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SENSITIVITY OF THE GENERAL LINEAR MODEL

by

Dale W. Stager

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WASHINGTON UNIVERSITY
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ABSTRACT

SENSITIVITY OF THE GENERAL LINEAR MODEL

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The sensitivity of the general linear model to violations of its normality assumptions was evaluated using computer simulation. Input data was developed from source models that have random errors generated from the normal, uniform, triangular and logistic distributions. In addition, the random errors were altered by rounding; heteroscedasticity and serial correlation. The general linear model was found to be sensitive to some of the cases studied and, in particular, very sensitive to serially correlated heteroscedastic random errors from the normal distribution.

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SENSITIVITY OF THE GENERAL LINEAR MODEL

1. INTRODUCTION

One of the aims of science is to describe and predict events and values occurring in the real world. To accomplish this, the scientist has relied heavily on mathematical models and the most popular method of developing these models has been linear regression. All mathematical models, of course, use mathematical terms to represent the real quantities and formulas to describe the relationships between those quantities. In general, a model could be expanded until it describes the real events and quantities to any degree of accuracy desired. This has practical limits, of course, as the increase in accuracy is invariably obtained by greatly increasing the complexity of the model. In practice, some simplification in the form of assumptions is always present; in this paper a commonly practiced simplification is described and its impact on the general linear regression model is investigated.

1.1 LINEAR REGRESSION ANALYSIS

Regression analysis starts with the formulation of a mathematical model that will describe the relationship

between the real quantities after certain unknown coefficients are determined. In the model, the known quantities are represented by independent variables, the unknown relationships by coefficients of the independent variables and the outcome of the relationship is represented by the dependent variable. In linear regression, the model must be linear in the coefficients and in the random error term that is added to complete the model (1)*, (2). The random errors are a stratagem used to account for the unknown influences that prevent exact formulation. The reasons for the random errors are classified as follows (2):

- (a) the relationship determined is only approximate,
- (b) the dependent variable cannot be accurately measured,
- (c) for repeated observations the independent variables act as random variables.

The general linear model is used if the random errors are required because of case (a) or (b) or both. Case (c) does not fit the general linear model and will not be covered here, although the following analysis may be extended to include it.

* The numbers in parentheses in the text indicate references in the Bibliography.

1.2 OBJECTIVE AND SCOPE

If the random error term was not needed the linear regression problem would reduce to a system of simultaneous equations. It is not surprising then, that the characteristics of the random error's probability distribution and the correlation between repeated observations determine the approach to be used to estimate the unknown coefficients. The general linear model assumes that the random errors are uncorrelated observations from the normal distribution. Other assumptions lead to different approaches (3), (4) and (5). In any case, the random errors cannot be observed directly, and the experimenter is, in general, forced to expend additional effort to indicate a course of action or make some possibly unsupported assumptions about the random errors. The most convenient assumption is to assume that the characteristics fit the general linear model, i.e., the errors are normal and uncorrelated. These assumptions permit the experimenter to make easily calculated probability statements about his regression model. In addition, there is an abundance of supporting aids, in the form of computer regression programs, available which almost exclusively assume normal and uncorrelated random errors. In summary, it has almost become a matter of unquestioned practice to assume normal and uncorrelated errors either directly or by default by using a computer regression program.

This study will investigate the consequences of assuming the random errors are uncorrelated observations from the same normal distribution when in actuality the random errors do not have those specific characteristics. The study, while staying within the scope of the general linear model, uses computer simulation to generate the input data to a regression program.

2. SUMMARY OF THE ASSOCIATED THEORY

2.1 GENERAL LINEAR MODEL

In linear regression, the relationships between the dependent variable, the independent variables and the random error can be written

$$y_j = \sum_{i=1}^p b_i x_{ji} + e_j \quad j = 1, 2, \dots, n,$$

or in vector form

$$y = Xb + e,$$

where

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad X = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1p} \\ x_{21} & x_{22} & \dots & x_{2p} \\ \vdots & \vdots & \dots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{np} \end{bmatrix}$$
$$b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_p \end{bmatrix}, \quad e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}.$$

The quantities given in the model have the following interpretation: y is a random $n \times 1$ observed vector, e is a

$n \times 1$ random error vector, X is an $n \times p$ matrix with rank p of known fixed quantities and b is a $p \times 1$ vector of unknown coefficients of the independent variables.

In the experimental procedure first a set of x_{1i} 's is selected and then an observation of the random variable y_i is recorded. This is continued until n , with n greater than p , observations are obtained. On the basis of the known matrix X and the observed random variable vector y , estimates for the coefficient vector b and the variance σ^2 of y can be developed.

2.1.1 Estimation of b and σ^2 under Normal Theory

It is assumed here that the elements of the vector of errors, e , are uncorrelated observations from the same normal distribution with mean zero and unknown variance σ^2 . Thus, e is distributed $n(0, \sigma^2 I)$ where 0 is the n -vector of zeroes, I is the identity matrix, and $\sigma^2 I$ is called the covariance matrix.

The maximum likelihood method will be used to develop the estimators of b and σ^2 . The likelihood function is

$$\begin{aligned} g(e; b, \sigma^2) &= (2\pi\sigma^2)^{-\frac{n}{2}} \exp[-e'e/2\sigma^2] \\ &= (2\pi\sigma^2)^{-\frac{n}{2}} \exp\left[-\frac{(y-Xb)'(y-Xb)}{2\sigma^2}\right] \end{aligned}$$

where the single quote mark ($'$) denotes the transpose.

The maximum likelihood estimators are found by setting equal to zero the derivatives with respect to b and σ^2 of the natural logarithm of $g(e;b,\sigma^2)$:

$$\begin{aligned}\frac{\partial}{\partial b} [\ln g(e;b,\sigma^2)] &= - \frac{\partial}{\partial b} \left[\frac{(y-Xb)'(y-Xb)}{2\sigma^2} \right] \\ &= - \frac{\partial}{\partial b} \left[\frac{y'y - y'Xb - b'X'y + b'X'Xb}{2\sigma^2} \right] \\ &= - \frac{X'y - X'Xb}{\sigma^2} = 0\end{aligned}$$

since $b'X'y$ and $y'Xb$ are scalars and therefore equal. Then

$$X'X\tilde{b} = X'y \quad (2.1)$$

The vector equation (2.1) is called the normal equations. The tilde is being used to identify the maximum likelihood estimator. Also, since

$$\frac{\partial}{\partial \sigma^2} [\ln g(e;b,\sigma^2)] = - \frac{n}{2\sigma^2} + \frac{(y-Xb)'(y-Xb)}{2\sigma^4} = 0,$$

it follows that

$$\tilde{\sigma}^2 = \frac{(y-X\tilde{b})'(y-X\tilde{b})}{n}$$

2.1.2 Estimation of b when the Error Distribution is Unspecified

The method of least squares is commonly used for the estimation of b if the error distribution is unspecified. The random error vector e is assumed to have mean zero and covariance matrix $\sigma^2 I$. In other words, the elements of e, e_j , will have mean zero and variance σ^2 and be uncorrelated random variables.

The least squares estimate of b minimizes the sum of the squares

$$\left(\sum_{j=1}^n y_j - \sum_{i=1}^p b_i x_{ji} \right)^2 = (y - Xb)'(y - Xb) \quad .$$

In vector notation the estimate satisfies

$$2X'y - 2X'Xb = 0 \quad .$$

Then,

$$X'Xb = X'y$$

yields the least squares estimate of b, which, therefore, is the same as the maximum likelihood estimate developed in paragraph 2.1.1.

2.1.3 Unbiased estimates of b and σ^2

Unbiased estimates are generally desirable and commonly used in regression analysis. In equation (2.1) X is of rank

p so that $S = X'X$ has an inverse and

$$\hat{b} = S^{-1}X'y$$

$\hat{b}$ is unbiased as the expectation of $\hat{b}$ is

$$\begin{aligned} E(\hat{b}) &= E(S^{-1}X'y) = S^{-1}X'E(y) \\ &= S^{-1}X'E(Xb+e) = S^{-1}X'Xb = b \end{aligned}$$

Then, we will now denote this estimate by $\hat{b}$.

That $\hat{\sigma}^2$ is biased is seen as follows.

$$\begin{aligned} E(\hat{\sigma}^2) &= E\left[\frac{(y-X\hat{b})'(y-X\hat{b})}{n}\right] \\ &= \frac{1}{n} E[y'(I-XS^{-1}X')'(I-XS^{-1}X')y] \end{aligned}$$

Now,

$$\begin{aligned} [I-X(S^{-1})'X'] [I-XS^{-1}X'] &= I-XS^{-1}X'-X(S^{-1})'X'+X(S^{-1})'X'XS^{-1}X' \\ &= I-XS^{-1}X' \end{aligned}$$

since $(S^{-1})' = S^{-1}$.

