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A NEW DERIVATION OF MULTIGROUP CROSS SECTIONS
FOR BeO- AND GRAPHITE-MODERATED SYSTEMS

Livermore, California

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Lawrence Radiation Laboratory, University of California
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ABSTRACT

Multigroup diffusion equation cross sections are rederived using updated numerical techniques and recent basic cross-section measurements. The basic series of Spade (BeO-moderated) and Snoopy (graphite-moderated) critical assemblies are calculated using these group constants, and the results are compared with those from earlier compilations.

*Work done under the auspices of the U. S. Atomic Energy Commission.

INTRODUCTION

Along with the development of finer analytical techniques has come a considerable amount of improved experimental data on neutron cross sections of interest to the Tory IIC design.^{*} Since certain shortcomings of the presently used cross-section library¹ have been apparent for some time, a program was launched to develop a more accurate and self-consistent set of cross sections for use in the production codes. This development has been oriented towards the simple-geometry Spade and Snoopy critical experiments, in order that the effects of the various refinements can be more easily interpreted and the usefulness of the complete set evaluated.

This report covers the development of new 18-group cross sections and their application to beryllium oxide- and graphite-moderated homogeneous critical experiments. These cross sections, for basic moderators and fuel, will next be used in conjunction with other experiments at N Division to ascertain the effective cross sections of other structural materials for use in a re-evaluation of the Tory IIC design; self-shielded cross sections for the heterogeneous crits have been included for this purpose.

The principal improvements over the library tape cross sections include the following, listed approximately in order of importance:

- 1) Generally updated cross-section data.
- 2) Generation of a separate set of cross sections for each M/U ratio of interest.
- 3) Incorporation of experimental angular scattering distributions.
- 4) Crystalline effects in neutron thermalization.
- 5) The use of fine-grouping for the initial cross section calculation.
- 6) Improved treatment of downscatter in the Be(n,2n) reaction.
- 7) New values of $\nu(E)$.
- 8) Treatment of inelastic scattering.

This new SUPERNOVA set of 18 group cross sections, transfer matrices, etc. is designed for use in the standard ZOOM² diffusion code. (A slight modification renders it suitable for the ANGIE³ diffusion code.) The techniques used in generating the SUPERNOVA set are an expansion of those previously

^{*}The Tory IIC is a full-scale reactor which will demonstrate the feasibility of the Pluto ramjet reactor system.

used in generating the NOVA⁴ set of cross sections which are superseded by this report. Specific details are discussed in the remainder of this report but the essence is this:

The basic experimental and/or theoretical cross-section data is converted to 189 energy group data by various neutronics codes developed at LRL. This fine-group data is fed into WHIZ,⁵ a zero-dimensional multigroup diffusion code which calculates flux spectra typical of the various reactors in question. WHIZ uses these spectra to weight these fine-group cross sections (as described in Ref. 5) and calculate 18 group numbers suitable for use in ZOOM.

THE CRITICAL EXPERIMENTS

Kloverstron et al.⁶ measured the critical height of a series of BeO-moderated oralloy fueled assemblies with BeO/U²³⁵ ratios of 247, 493, 986, 1920, 3826, and 7660. Recently, the measurements were extended to include a BeO/U²³⁵ = 120 assembly (private communication from Jack Morton). These spade crits employed fuel foils of varying thicknesses and separations, from which extrapolations to homogeneous geometry can be made. Table I lists the experimental crits, extrapolated heights (see following), and the material number densities used in the machine calculations.

The critical heights are quoted to ± 0.1 in., corresponding to about $\pm 5\%$ in BeO/U²³⁵. Each of the 493:1, 986:1, 1920:1 and 3826:1 crits were repeated with different foil thicknesses so that extrapolation to zero foil thickness is straightforward (see Fig. 5 of Ref. 6). The 120:1, 247:1 and 7660:1 crits were each run with only one foil thickness; to "extrapolate" them to the homogeneous limit recourse must be made to the systematic changes in the extrapolation. Figure 1 of this paper plots $\Delta B^2/\text{mil}$ of foil vs BeO/U²³⁵; it indicates no extrapolation is needed for the 120:1 and 247:1 systems, and shows that the extrapolation for the 7660:1 is ill defined, so that the extrapolated height in Table I is very approximate.

The data in Table I includes corrections for the diaphragm. Neutron room return was not observable within experimental accuracy in the 493:1 Spade experiment; therefore no room return corrections have been made.

An effective buckling decrement due to neutron reflection from the pedestal on which the lower half of the Spade assembly rests has been measured for the 247:1 Spade, and has been calculated for the other Spades.⁷ These corrections, in the form $\Delta B^2/B^2$ were obtained from calculated values

Fig. 1. Slopes of buckling vs foil thickness curves for Spade critical assemblies as a function of BeO/U²³⁵ ratio.

Table I. Homogeneous Spades.

BeO/ U ²³⁵	Extrap- olated Height (cm)	Base (cm × cm)	10 ²⁴ atoms/cc		
			U ²³⁵	U ²³⁸	Be ⁹ and O ¹²
120	49.66	(60.96) ²	5.559 × 10 ⁻⁴	4.145 × 10 ⁻⁵	0.06672
247	51.30	(60.96) ²	2.7878 × 10 ⁻⁴	2.0786 × 10 ⁻⁵	0.06886
493	55.12	(60.96) ²	1.3967 × 10 ⁻⁴	1.0414 × 10 ⁻⁵	↓
986	61.47	(60.96) ²	0.6983 × 10 ⁻⁴	0.5207 × 10 ⁻⁵	
1920	52.32	(76.20) ²	0.3587 × 10 ⁻⁴	0.2674 × 10 ⁻⁵	
3826	54.36	(91.44) ²	0.1800 × 10 ⁻⁴	0.1342 × 10 ⁻⁵	
7660	76.8	(91.44) ²	0.08990 × 10 ⁻⁴	0.06698 × 10 ⁻⁵	

Homogeneous Snoopys

C/U ²³⁵	Extrap- olated Height (cm)	Base (cm × cm)	10 ²⁴ atoms/cc		
			U ²³⁵	U ²³⁸	C
301	112.78	(107.95) ²	2.7335 × 10 ⁻⁴	1.9135 × 10 ⁻⁵	0.08228
603	95.50	(123.19) ²	1.368 × 10 ⁻⁴	0.9525 × 10 ⁻⁵	0.08249
1206	102.36	(123.19) ²	0.6840 × 10 ⁻⁴	0.4788 × 10 ⁻⁵	0.08249
2355	113.54	(123.19) ²	0.3503 × 10 ⁻⁴	0.2452 × 10 ⁻⁵	0.08249

Heterogeneous Crits

M/U ²³⁵	Foil thick./sep. (mil/in.)	H _c (cm)	Base (cm × cm)
120	16/1	49.66	60.96 × 60.96
247	8/1	51.30	60.96 × 60.96
493	4/1	55.88	↓
493	8/2	56.64	
986	2/1	63.50	
↓	4/2	65.532	
↓	6/3	69.85	
1920	1/1	53.594	76.20 × 76.20
↓	2/2	54.864	↓
-A	2/2A	66.04	
↓	3/3	69.342	
3826	1/1 ^a	58.166	91.44 × 91.44
↓	2/2 ^a	61.468	↓

Table I. (Con't)

Heterogeneous Crits			
M/U ²³⁵	Foil thick./sep. (mil/in.)	H _c (cm)	Base (cm × cm)
3826	1/2	57.658	91.44 × 91.44
↓	2/4	61.468	↓
7660	1/1 ^b	91.948	↓
301	4/½	113.28	107.95 × 107.95
603	2/½	96.01	123.19 × 123.19
1206	2/1	102.87	↓
2355	1/1	114.05	↓

^a Half-area triangular fuel foils.

^b Quarter-area triangular fuel foils.

(Note: Because of the lack of a straightforward approach to the self-shielding of the triangular foils, these crits are omitted in the calculations).

of $\Delta K/K$ as explained in Ref. 7. The corrections, converted back to $\Delta K/K$ in similar manner, are listed in Table II. They have been added to the calculated values of K to give all final values of K quoted in this report. (Plots of K vs M/U for the library and NOVA data which have appeared previously do not include this correction.)

Table II. Pedestal Corrections

BeO/U ²³⁵	$\Delta k/k$ (%)
120	0.480
247	0.380
493	0.295
986	0.172
1920	0.204 ^a
3826	.0
7660	0

^a (0.102 for 30 in. × 24 in. base crits)

A similar series of crits (Snoopy's) were done for graphite moderated systems.⁸ These results are also shown in Table I. The dimensions shown

include corrections for block porosity, room return, control voids, and nonhomogeneous fuel as per Ref. 8.

BASIC CROSS-SECTION DATA

Sources

Basic cross-section data were drawn primarily from three sources: the barn book⁹ and its supplement,¹⁰ and a compilation by R. J. Howerton.¹¹ The breakdown is given in Table III. Reactions with such high thresholds as to be insignificant in reactor calculations have been ignored.

Table III. Sources of cross-section data.

Element	Reaction ^a	Source (Ref. No.)
Oxygen	$\sigma_{n,p}(E)$	9
	$\sigma_{n,\alpha}(E)$	9
	$\sigma_{n,n}(E)$	10
	$\sigma_{n,n}(E, \ominus)$	11
Beryllium	$\sigma_{n,\alpha}(E)$	9
	$\sigma_{n,n}(E)$	10
	$\sigma_{n,2n}(E)$	12
	$\sigma_{n,\gamma}(E)$	10
	$\sigma_{n,n}(E, \ominus)$	12
U ²³⁵	$\sigma_{n,n}(E)$	13
	$\sigma_{n,n}(E, \ominus)$	9
	$\sigma_{n,n'}(E)$	9
	$\sigma_{n,f}(E)$	13 0.01-0.25 eV
		14 0.25-10 eV
		15 10 eV-20 keV
		16 20 keV-10 MeV
	$\sigma_{n,\gamma}(E)$	13 0.01-1.0 eV
		17 1.0-10 eV
		18 10.0 eV-20 keV
U ²³⁸		16 20 keV-10 MeV
	$\sigma_{n,n}(E)$	10
	$\sigma_{n,n}(E, \ominus)$	Same as U ²³⁵ $\sigma_{n,n}(E, \ominus)$

Table III. Sources of cross-section data. (Con't.)

