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AUGUST, 1970

Contract No. AT (11-1)-264

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DUAL RESONANCE MODEL WITH QUARK SPIN⁺

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ABSTRACT

We present a dual resonance model with nontrivial quark spin factors. An amplitude is found which satisfies factorization and eliminates the parity doubling ghosts. An application to $\pi\pi$ elastic scattering indicates that the positivity condition is not met on the first daughter trajectory if one assumes realistic values for the mass and intercept.

⁺Supported in part by the U. S. Atomic Energy Commission.

^ø Submitted to the Department of Physics, The University of Chicago, in partial fulfillment of the requirements for the Ph. D. degree.

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1. Introduction

Recent developments of the dual resonance model^{1,2} have revealed a close connection of the duality concept with the quark model.^{3,4,5} It is now possible to embark on construction of a hadron model out of quarks in the manner represented by the Harari-Rosner quark diagram.^{6,7}

A crucial step in this program is proof of the factorization property of the dual resonance model. The proof of factorization has been extended to all the daughter trajectories.^{8,9} The resonance spectrum in the model has been greatly clarified using the simple device of the harmonic oscillator.^{10,11,12} Roughly speaking, mesons appear to be bound states of the quark and antiquark with a relativistic string between them.

Previous attempts^{4,5} to incorporate quark spin into the dual model have suffered from a serious disease. Consistent factorization of the spin factor considered in that approach demands existence of ghosts associated with negative parity quarks. On the other hand, a recent work of Carlitz and Kislinger¹³ provided a new way to avoid parity doubling of the fermion trajectory in the Van Hove model. Motivated by this work many people have proposed to dualize the projection operator to eliminate parity doubling ghosts.^{14,15,16} We will present in this paper a different, but closely related approach to treat the quark spin correctly.

Our guiding principle to select a spin factor is the simple overall picture of the dual resonance model of Ref.

(10)-(12). After constructing a cyclically symmetric amplitude of mesons we proceed to check factorization of the whole amplitude. A simple quark propagator considered in Sec. 2 turns out to eliminate the parity doubling ghosts from the leading trajectory only. A generalization of the propagator is then suggested, and elimination of ghosts from all the trajectories, as well as complete factorization, is proved in Sec. 3. The generalized amplitude resembles a recent model of Carlitz, Ellis, Freund and Matsuda.¹⁶ The main difference lies in that we insist on the original form of the quark projector, and therefore factorize the meson projection operator into a product of quark and antiquark parts.

In applying our general construction to multimeson processes we have to introduce the π - ρ splitting more or less by hand. Since our model gives the spectral pattern of static SU(6) and the vertex structure of SU(6)_w, a simple prescription of breaking the ideal symmetry will be to let only the intercepts be different from the degenerate value, like in the broken SU(6). We explicitly calculate the $\pi\pi$ elastic amplitude from the quark model under this prescription. Unfortunately, this idea of symmetry breaking turns out not to work satisfactorily in this specific application, and new ghosts with imaginary coupling are encountered on the daughter levels. This new difficulty is most probably a common defect of any dual quark model based on the known scalar amplitude.

Throughout this paper we consider the meson sector of the hadron model, and assume external mesons in the ground state although generalization to mesons in excited states is straightforward if we neglect the fundamental π - ρ splitting. Contribution of the Pomeron exchange is separated out in the fashion suggested in Ref. (17).

2. Simple Spin Factor

Let us first write the dual amplitude with spin factors in a form appropriate to our discussion. Consider the $2N$ -point quark amplitude corresponding to a definite ordering of external particles, as in Fig. 1:

$$F_N = \int \prod_{i=2}^{N-2} dw_{i2} \prod_{1 \leq i < j \leq N-1} w_{ij}^{-\alpha(ij)-1} \Delta_{cd}^{ab}(\gamma \cdot p_{ij}, w_{ij}) \quad (1)$$

An external pair of neighboring quark and antiquark is assumed to compose a ground state meson. Except for the spin factor Δ the integrand is in the well known form of the dual resonance model.² A combination (ij) labels a mesonic channel corresponding to a partition $(i, i+1, \dots, j)$ $(j+1, j+2, \dots, i-1)$ of N external mesons, and $P_{ij} = p_i + p_{i+1} + \dots + p_j$, where p_i is momentum of i -th meson. The Regge trajectory of the orbital part is assumed linear, and the momenta are measured in unit of the common trajectory slope ($\approx 1(\text{Gev}/c)^2$). $\alpha(ij) = p_{ij}^2 + \alpha_0$. Those spinor indices, $a \sim d$, which belong to external quarks, are contracted together with new dummy indices in the manner specified below. We take the metric, $(+---)$, and the Dirac matrices with γ_0 hermitian and γ antihermitian.

We will suppress the $SU(3)$ indices of quarks, understanding that a factor proposed in Ref. (4) and (5), $I = \prod_i \delta_{\lambda(i), \mu(i)}$, can be inserted whenever necessary in front of the integral of Eq. (1), where the product runs over all continuous quark lines.

Among many conceivable spin factors particularly interesting is a simple quark propagator, $w^{\gamma \cdot p}$, where w is an integration variable corresponding to a channel with momentum p . In the light of the underlying physical picture presented in Ref. (10)-(12) quarks are regarded as propagating on the periphery of the quark diagram with simultaneous exchange of "oscillons" inside.¹⁸ Specifically we consider the following structure for the spin factor:

$$\Delta_{ca}^{ab}(\gamma p, w) = d_a^a(\gamma p, w) d_c^{\dagger}(-\gamma p, w)$$

$$d(\gamma p, w) = w^{\gamma(1-\gamma p/\sqrt{p^2})} \quad (2)$$

The spinor indices associated with the quark lines are shown in Fig. 2(a) where a directed line corresponds to a quark and the other one to an antiquark. g is an arbitrary positive constant which characterizes the strength of the spin coupling.

