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A Division of North American Aviation, Inc.

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ATOMICS INTERNATIONAL A Division of North American Aviation, Inc.		NAA-SR TDR NO 4565	APPROVALS
TECHNICAL DATA RECORD		PAGE 1 OF 18	
AUTHOR J. Susnir		DEPT & GROUP NO 795-52	DATE 3-1-61
			GO NO 7508
TITLE Reactor Vessel Bellows		S A NO 4963	RECOMMENDED FOR OUTSIDE DISTRIBUTION ☐ YES ☐ NO SIGNATURE _____
		TWR 2318	
PROGRAM HMPF	SUBACCOUNT TITLE Reactor Structure		
DISTRIBUTION USE AN ASTERISK (*) TO INDICATE THOSE WHO ARE TO RECEIVE COMPLETE COPIES	STATEMENT OF PROBLEM Establish a uniform basis for evaluating various bellows designs proposed for the HMPF Reactor Vessel.		
*R.C. Gerber *W.T. Morgan *J.E. Mahlwister *T. Ricci *W.V. Haberer *W.J. Sanders *M.P. Heisler *W. Peck *H. Holz *C. Conners *Engineering Mech. Unit (10) *HMPF Project File (7508-4963) *TDR Central File R. Holland-798	ABSTRACT <p>Two types of bellows configurations have been proposed for the HMPF Reactor Vessel. It has been established that the only logical manner of comparing the two designs is on an equal stress basis which implies equal life for each bellows. Results are presented giving the minimum number of convolutions required for each design.</p> <p>The first several copies of page 1 of this TDR are for tear-off and retention. The TDR can then be passed on to others who wish to see it. Please do NOT remove the final copy of page 1.</p>		

I Determination of Stresses Due to Axial Movement

When the ratio of the maximum to the minimum diameter of an annular plate approaches unity, Timoshenko (Ref. 1, Page 68) states that the structure can be analyzed with sufficient accuracy by considering an elemental strip. Therefore, the analysis presented in this report has been based on strip theory.

Configuration A

Consider first the bellows design shown in Fig. 5. A unit width strip of each half convolution can be treated as a guided cantilever beam as shown in Fig. 1.

Figure 1

Using strain-energy relationships, the axial displacement is,

$$\delta = -\frac{\partial U}{\partial F} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial F} ds \quad (1)$$

The rotation at the free end is,

$$\psi = \frac{\partial U}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} dx \quad (2)$$

$$\psi = 0 \text{ for the case being considered}$$

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For convenience, the moment, M , is separated into three parts; i.e.,

$$M_{\theta} = M_0 + FR (1 - \cos \theta) \quad 0 \leq \theta \leq \frac{\pi}{2}$$

$$M_x = M_0 + FR + Fx \quad 0 \leq x \leq L$$

$$M_{\phi} = M_0 + F(R + L) + FR \sin \phi \quad 0 \leq \phi \leq \frac{\pi}{2}$$

Considering EI to be a constant and substituting the above values of M into equations (1) and (2) results in the following:

$$\begin{aligned} EIS = & \int_0^{\pi/2} \left[M_0 + FR (1 - \cos \theta) \right] (1 - \cos \theta) R^2 d\theta \\ & + \int_0^L (M_0 + FR + Fx)(R + x) dx \\ & + \int_0^{\pi/2} \left[M_0 + F(R + L) + FR \sin \phi \right] \left[(R + L) + R \sin \phi \right] R d\phi \quad (3) \end{aligned}$$

and

$$\begin{aligned} 0 = & \int_0^{\pi/2} \left[M_0 + FR (1 - \cos \theta) \right] R d\theta \\ & + \int_0^L (M_0 + FR + Fx) dx \\ & + \int_0^{\pi/2} \left[M_0 + F(R + L) + FR \sin \phi \right] R d\phi \quad (4) \end{aligned}$$

