FINAL REPORT

RESEARCH IN METHODS OF NONLINEAR AND COMBINATORIAL PROGRAMMING

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In a published paper\(^1\), which reported on some of the research supported by this research contract, a generalization of the Hitchcock transportation problem and the "location-allocation" problem was presented. During the past year, a great deal of effort has been expended on thinking about and exploring some approaches to this problem. The results are presented in Report No. COO-1493-30, "The Transportation-Location Problem". Major results obtained include a proof of the fact that an optimal solution to the problem occurs at an extreme point of the convex set of feasible solutions despite the fact that the objective function is neither convex nor concave. The report contains a summary of the present state of attempts to deal with the problem. Further work to develop computationally effective algorithms is in progress.

Report No. COO-1493-31, "An Algorithm for Solving Integer Linear Programming Problems" represents some early work that was never published but has had impact on the work of several investigators over the past several years. It appears to be a fruitful approach to integer linear programming. The preface to the report discusses the reasons for issuing the report.

A major piece of research was concluded and is represented by the doctoral dissertation of Carl F. Meyer entitled "Strategic

Risk and Capital Investment. In this work, Prof. Meyer (now at the University of Missouri at St. Louis) has developed an optimality criterion for capital investment problems which involves the creation of a utility function that enables simultaneous optimization of both performance and risk. He presents detailed computational procedures for the solution of capital investment problems and techniques to construct these empirical utility response functions as well as applications, sample problems and computational details.

Another major area of research has been concentrated on methods of solving continuous constrained nonlinear programming problems. The report "Nonlinear Programming: the Global Use of the Lagrangian", (COO-1493-29) shows how the best local optimum can sometimes be recognized by looking at the associated Lagrangian. It also presents new classes of problems for which a local optimum must necessarily be the global optimum. The report on counterexamples (COO-1493-32) shows that two previously published global optimization "algorithms", (one by Ritter in Wharscheinlichkeitsthearie and the other by Tui in Pokl. Akad. Naul SSR) are not convergent. The report concerned with minimization involving economies of scale (COO-1493-33) shows how real global optimization problems can arise, and presents an accelerated method of solution.

Another area that has been investigated is that of "understanding" multidimensional data. The report on multidimensional piecewise polynomial curve fitting (COO-1493-34) presents a surprisingly simple approach which is calculationally feasible. This work has value, not only in the synthesis of mathematical
models, but also possibly in the development of global optimization algorithms for problems involving piecewise linear functions of several variables.

Work has also proceeded in several directions on those aspects of dynamic programming arising in stochastic inventory control. The major emphasis has been placed on obtaining practical solutions to problems whose optimal solutions are known to be complicated. Both theoretical and empirical analyses have been involved in the studies.

One direction has been toward the reduction of the dimension of the state space in multidimensional dynamic programming problems. Particular effort has been expended on a multiproduct inventory problem when one of the decision variables is constrained to lie on a specified curve. In certain cases then the problem was shown to reduce to one of essentially a single dimension and to be substantially easier to solve than the unconstrained problem. The results appear particularly useful when a linear constraint may be imposed. The report is in final preparation and is entitled "Optimal Policies for Proportionally Stocked Products", (COO-1493-36. It will be sent about one month from now.

Complementing this theoretical analysis has been the development of a computer program to study numerically the use of these policies when the constraint is linear and to compare them with those found by Wheeler and Witherspoon in a report sponsored by this contract last year. Interesting difficulties arose in even
formulating the constrained problem in discrete space, and these have led to further considerations of differences between discrete and continuous optimization. The Wheeler and Witherspoon examples are currently being run on this program.

Another direction has been toward the use of stationary policies in dynamic programming problems. Specifically, in the finite-horizon stochastic (s,S) inventory model with periodic review the parameters of the optimal policy generally vary with the length of the horizon even when each period's contribution to the objective function does not. Stationary policies, however, are easier to implement and it was found, generally easier to calculate. Work has been done relating optimal stationary policies to optimal non-stationary policies through the relations both have with optimal stationary policies for an infinite horizon. The preliminary work was presented at the 37th National ORSA meeting in Washington, D.C., April 1970.

Again, there has been a concomitant empirical effort through the development of a computer program. To date, however, only a few examples have been run, partly because of a need to strengthen the theoretical bounds on the policy parameters to make the running times faster.

A third direction has been toward the approximation of optimal policies when certain of the policy parameters may be assumed to be very large. This work is part of a doctoral dissertation now being written up and was first mentioned in last year's report. Certain conditions have been found under which the generally
complicated equations yielding optimal policies are well approximated by much simpler ones. Further conditions also yield essentially a reduction in the dimensionality of the state space.

A fourth direction has been toward obtaining exact optimal solutions more efficiently. A computer program has been developed which is more efficient than the Wheeler and Witherspoon program developed a year ago and far more general. The program handles up to five products easily, and a major advance comes from its ability to solve problems having piecewise linear ordering cost functions efficiently. The work was performed for a master's thesis.

The final direction of effort has been quite general and has related to the problem of formulating objective functions and constraints in optimization problems. A particular result included here (COO-1493-35) is a master's thesis studying by simulation the use of regression when the assumptions underlying the general linear hypothesis are violated in some fashion. The work showed the usual tests on the coefficients to be remarkably robust.