EVALUATION OF BURST PROBABILITY FOR TUBES
BY WEISULL DISTRIBUTIONS

October 1975

Samuel Kao
Applied Mathematics Department
Brookhaven National Laboratory
Upton, New York 11973

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1. SUMMARY AND CONCLUSION

The investigations of candidate distributions that best describe the burst pressure failure probability characteristics of nuclear power steam generator tubes has been continued. To date it has been found that the Weibull distribution provides an acceptable fit for the available data from both the statistical and physical viewpoints.

In this report the reasons for the acceptability of the Weibull distribution are stated together with the results of tests for the suitability of fit. In exploring the acceptability of the Weibull distribution for the fitting, a graphical method to be called the "density-gram" is employed instead of the usual histogram. With this method a more sensible graphical observation on the empirical density may be made for cases where the available data is very limited.

Based on these methods estimates of failure pressure are made for the left-tail probabilities.

2. INTRODUCTION

In a report issued by Frankel [3] it was assumed that burst pressure statistics for non-degraded and degraded tubes could be described by a type I Extreme Value Distribution. The reason for this assumption was that the available data seemed to fit this type of distribution. From a standpoint of safety however, this type of distribution, because of its short tail section to the left, yields non-conservative results. A more conservative distribution seems to be the Weibull distribution. Although this distribution is also of the extreme value type, it has larger weight to the left side and thus yields more conservative results. The logic for applying this distribution and goodness of fit are outlined in the sections that follow.

3. LOGIC FOR APPLYING THE WEIBULL DISTRIBUTION

Let $X_{(n)}$ denote the largest (or smallest) in a random sample of size "n" from a population with a cumulative distribution function, say $p(x)$. It is known from Gnedenko (1943) that for an arbitrary parent distribution $p(x)$, $X_{(n)}$ will not in general possess a limiting distribution, even after suitable standardization. However, if $p(x)$ is such that a limiting distribution exists, then this limiting distribution must be a Extreme Value Distribution. Of the three types of Extreme Value Distributions, the Weibull distribution seems to be the most appropriate. The Weibull distribution is defined as an extreme value distribution wherein n approaches infinity for the distribution of the minimum, $X_{(n)}$, of n values for various initial distributions bounded at the left.

For a tube, the surface may be considered to contain numerous flaws of different infinitesimal size randomly distributed. Tube failure will be initiated at the flaw that causes the tube to have the lowest strength value. Therefore, the burst pressure is equivalent to the strength of the weakest spot. Accordingly, the argument in the context plus the fact that the strength is non-negative (bounded from left by zero) significantly justifies the use of the Weibull distribution as the distribution for the burst pressure of tubes.

4. PLOTTING DENSITY-GRAMS

The procedure for plotting a density-gram is described in what follows.

Let $X_1, X_2, \dots, X_n$ be the values from a simple sample of size n . These may be rearranged and ordered such that

$X_{(1)} \leq X_{(2)} \leq \dots \leq X_{(n)}$. To be exact, let the order statistics be

$$X_{(1)} = \dots = X_{(k_1)} < X_{(k_1+1)} = \dots = X_{(k_1+k_2)} < \dots < X_{(k_1+\dots+k_{M-1}+1)} = \dots =$$

$X_{(k_1 + \dots + k_M)}$ such that $\sum_{i=1}^M k_i = n$. Thus, there are M distinct values

$y_1 < y_2 < \dots < y_M$ among the order statistics.

Define $d_j = (y_{j+1} - y_{j-1})/2$, $2 \leq j \leq M-1$, $d_1 = y_2 - y_1$, and $d_M = y_M - y_{M-1}$,

and let $f_j = k_j / (n d_j)$, $1 \leq j \leq M$, so $\sum_{j=1}^M d_j f_j = 1$.

In addition, let $Y_0 = y_1 - d_1/2$, $Y_M = y_M + d_M/2$, and $Y_j = (y_j + y_{j+1})/2$

for $1 \leq j \leq M-1$.

Then for value $x < Y_0$ or $> Y_M$, define $f(x) = 0$, and for x in the half-open

interval $[Y_j, Y_{j+1}]$, define $f(x) = f_j$, $0 \leq j \leq M-1$. The diagram $(x, f(x))$ may

be plotted in the usual way, and such a diagram is called the density-gram of

X_i , $1 \leq i \leq n$.

