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Just Where Exactly is the Radar? (a.k.a. The Radar Antenna Phase Center)

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Abstract

The “location” of the radar is the reference location to which the radar measures range. This is typically the antenna’s “phase center.” However, the antenna’s phase center is not generally obvious, and may not correspond to any seemingly obvious physical location, such as the focal point of a dish reflector. This report calculates the phase center of an offset-fed dish reflector antenna.

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Foreword

This report details the results of an academic study. It does not presently exemplify any modes, methodologies, or techniques employed by any operational system known to the author.

Classification

The specific mathematics and algorithms presented herein do not bear any release restrictions or distribution limitations.

This distribution limitations of this report are in accordance with the classification guidance detailed in the memorandum “Classification Guidance Recommendations for Sandia Radar Testbed Research and Development”, DRAFT memorandum from Brett Remund (Deputy Director, RF Remote Sensing Systems, Electronic Systems Center) to Randy Bell (US Department of Energy, NA-22), February 23, 2004. Sandia has adopted this guidance where otherwise none has been given.

This report formalizes preexisting informal notes and other documentation on the subject matter herein.

1 Introduction

The most fundamental task of a radar is to measure range from itself to a target of interest. It does so by measuring the echo delay time against its internal time base, and making assumptions about the velocity of propagation.

Nevertheless, for accurate and precise range measurements, a reasonable question (albeit often overlooked question) is “Just exactly from where is the range measurement being made?” In other words “Just where is the reference location on the radar to which range is measured?”

The knee-jerk response to this question is typically “The reference location is the ‘antenna phase center’ of the radar’s antenna.” This naturally leads to the follow-up question “So exactly where is the ‘antenna phase center’ of the radar’s antenna?” the response to which is often a blank stare.

We desire knowledge of the location of the antenna phase center for the following reasons.

1. This is, or should be, the reference location of the radar for range measurements.
2. This is, or should be, the location to which system delays need to be calibrated and compensated.
3. This is, or should be, the location for which we desire motion information from the radar’s motion measurement subsystem, including position, velocity, and angular orientation.

Unfortunately, for all too many radar systems, none of these locations actually coincide.

The IEEE¹ defines the antenna phase center as

“The location of a point associated with an antenna such that, if it is taken as the center of a sphere whose radius extends into the far-field, the phase of a given field component over the surface of the radiation sphere is essentially constant, at least over that portion of the surface where the radiation is significant.”

Essentially, the “phase center” for an antenna is the apparent point source of a constant-phase wavefront emanating from the antenna. This is a common viewpoint in the literature.²

The problem is that for most practical radar antennas, there is some difficulty with locating, or even defining such a ‘point.’ This location may

1. be frequency-dependent,
2. be polarization-dependent,

3. be different for different parts of the antenna,
4. depend on the environment of the antenna (e.g. radome characteristics), or
5. be quite different from other antenna characteristics (e.g. focal point, gimbal axis, etc.).

Finding such a reference location can be a decidedly non-trivial task. For example, Fridén and Kristensson³ seek to calculate the radiation center of an antenna, using the Spherical Wave Expansion (SWE) of the far field, based on angular momentum.

We will seek in this report a somewhat simpler approach, even if less rigorous, but nevertheless adequate for our purposes. We will answer a slightly different question, namely “What is the location about which we may rotate the radar antenna to minimize the variation in average echo delay time?” If necessary, we will confine ourselves to rotation angles that are on the order of a beamwidth for antennas that have dimensions large with respect to wavelength. Furthermore, we will also attempt to answer “For what additional system delays do we need to account?”

We confine our analysis herein to the parabolic dish reflector antenna. Such antennas, although seemingly ancient by comparison to modern sexy phased-array antennas, nevertheless still offer some performance attributes (e.g. efficiency, bandwidth, etc.) that are hard to beat.

2 Dish Reflector – Simple One-Dimensional Analysis

Consider a simple parabolic dish reflector. We will consider a one-dimensional analysis.

