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A COMPUTER PROGRAM FOR THE
SMOOTHING AND DIFFERENTIATION OF DATA FROM
MULTICHANNEL ANALYZERS

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A COMPUTER PROGRAM FOR THE SMOOTHING AND DIFFERENTIATION
OF DATA FROM MULTICHANNEL ANALYZERS *

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In the interpretation of spectra taken on multichannel analyzers it is frequently helpful, especially in cases where the statistics are poor, to smooth the data in order to reduce the statistical fluctuations.

When presented with such a spectrum one would normally draw through the points a smooth curve which best fits them. This can be done numerically once we define what we mean by best fit. The most common criterion is to minimize the sum of the squares of the residuals between the actual and fitted points.

This method has been described by Savitzky and Golay¹ who fit a polynomial over a range of $(2m + 1)$ data points and use that to evaluate the smoothed value at the mid-point. The data points taken are then shifted by one, dropping the point at the left and picking up one at the right. This process is repeated to cover the entire spectrum.

Upon doing one least squares analysis one obtains a set of coefficients, one for each point over which the fit is performed. The value at the mid-point of one interval of $2m + 1$ points is found by summing the products of the counts in each channel and the particular coefficient for that channel.

$$\text{i.e. } \bar{Y}_j = \sum_{i=-m}^{+m} C_i Y_{i+j} ,$$

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where the Y_{i+j} are the counts in channel $i + j$, the C_i are a set of convoluting factors and $\bar{Y}_j$ is the best calculated value for the number of counts in channel j . j is an index which runs over all of the channels in the spectrum.

The main part of the least-squares analysis of Savitzky and Golay is reproduced here along with some corrections and additions.

Convolution Using a Least Squares Procedure

A set of $2m + 1$ consecutive values are to be used to determine the best least squares fit through the values of a polynomial of degree $n < 2m + 1$. The polynomial has the form:

$$f_i = \sum_{k=0}^n b_{nk} i^k = b_{n0} + b_{n1} i + b_{n2} i^2 + \dots + b_{nn} i^n \quad (I)$$

The derivatives of this polynomial are

$$\frac{df}{di} = b_{n1} + 2b_{n2} i + 3b_{n3} i^2 + \dots + nb_{nn} i^{n-1} \quad (IIa)$$

$$\frac{d^2 f}{di^2} = 2b_{n2} + 6b_{n3} i + \dots + n(n-1)b_{nn} i^{n-2} \quad (IIb)$$

$$\frac{d^n f}{di^n} = n! b_{nn} \quad (IIc)$$

The value of i ranges from $-m$ to $+m$, and i is equal to zero at the central point of the set of $2m + 1$ values. Therefore the value of the s^{th} derivative at the point $i = 0$ is:

$$\left(\frac{d^s f_i}{di^s} \right)_{i=0} = s! b_{ns} = a_{ns} , \quad (III)$$

where

$$f_0 = b_{n0} = a_{n0} \quad (IVa)$$

$$\frac{df_0}{di} = b_{n1} = a_{n1} \quad (IVb)$$

$$\frac{d^2 f_0}{di^2} = 2b_{n2} = a_{n2} . \quad (IVc)$$

The least squares criterion requires that the sum of the squares of the differences between the observed values Y_i and the calculated ones f_i be a minimum.

$$\text{i.e.} \quad \frac{\partial}{\partial b_{nk}} \left[\sum_{i=-m}^{+m} (f_i - y_i)^2 \right] = 0 . \quad (V)$$

Minimizing with respect to b_{n0} we have

$$\begin{aligned} & \frac{\partial}{\partial b_{n0}} \left[\sum_{i=-m}^{+m} (b_{n0} + b_{n1}i + \dots + b_{nn}i^n - y_i)^2 \right] \quad (VIa) \\ & = 2 \sum_{i=-m}^{+m} (b_{n0} + b_{n1}i + \dots + b_{nn}i^n - y_i) = 0 . \end{aligned}$$

Minimizing with respect to b_{n1} we have

$$\frac{\partial}{\partial b_{nl}} \left[\sum_{i=-m}^{+m} (b_{n0} + b_{nl}i + \dots + b_{nn}i^n - y_i)^2 \right] \quad (\text{VIb})$$

$$= 2 \sum_{i=-m}^{+m} (b_{n0} + b_{nl}i + \dots + b_{nn}i^n - y_i)i = 0$$

With respect to the general b_{nr} we get

$$\frac{\partial}{\partial b_{nr}} \left[\sum_{i=-m}^{+m} (b_{n0} + b_{nl}i + \dots + b_{nn}i^n - y_i)^2 \right] \quad (\text{VIc})$$

$$= 2 \sum_{i=-m}^{+m} (b_{n0} + b_{nl}i + \dots + b_{nr}i^n - y_i)i^r = 0$$

$$2 \sum_{i=-m}^{+m} \left[\left(\sum_{k=0}^n b_{nk}i^k \right) - y_i \right] i^r = 0$$

$$\sum_{i=-m}^{+m} \sum_{k=0}^n b_{nk}i^{k+r} = \sum_{i=-m}^{+m} y_i i^r, \quad (\text{VII})$$

r is an index representing the equation number which runs from 0 to n .

Since b_{nk} is independent of i we can write the equation as

$$\sum_{k=0}^n b_{nk} \sum_{i=-m}^{+m} i^{k+r} = \sum_{i=-m}^{+m} y_i i^r = F_r^{\dagger}$$

or

$$\sum_{k=0}^n b_{nk} S_{r+k} = F_r^{\dagger} \quad , \quad (\text{VIIIa})$$

where

$$S_{r+k} = \sum_{i=-m}^{+m} i^{r+k} \quad , \quad (\text{VIIIb})$$

and

$$F_r^{\dagger} = \sum_{i=-m}^{+m} i^r y_i \quad . \quad (\text{VIIIc})$$

When $r+k$ is odd $S_{r+k} = 0$. Since S_{r+k} exists for even values of $r+k$ only, the set of $n+1$ equations can be separated into two sets, one for even values of k and one for odd values.

[†]Note: In the paper of Savitzky and Golay this is given as F_k , when it should be F_r .

For n even the secular equations VIIIa are

$$(IXa) \left\{ \begin{array}{l} S_0^b n_0 + S_2^b n_2 + S_4^b n_4 + \dots + S_n^b n_n = F_0 \\ S_2^b n_0 + S_4^b n_2 + S_6^b n_4 + \dots + S_{n+2}^b n_n = F_2 \\ S_4^b n_0 + S_6^b n_2 + S_8^b n_4 + \dots + S_{n+4}^b n_n = F_4 \\ \vdots \\ S_n^b n_0 + S_{n+2}^b n_2 + S_{n+4}^b n_4 + \dots + S_{2n}^b n_n = F_n \end{array} \right.$$

$$(IXb) \left\{ \begin{array}{l} S_2^b n_1 + S_4^b n_3 + S_6^b n_5 + \dots + S_n^b n_{n-1} = F_1 \\ S_4^b n_1 + S_6^b n_3 + S_8^b n_5 + \dots + S_{n+2}^b n_{n-1} = F_3 \\ S_6^b n_1 + S_8^b n_3 + S_{10}^b n_5 + \dots + S_{n+4}^b n_{n-1} = F_5 \\ \vdots \\ S_n^b n_1 + S_{n+2}^b n_3 + S_{n+4}^b n_5 + \dots + S_{2n-2}^b n_{n-1} = F_{n-1} \end{array} \right.$$

For n odd they are:

$$(IXc) \left\{ \begin{array}{l} S_0^b n_0 + S_2^b n_2 + S_4^b n_4 + \dots + S_{n-1}^b n_{n-1} = F_0 \\ S_2^b n_0 + S_4^b n_2 + S_6^b n_4 + \dots + S_{n+1}^b n_{n-1} = F_2 \\ S_4^b n_0 + S_6^b n_2 + S_8^b n_4 + \dots + S_{n+3}^b n_{n-1} = F_4 \\ \vdots \\ S_{n-1}^b n_0 + S_{n+1}^b n_2 + S_{n+3}^b n_4 + \dots + S_{2n-2}^b n_{n-1} = F_{n-1} \end{array} \right.$$

$$(IXd) \left\{ \begin{array}{lcl} S_2 b_{n1} + S_4 b_{n3} + S_6 b_{n5} + \dots + S_{n+1} b_{nn} & = & F_1 \\ S_4 b_{n1} + S_6 b_{n3} + S_8 b_{n5} + \dots + S_{n+3} b_{nn} & = & F_3 \\ S_6 b_{n1} + S_8 b_{n3} + S_{10} b_{n5} + \dots + S_{n+5} b_{nn} & = & F_5 \\ \vdots & & \vdots \\ S_{n+1} b_{n1} + S_{n+3} b_{n3} + S_{n+5} b_{n5} + \dots + S_{2n} b_{nn} & = & F_n \end{array} \right.$$

The convoluting factors are the $a_{ns} (= s!b_{ns})$ and to find them we must solve the secular equations (IX) for the b_{ns} .

Example

For a third degree polynomial ($n=3$) the secular equations are:

$$S_0 b_{30} + S_2 b_{32} = F_0$$

$$S_2 b_{30} + S_4 b_{32} = F_2$$

$$S_2 b_{31} + S_4 b_{33} = F_1$$

$$S_4 b_{31} + S_6 b_{33} = F_3$$

Solving for the coefficients b we have

$$b_{30} = \frac{F_0 S_4 - F_2 S_2}{S_0 S_4 - S_2^2}$$

$$b_{32} = \frac{F_2 S_0 - F_0 S_2}{S_0 S_4 - S_2^2}$$

$$b_{31} = \frac{F_1 S_6 - F_3 S_4}{S_2 S_6 - S_4^2}$$

$$b_{33} = \frac{F_3 S_2 - F_1 S_4}{S_2 S_6 - S_4^2}$$

If $m = 3$; $2m + 1 = 7$. Then from Eqs. (VIIIb) and (VIIIc)

$$S_0 = 7, \quad S_2 = 28, \quad S_4 = 196, \quad S_6 = 1588$$

$$F_0 = Y_{-3} + Y_{-2} + Y_{-1} + Y_0 + Y_1 + Y_2 + Y_3$$

$$F_1 = -3Y_{-3} - 2Y_{-2} - Y_{-1} + Y_1 + 2Y_2 + 3Y_3$$

$$F_2 = 9Y_{-3} + 4Y_{-2} + Y_{-1} + Y_1 + 4Y_2 + 9Y_3$$

$$F_3 = 27Y_{-3} - 8Y_{-2} - Y_{-1} + Y_1 + 8Y_2 + 27Y_3$$

$$b_{30} = \frac{1}{588} [196F_0 - 28F_2]$$

$$= \frac{1}{588} [-56Y_{-3} + 84Y_{-2} + 168Y_{-1} + 196Y_0 + 168Y_1 + 84Y_2 - 56Y_3]$$

$$b_{32} = \frac{1}{588} [7F_2 - 28F_0]$$

$$= \frac{1}{588} [35Y_{-3} - 21Y_{-1} - 28Y_0 - 21Y_1 + 35Y_3]$$

$$b_{31} = \frac{1}{6048} [1588F_1 - 196F_3] = \frac{1}{1512} [397F_1 - 49F_3]$$

$$b_{31} = \frac{1}{1512} [132Y_{-3} - 402Y_{-2} - 348Y_{-1} + 348Y_1 + 402Y_2 - 132Y_3]$$

$$b_{33} = \frac{1}{6048} [28F_3 - 196F_1] = \frac{1}{1512} [7F_3 - 49F_1]$$

$$= \frac{1}{1512} [-42Y_{-3} + 42Y_{-2} + 42Y_{-1} - 42Y_1 - 42Y_2 + 42Y_3]$$

From Eq. (III) the value of the s^{th} derivative at the point $i = 0$ is given by
 a_{ns}

$$a_{30} = b_{30} = -0.09524Y_{-3} + 0.14286Y_{-2} + 0.28571Y_{-1} + 0.33333Y_0 + 0.28571Y_1 \\ + 0.14286Y_2 - 0.09524Y_3$$

$$a_{31} = b_{31} = 0.08730Y_{-3} - 0.26587Y_{-2} - 0.23016Y_{-1} + 0.0Y_0 + 0.23016Y_1 \\ + 0.26587Y_2 - 0.08730Y_3$$

$$a_{32} = 2b_{32} = 0.11905Y_{-3} - 0.07143Y_{-1} - 0.095238Y_0 - 0.07143Y_1 + 0.11905Y_3$$

$$a_{33} = 6b_{33} = -0.16667Y_{-3} + 0.16667Y_{-2} + 0.16667Y_{-1} - 0.16667Y_1 - 0.16667Y_2 \\ + 0.16667Y_3$$

The coefficients of the Y's are the convoluting factors which give the value of the s^{th} derivative at the point $i = 0$. They are given in tabular form in Appendix B, where $N = 3$, $M = 7$. As an example of the use of these factors take the Gaussian array $Y_i = 1000e^{-0.09(i-8)^2}$ in Table I.