The meaning of Eq. (2) becomes clear when one notes an identity:

$$d(\gamma p, w) = \frac{1}{2}(1+w^2)^{\gamma} + \frac{1}{2}(1-w^2)^{\gamma} \gamma p/\sqrt{p^2} \quad (3)$$

The right hand side reduces to $\frac{1}{2}(1+\gamma \cdot p/\sqrt{p^2})$ when $w = 0$ and 1 when $w = 1$. This means that when one expands the integrand of (1) in terms of powers of w and looks at poles in p^2 , (i.e., $w \rightarrow 0$) one obtains an extra projection operator, $\frac{1}{2}(1+\gamma \cdot p/\sqrt{p^2})$, for each quark, at least on the level of

leading poles. The behavior near $w = 1$ is needed to avoid picking up any irrelevant factor from the dual channels.

It is necessary to rearrange indices in (2) in order to exhibit the spin structure of intermediate mesonic states:

$$\Delta_{cd}^{ab} = \sum_{\xi, \xi'}^{16} (\Gamma_{\xi})_c^a (\tilde{\Gamma}_{\xi'})_d^b f_{\xi\xi'} \\ f_{\xi\xi'} = \text{Tr} [\tilde{\Gamma}_{\xi} d(\gamma p) \Gamma_{\xi'} d(-\gamma p)]^{19} \quad (4)$$

where we have used the sixteen Dirac matrices Γ_{ξ} with the normalization $\text{Tr} \Gamma_{\xi} \tilde{\Gamma}_{\xi'} = g_{\xi\xi'}$ and the definition $\tilde{\Gamma} = \gamma_0 \Gamma^{\dagger} \gamma_0$ ($\Gamma^{\dagger}$ = hermitian conjugate of Γ). When one diagonalizes so that $f_{\xi\xi'} = 0$ for $\xi \neq \xi'$, f_{ξ} represents projection on the meson of type ξ , and $(\Gamma_{\xi})_c^a$ expresses coupling of the initial state (ac) to the intermediate state (ξ). This situation is shown schematically in Fig. 2(b).

The diagonalization can be performed directly, and we find the following bases for Γ'_{ξ} and the functions f :

$$\begin{aligned} \text{(I)} \quad \Gamma_{\xi} &= \gamma_5 (1 - \gamma p / \sqrt{p^2}) / \sqrt{2}, \quad (\gamma_{\mu} - p_{\mu} \gamma / p^2 + i \sigma_{\mu\nu} p^{\nu} / \sqrt{p^2}) / \sqrt{2} \\ \text{and} \quad f_{\xi} &= -1, \quad g_{\mu\nu} - p_{\mu} p_{\nu} / p^2, \quad \text{respectively} \\ \text{(II)} \quad \Gamma_{\xi} &= \gamma_5 (1 + \gamma p / \sqrt{p^2}) / \sqrt{2}, \quad (\gamma_{\mu} - p_{\mu} \gamma / p^2 - i \sigma_{\mu\nu} p^{\nu} / \sqrt{p^2}) / \sqrt{2} \\ \text{and} \quad f_{\xi} &= -w^{\dagger} \gamma, \quad (g_{\mu\nu} - p_{\mu} p_{\nu} / p^2) w^{\dagger} \gamma, \quad \text{respectively} \\ \text{(III)} \quad \Gamma_{\xi} &= 1, \quad p \gamma / \sqrt{p^2}, \quad \gamma_5 (\gamma_{\mu} - p_{\mu} \gamma / p^2) / \sqrt{2}, \quad i \gamma_5 \sigma_{\mu\nu} p^{\nu} / \sqrt{p^2} \\ \text{and} \quad f_{\xi} &= w^{\dagger} \gamma \end{aligned} \quad (5)$$

Thus, the intermediate particles we encounter are: (I), (II) a pseudoscalar and a vector with different couplings to quarks in the two cases; (III) two scalars with normal ($C = +1$) and abnormal ($C = -1$) charge conjugation and two axial vectors with $C = +1$ (like A_1) and $C = -1$ (B). By comparing the signs of these couplings with those obtained from the Feynman rules we know that all scalar and axial vector particles are ghosts with imaginary coupling, whereas pseudoscalars and vectors are real particles.²¹

Before proceeding to elimination of these ghosts, let us notice that the wave functions we have obtained in (I) satisfy the Bargman-Wigner equation²²:

$$(\gamma p - \sqrt{p^2}) \Gamma_i = \Gamma_i (\gamma p + \sqrt{p^2}) = 0 \quad (6)$$

This is directly proved by rewriting the expressions of (I) as follows:

$$\Gamma_i = \frac{1}{\sqrt{2}} (1 + \gamma p / \sqrt{p^2}) \gamma_5 P + \frac{1}{\sqrt{2}} (1 + \gamma p / \sqrt{p^2}) (\gamma_r - p_r / \sqrt{p^2}) V^r \quad (7)$$

Combining the $SU(3)$ indices in the way mentioned we immediately observe that the states in the first class represent the nonets of pseudoscalar P and vector meson V compatible with the $SU(6)$ symmetry pattern. Other states in the class II or III are obtained by reversing the parities of the constituent quark and/or antiquark. These particles in II and III do not seem to exist in nature. (Note that the A_1 meson, for example, is regarded as a Regge recurrence of ρ with $l = 1$.)

Now we remark a significant role of our quark propagator. It shifts the trajectories of unwanted states downward parallelly²³ and makes the contributions of those trajectories less significant provided g is positive and sufficiently large ($g \gg 1$). It is therefore expected that ghosts are eliminated at least from the leading trajectory. The elimination from the leading trajectory may be sufficient for phenomenological purpose although it does not assure complete elimination. This point must however be checked in consistency with factorization.

