the melting but were not available within the contract period. The interaction of the BeO and Al_2O_3 indicates the area of high temperature material compatibility that must be investigated thoroughly if the run length of MHD generators is to be significantly increased.

28, 29, 30 November 1962
(Build 14403-L-2)

The chase-around rig was fired five separate times on these dates. The A/A* 3.75 nozzle was used on 28 November for the first firing which lasted 8.9 minutes. This firing met the test objectives and is described in another section of this volume.

On 29 November three firings were made. Because of various problems, the runs were unsuccessful. The reasons were:

- first firing - chamber pressure did not record
- second firing - ammeter switches burned out
- third firing - loss of electrical contact between the electrodes and the contact pins. This was caused by excessive arcing and burning of the contact pins

The contact pins were replaced and another firing was attempted on 30 November. This run was terminated because of a loss of electrical integrity in the rig.

*Phase Diagrams for Ceramists, Lavin, McMurdie and Hall, The American Ceramic Society, Columbus, Ohio

This rig accumulated 35 minutes firing time on five separate firings. Although the last firing demonstrated that the electrical integrity of the rig had been destroyed, its mechanical integrity was still good. This demonstrates that the chase-around design incorporates the ability to withstand the severe thermal shocks associated with startup transients.

19 December, 1962

Rig 14403-L-3

This first power run of the chase-around rig using hydrogen and oxygen was built as follows:

- 1) The electrodes were ramped SiC - 25 per cent graphite
- 2) The electrode backups were grooved boron nitride arranged to adjust electrode temperature to T_4 (see Figure 78)
- 3) Contact with the electrodes was made with tantalum pins

Excluding the above modifications the rig was standard.

Approximately 18,000 watts* was generated in spite of the fact that, theoretically, H_2 is not as good a fuel for MHD purposes as JP4. A detailed analysis of the electrical portion of this run is made in another section of this volume.

The total run time on this rig was 32.6 minutes, Figures 132 and 133 are photographs of the rig after teardown. As can be seen, damage to the sidewalls was extensive. There are two possible reasons for the heavy damage: 1) the high temperatures melted the Al_2O_3 pins which combined to form a lower melting point complex, or 2) literature** suggests that at high temperatures beryllia combines with water to form the hydroxide $Be(OH)_2$ in the reaction

*This represents an output power density of 0.81 megawatts/ft³.

This level was closely approached on some of the JP4 fueled runs with this rig, but due apparently to less efficient combustion did not exceed it

**Inorganic Chemistry, E. deDarcy Barnett and C. L. Wilson, Longmans Green & Co.

Since the product of combustion of hydrogen and oxygen is superheated steam, the reaction is possible and may be one of the causes of the deterioration of the sidewalls.

4, 7, and 8 January 1963
Build 14422-L-2

A second long-duration run was made with a chase-around rig that was the same as Build 14422-L-1, except that flat electrodes were used and the aluminum-oxide dowel pins between the side-wall pieces were not used. Propane was used as a fuel.

The purposes of this run were:

- 1) The first long duration run (15 November, 1962) was made with the Al_2O_3 dowel pins. This run and subsequent runs and investigations showed that the Al_2O_3 pins were melting and contaminating the BeO and causing the BeO pieces to melt at a much lower temperature. Thus one purpose of this run was to demonstrate the extended endurance of the sidewalls with the Al_2O_3 pins removed.
- 2) The high power levels achieved with H_2 and O_2 even though the measured η_c 's were very low, indicated that the combustion was much more efficient with gaseous propellants. The H_2/O_2 runs produced more power than comparable JP4/ O_2 runs even though theoretical performance with hydrogen fuel is not as good as theoretical performance with JP4 fuel (because of the relatively high OH concentration in the H_2/O_2 products of combustion). This run was made using propane as a fuel with the idea that the propane would vaporize easily upon entering the combustion chamber, thus combining the high combustion efficiency of a gaseous fuel with the higher theoretical conductivity of a lower hydrogen content fuel.

Because the equilibrium pressure of propane was so high at the ambient temperatures at the time of firing, the propane flashed to vapor upon entering the pump. This caused cavitation in the pump and made it impossible to achieve the desired flow rates.

The turbine meters used to measure the fuel flow operate on volume displacement and because of the low density of the propane gas, they were overspeeded and failed. Thus no calculation of the run conditions can be made. The combustion was quite unsteady.

Because of the inability to achieve the desired propane flow rate, the chamber pressure achieved was low (37-42 psia), the O/F was probably very high (severe oxidation was noted on the copper nozzle).

Although the conditions were far from ideal, three separate firings were made. The maximum power achieved on these firings was 14.3 KW.

Three firings were necessary because of problems with the propane supply. The total accumulated run time was 88 minutes. The sidewalls performed very well as can be seen from Figures 177 and 178. Note that the one Al_2O_3 pin which was inadvertently left in caused extensive erosion. The electrodes eroded quite badly. It appears that more cooling is necessary on the electrodes.

3. Conclusions

An MHD generator of practical and durable design was built, tested and used to satisfy contract requirements. An extensive development program was conducted in an effort to produce a design philosophy that would allow the construction of a satisfactory experimental rig and of a much larger practical generator. As the program progressed, tests indicated that, to keep the boundary layer voltage drop small and to minimize deposits on the walls, the generator walls must be run very hot. Refractory ceramics were used to meet the high temperature requirement and it became apparent that presently available ceramics have a low resistance to thermal stress and low tensile strength. They had to be retained by controlled compression and redundant supports if they were to perform satisfactorily. The chase-around rig demonstrated the soundness of the philosophy and the methods used to implement it.

4. Recommendations

An experimental MHD generator of satisfactory design was developed and used to meet the contract requirements. A rig of superior design could be realized with improvements in several areas. The more obvious improvements needed are noted here.

Electrodes

The SiC-25 per cent graphite electrodes do not have sufficient durability for run durations much longer than 1 hour. If the electrodes were to be run cooler, they might last longer but there may be too much of a penalty paid in the rig's electrical performance. Since the plasma subjects the electrodes to an oxidizing atmosphere, probably no carbide or any other oxidizable material (one that does not form a protective oxide film) will be satisfactory for long duration runs. Recently the Carborundum Company developed a new material, Boride Z, which from the literature appears to be a potential electrode material. It requires testing.

Better control of the electrode temperature is a necessity and should be further investigated.

A more positive contact between the ceramic electrode and the power take-off pin is required. Refractory metals could be reconsidered if loss of strength due to recrystallization can be minimized.

A design was proposed to enlarge the contact between the pin and electrode surface. The essential features are shown in Figure 179. Further work should be done to exploit this concept.

Advanced refractory metal electrodes should be investigated and further work should be done on methods to reduce electrode boundary-layer voltage drop.

Sidewalls

The sidewall design of the chase-around rig, with some modifications, appears to be quite adequate for rigs of this size. Future versions of this sidewall concept should be built with either BeO dowel pins or no dowels at all. Running the walls slightly cooler may also help to improve their performance.

A decrease in the thickness of the block in the sidewall may decrease cracking and improve performance. To maintain electrical integrity for extended periods of time, no cracking can be tolerated because it allows cooled plasma and seeding materials to collect and it lessens the effectiveness of the insulating properties.

For larger-sized generators other schemes should be tested (for example, see the sidewall design described in Volume VI of the final report, which is based on the design principles successfully tested in the chase-around rig).

Materials

As pointed out in the long-duration runs, the area of materials and thermal-stress resistance must be considered carefully in the design of future rigs. An excellent paper* in this area can be found in the literature.

*Equilibrium Considerations for Interaction of High Temperature Materials with Their Environment, Symposium, Stanford Research Institute, June 1956

C. Combustion Phenomena

1. Introduction

This section reports work done on combustion and mass transfer as part of the MHD environmental testing effort. Combustion test work directed toward obtaining design information to support the 1000-megawatt and 250-megawatt MHD generator designs, and calibration of gas generators for the MHD channel test program is presented. The gas generators were designed to burn JP4 and gaseous oxygen with a 75 per cent KNO_2 -25 per cent H_2O solution introduced as seed material. Desired gas generator properties for the test program included dependable operation and high combustion efficiency over a combustion-chamber pressure range from 30 to 120 psia and an oxygen-to-fuel mass flow ratio range from 2.0 to 3.3. Combustion tests using 3-inch diameter (nominal dimension) chambers over a range of chamber lengths, pressure and oxygen-to-fuel flow rate ratios are reported. Supplementary work done to determine the distribution of fuel from concentric-type elements by using water and air to simulate the fuel and oxygen, respectively, is also included in this section. The development of several methods of introducing easily ionized seed materials into the combustion chamber is described. The relationship between MHD power output and combustion efficiency was calculated for several possible mechanisms of incomplete combustion.

The need for a basic understanding of combustion and mass transfer phenomena as they affect MHD generator performance is pointed out elsewhere in this volume. This understanding can manifest itself in either the form of theoretical predictions of the various phenomena or as correlations of test results on a variety of test devices.

The complex manner in which the various phenomena can affect combustion performance makes the correlation method more practical in the engineering solution of MHD power generation. This section contains the test results and correlation thereof as well as a report of tests conducted to increase the understanding of the fundamental processes.

2. Discussion

This section contains a discussion of the various combustion phenomena studied. These studies include gas generator tests of various injector,

combustion chamber and nozzle configurations, intended to provide a correlation of combustion efficiencies with the significant variables. Methods for improved seeding techniques are presented along with representative experimental data. Several incomplete combustion phenomena and their effects on MHD performance are considered as are several possible diagnostic techniques for studying these phenomena.

Gas Generator Tests - Tests were conducted with eight 3-inch diameter configurations of a gas generator for use with the 15 KW MHD channel. Pertinent geometric values are given in Table 3, and the general construction is shown in Figure 180, which also shows the three types of injectors tested. The showerhead type injector had numerous holes through the injector face through which the oxygen entered the combustion chamber. The fuel passed through the injector inside a number of tubes, which were brazed to the injector face. In the concentric type injector, the fuel entered the combustion region through tubes, but the oxygen ports were annuli concentric with the fuel tubes. The third injector type called like-on-like shower, consisted of shower-type oxygen injection ports supplied through tubes, while the fuel was supplied to several pairs of holes inclined towards each other. Each pair of fuel streams impinged upon each other to produce droplets.

The gas generators designed for possible use on the MHD generators were tested on a rocket engine type test stand. The results of these tests are presented in Tables 4 to 11. The parameters measured or calculated which are included in these tables are: chamber pressure at the injector end, (P_{c0}); oxidant to fuel ratio, (O/F); weight flow rate of seed, ($\dot{W}_s$); total flow rate of fuel, oxidant, and seed, ($\dot{W}_p$); heat loss per pound of reactant flow, ($\dot{Q}/\dot{W}_p$); and three parameters C^* , η_{c^*} , and L_e/K_i which will be explained below.

The parameter C^* is a characteristic velocity and is obtained from test data by the relation

$$C^* = \frac{P_c A_t g_o}{\dot{W}_p}$$

where $\dot{W}_p$ was defined above, P_c is the total pressure at the nozzle throat, g_o is the gravitational constant, and A_t is the nozzle throat area.

A correction was made for the loss of total pressure between the measured value near the injector to the value at the nozzle throat, based on an approximation fitted to data from a more accurate calculation. The approximation used was

$$\frac{P_{c_0} - P_c}{P_c} = \frac{0.15}{(A_c/A_t)^2} + 0.01$$

where A_c is the chamber cross-sectional area.

η_c^* is a common measure of rocket combustor performance and is defined as

$$\eta_c^* = \frac{C^*_{\text{MEASURED}}}{C^*_{\text{IDEAL}}}$$

where C^* ideal is determined by

$$C^*_{\text{IDEAL}} = \frac{P_c g_0}{(\rho u)_{\text{IDEAL}}}$$

η_c^* is referred to as C^* efficiency.

The measured quantities mentioned above were obtained in the following manner. The fuel and seed material flows were measured by turbine flowmeters whose accuracy was ± 1 per cent of the scale reading. The output signals were recorded on strip charts accurate within 1/2 per cent full scale reading. The oxygen flow was calculated from the expression

$$\dot{W}_{ox} = C_1 \frac{P_t}{\sqrt{T_t}}$$

where C_1 = a calibration constant

P_t = measured pressure upstream of a venturi having sonic flow velocity at the minimum cross-section

T_t = oxygen total temperature

The pressure was sensed by a pressure transducer and the signal recorded on a strip chart. The overall measurement accuracies for the fuel flow were expected to be about 2.6 per cent at 30 psia and 1.2 per cent at 120 psia. Corresponding accuracies for the seed flows were 5.2 and 1.3 per cent and for the oxygen flow 5.5 and 1.4 per cent.

Water flows for chamber and nozzle cooling were measured by a turbine meter and the output was read on a voltmeter. The cooling water flow was set before the run, and not continuously recorded. Overall accuracy was expected to be within 10 per cent. The chamber pressure was measured from a tap in the injector face by a 0-250 psi pressure transducer of ± 1 per cent full scale accuracy, and recorded on a strip chart.

The values of C^* efficiency are plotted in Figures 181, 182 and 183 for test configurations 42, 54, and 61 respectively. Typical data are plotted. The solid lines shown were fitted to much larger amounts of data some of which was omitted from the figures and tables in the interests of clarity and brevity. The dashed lines shown in Figure 181 were obtained by interpolation and extrapolation from other lines fitted to data at intermediate pressures. Two trends which can be seen from these curves and from the tables are that C^* efficiency increases with increasing pressure and with increasing oxygen-to-fuel mass ratio, O/F . The data scatter for these runs is given in Table 3.

Three configurations were run using the same showerhead injector and contraction ratio nozzle but with different lengths (configurations 48, 55, and 58). Since there was a very large amount of scatter in the heat-loss data in these tests, the heat loss was ignored in computing C^* efficiency values. Corrected efficiencies would be 1 or 2 per cent higher. Efficiency data scatter for configuration 48 varied from ± 6 per cent at 60 psia to ± 2.7 per cent at 75 psia. For configuration 55, data scatter varied from ± 1 per cent at 120 psia to ± 1.5 per cent at 60 psia and ± 3 per cent at 30 psia, while the configuration 58 data scatter was ± 1 per cent or less at all pressures. The data for the short chamber length, configuration 55, showed a great deal more variation with both O/F ratio and chamber pressure than the data for the long chamber length. This trend was expected, since the residence time of the burning reactants in the combustion chamber was about three times longer for configuration 58. What was not expected, however, was the low level of C^* efficiency for the long chamber. The small variation of C^* efficiency with either chamber pressure or O/F is inconsistent with the low level of C^* efficiency observed for configuration 58, which suggests that this level is in error. The small variations observed are characteristic of much higher levels of efficiency. The configuration 55 data appears, however, to be consistent with the short residence times present. At an O/F ratio of 2.6, the C^* efficiency was about 74 per cent at a pressure of 30 psia,

79.5 per cent at 60 psia, and 82 per cent at 120 psia. The above reasoning is supported by the higher value of C* efficiency, about 92 per cent for the intermediate length configuration 48 at a pressure of 75 psia. Through previous experience with small scale rocket motors, Pratt & Whitney Aircraft has developed a correlation in which the C* efficiency is a function of an effective length parameter, given by

$$L_e = L_c \left[\frac{P_c}{(P_c)_{REF}} \right]^n \sqrt{\frac{N}{\dot{W}_p}} \quad (16)$$

where

- L_e = effective length
- L_c = combustion chamber length
- $(P_c)_{REF}$ = an arbitrary reference chamber pressure
- n = an empirical exponent ranging from 0.5 to 1.0
- N = number of injection elements

It was realized that this correlation did not allow for variations in the effectiveness of different injector types, or for variations in efficiency due to off-design values of O/F ratio. Therefore, a modification of the previous correlation was developed which took into account both of these factors. For greater flexibility, the form of the pressure effect was changed to a coefficient that was not necessarily exponentially related to the pressure. The correlation method is described below.

The form of the effective length was chosen to be

$$L_e = K_i K_p K_f L_c \sqrt{\frac{N}{\dot{W}_p}} \quad (17)$$

where

- K_f is a coefficient depending only on O/F ratio
- K_p is a coefficient depending only on chamber pressure
- K_i is a coefficient depending on all other factors, mainly injector type and chamber geometry

Values of C* efficiency were plotted vs O/F ratio, and lines of constant chamber pressure drawn through the data, as illustrated in Figure 184a. The C* test values were also plotted as in Figure 184b as a function of O/F and constant pressure lines drawn. From these two plots, graphs of C* efficiency and C* were made vs chamber pressure at a reference O/F ratio of 2.7. By assuming that K_f had a value of 1.0 at the O/F ratio of 2.7, the coefficient K_p could be expressed as:

$$K_P = \frac{(L_e/K_i)}{K_f L_c \sqrt{N/\dot{W}_P}} \quad (18)$$

Using $P_c = 100$ psia as a reference value of chamber pressure,

$$\frac{(K_P)_{P_c}}{(K_P)_{P_c=100}} = \frac{\left(\frac{L_e}{K_i}\right)_{P_c}^{O/F=2.7} L_c \sqrt{\left(\frac{N}{\dot{W}_P}\right)_{P_c=100}^{O/F=2.7}}}{\left(\frac{L_e}{K_i}\right)_{P_c=100}^{O/F=2.7} L_c \sqrt{\left(\frac{N}{\dot{W}_P}\right)_{P_c}^{O/F=2.7}}} \quad (19)$$

or, for a given configuration,

$$\frac{(K_P)_{P_c}}{1.0} = \frac{\left(\frac{L_e}{K_i}\right)_{P_c}^{O/F=2.7} \sqrt{\left(\frac{\dot{W}_P}{P_c}\right)_{P_c}^{O/F=2.7}}}{\left(\frac{L_e}{K_i}\right)_{P_c=100}^{O/F=2.7} \sqrt{\left(\frac{\dot{W}_P}{P_c}\right)_{P_c=100}^{O/F=2.7}}} \quad (20)$$

Since

$$C^* = \frac{P_c A_T g_o}{\dot{W}_P}, \quad \dot{W}_P = \frac{P_c A_T g_o}{C^*} = \text{CONSTANT} \times \left(\frac{P_c}{C^*}\right) \quad (21)$$

Then

$$K_P = \frac{\left(\frac{L_e}{K_i}\right)_{P_c}^{O/F=2.7}}{\left(\frac{L_e}{K_i}\right)_{P_c=100}^{O/F=2.7}} \sqrt{\left(\frac{P_c}{100}\right) \left(\frac{C^*}{C^*}\right)_{P_c=100}^{O/F=2.7}} \quad (22)$$

With the reference correlation curve of C^* efficiency vs L_e/K_i shown in Figure 184c the test values of C^* efficiency from Figure 184c were used to obtain values of (L_e/K_i) . A reference value of (L_e/K_i) was obtained in the same way by using the C^* efficiency for $P_c = 100$ and $O/F = 2.7$. These values of (L_e/K_i) were substituted into Equation (22), along with C^* values from the smoothed curve in Figure 184d, to obtain values of K_P . The K_P values were plotted vs P_c and a smooth curve drawn through the points, as shown in Figure 184f.

In a manner similar to that for determining K_p , K_f was found from

$$K_f = \frac{(K_f)_{P_c=100}^{O/F}}{(K_f)_{P_c=100}^{O/F=2.7}} = \frac{\left(\frac{L_e}{K_i}\right)_{P_c=100}^{O/F}}{\left(\frac{L_e}{K_i}\right)_{P_c=100}^{O/F=2.7}} \sqrt{\frac{(C^*)_{P_c=100}^{O/F}}{(C^*)_{P_c=100}^{O/F=2.7}}}$$

A smooth curve was drawn through the calculated points as shown in Figure 184g. The final curves of K_p and K_f are shown as Figures 185 and 186, respectively.

Since some of the data showed large scatter, as previously stated, only data that was reasonably consistent within itself was used for the final correlation. All the data at any nominal pressure for a given configuration was eliminated if the scatter for the set was greater than ± 2 per cent. For the data that remained, the curves of K_p and K_f were used to calculate the (L_e/K_i) values shown in Figure 187. Although the points scattered a good deal around the reference curve, the points for a given configuration seemed to fall roughly parallel to the reference curve. Accordingly, a value for K_i was chosen for each configuration which made the data fall about the reference curve, as shown in Figure 188. Since the chamber lengths of 11.75 and 30.32 inches, respectively, made the only difference between configurations 55 and 58, the value of K_i used for configuration 58 was taken to be the same as for configuration 55. This made the data for the longer chamber fall far below the reference curve, and indicates that there probably was a consistent error in the test measurements for configuration 58 which caused the calculated efficiency to be much lower than the actual efficiency. Although it cannot be proved that the low efficiencies are incorrect, it is difficult to conceive of a mechanism that would allow only a negligible increase in C^* efficiency for a threefold increase in chamber length, unless the efficiency level were above 90 per cent.

Based on the most valid data obtained and the correlation achieved, the information is now available for designing gas generators using JP4 and oxygen having efficiencies at least as high as 97 per cent by extrapolation of the correlating parameter.

The designs of the 1000 and 250-megawatt MHD generator combustors were evaluated by this correlation, and the C^* efficiencies, based on theoretical shifting equilibrium C^* values reduced by the effect of chamber heat loss, estimated to be 97 per cent. Since the small-scale data was obtained at much lower chamber pressure values than those chosen for the 1000 and 250-megawatt designs, a value for K_p was obtained by using an exponential form, i.e., $K_p = (P_c/100)^n$. The exponent was estimated from Figure 185 to be 1.0 which is higher than the exponent of 0.66 found by other investigators* for liquid oxygen and heptane combustors with like-on-like elements. Since the mechanism of atomization of the fuel for concentric type elements is basically different from that of like-on-like elements, the difference in the pressure coefficients appears reasonable. However, small scale gas generators should be tested at the chamber pressure levels of the 1000 and 250-megawatt MHD gas generators to confirm pressure coefficient extrapolation.

Seeding Methods Development - Introduction of potassium into the hot plasma to increase its conductivity is generally referred to as seeding the plasma, and the particular potassium compound or solution used is called the seed material or the seed. There are a number of essentially different methods of introducing the seed material into the plasma and each has certain advantages. The aim of any seeding method, however, should be to distribute the seed material (and therefore the potassium and the free electrons it produces) as evenly as possible throughout the plasma that is entering the MHD channel. Because of the high velocities involved at the nozzle throat and beyond, the seed distribution attained at the nozzle throat is not much changed at the MHD channel entrance.

* Dobbins, R. A., Light Scattering and Transmission Properties of Sprays, A Technical Report on Contract AF 18 (600) - 1527, Dept. of Aero. Engr., Princeton, N.J., November 23, 1960

Lewis and Von Elbe, Combustion, Flames and Explosions of Gases, Academic Press, Inc., 1951

Loeb, L. B., Basic Processes of Gaseous Electronics. University of California Press, 1960

One method of seed introduction is through a separate set of injection holes into the combustion chamber. The seeding holes are fed from a manifold, so that three manifolds are required for the total injection system, one each for fuel, oxidant, and seed. For the range of seed material flows normally required, the number of injection holes is severely limited compared to the number of fuel or oxidant holes, because of practical limits on minimum hole size. To obtain reasonably uniform seed distribution under these conditions, long mixing lengths must be used. The additional heat losses to the wall of the long combustion chamber penalize the MHD performance, as discussed below.

In all of the combustion testing for MHD gas generators reported here, the seed material was a solution of 75 per cent by weight potassium nitrite (KNO_2) and 25 per cent water. The solution was injected into the oxygen supply pipe upstream of the gas generator. The large velocity differences between the oxygen and the seed solution resulted in shear forces that atomized the solution to a fog. This fog of seed material was swept into the combustion chamber by the oxygen, and the relatively large injection area thus utilized minimized the length of chamber needed for complete mixing. The short length necessary for mixing after injection is indicated by the test shown in Figure 189 in which water simulated the seed solution and air simulated the oxygen flow through six showerhead-type holes. This method of seed injection is simple and effective and gives good values of power per unit area in an MHD generator as discussed in the section concerning theoretical plasma performance. However, the water necessary for the solution reduces MHD performance from that available using several dry seed materials because of the electron affinity of the OH ion. Preliminary tests were performed in which a fluidized dry powder was injected into the oxygen supply pipe, producing an apparently uniform distribution of seed material out of the injector. The powder did not clog in the injector, and the uniformity of the seed distribution can be seen in Figure 190.

A different method of injecting the seed material into the combustion chamber is to premix it with the fuel storage tank. A number of methods were investigated, with varied amounts of success. The first of these methods tried used potassium carbonate, K_2CO_3 , ground very fine in a ball mill with 3 per cent potassium soap and an emulsifier, Co-mulser-A. The milled seed, soap, and an emulsifier were then mixed into JP4 and allowed to stand in a test tube. The seed tended to settle after

a time, and left a region of clear fuel at the top of the fluid column, as shown in Figure 191. Also shown are trapped pockets of JP4 all the way down the tube. The amount of settling, expressed as the height of the settling border ratioed to the full height of the fluid column, L_0 , is shown in Figure 192 as a function of time up to 300 hours. For this system, the use of 4 per cent Co-mulse-A increases the time before appreciable settling begins from 0.7 hour to 10 hours.

For applications where some delay in remixing could be accepted, allowing the settling level to drop to 0.9 of the original height would increase the time between mixing to 11 hours if no Co-mulse-A is used, and to 200 hours if 4 per cent Co-mulse-A is incorporated into the mixture.

Additional tests were performed using K_2CO_3 milled with various amounts of the emulsifier, but without the soap. For each amount of the emulsifier, the settling border fell rather soon to a level which then stayed constant for the duration of the test (30 to 75 hours) as shown in Figure 193. With zero per cent Co-mulse-A, the settling border fell within an hour to a steady level of about .92 of the original fluid height. With 2 per cent emulsifier added, the relative settling border fell to .45 after about 2 1/2 hours, and with 4 per cent emulsifier, the border fell to .47 after about 2 1/2 hours. The stable level achieved by the K_2CO_3 suspension without either soap or emulsifier was found to be a function of the percentage of seed material in the mixture. This effect is shown in Figure 194. The maximum stable settling border relative height of .92 was obtained with 26.0 per cent K_2CO_3 , with 19.9 per cent seed giving a stable relative height of .61 and 28.6 per cent seed producing a stable height of about .9 of the initial fluid height.

A gelled fuel approach was also investigated. Conventional aluminum-soap gelling agents were found to be useless because of a chemical reaction with the K_2CO_3 . Acetylene-black powder, however, was found to be effective in stabilizing JP4 - K_2CO_3 suspensions. Table 12 shows that a mixture containing about 6 per cent by weight of acetylene-black is sufficiently gelled to suspend 2.8 per cent by weight of K_2CO_3 . This would provide 1.5 per cent K in the plasma at O/F = 3.0. This mixture is thixotropic, exhibiting a heavy, ketchup-like consistency when at rest, but having a low apparent viscosity under shear forces, as when stirred or flowed through a pipe. Increases in the concentration of K_2CO_3 result in reductions in mixture viscosity,

necessitating increases in acetylene-black concentration to thicken the mixture.

The ideal seed-in-fuel system would be a true solution of a potassium compound in JP4. A thorough literature search and preliminary experimentation indicated the improbability of dissolving a potassium salt in JP4 to a high concentration, but as interest in low seeding rates increased, the search was renewed. Experimentation with alkanyl-succinic acids of potassium salts, conducted by Humphrey and Wilkinson, Inc., under contract to Pratt & Whitney Aircraft, produced a compound that contains approximately 14 per cent potassium by weight, and is soluble in JP4 in any proportion. Although the compound contains mainly hydrogen and carbon, the heating value was found to be lower than expected and theoretical performance at 2 per cent potassium in the plasma was calculated to be 53 per cent lower than with the KNO_2 -water solution. At a potassium concentration of 1 per cent in the plasma, however, the relative performance was only 7 per cent below that of the KNO_2 -water solution, and below 0.7 per cent concentration, the alkanyl-succinic acid had equal or better performance, as shown in Figure 195. The compound prepared is one of a family of compounds that can theoretically be made to contain up to 17 per cent potassium. This potential compound would have performance equal or superior to the KNO_2 -water solution at seeding concentrations up to 1.8 per cent potassium in the plasma. Figure 195 compares three alkanyl-succinic acid compounds with the 75 per cent KNO_2 -water solution in terms of power per unit area and potassium concentration in the plasma.

Incomplete Combustion Effects - In an actual combustion system inefficiencies exist which result in a reduction in both combustion and MHD performance. Although the relative contributions of the mechanisms causing inefficiencies in any particular combustion system are unknown, several mechanisms were postulated and an effort made to determine, theoretically, whether a general correlation exists between combustion and MHD performance. To this end it was convenient to define combustion efficiency as being identical with C^* efficiency which was previously defined and MHD performance as:

$$\eta_{\text{MHD}} = \frac{(\sigma V^{\frac{1}{2}}/\rho^{\frac{1}{2}})_{\text{ACT}}}{(\sigma V^{\frac{1}{2}}/\rho^{\frac{1}{2}})_{\text{TH}}}$$

Where the expression $(\sigma V^{\frac{1}{2}}/\rho^{\frac{1}{2}})$ is proportional to the power generated per unit area.* The quantities σ , V and ρ represent respectively, the conductivity, velocity and density of the plasma.

* This term is derived and discussed in Report PWA-1940, RADC-TR-61-104

A general correlation between η_{MHD} and η_{C^*} was anticipated due to the fact that both C^* and $\sigma V^{1/2}/\rho^{1/2}$ depend on the characteristics of the reactions involved, particularly on the combustion temperature and the molecular weight of the combustion products. In both cases propellants yielding low molecular weight gases and high combustion temperatures are desirable. The effect of these parameters on C^* is well known*. The effect on power per unit area (ie, $\sigma V^{1/2}/\rho^{1/2}$) may be observed in Figure 196 and anticipated by a consideration of the variation of V and ρ with m , the molecular weight of the combustion products.

To determine the effect of nonideal conditions, an $O_2/JP4$, potassium seed system was investigated assuming the following combustion inefficiencies:

- 1) Incomplete combustion of the reacting mixture with either fuel or oxidant unreacted,
- 2) Heat loss from the combustion chamber, and
- 3) Liquid droplets of unreacted fuel in the exhaust gases.

Of the above postulated mechanisms, 2), and particularly 3) may be considered realistic representations of actual conditions. The presence of mechanism 1) is questionable due to the rapid rates of reaction encountered at the elevated temperatures under consideration. The results of the investigation are discussed below.

Incomplete Reaction, Fuel or Oxidant Unreacted - The first mechanism of inefficiency studied considered that a portion of the fuel does not react as it goes through the gas generator, but is heated until it is in thermal equilibrium with the gaseous combustion products. Using these assumptions, it was determined that there is an optimum O/F ratio for each value of combustion efficiency as shown in Figure 197. At O/F's greater than 2, the MHD efficiency continually decreases as C^* efficiency decreases. The relationship for a typical O/F is shown in Figure 198 by the curve labeled JP4 Unreacted. In evaluating MHD the quantity $(\sigma V^{1/2}/\rho^{1/2})$, i.e. relative $\bar{P}/A$, power per unit area, is taken as that value at η_{C^*} equal to 1. A C^* efficiency of 95 per cent would cause an MHD efficiency of 82 per cent by this mechanism. Below an O/F of 2, the MHD efficiency passes through a maximum due to the fact that as the amount of JP4 unburned is increased, a larger fraction of

* Barrere, M., Jaumotte, A., et al., Rocket Propulsion, Elsevier Publishing Company, New York, 1960

the carbon from the JP4 that does burn is converted to CO_2 rather than CO. This results in a larger heat release. Operation at these reduced O/F's should however, be avoided due to the low values of P/A obtained.

The second combustion inefficiency mechanism assumed was that involving the unreacted JP4 cracked to lower hydrocarbons, carbon and hydrogen as it was heated to the equilibrium temperature. The composition of the cracked JP4 was obtained from a thermodynamic equilibrium calculation for JP4 and the cracked products over the required temperature range. Figure 199 shows the enthalpy of the cracked and uncracked JP4 as a function of temperature. It should be noted, however, that in an actual combustion chamber the residence time of the unreacted JP4 would be extremely short and it is not known, due to lack of kinetic data, to what extent JP4 would crack under these conditions.

The results of this second assumption are also presented in Figure 199 by the curve labeled Cracked JP4. For a C* efficiency of 95 per cent the cracked fuel mechanism would produce an MHD efficiency of 92 per cent. The higher performance of the cracked JP4 system may be attributed to several factors, but primarily to the reduction in M of the burned gases due to the presence of the low molecular weight cracked products as compared with the high molecular weight of uncracked JP4. Another factor is the similarity of the enthalpy versus temperature curves (see Figure 199) which indicates that the heat required to raise the cracked and uncracked JP4 to the combustion temperature should be similar for both cases.

An analysis similar to the first mechanism described above was performed assuming that a portion of the oxygen passed through the gas generator without reaction but was heated to thermal equilibrium with the burned gases. This caused a very sharp reduction in MHD efficiency since a 95 per cent C* efficiency would produce only a 13.5 per cent MHD efficiency, as shown in Figure 198. This pronounced effect may be attributed to the sharp drop in exhaust temperature as the unreacted O_2 varied from 0 to 40 per cent of that initially present. The large drop in temperature was caused by a reduction in the CO_2 formed, the formation of CO_2 yielding a large heat release, as well as the heat required to raise the O_2 to the combustion temperature. Since O_2 has a lower heat capacity than JP4 it would require a greater temperature drop to absorb the same amount of heat as JP4. There is also a reduction in the amount of water formed, which is a heat-releasing reaction.

Heat Loss - Heat losses to the walls of the gas generator and nozzle contribute to the inefficiency of the process. The heat loss was simulated by analytically withdrawing heat from the gas generator. This

resulted in a final combustion temperature lower than that for adiabatic operation. For a typical O/F (i. e. 2.4) the resultant MHD efficiency lies between that calculated for unreacted JP4 and unreacted oxygen, giving an MHD efficiency of 54 per cent for a 95 per cent C* efficiency (see Figure 198).

Liquid Droplets of Unreacted Fuel - The final mechanism considered as affecting MHD efficiency was that liquid JP4 droplets leaving the gas generator represent a momentum loss in the plasma, and a conductivity loss due to electron capture by fuel drops and composition changes resulting from the presence of unreacted fuel. These two effects were calculated as follows:

Power per unit area for an MHD generator may be expressed as:

$$\frac{P}{A} = \sigma \beta$$

where

$$\beta = V^{\frac{1}{2}} / \rho^{\frac{1}{2}}$$

and the other symbols are as previously defined.

Now, the loss in power per unit area may be computed by the expression:

$$\Delta \left(\frac{P}{A} \right) = \sigma \Delta \beta + \beta \Delta \sigma$$

The second term of the righthand member of the above equation, $\beta \Delta \sigma$, represents the loss due to the conductivity decrease. Therefore, the loss due to momentum effects can be computed from

$$\sigma \Delta \beta = \sigma_{IDEAL} \left[\left(\frac{V^{\frac{1}{2}}}{\rho^{\frac{1}{2}}} \right)_{IDEAL} - \left(\frac{V^{\frac{1}{2}}}{\rho^{\frac{1}{2}}} \right)_{ACTUAL} \right]$$

and the loss due to conductivity effects from

$$\beta \Delta \sigma = \left(\frac{V^{\frac{1}{2}}}{\rho^{\frac{1}{2}}} \right)_{IDEAL} \left(\sigma_{IDEAL} - \sigma_{ACTUAL} \right)$$

Computer calculations were made assuming the unburned fuel to be JP4 liquid at the fuel inlet temperature to the chamber for O/F ratios of 1.6 to 3.6 and up to 40 per cent of the fuel unburned. Ideal values of conductivity, velocity and density were obtained from calculations assuming complete burning, and the actual values were from the unburned fuel calculations.

Contributing to the reduction in conductivity is the number of electrons captured by the liquid droplets. This effect was calculated from the following relationships

$$\phi = f(T_e) = \frac{k T_e}{q} \ln \sqrt{\frac{M_i}{M_e}}$$

$$\sigma_s = \frac{\epsilon \phi}{r} = \frac{Q}{4 \pi r^2}$$

$$n_e = \frac{Q}{q}$$

$$N = \frac{\psi \gamma \rho_L}{4 \pi r^3/3}$$

where

ϕ = floating potential
 k = Boltzmann's constant
 T_e = electron temperature
 q = charge on electron
 M_i = mass of ion
 M_e = mass of electron
 ϵ = permittivity of free space
 r = radius of droplet

σ_s = surface charge density of droplet
 Q = total charge in droplet
 n_e = number of electrons captured per droplet
 N = number of droplets per pound of system
 γ = mass fraction of unburned fuel
 ρ_L = density of droplets
 ψ = pounds of fuel per pound of system

Using the above equations, it can be shown that the mole fraction of electrons removed, λ , can be expressed as

$$\lambda = 1.483 \times 10^{-12} \frac{\psi T_e \gamma M}{\rho_L r^2} \ln \sqrt{\frac{M_i}{M_e}}$$

where M = molecular weight of the products of combustion
and r = droplet radius in microns.

