THE EFFECT OF SOIL PARAMETERS ON EARTH PENETRATION OF PROJECTILES

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a TECHNICAL report

from the Texas A & M
RESEARCH FOUNDATION

College Station, Texas

EARTH PENETRATION AND DYNAMIC SOIL MECHANICS

This work performed for
Sandia Laboratories
Albuquerque, New Mexico
Sandia Laboratories Purchase Contract No. 48-7432
Buyer A. J. Phillips
Requestor J. L. Colp 9327

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From the
Texas A & M
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EARTH PENETRATION

AND

DYNAMIC SOIL MECHANICS

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Department of Civil Engineering
College of Engineering
Texas A & M University
College Station, Texas 77840

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CHAPTER I

BACKGROUND

One of man's first concerns with the problem of earth penetration by a projectile developed from the required thickness of embankments to protect facilities and personnel from artillery fire. This problem received attention as early as 1742, in Robin's New Principles of Gunnery. The problem was considered by Euler in 1745, Poncelet in 1835, and Petry in 1910. From 1910 to 1943, Peres, Probert, Morin and Didion, H. P. Robertson, Nobile de Giori, and others reported work which evaluated the constants in the formulas previously developed. The significant findings of each of these men is summarized in Dynamic Penetration of Selected Projectiles into Particulate Media, by Louis J. Thompson (3).¹

In most of this work done prior to 1966, the investigator made an intuitive assumption for the soil force resisting the projectile's penetration. The assumed forcing function was equated to the mass of the projectile times its deceleration, as required by Newton's equation of motion. This equation was integrated twice to

¹Numerals in parentheses refer to corresponding items in the references at the end of this paper.

Note.-The Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers is used as a pattern for format and style for this work.

determine the final depth of penetration when the velocity was zero. Field tests were performed and the constants in the assumed soil resisting function evaluated. In all of the work that followed the above approach, the "constants" were found to be variables as the test conditions such as velocity of impact, target type, projectile area, etc., were varied. This indicates that the soil resisting functions were not assumed correctly. But the majority of the work done was before the days of high speed computers, so that realistically complicated soil resisting functions could not be considered, and before the days when telemetry and instrumentation were available to measure the soil resisting forces on the projectile.

The problem of earth projectile penetration may be stated in a general way as follows:

Given a projectile of specified configuration and physical composition, moving in a given trajectory, and impacting an earth target of known composition and boundary geometry, determine the time-dependent behavior of the projectile and the target during the penetration event.

There are many reasons to study earth penetration, both military and civil.

A few programs in which earth penetration technology is important are the following:

- 1) disposal methods for aero-space nuclear power supplies;
- 2) diagnostic studies of site conditions (bearing capacity,

mobility characteristics, etc.) through the use of penetrometer deceleration-time traces on the earth and extraterrestrial locations;

3) military use of earth-penetration weapons and the enhancing of the effectiveness of weapons by earth penetration;

4) design of buried protective structures and computation of the depth required for defense against earth penetrating weapons;

5) design of sampling devices that will allow very long cores of undisturbed soil to be obtained on land and in undersea locations; and

6) design of penetrometer anchors for both land and undersea use.

Sandia Laboratories, prime contractor to the Atomic Energy Commission, became concerned with the study of earth impact as early as 1960. The direct primary effort of Sandia Laboratories in the area has been in projectile design, instrumentation and telemetry, and full scale prototype testing.

Sandia Laboratories has spent much effort and money in full scale field testing of prototype projectiles, penetrating a large variety of natural soil and rock formations (4). It has been very difficult to obtain reliable data in these full scale tests, because it is very difficult to hit a given target from either an airplane or helicopter, and it is difficult to find targets that are non-layered, uniform, and homogeneous. In effect the experiment vari-

ables are difficult to control in outside, full scale tests using natural targets. Also it is difficult to measure the target's physical properties after the event. This full scale work has been summarized by Wayne Young of Sandia Laboratories (7). His method of data analysis assumes that the relationship for the final depth of penetration has the following form where

D = depth of penetration,

$$D = 0.53 SN \left(\frac{W}{A}\right)^2 \ln (1 + 2V_0^2 10^{-5}) \text{ for } V_0 \leq 200 \text{ ft/sec, and}$$

$$D = 0.0031 SN \left(\frac{W}{A}\right)^2 \ln (V_0 - 100) \text{ for } V_0 \geq 200 \text{ ft/sec.}$$

In each case

$$N \cong 0.56 + 1.83 (L/D) \text{ for tangent ogive noses, or}$$

$$N \cong 0.56 + 0.23 (L/D) \text{ for cone and inverse ogive noses.}$$

The impact velocity, V_0 , is in feet per second; the weight to area ratio of the projectile, W/A , is in pounds per square inch. L/D is the length to diameter ratio of the nose. By performing tests in which D , V_0 , W/A , and L/D are measured, the unknown soil "constant" s can be calculated. The "constant" s varies from 1 to 50 and denotes the penetrability of the soil.

Because of the above mentioned difficulties in full scale empirical testing, and the difficulties in analysis of data where all the parameters could not be measured, Sandia Laboratories decided to sponsor a terradynamics laboratory at Texas A&M University, where

part of the mission would be to perform laboratory penetration tests into carefully constructed earth targets of specified and known properties, according to an experimental design that minimizes the number of tests required and maximizes the effectiveness of the data. The purposes of the experiments that are performed in the Terramechanics Laboratory are as follows:

- 1) to empirically devise means of predicting forces on a penetrating projectile;
- 2) to generate methods of predicting the total depth of penetration for an event when all the parameters are known;
- 3) to provide understanding and data for use in the formulation of a fundamental penetration equation founded on the field equations of mechanics and a constitutive equation for soils and rock; and
- 4) to delineate the soil and projectile properties that affect penetration.

During 1967-1968 a conceptual design for the Terramechanics Laboratory was formulated (1), the laboratory experiment was designed (3), a part of the facilities was developed and constructed, and pilot tests were made. During 1968-1969 the facilities were further developed, the first laboratory data obtained, and the method of analysis promulgated.

The work reported herein as well as five other separate reports

describe the work for the year 1968-1969. The other reports are the following:

- 1) A Dynamic Constitutive Relationship for Earth Materials;
- 2) A Dynamic Elasto-Plastic Finite Element Analysis of Projectile Penetration of a Half-Space;
- 3) The Dynamic Shearing Resistance of Clay as a Function of the Rate of Shear Deformation;
- 4) The Dynamic Response of Projectiles to Various Forcing Functions; and
- 5) Summation and Final Report for the Earth Penetration and Dynamic Soil Mechanics Project for the 1968-1969 year.

CHAPTER I-I

PURPOSE

The purpose of this study can be divided into the three following parts:

- 1) to present complete sets of data obtained for projectile penetration into soil targets of precisely known properties, and to describe how to build and test these targets;
- 2) to present a numerical method of data analysis for empirically determining the force on a projectile, during penetration; and
- 3) to demonstrate how physical soil parameters, as well as impact velocity and a projectile property influence penetration.

CHAPTER III

PLAN OF ANALYSIS AND EXPERIMENT DESIGN

The following is a description of the method to be used for analyzing the test data obtained.

Newton's equation of motion is the starting point for the solution. The equation of motion of the projectile is (Fig. 3.1)

$$(M_p + M_c) A = g (M_p + M_c) - F \quad (\text{Eq. 3.1a})$$

or

$$(M_p + M_c) v \frac{dv}{dz} = g (M_p + M_c) - F \quad (\text{Eq. 3.1b})$$

or

$$(M_p + M_c) \frac{d^2 z}{dt^2} = g (M_p + M_c) - F \quad (\text{Eq. 3.1c})$$

where

M_p = mass of the projectile,

M_c = mass of the soil moved by the projectile,

A = acceleration of the projectile,

v = velocity of the projectile = $\int A dt$, (Eq. 3.2)

v_i = velocity at impact,

z = depth of the projectile = $\int v dt = \iint A dt dt$, (Eq. 3.3)

d = total depth of penetration,

FIGURE 3.1

DYNAMIC FREE BODY DIAGRAM OF THE
PROJECTILE AFTER IT HAS ENTERED SOIL

t = time,

g = gravitational constant, and

F = soil resistance to projectile motion.

The boundary conditions are the initial conditions $z = 0$ when $v = v_i$ and $t = 0$, and the final conditions $z = d$ when $v = 0$ and $t = \infty$.

F is the unknown soil resisting function, and if this function can be predicted, determined, or measured, for a target projectile system, the following acceleration, velocity, and depth can be directly calculated from Eqs. 3.1, 3.2, and 3.3 (Fig. 3.2).

$$A = \frac{F}{M_p + M_c} + g \quad (\text{Eq. 3.4})$$

$$v = \int \left(\frac{F}{M_p + M_c} + g \right) dt \quad (\text{Eq. 3.5})$$

$$z = \int \left(\int \left(\frac{F}{M_p + M_c} + g \right) dt \right) dt \quad (\text{Eq. 3.6})$$

Therefore, once F is known, the entire problem is solved.

A decision had to be made as to what parameters would be considered to affect F , so that they could be measured during the testing. Any number of variables could have been considered. The following list of variables represents those thought to affect F .

U_1 = Velocity

U_2 = Depth

U_3 = Bulk density

FIGURE 3.2
RELATIONSHIPS AMONG FORCE, ACCELERATION, VELOCITY, DEPTH, & TIME

- U_4 = Degree of saturation
- U_5 = Cohesive yield strength
- U_6 = Angle of internal friction
- U_7 = $\frac{\text{Strain energy density to failure}}{\text{Mean principle stress at failure}}$
- U_8 = Projectile diameter
- U_9 = $\frac{\text{Afterbody length}}{\text{Area}}$
- U_{10} = $\frac{\text{Afterbody length}}{\text{Diameter}}$
- U_{11} = $\frac{\text{Nose length}}{\text{Diameter}}$
- U_{12} = $\frac{\text{Nose surface area}}{\text{Area projectile}}$
- U_{13} = Nose sharpness angle
- U_{14} = $\frac{\text{Nose volume}}{\text{Area} \times \text{nose length}}$
- U_{15} = $\frac{\text{Nose weight}}{\text{Area}}$
- U_{16} = Angle of target surface from horizontal
- U_{17} = Angle of attack between projectile axis and trajectory
- U_{18} = Angle of incidence of trajectory as measured from direction of gravity

The variables velocity and depth are not constant for a given test, but vary throughout the test. The other variables, U_3 through U_{18} are constant for a given test, but vary from test to test.

Any continuous function can be represented in the form of a generalized series. One simple representation would be a generalized power series.

If a solution is desired for the force on the projectile as a function of any variables that one wishes to consider, this function can be represented in the following form.

$$F = A_0 + \sum_{i=1}^n u_i A_i + \sum_{i=1}^n \sum_{j=1}^n u_i u_j A_{ij} + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n u_i u_j u_k A_{ijk} + \dots \quad (\text{Eq. 3.7})$$

F = the force on the projectile

u = variables to be considered

n = number of variables to be considered

A = linear constants

When the acceleration experienced by the projectile as a function of time is known, and since the force, velocity, and depth of penetration are known functions of the acceleration at any instant of time in any test one can obtain as many force, velocity, and depth of penetration values as desired from each test. The soil parameters are considered to remain constant throughout each test, but vary from test to test. In the primary series of tests performed for this thesis, the projectile parameters were not considered, and were held constant throughout all tests by using the same projectile.

This assumed function represents all of the variables in n -space, where all axes are orthogonal. Therefore, the values of

all variables for one instant of time, u_1 through u_n , represent one data point in n -space. One test will be considered to have one set of data points, with velocity ranging from impact velocity to zero, and depth of penetration ranging from zero to total depth of penetration.

It can be seen that the number of terms in Eq. 3.7 rapidly becomes excessive. The total number of terms equals $\sum_{i=0}^n n^i$. For example, if the number of variables to be considered is 7, the total number of terms generated is $7^0 + 7^1 + 7^2 + 7^3 + 7^4 + 7^5 + 7^6 + 7^7 = 960,800$. If of the terms that are repetitive, all are omitted except one (i.e., $A_{12}u_1u_2$ and $A_{21}u_2u_1$, omit one term; $A_{123}u_1u_2u_3$, $A_{132}u_1u_3u_2$, $A_{213}u_2u_1u_3$, $A_{231}u_2u_3u_1$, $A_{312}u_3u_1u_2$, and $A_{321}u_3u_2u_1$, omit five terms; etc.) this number is reduced to 13,700 terms.

However, all terms of the series do not have to be considered. Starting with x data points, summed over all sets in n -space, a "best fit" through these points will be determined for a curve which has the general shape of the assumed function. For instance, if only the linear terms (i.e., $A_0 + A_1u_1 + \dots + A_nu_n$) are considered, a straight line will be fit through the data points. If the quadratic terms with their interactions (i.e., $A_{11}u_1u_1 + A_{12}u_1u_2 + \dots + A_{22}u_2u_2 + A_{23}u_2u_3 + \dots + A_{nn}u_nu_n$) are also considered, the curve will have a quadratic form. If the cubed terms are considered with all of their interactions, the curve will have the form of a cubic, and so forth.

In order to obtain a "best fit" for the assumed function through the data points, a method of least squares was used (5). Stated simply, this method minimizes the sum of the squares of the straight line distance between the assumed curve and each data point. This is accomplished by multiplying the assumed function in turn by the partial derivative of the assumed function with respect to each constant.

$$\frac{\partial}{\partial A_0} (A_0 u_0 + A_1 u_1 + \dots + A_{(n)(n)} u_{(n)(n)}) = u_0 \quad (\text{Eq. 3.8a})$$

$$u_0 = 1 = \text{dummy variable}$$

$$\frac{\partial}{\partial A_{(n)(n)}} (A_0 u_0 + A_1 u_1 + \dots + A_{(n)(n)} u_{(n)(n)}) = u_1 \quad (\text{Eq. 3.8b})$$

etc.

$$\frac{\partial}{\partial A_{(n)(n)}} (A_0 u_0 + A_1 u_1 + \dots + A_{(n)(n)} u_{(n)(n)}) = u_{(n)(n)} \quad (\text{Eq. 3.8c})$$

By summing over the total number of data points, this method generates a set of simultaneous linear equations that can be solved by simply applying the measured u -values, inverting the matrix, and multiplying by the force vector.

$$\sum F u_0 = A_0 \sum u_0 u_0 + A_1 \sum u_0 u_1 + \dots + A_{(n)(n)} \sum u_0 u_{(n)(n)} \quad (\text{Eq. 3.9a})$$

$$\sum F u_1 = A_0 \sum u_1 u_0 + A_1 \sum u_1 u_1 + \dots + A_{(n)(n)} \sum u_1 u_{(n)(n)} \quad (\text{Eq. 3.9b})$$

"

"

$$\sum F u_{(n)(n)} = A_0 \sum u_{(n)(n)} u_0 + A_1 \sum u_{(n)(n)} u_1 + \dots + A_{(n)(n)} \sum u_{(n)(n)} u_{(n)(n)} \quad (\text{Eq. 3.9c})$$

Once the A-constants are known, an equation is obtained for the force on the projectile at any time during the penetration event as a function of the variables assumed to affect the force. For example, if

$$A_0 = 3, A_1 = -2, A_2 = 6, \text{ etc.}$$

$$F = 3 - 2u_1 + 6u_2 + \dots, \text{ etc.} \quad (\text{Eq. 3.10})$$

This method can be applied to any number of tests with any number of data points for any number of variables which are measured. The more tests and sets of variable values used as input, the closer the determined equation will be able to predict the force on a projectile in a future test. Generally, the more terms taken in the assumed function, the better the equation will fit the input data.