Next we describe how to couple the spinor indices of the N meson amplitude. The coupling scheme we consider is very general and may be applicable beyond our special spin factor. We associate a spin factor $\Delta_{cd}^{ab}(ij)$ with any pair (ij) of $\frac{1}{2} N(N-3)$ internal meson channels made of quark and antiquark pairs. Since there are only $2N$ available spinor indices of the external quarks and antiquarks, we have to introduce more indices and contract them. (The case with $N = 4$ is the only exception in which we do not need these new indices.) This is visualized most conveniently with the aid of the multiperipheral diagram in Fig. 3. In this diagram the vertical lines are not meant to represent the external mesons, but are to be interpreted as spurions attached to a quark line. The rule is as follows: (i) Arrange the external meson wave functions M_i in a definite order already specified. (ii) Insert $(N-3)$ quark propagators d between two neighboring M 's (M_i and M_{i+1} in the case of Fig. 3). The contraction

scheme follows the labels in the diagram, and reads

$$\dots (M_i)_{a(i)}^{b(i)} d_{b(i)}^{c(i)}(i-1, i) d_{c(i)}^{c(i+1)}(i, i+1) \dots d_{c(i+1)}^{a(i+1)}(i+1, i+2) (M_{i+1})_{a(i+1)}^{b(i+1)} \dots$$

We have used a's and b's for external spinors and c's for new indices. $d(i, j)$ is the quark propagator in the (ij) channel, that is, $d(\gamma \cdot p_{ij}, w_{ij})$. It is easy to see that we have exhausted all the spin factors that were introduced and the final result is just a trace over the indices. It will be instructive to give an example of the six point function to understand how many factors are involved.

$$S_6 = \text{Tr} [M_1 d(61) d(51) d(41) M_2 d(12) d(62) d(52) M_3 d(23) d(13) d(63) \\ \times M_4 d(34) d(24) d(14) M_5 d(45) d(35) d(25) M_6 d(56) d(46) d(36)] \quad (9)$$

The substitution of the spin factor so constructed in (1) completes the model. It is obvious that the full amplitude satisfies the duality in the sense that it is cyclically symmetric under the permutation, $M_i \rightarrow M_{i+1}$ and $p_i \rightarrow p_{i+1}$, and also under the anticyclic permutation. This is because one can cyclically permute terms within a trace without any change, and the remaining integrand of (1) shares the same symmetry. Proof of the invariance under the anticyclic permutation is also easy if one exploits the charge conjugation matrix.

Application of the above general rules to our specific propagator (2) gives our first model. We have to check

properties such as factorization, analyticity etc..before accepting it as a physical amplitude. First we investigate the factorization property (particle content). Corresponding to the division of $(r+s)$ legs and choice of independent variables in Fig. 4 we carry out the known decomposition^{8,9} leaving the spin factor S_N :

$$F_N = \int dx \varphi_{n+1}(x, p_L) \int dy \varphi_{n+1}(y, p_R) \int_0^1 dz z^{-d(1,n)-1} (1-z)^{d_0-1} \times \prod_{i,j} (1-x_i \dots x_{n-2} z y_{n-2} \dots y_j)^{-2p_{i+1} p_{n-j}} S_N \quad (10)$$

The φ 's are the scalar dual amplitudes on the left and right. The factorization is achieved when one expands the last integrand of (10) in powers of z to exhibit poles in $d(1,r)$ and succeeds in expressing the residue as a finite sum of left and right variables. That this is the case for the scalar model without S_N is proved in Ref. (8) and (9).

To enforce factorization of S_N we exploit the simple diagonalization property (5) and perform the diagonalization for all meson channels. For the channel (ij) , where i belongs to the left and j to the right, we pick up $d(j+1, i-1)$ between M_{i-1} and M_i (as in (8)) and $d(i, j)$ between M_j and M_{j+1} , and rearrange indices using (4) and (5):

$$d_{c_1}^{c_2}(i, j) d_{c_4}^{c_3}(j+1, i-1) = \Delta_{c_4 c_1}^{c_2 c_3}(i, j) = \sum_s (\Gamma_{ij}^s)_{c_4}^{c_2} (\tilde{\Gamma}_{ij}^s)_{c_1}^{c_3} f_s(i, j)$$

Since dependence on the integration variables is separated out in the f 's, we are able to deal with the orbital excitation alone. Just writing factors which depend both on the left and right variables we have a sum of the following forms with complicated coefficients for S_N :

$$(1 - x_i \cdots x_{n-2} \sum y_{s-2} \cdots y_j)^{\varepsilon(ij)}$$

$$\varepsilon(ij) = \delta_{s(i+2, n-j-1)} + \delta_{s(i+1, n-j)} - \delta_{s(i+2, n-j)} - \delta_{s(i+1, n-j-1)}^{(11)}$$

$\mathfrak{J}$'s denote the class of trajectories in (5) for the relevant channels, and δ_s is 0, 4g or 2g correspondingly whether $\mathfrak{J}$ belongs to I, II or III. It is then necessary to express $\varepsilon(ij)$ as a sum of products of left and right variables in order to maintain the factorization on all trajectories. This is impossible in our general case since the cross channels appearing in (11) choose their class of trajectories in a disorderly fashion. We also note that the coefficients in front of (11) do not vanish in this case.

It is, however, possible to show the factorization consistently on the leading trajectory provided $g > 1$. For the leading trajectory we are not interested in the z dependence except for $z^{-\alpha(1,r)-1}$. The spin factor therefore reduces to a product of the left, right parts and the projection operator for the intermediate states of I. In the example of the six point function the factors in Eq. (9) get simplified by factorizing on the line (13):

$$\begin{aligned} S_6 &= \left[d(-12) M_1, d(-23) M_2, d(12) M_3, d(13) \right]_a^t \Delta_{b,d}^{ac(13)} \\ &\quad \times \left[d(-45) M_4, d(-56) M_5, d(45) M_6, d(56) \right]_c^d \\ &= S_4(M_1, M_2, M_3, \Gamma_1) S_4(M_4, M_5, M_6, \Gamma_1) \end{aligned} \quad (12)$$

The unwanted trajectories, II and III, corresponding to the "line z" have been pushed away downward by more than one unit and eliminated from the leading trajectory. Thus our model, when limited to the leading trajectory, satisfies factorization as well as duality, and is free from ghosts associated with negative parity quark.