Then,

$$E(\hat{\sigma}^2) = \frac{1}{n} E[y'(I-XS^{-1}X')y]$$

Substituting $y = Xb+e$, multiplying and collecting terms

$$E(\hat{\sigma}^2) = \frac{1}{n} E[e'(I-XS^{-1}X')e]$$

The terms within the brackets on the right hand side are a quadratic form. The expectation of a quadratic form, $\omega A \omega$, where ω is distributed with mean zero and covariance matrix $\sigma^2 I$ is equal to $\sigma^2 \text{trace } [A]$ (2). Then,

$$\begin{aligned} E(\hat{\sigma}^2) &= \frac{1}{n} \sigma^2 \text{trace} [I - X S^{-1} X'] \\ &= \frac{1}{n} \sigma^2 [\text{trace}(I) - \text{trace}(X S^{-1} X')] \\ &= \sigma^2 \frac{(n-p)}{n} \end{aligned}$$

An unbiased estimate of σ^2 is therefore

$$\hat{\sigma}^2 = \frac{n \hat{\sigma}^2}{n-p} = \frac{(y - X \hat{b})' (y - X \hat{b})}{n-p}$$

The square root of the above estimate is called the standard error of y . Since the general linear model satisfies the conditions of the Gauss-Markoff theorem, it follows that $\hat{b}$ is the best linear unbiased estimate of b .

2.2 DEFINITION OF TEST STATISTICS

The general linear model develops coefficients based on the input data and the source model. These coefficients can then be used to test the hypothesis that $b_i = b^*_i$, b^*_i being the known value of the i^{th} coefficient, and the simultaneous hypothesis $b = b^*$, b^* being the vector of known coefficient values.

The first hypothesis, since the random errors are assumed to be $n(0, \sigma^2 I)$, can be tested using the t-distribution. The acceptance region of the test at the α level consists of the values satisfying $\left| \frac{\hat{b}_i - b^*_i}{(c_{ii} \hat{\sigma}^2)^{1/2}} \right| \leq t_{\frac{\alpha}{2}}$ where c_{ii} is the i^{th} diagonal element of $(X'X)^{-1}$ (2).

The normality assumption also permits the use of the F-distribution in testing the second hypothesis that $b = b^*$. The quantity $u = \frac{n-p}{p} \frac{(y - X\hat{b})' X S^{-1} X' (y - X\hat{b})}{(y - X\hat{b})' (y - X\hat{b})}$ will have an F-distribution with p and $(n-p)$ degrees of freedom (2). Then, the simultaneous hypothesis is accepted at the α level when $u \leq F_{\alpha; p, n-p}$ (2).

3. DESCRIPTION OF THE ANALYSIS PROGRAM

The analysis program uses computer simulation to develop input data for a regression analysis algorithm and then performs various statistical analyses on the algorithm's output. As stated in the introduction the input data is assumed to meet the requirements of the general linear model. The input data generated violates by design the assumption that the random errors are normally distributed with mean zero and covariance matrix $\sigma^2 I$. The output from the program then will reflect the general linear model's sensitivity to the violations simulated. Basically, the program generates values from a source model giving the exact relationship between the variables and then adds a random error term. The random errors are first generated as independent and identically distributed observations from specific probability distributions and are then, in most cases, altered in a controlled manner to give them realistic properties. This procedure is repeated at different variance levels for the specific probability distributions. In all, 324 combinations of source model, probability distribution, variance values and error alterations were evaluated.

3.1 SOURCE MODELS EXAMINED

$$\text{I. } y_j = 5 + 2x_{j2} + 3x_{j3} + e_j,$$

$$\text{II. } y_j = 5 + 2x_{j2} + 3x_{j3} + 9x_{j2}x_{j3} + e_j,$$

$$\text{III. } y_j = 5 + 2x_{j2} + 3x_{j3} + 9x_{j2}^2 + 2.5x_{j3}^2 + e_j,$$

$$\text{IV. } y_j = 5 + 2x_{j2} + 3x_{j3} + 9x_{j2}^2 + 2.5x_{j3}^2 + 3x_{j2}x_{j3} + e_j.$$

For all models $j = 1, 2, \dots, 30$. The coefficients were developed with the aid of a table of random numbers. x_{j1} was selected to be equal to one to simplify the notation from the theoretical discussion. x_{j2} and x_{j3} are numbers selected randomly from the intervals $[1, 11]$ and $[1, 21]$ respectively. Each y_j is developed from newly generated x_{j2} and x_{j3} .

3.2 PROBABILITY DISTRIBUTIONS

Random errors are generated from the uniform, triangular, logistic and normal probability distributions. Parameters for each distribution were adjusted so that each had mean zero and the same variance, which for this study is either 1, 4 or 25. The above distributions were selected since with the same mean and variance the curves of the associated probability density functions, starting with the uniform, approximate the shape of the normal.

3.3 RANDOM ERROR GENERATION

Random errors were generated in four ways that violated in at least one manner the assumption required by the general linear model, i.e., that the random errors are normally distributed, with covariance matrix $\sigma^2 I$. The four generation schemes are described in the following paragraphs.

3.3.1 Case 1

Random errors are generated that violate the assumption of normality. This is accomplished by providing random errors from the uniform, triangular and logistic distributions with the same mean and variance as the normal.

3.3.2 Case 2

This case simulates imprecise measurement due to rounding for each of the distributions in Case 1 plus the normal distribution. The decimal position to which a random error is rounded is, of course, a matter of scale and in this study rounding is to the nearest integer. This choice is arbitrary and it is anticipated that this will be a more severe distortion for the random errors when the variance is one as opposed to a variance of twenty-five.

3.3.3 Case 3

The third case results in random errors that are uncorrelated but do not have a constant variance. This

is accomplished by generating observations from each of the above distributions with a standard deviation that is a linear function of the x_{ji} and equal to $\sigma + (x_{j2}^2 + x_{j3}^2)^{\frac{1}{2}}$ for $\sigma = 1, 2$ or 5 and for each σ a $\Delta = .2, .4$ or $.8$.

3.3.4 Case 4

In many situations, for example in forecasting, the random errors are serially correlated. The fourth case provides serially correlated errors using an exponential smoothing formula (6) where an observation is generated from each of the above distributions with a standard deviation equal to $\frac{\sigma(1+\Delta)}{(1-\Delta)}$ for $\sigma = 1, 2$ or 5 and for each σ a $\Delta = .2$ or $.4$. If r_t is the t^{th} such observation generated, then the first error is

$$e_1 = (1-\Delta)r_1,$$

and successive errors are

$$e_t = (1-\Delta)r_t + \Delta e_{t-1}, \quad t = 2, 3, \dots, n.$$

Table 1 summarizes the combinations of cases, variance values, and deltas (Δ) used for each source model and probability distribution.

A total of 324 combinations are developed in the following manner:

Case 1:

$$3 \text{ (dist.)} \times 4 \text{ (source models)} \times 3 \text{ (var.)} = 36$$

Case 2:

$$4 \text{ (dist.)} \times 4 \text{ (source models)} \times 3 \text{ (var.)} = 48$$

Case 3:

$$4 \text{ (dist.)} \times 4 \text{ (source models)} \times 3 \text{ (var.)} \\ \times 3 \text{ (deltas)} = 144$$

Case 4:

$$4 \text{ (dist)} \times 4 \text{ (source models)} \times 3 \text{ (var.)} \\ \times 2 \text{ (deltas)} = 96 \\ \hline 324$$

Note that using the normal distribution with Case 1 does not violate the general linear model assumptions. Table 1 summarizes the above information.

3.4 CONSTRUCTION OF AN ACCEPTANCE REGION

If the random errors were indeed $n(0, \sigma^2 I)$ the hypothesis $b = b^*$, where b^* is the vector of the known coefficient values, would always be true, but the F-test would falsely reject it with some probability α . The number of times the hypothesis is accepted in numerous trials could then be viewed as a binomial random variable with $p = 1 - \alpha$. Acceptance of the hypothesis that $p = 1 - \alpha$ indicates that the general linear model is insensitive to the violations present in a combination in the sense that the Type I error of the test on the parameters does not differ significantly from α , the Type I error if the assumptions were all satisfied. The acceptance region can then be represented by the interval $n(p \pm h_F)$, where F is the number of trials, and h_F is the amount that the relative frequency

TABLE 1. Summary of Cases for Random Error Alterations,
Variance Values and Delta (Δ)

CASES	σ^2	DELTA (Δ) FOR EACH σ^2 IF APPLICABLE
1. Non-normal	1 4 25	
2. Rounded to the nearest integer	1 4 25	
3. Linear function	1 4 25	.2 .4 .8 .2 .4 .8 .2 .4 .8
4. Serial correlation	1 4 25	.2 .4 .2 .4 .2 .4

may differ from the true value of p . Since the number of acceptances is a binomial random variable the relationship between $\hat{n}$, p and h_F for a γ level test on p is found from (7) to be

$$n = \frac{k^2(\gamma)pg}{h_F^2},$$

where

$$\int_{-k(\gamma)}^{k(\gamma)} \phi(y) dy = \alpha$$

and $\phi(y)$ is the standardized normal density function. The F-acceptance region will be $n(p \pm h_F)$.

The t-test tests the hypothesis $b_i = b_i^*$ for each of three to six coefficients depending on the source model. The t-acceptance numbers used are the average number accepted for each combination. The new statistic has a variance equal to $\frac{npq}{c}$ where c is the number of coefficients. An analysis similar to the above for the new statistic shows that the acceptance region for the t-acceptance numbers is $n(p \pm h_t)$ where $h_t = h_F/\sqrt{c}$. There is, then, a different acceptance region for the t-acceptance numbers for each source model. Acceptance regions for the F and t-acceptance numbers with $n = 200$, $\gamma = .05$, $p = .95$ ($\alpha = .05$) are listed in Table 2.