Let the feed for the reflector be located at some point

$$(x_f, z_f) = \text{antenna feed location.} \quad (1)$$

We now define the location of the reflector with the curve

$$z - z_f = -f + \frac{1}{4f}(x - x_f)^2, \quad (2)$$

where

$$f = \text{the focal length of the reflector,} \quad (3)$$

and the reflector is constrained to the interval

$$x_1 \leq x \leq x_2, \quad (4)$$

where

$$\begin{aligned} x_1 &= \text{left edge of the reflector, and} \\ x_2 &= \text{right edge of the reflector.} \end{aligned} \quad (5)$$

The boresight of the beam is in the direction of the positive z -axis. We now also define a reference position in the vicinity of the antenna with an offset from the feed as

$$(x_r, z_r) = \text{offset position from the feed for the antenna reference location.} \quad (6)$$

We furthermore define some constant target range much larger than the antenna dimension as

$$r_c = \text{target range from the reference position.} \quad (7)$$

This lets us sweep a circular arc of target positions where the coordinates of the target location are

$$\begin{aligned} x_c(\theta) &= (x_f + x_r) + r_c \sin \theta, \text{ and} \\ z_c(\theta) &= (z_f + z_r) + r_c \cos \theta, \end{aligned} \quad (8)$$

Figure 1. Geometry definitions for parabolic dish reflector.

where

$$\theta = \text{the direction of a particular target position.} \quad (9)$$

In general, we will be interested in a fairly small arc of angles. These parameters are illustrated in Figure 1.

The range from the target position to the feed, by way of reflection at the parabolic dish reflector, is calculated as

$$r_t(x, z, \theta) = \sqrt{(x - x_f)^2 + (z - z_f)^2} + \sqrt{(x - x_c(\theta))^2 + (z - z_c(\theta))^2}. \quad (10)$$

What we desire is that this range is constant over all relevant angles θ . More explicitly, we define a range error as

$$r_{\varepsilon}(x, z, \theta) = r_t(x, z, \theta) - r_c - r_{\Delta}, \quad (11)$$

where

$$r_{\Delta} = \text{a range calibration constant.} \quad (12)$$

Consequently, we desire that the range error is zero for an appropriate r_{Δ} , over all angles θ . We will settle for minimized in some fashion.

We can combine these equations and expand the range error to

$$r_{\varepsilon}(x, z, \theta) = \sqrt{(x - x_f)^2 + (z - z_f)^2} + \sqrt{(x - x_c(\theta))^2 + (z - z_c(\theta))^2} - r_c - r_{\Delta}. \quad (13)$$

We note that for a parabola, we may equate

$$\sqrt{(x - x_f)^2 + (z - z_f)^2} = 2f - |z - z_f|. \quad (14)$$

Consequently, we may expand the range error further to

$$r_{\varepsilon}(x, z, \theta) = \left(\frac{2f - |z - z_f|}{+\sqrt{(x - x_c(\theta))^2 + (z - z_c(\theta))^2}} \right) - r_c - r_{\Delta}, \quad (15)$$

and further yet to

$$r_{\varepsilon}(x, z, \theta) = \left(\frac{2f - \left| -f + \frac{1}{4f}(x - x_f)^2 \right|}{+\sqrt{\left(x - \left(x_f + x_r \right) - r_c \sin \theta \right)^2 + \left(z_f - f + \frac{1}{4f}(x - x_f)^2 \right)^2}} \right) - r_c - r_{\Delta}, \quad (16)$$

and, given the geometry of typical reflector antenna, further yet to

$$r_{\mathcal{E}}(x, z, \theta) = \left(f + \frac{1}{4f}(x - x_f)^2 + \sqrt{\left(x - x_f - x_r - r_c \sin \theta \right)^2 + \left(-f + \frac{1}{4f}(x - x_f)^2 \right)^2} \right) - r_c - r_{\Delta}. \quad (17)$$

We can simplify our model by assuming that the focal point is at (0,0). After all, we are looking for relative coordinates (x_r, z_r) , as well as calibration parameter r_{Δ} . This allows us to write

$$r_{\mathcal{E}}(x, z, \theta) = \left(f + \frac{1}{4f}x^2 - r_{\Delta} + \sqrt{\left(x - x_r - r_c \sin \theta \right)^2 + \left(-f + \frac{1}{4f}x^2 \right)^2} \right) - r_c. \quad (18)$$

In general, at nonzero angles θ , this error will vary with the ‘reflecting’ position on the dish reflector, that is, with parameter x . Consequently, no constant reference position (x_r, z_r) or calibration parameter r_{Δ} can be found to satisfy our desire for zero range error over all θ . This is an artifact of the focal point moving as off-boresight angle changes. This is also what causes nulls and sidelobes in the antenna far-field pattern, i.e. summation of phases that add or cancel as off-boresight angle changes.