The calculations indicate that electron capture by the liquid droplets has a negligible effect on MHD performance. The results of the calculation are tabulated below for various C/F ratios. The equilibrium concentration is included for comparison.

Mole Fraction of Electrons Captured by Droplets

$O_2/JP4$	λ , mole fraction of electrons captured by droplets		e^- , equilibrium mole fraction of electrons
	r 10 microns	r 20 microns	
1.6	1.55×10^{-12}	3.86×10^{-13}	1.72×10^{-5}
2.0	1.80×10^{-12}	4.51×10^{-13}	1.208×10^{-4}
2.4(nominal)	1.81×10^{-12}	4.53×10^{-13}	1.89×10^{-4}
2.8	1.72×10^{-12}	4.31×10^{-13}	1.985×10^{-4}
3.2	1.61×10^{-12}	4.02×10^{-13}	1.87×10^{-4}
3.6	1.44×10^{-12}	3.60×10^{-13}	1.69×10^{-4}

The overall effect of unreacted fuel droplets on MHD and combustion efficiency is shown in Figure 200 for O/F's ranging from 2.4 to 3.6. At O/F's less than 2.0 the levels of absolute power per unit area are too low to warrant consideration. For purposes of comparison the relationship between η MHD and η C* at an O/F of 2.4 is shown in Figure 198. For a C* efficiency of 95 per cent, the droplet effect will produce an MHD efficiency of 75 per cent.

The summary plot of the effect on MHD efficiency of the various mechanisms studied (Figure 198) shows that there is a wide variation in results for the different mechanisms. To predict with any reasonable accuracy the relationship between C* efficiency and MHD efficiency, the actual mechanisms of incomplete combustion must be known. If several mechanisms contribute to the inefficiencies, as is certainly the case, then the relative degree of importance of each must be known also. The penalty in MHD performance due to C* inefficiency is so severe for some of the mechanisms, however, that a strong effort would be justified to determine the valid incomplete combustion mechanisms.

Incomplete Combustion Effects, Advanced Analysis and Evaluation -

The previous analysis of incomplete combustion mechanisms strongly indicated that for the combustion systems contemplated for MHD generators under this program, unvaporized fuel droplets were the major cause of performance loss due to the combustion system. Therefore, it was planned to use a detailed calculation for the combustion system performance in which the vaporization of fuel droplets was the controlling factor. In order to utilize this calculation, several kinds of tests were required to obtain basic input information. One test determined the fuel distribution spatially as it spread from the orifice of a single injection element. Another test was designed to determine the volume-surface mean drop size for the liquid spray from an injector element. Mean drop size has been defined as the drop diameter that has the same ratio of liquid volume to surface area as the complete spray. Since the vaporization rate for a liquid drop is a function of the surface area, the definition of mean drop size was especially useful. The third test was chosen to provide values of vaporization rate for fuel drops under actual gas generator combustion chamber conditions. Description of the three tests follows.

Spray Distribution Tests - For the liquid spray distribution tests, a concentric type injector element was tested using water and air to simulate fuel and oxygen, respectively. The concentric element produced a liquid jet surrounded by an annulus of air. A probe system consisting of a row of twenty adjacent tubes was used to collect water along a diameter across the center of the spray downstream of the injector element. The water was collected in bottles and measured, and the amounts used to calculate values of water flow per unit area, as a function of radius from the spray centerline. Plots of this data for three typical runs are shown in Figure 201, showing a rather bell-shaped distribution, but with a highly peaked value on the spray centerline. The test data for all runs is presented in Table 13, where the nondimensionalized liquid flow per unit area maximum values are given for two probe sizes, and the values at the 50 per cent flow position are given for the standard probe size. The nondimensional values were defined by dividing local values by values for the liquid jet based on the orifice diameter. The position of the 50 per cent flow point was also nondimensionalized by dividing the spreading radius within which was 50 per cent of the liquid flow, S_{50} , by the liquid orifice diameter. Also presented in the table are nondimensional values for the spreading radius within which were 97 per cent of the liquid flow.

To determine possible correlating parameters for the data, an analysis of jet atomization was conducted. It was postulated that the mechanism of atomization was that protuberances formed on the liquid surface by jet instability or wave formation by the adjacent gas stream would be sheared off because of relative velocities between the liquid jet and the gaseous annulus. Expressions for the drag force on a liquid wave around the circumference of the jet

$$F = C_D \frac{\rho_g (V_g - V_w)^2}{2} \pi D_L \delta$$

and the liquid mass in the wave

$$M = \rho_L \pi D_L \delta^2$$

were substituted into the equations of motion to give

$$\frac{dV_w}{d\tau} = \frac{\rho_g}{\rho_L} \frac{(V_g - V_w)^2}{2 \delta} C_D$$

where

C_D	= drag coefficient	D_L	= liquid jet diameter
ρ_g	= gas density in annulus	δ	= height of wave on jet surface
V_g	= annular gas velocity	ρ_L	= liquid density
V_w	= liquid wave velocity	τ	= time

For the small displacements ($x \approx 0$) necessary to shear off the wave, it was assumed that the wave velocity was approximately equal to the liquid velocity. Integrating the equation of motion twice resulted in the equation for wave position during the shearing operation:

$$X_{WAVE} = \frac{\rho_g}{\rho_L} \frac{(V_g - V_L)^2}{2 \delta} C_D \frac{\tau^2}{2} + V_L \tau$$

It was assumed that the wave height, δ , is proportioned to the jet diameter and that the time involved can be expressed as $\tau = X_{jet}/V_L$. Also, for a constant distance the injector, $X_{jet}/D_L = \text{constant} = C_1$. Using these relations in the above expression, a liquid displacement factor, $\phi = X_{WAVE}/D_L$, can be expressed as

$$\phi = C_1 \left\{ \left[\frac{\rho_g (V_g - V_L)^2}{\rho_L V_L^2} \right] C_2 C_1 + 1 \right\}$$

where

$$C_2 = \left(\frac{C_D}{4} \right) \left(\frac{X_{jet}}{\delta} \right)$$

The dynamic pressure of the gas relative to the liquid

$$q_{gr} = \frac{\rho_g (V_g - V_L)^2}{2 q_o}$$

where q_o is the gravitational constant, and the dynamic pressure of the liquid, $q_L = \rho_L (V_L^2 / 2 q_o)$ were substituted into the equation for ϕ to give

$$\phi = C_1 \left[\left(\frac{q_{gr}}{q_L} \right) C_2 C_1 + 1 \right]$$

By using the definitions of C_1 and ϕ , this expression was manipulated to obtain a ratio between the positions of the wave being torn off the jet and the position of the jet

$$\frac{X_{WAVE}}{X_{JET}} = \left(\frac{q_{gr}}{q_L} \right) C_2 C_1 + 1$$

This equation indicates that the ratio of the gas relative velocity head to the liquid velocity head might be a correlating parameter for spray distributions from a concentric injector.

A small value of the liquid displacement factor, ϕ or of (X_{wave}/X_{jet}) indicates a small liquid shearing rate and therefore low spreading rate of the spray. A high value of ϕ indicates a high shearing rate, but also implies severe axial gas forces that will limit the radial spreading rate. At a constant value of the velocity head ratio, the shearing rate per unit time should be constant also, but the spreading will still vary at a given distance from the injector because of the changing velocities.

Values of (q_{gr}/q_L) for the test runs are recorded in Table 13.

Figure 202 substantiates the above reasoning by showing that for a given liquid injection velocity, increasing the relative velocity head ratio from zero results in a rapid rise in the radial spreading distance, up to a maximum value of spreading. Further increases in velocity head ratio result in a gradual lowering of the amount spreading at the probe position. Increasing the liquid injection velocity decreases the spreading for any value of the velocity head ratio. The spreading of the spray at the midpoint of the mass distribution (S_{50}) agrees with the trends for the total spreading (S_{97}). As the liquid droplets spread radially, the liquid concentration on the jet centerline decreases to a minimum as the velocity head ratio increases to a value of about two. At higher values of the velocity head ratio, the centerline liquid concentration increases slowly, apparently due to the shorter time available for spreading. This effect is shown in Figure 203.

Rough correlations were obtained for the relationship between liquid spreading and liquid concentration. As shown in Figure 204, a plot for the various injection conditions of the nondimensional liquid spreading for the 50 per cent liquid flow point, S_{50}/D_L , against the nondimensional liquid concentration at that point, G_{50}/G_i , can be represented by a straight line of negative slope. Plots of S_{97}/D_L vs G_{max}/G_i and S_{50}/D_L vs G_{max}/G_i show the same kind of trend. The scatter of some points from the general trend is considerable, and is probably due to nonsymmetry of the actual sprays, where complete symmetry was assumed in the analysis.

The second type of concentric element tested had a small spray nozzle for fuel injection. When no gas was flowing through the oxidant annulus, the liquid spray angle was 90 degrees. The liquid spray distribution during the tests using air and water, was found to be similar to that obtained for the plain concentric element, but with lower peak values of concentration on the centerline and slightly higher concentration at the outer radii. Typical liquid concentration profiles are shown in Figure 205, and the pertinent test data is given in Table 13. The initial value of liquid flow per unit area, G_i , was based on the total liquid orifice area, because the discharge coefficient of the orifice was not known. The data plotted in Figure 204 shows that the nondimensional spread of the liquid from the spray nozzle element is more nearly constant for variations in injection conditions than that from the plain concentric element.

In order to increase the initial spreading of the liquid, a third element type was designed and tested in which the liquid was injected through an annulus and the gas through a central hole. The expansion of the gas stream diameter after injection was expected to aid in the droplet formation process, and to distribute more of the liquid outward from the element centerline. Although only two tests were made of this configuration, the liquid concentration profile obtained for the inverted concentric element are much flatter and thus more spread out than those of the normal concentric element. Two tests of normal concentric elements were made at injection conditions close to those of the inverted concentric tests, and the concentration profiles are compared in Figure 206. The profiles are somewhat nonsymmetrical, probably because of assembly tolerances on concentricity, and the probe device was not wide enough to reach the edges of the spray, but the very large reduction in peak concentration is evident. It is apparent that the inverted concentric element should provide uniform combustion products

within considerably shorter chamber lengths than necessary for normal concentric elements. The lower heat loss of shorter chambers would provide increased gas temperature and therefore increased gas conductivity in the MHD generator. The heat transfer rate to the injector face would probably be increased with this injector element type because the shorter length required for mixing would allow burning to commence nearer the injector face.

Droplet Size Determination - For determining the volume-surface mean diameter for liquid sprays, an optical system was constructed based on work done by R. A. Dobbins* at Princeton University. This method of droplet size measurement depends upon the fact that for droplets larger than about 10 microns, the portion of a parallel light beam that scatters at small angles is scattered almost entirely by diffraction. The intensity of the light scattered diffractively is described as a function of the scattering angle by the expression

$$I_{\theta} = E_0 \left(\frac{\pi D_i^2}{4 \lambda} \right)^2 \left[\frac{2 J_1(\alpha_i \theta)}{\alpha_i \theta} \right]^2$$

where θ = light scattering angle from the optic axis
 I_{θ} = intensity of scattered light at angle θ
 E_0 = flux of incident light
 D_i = droplet diameter
 λ = wavelength of incident light
 $\alpha_i = \pi D_i / \lambda$
 J_1 = first order Bessel function

Based on thin lens theory, the droplet diffraction pattern will be independent of the position or motion of the droplet within the light beam. For more than one droplet in the optical field, the intensity of the ringed pattern will be increased, but the pattern will remain the same as long as the drop sizes are the same. For liquid sprays, however, a range of drop diameters are present. In this case, the diffraction patterns for the different drop sizes overlap and merge to form a symmetrical, continuous distribution of light intensity varying from a maximum at the center, on the optic axis, to a negligible amount at some radius from the optic axis.

* Dobbins, R. A., Light Scattering and Transmission Properties of Sprays, A Technical Report on Contract AF 18(600)-1527, Dept. of Aero. Engr, Princeton, N. J., November 23, 1960

If the local light intensity on the focal plane is normalized with respect to the intensity on the optical axis, and the fraction of the total number of drops having a particular diameter D_i is given by

$$x_i = \frac{M_i}{N} = \frac{\text{no. of drops of diameter } D_i}{\text{total number of drops}}$$

then the distribution of light intensity can be expressed as

$$I(\theta) = \frac{I_\theta}{I_{\theta=0}} = \frac{\sum x_i D_i^4 \left[\frac{2 J_1(\alpha_i \theta)}{\alpha_i \theta} \right]^2}{\sum x_i D_i^4}$$

For a typical continuous spray distribution the relative intensity $I(\theta)$ can be related directly to the volume-surface mean diameter, D_{32} , of a group of droplets, where

$$D_{32} = \frac{\sum x_i D_i^3}{\sum x_i D_i^2}$$

The relationship is of the form

$$I(\theta) = f\left(\frac{\pi D_{32} \theta}{\lambda}\right) = f(\alpha_{32}), \text{ as shown in Figure 207.}$$

If a photographic plate is placed at the focal plane of the receiver lens, as shown in Figure 208, exposed to the scattered light, and developed, it will show a highly exposed, or dense dot, surrounded by a symmetrical region with decreasing density at increasing radii from the center. The film can be calibrated by exposing a portion of the film to the unscattered monochromatic light through a commercial step tablet. The film density can be determined for the scattered light area and for the calibration regions with a microphotometer. Since the transmittance values for the calibration step tablet are known, the film density values in the test region can be converted to values of relative light intensity vs radius from the center of the pattern on the film. Values of the light scattering angle, θ , are calculated from the system geometry, since

$$\sin \theta = \frac{\text{radius from optic axis on film}}{\text{focal length of receiver lens}}$$

Because the unscattered light does not focus at a point, but over a small region, the scattered light near the optic axis is obscured by the dot formed by the unscattered light, and the intensity profile must be extrapolated to the optic axis.

If the test intensity curve is normalized by dividing each intensity value by the intensity at the optic axis, the resulting curve can be compared to the theoretical curve (Figure 207) and a value of D_{32} chosen that will make the experimental and theoretical curves correspond the best. This value of D_{32} is then the volume-surface mean diameter for the spray. Analysis of all the errors that can affect the technique shows that the value of D_{32} is accurate within ± 10 per cent over a wide range of distributions of liquid mass vs drop size.

The optical rig described above was calibrated using polystyrene beads in salt solution to simulate the drops in a spray. The rig is pictured in Figure 209 and test intensity profiles obtained such as that shown in Figure 210.

It was planned to determine the fuel vaporization rate under actual combustion chamber conditions by analyzing streak photographs of burning fuel drops in a transparent two-dimensional combustion chamber. Use of a high speed motion picture camera without the framing prism allows the recording of burning fuel drops as streaks of light on the film. Knowledge of the film speed permits determination of the speed of the burning drop at each position along the chamber. Since the very small drops will travel at essentially the gas velocity because of their high drag-to-weight ratio, determination of the highest particle speed at each position along the combustion chamber allows a close estimation of the local gas velocities. By making the reasonable assumption that vaporized fuel mixes and burns in a very short time compared to the rate of fuel vaporization, the fuel vaporization rate can be calculated.

Since the necessity of using advanced combustion performance calculations for the MHD system gas generators was not apparent until relatively late in the contract period, it was not possible to obtain information on mean drop size for an injector element, or to run the transparent combustion chamber to obtain droplet trajectories through the chamber. Therefore, without most of the essential input information, no combustion performance calculations could be made.

It is recommended that the combustion performance calculation be done for the normal concentric element and for several other elements that might allow smaller combustion chambers. To support this calculation for the element types considered, data should be obtained on liquid spray distribution, volume-surface mean drop size, and droplet trajectories in a transparent combustion chamber.

Electrostatic Probe - Because of the strong dependence of MHD generator performance on combustor performance, due to variations in plasma properties the need became evident for instrumentation to measure plasma properties to compare with analytical predicted properties. Temperature is one of the most important properties of the plasma.

The elevated temperature of the JP4/O₂ plasma (5000°F) precludes the use of conventional temperature measuring devices such as thermocouples, thermometers, etc. At these high temperatures the gas composition will consist of species that will probably not be in thermal equilibrium with each other. The concept of temperature may be assigned to a species or group of species in a particular energy state if they obey the Maxwell-Boltzmann distribution law. Therefore, it is possible to have several "temperatures" associated with a particular gas mixture. The sodium D line reversal technique only averages the temperature across the flame and does not give a measurement at a point. As Lewis and Von Elbe¹ have pointed out, such methods require careful control of flame environment and thickness. The electrostatic probe method has been used in gaseous discharges^{2, 3, 4} to obtain electron temperatures. This method allows measurements to be made over a small region in a plasma. Recent work by Calcote^{5, 6} has indicated that the electrostatic probe may be used for measurements in flames.

- 1 Lewis and Von Elbe, Combustion, Flames and Explosions of Gases, Academic Press, Inc., 1954
- 2 Loeb, L. B., Basic Processes of Gaseous Electronics, University of California Press, 1960
- 3 Langmuir, D., and Mott-Smith, H. M. General Electric Review 27, 449, 538, 616, 762, 810, (1924)
- 4 Johnson, E. O., and L. Malter, A Floating Double Probe Method for Measurements in Gas Discharges, Physical Review, Vol. 80, No. 1, October, 1950
- 5 Calcote, H. F., Ion and Electron Profiles in Flames, 9th Combustion Symposium, August 1962
- 6 Calcote, H. F., and King, I. R., Studies of Ionizations in Flames by Means of Langmuir Probes, Fifth Symposium on Combustion p. 423, Reinhold Publishing Corporation, New York, 1955

From Langmuir's theory for the glow discharge plasma the current equations for a cylindrical probe may be obtained. The two equations for charged particles if their velocities form a Maxwell-Boltzmann distribution are as follows

$$i = 2 \pi r L I \left\{ \frac{a}{r} \left[1 - \operatorname{Erf} \sqrt{\frac{r^2 \eta}{a^2 - r^2}} \right] + \operatorname{Erf} \sqrt{\frac{a^2 \eta}{a^2 - r^2}} \right\} \text{ for acceleration potentials}$$

$$i = 2 \pi r L I e^{\eta} \quad \text{for retarding potentials}$$

where

$$\eta = \frac{eV}{kT}$$

and

$$I = N e \sqrt{\frac{kT}{2\pi m}}$$

Also,

- i = current received by probe
- r = radius of probe
- L = length of probe in plasma
- a = charged particle sheath radius
- I = random current density
- V = potential difference between probe and plasma
- k = Boltzmann's constant
- T = temperature of particle
- e = charge on particle
- N = particle concentration
- m = mass of particle

From the equation for retarding potentials we obtain by taking the logarithm of both sides

$$\log_e i = \phi V + \text{CONSTANT} \quad \text{where} \quad \phi = \frac{e}{kT}$$

since the positive ion contribution to the total current is considered small in the slightly negative region, a plot of $\log_e i$ versus v should yield the temperature of the electrons as shown in Figure 211.

$$\text{Temperature of electrons} = \frac{11,606}{\phi}$$

For large saturation voltages, the acceleration potential equations are applicable. Making the simplifying assumption that the quantity in the brackets does not change appreciably

$$i \sim I \quad \text{or} \quad i \sim N \sqrt{T}$$

For changes in temperature from 1000-3000°K, $\sqrt{T}$ doesn't vary more than a factor of 2, so the current is roughly a measure of the concentration.

A double probe theory has been developed by Johnson and Malter. This has the advantage that the current drawn is limited to the positive ion current which is many times smaller than the electron current. The theoretical voltage-current characteristics are shown in Figure 212. Their final equation is

$$\log_e \left(\frac{i_p}{i_{e2}} \right) = -\phi V_d + \log_e \left(\frac{A_1 j_{o1}}{A_2 j_{o2}} e^{\phi V_c} \right)$$

where v_d = potential difference between two probes
 v_c = contact potential between two probes
 A_1, A_2 = areas of probes
 j_{o1}, j_{o2} = random current densities to respective probes
 $i_p = i_{p1} + i_{p2}$ = total positive ion current to both probes
 i_{e2} = electron current to second probe

From a plot of $\log_e (i_p/i_{e2})$ versus v_d the slope and thus the electron temperature can be obtained.

After initial laboratory testing of materials, a probe was constructed using stainless steel tubing. The probe was mounted on a traversing mechanism which rotated the probe through the flame to be studied. The probe sweep speed was chosen to be fast enough to prevent overheating of the probe by the flame, but slow relative to the combustion gas velocity. Laboratory tests were made using both single and double probes and a Meker burner utilizing propane and air. Current vs voltage curves were similar in shape to those obtained from theory for the single probe, but saturation currents varied and for most of the tests no linear portion was found in the plots of the natural log of the current vs the voltage. The former effect was probably due to changes in the thickness of the charged particle sheath around the probe, and the latter effect was probably due to an effect of probe size, or to effects of drift velocity between the plasma and the probe. Double probe measurements gave curves similar in shape to that obtained by Johnson and Malter, but shifted with respect to the voltage axis (that is, the probe current was not zero at zero probe voltage).

Calculated electron temperatures for the few tests where a slope could be determined for the plot of log current vs voltage were about 250° Kelvin or 450° Rankine higher than the electron temperature measured by Calcote for peak fuel/air ratios. A good comparison was obtained, however, between the shapes of the curves of the random electron current in the saturated region and temperatures obtained by the sodium D line reversal apparatus for a range of fuel/air ratios.

During the testing of the probe in the exhaust of a gas generator, ionization profiles were obtained in a plane normal to the exhaust stream. Typical traces are shown in Figure 213 for several O/F ratios. The ionization profile shown at the bottom of Figure 213 was taken across a seeded exhaust flame. The sensitivity of the probe is shown by the clear definition of large variations in the ionization level, representing between 500 and 1000 cycles per second at the probe sweep speeds used. Comparison of the shape of a plot of the square of the saturation current vs fuel/air ratio for an unseeded flame with the shape of the temperature curve obtained by the sodium-line reversal technique shows quite good agreement.

From the work done, it was concluded that relative ionization levels can be measured. The swept single probe will measure nonuniformity in ionization levels. However, care must be taken to insure that the electron return is not changed from one test to another in order to allow valid comparisons between tests. Absolute ionization level measurements are dependent upon a knowledge of free electron temperatures.

Measurement of free electron temperatures requires further investigation in the laboratory. Work to date indicates that the double probe technique, because it disturbs the plasma least, is the more reasonable apparatus. But a probe must be developed that is small enough to cause negligible hydrodynamic distortion.

Electron temperature measurements on high velocity, high temperature rocket exhaust will require perfecting a voltage pulsing technique and a method for evaluating the data statistically. This is necessary because, as has been shown by high speed moving pictures, the rocket exhaust is nonhomogeneous. A measurement at any one point will not be representative of what is happening at another point or at the same point at a later time. Statistical methods can combine the results of a number of probe traverses at the same conditions to describe the plasma conditions more completely.

Conclusions

1. Combustion efficiency increases with increasing chamber pressure, oxidant-to-fuel ratio, combustion chamber length and residence time. The effect of pressure and oxidant-to-fuel ratio on combustion efficiency decreases with increasing length or residence time. Heat losses to the injector face, combustion chamber wall, and nozzle, lower combustion efficiency.
2. The correlation procedures developed appear to be valid within the range of test pressures. However the existence of some poor test data limits the conclusions that can be drawn from the correlation results. Quantitative extrapolations of the correlated data to higher pressures than were run would be questionable and would require more test results at the desired pressures. However, qualitative use of the results is a significant aid in estimating performance of higher pressure combustion chambers.
3. The injector element type tested that achieves the most complete mixing of oxidant and fuel in the shortest distance is the inverted concentric. The use of this element with its early mixing characteristics could result in injector face-cooling problems. However its use would probably result in the highest combustion efficiency for a fixed length or in substantial length savings to achieve a given combustion efficiency.
4. The effect nonideal combustion could have on MHD generator performance is serious regardless of the nonideal condition assumed. Heat loss from the combustion chamber is an obvious loss mechanism that will exist. The existence of liquid droplets is believed to be the most powerful loss occurring in present combustion chambers. It is shown theoretically that unreacted O_2 would result in a very serious degradation of MHD generator performance. However, it is believed that present designs have solved this problem.
5. Significantly better design procedures could be developed using a calculation which assumes that fuel droplet vaporization is the controlling mechanism for combustion performance. This calculation procedure is available but requires information from further tests for its use as a design tool.

IV. SUMMARIES OF VOLUMES II - VI

This section contains summaries of the other volumes of this final engineering report. The summaries are included so that a comprehensive report on all aspects of this study are available within one volume. A breakdown of the problem considered in these volumes is included in Section I of this volume. The conclusions reached in this volume as well as in the five summarized volumes are reported in Section V of this volume.

Summary of Volume II, Integration With Crossed-Field Devices

Volume II presents the results of an investigation of the generation of large amounts of microwave power in conjunction with the use of a magnetohydrodynamic generator as a source of primary power. This study was performed by Raytheon Company on a subcontract from Pratt & Whitney Aircraft Division.

The study included an evaluation of all classes of crossed-field devices. The objective of the contract was to yield a design of a crossed field rf generator in the superpower range that would be compatible with MHD power generation.

The large amount of DC power available for conversion to rf power (1000 megawatts) and the existence of an intense magnetic field over a large volume (that portion of the MHD generator's field lying outside the generator itself) were determining factors in selecting the approach to be followed. It was decided to consider approaches capable of generating rf power several orders of magnitude greater than the existing state of the art. This approach is in sharp contrast to the usual approach of extrapolating existing techniques by one order of magnitude or less.

This approach resulted in the conception of a new class of tube known as the electromagnetic amplifying lens (EAL). This tube concept has features that a conventional tube does not have, but also utilizes many principles based on prior experience with tubes such as the magnetron and the amplatron.

A detailed investigation of the EAL concept therefore constituted the major portion of the work under the study contract. This work

consisted of several principle parts. The first part consisted of a theoretical investigation of many of the properties of this device. The second portion consisted of a limited amount of cold test work on an EAL anode of respectable electrical size and which would have a reasonable physical size if it were reduced to a hot tube of high dissipation density design (several megawatts of average rf power output). A third part dealt with the paper design of an extremely high powered X-band tube of the layer cake type of construction. The average power-handling capability of this design lies in the 1000 megawatt class. Still another part deals with the economic implications of large tube development and subsequent manufacture. Included also is a discussion of the distinct possibility of building superpower tubes in the millimeter region.

All phases of the studies were promising and it appears that an rf device capable of producing 1000 megawatts of rf energy can be designed within a single envelope. The economic implications of this study are that this device could be produced for a cost of less than \$1 per kilowatt, assuming that development costs have been written off and that power supply test equipment is not included. This low cost brings into sharp focus the fact that primary power rather than tube cost will be the major bottleneck in any microwave system calling for large amounts of microwave power.

Summary of Volume III, Integration With Crossed-Field and Linear-Beam Microwave Devices

Volume III summarizes the results of a study program investigating superpower microwave generation performed by the Litton Electron Tube Corporation on a subcontract from Pratt & Whitney Aircraft Division.

The objective of this subcontract was to design a microwave generation system which yields optimum operational performance in conjunction with a given MHD prime power generator and lends itself to electrical integration with the MHD generator.

The evaluation was based on the anticipated state of the art in microwave tube manufacture five years from now. This restriction excludes all concepts which have not progressed at least to the experimental model stage.

It was also assumed that the tube would be operated continuous wave (CW) at X-band frequency. This assumption gives a basis for comparison of the various tube types considered.

The study was divided into two phases. The first phase was concerned with establishing the basic feasibility of such a system, from the rf tube designer's point of view, and an evaluation of the relative merits of different superpower microwave generation schemes. It was shown that both crossed-field and linear-beam devices are feasible in such a system. After careful evaluation of the desired system characteristics it was concluded that a linear-beam device offered the better solution. It was further concluded that an extended interaction circuit klystron tube would be the best of these devices.

The second phase of the project was a detailed study of specific klystron designs and their compatibility with the MHD generator. In this phase of the study a 200 kilowatt klystron was selected as the final choice for a number of practical reasons. First, 200 KW appears to be the limit for reliable handling of CW power in an X-band system. Even though the state of the art will eventually be extended in a number of components, 200 KW appears to be a practical limit at present. The most important component is the output window of the tube; and known window materials cannot be extrapolated beyond approximately 200 KW at X-band. Another reason for the 200 KW power level is given by the amount of beam power that can be generated in a gun and focused through the beam tunnel. Permanent magnet focusing is an essential requirement for a system that uses so many tubes and because permanent magnets are limited in the value of magnetic field which they can produce, this also gives an upper limit for the beam power.

Some of the pertinent characteristics of the design tube are:

1. DC Parameters

Total DC input power	333.3 KW
Regulated DC input power (for phase stability)	13.3 KW
Unregulated DC input power (collector)	320 KW
Beam voltage	30 KV
Beam current	11.11 amps
Beam perveance	$2.12 \times 10^{-6} \text{ A/V}^{3/2}$
Heater power approximately	30 W

2. RF Characteristics

Rf output power CW	200 KW
Gain	50 db
Conversion efficiency	60%
Center frequency	10 Kmc
Phase stability	.06 °/volt
Focusing field	1000 gauss

3. Mechanical Characteristics

Total length	18 in.
Max. diameter without magnets	3.5 in.
Max. diameter with magnets	4.5 in.
Total weight without magnets	25 lb
Total weight with magnets	50 lb
Cooling collector (liquid)	16 gpm
pressure drop	40 psi
body (liquid)	5 gpm
pressure drop	40 psi
Focusing	permanent magnet

The collector is designed to take only 130 KW of dissipation power. This assumes that the full beam power is never on without the rf drive.

An economic evaluation of the klystron approach was made in addition to the technical considerations. Only manufacturing costs were considered since development and engineering costs will be negligible in comparison. It is estimated that the cost of the 200 KW microwave generators will approach, depending on volume of production, 0.6 cents per watt of rf power, or 3.6 million dollars per 600 megawatts of rf output.

Summary of Volume IV, Materials

Volume IV presents the results of a Pratt & Whitney Aircraft study of the problem area of MHD generator materials. The object of the study was to test and evaluate existing and new materials for use in the power generating section of a magnetohydrodynamic generator.

The problem of selecting materials for use in the power-generating section of the MHD generator is very complex. This is a result of the environmental conditions encountered in this area, the diversity in

required properties, and the lack of information on material properties under the operating conditions. The desired materials fall into two categories, 1) highly refractory electrically-nonconducting materials, and 2) highly refractory electrically-conducting materials.

The prime requirement for insulator materials is an electrical resistivity significantly higher than that of the plasma so that the generator will not short out. Since the resistivity of insulator materials decreases rapidly as the temperature of the materials is increased, this requirement is difficult to meet. Very little information is available in the literature on electrical resistivity of materials at very high temperature, and much of the available information is useless because of inadequate description of the material being tested. Conversely the prime requirement for the electrode materials is that the electrical conductivity be greater than that of the plasma at elevated temperatures. In general, the conductivity of conductor materials decreases slowly with increasing temperatures. The conductivity of materials at high temperatures cannot be predicted with any degree of accuracy from their low temperature values and therefore must be measured. As is the case with insulator materials this information is not usually found in the literature. The materials evaluation program was divided into three phases, 1) a literature survey and preliminary analysis, 2) laboratory investigation, and 3) an environmental testing program.

The purpose of the literature survey was to select materials for the laboratory investigation. Both the published literature and unpublished data from various laboratories was surveyed for high temperature properties of materials.

The laboratory investigation included the design and construction of specialized high temperature testing equipment, the production of test data, and the evaluation of these results. First the electrical resistance of insulating materials and the electrical conductivity of ceramic conductors were measured. Those materials having promising electrical properties were then tested for chemical compatibility with, and erosion resistance to, plasmas of the types found in an MHD generator. The third part of the laboratory investigation was the determination of the high temperature mechanical properties. Those measured were the linear thermal expansion and modulus of rupture.

The specific electrical resistivity as a function of temperature has been measured for eleven refractory insulating materials, aluminum oxide, beryllium oxide, magnesium oxide, thorium oxide, zirconium

silicate, magnesium aluminate, strontium zirconate, barium zirconate, calcium zirconate, thorium zirconate, and boron nitride. In making the electrical measurements, samples in the form of short rods were heated in a tungsten resistance furnace, either in a high vacuum or in a purified argon atmosphere.

Of the materials tested, beryllium oxide, magnesium oxide, strontium zirconate and calcium zirconate were the best. The best of the commercial beryllium oxide samples was hot-pressed to a high theoretical density from a very high purity starting material. The electrical resistance of this material at 2000°C was 400 ohm-cm, a value higher than any other commercial material tested at this temperature.

The electrical resistivity of ten ceramic conducting materials for MHD electrodes was measured as a function of temperature. These materials were columbium diboride, titanium diboride, titanium nitride, zirconium nitride, titanium carbide, molybdenum disilicide, high density graphite, zirconium carbide + 25 per cent graphite by volume, zirconium carbide + 45 per cent graphite by volume and thorium oxide + 5 per cent yttrium oxide*. To measure the electrical conductivity of conducting ceramics or metals at high temperatures the samples were heated in either the tungsten resistance furnace in vacuum or in an argon atmosphere, or in the graphite resistance furnace in a purified argon atmosphere. The two best conductors of those tested, titanium diboride and columbium diboride, are almost as good conductors as tantalum.

The compatibility test burner consisted of a small water-cooled rocket motor to which the test piece was clamped, and a preheating furnace to safeguard the test piece against premature failure by thermal shock. The chemical compatibility test imposed an exhaust of a minimum of 1.76 Mach number at a minimum temperature of 4920°F and contained potassium nitride seeding equal to the highest value of potassium content suggested for the MHD generator. Materials which gave good performance in this test were beryllium oxide, magnesium oxide, hafnium oxide, and zirconium diboride.

The linear thermal expansion of twenty-five refractory insulator and conductor materials was measured during this program. Many of these were measured to temperatures above 2400°C using a graphite resistance furnace and spectroscopic graphite dilatometer rods. Measurements up to about 1800°C were made in an oxide resistance furnace with sapphire dilatometer rods.

* The electrical resistivity of silicon carbide + 25 per cent graphite by volume was measured but was not presented due to a lack of reproducibility in the data

Hot modulus of rupture tests were made only on those materials which had shown definite promise of usefulness in MHD generators as a result of previous tests in this program. Measurements were made on three forms of beryllium oxide, magnesium oxide, high density graphite, zirconium carbide with 30 per cent graphite, and silicon carbide with 20 per cent graphite.

To obtain additional information on MHD materials for use in design work, a program, funded by Pratt & Whitney Aircraft, was instituted at Southern Research Institute for the determination of properties not included under the in-house program. These properties were ultimate tensile strength, modulus of elasticity, compression strength, shear modulus, and thermal conductivity. Materials tested were those which had shown definite promise for MHD use. These included two hot-pressed beryllium oxides of different densities and two composite materials, zirconium carbide with graphite and silicon carbide with graphite.

The environmental program consisted of the testing of several materials as electrodes and sidewalls in MHD rig sections. During preliminary testing in this program, many materials were tested in a subsonic rig to determine their ability to withstand the high temperature and corrosive atmosphere of an operating MHD unit. The best insulator and conductor materials as indicated by these determinations and by the results of the laboratory investigation were tested further in a supersonic MHD rig which more closely approximated the temperature and erosion characteristics of an operating MHD unit. The best electrode and insulating materials were mounted in the supersonic rig and power was generated.

During preliminary testing 18 refractory electrode materials and nine insulator materials were run in a simulated MHD environment with a subsonic plasma velocity. Of the electrode materials, dense graphite, stannic oxide, and titanium nitride eroded severely. Materials which had adequate erosion resistance but which cracked severely during testing included columbium carbide, molybdenum disilicide, silicon carbide, tantalum carbide, tantalum diboride, titanium boronitride, titanium diboride, and zirconium diboride. Two materials which did not crack and had only moderate erosion were high density grain-oriented graphite and pyrolytic graphite. Other materials which stood up well in testing were the composite materials consisting of a carbide material containing dispersed graphite particles. The carbide composites tested were silicon carbide, tantalum carbide, tantalum car-

bide-zirconium carbide, and zirconium carbide. Of the insulator materials tested aluminum oxide melted and boron nitride, magnesium oxide, and strontium zirconate eroded badly. Hafnium oxide, thorium oxide, and zirconium silicate cracked very badly. Beryllium oxide held up very well during testing with no measurable erosion and only minor cracking.

For the next phase of testing using a supersonic MHD environmental rig, four electrode materials and two insulator materials were included. The zirconium carbide-graphite material was found to have inadequate erosion resistance in these tests but a graded electrode was developed having a zirconium carbide surface with the amount of graphite particles increasing toward the back of the electrode. This electrode held up very well in subsequent tests. Silicon carbide-graphite electrodes held up well in the supersonic plasma. The high density, grain-oriented graphite had no cracking and only moderate erosion but was considered inferior to the electrodes mentioned above. The tantalum carbide-graphite electrodes, however, eroded badly in the supersonic plasma. The high purity, high density beryllium oxide insulators held up very well with no evidence of erosion occurring, although some cracking resulted. Boron nitride eroded very badly and has MHD application as backup spacers only.