The empirical density of p_4 , σu , and the burst test results discussed by Frankel [3], are shown in TABLE 2 and FIGURE 1. It is apparent that for the three examples only unacceptably crude information about the true density may be inferred from the usual histogram. However, their density-grams (Figure 1) seem to render us more definitive information about their corresponding densities. Although there is some lack of information at the left tails, the density-grams for p_4 and p appear to be Weibull.

5. TESTS ON GOODNESS OF FIT

In order to justify that the application of the Weibull distribution for the fitting is statistically correct, a Chi-square test and a Kolmogorov-Smirnov test are made with the data of p_4 .

The Weibull distribution has the three-parameter probability density function

$$f(x; \eta, \sigma, \mu) = \begin{cases} \frac{\eta}{\sigma} \left(\frac{x-\mu}{\sigma}\right)^{\eta-1} \exp \left[- \left(\frac{x-\mu}{\sigma}\right)^{\eta} \right] & \text{for } x \geq \mu, -\infty < \mu < \infty, \\ 0 & \text{otherwise,} \end{cases} \quad \sigma > 0, \eta > 0,$$

which is equivalent to having the cumulative distribution function

$$F(x; \eta, \sigma, \mu) = 1 - \exp \left[- \left(\frac{x-\mu}{\sigma}\right)^{\eta} \right] \quad \text{for } x \geq \mu.$$

By following the estimation procedure as described in Hahn and Shapiro's (1967), is found that the estimates of the parameters are $\hat{\eta} = 16.4654$ and $\hat{\sigma} = 10.2410$, when $\mu = 0$ is assumed. Since there is no reason to claim $\mu > 0$, the conservative assumption of $\mu = 0$ is well-founded. Results of the numerical computations, are shown in table 3. Using this data the tests are as follows:

(1) Chi-Square Test

	[0, 9.495]	[9.495, 10.016]	[10.016, 10.446]	[10.446, ∞]
Expected Frequency	4.25	4.25	4.25	4.25
Observed Frequency	6	5	4	2

$$\chi^2 = \sum \frac{(\text{obs.} - \text{Expt.})^2}{\text{Expt.}} = 2.05$$

Since 2.05 is less than 3.84, the critical value at 5% significance level for Chi-square with one degree of freedom, the fitting is acceptable.

(2) Kolmogorov-Smirnov Test

From the result of $F_n(x) - F(x)$ in TABLE 3 it is seen that

$$\sqrt{n} \sup | F_n(x) - F(x) | = \sqrt{17} \times 0.23 = 0.948. \text{ Since } 0.948 \text{ is less than the}$$

critical value 1.36 of 5% significance level for the Kolmogorov-Smirnov test, the fitting is acceptable.

From the result of the statistical tests, it is seen that the Weibull distribution is of good fit for p_4 . In addition, it is expected that the same will hold for p_1 and p_2 when similar tests are carried out with the data available in Frankel's [3] report.

6. ESTIMATES OF LEFT-TAIL PROBABILITY

Since it is important to know the Prob {Burst pressure < c} for c that is much to the left of the distribution, an evaluation will now be made based on the calculated values of p_4 .

Substituting the parameters, the cumulative distribution has the following estimate:

$$F(x) = 1 - \exp \left[- \left(\frac{x}{10.24} \right)^{16.47} \right] \quad \text{for } x \geq 0$$

Based on this expression, the following results are obtained:

$$\text{Prob \{B.P. (Burst Pressure) < 3 kpsi\}} \quad 1.60 \times 10^{-9};$$

$$\text{Prob \{B.P. < 3.5 kpsi\}} \quad 2.07 \times 10^{-8};$$

$$\text{Prob \{B.P. < 4 kpsi\}} \quad 1.87 \times 10^{-7};$$

$$\text{Prob \{B.P. < 5 kpsi\}} \quad 7.4 \times 10^{-6};$$

$$\text{Prob \{B.P. < 7 kpsi\}} \quad 1.89 \times 10^{-3}.$$

It should be noted that the order of magnitude of the left-tail probability is essentially unchanged if μ is replaced by its estimate instead of $\mu = 0$.

7. FINAL REMARKS

Although there may be some reasons to apply Gumbel's (Type I) extreme value distribution for statistical fitting of burst pressure of tubes, the Weibull distribution is more appropriate than Gumbel's, since it does not have the deficiencies indicated in [6]. Regarding the evaluation of the left-tail probability, the result in Section 5 appears to be more reasonable than those obtained by Frankel in [2,3]. Nevertheless, further work needs to be undertaken. Degradation for tubes actually starts as soon as the tubes are used, therefore, the accuracy of evaluation for the left-tail probability of tube bursting in a reactor may be improved if the time dependence of the degradation process is taken into account. Perhaps, the discussions in Lamperti's [7] and in Dwass' [1] will offer a good basis for these investigations.