Instead, we will calculate the mean error from all positions along the reflector, and minimize the error in the mean. We define the mean range error as

$$\bar{r}_{\mathcal{E}}(\theta) = \frac{1}{(x_2 - x_1)} \int_{x_1}^{x_2} r_{\mathcal{E}}(x, z, \theta) dx. \quad (19)$$

We then will take the limit as we push range to infinity, namely

$$\bar{r}_{\mathcal{E},\infty}(\theta) = \lim_{r_c \rightarrow \infty} \bar{r}_{\mathcal{E}}(\theta). \quad (20)$$

Before we proceed, however, we first examine the second line of Eq. (18), specifically the remaining square-root term. If we expand this into a polynomial series in x , about $x = 0$, we observe that the n^{th} derivative of the square-root term which make up the coefficients of x^n where $n \geq 3$ have larger powers of r_c in the denominator than in the numerator, such that as $r_c \rightarrow \infty$, in the limit these terms go to zero. Consequently, we

may reasonably expand the series into only a second-order polynomial and ignore higher order terms in finding mean range error. This is equivalent to assuming planar wavefronts from target echoes.

This yields the expression for range error as

$$\bar{r}_{\mathcal{E},\infty}(\theta) = \frac{\begin{pmatrix} \left(12f^2 - 12fr_{\Delta} + x_1^2 + x_1x_2 + x_2^2\right) \\ + \left(12f^2 + 12fz_r - x_1^2 - x_1x_2 - x_2^2\right)\cos\theta \\ - 6f(x_1 + x_2 - 2x_r)\sin\theta \end{pmatrix}}{12f}. \quad (21)$$

Recall that we want this to be zero for all angles. We can achieve this by setting

$$\begin{aligned} x_r &= \frac{x_1 + x_2}{2}, \\ z_r &= \frac{x_1^2 + x_1x_2 + x_2^2 - 12f^2}{12f}, \text{ and} \\ r_{\Delta} &= \frac{x_1^2 + x_1x_2 + x_2^2 + 12f^2}{12f} = z_r + 2f. \end{aligned} \quad (22)$$

We make several observations.

- We have made the tacit assumption that the reflector is illuminated uniformly with position x . More accurately, we have disregarded any effects of non-uniform illumination of the reflector or other feed-based illumination effects.
- These parameters all depend on the focal length of the reflector, and the locations of the edges of the reflector. Consequently, even if the focal length remains constant, merely changing the size of the reflector can change the location of the optimum antenna reference position.
- The optimum antenna reference position is
 - 1) not at the feed position,
 - 2) not necessarily on the feed axis (horizontal or vertical), and
 - 3) not necessarily even on the dish surface.
 It is located in ‘space’ near the antenna.
- There is an added system delay that must be accounted for to make a proper range measurement.

Example 1.

Consider an offset-fed parabolic dish reflector with the following parameters

$$\begin{aligned} f &= 0.3048 \text{ m,} \\ x_1 &= 0.0635 \text{ m, and} \\ x_2 &= 0.3810 \text{ m.} \end{aligned} \tag{23}$$

where the feed point is at the origin. We shall also assume a 0.036 m wavelength, indicating that the antenna dimensions are large with respect to wavelength. From these parameters we calculate

$$\begin{aligned} x_r &= 0.2223 \text{ m,} \\ z_r &= -0.2574 \text{ m, and} \\ r_\Delta &= 0.3522 \text{ m.} \end{aligned} \tag{24}$$

This is illustrated in Figure 2.

Figure 2. Results of Example 1. The reference phase center is at the '+' location.

3 Dish Reflector – Simple Two-Dimensional Analysis

Advancing from the analysis of the previous section, we now consider a two-dimensional parabolic dish reflector fed from a point source. Consequently, the dish would be a section of a circular paraboloid described by

$$z - z_f = -f + \frac{1}{4f}(x - x_f)^2 + \frac{1}{4f}(y - y_f)^2, \quad (25)$$

where, as before

$$\begin{aligned} (x_f, y_f, z_f) &= \text{antenna feed location, and} \\ f &= \text{the focal length of the reflector,} \end{aligned} \quad (26)$$

but now the reflector is constrained to the two-dimensional rectangular aperture with

$$\begin{aligned} x_1 &\leq x \leq x_2, \text{ and} \\ y_1 &\leq y \leq y_2, \end{aligned} \quad (27)$$

where

$$\begin{aligned} x_1 &= \text{left edge of the reflector,} \\ x_2 &= \text{right edge of the reflector,} \\ y_1 &= \text{bottom edge of the reflector,} \\ y_2 &= \text{top edge of the reflector.} \end{aligned} \quad (28)$$

The boresight of the beam is in the direction of the positive z -axis. Now we define a reference position in the vicinity of the antenna with an offset from the feed in three dimensions as

$$(x_r, y_r, z_r) = \text{offset position from the feed for the antenna reference location.} \quad (29)$$