Solving Eq. (4) for F ,

$$F = \frac{-2M_0}{2R + L} \quad (5)$$

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After integration, Eq. 3 reduces to,

$$EI \delta = M_o \left[\pi R^2 + RL \left(1 + \frac{\pi}{2} \right) + \frac{L^2}{2} \right] + P \left[R^3 \left(\frac{3\pi}{2} \right) + R^2 L (3 + \pi) + RL^2 \left(1 + \frac{\pi}{2} \right) + \frac{L^3}{12} \right] \quad (6)$$

Substituting Eq. (5) into Eq. (6),

$$\frac{EI \delta}{M_o L^2} = - \frac{\frac{\pi}{2} \left(\frac{R}{L} \right)^3 + 2 \left(\frac{R}{L} \right)^2 + \frac{\pi}{4} \left(\frac{R}{L} \right) + \frac{1}{12}}{\frac{R}{L} + \frac{1}{2}} \quad (7)$$

Therefore,

$$M_o = - EI \delta C_1 \quad (8)$$

Where

$$C_1 = \frac{\frac{1}{L^2} \left[\frac{R}{L} + \frac{1}{2} \right]}{\frac{\pi}{2} \left(\frac{R}{L} \right)^3 + 2 \left(\frac{R}{L} \right)^2 + \frac{\pi}{4} \left(\frac{R}{L} \right) + \frac{1}{12}}$$

$$= \frac{1}{L^2} \frac{\frac{R}{L} + 0.5}{1.57 \left(\frac{R}{L} \right)^3 + 2 \left(\frac{R}{L} \right)^2 + 0.785 \left(\frac{R}{L} \right) + 0.0834}$$

Since this analysis is based on strip theory, the maximum bending stress in the bellows due to an axial displacement is,

$$\sigma = \frac{M_o t}{2I}$$

$$\sigma = \frac{Et \delta}{2} C_1 \quad (9a)$$

Note that in Eq. (9a) the minus sign has been dropped since it simply represents which side of the bellows is in tension or compression. For a bellows with N convolutions subject to a displacement d , the peak bending stress is,

$$\sigma = \frac{Et d}{4N} C_1 \quad (9b)$$

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The maximum stress in the bellows is a combination of bending and direct stresses and is equal to,

$$\sigma_{\max} = \frac{Etd}{4N} C_1 + \frac{F}{t}$$

or

$$\sigma_{\max} = \frac{Etd}{4N} C_1 \left[1 + \frac{t}{3(2R + L)} \right] \quad (10)$$

Eq. (10) demonstrates that the direct stress component is usually very small (i.e. t is much smaller than R and L) and can be disregarded.

Configuration B

The bellows design shown in Fig. 6 can be analyzed in a manner similar to that used for configuration A. Consider the convolution fixed at the end of the flat portion as shown in Fig. 2, equations (3) and (4) are reduced to,

$$\begin{aligned} EI\delta = & \int_0^{\pi/2} \left[M_o + FR(1 - \cos \theta) \right] (1 - \cos \theta) R^2 d\theta \\ & + \int_0^L (M_o + FR + Fx)(R + x) dx \end{aligned} \quad (11)$$

and

$$\begin{aligned} 0 = & \int_0^{\pi/2} \left[M_o + FR(1 - \cos \theta) \right] R d\theta \\ & + \int_0^L (M_o + FR + Fx) dx \end{aligned} \quad (12)$$

Solution of the equations (11) and (12) result in,

$$F = -\frac{M_o}{L} \left[\frac{\frac{\pi}{2} \left| \frac{R}{L} \right| + 1}{\left| \frac{\pi}{2} - 1 \right| \left| \frac{R}{L} \right|^2 + \left| \frac{R}{L} \right| + \frac{1}{2}} \right]$$