Element	Reaction ^a	Source (Ref. No.)
Carbon	$\sigma_{n,n'}(E)$	9
	$\sigma_{n,f}(E)$	9
	$\sigma_{n,\gamma}(E)$	{ 10 below 1 keV 11 above 1 keV
	$\sigma_{n,n}(E, \Theta)$	13
	$\sigma_{n,n}(E)$	9
	$\sigma_{n,\gamma}(E)$	1/v determined experimentally to be 13 ppm boron equiv.
	$\sigma_{n,\alpha}(E)$	11

^a The cross section notation used here conforms to that suggested by the AEC Nuclear Cross Section Advisory Group.¹²

Anisotropic Elastic Scattering

The angular distributions of neutrons scattered in elastic collisions have been measured for many of the common reactor materials. Much of this data is compiled in Ref. 19. Joanou *et al.*¹³ have analyzed the existing experimental data for a number of elements using a Legendre expansion through $l = 6$. Data is also presented which has been "smoothed." This latter set for Be, O, and C was used in SOPHIST-5,²⁰ an IBM-7090/7094 code which converts data in this form to transfer matrix elements appropriate to the ZOOM and ANGIE formulation, and also computes anisotropic scatter matrices appropriate for use in DSN.²¹

The inclusion of these distributions has a profound effect on the moderation of fast neutrons. It results in much stronger downscatter terms (particularly for oxygen) than existed in the library cross section set. This increases K by 3-5% for all the Spade crits and thus represents a significant improvement over the library set. In fact, the strong backward peaking in the oxygen at certain "resonances" causes such perturbation on the flux spectrum as to cause considerable deviation from "fission tail" and " $1/E$ " behavior in the 0.3-2.5-MeV range, even to causing reversals in the slope of the $E\phi$ -vs- E curve. (See Fig. 7.) Therefore, the analytic flux representation used throughout the fine-group data compilation had to be regarded as not suitable over this energy range — particularly as regards downscatter, which

is very dependent on flux shape within a group. The flux generated in WHIZ, therefore, was used as input to a second round of SOPHIST-5 calculations for beryllium and oxygen, to generate a new set of transfer coefficients. This iteration proved sufficient, and was the only instance where such a step was found to be necessary.

Inelastic Scattering

According to the Weiskopf level density formula,²² the probability density for the inelastic scatter of a neutron at energy E' to energy E is;

$$P(E' \rightarrow E) = CE e^{-E/\Theta_j}$$

where Θ_j = compound nucleus temperature

$$= \frac{\sqrt{E'}}{0.31\sqrt{A}}$$

where A = atomic mass of the scatterer, and

C = constant such that

$$\int_0^{E'} P(E' \rightarrow E) dE = 1.$$

Transfer matrices based on the above distribution have been determined for the inelastic scatter reaction of U^{235} and U^{238} with the FORTRAN code UTRANS.²³ Inelastic scatter for the moderators is insignificant to these applications because of the high threshold energies involved, and was not calculated; even the uranium $\sigma_{n,n'}$ with its lower threshold is rather insignificant here because of the high M/U ratios of the crits.

Be(n, γ)

This reaction is listed as a $1/v$ cross section, but several sources (e. g., Refs. 11 and 24) indicate a constant cross section at 1 mb when the $1/v$ representation reaches this value — this is merely to imply that there should be some nonvanishing probability for this reaction even at very high energies, but does not represent known experimental values of $\sigma_{n,\gamma}(E)$ at these energies. If the 1-mb value had been used, the 247:1 BeO crit would suffer a reduction in K of about -0.58%, and the 7660:1 case -0.74%, but we see no justification in such a representation.

Be(n,2n) Reaction

The experimental Be(n,2n) data and the fit to the data used by General Atomics¹³ is reproduced in Fig. 2. The SUPERNOVA calculations use the same fit except that (as indicated by the dashed curve) the values near the experimental threshold have been increased so that the three data points below the GA curve cutoff at 2.7 MeV are not completely ignored (the theoretical threshold is 1.85 MeV). The rapid fall-off of the fission spectrum above 3-4 MeV causes the reactivity calculations to be quite sensitive to the values of the cross section assumed below 3 or 4 MeV, and no high degree of accuracy can be claimed for the SUPERNOVA choice of (n,2n) cross section (which contributes about 10% to the reactivity of Spade crit calculations). Other "reasonable" fits to the (n,2n) cross section data were tried. It was found easy to raise or lower the K-vs-M/U curve for the Spade crits by a few percent, with almost no resulting change in slope.

The spectrum of the two neutrons emerging from the (n,2n) reaction is even less well known than is the cross section and recourse must be made to theoretical arguments. The SUPERNOVA numbers include the following assumptions:

- (1) For reactions proceeding through the 2.43 MeV Be^{9*} level (2.7 MeV in lab), the two neutrons emerge with energies

$$E_1 = 0.83 E' - 2.187$$

$$E_2 = 0.0189 E' + 0.473.$$
- (2) When energetically possible, 50% of the (n,2n) reactions go through this Be^{9*} level.
- (3) For all other modes, all neutrons come off with equal probabilities in all energy intervals $0 < E < E' - 1.7$.
- (4) All emerging neutrons have average lab cosines, $\mu = 0.11$.

The arguments leading to (1) above are outlined in Appendix A.

Assumption (2) approximates the branching ratios measured by Marion et al.,²⁵ and seems to be assumed, though not explicitly stated, in the British "Barn Book,"²⁴ but disagrees somewhat from APEX-467,²⁶ which allows up to 80% of reactions through the 2.43 Be^{9*} level. Assumption (3) agrees with APEX-467 and with the British. Assumption (4) is also used by the British, and is consistent with the crude arguments in Appendix A. To recapitulate, for reactions through the 2.43 Be^{9*} level

Fig. 2. Experimental (n,2n) cross-section data for Be^9 . We use the dashed line near threshold and the solid line (General Atomics' fit) elsewhere.

$$P_{2.43}(E' \rightarrow E)dE = \frac{dE}{2} [\delta(E - 0.83E' + 2.187) + \delta(E - 0.0189E' - 0.473)]$$

and for all other reactions

$$P_{\text{other}}(E' \rightarrow E)dE = \frac{dE}{E' - 1.7} \quad 0 < E \leq E' - 1.7$$

$$= 0 \quad E > E' - 1.7.$$

For incident energies $E' > 2.7$,

$$P(E' \rightarrow E)dE = \frac{1}{2} P_{\text{other}}(E' \rightarrow E)dE + \frac{1}{2} P_{2.43}(E' \rightarrow E)dE.$$

For $E' < 2.7$

$$P(E' \rightarrow E)dE = P_{\text{other}}(E' \rightarrow E)dE.$$

Calculations of the (n,2n) transfer matrix on the basis of the above distributions were done numerically with the FORTRAN code INTUNE.²³

Krumbein²⁷ seems to indicate a somewhat lower effective threshold than we have used here, and also offers an alternative scheme for calculating the neutron energy distributions.

The problem of handling the production of neutrons by the (n,2n) process in codes like ZOOM and WHIZ is best met through the use of a fictitious negative group absorption cross section equal in magnitude to the (n,2n) cross section and a positive scatter cross section equal in magnitude to twice this value. The appropriate transfer matrix is then associated with the (n,2n) events by considering this reaction as a separate material in the fine group WHIZ problems; after the collapsing to the Safonov grouping has occurred, the (n,2n) "material" is combined with the Be, the transfer matrices being weighted with their respective scatter cross sections.

U²³⁵ Fission

A plot of the data used for the capture and fission of U²³⁵ (from Wikner¹⁶) above 20 keV is shown in Fig. 3.

The resonance region represents a more complex situation; several sets of recent experimental fission data^{14,15,28} exist which agree fairly well in gross, but not as much in specific, details. Moreover, these sets are all analyzed by their respective authors in terms of resonance fission

Fig. 3. Fission and capture cross sections above 0.01 MeV for U^{235} .

parameters, as determined by area analysis of smooth curves fitted to the pointwise data over each fission resonance. Choosing an adequate smooth curve fit becomes difficult when the data has appreciable scatter and/or poor resolution. In addition, subjective errors are introduced when an ill-defined non-resonance background is subtracted out, and when a "wing" correction is estimated. Nevertheless, fission cross sections computed from resonance parameters thus obtained adequately reproduce the resonance peaks in the original cross section data. But the computed values of the valleys come out much too low. A set of group cross sections computed from resonance parameters by the SOPHIST-2²⁹ code results in partial resonance integrals about one-half as large as those determined from integration under the original data points. (The SOPHIST-2 equations are formulated on the basis that the single-level Breit-Wigner assumption holds; parametric data are also generally determined on the basis.) Some of the discrepancy could be removed by the formal introduction of one or more negative energy (i. e., bound level) resonances, but SOPHIST-2 is not equipped to handle these. The remainder cannot be explained by some missing $1/v$ contribution to the fission cross section, and is usually ascribed to interference effects between neighboring resonances.

Since the valleys do contribute an appreciable part to the fission in the resonance region, the SUPERNOVA cross sections are not calculated from resonance parameters. Instead, the pointwise data (not published in the original reports) were obtained from the LRL¹⁴ experimenters (0.03-60 eV) and from the Saclay (France)¹⁵ group (4.4 eV - 20 keV). Each of these sets of data were numerically integrated in SOPHIST-2 (which was modified so as to handle data from tape in blocks of up to 1500 data points) over a number of small energy intervals which span the region of overlap (4.4-60 eV) of the two sets. The fine-group average Saclay data cross sections for each interval,

$$\bar{\sigma}_F = \frac{\int \sigma_F(E) dE}{\int dE}$$

are plotted in Fig. 4. Also plotted is the ratio, $[\bar{\sigma}_F(\text{Saclay}) - \bar{\sigma}_F(\text{LRL})] / \bar{\sigma}_F(\text{Saclay})$. It is seen that below 10 eV, the Saclay data yield higher cross sections than the LRL data does. We find

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Fig. 4. Histogram of average values of the Saclay U^{235} fission cross sections, $\bar{\sigma}_F(\text{Saclay})$, for 24 fine-energy groups between 4.5 and 60 eV. Also shown is the ratio $[\bar{\sigma}_F(\text{Saclay}) - \bar{\sigma}_F(\text{LRL})] / \bar{\sigma}_F(\text{Saclay})$.

$$\frac{\int_{4.5}^{10} dE \sigma_F(\text{LRL})}{\int_{4.5}^{10} dE \sigma_F(\text{Saclay})} = 0.822$$

whereas, above 10 eV, we find

$$\frac{\int_{10}^{60} dE \sigma_F(\text{LRL})}{\int_{10}^{60} dE \sigma_F(\text{Saclay})} = 0.987.$$

In view of the degree of uncertainty evidenced by the plot of fine-group ratios, this latter result seems to indicate satisfactory agreement between the two sets of data above 10 eV as they stand.