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TABLE 1

<u>Sample</u>	<u>σ_u - kpsi</u>	<u>ϵ_u</u>	<u>P_1 - kpsi</u>	<u>P_2 - kpsi</u>	<u>P_4 - kpsi</u>
6578W	88.0	.37	12.55	11.69	9.35
6581W	89.0	.36	12.57	11.71	9.36
6590W	90.0	.42	12.65	11.79	9.37
NX	90.2	.40	12.68	11.81	9.43
6586W	90.5	.42	12.71	11.84	9.44
6579W	90.5	.37	12.71	11.84	9.46
6562W	90.6	.42	12.76	11.89	9.58
6585W	91.7	.44	12.92	12.04	9.60
6593W	92.7	.42	12.96	12.08	9.70
6578W	93.0	.40	13.11	12.21	9.75
5086W	93.4	.39	13.21	12.31	9.86
5046W	94.3	.37	13.46	12.54	10.10
6376W	96.2	.36	13.67	12.76	10.20
5045W	96.6	.39	13.78	12.81	10.37
4986W	97.5	.38	13.86	12.91	10.38
4973W	97.5	.30	14.31	13.33	11.00
5026W	103.0	.30	15.12	14.09	11.63
<hr/>					
$\bar{x}$	93.2	.383	13.24	12.33	9.92
$\Sigma (x-\bar{x})^2$	242	.02	8.17	7.08	6.69
$s = \left(\frac{\Sigma (x-\bar{x})^2}{n-1} \right)^{1/2}$	3.89	.034	.71	.67	.65

TABLE 2

Calculations for Density-Grams

Index	(1)				(2)			
	<u>P₄</u>	<u>y</u>	<u>d</u>	<u>f</u>	<u>σ_u</u>	<u>y</u>	<u>d</u>	<u>f</u>
0		9.345				87.5		
1	9.35	9.355	0.01		88.0	88.5	1.0	0.059
2	9.36	9.365	0.01	3.21	89.0	89.5	1.0	0.059
3	9.37	9.40	0.035		90.0	91.0	0.6	0.138
4	9.43	9.435	0.035	2.35	90.2	90.25	0.25	
5	9.44	9.45	0.015		90.5	90.55	0.2	
6	9.46	9.52	0.07	0.84	90.5			.2206
7	9.58	9.59	0.07	0.84	90.6	91.15	0.6	
8	9.60	9.65	0.06	0.98	91.7	92.2	1.05	0.056
9	9.70	9.725	0.075	0.78	92.7	92.85	0.65	0.090
10	9.75	9.805	0.08	0.74	93.0	93.2	0.35	0.168
11	9.86	9.98	0.175	0.34	93.4	93.85	0.65	0.090
12	10.10	10.15	0.17	0.35	94.3	95.25	1.4	0.042
13	10.20	10.285	0.135	0.44	96.2	96.4	1.15	0.051
14	10.37	10.375	0.09	0.65	96.6	97.05	0.65	0.090
15	10.38	10.69	0.315	0.19	97.5	100.25	3.2	0.036
16	11.00	11.315	0.125	0.47	97.5			
17	11.63	11.945	0.63	0.09	103.0	105.75	5.5	0.011

(3) Burst Test Results, Non-Degraded Tube

Index	<u>p</u>	<u>y</u>	<u>d</u>	<u>f</u>
0		9.525		
1	9.55	9.575	.05	2.86
2	9.60	9.725	.10	1.43
3	9.85	10.00	.225	0.63
4	10.15	10.175	.14	2.04
5	10.15			
6	10.20	10.275	.175	0.82
7	10.35	10.425	.15	0.95