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$$M_0 = - \frac{EI\delta}{L^2} \left[\frac{\left(\frac{\pi}{2} - 1\right) \left(\frac{R}{L}\right)^2 + \frac{R}{L} + \frac{1}{2}}{\left(\frac{\pi^2}{8} - 1\right) \left(\frac{R}{L}\right)^4 + \frac{\pi}{4} \left(\frac{R}{L}\right)^3 + \left(\frac{R}{L}\right)^2 + \frac{\pi}{6} \left(\frac{R}{L}\right) + \frac{1}{12}} \right]$$

Figure 2

The equation relating stress to displacement is in a form similar to Eq. (9b) except that C_1 is replaced by C_2 ,

$$\sigma = \frac{Etd}{4N} C_2 \quad (13)$$

Where

$$C_2 = \frac{\frac{1}{L^2} \left[\left(\frac{\pi}{2} - 1\right) \left(\frac{R}{L}\right)^2 + \frac{R}{L} + \frac{1}{2} \right]}{\left(\frac{\pi^2}{8} - 1\right) \left(\frac{R}{L}\right)^4 + \frac{\pi}{4} \left(\frac{R}{L}\right)^3 + \left(\frac{R}{L}\right)^2 + \frac{\pi}{6} \left(\frac{R}{L}\right) + \frac{1}{12}}$$

$$C_2 = \frac{1}{L^2} \frac{0.571 \left(\frac{R}{L}\right)^2 + \frac{R}{L} + 0.5}{0.231 \left(\frac{R}{L}\right)^4 + 0.785 \left(\frac{R}{L}\right)^3 + \left(\frac{R}{L}\right)^2 + 0.524 \left(\frac{R}{L}\right) + 0.0834}$$

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Since it has been demonstrated that the direct stress component is relatively small, Eq. (13) can be used directly to obtain the maximum displacement stress in the bellows.

II Determination of Stresses Due to Pressurization

Configuration A

the

As in/ previous analysis, a unit width strip is considered as a guided cantilever beam as shown in Fig. 3.

Figure 3

Since the horizontal components of the pressure loading result in hoop tension, or compression, these forces do not affect the analysis of the elemental strip. From symmetry,

$$F = p(R + \frac{L}{2}) \quad (14)$$

Using strain energy relations and noting that for compatibility the rotation at the free end is zero, the following relationship is obtained:

$$\gamma = \frac{\partial U}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} ds = 0 \quad (15)$$

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As was done previously the moment, M , is separated into three parts; i.e.,

$$M_{\theta} = FR (1 - \cos \theta) - M_0 - \frac{pR^2}{2} (1 - \cos \theta)^2 \quad 0 \leq \theta \leq \frac{\pi}{2}$$

$$M_x = F(R + x) - M_0 - \frac{p}{2} (R + x)^2 \quad 0 \leq x \leq L$$

$$M_{\phi} = \left[F - p(R + L) \right] R \sin \phi - \frac{pR^2}{2} \sin^2 \phi + F(R + L) - M_0 - \frac{p}{2} (R + L)^2 \quad 0 \leq \phi \leq \frac{\pi}{2}$$

Substituting the above values of M into Eq. (15), and using the relationship given in Eq. (14), results in the following:

$$\begin{aligned} 0 = & \int_0^{\pi/2} \left[pR \left(R + \frac{L}{2} \right) (1 - \cos \theta) - M_0 - \frac{pR^2}{2} (1 - \cos \theta)^2 \right] R d\theta \\ & + \int_0^L \left[p \left(R + \frac{L}{2} \right) (R + x) - M_0 - \frac{p}{2} (R + x)^2 \right] dx \\ & + \int_0^{\pi/2} \left[- \frac{pRL}{2} \sin \phi - \frac{pR^2}{2} \sin^2 \phi + p \left(R + \frac{L}{2} \right) (R + L) - M_0 - \frac{p}{2} (R + L)^2 \right] R d\phi \end{aligned} \quad (16)$$