This is surprising in view of the conclusions reached by the LRL group¹⁴ in their analysis of the parameters from the LRL, Saclay, and Columbia experiments. They found that when all three sets of data were normalized to the parametric data of Shore and Sailor³⁰ at the 4.9-, 6.4-, and 7.1 eV resonances, the Saclay parameters in the 5-40-eV range averaged 8% higher than the LRL parameters (the standard deviation of individual resonances for each set was about $\pm 14\%$ from the average of the three sets, though the larger resonances agreed better than this). Next, LRL renormalized their experiment by carrying down in energy to tie on to the BNL-325 U²³⁵ fission cross section curve in the 0.03-0.15-eV range. This caused them to reduce their original cross sections and values of Γ_F by 18%, and they suggest that the Saclay and Columbia data be similarly reduced.

Indeed, below 10 eV we have found the integrated Saclay data to be some 18% higher than the integrated LRL data. However, above 10 eV we have seen that the Saclay results as they stand are in essential agreement with LRL results, instead of being 18% larger (or 26% larger if one allows for the average 8% higher Saclay parameters prior to renormalization of the LRL data).

About all we can really conclude from all this is that parametric representation of U²³⁵ fission data is of rather limited value. For our purposes, the pointwise Saclay data was used as it stands above 10 eV, where

it appeared to possess less scatter than the LRL data; below 10 eV, and down to 0.25 eV, the pointwise LRL data appeared to be the stronger, and was therefore adopted.

U²³⁵ Capture

A significant departure from the ZOOM library cross section set occurs in the use of more recent data for U²³⁵ capture, particularly in the resonance regions. The origins of the cross sections used in the library set are lost in antiquity. The NOVA⁴ cross sections, based on the curves in BNL-325, Second Edition, Supplement 1 (below 1.8 eV), on the experiment of Shore and Sailor³⁰ (1.8-3.2 eV), and estimated from BNL-325 resonance parameters (above 3.2 eV) were much more realistic, and much higher than those of the library set. This is apparently the primary cause for the 7% drop in the calculated value of K for the 247:1 Spade crit in going from the library set to the NOVA cross sections.

Uncertainty in the radiative capture of U²³⁵ is even greater than that for fission. The capture is difficult to measure directly. Values obtained by subtracting fission and scattering from total cross sections are uncertain, because errors in the original data lead to larger relative errors in the difference. And as we have seen above, data from different sources may differ in normalization, accuracy, etc.

Most effort of late is focused on determination of $\alpha = \sigma_c / \sigma_F$, the ratio of radiative capture to fission. In resonances, one expects $\alpha = \Gamma_\gamma / \Gamma_F$, but values of Γ_γ are very approximate, and the technique is of little use in the valleys.

SUPERNOVA values of σ_c above 20 keV are based on the data in Ref. 16, which fits the experimental values of α of Divens and Hopkins.³¹ From .010-20 keV, the α curves of Ref. 18 are used. These assume $\alpha = 0.5$ from 50 eV to 1 keV with a smooth extrapolation from 1 keV to the 20-keV data. From 10-50 eV, the curves assume that the parametric ratio Γ_γ / Γ_F , gives α in the resonances (often average values of Γ_γ are used, since measured values are scarce) and that $\alpha = 0.174$ (the thermal value) in the valleys.

Below 1 eV, α was obtained from BNL-325. In the important range from 1-10 eV, we use a measurement by Brooks¹⁷ of $\eta = \nu / (1 + \alpha)$, the average number of prompt neutrons emerging per total absorption event.

Brook's data cannot be used as it stands; he presents only η/ϕ , where ϕ is an unmeasured scattering correction to the data. Therefore, a calculation of this correction factor was done with the IBM 7090 YOGI-N³² Monte Carlo code (see Appendix B for the details of the calculation). The calculated values of the scattering correction (which are appreciable) and the resultant values of η are shown in Figs. 5-a, b. These values of η are the basis for the SUPERNOVA U²³⁵ capture cross sections from 1-10 eV. It is seen that even after raising Brook's data by correction factors approaching two in some regions, the resultant η curve is disturbingly low, rarely approaching the thermal value, $\eta = 2.07$.

That portion of Brook's data from 10-50 eV was not used in generating the SUPERNOVA numbers. Its use would have further depressed the already low calculated values of K for the faster (lowest M/U ratio) systems by several percent. This was undoubtedly the motivation behind the arbitrary decision to not use this data; the rationalization was that the sudden drop in η above 30 eV made the data suspect.

Unfortunately, reports of even higher values of α have now appeared, too recently to include in detail in this report. De Saussure, Weston, Givin, Russell and Hockenburg (to be published in Nuclear Science and Engineering) have made direct measurement of α for U²³⁵ from 4 eV to 2 keV. Preliminary analysis of their data leads to capture cross sections considerably larger than those derived from Brook's data over the range of overlap below 30 eV. It seems that in the future, SUPERNOVA cross sections above will have to be substantially increased, wreaking further havoc with our calculations of the faster crits.

The Fission Spectrum

The spectrum of neutrons produced from either U²³⁵ or U²³⁸ fissions is assumed to be that fit to experimental U²³⁵ data given by Cranberg et al.³³

$$N(E)dE = ke^{-E/0.965} \sinh\sqrt{2.29 E} dE$$

where k is a normalizing constant.

The SUPERNOVA f^i , which sum to unity, are just

$$f^i = \int_{E_{i-1}}^{E_i} N(E) dE.$$

Figs. 5a and 5b. Values of $\eta = 2.414/(1 + a)$ derived from Brook's U^{235} data for 1-50 eV. Also shown are the Monte Carlo values of β , the scattering correction used to derive η .

(This spectrum was used for fine group flux weighting calculations for energies > 1 MeV, and was used also for the fine group fission spectrum.)

Neutrons per Fission: $\nu(E)$

The number of neutrons given off per fission has long been known to be a function of the energy of the neutron causing the fission. For U^{235} , Meadows and Whalen³⁴ give for prompt neutrons

$$\nu(E) = 2.414 + (0.097 \pm 0.008)E \quad (E \text{ in MeV}).$$

We add 0.016 delayed neutron per fission and use

$$\nu(E) = 2.43 + 0.1 E \quad (E \text{ in MeV}).$$

WHIZ, ZOOM, and ANGIE have provisions for a different ν for each group

$$\nu^j = (C + DE^j) \text{ ZOOM and WHIZ}$$

$$= (C_1 + C_2 E^j) \text{ ANGIE.}$$

Our $\nu(E)$ gives $C = C_1 = 2.43$

$$D = C_2 = 0.1$$

$$E^j = C^j = \text{average energy for fissions in group "j."}$$

This energy dependence increases K by only a few tenths of 1% in the fastest Spade or Snoopy, and by even less in the others, but is included for completeness.

Notice that the new $\nu = 2.43$ is some 2% lower than the $\nu = 2.4858$ currently in use in conjunction with the existing library tape numbers. A -2% change in ν means -2% in K .

Recent measurements³⁵ for U^{238} at 1.5 MeV give $\nu_{238}(1.5) = 2.65 \pm 0.09$, whereas our $\nu_{235}(1.5) = 2.58$. ZOOM, ANGIE, and WHIZ have provisions for only one set of ν values. In order to get the correct fission contribution from U^{238} , the "raw" U^{238} cross sections from WHIZ have been adjusted as follows to give the SUPERNOVA set.

$$\sigma_F(238) = \frac{2.65}{2.58} \sigma_F(\text{raw})$$

$$\sigma_C(238) = \sigma_C(\text{raw}) + \left(1 - \frac{2.65}{2.58}\right) \sigma_F(\text{raw}).$$

These corrections keep the total absorption cross section invariant, but increase the neutrons per actual U^{238} fission by the desired amount.

It is interesting to note that there is no experimental evidence (so far as we can determine) to refute the possibility that $\nu(E)$ might depart markedly from the postulated average $\nu(E) = 2.43 + 0.1E$ dependence in the resonance region.

The Transport Cross Section

In monoenergetic asymptotic diffusion theory with linear anisotropic scattering, the diffusion constant is given by

$$D = \frac{\Sigma_a}{\kappa^2}$$

where κ is determined from

$$\frac{\Sigma_s}{2\kappa} \ln \frac{\Sigma + \kappa}{\Sigma - \kappa} = \frac{\kappa^2 + 3\Sigma_a \Sigma_s \bar{\mu}}{\kappa^2 + 3\Sigma \Sigma_s \bar{\mu}}$$

and $\bar{\mu}$ = average value of lab scattering cosine, $\Sigma = \Sigma_a + \Sigma_s$ = total cross section. (The above expression is derived, except for a typographical error, on page 396 of Glasstone and Edlund.³⁶)

The validity of applying these formulae to individual groups in multi-group calculations is open to question; nevertheless it is commonly done and is done herein.

In the ZOOM and ANGIE codes, the quantity called the "group macroscopic transport cross section" is defined in terms of the diffusion constant for that group such that

$$3\Sigma_{TR} D \equiv 1$$

(dropping energy indices for convenience). This leads to

$$\Sigma_{TR} = \frac{1}{3D} = \frac{\kappa^2}{3\Sigma_a}$$

If the characteristic equation for κ is expanded in powers of Σ_a/Σ the above definition gives

$$\Sigma_{TR} = \Sigma \left[1 - \bar{\mu} - \left(\frac{4 - 9\bar{\mu}}{5} \right) \frac{\Sigma_a}{\Sigma} + \frac{4(1 - 9\bar{\mu} - 27\bar{\mu}^2)}{175} \left(\frac{\Sigma_a}{\Sigma} \right)^2 + \dots \right]$$

Alternatively, as $\Sigma_a \rightarrow \Sigma$ (i. e., pure absorber)

$$\Sigma_{TR} = \frac{\Sigma}{3}$$

[Some confusion arises in the literature because various authors define

$$\Sigma_{TR} \equiv \Sigma(1 - \bar{\mu}) \text{ or } \Sigma_s(1 - \bar{\mu}) \text{ or } \Sigma - \bar{\mu}\Sigma_s.$$

Then in the general case they cannot (and do not) use the simple relationship $D = 1/3\Sigma_{TR}$. Only in the case of pure scattering does $D \equiv \Sigma_a/\kappa^2 = 1/3\Sigma_{TR}$ for these definitions of Σ_{TR} , for only then do all the variously defined Σ_{TR} become identical.]

It is obvious that the diffusion constant appropriate for a homogeneous mixture is a function of the composite cross sections for the mixture. And only in the limiting cases of pure scattering or pure absorption illustrated two paragraphs above is our Σ_{TR} a linear function of these cross sections. Unfortunately, ZOOM and ANGIE are coded to calculate a Σ_{TR} for a mixture as a linear sum of (macroscopic) transport cross sections which are input for the individual materials. If the individual material Σ_{TR} is based on the properties of that material alone (as is the case with the current library tape numbers) the mixture Σ_{TR} thus obtained will in general be in error. The SUPERNOVA transport cross sections for each material have been modified (as explained below) so that the linear summation in ZOOM or ANGIE will give the "correct" Σ_{TR} (so far as the theory discussed in this section applies) for the particular mixture for which the cross sections are quoted.