This empirically determined equation is limited to the ranges of the variables chosen for study. If variable x was allowed to

range from 50 to 100 during testing, the equation which was determined could not be interpolated to accept a value of 500 for x , to be used as input data. This is due to the many interactions of the variables considered.

This same method could have been used to expand any measured variable as a function of the others. However, force was chosen because of its unique relationship with acceleration, velocity, and depth of penetration (Eqs. 3.4, 3.5, and 3.6).

Given an instantaneous velocity and corresponding depth, for a unique set of soil parameters, the equation will predict the force on the projectile for that instant of time. However, given an impact velocity, and applying the corresponding boundary condition that depth of penetration equals zero, the acceleration, $A = \text{force/mass}$, can be repeatedly integrated over a chosen time increment to obtain the velocity and depth as a function of time (Fig. 3.2). When the velocity goes to zero, the corresponding depth equals the total depth of penetration.

From $F = mA$,

$$F = \frac{W}{g} A - W \quad (\text{Eq. 3.11a})$$

or

$$A = g \left(\frac{F}{W} - 1 \right) \quad (\text{Eq. 3.11b})$$

where

F = force on projectile

W = weight of projectile

A = acceleration of projectile

g = acceleration due to gravity

The force is known as a function of instantaneous velocity and depth, and all soil parameters being considered, by determining the A -constants of the assumed function.

Therefore, by applying the boundary conditions, v_0 = impact velocity and z_0 (instantaneous depth of penetration) = 0, the instantaneous acceleration of the projectile can be determined.

By multiplying this value times a chosen time interval, the change in velocity for that time interval can be determined.

$$\Delta v = (A)(\Delta t) \quad (\text{Eq. 3.12})$$

where

Δv = change in velocity, and

Δt = change in time.

The velocity versus time relationship can be assumed to take any form (i.e., linear, quadratic, cubic, etc.), but if it is assumed to vary linearly over a very small time increment, the velocity at the end of Δt is

$$v_1 = v_0 - \Delta v \quad (\text{Eq. 3.13})$$

and the change in depth of penetration, Δz , is

$$\Delta z = \left(\frac{v_0 + v_1}{2} \right) \Delta t.$$

If these integrations are repeated until the velocity of the projectile is zero, the depth, z , at that time is the total depth of penetration (Fig. 3.2). It can be seen that by expanding the assumed solution for the force on the projectile, an expression is obtained which is extremely flexible, and the problem is, in fact, solved.

For a given test, the force on the projectile can be expressed as a function of instantaneous velocity and depth, grouping the effects of all other variables into a single constant, C_0 .

$$F = C_0 + C_{11}z + C_{12}z^2 + C_{13}z^3 + \dots + V(C_{21} + C_{22}z + C_{23}z^2 + \dots) + V(C_{31} + C_{32}z + \dots) \quad (\text{Eq. 3.14})$$

A second method of solution was adopted, which could accept the data from penetration tests in which all values were measured, except an acceleration versus time relationship. In this method, all variables were expanded for the total depth of penetration. The velocity was considered as impact velocity only. Therefore, all variables were considered single valued for each test, and one data point was obtained for each test.

Only the linear terms were considered, plus the impact velocity squared, and in one solution, the impact velocity squared times the bulk density.

Using this method, the parameters cannot be determined as a function of time, but rather, given a set of soil parameters and an impact velocity, a prediction can be made for the total depth of penetration.

The physical, experimental, and statistical design of the experiment are presented in detail in The Conceptual Design of a Terradynamics Laboratory (1) and Earth Penetration and Dynamic Soil Mechanics, Statistical Design Report (3).

There is a total of 188 tests to be conducted--47 tests for each of four different target materials. Table I lists these 47 tests.

TABLE I

EXPERIMENTAL LEVELS FOR THE 47 DESIGN POINTS FOR ONE SOIL

Treatment Code	Independent design variables						
	a = nose shape	b = projectile length	c = compaction energy in soil pre-paration	d = degree of saturation	e = impact velocity	f = projectile diameter	
<u>1/8 replicate</u> <u>in steel</u>			in.	%	fps	in.	
	short	cone	S	low	L-1	100	1.56
	short	cone	L	high	L-1	100	1.56
	short	cone	S	low	L-1	800	3.06
	short	cone	L	high	L-1	800	3.06
	short	ogive	L	low	H-1	800	1.56
	short	ogive	S	high	H-1	800	1.56
	short	ogive	L	low	H-1	100	3.06
	short	ogive	S	high	H-1	100	3.06
	long	cone	S	low	H-1	100	3.06
	long	cone	L	high	H-1	100	3.06
	long	cone	S	low	H-1	800	1.56
	long	cone	L	high	H-1	800	1.56
	long	ogive	L	low	L-1	800	3.06
	long	ogive	S	high	L-1	800	3.06
	long	ogive	L	low	L-1	100	1.56
	long	ogive	S	high	L-1	100	1.56

TABLE I (continued)

Independent design variables

Treatment
Code

a = nose shape	b = projectile length	c = compaction energy in soil preparation	d = degree of saturation	e = impact velocity	f = projectile diameter
----------------	-----------------------	---	--------------------------	---------------------	-------------------------

1/8 replicate
in aluminum

			in.	%	fps	in.
long	cone	L	high	L-I	800	3.06
long	cone	S	low	L-I	800	3.06
long	cone	L	high	L-I	100	1.56
long	cone	S	low	L-I	100	1.56
long	ogive	S	high	H-I	100	3.06
long	ogive	L	low	H-I	100	3.06
long	ogive	S	high	H-I	800	1.56
long	ogive	L	low	H-I	800	1.56
short	cone	L	high	H-I	800	1.56
short	cone	S	low	H-I	800	1.56
short	cone	L	high	H-I	100	3.06
short	cone	S	low	H-I	100	3.06
short	ogive	S	high	L-I	100	1.56
short	ogive	L	low	L-I	100	1.56
short	ogive	S	high	L-I	800	3.06
short	ogive	L	low	L-I	800	3.06

TABLE I (continued)

Independent design variables

Treatment Code

a	b	c	d	e	f
= nose shape	= projectile length	= compaction energy in soil preparation	= degree of saturation	= impact velocity	= projectile diameter

Star design points
(in steel)

		in.	%	fps	in.
blunt		I	int.	I	333 2.18
long	ogive	I	int.	I	333 2.18
long	cone	I	int.	I	333 2.18
blunt		L	int.	I	333 2.18
blunt		S	int.	I	333 2.18
blunt		I	high	I	333 2.18
blunt		I	low	I	333 2.18
blunt		I	int.	H	333 2.18
blunt		I	int.	L	333 2.18
blunt		I	int.	I	800 2.18
blunt		I	int.	I	100 2.18
blunt		I	int.	I	333 3.06
blunt		I	int.	I	333 1.56
short	ogive	I	int.	I	333 2.18
short	cone	I	int.	I	333 2.18

Code for Projectile Length

S = Short
I = Intermediate
L = Long

Code for Degree of Saturation

L = Low
L-I = Low-Intermediate
I = Intermediate
H-I = Intermediate-High
H = High (100%)

Because the large, mobile gas gun, which was capable of delivering the projectile at the impact velocities required in the statistical design, was being used by Sandia Corporation when testing began, a modified series of tests were chosen to be performed. These tests were not called for in the statistical design, but were chosen to yield as much information as possible with the available equipment.

The design variables of primary interest in these tests were the degree of saturation of the soil and the compactive effort delivered to the soil. All 18 variables, u_1 through u_{18} , were measured for each test. However, only variables u_1 through u_7 were allowed to vary from test to test. The other variables measured in the primary series of tests were arbitrarily chosen to remain constant at the following values:

- 1) projectile diameter--1 9/16 inches,
- 2) projectile weight--5.522 pounds,
- 3) projectile length--20 1/8 inches,
- 4) projectile nose shape--blunt,
- 5) projectile impact velocity--200 fps,
- 6) target surface--horizontal,
- 7) angle of incidence--perpendicular to target surface,
- 8) angle of attack--same as velocity vector of projectile,
- 9) target size--cylindrical, 4 ft. in diameter, 4 ft. in height, and

10) type of soil--kaolin clay.

With all the values mentioned above held constant, the primary series of tests to be run consisted of the five following combinations of compactive effort and degree of saturation of the soil.

TABLE 2
PRIMARY TESTS

Target	Compactive Effort	Degree of Saturation
1	low	medium
2	medium	medium
3	high	medium
4	medium	low
5	medium	high

The methods used to achieve these levels will be discussed in more detail in the following chapter.

During the period of testing of the targets which were built, it was decided that since a large amount of time and effort went into the construction of each target, additional information could be obtained by penetrating each target more than once. These extra tests were shot off-center into the targets, and varied in projectile velocity, nose shape (Fig. 3.3), and projectile weight. It is acknowledged that these extra tests were not performed under exacting conditions, since some disturbance of the targets occurred during

the first penetration, but it is felt that in most cases this disturbance was slight, and valuable information was obtained.

The kaolin clay was obtained from Kosse, Texas. It has the following physical and chemical properties:

- 1) specific gravity of the solids = 2.62
- 2) liquid limit = 40%
- 3) plasticity index = 10%
- 4) All material passes a number 270 sieve.

An x-ray diffraction test was performed on this clay (Fig. 3.4). The material was fractionated into the following groups of particle sizes: $50 - 5\mu$, $5 - 2\mu$, $2 - 0.2\mu$, and $<0.2\mu$. The results of these tests indicate that the material is highly crystalline kaolinite, containing only a trace of illite and quartz in the larger fractions.

The results of compaction tests are presented in Fig. 3.5.

- 1) Short cone
- 2) Long cone
- 3) 1.25 caliber ogive
- 4) 6.50 caliber ogive
- 5) Blunt

FIGURE 3.3

THE FIVE LEVELS OF PROJECTILE NOSES

X-RAY DIFFRACTION

FIGURE 3.5
COMPACTION TESTS

CHAPTER IV

TESTING PROCEDURES

General

The methods of testing can be separated into two main categories. The first of these involves the construction of uniform soil targets. If the targets are not uniform with respect to the measured soil properties, and if these properties are not known precisely, information obtained from penetrating these targets will be of little value. The second of these is the penetration of these targets by projectiles.

A large effort was made prior to the beginning of testing in both of these categories to determine the most accurate and efficient methods to use. However, problems arose and better procedures were discovered, both of which led to modifications of the original testing plan during the period of testing.

Soil Targets

All of the clay targets were constructed of kaolin clay. Before the building of full scale targets began, a series of compaction tests were run in six-inch diameter steel molds. The moisture content, number of layers, and number of blows were varied in an effort to obtain relationships to be used in the construction of

the full scale targets (Fig. 3.5).

The full scale targets were constructed in steel cylindrical molds. There were two molds, measuring four feet in diameter and four feet in height. They had a wall thickness of 1/4 inch. Both molds were split lengthwise, and flanged all around with 2 1/2 inch by 2 1/2 inch by 1/4 inch angles (Fig. 4.1). The molds could be used separately for building two targets which are four feet tall, or bolted together for building one target, eight feet in height. These molds were assumed to give infinite lateral resistance to the soil targets.

The compacting of the clay targets was done with a Cleco pneumatic floor rammer, model 5FB (Fig. 4.1). This floor rammer is rated at 550 cycles per minute using an air pressure of 150 psi.

Control was maintained over the compactive effort delivered to the target by the floor rammer by passing evenly over the surface of the target using a spiral technique. That is, each successive pass over the target was alternately spiraled from the perimeter to the center, and then from the center to the perimeter. Each pass was made in forty-five to fifty seconds, hand holding the floor rammer as vertically as possible. Each pass was begun when the air pressure reached 150 psi, and the pressure was not allowed to drop below 125 psi.

In the experimental design, the thickness of each compacted layer was arbitrarily chosen to be four inches. When running the

FIGURE 4.1

4FT. $\times$ 4FT. DIAMETER STEEL CYLINDRICAL SOIL MOLD AND
PNEUMATIC FLOOR RAMMER

compaction tests in the six-inch diameter molds, it was discovered that layers over two inches thick would not compact to a uniform density. That is, each layer had a density gradient, being denser at the top of the layer than at the bottom.

For this reason, the first four foot diameter target was constructed using layers consisting of 200 pounds of clay and water mixture. This amount of material compacted to a thickness of approximately 1 1/2 inches. When the triaxial samples taken from this target were examined, it was discovered that there still existed a slight density gradient within each layer. Every target after the first one was compacted in layers consisting of 100 pounds of clay and water mixture, hereafter referred to as a lift.

A problem was encountered in obtaining a good bond between layers. That is, the compacted layers did not bond well to each other, causing the triaxial samples to sometimes break between layers when extruded from the sampling tube. This problem was partially eliminated by scarifying or scratching the top of each compacted layer before the next lift was added.

The clay and water were mixed together in a J. C. Steele Model 200E pugmill (Fig. 4.2). This pugmill was equipped at the Terra-mechanics Laboratory with a material hopper, which is adjustable to control the rate of flow of material into the pugmill. The water was added through two spray bars, having ten adjustable nozzles on each bar. The quantity of water added was measured with a water

FIGURE 4.2
OPEN END PUGMILL

meter.

The clay and water were mixed and recycled through the pugmill until a uniform batch was obtained. A well mixed batch had particles ranging in size from approximately 1/2 inch in diameter to less than 1/64 inch in diameter.

Many variations were tried in the mixing procedure in an attempt to minimize the time and effort spent in mixing a batch of material. These variations included changing the pitch of the blades on the pugmill shaft, different procedures and orders of introducing clay and water to the pugmill, sealing off the exit end of the pugmill, and changing the size of each batch. Originally, batches were mixed using 800 pounds of dry clay. The batches increased in size up to 3300 pounds of dry clay per batch.

Testing of Target Properties

The following properties were measured in the soil targets both during construction of the targets and after the targets were penetrated:

- 1) moisture content (the weight of the water divided by the weight of the dry solids),
- 2) bulk or wet density (the weight divided by the volume of a given sample), and
- 3) soil strength parameters, cohesion (c) and the angle of internal friction (ϕ), at failure.

The primary purpose of measuring these properties during the construction of the targets was to maintain a check on uniformity. If excessive variations were detected, those layers were removed and rebuilt.

The measuring of these properties after penetration of the targets was performed in order to determine if any changes in these properties had taken place, and to be used as more nearly the correct values at the time of penetration.

Moisture content determinations were made using an electric balance, accurate to 0.01 grams, and an electric oven. The moisture content samples were allowed to dry overnight at 105 degrees Centigrade.

Bulk density measurements were performed by several methods, including weight and volume measurements of triaxial samples, a balloon density meter (Fig. 4.3), and the sand cone method of dropping 20-30 Ottawa sand from a known height into a hole from which the soil removed has a known weight. Of these methods, the in-place density measurement, using the balloon density meter, proved to be the most consistent.