The smoothed value at $i = 6$ is given by the derivative of zero order.

$$\begin{aligned} \bar{Y}_6 = & -0.095238Y_3 + 0.142857Y_4 + 0.285714Y_5 + 0.333333Y_6 + 0.285714Y_7 \\ & + 0.142857Y_8 - 0.095238Y_9 = 700 \end{aligned}$$

The value of the first derivative of Y_i at $i = 9$ is given by

$$\begin{aligned} Y'_9 = & 0.087302Y_6 - 0.265873Y_7 - 0.230159Y_8 + 0.230159Y_{10} + 0.265873Y_{11} \\ & - 0.087302Y_{12} = -153 \end{aligned}$$

Table I. Effect of a seven point cubic convolution on a Gaussian

i	Y _i		First Derivative		Second Derivative		Third Derivative	
	Actual	Calc ^a	Actual ^b	Calc	Actual ^b	Calc	Actual ^b	Calc
1	12							
2	39							
3	105							
4	237	253	171	174	80	48	-4	-16
5	445	462	240	237	50	14	-60	-51
6	698	700	251	240	-35	-42	-103	-70
7	914	894	165	153	-135	-97	-84	-51
8	1000	970	0	0	-180	-119	0	0
9	914	894	-165	-153	-135	-97	84	51
10	698	700	-251	-240	-35	-42	103	70
11	445	462	-240	-237	50	14	60	51
12	237	253	-171	-174	80	48	4	16
13	105							
14	39							
15	2							

^aThis is the smoothed value of Y_i.

^bThe actual values are calculated by differentiation of the function

$$Y = 1000e^{-0.09(i-8)^2}$$

Table I lists the value of the function Y_i , the smoothed value, and the first, second and third derivatives. They are plotted in Fig. 1. The derivatives are useful in computerized peak finding routines (1, 2, 3).

A computer program has been written to calculate these convolution factors for any degree of polynomial and taking any number of points. Appendix A contains a listing of the program and explanation as to its use. Some tables of the more commonly used convolution factors are given in Appendix B.

Repeated Convolution

We may wish to perform two or more convolutions in succession. For example, a smoothing using p points with a polynomial of degree n followed by a differentiation using q points and a polynomial of degree m .

For the first convolution we have

$$\left(\frac{d^{s_1} f_i}{d i^{s_1}} \right)_{i=0} = S_1! b_{ns_1} = a_{ns_1} = \sum_{i=-p}^{+p} C_{is_1}^n Y_i$$

to produce the new array $Y_i^{(s_1)}$.

For the second convolution

$$\begin{aligned} \left[\frac{d^{s_2}}{d j^{s_2}} \left(\frac{d^{s_1} f_i}{d i^{s_1}} \right)_{i=0} \right]_{j=0} &= \sum_{j=-q}^{+q} C_{js_2}^m Y_j^{(s_1)} \\ &= \sum_{j=-q}^{+q} C_{js_2}^m \sum_{i=-p}^{+p} C_{is_1}^n Y_{i+j} \end{aligned}$$

$$= \sum_{j=-q}^{+q} \sum_{i=-p}^{+p} c_{js_2}^m c_{is_1}^n y_{i+j} \quad (XI)$$

If we set $h = i+j$ then we have

$$a_{ns_1, ms_2} = \sum_{h=-(p+q)}^{p+q} d_h y_h \quad \text{where ,}$$

$$d_h = \sum_j \sum_i c_{js_2}^m c_{is_1}^n, \quad (XII)$$

where the sum is over all values of i and j such that $i + j = h$ and $-q \leq j \leq q$, $-p \leq i \leq p$. As an example take the case where we have a five point quadratic smooth, followed by a seven point convolution to obtain the quadratic first derivative. Then $p = 2$, and $q = 3$, and we have

$c_{-2} = -0.08571$	$c'_{-3} = -0.10714$
$c_{-1} = 0.34286$	$c'_{-2} = -0.07143$
$c_0 = 0.48571$	$c'_{-1} = -0.03571$
$c_1 = 0.34286$	$c'_0 = 0$
$c_2 = -0.08571$	$c'_1 = 0.03571$
	$c'_2 = 0.07143$
	$c'_3 = 0.10714$
$d_{-5} = c_{-2} c'_{-3} = + 0.009183$	
$d_{-4} = c_{-2} c'_{-2} + c_{-1} c'_{-3} = -0.03061$	

$$d_{-3} = c_{-2} c'_{-1} + c_{-1} c'_{-2} + c_0 c'_{-3} = -0.07347$$

$$d_{-2} = c_{-2} c'_0 + c_{-1} c'_{-1} + c_0 c'_{-2} + c_1 c'_{-3} = -0.08367$$

$$d_{-1} = c_{-2} c'_1 + c_{-1} c'_0 + c_0 c'_{-1} + c_1 c'_{-2} + c_2 c'_{-3} = -0.03571$$

$$d_0 = c_{-2} c'_2 + c_{-1} c'_1 + c_0 c'_0 + c_1 c'_{-1} + c_2 c'_{-2} = 0$$

$$d_1 = c_{-2} c'_3 + c_{-1} c'_2 + c_0 c'_1 + c_1 c'_0 + c_2 c'_{-1} = 0.03571$$

$$d_2 = c_{-1} c'_3 + c_0 c'_2 + c_1 c'_1 + c_2 c'_0 = 0.08367$$

$$d_3 = c_0 c'_3 + c_1 c'_2 + c_2 c'_1 = 0.07347$$

$$d_4 = c_1 c'_3 + c_2 c'_2 = 0.03061$$

$$d_5 = c_2 c'_3 = -0.009183$$

Use of the Convolution Factors

1. Smoothing

Figure II shows an alpha spectrum of $^{239}\text{Am}^4$ and ^{241}Am taken with a 6 mm Au-Si detector and recorded in a 400 channel pulse-height analyzer. In IIa is plotted the raw data and IIb is the spectrum after performing a smoothing operation using a quadratic polynomial and smoothing over eleven points at a time.

The smoothing operation has not changed either the peak heights or the peak shapes but it has particularly accentuated the two α_0 's which in the raw data are barely discernable above the statistical fluctuations of the background. It is much easier to perform a stripping operation on the smoothed spectrum than on the raw data.

Figure IIIa shows an $^{239}\text{Am}^5$ alpha spectrum taken on a magnetic alpha particle spectograph. The statistics are quite poor. The improvement after smoothing (over eleven points using a quadratic) shown in IIIb is remarkable. The computer program which performs the smoothing using the convolution factors is listed in Appendix C.

2. Peak Location

Figure IVa shows a gamma spectrum of ^{241}Am and ^{57}Co . In Fig. IVb is plotted the first derivative of this spectrum formed from a five point quadratic convolution. The peak maximum is the point at which the first derivative is zero.

In Fig. VIa is plotted an alpha spectrum of ^{239}Am and ^{241}Am . The first and second derivatives of this spectrum are shown in Vb and Vc. The first derivative is more useful than the second derivative for determining peak locations.

Appendix A

The computer program for calculating convolution factors

A complete listing of the program is given below.

For each set of factors one data card is required giving the values of POWER and NPOINT in a 2I10 format. POWER is the degree of the polynomial (i.e. 2 for a quadratic) and NPOINT is the total number of points ($2m + 1$) over which the convolution is to be performed. Reading is terminated by a blank card.

In accordance with the dimensions of the matrices

POWER $\leq$ 20 and NPOINT $\leq$ 30 .

The program runs in a field length of 51000.

The subroutine MATIV was written by Burton S. Garbow of the Argonne National Laboratory.

```

PROGRAM CONVOL (INPUT,OUTPUT,TAPE2=INPUT,TAPE3=OUTPUT)
C
C
C
CONVOL CALCULATES SETS OF COAVOLUTING FACTORS FOR SMOOTHING AND
CALCULATING DERIVATIVES BY A LEAST SQUARES PROCEDURE
C
C
000043 DIMENSION S(30),FO(30),F(20,30),A(20,30),R(20,30),C(20,30),D(20,30)
1)
CC0043 DIMENSION OARR(10,30)
000043 INTEGER PCNFR,POW,SUMS,SO,PC,S,F,C,D
C
C
C
POWER IS THE DEGREE OF THE FITTING POLYNOMIAL
NPPOINT IS THE NUMBER OF POINTS USED TO FIT THE POLYNOMIAL
NPCINT MUST BE ODD
C
C
000043 5 READ(2,1000) PCNFR,NPCINT
000052 DO 6 I = 1,NPCINT
000061 6 B(2,I) = 0
000063 IF(POWER.LE.0) CALL EXIT
000065 MPT = (NPCINT - 1)/2
000070 POW = 2 + POWER
000071 IPWR = POWER + 1
000072 SUMS = 0
000073 SO = NPCINT
000074 NM1 = NPCINT - 2
C
C
C
SET S(I) = 0 FOR ODD I
C
C
000075 DO 10 I = 1,NM1,2
000102 10 S(I) = 0
C
C
C
CALCULATE S(I) FOR EVEN I
C
C
000104 DO 30 I = 2,POW,2
000105 DO 20 J = 1,PP
000111 20 SUMS = SUMS + 2*(J**I)
000115 S(I) = SUMS
000117 30 SUMS = 0
C
C
C
SET UP F MATRIX
C
C
000122 DO 40 I = 1,NPCINT
000126 40 F(I) = 1
000130 DO 50 J = 1,PCNFR
000131 DO 50 I = 1,NPCINT
000140 50 F(J,I) = (-MPT + I - 1)**J
C
C
C
TEST TO SEE IF POWER EVEN OR ODD
C
C
CC0150 POW = POWER/2
CC0151 PW2 = .POW * 2
000153 IF(PW2.NE.POWER) GO TO 160
C
C
C
IF POWER IS EVEN DO THIS SECTION
SET UP MATRIX FOR EVEN DERIVATIVES
C
C
000155 MXXDIM = (POWER + 2)/2
000157 C(1,1) = SO
000161 DO 60 J = 2,MXXDIM
000167 60 C(1,J) = S((2*J) - 2)
000173 DO 70 I = 2,MXXDIM
000174 DO 70 J = 1,MXXDIM
000204 70 C(1,J) = S((2*I) + (2*J) - 4)
000212 DO 80 J = 1,NPOINT
000222 80 D(1,J) = FO(J)
000224 DO 90 I = 2,MXXDIM
000226 DO 90 J = 1,NPCINT
000241 90 D(1,J) = F((2*I - 2),J)
000246 DO 100 I = 1,MXXDIM
000247 DO 100 J = 1,MXXDIM
000256 100 A(1,J) = FLOAT(C(1,J))
000260 DO 110 I = 1,MXXDIM
000264 DO 110 J = 1,NPOINT
000273 110 D(I,J) = FLOAT(A(I,J))
000300 CALL MATINVA(MXXDIM,R,NPOINT,DETERM)
000303 WRITE(3,3001)
000316 WRITE(3,3002)
000321 WRITE(3,3003)
000324
C
C
C
MULTIPLY THE B MATRIX BY THE FACTORIAL OF THE DERIVATIVE NUMBER
C
C
000327 DO 115 I = 2,MXXDIM
000331 DO 115 J = 1,NPOINT
000332 KX = 2*I - 1
000334 115 B(1,J) = B(1,J) * FACTR(KX)
000346 DO 110 I = 1,MXXDIM
000347 IMZ = 2*I - 1
000351 DO 118 J = 1,NPOINT
000363 118 OARR(IMZ,J) = B(1,J)
C
C
C
SET UP MATRIX FOR ODD DERIVATIVES
C
C
000370 MXXDIM = POWER/2
000372 DO 120 I = 1,MXXDIM
000373 DO 120 J = 1,MXXDIM
000403 120 C(1,J) = S((2*I) + (2*J) - 2)
000411 DO 130 I = 1,MXXDIM
000412 DO 130 J = 1,NPCINT
000425 130 D(1,J) = F((2*I - 1),J)
000432 DO 140 I = 1,MXXDIM
000433 DO 140 J = 1,MXXDIM
000442 140 A(1,J) = FLOAT(C(1,J))
000447 DO 150 I = 1,MXXDIM
000450 DO 150 J = 1,NPOINT
000457 150 B(1,J) = FLOAT(A(1,J))
000464 CALL MATINVA(MXXDIM,R,NPOINT,DETERM)
C
C
C
MULTIPLY THE B MATRIX BY THE FACTORIAL OF THE DERIVATIVE NUMBER
C
C
000467 DO 155 I = 2,MXXDIM
000471 DO 155 J = 1,NPOINT
000472 KX = 2*I - 1
000474 155 B(1,J) = B(1,J) * FACTR(KX)
000506 DO 158 I = 1,MXXDIM
000507 IMZ = 2*I
000510 DO 158 J = 1,NPOINT
000522 158 OARR(IMZ,J) = B(1,J)
000527 JPWR = IPWR
000531 IF(JPWR.GT.7) JPWR = 7
000534 DO 159 N = 1,NPOINT
000536 MIN = (NPCINT - 1)/2
000540 NIT = -MIN - 1 + N
000542 159 WRITE(3,3004) NIT,OARR(J,N),J=1,JPWR)