In final MHD environmental testing, runs were made using beryllium oxide insulators and either zirconium carbide-graphite, graded electrodes or silicon carbide-graphite electrodes. These runs were made with power being generated for from 10 to 30 minutes.

As a result of the materials study the following conclusions were drawn:

- 1) The outstanding insulator material is high purity, high density, hot-pressed beryllium oxide.
- 2) Two outstanding electrode materials were found, silicon carbide-graphite and zirconium carbide-graphite. The thermal shock resistance of both of these materials was improved markedly by the controlled addition of dispersed graphite particles within the carbide matrix. Both materials showed good erosion resistance. However the silicon carbide material exhibited better oxidation resistance than the zirconium carbide.

- 3) The results of the final phase of environmental testing indicated that significant electrical power can be generated for extended periods of time with electrodes of either of the above electrode materials.

Summary of Volume V, Theoretical Plasma Conductivity Studies

Volume V presents the results of a Pratt & Whitney Aircraft Division analytical investigation of plasma conductivity under varying plasma conditions. Improved analytical methods were developed for determining the plasma and boundary layer conductivities in the MHD generator.

The purpose of the analysis was to investigate the relationships between the current, the electric field, and other current driving forces in MHD generators. These relationships were first derived from basic principles and then applied to determining the plasma conductivity in the uniform main stream, and to the very much more complicated problem of determining voltage drop in the electrode boundary layers. The study was restricted to conditions occurring in MHD generators, but not necessarily to those found only in hydrocarbon-fueled open-cycle generators. The first part of this investigation dealt with the problem of the main stream conductivity. In this part of the investigation the formulas for the electrical conductivity and diffusion coefficients of a magnetohydrodynamic generator plasma, including ion flow, are derived. The electron energy equation is developed, leading to an expression for electron temperature. These formulas are applied to calculating these quantities in segmented and unsegmented MHD generators. It is shown that in segmented generators these quantities have much more favorable values. The expressions derived are also compared to the approximate relations in the literature based on constant electron collision frequency. It is shown that the approximate relations can lead to inaccurate results, especially in the Hall mobility ratio.

It was necessary to restrict the range of conditions of the plasma in order to obtain a solution for the conductivity and other transport properties. This is because for each range of conditions, different sets of assumptions on the relative magnitudes of important parameters are valid. Plasmas used as the working fluid for MHD generators have the characteristic of high total particle density with a low proportion of electrons. However, because of the high overall density, the density of electrons is still higher than usual for most plasmas.

In more explicit terms, these general plasma properties lead to the following assumptions on the relative magnitude of parameters. The electron-electron collision frequency is much less than the electron-neutral collision frequency. However, the energy transfer among electrons is much greater than the energy transfer to the molecules. An electron can transfer all its energy to another electron in a collision, but it can only transfer a small proportion of its energy to an atom because of the great mass difference. Therefore it can be assumed that the electron-electron collision frequency is much greater than the product of the energy loss parameter and the electron-neutral collision frequency, except when the neutrals are multi-atom type molecules. The energy loss parameter is the fraction of excess energy electrons lose per collision with the neutrals.

Because of their much greater mass, the velocities of the ions can be neglected compared to those of the electrons, even though the current due to the ions has to be considered because of their possibly greater density.

Another very important assumption that has been made in the analysis is that the isotropic part of the distribution function of the electrons is Maxwellian. This assumption can be justified on more basic grounds under two circumstances. When there are only monatomic atoms in the plasma the energy transfer between electrons is much greater than the energy transfer to the atoms. Since the energy the electrons gain from the electric field is exactly equal to that lost to the atoms, this energy transfer rate is also much smaller than the electron-electron energy transfer rate. Thus, in this case the electrons are virtually only in thermal contact with themselves and form a canonical ensemble which is sufficient to insure a Maxwell distribution.

When a plasma contains a large proportion of multi-atomic molecules the electrons can transfer energy more easily to the molecules. This is because of collisions where the rotational and vibrational states of the molecules are excited. However, even in this case, while the inequality is greatly weakened, the electrons still transfer more energy among themselves. A Maxwell distribution also results from the opposite effect. The large electron-molecule energy transfer rate, even for small excess of electron energy, prevents the electron temperature from rising significantly above the gas temperature, when the electric fields are of a magnitude found in the main stream of MHD generators. Under these circumstances, the electrons are practically in equilibrium with the gas and maintain an almost exact

Maxwell distribution, independent of the electron-electron energy transfer mechanism. It is shown that in a segmented electrode MHD generator the effective conductivity $\sigma = (\sigma_a + \sigma_e) / \sigma_a$ is in general not equal to the conductivity without a magnetic field σ_0 . They are equal only when the electron collision frequency is constant and independent of electron speed. In the potassium-seeded hydrocarbon combustion product plasma the effective conductivity varies only slightly with the magnetic field. This is true also for a pure cesium plasma. However for a cesium-seeded plasma the effective conductivity changes greatly with magnetic field from its zero field amount.

The Hall mobility ratio α is also, in general, not directly proportional to the magnetic field. This is again true only when the electron collision frequency is independent of speed. The Hall mobility ratio is much more sensitive to variable collision frequency than the effective conductivity and in almost all cases must be calculated by correctly averaging the collision frequency in order to get accurate results.

In order to obtain a highly elevated electron temperature in an MHD generator it is necessary that the electrodes be segmented.

Typical negative ion densities in seeded hydrocarbon combustion product plasmas (due to the formation of OH^-) are too low to make the ion densities so much in excess of the electron densities that ion conductivity is important. However if the seed is a potassium halide salt the halogen forms a large enough density of negative ions that the ion current cannot be neglected.

The second part of the study was concerned with the electrode boundary layer. The ordinary momentum equation for a turbulent boundary layer was analyzed with the additional Lorentz force present in MHD generators. The turbulent energy equation of the neutral species was considered with the inclusion of the energy transferred by the electrons and the enthalpy generated by the changing chemical species across the boundary layer, as well as the heat transferred to the wall. The velocity and stagnation enthalpy profile forms were postulated and the rate of growth of the boundary layer thickness was found by solving the integrated momentum and energy equations.

The electron continuity equation was then set up allowing for electron diffusion, mobility, formation, and recombination. It was assumed that while the boundary layer is turbulent, the electron particle diffusion far exceeds the turbulent diffusion of the electrons, because of

their low mass. Because of the plasma condition that the net charge density is zero, the electron plus the negative ion density must be balanced by the positive ion density. Also, the divergence of the total electric current must be zero. These conditions cause the electron and ion fluxes to be tightly interrelated, thus the ion continuity equations must be solved simultaneously with the electron equation.

The diffusion and mobility were calculated using the coefficients calculated in the study. Next the formation and recombination rates are analyzed. These are extremely dependent (especially the formation rate) on the electron temperature. Thus to solve the continuity equation the electron energy equation must also be solved for the electron temperature. These must be solved simultaneously as the electron temperature is dependent on the electron density and flux. In solving the electron energy equation, the energy transferred to the neutrals, which is used in the neutral species boundary layer energy equation, is found.

The main external effect solved for was the boundary layer voltage deficiency. This deficiency is the difference between the actual voltage output and that which would be present if the main stream velocity and conductivity continued all the way to the electrode surface. The voltage deficiency comes from two effects: the drop in the electromotive force due to the velocity being smaller in the boundary layer and, more important, the voltage drop resulting from the increase of the electric field compensating for the decline of the conductivity in the boundary layer.

The main results of the second part of this study are that the anode voltage drop is approximately one-third the cathode drop, that the voltage drop decreases substantially with increasing wall temperature and that there is a decrease in voltage drop with increasing initial temperature gradient. It is also shown that the calculated boundary layer voltage drop levels out at high current densities but still has not become constant for current densities as high as 20,000 amps/meter².

Summary of Volume VI, Preliminary Design of 1000 and 250 Megawatt MHD Generators

Volume VI presents the pertinent results of the conceptual engineering studies conducted on two sizes of MHD generators. The studies included both analytical and design efforts. Design layouts, concepts, and supporting data are presented and discussed as regards perfor-

mance capabilities from both the structural and operational points of view.

One MHD generator was designed to produce 1000 megawatts of DC electric power for utilization in continuous-wave, radio-frequency energy conversion systems in the high power range, at frequencies compatible with present Air Force applications and requirements. The other was sized according to potential Air Force requirements, resulting in a unit producing a net power of 250 megawatts. They were designed, on the basis of present knowledge, to have twenty-hour operating life, to be capable of run durations of up to thirty minutes and to have startup times of the order of one minute. The generator operating parameters were based on systems studies performed under the contract and described in Volume I of the final report, on analytical studies of the microwave generators, (Volumes II and III), on analytical studies and experimental tests conducted to evaluate various materials for use in the fabrication of MHD generator electrodes and insulating sidewalls (Volume IV) and on the results of analytical studies (Volume V) and experimental tests (Volume I) to investigate plasma core-flow and electrode boundary layer conductivity under varying conditions. The design concepts formulated are considered to be of a preliminary nature and serve to provide the basis for eventual project activity leading to the development of fully dependable generators.

For design and discussion purposes, the MHD generator system was divided into the following major subsystems:

- 1) Combustion section to convert the chemical energy of the reactants to thermal energy
- 2) Nozzle section to convert thermal energy to kinetic energy
- 3) MHD generator channel to convert kinetic energy to electrical energy
- 4) Diffuser-splitter section to reduce losses due to MHD channel end effects and to recover some of the unused kinetic energy
- 5) Mixer section to prevent losses due to Hall current leakage to the scrubber section or other grounded structure by destroying the conductivity of the plasma with water-injection cooling
- 6) Scrubber section to remove seeding compounds and erosion

products from the exhaust gases

- 7) Magnet to provide the magnetic field with which the plasma interacts to produce power
- 8) Controls to start, and operate the unit, maintaining power output at the required level
- 9) Propellant and coolant storage, handling and disposal system

The efficient performance of the MHD generators depends on the proper functioning of all of these subsystems

The effort revealed no insurmountable or mutually inconsistent situations although technical problems remain for solution by a power-plant development program. In particular, the need exists for additional knowledge of: fundamental properties of some materials in the temperature ranges anticipated (from -273°F to $+5,200^{\circ}\text{F}$), precise flow and heat transfer data where the combined effects of current flow, ionization, recombination and possible catalysis are present, or under cryogenic, two-phase conditions, of coolant electrical properties including effects of impurities, and of two-phase coolant flow stability. In addition, the need exists for information on the effect of conductive deposits on the dielectric properties and performance of insulators, the effect of insulating deposits on electrode surfaces and the dielectric properties of water spray, consisting of discrete droplets containing contaminants.

V. CONCLUSIONS OF THE FINAL REPORT

Based on the studies and investigations of MHD power generators and rf superpower converters reported in the six volumes of this final report the following principal conclusions have been reached.

1. No major areas of incompatibility exist in the integration of MHD power generators and rf superpower converters. It appears to be more advantageous to physically separate the MHD generator and the rf converter rather than to integrate the two to utilize the fringing magnetic field of the MHD generator for the rf converter. The basis for this conclusion is that it is more efficient and less costly to distribute power supply to the rf generator rather than the rf energy. An extensive development program involving both extended analysis and testing would be necessary to achieve a reliable MHD generator. Somewhat less development effort would be necessary on the rf components of the proposed system. The smaller size generator (250 megawatts) would appear to be less difficult to develop than the 1000-megawatt generator.
2. The environmental testing program indicated several major problem areas. The need for and the difficulty in obtaining high efficiency, stable, consistent, and predictable combustion of high pressure JP4 and oxygen with the addition of a seeding material was demonstrated. The problem of obtaining and maintaining mechanical and electrical integrity of the MHD generator section was solved for the range of voltages encountered in the test program*. However, maintaining electrical integrity at voltages which are typical of the 1000 MW and 250 MW generators will be a much more severe problem.
3. Two methods were found in the environmental testing program for decreasing the boundary layer voltage drop. Ramped electrodes were found to decrease the boundary-layer voltage drop by increasing the temperature through induced boundary layer mixing. The existence of high oxidant-to-fuel ratio in the boundary layer through the addition of excess O_2 around the periphery of the injector also tended to decrease the voltage drop when ceramic carbide electrodes were used. The free oxygen apparently reacts with the carbides, oxidizing them, adding thermal energy to the boundary layer and thereby decreasing its resistance.

* Output power densities up to 0.81 megawatts/ft³ were produced for periods up to 1 1/2 hours

4. Correlation procedures have been developed which can be used to predict combustion efficiency of combustors operating within the pressure range tested. Combustion efficiency was shown to increase with increasing chamber pressure, oxidant-to-fuel ratio, combustion chamber length and residence time. The effects pressure and oxidant-to-fuel ratio have on combustion efficiency decrease with increasing chamber length or residence time. It was also shown that heat loss to the injector face, combustion chamber walls and nozzle, lowers combustion efficiency and results in a greater than linear degradation of MHD performance.
5. The feasibility of using either crossed-field or linear-beam rf converters was established. Both tube types would require development effort to achieve working status, these efforts probably being somewhat less for the klystron tube than for the new EAL crossed-field device.
6. High density, high purity, hot-pressed beryllium oxide was found to have acceptable properties for use as a sidewall material in MHD generators except for its resistance to thermal stresses. The best ceramic electrode material found was silicon carbide graphite. The durability of this material for long durations was insufficient at the conditions encountered in the MHD environmental testing. Therefore effort toward improvement of electrode materials and electrode cooling should be continued and development of BeO as a sidewall material should be extended.
7. Improved analytical methods were developed for the problem area of bulk and boundary layer conductivity. It was shown that the effective conductivity resulting from the use of segmented electrodes is in general not equal to the conductivity without a magnetic field. It was also shown that the Hall mobility ratio is not directly proportional to the magnetic field strength. A further result is that segmented electrodes are essential to the production of highly elevated electron temperatures. The main conclusions regarding the boundary layer are that the anode voltage drop is about one third the cathode boundary layer and that the voltage drop decreases substantially with increasing wall temperature.
8. Structural concepts and design procedures were developed which should advance the state of the art of MHD generators. The design and construction of sidewalls and electrode walls with both mechanical and electrical integrity was shown to be a major problem. However, designs using existing materials were formulated which, with intensive developmental work, should be capable of meeting the demands of the environment.

VI. RECOMMENDATIONS OF THE FINAL REPORT

Based on the studies resulting from this contract and other work performed at Pratt & Whitney Aircraft, the following recommendations are made:

1. Parallel efforts should be pursued in the following areas,
 - a. Further investigations of both ceramic and metallic electrode materials
 - b. MHD generator structural concepts testing
 - c. Combustion studies
 - d. Development of diagnostic techniques and instrumentation
 - e. Determination of more accurate plasma data such as collision cross-sections of the plasma constituents in the range of interest
 - f. Adoption and modification of the theoretical predictions to include various loss mechanisms
2. With development of more refined rigs, instrumentation and calculation methods, a more extensive determination of experimental values of plasma conductivities, both bulk and boundary layer, and power generation parameters should be undertaken.
3. Following completion of the above a demonstration of the self-sustaining ability of MHD for reasonable periods of time should be pursued, including more extensive studies of the dynamic characteristics and problems of the MHD control system.
4. Concurrently with the above efforts, development of superpower rf converters should be pursued.

All of these efforts would contribute to the overall advancement of MHD power generation and rf conversion in an orderly fashion.

APPENDIX I

Derivation of Basic Equations

APPENDIX 1

Derivation of Basic Equations

The performance of the MHD generator presented in this report is based primarily on the theory of one-dimensional fluid flow. The equations pertinent to the solution are those of continuity, momentum, energy, Maxwell's law, Ohm's law, and the perfect gas law. In a most general three-dimensional solution these six equations are actually 23 equations.

Coordinate System

The coordinate system used in the following development with superimposed vectors indicating the directions considered is:

General Equations

The equations used before any assumptions regarding one-dimensionality, viscosity, steady state, etc., are:

Continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho V_j)}{\partial x_j} = 0 \quad (1 \text{ equation})$$

Equation of motion (per unit volume)

$$\rho \frac{dV_i}{dt} = - \frac{\partial P}{\partial x_i} + \frac{\eta}{3} \frac{\partial^2 V_j}{\partial x_i \partial x_j} + \eta \frac{\partial^2 V_i}{\partial x_i \partial x_j} + \epsilon_{ijk} J_j B_k \quad (3 \text{ equations})$$

Maxwell's equations

$$\epsilon_{ijk} \frac{\partial E_k}{\partial x_j} = - \frac{\partial B_i}{\partial t} \quad (3 \text{ equations})$$

$$\epsilon_{ijk} \frac{\partial H_k}{\partial x_j} = J_i \quad (3 \text{ equations})$$

$$\frac{\partial B_i}{\partial x_i} = 0 \quad (1 \text{ equation})$$

$$\frac{\partial D_i}{\partial x_i} = \rho^e \quad (1 \text{ equation})$$

$$B_i = \mu H_i \quad (3 \text{ equations})$$

$$D_i = \epsilon E_i \quad (3 \text{ equations})$$

$$\text{Ohm's law} \quad J_i = \sigma_{ij} (E_j + \epsilon_{imn} V_m B_n) \quad (3 \text{ equations})$$

$$\text{Perfect gas law} \quad P = \rho R T \quad (1 \text{ equation})$$

Energy equation

$$\rho \frac{d(C_v T)}{dt} = \frac{P_{ij} \partial V_i}{\partial x_j} + \frac{J_i^2}{\sigma} + \kappa \frac{\partial^2 T}{\partial x_j \partial x_j} \quad (1 \text{ equation})$$

$$\text{where } P_{ij} = -P\delta_{ij} - \frac{2\eta}{3} \frac{\partial V_k}{\partial x_k} \delta_{ij} + \eta \left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right)$$

In analyzing this system of equations the unknowns are V_i , P , ρ , T , ρ^ϵ , J_i , E_i , B_i , D_i , H_i .

Therefore, there are 22 unknown quantities and 23 equations. How-

ever, one equation $\frac{\partial B_i}{\partial x_i} = 0$ is not independent, reducing to 22

the number of equations. The conductivity tensor σ_{ij} is determined outside of this system of equations as a function of temperature, pressure, and gas composition.

Assumptions

It was assumed that:

- 1) The flow is nonviscous which implies that $\eta = 0$ in the equations of energy and motion,
- 2) The system is in steady state which implies that $\frac{\partial}{\partial t} = 0$
- 3) There is no heat conduction, therefore $K = 0$.

Applying these assumptions to the system of equations and defining the substantial derivative $\frac{d}{dt} = \frac{\partial}{\partial t} + V_j \frac{\partial}{\partial x_j}$, the equations can be rewritten.

Basic Equations

Continuity equation

$$\frac{\partial(\rho V_j)}{\partial x_j} = 0 \quad (1-1)$$

Equation of motion

$$\rho V_j \frac{\partial V_i}{\partial x_j} = - \frac{\partial P}{\partial x_i} + \epsilon_{ijk} J_j B_k \quad (1-2)$$

Maxwell's equations

$$\epsilon_{ijk} \frac{\partial E_k}{\partial x_j} = 0 \quad (1-3a)$$

$$\epsilon_{ijk} \frac{\partial H_k}{\partial x_j} = J_i \quad (1-3b)$$

$$\frac{\partial D_i}{\partial x_i} = \rho^\epsilon \quad (1-3c)$$

$$B_i = \mu H_i \quad (1-3d)$$

$$D_i = \epsilon E_i \quad (1-3e)$$

Ohm's law

$$J_i = \sigma_{ij} (E_j + \epsilon_{jmn} V_m B_n) \quad (1-4)$$

Perfect gas law

$$P = \rho R T \quad (1-5)$$

Energy equation

$$\rho V_j \frac{\partial (C_v T)}{\partial x_j} + P \delta_{ij} \frac{\partial V_i}{\partial x_j} = \frac{J_i^2}{\sigma} \quad (1-6)$$

Expansion of Ohm's Law for Tensor Conductivity

Consider Equation (1-4) Ohm's law, $J_i = \sigma_{ij} (E_j + \epsilon_{jmn} V_m B_n)$. The conductivity tensor used in the expansion is

$$\sigma_{ij} = \begin{bmatrix} \sigma_R & \sigma_n & 0 \\ -\sigma_H & \sigma_R & 0 \\ 0 & 0 & \sigma_o \end{bmatrix}$$

Expanding on j , where $j = 1, 2, 3$

$$J_i = \sigma_{i1} (E_1 + \epsilon_{1mn} V_m B_n) + \sigma_{i2} (E_2 + \epsilon_{2mn} V_m B_n) + \sigma_{i3} (E_3 + \epsilon_{3mn} V_m B_n)$$

$$\epsilon_{jmn} = \pm 1 \quad \text{if } j \neq m \neq n$$

$$\epsilon_{jmn} = 0 \quad \text{if any two indexes are the same.}$$

For one-dimensional flow

$$V_2 = V_3 = 0 \quad \therefore m = 1$$

$$B_1 = B_2 = 0 \quad \therefore n = 3$$

Ohm's law can now be written

$$J_i = \sigma_{i1}(E_1 + \epsilon_{113}V_1B_3) + \sigma_{i2}(E_2 + \epsilon_{213}V_1B_3) + \sigma_{i3}(E_3 + \epsilon_{313}V_1B_3)$$

$$\epsilon_{113} = 0 \quad \epsilon_{313} = 0$$

$$J_i = \sigma_{i1}E_1 + \sigma_{i2}(E_2 - V_1B_3) + \sigma_{i3}E_3$$

Expanding on i where $i=1,2,3$ gives

$$J_1 = \sigma_{11}E_1 + \sigma_{12}(E_2 - V_1B_3) + \sigma_{13}E_3$$

$$J_2 = \sigma_{21}E_1 + \sigma_{22}(E_2 - V_1B_3) + \sigma_{23}E_3$$

$$J_3 = \sigma_{31}E_1 + \sigma_{32}(E_2 - V_1B_3) + \sigma_{33}E_3$$

Replacing 1, 2, 3, by x, y, and z and introducing the σ_{ij} tensor

$$J_x = \sigma_R E_x + \sigma_H (E_y - V_x B_z) \quad (1-7a)$$

$$J_y = -\sigma_H E_x + \sigma_R (E_y - V_x B_z) \quad (1-7b)$$

$$J_z = \sigma_o E_z \quad (1-7c)$$

Since the electrodes or insulators are solid in the Z direction there is no Z component of electric field. Hence $E_z = 0$ which implies that $J_z = 0$.

For the case where Hall currents are precluded, $J_x = 0$, then

Equation (1-7a) becomes, for the directions indicated on the coordinate system

$$J_x = -\sigma_R E_x + \sigma_H (-E_y + V_x B_z) = 0$$

$$E_x = \frac{\sigma_H}{\sigma_R} (V_x B_z - E_y)$$

then

$$J_y = \frac{\sigma_H^2}{\sigma_R} (V_x B_z - E_y) + \sigma_R (V_x B_z - E_y)$$

$$J_y = (V_x B_z - E_y) \left(\frac{\sigma_H^2 + \sigma_R^2}{\sigma_R} \right)$$

but

$$\frac{\sigma_H^2 + \sigma_R^2}{\sigma_R} = \sigma' = \sigma_o^*$$

$$J_y = \sigma_o (V_x B_z - E_y)$$

This is the value of current density when the electrodes are segmented and the Hall current J_x is precluded. It is of interest to explore the reasons for segmentation. For the case where the electrodes are solid $E_x = 0$.

$$J_x = \sigma_H (E_y - V_x B_z)$$

$$J_y = \sigma_R (E_y - V_x B_z)$$

$$\frac{J_x}{\sigma_H} = \frac{J_y}{\sigma_R} \quad \text{or} \quad \frac{J_x}{J_y} = \frac{\sigma_H}{\sigma_R}$$

σ_H / σ_R is defined as the Hall coefficient $\omega \tau$. From the relationship

$$\sigma_o = \frac{\sigma_H^2 + \sigma_R^2}{\sigma_R} = \frac{\sigma_R^2 \left(\frac{\sigma_H^2}{\sigma_R^2} + 1 \right)}{\sigma_R}$$

then $\sigma_o = \sigma_R [(\omega \tau)^2 + 1]$ and $\sigma_o = \frac{\sigma_o}{[(\omega \tau)^2 + 1]}$

Therefore for values of $\omega \tau$ much greater than 1, σ_R

*Volume V of final MHD report, TR-62 464 (For the cases considered, σ' was approximately equal to σ_o)

is much less than σ_0 and J_y is less than J_y for the segmented case. This results in less power.

In this section the following assumptions are applied to Maxwell's equations, the equation of motion, and the energy equation.

- 1) Quasi one-dimensional flow whereby $\frac{\partial}{\partial z} = \frac{\partial}{\partial y} = 0$
- 2) The applied magnetic field B is in the Z direction only.

Consequence of $\nabla \times \vec{E} = 0$ and One-Dimensionality

Equation (1-3a), $\epsilon_{ijk} \frac{\partial E_k}{\partial x_j} = 0$, can also be written

$\nabla \times \vec{E} = 0$. This can be expanded out using spherical coordinates natural for a diverging channel. In this coordinate

system $\frac{\partial}{\partial \theta} = \frac{\partial}{\partial \phi} = 0$, $E_\phi = 0$

The position vector $\vec{r} = x_i \vec{i} + y_j \vec{j} + z_k \vec{k}$

In general orthogonal coordinates

$$\nabla \times \vec{F} = \frac{1}{h_1 h_2 h_3} \begin{bmatrix} h_1 u_1 & h_2 u_2 & h_3 u_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 F_1 & h_2 F_2 & h_3 F_3 \end{bmatrix}$$

u_1 , u_2 and u_3 as the orthogonal unit vectors and

$$h_1 = \frac{\partial \bar{r}}{\partial u_1}, \quad h_2 = \frac{\partial \bar{r}}{\partial u_2}, \quad h_3 = \frac{\partial \bar{r}}{\partial u_3}$$

In spherical coordinates

$$X = r \cos \theta \sin \phi$$

$$Y = r \sin \theta \sin \phi$$

$$Z = r \cos \phi$$

and

$$\bar{u}_1 = \bar{r}_1, \quad \bar{u}_2 = \bar{\theta}_1, \quad \bar{u}_3 = \bar{\phi}_1$$

$$\bar{r} = r \cos \theta \sin \phi \bar{i} + r \sin \theta \sin \phi \bar{j} + r \cos \phi \bar{k}$$

$$\frac{\partial \bar{r}}{\partial r} = \cos \theta \sin \phi \bar{i} + \sin \theta \sin \phi \bar{j} + \cos \phi \bar{k}$$

$$\left| \frac{\partial \bar{r}}{\partial r} \right| = \left[\cos^2 \theta \sin^2 \phi + \sin^2 \theta \sin^2 \phi + \cos^2 \phi \right]^{1/2} = 1 = h_1$$

$$\frac{\partial \bar{r}}{\partial \theta} = -r \sin \theta \sin \phi \bar{i} + r \cos \theta \sin \phi \bar{j}$$

$$\left| \frac{\partial \bar{r}}{\partial \theta} \right| = \left[r^2 \sin^2 \theta \sin^2 \phi + r^2 \cos^2 \theta \sin^2 \phi \right]^{1/2} = r \sin \phi = h_2$$

$$\frac{\partial \bar{r}}{\partial \phi} = r \cos \theta \cos \phi \bar{i} + r \sin \theta \cos \phi \bar{j} - r \sin \phi \bar{k}$$

$$\left| \frac{\partial \bar{r}}{\partial \phi} \right| = \left[r^2 \cos^2 \theta \cos^2 \phi + r^2 \sin^2 \theta \cos^2 \phi + r^2 \sin^2 \phi \right]^{1/2} = r = h_3$$

$$\frac{\partial}{\partial u_2} = \frac{\partial}{\partial \theta} = 0 \quad \text{and} \quad \frac{\partial}{\partial u_3} = \frac{\partial}{\partial \phi} = 0$$

$$\bar{V} \times \bar{E} = \frac{1}{r^2 \sin \phi} \begin{vmatrix} \bar{r}_1 & r \sin \phi \bar{\theta}_1 & r \bar{\phi}_1 \\ \frac{\partial}{\partial r} & 0 & 0 \\ E_r & r \sin \phi E_\theta & 0 \end{vmatrix}$$

$$\bar{\nabla} \times \bar{E} = \frac{1}{r^2 \sin \phi} \left[r \bar{\phi}_i \frac{\partial}{\partial r} (r \sin \phi E_\theta) \right]$$

$$\bar{\nabla} \times \bar{E} = \frac{1}{r^2 \sin \phi} \left[r \bar{\phi}_i \left(\sin \phi E_\theta + r \sin \phi \frac{\partial E_\theta}{\partial r} + r E_\theta \frac{\partial}{\partial r} \sin \phi \right) \right]$$

But

$$\frac{\partial}{\partial r} \sin \phi = 0$$

$$\begin{aligned} \bar{\nabla} \times \bar{E} &= \frac{1}{r^2 \sin \phi} \left[r \sin \phi E_\theta \bar{\phi}_i + r^2 \sin \phi \frac{\partial E_\theta}{\partial r} \bar{\phi}_i \right] \\ &= \frac{E_\theta}{r} \bar{\phi}_i + \frac{\partial E_\theta}{\partial r} \bar{\phi}_i \end{aligned}$$

But

$$\begin{aligned} \bar{\nabla} \times \bar{E} &= 0 \quad \therefore \frac{E_\theta}{r} = - \frac{\partial E_\theta}{\partial r} \\ \frac{\partial r}{r} + \frac{\partial E_\theta}{E_\theta} &= 0 \end{aligned}$$

Integrating

$$\ln r + \ln E_\theta = \ln C$$

$$E_\theta r = \text{constant}$$

However, y the distance between the electrode walls, can be expressed as a function of r by the following relationship

$$y = C_1 + C_2 r$$

But at $r = 0$, $y = 0$, therefore $C_1 = 0$

Hence $y = C_2 r$ or $r = \frac{y}{C_2}$

Therefore $E_\theta y = \text{constant}$.

At the centerline E_y is exactly the same as E_θ . The greatest discrepancy between them occurs at the wall where E_θ is perpendicular to the wall but at an angle θ to E_y . For values of θ equal to 13° (the maximum angle used) the $\cos \theta \cong .97$. Therefore the discrepancy at the wall is only about 3 per cent. Hence $E_y y = \text{constant}$.

Result of Small Magnetic Reynolds Number

Introducing Equation (1-3d), $B_i = \mu H_i$ into (3b) yields

$$J_i = \frac{1}{\mu} \epsilon_{ijk} \frac{\partial B_k}{\partial x_j}$$

This equation can be equated to Equation (1-4), Ohm's law, giving

$$\frac{1}{\mu} \epsilon_{ijk} \frac{\partial B_k}{\partial x_j} = \sigma_{ij} (E_j + \epsilon_{ijk} V_j B_k)$$

Let $E = E'(UB_o)$

where U is an average velocity

B_o is the average applied magnetic field

$$E' = \frac{E}{UB_o}$$

E' is a nondimensionalized electric field

$$V = V'U$$

where V' is a nondimensionalized velocity

$$B = B'B_o$$

B' is a nondimensionalized magnetic field

$$x = x'L$$

x' is a nondimensionalized length

Then

$$\frac{1}{\mu} \epsilon_{ijk} \frac{\partial (B'_0)_k}{\partial (X'_L)_j} = \sigma_{ij} \left[(E' U B_0)_j + \epsilon_{ijk} (V' U)_j (B' B_0)_k \right]$$

$$\frac{1}{\mu} \frac{B_0}{L} \epsilon_{ijk} \frac{\partial (B')_k}{\partial X_j} = U B_0 \sigma_{ij} (E'_j + \epsilon_{ijk} V'_j B'_k)$$

The magnetic Reynolds number R_m is defined by $\mu L \sigma_{ij} U$

Therefore

$$\begin{aligned} \epsilon_{ijk} \frac{\partial B'_k}{\partial X_j} &= \mu L \sigma_{ij} U (E'_j + \epsilon_{ijk} V'_j B'_k) \\ &= R_m (E'_j + \epsilon_{ijk} V'_j B'_k) \end{aligned}$$

Now B'_k is the nondimensionalized total magnetic field

$$B'_k = B_{0k} + B'_{ik}$$

Then

$$\epsilon_{ijk} \frac{\partial B'_k}{\partial X_j} = \bar{\nabla} \times (\bar{B}'_0 + \bar{B}'_i) = \bar{\nabla} \times \bar{B}'_0 + \bar{\nabla} \times \bar{B}'_i$$

Since the curl of the applied magnetic field is zero and, since the exciting current is external to the flow

$$\bar{\nabla} \times \bar{B}'_i = R_m (\bar{E}' + \bar{V}' \times \bar{B}')$$

$$R_m = \frac{\bar{\nabla} \times \bar{B}'_i}{(\bar{E}' + \bar{V}' \times \bar{B}')}$$

For the MHD generator considered for this application the parameter $\mu L \sigma_{ij} U$ is found to be much less than 1.

$$\text{For } R_m \ll 1 \quad \frac{\bar{\nabla} \times \bar{B}'_i}{(\bar{E}' + \bar{V}' \times \bar{B}')} \ll 1$$

and since $\bar{E}'$, $\bar{V}'$, and $\bar{B}'$ are nondimensionless they are of the order of one and $\bar{\nabla} \times \bar{B}'_i \ll 1$, say 0.

Consequence of $\bar{\nabla} \times \bar{B} = 0$ and One-Dimensionality

$$\bar{B} = \bar{B}_o + \bar{B}_i$$

where $\bar{B}_o$ is the applied field

$\bar{B}_i$ is the induced field

It has been shown that the induced field is small. Considering

Equations (1-3b), (1-3d) and $\bar{B} = \bar{B}_o$ it follows that

$$\bar{\nabla} \times \bar{B} = \mu \bar{J}$$

$$\bar{\nabla} \times (\bar{B}_o + \bar{B}_i) = \mu \bar{J}$$

where $\bar{J}$ is the induced current.

Then $\bar{\nabla} \times \bar{B}_o = 0$

$$\bar{B}_o = B_\phi \bar{\Phi}, \text{ since } B_r = B_\theta = 0$$

Then

$$\bar{\nabla} \times \bar{B} = \frac{1}{r^2 \sin \phi} \begin{vmatrix} \bar{r}_i & r \sin \phi \bar{\theta}_i & r \bar{\Phi}_i \\ \frac{\partial}{\partial r} & 0 & 0 \\ 0 & 0 & r B_\phi \end{vmatrix}$$

$$= \frac{1}{r^2 \sin \phi} \left[r \sin \phi \bar{\theta}_i \frac{\partial}{\partial r} (r B_\phi) \right]$$

$$= \frac{\bar{\theta}_i}{r} \left(B_\phi + r \frac{\partial B_\phi}{\partial r} \right)$$

$$\therefore \frac{B_\phi}{r} + \frac{\partial B_\phi}{\partial r} = 0$$

Integrating $\ln B_\phi + \ln r = \ln C$

$$r B_\phi = \text{constant}$$

$Z = C_1 + C_2 r$ and since Z is zero at $r=0$, $C_1=0$.

Therefore $r = \frac{Z}{\text{constant}}$

and $B_\phi Z = \text{constant}$

In a manner similar to the proof that $E_y y$ is constant it can be shown that $B_\phi r = \text{constant}$ is the same as $B_z Z$ being constant.