As before, we define a constant target range much larger than the antenna dimension as

$$r_c = \text{target range from the reference position.} \quad (30)$$

This lets us sweep a spherical shell of target positions where the coordinates of the target location are

$$\begin{aligned}
x_c(\theta, \phi) &= (x_f + x_r) + r_c \sin \theta \cos \phi, \\
y_c(\theta, \phi) &= (y_f + y_r) + r_c \sin \phi, \text{ and} \\
z_c(\theta, \phi) &= (z_f + z_r) + r_c \cos \theta \cos \phi,
\end{aligned} \tag{31}$$

where

$$\begin{aligned}
\theta &= \text{the azimuth direction of a particular target position, and} \\
\phi &= \text{the elevation direction of a particular target position.}
\end{aligned} \tag{32}$$

As before, in general, we will generally be interested in a fairly small arc of angles.

The range from the target position to the feed, by way of reflection at the parabolic dish reflector, is calculated as

$$r_t(x, y, z, \theta, \phi) = \left(\frac{\sqrt{(x - x_f)^2 + (y - y_f)^2 + (z - z_f)^2}}{\sqrt{(x - x_c(\theta, \phi))^2 + (y - y_c(\theta, \phi))^2 + (z - z_c(\theta, \phi))^2}} \right). \tag{33}$$

More explicitly, we define a range error as

$$r_e(x, y, z, \theta, \phi) = r_t(x, y, z, \theta, \phi) - r_c - r_\Delta, \tag{34}$$

where, as before

$$r_\Delta = \text{a range calibration constant.} \tag{35}$$

Consequently, we desire that the range error is zero for an appropriate r_Δ , over all angles θ and ϕ . We will again settle for minimized in some fashion.

As before, we may place the origin at the feed, and then the error can be expanded and simplified to

$$r_e(x, y, z, \theta, \phi) = \left(\frac{f + \frac{1}{4f}x^2 + \frac{1}{4f}y^2 - r_\Delta - r_c}{\sqrt{(x - x_r - r_c \sin \theta \cos \phi)^2 + (y - y_r - r_c \sin \phi)^2} + \sqrt{\left(-f + \frac{1}{4f}x^2 + \frac{1}{4f}y^2 - z_r - r_c \cos \theta \cos \phi\right)^2}} \right). \tag{36}$$

In general, at nonzero angles θ or ϕ , this error will vary with the position on the dish reflector, that is, with parameters x and y . Consequently, as with the one-dimensional case, no constant reference position (x_r, y_r, z_r) or calibration parameter r_Δ can be found to satisfy our desire for zero range error over all angles θ and ϕ . This is again an artifact of the focal point moving as off-boresight angle changes in any direction.

Instead, as before, we will calculate the mean error from all positions along the reflector, in both dimensions, and minimize the error in the mean. We now define the mean error for this particular rectangular aperture as

$$\bar{r}_\varepsilon(\theta, \phi) = \frac{1}{(x_2 - x_1)(y_2 - y_1)} \int_{y_1}^{y_2} \int_{x_1}^{x_2} r_\varepsilon(x, y, z, \theta, \phi) dx dy. \quad (37)$$

More generally, for other aperture shapes, the integral may be written as

$$\bar{r}_\varepsilon(\theta, \phi) = \frac{1}{A} \iint_A r_\varepsilon(x, y, z, \theta, \phi) dA, \quad (38)$$

where

$$A = \text{the area of the aperture projected in the } xy \text{ plane.} \quad (39)$$

We then will take the limit as we push range to infinity, namely

$$\bar{r}_{\varepsilon, \infty}(\theta, \phi) = \lim_{r_c \rightarrow \infty} \bar{r}_\varepsilon(\theta, \phi). \quad (40)$$

Before we proceed, however, we first examine the second line of Eq. (36), specifically the remaining square-root term. If we expand this into a polynomial series in x , about $x = 0$, we observe that the n^{th} derivative of the square-root term which make up the coefficients of x^n where $n \geq 3$ have larger powers of r_c in the denominator than in the numerator, such that as $r_c \rightarrow \infty$, in the limit these terms go to zero. The same is true if we expand this into a polynomial series in y , about $y = 0$, where we also observe that the n^{th} derivative of the square-root term which make up the coefficients of y^n where $n \geq 3$ have larger powers of r_c in the denominator than in the numerator, such that as $r_c \rightarrow \infty$, in the limit these terms go to zero. Consequently, we may reasonably expand the series into only a second-order polynomial in x and y , and ignore higher order terms in finding mean range error. This is again equivalent to assuming planar wavefronts from target echoes.