TABLE 2. Acceptance Regions for $n = 200$ with $p = .95$

F-Acceptance Regions

	$\underline{h_F}$	<u>Acceptance Region</u>
$\gamma = .05$	.030	184 - 196
$\gamma = .10$	.025	185 - 195

t-Acceptance Regions

	<u>Source Model</u>	$\underline{h_t}$	<u>Acceptance Region</u>
$\gamma = .05$	I	.017	186.6 - 193.4
	II	.015	187.0 - 193.0
	III	.014	187.2 - 192.8
	IV	.012	187.6 - 192.4
$\gamma = .10$	I	.015	187.0 - 193.0
	II	.013	187.4 - 192.6
	III	.011	187.8 - 192.2
	IV	.010	188.0 - 192.0

3.5 NUMBER OF ITERATIONS

An iteration is the completion of the following steps:

- 1) selection of a source model, probability family, variance and random error generation scheme;
- 2) generation of thirty y_j using the known values of b and the generated x_{j2} and x_{j3} ;
- 3) addition to each y_j of a random error generated in accordance with the probability distribution, variance and generation scheme selected;
- 4) the calculation of $\hat{b}$ and $\hat{\sigma}^2$ using the generated y_j 's as input data to the regression algorithm;
- 5) tabulation of the statistical acceptability of each individual hypothesis that b_i equals its known value and of the joint hypothesis that all b_i 's equal their known values.

Other authors in simulation have used 50 iterations (8); 100 iterations (9); and 400 iterations (10). In the latter, the authors stated that their statistics were fairly stable when the number of iterations were between 100 and 200. The decision to use 200 iterations for all experiments in this study was a compromise between computer time and accuracy as step 5 performs the binomial test discussed in the preceeding section. It was felt the compromise was reasonable in comparison with (7), (8), and (9).

3.6 VALIDATION OF THE ANALYSIS PROGRAM

One check on the Analysis Program is to compare the generated average standard error with the expected value of $\hat{\sigma}$. The expected value of $\hat{\sigma}$ is easily calculated for the normal distribution with case 1 random errors. In addition, it can be shown by the law of large numbers that $\hat{\sigma}$ is asymptotically unbiased for σ , 200 iterations more than qualifies, so that the other three distributions with case 1 random errors will have approximately the same expected value of $\hat{\sigma}$. The expected values of $\hat{\sigma}$ based on the normal distribution for σ^2 equal to 1, 4, 25 are .991, 1.982 and 4.957 respectively. All of the distributions provided an average standard error quite close to the expected value. The ranges for the average standard error for $\sigma^2 = 1, 4, 25$ were .98 to 1.00, 1.97 to 2.00 and 4.92 to 5.00, respectively.

Two other similar checks can be made by comparing the average F-statistic and the average t-statistic used in the F and t-test respectively with their expected values. The average F-statistic was quite close to the expected value, 1.07, in that the total range for all distributions was 1.00 to 1.13. The average t-statistic compared well to the expected value of 0 as the total range was -.13 to 13.

4. OBSERVATIONS AND CONCLUSIONS

4.1 THE RESULTS OF THE SENSITIVITY ANALYSIS

This study, as stated earlier, is investigating the consequences of incorrectly assuming that the random errors are distributed $n(0, \sigma^2 I)$. The null hypothesis for each combination is that the probability of the coefficients being accepted even when the assumptions are not satisfied still equals .95. If a certain measured value for a combination falls outside the acceptance region (see Table 2) the hypothesis is rejected. The measured values for a combination are both the number of times out of 200 iterations that the vector of coefficients was accepted by the F-test and, as the t-test evaluates individually three to six coefficients each iteration depending on the source model, the average number of times a coefficient was accepted by the t-test.

Five combinations had F-acceptance numbers outside the F-acceptance region with $\gamma = .05$. The random error characteristics of the five follow:

- 2 from the normal distribution;
- 3 from the uniform distribution;
- 2 with case 2 alterations;
- 3 with case 4 alterations.

Fifty-two combinations were outside the t-acceptance regions with $\gamma = .05$. The random error characteristics follow:

- 30 from the normal distribution;
- 10 from the uniform distribution;
- 7 from the triangular distribution;
- 5 from the logistic distribution;
- 3 had case 1 alterations;
- 10 had case 2 alterations;
- 18 had case 3 alterations;
- 21 had case 4 alterations.

Eleven of the normally distributed random errors had case 3 alterations out of a possible 36 and 12 had case 4 alterations out of a possible 24.

The F-acceptance region with $\gamma = .10$ had twenty-two combinations outside its boundary. Their random error characteristics follow:

- 16 from the normal distribution;
- 5 from the uniform distribution;
- 1 from the triangular distribution;
- 4 with case 2 alterations;
- 6 with case 3 alterations;
- 12 with case 4 alterations.

Nine of the sixteen combinations from the normal distribution had case 4 alterations out of a possible 24.

The t-acceptance regions with $\gamma = .10$ rejected seventy-two combinations. Their random error characteristics follow:

- 33 from the normal distribution;
- 16 from the uniform distribution;
- 15 from the triangular distribution;
- 8 from the logistic distribution;
- 9 had case 1 alterations;
- 15 had case 2 alterations;
- 24 had case 3 alterations;
- 24 had case 4 alterations.

Eleven of the combinations from the normal distribution had case 3 alterations out of a possible 36 and 12 had case 4 alterations out of 24.

4.2 COMPARISONS OF VARIOUS CHARACTERISTICS

The Kruskal-Wallis H-test, the Mann-Whitney U-test, one-way analysis of variance and the t-test for differences in means were used to analyze the effects of distribution, case number, source model and variance parameters on the F and t-acceptance numbers. Because of the relation of the F and t-acceptance numbers to the binomial the normality requirements of the last two tests are satisfied by the normal approximation to the binomial (7). The Kruskal-Wallis H-test and the one-way analysis of variance were used to test simultaneously for significant group

differences. The Mann-Whitney U-test and the t-test for differences in means were used to test for any pairwise differences.

The average rank (Table 3) developed for the H-test can be referred to for ranking the characteristics. The H-test reported at the 5% and 10% levels significant effects on the F and t-acceptance numbers by distributions, case numbers and source models but not by the variance parameters (Table 3).

In order to run the analysis of variance it was necessary to consider each different delta (Δ) for case numbers 3 and 4 as a separate case, i.e., case 4 yields two separate cases: 4 and $\Delta = .2$, 4 and $\Delta = .4$. At the 10% level, distributions, case numbers, and source models but not variance parameters had an effect on the F-acceptance number (Table 4). Distribution and source model both had a significant effect on the t-acceptance numbers at the 10% level. The lack of influence by the variance parameters was not expected. Cases 1 and 2 do generate homoscedastic distributions and the F and t-acceptance numbers should not be affected by a variance change. Cases 3 and 4, however, generate heteroscedastic distributions so that there probably is a variance effect. Due to the unknown characteristic of the generated F and t-statistics associated with cases 3 and 4, it can only be said that on an empirical basis there is no variance parameter effect.

The U-test indicates that all pairings of the distributions (Table 5a) have a significant difference at the 5% level between the F or t-acceptance numbers or both except the following:

F and t-acceptance numbers for the uniform and triangular distributions at the 5% level and the triangular and logistic distributions at the 10% level;

F-acceptance numbers for the uniform and logistic distributions at the 10% level.

The associated t-test for differences in means indicates that all pairings of the distributions (Table 5a) have a significant difference at the 5% level between the F or t-acceptance numbers or both except the following:

F-acceptance numbers for the uniform and triangular distributions at the 10% level and the uniform and logistic distributions at the 5% level.

The U-test indicates that all the pairings of case numbers (Table 5b) have a significant difference at the 5% level between the F or t-acceptance numbers or both except the following:

F and t-acceptance numbers for cases 1 and 2 at the 10% level;

F-acceptance numbers for cases 2 and 4 at the 10% level;

t-acceptance numbers for cases 2 and 3 at the 10% level.

The t-test for differences in means indicates that all the case number pairings have a significant difference between the F or t-acceptance numbers or both except the following:

F-acceptance numbers for cases 1 and 2, and cases 2 and 4 at the 10% level;

t-acceptance numbers for cases 1 and 3, and cases 2 and 3 at the 10% level.

The U-test indicates that all the source model pairings (Table 5c) have a significant difference at the 10% level between the F or t-acceptance numbers on both except the following:

F-acceptance numbers for source model pairs I and II, and II and III at the 10% level;

t-acceptance numbers for source model pairs I and III, I and IV, and III and IV at the 10% level.

The t-test for differences in means indicates that all the source model pairings have a significant difference at the 10% level between the F or t-acceptance numbers or both except the following:

F and t-acceptance numbers for source models I and IV at the 10% level;

F-acceptance numbers for source models II and III at the 10% level and source models I and II at the 5% level;

t-acceptance numbers for source models I and III and III and IV at the 10% level.

The U-test indicates for the variance parameter that there is a significant difference only between the F-acceptance numbers for variance 4 and 25 at the 10% level. The t-test for differences in means did not indicate any significant differences.

Some additional work was done with the combinations having rounded (case 2) random errors. The actual variance was calculated for each distribution and each variance parameter. The results were close to the variance parameters as can be seen below:

Distribution	$\sigma^2=1$	$\sigma^2=4$	$\sigma^2=25$
Normal	1.083	4.083	25.083
Uniform	1.113	3.948	25.055
Triangular	1.084	4.080	25.086
Logistic	1.083	4.074	24.806

The average standard errors for these combinations were then compared using the U-test with those combinations having case 1 random errors (the normal distribution was excluded). The results indicated that there was no significant difference between the two groups. The square root of the

appropriate variance parameter was then divided into the average standard error and the test performed again. This time a significant difference was reported at the 10% level. Hence, there is a difference in the average standard error due to rounding, although the effect of leaving out the normal case 1 combinations is unknown.