One-Dimensional Flow Equations $J_x = 0$

Consider the equation of motion

$$\rho V_j \frac{\partial V_i}{\partial V_j} = - \frac{\partial P}{\partial x_i} + \epsilon_{ijk} J_j B_k$$

$$\frac{\partial}{\partial x_i} = \frac{\partial}{\partial x_j} = \frac{\partial}{\partial x_k} \text{ only}$$

V is only in the x or 1 direction

J is only in the y or 2 direction

B is only in the z or 3 direction

$$\therefore K = 3$$

Therefore

$$\rho V_i \frac{\partial V_i}{\partial x_i} = - \frac{\partial P}{\partial x_i} + \epsilon_{i23} J_2 B_3$$

Expanding on i

$$i=1 \quad \rho V_1 \frac{\partial V_1}{\partial X_1} = -\frac{\partial P}{\partial X_1} + J_2 B_3$$

$$\rho V_x \frac{\partial V_x}{\partial X} = -\frac{\partial P}{\partial X} + J_y B_z$$

$$i=2 \quad V_1 = V_2 = 0 \quad \frac{\partial V_2}{\partial X_1} = 0$$

Thus

$$\frac{dP}{dX_2} = 0 \quad \frac{dP}{dy} = 0$$

$$i=3 \quad V_1 = V_3 = 0 \quad \frac{\partial V_3}{\partial X_1} = 0$$

$$\frac{dP}{dz} = 0$$

Consider Equation (1-6), the energy equation

$$\rho V_j \frac{\partial (C_v T)}{\partial X_j} + P \delta_{ij} \frac{\partial V_i}{\partial X_j} = \frac{J^2}{\sigma}$$

Expanding on j and i

$$\rho \left[V_1 \frac{\partial (C_v T)}{\partial X_1} + V_2 \frac{\partial (C_v T)}{\partial X_2} + V_3 \frac{\partial (C_v T)}{\partial X_3} \right] + P \left[\frac{\partial V_1}{\partial X_1} + \frac{\partial V_2}{\partial X_2} + \frac{\partial V_3}{\partial X_3} \right] = \frac{J^2}{\sigma}$$

$$V_2 = V_3 = 0$$

$$\rho V_x \frac{\partial (C_v T)}{\partial X} + P \frac{\partial V_x}{\partial X} = \frac{J^2}{\sigma}$$

$$R = C_p - C_v$$

$$C_v T = C_p T - RT$$

Then

$$\rho V \frac{d}{dx} (C_p T - RT) + P \frac{dV}{dx} = \frac{J^2}{\sigma}$$

$$J_y = \sigma (V_x B_z - E_y)$$

$$\begin{aligned} \frac{J_y^2}{\sigma} &= J_y (V_x B_z - E_y) \\ &= J_y V_x B_z - J_y E_y \end{aligned}$$

$$\rho V \frac{d(C_p T)}{dx} - \rho V \frac{d}{dx} (RT) + P \frac{dV}{dx} = J_y V_x B_z - J_y E_y$$

As above the momentum equation is

$$\rho V_x \frac{dV_x}{dx} = - \frac{dP}{dx} + J_y B_z$$

The coordinate system shows B_z to be in the negative direction.

Therefore momentum equation when multiplied by V_x yields

$$\rho V_x^2 \frac{dV_x}{dx} + V_x \frac{dP}{dx} = - J_y B_z V_x \quad \text{and} \quad d(C_p T) = dh$$

Then

$$\rho V \frac{dh}{dx} - \rho V \frac{d(RT)}{dx} + P \frac{dV}{dx} = - \rho V^2 \frac{dV}{dx} - V \frac{dP}{dx} - J_y E_y$$

From state

$$P = \rho RT$$

and from continuity

$$\dot{m} = \rho VA \quad V = \frac{\dot{m}}{\rho A}$$

then

$$PV = \frac{\dot{m} RT}{A}$$

$$\rho V \frac{dh}{dx} - \rho V \frac{d(RT)}{dx} + \rho V^2 \frac{dV}{dx} + P \frac{dV}{dx} + V \frac{dP}{dx} = -J_y E_y$$

$$\frac{d(PV)}{dx} = P \frac{dV}{dx} + V \frac{dP}{dx}$$

$$\rho V \frac{dh}{dx} - \rho V \frac{d(RT)}{dx} + \rho V^2 \frac{dV}{dx} + \frac{d(PV)}{dx} = -J_y E_y$$

But

$$PV = \frac{\dot{m} RT}{A}$$

and

$$\frac{d(PV)}{dx} = \frac{d}{dx} \left(\frac{\dot{m} RT}{A} \right)$$

From Equation (1-1) $\frac{d(\rho V)}{dx} = 0$

$$\rho V = \text{constant} \quad \therefore \frac{\dot{m}}{A} = \text{constant and it}$$

can be taken out of the derivative.

Then

$$\frac{d}{dx} \left(\frac{\dot{m} RT}{A} \right) = \frac{\dot{m}}{A} \frac{d(RT)}{dx} = \rho V \frac{d(RT)}{dx}$$

The equation finally reduces to

$$\rho V \frac{dh}{dx} + \rho V^2 \frac{dV}{dx} = -J_y E_y$$

This equation will henceforth be referred to as the energy equation.

Summary

The result of this appendix can be summarized as follows

Momentum equation

$$\rho V_x \frac{dV_x}{dx} = -J_y B_z - \frac{dP}{dx} \quad (1-8)$$

Energy equation

$$\rho V \frac{dh}{dx} + \rho V^2 \frac{dV}{dx} = -J_y E_y \quad (1-9)$$

Maxwell's equation $E_y y = \text{constant} \quad (1-10)$

$$B_z Z = \text{constant} \quad (1-11)$$

Small magnetic Reynolds number

$$B_i \ll B_o \quad (1-12)$$

APPENDIX 2

Derivation of MHD Generator Performance Equations

APPENDIX 2

Derivation of MHD Generator Performance Equations

A. Specified Thermodynamics

In these types of solutions a chemical thermodynamics program is used as a subroutine and the composition of the plasma is calculated using shifting equilibrium theory at specific points in the generator duct.

Constant Enthalpy

Among the solution types are those of constant velocity, constant enthalpy, and constant mach number. As an example, consider the case of constant enthalpy.

Energy equation

$$\rho V \frac{dh}{dx} + \rho V^2 \frac{dV}{dx} = -J_y E_y \quad (2-1)$$

$$h = \text{constant}, \quad dh = 0.$$

Then

$$\rho V^2 \frac{dV}{dx} = -J_y E_y$$

or

$$\frac{\rho V^2}{E_y} \frac{dV}{dx} = -J_y dx \quad (2-2)$$

Introducing the momentum equation

$$\rho V \frac{dV}{dX} + \frac{dP}{dX} = -J_y B_z \quad (2-3)$$

$$\frac{\rho V dV}{B_z} + \frac{dP}{B_z} = -J_y dX \quad (2-4)$$

Equating Equation (2-4) to (2-2) yields

$$\frac{\rho V dV}{B_z} + \frac{dP}{B_z} = \frac{\rho V^2 dV}{E_y} \quad (2-5)$$

As a further restriction take $\frac{E_y}{V B_z}$ as a given constant. (2-6)

From the state equation $P = \rho RT$ (2-7)

Then $\rho = \frac{P}{RT}$ (2-8)

Substituting Equations (2-8) and (2-5)

$$\frac{P}{RT} V dV + dP = \frac{P B_z}{RT} \frac{V^2 dV}{E_y} \quad (2-9)$$

or

$$RT \frac{dP}{P} = \frac{B_z V}{E_y} V dV - V dV = V dV \left(\frac{B_z V}{E_y} - 1 \right) \quad (2-10)$$

$$V dV = \frac{RT \frac{dP}{P}}{\frac{B_z V}{E_y} - 1} \quad (2-11)$$

This can be integrated if the quantity RT is assumed constant. For constant enthalpy, temperature is close to constant over a narrow range.

Integrating (2-11) yields

$$\frac{V^2}{2} \Big|_1^2 = \frac{RT}{\left(\frac{B_z V}{E_y} - 1\right)} \ln P \Big|_1^2 \quad (2-12)$$

$$\frac{V_2^2}{2} - \frac{V_1^2}{2} = \frac{RT}{\left(\frac{B_z V}{E_y} - 1\right)} \ln \left(\frac{P_2}{P_1}\right) \quad (2-13)$$

$$V_2 = \left[\frac{2RT}{\left(\frac{B_z V}{E_y} - 1\right)} \ln \left(\frac{P_2}{P_1}\right) + V_1^2 \right]^{.5} \quad (2-14)$$

In this computer program pressure is the independent variable.

As a function of enthalpy and pressure the gas composition, density, temperature, and gas constant are determined. The initial value of velocity for the generator is given and average values of R and T used for calculating V_2 in Equation (2-14).

The energy equation can be written using stagnation conditions as

$$\rho V \frac{dh_T}{dX} = -J_y E_y$$

now

$$\rho V = \frac{\dot{m}}{A}$$

$$\dot{m} dh_T = -J_y E_y A dX = \text{power}$$

For a specific power output and total enthalpy change the mass flow is determined. The MHD duct area variation is found by using the continuity equation, since $\dot{m}$ is a constant, and ρ and V are functions of pressure and enthalpy.

From the chemical thermodynamics part of the program the stagnation enthalpy and pressure are determined for various values of static pressure. A governing criterion for the generator exit in this design point type generator is total pressure.

From Maxwell's equations $\nabla \times \vec{B} = \mu \vec{J}$ and $\nabla \times \vec{E} = 0$. It has been shown that B_z times Z (the insulator distance) is constant, and that E_y times y (the electrode distance) is constant.

The geometry of the generator, variation of B field and cross-stream electric field E_y can be determined from Maxwell's equations and the restriction that $\frac{E}{VB}$ is constant. The geometry resulting from these relations has curved walls and a nonconstant value of the aspect ratio, $\frac{y}{Z}$. In other instances of constant enthalpy cases $\frac{y}{Z}$ was kept constant and (E_y) (y) relaxed. This too resulted in curved walls with more accentuated curvatures.

The current density J_y is calculated from the Ohm's law relation, Equation (1-4), and from J_y , the current I is obtained.

B. Specified Geometric and Electrodynamic Variables (Froson Neutral Constituents)

In order to eliminate wall curvature, (more realistic and practical from a construction standpoint) and to study off-design performance,

a program was set up whereby certain geometric parameters could be specified in conjunction with several electromagnetic parameters. As became evident in the constant thermodynamic derivation, certain boundary conditions or restrictions had to be imposed for a meaningful solution. Six parameters were considered for use as boundary conditions. These parameters are: area, $\frac{y}{z}$, $E_y y$, $B_z z$, B and $\frac{E}{\sqrt{B}}$. To solve the set of equations it is necessary to specify four of the parameters as functions of the independent variable, length. These parameters could be held constant or allowed to vary in some predetermined manner. This made it possible to adhere strictly to the relations derived from the Maxwell equations $\nabla \times B = \mu J$ and $\nabla \times E = 0$, relax these relations if desired, operate at some desired value of cross-stream potential, or with some variation in $\frac{E}{\sqrt{B}}$.

In this program the gas composition, gas constant, and specific heat are frozen at the MHD inlet. However, changes in electron concentration are taken into account for the purpose of calculating conductivity by using the Saha equation. The initial value of electron concentration is computed by the chemical thermodynamics program which considers the possibility of K^+ , O^- , OH^- as well as many other ions existing. The Saha equation is used in the program, considering only the ionic reaction $K \rightleftharpoons K^+ + e^-$ to scale the initial value of electron concentration.

Following is the derivation of equations of flow used to compute the performance of the specified geometric and electrodynamic variable program.

$$\text{Continuity} \quad \dot{m} = \rho VA \quad (2-15)$$

$$\text{State} \quad P = \rho RT \quad (2-16)$$

Substitution for ρ in Equation (2-15) gives

$$\dot{m} RT = PVA \quad (2-17)$$

Differentiating with respect to x

$$\dot{m} R \frac{dT}{dx} + \dot{m} T \frac{dR}{dx} + TR \frac{d\dot{m}}{dx} = \rho V \frac{dA}{dx} + P A \frac{dV}{dx} + VA \frac{dP}{dx} \quad (2-18)$$

$$R = \text{constant} \quad \frac{dR}{dx} = 0$$

$$\dot{m} = \text{constant} \quad \frac{d\dot{m}}{dx} = 0$$

Dividing by A

$$\frac{\dot{m}}{A} R \frac{dT}{dx} = \frac{PV}{A} \frac{dA}{dx} + P \frac{dV}{dx} + V \frac{dP}{dx} \quad (2-19)$$

$$\text{now} \quad R = C_p - C_v \quad \text{and} \quad \gamma = C_p / C_v$$

then

$$R = C_p \left(1 - \frac{C_v}{C_p}\right) = C_p \left(1 - \frac{1}{\gamma}\right) = C_p \left(\frac{\gamma-1}{\gamma}\right)$$

$$\frac{\dot{m}}{A} C_p \left(\frac{\gamma-1}{\gamma}\right) \frac{dT}{dx} = \frac{PV}{A} \frac{dA}{dx} + P \frac{dV}{dx} + V \frac{dP}{dx} \quad (2-20)$$

$$\text{from continuity} \quad \dot{m}/A = \rho V$$

$$\rho V C_p \frac{dT}{dx} = \frac{\gamma}{\gamma-1} \left[\frac{PV}{A} \frac{dA}{dx} + P \frac{dV}{dx} + V \frac{dP}{dx} \right] \quad (2-21)$$

Noting that $C_p dT = dh$ this expression can be substituted into the energy equation.

$$\frac{\gamma}{\gamma-1} \left[\frac{PV}{A} \frac{dA}{dx} + P \frac{dV}{dx} + V \frac{dP}{dx} \right] + \rho V^2 \frac{dV}{dx} = -J_y E_y \quad (2-22)$$

$$\text{From Ohm's law } J_y = \sigma_o (V_x B_z - E_y) \quad (2-23)$$

Then

$$-J_y E_y = \sigma_o (E_y^2 - E_y V_x B_z) \quad (2-24)$$

Consider the momentum equation multiplied by V

$$\rho V^2 \frac{dV}{dx} + V \frac{dP}{dx} = -J_y V B_z \quad (2-25)$$

Solving for $V \frac{dP}{dx}$

$$V \frac{dP}{dx} = -J_y V B_z - \rho V^2 \frac{dV}{dx} \quad (2-26)$$

Substitution of Equations (2-26) and (2-24) into Equation (2-22) gives

$$\frac{\gamma}{\gamma-1} \left[\frac{PV}{A} \frac{dA}{dx} + P \frac{dV}{dx} - J_y V B_z - \rho V^2 \frac{dV}{dx} \right] + \rho V^2 \frac{dV}{dx} = \sigma_o (E_y^2 - V E_y B_z) \quad (2-27)$$

Dividing through by P

$$\frac{\gamma}{\gamma-1} \left[\frac{V}{A} \frac{dA}{dx} + \frac{dV}{dx} - \frac{J_y V B_z}{P} - \frac{\rho V^2}{P} \frac{dV}{dx} \right] + \frac{\rho V^2}{P} \frac{dV}{dx} = \frac{\sigma_o}{P} (E_y^2 - V E_y B_z) \quad (2-28)$$

From state $P = \rho RT$ and $\frac{P}{\rho} = RT$

Now the sound velocity $a = \sqrt{\gamma RT}$

$$M_N = \frac{V}{a} \quad M_N^2 = \frac{V^2}{a^2} = \frac{V^2}{\gamma RT}$$

The term

$$\frac{\rho V^2}{P} \frac{dV}{dx} = \frac{M_N^2 (\gamma RT)}{RT} \frac{dV}{dx} = \gamma M_N^2 \frac{dV}{dx}$$

Equation (2-28) now becomes

$$\frac{\gamma}{\gamma-1} \left[\frac{V}{A} \frac{dA}{dX} + \frac{dV}{dX} - \frac{J_y V B_z}{P} - \gamma M_N^2 \frac{dV}{dX} \right] + \gamma M_N^2 \frac{dV}{dX} = \frac{\sigma_o (E_y^2 - V E_y B_z)}{P} \quad (2-29)$$

From Ohm's law $J_y = \sigma_o (V_x B_z - E_y)$

$$J_y V B_z = \sigma_o (V^2 B_z^2 - V E_y B_z) \quad (2-30)$$

$$\begin{aligned} \frac{V dA}{A dX} + \frac{dV}{dX} + \frac{\sigma_o (V E_y B_z - V^2 B_z^2)}{P} - \gamma M_N^2 \frac{dV}{dX} + (\gamma-1) M_N^2 \frac{dV}{dX} &= \frac{\gamma-1}{\gamma} \left[\frac{\sigma_o}{P} (E_y - V E_y B_z) \right] \\ \frac{dV}{dX} - \gamma M_N^2 \frac{dV}{dX} + \gamma M_N^2 \frac{dV}{dX} - M_N^2 \frac{dV}{dX} &= \frac{\gamma-1}{\gamma} \left[\frac{\sigma_o}{P} (E_y - V E_y B_z) \right] - \frac{\sigma_o (V E_y B_z - V^2 B_z^2)}{P} - \frac{V}{A} \frac{dA}{dX} \end{aligned} \quad (2-31)$$

$$\frac{dV}{dX} (1 - M_N^2) = \frac{\sigma_o}{P} \left[\frac{\gamma-1}{\gamma} (E_y - V E_y B_z) - (V E_y B_z - V^2 B_z^2) \right] - \frac{V}{A} \frac{dA}{dX} \quad (2-32)$$

Finally

$$\frac{dV}{dX} = \frac{1}{1 - M_N^2} \left\{ \frac{\sigma_o}{P} \left[\frac{\gamma-1}{\gamma} (E_y - V E_y B_z) - (V E_y B_z - V^2 B_z^2) \right] - \frac{V}{A} \frac{dA}{dX} \right\} \quad (2-33)$$

Now the momentum equation

$$\rho V \frac{dV}{dX} + \frac{dP}{dX} = \sigma_o (E_y B_z - V B_z^2) \quad (2-34)$$

$$\frac{dP}{dX} = -\rho V \frac{dV}{dX} + \sigma_o (E_y B_z - V B_z^2) \quad (2-35)$$

The generator performance is found by solving Equations (2-34) and (2-36). As in the specified thermodynamics program the initial values of all the variables are known. Since the geometry has been specified as a function of X , from the four parameters mentioned previously, the electric field E and magnetic field B can be determined as a function of X . The only variables needed for a complete solution then are the thermodynamic quantities, temperature,

pressure and density, and the velocity and Mach number. Conductivity is computed locally as a function of X . In the solution of the equations the Runge-Kutta method utilizing fourth-order accuracy is used to find $\frac{dV}{dX}$ and $\frac{dP}{dX}$ over an increment of length.

The procedure used is as follows, where the subscript i indicates the initial value and the subscript l the last or final value.

$$V_l = V_i + 1/6 (K_1 + 2K_2 + 2K_3 + K_4)$$

and

$$P_l = P_i + 1/6 (\ell_1 + 2\ell_2 + 2\ell_3 + \ell_4)$$

Define the following

$$X_i \equiv X_o, \quad X_i + \frac{\Delta X}{2} \equiv X_M \quad \text{and} \quad X_i + \Delta X \equiv X_L$$

Then

$$K_1 = \left(\frac{dV}{dX} \right)_o \Delta X$$

$$\ell_1 = \left(\frac{dP}{dX} \right)_o \Delta X$$

Where all variables in the $\frac{dV}{dX}$ and $\frac{dP}{dX}$ relationships are evaluated at X_o .

$$K_2 = \left(\frac{dV}{dX} \right)_M \Delta X$$

$$\ell_2 = \left(\frac{dP}{dX} \right)_M \Delta X$$

where $V = V_0 + \frac{K_1}{2}$ and $P = P_0 + \frac{l_1}{2}$; all other variables are evaluated at X_M .

$$K_3 = \left(\frac{dV}{dX} \right)_M \Delta X$$

$$l_3 = \left(\frac{dP}{dX} \right)_M \Delta X$$

where $V = V_0 + \frac{K_2}{2}$ and $P = P_0 + \frac{l_2}{2}$; all other variables are evaluated at X_M .

$$K_4 = \left(\frac{dV}{dX} \right)_L \Delta X$$

$$l_4 = \left(\frac{dP}{dX} \right)_L \Delta X$$

where $V = V_0 + K_3$ and $P = P_0 + l_3$; all other variables are evaluated at X_L .

This procedure is repeated over all intervals of X .

Utilization of this procedure and the $\frac{dV}{dX}$ and $\frac{dP}{dX}$ expressions as written results in difficulties near Mach number = 1.0. In Equation (2-34) the term $\frac{1}{(1 - M_N^2)}$ becomes increasingly large as the Mach number approaches 1.0. Consider the case where Mach number is close to 1.0, say 1.1. $\frac{dV}{dX}$ will be negative if the large term inside the parenthesis is positive, since $\frac{1}{(1 - M_N^2)}$ is negative. This change in $\frac{dV}{dX}$ will lower the velocity, possibly to less than Mach 1.0, to .9 for instance. Now $\frac{1}{(1 - M_N^2)}$ is greater than zero and

$\frac{dV}{dX}$ will be positive. This oscillation around Mach 1.0 will therefore cause instabilities in the computer computations. One way to circumvent this problem is to reduce the increment size for values of Mach number in this regime. Another method would be to change the form of the equations so that the mathematical instability would no longer exist.

At present a constant value of ΔX is used except for the last increment. Since the flow is generally not near Mach 1.0 a varying increment size within the generator will not be incorporated.

In this program all of the generator inlet conditions are specified, including area and mass flow. Hence the power output for a particular exit total pressure cannot be specified. The primary purpose of the program is to calculate off-design performance for a specific duct, letting the power and stagnation pressure be dependent variables. However, as mentioned previously on the discussion of the specified thermodynamics program, the stagnation pressure is an important consideration for determination of the exit conditions of a design-point generator. This program has been modified to optionally include design-point performance. In this option the power output and exit stagnation pressure are specified and the length is

treated as the dependent variable. At the desired pressure the actual power is compared with the design power and the performance recomputed on the basis of a new inlet area. The wall shape is then determined by keeping the angle the same as in the original guess or keeping the original exit-to-inlet area ratio the same.

The conductivity calculation in this program uses the method whereby the collision cross-section Q is kept constant for some species and the collision frequency constant for others*.

The main criterion for optimizing an engine is cost. Because of this, various additions have been made to the program, including magnet weight and cost, diffuser cost, generator cost, mass flow cost, and total cost. The method used for optimizing a generator is discussed in Section B.2, Part II of this volume.

C. Specified Geometric and Electrodynamic Variables (Shifting Equilibrium)

Another program in which certain geometric and electrical variables were specified was completed late in the contract period. The program is being used to validate test results, but it was not finished

* Pratt & Whitney Aircraft Division Report PWA-1940, RADC TR 61-104

in time to use in the analytical optimization of the generator. This program is similar to the one previously discussed in Appendix 2 above except for:

- 1) An improved method of calculating conductivity has been incorporated where it is not necessary to consider either collision cross-section Q or collision frequency ν , constant. Instead, curves of collision frequency versus electron velocity are integrated at each value of temperature for each constituent (see Volume V of this final report).
- 2) Shifting equilibrium has been assumed and incorporated into the program in the form of tables of the thermodynamic properties and gas composition.

For a particular seed, oxidant, and fuel proportion, tables of enthalpy, entropy, sonic velocity, density, gas constant, specific heat, electron concentration, conductivity, and changes of gas constant with respect to pressure and temperature are utilized. These tables permit very accurate computation of Mach number, stagnation temperature, and stagnation pressure. In the frozen equilibrium program it was necessary to use an expression such as $(1 + \frac{\gamma-1}{2} M_N^2)^{\frac{\gamma}{\gamma-1}}$ with constant γ to compute stagnation pressure.

If the composition is allowed to vary the expression is innaccurate due to difficulties in defining a value of γ to fit the process. The sensitivity of total pressure to γ is apparent when values of γ less than 1.2 are used and the exponent can change significantly.

- 3) Boundary layer voltage drop has been included.
- 4) Performance of a nonsegmented electrode generator has been included.

The basic method of solution used in this program is the same as the one used in the frozen program. The same options regarding exit total pressure, power, and length exist, and the calculations involving weight and cost are also included. Due to the fact that the composition is assumed to be in chemical equilibrium in the duct at all times, the values of gas constant and specific heat change. Because of this, expressions such as $dR=0$ and $dh=c_p dT$ are not incorporated into the solution.

Following is the derivation of the final flow equations used in the machine computation, for $J_x=0$.

Momentum

$$\rho V \frac{dV}{dX} + \frac{dP}{dX} = -J_y B_z \quad (2-37)$$

Energy

$$\rho V \frac{dh}{dX} + \rho V^2 \frac{dV}{dX} = -J_y E_y \quad (2-38)$$

Continuity

$$\dot{m} = \rho AV \quad (2-39)$$

Ohm's law

$$J_y = \sigma_0 (V_x B_z - E_y) \quad (2-40)$$

State equation

$$P = \rho RT \quad (2-41)$$

Substituting Equation (2-41) into (2-39)

$$\dot{m} = AV \left(\frac{P}{RT} \right) \quad (2-42)$$

$$\dot{m} RT = PAV \quad (2-43)$$

Differentiating Equation (2-43) with respect to X

$$\dot{m} R \frac{dT}{dX} + \dot{m} T \frac{dR}{dX} = PA \frac{dV}{dX} + PV \frac{dA}{dX} + AV \frac{dP}{dX} \quad (2-44)$$

Introducing the momentum Equation (2-37)

$$\rho V \frac{dV}{dX} + \frac{dP}{dX} = -JB \quad B=B_z \text{ and } J=J_y \quad (2-45)$$

Multiplying by V

$$\rho V^2 \frac{dV}{dX} + V \frac{dP}{dX} = -VJB \quad (2-46)$$

$$V \frac{dP}{dX} = -VJB - \rho V^2 \frac{dV}{dX} \quad (2-47)$$

Substituting Equation (2-47) into (2-44)

$$\dot{m} R \frac{dT}{dx} + \dot{m} T \frac{dR}{dx} = PA \frac{dV}{dx} + PV \frac{dA}{dx} - A(VJB + \rho V^2 \frac{dV}{dx}) \quad (2-48)$$

Now to eliminate dR consider the state equation (2-41) $PV = RT$

Differentiating at constant P

$$P \partial V)_P = R \partial T)_P + T \partial R)_P \quad (2-49a)$$

Dividing by $R \partial T)_P$

$$\frac{P}{R} \frac{\partial V}{\partial T})_P = 1 + \frac{T}{R} \frac{\partial R}{\partial T})_P \quad (2-49b)$$

From the state equation

$$\frac{P}{R} = \frac{T}{V}$$

Then

$$\frac{T}{V} \frac{\partial V}{\partial T})_P = 1 + \frac{T}{R} \frac{\partial R}{\partial T})_P \quad (2-49c)$$

$$\frac{T}{V} \frac{\partial V}{\partial T})_P \equiv \alpha$$

Therefore

$$\alpha = 1 + \frac{T}{R} \frac{\partial R}{\partial T})_P \quad (2-49d)$$

$$\alpha - 1 = \frac{T}{R} \frac{\partial R}{\partial T})_P \quad (2-49e)$$

$$\frac{\partial R}{\partial T})_P = \frac{R}{T} (\alpha - 1) \quad (2-49f)$$

Again consider Equation (2-41) $PV = RT$

Differentiating at constant T

$$P \partial V)_T + V \partial P)_T = T \partial R)_T \quad (2-49g)$$

Dividing by $V \partial P)_T$

$$\frac{P}{V} \frac{\partial V}{\partial P})_T + 1 = \frac{T}{V} \frac{\partial R}{\partial P})_T \quad (2-49h)$$

Now

$$\frac{T}{V} = \frac{P}{R}$$

Therefore

$$\begin{aligned} \left(\frac{P}{V} \frac{\partial V}{\partial P} \right)_T + 1 &= \left(\frac{P}{R} \frac{\partial R}{\partial P} \right)_T \\ - \frac{P}{V} \frac{\partial V}{\partial P} \Big|_T &\equiv \beta \end{aligned} \quad (2-49i)$$

Then

$$\left(\frac{P}{R} \frac{\partial R}{\partial P} \right)_T = 1 - \beta \quad (2-49j)$$

$$\left(\frac{\partial R}{\partial P} \right)_T = \frac{R}{P} (1 - \beta) \quad (2-49k)$$

Now consider $R = R(T, P)$

Differentiating gives

$$dR = \left(\frac{\partial R}{\partial T} \right)_P dT + \left(\frac{\partial R}{\partial P} \right)_T dP \quad (2-49l)$$

But from Equation (2-49k)

$$\left(\frac{\partial R}{\partial P} \right)_T = \frac{R}{P} (1 - \beta)$$

and from Equation (2-49f)

$$\left(\frac{\partial R}{\partial T} \right)_P = \frac{R}{T} (\alpha - 1)$$

Where α and β are determined in the chemical thermodynamic program.

Then

$$dR = \frac{R}{T} (\alpha - 1) dT + \frac{R}{P} (1 - \beta) dP \quad (2-50)$$

Substituting for dR in Equation (2-48) gives

$$\dot{m} R \frac{dT}{dx} + \dot{m} T \left[\frac{R}{T} (\alpha - 1) \frac{dT}{dx} + \frac{R}{P} (1 - \beta) \frac{dP}{dx} \right] = PA \frac{dV}{dx} + PV \frac{dA}{dx} - A(VJB + \rho V^2 \frac{dV}{dx}) \quad (2-51)$$

Expanding

$$\begin{aligned} \dot{m} R \frac{dT}{dx} + \dot{m} R \alpha \frac{dT}{dx} - \dot{m} R \frac{dT}{dx} + \dot{m} R \frac{T}{P} \frac{dP}{dx} - \dot{m} R \frac{T}{P} \beta \frac{dP}{dx} \\ = PA \frac{dV}{dx} + PV \frac{dA}{dx} - AVJB - A\rho V^2 \frac{dV}{dx} \end{aligned} \quad (2-52)$$

From the momentum equation

$$\frac{dP}{dx} = -JB - \rho V \frac{dV}{dx} \quad (2-53)$$

Introducing Equation (2-53) into (2-52)

$$\begin{aligned} \dot{m} R \alpha \frac{dT}{dx} + \frac{\dot{m} RT}{P} (-JB - \rho V \frac{dV}{dx}) + \frac{\dot{m} RT}{P} \beta (JB + \rho V \frac{dV}{dx}) \\ = PA \frac{dV}{dx} + PV \frac{dA}{dx} - AVJB - A\rho V^2 \frac{dV}{dx} \end{aligned} \quad (2-54)$$

Introducing energy Equation (2-38)

$$\rho V \frac{dh}{dx} + \rho V^2 \frac{dV}{dx} = -JE \quad (2-55)$$

The enthalpy h of a chemical system can be written $h = \sum_i m_i h_i$

where m_i is the mass fraction of the i th species and h_i , the enthalpy of the i th species

Also

$$h = h(T, P)$$

Differentiating for a reacting gas where changes in m_i are permitted

$$dh = \sum_i \left[m_i \left(\frac{\partial h_i}{\partial T} \right)_P dT + m_i \left(\frac{\partial h_i}{\partial P} \right)_T dP + h_i \left(\frac{\partial m_i}{\partial T} \right)_P dT + h_i \left(\frac{\partial m_i}{\partial P} \right)_T dP \right] \quad (2-56a)$$

Rearranging the terms

$$dh = \sum_i \left[m_i \left(\frac{\partial h_i}{\partial T} \right)_P + h_i \left(\frac{\partial m_i}{\partial T} \right)_P \right] dT + \sum_i \left[m_i \left(\frac{\partial h_i}{\partial P} \right)_T + h_i \left(\frac{\partial m_i}{\partial P} \right)_T \right] dP \quad (2-56b)$$

This can be rewritten using C_P , the specific heat at constant pressure, and C_T the constant temperature coefficient, where the C_P and C_V values indicate reacting quantities.

If the expression $h = h(T, P)$ is considered and differentiated the following expression is obtained

$$dh = \left(\frac{\partial h}{\partial T} \right)_P dT + \left(\frac{\partial h}{\partial P} \right)_T dP \quad (2-57a)$$

In this expression $h = \sum_i m_i h_i$ therefore the two equations for dh are equivalent, that is, $\left(\frac{\partial h}{\partial T} \right)_P = C_P$ (for a reacting gas) and $\left(\frac{\partial h}{\partial P} \right)_T = C_T$ (also for a reacting gas).

$$dh = T ds + v dP \quad (2-57b)$$

Then

$$\frac{dh}{dP} = T \frac{ds}{dP} + v \quad (2-57c)$$

$$\left(\frac{\partial h}{\partial P} \right)_T = T \left(\frac{\partial s}{\partial P} \right)_T + v \quad (2-57d)$$

$$\left(\frac{\partial s}{\partial P} \right)_T = - \left(\frac{\partial v}{\partial T} \right)_P \quad (2-57e)$$

Then

$$\left(\frac{\partial h}{\partial P} \right)_T = v - T \left(\frac{\partial v}{\partial T} \right)_P \quad (2-57f)$$

$$\left(\frac{\partial h}{\partial P} \right)_T = v \left(1 - \frac{T}{v} \left(\frac{\partial v}{\partial T} \right)_P \right) \quad (2-57g)$$

$$\alpha \equiv \frac{T}{v} \left(\frac{\partial v}{\partial T} \right)_P$$

$$\left(\frac{\partial h}{\partial P} \right)_P = \frac{1}{\rho} (1 - \alpha) \quad (2-57h)$$

Equation (2-57a)

$$dh = C_p dT + \frac{(1 - \alpha) dP}{\rho} \quad (2-57i)$$

Substituting this into Equation (2-55)

$$\rho V C_p \frac{dT}{dx} + V(1 - \alpha) \frac{dP}{dx} + \rho V^2 \frac{dV}{dx} = -JE \quad (2-58)$$

$$\frac{dT}{dx} = - \frac{JE}{\rho V C_p} - \frac{V}{C_p} \frac{dV}{dx} - \frac{(1 - \alpha) dP}{\rho C_p dx} \quad (2-59)$$

Introducing Equation (2-53) into (2-54)

$$\frac{dT}{dx} = - \frac{JE}{\rho V C_p} - \frac{V}{C_p} \frac{dV}{dx} + \frac{(1 - \alpha) JB}{\rho C_p} + \frac{(1 - \alpha) V}{C_p} \frac{dV}{dx} \quad (2-60)$$

$$\frac{dT}{dx} = - \frac{JE}{\rho V C_p} - \frac{\alpha V}{C_p} \frac{dV}{dx} + \frac{(1 - \alpha) JB}{\rho C_p} \quad (2-61)$$

Introducing Equation (2-61) into (2-54)

$$\begin{aligned} \dot{m} R \alpha \left[-\frac{J E}{\rho V C_p} - \frac{\alpha V dV}{C_p dx} + \frac{(1-\alpha) J B}{\rho C_p} \right] + \frac{\dot{m} R T}{P} (-J B - \rho V \frac{dV}{dx}) \\ + \frac{\dot{m} R T}{P} \beta \left[J B + \rho V \frac{dV}{dx} \right] = P A \frac{dV}{dx} + P V \frac{dA}{dx} - A V J B - A \rho V^2 \frac{dV}{dx} \end{aligned} \quad (2-62)$$

Expanding

$$\begin{aligned} -\frac{\dot{m} R \alpha J E}{\rho V C_p} - \frac{\dot{m} R V \alpha^2 dV}{C_p dx} + \frac{\dot{m} R \alpha J B}{\rho C_p} - \frac{\dot{m} R \alpha^2 J B}{\rho C_p} - \frac{\dot{m} R T J B}{P} \\ - \frac{\dot{m} R T}{P} \rho V \frac{dV}{dx} + \frac{\dot{m} R T \beta J B}{P} + \frac{\dot{m} R T \beta \rho V dV}{P dx} = P A \frac{dV}{dx} + P V \frac{dA}{dx} \\ - A V J B - A \rho V^2 \frac{dV}{dx} \end{aligned} \quad (2-63)$$

Collecting $\frac{dV}{dx}$ terms

$$\begin{aligned} \frac{dV}{dx} \left[-\frac{\dot{m} R V \alpha^2}{C_p} - \frac{\dot{m} R T}{P} \rho V + \frac{\dot{m} R T}{P} \rho V \beta - P A + A \rho V^2 \right] = \frac{\dot{m} R \alpha}{\rho V C_p} J E \\ - \frac{\dot{m} R \alpha J B}{\rho C_p} + \frac{\dot{m} R \alpha^2 J B}{\rho C_p} + \frac{\dot{m} R T J B}{P} - \frac{\dot{m} R T \beta J B}{P} + P V \frac{dA}{dx} - A V J B \end{aligned} \quad (2-64)$$

Dividing through by $\dot{m}$ and noting that $\frac{R T}{P} = \frac{1}{\rho}$

$$\frac{dV}{dx} \left[-\frac{R V \alpha^2}{C_p} - V + V \beta - \frac{R T}{V} + V \right] = \frac{R \alpha J E}{\rho V C_p} + \frac{J B R \alpha (1-\alpha)}{\rho C_p} - \frac{\beta}{\rho} J B + \frac{R T}{A} \frac{dA}{dx} \quad (2-65)$$

Solve for dV/dx

Solve Equation (2-53)

$$\frac{dP}{dx} = -J B - \rho V \frac{dV}{dx} \quad (2-66)$$

Introducing Equation (2-59)

$$\frac{dT}{dx} = -\frac{1}{C_p} \left[\frac{J E}{\rho V} + V \frac{dV}{dx} + \frac{(1-\alpha) dP}{\rho dx} \right] \quad (2-67)$$

Equations (2-65), (2-66), and (2-67) are the flow equations solved in the computer program, and will be made correct dimensionally in the next section.