```

```

000563      IF(IPWR.LE.7) GO TO 5
000565      WRITE(3,3000) POWER,APCINT,POWER,MPT
000600      WRITE(3,3001)
000603      WRITE(3,3005)
000606      WRITE(3,3003)
000611      DO 300 N = 1,NPOINT
000613      MIN = (NPOINT-1)/2
000615      NIT = -MIN - 1 + N
000617      300 WRITE(3,3004) NIT,(OARR(J,N),J=8,IPWR)
000640      GO TO 5

C
C      IF POWER IS ODD DO THIS SECTION
C      SET UP MATRIX FOR EVEN DERIVATIVES
C

000640      160 MXDIM = (PCWER + 1)/2
000643      C(1,1) = 50
000644      DO 170 J = 2,MXDIM
000655      170 C(1,J) = S(2*J - 2)
000657      DO 180 I = 2,MXDIM
000661      DO 180 J = 1,MXDIM
000672      180 C(I,J) = S(2*I + 2*J - 4)
000677      DO 190 J = 1,NPOINT
000707      190 D(I,J) = FC(J)
000711      DO 200 I = 2,MXDIM
000713      DO 200 J = 1,NPOINT
000726      200 D(I,J) = F((2*I - 2),J)
000733      DO 210 I = 1,MXDIM
000734      DO 210 J = 1,MXDIM
000743      210 A(I,J) = FLOAT(C(I,J))
000750      DO 220 I = 1,MXDIM
000751      DO 220 J = 1,APCINT
000760      220 B(I,J) = FLOAT(D(I,J))
000765      CALL MATINV(A,MXDIM,B,NPOINT,DETERM)
000770      WRITE(3,3000) POWER,APCINT,PCWER,MPT
001003      WRITE(3,3001)
001006      WRITE(3,3002)
001011      WRITE(3,3003)

C
C      MULTIPLY THE B MATRIX BY THE FACTORIAL OF THE DERIVATIVE NUMBER
C

001014      DO 225 I = 2,MXDIM
001016      DO 225 J = 1,NPOINT
001017      KX = 2*I - 2
001021      225 B(I,J) = B(I,J) * FACTCR(KX)
001033      DO 228 I = 1,MXDIM
001034      IMZ = 2*I - 1
001036      DO 228 J = 1,NPOINT
001050      228 OARR(IMZ,J) = B(I,J)

C
C      SET UP MATRIX FOR ODD DERIVATIVES
C

001055      MXDIM = (POWER + 1)/2
001057      DO 230 I = 1,MXDIM
001061      DO 230 J = 1,MXDIM
001072      230 C(I,J) = S(2*I + 2*J - 2)
001077      DO 240 I = 1,MXDIM
001100      DO 240 J = 1,NPOINT
001113      240 C(I,J) = F((2*I - 1),J)
001120      DO 250 I = 1,MXDIM
001121      DO 250 J = 1,MXDIM
001130      250 A(I,J) = FLOAT(C(I,J))
001135      DO 260 I = 1,MXDIM
001136      DO 260 J = 1,NPOINT
001145      260 B(I,J) = FLOAT(D(I,J))
001152      CALL MATINV(A,MXDIM,B,NPOINT,DETERM)

C
C      MULTIPLY THE B MATRIX BY THE FACTORIAL OF THE DERIVATIVE NUMBER
C

001155      DO 265 I = 2,MXDIM
001157      DO 265 J = 1,NPOINT
001160      KX = 2*I - 1
001162      265 B(I,J) = B(I,J) * FACTCR(KX)
001174      DO 268 I = 1,MXDIM
001175      IMZ = 2*I
001176      DO 268 J = 1,NPOINT
001210      268 OARR(IMZ,J) = B(I,J)
001215      JPWR = IPWR
001217      IF(JPWR.GT.7) JPWR = 7
001222      DO 269 N = 1,NPOINT
001224      MIN = (NPOINT - 1)/2
001226      NIT = -MIN - 1 + N
001230      269 WRITE(3,3004) NIT,(OARR(J,N),J=1,JPWR)
001251      IF(IPWR.LE.7) GO TO 5
001253      WRITE(3,3000) POWER,APCINT,POWER,MPT
001266      WRITE(3,3001)
001271      WRITE(3,3005)
001274      WRITE(3,3003)
001277      DO 400 N = 1,NPOINT
001301      MIN = (NPOINT-1)/2
001303      NIT = -MIN - 1 + N
001305      400 WRITE(3,3004) NIT,(OARR(J,N),J=8,IPWR)
001326      GO TO 5

C
001326      1000 FORMAT(21I0)
001326      3000 FORMAT(1H1,29X,*DEGREE*,13,* POLYNOMIAL*,2X,13,* POINTS*,5X,*N=*,1
13,5X,*M=*,13)
001326      3001 FORMAT(1H0,5X,6HPICNTS,45X,10HDERIVATIVE)
001326      3002 FORMAT(1H ,20X,1H0,15X,1H1,15X,1H2,15X,1H3,15X,1H4,15X,1H5,15X,1H6
1)
001326      3003 FORMAT(1H )
001326      3004 FORMAT(1H ,19,4X,7(F12.8,4X))
001326      3005 FORMAT(1H ,20X,1H7,15X,1H8,15X,1H9,14X,2H10,14X,2H11,14X,2H12,14X,
12H13)
001326      END

```

```

FUNCTION FACTOR(I)
C
C THIS FUNCTION COMPUTES THE FACTORIAL OF I
C
010111 IF(I.EQ.0 .OR. I.EQ.1) GO TO 30
010117 IPRCD = I
010120 1C I = I - 1
010122 IF(I.EQ.1) GO TO 20
010123 IPROD = IPROD * I
010125 GO TO 10
010125 20 FACTOR = IPRCD
010127 RETURN
010127 30 FACTOR = 1.
010131 RETURN
010131 END

FORTRAN IV SUBROUTINE PATINV(A,N,B,M,CETERM)
C
C MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS
C
010161 DIMENSION IPIVCT(20),A(20,30),B(20,1),INDEX(20,2),PIVCT(20)
010161 COMMON /LSP/ PIVOT,IPIVCT,INDEX
010161 EQUIVALENCE (IROW,JROW), (ICOL,JCOL), (AMAX, T, SWAP)
C
C INITIALIZATION
C
010161 10 DETERM=1.0
010162 15 DO 20 J=1,N
010167 20 IPIVCT(J)=0
010174 30 DO 550 I=1,M
C
C SEARCH FOR PIVOT ELEMENT
C
010176 40 AMAX=0.0
010177 45 DO 105 J=1,N
010201 50 IF (IPIVCT(J)-1) 60, 105, 60
010204 60 DO 100 K=1,N
010206 70 IF (IPIVCT(K)-1) 80, 100, 740
010211 80 IF (ABS(AMAX)-ABS(A(J,K))) 85, 100, 100
010220 85 IROW=J
010222 90 ICOL=K
010224 95 AMAX=A(J,K)
010230 100 CONTINUE
010233 105 CONTINUE
010236 IF(AMAX) 110,800,110
010237 110 IPIVCT(ICOL)=IPIVCT(ICOL)+1
C
C INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
010242 130 IF (IROW-ICOL) 140, 260, 140
010244 140 CETERM=-CETERM
010245 150 DO 200 L=1,N
010247 160 SWAP=A(IROW,L)
010253 170 A(IROW,L)=A(ICOL,L)
010261 200 A(ICOL,L)=SWAP
010265 205 IF(M) 260, 260, 210
010267 210 DO 250 L=1, M
010271 220 SWAP=B(IROW,L)
010275 230 B(IROW,L)=B(ICOL,L)
010303 250 A(ICOL,L)=SWAP
010307 260 INDEX(I,1)=IROW
010311 270 INDEX(I,2)=ICOL
010313 310 PIVOT(I)=A(ICOL,ICOL)
010320 320 DETERM=DETERM*PIVOT(I)
C
C DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
010322 330 A(ICOL,ICOL)=1.0
010326 340 DO 350 L=1,N
010342 350 A(ICOL,L)=A(ICOL,L)/PIVOT(I)
010347 355 IF(M) 380, 380, 360
010351 360 DO 370 L=1,M
010364 370 B(ICOL,L)=B(ICOL,L)/PIVOT(I)
C
C REDUCE NON-PIVCT ROWS
C
010371 380 DO 550 L=1,N
010373 390 IF(L-ICOL) 400, 550, 400
010375 400 T=A(L,ICOL)
010401 420 A(L,ICOL)=0.0
010405 430 DO 450 L=1,N
010417 450 A(L,L)=A(L,L)-A(ICOL,L)*T
010425 455 IF(M) 550, 550, 460
010427 460 DO 500 L=1,M
010440 500 B(L,L)=B(L,L)-B(ICOL,L)*T
010446 550 CONTINUE
C
C INTERCHANGE COLUMNS
C
010453 600 DO 710 I=1,N
010455 610 L=N+1-I
010460 620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
010463 630 JROW=INDEX(L,1)
010465 640 JCOL=INDEX(L,2)
010467 650 DO 705 K=1,N
010471 660 SWAP=A(K,JROW)
010475 670 A(K,JROW)=A(K,JCOL)
010504 700 A(K,JCOL)=SWAP
010511 705 CONTINUE
010513 710 CONTINUE
010516 740 RETURN
010517 800 DETERM = 0.
010520 RETURN
010520 END

```

F4020003

F4C20002

ANF40201

F4020004

F4C20007

F4020008

F4020009

F4020010

F4020011

F4020012

F4020013

F4020014

F4020015

F4020016

F4C20017

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F4020019

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F4020021

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F4020026

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F4020028

F402REV.

F4C20029

F4020030

F4020031

F4020032

F4020033

F4C20034

F4020035

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F4020039

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F4C20047

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F4C20050

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F4C20078

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F4C20080

F4020081

F4020082

F4C20083

F4020084

F402REV.

F402REV.

Appendix B

Convolution Factors

In this appendix are given the convolution factors calculated using fitting polynomials from first to sixth degree and smoothing up to 23 points.

PCINTS		DEGREE 1 PCLYACMIAL	3 PCINTS	N= 1	M= 1
	0	1	2	3	4
-1	.33333333	-.5000C000			
0	.33333333	C.			
1	.33333333	.5000C000			

POINTS		DEGREE 1 PCLYACMIAL	5 PCINTS	N= 1	M= 2
	0	1	2	3	4
-2	.20000000	-.20C00000			
-1	.20C00000	-.10CC0000			
0	.20000000	C.			
1	.20C00000	.10CC0000			
2	.20000000	.20CC0000			

POINTS		DEGREE 1 PCLYACMIAL	7 POINTS	N= 1	M= 3
	0	1	2	3	4
-3	.14285714	-.10714286			
-2	.14285714	-.07142857			
-1	.14285714	-.03571429			
0	.14285714	0.			
1	.14285714	.03571429			
2	.14285714	.07142857			
3	.14285714	.10714286			

POINTS		DEGREE 1 POLYNOMIAL	9 PCINTS	N= 1	M= 4
	0	1	2	3	4
-4	.11111111	-.06666667			
-3	.11111111	-.05C00000			
-2	.11111111	-.03333333			
-1	.11111111	-.01666667			
0	.11111111	0.			
1	.11111111	.01666667			
2	.11111111	.03333333			
3	.11111111	.05C00000			
4	.11111111	.06666667			

POINTS		DEGREE 1 POLYNOMIAL	11 POINTS	N= 1	M= 5
	0	1	2	3	4
-5	.09090909	-.04545455			
-4	.09090909	-.03636364			
-3	.09090909	-.02727273			
-2	.09090909	-.01818182			
-1	.09090909	-.00909091			
0	.09090909	0.			
1	.09090909	.00909091			
2	.09090909	.01818182			
3	.09090909	.02727273			
4	.09090909	.03636364			
5	.09090909	.04545455			

POINTS		DEGREE 1 PCLYACMIAL	13 POINTS	N= 1	M= 6
	0	1	2	3	4
-6	.07692308	-.03296703			
-5	.07692308	-.02747253			
-4	.07692308	-.02197802			
-3	.07692308	-.01648352			
-2	.07692308	-.01098901			
-1	.07692308	-.00549451			
0	.07692308	0.			
1	.07692308	.00549451			
2	.07692308	.01098901			
3	.07692308	.01648352			
4	.07692308	.02197802			
5	.07692308	.02747253			
6	.07692308	.03296703			

PCINTS		DEGREE 1 POLYNOMIAL	15 POINTS	N= 1	M= 7
	0	1	2	3	4
-7	.06666667	-.02500000			
-6	.06666667	-.02142857			
-5	.06666667	-.01785714			
-4	.06666667	-.01428571			
-3	.06666667	-.01071429			
-2	.06666667	-.00714286			
-1	.06666667	-.00357143			
0	.06666667	0.			
1	.06666667	.00357143			
2	.06666667	.00714286			
3	.06666667	.01071429			
4	.06666667	.01428571			
5	.06666667	.01785714			
6	.06666667	.02142857			
7	.06666667	.02500000			