4.3 CONCLUSIONS AND RECOMMENDATIONS

The general linear model is sensitive to normally distributed case 4 (serially correlated) random errors. Most combinations with those characteristics had a significantly higher probability of being accepted and could result in a larger probability of a Type II error. In addition, the general linear model in conjunction with the t-test is sensitive to a broader spectrum of violations. It is recommended that if the general linear model is to be used in regression analysis with random errors that violate the distribution requirements, that only the F-test be used to evaluate any source model assumptions and that it not be used if the random errors are normally distributed with case 4 random errors.

TABLE 3 Average Rank for Distribution,
Case Number, Source Model and Variance Parameter

AVERAGE RANK

Distribution

F¹

t

Case Number

F

t

Source Model

F

t

Variance Parameter

F

t

normal	uniform	tri- angular	logistic
205.4	153.2	165.5	130.5
214.2	172.1	141.8	128.8
1	2	3	4
140.5	186.8	126.9	209.0
114.2	150.2	163.1	186.1
I	II	III	IV
177.0	144.1	133.9	194.0
197.2	84.2	194.5	173.6
1	4	25	
219.9	211.7	218.6	
224.1	216.2	209.0	

¹Average rank for the F-acceptance number or the
t-acceptance number.

TABLE 4 Kruskal-Wallis H-test and Analysis of
Variance F-test Statistics for Differences among Distributions,
Case Numbers, Source Models, and Variance Parameters

	H-value ¹	F-value ²
Distribution		
F	25.89	14.63
t	37.77	21.16
Case Number		
F	53.23	11.09
t	16.62	1.37
Source Model		
F	22.36	5.56
t	78.79	41.15
Variance Parameter		
F	0.28	.90
t	0.80	.25

¹An H-value greater than 7.81 or 6.25 indicates significance at the .05 or .10 level respectively except for the Variance Parameter where the critical numbers are 5.99 or 4.60 respectively.

²An F-value greater than the following critical values indicate significance at the .05 or .10 level respectively:

Distribution: 2.60; 2.08
Case number: 2.10; 1.77
Source model: 2.60; 2.08
Variance parameter: 3.00; 2.30.

TABLE 5a U-test and t-test Statistics
for Pairwise Differences in Distributions

DISTRIBUTIONS	acceptance number	U-test ¹	t-value ²
Normal/ Uniform	F	4.216	3.209
	t	3.479	4.904
Normal/ Triangular	F	3.734	2.578
	t	4.712	5.634
Normal/ Logistic	F	5.677	5.673
	t	5.542	6.543
Uniform/ Triangular	F	-1.818	-1.023
	t	1.909	2.095
Uniform/ Logistic	F	.941	1.798
	t	2.951	2.707
Triangular/ Logistic	F	.862	3.436
	t	.608	3.221

¹The entry is the standardized approximately normal deviate for the U-test. If the absolute value is greater than 1.96 or 1.645 a difference is indicated at the .05 or .10 level respectively.

²If the absolute value of this t-statistic is greater than 1.96 or 1.645 a difference is indicated at the .05 or .10 level respectively.

TABLE 5b U-test and t-test Statistics for Pairwise
Differences in Case Numbers

CASE NUMBER	acceptance number	U-test	t-value
1 and 2	F	-1.555	-2.697
	t	-1.442	-1.393
1 and 3	F	2.090	1.212
	t	-2.564	-3.044
1 and 4	F	-3.408	-4.032
	t	-3.719	-3.783
2 and 3	F	5.042	4.459
	t	- .584	-1.405
2 and 4	F	-1.202	-1.601
	t	-1.995	-2.262
3 and 4	F	-7.899	-7.476
	t	-2.288	-1.352

TABLE 5c U-test and t-test Statistics for Pairwise
Differences in Source Model and Variance Parameter

SOURCE MODEL	acceptance number	U-test	t-value
I and II	F	1.401	1.849
	t	7.306	8.854
I and III	F	2.252	3.003
	t	.027	.567
I and IV	F	-2.757	-1.169
	t	1.448	1.611
II and III	F	.844	.923
	t	-7.246	-8.654
II and IV	F	-3.769	-2.685
	t	-6.882	-8.469
III and IV	F	-4.967	-3.726
	t	1.643	1.043
VARIANCE PARAMETER			
1 and 4	F	.812	.496
	t	.047	.484
1 and 25	F	1.237	-.258
	t	.533	1.115
4 and 25	F	-1.773	-.755
	t	.051	.657

5. ACKNOWLEDGEMENT

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6. APPENDICES

APPENDIX 6.1

Data Summary Listing

The output data from the Analysis Program for each combination was summarized and listed. The combinations and the data are listed by delta (Δ) within case number within source model within distribution. The report headings are defined as follows:

DIST	distribution
MODEL/CASE	Source model/case number
S.D. GIVEN	Square root of the variance parameter
DELT I/A	Delta (Δ) if applicable
STND ERROR	Average standard error
NO. ACCEPTED	Number accepted by the t or F-test
RANKINGS	The rank in ascending order for the specified values. The average rank is assigned in case of ties, e.g., if three values are tied for a rank of 3 all receive a rank of 4.

DIST.	MODEL/	S.D.	DELTA	STND	NO.	ACCEPTED	-----RANKINGS-----			
	CASE	GIVEN	I/A	ERROR	T-TEST	F-TEST	S.E.	T-TEST	F-TEST	
NORM	1/2	1		1.04	192.3	192	20.50	267.50	206.00	
	1/2	2		2.01	191.0	190	85.50	175.50	126.50	
	1/2	5		4.97	190.7	191	150.00	144.50	174.00	
	1/3	1	.2	3.74	195.3	194	132.00	323.50	252.50	
	1/3	1	.4	6.52	194.7	193	194.50	317.50	229.50	
	1/3	1	.8	12.08	194.7	192	244.00	317.50	206.00	
	1/3	2	.2	4.71	194.3	196	140.00	308.00	312.00	
	1/3	2	.4	7.47	195.3	194	204.00	323.50	252.50	
	1/3	2	.8	13.03	194.7	193	248.00	317.50	229.50	
	1/3	5	.2	7.65	194.7	195	224.00	317.50	286.00	
	1/3	5	.4	10.39	194.0	196	236.00	303.50	312.00	
	1/3	5	.8	15.91	195.0	196	252.00	322.00	312.00	
	1/4	1	.2	1.22	190.7	190	38.50	144.50	126.50	
	1/4	1	.4	1.52	189.3	191	52.50	73.50	174.00	
	1/4	2	.2	2.44	190.7	190	96.50	144.50	126.50	
	1/4	2	.4	3.04	189.3	191	112.50	73.50	174.00	
	1/4	5	.2	6.09	190.7	190	177.00	144.50	126.50	
	1/4	5	.4	7.60	189.3	191	208.50	73.50	174.00	
	2/2	1		1.04	190.8	194	20.50	158.00	252.50	
	2/2	2		2.01	191.8	192	85.50	228.50	206.00	
	2/2	5		4.98	191.8	195	156.00	228.50	286.00	
	2/3	1	.2	3.73	192.8	194	131.00	281.00	252.50	
	2/3	1	.4	6.50	194.0	194	191.00	303.50	252.50	
	2/3	1	.8	12.05	194.8	193	243.00	320.50	229.50	
	2/3	2	.2	4.70	192.0	195	139.00	246.50	286.00	
	2/3	2	.4	7.46	192.8	194	203.00	281.00	252.50	
	2/3	2	.8	13.00	194.0	194	247.00	303.50	252.50	
	2/3	5	.2	7.64	191.5	196	223.00	207.50	312.00	
	2/3	5	.4	10.37	191.8	195	235.00	228.50	286.00	
	2/3	5	.8	15.88	194.8	194	251.00	320.50	252.50	
	2/4	1	.2	1.22	191.8	195	38.50	228.50	286.00	
	2/4	1	.4	1.52	190.8	194	52.50	158.00	252.50	
	2/4	2	.2	2.44	191.8	195	96.50	228.50	286.00	
	2/4	2	.4	3.05	190.8	194	120.00	158.00	252.50	
	2/4	5	.2	6.11	191.8	195	183.00	228.50	286.00	
	2/4	5	.4	7.62	190.8	194	217.50	158.00	252.50	
	3/2	1		1.04	192.6	194	20.50	272.00	252.50	
	3/2	2		2.01	193.0	194	85.50	287.50	252.50	
	3/2	5		4.98	194.2	198	156.00	306.00	324.00	
	3/3	1	.2	3.72	189.0	189	130.00	66.50	78.50	
	3/3	1	.4	6.48	189.0	188	190.00	66.50	48.00	
	3/3	1	.8	12.02	188.4	188	242.00	49.00	48.00	
	3/3	2	.2	4.68	188.2	187	137.50	44.50	26.50	
	3/3	2	.4	7.44	189.0	189	202.00	66.50	78.50	
	3/3	2	.8	12.97	189.0	188	246.00	66.50	48.00	
	3/3	5	.2	7.62	188.2	189	217.50	44.50	78.50	
	3/3	5	.4	10.34	188.0	188	234.00	39.00	48.00	
	3/3	5	.8	15.84	189.0	187	250.00	66.50	26.50	
	3/4	1	.2	1.22	194.4	196	38.50	312.50	312.00	