WHIZ has been modified so as to calculate values of Σ_{TR} by solving the characteristic equation for κ and now requires values of $\bar{\mu}$ (instead of σ_{TR}) in the fine group input for each material. (Values of $\bar{\mu}$ for elastic scattering come from SOPHIST-5; for (n,2n), (n,n'), and crystalline scattering they come from theoretical estimates). WHIZ calculates fine group Σ_{TR} 's for the mixture, solves the diffusion equation, and calculates collapsed group values of $\bar{\mu}$ and the various cross sections for each material and for the mixture. From these values for the mixture WHIZ calculates Σ_{TR}^i for the mixture. From the cross sections, $\bar{\mu}$'s and number densities it calculates

Σ_{TR}^i for each material, sums these to get values of Σ_{TR}^i for the mixture as is done in ZOOM and ANGIE, and finds the ratio of the "correct" to the "summation" Σ_{TR}^i for the mixture. The individual material microscopic transport cross sections are then multiplied by this ratio (a correction of several percent at times) and are presented as the SUPERNOVA transport cross sections. These corrected cross sections, when used in ZOOM and ANGIE insure that the group diffusion constants for the mixture are what the theory outlined at the beginning of this section predicts.

Crystal Scattering

One well-recognized weakness in earlier treatments of thermal scattering has been the assumption of a gas model for the moderator, even though the moderators of interest to us are crystalline, with bonds that profoundly affect the scattering of neutrons with incident energies comparable to or less than these crystalline bonds.

CRYSTAL SOPHIST is an unpublished IBM 7090/7094 code developed at LRL to calculate scattering matrices for simple cubic Bravais lattices using Debye Model theory (See Ref. 37 for theory). Neither BeO nor graphite are Debye crystals (nor are they simple cubics), so the best choice of Debye θ for each is difficult.

For graphite, a Debye $\theta = 1020^\circ\text{K}$ was used. The model was checked by a WHIZ calculation of the flux spectrum for one of the poisoned graphite assemblies studied by Parks et al.³⁸ The results agreed well with the experimental data and the calculations of Parks.

The BeO Debye model is further complicated by the transition from the average crystalline atomic mass of 12.5 to the higher-energy gas model mass of 9 (Be) or 16 (O). CRYSTAL SOPHIST postulates a smooth transition of the form

$$M = (M_c - M_G) \exp \left\{ -Z_m \frac{E + E'}{k\theta_D} \right\} + M_G$$

where

- M_c = crystalline mass = 12.5 for BeO
- M_G = gas mass = 9 or 16
- $E + E'$ = sum of initial and final neutron lab energies
- $k\theta_D$ = Debye temperature in energy units
- Z_m = unknown parameter.

Some experimental values of inelastic (in the lab) scattering cross sections for BeO at various temperatures have been reported by Begum et al.³⁹ Calculated values for three temperatures were obtained from CRYSTAL SOPHIST for various choices of θ_D and Z_m . The requirement of a fit to the experimental data pins down θ_D to within 2 or 3% and Z_m to about 20%. The excellent fit obtained with $\theta_D = 1000^\circ\text{K}$ and $Z_m = 1.3$ shown in Fig. 6, was the basis of our choice of these values for BeO.

To calculate transport cross sections for the crystalline energy region one needs the appropriate average values of the lab scattering cosines. The theory embodied in CRYSTAL SOPHIST (private communication from R. N. Stuart) suggests that $\bar{\mu}$ be calculated from;

$$\bar{\mu}(E) = \coth C(E) - \frac{1}{C(E)}$$

where
$$C(E) = \frac{6E}{Ak\theta_D} \Phi_1\left(\frac{T}{\theta_D}\right)$$

kT = moderator temperature (energy units)

$k\theta_D$ = Debye temperature (energy units)

A = effective atomic mass

$\Phi_1\left(\frac{T}{\theta_D}\right)$ = function tabulated in Ref. 40.

This relation was used up to that energy where $\bar{\mu}$ reached the free gas value, $2/3A$.

The use of CRYSTAL SOPHIST represents another major departure of the SUPERNOVA cross section set from that of the ZOOM library. The crystal model makes little change in the calculated K for fast systems (e. g., 247:1) but lowers the more thermal systems (e. g., 3826:1) by about 1%. For a very thermal system there would of course be no effect: the thermal flux will be Maxwellian for any moderation mechanism sophisticated enough to impose detailed balance.

THE SUPERNOVA CROSS SECTIONS

Cross Section Calculations

Essentially one must know the final flux distribution in order to properly flux weight the raw cross section data and come up with multigroup

Fig. 6. Experimental and calculated values of the BeO inelastic (lab system) scattering cross section for incident neutron energy $E = 0.00265 \text{ eV}$, and temperatures ranging from 100-900 $^\circ\text{K}$.

constants with which to determine the flux distribution (and reactivity) in the standard multigroup formulations of a reactor. Thus one is really faced with an iterative problem; its severity may, however, be reduced through the use of finer groupings. A very fine grouping, such as was used in the generation of the NOVA cross section set,⁴ will almost completely eliminate the need for any prescience in setting the first flux guess. This procedure, however, becomes costly of time and effort, so we have tried here to match some reasonable guess of the initial spectrum with a moderately fine group scheme (189 groups total). In only one instance was a second iteration on the flux necessary, as described below.

One very useful tool which eases machine time requirements on many-group problems is the WHIZ code,⁵ a zero-dimensional diffusion program with multigroup buckling which flux weights cross sections, then collapses them into any desired coarser grouping. SUPERNOVA numbers were obtained for each Safonov group by collapsing from 10 or 11 subgroups of roughly equal lethargy width.

Since WHIZ handles only 130 groups it was necessary to run tandem problems in order to cover the entire energy range. For convenience the energy range was divided into three parts: fast (10.5 MeV-3.1622 keV), intermediate (3.1622 keV-10 eV), and thermal (10-0.01 eV). Separate WHIZ tapes were prepared for each of these ranges. Except as otherwise noted herein, fine group cross sections for WHIZ were generated by SOPHIST-2²⁹ by weighting experimental cross section data with "reasonable" flux guesses. (Only for U-238 were cross sections calculated from resonance parameters by SOPHIST-2.) The requisite large energy transfer arrays for elastic scattering were entered directly from tapes prepared by SOPHIST-5²⁰ (high-energy) and CRYSTAL SOPHIST³⁷ (low-energy).

The high-energy-group WHIZ was run first, with fine grouping extending down one Safonov group into the intermediate energy region so as to provide sources for that region, and SUPERNOVA numbers were obtained for groups 12-18. These were used to make the WHIZ tape for the intermediate region (again with fine grouping extending into the lower energy range), and SUPERNOVA numbers were obtained for groups 7-11. Finally, WHIZ tapes were run for the thermal region, using SUPERNOVA numbers in the higher energy groups.

It was found that the cross sections obtained in the top two energy regions were independent of the cross sections used for lower energies, so that no recycling of this kind was needed.

A look at the flux distributions obtained from the various WHIZ problems (Figs. 7-10) suggests the possibility of improving the SUPERNOVA set by using these fluxes as better weighting functions for the various fine group input quantities to WHIZ. This was found necessary only for the high-energy region in the BeO systems where large anisotropic scattering effects gave marked, unexpected changes in the flux distribution, even causing reversal of the flux slope. A single iteration proved sufficient and was the only instance where such a step was necessary. In the thermal region the resultant flux spectrum is quite different for each M/U ratio. To check the importance of the initial flux guess, WHIZ problems were run with tapes based on different thermal spectra. The results agreed, showing that the WHIZ groupings were fine enough so that a single set of fine group cross sections and transfer arrays sufficed for getting SUPERNOVA numbers for the various M/U ratios. Therefore we decided to make and use only one WHIZ tape for each energy region, a great saving in time and effort.

This decision does involve some inaccuracy in the resonance region. The fine-group U^{235} capture and fission cross sections used in making the WHIZ tape were calculated in SOPHIST-2 using its resonance flux depression option for homogeneous mixtures of moderator and absorber. The flux used as a weighting function in the fine group calculation is

$$\Phi = \frac{\Phi(E)}{1 + \frac{\sigma(E)}{\sigma_s}}$$

where $\Phi(E)$ = input flux to SOPHIST-2,

$\sigma(E)$ = cross section being weighted,

σ_s = moderator scattering cross section per atom of absorber.

The SOPHIST-2 was run with numbers hopefully appropriate to the BeO/ U^{235} = 247 Spade, the fastest Spade crit assembled at the time. The use of these 247:1 fine group cross sections (which incorporate flux depression factors of several percent in the more prominent resonances) in WHIZ calculations for more dilute systems was excused on the basis of the decreasing importance of the resonance region for these more thermal systems. Subsequently, these

Figs. 7a and 7b. Scalar flux spectra for the various Spade (BeO) crits plotted from the fine group WHIZ output as $E\Phi(E)$ vs E .

Figs. 8a and 8b. Scalar flux spectra for the various Snoopys (graphite) crits plotted from the fine group WHIZ output as $E\Phi(E)$ vs E .

Fig. 9. Low-energy scalar flux spectra for the various Spade (BeO) crits plotted from the fine groups WHIZ output as $\Phi(E)$ vs E .

Fig. 10. Low-energy scalar flux spectra for the various Snoopy (graphite) crits plotted from the fine group WHIZ output as $\Phi(E)$ vs E .

numbers were also used to calculate cross sections for the more recent 120:1 Spade where the resonance region is quite important. Recalculations with a more appropriate flux depression factor might yield somewhat smaller U^{235} capture and fission cross sections for the 120:1 Spade and the 301:1 Snoopy.

Table IVa gives the fine group U^{235} fission and capture cross sections (and α) used in WHIZ for all SUPERNOVA calculations. Table IVb shows the resonance integrals calculated for a very dilute (33,000:1 C/ U^{235}) system, and experimental values for the fission resonance integral from Ref. 41. (Unfortunately, no such data is available for capture.)

A single set of fine-group cross sections might suffice as input to WHIZ, but the standard 18 ZOOM groups are too coarse to expect a single set of 18 group numbers to be adequate for all M/U ratios. Therefore SUPERNOVA cross sections were calculated for each of the various Spade and Snoopy M/U ratios. This is a major and important departure from the present library set which uses one set of cross sections for all systems. Using the 247:1 numbers to calculate the 3826:1 crit gives a value of K nearly 5% below that calculated from the 3826:1 set. It is strongly recommended that cross sections applicable to the M/U ratio of interest be used wherever possible.