		DEGREE 1 POLYNOMIAL	17 POINTS	N= 1	M= 8		
POINTS			DERIVATIVE				
	0	1	2	3	4	5	6
-8	.05882353	-.01566784					
-7	.05882353	-.01715686					
-6	.05882353	-.01470588					
-5	.05882353	-.01225490					
-4	.05882353	-.00980392					
-3	.05882353	-.00735294					
-2	.05882353	-.00490196					
-1	.05882353	-.00245098					
0	.05882353	C.					
1	.05882353	.00245098					
2	.05882353	.00490196					
3	.05882353	.00735294					
4	.05882353	.00980392					
5	.05882353	.01225490					
6	.05882353	.01470588					
7	.05882353	.01715686					
8	.05882353	.01566784					

		DEGREE 1 POLYNOMIAL	19 POINTS	N= 1	M= 9		
POINTS			DERIVATIVE				
	0	1	2	3	4	5	6
-9	.05263158	-.01578947					
-8	.05263158	-.01403509					
-7	.05263158	-.01226070					
-6	.05263158	-.01052632					
-5	.05263158	-.00877193					
-4	.05263158	-.00701754					
-3	.05263158	-.00526316					
-2	.05263158	-.00350877					
-1	.05263158	-.00175439					
0	.05263158	C.					
1	.05263158	.00175439					
2	.05263158	.00350877					
3	.05263158	.00526316					
4	.05263158	.00701754					
5	.05263158	.00877193					
6	.05263158	.01052632					
7	.05263158	.01226070					
8	.05263158	.01403509					
9	.05263158	.01578947					

		DEGREE 1 POLYNOMIAL	21 POINTS	N= 1	M= 10		
POINTS			DERIVATIVE				
	0	1	2	3	4	5	6
-10	.04761905	-.01298701					
-9	.04761905	-.01168831					
-8	.04761905	-.01038961					
-7	.04761905	-.00909091					
-6	.04761905	-.00779221					
-5	.04761905	-.00649351					
-4	.04761905	-.00519481					
-3	.04761905	-.00389610					
-2	.04761905	-.00259740					
-1	.04761905	-.00129870					
0	.04761905	C.					
1	.04761905	.00129870					
2	.04761905	.00259740					
3	.04761905	.00389610					
4	.04761905	.00519481					
5	.04761905	.00649351					
6	.04761905	.00779221					
7	.04761905	.00909091					
8	.04761905	.01038961					
9	.04761905	.01168831					
10	.04761905	.01298701					

		DEGREE 1 POLYNOMIAL	23 POINTS	N= 1	M= 11		
POINTS			DERIVATIVE				
	0	1	2	3	4	5	6
-11	.04347826	-.01086957					
-10	.04347826	-.00988142					
-9	.04347826	-.00889328					
-8	.04347826	-.00790514					
-7	.04347826	-.00691700					
-6	.04347826	-.00592885					
-5	.04347826	-.00494071					
-4	.04347826	-.00395257					
-3	.04347826	-.00296443					
-2	.04347826	-.00197628					
-1	.04347826	-.00098814					
0	.04347826	C.					
1	.04347826	.00098814					
2	.04347826	.00197628					
3	.04347826	.00296443					
4	.04347826	.00395257					
5	.04347826	.00494071					
6	.04347826	.00592885					
7	.04347826	.00691700					
8	.04347826	.00790514					
9	.04347826	.00889328					
10	.04347826	.00988142					
11	.04347826	.01086957					

		DEGREE 2 POLYNOMIAL		3 POINTS	N= 2	M= 1
POINTS				DERIVATIVE		
	0	1	2	3	4	5
-1	.00000000	-.50000000	1.00000000			
0	1.00000000	0.	-2.00000000			
1	.00000000	.50000000	1.00000000			

		DEGREE 2 POLYNOMIAL		5 POINTS	N= 2	M= 2
POINTS				DERIVATIVE		
	0	1	2	3	4	5
-2	-.08571429	-.20000000	.28571429			
-1	.34285714	-.10000000	-.14285714			
0	.48571429	0.	-.28571429			
1	.34285714	.10000000	-.14285714			
2	-.08571429	.20000000	.28571429			

		DEGREE 2 POLYNOMIAL		7 POINTS	N= 2	M= 3
POINTS				DERIVATIVE		
	0	1	2	3	4	5
-3	-.09523810	-.10714286	.11904762			
-2	.14285714	-.07142857	.00000000			
-1	.28571429	-.03571429	-.07142857			
0	.33333333	0.	-.09523810			
1	.28571429	.03571429	-.07142857			
2	.14285714	.07142857	.00000000			
3	-.09523810	.10714286	.11904762			

		DEGREE 2 POLYNOMIAL		9 POINTS	N= 2	M= 4
POINTS				DERIVATIVE		
	0	1	2	3	4	5
-4	-.09090909	-.06666667	.06060606			
-3	.06060606	-.05000000	.01515152			
-2	.16883117	-.03333333	-.01731602			
-1	.23376623	-.01666667	-.03679654			
0	.25541126	0.	-.04324004			
1	.23376623	.01666667	-.03679654			
2	.16883117	.03333333	-.01731602			
3	.06060606	.05000000	.01515152			
4	-.09090909	.06666667	.06060606			

		DEGREE 2 POLYNOMIAL		11 POINTS	N= 2	M= 5
POINTS				DERIVATIVE		
	0	1	2	3	4	5
-5	-.08391608	-.04545455	.03496503			
-4	.02097902	-.03436364	.01398601			
-3	.10256410	-.02727273	-.00233100			
-2	.16083916	-.01818182	-.01398601			
1	.19940420	-.00909091	-.02097902			
0	.20745921	0.	-.02331002			
1	.19940420	.00909091	-.02097902			
2	.16083916	.01818182	-.01398601			
3	.10256410	.02727273	-.00233100			
4	.02097902	.03436364	.01398601			
5	-.08391608	.04545455	.03496503			

		DEGREE 2 POLYNOMIAL		13 POINTS	N= 2	M= 6
POINTS				DERIVATIVE		
	0	1	2	3	4	5
-6	-.07692308	-.03296703	.02197802			
-5	0.	-.02747253	.01098901			
-4	.06293706	-.02197802	.00149800			
-3	.11188811	-.01648352	-.00499500			
-2	.14685315	-.01098901	-.00999001			
-1	.16783217	-.00549451	-.01298701			
0	.17432517	0.	-.01398601			
1	.16783217	.00549451	-.01298701			
2	.14685315	.01098901	-.00999001			
3	.11188811	.01648352	-.00499500			
4	.06293706	.02197802	.00149800			
5	0.	.02747253	.01098901			
6	-.07692308	.03296703	.02197802			

		DEGREE 2 POLYNOMIAL		15 POINTS	N= 2	M= 7
POINTS				DERIVATIVE		
	0	1	2	3	4	5
-7	-.07058824	-.02500000	.01470588			
-6	-.01176471	-.02142857	.00840336			
-5	.03800905	-.01785714	.00307046			
-4	.07873303	-.01428571	-.00129282			
-3	.11040724	-.01071429	-.00468649			
-2	.13303167	-.00714286	-.00711054			
-1	.14660633	-.00357143	-.00856496			
0	.15113122	0.	-.00904977			
1	.14660633	.00357143	-.00856496			
2	.13303167	.00714286	-.00711054			
3	.11040724	.01071429	-.00468649			
4	.07873303	.01428571	-.00129282			
5	.03800905	.01785714	.00307046			
6	-.01176471	.02142857	.00840336			
7	-.07058824	.02500000	.01470588			

DEGREE 2 POLYNOMIAL 17 POINTS N= 2 M= 8						
POINTS	DERIVATIVE					
	0	1	2	3	4	5
-8	-.06501548	-.01960784	.01031992			
-7	-.01857585	-.01715686	.00644995			
-6	.02167183	-.01470588	.00309598			
-5	.05572755	-.01225490	.00025800			
-4	.08359133	-.00980392	-.00206398			
-3	.10526316	-.00735294	-.00386997			
-2	.12074303	-.00490196	-.00515996			
-1	.13003096	-.00245098	-.00593395			
0	.13312493	0.	-.00619195			
1	.13003096	.00245098	-.00593395			
2	.12074303	.00490196	-.00515996			
3	.10526316	.00735294	-.00386997			
4	.08359133	.00980392	-.00206398			
5	.05572755	.01225490	.00025800			
6	.02167183	.01470588	.00309598			
7	-.01857585	.01715686	.00644995			
8	-.06501548	.01960784	.01031992			

DEGREE 2 POLYNOMIAL 19 POINTS N= 2 M= 9						
POINTS	DERIVATIVE					
	0	1	2	3	4	5
-9	-.06015038	-.01578947	.00751380			
-8	-.02255639	-.01403509	.00501253			
-7	.01061477	-.01228070	.00280112			
-6	.03936311	-.01052632	.00088456			
-5	.06368863	-.00877193	-.00073714			
-4	.08359133	-.00701754	-.00206398			
-3	.09907121	-.00526316	-.00309598			
-2	.11012826	-.00350877	-.00383311			
-1	.11676249	-.00175439	-.00427539			
0	.11897391	0.	-.00442282			
1	.11676249	.00175439	-.00427539			
2	.11012826	.00350877	-.00383311			
3	.09907121	.00526316	-.00309598			
4	.08359133	.00701754	-.00206398			
5	.06368863	.00877193	-.00073714			
6	.03936311	.01052632	.00088456			
7	.01061477	.01228070	.00280112			
8	-.02255639	.01403509	.00501253			
9	-.06015038	.01578947	.00751380			

DEGREE 2 POLYNOMIAL 21 POINTS N= 2 M= 10						
POINTS	DERIVATIVE					
	0	1	2	3	4	5
-10	-.05599062	-.01298701	.00564653			
-9	-.02484472	-.01168831	.00395257			
-8	.00294214	-.01038961	.00243692			
-7	.02745995	-.00909091	.00109959			
-6	.04870873	-.00779221	-.00005944			
-5	.06668846	-.00649351	-.00104015			
-4	.08139915	-.00519481	-.00184255			
-3	.09284080	-.00389610	-.00246664			
-2	.10101340	-.00259740	-.00291242			
-1	.10591697	-.00129870	-.00317989			
0	.10755149	0.	-.00326904			
1	.10591697	.00129870	-.00317989			
2	.10101340	.00259740	-.00291242			
3	.09284080	.00389610	-.00246664			
4	.08139915	.00519481	-.00184255			
5	.06668846	.00649351	-.00104015			
6	.04870873	.00779221	-.00005944			
7	.02745995	.00909091	.00109959			
8	.00294214	.01038961	.00243692			
9	-.02484472	.01168831	.00395257			
10	-.05599062	.01298701	.00564653			

DEGREE 2 POLYNOMIAL 23 POINTS N= 2 M= 11						
POINTS	DERIVATIVE					
	0	1	2	3	4	5
-11	-.05217391	-.01086957	.00434783			
-10	-.02608696	-.00998142	.00316206			
-9	-.00248447	-.00889328	.00208922			
-8	.01863354	-.00790514	.00112931			
-7	.03726708	-.00691700	.00020233			
-6	.05341615	-.00592885	-.00045172			
-5	.06708075	-.00494071	-.00107284			
-4	.07826087	-.00395257	-.00158103			
-3	.08695652	-.00296443	-.00197628			
-2	.09316770	-.00197628	-.00225861			
-1	.09689441	-.00098814	-.00242801			
0	.09813665	0.	-.00248447			
1	.09689441	.00098814	-.00242801			
2	.09316770	.00197628	-.00225861			
3	.08695652	.00296443	-.00197628			
4	.07826087	.00395257	-.00158103			
5	.06708075	.00494071	-.00107284			
6	.05341615	.00592885	-.00045172			
7	.03726708	.00691700	.00020233			
8	.01863354	.00790514	.00112931			
9	-.00248447	.00889328	.00208922			
10	-.02608696	.00998142	.00316206			
11	-.05217391	.01086957	.00434783			

		DEGREE 3 POLYNOMIAL		3 POINTS	N= 3	M= 1
POINTS				DERIVATIVE		
	0	1	2	3	4	5
-1	.00000000	-.50000000	1.00000000	0.		
0	1.00000000	0.	-2.00000000	0.		
1	.00000000	.50000000	1.00000000	0.		