Derivation of the Flow Equations : $E_x = 0$

In Appendix 1 Ohm's law for $E_x = 0$ was derived. Although $J_x \neq 0$

$\vec{J} \cdot \vec{E} = J_y E_y$ as in the case of $J_x = 0$. However J_y does not

equal $\sigma_o (V_x B_z - E_y)$ but rather $\sigma_R (V_x B_z - E_y)$.

$$\sigma_o = \frac{\sigma_H^2 + \sigma_R^2}{\sigma_R} \quad \text{for} \quad \omega \tau = \frac{\sigma_H}{\sigma_R} *$$

$$\sigma_o = \sigma_R [(\omega \tau)^2 + 1]$$

J_y can now be written $J_y = \frac{\sigma_o}{1 + (\omega \tau)^2} (V_x B_z - E_y)$

Therefore for the case $E_x = 0$ the only change in the 3 final equations

of flow, is the inclusion of $1 + (\omega \tau)^2$ in the denominator of those

terms containing J_y .

For this case $\vec{J} \times \vec{B}$ has a j component as well as an i component.

Therefore

$$\vec{J} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ J_x & J_y & 0 \\ 0 & 0 & -B_z \end{vmatrix} = -\vec{i} J_y B_z + \vec{j} J_x B_z$$

* Pratt & Whitney Aircraft Division Report PWA-1940, RADQ TR 61-104, p. 13

This yields an additional equation of momentum

$$\rho V_x \frac{\partial V_y}{\partial x} + \rho V_z \frac{\partial V_y}{\partial z} + \rho V_y \frac{dV_y}{dy} + \frac{dP_y}{dy} = J_x B_z$$

However V_y was assumed equal to 0 in Appendix 1.

$$\text{Then } dP_y = J_x B_z dy$$

$$\text{and } J_x = \omega \tau J_y$$

$$\text{Therefore } dP_y = \omega \tau J_y B_z dy$$

The three flow equations for $E_x=0$ are:

$$\begin{aligned} \frac{dV}{dx} \left[\frac{-RV\alpha^2}{jgC_p} + \frac{V\beta}{g} - \frac{RT}{V} \right] &= \frac{R\alpha\sigma_o \left(\frac{VEB}{3.28} - E^2 \right)}{[1+(\omega\tau)^2] \rho VC_p (.454)(3.28)(g)(j)} + \frac{RT}{A} \frac{dA}{dx} \\ &+ \frac{R\alpha(\alpha-1)\sigma_o \left(\frac{VB^2}{3.28} - EB \right)}{\rho C_p [1+(\omega\tau)^2] (3.28)^2 (.454)(g)(j)} - \frac{\beta\sigma_o \left(\frac{VB^2}{3.28} - EB \right)}{\rho [1+(\omega\tau)^2] (.454)(3.28)^2 (g)} \end{aligned} \quad (2-68)$$

$$\frac{dP}{dx} = - \left[\frac{\sigma_o \left(\frac{VB^2}{3.28} - EB \right)}{g [1+(\omega\tau)^2] (3.28)^2 (144)(.454)} + \frac{\rho V}{(144)(g)} \frac{dV}{dx} \right] \quad (2-69)$$

$$\frac{dT}{dx} = - \frac{1}{jC_p} \left[\frac{\sigma_o \left(\frac{VEB}{3.28} - E^2 \right)}{\rho V [1+(\omega\tau)^2] g (.454)(3.28)} + \frac{VdV}{g dx} + \frac{(144)(1-\alpha)}{\rho} \frac{dP}{dx} \right] \quad (2-70)$$

These same three equations are the dimensionally correct equations for $J_x=0$ if all the $[1+(\omega\tau)^2]$ terms are replaced by 1. The solution of the three final flow equations for $E_x=0$ or $J_x=0$ is effected by the Runge-Kutta method. Values of the thermodynamic and electrical variables required in the equations are obtained from the table for every pass through the three equations.

APPENDIX 3

Nomenclature

APPENDIX 3

Nomenclature

a	sound velocity
A	area
AR	aspect ratio
B	magnetic field
C	conductor length
C_p	specific heat at constant pressure
C_v	specific heat at constant volume
c/c_g	conductor fraction of electrode length
$C/g/h$	electrode length-to-channel width ratio
D	electric displacement
E	electric field
E_o	ideal electric field
E/NB	electrical efficiency
$E_j Y$	cross-stream voltage
E_x/E_o	actual axial electric field-to-ideal cross-stream electric field ratio
g	gravitational constant
g	insulator length
h	enthalpy
h	channel width
I	current
J	current density
$\dot{m}$	mass flow rate
M_N	Mach number
NI	number of ampere turns
O/F	oxidant-to-fuel ratio
P	pressure
P_c	chamber pressure
Q	collision cross-section
R	gas constant
R_m	magnetic Reynolds number
T	temperature
V	velocity
x	axial distance
Y	dimension in Y direction
Z	dimension in Z direction

α	Hall mobility ratio
γ	segmentation factor
γ	ratio of specific heats
ϵ	effectiveness
ϵ_0	dielectric permittivity
η	electrical efficiency
η_0	ideal electrical efficiency
θ	generator wall divergence angle
κ	thermal conductivity
Λ	Hall current leakage factor
μ	magnetic permeability
ρ	density
ρ^e	electric charge density
σ	plasma electrical conductivity
σ_0	plasma electrical conductivity in absence of magnetic field
σ_e	effective plasma electrical conductivity
σ_u	plasma electrical conductivity perpendicular to electric field and magnetic field
σ_R	plasma electrical conductivity transverse to electric field and perpendicular to magnetic field
σ_{ij}	plasma electrical conductivity tensor
σ_e/σ_0	effective-to-scalar conductivity ratio
τ_{ij}	electron collision time tensor
ϕ	voltage
ψ	current stream function
ω	cyclotron frequency

Subscripts

x	-	x direction
y	-	y direction
z	-	z direction
1	-	x direction
2	-	y direction
3	-	z direction
0	-	initial conditions

APPENDIX 4

Tables

TABLE 1

1000 MW MHD Generator Characteristics

Gross power	1300 MW
Net power	1000 MW
Inlet total pressure (nominal combustion chamber pressure)	350 psia
Nominal mass flow rate	1650 lbs/sec
O/F (O ₂ + JP4)	2.9
O ₂ flow rate	1225 lb/sec
JP4 flow rate	420 lb/sec
Weight % seed (potassium)	0.1
Seed flow rate (KNO ₃ + H ₂ O)	4.78 lb/sec
MHD generator inlet Mach number	3.0
Inlet E/VB (electrical efficiency)	0.60
$\omega \tau$ max.	4.6
Cross-stream voltage (ideal)	14,600 volts
Axial voltage (Hall potential)	40,800 volts
Exit E/VB (electrical efficiency)	0.81
Exit Mach number	1.52
Exit total pressure	22.0 psia
MHD generator axial length	13.31 ft
Inlet height, Y ₀	9.43 ft
Inlet width, Z ₀	5.58 ft
Inlet aspect ratio, Y ₀ /Z ₀	1.70
Exit height, Y _E	15.63 ft
Exit width, Z _E	6.74 ft
Exit aspect ratio, Y _E /Z _E	2.32
Electrode wall divergence	13°
Insulating wall divergence	2.5°
Plasma conductivity, inlet	2.8 mhos/meter
Plasma conductivity, exit	10.4 mhos/meter
Magnet (overlap at both ends to reduce end effects)	
Field strength	3 web/meters ²
Field power	68.7 MW
O ₂ flow	612 lbs/sec-coil
Current (coils in series)	7600 amps

TABLE 1 (Cont'd.)

NI/coil	9.57×10^6 ampere-turns
Axial length (centerline dimension)	19.48 ft
Average height (nominal)	15.56 ft
Average coil separation (nominal)	10.17 ft
Number of turns per coil	1,260
Wire dimensions (square cross-section)	0.772 in x 0.772 in
Wire hole diameter	0.386 in
Field weight	305,000 lbs
Exit field strength	2.5 webers/meters ²

TABLE 2

250 MW MHD Generator Characteristics

Gross power	325 MW
Net power	250 MW
Inlet total pressure (nominal combustion chamber pressure)	350 psia
Nominal mass flow rate	387 lbs/sec
O/F (JP4 + O ₂)	2.9
O ₂ flow rate	282 lb/sec
JP4 flow rate	96 lb/sec
Weight % seed (potassium)	0.8
Seed flow rate (KNO ₃ + H ₂ O)	9 lb/sec
MHD generator inlet Mach number	3.0
Inlet E/VB (electrical efficiency)	0.7
$\omega\tau_{max}$	4.4
Cross-stream voltage (ideal)	8,066 volts
Axial voltage (Hall potential)	17,150 volts
Exit E/VB (electrical efficiency)	0.86
Exit Mach number	1.46
Exit total pressure	22 psia
MHD generator axial length	7.06 ft
Inlet height, Y_o	4.53 ft
Inlet width, Z_o	2.66 ft
Inlet aspect ratio, Y_o/Z_o	1.7
Exit height, Y_E	7.79 ft
Exit width, Z_E	2.91 ft
Exit aspect ratio, Y_E/Z_E	2.68
Electrode wall divergence	13°
Insulator wall divergence	1°
Plasma conductivity, inlet	6.8 mhos/meter
Plasma conductivity, exit	22.0 mhos/meter
Magnet (Overlap at both ends to reduce end effects)	
Field strength, inlet	3 webers/meter ²
Total field power	21.1 megawatt
O ₂ flow	141 lbs/sec-coil
Current, coils in series	3,580 amps

TABLE 2 (Cont'd.)

NI/coil	6.59×10^6 amp-turns
Axial length - (centerline dimension)	13.62 ft
Average height (nominal)	9.16 ft
Average coil separation (nominal)	6.79 ft
Number of turns per coil	1,840
Wire dimensions (square cross-section)	0.688 in
Wire hole diameter	0.344 in
Field weight	230,000 lbs
Exit field strength	2.7 webers/meter ²

Gas Generat

<u>Configuration</u>	<u>Injector type</u>	<u>N_{ox}</u>	<u>N_F</u>	<u>D_{ox}</u>	<u>D_F</u>	<u>A_{ox} To</u>
42	like-on-like fuel shower- head	6	6	.1630	.0240	.125
46	concentric	6	6	-	.1040	.302
48	showerhead	54	37	.0935	.0410	.370
53	concentric	6	6	-	.1040	.302
54	showerhead	96	61	.0700	.0330	.369
55	showerhead	54	37	.0935	.0410	.370
58	showerhead	54	37	.0935	.0410	.370
61	showerhead	54	37	.0935	.0410	.370

1

TABLE 3

Gas Generator Configurations Tested

D_F	A_{ox} Total	A_F Total	L_c	A_t	A_c/A_t	Nominal P_c , psia	Data Scatter %	η_C *average at $O/F = 2.6$
.0240	.1252	.0027	19.25	1.685	4.20	30	$\pm 5.$	.85
						75	± 0.5	.89
.1040	.3028	.0510	20.75	1.685	4.20	30	± 0.5	.835
						60	± 2.3	.895
.0410	.3708	.0488	20.75	2.717	2.60	60	$\pm 6.$	-
						75	± 2.7	~ .92 (extra- polated)
.1040	.3028	.0510	20.75	2.765	2.59	50	$\pm 6.$	-
						100	± 1.0	.94
.0330	.3695	.0522	20.75	2.717	2.60	30	± 1.5	.895
						60	± 0.8	.92
						120	± 0.6	.93
.0410	.3708	.0488	11.75	2.717	2.60	30	$\pm 3.$	~ .75 (extrapo- .775 lated)
						60	± 1.5	
						120	± 1.0	~ .82 (extrapolated)
.0410	.3708	.0488	30.32	2.717	2.60	30	± 1.0	~ .83
						60	± 0.5	.83
						120	± 1.0	.835
.0410	.3708	.0488	18.75	2.405	1.84	60	$\pm 4.$	.895
						120	$\pm 2.$	.905

TABLE 4

Test Data for Configuration 42

	P_{CO} psia	O/F	W_S lb/sec	W_P lb/sec	C^* ft/sec	Q/W_P Btu/lb	η_C^*	L_e/K_i
227	28.6	2.248	0	.3352	4534	209.2	80.72	26.0
	34.6	2.033		.3730	4930	219.6	87.04	26.4
	30.5	2.229		.3390	4781	229.2	85.15	26.8
	36.4	3.108		.4190	4617	225.5	85.37	34.7
	38.4	3.298		.4470	4565	226.5	85.04	37.0
	32.8	2.523		.3370	4624	226.8	82.72	28.7
	28.4	2.125		.3250	4652	377.8	84.29	25.5
	27.3	2.099		.3130	4635	392.3	83.94	25.2
	29.3	2.252		.3350	4656	401.2	84.20	26.4
	28.0	2.242	0	.3355	4435	209.1	78.96	25.5
	29.1	2.229		.3390	4562	229.2	81.52	24.8
	30.0	2.229		.3390	4703	229.2	83.76	26.5
	35.1	3.137		.4220	4420	218.6	81.81	34.2
	35.9	3.137		.4220	4521	218.6	83.68	34.6

TABLE 4 Cont'd

Test Data for Configuration 42

<u>P_CO_{psia}</u>	<u>O/F</u>	<u>W_Slb/sec</u>	<u>W_Plb/sec</u>	<u>C* ft/sec</u>	<u>Q/W_P Btu/lb</u>	<u>η_C*</u>	<u>L_e/K_i</u>
70.9	1.979	0	.7332	5019	141.2	86.55	30.3
71.0	2.319		.7370	5000	145.5	88.07	33.2
78.5	2.664		.8141	5007	210.6	89.47	37.7
75.5	2.558		.7802	5023	206.2	89.28	36.2
73.6	2.495		.7595	5031	190.0	89.30	35.2

TABLE 5

Test Data for Configuration 46

P_{CO}	O/F	W_S	W_P	C^*	Q/W_P	η_{C^*}	L_e/K_i
60.9	1.864	0	.6656	5570	334.4	87.56	34.1
64.0	1.946		.6847	5563	410.9	89.57	35.3
61.0	2.000		.6678	5679	282.0	85.74	34.7
63.7	2.146		.6852	5539	508.7	89.46	36.2
62.5	2.045		.6694	5694	360.7	87.39	35.6
64.0	2.151		.6834	5597	427.1	89.19	36.4
72.8	2.605		.7816	5598	408.4	88.69	39.7
73.1	2.558		.7924	5649	233.2	87.04	39.3
74.5	2.589	.0228	.8044	5530	419.1	89.26	40.3
30.9	1.998	0	.3627	5570	318.4	81.53	29.8
30.2	2.320	.0102	.3537	5461	403.5	83.34	31.4
64.6	1.983	0	.6932	4967	396.3	88.87	35.9
62.5	2.076	0	.6730	4950	308.9	87.30	35.6
63.5	2.085	0	.6720	5036	408.8	89.83	36.1
72.8	2.570	0	.7775	4991	434.8	89.17	39.7

TABLE 6

Test Data Configuration 48

	<u>O/F</u>	<u>W_S</u>	<u>W_P</u>	<u>C*</u>	<u>Q/W_P</u>	<u>η_{C^*}</u>	<u>L_e/K_i</u>
61.5	3.428	.0392	1.1152	4666		85.43 ²	94.0
62.2	3.489	0	1.0550	4993		90.16	97.5
65.4	3.587	0	1.1100	4985		91.41	97.6
63.0	3.305	.0442	1.0732	4967		90.42	96.5
60.0	2.454	.0467	.9897	5130		89.29	92.0
58.0	3.500	.0353	1.1153	4400		80.87	91.3
56.4	2.511	.0873	1.0843	4401		77.36	85.4
78.0	3.576	.0690	1.3320	4955		91.34	98.3
78.0	3.576	.0804	1.3434	4913		90.68	97.7
60.3	3.253	.0392	1.0982	4646		84.32	93.2
61.5	3.361	.0392	1.1032	4717		86.07	94.3
62.1	3.458	.0392	1.1092	4737		86.85	94.5
59.7	3.449	.0392	1.1202	4509		82.68	92.3
54.6	3.403	.0289	1.0769	4290		78.47	89.5

TABLE 6 Cont'd

Test Data Configuration 48

<u>P_{CO}</u>	<u>O/F</u>	<u>W_S</u>	<u>W_P</u>	<u>C*</u>	<u>Q/W_P</u>	<u>η_C*</u>	<u>L_e/K_i</u>
55.5	2.511	.0873	1.0843	4331		76.15	84.8
54.0	2.511	.0873	1.0843	4214		74.11	83.6
76.0	3.576	0	1.2630	5092		93.12	99.1
78.0	3.576	0	1.2630	5226		95.54	100.8
79.0	3.591	.0848	1.3518	4945		91.36	98.0
79.0	3.591	.0848	1.3518	4945		91.36	98.0
77.0	3.591	0	1.2670	5142		94.10	99.9

1. Theoretical C* value uncorrected for heat loss since measured heat loss not considered correct

2. Theoretical C* uncorrected for heat loss, heat loss data unavailable

TABLE 7

Test Data for Configuration 53

P_{CO}	O/F	W_S	W_P	C^*	Q/W_P	η_{C^*}	L_e/K_i
48.2	2.592	0	.8190	5520	478	91.12	26.3
49.8	2.592	.0350	.8540	5407	508	91.99	26.3
45.0	2.909	0	.9030	5484	345	77.96	25.9
51.3	2.909	0	.9030	5481	371	88.69	28.5
51.3	2.952	.0263	.9393	5370	502	86.88	28.3
46.5	3.812	0	1.0250	5308	292	73.38	37.4
55.5	3.812	0	1.0250	5206	402	89.05	52.1
56.4	3.812	.0360	1.0610	5102	476	89.02	53.1
96.6	2.175	0	1.5940	5670	359	91.37	65.9
97.8	2.187	.0452	1.6452	5581	383	91.05	66.9
98.4	2.191	.0452	1.6472	5581	383	91.52	67.7
99.9	2.719	0	1.6030	5602	382	95.19	92.7
100.2	2.738	.0262	1.6262	5557	401	95.37	94.4
100.5	2.738	.0518	1.6518	5519	409	94.80	94.4
100.5	2.738	.0625	1.6625	5496	427	93.91	94.2

TABLE 8

Test Data Configuration 54

P_{CO}	O/F	W_S	W_P	C^*	Q/W_P	η_{C^*}	L_e/K_i
63.2	2.279	.0292	1.0292	5191	102.2	91.10	114.7
65.8	2.903	.0320	1.0820	5143	109.2	92.99	119.6
66.4	3.333	.0580	1.1370	4939	104.5	92.08	118.8
33.2	3.452	.0300	.5910	4746	226.2	90.66	108.6
32.6	2.690	.0240	.5590	4936	147.8	89.77	108.4
35.0	2.857	.0392	.6062	4880	146.0	90.24	109.6
65.0	2.852	.0420	1.0820	5076	135.7	92.06	119.1
106.4	2.587	.0812	1.7312	5197	141.4	92.75	122.9
124.3	2.441	.0832	1.9862	5290	133.4	93.37	124.5
124.3	2.752	.0980	2.0340	5166	111.9	92.60	126.4
121.8	3.477	.0812	2.0512	5021	171.4	93.61	126.4
29.5	3.981	0	.5380	4643	258.4	89.55	105.7
33.8	3.271	.0380	.6060	4712	203.6	89.43	108.3

TABLE 8 Con't

Test Data Configuration 54

P_{CO}	O/F	W_S	W_P	C^*	Q/W_P	η_{C^*}	L_e/K_i
31.4	2.889	0	.5250	5051	166.9	91.48	110.5
32.7	2.690	.0320	.5670	4879	145.7	89.21	107.6
32.9	2.690	.0380	.5730	4848	144.2	88.96	107.3
34.5	2.857	.0412	.6082	4795	145.5	88.78	108.6
31.8	3.440	0	.5550	4846	247.2	90.90	109.2
60.5	2.447	0	.9790	5228	112.7	91.39	116.4
65.7	2.258	.0600	1.0700	5193	119.7	92.22	114.6
66.5	2.857	0	1.0800	5204	110.0	92.77	120.6
66.5	2.951	.0680	1.1150	5044	117.2	92.64	119.3
63.5	3.011	0	1.0470	5127	116.6	92.75	119.9
66.3	3.146	0	1.1070	5066	104.3	91.67	119.9
64.8	3.255	0	1.0850	5050	101.6	91.91	119.6
66.2	3.320	.0420	1.1220	4990	105.9	92.48	119.5
64.0	3.320	0	1.0800	5009	102.7	91.51	119.1

TABLE 8 Cont'd

Test Data Configuration 54

P_{CO}	O/F	W_S	W_P	C*	Q/W_P	η_{C^*}	L_e/K_i
100.0	2.796	0	1.6020	5276	140.0	93.45	125.1
104.3	2.615	0	1.6450	5362	112.4	93.84	125.4
106.9	2.626	.1020	1.7590	5139	150.7	92.36	122.7
104.6	2.626	0	1.6570	5338	122.3	93.53	124.9
107.1	3.466	0	1.7910	5056	161.1	92.82	125.1
123.9	2.441	.1020	2.0050	5224	136.5	92.58	123.8
120.8	2.455	0	1.9000	5376	122.5	93.27	125.4
117.8	2.881	0	1.8860	5282	138.1	93.75	127.9
123.6	2.752	.0560	1.9920	5245	115.9	93.30	127.4
121.5	2.757	0	1.9350	5308	164.9	93.77	128.1
125.8	2.961	0	2.0200	5264	114.3	93.58	129.1
125.8	3.056	0	2.0440	5202	115.0	92.96	128.6
118.6	3.485	0	1.9690	5094	136.8	93.29	127.0
122.8	3.446	.1292	2.1032	4936	183.3	92.79	125.3

TABLE 9

Test Data Configuration 55

P_{CO}	O/F	W_S	W_P	C^*	Q/W_P	ηC^*	L_e/K_i
57.1	2.412	0	1.0610	4552		78.46 ¹	45.8
57.3	2.550	0	1.0580	4576		79.45	47.5
63.2	2.793	.0750	1.1750	4544		80.78	49.6
28.9	3.000	.0380	.5980	4089		74.29	41.5
28.8	2.821	.0870	.5720	4262		76.69	41.3
30.5	3.680	0	.5990	4304		80.11	44.4
59.0	3.026	0	1.0910	4751		81.49	50.4
57.8	3.149	0	1.0830	4511		80.99	50.5
118.8	2.815	.0665	2.1265	4723		82.87	54.4
123.0	2.917	.0730	2.2080	4709		83.10	55.0
56.8	2.380	0	1.0650	4507		77.56	45.2
59.7	2.412	.0760	1.1370	4442		77.42	45.5
60.2	2.508	.0680	1.1310	4502		78.77	46.9

TABLE 9 Cont'd

Test Data Configuration 55

P_{CO}	O/F	W_S	W_P	C^*	Q/W_P	η_{C^*}	L_e/K_i
57.5	2.475	0	1.0530	4617		79.84	47.0
59.6	2.898	0	1.1030	4571		80.89	49.8
59.2	2.842	0	1.0950	4574		80.69	49.4
62.7	2.793	.0690	1.1690	4535		80.56	49.4
29.9	2.979	.0530	.6140	3983		72.55	41.0
28.5	2.821	.0340	.5690	4241		76.26	40.9
64.8	3.250	0	1.1900	4603		82.96	51.9
60.5	3.119	0	1.1080	4616		82.67	51.0
61.4	3.160	.0735	1.1675	4445		80.63	50.4
59.5	3.134	0	1.1080	4539		81.39	50.5
56.9	3.184	0	1.0710	4491		80.79	50.2
59.5	3.202	.0720	1.1350	4431		80.59	50.1
116.6	2.774	0	2.0680	4765		82.98	54.3

TABLE 9 Cont'd

Test Data Configuration 55

<u>P_{CO}</u>	<u>O/F</u>	<u>W_S</u>	<u>W_P</u>	<u>C*</u>	<u>Q/W_P</u>	<u>η_{C*}</u>	<u>L_e/K_i</u>
120.1	2.791	.1045	2.1705	4679		82.23	54.0
121.0	2.944	0	2.1300	4804		84.42	55.6
124.5	2.882	.1150	2.2500	4677		82.62	54.9
119.5	2.882	0	2.1350	4733		82.90	54.8

TABLE 10

Test Data Configuration 58

P_{CO}	O/F	W_S	W_P	C^*	Q/W_P	η_{C^*}	L_e/K_i
55.8	2.355	.0690	1.0035	4715		82.04 ¹	134.0
55.1	2.659	0	.9770	4785		83.62	137.5
55.3	3.091	.0730	1.0220	4592		83.22	137.4
54.7	3.326	0	1.0015	4635		84.07	139.8
96.6	2.850	.0890	1.7425	4701		83.16	136.3
110.6	2.492	0	1.9225	4879		83.73	135.3
119.4	2.514	.1000	2.0960	4830		83.61	135.2
114.0	2.999	0	2.0175	4791		84.52	138.2
114.7	2.972	0	2.0315	4789		84.35	138.7
118.3	3.472	.1260	2.2010	4559		83.23	137.8
56.2	2.361	.0605	1.0000	4769		82.89	134.4
56.4	2.361	.0780	1.0175	4700		81.91	133.0
57.1	2.654	.0705	1.0315	4698		83.04	136.0

TABLE 10 Cont'd

Test Data Configuration 58

P_{CO}	O/F	W_S	W_P	C^*	Q/W_P	η_{C^*}	L_e/K_i
54.1	2.660	.0	.9590	4788		83.70	137.5
52.7	3.103	.0	.9520	4696		84.21	139.9
55.6	2.950	.0730	1.0250	4604		82.79	136.9
57.1	3.289	.0705	1.0655	4548		83.17	138.0
54.1	3.241	0	.9840	4666		84.26	139.0
96.6	2.858	.0890	1.7440	4697		83.12	136.0
93.7	2.875	0	1.6525	4810		84.49	138.0
114.0	2.492	.0965	2.0190	4788		82.83	134.3
114.0	2.504	.0965	2.0165	4794		82.98	134.6
119.4	2.523	.1200	2.1140	4789		83.07	134.8
114.0	2.544	0	1.9920	4852		83.47	135.7
117.6	2.964	.0955	2.1150	4715		83.63	137.2
117.7	2.981	.1030	2.1235	4702		83.53	137.2

240

PWA-2170

TABLE 10 Cont'd

Test Data Configuration 58

<u>P_{CO}</u>	<u>O/F</u>	<u>W_S</u>	<u>W_P</u>	<u>C*</u>	<u>Q/W_P</u>	<u>η_C*</u>	<u>L_e/K_i</u>
117.4	2.994	.1130	2.1340	4667		83.03	136.8
117.9	2.979	.1190	2.1465	4657		82.83	136.7
118.2	3.465	.1260	2.2020	4551		83.06	137.9
114.0	3.465	0	2.0760	4656		84.22	139.0

TABLE 11

Test Data Configuration 61

P_{CO}	O/F	W_S	W_P	C^*	Q/W_P	η_{C^*}	L_e/K_i
53.5	2.320	0	.8300	4727	85.3	81.74	76.3
61.3	2.408	0	.9270	4849	124.3	84.30	80.2
72.1	3.527	.0710	1.0850	4867	132.6	90.77	103.0
30.1	2.939	.0400	.4890	4517	118.5	85.21	90.6
123.6	3.547	0	1.7960	5042	144.9	92.56	105.0
129.0	3.058	.230	1.8720	5048	130.4	92.68	102.2
132.7	2.596	.1220	1.9090	5095	97.8	89.40	89.2
62.7	2.735	.0620	.9060	5072	96.7	90.62	92.3
67.4	2.806	.0550	.9570	5160	106.8	92.37	96.1
66.2	3.478	.0660	.9750	4974	137.8	92.70	103.8
62.8	2.721	.0530	.8940	5146	98.0	91.75	92.4
62.8	2.942	0	.8870	5187	99.5	92.65	99.5
66.4	2.947	.0590	.9590	5076	115.0	91.72	99.4

TABLE 11 Cont'd

Test Data Configuration 61

P_{CO}	O/F	W_S	W_P	C^*	Q/W_P	η_{C^*}	L_e/K_i
62.2	3.520	0	.9040	5041	132.4	93.16	103.9
61.3	3.520	0	.9040	4967	132.4	91.82	103.0
129.2	3.589	.1180	1.9170	4938	158.4	91.83	103.8
129.4	3.011	.1160	1.8770	5049	132.2	90.89	101.2
134.1	2.486	.1120	1.9210	5115	91.2	88.97	85.7
59.8	2.746	0	.9140	4797	117.5	84.92	90.0
63.9	2.649	.0720	.9880	4738	116.6	84.44	86.7
73.4	3.584	.0810	1.1170	4816	154.4	90.32	102.1
72.8	3.578	.0770	1.1070	4820	155.8	90.36	102.2
124.5	3.520	0	1.7990	5070	121.6	92.77	105.3
124.3	3.535	0	1.7960	5072	123.6	92.90	105.3
124.0	3.070	0	1.7500	5193	132.0	95.34	103.9
128.1	2.600	0	1.7890	5246	80.0	90.94	90.5

TABLE 11 Cont'd

Test Data Configuration 61

P_{CO}	O/F	W_S	W_P	C^*	$\Omega_i W_P$	η_{C^*}	L_e/K_i
62.7	2.730	.6550	.8980	5117	97.5	91.29	92.6
62.3	2.794	0	.9030	5052	97.0	89.49	94.0
66.0	3.530	.0585	.9645	5016	148.4	93.72	104.0
59.8	2.720	0	.8370	5232	83.7	92.26	93.2
66.7	2.907	.0605	.9475	5160	117.1	93.09	99.0
66.4	2.917	.0595	.9525	5110	115.8	92.20	98.8
123.3	3.000	0	1.7600	5131	101.2	91.14	101.7
128.2	2.490	0	1.7870	5255	70.3	90.50	87.0

TABLE 12

Settling of K_2CO_3 In Gels of JP4 With Acetylene-Black

<u>% C_2H_2-black</u>	<u>%K_2CO_3 at top</u>	<u>% K_2CO_3 at bottom (after 18 days)</u>	<u>Suspension Ra- tio - % at top % at bottom</u>
3	.655	3.21	.204
5	2.24	2.82	.794
5.5	2.37	2.92	.812
6	2.66	2.88	.925
6.5	2.50	2.86	.874
7	2.59	2.70	.959

TABLE

Test Operating Condi

Injector	Test Number	D_L	D_{LG}	D_G	AVG. ΔP_{H_2O}	AVG. ΔP_{AIR}	Probe Distance From Face	G_i	G_{max} .044 p
Concentric	1	0.067	0.13	0.189	1	2	2 in.	5.278	
"	2	"	"	"	"	5	"	"	
"	3	"	"	"	"	10	"	"	
"	4	"	"	"	"	15	"	"	
"	5	"	"	"	2	2	"	7.391	
"	6	"	"	"	"	5	"	"	
"	7	"	"	"	"	10	"	"	
"	8	"	"	"	"	15	"	"	
"	9	"	"	"	5	2	"	11.613	
"	10	"	"	"	"	5	"	"	
"	11	"	"	"	"	10	"	"	
"	12	"	"	"	"	15	"	"	
"	13	"	"	"	10	2	"	16.891	
"	14	"	"	"	"	5	"	"	
"	15	"	"	"	"	10	"	"	
"	16	"	"	"	"	15	"	"	
"	17, 33	"	"	"	1	2	5 in.	5.278	.0170
"	18, 34	"	"	"	"	5	"	"	.0160
"	19, 35	"	"	"	"	10	"	"	.0243
"	20, 36	"	"	"	"	15	"	"	.0252
"	21, 37	"	"	"	2	2	"	7.391	.0234
"	22, 38	"	"	"	"	5	"	"	.0240
"	23, 39	"	"	"	"	10	"	"	.0285
"	24, 40	"	"	"	"	15	"	"	.0362
"	25, 41	"	"	"	5	2	"	11.613	.3217
"	26, 42	"	"	"	"	5	"	"	.1069
"	27, 43	"	"	"	"	10	"	"	.0548
"	28, 44	"	"	"	"	15	"	"	.0653
"	29, 45	"	"	"	10	2	"	16.891	1.1507
"	30, 46	"	"	"	"	5	"	"	.5019
"	31, 47	"	"	"	"	10	"	"	.2618
"	32, 48	"	"	"	"	15	"	"	.1421

1

TABLE 13

PWA-2170

ating Conditions and Geometry

G_i	$G_{\max}$.044 probe	$G_{\max}$.19 probe	$G_{\max}/G_i$.044 probe	$G_{\max}/G_i$.19 probe	G_{50} .19 probe	G_{50}/G_i .19 probe	$\frac{qgR}{qL}$	S_{50}/D_I	S_{97}/D_I
.278		.02065		.00391	.0075	.00143	1.81	11.35	28.806
"		.03258		.00617	.0125	.00237	4.32	8.81	25.373
"		.04240		.00803	.0180	.00341	7.98	6.87	21.403
"		.04430		.00934	.0225	.00426	10.05	5.97	19.403
.391		.05220		.00706	.01975	.00267	.90	8.96	26.09
"		.04639		.00628	.0175	.00237	2.17	8.96	28.000
"		.05230		.00708	.01975	.00267	4.03	7.76	24.000
"		.05240		.00709	.01275	.001725	5.07	6.72	22.567
.613		.39890		.03435	.175	.01507	.35	4.03	12.657
"		.17930		.01544	.130	.0112	.86	5.225	18.478
"		.09290		.00800	.028	.00241	1.60	7.47	21.164
"		.08570		.00738	.023	.00198	2.02	6.865	20.090
.891		.02300		.06056	.510	.0302	.16	2.985	9.164
"		.43200		.02558	.205	.01213	.39	3.73	13.015
"		.27200		.01610	.095	.00562	.73	5.52	15.731
"		.13990		.00828	.050	.00296	.93	6.12	14.836
.278	.01763	.00491	.00334	.00093	.0017	.000322	1.81	21.5	41.045
"	.01605	.00611	.00304	.00116	.001975	.000374	4.32	19.25	40.149
"	.02434	.00658	.00161	.00125	.00245	.000464	7.98	15.83	37.91
"	.02526	.00542	.00468	.00103	.0024	.000455	10.05	14.18	33.91
.391	.02349	.01467	.00318	.00198	.00425	.000575	.90	18.8	48.06
"	.02408	-	.00326	-	.0044	.000595	2.17	16.12	45.134
"	.02855	.01686	.00386	.00228	.0058	.000785	4.03	13.73	39.851
"	.03625	.01658	.00490	.00224	.0054	.00073	5.07	13.43	38.985
.613	.32170	.15100	.02770	.01300	.0415	.00357	.35	7.16	26.06
"	.10691	.06350	.00920	.00547	.01725	.001485	.86	12.1	31.343
"	.05480	.03540	.00472	.00305	.013	.00112	1.60	12.54	29.403
"	.06539	.03410	.00563	.00293	.0115	.00099	2.02	11.64	28.209
.891	1.15070	.50500	.06811	.02990	.1975	.0117	.16	4.03	9.522
"	.50197	.31600	.02972	.01871	.075	.00444	.39	6.865	23.075
"	.26184	.15700	.01550	.00929	.0475	.00281	.73	8.01	27.045
"	.14210	.09060	.00841	.00536	.0375	.00222	.93	8.96	23.761

TABLE 13 (

Test Operating Condi

Injector	Test Number	D_L	D_{LG}	D_G	AVG. ΔP_{H_2O}	AVG. ΔP_{AIR}	Probe Distance From Face	G_i	G_{max} .044 p
BETE(L)	1	0.055	0.25	0.29	1	6	5 in.	2.72	
"	2	"	"	"	"	20	"	"	
"	3	"	"	"	"	50	"	"	
"	4	"	"	"	10	6	"	6.15	
"	5	"	"	"	"	20	"	"	
"	6	"	"	"	"	50	"	"	
BETE(H)	7	"	"	"	15	6	5 in.	3.615	
"	8	"	"	"	"	20	"	"	
"	9	"	"	"	"	50	"	"	
"	10	"	"	"	80	6	"	6.25	
"	11	"	"	"	"	20	"	"	
"	12	"	"	"	"	50	"	"	
Inverted	1	0.07	0.19	0.204	1.6	15	5 in.	6.715	
"	2	"	"	"	4	15	"	10.58	
Concentric	1	0.07	0.19	0.292	2	15	5 in.	7.58	
"	2	"	"	"	5	15	"	11.8

1

BLE 13 Cont'd

g Conditions and Geometry

$G_{\max}$.044 probe	$G_{\max}$.19 probe	$G_{\max}/G_i$.044 probe	$G_{\max}/G_i$.19 probe	G_{50} .19 probe	G_{50}/G_i .19 probe	$\frac{qgR}{qL}$	S_{50}/D_L	S_{97}/D_L
43.1 $\times 10^{-4}$			15.88 $\times 10^{-4}$	.0016	.000588		14.2	31.182
62.2 $\times 10^{-4}$			22.87 $\times 10^{-4}$	.00225	.000828		14.42	35.091
49.5 $\times 10^{-4}$			18.2 $\times 10^{-4}$	.002025	.000745		14.22	33.455
31.6 $\times 10^{-4}$			51.38 $\times 10^{-4}$	.0084	.001367		11.05	28.982
25.9 $\times 10^{-4}$			42.11 $\times 10^{-4}$	.0079	.001285		13.15	29.473
18.8 $\times 10^{-4}$			30.57 $\times 10^{-4}$	.0065	.001058		13.55	35.636
10.3 $\times 10^{-4}$			28.49 $\times 10^{-4}$	.0033	.000914		14.37	35.873
92.2 $\times 10^{-4}$			25.505 $\times 10^{-4}$	.0030	.00083		13.23	32.255
92.2 $\times 10^{-4}$			25.505 $\times 10^{-4}$	.00379	.00105		11.81	23.327
184.3 $\times 10^{-4}$			35.87 $\times 10^{-4}$	.0097	.001552		12.39	30.418
392.5 $\times 10^{-4}$			62.8 $\times 10^{-4}$	.014	.00224		9.91	26.927
2.72 $\times 10^{-4}$			43.52 $\times 10^{-4}$	.0092	.001472		12.10	29.818
3.49 $\times 10^{-3}$			5.2 $\times 10^{-4}$					
6.08 $\times 10^{-3}$			5.75 $\times 10^{-4}$					
3.947 $\times 10^{-2}$			5.21 $\times 10^{-3}$			4.82		
4.434 $\times 10^{-2}$			3.81 $\times 10^{-3}$			1.96

2

APPENDIX 5

Experimental Determination of Conductivity

APPENDIX 5

Experimental Determination of Conductivity

A. MHD Equations and Assumptions Used in Determining Plasma Conductivity

Several experimental methods are currently used for the determination of plasma conductivity. The fundamental equations and the assumptions used to form the basis of these test techniques are presented below. The conductivity test techniques are divided into two basic types: 1) methods used in the presence of a magnetic field, the interaction of the magnetic field with the moving plasma providing the driving EMF, and 2) methods used where no magnetic field is present and the EMF to drive the current is supplied external to the test section. The presentation of the equations is divided into these two areas.