In addition to the cross sections calculated for the "extrapolated height" homogeneous crits, SUPERNOVA numbers were generated for each of the crits using the flux disadvantage factor feature of WHIZ (which now incorporates the SLLAB⁴² routine). For each of the crits, WHIZ calculated fine-group flux disadvantage factors appropriate to the foil thickness and spacing, (excluding the three triangular foil cases which we do not attempt to calculate), multiplied the Oy cross sections by these factors, and proceeded as before. The final SUPERNOVA numbers obtained are to be used as effective homogeneous cross sections; the self-shielding factors are incorporated in them and are not available separately.

All of the SUPERNOVA 18-group cross sections and transfer coefficients are now available on the SUPERNOVA library tape. Macroscopic numbers are listed for mixtures corresponding to each of the homogeneous and homogenized-foil Spade and Snoopy crits of Table V (the mixtures are calculated from the number densities of Table I). Microscopic numbers are given for the individual elements for all the homogeneous crits, and for the 120:1, 247:1, and 493:1 foil Spades.

Table IVa. Fine-group structure and U^{235} group data.

Group	$E_{\max}$	σ_c	σ_F	$\alpha = \sigma_c / \sigma_F$
189	10.5×10^6	0.0	1.876	0.0
188	9.0	0.0	1.847	
187	8.0	0.0	1.696	
186	7.0	0.0	1.337	
185	6.0	0.0	1.192	
184	5.5	0.0	1.177	
183	5.0	0.0	1.183	
182	4.5	0.0	1.208	
181	4.0	0.00705	1.241	0.00568
180	3.6	0.02095	1.283	0.01633
179	3.1622	0.03246	1.317	0.02465
178	2.9	0.04007	1.336	0.02999
177	2.7	0.04663	1.352	0.03449
176	2.5	0.05324	1.36	0.03915
175	2.3	0.06	1.36	0.04412
174	2.1	0.06725	1.357	0.04956
173	1.9	0.07744	1.333	0.05809
172	1.7	0.08803	1.306	0.06740
171	1.5	0.09848	1.280	0.07694
170	1.3	0.1089	1.253	0.08691
169	1.1	0.1168	1.233	0.09473
168	1.0×10^6	0.1223	1.220	0.1002
167	9.0×10^5	0.1275	1.206	0.1057
166	8.0×10^5	0.1384	1.208	0.1146
165	7.0×10^5	0.155	1.225	0.1265
164	6.0	0.1677	1.238	0.1355
163	5.5	0.1758	1.246	0.1411
162	5.0	0.1918	1.265	0.1516
161	4.5	0.2163	1.295	0.1670
160	4.0	0.2381	1.323	0.1800
159	3.6	0.2581	1.348	0.1915
158	3.1622	0.2746	1.368	0.2007
157	2.9	0.2857	1.382	0.2067

Table IVa. Fine-group structure and U^{235} group data. (Continued)

Group	$E_{\max}$	σ_c	σ_F	$\alpha = \sigma_c / \sigma_F$
156	2.7	0.2953	1.394	0.2118
155	2.5	0.3162	1.428	0.2214
154	2.3	0.3482	1.484	0.2346
153	2.1	0.3802	1.540	0.2469
152	1.9	0.4123	1.597	0.2582
151	1.7	0.4443	1.653	0.2688
150	1.5	0.4765	1.709	0.2788
149	1.3	0.5084	1.765	0.2880
148	1.1	0.5321	1.806	0.2946
147	1.0×10^5	0.5603	1.863	0.3008
146	9.0×10^4	0.6004	1.948	0.3082
145	8.0	0.6404	2.033	0.3150
144	7.0	0.6805	2.119	0.3211
143	6.0	0.7253	2.206	0.3288
142	5.5×10^4	0.7754	2.296	0.3377
141	5.0	0.8254	2.386	0.3459
140	4.5	0.8755	2.476	0.3536
139	4.0	0.9203	2.557	0.3599
138	3.6	0.9623	2.632	0.3656
137	3.1622×10^4	0.9976	2.701	0.3694
136	2.9	1.026	2.801	0.3663
135	2.7	1.051	2.901	0.3623
134	2.5	1.077	3.001	0.3589
133	2.3	1.102	3.101	0.3554
132	2.1	1.075	2.6	0.4135
131	1.9	1.050	2.555	0.4110
130	1.7	1.137	2.722	0.4177
129	1.5	1.216	2.862	0.4249
128	1.3	1.303	3.012	0.4326
127	1.1	1.426	3.246	0.4393
126	1.0×10^4	1.522	3.426	0.4442
125	9.0×10^3	1.576	3.507	0.4494
124	8.0	1.625	3.572	0.4549

Table IVa. Fine-group structure and U^{235} group data. (Continued)

Group	$E_{\max}$	σ_c	σ_F	$\alpha = \sigma_c / \sigma_F$
123	7.0	1.750	3.795	0.4611
122	6.0	2.096	4.503	0.4655
121	5.5	1.897	4.041	0.4694
120	5.0	2.083	4.406	0.4728
119	4.5	2.213	4.64	0.4769
118	4.0	2.306	4.799	0.4805
117	3.6	2.369	4.896	0.4839
116	3.1622	2.587	5.307	0.4875
115	2.9	2.508	5.121	0.4897
114	2.7×10^3	2.994	6.1	0.4908
113	2.5	2.871	5.828	0.4926
112	2.3	2.984	6.035	0.4944
111	2.1	3.18	6.405	0.4965
110	1.9	3.457	6.938	0.4983
109	1.7	3.286	6.583	0.4992
108	1.5	4.084	8.168	0.5000
107	1.3	4.44	8.88	0.5000
106	1.1	3.857	7.714	0.5000
105	1.0×10^3	3.958	7.916	0.5000
104	9.0×10^2	4.304	8.608	0.5000
103	8.0	5.45	10.9	0.5000
102	7.0	5.891	11.78	0.5008
101	6.0	6.892	13.78	0.5001
100	5.5	7.707	15.4	0.5005
99	5.0	6.516	13.03	0.5001
98	4.5	6.664	13.33	0.4999
97	4.0	6.189	12.38	0.4999
96	3.6	8.583	17.16	0.5002
95	3.1622	6.667	13.35	0.4994
94	2.9	10.45	20.89	0.5002
93	2.7	10.90	21.78	0.5005
92	2.5	10.42	20.82	0.5005
91	2.3	10.73	21.44	0.5005

Table IVa. Fine-group structure and U^{235} group data. (Continued)

Group	$E_{\max}$	σ_c	σ_F	$a = \sigma_c / \sigma_F$
90	2.1	9.768	19.52	0.5004
89	1.9	9.358	18.69	0.5007
88	1.7	9.507	19.01	0.5001
87	1.5	11.61	23.20	0.5004
86	1.3×10^2	10.88	21.72	0.5009
85	1.1	9.678	19.34	0.5004
84	1.0×10^2	11.18	22.34	0.5004
83	90×10^0	12.79	25.53	0.5010
82	80	14.78	29.45	0.5019
81	70	8.177	16.34	0.5004
80	60	31.52	61.60	0.5117
79	55	28.12	49.70	0.5658
78	50	22.94	29.61	0.7747
77	45	21.41	31.87	0.6718
76	40	13.83	32.24	0.4290
75	36	58.62	76.09	0.7704
74	31.622	16.24	13.5	1.203
73	29	25.84	27.8	0.9295
72	27	36.09	43.88	0.8225
71	25	40.43	48.89	0.8270
70	23	33.41	33.11	1.0091
69	21	73.94	95.76	0.7721
68	19	12.32	25.69	0.4796
67	17	34.24	29.19	1.173
66	15	11.70	39.81	0.2939
65	13	98.37	52.74	1.865
64	11	8.617	22.17	0.3887
63	10	32.816	68.99	0.4757
62	9	51.817	102.5	0.5056
61	8	25.867	19.04	1.359
60	7	103.4	31.41	3.292
59	6	5.432	15.24	0.3564
58	5.5×10^0	51.02	9.519	5.360

Table IVa. Fine-group structure and U^{235} group data. (Continued)

Group	$E_{\max}$	σ_c	σ_F	$\alpha = \sigma_c / \sigma_F$
57	5.0	43.1	9.558	4.509
56	4.5	5.884	7.715	0.7627
55	4.0	26.81	28.23	0.9497
54	3.6	13.29	33.14	0.4010
53	3.1622	8.755	30.78	0.2844
52	2.9	4.247	15.08	0.2816
51	2.7	4.909	11.03	0.4451
50	2.5	6.706	13.23	0.5069
49	2.3	8.638	15.15	0.5702
48	2.1	30.59	21.57	1.418
47	1.9	6.699	18.22	0.3677
46	1.7	8.141	21.30	0.3822
45	1.5	8.836	27.62	0.3199
44	1.3	21.804	74.85	0.2913
43	1.1	29.42	83.39	0.3528
42	1.0	18.77	64.75	0.2899
41	0.9	9.293	61.08	0.1521
40	0.8	8.694	61.67	0.1426
39	0.7	9.554	67.58	0.1414
38	0.6	11.11	77.04	0.1442
37	0.55	12.47	83.94	0.1486
36	0.5	15.22	97.59	0.1560
35	0.45	18.96	112.8	0.1681
34	0.4	25.39	138.3	0.1836
33	0.36	36.34	172.1	0.2112
32	0.31622	46.04	192.1	0.2397
31	0.29	48.97	195.8	0.2501
30	0.27×10^0	46.38	188.6	0.2459
29	0.25	44.98	179.9	0.2500
28	0.23	42.71	173.8	0.2457
27	0.21	40.69	175.1	0.2324
26	0.19	40.50	180.0	0.2250
25	0.17	39.17	190.6	0.2055

Table IVa. Fine-group structure and U^{235} group data. (Continued)

Group	$E_{\max}$	σ_c	σ_F	$a = \sigma_c / \sigma_F$
24	0.15	39.91	204.3	0.1953
23	0.13	42.02	223.9	0.1877
22	0.11	44.25	243.3	0.1819
21	0.10	45.62	260.6	0.1751
20	0.09	47.92	279.8	0.1713
19	0.08	52.67	303.4	0.1736
18	0.07	56.31	332.8	0.1692
17	0.06	60.43	359.0	0.1683
16	0.055	63.90	379.9	0.1682
15	0.05	68.11	404.1	0.1685
14	0.045	73.23	432.0	0.1695
13	0.04	78.60	461.6	0.1703
12	0.036	84.40	494.5	0.1707
11	0.031622	90.29	526.4	0.1715
10	0.029	94.76	550.7	0.1721
9	0.027	99.09	574.2	0.1726
8	0.025	103.9	600.8	0.1729
7	0.023	109.3	631.0	0.1732
6	0.021	115.5	665.4	0.1736
5	0.019	122.7	704.8	0.1741
4	0.017	131.1	751.0	0.1746
3	0.015	141.1	806.9	0.1749
2	0.013	153.5	876.3	0.1752
1	0.011	222.9	1233.3	0.1807

Table IVb. Integral data.