		DEGREE 3 POLYNOMIAL		5 POINTS	N= 3	M= 2
POINTS				DERIVATIVE		
	0	1	2	3	4	5
-2	-.08571429	.08333333	.28571429	-.50000000		
-1	.34285714	-.66666667	-.14285714	1.00000000		
0	.48571429	0.	-.28571429	0.		
1	.34285714	.66666667	-.14285714	-1.00000000		
2	-.08571429	-.08333333	.28571429	.50000000		

		DEGREE 3 POLYNOMIAL		7 POINTS	N= 3	M= 3
POINTS				DERIVATIVE		
	0	1	2	3	4	5
-3	-.09523810	.08730159	.11904762	-.16666667		
-2	.14285714	-.26587302	.00000000	.16666667		
-1	.28571429	-.23015873	-.07142857	.16666667		
0	.33333333	0.	-.09523810	0.		
1	.28571429	.23015873	-.07142857	-.16666667		
2	.14285714	.26587302	.00000000	-.16666667		
3	-.09523810	-.08730159	.11904762	.16666667		

		DEGREE 3 POLYNOMIAL		9 POINTS	N= 3	M= 4
POINTS				DERIVATIVE		
	0	1	2	3	4	5
-4	-.09090909	.07239057	.06060606	-.07070707		
-3	.00000000	-.11982862	.01515152	.03535354		
-2	.16883117	-.16245791	-.01731602	.06565657		
-1	.23376623	-.10606061	-.03679654	.04545455		
0	.25541126	0.	-.04329004	0.		
1	.23376623	.10606061	-.03679654	-.04545455		
2	.16883117	.16245791	-.01731602	.06565657		
3	.06060606	.11982862	.01515152	-.03535354		
4	-.09090909	-.07239057	.06060606	.07070707		

		DEGREE 3 POLYNOMIAL		11 POINTS	N= 3	M= 5
POINTS				DERIVATIVE		
	0	1	2	3	4	5
-5	-.08391608	.05827506	.03496503	-.03496503		
-4	.02097902	-.05710956	.01398601	.00699301		
-3	.10256410	-.10334110	-.00233100	.02564103		
-2	.16083916	-.09770785	-.01398601	.02680653		
-1	.19580420	-.05749806	-.02097902	.01631702		
0	.20745921	0.	-.02331002	0.		
1	.19580420	.05749806	-.02097902	-.01631702		
2	.16083916	.09770785	-.01398601	.02680653		
3	.10256410	.10334110	-.00233100	.02564103		
4	.02097902	.05710956	.01398601	-.00699301		
5	-.08391608	-.05827506	.03496503	.03496503		

		DEGREE 3 POLYNOMIAL		13 POINTS	N= 3	M= 6
POINTS				DERIVATIVE		
	0	1	2	3	4	5
-6	-.07692308	.04716117	.02197802	-.01923077		
-5	0.	-.02747253	.01098901	-.00000000		
-4	.06293706	-.06568432	.00199800	.01048951		
-3	.11188811	-.07475857	-.00499500	.01398601		
-2	.14685315	-.06197969	-.00999001	.01223776		
-1	.16783217	-.03463203	-.01298701	.00699301		
0	.17482517	0.	-.01398601	0.		
1	.16783217	.03463203	-.01298701	-.00699301		
2	.14685315	.06197969	-.00999001	.01223776		
3	.11188811	.07475857	-.00499500	.01398601		
4	.06293706	.06568432	.00199800	-.01048951		
5	0.	.02747253	.01098901	.00000000		
6	-.07692308	-.04716117	.02197802	.01923077		

		DEGREE 3 POLYNOMIAL		15 POINTS	N= 3	M= 7
POINTS				DERIVATIVE		
	0	1	2	3	4	5
-7	-.07058824	.03867102	.01470588	-.01143791		
-6	-.01176471	-.01233271	.00840336	-.00163399		
-5	.03800905	-.04234600	.00307046	.00439920		
-4	.07873303	-.05486725	-.00129282	.00729010		
-3	.11040724	-.05339486	-.00468649	.00766717		
-2	.13303167	-.04142725	-.00711054	.00615887		
-1	.14660633	-.02246283	-.00856496	.00339367		
0	.15113122	0.	-.00904977	0.		
1	.14660633	.02246283	-.00856496	-.00339367		
2	.13303167	.04142725	-.00711054	.00615887		
3	.11040724	.05339486	-.00468649	.00766717		
4	.07873303	.05486725	-.00129282	.00729010		
5	.03800905	.04234600	.00307046	-.00439920		
6	-.01176471	.01233271	.00840336	.00163399		
7	-.07058824	-.03867102	.01470588	.01143791		

DEGREE 3 POLYNOMIAL 17 POINTS N= 3 M= 8						
POINTS	0	1	2	3	4	5
-8	-.06501548	.03216374	.01031992	-.00722394		
-7	-.01857585	-.00421397	.00644995	-.00180599		
-6	.02167183	-.02764878	.00309598	.00180599		
-5	.05572755	-.03998968	.00025800	.00386997		
-4	.08359133	-.04308566	-.00206398	.00464396		
-3	.10526316	-.03878569	-.00386997	.00438596		
-2	.12074303	-.02893877	-.00515996	.00335397		
-1	.13003096	-.01539388	-.00593395	.00180599		
0	.13312693	0.	-.00619195	0.		
1	.13003096	.01539388	-.00593395	-.00180599		
2	.12074303	.02893877	-.00515996	-.00335397		
3	.10526316	.03878569	-.00386997	-.00438596		
4	.08359133	.04308566	-.00206398	-.00464396		
5	.05572755	.03998968	.00025800	-.00386997		
6	.02167183	.02764878	.00309598	-.00180599		
7	-.01857585	.00421397	.00644995	.00180599		
8	-.06501548	-.03216374	.01031992	.00722394		

DEGREE 3 POLYNOMIAL 19 POINTS N= 3 M= 9						
POINTS	0	1	2	3	4	5
-9	-.06015038	.02711324	.00751880	-.00478469		
-8	-.02255639	.00026582	.00501253	-.00159490		
-7	.01061477	-.01816931	.00280112	.00065672		
-6	.03936311	-.02924368	.00088456	.00208744		
-5	.06368863	-.03400882	-.00073714	.00281452		
-4	.08359133	-.03351628	-.00206398	.00295525		
-3	.09907121	-.02881759	-.00309598	.00262689		
-2	.11012826	-.02096429	-.00383311	.00194671		
-1	.11676249	-.01100791	-.00427539	.00103199		
0	.11897391	0.	-.00442282	0.		
1	.11676249	.01100791	-.00427539	-.00103199		
2	.11012826	.02096429	-.00383311	-.00194671		
3	.09907121	.02881759	-.00309598	-.00262689		
4	.08359133	.03351628	-.00206398	-.00295525		
5	.06368863	.03400882	-.00073714	-.00281452		
6	.03936311	.02924368	.00088456	-.00208744		
7	.01061477	.01816931	.00280112	-.00065672		
8	-.02255639	-.00026582	.00501253	.00159490		
9	-.06015038	-.02711324	.00751880	.00478469		

DEGREE 3 POLYNOMIAL 21 POINTS N= 3 M= 10						
POINTS	0	1	2	3	4	5
-10	-.05590062	.02313508	.00564653	-.00329381		
-9	-.02484472	.00276052	.00395257	-.00131752		
-8	.00294214	-.01191054	.00243692	.00013869		
-7	.02745995	-.02151184	.00109959	.00113261		
-6	.04870873	-.02667709	-.00005944	.00172203		
-5	.06668846	-.02804002	-.00104015	.00196473		
-4	.08139915	-.02623434	-.00184255	.00191850		
-3	.09284080	-.02189378	-.00246664	.00164113		
-2	.10101340	-.01565205	-.00291242	.00119039		
-1	.10591697	-.00814289	-.00317989	.00062409		
0	.10755149	0.	-.00326404	0.		
1	.10591697	.00814289	-.00317989	-.00062409		
2	.10101340	.01565205	-.00291242	-.00119039		
3	.09284080	.02189378	-.00246664	-.00164113		
4	.08139915	.02623434	-.00184255	-.00191850		
5	.06668846	.02804002	-.00104015	-.00196473		
6	.04870873	.02667709	-.00005944	-.00172203		
7	.02745995	.02151184	.00109959	-.00113261		
8	.00294214	.01191054	.00243692	-.00013869		
9	-.02484472	-.00276052	.00395257	.00131752		
10	-.05590062	-.02313508	.00564653	.00329381		

DEGREE 3 POLYNOMIAL 23 POINTS N= 3 M= 11						
POINTS	0	1	2	3	4	5
-11	-.05217391	.01995541	.00434783	-.00234114		
-10	-.02608896	.00412993	.00316206	-.00106415		
-9	-.00248447	-.00769231	.00208922	-.00009121		
-8	.01841354	-.01591162	.00112931	.00060809		
-7	.03726708	-.02092835	.00024233	.00106415		
-6	.05341615	-.02314280	-.00045172	.00130739		
-5	.06708075	-.02295531	-.00107284	.00136820		
-4	.07826087	-.02076619	-.00158103	.00127698		
-3	.08645652	-.01697578	-.00197628	.00106415		
-2	.09316770	-.01198439	-.00225861	.00076011		
-1	.09689441	-.00619236	-.00242801	.00039526		
0	.09813665	0.	-.00248447	0.		
1	.09689441	.00619236	-.00242801	-.00039526		
2	.09316770	.01198439	-.00225861	-.00076011		
3	.08645652	.01697578	-.00197628	-.00106415		
4	.07826087	.02076619	-.00158103	-.00127698		
5	.06708075	.02295531	-.00107284	-.00136820		
6	.05341615	.02314280	-.00045172	-.00130739		
7	.03726708	.02092835	.00024233	-.00106415		
8	.01841354	.01591162	.00112931	-.00060809		
9	-.00248447	.00769231	.00208922	-.00009121		
10	-.02608896	-.00412993	.00316206	.00106415		
11	-.05217391	-.01995541	.00434783	.00234114		

		DEGREE 4 POLYNOMIAL		5 POINTS	N= 4	M= 2
POINTS		DERIVATIVE				
	0	1	2	3	4	5
-2	-.00000000	.08333333	-.08333333	-.50000000	1.00000000	
-1	.00000000	-.66666667	1.33333333	1.00000000	-.40000000	
0	1.00000000	0.	-.25000000	0.	6.00000000	
1	.00000000	.66666667	1.33333333	-1.00000000	-4.00000000	
2	-.00000000	-.08333333	-.08333333	.50000000	1.00000000	

		DEGREE 4 POLYNOMIAL		7 POINTS	N= 4	M= 3
POINTS		DERIVATIVE				
	0	1	2	3	4	5
-3	.02164502	.08730159	-.09848485	-.16666667	.27272727	
-2	-.12987013	-.26587302	.50757576	.16666667	-.63636364	
-1	.32467532	-.23015873	-.14393939	.16666667	.09090909	
0	.54709957	0.	-.53030303	0.	.54545455	
1	.32467532	.23015873	-.14393939	-.16666667	.09090909	
2	-.12987013	-.26587302	.50757576	-.16666667	-.63636364	
3	.02164502	.08730159	-.09848485	.16666667	.27272727	

		DEGREE 4 POLYNOMIAL		9 POINTS	N= 4	M= 4
POINTS		DERIVATIVE				
	0	1	2	3	4	5
-4	.03496503	.07239057	-.07342657	-.07070707	.09790210	
-3	-.12320513	-.11952862	.21620047	.03535354	-.14685315	
-2	.06493007	-.16245791	.08799534	.06565657	-.07692308	
-1	.31468531	-.10606061	-.12296037	.04545455	.06293706	
0	.41724942	0.	-.21561772	0.	.12587413	
1	.31468531	.10606061	-.12296037	-.04545455	.06293706	
2	.06493007	-.16245791	.08799534	-.06565657	-.07692308	
3	-.12320513	.11952862	.21620047	.03535354	-.14685315	
4	.03496503	.07239057	-.07342657	.07070707	.09790210	

		DEGREE 4 POLYNOMIAL		11 POINTS	N= 4	M= 5
POINTS		DERIVATIVE				
	0	1	2	3	4	5
-5	.04195804	.05827506	-.05244755	-.03496503	.04195804	
-4	-.10489510	-.05710956	.10139860	.00699301	-.04195804	
-3	-.02331002	-.10334110	.08508159	.02564103	-.04195804	
-2	.13986014	-.09770785	.06058275	.02640653	-.00699301	
-1	.27972028	-.05749806	-.07925408	.01631702	.02797203	
0	.33333333	0.	.11072261	0.	.04195804	
1	.27972028	.05749806	-.07925408	-.01631702	.02797203	
2	.13986014	-.09770785	.06058275	.02640653	-.00699301	
3	-.02331002	.10334110	.08508159	-.02564103	-.04195804	
4	-.10489510	.05710956	.10139860	.00699301	-.04195804	
5	.04195804	.05827506	-.05244755	.03496503	.04195804	

		DEGREE 4 POLYNOMIAL		13 POINTS	N= 4	M= 6
POINTS		DERIVATIVE				
	0	1	2	3	4	5
-6	.04524887	.04716117	-.03789593	-.01923077	.02036199	
-5	-.08144796	-.02747253	.05090498	-.00000000	-.01357466	
-4	-.05553270	-.06568432	.06005759	.01048951	-.01974496	
-3	.04524887	-.07478867	.02766351	.01398601	-.01110654	
-2	.16042781	-.06197969	-.01664267	.01223776	.00226244	
-1	.24681201	-.03463203	-.05169340	.00699301	.01316331	
0	.27848622	0.	-.06478815	0.	.01727684	
1	.24681201	.03463203	-.05169340	-.00699301	.01316331	
2	.16042781	-.06197969	-.01664267	.01223776	.00226244	
3	.04524887	.07478867	.02766351	-.01398601	-.01110654	
4	-.05553270	.06568432	.06005759	-.01048951	-.01974496	
5	-.08144796	.02747253	.05090498	.00000000	-.01357466	
6	.04524887	.04716117	-.03789593	.01923077	.02036199	