The fundamental equations and methods of measuring plasma conductivity in the presence of a magnetic field consider τ as a tensor. A Hall mobility ratio $\alpha' = \tau_H / \tau_T$ is therefore defined which recognizes that τ_H and τ_T differ in magnitude. Ignoring this difference can result in appreciable error.

Notation

$\bar{E}$ = electric field felt by stationary positive charge
 $\bar{E}'$ = electric field felt by moving positive charge in presence of a magnetic field, = $(\bar{u} \times \bar{B} + \bar{E})$

$$E_y = \frac{\partial V}{\partial y}$$

$$E_x = \frac{\partial V}{\partial x}$$

e = charge of an electron

H = distance between opposite electrodes

J = current density

L = generator length

m = mass of an electron

n = number density of electrons

V_T = voltage at electrode terminals

V_{oc} = voltage at electrode terminals under open circuit, power generation conditions

δV = electrode voltage drop (includes both anode and cathode drops)

α' = Hall mobility ratio

B = applied magnetic field strength

σ_{ij} = conductivity tensor

σ_o = scalar conductivity $= \frac{ne^2}{m} \tau_o$

σ' = generalized scalar conductivity $= \frac{ne^2}{m} \left(\frac{\tau_r^2 + \tau_n^2}{\tau_r} \right)$

σ'_Λ = effective conductivity $= \frac{\sigma'}{1 + \alpha' \Lambda}$

Λ = Hall leakage current ratio, $= \frac{J_x}{J_y}$

ω = cyclotron frequency

τ = reciprocal of the collision frequency, ν

$\bar{}$ = average quantity

Barred symbols are vector quantities.

The coordinate system used with the equations is as follows:

The equations presented are for one-dimensional steady flow. In this system the equation for conductivity is

$$\sigma_{ij} = \left(\frac{ne^2}{m} \right) \tau_{ij}$$

Or, writing out the tensor for τ_{ij}

$$\sigma_{ij} = \frac{ne^2}{m} \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}$$

$$\tau_{11} = \tau_{22} \equiv \tau_T$$

$$\tau_{12} = -\tau_{21} \equiv \tau_H$$

$$\tau_{33} \equiv \tau_o$$

$$\tau_{13} = \tau_{23} = \tau_{31} = \tau_{32} \equiv 0$$

and

$$\tau_T = \widehat{\left(\frac{1}{v} \right) \left(\frac{1}{1 + (\omega/v)^2} \right)}; \quad \tau_H = \widehat{\left(\frac{1}{v} \right) \left(\frac{\omega}{v} \right) \left(\frac{1}{1 + \omega^2/v^2} \right)}; \quad \tau_o = \widehat{\left(\frac{1}{v} \right)}$$

The expansion of the steady state form of Ohm's law, $J_i = \sigma_{ij} E_j$ yields these expressions for the current density

$$J_x = \frac{ne^2}{m} \left[-\tau_T E_x + \tau_H (uB - E_y) \right] \quad (5-1)$$

$$J_y = \frac{ne^2}{m} \left[\tau_H E_x + \tau_T (uB - E_y) \right] \quad (5-2)$$

If Equations (5-1) and (5-2) are solved for E_x

$$E_x = -\frac{m}{ne^2} \frac{J_x}{\tau_T} + \frac{\tau_H}{\tau_T} (uB - E_y) \quad (5-3)$$

$$E_x = \frac{m}{ne^2} \frac{J_y}{\tau_H} - \frac{\tau_T}{\tau_H} (uB - E_y) \quad (5-4)$$

Equating Equations (5-3) and (5-4) yields

$$J_y = \frac{ne^2}{m} \left(\frac{\tau_n^2 + \tau_r^2}{\tau_r} \right) (uB - E_y) - \frac{\tau_n}{\tau_r} J_x \quad (5-5)$$

Defining a generalized scalar conductivity $\sigma' = \frac{ne^2}{m} \left(\frac{\tau_n^2 + \tau_r^2}{\tau_r} \right)$ (5-6)

and a generalized Hall mobility ratio $\alpha' = \frac{\tau_n}{\tau_r}$ (5-7)

then $J_y = \sigma' (uB - E_y) - \alpha' J_x$ (5-8)

where $E_y = \frac{V_1 + \delta V}{H}$ (5-9)

substituting Equation (5-9) into Equation (5-5), using Equation (5-7) and rearranging gives

$$V_T = (uBH - \delta V) - H J_y \left(\frac{1 + \alpha' \Lambda}{\sigma'} \right) \quad (5-10)$$

where $\Lambda = \frac{J_x}{J_y}$ (5-11)

Defining an effective conductivity,

$$\sigma'_\Lambda = \left(\frac{\sigma'}{1 + \alpha' \Lambda} \right) \quad (5-12)$$

then $V_1 = (uB - \delta V) - H \left(\frac{J_y}{\sigma'_\Lambda} \right)$ (5-13)

and $\frac{dV_T}{dJ_y} = \frac{d(uBH - \delta V)}{dJ_y} - H \frac{d}{dJ_y} \left(\frac{J_y}{\sigma'_\Lambda} \right)$ (5-14)

if $J_y \frac{d\Lambda}{dJ_y} \ll \Lambda$ and if the changes in σ' , α' , uBH and δV

with J_y are insignificant, in the range of interest, then, from Equation (5-14)

$$\sigma'_\Lambda = - \frac{H}{dV_T/dJ_y} \quad (5-15)$$

and, from Equations (5-12) and (5-15)

$$\sigma' = \frac{-(1 + \alpha' \Lambda) H}{dV_T / dJ_y} \quad (5-16)$$

now, as $B \rightarrow 0$, $\omega \rightarrow 0$; therefore $T_H \rightarrow 0$ and $T_T \rightarrow T_0$ so that $\alpha' \rightarrow 0$ and $\sigma' \rightarrow \sigma_0$

Then, from Equation (5-6)

$$\sigma_0 = \frac{ne^2}{m} T_0 \quad (5-17)$$

solving for $\frac{ne^2}{m}$ from Equation (5-6) and using Equation (5-7)

$$\sigma_0 = \sigma' \frac{T_0}{T_T} \left(\frac{1}{1 + \alpha'^2} \right) \quad (5-18)$$

From Equations (5-6), (5-7), and (5-10)

$$\sigma' = \frac{(J_y + \alpha' J_x)}{(\mu B H - V_T - \delta V) / H} \quad (5-19)$$

Now

$$\frac{(\mu B H - V_T - \delta V)}{H} \equiv E_y = \mu B - \frac{\partial V}{\partial y} \quad (5-20)$$

Combining Equations (5-3) and (5-4) to cancel out $(\mu B - E_y)$, using Equations (5-6) and (5-7) and rearranging, we obtain

$$J_x = \alpha' J_y - \sigma' E_x \quad (5-21)$$

Then from Equations (5-19), (5-20), and (5-21)

$$\sigma' = \frac{J_y (1 + \alpha'^2)}{\frac{V_{oc}}{H} - \frac{\partial V}{\partial y} + \alpha' \frac{\partial V}{\partial x}} \quad (5-22)$$

Substituting this into Equation (5-18) we have

$$\sigma_0 = \frac{T_0}{T_T} \left(\frac{J_y}{\frac{V_{oc}}{H} - \frac{\partial V}{\partial y} + \alpha' \frac{\partial V}{\partial x}} \right) \quad (5-23)$$

If no magnetic field is present but an external voltage is applied across the plasma, Equation (5-10) takes the following form

$$V_T = \frac{H J_y}{\sigma_0} + \delta V \quad (5-24)$$

From Equation (5-24)

$$\frac{dV_T}{dJ_y} = \frac{H}{\sigma_o} + \frac{d\delta V}{dJ_y} \quad (5-25)$$

if σ_o is constant.

If the last term in Equation (5-24) is negligible for the range of interest, then

$$\sigma_o = \frac{H}{dV_T/dJ_y} \quad (5-26)$$

From Equation (5-21)

$$\sigma_o = \frac{H J_y}{V_T - \delta V} \quad (5-27)$$

and, in the free stream, at a point away from the influence of the channel walls

$$\sigma_o = \frac{J_x}{E_y} = \frac{J_y}{\partial V / \partial y} \quad (5-28)$$

B. Experimental Techniques

1. Measurement of Plasma Conductivity with a Magnetic Field Present

In an MHD generator, the interaction of the magnetic field with the moving plasma produces a condition within the plasma where the voltage vector and current vector are not parallel, the angle between them being a function of α' . The measurement of plasma conductivity under these conditions results in the determination of an effective conductivity the limits of which are illustrated by Equations (5-11) and (5-12)

$$\Lambda = \frac{J_x}{J_y} \quad (5-11)$$

$$\sigma_\Lambda' = \frac{\sigma'}{1 + \alpha' \Lambda} \quad (5-12)$$

substituting Equations (5-3), (5-4) and (5-7) into (5-11) and rearranging

$$\Lambda = \frac{\alpha' (uB - E_y) - E_x}{(uB - E_y) + \alpha' E_x} \quad (5-12a)$$

For $J_x = 0$, $\Lambda = 0$, and

$$\sigma'_\Lambda = \sigma' \quad (5-12b)$$

For $E_x = 0$, $\Lambda = \alpha'$, and

$$\sigma'_\Lambda = \frac{\sigma'}{1 + \alpha'^2} \quad (5-12c)$$

which has been defined as σ'_τ , transverse conductivity.

Though methods of determining the value of Λ for a given run have been developed, further refinement is necessary so that it is better to operate at one of the limits illustrated above when measuring conductivity. The first, expressed by Equation (5-12b) implies excellent generator electrical integrity so that $\alpha' J_x \ll J_y$. The condition for the latter method can theoretically be achieved by shorting out the Hall field E_x measuring the shorted Hall current, $I_x = J_x A_x$, determining α' from the relation $\alpha' \Big|_{E_x=0} = \frac{J_x}{J_y}$ and using Equation (5-12c).

The development, at Pratt & Whitney Aircraft, of MHD test generators meeting the first condition through good electrical integrity and proper electrode segmentation permits testing at the condition $\alpha' J_x \ll J_y$.

By applying the equations presented in the preceding section and using calculated values*, test methods can be employed to evaluate the average scalar stream conductivity (σ_o) of the plasma. These values may then be compared with the values of σ_o obtained at similar flow conditions by the test methods employed when a magnetic field is not present and with the theoretical calculations.

The application of the magnetic field may result in other phenomena not present otherwise, such as induced leakage currents in both the x and y directions, as well as changes in the boundary layer and its effects, and changes in the current flow patterns near the electrodes**. Leakage current in the y direction during power generation conditions may be detected by observing the Hall voltage V_x at open circuit.

* Presumably, accurate values of α' , τ_o , τ_T and τ_H can be theoretically calculated with higher accuracy than can n . (Reports PWA-1940 and 2006)

** Analysis of Magneto Gasdynamic Generators having Segmented Electrodes and Anisotropic Conductivity; UAC R-1852-2, J.C. Crown, February, 1961

From Equations (5-3) and (5-4) we can get

$$E_x = \frac{\alpha'}{\sigma'} \left[J_y - \frac{J_x}{\alpha'} \right] \quad (5-29)$$

Thus, unless the Hall current J_x is at its limit, $\alpha' J_y$, for which case the electrical integrity is nonexistent, any slight amount of leakage current in the y direction (at open-circuit conditions) will result in a finite Hall voltage where $V_x = L E_x$. Good electrical integrity in the y direction indicates good integrity in the x direction within the generator proper although external defects such as the shorting to ground* of downstream structure exposed to the plasma may provide another path for the Hall leakage current.

Method 1 - The average scalar conductivity of the plasma is obtained from Equations (5-16) and (5-18), assuming $\alpha' J_x \ll J_y$

$$\sigma = -\frac{H}{dV_T/dJ_y} \left(\frac{\tau_o}{\tau_T} \right) \left(\frac{1}{1 + \alpha'^2} \right) \quad (5-30)$$

An experimental setup like that shown in Figure 214 is used to obtain output voltage V_T versus current density, for constant plasma flow conditions and constant magnetic field. Segmentation is used not only to reduce blooming effects (Figure 215) but also to minimize shorting of the Hall potential E_x . Thus each load circuit including electrodes must be electrically isolated from the others. Variable load resistors permit data to be obtained over the full range from open circuit to short circuit.

A plot of V_T versus output current I_y like that shown in Figure 216 is made, preferably from the center electrode pair to minimize the effect of blooming**. A straight line least squares fit is drawn through the data in the range of interest, (Region 3). The slope of this line $\Delta V_T / \Delta I_y$ is measured. Using Equation (5-30) and the relation, $I_y = J_y A_y$ (5-31)

* Assuming that the combustion chamber-nozzle assembly is grounded

** Blooming refers to the divergence of the current flow lines in the plasma resulting in a reduced current density in the plasma core flow as compared to that at the electrode surface, Figure 215.

we have,

$$\sigma' = -\frac{H/A_{yz}}{\Delta V_z/\Delta I_z} \left[\frac{\gamma_o}{\gamma_r} \left(\frac{1}{1+\alpha} \right) \right] \quad (5-32)$$

Care must be exercised in the application of this method since the data corresponding to Regions 1 and 2 in Figure 217 will generally not fall on the line drawn through the data corresponding to Region 3. This is due to the fact that $\delta V'$ varies considerably in Regions 1 and 2, while, presumably, it is nearly constant in Region 3.

The merit of this method is that the actual value of $\delta V'$ need not be known, only that it be constant, or nearly so over the region where the slope is determined.

To simplify analysis, the data reduction program calculates both σ_o and σ' . Thus, the relation from Equation (5-16) is used.

$$\sigma' = -\frac{H/A_{yz}}{\Delta V_z'/\Delta I_z} \quad (5-33)$$

Method 2

The average scalar conductivity of the plasma is obtained using voltage probes and Equations (5-23) and (5-31).

The test setup shown in Figure 218 is used to obtain $\partial V/\partial y$ and $\partial V/\partial x$. The probing technique employs voltage probes. Assumptions upon which its successful use are based are similar to those discussed for Method 5 below. The assumption is made that the voltage gradients which the voltage probes experience are linear with x and y , represent those in the core of the stream, and are not influenced by end effects. It is also assumed that V_{oc} which is αBH does not change as load is applied.

Fine electrode segmentation helps to approach this condition as does good electrical integrity within the rig. Adjacent electrodes and load circuits are electrically isolated.

2. Measurement of Plasma Conductivity with No Magnetic Field Present

The test methods involve use of an experimental setup similar to that shown in Figure 219. A voltage is applied across the electrodes of an MHD channel containing a moving and electrically-conducting plasma. The applied voltage V_T is varied, and the resulting current flow is measured. Electrode segmentation in the axial direction, as shown in Figure 220 is used to reduce the current blooming for the center electrode. Lateral electrode segmentation, shown in Figure 220 can be used to isolate wall effects. In all cases, the same voltage is applied across all electrode sections to eliminate current leakage between adjacent electrodes. Modifications to this basic setup are made according to the requirements of the particular method being used.

Method 3 - The average scalar conductivity of the plasma is obtained from Equations (5-26) and (5-31).

$$\sigma_o = \frac{H/A_y}{dV_T/dI_y} \quad (5-34)$$

For a given test, a plot is made of applied voltages versus current similar to that shown in Figure 217. The average scalar conductivity, σ_o , is obtained by measuring the slope, dV_T/dI_y , of a straight line fitted by the least squares method to the data in the range of interest (region 3) and using Equation (5-34). This method implies that $\frac{dV_T}{dI_y} \neq 0$. In Region 3 of Figure 217 this appears to be a reasonable assumption. The current density J_y is based upon the measured current and assumes no blooming. The middle electrode of a segmented-electrode test section would best satisfy this requirement, but the end electrodes could introduce appreciable error. The measured current flow I_y to the electrode will be that flowing through the plasma only if the rig has ideal electrical insulation. The amount of current leakage for given flow conditions and applied voltage can be determined approximately by operating the rig without seeding and recording any current flows. Corrections described in Appendix 7 can then be made. This implies zero conductivity for the unseeded plasma. Lateral segmentation of the middle electrode will serve to isolate leakage losses.

Method 4 - The average scalar conductivity of the plasma is obtained from Equations (5-27) and (5-31)

$$\sigma_o = \frac{H \cdot I_y}{A_y (V_T - \delta V)} \quad (5-35)$$

This method*, employing variable distance between electrodes, eliminates δV . This technique is applied to an MHD test section having a rectangular cross-section. Two different builds are used, one having the electrodes in the narrow sides, the other having the electrodes in the wide side. The other sides in both cases utilize electrically-insulating materials. Electrode segmentation and circuitry, as shown in Figure 221, is used to minimize blooming and wall leakage effects. Successive runs with the different rig builds are made under identical test conditions with applied voltages and currents recorded. By assuming δV to be identical in both runs and taking identical current conditions, σ_o is obtained from

$$\sigma_o = \frac{I_y / A_y}{\Delta V / \Delta H} \quad (5-36)$$

where $\Delta V = |V_{T_2} - V_{T_1}|$ and $\Delta H = |H_2 - H_1|$

Having determined σ_o , δV can be evaluated from Equation (5-35). The precision of the results obtained by this method depends on reproducing plasma conditions for successive builds. The assumption that δV is the same in both runs may be controversial. Supporting factors are that the same combustion chamber, nozzle, electrode materials, and electrode temperatures will be used. Since axial and lateral electrode segmentation exists, leakage and corner boundary layer effects are isolated.

Method 5 - The average scalar conductivity of the plasma is obtained from Equations (5-28) and (5-31) utilizing voltage probes to measure the plasma voltage gradient

$$\sigma_o = \frac{I_y / A_y}{\partial V / \partial y} \quad (5-37)$$

* UAC Research Laboratories Report P-2000-135, by R. Stoeffler

This method can use a test setup similar to that shown either on Figure 219 or Figure 222. With the plasma flowing, applied voltages and currents are recorded as well as the voltages of the probes with respect to an electrode. From this data and the known geometry of the rig, the probe voltages are plotted on voltage-distance coordinates. From the slope of a straight line drawn through the points, $\partial V / \partial y$ is evaluated. Using the measured current density, σ_o is evaluated from Equation (5-37).

This technique is experimentally difficult because durable, high-temperature conducting materials must be used. Because a penetration depth may be required to sense the voltage gradient at the plasma core, the probes may require an insulating sheath to avoid voltage differences in the boundary layer along the insulated sidewall. Once σ_o is determined, δV can be computed by

$$\delta V = V_r - \frac{H I_z}{\sigma_o A_y} \quad (5-38)$$

3. Measurement of Plasma Conductivity by Other Methods (AC Conductivity Probe)

Other methods of determining plasma conductivity which do not rely on the use of an externally-applied or internally-generated transverse electric field are desirable in order to avoid some of the sources of error inherent in the methods described above. A method which has been studied extensively is discussed below.

Studies have been made and are underway to develop improved methods of evaluation of local scalar plasma conductivity. The following describes a technique for determining the scalar conductivity of a plasma utilizing an AC conductivity probe developed by the United Aircraft Research Laboratories*. A high temperature version of this probe is being developed for MHD plasma conductivity measurements. The probe consists of a cooled protective tubular sheath containing a cooled coil.

The AC probe is designed to be immersed in the plasma and to permit a direct measurement of scalar conductivity. This differs from other

* Reported in UAC Research Report M-1282-1, R. A. Olson and E. C. Lary

AC techniques mentioned in the literature in that the probe is immersed in the plasma and produces an oscillating magnetic field external to the probe. The oscillating magnetic field induces azimuthal currents in the plasma which dissipate power in an amount dependent on the plasma conductivity and the affected plasma volume. Electrodes are not used and thus sheath effects are not present. By keeping the probe temperature boundary layer thin, the effects of boundary layer conductivity may be eliminated and the probe measures the true scalar conductivity of the free-stream plasma. The probe may be operated relatively close to a wall without adverse effects. It is discussed further in Appendix 8.

APPENDIX 6

Test Equipment and Procedures

APPENDIX 6

Test Equipment and Procedures

The discussion of test equipment and associated procedures used during the investigation is divided into the three basic areas of facilities, rigs, and instrumentation.

A. Facilities1. Test Stand

Most of the contract testing was done with a rocket engine type test stand (X-197C) at East Hartford, Connecticut. A picture of the stand with an MHD test rig in place is shown in Figure 223. A picture of the rig and the magnet is shown in Figure 224. A drawing of a typical rig and its auxiliary components is shown in Figure 225. The rig and the components are described later. A plumbing schematic of the propellant feed system and cooling water system is shown in Figure 226. The storage capacities and flow rates and other data are given below.

Test Stand Capacities

Magnetic field (continuous)	22,000 gauss
Magnetic field volume	9 inch dia. x 9 inch gap
Hydrogen flow	1.4 lbs/sec
Hydrogen storage	93,000 SCF
Oxygen flow	7.0 lbs/sec
Oxygen storage	76,700 SCF
Nitrogen flow	7.0 lbs/sec
Nitrogen storage	50,000 SCF
JP4 flow (pressurized)	0.09-3.0 lbs/sec at 100-1200 psi
JP4 storage (pressurized)	25 gals
JP4 flow (JT-3 pump)	0.1-3.0 lbs/sec at 300 psi
JP4 storage (JT-3 pump)	275 gals
Seeding flow	0.01 - 1.0 lbs/sec
Seeding storage	30 gals
Water	63 lbs/sec at 90 psi (city)
Centrifugal pump	250 GPM - 100 psi
Load circuits	5
Load circuits capacity	5,000 watts per circuit

The propellants are controlled semi-automatically. Once the tank pressures and venturi pressures have been set, the firing sequence is controlled automatically. When a warmup is used, the switch from H_2/O_2 to $JP4/O_2$ is made manually. The switch to H_2/O_2 for the cool-down is made manually, shutdown and purging are accomplished automatically.

The stand has several safety interlocks which initiate an automatic shutdown if departures from routine operation occur.

The oxygen flow rate and hydrogen flow rate are measured with choked venturis. The $JP4$ and KNO_3 solution flow rates are measured with turbine meters. The flowmeter outputs, venturi pressures, chamber pressures and rig static pressures are all recorded on strip chart recorders.

The coolant water flow rates are measured with meters. The cooling water ΔT 's are measured with chromel-alumel thermocouples. The flowmeter outputs and thermocouple outputs are recorded on an oscillograph.

The stand can be used for either applied voltage runs or power runs. For the applied voltage runs, voltage is applied to the rig with a DC generator having a rating of 100 amperes at 250 volts. The five electrode pairs of the rig are connected in parallel with one another and in series with the generator. As another alternative, voltage can be applied with 6, 12, 18, and 24-volt batteries.

For power runs, a water-cooled magnet is utilized which is capable of continuous operation at field strengths up to 25,000 gauss. The magnet pole faces are 7 inches in diameter. The magnet halves are mounted on wheeled carriages which are placed on rails: the magnet can thus be moved apart to facilitate work on the rig. Power to the magnet is supplied by two 65 KW motor-generator sets which deliver up to 500 amperes apiece. A picture of the stand which shows the magnet is shown in Figure 224.

A diagram of the electrical circuit for one electrode pair is shown in Figure 227. All the circuits are isolated from one another in a power run. Figure 228 is a schematic of electrical and instrumentation wiring for the test stand. For a power run, switch A (Figure 227) is thrown to the position indicated for a

power run and the relays are all open. The load condition is then open circuit. The relays are closed one by one starting at the right of Figure 227. The relays are thrown at the same time for all electrodes. After contact is made at the newest setting, contact is broken for the previous setting.

For an applied-voltage run, the relay on the left as shown in Figure 227 is closed so that the current is shorted around the loads and switch A is thrown to the position indicated for a conductivity run. Switch B provides a choice between battery-applied voltage and generator-applied voltage.

All five cross-stream voltages are measured. The voltmeter leads are connected to the electrode taps at the rig. This eliminates error in voltage measurement induced by resistance drop over the high current load lines.

The currents are measured for each electrode circuit at the position indicated on Figure 227.

The Hall voltages between adjacent electrodes on each electrode wall (top and bottom) are measured along with the total Hall voltage between flame shield and combustion chamber (a total of 9 voltage points). The interelectrode Hall voltages are measured across the voltmeter leads.

The electrical data is measured and recorded with meters on a photopanel. Continuous movies are taken of this panel during a run. The meters are damped to reduce scatter due to minor instabilities within the rig.

Timing on the strip charts and the oscillograph is done via a separate (one-second timing pulse) trace. Timing on the photopanel is accomplished with a digital readout counter, the smallest division of which is 0.1 second. The counter is started automatically when JP4 flow is started so that synchronization among all three recording devices can be realized.

2. Run Procedure

The normal run procedures followed in the operation of the test stand are outlined below:

Prerun

- 1) The rig is mounted and instrumentation and power leads are connected. Cooling water is then started just prior to the run, to all coolant channels which require water.
- 2) The electrical integrity of the rig is checked with a 1000-volt ohmmeter. The continuity of the power circuits is checked.
- 3) After these checks have been made, if a run with applied magnetic field is to be made, the magnet halves are closed and secured.
- 4) All valves for the tank farm are opened (O_2 , N_2 , H_2).
- 5) If JP4 is to be used as a fuel, the isolation valve on the JP4 tank is opened.
- 6) The vent valve and the isolation valve on the KNO_2 tank are opened.
- 7) If nitrogen or hydrogen is to be used as a coolant, the flows of these coolants are set.
- 8) The switches and valves are put in the correct position for the desired fuel (JP4, hydrogen or propane).
- 9) The electrical switches shown in Figure 227 are put in the correct position for the desired run type.
- 10) Following a check of all previously mentioned items, a dry run is made to familiarize test personnel with the run schedule.

Firing, Warmup and Setting Conditions

Immediately prior to ignition the following steps are taken:

- 1) The hydrogen regulator and oxygen regulator are set to provide the proper flows for warmup.
- 2) The photopanel lights are turned on.
- 3) The stripcharts and oscillograph are turned on.
- 4) The timing pulse recording pins on the strip charts are turned on.

- 5) After a check to make sure the recording devices are all operating, the oxygen valve is opened, the spark plug is activated and hydrogen is turned on.
- 6) After ignition has been accomplished, the oxygen and hydrogen are brought to the proper conditions for warmup. The O/F during warmup is kept fairly low (1-2.5) in order to prevent the possibility of a hot oxidizing atmosphere which could accelerate rig deterioration.
- 7) Warmup is continued for approximately one minute. Then the oxygen flow is increased and flow of the fuel selected for the run is started. After ignition on the fuel for the run, the hydrogen flow to the injector is shut off and fuel flow and oxygen flow are increased to the values selected for the run. These conditions are maintained for approximately 1 to 1 1/2 minutes so that the rig insulating wall and electrode temperatures can stabilize before electrical data is taken.
- 8) The photopanel camera is turned on at this point.
- 9) After the one minute elapsed time burning with the chosen fuel for the run, the seed is turned on and brought to the desired flow rate.
- 10) At this time leads from instrumentation such as thermocouples exposed to the high potentials are disconnected from sensitive recording instruments.
- 11) After the flow rates of the propellants and chamber pressure have been checked, the rig is ready for the diagnostic portion of the test.
- 12) The motor-generator set or sets are turned on (either for the magnet or for applied voltage). If the run involves an applied magnetic field, the magnet current is increased to the desired level.

Conductivity Test

At this point, a run is made which involves either an applied magnetic field (power run) or no applied magnetic field (applied voltage run). The two types of tests are discussed separately below.

Applied Magnetic Field (Power Run)

At this point the rig is at open circuit. The sequence through the load resistances can be made with an automatic system which dwells at open circuit and each of 6 preselected loads for about five seconds. As another alternative, the loads can be sequenced manually. Manual sequencing allows an arbitrary dwell time at each load and all 10 loads plus short-circuit can be included in the sequence. In either case, the load sequence progresses from high resistance loads to low resistance loads to prevent opening relays carrying high current. Current is minimized by momentarily stopping seed flow in order to open the relays and begin another cycle.

No Applied Magnetic Field (Applied-Voltage Run)

For this type of run, the voltage applied with the applied-voltage generator is increased slowly until either the ammeters peg or the voltage is at the maximum permitted by the generator (250 volts). The voltage is then decreased slowly to the lowest value possible (about 30 volts). An alternative can be chosen. The voltage can be applied in steps at 6, 12, 18, and 24 volts by using batteries. The voltage is usually decreased from 24 volts to zero and then increased again to 24 volts, to permit a check of the electrical integrity of the rig before seed is turned on. If both types of runs (power and applied-voltage) are desired during one firing, the first cycle is made, then the appropriate generator is shut off, seed flow rate is shut off and all switches are thrown to the correct position for the second cycle. The generator appropriate to the second cycle is then started and brought to the desired starting conditions and seed flow is turned back on. The second cycle is then made following the sequence described above.

If it is necessary to change conditions between cycles this can be done without disturbing any of the electrical settings. The flow rates are simply adjusted to the new desired values. This is usually done when the rig is at open circuit condition if the second cycle involves a run with applied magnetic field.

After the electrical cycle is completed the generators are turned off.

Shutdown

After the electrical data required from a particular firing has been obtained, seed flow is shut off. The fuel flow rate and oxygen flow rate are decreased to the warmup values. Hydrogen flow to the injector is started and the primary fuel is shut off. The fuel line is purged. After a minute of cool down on hydrogen and oxygen, the propellant flow is stopped (oxygen first), and the propellant lines are all purged.

Postrun

All supply line valves are closed and the cooling water is turned off after a short period during which the rig liner cools to acceptable levels. This is done to avoid heat damage to structures and seals.

Instrumentation

The accuracy of the basic stand instrumentation is summarized in the table below. The accuracies stated in the table are those which have been established by calibration or manufacturer's specifications. The error has been given as a percentage of the full scale reading.

In addition to these predictable instrument errors, sensing element errors and reading errors, random errors may occur in the system or in the instruments themselves. For example, changes in temperature between the time of calibration and the time of the test can cause changes in the pressure transducer response.

Another source of error could be the nozzle throat area. The throat area changes from run to run because of warping of the nozzle during firing, erosion, and the differences in refractory coating thicknesses for each run.

Although, for some of the runs, the nozzle throat area was measured both before and after the run, the area, A^* , can change appreciably during the run. For a temperature rise of 700°F, for example, the throat area could change on the order of 1 per cent due to thermal expansion. Loss of Rokide coating could result in another 1 per cent change in the throat area.

It is seen that the error in measurement of C^* ($C^* = \frac{P_c A^* g C_p}{W_t}$) could easily be ± 5 per cent.

Accuracy In Determination Of Run Parameters

<u>Parameter</u>	<u>Sensing Element</u>	<u>Accuracy</u>	<u>Reading Error</u>	<u>Recorder</u>	<u>Reading</u>	<u>Total</u>
Chamber Pressure, P_c	$\pm 1/4\%$	$\pm 1\%$	$\pm 0.1\%$	$\pm 1/4\%$	$\pm 1/4\%$	$\pm 1.85\%$
O ₂ Flow, $\dot{W}_O$	$\pm 1\%$	$\pm 1\%$	$\pm 0.25\%$	$\pm 1/4\%$	$\pm 1/4\%$	$\pm 2.3/4\%$
H ₂ Flow, $\dot{W}_H$	$\pm 1\%$	$\pm 1\%$	$\pm 0.25\%$	$\pm 1/4\%$	$\pm 1/4\%$	$\pm 2.3/4\%$
JP4 Flow, $\dot{W}_f$	$\pm 0.5\%$	$\pm 0.5\%$	Digital	$\pm 1/4\%$	$\pm 1/4\%$	$\pm 1.1/2\%$
KNO ₂ Flow, $\dot{W}_s$	$\pm 0.5\%$	$\pm 0.5\%$	Digital	$\pm 1/4\%$	$\pm 1/4\%$	$\pm 1.1/2\%$
Terminal Voltage, V_T	$\pm 1\%$				$\pm 1\%$	$\pm 2\%$
Current, I	$\pm 1\%$				$\pm 1\%$	$\pm 2\%$

Error is expressed as % of full-scale reading

3. MHD Generator Auxiliary Equipment

Injectors

Because of combustion instability associated with the higher performance showerhead injector, much of the program was run using a 6-element, like-on-like impinging injector in the combustion chamber. In all, four basic injector configurations were used. These configurations are shown in Figure 77 and are described below.

37-Element Showerhead Injector - This injector has 37 fuel holes whose axes are parallel to the axis of the combustion chamber and which are 0.040 - 0.042 inch in diameter. There are 54 oxygen holes, all having a parallel axis, which are 0.066 - 0.068 inch in diameter. The injector face is 2-3/8 inches in diameter*.

61-Element Showerhead Injector - This injector has 61 fuel holes with parallel axis which are 0.032 - 0.034 inch in diameter. There are 104 oxygen parallel axis holes which are 0.066 - 0.068 inch in diameter. The injector face is 2-3/8 inches in diameter*.

48-Element Showerhead Injector - 13 of the outer fuel holes on the 61-element injector were plugged, as well as 5 inner oxygen holes. The holes were plugged to obtain a more uniform distribution of O/F and per cent K over the injector face.

6-Element, Like-on-Like Impinging - This injector has 6 pairs of impinging fuel holes. The fuel holes are 0.030 - 0.032 inch in diameter. There are 6 oxygen holes with axis parallel to the axis of the combustion chamber and which are 0.356 - 0.358 inch in diameter. The injector face is 3 inches in diameter.

Combustion Chamber

The combustion chamber for MHD tests was copper and it was cooled with water. The chamber was 3 inches in inside diameter and was 18 inches long. Immediately downstream of the chamber, an igniter ring supported a spark plug. The igniter ring was 1 1/2 inches long and 3 inches in inside diameter. The igniter

* The 2-3/8 inch diameter injectors were provided with filler rings to occupy the space between them and the inside surface of the 3-inch diameter combustion chamber

ring was water-cooled (see Figure 225).

Nozzles

The nozzles were water-cooled, cast copper. They were coated by the flame spray method with a 0.010 inch thickness of either Rokide A or Rokide Z. The purpose of the coating was to reduce the possibility of electrical shorting to the nozzle and to reduce heat loss through the nozzle walls. The diverging portion of the nozzles was a two-dimensional ideal nozzle contour. The converging portion of the nozzles made the transition from 3 inches inside diameter to the rectangular throat. The nozzle throat-to-exit area ratios used were 1.5, 2.0, and 3.75. A photograph of the nozzles is shown in Figure 229.

Flame Shield

In a power run, the magnet is closed and is in close proximity to the exhaust from the MHD generator. To protect the magnet from the high temperature exhaust, a water-cooled flameshield is attached to the MHD rig exit. The flameshield cross-sectional area is greater than that of the rig so that it acts as a diffuser. The inside of the flame shield was coated with Rokide A. This coating insulated the plasma from possible leakage paths to ground, if the electrical isolation of the flame shield itself should fail.

B. Rigs

The various MHD generator channel sections, hereinafter referred to as rigs, used in the investigation are described in this section in the order in which they were developed. The extreme conditions which an MHD rig encounters require exceptional materials and designs to successfully cope with them. Several rigs were conceived, designed, built and tested before the final, successful design was achieved. Although the last design proved competent to fulfill the requirements of the contract, it too was revealed to have many areas where improvements could be made. The experience gained from the operation of this last rig provided valuable information and ideas for the design of the 250-megawatt and 1000-megawatt MHD generators discussed in Volume VI of the final report.