The above argument suggests taking a limit, $g \rightarrow \infty$ to obtain complete factorization. This procedure poses a delicate mathematical problem of interchange of the limit and integration, however. The limit does not seem to be easily implemented. It is worth mentioning, nevertheless, that in spite of our failure the correct projection may come out as a limit of strong coupling. Alternatively, we would like to suggest in the next section more abstract approach to modify the quark propagator (3) using the technique of neutralizer.

Another property easy to establish is the Regge behavior in one Mandelstam variable. The leading high energy behavior is essentially the same as in the scalar amplitude aside from a multiplicative spin factor. For the four point case the amplitude behaves like

$$F_4 \sim \Gamma(-\alpha_t) \Gamma(-\alpha_s)^{d_s} \text{Tr}[M_2 M_3 \Gamma_2(p_t)] \text{Tr}[M_4 M_1 \Gamma_1(-p_t)] \quad (13)$$

as $|s| \rightarrow \infty$ except on the real axis

The more detailed analytical structure in the complex l plane will be discussed in the next section, in connection with the presence of fixed cuts.

A problem of our amplitude is existence of the spurious cut $(p^2)^{-1/2}$ which can potentially produce ancestors in the dual channels. This is cured easily by modifying the $(p^2)^{-1/2}$ in the definition of the quark propagator in the following manner. Focusing on the relevant singularity we want to alter

$$\begin{aligned} & \int dw_i' \int_0^1 dw (p^2)^{-1/2} w^{-d(p^2)-1} f(\dots, w, \dots) \\ &= \int dw_i' (p^2)^{-1/2} \sum_{n=0}^{\infty} \frac{1}{n! [n-d(p^2)]} \left[\frac{\partial^n f}{\partial w^n} \right]_{w=0} \end{aligned}$$

to a different form

$$\int dw_i' \sum_{n=0}^{\infty} \frac{1}{n! [n-d(p^2)] (n-d_0)^{1/2}} \left[\frac{\partial^n f}{\partial w^n} \right]_{w=0}$$

The square root factors appear only at the resonance poles and give smooth behavior elsewhere. The extra square root is transformed into a power by use of the integral:

$$A^{-1/2} = \frac{2}{\sqrt{\pi}} \int_0^{\infty} dy e^{-Ay^2}$$

We then have

$$\begin{aligned} & \frac{2}{\sqrt{\pi}} \int dw_i' \int_0^1 dw \int_0^{\infty} dy w^{-d(p^2)-1} f(\dots, w e^{-y^2}, \dots) e^{d_0 y^2} \\ &= \frac{2}{\sqrt{\pi}} \int dw_i' \int_0^{\infty} dy \int_0^{e^{-y^2}} dw w^{-d(p^2)-1} e^{-p^2 y^2} f(\dots, w, \dots) \\ &= \frac{2}{\sqrt{\pi}} \int dw_i' \int_0^1 dw w^{-d(p^2)-1} f(\dots, w, \dots) (p^2)^{-1/2} \int_0^{(-p^2 \ln w)^{1/2}} dy e^{-y^2} \end{aligned}$$

We can therefore set up a substitution rule: whenever $(p^2)^{-1/2}$ appears, this is replaced by $\frac{2}{\sqrt{\pi}} (p^2)^{-1/2} \text{Erf} [(-p^2 \ln w)^{1/2}]$ or $\frac{2}{\sqrt{\pi}} (-\ln w)^{1/2} {}_1F_1(1/2, 3/2; p^2 \ln w)$ where ${}_1F_1(\alpha, \beta; z)$ denotes the confluent hypergeometric function. For later convenience we define the new function:

$$\mu^-(w, p^2) \equiv \frac{2}{\sqrt{\pi}} (-\ln w)^{1/2} {}_1F_1\left(\frac{1}{2}, \frac{3}{2}; p^2 \ln w\right) \quad (14)$$

From the asymptotic behavior of ${}_1F_1$, $\mu^{-1} \sim (p^2)^{-1/2}$ when $w \approx 0$. Furthermore using the series expansion we obtain near $w = 1$

$$\mu^-(w, s) = \sum_{n=0}^{\infty} \gamma_n(s) (1-w)^{n+\frac{1}{2}}$$

where $\gamma_n(s)$ is a n -th order polynomial of s . This assures that we do not introduce any ancestors.

3. Generalization and Application to Multipion Processes

The failure to maintain the factorization on all the trajectories in our previous model is traced back to the fact that we picked up powers of z from the dual channels as shown in (11) when we factorized on the channel " z ." This is remedied by generalizing the quark propagator (3):

$$d(xp, w) = f(w) + xp \mu'(w, p^2) g(w) \quad (15)$$

The conditions which the functions f and g have to satisfy are²⁴

$$\begin{aligned} f(0) = g(0) &= \frac{1}{2}, \quad f(1) = 1, \quad g(1) = 0 \\ f^{(n)}(w) = g^{(n)}(w) &= 0 \quad \text{at } w = 0, 1; \quad n = 1, 2, 3, \dots \end{aligned} \quad (16)$$

