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Quasilinear Theory of Ion-Cyclotron Resonance Heating of Plasmas and Associated Longitudinal Cooling

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ABSTRACT

It is shown from quasilinear theory that an initially isotropic magnetized plasma will be forced into an anisotropic state in ion-cyclotron resonance heating. Strong heating of perpendicular ion temperature and strong cooling of longitudinal temperature should occur simultaneously. The maximum temperature ratio predicted by quasilinear theory is in exact agreement with that predicted from basic thermodynamic arguments by Busnardo-Neto, Dawson, Kamimura and Lin. Heating by fast hydromagnetic wave is also examined.

Very recently Busnardo-Neto, Dawson, Kamimura and Lin[1] have reported on some studies of the absorption of ion-cyclotron waves which involve computer simulation and theoretical analysis. As opposed to previous reports, they find that an initially isotropic plasma is forced into an anisotropic state by means of an applied external pump field near the ion-cyclotron frequency. Their studies showed that in addition to a strong heating of the ions in the perpendicular direction (perpendicular to the external magnetic field $\vec{B} = B\hat{i}_z$) there was a strong cooling of the parallel ion temperature. They used very basic thermodynamic arguments to predict the nonlinear saturation of the heating-cooling process and the simulation results confirmed their predictions. Their argument runs as follows: An ion-cyclotron wave of frequency ω and wave number k propagating along an external magnetic field resonates with particles of velocity v_R such that $kv_R = \omega - \omega_c$, where ω_c is the ion-cyclotron frequency. Since the frequency of these waves is always less than the ion-cyclotron frequency the resonant particles and the wave propagate in opposite directions. The resonant particles either continually gain or lose energy to the wave. Let us assume that the particle absorbs n quanta from the wave. Its perpendicular energy will increase by $n\hbar\omega_c$. The wave supplies only $n\hbar\omega$ of this energy; thus the remainder $n\hbar(\omega_c - \omega)$ must come from the parallel motion of the particle. That is, in absorbing energy dW from the wave the perpendicular energy increases by $dQ_\perp = dW\,\omega_c/\omega$, and the parallel energy decreases by $dQ_\parallel = dW(\omega_c-\omega)/\omega$. From the thermodynamic point of view one can regard the parallel and perpendicular motions as two independent systems with two temperatures $T_\parallel$ and $T_\perp$. Since the entropy S must increase or at best remain the same for a reversible process, one finds that $dS = - dQ_\parallel/T_\parallel + dQ_\perp/T_\perp \geq 0$. That is, $\omega_c/(\omega_c - \omega) \geq T_\perp/T_\parallel$. This is an upper limit on the temperature anisotropy that can be achieved. It is our aim in this paper to give a rigorous proof of these results from quasilinear plasma kinetic theory.[2,3]

According to the quasilinear theory (for example, see Eq. 52 of Ref. 2 and Eq. 11-33 of Ref. 3) one can show that the time rate of change of the particle distribution function due to emission and absorption of cyclotron waves of frequency ω and wave vector $\vec{k}$ in a box of volume L^3 is given by

$$\frac{\partial F(E_\perp, v_z)}{\partial t} = \sum_{\vec{k}} \frac{2\pi^2 q^2 \omega_c}{L^3 \mu\omega}$$

$$\times \left[(\omega_c \varepsilon_k/\omega)\{(1 + E_\perp P)[\delta(\omega - \omega_c - k_z v_z) P\, F(E_\perp, v_z)]\} + \{(1 + E_\perp P)[\delta(\omega - \omega_c - k_z v_z) F(E_\perp, v_z)]\}\right] , \quad (1)$$

where the linear differential operator $P = [(\partial/\partial E_\perp)+(k_z/\mu\omega_c)(\partial/\partial v_z)]$, q and μ are the charge and mass of the particle, respectively, $E_\perp = (\mu v_\perp^2/2)$ is the particle perpendicular energy, and ε_k represents the energy in the cyclotron wave of frequency ω and wave vector $\vec{k}$. It may be noted that Eq. (1) is a Fokker-Planck equation whose first and second terms represent a "diffusion" and "dynamical friction," respectively. The velocity space diffusion term is proportional to the energy ε_k of the cyclotron wave and is a consequence of a balance between induced or stimulated emission and absorption of the cyclotron waves. On the other hand, the dynamical friction term (i.e., the term which is independent of ε_k on the right hand side of Eq. 1) is a consequence of the spontaneous emission of the cyclotron waves. In the usual formulation of the classical quasi-linear plasma kinetic theory based on the Vlasov-Maxwell's equations it is extremely difficult to obtain the dynamical friction term (which is for example absent in Eq. 11-33 of Ref. 3). If the energy $\varepsilon_k >> \kappa T_\perp$ one can indeed neglect the dynamical friction term in Eq. (1). Here κ is the Boltzmann constant.