Solving Eq. (16) for M_0 ,

$$M_0 = pL^2 \frac{\frac{\pi}{4} \left(\frac{R}{L} \right)^3 + \frac{\pi-1}{2} \left(\frac{R}{L} \right)^2 + \frac{1}{2} \left(\frac{R}{L} \right) + \frac{1}{12}}{\pi \left(\frac{R}{L} \right) + 1} \quad (17a)$$

Let,

$$K_1 = L^2 \frac{\frac{\pi}{4} \left(\frac{R}{L} \right)^3 + \frac{\pi-1}{2} \left(\frac{R}{L} \right)^2 + \frac{1}{2} \left(\frac{R}{L} \right) + \frac{1}{12}}{\pi \left(\frac{R}{L} \right) + 1}$$

$$K_1 = L^2 \frac{0.785 \left(\frac{R}{L} \right)^3 + 1.07 \left(\frac{R}{L} \right)^2 + 0.5 \left(\frac{R}{L} \right) + 0.0834}{3.14 \left(\frac{R}{L} \right) + 1}$$

Then,

$$M_0 = pK_1 \quad (17b)$$

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The maximum bending stress in the bellows is,

$$\sigma = \frac{6M_o}{t^2}$$
$$\sigma = \frac{6p}{t^2} K_1 \quad (18)$$

The maximum stress in the bellows is the sum of the bending and direct components,

$$\sigma_{\max} = \frac{6p}{t^2} K_1 + \frac{p}{t} P_1 \quad (19)$$

Where

$$P_1 = L \left[\left(\frac{R}{L} \right) + \frac{1}{2} \right]$$

The direct stress is relatively small for a typical bellows design (i.e. $t \ll 1$) and can usually be neglected. Therefore, for engineering applications, Eq. (18) can be used to obtain the maximum stress in the bellows.

Configuration B

Considering the convolution to be fixed at one end of the flat portion as shown in Fig. 4, Eq. (16) is reduced to

$$0 = \int_0^{\pi/2} \left[FR (1 - \cos \theta) - M_o - \frac{pR^2}{2} (1 - \cos \theta)^2 \right] R d\theta$$
$$+ \int_0^L \left[F (R + x) - M_o - \frac{p}{2} (R + x)^2 \right] dx \quad (20)$$

Figure 4

To find the force F , the relationship given in Eq. (1) is used. Substituting the values of M_0 and M_x on Page 8 into Eq. (1), and noting that the deflection, δ , is zero results in,

$$\delta = 0 = \int_0^{\pi/2} \left[FR(1-\cos \theta) - M_0 - \frac{pR^2}{2} (1-\cos \theta)^2 \right] (1-\cos \theta) R^2 d\theta$$

$$+ \int_0^L \left[F(R+x) - M_0 - \frac{p}{2} (R+x)^2 \right] (R+x) dx \quad (21)$$

Eliminating M_0 from equations (20) and (21) and solving for F ,

$$F = pL \frac{\left(\frac{\pi^2}{8} - \frac{\pi}{24} - 1 \right) \left(\frac{R}{L} \right)^5 + \left(\frac{\pi}{4} + \frac{1}{3} \right) \left(\frac{R}{L} \right)^4 + \left(1 - \frac{\pi}{16} \right) \left(\frac{R}{L} \right)^3 + \frac{\pi+1}{6} \left(\frac{R}{L} \right)^2 + \left(\frac{\pi}{16} + \frac{1}{12} \right) \left(\frac{R}{L} \right) + \frac{1}{24}}{\left(\frac{\pi^2}{8} - 1 \right) \left(\frac{R}{L} \right)^4 + \frac{\pi}{4} \left(\frac{R}{L} \right)^3 + \left(\frac{R}{L} \right)^2 + \frac{\pi}{6} \left(\frac{R}{L} \right) + \frac{1}{12}}$$