The soil strength parameters, cohesion and the angle of internal friction, were measured by two methods. One was a direct measurement of these values at failure, using a Cohron Sheargraph (Fig. 4.4). This device has a vaned foot and a calibrated spring which resists torque and normal force. It actually plots a failure envelope.

FIGURE 4.3
BALLOON DENSITY METER

FIGURE 4.4
COHRON SHEARGRAPH

lope by marking on a pre-calibrated, pressure-sensitive paper.

The other method of measuring these strength parameters was by taking triaxial samples, 2.775 inches in diameter, using a steel, thin-walled tube, and testing those samples in compression at different confining pressures in a triaxial testing machine.

Penetration Tests

The primary information sought from the projectile penetration tests was the following:

- 1) the acceleration versus time relationship for the entire event,
- 2) the impact velocity,
- 3) the angles of incidence and attack of the projectile,
- 4) surface heave of the target, and
- 5) total depth of penetration.

The acceleration versus time relationship was the most difficult to obtain. Frequent modifications were required to either correct or improve the system which gathered this information.

The projectiles were equipped with on-board instrumentation, including either piezoelectric or piezoresistive accelerometers, with the necessary amplifiers and circuits to enable the instrument packages to transmit the acceleration versus time information. Two types of transmitting systems were used. At first, the instrumentation package contained, along with the accelerometer and cir-

cultry, a transmitter, an antenna, and a "one shot," water activated battery. This battery proved unreliable, and was replaced with two nickle-cadmium, rechargeable batteries (Fig.4.5). This system also proved unreliable, and the entire system was replaced with a hard-wire system. The hard-wire system eliminated the transmitter, batteries, and antenna, and used instead an extremely small diameter, high strength, stranded steel wire, which was attached directly to the receiving equipment. The wire consisted of four insulated wires, which were wound on a tapered bobbin, and trailed out of the rear of the projectile during flight (Fig.4.6). This system was used on all tests except the first one.

The acceleration versus time record was recorded on magnetic tape, to be played back on a Honeywell 906C visicorder for viewing and evaluating.

Impact velocity was measured by three different methods. A chronograph, consisting of an aluminum tube which held two pre-focused light sources and two corresponding light sensitive photo cells, placed precisely six inches apart, was used for two methods of velocity measurement. The most dependable velocity measurement was made by recording the voltage output of the two photo cells on magnetic tape along with time marks. (The rise time of the photo cell was approximately five microseconds.) Then, by knowing the distance between the two light beams and the time elapsed between the breaking of these light beams, the projectile velocity could be

1) Antenna	5) Nickel-cadmium rechargeable batteries
2) Metal sleeve	6) Amplifiers and circuitry
3) Hollow projectile afterbody	7) Accelerometer
4) Transmitter	

FIGURE 4.5
RADIO INSTRUMENT PACKAGE

FIGURE 4.6
HARD-WIRE INSTRUMENT PACKAGE

determined.

A more accurate, but less dependable velocity measurement was made by using the voltage output of these same photo cells to trigger a Beckman/Berkeley counter, which measured the elapsed time in microseconds, between the breaking of the light beams.

The third method of measuring velocity was primarily a check on the others. Back drops, which were marked with horizontal and vertical lines, were placed perpendicular to the surface of the target. High speed photographs were taken of the penetration event, with the camera aimed parallel to the target surface. Time marks were put on the edge of the film, thereby permitting distance traveled and elapsed time measurements to be made, from which the velocity could be determined.

The high speed cameras used were Redlake Hycam, sixteen millimeter movie cameras. These cameras were rated at a maximum speed of 9000 frames per second, but performed at slightly more than 8000 frames per second. The test setup incorporated a mirror so that one camera could photograph the projectile entering the target simultaneously from orthogonal directions. The primary purpose of these high speed photographs was to determine the incident and attack angles of the projectile. All tests were intended to be shot vertically into a horizontal target surface (Fig. 4.7).

An attempt was made to measure the motion of the target surface during penetration using three methods. First, markers, including

FIGURE 4.7 - HIGH SPEED SEQUENCE SHOWING PROJECTILE ENTERING TARGET FROM TWO ANGLES

either precisely placed glass beads or inked lines, were measured before and after penetration. Another method used was still photographs before and after penetration, and finally the other high speed Hycam camera was mounted above and aimed nearly perpendicular to the target surface in an effort to obtain target surface motion as a function of time during penetration.

The total depth of penetration was measured directly.

The gun for propelling the projectiles into the soil targets operated with compressed nitrogen gas. This gas gun had a pressure tank, which housed a double-acting piston, mounted directly over a ten foot long barrel. Different pressures were used to first set the piston in a down position, sealing off the pressure tank from the barrel, then to fill the pressure tank to the required pressure, and finally, to be released suddenly onto the piston by a quick-acting, electrically operated solenoid valve, throwing the piston up, and allowing all of the compressed gas in the pressure tank to be released into the barrel.

The gun was originally mounted on a large tripod, which was adjustable in height. Later it was bolted to a beam in the hangar ceiling (Fig. 4.8), and the tripod was used as a working platform.

All of the required power outlets, recording equipment, and electronic timing equipment were mounted on and centralized around a test console to one side of the tripod (Fig. 4.9). The entire firing event, including the starting of the cameras, firing of the

FIGURE 4.8

GAS GUN MOUNTED TO BEAM IN HANGAR CEILING

FIGURE 4.9

ELECTRONICS CONSOLE AND RECORDING EQUIPMENT FOR
PENETRATION TESTS

gun, and triggering of the flashbulbs, was set up in a timing circuit, in which the delay times between each of the steps mentioned above could be pre-set, and varied in increments of one millisecond.

The entire test setup underwent several evolutions, starting out with a single painted plywood backdrop, which had yardsticks taped to it, stop watch determined delay times, and single camera coverage using 1000 watt photo flood lighting, and 1000 frames per second camera speed, to a fully automated test using roll-into-place, hinged backdrops, triple camera coverage at film speeds of 8000 plus frames per second, and flashbulb banks for lighting (Fig. 4.10 and 4.11).

PLAN VIEW

ELEVATION

FIGURE 4.10

TEST SETUP

FIGURE 4.11

ELECTRONICS FOR TEST SETUP

CHAPTER V

DATA OBTAINED

Test Results

The results presented in Table 3 include the measured values from the five primary tests, to be used in determining the effects of selected soil parameters. In these tests, all variables were held constant except those related to soil parameters. Also included in this table are the results of three additional tests, in which several projectile parameters were varied.

Table 4 includes the results of nine more tests, in which all the desired information was successfully obtained, except an acceleration versus time relationship. These tests will be used in expanding the test results about the total depth of penetration, and as input to check the relationships generated when considering only the five primary tests.

Directly following these tables are the figures which show the acceleration and velocity of the projectiles, the depth of penetration, and the force on the projectile, all as a function of time. The moment of impact is considered to be time equal to zero.

The acceleration of the projectile as a function of time is a measured result. As seen from Eqs. 3.4, 3.5, and 3.6, by integrating this function, or curve, with respect to time, the projectile veloc-

TEST NUMBER		1	2	3	4	5	6	7	8
TARGET NUMBER		2	3	6	7	8	8	8	2
U ₃	BULK DENSITY (pcf)	124	100	112	113	102	102	102	124
U ₄	DEGREE OF SATURATION (%)	94	25	81	79	65	65	65	94
U ₅	COHESIVE YIELD STRENGTH (psf)	4320	288	2880	5472	2880	2880	2880	4320
U ₆	ANGLE OF INTERNAL FRICTION, ϕ (°)	7.0	10.0	5.0	6.4	2.3	2.3	2.3	7.0
U ₇	STRAIN ENERGY DENSITY TO FAILURE MEAN PRINCIPLE STRESS AT FAILURE	0.15	0.03	0.35	0.15	0.10	0.10	0.10	0.15
U ₈	PROJECTILE DIAMETER (ft)	0.1302	0.1302	0.1302	0.1302	0.1302	0.1302	0.1302	0.1302
U ₉	AFTERBODY WEIGHT AREA $\frac{W}{A}$ (psf)	414.72	414.72	414.72	414.72	414.72	276.48	552.96	414.72
U ₁₀	AFTERBODY LENGTH DIAMETER $\frac{L}{D}$	12.86	12.86	12.86	12.86	12.86	13.90	12.86	12.05
U ₁₁	NOSE LENGTH DIAMETER	0	0	0	0	0	0	0	2.5
U ₁₂	NOSE SURFACE AREA AREA PROJECTILE	1.00	1.00	1.00	1.00	1.00	1.00	1.00	5.10
U ₁₃	NOSE SHARPNESS ANGLE (°)	180	180	180	180	180	180	180	22.63
U ₁₄	NOSE VOLUME AREA x NOSE LENGTH	0	0	0	0	0	0	0	0.33
U ₁₅	NOSE WEIGHT AREA (psf)	0	0	0	0	0	0	0	53.06
U ₁₆	ANGLE OF TARGET SURFACE FROM HORIZONTAL (°)	0	0	0	0	0	0	0	0
U ₁₇	ANGLE OF ATTACK BETWEEN PROJECTILE AXIS AND TRAJECTORY (°)	0	0	0	0	0	0	0	0
U ₁₈	ANGLE OF INCIDENCE OF TRAJECTORY AS MEASURED FROM DIRECTION OF GRAVITY (°)	0	0	0	0	0	0	0	0

TABLE 3 TEST RESULTS (Primary Tests)

TEST NUMBER		10	11	13	14	15	16	17	18	19
TARGET NUMBER		4	5	2	3	1	7	7	6	7
U ₃	BULK DENSITY (pcf)	95	116	124	100	118	113	113	112	113
U ₄	DEGREE OF SATURATION (%)	21	78	94	25	97	79	79	81	79
U ₅	COHESIVE YIELD STRENGTH (psf)	244.8	—	4320	288	1440	5472	5472	2880	5472
U ₆	ANGLE OF INTERNAL FRICTION, ϕ (°)	10.0	—	7.0	10.0	4.0	6.4	6.4	5.0	6.4
U ₇	STRAIN ENERGY DENSITY TO FAILURE MEAN PRINCIPLE STRESS AT FAILURE	0.03	—	0.15	0.03	0.48	0.15	0.15	0.35	0.15
U ₈	PROJECTILE DIAMETER (ft)	0.1302	0.1302	0.1302	0.1302	0.1302	0.1302	0.1302	0.1302	0.1302
U ₉	AFTERBODY WEIGHT AREA $\frac{W}{A}$ (psf)	414.72	414.72	414.72	414.72	414.72	414.72	414.72	414.72	414.72
U ₁₀	AFTERBODY LENGTH DIAMETER $\frac{L}{D}$	12.86	12.86	12.86	12.86	13.97	12.86	12.86	12.86	12.86
U ₁₁	NOSE LENGTH DIAMETER	0	0	0	0	0	0	0	0	0
U ₁₂	NOSE SURFACE AREA AREA PROJECTILE	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
U ₁₃	NOSE SHARPNESS ANGLE (°)	180	180	180	180	180	180	180	180	180
U ₁₄	NOSE VOLUME AREA x NOSE LENGTH	0	0	0	0	0	0	0	0	0
U ₁₅	NOSE WEIGHT AREA (psf)	0	0	0	0	0	0	0	0	0
U ₁₆	ANGLE OF TARGET SURFACE FROM HORIZONTAL (°)	0	0	0	0	0	0	0	0	0
U ₁₇	ANGLE OF ATTACK BETWEEN PROJECTILE AXIS AND TRAJECTORY (°)	0	0	0	0	0	0	0	0	0
U ₁₈	ANGLE OF INCIDENCE OF TRAJECTORY AS MEASURED FROM DIRECTION OF GRAVITY (°)	0	0	0	0	0	0	0	0	0

TABLE 4 TEST RESULTS (Additional Tests)

TABLE 5
TEST DATA (ALL TESTS)

THE ACCELEROMETERS USED ON ALL TESTS WERE PIEZOELECTRIC AND TYPE 2225M2.

† The gun fired prematurely.

* Estimated from measured velocity.

TEST NUMBER	TARGET NUMBER	DATE OF TEST	ACCELEROMETER SERIAL NUMBER	GAS PRESSURE (PSI)	IMPACT VELOCITY (FPS)	TOTAL DEPTH OF PENETRATION (FT)
1	2	4/14	LF09	200	203	0.95
2	3	4/25	LF09	200	185	1.04
3	6	5/20	NAOI	200	183	1.71
4	7	5/22	LF09	200	200	0.83
5	8	5/29	LF23	200	203	1.37
6	8	5/30	LF23	200	271	1.30
7	8	5/30	LF23	200	194	1.55
8	2	4/20	LF09	200	170	0.98
10	4	4/30	LF09	200	180 *	0.97
11	5	5/8	LF09	750	360*	3.25
13	2	4/16	LF09	200	175 *	0.96
14	3	4/25	LF09	200	186 *	0.94
15	1	4/3	LF09	180	168 *	2.74
16	7	5/23	LF09	500	290*	2.26
17	7	5/23	LF09	300	235*	0.83
18	6	5/14	NAOI	200	187 *	1.71
19	7	5/23	LF09	750†	202*	0.91

ity can be obtained as a function of time. By integrating this velocity versus time relationship with respect to time, the depth of penetration can be obtained as a function of time (Fig. 3.2).

From Newton's equation of motion, force equals mass times acceleration. Therefore, the force on the projectile, as a function of time, is the same as the acceleration versus time relationship, changed by a scale factor equal to the mass of the projectile.

The above integrations were performed by summing incremental areas under the acceleration, and then the velocity curves. The time increments used were 0.0001 seconds in the region of the initial acceleration spike, and 0.001 seconds until acceleration equaled zero. A linear average of acceleration and velocity was taken over each time interval.

Each of these relationships is presented in tabular form, accompanying the corresponding plots. These tabular values were used as input for determining the final empirical relationships.

Table 5 lists other recorded information for all of the tests.

The data and relationships used in determining the soil parameters reported in Tables 3 and 4 are presented in Appendix II-Target Data, at the end of this paper.

Accuracy of Data

Although all measurements were intended to represent true values, some error is always inherent. In most cases, the amount

of error is not known, but can sometimes be estimated.

For example, several of the targets began drying immediately upon completion of construction, and continued to do so until examination after penetration. By knowing the initial and final moisture contents, and the relative times between end of construction, penetration, and final examination, a good estimate can be made of what the moisture content was at the time of penetration.

Similarly, by considering the modes of failure of the triaxial samples, some tests can be weighed more heavily than others when determining soil parameters.

In all cases, the values reported are those thought to be most nearly correct at the time of penetration, for the depth that the projectile penetrated.

The chronograph yielded a measurement for projectile velocity, just as the projectile cleared the barrel. The end of the barrel was located approximately eight feet above the surface of the target.

It was discovered that the values obtained for impact velocity from integration of the acceleration trace were 15 to 20 fps faster than the corresponding values measured by the chronograph. However, if the values obtained from the integration of the acceleration trace were assumed to be correct, and integrated again to obtain depth of penetration, the total depth of penetration obtained corresponded very closely to the measured depth.

This indicated that the impact velocity obtained from integration of the acceleration was more nearly correct, and the air blast from the gun must have accelerated the projectile between the end of the barrel and the surface of the target.