We now wish to calculate the time rate of change of the perpendicular and parallel ion temperatures $T_\perp$ and $T_\parallel$. It is relatively easy to see that in ion-cyclotron resonance heating the time rate of change of the average perpendicular and parallel particle energies $\langle E_\perp \rangle$ and $\langle E_\parallel \rangle$ are given by

$$\frac{\partial\langle E_\perp\rangle}{\partial t} = \int_{-\infty}^{\infty} dv_z \int_0^{\infty} dE_\perp E_\perp \frac{\partial F(E_\perp, v_z)}{\partial t} = \frac{\partial(\kappa T_\perp)}{\partial t} \qquad (2)$$

and

$$\frac{\partial\langle E_\parallel\rangle}{\partial t} = \int_{-\infty}^{\infty} dv_z \int_0^{\infty} dE_\perp (\mu v_z^2/2) \frac{\partial F(E_\perp, v_z)}{\partial t} = \frac{\partial(\kappa T_\parallel/2)}{\partial t} \quad , \qquad (3)$$

respectively. Here we have written $\langle E_\perp\rangle = \langle\mu v_\perp^2/2\rangle = \langle\mu(v_x^2 + v_y^2)/2\rangle = \kappa T_\perp$ and $\langle E_\parallel\rangle = \langle\mu v_z^2/2\rangle = \kappa T_\parallel/2$ since the perpendicular motion has two degrees of freedom while the parallel motion has only one degree of freedom and by equipartition theorem the average energy per degree of freedom is $\kappa T/2$. As is usually done in the classical quasilinear plasma kinetic theory, let us now assume that the ion-cyclotron wave energy $\varepsilon_k \gg \kappa T_\perp$, and consequently we can neglect the dynamical friction term in Eq. (1). Then on making use of Eq. (1), Eqs. (2) and (3) become

$$\frac{\partial(\kappa T_\perp)}{\partial t} = A\left[F_\parallel(v_z) - \left(\frac{\kappa T_\perp k_z}{\mu\omega_c}\right)\frac{\partial F_\parallel(v_z)}{\partial v_z}\right]_{v_z = V_R} \qquad (4)$$

and

$$\frac{\partial(\kappa T_\parallel/2)}{\partial t} = -\left(\frac{\omega_c - \omega}{\omega_c}\right) A\left[F_\parallel(v_z) - \left(\frac{\kappa T_\perp k_z}{\mu\omega_c}\right)\frac{\partial F_\parallel(v_z)}{\partial v_z}\right]_{v_z = V_R} \qquad (5)$$

respectively. Here

$$A = \sum_{\vec{k}} \left(\frac{2\pi^2 q^2 \omega_c^{\,2} \varepsilon_k}{L^3 \mu\omega^2}\right) ,$$

$F_\parallel(v_z) = \int_0^\infty dE_\perp F(E_\perp, v_z)$, and $V_R = (\omega - \omega_c)/k_z$. The somewhat lengthy and tedious algebra involved in obtaining Eqs. (4) and (5) only makes use of integrations by parts over $dE_\perp$ and over dv_z, and the constant terms resulting from the parts integrations vanish at $E_\perp = 0$ and $E_\perp = \infty$ and at $v_z = \pm\infty$.