This yields the expression for range error as

$$\bar{r}_{\mathcal{E},\infty}(\theta) = \frac{\begin{pmatrix} \left(12f^2 - 12fr_{\Delta} + x_1^2 + x_1x_2 + x_2^2 + y_1^2 + y_1y_2 + y_2^2\right) \\ + \left(12f^2 + 12fz_r - x_1^2 - x_1x_2 - x_2^2 - y_1^2 - y_1y_2 - y_2^2\right)\cos\theta \\ - 6f(x_1 + x_2 - 2x_r)\sin\theta\cos\phi \\ - 6f(y_1 + y_2 - 2y_r)\sin\phi \end{pmatrix}}{12f}. \quad (41)$$

Recall that we want this to be zero for all angles. We can achieve this by setting

$$\begin{aligned} x_r &= \frac{x_1 + x_2}{2}, \\ y_r &= \frac{y_1 + y_2}{2}, \\ z_r &= \frac{x_1^2 + x_1x_2 + x_2^2 + y_1^2 + y_1y_2 + y_2^2 - 12f^2}{12f}, \text{ and} \\ r_{\Delta} &= \frac{x_1^2 + x_1x_2 + x_2^2 + y_1^2 + y_1y_2 + y_2^2 + 12f^2}{12f} = z_r + 2f. \end{aligned} \quad (42)$$

We elaborate on several earlier observations.

- We have again made the tacit assumption that the reflector is illuminated uniformly with position x and y . More accurately, we have disregarded any effects of non-uniform illumination of the reflector, or other feed-based illumination effects.
- These parameters all depend on the focal length of the reflector, and the locations of the edges of the reflector in both dimensions. Consequently, even if the focal length remains constant, merely changing the size of the reflector in either dimension can change the location of the optimum antenna reference position.
- The optimum antenna reference position is
 - 1) not at the feed position,
 - 2) not necessarily in any cardinal direction from the feed, and
 - 3) not necessarily even on the dish surface.
 It is located in ‘space’ near the antenna.
- There is an added system delay that must be accounted for to make a proper range measurement.

Example 2.

Consider an offset-fed parabolic dish reflector with the following parameters

$$\begin{aligned} f &= 0.3048 \text{ m,} \\ x_1 &= 0.0635 \text{ m, and} \\ x_2 &= 0.3810 \text{ m.} \\ y_1 &= -0.4191 \text{ m, and} \\ y_2 &= 0.4191 \text{ m,} \end{aligned} \tag{43}$$

where the feed point is at the origin. We shall also assume a 0.036 m wavelength, indicating that the antenna dimensions are large with respect to wavelength. From these parameters we calculate

$$\begin{aligned} x_r &= 0.2223 \text{ m,} \\ y_r &= 0 \text{ m,} \\ z_r &= -0.2094 \text{ m, and} \\ r_\Delta &= 0.4002 \text{ m.} \end{aligned} \tag{44}$$

This is illustrated in Figure 3.

Figure 3. Results of Example 2. The reference phase center is at the small red dot position.

“Risk comes from not knowing what you're doing.”
-- Warren Buffett

4 Comments & Conclusions

For a well-designed radar system, we desire that the following points all coincide.

1. The reference location for the antenna as calculated in previous sections, i.e. the antenna phase center.
2. The point to which system delays are calculated and compensated.
3. The point to which the radar system calculates its motion, i.e. its navigation point.

When we don't achieve coincidence, we degrade system performance.

For example,

- If the range calibration is in error with respect to the navigation point, then the radar echo data will exhibit a range error.
- If the antenna reference location doesn't coincide with the navigation point, then an antenna rotation about the reference will modulate the range measurement.
- If the range calibration is in error with respect to the antenna reference location, then at least one of the above, and perhaps both, errors will manifest.

A byproduct of this is that if the radar uses a constant waveform without any motion compensation, and the antenna rotates about a point other than the antenna reference point, then simply mechanically scanning the antenna will result in an apparent range modulation, i.e. phase modulation of the echo signal. Its significance depends on the requirements of the radar mode implemented as well as the specific offset.

A Note About the Feed

We have also assumed a point feed for the reflector, which might seem somewhat optimistic. A practical feed has its own issues with its phase center in addition to its non-isotropic nature. As the feed's own focal point moves, so too will the pattern of the complete antenna be affected. We note that this can sometimes be compensated by adjusting the curvature of the reflector. Such a system would necessitate a more complete and complex analysis.

“Doing what's right isn't the problem. It is knowing what's right.”
-- Lyndon B. Johnson

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