1. Keystone Electrode Rig

The initial configuration in the series of keystone electrode rigs is

shown in Figures 230, 155, and 156. It used gas cooled segmented nickel electrodes (i.e. "cold" electrodes) and gas-cooled aluminum oxide-coated sidewalls. Poor electrical performance as well as gas sealing problems were experienced in the initial tests of this rig.

To improve electrical performance as well as reduce the number of sealing surfaces, the generator was modified to allow the use of hotter running ceramic electrodes (see Figure 231). A coolant bypass was added providing cooling for the back side of the electrodes. The use of the bypass also reduced the number of gas seals per electrode assembly from 6 to 2. The electrode's axial length was reduced to allow the use of a wider (0.385 inch) interelectrode ceramic insulator. This improved resistance to leakage currents between electrodes. The electrical connection to the electrodes was made with carburized tantalum pins screwed into holes tapped into the back of the electrodes.

A third configuration of the keystone rig was used when the electrode material was something other than graphite. Drilling and tapping a blind hole into a material harder than graphite (i.e. SiC, ZrC, etc.) is very difficult and prohibitively expensive. In these cases, the electrical connection to the electrodes was made with carburized tantalum pins, spring-loaded against the back of the electrode. Figure 232 is a detail of the electrode contact configuration and Figure 233 is a photograph of an assembled rig.

In spite of the extensive modifications noted above, the performance of these rigs was impaired by the poor mechanical and electrical characteristics of the ceramic coatings used on the insulating sidewalls. Several methods were tried in an effort to develop a satisfactory insulating coating. Coatings with a base of Al_2O_3 and also with a base of ZrO_2 were flame-sprayed on, using the rod method. Sidewalls with a coating of Al_2O_3 sprayed on using the plasma spray method were tested. In some cases a second coat of Astroceram (a refractory ceramic cement) was applied in an effort to eliminate porosity. No one coating or process exhibited very satisfactory performance.

2. Alternating Sidewall Rig

The purpose of this rig was to permit measurement of the plasma conductivity independently of the effect of the boundary layer by the technique discussed under Method 4, Appendix 5. A picture of the rig is shown in Figure 159.

The electrodes in this rig could be placed on either the 1.65-inch wall or the 3.30-inch wall. The electrodes were zirconium carbide. Contact with the electrodes was accomplished by screwing zirconium carbide studs into holes tapped in the electrodes. The studs were kept in slight tension.

The electrode backups were of aluminum oxide. The electrodes were uncooled and the uncooled steel housing was shielded from the hot electrodes by the thick Al_2O_3 electrode backups. The interelectrode spacers and the end spacers on the 3.30-inch wall were of beryllium oxide.

The 1.65-inch wall, in the vicinity of the active electrode section of the rig, was made of beryllium oxide bars. The upstream and downstream 1.65-inch sections were Rokide A coated, gas-cooled nickel. The 1.65-inch wall housing was a modification of the electrode housing used in the keystone electrode rig, the main difference being that the electrodes could be replaced by the BeO bars.

This rig was run on 30 January, 1962, in the configuration shown in Figure 159. No warmup was used and chamber pressure rose to 60 psia immediately after ignition. Either at ignition or immediately thereafter, the two downstream electrodes and associated spacers failed. The run was terminated immediately with a total run time of 6 seconds.

Examination of the rig after shutdown revealed that 9 of the 10 electrodes broke in the vicinity of the electrode contact, that almost all the BeO pieces on the 3.30-inch wall were broken or cracked, and that the shorter BeO bars used in the 1.65-inch wall were not damaged.

Although the rig design was not satisfactory, the run pointed out several areas which should be avoided and helped to provide design criteria for the chase-around configuration discussed later. These criteria are discussed in the section on structural concepts.

3. Toadstool Electrode Rig

The general configuration of this rig is shown in Figure 234. It was designed with ceramic electrodes to allow high, controlled surface temperature. The sidewalls used are the same as those used on the keystone electrode rig.

The electrodes were hot-pressed ZrC containing graphito. These were backed up by ZrO_2 insulators for both thermal and electrical insulation.

Interelectrode insulators were BeO. These components were assembled on a stainless steel housing and held in place by spring-loaded carburized tantalum pins which also served as electrical connectors.

Two electrode pin configurations were used. The first was as shown in Figure 234, where the pins have heads that seat in matching holes through the electrodes. This configuration was not very successful. The electrodes cracked due to thermally and mechanically-induced stresses around the hole, which resulted in loss of the electrode. Some trouble with rapid erosion of the pin head after the carbide layer had worn off (sometimes in less than one minute) was also experienced.

The second configuration was the integral-pin design. The electrodes were hot-pressed with the carburized tantalum pins in place. This reduced the problems with pin erosion and electrode cracking, but the manufacturing process involved keeping the electrode and pin at a temperature above the recrystallization temperature of tantalum for about eight hours. The resulting grain growth caused the pins to be very brittle, in turn resulting in failure of the pin and loss of the electrode.

4. Button Electrode Rig

As stated in Appendix 5, the purpose of this rig was to eliminate the effects of fringing and leakage to the walls in the coated sidewall rigs for conductivity tests. The buttons were located in the center of the rig so as to involve only the high temperature, high conductivity core flow in the analysis. A picture of the rig electrodes after a test run is shown in Figure 69.

The sidewalls in this rig were water-cooled stainless steel coated with Rokide A. The Rokide A was stoned smooth.

The electrode housing and the dummy end electrode were hydrogen-cooled stainless steel. The streamside face was nickel coated with Rokide A.

A sketch of the electrode configuration for this rig is shown in Figure 235. The large electrode plate has two grooves cut lengthwise into their back into which were placed two tungsten wires. These wires were held in place with Astroceram. The wires served the purpose of maintaining electrical contact in the event that the plate should crack.

The electrical connection to the electrodes was made by screwing carburized tantalum pins into holes tapped in the back of the electrodes.

The electrode plate was separated from the electrode housing by a beryllium oxide spacer. There was no insulator between the electrode plate and the dummy end electrode.

Four SiC voltage probes were placed in the sidewall in the plane of the middle electrode button. Figure 236 shows some of the rig details.

5. Chase-Around Rig

The chase-around rig was designed to eliminate the problems associated with previous rigs in order to provide a reliable, durable, test rig with which to make plasma conductivity and power tests. While other improvements could be made (rig design was frozen because of contract time limitations) the chase-around design concept has performed well, especially as compared with previous rig designs.

Materials Durability - The chase-around rig has only ceramic materials in contact with the hot plasma. While the refractory metals (tantalum, tungsten, and molybdenum) melt at temperatures higher than they would operate at in the rig, they oxidize and /or recrystallize quite rapidly at these temperatures. Earlier rigs utilized tantalum which had been carburized to avoid the oxidation problem. This scheme proved to be impractical because the thin carburized layer wore away quite rapidly and because in the carburizing process, the tantalum was heated above its recrystallization temperature, making the final piece very brittle. By avoiding the use of nonceramics in contact with the plasma, the problems of melting and oxidation are drastically reduced. The BeO sidewalls do not appear to erode, react or melt at all during the runs made with this rig (up to 87 minutes) when a hydrocarbon fuel is used and when contact with contaminating materials is avoided*. The SiC - 25 per cent graphite electrodes oxidize, but at a slow rate. The erosion and oxidation of flat electrodes is negligible for run times up to twenty minutes. Check runs have shown that there is no appreciable shorting because of the molten SiO₂ which forms downstream of the electrodes, so for runs of the duration required for contract testing, SiC - 25 per cent graphite was quite acceptable as an electrode material.

* The BeO does exhibit less resistance to destructive reactions with the plasma when hydrogen is used as the fuel. It has been reported that beryllia reacts with superheated steam and this appears to have occurred

Structural Refinements - Ceramic materials in general are quite brittle and many of the hard, erosion-resistant ceramics are susceptible to thermal stress and thermal shocks. The choice of an appropriate structural design minimizes these problems. The design must allow the pieces to expand, warp and move during the transient warmup period. The design should keep all pieces in compression to avoid failures due to tension induced by thermal stress.

The chase-around rig incorporates all the aforementioned characteristics. The rig, which is shown in Figure 168, permits motion of all ceramic pieces with thermal expansion. The springs shown in diametrically opposing corners exert a compressive force on the beryllia sidewall pieces. The angles on the ends of the sidewall pieces and the electrode pieces transmit this loading so that the electrode pieces are also in compression. Thermal expansion thus results in the pieces being put into heavier compressive loading.

Variation of expansion in the axial direction is permitted by the fact that each sidewall piece is sprung independently of other sidewall pieces and axial clearance is deliberately left during assembly.

As seen in Section BB of Figure 168, the sidewall pieces can be interlocked with dowel pins. These pins provide redundancy because a piece that cracks will be held in place by the sidewall pieces adjacent to it.

Electrical Integrity - The resistivity of the sidewall material and of the material of the interelectrode spacers must be very high at the temperatures of operation in order to prevent leakage currents. The problem of maintaining high sidewall resistivity is complicated by the fact that heat transfer must be reduced in order to minimize heat loss from the plasma. This implies that sidewall temperatures will be very high (3000°F - 3500°F). At these high temperatures, the resistivity of sidewall coatings (flame-sprayed coating and plasma-sprayed coatings) is very low. Rokide A which was used on the sidewalls of previous rigs has a resistivity of 0.1 ohm-cm at 3200°F. The resistivity of the 99 per cent purity, 97 per cent density beryllia used in the chase-around rig sidewalls is 1000 ohm-cm at 3200°F. Since the resistivity of the plasma is of the order of 10 ohm-cm, the coating materials appear as a dead short to the plasma. The beryllia sidewalls have a resistivity two orders of magnitude higher than the plasma.

Thus, beryllia presents a great improvement over the sidewall materials used previously. The rig electrical integrity is further improved by the fact that the BeO sidewall pieces are 5/8-inch thick as opposed to the 0.010-inch thick Rokide A. Because the Rokide A was so thin, a very low resistance path was available through the thin coating to the metal housing. The rig avoids this path by the use of thick BeO pieces. These pieces are further backed up by boron nitride backup spacers which are 3/16-inch thick.

The BN pieces are also used behind the electrodes. They serve a dual purpose in that they insulate the electrodes from the metallic structure of the rig electrically and they provide an additional heat barrier which maintains a high electrode surface temperature.

The low coefficient of friction between the BN and BeO or SiC permits sliding of the sidewall pieces and electrode pieces due to thermal expansion to occur readily.

The BN pieces have serrations perpendicular to the plasma flow and the axial electric field. This helps to prevent shorting of the Hall field because it prevents flow leakage and circulation due to axial pressure gradients. Shorting could occur in any plasma which leaked behind the electrodes or sidewalls if this orientation did not prevent leakage and act as an insulator against current flow. The rig is further isolated electrically by a coating of silicone varnish on all the metal housing pieces. The springs which provide the compressive force on the sidewall pieces are isolated from them by zirconia blocks between the spring and the sidewall piece.

From the foregoing paragraphs it is seen that the portions of the rig in contact with the plasma are well isolated electrically from the metallic housing pieces. The electrode housing pieces are insulated from the sidewall housing pieces by either Silastic or Teflon gaskets. Thus the electrical integrity of the rig is assured.

Cooling - Cooling of the chase-around rig is accomplished by water flowing in tubes drilled in the housing pieces. This method eliminates the brazing and welding used in previous rigs. The welded construction cracked and leaked quite frequently. This was especially troublesome because hydrogen was used for cooling in the previous rigs and quite possibly caused embrittlement.

Because the metal structure runs fairly cool, the problem of housing warping is minimized.

The compression springs are cooled with nitrogen in order to prevent loss of strength at high temperatures.

Wall Temperature - The electrodes and sidewalls of the chase-around rig run hotter than in previous rigs because of the thermal insulation provided by the boron nitride and the thick beryllia pieces. This results in a reduction in heat loss from the plasma and it also leads to hotter boundary layers which should have higher conductivity, leading to more power production and more accurate determination of the core flow conductivity. The high wall temperature also minimizes condensation of conductive deposits of seed compounds and carbon, noted in the past with lower wall temperatures, thus reducing leakage paths.

The electrode face temperature is controlled by the configuration and orientation of the BN backup pieces. There are two basic configurations of the BN backup pieces. The pieces are available serrated on one surface and flat on the other surface or flat on both surfaces. When the serrated pieces are used, plasma becomes trapped by the serrations. The thermal conductivity of the plasma increases rapidly with temperature. The contribution of radiation heat transfer also increases rapidly with temperature. Thus, when the serrated pieces are placed so that the serrations face toward the stream, the serrations are at higher temperatures and more heat transfer results than when they face away from the stream. Therefore, highest temperature is achieved by running the rig with the serrated edges away from the plasma. The second highest temperature is achieved by having the serrated edges facing toward the plasma. The two lower temperatures are achieved with nonserrated pieces. The thermal conductivity of hot-pressed boron nitride is anisotropic; it is highest in the direction perpendicular to molding pressure. The lowest temperature is achieved by orienting the backup blocks so that the heat flux is perpendicular to the direction of pressing. The second lowest temperature is achieved by orienting the backup blocks so that the heat flux is parallel to the direction of pressing. These configurations are shown on Figure 78.

In preliminary runs with this rig it was discovered that the hottest electrode temperature achieved with BN spacer serrations facing away from the stream resulted in very rapid erosion of the electrodes. After this, the hottest electrode temperature configuration used during contract running was with the serrations facing toward the stream. This temperature configuration is designated T4. The electrode temperature configuration designated T3 was achieved

by grit blasting the backup pieces through a 1/4-inch mesh screen. The grit blasting produces a corrugated effect. The two lower temperatures are obtained by the method described above and are designated T_1 and T_2 , T_1 being the lower temperature. All the configurations are shown in Figure 78.

Electrode Contact - Contact is established with the electrodes through spring-loaded carbon or tantalum pins. The portion of the electrode which is in contact with the pin is rhodium plated to insure good electrical contact: the pins themselves are sometimes tipped with rhodium for the same purpose. This method of electrode contact eliminates the problems of electrode cracking because of rigid confinement or because of high stresses induced through the tapped holes necessary in other methods of contact. The integral pin electrode design in which the electrode was molded around the contact reduced these problems. However, because the stem materials available recrystallized at temperatures experienced in the rig, this design was not acceptable. The method of electrode contact used in the chase-around rig does present a problem in that the sliding point contact between the pin and the electrode could lead to a high and/or variable resistance in series with the plasma.

Instrumentation - The normal instrumentation used in MHD runs is covered in the section which describes the test stand. In addition, provisions are made in the chase-around rig for voltage probe measurements, static pressure measurements, and electrode temperature measurements. The provisions for these measurements can be seen in Figure 168.

The voltage probes are made of silicon carbide. The surface of the probes exposed to the stream is flush with adjacent insulating sidewall surface. They are shouldered to prevent motion into the stream. Contact with the probes is made by wrapping thoriated tungsten wires around the end of the probe. The probes and the wires are then cemented in place with Astroceram. The leads are taken out through an aluminum oxide tube which is placed inside one of the compressive springs.

The static pressure is measured at the taps shown in Figure 168. The tantalum tube is inserted in the BeO block through the spring wells.

The electrode temperature is measured at a slot cut in the back face of the electrode. A platinum-platinum 10 per cent rhodium thermocouple is inserted in the groove. The thermocouple is

sheathed in tantalum and is cemented in place with Astroceram. The Astroceram insures good thermal contact with the electrode and also protects the tantalum from oxidation.

Note: For the electrode and electrode spacer size used, a current of 15 amperes to electrode 3 (least blooming effect) corresponds to a current density, J_y , in the plasma of 10,000 amps/m².

APPENDIX 7

Theoretical Prediction of Conductivity
Based on Experimental Conditions

APPENDIX 7

Theoretical Prediction of Conductivity
Based on Experimental Conditions

The comparison of experimental and theoretically predicted values of plasma conductivity requires that the composition, temperature, and density of the plasma in the test rig be known. Ideally, these quantities would be measured directly. However, the environment and the state of the instrumentation art preclude this without engaging in an extensive development program. These quantities must therefore be determined indirectly. This is done with a thermochemical calculation procedure which assumes infinite reaction rates between species, and uses for its starting point conditions* at the combustion chamber. It then calculates the pertinent changes that occur as this plasma is isentropically expanded through a nozzle of prescribed area ratio. Modifications to the procedure permit the arbitrary freezing of the thermochemical reactions and/or electron quantity at the nozzle inlet or throat. Values of the following variables may be calculated at the exit of the prescribed area ratio, A/A^* , nozzle.

- Static temperature, T ,
- Static pressure, p ,
- Composition of the plasma, including electron density, n ,
- Entropy, S
- Enthalpy, h
- Ratio of specific heats, γ
- Specific heat, C_p
- Average molecular weight, M_w
- Gas constant, R
- Density, ρ

The plasma composition, temperature, and density can then be used, along with a prescribed magnetic field strength, to calculate plasma

*Stagnation pressure at nozzle inlet, P_t

- Fuel composition
- Fuel flow rate, $\dot{w}_f$
- Oxygen flow rate, $\dot{w}_o$
- Seed solution flow rate, $\dot{w}_s$
- Seed solution composition
- Heat loss

conductivity and Hall mobility ratio according to the procedures outlined in Volume V of the final report. This theoretical prediction can then be compared with experimental data.

The equations developed in Appendix 5 provide a basis of experimentally determining plasma conductivity and the equations and data used to theoretically predict conductivity necessarily involve certain simplifying assumptions. The degree to which the test conditions approach these assumptions strongly influences the usefulness of the results. In order to reconcile theoretical predictions with experimental data, the departure from ideal conditions must first be experimentally and analytically determined and then suitable analytical corrections made. Both of these steps can involve great difficulty and uncertainty. Obviously, it is desirable to operate at or near ideal conditions to avoid errors due to inexact correction procedures. However in some cases the effort necessary to do so was beyond the scope of the contract. Some of the major factors are discussed in this section. They are:

- 1) Rig mechanical and electrical integrity,
- 2) Combustor efficiency,
- 3) Nozzle efficiency,
- 4) Measurement errors,
- 5) Instrumentation accuracy, and
- 6) Thermochemical equilibrium of flow.

A. Rig Mechanical and Electrical Integrity

The mechanical and electrical integrity of MHD test rigs is vital to the acquisition of good data. Mechanical integrity and durability is desirable in order to permit test runs of reasonable length, thus allowing conditions to be adjusted or varied without shutdown. Long runs also minimize demands on instrumentation response. It is also important in that changes in the internal dimensions and contours of the rig can lead to unknown conditions. Electrical integrity is obviously important in that any leakage can distort fields, alter probe readings, and change plasma characteristics. Because of the extreme conditions under which a rig of practical configuration must operate, the job of obtaining and maintaining this integrity assumes major proportions. Some of the conditions are listed below:

- 1) Plasma velocity, 5,500 to 6,700 ft/sec
- 2) Plasma temperature, 4,500 to 5,700°R

- 3) Heat flux to MHD channel walls, 50 to 150 Btu/ft²sec
- 4) Heat flux to nozzles, 100 to 400 Btu/ft²sec
- 5) Electric fields of up to 1,500 volts/ft
- 6) Chemically reactive, oxidizing plasma
- 7) MHD channel liner surface temperature up to 4000°R
(to avoid condensation of conductive deposits, to avoid excessive heat loss and to minimize thermal stress by reducing heat flux)

Because of the extreme thermal effects and the nature of the materials that must be used to line the MHD channel, allowance must be made for thermally-induced expansion and warpage to avoid inducing destructive stresses. This results in several major effects. The internal dimensions of the channel will change as the rig warms up, and, since the flow is supersonic, the decrease thus induced in the channel cross-sectional area, A_x , may change the Mach number. The nozzle throat area, A^* , may also increase* and the combined effect will be to reduce channel Mach number from that calculated based on the ratio, A_x/A^* , determined from measurements made with the rig cold. The change in Mach number can induce changes in temperature, density and velocity, the first two changes affecting plasma conductivity, the latter, power output.

Because of the necessity of allowing for thermal expansion, small cracks and irregularities may induce weak, oblique shock waves, evidence of which has been observed experimentally, which can also alter the Mach number.

The buildup of tolerances in liner parts can affect Mach number in another way. The alignment of the nozzle exit with the MHD channel entrance may not be perfect. To minimize the undesirable effects of exposing leading-edge corners of the liner at the nozzle exit to stagnation heating, the MHD channel inlet liner pieces must be slightly chamfered. This chamfer, when misalignment occurs, can also induce oblique shock waves as can irregularities in the liner surface, due to erosion and thermal stresses. All of the above effects tend to reduce MHD channel Mach number.

*A decrease in nozzle throat area has been observed in some cases due, apparently, to inward thermal bowing of the throat walls. The walls take on a permanent set

The net reduction in Mach number can be deduced by measuring the stream velocity and temperature, or by indirect methods such as measuring stream total and static pressure and using suitable relations to back-calculate a value, or by measuring the angle of an oblique shock wave induced by a ramp of known angle, and using a computed value of γ to determine the resulting Mach number. All of these methods have been investigated analytically and some experimentally. However, extensive additional work is necessary before the results can be used with certainty.

Methods of determining stream velocity investigated include a device which detects the time needed for tracer particles to travel a known length of MHD channel, voltage probes which detect velocity from the relation

$$u = \frac{\Delta V}{B \Delta y} \quad (7-1)$$

and observation of the open circuit voltage, V_{oc} during a power generation run, i.e.

$$u = \frac{V_{oc}}{BH} \quad (7-2)$$

The latter two methods depend for their success on excellent electrical integrity within the rig and negligible current leakage within the boundary layer due to a deficiency in induced voltage. The probe method is questionable because of the uncertainty of the effects of eddy currents within the stream.

If the stream velocity can be determined, average plasma density can be determined simply from the continuity equation.

$$\rho = \frac{\dot{W}}{u A_x} \quad (7-3)$$

By measuring static pressure and using a plot of p/ρ vs T obtained from the thermochemical calculation previously mentioned for the overall composition derived from the propellant flow measurements, static temperature can be determined. Assumptions necessary are:

$$\left. \frac{p}{\rho} \frac{\partial R}{\partial p} \right|_{T=C} < < \left. \frac{T}{\rho} \frac{\partial R}{\partial T} \right|_{p=C}$$

and that the flow is in thermochemical equilibrium. Both assumptions appear reasonable. Figure 237 is a plot of T vs p/ρ for a typical composition and for pressures of 10 and 20 psia. As can be seen, the data for 10 and 20 psia nearly coincide, illustrating that the first assumption is true. The fact that the line curves, however, illustrates that the gas constant, R , does vary with temperature over the ranges experienced in these MHD generators. The assumption of equilibrium flow is justified by published rocket data which indicates flow equilibrium to expansion ratios of at least 2 and possibly for three.

The temperature thus obtained, along with the pressure and average density already known, the average composition can then be used to calculate conductivity from the theoretical relations.

Rig electrical integrity is necessary to obtain good conductivity data. Equations (5-11) and (5-12) illustrate the effect that axial current leakage can have during power runs. Either the value of Λ must be minimized or else it must be determined. If any appreciable axial leakage occurs, it is better to short out the Hall potential entirely in which case $\Lambda \rightarrow \alpha'$. The rigs used, however, exhibited good internal electrical integrity. Transverse current leakage can introduce errors since current density can be determined only from the measured current, I_L , from the electrode, that is

$$J_y = \frac{I_y}{A_y} \quad (7-4)$$

If leakage occurs around the plasma, then the current density will be in error since

$$I_y = I_L + I_\Lambda \quad (7-5)$$

This leakage can be determined, during an applied-voltage conductivity run ($B=0$), by applying voltage during seed-off conditions (assuming zero conductivity of unseeded plasma) and measuring the leakage current, if any. From this data, the value of the transverse leakage path resistance may be computed. This assumes that the presence of seed in the plasma does not change the leakage path resistance, R_Λ , and that the leakage path resistance does not change during the switchover to a power run ($B \neq 0$). Corrections to observed conductivity can then be made using the following relation:

$$\sigma'_\Lambda = \frac{H}{A_y} \left[\frac{1}{|dV_T/dI_L|} - \frac{1}{R_\Lambda} \right] \quad (7-6)$$

This relation is derived from Equation (5-10) which can be written

$$V_T = (uBH - \delta V) - I_L R_{g\Lambda} \left(1 + \frac{R_L}{R_\Lambda}\right) \quad (7-7)$$

by using the parallel circuit relation

$$I_\Lambda R_\Lambda = I_L R_L \quad (7-8)$$

and equation (7-5).

Assuming that:

- 1) R_Λ is a constant,
- 2) σ'_Λ is a constant ($R_{g\Lambda}$ is a constant),
- 3) The leakage current sees the boundary layer voltage drop as does the load, R_L , and
- 4) The boundary layer voltage drop, δV , is a constant in the range of interest, we get

$$\frac{dV_T}{dI_L} = - \left[\frac{R_{g\Lambda}}{1 + R_{g\Lambda}/R_\Lambda} \right] \quad (7-9)$$

Recalling that dV_T/dI_L is negative for a power run, and solving for $R_{g\Lambda}$, we get

$$R_{g\Lambda} = \left[\frac{|dV_T/dI_L|}{1 - |dV_T/dI_L|} \right] R_\Lambda \quad (7-10)$$

Then, using the definition

$$R_{g\Lambda} = \frac{H}{\sigma'_\Lambda A_g} \quad (7-11)$$

and Equation (7-10) we get, after rearranging

$$\sigma'_\Lambda = \frac{H}{A_g} \left[\frac{1}{|dV_T/dI_L|} - \frac{1}{R_\Lambda} \right] \quad (7-12)$$

Using the same procedure and assumptions, we may obtain a similar equation to use to correct for transverse leakage current during applied voltage ($B=0$) conductivity tests. Again, the leakage path resistance (R_Λ) must first be determined with seed off. The equation is

$$\sigma_o = \frac{H}{A_g} \left[\frac{1}{dV_T/dI} - \frac{1}{R_\Lambda} \right] \quad (7-13)$$

where I , in this case, is the current measured to the electrode pair of interest.

Resistance in series with the plasma and between the voltmeter taps* can introduce errors to conductivities determined by Methods 1 and 3. At present there is no way of determining this accurately during a run since all evidence points to large changes with temperature. Thus the only course of action is to minimize the causes of this source of error. When this does occur, the conductivities determined from the dV_T/dI_y data are lower than actually exist in the plasma.

B. Combustor Efficiency

The combustor efficiency, through its effect on plasma temperature, density and composition, can have a very powerful effect on plasma conductivity. If plasma temperature, density and composition cannot be measured at the location where the experimental measurement of conductivity is made (in the MHD channel) then an indirect method of determining these parameters, in order to calculate a theoretical conductivity with which to compare the measured value, must be used.

A method is available which uses an equilibrium thermochemical calculation to calculate plasma properties at any point of an isentropic expansion, based on conditions at the beginning of this expansion. In general, corrections must be made to this procedure to allow for various loss mechanisms since the properties predicted, based on measured combustion chamber pressure (P_c) at the injector face, measured flow rates, $\dot{w}_t$, and fuel, oxidizer and seed solution composition, will be enough in error to seriously alter the predicted conductivity. The loss mechanisms include:

- 1) Momentum pressure loss within the combustion chamber due to heat release within the plasma and to frictional effects. The result is that $p_T^* \neq p_c$ where p_T^* = total pressure at the nozzle throat
- 2) Heat loss from the plasma to the combustion chamber and nozzle inlet walls by radiation, convection and conduction
- 3) Combustion inefficiencies such as:
 - a) incomplete combustion of the reactants

*Contact resistance between electrodes and power taps, oxide coating on the electrode surface, and electrode internal resistance are examples

- 1) fuel unreacted and unvaporized (droplets)
- 2) fuel unreacted, vaporized but uncracked
- 3) fuel unreacted, vaporized and cracked
- 4) oxidizer unreacted
- b) Nonuniform distribution of the reactants with time and location
- c) Various combinations of the above effects

Efforts to analytically evaluate these effects produced usable results, some of which are presented below. However, an evaluation of the combined effect of those inefficiencies and of ways to experimentally determine to what degree each mechanism was present was beyond the scope of the contract and was not completed. Thus, the methods used are all approximations and give, at best, an envelope into which the experimental data should fall. More precise comparisons await better methods of detecting and correcting for inefficiencies.

The momentum pressure loss was corrected using an empirical relation giving

$$\frac{P_T^*}{P_C} \text{ vs } \frac{A_{\text{CHAMBER}}}{A^*}$$

which agrees quite closely for values of δ in the range of interest, with theoretical data presented in Rocket Propulsion Elements, by Sutton. The total pressure loss, assuming that the pressure measured at the injector face, P_C , corresponded to the total pressure at that point, was approximately 3 per cent, depending on nozzle area ratio. The calculated nozzle total pressure (P_T^*) was then used for further computations including both measured and ideal-adiabatic C^* .

η_{C^*} is a common measure of rocket combustor performance

$$\eta_{C^*} = \frac{C^* \text{ measured}}{C^* \text{ ideal}} \quad (7-14)$$

for

$$C^* \text{ measured} = \frac{P_T^* A^* C_D g}{\dot{W}_T} \quad (7-15)$$

and

$$C^*_{\text{ideal}} = \frac{p_r^* g}{(\rho^* u^*)_{\text{ideal}}} \quad (7-16)$$

where $(\rho^* u^*)_{\text{ideal}}$ is obtained from the equilibrium thermochemical calculation for isentropic expansion using the value of p_r^* calculated from measured chamber pressure along with the measured flow rate $\dot{w}_t$ and propellant compositions. That is

$$\eta_{C^*} = \frac{\rho^* u^*}{\dot{w}_t / A^* C^*_D} \bigg|_{p_r^*} \quad (7-17)$$

Two values of η_{C^*} can be defined:

$\eta_{C^* \text{ ideal-adiabatic}}$

and

$\eta_{C^* \text{ ideal-corrected}}$

The latter uses a corrected value of $(\rho^* u^*)_{\text{ideal}}$ based on the measured heat removed from the combustion chamber in Btu/lb of plasma. That is

$$\dot{q}_{\text{MEAS}} = \frac{\dot{Q}/A}{\dot{w}_t} \quad (7-18)$$

and is obtained by reducing the initial enthalpy of the combustion products at constant p_r^* increasing the density, ρ^* and decreasing u^* such that the product decreases for the conditions of interest.

Since for equilibrium conditions the value of plasma conductivity is extremely sensitive to temperature,

$$\sigma \sim T^{10} \quad (7-19)$$

the first definition of η_{C^*} is used which includes not only the effect of combustion inefficiencies but also the effect of combustion chamber inefficiency (that is, for high MHD generator performance, heat loss from the plasma to the chamber walls must be minimized). Thus,

$$\eta_{C^* \text{ ideal-adiabatic}} = \frac{C^*_{\text{measured}}}{C^*_{\text{ideal-adiabatic}}} \quad (7-20)$$

A correction that may be used analytically reduces the initial enthalpy of the plasma in the combustion chamber, to reduce predicted C^* to match C^*_{measured} . The resulting plasma is then (analytically)

expanded isentropically* through the nozzle of specified area ratio and the composition, temperature and pressure obtained used to compute conductivity.

Another correction that may be used follows the above procedure but reduces the initial enthalpy only by the measured heat out. In general, C^*_{measured} is not matched.

An attempt to simultaneously correct for two of the previously discussed inefficiencies involves a correction identical to that just described followed by a reduction in the amount of fuel flowing which increases O/F so that the combined effect is to approximate C^*_{measured} , see Figure 63. This iterative method encounters difficulties when the initial O/F is high since the amount of fuel removed to match C^* (by reducing $\dot{w}_f/A^*$) may exceed the amount initially present.

Admittedly, all of the above methods are approximations whose accuracy very likely decreases rapidly as $\eta C^*_{\text{ideal-adiabatic}}$ decreases.

At least three approaches may be used, in future work, to correct the situation:

- 1) Develop a method of correcting for the combined effect of these and other inefficiencies and a method of determining the degree to which each is present regardless of the value of ηC^*
- 2) Improve combustor performance to the point where errors due to approximate methods of correction are reduced to second order or less, that is, achieve high $\eta C^*_{\text{ideal-adiabatic}}$
- 3) Develop methods of measuring the necessary parameters (temperature, pressure, composition) directly at the point where the experimental determination of plasma conductivity is made

*See discussion on flow equilibrium

It should be noted that nonisentropic conditions within the nozzle are not considered. These, too, should be examined in any future work.

C. Nozzle Efficiency

Nozzle performance must be evaluated in order to correct, if necessary, predicted MHD channel conductivities based on theoretical relations using combustion chamber conditions. The corrections described in the section on combustion efficiency based on a computed η_{C*} only compensate for losses up to the nozzle throat. Poor nozzle performance may result in variations in plasma velocity and Mach number and in nonhomogeneous plasma composition, temperature and density at the nozzle exit. Factors contributing to these effects besides those due to influences upstream of the nozzle may include boundary layer growth different than predicted, burning in the plasma, recombination, variable γ , and thermal distortion of the nozzle.

The nozzles used were evaluated in cold-flow tests, a discussion of which is presented below. They were found to have good performance at the cold-flow condition but their behavior under actual MHD test conditions is indeterminate with the present state of instrumentation and calibration technology.

In the open-cycle MHD power generation tests made to date, it has been observed that the open-circuit voltage is considerably lower than predicted. Since open-circuit voltage is a linear function of gas velocity for a constant applied magnetic field, it follows that the observed open-circuit voltage loss is a direct result of a gas velocity that is lower than theoretical. Gas velocities less than theoretical are not unexpected and losses of a few per cent could be tolerated. However, the open-circuit voltage measurements indicate reductions of 20 to 30 per cent in gas velocity which, since power output is proportional to the square of velocity, gives losses in power output of 35 to 50 per cent.

One possible reason for low gas velocities is inefficient nozzle performance. An imperfect nozzle design or failure to hold tolerances when building the nozzle could produce poor performance characteristics. The performance of a particular nozzle can be evaluated by cold flowing the nozzle with air and measuring thrust and mass flow rate for various nozzle pressure ratios.

Three converging-diverging nozzles, representative of those used in MHD testing, were cold flowed with air at the UAC cold-flow test facility. The results of these tests are presented in graphical form in Figures 238 through 240. Each figure contains a plot of C_v (measured thrust/ideal thrust) vs nozzle pressure ratio (chamber pressure/exhaust pressure) and a plot of C_d (measured mass flow/ideal mass flow), also vs nozzle pressure ratio. An explanation of these data is presented at the end of this appendix. The pertinent results derived from the cold flow tests are presented in the following table.

Nozzle area ratio A/A^*	1.5	2.0	3.75
Velocity ratio ($V_{\text{actual}}/V_{\text{ideal}}$)	0.97	0.98	0.97
Average discharge coeff. ($M_{\text{actual}}/M_{\text{ideal}}$)	0.99	0.98	0.98

Inspection of the above table shows that the gas velocity at the nozzle exit is only 2 to 3 per cent lower than theoretical. The average discharge coefficients for the three nozzles tested indicate mass flow rates within 1 to 2 per cent of theoretical. It is thus apparent that the nozzles tested show good performance characteristics when cold flowed with air.

A factor that cannot be taken into account in cold flow tests is nozzle expansion and distortion due to the high temperature of the combustion gases used for MHD testing. Thermal expansion of the nozzle could cause changes in nozzle area ratio and distortion of the nozzle walls. Either or both of these occurrences could result in poor nozzle performance as well as a nonuniform velocity distribution at the nozzle exit.

The combustion products used in MHD tests are obtained by burning a hydrocarbon fuel in a combustion chamber and expanding the evolved gases through a nozzle into the MHD test section. The combustion so achieved is not complete, which presents the problem of evaluating the nozzle exit velocity, for a set of test conditions which are unknown. The thermochemical calculation procedure mentioned in Appendix 5 can be used to determine the nozzle exit velocity, assuming ideal combustion. At present there is no suitable method available for solution of the nonideal combustion case.

One measure of combustion efficiency is the ratio of measured characteristic velocity to the ideal characteristic velocity. The measured characteristic velocity is calculated for each MHD test from the following relation.

$$C^*(\text{Measured}) = \frac{(\text{chamber pressure}) (\text{nozzle throat area}) C_D}{(\text{mass flow rate})}$$

For MHD tests the ideal C^* is calculated for ideal combustion at the measured chamber pressure and measured oxygen-to-fuel ratio.

C^* efficiencies were calculated for a number of MHD power tests where open-circuit voltage was measured. The gas velocity in the test section was determined from the open-circuit voltage and ideal velocity was calculated with the thermochemical procedure. A plot of the ratio of measured velocity to ideal velocity vs C^* efficiency is presented in Figure 241. The points plotted are for chamber pressures of approximately 60 psia and oxygen-to-fuel ratios of about 2.6. All test points are for the area ratio 2.0 nozzle.