Since the frequency ω of the ion-cyclotron waves is always less than the ion-cyclotron frequency ω_c it is clear from Eqs. (4) and (5) that, regardless of the nature of the particle distribution function $F(E_\perp, v_z)$, in ion-cyclotron resonance heating a strong heating of the perpendicular ion temperature $T_\perp$ must necessarily result in a strong cooling of the longitudinal ion temperature $T_\parallel$. However, the time rate of change of this heating-cooling process depends on the ion-cyclotron wave energy ε_k, the instantaneous value of $T_\perp/T_\parallel$ (as we shall see later), the ratio $(\omega_c - \omega)/\omega_c$, and the nature of the parallel velocity distribution function $F_\parallel(v_z)$ in the neighborhood of the particle-wave resonance velocity $v_z = (\omega - \omega_c)/k_z$. Further, it is seen from Eqs. (4) and (5) that at any instant of time regardless of the nature of the particle distribution function $(\partial\langle E_\perp\rangle/\partial t)/(\partial\langle E_\parallel\rangle/\partial t) = 2(\partial T_\perp/\partial t)/(\partial T_\parallel/\partial t) = -\omega_c/(\omega_c - \omega)$. As shown earlier in this paper from a quantum view point this result is a consequence of the conservation law of energy in the interaction between the photon (i.e., the quanta of ion cyclotron wave) and the particles. This result was assumed and used in the thermodynamic derivation of the upper limit on the temperature anisotropy that can be achieved in ion-cyclotron resonance heating.[1]

Let us now assume that $F_\parallel(v_z)$ is a Maxwell-Boltzmann distribution function. That is, $F_\parallel(v_z) = (\mu/2\pi\kappa T_\parallel)^{1/2}\exp(-\mu v_z^2/2\kappa T_\parallel)$. Then $\partial F_\parallel/\partial v_z = -(\mu v_z/\kappa T_\parallel)F_\parallel$. Thus, Eqs. (4) and (5) become

$$\partial(\kappa T_\perp)/\partial t = A[1 - (T_\perp/T_\parallel)(\omega_c - \omega)/\omega_c]F_\parallel(v_z = V_R) \,, \qquad (6)$$

and

$$\partial(\kappa T_\parallel/2)/\partial t = -\{(\omega_c - \omega)/\omega_c\}A[1 - (T_\perp/T_\parallel)(\omega_c - \omega)/\omega_c]F_\parallel(v_z=V_R) \,, \qquad (7)$$

respectively. Hence, at any instant of time the heating-cooling process can occur if and only if $\partial T_\perp/\partial t \geqslant 0$ and $\partial T_\parallel/\partial t \leq 0$. That is, if and only if $T_\perp/T_\parallel \leq \omega_c/(\omega_c - \omega)$. Thus we see that in ion-cyclotron resonance heating the upper limit on the temperature anisotropy that can be achieved as predicted by the quasilinear plasma kinetic theory is in exact agreement with that predicted from basic thermodynamic arguments by Busnardo-Neto, Dawson, Kamimura and Lin.[1] However, Eqs. (6) and (7) of quasilinear kinetic theory not only shows how this heating-cooling process evolves in time and leads to the upper limit on the temperature anisotropy that can be achieved but also shows that this upper limit is indeed a quasilinear steady state in ion-cyclotron resonance heating. It may be noted from the coupled set of nonlinear Eqs. (6) and (7) that at any instant of time the rate of change of the perpendicular temperature depends on the instantaneous values of $T_\perp$ and $T_\parallel$, and the rate of change of the longitudinal temperature in turn depends on the instantaneous values of $T_\perp$ and $T_\parallel$.

Let us now examine how close to this upper limit on the temperature anisotropy can really be achieved in ion-cyclotron resonance heating of a collisionless plasma. We shall do this within the framework of the quasilinear approximation. That is, we will neglect all other nonlinear interactions such as mode-coupling, nonlinear landau damping, nonlinear decay interactions, nonlinear particle-wave resonance broadening, Karplus-Schwinger nonlinear saturation mechanism, particle trapping, etc. For this purpose we must retain the dynamical friction term in Eq. (1). This will also teach us under what conditions one can neglect the dynamical friction term in comparison to the velocity space diffusion term in Eq. (1) as is usually done in classical quasilinear plasma kinetic theory (see Eq. 11-33 of Ref. 3). Thus for a Maxwell-Boltzmann distribution of $F_{\parallel}(v_z)$ one can show (after somewhat lengthy and tedious algebra) from Eqs. (1), (2), and (3) that

$$\partial(\kappa T_{\perp}/\partial t) = -\{\omega_c/(\omega_c - \omega)\}\partial(\kappa T_{\parallel}/2)/\partial t$$
$$= A[1 - (T_{\perp}/T_{\parallel})(\omega_c-\omega)/\omega_c - \omega\kappa T_{\perp}/\omega_c\varepsilon_k]\ F_{\parallel}(v_z = V_R). \tag{8}$$