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Substituting the above value for F into Eq. (20) and solving for M_o ,

$$M_o = pL^2 \left[\left(\frac{\pi^3}{64} - \frac{\pi^2}{48} - \frac{\pi}{12} \right) \left(\frac{R}{L} \right)^7 + \left(\frac{3\pi^2}{32} - \frac{5\pi}{24} - \frac{1}{6} \right) \left(\frac{R}{L} \right)^6 \right. \\ \left. + \left(\frac{7\pi}{24} - \frac{\pi^2}{32} - \frac{1}{3} \right) \left(\frac{R}{L} \right)^5 + \left(\frac{\pi}{48} + \frac{1}{3} \right) \left(\frac{R}{L} \right)^4 + \left(\frac{\pi^2}{32} - \frac{\pi}{16} + \frac{1}{6} \right) \left(\frac{R}{L} \right)^3 \right. \\ \left. + \left(\frac{\pi}{12} - \frac{1}{12} \right) \left(\frac{R}{L} \right)^2 + \left(\frac{1}{24} + \frac{\pi}{288} \right) \left(\frac{R}{L} \right) + \frac{1}{144} \right] \left[1 / \left\{ \left(\frac{\pi^2}{8} - 1 \right) \left(\frac{R}{L} \right)^4 \right. \right. \right. \\ \left. \left. + \frac{\pi}{4} \left(\frac{R}{L} \right)^3 + \left(\frac{R}{L} \right)^2 + \frac{\pi}{6} \left(\frac{R}{L} \right) + \frac{1}{12} \right\} \left\{ \frac{\pi}{2} \left(\frac{R}{L} \right) + 1 \right\} \right]$$

The maximum stress at the apex of the bellows is,

$$\sigma_{\max} = \frac{6p}{t^2} K_2 + \frac{p}{t} P_2 \quad (22a)$$

Where:

$$K_2 = L^2 \left[0.017 \left(\frac{R}{L} \right)^7 + 0.104 \left(\frac{R}{L} \right)^6 + 0.275 \left(\frac{R}{L} \right)^5 + 0.399 \left(\frac{R}{L} \right)^4 + 0.279 \left(\frac{R}{L} \right)^3 + 0.179 \left(\frac{R}{L} \right)^2 \right. \\ \left. + 0.0525 \left(\frac{R}{L} \right) + 0.00694 \right] \left[1 / \left\{ 0.231 \left(\frac{R}{L} \right)^4 + 0.785 \left(\frac{R}{L} \right)^3 + \left(\frac{R}{L} \right)^2 + 0.524 \left(\frac{R}{L} \right) + 0.0834 \right\} \left\{ 1.57 \left(\frac{R}{L} \right) + 1 \right\} \right] \\ P_2 = L \frac{0.100 \left(\frac{R}{L} \right)^5 + 0.452 \left(\frac{R}{L} \right)^4 + 0.804 \left(\frac{R}{L} \right)^3 + 0.691 \left(\frac{R}{L} \right)^2 + 0.279 \left(\frac{R}{L} \right) + 0.0416}{0.231 \left(\frac{R}{L} \right)^4 + 0.785 \left(\frac{R}{L} \right)^3 + \left(\frac{R}{L} \right)^2 + 0.524 \left(\frac{R}{L} \right) + 0.0834}$$

As was demonstrated before, the direct stress component in Eq. (22a) can be neglected and the maximum stress can be adequately represented by,

$$\sigma_{\max} = \frac{6p}{t^2} K_2 \quad (22b)$$

The maximum moment at the root of the bellows is,

$$M_r = M_o + \frac{p}{2} (R + L)^2 - F (R + L) \quad (23)$$

Consequently, the maximum stress at the root of the bellows is,

$$\sigma_{\max} = \frac{6p}{t^2} K_2 + \frac{3p}{t^2} (R + L)^2 - \frac{6p}{t^2} P_2 (R + L)$$

or

$$\sigma_{\max} = \frac{6p}{t^2} \left[K_2 + \frac{1}{2} (R + L)^2 - P_2 (R + L) \right] \quad (24)$$

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III Evaluation of HMPF Reactor Vessel Bellows

Three bellows configurations were proposed for the HMPF Reactor Vessel. Two were of the type shown in Fig. 5 with the following dimensions:

No. 1: $L = 2.187$ in., $R = 0.688$ in., $t = 0.093$ in.