DIST.	MODEL	S.D.	DELTA	STND	NO. ACCEPTED		-----RANKINGS-----		
		CASE GIVEN	I/A	ERROR	T-TEST	F-TEST	S.E.	T-TEST	F-TEST
NORM	3/4	1	.4	1.52	194.4	195	52.50	312.50	286.00
	3/4	2	.2	2.44	194.4	196	96.50	312.50	312.00
	3/4	2	.4	3.05	194.4	195	120.00	312.50	286.00
	3/4	5	.2	6.11	194.4	196	183.00	312.50	312.00
	3/4	5	.4	7.62	194.4	195	217.50	312.50	286.00
	4/2	1		1.04	193.2	194	20.50	293.50	252.50
	4/2	2		2.01	192.8	196	85.50	281.00	312.00
	4/2	5		4.99	193.5	197	163.00	298.00	321.50
	4/3	1	.2	3.71	190.5	186	129.00	128.50	13.50
	4/3	1	.4	6.47	190.7	185	189.00	144.50	5.50
	4/3	1	.8	12.00	190.7	185	241.00	144.50	5.50
	4/3	2	.2	4.68	190.3	188	137.50	102.50	48.00
	4/3	2	.4	7.43	190.5	186	201.00	128.50	13.50
	4/3	2	.8	12.95	190.7	185	245.00	144.50	5.50
	4/3	5	.2	7.62	189.8	190	217.50	84.00	126.50
	4/3	5	.4	10.33	190.2	190	233.00	90.50	126.50
	4/3	5	.8	15.82	190.8	188	249.00	158.00	48.00
	4/4	1	.2	1.22	194.0	196	38.50	303.50	312.00
	4/4	1	.4	1.53	193.7	196	60.00	299.50	312.00
	4/4	2	.2	2.45	194.3	196	102.50	308.00	312.00
	4/4	2	.4	3.05	193.8	196	120.00	301.00	312.00
	4/4	5	.2	6.11	194.3	196	183.00	308.00	312.00
	4/4	5	.4	7.63	193.7	196	222.00	299.50	312.00
UNIF	1/1	1		1.00	190.3	191	9.50	102.50	174.00
	1/1	2		2.00	190.3	191	80.00	102.50	174.00
	1/1	5		5.00	190.3	191	167.00	102.50	174.00
	1/2	1		1.05	189.3	192	26.50	73.50	206.00
	1/2	2		1.99	188.3	192	73.00	47.00	206.00
	1/2	5		5.00	191.3	190	167.00	195.50	126.50
	1/3	1	.2	2.57	188.7	193	107.50	59.00	229.50
	1/3	1	.4	4.18	192.0	192	136.00	246.50	206.00
	1/3	1	.8	7.39	192.3	192	200.00	267.50	206.00
	1/3	2	.2	3.56	191.3	194	128.00	195.50	252.50
	1/3	2	.4	5.15	192.0	193	172.00	246.50	229.50
	1/3	2	.8	8.35	192.0	192	232.00	246.50	206.00
	1/3	5	.2	6.53	190.3	191	196.00	102.50	174.00
	1/3	5	.4	8.10	191.3	193	228.00	195.50	229.50
	1/3	5	.8	11.27	191.0	193	240.00	175.50	229.50
	1/4	1	.2	1.22	193.0	194	38.50	287.50	252.50
	1/4	1	.4	1.52	192.0	197	52.50	246.50	321.50
	1/4	2	.2	2.45	193.0	194	102.50	287.50	252.50
	1/4	2	.4	3.05	192.0	197	120.00	246.50	321.50
	1/4	5	.2	6.12	193.0	194	187.50	287.50	252.50
	1/4	5	.4	7.62	192.0	197	217.50	246.50	321.50
	2/1	1		1.00	186.3	189	9.50	4.50	78.50
	2/1	2		1.99	186.3	189	73.00	4.50	78.50
	2/1	5		4.98	186.5	189	156.00	8.00	78.50
	2/2	1		1.05	187.8	192	26.50	32.50	206.00
	2/2	2		1.98	186.8	189	66.00	12.50	78.50

DIST.	MODEL/ CASE GIVEN	S.D. I/A	DELTA I/A	STND ERROR	NO. ACCEPTED		-----RANKINGS-----		
					T-TEST	F-TEST	S.E.	T-TEST	F-TEST
UNIF	2/2	5		4.98	186.8	188	156.00	12.50	48.00
	2/3	1	.2	2.56	190.5	188	105.50	128.50	48.00
	2/3	1	.4	4.16	190.5	186	134.00	128.50	13.50
	2/3	1	.8	7.36	191.3	184	198.00	195.50	1.50
	2/3	2	.2	3.54	190.0	188	125.50	87.50	48.00
	2/3	2	.4	5.13	190.3	188	170.00	102.50	48.00
	2/3	2	.8	8.32	190.5	186	230.00	128.50	13.50
	2/3	5	.2	6.51	189.3	189	192.50	73.50	78.50
	2/3	5	.4	8.07	189.8	188	225.50	84.00	48.00
	2/3	5	.8	11.23	187.8	188	238.00	32.50	48.00
	2/4	1	.2	1.22	188.5	194	38.50	54.00	252.50
	2/4	1	.4	1.52	188.0	194	52.50	39.00	252.50
	2/4	2	.2	2.44	188.5	194	96.50	54.00	252.50
	2/4	2	.4	3.04	188.0	194	112.50	39.00	252.50
	2/4	5	.2	6.11	188.5	194	183.00	54.00	252.50
	2/4	5	.4	7.61	188.0	194	212.00	39.00	252.50
	3/1	1		1.00	190.4	189	9.50	118.00	78.50
	3/1	2		2.00	190.6	189	80.00	136.00	78.50
	3/1	5		4.99	190.6	189	163.00	136.00	78.50
	3/2	1		1.05	192.2	191	26.50	262.00	174.00
	3/2	2		1.98	191.2	191	66.00	190.00	174.00
	3/2	5		4.99	190.6	189	163.00	136.00	78.50
	3/3	1	.2	2.57	191.0	186	107.50	175.50	13.50
	3/3	1	.4	4.17	191.4	187	135.00	200.50	26.50
	3/3	1	.8	7.38	191.0	187	199.00	175.50	26.50
	3/3	2	.2	3.55	192.2	185	127.00	262.00	5.50
	3/3	2	.4	5.14	191.0	186	171.00	175.50	13.50
	3/3	2	.8	8.33	191.6	187	231.00	212.00	26.50
	3/3	5	.2	6.52	191.4	186	194.50	200.50	13.50
	3/3	5	.4	8.09	192.0	184	227.00	246.50	1.50
	3/3	5	.8	11.25	191.2	185	239.00	190.00	5.50
	3/4	1	.2	1.22	192.6	190	38.50	272.00	126.50
	3/4	1	.4	1.52	193.0	193	52.50	287.50	229.50
	3/4	2	.2	2.45	192.8	190	102.50	281.00	126.50
	3/4	2	.4	3.05	193.0	193	120.00	287.50	229.50
	3/4	5	.2	6.11	192.8	190	183.00	281.00	126.50
	3/4	5	.4	7.62	193.0	193	217.50	287.50	229.50
	4/1	1		1.00	191.8	192	9.50	228.50	206.00
	4/1	2		1.99	191.8	192	73.00	228.50	206.00
	4/1	5		4.98	191.8	192	156.00	228.50	206.00
	4/2	1		1.05	192.2	194	26.50	262.00	252.50
	4/2	2		1.98	190.5	193	66.00	128.50	229.50
	4/2	5		4.98	191.5	192	156.00	207.50	206.00
	4/3	1	.2	2.56	191.5	189	105.50	207.50	78.50
	4/3	1	.4	4.15	192.2	187	133.00	262.00	26.50
	4/3	1	.8	7.35	192.7	185	197.00	276.00	5.50
	4/3	2	.2	3.54	192.3	191	125.50	267.50	174.00
	4/3	2	.4	5.12	191.5	189	169.00	207.50	78.50
	4/3	2	.8	8.31	192.0	187	229.00	246.50	26.50