This led to another estimation. In all of the tests for which no acceleration versus time information was available, there was no way to obtain an accurate value for impact velocity. All of the measured values for velocity in these tests were scaled up to yield a set of estimated impact velocities.

TABLE 6
TEST 1 - TARGET 2

T (sec $\times 10^{-3}$)	A (fps ² $\times 10^4$)	V (fps)	D (ft)	F (lbs $\times 10^3$)
0	0	203	0	0
.3	8.630	190	.06	15.01
.6	.649	176	.11	1.13
1	.778	173	.18	1.35
2	1.427	162	.35	2.48
3	1.873	145	.50	3.26
4	2.120	125	.64	3.69
5	2.558	102	.75	4.45
6	2.682	76	.84	4.67
7	2.641	48	.90	4.60
8	1.538	21	.94	2.68
9	1.471	3	.95	2.56
9.3	0	0	.95	0

T = time
 A = acceleration of projectile
 V = velocity of projectile
 D = depth of penetration
 F = force on projectile

mass of projectile = 0.174 slugs

TEST 1

TARGET 2

FIGURE 5.1

TABLE 7
TEST 2 - TARGET 3

T (sec $\times 10^{-3}$)	A (fps ² $\times 10^4$)	V (fps)	D (ft)	F (lbs $\times 10^3$)
0	0	185	0	0
.3	3.248	180	.06	5.65
.5	1.577	175	.09	2.74
1	1.577	167	.16	2.74
2	1.592	151	.32	2.77
3	1.763	134	.48	3.07
4	1.733	116	.60	3.02
5	1.558	100	.71	2.71
6	1.506	85	.80	2.62
7	1.577	70	.88	2.74
8	1.547	54	.94	2.69
9	1.484	39	.98	2.58
10	1.410	26	1.01	2.45
11	1.217	13	1.03	2.12
12	.668	6	1.04	1.16
13	0	0	1.04	0

T = time
 A = Acceleration of projectile
 V = velocity of projectile
 D = depth of penetration
 F = force on projectile

mass of projectile = 0.174 slugs

TEST 2

TARGET 3

FIGURE 5.2

TABLE 8
TEST 3 - TARGET 6

T (sec $\times 10^{-3}$)	A (fps ² $\times 10^4$)	V (fps)	D (ft)	F (lbs $\times 10^3$)
0	0	183	0	0
.1	9.220	178	.02	16.04
.5	0.811	156	.09	1.41
1	0.601	153	.18	1.05
2	0.493	148	.37	.86
3	0.489	143	.51	.85
4	0.488	138	.65	.85
5	0.635	132	.79	1.11
6	0.586	126	.92	1.02
7	0.644	120	1.04	1.12
8	0.795	113	1.15	1.38
9	0.910	104	1.26	1.58
10	1.030	95	1.36	1.79
11	1.232	83	1.45	2.14
12	1.385	70	1.53	2.41
13	1.406	56	1.59	2.44
14	1.419	42	1.64	2.46
15	1.378	28	1.67	2.40
16	1.142	15	1.70	1.99
17	1.067	4	1.71	1.86
17.7	0	0	1.71	0

T = time

A = acceleration of projectile

V = velocity of projectile

D = depth of penetration

F = force on projectile

mass of projectile = 0.174 slugs

TEST 3

TARGET 6

FIGURE 5.3

TABLE 9
TEST 4 - TARGET 7

T (sec $\times 10^{-3}$)	A (fps ² $\times 10^4$)	V (fps)	D (ft)	F (lbs $\times 10^3$)
0	0	200	0	0
.2	5.790	194	.04	10.08
.6	2.898	177	.11	5.04
1	2.098	167	.18	3.64
2	2.298	145	.34	4.00
3	2.298	122	.47	4.00
4	2.198	99	.58	3.82
5	2.198	77	.67	3.82
6	1.997	56	.74	3.48
7	1.898	37	.78	3.30
8	1.698	19	.81	2.96
9	1.298	4	.82	2.26
9.6	0	0	.83	0

T = time

A = acceleration of projectile

V = velocity of projectile

D = depth of projectile

F = force on projectile

mass of projectile = 0.174 slugs

TEST 4

TARGET 7

FIGURE 5.4

TABLE 10
TEST 5 - TARGET 8

T (sec $\times 10^{-3}$)	A (fps ² $\times 10^4$)	V (fps)	D (ft)	F (lbs $\times 10^3$)
0	0	203	0	0
.1	5.900	200	.02	10.27
.2	1.041	197	.04	1.81
1	1.389	187	.19	21.42
2	1.877	171	.37	3.26
3	1.338	154	.53	2.32
4	1.093	142	.68	1.90
5	1.250	131	.82	2.18
6	1.198	118	.94	2.08
7	1.860	103	1.05	3.24
8	1.614	86	1.15	2.81
9	1.720	69	1.22	2.99
10	1.650	52	1.29	2.87
11	1.720	35	1.33	2.99
12	1.440	20	1.36	2.50
13	1.231	6	1.37	2.14
14	0	0	1.372	0

T = time
 A = acceleration of projectile
 V = velocity of projectile
 D = depth of projectile
 F = force on projectile

mass of projectile = 0.174 slugs

TEST 5

TARGET 8

FIGURE 5.5

TABLE II
TEST 6 - TARGET 8

T (sec $\times 10^{-3}$)	A (fps ² $\times 10^4$)	V (fps)	D (ft)	F (lbs $\times 10^3$)
0	0	271	0	0
.3	11.210	254	.08	13.00
.5	3.130	240	.13	3.62
1	3.115	224	.24	3.60
2	2.605	196	.45	3.02
3	2.435	171	.64	2.82
4	2.345	147	.80	2.72
5	2.315	123	.93	2.68
6	2.285	100	1.04	2.64
7	2.145	78	1.13	2.48
8	2.070	57	1.20	2.40
9	1.897	37	1.25	2.20
10	1.609	20	1.28	1.86
11	1.305	5	1.29	1.51
11.8	0	0	1.30	0

T = time

A = acceleration of projectile

V = velocity of projectile

D = depth of projectile

F = force on projectile

mass of projectile = 0.1158 slugs

TEST 6

TARGET 8

FIGURE 5.6

TABLE 12
TEST 7 - TARGET 8

T (sec $\times 10^{-3}$)	A (fps ² $\times 10^4$)	V (fps)	D (ft)	F (lbs $\times 10^3$)
0	0	194	0	0
.3	3.015	189	.06	6.99
1	1.390	174	.18	3.22
2	.836	163	.35	1.94
3	1.003	154	.51	2.33
4	1.321	142	.66	3.06
5	.920	131	.80	2.13
6	1.239	120	.92	2.87
7	.970	109	1.04	2.25
8	1.155	99	1.14	2.68
9	1.490	86	1.23	2.66
10	1.340	71	1.31	2.63
11	1.421	58	1.38	2.64
12	1.003	46	1.43	2.33
13	1.038	35	1.47	2.40
14	.854	26	1.50	1.98
15	.653	18	1.52	1.51
16	.753	11	1.53	1.74
17	.419	5	1.54	.97
18	.335	2	1.55	.77
19	0	0	1.55	0

T = time

A = acceleration of projectile

V = velocity of projectile

D = depth of projectile

F = force on projectile

mass of projectile = 0.232 slugs

TEST 7

TARGET 8

FIGURE 5.7

TABLE 13
TEST 8 - TARGET 2

T (sec $\times 10^{-3}$)	A (fps ² $\times 10^4$)	V (fps)	D (ft)	F (lbs $\times 10^3$)
0	0	170	0	0
1	.258	168	.169	0.45
2	1.094	162	.334	1.91
3	1.867	147	.483	3.25
4	1.932	128	.620	3.36
5	2.190	107	.737	3.81
6	2.415	84	.833	4.20
7	2.540	59	.905	4.42
8	2.415	35	.951	4.20
9	2.255	11	.974	3.92
10	0	0	.980	0

T = time
 A = acceleration of projectile
 V = velocity of projectile
 D = depth of projectile
 F = force on projectile

mass of projectile = 0.174 slugs

TEST 8

TARGET 2

FIGURE 5.8

CHAPTER VI

RESULTS OF DATA ANALYSIS

Presentation of Results

The method to be used for analyzing the data obtained was described in Chapter III, Plan of Analysis and Experiment Design.

The typical engineering approach for determining the effect of selected variables on an experiment is usually as follows. The tests are designed so that all variables can be held constant, except the one being studied. Then three or more tests are conducted, varying this parameter over a desired range. Using this method, three or more tests must be conducted for each variable to be considered.

If this method were applied to this experiment, considering all 18 variables, a total of 5400 tests would be required. If the tests were performed at the rate of one per week, the total time required would exceed 100 years. Upon completing all of these tests, the interaction terms would still be unknown.

Applying the above method to the data obtained for this paper, the impact velocity was plotted versus the corresponding total depth of penetration for three tests (Fig. 6.1). These tests were chosen so that all other parameters remained constant. This method of analysis could be carried further by attempting to fit a func-

FIGURE 6.1
MEASURED IMPACT VELOCITY vs TOTAL
DEPTH OF PENETRATION

tion to the curve in Figure 6.1.

For the solution expanded for the force on the projectile, as described in Chapter III, the following truncated series was assumed as a solution.

$$F = A_0 + \sum_{i=1}^n A_i u_i + \sum_{i=1}^n \sum_{j=1}^n A_{ij} u_i u_j + \sum_{i=1}^n A_{iii} u_i^3 \quad (\text{Eq. 6.1})$$

This solution includes all of the linear, quadratic, and pure cubic terms. The repeating quadratic terms were omitted.

Since the number of linear equations and constants generated by this method is excessively large to be solved by hand, a computer program was written to do this job. This program is presented in Appendix IIIA.

First a system of simultaneous linear equations is set up according to the method of least squares. The A-constants are common to every equation. Therefore, the corresponding values obtained from summing the variables and their interactions form a square matrix. By inverting this matrix, and multiplying by the force vector, the program solves for the A-constants, thereby generating a solution for the assumed form. The program then uses this equation to calculate a force for each of the sets of variables used as input, and compares these calculated forces to the observed forces, expressing the difference in percent error based on the observed value.

The generated equation for force can be expressed as:

$$F = C_0 + C_1d + C_2v + C_3vd + C_4d^2 + C_5v^2 + C_6d^3 + C_7v^3 = m \frac{dv}{dt} \quad (\text{Eq. 6.2})$$

By calculating the initial force, and using the initial velocity for each test, the program integrates numerically to obtain a corresponding total depth of penetration for each test. These values are compared to the observed total depths in the same manner as the forces.

For this method of solution, the results of tests 1 through 5 were used as input. The sets of variables obtained from these tests are presented in Tables 2, 5, 6, 7, 8, and 9 in Chapter V, Data Obtained.

Several combinations of variables were tried in an attempt to obtain a better fit of the assumed function to the data. Also, solutions were generated considering the deceleration spike in different manners. One method was to consider the deceleration spike as it was recorded. Another method was to consider only the decreasing side of the spike, assuming impact occurred at the top of the spike. Finally, the spike was not considered at all. Time was assumed to begin after the spike had taken place. These variations were tried because the generated function was not closely predicting the forces near the spike.

The inverted matrix was multiplied by the original matrix and written out. If the inverted matrix was exact, the product of these two matrices would be a unit matrix. By comparing the product

obtained to the unit matrix, a check was obtained for the condition of the original matrix.

The computer program was initially used with single precision. That is, approximately 7 decimal places were carried in all calculations. The program was converted to double precision, or twice as many decimal places, in an attempt to improve accuracy.

The first solution used single precision, considered the entire spike and the following variables.

- u_1 = instantaneous velocity (fps)
- u_2 = instantaneous depth of penetration (ft)
- u_3 = bulk density (pcf)
- u_4 = degree of saturation (%)
- u_5 = cohesion (psf)
- u_6 = angle of internal friction ($^{\circ}$)
- u_7 = strain energy/mean principle stress at failure.

The results were very poor. The product of the matrix and its inverse varied drastically from the unit matrix. The predicted forces were in error from 170% to 2500%, and differed in sign from the observed forces. This fit was considered worthless.

The second solution considered the same 7 variables, but used double precision, and considered initial time at the peak of the spike. The results were improved, but remained unacceptable.

Since the assumed solution could not obtain an acceptable fit for seven variables, the number of variables was reduced to four. The values for cohesion, angle of internal friction, and strain

energy divided by the mean principle stress at failure were omitted. Since these three values are fixed for a given density and degree of saturation of a particular soil, it was assumed that these two variables described the soil. Therefore, the following variables were considered to affect the force on the projectile.

u_1 = instantaneous velocity (fps)

u_2 = instantaneous depth (ft)

u_3 = bulk density (pcf)

u_4 = degree of saturation (%)

This solution used double precision, and considered initial time to be at the peak of the spike. The following is a presentation of the results obtained.

Empirical Equation:

$$\begin{aligned} \text{Force} = & 0.346560\text{D } 05 - 0.178821\text{D } 03(u_1) + 0.464025\text{D } 04(u_2) \\ & + 0.127125\text{D } 03(u_3) + 0.110000\text{D } 02(u_4) + 0.117245\text{D } 01(u_1^2) \\ & + 0.733005\text{D } 02(u_1 u_2) - 0.964711\text{D } 00(u_1 u_3) + 0.150141\text{D } 01(u_1 u_4) \\ & - 0.181800\text{D } 05(u_2^2) - 0.213167\text{D } 03(u_2 u_3) + 0.266132\text{D } 03(u_2 u_4) \\ & - 0.144507\text{D } 01(u_3^2) + 0.898438\text{D } -01(u_3 u_4) - 0.847266\text{D } 01(u_4^2) \\ & - 0.366002\text{D } -02(u_1^3) + 0.840008\text{D } 04(u_2^3) - 0.798035\text{D } -02(u_3^3) \\ & + 0.782928\text{D } -01(u_4^3) \end{aligned}$$

VERIFICATION OF CURVE FITTING

OBSERVED FORCE ON PROJECTILE	PREDICTED FORCE ON PROJECTILE	PERCENT ERROR BASED ON OBSERVED FORCE
-0.1501D 05	-0.6472D 04	56.88
-0.1130D 04	-0.4528D 04	-300.67
-0.1350D 04	-0.3522D 04	-160.91
-0.2480D 04	-0.1628D 04	34.37
-0.3260D 04	-0.6561D 04	79.87
-0.3690D 04	-0.5799D 03	84.28
-0.4450D 04	-0.1159D 04	73.96
-0.4670D 04	-0.2022D 04	56.71
-0.4600D 04	-0.2558D 04	44.39
-0.2680D 04	-0.2200D 04	17.92
-0.2560D 04	-0.1165D 04	54.49
-0.5650D 04	-0.4662D 04	17.48
-0.2740D 04	-0.3840D 04	-40.15
-0.2740D 04	-0.2764D 04	-0.89
-0.2770D 04	-0.1656D 04	40.20
-0.3070D 04	-0.1604D 04	47.74
-0.3020D 04	-0.1794D 04	40.60
-0.2710D 04	-0.2416D 04	10.84
-0.2620D 04	-0.2990D 04	-14.10
-0.2740D 04	-0.3448D 04	-25.82
-0.2690D 04	-0.3419D 04	-27.11
-0.2580D 04	-0.2916D 04	-13.04
-0.2450D 04	-0.2178D 04	11.11
-0.2120D 04	-0.9863D 03	53.48
-0.1160D 04	-0.2014D 03	82.64
-0.1604D 05	-0.5328D 04	66.78
-0.1410D 04	-0.2765D 04	-96.07
-0.1050D 04	-0.1770D 04	-68.55
-0.8600D 03	-0.5512D 03	35.90
-0.8500D 03	-0.1625D 03	80.88
-0.8500D 03	-0.1052D 03	87.63
-0.1110D 04	-0.2487D 03	77.59
-0.1020D 04	-0.4625D 03	54.66
-0.1120D 04	-0.6430D 03	42.58
-0.1380D 04	-0.7752D 03	43.83
-0.1580D 04	-0.8700D 03	44.94
-0.1790D 04	-0.8427D 03	52.92
-0.2140D 04	-0.8775D 03	59.00
-0.2410D 04	-0.8665D 03	64.04
-0.2440D 04	-0.9162D 03	62.45
-0.2460D 04	-0.8507D 03	65.42