TABLE 3

Calculations on p_4

x	$F_n(x)$	$\ln x$	$\ln \ln [1/(1-F_n(x))]$	$F_n(x) - F(x)$
9.35	1/34	2.235	-2.803	.23
9.36	3/34	2.236	-2.078	.17
9.37	5/34	2.237	-1.639	.12
9.43	7/34	2.244	-1.316	.08
9.44	9/34	2.245	-1.055	.02
9.46	11/34	2.247	-.832	-.03
9.58	13/34	2.260	-.634	-.04
9.60	15/34	2.262	-.453	-.10
9.70	17/34	2.272	-.283	-.11
9.75	19/34	2.277	-.120	-.15
9.86	21/34	2.288	.041	-.16
10.10	23/34	2.313	.202	-.10
10.20	25/34	2.322	.369	-.11
10.37	27/34	2.339	.551	-.08
10.38	29/34	2.340	.761	-.13
11.00	31/34	2.398	1.041	.05
11.63	33/34	2.454	3.106	.06

$$\Sigma \ln x = 38.9698 \quad \Sigma \ln \ln [1/(1-F_n(x))] = -9.5349$$

$$\Sigma (\ln x)^2 = 89.3937 \quad \Sigma \{ \ln \ln [1/(1-F_n(x))] \}^2 = 29.9162$$

$$\Sigma (\ln x) \ln \ln [1/(1-F_n(x))] = -20.8426$$

$$\hat{\eta} = 16.4654, \quad \hat{\sigma} = 10.2410$$

$$\sup_x |F_n(x) - F(x)| = .23$$

TABLE 4

Percentiles of F_4^C

<u>Percent</u>	<u>c = 1</u>	<u>c = .5</u>	<u>c = .4</u>	<u>c = .3</u>	<u>c = .2</u>	<u>c = .1</u>
50	10.0156	5.0078	4.0062	3.0047	2.0031	1.0013
10	8.9328	4.4663	3.5737	2.6803	1.7868	.8930
5	8.5507	4.2753	3.4210	2.5658	1.7104	.8548
2.5	8.1918	4.0958	3.2776	2.4582	1.6386	.8190
1	7.7448	3.8723	3.0990	2.3243	1.5493	.7743
0.5	7.4244	3.7121	2.9710	2.2282	1.4853	.7422
.1	6.7322	3.3660	2.6944	2.0208	1.3469	.6730
$\hat{\eta}$:	16.4654	16.4639	16.4897	16.4883	16.4788	16.4671
$\hat{\sigma}$:	10.2410	5.1205	4.0962	3.0722	2.0482	1.0238

FIGURE 1
p. 4

curve of interpolation

FIGURE 1 (Continued) σ_A

15

2

15

1

0.5

Curve of interpolation

98.6

94.3

105.25

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$X_{(k_1 + \dots + k_M)}$ such that $\sum_{i=1}^M k_i = n$. Thus, there are M distinct values

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and let $f_j = k_j/(nd_j)$, $1 \leq j \leq M$, so $\sum_{j=1}^M d_j f_j = 1$.

In addition, let $y_0 = y_1 - d_1/2$, $y_M = y_M + d_M/2$, and $y_j = (y_j + y_{j+1})/2$

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Then for value $x < y_0$ or $> y_M$, define $f(x) = 0$, and for x in the half-open

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Substituting the parameters, the cumulative distribution has the following estimate:

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TABLE 1

<u>Sample</u>	<u>σ'_u - kpsi</u>	<u>ϵ_u</u>	<u>P_1 - kpsi</u>	<u>P_2 - kpsi</u>	<u>P_4 - kpsi</u>
6578W	88.0	.37	12.55	11.69	9.35
6581W	89.0	.36	12.57	11.71	9.36
6590W	90.0	.42	12.65	11.79	9.37
NX	90.2	.40	12.68	11.81	9.43
6586W	90.5	.42	12.71	11.84	9.44
6579W	90.5	.37	12.71	11.84	9.46
6562W	90.6	.42	12.76	11.89	9.58
6585W	91.7	.44	12.92	12.04	9.60
6593W	92.7	.42	12.96	12.08	9.70
6578W	93.0	.40	13.11	12.21	9.75
5086W	93.4	.39	13.21	12.31	9.86
5046W	94.3	.37	13.46	12.54	10.10
6376W	96.2	.36	13.67	12.76	10.20
5045W	96.6	.39	13.78	12.81	10.37
4986W	97.5	.38	13.86	12.91	10.38
4973W	97.5	.30	14.31	13.33	11.00
5026W	103.0	.30	15.12	14.09	11.63
<hr/>					
$\bar{x}$	93.2	.383	13.24	12.33	9.92
$\Sigma(x-\bar{x})^2$	242	.02	8.17	7.08	6.69
$s = \left(\frac{\Sigma(x-\bar{x})^2}{n-1} \right)^{1/2}$	3.89	.034	.71	.67	.65