DIST.	MODEL/	S.D.	DELTA	STND	NO.	ACCEPTED	-----RANKINGS-----		
	CASE	GIVEN	I/A	ERROR	T-TEST	F-TEST	S.E.	T-TEST	F-TEST
UNIF	4/3	5	.2	6.51	192.5	194	192.50	270.00	252.50
	4/3	5	.4	8.07	190.8	192	225.50	158.00	206.00
	4/3	5	.8	11.22	191.7	189	237.00	217.50	78.50
	4/4	1	.2	1.22	191.5	195	38.50	207.50	286.00
	4/4	1	.4	1.52	191.8	195	52.50	228.50	286.00
	4/4	2	.2	2.44	191.5	195	96.50	207.50	286.00
	4/4	2	.4	3.05	191.5	195	120.00	207.50	286.00
	4/4	5	.2	6.11	191.7	195	183.00	217.50	286.00
	4/4	5	.4	7.61	191.5	195	212.00	207.50	286.00
TRIA	1/1	1		1.00	190.3	190	9.50	102.50	126.50
	1/1	2		2.00	190.3	190	80.00	102.50	126.50
	1/1	5		4.99	190.3	190	163.00	102.50	126.50
	1/2	1		1.04	190.3	193	20.50	102.50	229.50
	1/2	2		2.01	190.0	189	85.50	87.50	78.50
	1/2	5		5.00	190.7	190	167.00	144.50	126.50
	1/3	1	.2	19.89	192.7	190	256.00	276.00	126.50
	1/3	1	.4	38.90	193.3	190	268.00	296.50	126.50
	1/3	1	.8	76.93	193.0	190	292.00	287.50	126.50
	1/3	2	.2	20.78	192.7	190	260.00	276.00	126.50
	1/3	2	.4	39.79	192.7	190	272.00	276.00	126.50
	1/3	2	.8	77.81	193.3	190	296.00	296.50	126.50
	1/3	5	.2	23.50	191.7	190	264.00	217.50	126.50
	1/3	5	.4	42.46	192.3	190	278.00	267.50	126.50
	1/3	5	.8	80.46	192.7	190	300.00	276.00	126.50
	1/4	1	.2	1.22	191.7	192	38.50	217.50	206.00
	1/4	1	.4	1.52	192.0	195	52.50	246.50	286.00
	1/4	2	.2	2.45	191.7	192	102.50	217.50	206.00
	1/4	2	.4	3.05	192.0	195	120.00	246.50	286.00
	1/4	5	.2	6.12	191.7	192	187.50	217.50	206.00
	1/4	5	.4	7.62	192.0	195	217.50	246.50	286.00
	2/1	1		0.99	187.3	188	4.50	20.00	48.00
	2/1	2		1.99	187.3	188	73.00	20.00	48.00
	2/1	5		4.97	187.3	188	150.00	20.00	48.00
	2/2	1		1.04	187.3	192	20.50	20.00	206.00
	2/2	2		2.00	187.3	188	80.00	20.00	48.00
	2/2	5		4.98	185.8	189	156.00	3.00	78.50
	2/3	1	.2	19.80	187.5	188	254.00	26.50	48.00
	2/3	1	.4	38.73	187.5	189	266.00	26.50	78.50
	2/3	1	.8	76.58	188.3	188	290.00	47.00	48.00
	2/3	2	.2	20.69	186.8	187	258.00	12.50	26.50
	2/3	2	.4	39.61	187.5	188	270.00	26.50	48.00
	2/3	2	.8	77.46	187.5	189	294.00	26.50	78.50
	2/3	5	.2	23.40	185.5	188	262.00	2.00	48.00
	2/3	5	.4	42.27	186.5	188	274.00	8.00	48.00
	2/3	5	.8	80.09	187.5	187	298.00	26.50	26.50
	2/4	1	.2	1.22	186.5	194	38.50	8.00	252.50
	2/4	1	.4	1.52	187.5	195	52.50	26.50	286.00
	2/4	2	.2	2.44	186.5	194	96.50	8.00	252.50
	2/4	2	.4	3.04	187.5	195	112.50	26.50	286.00

DIST.	MODEL/	S.D.	DELTA	STND	NO.	-----RANKINGS-----			
						ACCEPTED	S.E.	T-TEST	F-TEST
	CASE	GIVEN	I/A	ERROR	T-TEST	F-TEST			
TRIA	2/4	5	.2	6.10	186.5	194	178.50	8.00	252.50
	2/4	5	.4	7.60	187.5	195	208.50	26.50	286.00
	3/1	1		1.00	191.4	190	9.50	200.50	126.50
	3/1	2		1.99	191.2	190	73.00	190.00	126.50
	3/1	5		4.98	191.4	190	156.00	200.50	126.50
	3/2	1		1.04	192.6	191	20.50	272.00	174.00
	3/2	2		2.01	190.4	189	85.50	118.00	78.50
	3/2	5		4.99	191.4	188	163.00	200.50	48.00
	3/3	1	.2	19.85	190.4	190	255.00	118.00	126.50
	3/3	1	.4	38.82	190.4	190	267.00	118.00	126.50
	3/3	1	.8	76.77	190.4	190	291.00	118.00	126.50
	3/3	2	.2	20.74	191.0	189	259.00	175.50	78.50
	3/3	2	.4	39.70	190.4	190	271.00	118.00	126.50
	3/3	2	.8	77.65	190.4	190	295.00	118.00	126.50
	3/3	5	.2	23.46	192.0	187	263.00	246.50	26.50
	3/3	5	.4	42.37	191.4	188	276.00	200.50	48.00
	3/3	5	.8	80.29	190.4	190	299.00	118.00	126.50
	3/4	1	.2	1.22	191.2	190	38.50	190.00	126.50
	3/4	1	.4	1.52	192.2	195	52.50	262.00	286.00
	3/4	2	.2	2.44	191.2	190	96.50	190.00	126.50
	3/4	2	.4	3.05	192.2	195	120.00	262.00	286.00
	3/4	5	.2	6.11	191.0	190	183.00	175.50	126.50
	3/4	5	.4	7.62	192.2	195	217.50	262.00	286.00
	4/1	1		0.99	192.0	195	4.50	246.50	286.00
	4/1	2		1.99	192.0	195	73.00	246.50	286.00
	4/1	5		4.97	191.8	195	150.00	228.50	286.00
	4/2	1		1.04	191.7	196	20.50	217.50	312.00
	4/2	2		2.00	191.7	189	80.00	217.50	78.50
	4/2	5		4.98	191.2	195	156.00	190.00	286.00
	4/3	1	.2	19.77	189.7	192	253.00	79.50	206.00
	4/3	1	.4	38.65	189.7	192	265.00	79.50	206.00
	4/3	1	.8	76.43	189.7	192	289.00	79.50	206.00
	4/3	2	.2	20.65	190.3	192	257.00	102.50	206.00
	4/3	2	.4	39.53	189.7	192	269.00	79.50	206.00
	4/3	2	.8	77.31	189.7	192	293.00	79.50	206.00
	4/3	5	.2	23.36	189.8	192	261.00	84.00	206.00
	4/3	5	.4	42.20	190.5	192	273.00	128.50	206.00
	4/3	5	.8	79.95	189.7	192	297.00	79.50	206.00
	4/4	1	.2	1.22	191.2	195	38.50	190.00	286.00
	4/4	1	.4	1.52	190.8	195	52.50	158.00	286.00
	4/4	2	.2	2.44	191.0	195	96.50	175.50	286.00
	4/4	2	.4	3.04	191.0	195	112.50	175.50	286.00
	4/4	5	.2	6.10	191.0	195	178.50	175.50	286.00
	4/4	5	.4	7.60	191.0	195	208.50	175.50	286.00
LOGI	1/1	1		0.99	189.0	191	4.50	66.50	174.00
	1/1	2		1.98	189.0	191	66.00	66.50	174.00
	1/1	5		4.95	189.0	191	147.00	66.50	174.00
	1/2	1		1.03	190.3	191	14.50	102.50	174.00
	1/2	2		1.99	189.3	192	73.00	73.50	206.00

DIST.	MODEL/ CASE GIVEN	S.D. I/A	DELTA I/A	STND ERROR	NO. ACCEPTED T-TEST	-----RANKINGS-----		
						F-TEST	S.E.	T-TEST F-TEST
LOGI	1/2	5		4.95	188.3	191	147.00	47.00 174.00
	1/3	1	.2	42.66	192.0	190	280.00	246.50 126.50
	1/3	1	.4	84.44	192.0	190	304.00	246.50 126.50
	1/3	1	.8	168.02	192.0	190	318.00	246.50 126.50
	1/3	2	.2	43.53	191.7	190	284.00	217.50 126.50
	1/3	2	.4	85.31	192.0	190	308.00	246.50 126.50
	1/3	2	.8	168.88	192.0	190	320.00	246.50 126.50
	1/3	5	.2	46.18	190.7	190	288.00	144.50 126.50
	1/3	5	.4	87.94	191.7	190	312.00	217.50 126.50
	1/3	5	.8	171.49	192.0	190	324.00	246.50 126.50
	1/4	1	.2	1.21	191.0	187	31.50	175.50 26.50
	1/4	1	.4	1.52	190.7	187	52.50	144.50 26.50
	1/4	2	.2	2.41	191.0	187	90.50	175.50 26.50
	1/4	2	.4	3.04	190.7	187	112.50	144.50 26.50
	1/4	5	.2	6.03	191.0	187	175.00	175.50 26.50
	1/4	5	.4	7.60	190.7	187	208.50	144.50 26.50
	2/1	1		0.98	187.0	189	1.50	16.00 78.50
	2/1	2		1.97	187.0	189	62.00	16.00 78.50
	2/1	5		4.92	187.0	189	141.50	16.00 78.50
	2/2	1		1.03	184.5	194	14.50	1.00 252.50
	2/2	2		1.99	187.8	191	73.00	32.50 174.00
	2/2	5		4.93	186.8	188	143.50	12.50 48.00
	2/3	1	.2	42.43	188.0	190	277.00	39.00 126.50
	2/3	1	.4	84.00	188.5	190	302.00	54.00 126.50
	2/3	1	.8	167.14	188.5	189	314.00	54.00 78.50
	2/3	2	.2	43.30	188.0	190	282.00	39.00 126.50
	2/3	2	.4	84.87	188.0	190	306.00	39.00 126.50
	2/3	2	.8	168.00	188.5	190	317.00	54.00 126.50
	2/3	5	.2	44.94	187.8	190	285.00	32.50 126.50
	2/3	5	.4	87.48	188.0	190	310.00	39.00 126.50
	2/3	5	.8	170.60	188.0	190	322.00	39.00 126.50
	2/4	1	.2	1.20	188.8	186	29.50	61.00 13.50
	2/4	1	.4	1.51	188.5	188	45.00	54.00 48.00
	2/4	2	.2	2.40	188.8	186	89.00	61.00 13.50
	2/4	2	.4	3.03	188.5	188	109.00	54.00 48.00
	2/4	5	.2	6.01	188.8	186	173.00	61.00 13.50
	2/4	5	.4	7.57	188.5	188	205.00	54.00 48.00
	3/1	1		0.99	190.8	189	4.50	158.00 78.50
	3/1	2		1.97	190.8	189	62.00	158.00 78.50
	3/1	5		4.94	190.8	189	145.00	158.00 78.50
	3/2	1		1.03	191.8	195	14.50	228.50 286.00
	3/2	2		1.99	190.8	191	73.00	158.00 174.00
	3/2	5		4.95	190.4	189	147.00	118.00 78.50
	3/3	1	.2	42.57	191.0	190	279.00	175.50 126.50
	3/3	1	.4	84.27	191.0	190	303.00	175.50 126.50
	3/3	1	.8	167.68	191.0	189	315.50	175.50 78.50
	3/3	2	.2	43.44	191.0	190	283.00	175.50 126.50
	3/3	2	.4	85.14	191.0	190	307.00	175.50 126.50
	3/3	2	.8	168.55	191.0	190	319.00	175.50 126.50