Inspection of Figure 241 shows that the plot is essentially a straight line. If a line is drawn through the data and extrapolated to a C^* efficiency of one, the velocity ratio intercept is about .86. This would indicate that the actual velocity in the MHD section is 14 per cent less than ideal. It is evident that the losses indicated by the cold flow tests cannot account for the loss noted in the MHD tests. The losses noted could be the result of oblique shocks at the entrance to the test section. The measured dimensions of a typical nozzle and the chase-around test rig are presented in the following table:

<u>Location of Measurement</u>	<u>Height</u>	<u>Width</u>
Nozzle exit	3.39 inch	1.69 inch
Rig entrance (beginning of chamfer)	3.46	1.80
Rig exit	3.24	1.58

The difference between the entrance and exit dimensions of the test rig is due to chamfered walls at the rig entrance. The walls were chamfered to prevent a sharp corner facing upstream if the nozzle were larger than the test rig or misaligned. In any event it appears

that the gas stream experiences a sudden expansion followed by a short length of converging channel, ending in a constant-area duct. Assuming perfect alignment between the rig and nozzle, the minimum change in any wall relative to the flow would be 0.055 inch between the nozzle exit and rig exit. The change in area going from the nozzle exit to rig exit is approximately 12 per cent. It is expected that such a configuration could produce oblique shocks and may in fact account for a portion of the velocity loss noted. Additional effects not accounted for but possibly present and accounting for velocity loss are combustion in the expansion portion of the nozzle, recombination, both contributing to variable γ (the nozzles were designed for constant γ flow) and unburned but reacting fuel running along the bottom of the nozzle which would induce asymmetric flow.

The cold flow test data presented in Figures 238 through 240 were obtained by flowing air from a variable pressure source through the test nozzles to the atmosphere. Thrust and mass flow rate were measured for a number of chamber pressures. Since total conditions and nozzle throat area are known, the ideal mass flow rate can be calculated for each set of chamber conditions. The discharge coefficient (C_d), which is defined as the ratio of measured mass flow rate to ideal mass flow rate, can thus be determined over a range of total pressures. A plot of C_d vs the ratio of total pressure to exhaust pressure (atmospheric in this case) can thus be constructed for each of these test nozzles.

The ratio of measured thrust to ideal thrust can be determined by evaluating ideal thrust for each chamber condition tested. The ideal thrust is defined as that thrust obtained in an ideal isentropic expansion from the measured total pressure to ambient. Physically this means that for a fixed pressure ratio, the nozzle area ratio is also fixed such that the static pressure at the nozzle exit is equal to atmospheric. The thrust for a given set of total conditions is a maximum when the nozzle area ratio is such that the static pressure at the exit is equal to the pressure to which the nozzle exhausts*.

The measured thrust for a nozzle of fixed area ratio discharging to a constant pressure increases as total pressure is increased but can never be greater than the ideal thrust for the same total pressure. It is also evident that at some total pressure, the static pressure at the

*Shapiro, Ascher H., The Dynamics and Thermodynamics of Compressible Fluid Flow, Vol. 1. The Ronald Press Co., New York, 1953

nozzle exit will be equal to the exhaust pressure. For this condition the measured thrust depends only on the weight flow and exit velocity of the nozzle. The ideal thrust for this condition is also a function of only weight flow and gas velocity. The ratio of measured to ideal thrust is thus a ratio of weight flows and velocities. Since weight flow can be rewritten in terms of gas density, nozzle exit area, and velocity, the ratio of measured to ideal thrust can be reduced to the ratio $V^2_{\text{actual}}/V^2_{\text{ideal}}$.

The discharge velocity of a supersonic nozzle becomes essentially constant when the nozzle pressure ratio becomes sufficiently high to prevent separation of the flow before the nozzle exit, i.e., no normal shocks occur in the nozzle. This means that the ratio of measured to ideal thrust, evaluated at the pressure ratio which gives static pressure equal to exhaust pressure, is a valid measure of nozzle exit velocity over a range of pressure ratios.

To obtain the value of V^2_a/V^2_i , the plots of C_v vs pressure ratio presented in Figures 238 through 240 must be extrapolated to the design pressure ratio of the nozzle being tested. The curves were extrapolated and the ratio of actual to ideal velocity calculated for each of the three nozzles tested.

It should be noted that, although the cold-flow tests invariably showed nozzle exhaust velocities of the order of 97 per cent of ideal, velocities determined from measurements of open-circuit voltage during power generation tests were down by as much as 30 per cent. The resultant changes in MHD channel Mach number, density and temperatures could be appreciable, but extensive instrumentation development effort would be required to evaluate the effect. Since the method of determining these parameters, described under rig mechanical and electrical integrity could not be used because the plasma velocity-measuring system requires extensive development, the values of conductivity theoretically predicted from combustion chamber conditions and ideal nozzle area ratios are subject to uncertainty.

D. Measurement Errors

Measurement errors can invalidate test data if not properly compensated for. Besides those already discussed, such as blooming and resistance in series with the plasma but between voltmeter taps, two sources of

error can be due to boundary layer effects. The first is due to the fact that the average velocity between electrodes for generation of EMF during power runs may be altered due to the velocity profile in the boundary layer. Another effect also tending to yield a decreased velocity, u , is that the boundary layer displacement thickness effectively decreases A_x/A^* , decreasing Mach number and velocity. Either effect would tend to give low values of open-circuit voltage because of the decrease in uBH due to lower u . Boundary layer thickness can be calculated and allowances made, if data on boundary layer growth at these conditions becomes available. Calculations using simplified procedures indicate that, for this size test rig, the effect is small.

The other and possibly most important uncertainty due to its potentially very powerful effect on the measurement of plasma conductivities is the assumption of the behavior of the boundary layer voltage drop δV . Various suppositions have been offered as to the current density or field gradient at which the boundary layer begins to act like a constant voltage drop. Presumably, the boundary layer at first acts like a constant resistance in series with the core flow plasma, then undergoes a transition during which its resistance is some function of current density and/or voltage gradient and then finally reaches a state where it acts like a constant voltage drop. That is

$$\delta V = I R_{bL} = \text{CONSTANT} \quad (7-20)$$

$$R_{bL} = \frac{\delta V}{I} = \frac{\text{CONSTANT}}{I} \quad (7-21)$$

Use of Methods 2 or 5 (Appendix 5) for determination of plasma conductivity which do not depend on the assumption of constant δV should provide dependable data not only on core flow conductivity but also on the variation of boundary layer voltage drop with current density. If dependable or consistent voltage probe data is not available, the only other alternative is to minimize the source of error. In this case reduce the δV to zero or as close thereto as possible. A method had been previously conceived to do this based on analysis and testing. It involves the use of ramp-like devices to induce high energy, stream-wise boundary layer vortices on the electrode which violently stir hot, high conductivity plasma down into the normally cooler boundary layer.

A potential source of appreciable electrical losses in MHD generators and appreciable errors in test rigs used to measure plasma conductivity and other MHD phenomena is the voltage drop that occurs at the electrodes due to the boundary layer that flows over them. For rigs of the size now used to investigate MHD generator plasma phenomena, this boundary-layer voltage drop is of the order of two to six times the voltage drop in the plasma core flow. Much of the research work now being performed relies on knowledge of this core-flow voltage drop. Generally, only the overall voltage drop across the channel is measured. Thus, uncertainties in the magnitude and behavior of the electrode voltage drop as a function of current density and other variables can introduce large errors in experimental data obtained where the effect is present. For the case of power generation by MHD, the electrode voltage drop is a direct loss and thus detracts from the efficiency of the generator. It is therefore desirable to minimize or eliminate this effect both for the case of power generation and for the case of MHD plasma research.

Experimental evidence shows that the effect can be essentially eliminated by inducing streamwise boundary-layer vortices to flow over the electrodes; their effect being to violently stir hot, core-flow plasma down into the boundary layer, increasing its temperature and thus its conductivity.

The methods so far used to induce the streamwise boundary layer vortices (and which have been electrically successful) rely on the interaction with the boundary layer of devices which protrude into it. The mechanism by which this interaction takes place and the vortices generated is described in Reference 1. These devices, though highly successful for short runs, are subject to very high heat flux and to high erosion. In addition, the very effect upon which they rely (see Reference 1) to produce the vortices induces very high heat flux and erosion in the wall from which the vortex generators protrude upstream of and to the side of the vortex generators. Although these effects may be reduced by materials development, another effect remains which is undesirable in MHD generators and research rigs now in use or under construction, and in those proposed such as the 1000-megawatt generator described in Reference 2. This is the production of shock waves by the vortex generators.

These MHD generators all are designed to have supersonic plasma flow in the generator channel section in accordance with optimization

studies shown in Reference 2. Thus any object protruding abruptly into the stream will induce shock waves in the plasma that can have undesirable effects both on the core flow and on the stability of the boundary layers on which these shock waves impinge (see Reference 3).

Since it is highly desirable to remove the effects of the electrode boundary layers in MHD generators and present methods either are not suitable for supersonic flows (protruding vortex generators) or require complex, untested and undeveloped schemes for sucking off the boundary layer upstream of the electrodes which would be subject to high heat flux, erosion, corrosion and would require elaborate ducting, it was suggested that either of several methods based on theoretical and experimental work described in References 4, 5, 6, and 7 be used which will generate energetic streamwise boundary-layer vortices without inducing disruptive shock waves in the plasma or causing excessive erosion and heat flux. These methods are described below.

The essence of the method (see Reference 4) is that, under a wide range of conditions, the boundary layer velocity profile common to any viscous fluid, such as a plasma, flowing along a wall which is curved concave to the flow will react to produce streamwise arrays of boundary-layer vortices that will continue downstream some distance from the location at which they were generated. The curvature of this wall need only be two-dimensional. References 5, 6 and 7 point out that the wall need not be solid but may, in fact, be a relatively stagnant portion of the fluid (i. e., the boundary layer may be separated as described in References 3, 5, 6, 7, 8, and 9). The radius of wall curvature necessary to produce these vortices may be quite large according to calculations performed in the preparation of Reference 7. Thus, a curvature gentle enough to produce essentially isentropic compression in the plasma (see pp. 94, 95, and 96 in Reference 10) within the confines of the MHD generator channel may be used to generate the vortices. That is, the compression wavelets would not coalesce into a strong oblique shock (which would disrupt the flow) within the confines of the generator. Since the wall may be a stagnant region of plasma, it is not necessary to use a specially contoured section of the wall to generate the vortices, thus simplifying the construction.

A rearward-facing two-dimensional step of the type shown in Figure 242 and discussed in References 3, 5, 6, 7, and 8 may be used to produce a region of separated flow. The necessary flow curvature occurs at the reattachment point for the rearward-facing step. These step-like contours may be repeated downstream at appropriate intervals.

A forward-facing two-dimensional step may also be used to generate the vortices as discussed in Reference 6, the necessary flow curvature occurring at the separation point, in this case as shown in Figure 243. Or, to avoid the possibility of a shock wave occurring at the forward lip of the step as well as to avoid the high heat flux that may occur there at the reattachment point (Reference 11), a two-dimensional, low, ramp-like device may be used. Figure 244 shows two methods of applying this device. Figure 244a shows the device followed by the electrodes at the same elevation as the tip of the ramp; Figure 244b shows the ramp followed by a simple step. Since the vortices generated by these devices may extend some distance downstream it is not absolutely necessary that the step or ramp be part of the electrode itself although this arrangement may be used. The spacers ahead of the electrodes may act as the vortex generator.

The effectiveness of this method of generating boundary-layer vortices over the electrodes in MHD generators has been demonstrated. The effectiveness of these vortices themselves in eliminating the large electrode voltage drop has also been demonstrated.

REFERENCES

1. The Effect on Transition of Isolated Surface Excrescences in the Boundary Layer, N. Gregory and W.S. Walker, British ARC, FM 1482, Perf. 698, Oct. 1950
2. Magnetohydrodynamic Power Generator Feasibility Study, RADC-TR-61-104, February 1961
3. Investigation of Separated Flows in Supersonic and Subsonic Streams with Emphasis on the Effect of Transition, Dean R. Chapman, Donald M. Kuehn and Howard K. Larson, NACA TN-3869, March 1957
4. On the Three-Dimensional Instability of Laminar Boundary Layers on Concave Walls, H. Gortler, NACA TM-1375, 1954

5. Forces and Pitching Moments on an Aspect-Ratio 3.1 Wing-Body Combination at Mach Numbers from 2.5 to 3.5 and Sublimation Studies of the Effect of Single Element Roughness on Boundary Layer Flow, Edward J. Hopkins, Stephen J. Keating, Jr., and Richard B. Muhl, NACA RM-A-58-E-21a, August 1958
6. Experimental Evidence of Three-Dimensional Perturbations in the Reattachment of a Two-Dimensional Laminar Boundary Layer at Mach = 2.05, Jean J. Ginoux, Training Center for Experimental Aerodynamics (Belgium) Tech. Note #1, Nov. 1958
7. Photographic Evidence of Streamwise Arrays of Vortices in Boundary Layer Flow, Edward J. Hopkins, Stephen J. Keating, Jr., and Angelo Bandettini, NASA TN-D-328, Sept. 1960
8. Transition Reynolds Numbers of Separated Flows at Supersonic Speeds, Howard K. Larson and Stephen J. Keating, Jr., NASA TN-D-349, Dec. 1960
9. Separation of Flow, Paul K. Chang, Journal of the Franklin Institute, Vol. 272, #6, Dec. 1961
10. Elements of Gasdynamics, Liepman and Roshko, John Wiley & Sons, 1957.
11. Heat Transfer in Separated Flows, H. K. Larson, Institute of Aeronautical Sciences, Report 59-37

E. Instrumentation Accuracy

The need for instrumentation accuracy in an investigation of this type is too well known to require elaboration. In order to minimize errors due to this cause, transducers and other instruments were periodically calibrated and the calibrations used in data reduction. The test stand itself was completely checked with an MHD rig simulator which imposed the voltage gradients both transverse and axial (Hall) which the rig would impose during a run, in order to test the integrity of the system in resisting common-mode voltage effects. Grounds and shielding were given careful attention to forestall errors due both to stray electric fields and to the very strong magnetic field present during power generation runs.

The magnet itself was calibrated with and without the rig in place and plots of field strength versus x, y, and z coordinates within the rig were obtained.

The calibrations and precautions previously mentioned presumably minimized systematic errors. Part of the function of the analytical group was to carefully inspect the data to search out and eliminate random errors due to mistakes in reading. Most of the data was recorded by one of three methods, film, oscillograph paper, or strip-chart sheets and was read by aides.

An analysis of instrumentation accuracy is given in Appendix 6.

F. Thermochemical Equilibrium of Flow

In systems involving high enthalpy, dissociated combustion products expanding to high Mach numbers and velocities, the rate of chemical reaction may not be fast enough to keep pace with the gases as they expand. As a result, the composition of the gas at any point in the flow device may not be that which is predicted by shifting equilibrium analyses. In particular, for the supersonic MHD plasmas being studied at Pratt & Whitney Aircraft there are many reactions occurring, each with its own reaction rate. Thus some of the reactions may be in equilibrium at the local static temperature and pressure while other reactions may freeze at some point early in the expansion.

Of principal interest for MHD devices are the reactions involving electrons. The thermochemical calculations available at Pratt & Whitney Aircraft have been used to calculate conductivity and plasma velocity, based on the assumptions below:

- 1) The reaction rates are infinite, that is, the composition of the plasma at any point in the expansion is given by shifting equilibrium predictions.
- 2) The composition is in shifting equilibrium in the subsonic flow region, but the composition freezes at the nozzle throat and remains constant at this value in the supersonic flow region.

- 3) All reactions are frozen except $K \rightleftharpoons K^+ + e^-$. Because there is relatively little energy connected with this reaction, the temperature and pressure at the nozzle exit are assumed to be equal to the values achieved with a frozen expansion (Assumption 2).

To calculate the conductivity for the third assumption, the Saha relationship is used to find the mole fraction of electrons (X_e) at the exit of the nozzle, as follows

$$\left(\frac{X_e}{X_{e*}}\right)^2 = \left(\frac{T}{T*}\right)^{\frac{3}{2}} e^{-\frac{\phi}{K}\left(\frac{1}{T} - \frac{1}{T*}\right)} \quad (7-22)$$

where X_{e*} is the mole fraction of electrons at the throat, T is temperature, ϕ ionization potential and K Boltzmann's constant. The number density of electrons at exit is then

$$n_e = X_e \frac{P}{KT} \quad (7-23)$$

The conductivity for Assumption 3 is assumed equal to the conductivity for Assumption 2 multiplied by the ratio of number densities.

$$\sigma_3 = \sigma_2 \frac{n_{e3}}{n_{e2}} \quad (7-24)$$

Because pressure and temperature are the same for Assumption 3 as for Assumption 2, the collision frequencies (i.e., τ_{ij} 's) will be the same.

Assumption 3 was used for only one point ($A/A* = 2.0$, $M = 2.07$). The results of this calculation are compared with the results for Assumptions 1 and 2 below:

<u>Assumption</u>	<u>σ' (mhos/m)</u>
1 (equilibrium)	23.5
2 (frozen)	24.6
3 (partial equilibrium)	0.77

Two plots illustrating the results of these calculations for Assumptions 1 and 2 are shown in Figures 245 and 246. Figure 245 demonstrates that, for the area ratios of interest in the current test program, there is no appreciable velocity difference for the two assumptions. Figure

246 illustrates that although the temperature is much less for the frozen composition case (Assumption 2) the value of conductivity is higher at Mach numbers above 1. The reason for the higher conductivity for case 2 is that the mole fraction of electrons was frozen at the throat. The mole fraction of electrons is thus higher for the frozen case than for the shifting equilibrium case, and since conductivity is proportional to the number density of electrons, the conductivity is higher also. The third assumption results in a much lower conductivity since the reaction is in equilibrium at a much lower temperature and the number density of electrons is therefore much lower.

Because of the number of reactions involved in hydrocarbon combustion and because of lack of sufficient data for reaction rates and the experimental difficulty in obtaining this data, it is not possible at this time to determine the degree of equilibrium of the reactions of interest.

There is considerable interest in the analysis of flow with finite reaction rates because of the large differences in thrust levels predicted for frozen and shifting equilibrium flows at high Mach numbers. Experimental results have shown that for jet fuel and oxygen, the thrusts measured experimentally do fall between frozen and shifting predictions*. Interest in the phenomena as it affects MHD performance has also been expressed and it has been shown that significant increases in electron conductivity may be realized from the fact that the reaction rates are finite**. It is hoped that data and methods will be available soon that will allow an assessment of the effect of finite reaction rates.

*Recombination and Condensation Processes in High Area-Ratio Nozzles by W. T. Olsen, ARS Space Flight Report to the Nation

**Ionization Nonequilibrium in Expanding Flows, Alan Q. Eschenroeder, ARS Journal, February, 1962

APPENDIX 8

Special Support Work

Appendix 8

Special Support Work

Special analytical and experimental work undertaken in support of the contract in an effort to provide a better understanding of the data, to increase the precision of the results and/or to provide new methods of obtaining the data not subject to the restrictions of other methods is discussed. Information is presented on voltage probes, the AC conductivity probe, and the velocity probe.

A. Voltage Probes1. Introduction

Several methods are available to measure plasma properties in an MHD generator. One of the more useful methods utilizes voltage probes to determine the magnitude of the electric field. At P&WA, voltage probes have been used in an attempt to measure the voltage gradient in a plasma and use it to determine the plasma's conductivity. The attempts to obtain useful and consistent data from voltage probes were frustrated by mechanical and electrical difficulties and it was not until the last portion of the test program that some useful results were obtained.

The physical theory of voltage probes in a low density plasma at moderate temperatures is fairly well understood. However, the use of probes in a MHD duct where a high density plasma is moving at supersonic velocities and is at extremely high temperatures is another matter entirely. It was hoped that a series of experiments could have been performed to determine some of the characteristics of voltage probes in an MHD generator and compare these characteristics to the properties of probes in low density and low temperature plasmas.

2. Theory

Theory shows that the probe curve of a thermal plasma will exhibit the characteristics shown by Cobine*.

*Gaseous Conductors, J. D. Cobine, Dover Publications, New York, 1958

When using the probes to measure the voltage gradient in the plasma, the probe circuit should be built in such a manner that the probe is always operating in the region A-B of the curve above. In this region the voltage can be more precisely determined because the current drawn by the probe can change significantly with little change in the voltage.

3. Experimental Work

During the series of conductivity tests that Pratt & Whitney conducted, one experiment was performed in an effort to obtain the voltage-current characteristics of a voltage probe during an applied-voltage conductivity test. The purpose of this experiment was to determine the probe characteristics and compare the results to the curves presented in a United Aircraft Research Laboratories report*.

4. Apparatus and Procedure

The rig used was the basic chase-around rig equipped with the array of voltage probes shown in Figure 247. Each individual probe was connected to the measuring galvanometers of an oscillograph through separate filter circuits. However, the test probe had a load box connected in parallel with the probe circuit. The schematic diagram of the probe and the probe circuit is shown in Figure 248.

*Plasma Probe Diagnostics, F. Salz and E. C. Lary, UAR-0820, Jan. 1961

The probe load box consisted of a set of resistors that could be switched into the circuit. The values of the resistances ranged from 4.2K to open circuit in nine steps (4.4K, 9.1K, 24.7K, 51.1K, 95K, 220K, 500K, 1 Meg, 2 Meg). The filter circuit resistance was measured and found to be 3.02 megohms.

The run procedure was as follows. During the applied-voltage conductivity run of 12 December, 1962 the applied voltage was held constant at about 215 volts while the resistances in the probe load box were switched into the circuit. The response was observed as a galvanometer deflection on the oscillograph calibrated in volts. The probe current was then calculated from the simple Ohm's law relation.

5. Results

The oscillograph readings for applied voltage, control probe voltage, and test probe voltage are shown in Figure 249 plotted against numbers which are roughly proportional to time. Numbers 1 through 5 are readings taken with the resistance of the probe load box at 4.4K, numbers 6 through 9 are with resistance of 9.1K, 10 through 13 are with resistance of 24.7K, and 14 through 16 are with a resistance of 51.1K. At resistances higher than 51.1K the test probe read the same as the control probe.

With the probe load box at open circuit, the probe sees a resistance of over 3 megohms, hence the probe current is in microamps. This current is almost negligible compared to the milliamp current drawn when the lower resistances are in the circuit. The probe voltage can therefore be considered as the floating potential, V_f (the potential that the probe assumes when it is drawing no net current). Figure 250 is a plot of probe current versus voltage above the floating potential. The point at the origin is for the probe load resistance infinite, that is, the 3 Meg filter circuit. The next points are for 51.1K, 24.7K, 9.1K and 4.4K, respectively.

The shape of the curve in Figure 250 does indeed appear to have some of the characteristics of the theoretical curves of the above sketch. However, the knee at the plasma potential, V_p , is not discernible. It is felt that the region of interest is below 6 milliamps. In this region the galvanometer deflections are rather small, thus increasing the error in the readings.

Figure 251 is a plot of the current versus the voltage above the floating potential. Here again the plot is similar to the theoretical curves but due to insufficient data points no conclusions should be drawn.

Because there are not sufficient data points in the region of interest, no conclusions can be drawn except to say that further experiments should be carried out to define more clearly the area near the origin including points below the floating potential.

It is also felt that the high resistance of the filter circuits adequately keeps the probe current low enough so that the probe is at a potential very near to the floating potential. If the sheath drop (the potential difference between the floating potential and the actual plasma potential) is small or at least uniform for all the probes, the data from voltage probes should yield realistic values of the electric field.

6. Practical Difficulties

Early in the test program it was found that the silicon carbide probes were subject to a great deal of high frequency hash. After a set of galvanometers in the CEC oscillograph were destroyed by this high frequency hash a set of high frequency filters was designed. The basic filter circuit for one probe is shown in Figure 252. These filters were reasonably effective and soon after they were installed there was some limited success in obtaining data.

The tests were still plagued with shorts and poor electrical continuity. Most of the trouble was a result of the difficulty of maintaining contact with the probes during the run. The combination of high temperature and severe vibration rule out brazing or cementing with Astroceram.

Recent tests indicate there is a good possibility of solving the contact problem. The proposed method is shown in the sketch below.

The welding of the lead to the probe eliminates the melting problems that occur when braze is used and the use of the tungsten-rhenium lead should reduce the brittleness of the weld joint usually found when tungsten is welded.

Figure 253 is a sketch showing a completed assembly of voltage probes in a sidewall block. Figure 153 is a photograph illustrating the probe erosion that takes place during the run.

7. Recommendations for Further Study

The voltage probe technique for studying the properties of a plasma in an MHD generator certainly has promise, but the characteristics of the probe and the probe theory should be explored more fully. As a first step it is felt that the experiment described in this report should be repeated with particular emphasis in the region near the origin. More sensitive galvanometers could be used and some accuracy could be gained by utilizing full deflections. A means of switching the probe reference from the anode to the cathode would allow observation of points below the floating potential.

The probe characteristics curve could also be displayed on an oscilloscope. In an Aeronautical Research Laboratories report* a technique is described wherein a voltage pulse is applied to the probe and the current response displayed on an oscilloscope. Further investigation should include exploring the effects of varying the surface area of the probe exposed to the stream. An MHD section could be equipped with various size probes side by side and an experiment similar to the one mentioned above could be performed.

*Studies of the Dynamic Properties of Langmuir Probes, H. V. Osbam, Aeronautical Research Laboratories, USAF, Aug. 1962

The construction of the probe and the material used should also be examined. Cooled nickel probes should be considered. The study of probe materials should also investigate the phenomena of thermionic emission and of secondary emission of electrons.

During all of the conductivity tests that used voltage probes, the probes were referenced to the high-current anode. Because of the high current drawn through the boundary layer, small changes in the boundary layer resistance could result in large changes in the voltage reading. This may account for much of the high frequency hash noticed in the probe readings. The referencing of a probe to another sidewall probe could eliminate the hash problem. This should be investigated.

When the theory of probes in high density, high temperature plasmas is understood more fully and when their application is perfected, the data obtained from them will yield much valuable information concerning plasma potential, plasma temperature, particle velocity distributions, and ion and electron number.

B. AC Conductivity Probe

The AC probe mentioned in Appendix 5 is designed to be immersed in the plasma and to permit a direct measurement of scalar plasma conductivity. This differs from other AC techniques mentioned in the literature in that the probe is immersed in the plasma and produces an oscillating magnetic field external to the probe. The oscillating magnetic field induces azimuthal currents in the plasma which dissipate power in an amount dependent on the plasma conductivity and the affected plasma volume. Electrodes are not used and thus sheath effects are not present. The probe consists of a cooled protective tubular sheath containing a cooled coil. By keeping the probe temperature boundary layer thin, the effects of boundary-layer conductivity may be eliminated and the probe measures the true scalar conductivity of the free-stream plasma. The probe may be operated relatively close to a wall without adverse effects. In tests with cold salt solutions, a one-half inch diameter probe with a coil current frequency of 13.5 megacycles was brought to within 0.8 inch of a conducting wall without affecting the conductivity reading.

The probe circuit is made part of a grid-dip meter circuit as shown in Figure 254. The triode vacuum tube supplies rf power to sustain

the radio frequency oscillation of a tank circuit. The measuring coil in the probe represents all or part of the inductance of this circuit depending on the particular circuit used. The azimuthal currents induced by the probe coil in the plasma surrounding it cause a resistive loading to the tuned circuit which changes the grid current of the triode. The grid current is very nearly linear with the conductivity of the plasma*. A microammeter indicates the grid current flowing. The grid current is calibrated by using solutions of known conductivities.

The attenuation distance of an electromagnetic wave in a plasma of finite resistivity when the impressed frequency is much less than the plasma frequency** is given by Lyman Spitzer in Physics of Fully Ionized Gases, Interscience Publishers, Inc., 1956, to be

$$d = \sqrt{\frac{1}{2\pi\omega\sigma}}$$

where ω is the frequency impressed and σ is the conductivity of the plasma. In addition to the restrictions imposed by this equation, the distance over which the influence of the coil is felt is further reduced by the fact the magnet field produced by the coil is dipolar. A dipolar field decreases as r^{-3} where r is the radial distance from the coil axis. The power dissipation in the plasma caused by the coil is proportional to $\sigma\omega^2B^2r^2$ and therefore attenuates as r^{-4} . From this discussion it is apparent that the probe will sample only a small volume of the plasma.

* The dependence of grid current on conductivity has been demonstrated in salt solutions with conductivities ranging from 0 to 35 mhos/m. Calibration curves for a low temperature probe have been obtained and are presented in UAC Report M-1282-1

** The plasma frequency is defined by Spitzer to be

$$\omega_p = \left(\frac{4\pi n_e e^2}{m_e} \right)^{\frac{1}{2}}$$

where n_e is the electron density, e is the electron charge and m_e the electron mass. For conditions commonly experienced in combustion product plasma, the plasma frequency is of the order of 1.5×10^{12} cycles per second

restriction is placed on the probe material by the skin effect equation which gives the attenuation distance for an electromagnetic field as

$$d = \sqrt{\frac{2}{\omega \mu_0 \sigma_m}}$$

where ω is the frequency of the field, μ_0 is the permeability of free space, and σ_m is the conductivity of the probe material. Evidently the conductivity of the material used to protect the coil must be almost zero at the conditions under which the probe is used. If this condition is not met, the field will never penetrate beyond the walls of the tube. Therefore the material chosen must have high resistivity at operating conditions.

To adapt the probe for use in the hot (5000 to 6000°R) plasma of an MHD generator, cooling must be provided and the probe must be supported in the plasma.

An analysis of the heat transfer and strength characteristics of the probe has been made including consideration of cooling requirements and stresses in the probe caused by coolant pressure and thermal gradients. The analysis leads to a maximum allowable pressure for each probe size and area ratio. This area ratio is defined as the area of the channel section in which the probe is placed, divided by the nozzle throat area, A^* . We may use the total channel area because the critical point of the probe for heat transfer considerations is the leading edge, and at this point the flow area is equal to the channel area. The results of a one-dimensional analysis are shown in Figure 255.

The probe must be used in subsonic flow because compression shocks will form in supersonic flow upstream of the probe, introducing conductivity errors.

The requirement of subsonic flow dictates that the probe be placed in the downstream end of a combustion chamber of an MHD rig where the area ratio A_c/A^* is high and the combustion is complete.

Stress and thermodynamic considerations specify a probe material with high thermal conductivity and high strength at high temperature. The electrical theory of the probe demands that the electrical resistivity of the material remain high at the operating temperatures it will encounter. Two configurations of the probe have been proposed (Figure 256).

One version of the probe discussed above has been designed and built. Because of lack of time and personnel, the device was not tested. A picture of the probe is shown in Figure 257. The configuration chosen was the stationary probe shown in Figure 256b. The reasons for choosing the stationary probe include the fact that the cantilevered probe presents more problems insofar as stress considerations are concerned.

It was decided to fix the diameter of the probe at $3/4$ inch as a compromise between the small coil size desired for local sampling and the larger size dictated by heat transfer and stress considerations. The probe was designed so that three different thicknesses of probe wall could be used (0.035, 0.070 and 0.100 inch). The thinnest wall thickness is the optimum thickness based on heat transfer considerations. The heavier walls are more desirable from the point of view of handling and being able to withstand the buffeting of the moving plasma stream. The material chosen for the protective tubular sheath was BeO. This material combines the qualities of high electrical resistivity at high temperatures, good resistance to erosion at the conditions encountered, and good characteristics for heat transfer and stress considerations.

The probe will be used in a specially designed section that can be placed between the combustion chamber and nozzle of a test rig. The conditions will then be subsonic. Hydrogen was chosen as a coolant because of its excellent heat transfer characteristics. With the specifications noted in the previous paragraphs, a heat transfer analysis was conducted. The analysis was made for various chamber pressures and probe hot wall temperatures. The results of this analysis are expressed in terms of predicted stress divided by allowable stress for various wall thicknesses, chamber pressures, and hot wall temperatures. These results are shown graphically in Figure 258.

From Figure 258 it is obvious that the probe hot wall temperature should be kept between 1000 and 2000°R, that chamber pressure should be kept as low as possible, and that the thinner tube is better from the standpoint of thermal stress considerations.

Coolant pressures and flow rates have been specified as a function of operating conditions.

The electrical design of the probe also presents problems. The probe is most valuable when it samples a very small volume of the plasma. This implies that the length of the coil should be as small as possible. The interaction of the coil with the plasma is a function of the inductance of the coil. The inductance of a coil is given by the formula

$$L = \frac{d^2 t^2}{18d + 40b}$$

where d is the coil diameter, t is the number of turns and b is the coil length. It is obvious that for maximum inductance the coil should be as short as possible, with a large number of turns on a large diameter. Because room must be left for cooling, the probe diameter was arbitrarily fixed at 3/8 inch. The variables then are the number of turns and the length of the coil. The length of the coil was arbitrarily fixed at 3/8 inch in order to keep the volume affected by the probe as small as possible.

By using different wire sizes, different numbers of turns can be made in the given length. Because the operating frequencies are on the order of 10-20 megacycles, the resistance of the coil will be affected by the skin effect. For maximum probe sensitivity, the ratio of probe resistance to probe inductance should be as low as possible. For high inductance, with the restrictions placed on coil size, a small diameter wire should be used. However, a small diameter wire leads to high values of R/L . Because the optimum wire size is a function of many variables, coils were constructed with 3 different wire diameters. The optimum wire size would then be determined by experiment.

Also, to insure that the change in coil resistance with temperature change does not affect the probe reading, a thermocouple should be placed near the coil and monitored.

The resulting probe design is shown in Figures 257 and 259. The center body around which the coil is wrapped is supported outside of the stream. The protective ceramic tube is not held rigidly. It is held in place by two steel supports which transmit a compressive force provided by the bellows. The bellows allow for thermal expansion and vibration without destroying the ceramic tube.

The probe should be run in a fixed location for the first few runs. However, the design can easily be adapted so that the probe coil can be traversed across the stream. The probe can then be used to determine conductivity profiles.

C. Plasma Velocity Measurements

1. Summary

A technique for direct measurement of the velocity of a plasma in an MHD generator duct is described. One test was run to determine the feasibility of the technique, but the results were inconclusive due to the predominance of a relatively low frequency signal which was not anticipated.

2. Introduction

The need for an independent measurement of velocity has been discussed previously in Appendix 5. It was also stated that some preliminary development work in that direction was done at Pratt & Whitney Aircraft. This work will be described in this section.

3. Theory

The theory of operation of the method of velocity measurement to be described here is based on the following premises:

- 1) The observed fact that inhomogeneities exist in high velocity plasmas which cause significant changes in the local conductivity,
- 2) The observation that these inhomogeneities can cause distinctive current waveforms when a voltage is impressed across the plasma by means of small probes, and
- 3) The assumption that some of these current waveforms can be identified at probes spaced far enough apart so that the time required for the inhomogeneity to pass from one pair of probes to the next can be accurately measured.

If all the above conditions are satisfied, the velocity, u , can be determined from the equation

$$u = \frac{D}{t}$$

where D is the distance between the pairs of probes, and t is the time required for a disturbance to go from Probe 1 to Probe 2 (see Figure 260).

4. Experimental Work

A rig was built, as shown in Figure 260, with probes approximately 5.5 inches apart. The channel was 1.65 inches square in cross-section, and was attached to a convergent-divergent nozzle with an area ratio of 2.0. This was set up at X-196-B test stand, and one run was made to investigate feasibility of the method. The nozzle used was designed to give a stream velocity of about 6000 ft/sec. This results in a time of approximately 80 microseconds for a disturbance carried by the stream to pass from one pair of probes to the second pair.

5. Instrumentation

A circuit was set up (Figure 261) using a DuMont Model 266 dual beam oscilloscope to observe the voltage at the probes, which is the inverse of the current waveform.

Instrumentation was also provided for measurement of reactant flows and chamber pressure. A tape recorder was connected across the scope inputs to record the signal.

6. Test Procedure

The rig was lit on JP4 and O_2 . Seed flow was turned on as soon as JP4 and oxygen flows were stable. Voltage was applied and the AC component of voltage on the probes was monitored on the oscilloscope.

7. Results and Discussion

The observed waveform showed that approximately 80 per cent of the peak to peak AC voltage was at about 800 cps. The remaining 20 per cent of the voltage appeared to be wideband noise, which was the type of signal expected. The high fraction of low frequency signal present made it impossible to examine the high frequency in detail during this run.

The attempt to record the signal was unsuccessful.

8. Conclusions

The high frequency waveforms observed appeared to be of the type expected. The feasibility of the method was not proved, because of inability to examine the high frequency component of the signals from both probes in detail.

9. Recommendations

Follow-up work on this method should be directed toward:

- 1) Using sharp cutoff high-pass filters and measuring time shift of waveforms directly as outlined, or
- 2) Using sharp cutoff low-pass filters and an accurate measurement of fundamental frequency and phase shift between the probes. This will not give a direct measurement of velocity unless the low frequency is due to a variation in one of the reactant flows.