		DEGREE 4 POLYNOMIAL		15 POINTS	N= 4	M= 7
POINTS		DERIVATIVE				
	0	1	2	3	4	5
-7	.04643963	.03867102	-.02799278	-.01143791	.01083591	
-6	-.06191950	-.01233271	.02670279	-.00163399	-.00464396	
-5	-.06358657	-.04234600	.04013853	.00439920	-.00940700	
-4	-.09357228	-.05486725	.02873700	.00729010	-.00762086	
-3	.08129641	-.05339486	.00593485	.00766717	-.00269545	
-2	.16237632	-.04142725	-.01781719	.00615887	.00271710	
-1	.21920804	-.02246283	-.03505434	.00339367	.00672238	
0	.23951590	0.	-.04129771	0.	.00818377	
1	.21920804	.02246283	-.03505434	-.00339367	.00672238	
2	.16237632	-.04142725	-.01781719	-.00615887	.00271710	
3	.08129641	.05339486	.00593485	-.00766717	-.00269545	
4	-.09357228	.05486725	.02873700	-.00729010	-.00762086	
5	-.06358657	.04234600	.04013853	-.00439920	-.00940700	
6	-.06191950	.01233271	.02670279	.00163399	-.00464396	
7	.04643963	.03867102	-.02799278	.01143791	.01083591	

DEGREE 4 POLYNOMIAL 17 POINTS N= 4 M= 3						
POINTS	0	1	2	3	4	5
-8	.04643963	.03216374	-.02115583	-.00722394	.00619195	
-7	-.04643963	-.00421397	.01431889	-.00180599	-.00154799	
-6	-.06191950	-.02764878	.02670279	.00180599	-.00464396	
-5	-.02764378	-.03998968	.02386481	.00386997	-.00464396	
-4	.03215051	-.04308566	.01246328	.00464396	-.00285782	
-3	.09883306	-.03878569	-.00205406	.00438596	-.00035723	
-2	.15718028	-.02893877	-.01545011	.00335397	.00202429	
-1	.19647535	-.01539388	-.02469834	.00180599	.00369136	
0	.21028816	0.	-.02798285	0.	.00428673	
1	.19647535	.01539388	-.02469834	-.00180599	.00369136	
2	.15718028	.02893877	-.01545011	-.00335397	.00202429	
3	.09883306	.03878569	-.00205406	-.00438596	-.00035723	
4	.03215051	.04308566	.01246328	-.00464396	-.00285782	
5	-.02764378	.03998968	.02386481	-.00386997	-.00464396	
6	-.06191950	.02764878	.02670279	-.00180599	-.00464396	
7	-.04643963	.00421397	.01431889	.00180599	-.00154799	
8	.04643963	-.03216374	-.02115583	.00722394	.00619195	

DEGREE 4 POLYNOMIAL 19 POINTS N= 4 M= 9						
POINTS	0	1	2	3	4	5
-9	.04576659	.02711324	-.01633035	-.00478469	.00374454	
-8	-.03432494	.00026582	.00766244	-.00159490	-.00041606	
-7	-.05653520	-.01816931	.01792117	.00065672	-.00237399	
-6	-.03903621	-.02924368	.01853761	.00208744	-.00277169	
-5	.00242294	-.03400882	.01305796	.00281452	-.00216596	
-4	.05451609	-.03351628	.00448284	.00295525	-.00102791	
-3	.10634092	-.02881759	-.00473268	.00262689	.00025698	
-2	.14941446	-.02096429	-.01267912	.00194671	.00138891	
-1	.17768206	-.01100771	-.01799255	.00103199	.00215372	
0	.18750841	0.	-.01985462	0.	.00242294	
1	.17768206	.01100771	-.01799255	-.00103199	.00215372	
2	.14941446	.02096429	-.01267912	-.00194671	.00138891	
3	.10634092	.02881759	-.00473268	-.00262689	.00025698	
4	.05451609	.03351628	.00448284	-.00295525	-.00102791	
5	.00242294	.03400882	.01305796	-.00281452	-.00216596	
6	-.03903621	.02924368	.01853761	-.00208744	-.00277169	
7	-.05653520	.01816931	.01792117	-.00065672	-.00237399	
8	-.03432494	-.00026582	.00766244	.00159490	-.00041606	
9	.04576659	-.02711324	-.01633035	.00478469	.00374454	

DEGREE 4 POLYNOMIAL 21 POINTS N= 4 M= 10						
POINTS	0	1	2	3	4	5
-10	.04472050	.02313508	-.01284585	-.00329381	.00237154	
-9	-.02484472	.00276052	.00395257	-.00131752	.00000000	
-8	-.05001635	-.01191054	.01216975	.00013869	-.00124818	
-7	-.04315136	-.02151184	.01407669	.00113261	-.00166424	
-6	-.01515297	-.02667709	.01167721	.00172203	-.00150516	
-5	.02452935	-.02804002	.00670795	.00196473	-.00099365	
-4	.06784993	-.02623434	.00063837	.00191850	-.00031816	
-3	.10841682	-.02189378	-.00532924	.00164113	.00036711	
-2	.14099147	-.01565205	-.01025975	.00119039	.00094225	
-1	.16199065	-.00814289	-.01348524	.00062409	.00132160	
0	.16923254	0.	-.01460493	0.	.00145376	
1	.16199065	.00814289	-.01348524	-.00062409	.00132160	
2	.14099147	.01565205	-.01025975	-.00119039	.00094225	
3	.10841682	.02189378	-.00532924	-.00164113	.00036711	
4	.06784993	.02623434	.00063837	-.00191850	-.00031816	
5	.02452935	.02804002	.00670795	-.00196473	-.00099365	
6	-.01515297	.02667709	.01167721	-.00172203	-.00150516	
7	-.04315136	.02151184	.01407669	-.00113261	-.00166424	
8	-.05001635	.01191054	.01216975	-.00013869	-.00124818	
9	-.02484472	.00276052	.00395257	.00131752	.00000000	
10	.04472050	-.02313508	-.01284585	.00329381	.00237154	

DEGREE 4 POLYNOMIAL 23 POINTS N= 4 M= 11						
POINTS	0	1	2	3	4	5
-11	.04347826	.01995541	-.01027499	-.00234114	.00156076	
-10	-.01739130	.00412993	.00183271	-.00106415	.00014189	
-9	-.04347826	-.00769231	.00835614	-.00009121	-.00066890	
-8	-.04347826	.01591162	.01062464	.00060809	-.00101348	
-7	-.02517162	-.02092835	.00982764	.00106415	-.00101881	
-6	.00457666	-.02314280	.00701461	.00130739	-.00079691	
-5	.03981693	-.02295531	.00309511	.00136820	-.00044486	
-4	.07551487	-.02076619	-.00116123	.00127698	-.00004481	
-3	.10755149	-.01697578	-.00512474	.00106415	.00033605	
-2	.13272311	-.01198439	-.00830564	.00076011	.00064543	
-1	.14874142	-.00619236	-.01035411	.00039526	.00084599	
0	.15423341	0.	-.01106026	0.	.00091533	
1	.14874142	.00619236	-.01035411	-.00039526	.00084599	
2	.13272311	.01198439	-.00830564	-.00076011	.00064543	
3	.10755149	.01697578	-.00512474	-.00106415	.00033605	
4	.07551487	.02076619	-.00116123	-.00127698	-.00004481	
5	.03981693	.02295531	.00309511	-.00136820	-.00044486	
6	.00457666	.02314280	.00701461	-.00130739	-.00079691	
7	-.02517162	.02092835	.00982764	-.00106415	-.00101881	
8	-.04347826	.01591162	.01062464	-.00060809	-.00101348	
9	-.04347826	.00769231	.00835614	-.00009121	-.00066890	
10	-.01739130	.00412993	.00183271	.00106415	.00014189	
11	.04347826	-.01995541	-.01027499	.00234114	.00156076	

DEGREE 5 POLYNOMIAL 5 POINTS N= 5 M= 2						
POINTS	DERIVATIVE					
	0	1	2	3	4	5
-2	-.00000000	.07109333	-.08333333	-.40812500	1.00000000	-.36750000
-1	-.00000000	-.63200000	1.33333333	.74000000	-.4.00000000	1.04000000
0	1.00000000	0.	-.2.50000000	0.	6.00000000	0.
1	.00000000	.63200000	1.33333333	-.74000000	-.4.00000000	-1.04000000
2	-.00000000	-.07109333	-.08333333	.40812500	1.00000000	.36750000

DEGREE 5 POLYNOMIAL 7 POINTS N= 5 M= 3						
POINTS	DERIVATIVE					
	0	1	2	3	4	5
-3	.02164502	-.01666667	-.09848485	.12500000	.27272727	-.50000000
-2	-.12987013	.15000000	.50757576	-1.00000000	-.63636364	2.00000000
-1	.32467532	-.75000000	-.14393939	1.62500000	.09090909	-2.50000000
0	.56709957	0.	-.53030303	0.	.54545455	0.
1	.32467532	.75000000	-.14393939	-1.62500000	.09090909	2.50000000
2	-.12987013	-.15000000	.50757576	1.00000000	-.63636364	-2.00000000
3	.02164502	.01666667	-.09848485	-.12500000	.27272727	.50000000

DEGREE 5 POLYNOMIAL 9 POINTS N= 5 M= 4						
POINTS	DERIVATIVE					
	0	1	2	3	4	5
-4	.03496503	-.02960373	-.07342657	.08741259	.09790210	-.15384615
-3	-.12820513	.16095571	.21620047	-.39947552	-.14685315	.42307692
-2	.06700007	.26445221	.08799334	.22377022	-.07892308	-.15384615
-1	.31468531	-.33554779	-.12296037	.40122378	.06293706	-.34615385
0	.41724942	0.	-.21561772	0.	.12587413	0.
1	.31468531	.33554779	-.12296037	-.40122378	.06293706	.34615385
2	.067993007	.26445221	.08799334	-.22377622	-.07892308	.15384615
3	-.12820513	-.16095571	.21620047	.39947552	-.14685315	-.42307692
4	.03496503	.02960373	-.07342657	-.08741259	.09790210	.15384615

DEGREE 5 POLYNOMIAL 11 POINTS N= 5 M= 5						
POINTS	DERIVATIVE					
	0	1	2	3	4	5
-5	.04195804	-.03339161	-.05244755	.05638112	.04195804	-.05769231
-4	-.10489510	.12622378	.10139060	-.17569930	-.04195804	.11538462
-3	-.02331002	-.07278555	.08508159	-.00480769	-.04195804	.01923077
-2	.13986014	-.21993007	.00058275	.14860140	-.00699301	-.07692308
-1	.27972028	-.17972028	-.07925408	.13811189	.02797203	-.07692308
0	.33711133	0.	-.11072261	0.	.04195804	0.
1	.27972028	.17972028	.07925408	-.13811189	.02797203	.07692308
2	.13986014	.21993007	.00058275	.14860140	-.00699301	.07692308
3	-.02331002	-.07278555	.08508159	.00480769	-.04195804	-.01923077
4	-.10489510	.12622378	.10139060	.17569930	-.04195804	-.11538462
5	.04195804	.03339161	-.05244755	-.05638112	.04195804	.05769231

DEGREE 5 POLYNOMIAL 13 POINTS N= 5 M= 6						
POINTS	DERIVATIVE					
	0	1	2	3	4	5
-6	.04524887	-.03306938	-.03789593	.03676471	.02036199	-.02498688
-5	-.08144796	.09287330	.05090498	-.08399321	-.01357466	.03733032
-4	-.05553270	-.00004114	.06005759	-.03532497	-.01974496	.02036199
-3	.04524887	-.11497385	.02766351	.04198375	-.01110654	-.01244344
-2	.16242781	-.15679761	-.01664267	.07841423	.00226244	-.02941176
-1	.24681231	-.10756890	-.05169340	.05789798	.01316331	-.02262443
0	.27848622	0.	-.06478815	0.	.01727684	0.
1	.24681231	.10756890	-.05169340	-.05789798	.01316331	.02262443
2	.16242781	.15679761	-.01664267	.07841423	.00226244	.02941176
3	.04524887	.11497385	.02766351	-.04198375	-.01110654	.01244344
4	-.05553270	.00004114	.06005759	.03532497	-.01974496	-.02036199
5	-.08144796	-.09287330	.05090498	.08399321	-.01357466	-.03733032
6	.04524887	.03306938	-.03789593	-.03676471	.02036199	.02498688