-0.2400D 04	-0.7816D 03	67.43
-0.1990D 04	-0.4008D 03	79.86
-0.1860D 04	-0.4789D 02	97.43
-0.1008D 05	-0.8374D 04	16.93
-0.5040D 04	-0.5676D 04	-12.61
-0.3640D 04	-0.4212D 04	-15.71
-0.4000D 04	-0.2217D 04	44.57
-0.4000D 04	-0.1591D 04	60.22
-0.3820D 04	-0.1718D 04	55.04
-0.3820D 04	-0.2097D 04	45.10
-0.3480D 04	-0.2341D 04	32.74
-0.3300D 04	-0.2122D 04	35.70
-0.2960D 04	-0.1427D 04	51.80
-0.2260D 04	-0.2688D 03	88.11
-0.1027D 05	-0.6884D 04	32.97
-0.1810D 04	-0.6176D 04	-241.24
-0.2420D 04	-0.3348D 04	-38.34
-0.3260D 04	-0.9528D 03	70.77
-0.2320D 04	0.1402D 03	106.04
-0.1900D 04	0.2343D 03	112.33
-0.2180D 04	-0.6304D 02	97.11
-0.2080D 04	-0.5227D 03	74.87
-0.3240D 04	-0.1115D 04	65.59
-0.2810D 04	-0.1763D 04	37.24
-0.2990D 04	-0.2277D 04	23.84
-0.2870D 04	-0.2602D 04	9.33
-0.2990D 04	-0.2597D 04	13.14
-0.2500D 04	-0.2254D 04	9.83
-0.2140D 04	-0.1554D 04	27.39

OBSERVED DEPTH
OF PROJECTILEPREDICTED DEPTH
OF PROJECTILEPERCENT ERROR
BASED ON OBSERVED DEPTH

0.1040D 01	0.1076D 01	-3.42
0.1710D 01	0.8276D 01	-383.99
0.8200D 00	0.1036D 01	-26.30
0.1370D 01	0.8384D 01	-511.97
0.9500D 00	0.8466D 01	-791.14

The product of the matrix with its inverse was considerably improved, but not of acceptable error. The fit is considered to be unacceptable.

In an effort to further improve the fit of the assumed function to the data, the number of variables was decreased to three. An attempt was made to describe the soil with one variable. It was decided that a variable which was the product of strain energy divided by the mean principle stress at failure times the bulk density might best perform this function.

As a result, the following variables were considered to affect the force on the projectile.

u_1 = instantaneous velocity (fps)

u_2 = instantaneous depth (ft)

u_3 = (strain energy/mean principle stress at failure)(bulk density) (pcf)

This solution used double precision, and considered initial time to be at the peak of the deceleration spike. The following is a presentation of the results obtained.

Empirical Equation

$$\begin{aligned} \text{Force} = & -0.143257D\ 04 - 0.976479D\ 02(u_1) + 0.706168D\ 04(u_2) \\ & + 0.104124D\ 04(u_3) + 0.151487D\ 01(u_1^2) - 0.454577D\ 02(u_1u_2) \\ & + 0.183480D\ 01(u_1u_3) - 0.116129D\ 05(u_2^2) + 0.332799D\ 03(u_2u_3) \\ & - 0.105445D\ 03(u_3^2) - 0.654964D\ 02(u_1^3) + 0.265835D\ 04(u_2^3) \\ & + 0.180107D\ 01(u_3^3) \end{aligned}$$

The percent errors for the predicted forces were, on the average, 1/3 to 1/2 those for 4 variables. The depth calculations were much improved.

OBSERVED DEPTH OF PROJECTILE	PREDICTED DEPTH OF PROJECTILE	PERCENT ERROR BASED ON OBSERVED DEPTH
0.9500D 00	0.7950D 00	16.32
0.1040D 01	0.1055D 01	-1.40
0.1710D 01	0.6391D 00	62.63
0.8200D 00	0.1016D 01	-23.94
0.1370D 01	0.1358D 01	0.86

The product of the matrix and its inverse was acceptably close to the unit matrix. All diagonal terms were exactly equal to one, and the off-diagonal terms ranged from 10^{-5} to zero. The fit was acceptable; however, the function did not closely define the deceleration spike.

In an attempt to improve the fit, the same variables were considered with initial time considered to be immediately following the spike. In effect, the initial spike was ignored. The results were essentially the same, with only a slight improvement of fit.

The second method that was used to analyze the test data expanded the measured parameters for the total depth of penetration, as explained in Chapter III.

Since an acceleration versus time relationship was not required for these solutions, the results from 16 tests could be used, instead

of the results from 5 tests. The measured parameters for these tests are presented in Chapter V, Data Obtained.

By adding these extra tests, a projectile parameter, W/A , could be considered.

A computer program was available from Dr. H. O. Hartley and Dr. C. E. Gate of the Institute of Statistics at Texas A&M University to generate this solution. This program is presented in Appendix IIIB.

Solutions were generated for two sets of variables. In the first solution, the following parameters were considered to affect the total depth of penetration.

- 1) bulk density (pcf)
- 2) degree of saturation (%)
- 3) cohesion (psf)
- 4) angle of internal friction ($^{\circ}$)
- 5) strain energy/mean principle stress at failure
- 6) projectile weight/cross-sectional area, W/A (psf)
- 7) measured impact velocity (fps)
- 8) (measured impact velocity)² (fps)²

The second solution used input from the same 16 tests, but added one parameter to be considered.

- 9) (measured impact velocity)² (bulk density) (lb/ft-sec²)

The computer program incorporated two statistical tests for the fit of the function to the data. The t-test gives a term by term

relative significance of the generated coefficients, and the F-test represents the overall ability of the generated solution to predict total depth (2).

If 95% probability is desired, four of the eight coefficients met the minimum t-test requirements in the first solution, and only two of the nine coefficients met these requirements in the second solution. However, the F-test values indicate that both solutions should be excellent predictors of total depth of penetration.

The following is a presentation of the results of these two solutions.

Solutions for 16 tests and 8 variables:

Coefficients	t-values
Total depth = -1.663	
+0.465(γ)	3.151
+0.278(S)	0.290
-1.052(c)	-1.691
-0.531(ϕ)	-1.630
+0.973(E/σ_m)	2.617
+1.835(W/A)	1.824
+2.012(v_0)	0.248
+1.391(v_0^2)	0.248

F = 67.813

Observed Depth	Predicted Depth	Difference
0.95	1.07	-0.12
1.04	1.06	-0.02
1.71	1.92	-0.21
0.83	0.79	0.04
1.37	1.30	0.07
1.30	1.23	0.07
1.55	1.55	0.00
0.70	0.70	0.00
3.25	3.38	-0.13
0.96	0.87	0.09
0.94	0.91	0.03
2.74	2.53	0.21
2.26	1.84	0.42
0.83	1.19	-0.36
1.71	1.84	-0.13
0.91	0.86	0.05

Solutions for 16 tests and 9 variables:

Coefficients

t-values

Total depth = -1.664

+0.465(γ)	2.999
+0.278(S)	0.276
-1.052(c)	-1.609
-0.531(ϕ)	-1.552
+0.973(E/σ_m)	2.491
+1.835(W/A)	1.736
+2.012(v_0)	0.236
+1.391(v_0^2)	0.236
+0.001(v_0^2)(γ)	1.104

F = 54.630

Observed Depth	Predicted Depth	Difference
0.95	1.07	-0.12
1.04	1.06	-0.02
1.71	1.92	-0.21
0.83	0.79	+0.04
1.37	1.30	+0.07
1.30	1.23	+0.07
1.55	1.55	0.00
0.70	0.70	0.00
3.25	3.38	-0.13
0.96	0.87	+0.09

0.94	0.91	+0.03
2.74	2.53	+0.21
2.26	1.84	+0.42
0.83	1.19	-0.36
1.71	1.84	-0.13
0.91	0.86	+0.05

Discussion of Results of Data Analysis

The method presented in this paper for empirically determining the force on a projectile as a function of velocity, depth, and soil parameters is a very general method.

Because of the limited number of tests, and because the ranges of analysis variables within these tests are relatively small, it was difficult to fit an empirical equation to the measured data. For five tests, the maximum number of variables that could be considered, and still maintain a reasonably close fit, was three.

Since only a small number of interaction terms could be considered, the exact contribution of individual variables on either force or total depth of penetration cannot be determined. However, for the solutions expanded for the total depth of penetration, the relative contributions of the individual variables can be determined by comparing the relative magnitudes of the statistical t-test values. (See solutions on pages 85 and 87.)

As more and wider ranges of variables are introduced in future

testing, the closeness of fit will improve. Not only can more variables be considered, but all of the cubic and even higher order terms can be considered. This will also improve the fit, especially near the deceleration spike.

CHAPTER VII

CONCLUSIONS

Eight fully completed sets of data were obtained, including well constructed and tested targets, and a good acceleration versus time measurement for each test.

The numerical method of data analysis for empirically determining the force on a projectile was limited by the amount of data available. However, the method is extremely general, and can become a powerful tool when supplied with more data from future tests.

It is impossible to determine the exact contribution of each variable on either force or depth of penetration because all of the interaction terms are not known. However, by comparing the relative contributions of the individual terms, it is demonstrated that the soil parameters have a strong influence on the depth of penetration, and must be considered in a reasonable solution.

The tests performed for this study were highly limited in the ranges and number of variables, but represent a starting point from which a very powerful equation can be obtained.

APPENDIX I

LIST OF NOTATIONS

fps	= feet per second
pcf	= pounds per cubic foot
psi	= pounds per square inch
psf	= pounds per square foot
ft	= feet
lbs	= pounds
%	= per cent
μ	= microns
$\text{\AA}$	= angstroms
($^{\circ}$)	= degrees
sec	= seconds
c	= cohesion
ϕ	= angle of internal friction
γ_b	= bulk density
γ_d	= dry density
w	= moisture content
G_s	= specific gravity of the solids
γ_w	= density of water
e	= void ratio
S	= degree of saturation

E	= strain energy density
ϵ	= strain
σ	= normal stress
σ_v	= vertical stress
σ_h	= horizontal stress
σ_m	= mean principle stress
τ	= shearing stress
V	= volume
M_p	= mass of projectile
M_i	= mass of soil moved by projectile
A or a	= acceleration of the projectile
v	= velocity of the projectile
v_i	= impact velocity of the projectile
z	= depth of penetration
D or d	= total depth of penetration
T or t	= time
g	= acceleration due to gravity
F	= force resisting the projectile
W/A	weight of projectile/cross-sectional area of projectile
L/D	= afterbody length of the projectile/diameter of the projectile
u	= test variable
Eq.	= equation
App.	= appendix

APPENDIX II

TARGET DATA

General

The methods used to obtain values for the target parameters were discussed briefly in Chapter IV-Testing Procedures. This appendix is a detailed presentation of the values obtained.

The values which were measured directly were moisture content, bulk density, and the soil strength parameters. From these values, and from the results of the triaxial tests, all other parameters of interest can be determined.

If

γ_b = bulk density,

γ_d = dry density,

w = moisture content,

G_s = specific gravity of the solids = 2.62 pcf,

γ_w = density of water = 62.5 pcf,

e = void ratio,

S = degree of saturation,

σ_v = vertical stress,

σ_h = horizontal stress,

σ_m = mean principle stress,

V = volume,

ΔV = change in volume,

ϵ_v = vertical strain,

E = strain energy, and

t = time,

the following relationships are valid.

$$\gamma_d = \frac{\gamma_b}{1 + w} \quad (\text{Eq. App. 2.1})$$

$$S = \frac{G_s (\gamma_b - \gamma_d)}{(G_s \gamma_w - \gamma_d)} \quad (\text{Eq. App. 2.2})$$

$$e = \frac{G_s \gamma_w}{\gamma_d} - 1 \quad (\text{Eq. App. 2.3})$$

$$\sigma_m = 1/3 (\sigma_v - 2\sigma_h) \quad (\text{Eq. App. 2.4})$$

$$E = \int_0^t (\sigma_v - \sigma_h) d\epsilon_v + \sigma_h \left(\frac{\Delta V}{V} \right) \quad (\text{Eq. App. 2.5a})$$

The change in volume, ΔV , was measured for only one set of triaxial tests. In the remaining tests, $\frac{\Delta V}{V}$ was assumed to be zero.

This assumption reduces the above equation to the following:

$$E = \int_0^t (\sigma_v - \sigma_h) d\epsilon_v \quad (\text{Eq. App. 2.5b})$$

The time interval can be equated to a corresponding strain interval. This interval was arbitrarily chosen to be terminated by either $(\sigma_v - \sigma_h) \rightarrow 0$, or $\epsilon_v \rightarrow 20\%$, whichever occurred first.

The ratio of strain energy to the mean principle stress at fail-

ure was chosen as an analysis variable to maintain dependence of the generated empirical equations on the mean principle stress, and because it becomes a dimensionless number.

The soil strength parameters were determined by two methods, as mentioned in Chapter IV-Testing Procedures. One set of parameters was determined by using the Cohron Sheargraph. This instrument was limited to a low range of stress values, having a maximum normal stress capability of approximately 17 psi. It is believed, however, that the Sheargraph is accurate within this limited range, and yields a more correct value for cohesion than the value determined from the triaxial tests.

Soil strength parameters, determined from triaxial tests, considered for compression only, are limited in the lower stress range to one half of $(\sigma_v - \sigma_h)$ at failure for the unconfined case. They are not limited above that value, but can be determined for as high as one wishes to carry the confining pressure, σ_h .

The values reported rely on the triaxial test results for determining the angle of internal friction through 100 psi confining pressure, and on the Sheargraph results as more nearly correct in the low stress ranges.

Target I

Desired moisture content--28.5%

Size of lifts--200 lbs.

(a)

TARGET 1 ($w = 28.5\%$)

(b)

TARGET 2 ($w = 22\%$)

FIGURE APP 1

TARGET SURFACES AFTER PENETRATION

— VALUES MEASURED DURING TARGET CONSTRUCTION
 --- VALUES MEASURED AFTER PROJECTILE PENETRATION
 * VALUES NOT MEASURED DIRECTLY

TARGET 1
 FIGURE APP 2

Passes per lift--2

End of construction until first penetration--43 days

End of construction until final examination--45 days

Size of batches--800 lbs.