DIST.	MODEL/ CASE GIVEN	S.D. GIVEN	DELTA I/A	STND ERROR	NO. ACCEPTED		-----RANKINGS-----		
					T-TEST	F-TEST	S.E.	T-TEST	F-TEST
LOGI	3/3	5	.2	46.09	191.0	191	287.00	175.50	174.00
	3/3	5	.4	87.76	191.0	190	311.00	175.50	126.50
	3/3	5	.8	171.15	190.8	190	323.00	158.00	126.50
	3/4	1	.2	1.21	192.0	191	31.50	246.50	174.00
	3/4	1	.4	1.52	193.2	192	52.50	293.50	206.00
	3/4	2	.2	2.42	192.0	191	92.00	246.50	174.00
	3/4	2	.4	3.05	193.2	192	120.00	293.50	206.00
	3/4	5	.2	6.04	192.0	191	176.00	246.50	174.00
	3/4	5	.4	7.61	193.2	192	212.00	293.50	206.00
	4/1	1		0.98	190.3	193	1.50	102.50	229.50
	4/1	2		1.97	190.3	193	62.00	102.50	229.50
	4/1	5		4.92	190.3	193	141.50	102.50	229.50
	4/2	1		1.03	190.8	195	14.50	158.00	286.00
	4/2	2		1.98	190.3	191	66.00	102.50	174.00
	4/2	5		4.93	190.0	193	143.50	87.50	229.50
	4/3	1	.2	42.35	190.3	191	275.00	102.50	174.00
	4/3	1	.4	83.84	190.5	191	301.00	128.50	174.00
	4/3	1	.8	166.82	190.5	191	313.00	128.50	174.00
	4/3	2	.2	43.22	190.7	191	281.00	144.50	174.00
	4/3	2	.4	84.71	190.3	191	305.00	102.50	174.00
	4/3	2	.8	167.68	190.5	191	315.50	128.50	174.00
	4/3	5	.2	45.86	190.0	191	286.00	87.50	174.00
	4/3	5	.4	87.32	190.2	191	309.00	90.50	174.00
	4/3	5	.8	170.28	190.3	191	321.00	102.50	174.00
	4/4	1	.2	1.20	190.3	190	29.50	102.50	126.50
	4/4	1	.4	1.52	190.7	189	52.50	144.50	78.50
	4/4	2	.2	2.41	190.5	190	90.50	128.50	126.50
	4/4	2	.4	3.04	190.3	189	112.50	102.50	78.50
	4/4	5	.2	6.02	190.3	190	174.00	102.50	126.50
	4/4	5	.4	7.59	190.5	189	206.00	128.50	78.50

APPENDIX 6.2

Analysis Program Fortran Listing

FLAG FORTRAN

C

C

GENERAL LINEAR ANALYSIS PROGRAM

C

DIMENSION XX(30,2),SBEST(6),QBEST(6),ITEST(6)
DIMENSION BEST(6),WA(6),WB(30),WC(1),Q2(1)
DIMENSION X(30,6),Y(30),B(6),E(30),S(6,6),SIN(6,6)
DIMENSION R(5),WD(6),TV(6),STV(6)
DIMENSION A(30,5),STD(5),RX(15),D(5),SRX(5,5),SI(36)
EQUIVALENCE (X(31),A),(SIN(1),SI)