DEGREE 5 POLYNOMIAL 15 POINTS N= 5 M= 7						
POINTS	DERIVATIVE					
	0	1	2	3	4	5
-7	.046643963	-.03109907	-.02799278	.02465170	-.01083591	-.01131950
-6	-.06191950	.06740454	.02670279	-.04287926	-.00464396	.01362229
-5	-.00350657	.02509067	.04013053	-.03009724	-.00940700	.01165754
-4	-.00357228	-.05180043	.02873700	.00570374	-.00762086	.00052393
-3	.08129641	-.10573986	.00593485	.03474339	-.00269545	-.00894261
-2	.16237632	-.11112765	-.01781719	.04221243	.00271710	-.01190760
-1	.21920804	-.06951060	-.03505434	.02772982	.00672238	-.00903763
0	.23951590	0.	-.04129771	0.	.00818377	0.
1	.21920804	.06951060	-.03505434	-.02772982	.00672238	.00803763
2	.16237632	.11112765	-.01781719	-.04221243	.00271710	.01190760
3	.08129641	.10573986	.00593485	-.03474339	-.00269545	.00894261
4	-.00357228	.05180043	.02873700	.00570374	-.00762086	-.00052393
5	-.00350657	-.02509067	.04013053	.03009724	-.00940700	-.01165754
6	-.06191950	-.06740454	.02670279	-.04287926	-.00464396	-.01362229
7	.046643963	.03109907	-.02799278	-.02465170	.01083591	.01131950

DEGREE 5 POLYNOMIAL 17 POINTS N= 5 M= 8						
POINTS	DERIVATIVE					
	0	1	2	3	4	5
-8	.04643963	-.02858617	-.02115583	.01702786	.00619195	-.00619195
-7	-.04643963	.04844221	.01431889	-.02302632	-.00154799	.00541796
-6	-.06191950	.03310114	.02670279	-.02244582	-.00464396	.00619195
-5	-.02786378	-.01720846	.02386481	-.00522446	-.00464396	.00232198
-4	.03215051	-.06411447	.01246328	.01303882	-.00285782	-.00214337
-3	.09483306	-.08726879	-.00205406	.02374077	-.00035723	-.00494165
-2	.15718028	-.08034254	-.01545011	.02387473	.00202429	-.00523934
-1	.19647535	-.04752124	-.02469834	.01463146	.00369136	-.00327459
0	.21028816	0.	-.02798285	0.	.00428673	0.
1	.19647535	.04752124	-.02469834	-.01463146	.00369136	.00327459
2	.15718028	.08034254	-.01545011	-.02387473	.00202429	.00523934
3	.09483306	.08726879	-.00205406	-.02374077	-.00035723	.00494165
4	.03215051	.06411447	.01246328	-.01303882	-.00285782	.00214337
5	-.02786378	.01720846	.02386481	.00522446	-.00464396	-.00232198
6	-.06191950	.03310114	.02670279	.02244582	-.00464396	-.00619195
7	-.04643963	-.04844221	.01431889	.02302632	-.00154799	-.00541796
8	.04643963	.02858617	-.02115583	-.01702786	.00619195	.00619195

DEGREE 5 POLYNOMIAL 19 POINTS N= 5 M= 9						
POINTS	DERIVATIVE					
	0	1	2	3	4	5
-9	.04576659	-.02601415	-.01633035	.01209174	.00374454	-.00343249
-8	-.03432494	.03568407	.00766244	-.01284585	-.00041606	.00228833
-7	-.05653520	.03287465	.01792117	-.01555789	-.00237399	.00327789
-6	-.03903621	.00096601	.01853761	-.00750896	-.00277169	.00195181
-5	.00242294	-.03557139	.01305796	.00331089	-.00216596	-.00010096
-4	.05451609	-.06164254	.00448284	.01188983	-.00102791	-.00181720
-3	.10534002	-.06996527	-.00473268	.01569785	.00025698	-.00265850
-2	.14941446	-.05950768	-.01267912	.01419040	.00138891	-.00249024
-1	.17768206	-.03392561	-.01799255	.00831202	.00215372	-.00148068
0	.18750841	0.	-.01985462	0.	.00242294	0.
1	.17768206	.03392561	-.01799255	-.00831202	.00215372	.00148068
2	.14941446	.05950768	-.01267912	-.01419040	.00138891	.00249024
3	.10534002	.06996527	-.00473268	-.01569785	.00025698	.00265850
4	.05451609	.06164254	.00448284	-.01188983	-.00102791	.00181720
5	.00242294	.03557139	.01305796	-.00331089	-.00216596	.00010096
6	-.03903621	-.00096601	.01853761	.00750896	-.00277169	-.00195181
7	-.05653520	-.03287465	.01792117	.01555789	-.00237399	-.00327789
8	-.03432494	.03568407	.00766244	.01284585	-.00041606	-.00228833
9	.04576659	.02601415	-.01633035	-.01209174	.00374454	.00343249

DEGREE 5 POLYNOMIAL 21 POINTS N= 5 M= 10						
POINTS	DERIVATIVE					
	0	1	2	3	4	5
-10	.04472050	-.02358468	-.01284585	.00880207	.00237154	-.00200669
-9	-.02484472	.02612040	.00395257	-.00736546	.00000600	.00100334
-8	-.05001635	.02989134	.01216975	-.01068394	-.00174818	.00179546
-7	-.04315136	.01004449	.01407669	-.00703741	-.00166424	.00135540
-6	-.01515297	-.01717885	.01167721	-.00073709	-.00150516	.00040796
-5	.02452935	-.04085299	.00670795	.00528204	-.00094365	-.00055034
-4	.06789993	-.05460851	.00068337	.00926465	-.00031816	-.00121872
-3	.10841682	-.05587288	-.00532924	.01043841	.00036711	-.00145946
-2	.14099137	-.04511105	-.01025975	.00881741	.00094225	-.00126531
-1	.16199065	-.02506614	-.01348524	.00500557	.00132160	-.00072688
0	.16923254	0.	-.01460493	0.	.00145376	0.
1	.16199065	.02506614	-.01348524	-.00500557	.00132160	.00072688
2	.14099137	.04511105	-.01025975	-.00881741	.00094225	.00126531
3	.10841682	.05587288	-.00532924	-.01043841	.00036711	.00145946
4	.06789993	.05460851	.00068337	-.00926465	-.00031816	.00121872
5	.02452935	.04085299	.00670795	.00528204	-.00094365	.00055034
6	-.01515297	-.01717885	.01167721	.00073709	-.00150516	-.00040796
7	-.04315136	-.01004449	.01407669	.00703741	-.00166424	-.00135540
8	-.05001635	-.02989134	.01216975	.01068394	-.00174818	-.00179546
9	-.02484472	.02612040	.00395257	.00736546	.00000600	-.00100334
10	.04472050	.02358468	-.01284585	-.00880207	.00237154	.00200669

DEGREE 5 POLYNOMIAL 23 POINTS N= 5 M= 11						
POINTS	DERIVATIVE					
	0	1	2	3	4	5
-11	.04347826	-.02137124	-.01027499	.00654961	.00156076	-.00122631
-10	-.01739130	.01915780	.00183271	-.00429715	.00014189	.00044593
-9	-.04347826	.02612040	.00835614	-.00736546	-.00066890	.00100334
-8	-.04347826	.01414412	.01062464	-.00585791	-.00101348	.00049186
-7	-.02517162	-.00570274	.00982784	-.00221139	-.00101881	.00045180
-6	.00457666	-.02551562	.00701461	.00181786	-.00079691	-.00007041
-5	.03981693	-.04015826	.00309511	.00506913	-.00044486	-.000351047
-4	.07551487	-.04686723	-.00116123	.00689219	-.00004481	-.00077451
-3	.10755149	-.04485643	-.00512474	.00706222	.00033605	-.00082732
-2	.13272311	-.03492167	-.00830564	.00569469	.00066453	-.00068063
-1	.14874142	-.01904514	-.01035411	.00316032	.00084599	-.00038139
0	.15423341	0.	-.01106026	0.	.00091533	0.
1	.14874142	.01904514	-.01035411	-.00316032	.00084599	.00038139
2	.13272311	.03492167	-.00830564	-.00569469	.00066453	.00068063
3	.10755149	.04485643	-.00512474	-.00706222	.00033605	.00082732
4	.07551487	.04686723	-.00116123	-.00689219	-.00004481	.00077451
5	.03981693	.04015826	.00309511	.00506913	-.00044486	.000351047
6	.00457666	.02551562	.00701461	.00181786	-.00079691	.00007041
7	-.02517162	-.00570274	.00982784	.00221139	-.00101881	-.00045180
8	-.04347826	-.01414412	.01062464	.00585791	-.00101348	-.00049186
9	-.04347826	.02612040	.00835614	.00736546	-.00066890	-.00100334
10	-.01739130	.01915780	.00183271	.00429715	.00014189	.00044593
11	.04347826	.02137124	-.01027499	-.00654961	.00156076	.00122631

DEGREE 6 POLYNOMIAL 7 POINTS N= 6 M= 3							
POINTS	DERIVATIVE						
	0	1	2	3	4	5	6
-3	.00000000	-.01666667	.01111111	.12500000	-.16666667	-.50000000	1.00000000
-2	-.00000000	.15000000	-.15000000	-1.00000000	2.00000000	2.00000000	-6.00000000
-1	.00000000	-.75000000	1.50000000	1.62500000	-6.50000000	-2.50000000	15.00000000
0	1.00000000	0.	-2.72222222	0.	9.33333333	0.	-20.00000000
1	.00000000	.75000000	1.50000000	-1.62500000	-6.50000000	2.50000000	15.00000000
2	-.00000000	-.15000000	-1.50000000	1.00000000	2.00000000	-2.00000000	-6.00000000
3	.00000000	.01666667	.01111111	-.12500000	-.16666667	.50000000	1.00000000

DEGREE 6 POLYNOMIAL 9 POINTS N= 6 M= 4							
POINTS	DERIVATIVE						
	0	1	2	3	4	5	6
-4	-.00543701	-.02960373	.02852629	-.08741259	-.11623932	-.15384615	.26666667
-3	.04351204	.16095571	-.21709920	-.39947552	.76324786	.42307692	-1.13333333
-2	-.15229215	-.26445221	.64873608	.22377622	-1.25470085	-.15384615	1.46666667
-1	.30658630	-.33554779	-.09747216	.40122378	.00940171	-.34615385	.06666667
0	.61420462	0.	-.72538203	0.	1.19658120	0.	-1.33333333
1	.30658630	.33554779	-.09747216	-.40122378	.00940171	.34615385	.06666667
2	-.15229215	.26445221	.64873608	-.22377622	-1.25470085	.15384615	1.46666667
3	.04351204	-.16095571	-.21709920	.39947552	.76324786	-.42307692	-1.13333333
4	-.00543701	.02960373	.02852629	-.08741259	-.11623932	.15384615	.26666667

DEGREE 6 POLYNOMIAL 11 POINTS N= 6 M= 5							
POINTS	DERIVATIVE						
	0	1	2	3	4	5	6
-5	-.01151789	-.03339161	.03142054	.05638112	-.06900452	-.05769231	.08823529
-4	.06627789	.12622378	-.16697930	-.17569930	.31312217	.11538462	-.28235294
-3	-.12669683	-.07278555	.24722656	-.00480769	-.25040567	.01923077	1.70588824
-2	.01151789	-.21993007	.20186617	.14960140	-.27330317	-.07692308	.21176471
-1	.32250103	-.17972028	-.14634855	.13811189	.11674208	-.07692308	-.07058824
0	.47593583	0.	-.33437086	0.	.33785822	0.	-.23529412
1	.32250103	.17972028	-.14634855	-.13811189	.11674208	.07692308	-.07058824
2	.01151789	.21993007	.20186617	-.14960140	-.27330317	.07692308	.21176471
3	-.12669683	.07278555	.24722656	.00480769	-.25040567	-.01923077	1.70588824
4	.06627789	-.12622378	-.16697930	.17569930	.31312217	-.11538462	-.28235294
5	-.01151789	.03339161	.03142054	-.05638112	-.06900452	.05769231	.08823529

DEGREE 6 POLYNOMIAL 13 POINTS N= 6 M= 6							
POINTS	DERIVATIVE						
	0	1	2	3	4	5	6
-6	-.01567064	-.03306938	.02904594	.03676471	-.04104152	-.02488688	.03405573
-5	.07335080	.09287330	-.11644968	-.08399321	.13943411	.03733032	-.08513932
-4	-.07335080	-.00004114	.08440009	-.03532497	-.04207351	.02036199	.01738390
-3	.07577562	-.11487385	.15850443	.06198375	-.13112249	-.01244344	.06656347
-2	.06350830	-.15679761	.05029919	.07841423	.04941410	-.02941176	.03405573
-1	.30310247	-.10756890	-.11254965	.05789798	.06898468	-.02262443	-.03095975
0	.49106714	0.	-.18650063	0.	.12891958	0.	-.06191950
1	.30310247	.10756890	-.11254965	-.05789798	.06898468	.02262443	-.03095975
2	.06350830	.15679761	.05029919	-.07841423	-.04941410	.02941176	.03405573
3	-.07577562	.11487385	.15850443	.06198375	-.13112249	.01244344	.06656347
4	-.07335080	.00004114	.08440009	-.03532497	-.04207351	.02036199	.01238390
5	.07335080	-.09287330	.11644968	.08399321	.13943411	-.03733032	-.08513932
6	-.01567064	.03306938	.02904594	-.03676471	-.04104152	.02488688	.03405573