After construction, this target was covered with a sheet of plastic, and allowed to stand for over six weeks. It was hoped that this curing period would cause the target to increase in uniformity, and the layers would bond together better. Although a better bond was produced between the layers, moisture from the atmosphere collected under the sheet of plastic, and was absorbed by the target.

This increase in moisture was accompanied by a slight decrease in void ratio, indicating that some of the air in the voids was forced out of the target or into solution, or both, by the overburden pressure of the material.

The surface of the target was struck off smooth just before penetration, and small glass heads were placed at 1/2 inch intervals to measure surface movement (Fig.App. 1).

Bulk density determinations were made from the trimmed triaxial samples.

Target 2

Desired moisture content--24.5%

Size of lifts--100 lbs.

Passes per lift--2

— VALUES MEASURED DURING TARGET CONSTRUCTION
 --- VALUES MEASURED AFTER PROJECTILE PENETRATION
 * VALUES NOT MEASURED DIRECTLY

TARGET 2
 FIGURE APP 3

TRIAXIAL TESTS

TARGET 2 **DURING CONSTRUCTION**

DISTANCE FROM BOTTOM OF TARGET TO TOP OF TRIAXIAL SAMPLE IN PLACE (inches)	σ_h CONFINING PRESSURE (psf)	MOISTURE CONTENT (%)	BULK DENSITY (psf)	INITIAL HEIGHT (inches)	INITIAL DIAMETER (inches)	MODE OF FAILURE
30	0	—	96.2	3.560	2.780
30	50	—	98.4	4.680	2.800
30	90	—	99.7	3.150	2.750
42	0	—	97.9	4.034	2.808
42	50	—	98.8	3.014	2.800
42	100	—	98.9	4.940	2.800

FIGURE APP 4

STRESS vs STRAIN FROM TRIAXIAL TESTS

FIGURE APP 5

FAILURE ENVELOPE

TARGET 2

DURING CONSTRUCTION

FIGURE APP 6

TARGET 5

FIGURE APP 7

End of construction until first penetration--14 days

End of construction until final examination--15 days

Size of batches--800 lbs.

After this target had been constructed to three feet in depth, it was decided that the uniformity was not acceptable, and 18 inches were removed. Construction then continued until the target was completed.

This target was also protected from drying by covering it with a plastic sheet.

The surface of this target was rough (Fig. Appl), and for this reason was not marked for measurement of surface movement.

Since this target was penetrated approximately one foot, the top 15 inches were removed and rebuilt, and the entire target was designated target 5.

The triaxial tests were performed using a load cell to measure force and an LVDT to measure displacement. The tests were recorded on magnetic tape, and printed on visicorder paper for evaluation. This system has the advantage of recording continuous measurements, rather than readings at intervals.

The bulk density determinations were made from trimmed triaxial samples.

Target 3

Desired moisture content--8%

Size of lifts--100 lbs.

Passes per lift--2

End of construction until first penetration--2 days

End of construction until final examination--11 days

Size of batch--entire target mixed in one batch

This target was constructed from the material removed from target number one. All of the material was run through the crusher as it came out of target number one. It was in the form of plate shaped particles, approximately 1/8 inch in thickness. Part of this material was air dried and the remainder was dried in a rotary kiln. Then all of the materials were run through the crusher again, and then through the pugmill for mixing. The resulting material contained particles ranging in size from dust to approximately 1/8 inch in diameter.

Compacting this material was difficult because the air discharge from the pneumatic compactor kept the dust stirred up inside the target. During construction, the top 2 to 3 inches of material remained loose, and began to compact below that depth.

The resulting material resembled a slightly cemented sand, having little or no cohesion. It was impossible to obtain triaxial samples. The soil strength parameters were determined from the shear-graph results.

Bulk density determinations were made by the sand cone method of dropping a very uniform Ottawa sand from a constant height into a

— VALUES MEASURED DURING TARGET CONSTRUCTION
 --- VALUES MEASURED AFTER PROJECTILE PENETRATION
 * VALUES NOT MEASURED DIRECTLY

TARGET 3
 FIGURE APP 8

- VALUES MEASURED DURING TARGET CONSTRUCTION
 --- VALUES MEASURED AFTER PROJECTILE PENETRATION
 * VALUES NOT MEASURED DIRECTLY

TARGET 4
 FIGURE APP 9

hole, the material from which had a known weight. Knowing the density and weight of the sand filling the hole, the volume of the hole could be determined.

Again, the top 15 inches of this target were removed and rebuilt to make target number 4.

Target 6

Desired moisture content--28%

Size of lift--100 lbs.

Passes per lift--2

End of construction until first penetration--6 days

End of construction until final examination--13 days

Size of batches--800 lbs.

This target was a duplication of target number one, except the size of lifts was decreased from 200 to 100 lbs.

The bulk density measurements were made with a balloon density meter.

The purpose of the target was marked with inked lines for measurements of surface movement.

The triaxial tests were performed using displacement gauges and a proving ring, with readings at 10 second intervals.

Target 7

Desired moisture content--24.5%

— VALUES MEASURED DURING TARGET CONSTRUCTION
 - - - VALUES MEASURED AFTER PROJECTILE PENETRATION
 * VALUES NOT MEASURED DIRECTLY

TARGET 6
 FIGURE APP 10

TRIAXIAL TESTS

TARGET 6

DURING CONSTRUCTION

DISTANCE FROM BOTTOM OF TARGET TO TOP OF TRIAXIAL SAMPLE IN PLACE (inches)	σ_h CONFINING PRESSURE (psf)	MOISTURE CONTENT (%)	BULK DENSITY (psf)	INITIAL HEIGHT (inches)	INITIAL DIAMETER (inches)	MODE OF FAILURE
18	0	26.9	118.0	5.034	2.775
18	50	33.5	120.9	5.023	2.754
18	100	30.9	120.3	4.996	2.750
18	100	31.1	118.8	5.100	2.773
30	0	26.0	119.3	4.986	2.773
30	50	34.8	119.5	5.059	2.770
30	100	24.8	119.8	4.649	2.765

FIGURE APP II

STRESS vs STRAIN FROM TRIAXIAL TESTS

FIGURE APP 12

FAILURE ENVELOPE

TARGET 6
DURING CONSTRUCTION

FIGURE APP 13

TRIAXIAL TESTS

TARGET 6 AFTER PENETRATION

DISTANCE FROM BOTTOM OF TARGET TO TOP OF TRIAXIAL SAMPLE IN PLACE (inches)	σ_h CONFINING PRESSURE (psf)	MOISTURE CONTENT (%)	BULK DENSITY (psf)	INITIAL HEIGHT (inches)	INITIAL DIAMETER (inches)	MODE OF FAILURE
30	0	26.1	120.9	5.025	2.765
30	50	25.4	120.9	4.515	2.775
30	100	25.8	123.3	4.735	2.750
42	0	24.6	122.0	5.050	2.790
42	50	28.5	124.0	5.007	2.775
42	100	24.2	124.0	5.008	2.776

FIGURE APP 14

STRESS vs STRAIN FROM TRIAXIAL TESTS

TARGET 6 AFTER PENETRATION

FIGURE APP 15

FAILURE ENVELOPE

TARGET 6
AFTER PENETRATION

FIGURE APP 16

Size of lifts--100 lbs.

Passes per lift-- 4

End of construction until first penetration--9 days

End of construction until final examination--17 days

Size of batches--3300 lbs. (2 batches for entire target)

This target was compacted twice as much as target number 2, which had a corresponding moisture content. It was hoped that a high level of density would be attained. However, even though the resulting target was very hard, the bulk density was not substantially increased.

The top of this target was scraped very smooth, 1/2 inch below the top of the steel mold. The surface was then coated with a film of oil in an attempt to retard evaporation.

Inked lines were placed on the surface before penetration for measurements of surface movements.

Triaxial tests were performed using displacement gauges and a proving ring (Fig. App. 17).

Density measurements were made with a balloon density meter.

Target 8

Desired moisture content--24.5%

Size of lifts--100 lbs.

Passes per lift--1

End of construction until first penetration--2 days

FIGURE APP. 17

TRIAXIAL SAMPLES AFTER TESTING FROM TARGETS 6 AND 7

— VALUES MEASURED DURING TARGET CONSTRUCTION
 --- VALUES MEASURED AFTER PROJECTILE PENETRATION
 * VALUES NOT MEASURED DIRECTLY

TARGET 7
 FIGURE APP 18

TRIAXIAL TESTS

TARGET 7

DURING CONSTRUCTION

DISTANCE FROM BOTTOM OF TARGET TO TOP OF TRIAXIAL SAMPLE IN PLACE (inches)	σ_h CONFINING PRESSURE (psf)	MOISTURE CONTENT (%)	BULK DENSITY (psf)	INITIAL HEIGHT (inches)	INITIAL DIAMETER (inches)	MODE OF FAILURE
18	0	23.6	124.0	4.980	2.779
18	50	27.5	123.0	5.097	2.792
18	100	27.5	122.1	5.052	2.800
30	0	24.0	124.9	4.900	2.783
30	50	26.5	124.0	5.050	2.794
30	100	26.7	121.3	5.096	2.800
42	0	23.6	123.0	4.932	2.788
42	50	27.2	123.0	5.095	2.796
42	100	26.3	121.0	5.074	2.810

FIGURE APP 19

STRESS vs STRAIN FROM TRIAXIAL TESTS

FIGURE APP 20

FAILURE ENVELOPE

TARGET 7

DURING CONSTRUCTION

FIGURE APP 21

TRIAXIAL TESTS

TARGET 7

AFTER PENETRATION

DISTANCE FROM BOTTOM OF TARGET TO TOP OF TRIAXIAL SAMPLE IN PLACE (inches)	σ_h CONFINING PRESSURE (psi)	MOISTURE CONTENT (%)	BULK DENSITY (psf)	INITIAL HEIGHT (inches)	INITIAL DIAMETER (inches)	MODE OF FAILURE
30	0	22.8	124.8	5.010	2.800
30	50	23.2	125.5	4.893	2.791
30	100	23.4	127.1	4.900	2.775
42	0	22.8	122.6	5.080	2.800
42	50	23.6	125.0	5.013	2.785
42	100	23.6	125.0	5.015	2.791

FIGURE APP 22

STRESS vs STRAIN FROM TRIAXIAL TESTS

TARGET 7

AFTER PENETRATION

FIGURE APP 23

FAILURE ENVELOPE

TARGET 7
AFTER PENETRATION

FIGURE APP 24

End of construction until final examination--6 days

Size of batches--(one batch for entire target)

This target was compacted 1/2 as much as target number two, which had a corresponding moisture content.

The material used in constructing this target was mixed in two 3300 pound batches, and combined to form one pile of mixed clay. The pile was covered with a sheet of plastic, and allowed to set until the target was built. The elapsed time between mixing and construction was 8 days.

This target was scraped smooth 1/2 inch below the top of the steel mold. The surface was covered with a mixture of melted parafin and carbon tetrachloride. The function of the carbon tetrachloride was to retard hardening of the parafin until the liquid mixture had sought a level surface. This was done in an attempt to retard or eliminate water evaporating from the target, and was very effective. Density measurements were made with a balloon density meter.

The triaxial samples were tested using displacement gauges and a proving ring. One set of triaxial tests for this target were performed with water as a confining medium, so that volume change measurements could be made.

Target 9

This target was constructed of 40-50 Ottawa sand. This is a very uniform, very clean sand.

— VALUES MEASURED DURING TARGET CONSTRUCTION
 - - - VALUES MEASURED AFTER PROJECTILE PENETRATION
 * VALUES NOT MEASURED DIRECTLY

TARGET 8

FIGURE APP 25

TRIAXIAL TESTS

TARGET 8

AFTER PENETRATION

DISTANCE FROM BOTTOM OF TARGET TO TOP OF TRIAXIAL SAMPLE IN PLACE (inches)	σ_h CONFINING PRESSURE (psi)	MOISTURE CONTENT (%)	BULK DENSITY (psf)	INITIAL HEIGHT (inches)	INITIAL DIAMETER (inches)	MODE OF FAILURE
30	0	23.5	110.5	4.600	2.794	
30	50	23.8	111.1	4.225	2.775	
30	100	23.4	109.5	5.020	2.775	
42	0	23.7	110.0	3.875	2.800	
42	50	23.6	108.9	4.475	2.800	
42	100	24.0	113.3	5.125	2.775	

FIGURE APP 26

STRESS vs STRAIN FROM TRIAXIAL TESTS

FIGURE APP 27

FAILURE ENVELOPE

TARGET 8

AFTER PENETRATION

FIGURE APP 28

—— VALUES DETERMINED FROM SPOT MEASUREMENTS

--- VALUES DETERMINED FROM TOTAL WEIGHT AND VOLUME MEASUREMENTS

* VALUES NOT MEASURED DIRECTLY

TARGET 9

FIGURE APP 30

(40-50 Ottawa Sand)

Uniformity of the density was controlled by dropping the sand from a constant height. A box with standard wire screen stretched over the bottom was constructed for this purpose.

The target mold was similar in design to those used for the clay targets, except for the dimensions. The steel cylindrical mold was seven feet tall and 30 inches in diameter.

Density measurements were made by placing a one pound coffee can in the target during construction and weighting it when it was full.

Surface Movement Measurements

As was previously mentioned, measurements of target surface movements were made using various types of markers on the surfaces. The measurements reported were made before and after penetration, using a mark on the steel mold as a fixed reference point.

These measurements were made along four perpendicular lines, which crossed at the center of the target (Tables 14 and 15, and Fig. App. 31).

Movement away from the center of the target is considered to be positive, and toward the center to be negative. These measurements are lateral measurements only.

Observations

The following is a brief discussion of things observed by the

TABLE 14

SURFACE MOVEMENT (TARGET 7)

Original Distance From Center of Target (Inches)	Line No. 1	Line No. 2	Line No. 3	Line No. 4
0	-	-	-	-
1/2	-	-	-	-
1	5*	6	-	6
1 1/2	4	5	6	6
2	3	4	5	5
2 1/2	2	3	5	5
3	2	3	5	4
3 1/2	2	3	5	3
4	2	2	4	2
4 1/2	2	3	3	1
5	2	2	3	1
5 1/2	2	2	3	1
6	1	1	2	1
6 1/2	1	1	1	0
7	0	0	1	0

- marker destroyed by crater

* 1 unit = 1/50 inch

All values are positive.

TABLE 15

SURFACE MOVEMENT (TARGET 8)

Original Distance From Center of Target (Inches)	Line No. 1	Line No. 2	Line No. 3	Line No. 4
0	-	-	-	-
1/2	-	-	-	-
1	3	-	-	4
1 1/2	3	-	4.5	4
2	3	-	4	3
2 1/2	3	-	3	2
3	3	-	2	2
3 1/2	2	5	2	2
4	2	5	2	1
4 1/2	1.5	5	1.5	1
5	1.5	5	1.5	1
5 1/2	1	4	1.5	1
6	1	4	1	0
6 1/2	1	4	1	0
7	1	3	1	0
7 1/2	1	3	1	0
8	1	2	1	0
8 1/2	1	1	1	0
9	1	1	1	0
9 1/2	0.5	0.5	0	0
10	0.5	0.5	0	0
10 1/2	0.5	0	0	0
11	0	0	0	0

- marker destroyed by crater

* 1 unit = 1/50 inch

All values are positive.