They are easily met by using a class of functions called van der Corput neutralizers.²⁵ Using such a neutralizer $h(w)$ we only need identify

$$f(w) = \frac{1}{2} [1 + h(w)], \quad g(w) = \frac{1}{2} [1 - h(w)] \quad (17)$$

where $h(0) = 0$, $h(1) = 1$, $h^{(n)}(w) = 0$ at $w = 0, 1$. An example sufficient for our subsequent purpose is given by

$$\begin{aligned} h(w) &= j(w) / j(1) \\ j(w) &= \int_0^w dz (-\ln z)^{\ln 2} [-\ln(1-z)]^{\ln(1-z)} k(z) \end{aligned} \quad (18)$$

where $k(z)$ is an arbitrary regular function in $0 \leq z \leq 1$ satisfying $k(z) = k(1-z)$. Although this choice is not unique, most of the basic properties of the amplitude turn out to be independent of the choice and determined solely by the conditions (16). A property of the formula (18), $h(w) = 1-h(1-w)$, will simplify some later calculations. With this new quark propagator we can show

$$\begin{aligned} \Delta_{c,d}^{ab}(w=0) &= \left[\frac{1}{2} (1 + \gamma P / \sqrt{p^2}) \right]_d^a \left[\frac{1}{2} (1 - \gamma P / \sqrt{p^2}) \right]_c^b \\ &= \left[\gamma_5 (1 - \gamma P / \sqrt{p^2}) / \sqrt{2} \right]_c^a \left[\gamma_5 (1 + \gamma P / \sqrt{p^2}) / \sqrt{2} \right]_d^b + (\gamma_{\mu\nu} - p_\mu p_\nu / p^2) \\ &\quad \times \left[(1 + \gamma P / \sqrt{p^2}) (\gamma^\mu - p^\mu / \sqrt{p^2}) / \sqrt{2} \right]_c^a \left[(1 - \gamma P / \sqrt{p^2}) (\gamma^\nu + p^\nu / \sqrt{p^2}) / \sqrt{2} \right]_d^b \end{aligned} \quad (19)$$

$$\Delta_{c,d}^{ab}(w=1) = 1$$

$$\Delta_{c,d}^{(n)ab}(w) = 0 \quad \text{at } w = 0, 1 \quad (20)$$

The second line of (19) indicates explicit decomposition into the Barman-Wigner bases, which is sufficient together with (20) to guarantee complete elimination of parity doubling ghosts from all the trajectories. It is also trivial to prove complete factorization since the condition (20) forbids any factor like (11) to come out from the channels dual to z near $z = 0$.

To advance construction of multimeson amplitudes one has to specify the wave functions of the external mesons. For external mesons in the ground state we may simply use

$$M_i = \gamma_5 (1 - \gamma \not{p}_i / m) \quad \text{for pseudoscalar} \quad (21)$$

$$M_i^\mu = (1 + \gamma \not{p}_i / m) (\gamma^\mu - \not{p}_i^\mu / m) \quad \text{for vector} \quad (22)$$

where m represents a common mass of external mesons. This wave function can be obtained by boosting appropriate direct product of two spinors of the quark and antiquark system from the rest frame. In the following we focus our attention upon four pion amplitude. This case is interesting since we can compare our result with the Lovelace amplitude.²⁶

Our 4π amplitude is explicitly

$$F_4 = \int_0^1 dx \, x^{-\alpha_4} (1-x)^{-\alpha_5} S_4$$

$$S_4 = \text{Tr} [M_1 d(-\not{p}_1, 1-x) M_2 d(\not{p}_1, x) M_3 d(\not{p}_2, 1-x) M_4 d(-\not{p}_2, x)] \quad (23)$$

where $p_s = p_1 + p_2$, $p_t = p_2 + p_3$. (Refer to Fig. 5 for the ordering of the spin factors.) We have put $\alpha + 1$ instead of α so that α means the total angular momentum of the $\pi\pi$ states rather than the orbital angular momentum of $q\bar{q}$. Substituting (15) and (21) in (23) we obtain after a long, but straightforward calculation

$$\begin{aligned}
 S_4 = & [f(x)f(1-x)]^2 [8-2(s+t)-\frac{1}{2}st] + 2f(x)\tilde{g}(x,s)f^2(1-x) \\
 & \times s(4-s-2t) + 2f^2(x)f(1-x)\tilde{g}(1-x,t)t(4-2s-t) - \tilde{g}^2(x,s)f^2(1-x) \\
 & \times s(-2s+2t+\frac{1}{2}s^2+\frac{1}{2}st) - f^2(x)\tilde{g}^2(1-x,t)t(2s-2t+\frac{1}{2}t^2+\frac{1}{2}st) \\
 & - 2f(x)\tilde{g}(x,s)f(1-x)\tilde{g}(1-x,t)st(s+t) - 2f(x)\tilde{g}(x,s)\tilde{g}^2(1-x,t) \\
 & \times st(s+t) - 2\tilde{g}^2(x,s)f(1-x)\tilde{g}(1-x,t)st(s+t) \\
 & + \frac{1}{4}\tilde{g}^2(x,s)\tilde{g}^2(1-x,t)st[st-2(s+t)^2]
 \end{aligned}$$

(24)

where $s = p_s^2$, $t = p_t^2$ and $\tilde{g}(x,s) = g(x)\mu^{-1}(x,s)$. We have omitted $1/m$ in front of each momentum as well as μ to simplify the expression, thus $s\mu^{-1}$ should be understood as $s\mu^{-1}/m$ etc. It is easy to check that S_4 explicitly fulfill the cyclic symmetry under the simultaneous interchange, $x \leftrightarrow 1-x$ and $s \leftrightarrow t$.

Let us first study the singularity structure in s of our amplitude. The only singularities of the integral representation (23) come from the end points where the integrand diverges. Near the point, $x = 0$, relevant to the s channel poles we may drop some of the terms in S_4 using the relations (16) and replace (24) by

$$\begin{aligned}
 S_4 &= 8 - 4t - 5t - 2\sqrt{3}(s+2t-4) - \frac{1}{2}s^2 \\
 &= \frac{1}{2}(\sqrt{3}+2)^2(-s-2t+4)
 \end{aligned}
 \tag{25}$$

The meaning of this factor at the s channel poles becomes transparent if we rewrite the momentum transfer squared in the C. M. frame:

$$\begin{aligned}
 t &= -2q_s^2(1-z) = -\frac{1}{2}(s-4m^2)(1-z) \\
 z &= \cos \theta_s
 \end{aligned}
 \tag{26}$$