TABLE 2

Calculations for Density-Grams

Index	(1)				(2)			
	<u>P₄</u>	<u>Y</u>	<u>d</u>	<u>f</u>	<u>σ_u</u>	<u>Y</u>	<u>d</u>	<u>f</u>
0		9.345				87.5		
1	9.35	9.355	0.01		88.0	88.5	1.0	0.059
2	9.36	9.365	0.01	3.21	89.0	89.5	1.0	0.059
3	9.37	9.40	0.035		90.0	91.0	0.6	0.138
4	9.43	9.435	0.035	2.35	90.2	90.25	0.25	
5	9.44	9.45	0.015		90.5	90.55	0.2	
6	9.46	9.52	0.07	0.84	90.5			.2206
7	9.58	9.59	0.07	0.84	90.6	91.15	0.6	
8	9.60	9.65	0.06	0.98	91.7	92.2	1.05	0.056
9	9.70	9.725	0.075	0.78	92.7	92.85	0.65	0.090
10	9.75	9.805	0.08	0.74	93.0	93.2	0.35	0.168
11	9.86	9.98	0.175	0.34	93.4	93.85	0.65	0.090
12	10.10	10.15	0.17	0.35	94.3	95.25	1.4	0.042
13	10.20	10.285	0.135	0.44	96.2	96.4	1.15	0.051
14	10.37	10.375	0.09	0.65	96.6	97.05	0.65	0.090
15	10.38	10.69	0.315	0.19	97.5	100.25	3.2	0.036
16	11.00	11.315	0.125	0.47	97.5			
17	11.63	11.945	0.63	0.09	103.0	105.75	5.5	0.011

(3) Burst Test Results, Non-Degraded Tube

Index	<u>p</u>	<u>Y</u>	<u>d</u>	<u>f</u>
0		9.525		
1	9.55	9.575	.05	2.86
2	9.60	9.725	.10	1.43
3	9.85	10.00	.225	0.63
4	10.15	10.175	.14	2.04
5	10.15			
6	10.20	10.275	.175	0.82
7	10.35	10.425	.15	0.95

TABLE 3

Calculations on p_4

x	$F_n(x)$	$\ln x$	$\ln \ln [1/(1-F_n(x))]$	$F_n(x) - F(x)$
9.35	1/34	2.235	-2.803	.23
9.36	3/34	2.236	-2.078	.17
9.37	5/34	2.237	-1.639	.12
9.43	7/34	2.244	-1.316	.08
9.44	9/34	2.245	-1.055	.02
9.46	11/34	2.247	-.832	-.03
9.58	13/34	2.260	-.634	-.04
9.60	15/34	2.262	-.453	-.10
9.70	17/34	2.272	-.283	-.11
9.75	19/34	2.277	-.120	-.15
9.86	21/34	2.288	.041	-.16
10.10	23/34	2.313	.202	-.10
10.20	25/34	2.322	.369	-.11
10.37	27/34	2.339	.551	-.08
10.38	29/34	2.340	.761	-.13
11.00	31/34	2.398	1.041	.05
11.63	33/34	2.454	3.106	.06

$$\Sigma \ln x = 38.9698 \quad \Sigma \ln \ln [1/(1-F_n(x))] = -9.5349$$

$$\Sigma (\ln x)^2 = 89.3937 \quad \Sigma \{ \ln \ln [1/(1-F_n(x))] \}^2 = 29.9162$$

$$\Sigma (\ln x) \ln \ln [1/(1-F_n(x))] = -20.8426$$

$$\hat{\eta} = 16.4654, \quad \hat{\sigma} = 10.2410$$

$$\sup_x |F_n(x) - F(x)| = .23$$

TABLE 4

Percentiles of F_4^c

<u>Percent</u>	<u>c = 1</u>	<u>c = .5</u>	<u>c = .4</u>	<u>c = .3</u>	<u>c = .2</u>	<u>c = .1</u>
50	10.0156	5.0078	4.0062	3.0047	2.0031	1.0013
10	8.9328	4.4663	3.5737	2.6803	1.7868	.8930
5	8.5507	4.2753	3.4210	2.5658	1.7104	.8548
2.5	8.1918	4.0958	3.2776	2.4582	1.6386	.8190
1	7.7448	3.8723	3.0990	2.3243	1.5493	.7743
0.5	7.4244	3.7121	2.9710	2.2282	1.4853	.7422
.1	6.7322	3.3660	2.6944	2.0208	1.3469	.6730
$\hat{\eta}$:	16.4654	16.4639	16.4897	16.4883	16.4788	16.4671
$\hat{\sigma}$:	10.2410	5.1205	4.0962	3.0722	2.0482	1.0238

FIGURE 1 (Continued)
ON