TARGET 7

TARGET 8

Figure App 31

TARGET SURFACE MOVEMENT

author that were thought to be significant.

It was expected that radial cracks would propagate from the point of impact toward the perimeter of the target during the penetration event. These cracks would indicate radial tension caused by the surface heave. In all targets which had a smooth, relatively dry surface before penetration, these radial cracks were definitely observed.

In most cases, the densified soil nose, which is known to form in front of the projectile during penetration, was easily removed and separated from the target during dismantling. It was observed that this nose does not form on the entire diameter of a blunt-nosed projectile, but rather has a diameter slightly less than that of the projectile. When the face of a blunt nose was painted before penetration, it could be seen upon retrieving the projectile that the paint was removed only from the outer perimeter of the nose.

Likewise, stripes were painted the full length of the projectile, which were not worn off at all. This indicates that there is no contact between the soil and the outer surface of the projectile, except at the nose, during penetration.

It was also observed that soil from the surface of the target was pulled down with the projectile, coating the full depth of the hole formed by the projectile.

APPENDIX III COMPUTER PROGRAMS

A

```

C   MAIN PROGRAM
      IMPLICIT REAL*8(A-G,O-Z)
      INTEGER*2 NZ,NZA,NZB,NZC,NSS,NPO,NNE,NBLNK
      DIMENSION AU(150,8),IT(7),COL(140),XKOE(20,20),DROW(140),
      *DU(400),LX(141),LY(141),NZ(8),NZA(64),NZB(64),NSS(64),NZC(64)
      DIMENSION CHEK(140),FARE(140),FN(8),RE(20,20),AN(20,20)
      DIMENSION FC(8)
      EQUIVALENCE (DU(1),XKOE(1,1))
      READ (5,10) NT,NP,ICV
10   FORMAT (20I4)
      EPS=0.00001
      READ (5,10) (IT(J),J=1,NT)
      READ(5,20) (FN(J),J=1,8)
      READ(5,20) (FN(J),J=1,8)
950  FORMAT (///,50X,'VERIFICATION OF CURVE FITTING',//,30X,
      *OBSERVED FORCE',16X,'PREDICTED FORCE',15X,'PERCENT ERROR',/,
      *31X,'ON PROJECTILE',17X,'ON PROJECTILE',12X,'BASED ON OBSERVED
      *FORCE',/)
960  FORMAT(29X,D14.4,16X,D14.4,19X,F9.2)
970  FORMAT(29X,D14.4,16X,D14.4,19X,'UNDEFINED')
910  FORMAT (1H1,/,40X,'THIS IS TEST NUMBER',16,/,10X,'FORCE',10X,
      *U1',8X,'U2',8X,'U3',8X,'U4',8X,'U5',8X,'U6',8X,'U7',/)
920  FORMAT (9X,F7.3,6X,F7.3,6(3X,F7.3))
25   FORMAT (40A2)
930  FORMAT(1H1,///,2X,'FORCE = ',5(A2,D14.6,1X,A2,A2,A2),/,50
      *(11X,5(A2,D14.6,1X,A2,A2,A2),/))
      DATA BB/'XKOF'/,CC/'PRNT'/,DD/'XINV'/.CNP/'JUNK'/
      NPM=NP-1
      ND=1+(NPM*(NPM+5))/2)
      DO 60 J=1,ND
      DO 70 K=1,ND
      XKOE(J,K)=0.0
70   CONTINUE
      COL(J) = 0.0
60   CONTINUE
      IS = 1
      IE=0
      DO 100 KA = 1,NT
      WRITE (6,910)KA
      READ (5,20) (AU(IS,J),J=1,NP)

```

```

20. FORMAT (8F10.3)
   WRITE (6,920) (AU(1S,J),J=1,NP)
   ISS = 1S + 1
   IE=IT(KA) + IE
   DO 200 KB=ISS,IE
   DO 250 KC=4,NP
   AU(KB,KC)=AU(1S,KC)
250 CONTINUE
   READ(5,20)AU(KB,1),AU(KB,2),AU(KB,3)
   WRITE (6,920) (AU(KB,J),J=1,NP)
200 CONTINUE
   IS=IE+1
100 CONTINUE
   NROWS = IE
   READ (5,25) (NZ(J),J=1,NP)
   READ (5,25) NPO,NNE,NBLNK
   CALL COEF(AU,IT,NT,NROWS,NP,COL,XKOE,ND,DROW)
666 FORMAT(/,10X,8D14.4,/,8(24X,7D14.4,/))
   WRITE(6,666)(COL(IJK),IJK=1,ND)
   CALL MATPRN(XKOE,ND,BB,CC)
   DO 688 I=1,ND
   DO 688 J=1,ND
   RE(I,J)=XKOE(I,J)
686 CONTINUE
   CALL MATINV(DU,LX,LY,ND,20,EPS,DET)
   CALL SIMALT(XKOE,COL,DROW,ND,ND)
   WRITE (6,666) (DROW(IJK),IJK=1,ND)
   CALL MULT(AN,RE,XKOE,ND,ND,ND)
   CALL MATPRN(AN,ND,DD,CC)
   JW=NP
   NZA(1)=NZ(1)
   DO 650 J=2,NP
   NZA(J)=NZ(J)
   NZB(J)=NBLNK
   NZC(J)=NBLNK
   JK=J+NPM*(NPM+1)/2
   NZA(JK)=NZ(J)
   NZB(JK)=NZ(J)
   NZC(JK)=NZ(J)
   DO 660 K=J,NP
   JW=JW+1
   NZA(JW)=NZ(J)
   NZB(JW)=NZ(K)
   NZC(JW)=NBLNK
660 CONTINUE
650 CONTINUE
   DO 680 J=1,ND

```

```

      NSS(J)=NPO
      FARE(J)=DROW(J)
      IF(DROW(J).LT.)GO TO 690
      FO TO 680
690  CONTINUE
      FARE(J)=DABS(DROW(J))
      NSS(J)=NNE
680  CONTINUE
      WRITE (6,940)
      WRITE (6,930) (NSS(J),FARE(J),NZA(J),NZB(J),NZC(J),J=1,ND)
      WRITE (6,940)
940  FORMAT(1H)
      WRITE(6,950)
      CHEK(1)=1.0
      DO 1250 J=1,NROWS
      CALL GEROW(CHEK,AU,ND,NROWS,NP,J)
      FORCE=0.0
      DO 1500 JS=1,ND
      FORCE=FORCE+DROW(JS)*CHEK(JS)
1500 CONTINUE
      IF(AU(J,1).EQ.0.0) GO TO 1400
      PERER=(AU(J,1)-FORCE)*100./AU(J,1)
      WRITE(6,960)AU(J,1),FORCE,PERER
      GO TO 1250
1400 CONTINUE
      WRITE(6,970) AU(J,1),FORCE
      CALL DEPTH(AU,DROW,IT,CHEK,NROWS,ND,NP,NT,ICV)
      WRITE(6,940)
      STOP
      END

      SUBROUTINE DEPTH(AU,DROW,IT,CHEK,MU,ND,NP,NT,ICV)
      IMPLICIT REAL*8(A-G,O-Z)
      DIMENSION AU(MU,NP),DROW(ND),CHEK(ND),IT(NT)
500  FORMAT(///,50X,'VERIFICATION OF CURVE FITTING',//,30X,
      *OBSERVED DEPTH',16X,'PREDICTED DEPTH',15X,'PERCENT ERROR',
      */,30X,'OF PROJECTILE'.17X,'OF PROJECTILE'.11X,'BASED ON
      *OBSERVED DEPTH'./)
510  FORMAT(29X,D14.4,16X,D14.4,19X,F9.2)
520  FORMAT(29X,D14.4,16X,D14.4,19X,'UNDEFINED')
530  FORMAT(//,10X,'*****', 'I HAVE TRIED TO ITERATE ',110,'
      *TIMES TO EVALUATE THE FINAL DEPTH OF PENETRATION WITHOUT
      *SUCCESS',//)
      DI=.0001
      ZF=0.0
      IE=1
      G=32.2

```

```

      WP=5.522
      IL=0
      WRITE(6,500)
      DO 100 NJ=1,NT
      VZ=AU(IE,2)
      ZZ=0.0
      IQQ=0
150  CONTINUE
      IQQ=IQQ+1
      CALL GEROW(CHEK,AU,ND,MU,NP,IE)
      FORCE=0.0
      DO 200 JS=1,ND
      FORCE=FORCE+DROW(JS)*CHEK(JS)
200  CONTINUE
      AC=(-1.0+FORCE/WP)*G
      DV=AC*DT
      VF=VZ+DV
      IF(VF.LE.0.0)GO TO 95
      DZ=(VF+VZ)*DT/2.0
      ZF=ZZ+DZ
      AU(IE,2)=VF
      AU(IE,3)=ZF
      VZ=VF
      ZZ=ZF
      IF(IQQ.GT.1CV) GO TO 94
      GO TO 150
94  CONTINUE
      WRITE(6,530) IQQ
95  CONTINUE
      IL=II(NJ)*IL
      ZOB=AU(IL,3)
      IF(ZOB.EQ.0.0) GO TO 98
      ZERR=(ZOB-ZF)*100./ZOB
      WRITE(6,510)ZOB,ZF,ZERR
      IE=IE+IT(NJ)
      GO TO 100
98  CONTINUE
      WRITE(6,520) ZOB,ZF
      IE=IE+II(NJ)
100 CONTINUE
      RETURN
      END

      SUBROUTINE COEF(AU,IT,NT,MU,NP,COL,XKOE,ND,DROW)
      IMPLICIT REAL*8(A-G,O-Z)
      DIMENSION AU(MU,NP),IT(NT),COL(ND),XKOE(ND,ND),DROW(ND)
      DROW(I)=1.0

```

```

      IS=1
      IE=0
      DO 100 J=1,NT
      CALL GEROW(DROW,AU,ND,MU,NP,IS)
555  FORMAT(/,10X,10D12.4)
      WRITE(6,555) (DROW(JKJ),JKJ=1,nd0)
      DO 200 JJ=1, ND
      COL(JJ)=AU(IS,1)*DROW(JJ)+CCL(JJ)
      DO 300 KK=1,ND
      XKOEJ(JJ,KK)=XKOEJ(JJ,KK)+DROW(JJ)*DROW(KK)
300  CONTINUE
200  CONTINUE
      IS=IS+1
      IE=IE(JJ)+IE
      DO 400 K=IS,IE
      CALL GEROW(DROW,AU,ND,MU,NP,K)
      DO 500 JJ=1,ND
      COL(JJ)=AU(K,1)*DROW(JJ)+COL(JJ)
      DO 600 KK=1,ND
      XKDEF(JJ,KK)=XKDEF(JJ,KK)+DROW(JJ)*DROW(KK)
600  CONTINUE
500  CONTINUE
400  CONTINUE
      IS=IE+1
100  CONTINUE
      RETURN
      END

```

```

SUBROUTINE GEROW(DROW,AU,ND,MU,NP,IS)
  IMPLICIT REAL*8(A-G,O-Z)
  DIMENSION DROW(ND),AU(MU,NP)
  NPM=NP-1
  JD=1+NPM*(NPM+3)/2
  JQ=NP
  DO 100 JP=2,NP
  DROW(JP)=AU(IS,JP)
  JD=JD+1
  DROW(JD)=AU(IS,JP)*AU(IS,JP)*AU(IS,JP)
  DO 200 J=JP,NP
  JQ=JQ+1
  DROW(JQ)=AU(IS,JP)*AU(IS,J)
200  CONTINUE
100  CONTINUE
  RETURN
  END

```

```

SUBROUTINE SIMALT ( A , B , C , M , N )

```

```

      IMPLICIT REAL*8(A-G,O-Z)
      DIMENSION A(M,N) , B(N) , C(M)
      DO 200 I=1 , M
      C(I) = 0.0
200  CONTINUE
      DO 500 I=1 , M
      DO 400 J = 1 , N
      C(I) = C(I) + A(I,J) * B(J)
400  CONTINUE
500  CONTINUE
      RETURN
      END

      SUBROUTINE MATINV(Z,IROW,ICOL,N,NDIM,SMLST,DET)
      IMPLICIT REAL*8(A-G,O-Z)
      DIMENSION A(I),IROW(I),ICOL(I)
      NPI=N+1
      DO 5 I=1,N
      ICOL(I)=I
5  IROW(I)=I
      DO 75 ITER=1,N
      MAXR=ITER
      MAXC=I
      TEMP=DABS(A(MAXR))
      LIMITC=NPI-ITER
      DO 15 I=ITER,N
      DO 15 J=1,LIMITC
      IJ=(J-1)*NDIM+I
      IF (TEMP-(DABS(A(IJ)))) 10,15,15
10  MAXR=I
      MAXC=J
      TEMP=DABS(A(IJ))
15  CONTINUE
      SMLST=-0.0
      IF (TEMP-SMLST) 20,20,25
20  IROW(NPI)=ITER
      WRITE(6,200)
200 FORMAT(/IHO,46X,3/HTHIS FRAME APPARENTLY HAS DEFLECTIONS,/,
      *47X, 34HWHICH ARE INDEPENDENT OF THE LOADS)
25  IF (MAXR-ITER) 30,40,30
30  DO 35 J=1,N
      MAXRJ=(J-1)*NDIM+MAXR
      ITJ=(J-1)*NDIM+ITER
      TEMP=A(MAXRJ)
      A(MAXRJ)=A(ITJ)
35  A(ITJ)=TEMP
      ITEMP=IROW(MAXR)
      IROW(MAXR)=IROW(ITER)

```

```

      IROW(MAXR)=ITEMP
40  IF (MAXC-1) 45,55,45
45  DO 50 I=1,N
      IMAXC=(MAXC-1)*NDIM+1
      TEMP=A(I)
      A(I)=A(IMAXC)
50  A(IMAXC)=TEMP
      ITEMP=ICOL(MAXC)
      ICOL(MAXC)=ICOL(I)
      ICOL(I)=ITEMP
55  TEMP=A(ITER)
      ITEMP=ICOL(I)
      DO 60 J=2,N
          ITJMI=(J-2)*NDIM+ITER
          A(ITJMI)=A(ITJ)/TEMP
60  ICOL(J-1)=ICOL(J)
      ITN=(N-1)*NDIM+ITER
      A(ITN)=1.0/TEMP
      ICOL(N)=ITEMP
      DO 75 I=1,N
          IF (I-ITER) 65,75,65
65  TEMP=A(I)
      DO 70 J=2,N
          IJMI=(J-2)*NDIM+I
          IJ=(J-1)*NDIM+I
          ITJMI=(J-2)*NDIM+ITER
          A(IJMI)=A(IJ)-A(ITJMI)*TEMP
70  CONTINUE
      IN=(N-1)*NDIM+1
      ITN=(N-1)*NDIM+ITER
      A(IN)=- (TEMP*A(ITN))
75  CONTINUE
      DO 100 I=1,N
          DO 80 J=1,N
              IF (IROW(J)-I) 80,85,80
80  CONTINUE
85  IF (I-J) 90,100,90
90  DO 95 L=1,N
          LI=(I-1)*NDIM+L
          LJ=(J-1)*NDIM+L
          TEMP=A(LI)
          A(LI)=A(LJ)
95  A(LJ)=TEMP
      IROW(J)=IROW(I)
100 CONTINUE
      DO 125 I=1,N
          DO 105 J=1,N