Taking into account the implicit factor $1/m$ we find

$$S_4(d_s = n) = -\frac{1}{2}(\sqrt{s_n}/m + 2)^2(s_n/m^2 - 4)z
 \tag{27}$$

The factor z clearly shows that the coupling between the external pions and the internal ρ trajectory is the one predicted by $SU(6)_w$ ²⁷ or more precisely $SU(2)_w$ symmetry (without $SU(3)$). In our dynamical approach based on the quark diagram the symmetry arises from the fact that a pair of $q\bar{q}$ is annihilated or created through the 3p_0 state. Combining this spin coupling with the orbital part (the rest of the integrand) we obtain a series of simple poles:

$$\frac{1}{2}(\sqrt{s_n}/m + 2)^2(s_n/m^2 - 4)z \frac{\Gamma(d_t + n - 1)}{(n-1)! \Gamma(d_t)} \frac{1}{n - d_s}
 \tag{28}$$

where $s_n = n - \alpha_0$, $n = 1, 2, 3, \dots$. All the other terms in (24) contribute as a background in the physical region of the s channel. Since those terms contain neutralizers $h(x)$ introduced in (17) it is possible in principle to make the contribution of background as small as one wishes by a suitable choice of $h(x)$. Although saturation of the finite energy sum rules by resonances alone is no longer valid in this model, the amplitude can still maintain approximate duality²⁸ if the background contribution is negligible.

Next, we make the angular momentum analysis of the full amplitude (23). We will follow closely a method applied in Ref. (29) to the Lovelace amplitude. Let us rewrite (23) in a more convenient form by substituting (17) in (24) and lumping the same power of $h(x)$:

$$F_+ = \sum_{i=0}^4 \int_0^1 dx \, x^{-d_s} (1-x)^{-d_t} h^i(x) c_i(s, t; x) \quad (29)$$

where $c_i(s, t; x)$ is a polynomial of s , and t multiplied by some μ^{-1} 's. We now define the partial wave amplitude in the s channel:

$$a_\ell(s) = \left(\ell + \frac{1}{2} \right) \int_{-1}^1 dz \, F_+(s, t) P_\ell(z) \quad (30)$$

The relation among s , t and z is given by (26).

For orientation we first discuss a term without any function of μ^{-1} . Introducing a new integration variable u by $x = 1 - e^{-u}$, we have

$$\begin{aligned}
 a_s^{(0)}(s) &= (l + \frac{1}{2}) \gamma(s) \int_{-1}^1 dz z P_l(z) \int_0^\infty du (1 - e^{-u})^{-d_s} \exp[2q^2(z - z_0)u - u] \\
 &= (l + \frac{1}{2}) \gamma(s) \int_0^\infty du (1 - e^{-u})^{-d_s} e^{-u(1 + 2q^2 z_0)} \int_{-1}^1 dz z P_l(z) e^{2q^2 z u}
 \end{aligned}
 \tag{31}$$

where $z_0 = 1 - \alpha_0/2q^2$, and the extra z in the integrand originates from a factor t . The integral representation of F_4 is valid only in the region,

$$d_s < 1, \quad 2q^2(z_0 - 1) + 1 > 0 \quad \text{and} \quad 2q^2(z_0 + 1) + 1 > 0$$

or³⁰

$$1 - d_0 > s > -1 + d_0 + 4m^2 \quad \text{and} \quad d_0 < 1 \tag{32}$$

A necessary condition derived from this, $1 - \alpha_0 > 2m^2$, is met in both the symmetric limit, $\alpha_0 = -m^2$ with $\alpha_0 > -1$, and for the actual value, $\alpha_0 = \alpha_p(0) \approx 1/2$ and $m^2 = m_\pi^2 \approx 0$. The $a_s(s)$ can be continued analytically in s from the region (32) at the last stage. Defining

$$\gamma(u, s) = [(1 - e^{-u})/u]^{-d_s} e^{-u(1 + 2q^2 z_0)}$$

and using the relations

$$(2l+1) z P_l(z) = (l+1) P_{l+1}(z) + l P_{l-1}(z)$$

$$P_l(z) = \frac{1}{2^l l!} \frac{d^l}{dz^l} (z^2 - 1)^l$$

we obtain

$$\begin{aligned}
 a_{\lambda}^{(0)}(s) &= \frac{1}{2} \gamma(s) \left(\lambda + \frac{1}{2}\right) \int_0^{\infty} du u^{-ds} r(u, s) \int_{-1}^1 dz e^{2\gamma^2 z u} \\
 &\quad \times \left[\frac{1}{2^{\lambda+1} \lambda!} \frac{d^{\lambda+1}}{dz^{\lambda+1}} (z^2 - 1)^{\lambda+1} + \frac{\lambda}{2^{\lambda-1} (\lambda-1)!} \frac{d^{\lambda-1}}{dz^{\lambda-1}} (z^2 - 1)^{\lambda-1} \right] \\
 &= \frac{1}{2} \gamma(s) \left(\lambda + \frac{1}{2}\right) \int_0^{\infty} du u^{-ds} r(u, s) \int_{-1}^1 dz e^{2\gamma^2 z u} \\
 &\quad \times \left[\frac{(2\gamma^2 u)^{\lambda+1}}{2^{\lambda+1} \lambda!} (1 - z^2)^{\lambda+1} + \frac{\lambda (2\gamma^2 u)^{\lambda-1}}{2^{\lambda-1} (\lambda-1)!} (1 - z^2)^{\lambda-1} \right] \\
 &= \frac{1}{2} \sqrt{\pi} \gamma(s) \left(\lambda + \frac{1}{2}\right) \frac{1}{\gamma^2} \int_0^{\infty} du u^{-ds-1} r(u, s) \\
 &\quad \times \left[\frac{\lambda+1}{2^{\lambda+4} \Gamma(\lambda + \frac{5}{2})} M_{0, \lambda + \frac{3}{2}}(4\gamma^2 u) + \frac{\lambda}{2^{\lambda+2} \Gamma(\lambda + \frac{1}{2})} M_{0, \lambda - \frac{1}{2}}(4\gamma^2 u) \right]
 \end{aligned} \tag{33}$$

where $M_{0, \lambda}(x)$ is the Whittaker function³¹. Using the formula (33) we make an analytic continuation in λ . Since $M_{0, \lambda}(x)/\Gamma(\lambda+1)$ is regular in λ , the only singularities come from the lower bound of the integral (33). From the behaviour of $M_{0, \lambda}(x)$ near $x = 0$ we know that there exist moving poles at $\lambda = \alpha_s - n$ ($n=0, 1, 2, \dots$).

