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Solution of the Monoenergetic Boltzmann
Equation for an Isotropic Point Source in an Infinite
Homogeneous Anisotropically Scattering Medium

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In an earlier work¹, the effect of anisotropic scattering upon the infinite medium diffusion length was examined. In the present note the effect of anisotropic scattering upon the total flux from an isotropic point source is determined and numerical comparisons of several approximations for anisotropic scattering are given for elastic scattering by hydrogen.

If x is measured in mean free paths and the differential cross section of the scatterer can be represented by a N -term Legendre expansion with coefficients b_n , the monoenergetic Boltzmann equation for an infinite homogeneous medium containing an isotropic point source and an anisotropic scatterer may be written:

$$\mu \frac{\partial f}{\partial x} + f = c \sum_{n=0}^N \frac{2n+1}{2} b_n P_n(\mu) \int_{-1}^1 P_n(\mu') f(x, \mu') d\mu' + \frac{\delta(x)}{4\pi}. \quad (1)$$

Above, $c = \Sigma_s / \Sigma_t$. The Fourier transform of $f(x, \mu)$,

$$\phi(\mu, \omega) = \int_{-\infty}^{\infty} f(x, \mu) e^{i\omega x} dx, \quad (2)$$

satisfies the equation

$$(1 - i\omega\mu) \phi = c \sum_{n=0}^N \frac{2n+1}{2} b_n P_n(\mu) \phi_n(\omega) + \frac{1}{4\pi}, \quad (3)$$

where

$$\phi_n(\omega) = \int_{-1}^1 \phi(\mu, \omega) P_n(\mu) d\mu. \quad (4)$$

The total flux, $\psi(x)$, is given by

$$\psi(x) = 2\pi \int_{-1}^1 f(x, \mu) d\mu = \int_{-\infty}^{\infty} e^{-i\omega x} \phi_0(\omega) d\omega. \quad (5)$$

Solving equation (3) for ϕ_0 gives

$$2\pi\phi_0(\omega) = \frac{\frac{\tan^{-1}\omega}{\omega} - \frac{c}{i\omega} \sum_{n=1}^N (2n+1)b_n a_n \left(\frac{1}{i\omega}\right) Q_n\left(\frac{1}{i\omega}\right)}{1 - \frac{c}{i\omega} \sum_{n=0}^N (2n+1)b_n h_n \left(\frac{1}{i\omega}\right) Q_n\left(\frac{1}{i\omega}\right)} \quad (6)$$

In (6) Q_n are the Legendre functions of the second kind and a_n and h_n are polynomials defined by the recursion relation

$$(n+1) R_{n+1} = (2n+1)(1-cb_n) R_n / i\omega - nR_{n-1}, \quad (7)$$

with initial conditions $h_0 = 1$, $a_0 = 0$, and $a_1 = 1/i\omega$. Performing the integration of equation (5) divides the flux into a so-called asymptotic portion due to the singularity of (6) at the root of the denominator, $\omega = i\kappa$,

$$\psi_{as}(x) = \frac{e^{-\kappa|x|} \left\{ \left[c \sum_{n=1}^N (2n+1)b_n a_n \left(\frac{1}{\kappa}\right) Q_n\left(\frac{1}{\kappa}\right) \right] - Q_0\left(\frac{1}{\kappa}\right) \right\}}{\kappa \frac{d}{d\kappa} \left[1 - \frac{c}{\kappa} \sum_{n=0}^N (2n+1)b_n h_n \left(\frac{1}{\kappa}\right) Q_n\left(\frac{1}{\kappa}\right) \right]} \quad (8)$$

and a transient portion due to the essential singularity of Q_0 at $\omega = i$,

$$\psi_{tran}(x) = \frac{1}{2} \int_1^{\infty} \frac{e^{-|x|t} [(1-PA)(t+WH) + (WA)(PH)] dt}{(t+WH - PHQ_0(t))^2 + (\frac{\pi PH}{2})^2}. \quad (9)$$

In (10),

$$PA = \sum_{n=1}^N p_n a_n P_n,$$

$$p_n = c(2n+1)b_n,$$

$$WA = \sum_{n=1}^N p_n a_n W_{n-1},$$

$$Q_n = P_n Q_0 - W_{n-1}, \quad (10)$$

$$WH = \sum_{n=1}^N p_n h_n W_{n-1},$$

$$PH = \sum_{n=0}^N p_n h_n P_n,$$

and all functions are evaluated at $1/t$.

It can be shown, that for a scattering function consisting of an isotropic, a forward, and a backward component,

$$\psi_{\text{FWD-BWD}}(x, c) = \frac{\sqrt{1-cb_1}}{\sqrt{1-cb_2}} \psi_{\text{ISOTROPIC}}(rx, c'), \quad (11)$$

where

$$r = \sqrt{(1-cb_1)(1-cb_2)}, \quad (12)$$

and

$$c' = c(1-b_2)/(1-cb_2). \quad (13)$$

The transport scattering approximation is obtained by setting $b_2 = b_1$.

For small absorption (c near unity), a good approximation² to the isotropic flux is

$$\psi_{\text{ISOTROPIC}}(x, c) = \frac{\kappa(1-\kappa^2) e^{-\kappa|x|}}{c(\kappa^2 - 1 + c)} + \frac{1}{2} E_1\left(\frac{5}{4}|x|\right), \quad (14)$$

which gives an immediate approximation for $\psi_{\text{FWD-BWD}}$. Provided $\psi_{\text{FWD-BWD}}$ is a reasonable approximation of the exact solution for anisotropic scattering, (14) permits a fairly simple representation of the flux which still includes the effect of anisotropic scattering.

Fig. 1 shows the percentage deviation of the flux of several approximations from a P_4 ($N=4$) solution for elastic scattering by hydrogen. The principal effect of anisotropic scattering in this case is to deplete the flux near the source and increase the flux at large distances. That is, because of the forward bias of scattering, neutrons penetrate further into the medium before being absorbed. In this extreme case of anisotropic scattering it is seen that the forward-backward approximation (curve E) deviates less than 2% from the P_4 solution except near the source and hence that it may be possible to include most of the effects of anisotropic scattering by using this simple approximation.

References

1. Lathrop, K. D., ANS Transactions, 6, No. 1, (1963), p. 18.
2. Weinberg, A. M. and Wigner, E. P., The Physical Theory of Neutron Chain Reactors, (1958), p. 243.

Figure Caption

Percentage deviations from P_4 solution. A - Isotropic scattering. B - P_1 scattering approximation. C - P_2 approximation. D - Transport approximation. E - Forward-backward approximation.