```



```

      IF (ICOL(J)-1) 105,110,105
105 CONTINUE
110 IF (1-J) 115,125,115
115 DO 120 L=1,N
      IL=(L-1)*NDIM+1
      JL=(L-1)*NDIM+J
      TEMP=A(IL)
      A(IL)=A(JL)
120 A(JL)=TEMP
      ICOL(J)=ICOL(L)
125 CONTINUE
      IROW(NPI)=0
      RETURN
      END

```

```

      SUBROUTINE MATPRN ( A , N , B , RITE )
      IMPLICIT REAL*8(A-G,O-Z)
      DIMENSION A ( N , N )
      5 FORMAT (1H1)
      6 FORMAT (10X)
      10 FORMAT(11(D(2.3)))
      22 FORMAT ( // , 24H PRESENT CONTENTS OF
      * 1211 MATRIX , // )
      DATA YAZ /'PRNT'/
      IF ( RITE .NE. YAZ ) GO TO 600
18 CONTINUE
      IF ( N .LE. 11 ) GO TO 30
      WRITE (6,10) ( A(I,J) , J=1 , N )
50 CONTINUE
      GO TO 500
60 NI = N - 11
      DO 200 K = 1 , NI , 11
      KK = K + 11
      DO 100 I = 1 , N
      WRITE (6,10) ( A(I,J) , J=K , KK)
100 CONTINUE
      WRITE (6,5)
      WRITE ( 6 , 22 ) B
200 CONTINUE
      DO 300 I = 1 , N
      WRITE (6,10) ( A(I,J) , J=K , N )
300 CONTINUE
500 CONTINUE
600 CONTINUE
      RETURN
      END

```

, A4,

```
SUBROUTINE MULT(A,B,C,K,M,N)
  IMPLICIT REAL*8(A-G,O-Z)
  DIMENSION A(K,N), B(K,M), C(M,N)
  DO 100 I=1,K
    DO 200 J=1,N
      Q = 0.0
      DO 300 L=1,M
        Q = Q + B(I,L)*C(L,J)
      300 CONTINUE
      A(I,J) = Q
    200 CONTINUE
  100 CONTINUE
  RETURN
END
```

B

```

      DIMENSION X(20,300),Y(300),XBAR(20),XPX(20,20),XPXC(20,20),
      *XPY(20),XPYC(20),BHAT(20),RAD(20),T(20),YHAT(300),CORR(300),
      *DIFF(300),LORK(20),MORK(20),FMT(180),Z(300)

      DATA END/3HEND/

929 READ(5,339) NVAR,INDEX,NVFC,NDATA
339 FORMAT (4I2)

      N = 0
      M = NVAR
      IF(INDEX.EQ.0) GO TO 340
      NVFC = NVFC*20
      READ(5,341) (FMT(I),I=1,NVFC)
341 FORMAT(20A4)
      DO 342 I = 1,300
      READ (5,FMT) (X(J,I),J=1,NVAR),Y(I),CHK
      IF(CHK.EQ.END) GO TO 778
      N = N + 1
342 CONTINUE
340 MI = M+1
      DO 333 J = 1,300
      CORR(J) =0.0

```

```

CALL RGREAD(Z,M,CHK)
IF (CHK.EQ.END) GO TO 778
N = N + 1
Y(J) = A(MI)
DO 333 I = 1,M
333 X(I,N) = Z(I)
778 M = NVAR
XN = N
YBAR = 0.0
DO 7 I = 1,N
7 YBAR = YBAR + Y(I)/XN
DO 2 J = 1,M
XBAR(J) = 0.0
DO 1 I = 1,N
1 XBAR(J) = XBAR(J) + X(J,I)/XN
2 CONTINUE
SST = 0.0
DO 13 J = 1,N
13 SST = SST + (Y(J) - YBAR)*(Y(J)-YBAR)
DO 3 J = 1,M
DO 3 I = 1,M
XPX(J,I) = 0.0
DO 4 K = 1,N
4 XPX(J,I) = XPX(J,I) + (X(I,K)-XBAR(I))*(X(J,K)-XBAR(J))
XPXC(J,I) = XPX(J,I)

```

```

3 CONTINUE
  DO 5 J=1,M
    XPY(J) = 0.0
    DO 6 I = 1,N
      6 XPY(J) = XPY(J) + (S(J,I)-XBAR(J))*(Y(I)-YBAR)
      XPYC(J) = XPY(J)
    5 CONTINUE
  DO 22 J = 1,M
    22 CORR(J) = XPY(J)/(SQRT(SST*XPX(J,J)))

    JK = M
    DO 23 J=1,M
      DO 23 I =1,M
        JK = JK + 1
      23 CORR(JK)=XPX(I,J)/(SQRT(XPX(I,I)*XPX(J,J)))
      DO 25 I = 1,JK
        25 WRITE (6,24)CORR(I)
      24 FORMAT (1H , E16.8)

      EPS = 1.0E-05
      DO 777 I = 1,M
        777 XPX(I,M+1) = XPY(I)

        CALL DIMEQN(XPX,M,EPS,D,I)
        DO 780 I = 1,M
          780 BHAT (I) = XPX(I,M+1)

        W = 0.0
        SSR = 0.0

```

```

      DO 8 I = 1,M
        W = W + BHAT(I)*XBAR(I)
        BSUBO = YBAR - W
      8 SSR = SSR + BHAT(I) * XPY(I)
        XM = M
        XMSR = SSR/XM
        SSE = SST - SSR
        XE = N - M - 1
        K = XE
        XMSE = SSE/XE
        F = XMSR/XMSE
        DO 10 I = 1,M
          RAD(I) = XPX(I,I)*XMSE
          T(I) = BHAT(I)/SQRT(RAD(I))
10 CONTINUE
        DO 11 I = 1,N
          YHAT(I) = BSUBO
          DO 12 J = 1,M
12 YHAT(I) = YHAT(I)+BHAT(J)*X(J,I)
          DIFF(I) = Y(I) - YHAT(I)
11 CONTINUE
        WRITE(6,100) M,N
        WRITE(6,104)
        DO 105 I = 1,M
105 WRITE(6,106) I,(XPXC(I,J),J=1,M)

```

```

WRITE(6,119)
DO 120 I=1,M
120 WRITE(6,121) I,XPYC(I)
WRITE (6,107)
DO 108 I = 1,M
108 WRITE (6,109) I,(XPX(I,J),J=1,M)
NI = N - I
WRITE(6,110) NI,SST,M,SSR,XMSR,F,K,SSE,XMSE
WRITE(6,111) BSUBO
DO 112 I = 1,M
112 WRITE (6,113) I, BHAT(I),JT(I)
WRITE (6,114)
DUB2=0.
DUBB=0.
DURB=0.
DO 222 I=2,N
JJ = I-1
222 DUB2=DUB2+((DIFF(I)-DIFF(JJ))*(DIFF(I)-DIFF(JJ)))
DO 115 I = 1,N
DUBB=DUBB+(DIFF(I)*DIFF(I))
115 WRITE(6,116) I,Y(I),YHAT(I),DIFF(I)
DURB=DUB2/DUBB
WRITE(6,223) DURB
223 FORMAT(1H0,'DURBAN-WATSON STATISTIC =',F7.4)
100 FORMAT(1H1,26X,24HMULTIPLE REGRESSION ON,12,17H

```

```

*VARIABLES WITH,13,13H OBSERVATIONS //////////)
103 FORMAT(1H,1X,4HROW,13,3X,6E18.8/(12X,6E18.8))
104 FORMAT(1H0,50X,20HCORRECTED X'X MATRIX///)
106 FORMAT(1H,1X,4HROW,13,3X,6E18.8/(12X,6E18.8))
107 FORMAT(1H1,10X,28HCORRECTED X'X INVERSE MATRIX///)
109 FORMAT(1H,1X,4HROW,13,3X,6E18.8/(12X,6E18.8))
110 FORMAT(1H1,/,44X,5HANOVA///11X,6HSOURCE,8X,2HDF,10X,2HSS,
*16X,2HMS,16X,1HF,17X,///11X,5HTOTAL,9X,13,2X,E18.8//11X,
*10HREGRESSION,4X,13,2X,3E18.8,///11X,8HRESIDUAL,6X,13,2X,
*2E18.8/////////)
111 FORMAT(1H,21X,12HCOEFFICIENTS,15X,8HT VALUES///11X,
*11HBSUBO . = ,E15.8)
113 FORMAT(1H,10X,5HBHAT(,12,4H) = ,E15.8,10X,2HT(,12,4H) =
*,E15.8)
114 FORMAT(1H1,10X,' OBSERVATION ',10X,' Y-OBSERVED ',10X,
*' Y-ESTIMATED ',10X,' DIFFERENCE ' , //////////)
116 FORMAT(1H,10X,13,19X,E15.8,5X,E15.8,5X,E15.8)
117 FORMAT(12,2X,13,2X,13)
119 FORMAT(1H,/////////,11X,20HCORRECTED X'Y MATRIX///)
121 FORMAT(1H,1X,4HROW,13,3X,6E18.8/(11X,6E18.8))
IF(NDATA.NE.0) GO TO 929
STOP
END

```



```

C  SUBROUTINE DIMEQN IS A DOUBLE PRECISION MATRIX INVERSION
C  ROUTINE THAT SEEKS MAXIMUM PIVIT ELEMENTS AND INVERTS IN
C  PLACE
C  A=ARRAY CONTAINING MATRIX OF COEFFICIENTS TO BE INVERTED
C  N=ORDER OF A
C  EPS=MINIMUM ALLOWABLE VALUE OF MATRIX PIVIT BEFORE MATRIX
C  IS TERMED SINGULAR
C  DET=VALUE OF DETERMINANT OF THE MATRIX
C  NSOLUT=NO SOLUTION VECTORS IN THE AUGMENTED MATRIX
SUBROUTINE DIMEQN(A,N,EPS,DET,NSOLUTE)
C  DOUBLE PRECISION A,EPS,DET,Y,BIGA,AJCK
  INTEGER*2 IPIV,ICJ
  DIMENSION IPIV(2,120),ICJ(120),Y(120),A(120,120)
  M=N+NSOLUTE
  DET= 1.0
  ASSIGN 205 TO IZERO
  DO 100 K=1,N
    KMI=K-1
    IF(KMI.GT.0) ASSIGN 95TO IZERO
    BIGA = 0.0
    DO 101 J=1,N
      DO 101 J=1,N
        GO TO IZERO ,(95,205)
95 DO 102 II=1,KMI

```

```

      IF(I.EQ.IPIV(1,11).OR.J.EQ.IPIV(2,11)) GO TO 101
102 CONTINUE
205 CONTINUE
      IF(ABS(A(I,J)).LT.BIGA) GO TO 101
      BIGA=ABS(A(I,J))
      IPIV(1,K)=I
      IPIV(2,K)=J
101 CONTINUE
      IF(BIGA.GE.EPS) GO TO 201
      DET= 0.0
      GO TO 200
201 IR= IPIV(1,K)
      JC= IPIV(2,K)
      BIGA= A(IR,JC)
      DET= DET*BIGA
      DO 103 LL= 1,M
103 A(IR,LL)= A(IR,LL)/BIGA
      A(IR,JC)= 1.0/BIGA
      DO 100 LLL= 1,N
      AJCK= A(LLI,JC)
      IF(LLI.EQ.IR) GO TO 100
      A(LLI,JC)= -AJCK/BIGA
      DO 104 L4= 1,M
      IF(L4.EQ.JC) GO TO 104

```

A(LLL,L4)= A(LLL,L4) - AJCK*A(IR,L4)

104 CONTINUE

100 CONTINUE

DO 105 I= 1,N

IR= IPIV(1,I)

ICJ(IR)= JC

105 CONTINUE

ICT= 0

NMI= N - 1

DO 106 I=1,NMI

IPI= I + 1

DO 106 J= IPI, N

IF(ICJ(J).GE.ICJ(I)) GO TO 106

ITEMP= ICJ(J)

ICJ(J)= ICJ(I)

ICJ(I)= ITEMP

ICT= ICT + 1

106 CONTINUE

IF((ICT/2)*2.NE.ICT) DET= -DET

DO 107 J=1,M

DO 108 I=1,N

JC= IPIV(2,I)

IR= IPIV(1,I)

108 Y(JC)= A(IR,J)

```
DO 107 K= 1,N
```

```
107 A(K,J)= Y(K)
```

```
DO 110 I= 1,N
```

```
DO 111 J= 1,N
```

```
IR= IPIV(1,J)
```

```
JC= IPIV(2,J)
```

```
111 Y(IR)= A(I,JC)
```

```
DO 110 K=1,N
```

```
110 A(I,K)= Y(K)
```

```
200 RETURN
```

```
END
```

```
SUBROUTINE RGREAD(Z,M,CHK)
```

```
DATA END/ 3HEND/
```

```
DIMENSION Z(300),X(7), Y(7),U(7),W(7),V(7),T(7),S(7)
```

```
DIMENSION CODE(8)
```

```
DATA CODE/124.,97.,5472.,10.,.35,553.,341.,360./
```

```
1 FORMAT(3X,F3.0,F2.0,F4.0,F3.1,F2.2,2F3.0,3X,F3.2,A3)
```

```
READ(5,1) (Z(I),I=1,8),CHK
```

```
IF(CHK.EQ.END) RETURN
```

```
DO 4 I=1,7
```

```
4 Z(I)= Z(I)/CODE(I)
```

```
EX= Z(8)
```

```
Z(8)= Z(7)*2
```

Z(9)= EX

3 RETURN

END

APPENDIX IV

REFERENCES

1. Benson, F. J., Thompson, L. J., Carpenter, S. H., and Ferguson, G. H. III, The Conceptual Design of a Terradynamics Laboratory, Final Technical Report to Sandia Corporation, June, 1968.
2. Cochran, Wm. G., and Cox, G. M., Experimental Designs, John Wiley and Sons, Inc., New York, Second Edition, 1957.
3. Hartley, H. O., Gates, C. E., and Berry, Wayne, Earth Penetration and Dynamic Soil Mechanics, Statistical Design Report, Sandia Laboratories, Albuquerque, New Mexico, SC-CR-68-3537.
4. Thompson, L. J., Dynamic Penetration of Selected Projectiles Into Particulate Media, Sandia Corporation, Albuquerque, New Mexico, SC-DR-67-60.
5. Wylie, C. R., Jr., Advanced Engineering Mathematics, McGraw-Hill Book Company, Inc., New York, 1960.
6. Young, C. W., The Development of Empirical Equations for Predicting Depth of an Earth-Penetrating Projectile, Sandia Corporation, Albuquerque, New Mexico, SC-DR-67-60.
7. Young, C. Wayne, "Depth Prediction for Earth-Penetrating Projectiles," The Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 95, No. SM3, Proc. Paper 6558, May, 1969, p. 803.