A less trivial piece is a term which contains $\mu^{-1}(x, s)$. This term is treated by invoking an integral representation of ${}_1F_1$.

$$\mu^{-1}(x, s) = \frac{2}{\sqrt{\pi}} (-\ln x)^{\frac{s}{2}} F\left(\frac{1}{2}, \frac{3}{2}; s \ln x\right)$$

$$= \frac{1}{\sqrt{2\pi}} x^{s/2} \int_{-1}^1 dy (1+y)^{-\frac{1}{2}} x^{sy/2} \quad (34)$$

Carrying out the same partial integration as before, we obtain

$$\begin{aligned} a_2^{(n)}(s) &= \frac{(l+\frac{1}{2})}{2^l \Gamma(l+1)} (2y^2)^l \int_0^\infty du r(u, s) u^{l-d_s} \\ &\times \int_{-1}^1 dz (1-z^2)^l e^{2y^2zu} [-\ln(1-e^{-u})]^{\frac{1}{2}} \int_{-1}^1 dy (1+y)^{-\frac{1}{2}} \\ &\times (1-e^{-u})^{s(1+y)/2} \end{aligned}$$

The singularities are again caused by the end point, $u = 0$. Near this point the integrand depends on u like $u^{l-\alpha_s+n} \times (-\ln u)^{1/2} u^{s(1+y)/2}$. By interchanging the order of u and y integration, we can see the singularity structure in the l -plane essentially from

$$\int_{-1}^1 dy (1+y)^{-\frac{1}{2}} \frac{A_n(y, s)}{l - d_s + n + 1 + s(1+y)/2}$$

This means that there is a fixed cut of the logarithmic type starting from $l = \alpha_0 - n - 1$. All the other terms can be analyzed in similar fashion.

In summary we establish that our model contains fixed cuts with branch points at $\ell = \alpha_0 - n$, as well as ordinary moving poles in the complex angular momentum plane. The fixed cuts are responsible for the elimination of parity doubling ghosts a la Carlitz and Kislinger¹³.

In applying the model to the realistic problem of $\pi\pi$ elastic scattering we have to introduce splitting of the degenerate π - ρ trajectory since our model in the present form treats the π and ρ sector on the equal footing. A way of breaking this symmetry is to make the intercepts of pseudo-scalar and vector different from the degenerate value without changing the $SU(6)_W$ symmetric vertex, which is in accordance with the conventional idea of the broken $SU(6)$ symmetry. In our scheme this is achieved apparantly at the expense of abandoning the product form of quark and antiquark for the internal meson projection.

To see how the idea works, we first use the nonrelativistic notation and then generalize it. The projections for the pseudoscalar and vector states are $\frac{1}{4} (1 + \sigma_1 \cdot \sigma_2)$ and $\frac{1}{4} (3 - \sigma_1 \cdot \sigma_2)$ respectively, where the operator with the subscript 1 refers to quark, and the operator with 2 to antiquark. Noting that β is the parity operator in the rest frame, we propose to replace the factor $h(w) + \frac{1}{4} (1 + \beta_1) (1 - \beta_2) (1 - h)$ by

$$\Delta(w) = h(w) + \frac{1}{16} (1 + \beta_1) (1 - \beta_2) [1 - h(w)] [1 + 3 w^{Ad} + \sigma_1 \cdot \sigma_2 (1 - w^{Ad})]$$

where $h(w)$ is a neutralizer and $\Delta\alpha = \alpha_\rho(0) - \alpha_\pi(0) \approx 1/2$. The covariant expression for this is easily obtained and reads

$$\begin{aligned} \Delta(p, w)_{cd}^{ab} = & h(w) \delta_d^a \delta_c^b + \frac{1}{16} [1 - h(w)] \{ (1 + 3w^{ab}) \\ & \times (1 + \gamma p \mu^{-1})_d^a (1 - \gamma p \mu^{-1})_c^b + (1 - w^{ab}) \frac{1}{2} [(1 + \gamma p \mu^{-1}) \sigma_{\mu\nu}]_d^a \\ & \times [(1 - \gamma p \mu^{-1}) \sigma^{\mu\nu}]_c^b - (1 - w^{ab}) [(1 + \gamma p \mu^{-1}) p_\alpha \sigma^{\alpha\mu}]_d^a \\ & \times [(1 - \gamma p \mu^{-1}) p^\beta \sigma_{\beta\mu}]_c^b \mu^{-2} \} \end{aligned} \quad (35)$$

The difference between the non-product form, $h + 1/4(1 + \beta_1)(1 - \beta_2)(1 - h)$ and the previous factor $[h + 1/2(1 + \beta_1)(1 - h)][h + 1/2(1 - \beta_2)(1 - h)]$, is minor and appears only in the background. Thus all the previous results in this section are not altered.

In the application to $\pi\pi$ elastic scattering we can explicitly verify that the intermediate π sector is decoupled, and the effect of the above symmetry breaking is to differentiate the external pion mass and the negative of ρ intercept. We now show that the $\pi\pi$ amplitude constructed in this manner contains an infinite number of states with negative probability. In particular all resonances on the first daughter trajectory are ghosts.