DEGREE 6 POLYNOMIAL 15 POINTS N= 6 M= 7							
POINTS	DERIVATIVE						
	0	1	2	3	4	5	6
-7	-.02063983	-.03109907	.02527577	.02465170	-.02516340	-.01191950	.01475748
-6	.07221942	.06740454	-.07983431	-.04287926	.06735466	.01362229	-.02951496
-5	-.03778677	.02589069	.01965062	-.03089724	.00443889	.01165754	-.00567595
-4	.08613162	-.05180043	.09429829	.00510374	-.09142771	.00052393	.01816305
-3	-.01111376	-.10573986	.07931880	.03474339	-.05228890	-.00894261	.02033024
-2	.13842197	-.11112765	.00080817	.04221243	-.00987007	-.01190760	.00515996
-1	.27784393	-.06951060	-.08161776	.02772982	.03819031	-.00803763	-.01289990
0	.33333333	0.	-.11579918	0.	.05853245	0.	-.02063983
1	.27784393	.06951060	-.08161776	-.02772982	.03819031	.00803763	-.01289990
2	.13842197	.11112765	.00080817	-.04221243	-.00987007	.01190760	.00515996
3	-.01111376	.10573986	.07931880	-.03474339	-.05228890	.00894261	.02033024
4	.08613162	-.05180043	.09429829	.00510374	-.09142771	.00052393	.01816305
5	-.03778677	.02589069	.01965062	-.03089724	.00443889	-.01165754	-.00567595
6	.07221942	-.06740454	-.07983431	.04287926	.06735466	.01362229	-.02951496
7	-.02063983	.03109907	.02527577	-.02465170	-.02516340	-.01191950	.01475748

DEGREE 6 POLYNOMIAL 17 POINTS N= 6 M= 8							
POINTS	DERIVATIVE						
	0	1	2	3	4	5	6
-8	-.02355633	-.02858617	.02150282	.01702786	-.01597344	-.00614195	.00699960
-7	.06730381	.04894221	-.05500142	-.02302632	.03447077	.00541796	-.01137434
-6	-.00942253	.03310114	-.00529120	-.02244582	.01198008	.00619195	-.00524970
-5	-.07151125	-.01720846	.05052647	-.00522446	-.01849733	.00232198	.00437475
-4	.05399836	-.06411447	.06496624	.01303882	-.03013830	-.00214337	.00861449
-3	.03624051	-.08726879	.03609262	.02374077	-.02017820	.00494165	.00625325
-2	.15583420	-.08034254	-.01462975	.02387473	.00159803	-.00523934	.00013461
-1	.25368359	-.04752124	-.05956358	.01463146	.02180730	-.00327459	-.00572082
0	.29105274	0.	-.07720437	0.	.02986218	0.	-.00807648
1	.25368359	.04752124	-.05956358	-.01463146	.02180730	.00327459	-.00572082
2	.15583420	.08034254	-.01462975	-.02387473	.00159803	.00523934	.00013461
3	.03624051	.08726879	.03609262	-.02374077	-.02017820	.00494165	.00625325
4	-.05399836	.06411447	.06496624	.01303882	-.03013830	-.00214337	.00861449
5	-.07151125	.01720846	.05052647	-.00522446	-.01849733	.00232198	.00437475
6	-.00942253	-.03310114	-.00529120	.02244582	.01198008	-.00619195	-.00524970
7	.06730381	-.04894221	-.05500142	.02302632	.03447077	-.00541796	-.01137434
8	-.02355633	.02858617	.02150282	-.01702786	-.01597344	.00614195	.00699960

DEGREE 6 POLYNOMIAL 19 POINTS N= 6 M= 9							
POINTS	DERIVATIVE						
	0	1	2	3	4	5	6
-7	-.02562929	-.02601415	.01816323	.01209174	-.01048055	-.00343249	.00356979
-8	.06086957	.03568407	-.03832901	-.01284585	.01855072	.00228833	-.00475973
-7	.00926100	.03287465	-.01386704	-.01555789	.01073541	.00329789	-.00328981
-6	-.05163548	.00096601	.02462471	-.00750896	-.00528200	.00195181	.00062996
-5	-.06407323	-.03557139	.04518434	.00331089	-.01541482	-.00010096	.00332481
-4	-.01827971	-.06164254	.03965277	.01198983	-.01553192	-.00181720	.00363979
-3	.06708844	-.06996527	.01423099	.01569785	-.00756360	-.00265850	.00196258
-2	.16093687	-.05950768	-.01824596	.01419040	.00368466	-.00249024	-.00057612
-1	.23217122	-.03392561	-.04431797	.00831202	.01301028	-.00148068	-.00272446
0	.25858124	0.	-.05419213	0.	.01658366	0.	-.00355364
1	.23217122	.03392561	-.04431797	-.00831202	.01301028	.00148068	-.00272446
2	.16093687	.05950768	-.01824596	-.01419040	.00368466	.00249024	-.00057612
3	.06708844	.06996527	.01423099	-.01569785	-.00756360	.00265850	.00196258
4	-.01827971	.06164254	.03965277	-.01198983	-.01553192	.00181720	.00363979
5	-.06407323	.03557139	.04518434	-.00331089	-.01541482	.00010096	.00332481
6	-.05163548	.00096601	.02462471	.00750896	-.00528200	-.00195181	.00062996
7	.00926100	-.03287465	-.01386704	.01555789	.01073541	-.00329789	-.00328981
8	.06086957	-.03568407	-.03832901	.01284585	.01855072	-.00228833	-.00475973
9	-.02562929	.02601415	.01816323	-.01209174	-.01048055	.00343249	.00356979

DEGREE 6 POLYNOMIAL 21 POINTS N= 6 M= 10							
POINTS	DERIVATIVE						
	0	1	2	3	4	5	6
-10	-.02705314	-.02358468	.01534134	.00880207	-.00709535	-.00200669	.00193237
-9	.05410628	.02612040	-.02705334	-.00736546	.01040258	.00100334	-.00212560
-8	.02100178	.02989134	-.01572073	-.01068394	.00810916	.00179546	-.00191203
-7	-.03295195	.01004449	.01007115	-.00703741	-.00032037	.00135540	-.00027460
-6	-.05955041	-.01717885	.02911312	-.00573709	-.00735496	.00040796	.00119532
-5	-.04296675	-.04085299	.03321525	.00528204	-.00988693	-.00055034	.00181720
-4	.01136986	-.05460851	.02283905	.00926465	-.00776656	-.00121872	.00152196
-3	.08618477	-.05587288	.00340181	.01043841	-.00256218	-.00145946	.00059856
-2	.16005743	-.04511105	-.01774725	.00881741	.00345433	-.00126531	-.00051330
-1	.21340991	-.02506614	-.03367878	.00500557	.00309660	-.00072688	-.00138436
0	.23278443	0.	-.03956324	0.	.00982735	0.	-.00171101
1	.21340991	.02506614	-.03367878	-.00500557	.00309660	.00072688	-.00138436
2	.16005743	.04511105	-.01774725	-.00881741	.00345433	.00126531	-.00051330
3	.08618477	.05587288	.00340181	-.01043841	-.00256218	.00145946	.00059856
4	.01136986	.05460851	.02283905	.00926465	-.00776656	.00121872	.00152196
5	-.04296675	.04085299	.03321525	-.00528204	-.00988693	.00055034	.00181720
6	-.05955041	.01717885	.02911312	.00073709	-.00735496	-.00040796	.00119532
7	-.03295195	-.01004449	.01007115	-.00703741	-.00032037	.00135540	-.00027460
8	.02100178	-.02989134	-.01572073	.01068394	.00810916	-.00179546	-.00191203
9	.05410628	-.02612040	-.02705334	.00736546	.01040258	-.00100334	-.00212560
10	-.02705314	.02358468	.01534134	-.00880207	-.00709535	.00200669	.00193237

DEGREE 6 POLYNOMIAL 23 POINTS N= 6 M= 11							
POINTS	DERIVATIVE						
	0	1	2	3	4	5	6
-11	-.02798601	-.02137124	.01300059	.00654961	-.00491934	-.00122631	.00109945
-10	.04757621	.01915780	-.01932691	-.00429715	.00603288	.00044593	-.00099950
-9	.02798601	.02612040	-.01491945	-.00736546	.00581120	.00100334	-.00109945
-8	-.01749125	.01414412	.00216079	-.00585791	.00134292	.00089186	-.00039980
-7	-.04944896	-.00570274	.01773465	-.00221139	-.00322018	.00045180	.00037350
-6	-.05081670	-.02551562	.02505597	.00181786	-.00581976	-.00007041	.00085221
-5	-.02022271	-.04015826	.02264975	.00506913	-.00588902	-.00051047	.00092369
-4	.03375866	-.04686723	.01243857	.00689219	-.00383109	-.00077451	.00064240
-3	.09747448	-.04485643	-.00184270	.00706222	-.00057769	-.00082732	.00015503
-2	.15617578	-.03492167	-.01594407	.00569469	.00277202	-.00068063	-.00036081
-1	.19711506	-.01904514	-.02610919	.00316032	.00523232	-.00038139	-.00074421
0	.21175883	0.	-.02979602	0.	.00613151	0.	-.00085801
1	.19711506	.01904514	-.02610919	-.00316032	.00523232	.00038139	-.00074421
2	.15617578	.03492167	-.01594407	-.00569469	.00277202	.00068063	-.00036081
3	.09747448	.04485643	-.00184270	-.00706222	-.00057769	.00082732	.00015503
4	.03375866	.04686723	.01243857	-.00689219	.00383109	.00077451	.00064240
5	-.02022271	.04015826	.02264975	.00506913	-.00588902	.00051047	.00092369
6	-.05081670	.02551562	.02505597	-.00181786	-.00581976	.00007041	.00085221
7	-.04944896	.00570274	.01773465	.00221139	-.00322018	.00045180	.00037350
8	-.01749125	-.01414412	.00216079	.00585791	.00134292	-.00089186	-.00039980
9	.02798601	-.02612040	-.01491945	.00736546	.00581120	-.00100334	-.00109945
10	.04757621	-.01915780	-.01932691	.00429715	.00603288	-.00044593	-.00099950
11	-.02798601	.02137124	.01300059	-.00654961	-.00491934	.00122631	.00109945

Appendix C

Computer program for smoothing and differentiation of data

The smoothing program, in the form of a subroutine, is reproduced below. It is a more generalized form of the program given by Savitzky and Golay.¹ NDATA is the input array of N points which is to be smoothed or differentiated. CONF is the array of NP convolution factors. These four variables are set by the main program for input to the subroutine. OUTDAT is the array of M smoothed (or differentiated) data points as calculated by the subroutine.

Because the dimension of the array IPOINT is set at 50, NP must be less than 50. If smoothing is desired over more than 50 points. The dimension of IPOINT can be increased.

```

SUBROUTINE SMOOTH(N, INDATA, M, OUTDAT, NP, CONF)
C
C THIS SUBROUTINE SMOOTHS DATA OVER NP POINTS
C
C N=NUMBER OF RAW DATA INPUT POINTS
C INDATA=ARRAY OF N INPUT DATA POINTS
C
C M=NUMBER OF SMOOTHED DATA POINTS=N-NP+1
C OUTDAT=ARRAY OF M SMOOTHED POINTS
C
C NP=NUMBER OF POINTS OVER WHICH CONVOLUTION IS PERFORMED
C CONF IS THE ARRAY OF CONVOLUTING FACTORS
C
002715 DIMENSION INDATA(N), OUTDAT(M), CONF(NP), IPCINT(50)
002715 INTEGER OUTDAT
002715 INIT = (NP-1)/2
002717 SUM = 0.
002720 M = N + 1 - NP
002721 DO 10 I = 2, NP
002727 J = I - 1
002730 10 IPCINT(I) = INDATA(J)
002733 DO 100 I = 1, M
002740 NPM1 = NP - 1
002741 J = I + NPM1
002743 DO 20 K = 1, NPM1
002751 KA = K + 1
002752 20 IPCINT(K) = IPCINT(KA)
002755 IPCINT(NP) = INDATA(J)
002763 DO 30 L = 1, NP
002771 30 SUM = SUM + CONF(L) * IPCINT(L)
003000 OUTDAT(I+INIT) = SUM
003003 SUM = 0.
003004 100 CONTINUE
003006 DO 120 I = 1, INIT
003012 120 OUTDAT(I) = 0
003014 NEXT = M + INIT + 1
003022 DO 140 I = NEXT, N
003026 140 OUTDAT(I) = 0
003030 RETURN
003030 END

```

FOOTNOTES AND REFERENCES

* Work performed under the auspices of the U. S. Atomic Energy Commission.

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3. V. Barnes, IEEE. Trans. Nucl. Sci. NS-15, No. 3, 437 (1968).
4. D. J. Gorman and F. Asaro, Lawrence Radiation Laboratory Annual Report, 1969 (unpublished).
5. D. J. Gorman and F. Asaro, unpublished data.

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