Reaction	E_c (eV)	Resonance integral	Source
Calculated σ_F	0.44	276.8	SUPERNOVA
Calculated σ_c	0.4	154.47	SUPERNOVA
Experimental σ_F	0.44	271	Ref. 42
Experimental σ_F	0.44	274 ± 11	Ref. 42

To illustrate the changes resulting from different M/U ratios, Appendix C lists the SUPERNOVA microscopic cross sections and transfer coefficients for the individual materials (Be^9 , O^{16} , U^{235} , and U^{238}) in the homogeneous 247:1 and 3826:1 Spades.

Table V. SUPERNOVA multiplication factors, K, for Spade and Snoopy crits.

Spade BeO/U-235	Foil size (mils)	Foil spacing (in.)	Base (cm × cm)	K	K (w/ Δv = 0.2 in groups 6-10)
120	Homogeneous		60.96 × 60.96	0.9620	1.0075
↓	16	1	60.96 × 60.96	0.9659	1.0079
247	Homogeneous		60.96 × 60.96	0.97107	1.00415
↓	8	1	60.96 × 60.96	0.97323	1.00395
493	Homogeneous		60.96 × 60.96	0.98768	1.00817
↓	4	1	60.96 × 60.96	0.98870	1.00825
↓	8	2	60.96 × 60.96	0.98467	1.00326
986	Homogeneous		60.96 × 60.96	0.99672	1.00843
↓	2	1	60.96 × 60.96	0.99879	1.01022
↓	4	2	60.96 × 60.96	0.99610	1.00722
↓	6	3	60.96 × 60.96	0.99828	1.00910
1920	Homogeneous		76.20 × 76.20	1.00240	1.00907
↓	1	1	76.20 × 76.20	1.00330	1.00989
↓	2	2	76.20 × 76.20	1.00062	1.00712
1920 - A	2	2	76.20 × 60.96	1.00430	1.01081
1920	3	3	76.20 × 60.96	1.00302	1.00944
3826	Homogeneous		91.44 × 91.44	1.00296	1.00665
↓	1	2	91.44 × 91.44	1.01356	1.01723
↓	2	4	91.44 × 91.44	1.01054	1.01417
7660	Homogeneous		91.44 × 91.44	1.01571	1.01776
301	Homogeneous		107.95 × 107.95	0.95871	1.00228
↓	4	0.5	107.95 × 107.95	0.95924	1.00171
603	Homogeneous		123.19 × 123.19	0.96218	0.99280
↓	2	0.5	123.19 × 123.19	0.96536	0.99531
1206	Homogeneous		123.19 × 123.19	0.98491	1.00363
↓	2	1	123.19 × 123.19	0.98377	1.00200
2355	Homogeneous		123.19 × 123.19	0.99518	1.00603
↓	1	1	123.19 × 123.19	0.99036	1.00103

Results

ZOOM calculations of K for each of the Spade and Snoopy crits of Table I are given in Table V and are plotted vs M/U^{235} in Fig. 11. The K for the extrapolated height "homogeneous" crits are replotted in Fig. 12, along with the corresponding values from the library cross sections and the values for Spade crits from the NOVA set. (The K for all Spades include the pedestal corrections of Table II.) Note that in going from the library numbers to the SUPERNOVA set the slopes of the curves have changed sign. This effect (evidenced also by the similar behavior of the NOVA curve) is caused in nearly equal parts from weighting the low energy U^{235} cross sections with fluxes appropriate to the M/U in question instead of using a single set for all M/U ratios, and from the increased resonance region U^{235} capture in the SUPERNOVA set.

Unfortunately, the new curve for K is not a constant 1.0 (the swing in K between fast and thermal systems remains at more than 4%). The similarity in slope of the Spade and Snoopy curves indicate that the likely chief villain is that material common to the two crits, Oy, and primarily U^{235} . The U^{235} thermal data is well documented, and the SUPERNOVA K 's for the more thermal crits are gratifyingly close to unity, so it is likely that the thermal group cross sections are pretty good.

The medium fission energy for the 247:1 Spade is in group 6; for the thermal Spades (3826:1 and 7660:1) it is in group 2. The corresponding shift in the Snoopys is from 8 to 2. In order to raise K by a few percent for the fast systems but without much change to the thermal K , it seems necessary to increase the effectiveness of U^{235} in the resonance energy region (from a few eV to a few hundred eV). It is just this energy range (as previously mentioned) where experiments are difficult and existing sets of data are often inconsistent.

The calculated K for the fast crits could be raised relative to the thermal systems by lowering the values of α (the ratio of radiative capture to fission) in the resonance region. Unfortunately, what experimental data there are indicate that the values of α used in generating the SUPERNOVA set (see listing in Table IV) are already too low in the resonance region.

In lieu of pushing the U^{235} capture cross section data below the believable limits, we offer an alternate means of achieving the same result. If ν (the

Fig. 11. Values of K for the various Spade and Snoopy crits calculated from the SUPERNOVA cross sections, plotted vs M/U^{235} (data from Table V).

Fig. 12. Values of K for the "homogenized" Spade and Snoopy crits calculated from the library, NOVA, and SUPERNOVA cross sections, plotted vs M/U²³⁵.

number of nascent neutrons per fission) is increased several percent in the resonance energy groups, the calculated K -vs- M/U curves for both Spades and Snoopys become very good. Table V and Fig. 13 give the calculated K for each of the crits considered (Spades and Snoopys, homogeneous and foil) when $\Delta\nu = 0.2$ ($\approx 8\%$ of 2.43) is added to groups 6-10. (This is easily done in ZOOM by setting $E^j = 2$ in the $C + DE^j = 2.43 + 0.1E^j$ term for $j = 6-10$.)

We can find no experimental measurement of ν in the resonance region to test the hypothesis that the average ν is higher there (or even different). Of interest, however, are recent experimental measurements by Cowan et al.⁴³ of relative abundances of a few fission fragments from various U^{235} resonance fissions. His limited results show that the fission fragment distribution differs from resonance to resonance and for the few isotopes measured was noticeably different from thermal fission results; but the measurements were nowhere near complete enough for a determination of ν .

Those reluctant to consider such an arbitrary change in ν as proposed may be happier to consider that identical results may be obtained in the following manner: the effect on K of changing ν for a group by a given percentage is exactly the same as would have resulted from changing the group fission cross section by the same percentage and adjusting the group capture cross section in the opposite direction so as to keep the total cross section invariant. Note, however, that this yields values of α for these groups which are lower than experimental data seem to warrant.

Comments and Future Efforts

One obvious shortcoming in the old library set of cross sections when used to calculate the Spade and Snoop crits is that only one set of U^{235} cross sections is available (a set whose thermal-group values seem designed for very fast crits). This defect is corrected in the NOVA and SUPERNOVA sets.

The explicit handling of anisotropic scattering in the SUPERNOVA set has increased K for Spades by 3-5%. Larger values of α (ratio of capture to fission in U^{235}) have decreased K for the fast Spades and Snoopys by about 6-7%. Lower Be ($n, 2n$) cross sections have reduced the Spades nearly 1%. A lower value for the thermal ν reduces all K by 2%. Crystalline effects lower K for the more thermal crits by about 1%. More consistent evaluation of transport cross sections have changed individual group leakages by a few percent, but the overall change in K is probably less than 1%. In general

Fig. 13. Values of K for the various Spade and Snoopy crits calculated from the SUPERNOVA cross sections, but with $\Delta v = 0.2$ added to groups 6-10 (data from Table V).

the basic cross section data, the theory, and the techniques used in generating the SUPERNOVA cross sections are substantially better than those used in preparing the library set.

It might reasonably be expected that all the modifications made in generating the SUPERNOVA cross section set would also result in improvements in the calculations of other parameters — preliminary calculations of a number of these effects have been done. Each of these areas will be reported on in the future in detail, but the following comments can be made here:

- 1) Neutron lifetime as compared with experimental prompt decay constant measurements with pulsed neutron sources on the Spades — the ZOOM library discrepancy has been halved.
- 2) Reactivity of sheet and bar experiments in Spade assemblies — again, the error resulting from use of the library tape cross sections is cut in half or less.
- 3) Hot Box⁴⁴ calculations of the temperature effect on reactivity — significant improvement in the calculated value of this effect; the most sensitive (1120:1) BeO/U-235 Hot Box is now in essential agreement with the experiment.

Further effort will be directed at obtaining cross sections of various materials which have been checked against the sheet and bar experiments; cross sections at elevated temperatures for use in dealing with Hot Boxes; and, finally, a complete set will be developed for a study of the Tory IIC configuration.

Effort is underway to improve transport cross sections by better theory, and by numerical comparisons of leakages and fluxes from ZOOM and DSN.

The details of fitting BeO and graphite to a Debye model crystal are still uncertain, and all crystalline effects are still somewhat tentative.

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APPENDIX A

(n,2n) Neutrons via the 2.43-MeV Be^{9*} Mode

Marion et al.²⁵ have measured angular distributions for the inelastic $\text{Be}^9(n,n_1)\text{Be}^{9*}$ reaction at 3.5, 4.1 and 5.0 MeV. Their results can be reasonably fit by an angular scattering probability

$$P(\mu_c)d\mu_c = \left(\frac{1}{2} + \frac{1}{4}\mu_c\right) d\mu_c$$

$$\therefore \bar{\mu}_c = \int_{-1}^1 d\mu_c \mu_c P(\mu_c) = 1/6 = \text{Average center-of-mass cosine of neutron.}$$

Upon converting to the lab frame one gets (to 1st order in μ)

$$P_{N_1}(\mu)d\mu = (1/2 + 0.359\mu)d\mu = \text{lab distribution for neutron}$$

and

$$P_{q*}(\mu)d\mu = (1/2 - 0.137\mu)d\mu = \text{lab distribution for } \text{Be}^{9*}.$$

Elementary calculations invoking conservation of energy and momentum give

$$E_{N_1} = 0.82 E' - 2.187 + 2\mu_c [0.81(E')^2 - 2.187 E']^{1/2} = \text{lab energy of neutron}$$

$$E_{q*} = 0.18 E' - 0.243 - 2\mu_c [0.81(E')^2 - 2.187 E']^{1/2} = \text{lab energy of } \text{Be}^{9*}$$

for an initial lab energy E' with the neutron emerging at center-of-mass cosine $-\mu_c$. We approximate these by evaluating the brackets at $E' \approx 3$ MeV (excitation of 2.43-MeV level is strongly peaked around $E' \sim 3$ for fission neutrons) and by assuming $\mu_c = \bar{\mu}_c = 1/6$. This gives our lab energy and angle distributions as:

$$E_{N_1} = 0.83 E' - 2.187 \quad P_{N_1}(\mu)d\mu = \left(\frac{1}{2} + 0.359\mu\right)d\mu$$

$$E_{q*} = 0.17 E' - 0.243 \quad P_{q*}(\mu)d\mu = \left(\frac{1}{2} - 0.137\mu\right)d\mu.$$

We must now consider the decay modes of the Be^{9*} 2.43-MeV level.