Let us evaluate the coupling of the n -th level with angular momentum J to the elastic $\pi\pi$ states

$$c_n^k = (l + \frac{1}{2}) \int_{-1}^1 dz R_n(z) P_l(z)$$

$$R_n(z) = \frac{\beta_n}{(n-1)!} d_t(d_t+1) \cdots (d_t+n-2)$$

(35)

For the moment we assume that $m^2 = m_\pi^2 = 0$, $\alpha_0 = \alpha_p(0) = 1/2$, and indicate later how things may be changed if we relax this canonical condition. In this case $\beta_n = (n-1/2)^2 z$, and we can calculate c_n^k explicitly for low values of n . The coefficients are listed in Table 1 for $l \leq n \leq 3$ in a relative scale. (We have also shown the corresponding values in the Lovelace model, in which one takes the amplitude $(1-\alpha_s-\alpha_t)$ $\times B_4(1-\alpha_s, 1-\alpha_t)$.) We immediately notice that there are ghosts with negative probability in our model at least for $(n, l) = (2, 1), (3, 2), (3, 0)$. More generally we use

$$d_t(d_t+1) \cdots (d_t+n-2) = \frac{1}{4} (2n-1) z - \frac{1}{4} (2n-3) \equiv c_n z + d_n$$

and evaluate

$$d_t(d_t+1) \cdots (d_t+n-2) = (c_n z)^{n-1} - \frac{1}{4} (n-1) (c_n z)^{n-2} + O(z^{n-3})$$

An explicit calculation leads to the formulas

$$c_n^n = \frac{n(n-\frac{1}{2})^{n+1}}{2^{n-1} (2n-1)!!} > 0 \quad (36)$$

$$c_n^{n-1} = - \frac{(n-1)(n-\frac{1}{2})^n}{2^n (2n-3)!!} \leq 0 \quad (37)$$

The negative sign in (37) means that particles on the first daughter trajectory are all ghosts except for $n = 1$.

Since the conclusion we have reached is important, we give a simple explanation for it. At the s channel poles our model is essentially

$$\begin{aligned} & \text{Tr} [M_1 M_2 \Gamma_\mu(p_3)] \text{Tr} [M_3 M_4 \Gamma^\mu(-p_3)] \\ & \times [\text{poles of } B_4(1-\alpha_s, 1-\alpha_t)] \end{aligned} \quad (38)$$

The $SU(6)_w$ symmetric vertices give rise to a factor z which is then coupled to the orbital part in B_4 . Since we know that there are infinite ghosts in the general four point function except for the Lovelace case³², we conclude that they also exist in our amplitude. We now want to vary the parameters, m^2 and $\alpha_p(0)$. A condition which inhibits emergence of the ghosts on the first few daughter trajectories has been given in Ref. (32). The condition reads for our betafunction:

$$3 \alpha_p(0) + 4 m^2 \geq 2 \quad (39)$$

This excludes the realistic case, $\alpha_p(0) \approx 1/2$ and $m^2 \approx 0$; in the case of the ideal symmetry, $\alpha_p(0) = -m^2$, it is equivalent to a condition $\alpha_p(0) \leq -2$. So a simple prescription for identifying the realistic parameters fails to evade the conclusion that there emerge an infinite number of ghosts. This important result is applicable to any model³³ factorizable at the particle poles in the way indicated in (38).

In the notable exception provided by the Lovelace model, one assumes a factor $1 + z$ for the spin part instead of z . However, the Lovelace factor is very difficult to understand from the viewpoint of the quark model, since the factor z is a result of the graphical rules of the quark diagram. In spite of the beautiful successes with observed facts, especially consistency with the soft pion theorem, the Lovelace model has little to do with the quark model. One may simply recall that a quark model compatible with the $SU(6)$ symmetry predicts $m_\rho = m_\pi$, whereas the soft pion theorem demands $m_\pi = 0$, thus conflicting with the canonical value $m_\rho^2 = 1/2$. In summary, we face the embarrassing situation that although we succeeded in eliminating parity doubling ghosts, we have introduced another kind of ghost on the lower trajectories.³⁴

4. Concluding Remarks

We have presented two dual models incorporating the quark spin, and derived the detailed properties of the amplitudes. Although our initial aim to eliminate the ghosts associated with negative parity quark was completely achieved in the second model, a detailed application to the $\pi\pi$ scattering has shown that another type of ghosts with negative probability emerges on the non-leading trajectories. The existence of the second type ghosts is due to the fact that the $SU(6)_w$ symmetric vertex among quarks does not match completely well the orbital excitation factor of the dual resonance model, as would be necessary for apparant ghost contributions to cancel. Our result suggest that a quark model based on the scalar dual amplitude is too restrictive to treat two different trajectories such as π and ρ . A fundamental question is whether the two types of ghosts are correlated, so that the complete elimination of both kinds is impossible. No general arguments have been given so far.

Acknowledgments

The author is indebted to Professor Y. Nambu for enlightening discussions and valuable suggestions.

Table 1 Partial wave residua c_n^l for the quark model (I) and the Lovelace model (II).

$\begin{matrix} n \\ l \end{matrix}$	(I)			(II)		
	1	2	3	1	2	3
0	0	9/4	-125/48	1	0	5/16
1	1	-9/4	75/8	1	3/2	5/16
2		9/2	-125/24		3/2	25/16
3			125/16			25/16

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$$\sum_{N=0}^{\infty} (2p_2 \cdot p_3 \sum_n x^n/n)^N / N! = (1-x)^{-2p_2 \cdot p_3}.$$
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Figure Captions

Fig. 1 Quark diagram for N mesons.

Fig. 2 Intermediate meson channel and diagonalization.

Fig. 3 Multiperipheral diagram for contraction of spinor indices.

Fig. 4 (a) Factorization of the quark diagram.

(b) Choice of the integration variables corresponding
to the diagram (a).

Fig. 5 Quark diagram for the $\pi\pi$ scattering.

Fig. 1

Fig. 2

Fig. 3

(a)
Fig. 4

(b)
Fig. 4

Fig. 5