C

C

HOUSE KEEPING

C

IT=0
LINES=42
90 READ(1,9000)NUMH,N,MOD,IERR,MX,DELTA,T,F,ESIG,B,IXI
IF(MOD-9)95,9999,9999
95 IF(IT)97,97,98
97 NUM=NUMH
IT=1
98 DO 100 I=1,N
Y(I)=0.
E(I)=0.
WB(I)=0.
DO 100 J=1,6
100 X(I,J)=0.
DO 120 I=1,6
DO 110 J=1,6
S(I,J)=0.
110 SIN(I,J)=0.
WA(I)=0.
SBEST(I)=0.
QBEST(I)=0.
ITEST(I)=0
TV(I)=0.
STV(I)=0.
WD(I)=0.
120 BEST(I)=0.
DO 130 I=1,5
DO 130 J=1,5
130 SRX(I,J)=0.
A(1,1)=0.
VB=0.
XMB=0.
IT1=0
IT2=0
SFC=0.
TSSR=0.
QSSR=0.
ICTR=0
IXX=1

```

      DET=0.
      SI(1)=0.
      WC(1)=0.
      Q1=0.
      Q2(1)=0.
      IDET=0
C
C      GENERATE X VARIABLES
C
150  Z=RANF(NUM)
      Z=Z
155  XT=MX
      DO 500 JX=1,MX
      GO TO (190,202),IXX
190  DO 200 I=1,N
      ITRN=RANF(0)*1000.0+100.0
      XX(I,1)=ITRN
      XX(I,1)=XX(I,1)/100.
      ITRN=RANF(0)*2000.0+100.0
      XX(I,2)=ITRN
200  XX(I,2)=XX(I,2)/100.
202  IF(IXI)203,203,204
203  IXX=2
      GO TO 205
204  IXX=1
205  GO TO (2100,2350,2450,2550),MOD
C
C      GENERATE STRUCTURE MATRIX BASED ON MODEL TYPE
C      STRUCTURE 1
C
2100 K=3
      DO 220 J=2,K
      DO 220 I=1,N
      X(I,1)=1.
220  X(I,J)=XX(I,J-1)
      GO TO 2600
C
C      STRUCTURE 2
C
2350 DO 2355 I=1,N
      X(I,1)=1.0
      X(I,2)=XX(I,1)
      X(I,3)=XX(I,2)
2355 X(I,4)=XX(I,1)*XX(I,2)
      K=4
      GO TO 2600
C
C      STRUCTURE 3
C
2450 DO 2455 I=1,N
      X(I,1)=1.0
```

```

      X(I,2)=XX(I,1)
      X(I,3)=XX(I,2)
      X(I,4)=XX(I,1)**2
2455  X(I,5)=XX(I,2)**2
      K=5
      GO TO 2600
C
C   STRUCTURE 4
2550  DO 2555 I=1,N
      X(I,1)=1.0
      X(I,2)=XX(I,1)
      X(I,3)=XX(I,2)
      X(I,4)=XX(I,1)**2
      X(I,5)=XX(I,2)**2
2555  X(I,6)=XX(I,1)*XX(I,2)
      K=6
      GO TO 2600
C
C   TEST FOR TYPE OF RANDOM ERROR
C
2600  GO TO (239,240,280,290),IERR
C
C   GEN STND NORMAL ERRPRS.
C
239   AVE=0.
      SIGMA=ESIG
      V=0.
      V2=0.
      DO 230 I=1,N,2
      CALL RAND2(AVE,SIGMA,V,V2)
      E(I)=V
230   E(I+1)=V2
      GO TO 350
C
C   ROUNDED STND NORMAL ERRORS - - E(I) TYPE 2
C
240   AVE=0.
      SIGMA=ESIG
      V=0.
      V2=0.
      DO 255 I=1,N,2
      CALL RAND2(AVE,SIGMA,V,V2)
      IF(V)247,248,248
247   V=V-1.0
248   V=V+.5
      IF(V2)250,251,251
250   V2=V2-1.0
251   V2=V2+.5
      ITRN=V
      E(I)=ITRN
      ITRN=V2
```

```
255 E(I+1)=ITRN
    GO TO 350
C
C   LINEAR CHANGE IN SIGMA -- E(I) TYPE 3
C
280 AVE=0.
    V=0.
    V2=0.
    DO 282 I=1,N
        DSQ=0.
        DO 281 J=1,2
281  DSQ=DSQ+XX(I,J)**2
        DSQ=SQRTF(DSQ)
        SIGMA=ESIG+DSQ*DELTA
        CALL RAND2(AVE,SIGMA,V,V2)
282  E(I)=V
        GO TO 350
C
C   SERIALLY CORRELATED ERRORS -- E(I) TYPE 4
C
290 AVE=0.
    V=0.
    V2=0.
    SIGMA=(ESIG*(1.+DELTA))/(1.-DELTA)
    IF(IT1)292,292,295
292  DO 293 I=1,N,2
        IT1=1
        CALL RAND2(AVE,SIGMA,V,V2)
        E(I)=(1.-DELTA)*V
293  E(I+1)=(1.-DELTA)*V2
        GO TO 350
295  DO 297 I=1,N,2
        CALL RAND2(AVE,SIGMA,V,V2)
        E(I)=(1.-DELTA)*V+DELTA*E(I)
297  E(I+1)=(1.-DELTA)*V2+DELTA*E(I+1)
        GO TO 350
C
C   GENERATE Y=X*B+E
C
350 CALL GMPRD(X,B,Y,N,K,1)
    DO 360 I=1,N
360  Y(I)=Y(I)+E(I)
C
C   COMPUTE S INVERSE = SIN=(X'X)INVERSE
C
395 CALL GTPRD(X,X,S,N,K,K)
    CALL INVER(K,S,K,SIN,DET)
    IF(DET)420,410,420
410  WRITE(2,9003)
        LINES=LINES+1
        IDET>IDET+1
```



```

IF(IDET-5)155,155,90
C
C   COMPUTE B HAT = BEST=SIN*X*Y
C
420 CALL GTPRD(X,Y,WA,N,K,1)
    CALL GMPRD(SIN,WA,BEST,K,K,1)
C
C   COMPUTE SIGMA HAT = SIG=(Y-X*BEST)*(Y-X*BEST)/N-K
C
    CALL GMPRD(X,BEST,WB,N,K,1)
    CALL GMSUB(Y,WB,WB,N,1)
    CALL GTPRD(WB,WB,WC,N,1,1)
    DF=N-K
    SIG=WC(1)/DF
C
C   COMPUTE Q2=(Y-X*B)*X*SIN*X*(Y-X*B)
C
    CALL GTPRD(X,E,WA,N,K,1)
    CALL GTPRD(WA,SIN,WD,K,1,K)
    CALL GMPRD(WD,WA,Q2,1,K,1)
    Q1=SIG*DF
    XK=K
    FC=(DF*Q2(1))/(XK*Q1)
    IF(F-FC)445,440,440
440 ICTR=ICTR+1
445 CONTINUE
    SFC=SFC+FC
    TSSR=TSSR+SQRTF(SIG)
    QSSR=QSSR+SIG
    DO 460 I=1,K
    SBEST(I)=SBEST(I)+BEST(I)
460 QBEST(I)=QBEST(I)+BEST(I)**2
C
C   CORRELATION COEFFICIENTS
C
    M=K-1
    CALL CORE(N,M,A,S,STD,RX,D)
    DO 468 I=1,M
    DO 468 J=1,M
    IF(I-J)462,464,464
462 IRX=I+(J*J-J)/2
    GO TO 466
464 IRX=J+(I*I-I)/2
466 SRX(I,J)=RX(IRX)+SRX(I,J)
468 CONTINUE
C
C   T - TEST
C
    IS=-K
    DO 500 I=1,K
    IS=IS+K+1

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TV(I)=      (BEST(I)-B(I))/SQRTF(SIG*SI(IS))
STV(I)=STV(I)+TV(I)
IF(T-ABSF(TV(I)))500,470,470
470  ITEST(I)=ITEST(I)+1
500  CONTINUE
C
C    PRINT STATISTICS FOR THIS ITERATION SET
C
    IF (LINES-42)620,600,600
600  WRITE(2,9010)
    WRITE(2,9011)
    LINES=0
620  IT2=0
    WRITE(2,9009)MOD,IERR
    ACFF=ICTR
    ACFF=1.0-ACFF/XT
    XMFC=SFC/XT
621  DO 622 I=1,M
    DO 622 J=1,M
622  SRX(I,J)=SRX(I,J)/XT
    DO 630 I=1,K
    VB=SQRTF((QBEST(I)-SBEST(I)**2/XT)/(XT-1.0))
    XMB=SBEST(I)/XT
    XMTT=STV(I)/XT
    ACTT=ITEST(I)
    ACTT=1.0-ACTT/XT
    VSR=SQRTF((QSSR-TSSR**2/XT)/(XT-1.0))
    XMSR=TSSR/XT
    IF(IT2)625,625,627
625  IT2=1
    WRITE(2,9012)I,B(I),XMB,VB,XMTT,ITEST(I),ACTT,(SRX(I,
1J),J=1,5),
    2ESIG,XMSR,VSR,XMFC,ICTR,ACFF
    GO TO 630
627  IF(I-(K-1))628,628,629
628  WRITE(2,9014)I,B(I),XMB,VB,XMTT,ITEST(I),ACTT,(SRX(I,
1J),J=1,5)
    GO TO 630
629  WRITE(2,9014)I,B(I),XMB,VB,XMTT,ITEST(I),ACTT
630  LINES=LINES+1
    GO TO (635,635,640,640,635),IERR
635  WRITE(2,9016)N,MX,IXI
    LINES=LINES+2
    GO TO 90
640  WRITE(2,9015)N,MX,IXI,DELTA
    LINES=LINES+2
    GO TO 90
C
C    FORMAT STMTS
C
9000  FORMAT(I10,I2,2I1,I4,F4.2,F4.3,8F4.2,I1)

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9003 FORMAT(5X,30HPREVIOUS X-MATRIX WAS SINGULAR)
9009 FORMAT(1H ,I2,1H/,I1,3X,11HNORMAL DIST)
9010 FORMAT(1H1,6HMODEL/,1X,4HCOEF,6H GIVEN,2X,4HMEAN,2X,
14HSTND,7X,
28HT - TEST,7X,23HCORRELATION COEFFICIENT,13X,10HSTND
3ERROR,
47X,8HF - TEST)
9011 FORMAT(1X,5HERROR,3X,4HNO. ,5HVALUE,2X,5HVALUE,2X,
13HDEV,3X,4HMEAN,
21X,4HACCT,1X,5HALPHA,3X,3H1,I,3X,3H2,I,3X,3H3,I,3X,
33H4,I,3X,3H5,I,
43X,5HGIVEN,2X,4HMEAN,5X,4HS.D.,3X,
34HMEAN,2X,4HACCT,1X,5HALPHA,/)
9012 FORMAT(1H ,8X,I2,1X,F5.2,1X,F6.2,1X,F6.2,1X,F5.2,1X,
114,2X,F5.3,
25(1X,F5.2),
21X,F5.2,1X,F7.2,1X,F7.2,2X,F5.2,1X,I4,2X,F5.3)
9014 FORMAT(1H ,8X,I2,1X,F5.2,1X,F6.2,1X,F6.2,1X,F5.2,1X,
114,2X,F5.3,
25(1X,F5.2))
9015 FORMAT(12X,8HNO. OBV=,I4,2X,15HNO. ITERATIONS=,I4,2X,
114HX(I) CODE IS =,I2,2X,10HDELTA I/A=,F5.2,/)
9016 FORMAT(12X,8HNO. OBV=,I4,2X,15HNO. ITERATIONS=,I4,2X,
114HX(I) CODE IS =,I2,/)
9999 CONTINUE
END
FLAG FORTRAN
SUBROUTINE GMPRD(A,B,R,N,M,L)
DIMENSION A(1),B(1),R(1)
IR=0
IK=-M
DO 10 K=1,L
IK=IK+M
DO 10 J=1,N
IR=IR+1
JI=J-N
IB=IK
R(IR)=0.
DO 10 I=1,M
JI=JI+N
IB=IB+1
10 R(IR)=R(IR)+A(JI)*B(IB)
RETURN
END
FLAG FORTRAN
SUBROUTINE GTPRD(A,B,R,N,M,L)
DIMENSION A(1),B(1),R(1)
IR=0
IK=-N
DO 10 K=1,L
IJ=0

```

```

    IK=IK+N
    DO 10 J=1,M
    IB=IK
    IR=IR+1
    R(IR)=0.
    DO 10 I=1,N
    IJ=IJ+1
    IB=IB+1
10   R(IR)=R(IR)+A(IJ)*B(IB)
    RETURN
    END
FLAG FORTRAN
SUBROUTINE GMSUB(A,B,R,N,M)
DIMENSION A(1),B(1),R(1)
NM=N*M
DO 10 I=1,NM
10   R(I)=A(I)-B(I)
    RETURN
    END
FLAG FORTRAN
SUBROUTINE CORE(N,M,A,S,STD,R,D)
DIMENSION A(1),S(1),STD(1),R(1),D(1)
K=(M*M+M)/2
DO 102 I=1,K
102  R(I)=0.
    FN=N
    L=0
    DO 115 I=1,N
    JK=0
    L=I-N
    DO 110 J=1,M
    L=L+N
110  D(J)=A(L)-S(J+1)/FN
    DO 115 J=1,M
    DO 115 K=1,J
    JK=JK+1
115  R(JK)=R(JK)+D(J)*D(K)
    JK=0
    DO 220 J=1,M
    JK=JK+J
220  STD(J)=SQRTF(ABSF(R(JK)))
    DO 230 J=1,M
    DO 230 K=J,M
    JK=J+(K*K-K)/2
    IF(STD(J)*STD(K))225,222,225
222  R(JK)=0.
    GO TO 230
225  R(JK)=R(JK)/(STD(J)*STD(K))
230  CONTINUE
    RETURN
    END

```

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