Mode (A): $\text{Be}^{9*} \rightarrow \text{Be}^8 + \text{N}$ ($Q = 0.768 \text{ MeV}$)

One easily establishes that the neutron lab energy is

$$E_{\text{NA}} = \frac{1}{9} [E_{\text{q}^*} + 8Q + \mu_c \sqrt{8QE_{\text{q}^*}}]$$

where E_{q^*} is the lab energy of the Be^{9*} , μ_c is cosine of angle of neutron in Be^{9*} frame. We use $E_{\text{q}^*} = 0.17 E' - 0.243$ as before and average over μ_c . Then

$$\bar{E}_{\text{NA}} = \frac{1}{9} [E_{\text{q}^*} + 8Q] = 0.0189 E' + 0.656.$$

We use this average as the lab energy distribution for the neutron emitted through mode (A).

Mode (B): $\text{Be}^{9*} \rightarrow \alpha + \text{He}^5 \rightarrow \alpha + \alpha + \text{N}$

Here things are complicated by the fact that the intermediate state is energetically forbidden classically. However, Heisenberg's uncertainty principle shows that the energy of such a short lived state can not be precisely pinned down.

$$\Delta E \Delta t \sim \hbar = 6.58 \cdot 10^{-22} \text{ MeV-sec.}$$

For $\Delta t = 2 \times 10^{-21}$ sec, $\Delta E \sim 0.33$ MeV is the uncertainty in the energy of the $\alpha + \text{He}^5$. We assume the α has $5/9$ (0.33) $\simeq 0.2$ MeV energy in the Be^{9*} frame. If the He^5 is emitted isotropically in the Be^{9*} frame, and the N is emitted isotropically in the He^5 frame, one can show that the average lab energy of the neutron

$$\overline{E}_{N_B} = \frac{1}{9} E_{q^*} + 0.432 = \frac{0.17E' - 0.243}{9} + 0.432 = 0.0189 E' + 0.459.$$

Henley and Kunz⁴⁴ find that approximately 7% of the decays proceed via mode (A), 93% via (B) and claim (C) very unlikely. Therefore, we get for the average lab energy of the neutron emerging from the 2.43-MeV Be^{9*} level

$$\begin{aligned} E_{N_2} &= 0.07 (0.0189 E' + 0.656) + 0.93 (0.0189 E' + 0.459) \\ &= 0.0189 E' + 0.473 \end{aligned}$$

Angular Distributions

For the angular distribution previously discussed we find the average lab cosine for the neutrons which excite the 2.43-MeV level ($1/4$ of the total when energy permits)

$$\overline{\mu}_{N_1} = \int_{-1}^1 d\mu \mu P_{N_1}(\mu) = \int_{-1}^1 d\mu \mu [1/2 + 0.359\mu] \simeq 0.24.$$

If we assume the other neutrons come off with the average $\overline{\mu} = 2/3A$ associated with isotropic elastic scattering, the average value of μ is

$$\overline{\mu} = \frac{1}{4} (0.24) + \frac{3}{4} \left(\frac{2}{27} \right) = 0.115.$$

This is perhaps the basis for the distribution used in the British "Barn Book."

APPENDIX B

Monte Carlo Correction to the Brooks Data

The experimental arrangement is sketched in Fig. 14-a. In the experiment two thick uranium disk targets are used, and three time-of-flight runs are made to measure:

a. BROOKS EXPERIMENTS

b. YOGI REGIONS

GLL-646-1631

Fig. 14. The Brooks experiment and the Monte Carlo representation.

- 1) Y_A , the count rate with the first target at position A;
- 2) Y_{AB} , the count rate with the first target at A and the second at B;
- 3) Y , the count rate with the second target at A.

The transmission ratio, Y_{AB}/Y_A , is a measure of the reactions in the second target. If N = number of incident neutrons which would hit A (if nothing at B) and N_F , N_C , N_S = number of these neutrons removed from beam by fissions, captures, and scatters at B,

$$\begin{aligned} \text{then } \frac{Y_{AB}}{Y_A} &= \frac{N - N_F - N_C - N_S}{N} = 1 - \frac{N_F}{N} \left(1 + \frac{N_C}{N_F} \right) \frac{N_F + N_C + N_S}{N_F + N_C} \\ &= 1 - \frac{N_F}{N} (1 + \alpha)\beta \end{aligned}$$

where $\beta \equiv (N_F + N_C + N_S)/(N_F + N_C)$ may be considered a scattering correction. Y is simply related to the number of prompt fission neutrons created in the second target by the detector efficiency K ,

$$Y = KvN_F.$$

Solving from above for N_F in terms of Y_{AB}/Y_A , one gets

$$\begin{aligned} Y &= \frac{KvN \left(1 - Y_{AB}/Y_A \right)}{(1 + \alpha)\beta} \\ \frac{\beta Y}{KN \left(1 - Y_{AB}/Y_A \right)} &= \frac{v}{1 + \alpha} \equiv \eta. \end{aligned}$$

This is the expression given by Brooks, but he makes little attempt to calculate β . Instead he presents values of η/β , with estimates of the energy regions in which one might expect $1 < \beta < 1.05$.

Since the target is not small in terms of mean free path (the transmission approaches zero at some of the resonances), a correction factor cannot be derived on the basis that a scattered neutron simply escapes. In order to render the data useful, a Monte Carlo representation of the transmission experiment was set up as sketched in Fig. 14-b and solved in the YOGI³² program. Region 5 approximates a void; regions 2, 3, and 4 were assigned essentially infinite capture cross sections. The cross sections in region 1 were from the sources used in this report for fission and scatter data; the

capture was calculated from the resonance parameters of Ref. 10 below 10 eV, and from Ref. 18 above 10 eV. Since the samples contained 93% U^{235} , 6% U^{238} , and 1% U^{234} , capture for U^{238} was included. (The η in this experiment includes the effect of U^{238} and U^{234} capture; this was later subtracted out to get the U^{235} capture cross sections determined with this approach.) Finite energy zones were used, the width being a compromise between problem running time and the fineness of the correction desired. The energy breakdown is detailed in Table VI, together with the number of particle histories per run and the largest observed fractional variance for the energy meshes of region 1.

Table VI. YOGI problems.

Energy Span (eV)	E(eV)	No. of Particle Histories	Largest Variance (reg. 1)
1-6	0.1	30,000	0.046
6-10	0.1	20,000	0.046
10-30	0.5	20,000	0.050
30-50	0.5	20,000	0.051

Figure 5 is a plot of the YOGI values for the scattering correction β and the resultant values of η .

$$\beta = 1 + \frac{C_2 + C_4}{C_1}$$

$$\eta = \beta \left(\frac{\eta}{\beta} \right)_{\text{Brooks}}$$

where C_i = capture fraction in region i from YOGI. The ~.5% variances in the C_i could be reduced by longer running times in the Monte Carlo calculation, but the extra effort seems unwarranted, since the above equation indicates that β is relatively insensitive to small errors in the C_i . The Brooks data error flags are generally much more than 5%, so little improvement in the resultant η curve could be obtained from better Monte Carlo statistics, or from the use of a finer energy mesh in YOGI.

From the values of η , we calculate

$$1 + a = \frac{2.414}{\eta}$$

$$\sigma_c = a\sigma_F$$

($\nu = 2.414$ since the experiment counts only prompt fission neutrons). It is obvious that uncertainties in η lead to greater uncertainties in a , particularly for small a . Fortunately, Brook's data appears strongest in the regions of large values of the cross sections and/or a , so that the energy regions where the derived data is most uncertain tend to be regions of lesser neutronic importance.

APPENDIX C

The SUPERNOVA Constants

I. Energy Groups and Fission Spectrum

$\nu^i = C + D \bar{E}^i$ $C = 2.43$ $D = 0.1$			
i	Upper energy (eV)	f_i	$\bar{E}^i$
18	$\geq 10^7$	0.1867	4.7
17	$\sqrt{10} \times 10^6$	0.5033	1.5
16	10^6	0.2382	0.6
15	$\sqrt{10} \times 10^5$	0.0579	0.2
14	10^5	0.0113	0.06
13	$\sqrt{10} \times 10^4$	0.0021	0.02
12	10^4	0.0004	0.006
11	$\sqrt{10} \times 10^3$	0	0.002
10	10^3	↓	2^a
9	$\sqrt{10} \times 10^2$		2^a
8	10^2		2^a
7	$\sqrt{10} \times 10^1$		2^a
6	10		2^a
5	$\sqrt{10}$		0
4	1	↓	↓
3	$\sqrt{10} \times 10^{-1}$		
2	10^{-1}		
1	$\sqrt{10} \times 10^{-2}$	↓	↓

^a Option which can be used to increase ν by $\Delta\nu = 0.2 \approx 8\%$ of 2.43, in groups 6-10.

II. Representative SUPERNOVA Cross Sections

On the following pages are the microscopic cross sections and transfer coefficients for U^{235} , U^{238} , Be^9 , and O^{16} for the 247:1 and 3826:1 Spade Critical assemblies. The complete set of SUPERNOVA cross sections, too lengthy to include herein, is available on the SUPERNOVA library tape.