No. 2: $L = 3.625$ in., $R = 0.688$ in., $t = 0.093$ in.

The third configuration was of the type shown in Fig. 6 with the following dimensions:

No. 3: $L = 2.125$ in., $R = 0.375$ in., $t = 0.050$ in.

The maximum stress in each type of bellows is in the form

$$\sigma = \sigma_d + \sigma_p$$

or

$$\sigma = \frac{E \Delta d}{4N} C + \frac{6p}{t^2} K$$

The principle bellows parameters, C and K, are plotted in Fig. 7 for convenience.

During normal reactor operation, the thermal expansion of the reactor vessel and supports will be approximately 2-3/4 in. at the bellows. Consequently, it was decided to cold-spring the bellows 2-3/4 in. so that the displacement stress during normal operations would be approximately zero. The design pressure at normal operating conditions is 5 psi.

Based on information given in Ref. 2, the limiting stress for the cold-spring was 90,000 psi (i.e. twice the yield strength). The stress due to pressure was limited to 18,750 psi which is 1-1/2 times the ASME Boiler Code Allowable for Type 304 SS at 1000 F. Table I summarizes the design information for the various bellows. Table II indicates the stress levels and the number of convolutions required. It can be seen in Table II that the bellows are stressed to approximately their limit for displacement stresses (σ_d) and a large margin exists for pressure stresses, (σ_p).

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TABLE I
Design Data

Bellows No.	Type	L	L ²	R	t	R/L	CL ²	C	K/L ²	K
1	A	2.187	4.80	0.594	0.093	0.271	1.57	0.327	0.172	0.826
2	A	3.625	13.2	0.688	0.093	0.190	2.14	0.162	0.142	1.87
3	B	2.125	4.53	0.375	0.050	0.176	3.28	0.724	0.110	0.498

TABLE II

Bellows No.	Type	Room Temperature						Operating Conditions (945 F outlet)				
		d (in.)	p (psi)	∇d^* (psi)	∇p (psi)	$\nabla d, \text{Allow.}$ (psi)	$N_{\text{req.}}$	d (in.)	p (psi)	∇d (psi)	∇p (psi)	$\nabla p, \text{Allow.}$ (psi)
1	A	2.75	0	83,500	0	90,000	7	0	5	0	2900	18,750
2	A	2.75	0	72,500	0	90,000	4	0	5	0	6500	18,750
3	B	2.75	0	87,000	0	90,000	8	0	5	0	6000	18,750

* ∇d was obtained using $E = 28 \times 10^6$ psi

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REFERENCES

1. Timoshenko, S., "Theory of Plates and Shells," McGraw-Hill Book Co., New York 1940.
2. Fenley, A. B., TDR 4826, "Physical Properties of Material from Zallea Brothers Test Bellows."

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NOMENCLATURE:

d	Total Bellows Displacement (in.)
E	Young's Modulus (psi)
F	Force (lb)
I	Section Moment of Inertia (in. ⁴)
L	Length of Straight Portion of Bellows (in.)
M	Bending Moment (in-lb)
N	Number of Convolutions in Bellows
p	Pressure (psi)
R	Mean Radius of Bellows Convolution (in.)
t	Thickness of Bellows Material
U	Strain Energy (in-lb)
ds	Incremental length
x, θ , ϕ	Coordinates
C_1 , C_2 , K_1	Bellow's Parameters
K_2 , P_1 , P_2	
δ	Displacement (in.)
γ	Rotation (radians)
σ	Direct Stress (psi)
σ_d	Displacement Stress (psi)
σ_p	Pressure Stress (psi)

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Configuration A
Roll Formed Expansion Joint
Figure 5

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Configuration B

Welded Expansion Joint

Figure 6

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FIG. 7