Hence, in ion-cyclotron resonance heating if a large amplitude wave packet is impressed on a Maxwellian collisionless plasma, this heating-cooling process can occur if and only if $\partial T_{\perp}/\partial t \geqslant 0$ and $\partial T_{\parallel}/\partial t \leqslant 0$. That is, if and only if the ion-cyclotron wave energy

$$\varepsilon_k \geqslant \omega\kappa T_{\perp}/\omega_c\Delta \quad , \tag{9}$$

where $\Delta = 1 - (T_{\perp}/T_{\parallel})(\omega_c - \omega)/\omega$ is a measure of the departure from the upper limit of the temperature anisotropy that can be achieved in ion-cyclotron resonance heating. That is, within the framework of the quasilinear plasma kinetic theory Eq. (9) represents the necessary and sufficient criterion for the heating-cooling process to occur in ion-cyclotron resonance heating of a collisionless plasma. It should be noted that the energy in the ion cyclotron wave $\varepsilon_k >> \omega\kappa T_\perp/\omega_c\Delta$ is also the necessary and sufficient condition that must be satisfied for us to neglect the dynamical friction term in comparison to the velocity space diffusion term in Eq. (1). Furthermore, Eq. (9) represents the threshold condition that must be satisfied by the energy in the ion-cyclotron wave ε_k before this heating-cooling process can occur in ion-cyclotron resonance heating.

The radio frequency heating experiments which are presently carried out in large toroidal plasma confinement devices such as the tokamaks are done not only by launching an ion-cyclotron wave but also by launching a fast hydromagnetic wave. The frequency of the fast hydromagnetic wave is near the second harmonic of the ion-cyclotron frequency (i.e., $\omega \simeq 2\omega_c$). Hence, we now wish to examine the upper limit on the temperature anisotropy that can be achieved in the fast hydromagnetic wave heating of plasmas. According to the quasilinear theory (for example, see Eq. 63 of Ref. 2) one can show that for fast hydromagnetic wave heating

$$\begin{aligned}\partial(\kappa T_\perp)/\partial t &= -\{2\omega_c/(2\omega_c - \omega)\}\partial(\kappa T_\parallel/2)/\partial t \\ &= C\kappa T_\perp[2 - (T_\perp/T_\parallel)(2\omega_c - \omega)/2\omega_c]F_\parallel\{v_z = (\omega - 2\omega_c)/k_z\} ,\end{aligned} \tag{10}$$

where

$$C = \sum_{\vec{k}} \frac{4\pi^2 q^2 k_\perp^2 \varepsilon_k}{L^3 \mu^2 \omega^2} .$$

Thus the upper limit on the temperature anisotropy that can be achieved in the fast hydromagnetic wave heating of plasmas is $T_\perp/T_\parallel \leq 4\omega_c/(2\omega_c - \omega)$.

In conclusion we have shown from quasilinear plasma kinetic theory that an initially isotropic magnetized plasma will be forced into an anisotropic state both in ion-cyclotron resonance heating and in the fast hydromagnetic wave heating. The quasilinear theory yields a coupled set of nonlinear rate equations one of which describes the time evolution of the perpendicular ion temperature while the other describes the time evolution of the longitudinal temperature. The maximum temperature ratio predicted from basic thermodynamic arguments by Busnardo-Neto, Dawson, Kamimura and Lin[1] is indeed the quasilinear steady state solution of this coupled set of nonlinear rate equations. For a given value of the pump wave energy the quasilinear theory tells us how close to this upper limit on the temperature anisotropy can really be achieved. Finally, the quasilinear theory predicts a threshold value of the pump wave energy which has to be exceeded for this heating-cooling process to occur. In summary the quasilinear theory presented in this paper describes the detailed time evolution of the intrinsically coupled heating-cooling process while the entropy considerations only yield the upper limit on the temperature anisotropy that can be achieved.

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