ZOOM SUPERNOVA ZOOM 247 - 1 U²³⁵

N 47,27877999

C SIGMA	F SIGMA	TR SIGMA	S SIGMA	CELL FACTOR	GROUP
48,59000000	51,12700000	51,38259999	51,63279999	51,10000000	18
49,80900000	51,13080000	51,40140000	51,53609999	51,10000000	17
50,17629999	51,12540000	51,44159999	51,55990000	51,10000000	16
50,40429999	51,15840000	51,74650000	51,86589999	51,10000000	15
50,75080000	51,22410000	51,98499999	52,10179999	51,10000000	14
51,10839999	51,28620000	52,11179999	52,10699999	51,10000000	13
51,19280000	51,41289999	52,12150000	52,10949999	51,10000000	12
51,34209999	51,68869999	52,13270000	52,11000000	51,10000000	11
51,61599999	52,12319999	52,15359999	52,11000000	51,10000000	10
52,10069999	52,20119999	52,18689999	52,11000000	51,10000000	9
52,21290000	52,35680000	52,27030000	52,11000000	51,10000000	8
52,37560000	52,39780000	52,33640000	52,11000000	51,10000000	7
52,30580000	52,34109999	52,30000000	52,11579999	51,10000000	6
52,12989999	52,32290000	52,25300000	52,13300000	51,10000000	5
52,15920000	52,90680000	52,45600000	52,14319999	51,10000000	4
52,42820000	53,19459999	52,88600000	52,14619999	51,10000000	3
52,56520000	53,33219999	53,13800000	52,15489999	51,10000000	2
53,11900000	53,68320000	53,27000000	52,16379999	51,10000000	1

ZOOM SUPERNOVA ZOOM 247 - 1 U²³⁸

N 46,20785999

C SIGMA	F SIGMA	TR SIGMA	S SIGMA	CELL FACTOR	
49,11199999	50,60780000	51,39770000	51,70610000	51,10000000	18
50,10950000	50,33400000	51,45719999	51,67159999	51,10000000	17
50,17159999	-	51,54199999	51,77710000	51,10000000	16
50,21440000	-	51,79569999	52,10079999	51,10000000	15
50,47579999	-	52,10949999	52,12250000	51,10000000	14
50,59020000	-	52,12900000	52,13329999	51,10000000	13
50,72189999	-	52,15810000	52,15859999	51,10000000	12
51,17710000	-	52,17639999	52,17320000	51,10000000	11
51,32600000	-	52,18540000	52,17919999	51,10000000	10
52,12429999	-	52,29810000	52,27250000	51,10000000	9
52,45729999	-	53,17600000	53,16700000	51,10000000	8
52,54780000	-	52,77000000	52,65089999	51,10000000	7
53,11260000	-	52,48400000	52,14969999	51,10000000	6
50,49470000	-	51,83699999	51,82999999	51,10000000	5
50,63360000	-	51,83800000	51,82999999	51,10000000	4
51,10329999	-	51,84099999	51,82999999	51,10000000	3
51,16800000	-	51,84799999	51,82999999	51,10000000	2
51,31400000	-	51,86099999	51,82999999	51,10000000	1

[illegible]

ZOOM SUPERNOVA ZOOM 247 - 1 Be⁹

N 49,68860000

C SIGMA	F SIGMA	TR SIGMA	S SIGMA	CELL FACTOR	
50-47629999	-	51,12289999	51,25639999	51,10000000	18
48,94000000	-	51,18010000	51,23960000	51,10000000	17
-	-	51,33820000	51,38730000	51,10000000	16
-	-	51,44270000	51,48739999	51,10000000	15
-	-	51,52889999	51,57230000	51,10000000	14
-	-	51,55140000	51,59559999	51,10000000	13
-	-	51,55549999	51,60000000	51,10000000	12
46,38279977	-	51,55549999	51,60000000	51,10000000	11
46,67819987	-	51,55549999	51,60000000	51,10000000	10
47,12039998	-	51,55549999	51,60000000	51,10000000	9
47,21329999	-	51,55549999	51,60000000	51,10000000	8
47,37820000	-	51,55549999	51,60000000	51,10000000	7
47,67539999	-	51,55489999	51,60000000	51,10000000	6
48,12039999	-	51,55529999	51,60000000	51,10000000	5
48,21259999	-	51,55420000	51,60000000	51,10000000	4
48,37279999	-	51,55169999	51,60000000	51,10000000	3
48,62520000	-	51,54960000	51,60000000	51,10000000	2
49,11639999	-	51,56109999	51,60000000	51,10000000	1

ZOOM SUPERNOVA ZOOM 247 - 1 Be⁹

TRANSFER COEFFICIENTS

[illegible]

ZOOM SUPERNOVA ZOOM 247 - 1

O¹⁶

N 49,68860000

C SIGMA	F SIGMA	TR SIGMA	S SIGMA	CELL FACTOR	
49,46400000	-	51,14600000	51,19360000	51,10000000	18
-	-	51,22600000	51,25400000	51,10000000	17
-	-	51,39529999	51,45479999	51,10000000	16
-	-	51,38069999	51,37020000	51,10000000	15
-	-	51,35520000	51,37000000	51,10000000	14
-	-	51,35459999	51,37000000	51,10000000	13
-	-	51,35459999	51,37000000	51,10000000	12
-	-	51,36420000	51,37999999	51,10000000	11
-	-	51,36420000	51,37999999	51,10000000	10
-	-	51,36420000	51,37999999	51,10000000	9
-	-	51,36420000	51,37999999	51,10000000	8
-	-	51,36420000	51,37999999	51,10000000	7
-	-	51,36370000	51,37999999	51,10000000	6
-	-	51,36400000	51,37999999	51,10000000	5
-	-	51,36330000	51,37999999	51,10000000	4
-	-	51,36170000	51,37999999	51,10000000	3
-	-	51,35980000	51,37999999	51,10000000	2
-	-	51,35820000	51,37999999	51,10000000	1

ZOOM . SUPERNOVA

ZOOM

247 - 1

 O^{16}

TRANSFER COEFFICIENTS

[illegible]

ZOOM SUPERNOVA ZOOM 3826 - 1 U²³⁵

N 46,18000000

C SIGMA	F SIGMA	TR SIGMA	S SIGMA	CELL FACTOR	
48,59000000	51,12700000	51,38220000	51,63239999	51,10000000	18
49,81000000	51,13080000	51,40150000	51,53589999	51,10000000	17
50,17629999	51,12540000	51,44159999	51,55999999	51,10000000	16
50,40479999	51,15850000	51,74709999	51,86639999	51,10000000	15
50,75179999	51,22419999	51,98529999	52,10189999	51,10000000	14
51,10850000	51,28629999	52,11179999	52,10699999	51,10000000	13
51,19310000	51,41339999	52,12150000	52,10949999	51,10000000	12
51,34290000	51,69000000	52,13270000	52,11000000	51,10000000	11
51,61829999	52,12359999	52,15380000	52,11000000	51,10000000	10
52,10089999	52,20149999	52,18699999	52,11000000	51,10000000	9
52,21889999	52,36379999	52,27449999	52,11000000	51,10000000	8
52,38470000	52,40200000	52,34070000	52,11000000	51,10000000	7
52,30320000	52,33370000	52,29699999	52,11579999	51,10000000	6
52,13280000	52,33340000	52,25699999	52,13300000	51,10000000	5
52,16800000	52,94930000	52,47400000	52,14319999	51,10000000	4
52,42220000	53,20409999	52,92100000	52,14619999	51,10000000	3
52,61980000	53,36539999	53,15300000	52,15489999	51,10000000	2
53,12969999	53,74079999	53,30000000	52,16379999	51,10000000	1

TRANSFER COEFFICIENTS

[illegible]

ZOOM SUPERNOVA ZOOM 3826 - 1 U²³⁸

N 45,13419999

C SIGMA	F SIGMA	TR SIGMA	S SIGMA	CELL FACTOR	
49,10900000	50,60839999	51,39739999	51,70570000	51,10000000	18
50,10970000	50,33329999	51,45729999	51,67140000	51,10000000	17
50,17159999	-	51,54199999	51,77719999	51,10000000	16
50,21459999	-	51,79579999	52,10079999	51,10000000	15
50,47649999	-	52,10949999	52,12250000	51,10000000	14
50,59029999	-	52,12900000	52,13329999	51,10000000	13
50,72279999	-	52,15819999	52,15869999	51,10000000	12
51,17770000	-	52,17639999	52,17329999	51,10000000	11
51,32559999	-	52,18540000	52,17919999	51,10000000	10
52,12650000	-	52,30310000	52,27699999	51,10000000	9
52,47730000	-	53,18580000	53,17630000	51,10000000	8
52,54250000	-	52,76200000	52,64410000	51,10000000	7
53,11470000	-	52,49300000	52,15089999	51,10000000	6
50,49470000	-	51,83699999	51,82999999	51,10000000	5
50,64330000	-	51,84099999	51,82999999	51,10000000	4
51,11189999	-	51,85399999	51,82999999	51,10000000	3
51,18150000	-	51,86899999	51,82999999	51,10000000	2
51,33559999	-	51,90300000	51,82999999	51,10000000	1

TRANSFER COEFFICIENTS

[illegible]

ZOOM SUPERNOVA ZOOM 3826 - 1 Be⁹

N 49,68860000

C SIGMA	F SIGMA	TR SIGMA	S SIGMA	CELL FACTOR	
50-47680000	-	51,12279999	51,25619999	51,10000000	18
48,98000000	-	51,18000000	51,23940000	51,10000000	17
-	-	51,33820000	51,38730000	51,10000000	16
-	-	51,44289999	51,48760000	51,10000000	15
-	-	51,52899999	51,57240000	51,10000000	14
-	-	51,55140000	51,59559999	51,10000000	13
-	-	51,55549999	51,60000000	51,10000000	12
46,38359983	-	51,55549999	51,60000000	51,10000000	11
46,68039983	-	51,55549999	51,60000000	51,10000000	10
47,12099999	-	51,55549999	51,60000000	51,10000000	9
47,21509998	-	51,55549999	51,60000000	51,10000000	8
47,38249997	-	51,55549999	51,60000000	51,10000000	7
47,68039997	-	51,55549999	51,60000000	51,10000000	6
48,12129999	-	51,55549999	51,60000000	51,10000000	5
48,21670000	-	51,55549999	51,60000000	51,10000000	4
48,40580000	-	51,55539999	51,60000000	51,10000000	3
48,67470000	-	51,55660000	51,60000000	51,10000000	2
49,12580000	-	51,57890000	51,60000000	51,10000000	1

TRANSFER COEFFICIENTS

[illegible]

ZOOM SUPERNOVA ZOOM 3826 - 1 O¹⁶

N 49,68860000

C SIGMA	F SIGMA	TR SIGMA	S SIGMA	CELL FACTOR	
49,46800000	-	51,14520000	51,19290000	51,10000000	18
-	-	51,22629999	51,25420000	51,10000000	17
-	-	51,39529999	51,45479999	51,10000000	16
-	-	51,38060000	51,37020000	51,10000000	15
-	-	51,35520000	51,37000000	51,10000000	14
-	-	51,35459999	51,37000000	51,10000000	13
-	-	51,35459999	51,37000000	51,10000000	12
-	-	51,36420000	51,37999999	51,10000000	11
-	-	51,36420000	51,37999999	51,10000000	10
-	-	51,36420000	51,37999999	51,10000000	9
-	-	51,36420000	51,37999999	51,10000000	8
-	-	51,36420000	51,37999999	51,10000000	7
-	-	51,36420000	51,37999999	51,10000000	6
-	-	51,36420000	51,37999999	51,10000000	5
-	-	51,36409999	51,37999999	51,10000000	4
-	-	51,36400000	51,37999999	51,10000000	3
-	-	51,36390000	51,37999999	51,10000000	2
-	-	51,36939999	51,37999999	51,10000000	1

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