NOVEMBER 1974

MATT-1088

TWO-ENERGY-COMPONENT TOROIDAL
FUSION DEVICES

BY

H. L. BERK, H. P. FURTH,
D. L. JASSBY, R. M. KULSRUD,
C. S. LIU, M. N. ROSENBLUTH,
P. H. RUTHERFORD, F. H. TENNEY,
T. JOHNSON, J. KILLEEN,
A. A. MIRIN, M. E. RENSINK AND
C. W. HORTON, JR.

PLASMA PHYSICS
LABORATORY

MASTER

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

This work was supported by U. S. Atomic Energy Commission Contract AT(11-1)-3073. Reproduction, translation, publication, use, and disposal, in whole or in part, by or for the United States Government is permitted.

DISTRIBUTION OF THIS DOCUMENT UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

TWO-ENERGY-COMPONENT TOROIDAL FUSION DEVICES*

H. L. Berk[†], H. P. Furth, D. L. Jassby, R. M. Kulsrud,
C. S. Liu, M. N. Rosenbluth, P. H. Rutherford, and F. H. Tenney

and

T. Johnson, J. Killeen, A. A. Mirin, and M. E. Rensink
Lawrence Livermore Laboratory, University of California,
Livermore, California 94550, USA

and

C. W. Horton, Jr.

The University of Texas at Austin,
Austin, Texas 78712, USA

ABSTRACT

Injection of a superthermal ion component into a toroidal plasma can raise the fusion power density to levels considerably higher than those attainable in ordinary one-component plasmas. As a result, energy break-even in D-T can be attained in bulk plasmas with $n\tau$ -values as low as $10^{13}\text{cm}^{-3}\text{sec}$, at temperatures of ~ 5 keV. The optimal energy of the superthermal ion component is in the range 100 - 200 keV; injection-energy requirements can be reduced by adiabatic compression prior to or during the reaction phase. The appropriate ratio $\bar{\Gamma}$ of fast-ion pressure to bulk-plasma pressure is about 0.5. The stability of two-component tokamak plasmas has been examined, and is found to be favorable for parameters of practical interest.

The basic theory of the two-component toroidal reactor has been described in Ref. [1]. A model regime has been demonstrated experimentally in Ref. [2]. The present paper includes a number of studies that are relevant to the practical implementation of the two-component reactor concept, particularly with regard to tokamak devices.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

DISTRIBUTION OF THIS DOCUMENT UNLIMITED

1 SLOWING-DOWN CALCULATIONS

The fusion energy gain of deuteron beams injected into a tritium plasma is

$$Q = \int_0^{W_0} [(n_T \sigma v E_f dW) / |dW/dt|] / (W_0) \quad (1)$$

where W_0 is the injection energy, σv is the fusion reactivity, n_T is the triton-target density, dW/dt is the instantaneous slowing-down rate, and $E_f = 17.6$ MeV. In order that Q may represent the energy gain of the entire plasma-beam system, the injection power density P_b must compensate all the plasma power loss:

$$P_b = \frac{n_b W_0}{\tau_s} = \frac{3n_e T_e}{\tau_E} \quad (2)$$

and if $\bar{W}_b$ represents the time-averaged energy of an injected ion during slowing-down, we have

$$\bar{\Gamma} = \frac{n_b \bar{W}_b}{3n_e T_e} = \frac{\tau_s \bar{W}_b}{\tau_E W_0} \sim \frac{\tau_s}{2\tau_E} \quad (3)$$

The values of $\bar{\Gamma} = 1.27Q$ in Ref. [1] were derived using a test-particle model for the fast ions and assuming classical slowing-down in a plasma with $T_i = 0$. In practice, one tends to have $T_i \approx T_e$, because of thermalization of electrons and ions and because of the direct input of beam power to the bulk ions. Solution of the Fokker-Planck equation with $T_i = T_e$, including energy diffusion terms, reveals somewhat larger Q -values. For $W_0 \lesssim 100$ keV, finite T_i also leads to a substantial increase in fusion reactivity. The resultant improvement in Q -values is illustrated in Fig. 1. Similar calculations are reported in Ref. [3].

The fast-ion energy can be "clamped" by techniques such as adiabatic compression or acceleration by the tokamak electric field, thus extending the slowing-down time through the energy range of maximum reactivity [4]. The ideal maximum fusion gain for ions clamped at energy W_0 is:

$$Q_c = \frac{n_T \sigma v E_f}{|dW/dt|_0} \quad (4)$$

To realize Q_c , the clamping period τ_c must be many times the slowing-down time τ_s , or else the superthermal energy must be recovered by magnetic decompression [4]. Figure 2 compares $Q_{\max}$ derived from Ref. [1] with Q_c for ions that are clamped at the energy W_c that gives maximum ratio of fusion reaction rate to plasma drag. Energy clamping enables Q to be enhanced by a factor of almost 2, and the optimal injection energy is reduced by 40 - 50%.

A two-dimensional multi-species Fokker-Planck code has been used to calculate the time-dependent distribution functions of the fast ions in energy and pitch angle, including the effects of energy clamping applied by major-radius compression. The Fokker-Planck equations are

$$\frac{\partial f_i}{\partial t} + \frac{\dot{R}}{R} \left(-\left(1 - \frac{1}{2} \sin^2 \theta\right) v \frac{\partial f_i}{\partial v} + \frac{1}{2} \sin \theta \cos \theta \frac{\partial f_i}{\partial \theta} \right) = \left(\frac{\partial f_i}{\partial t} \right)_c + S_i \quad (5)$$

for each ion species, including impurities, and

$$\frac{\partial f_e}{\partial t} + \frac{\dot{R}}{R} \left(-\frac{2}{3} v \frac{\partial f_e}{\partial v} \right) = \left(\frac{\partial f_e}{\partial t} \right)_c + S_e \quad (6)$$

for the electrons. Here S_i and S_e are particle source terms, which include particle and energy confinement times. The solution of Eqs. (5) and (6) is shown in Fig. 3, for the case of 60-keV deuterons injected tangentially into a $Z=1$ plasma of $n_0 = 2 \times 10^{13} \text{ cm}^{-3}$, and $T_e = T_i = 1.5 \text{ keV}$. An illustrative "break-even" experiment consists of the following set of operations: (1) Rapid compression through $C \equiv R_0/R_1 = 1.5$ ($t = 30 - 40 \text{ msec}$) increases the average beam-ion energy to 95 keV; (2) Slow compression through $C = 1.5$ ($t = 40 - 90 \text{ msec}$) "clamps" the beam energy; (3) Rapid decompression ($C = 2.3$, $t = 90 - 100 \text{ msec}$) returns most of the energetic-ion energy to the compression-field circuit; (4) Thermalization of the remaining beam energy takes place ($t > 100 \text{ msec}$). The overall Q of the cycle is 1.19. The average T_e during the slow compression is only 3.1 keV. The compression circuit does no net work directly on the plasma; as usual, the plasma transport and radiation losses are made up by energy transfer from the beam ions. (The energization of neutral-beam-injected ions by compression has been demonstrated experimentally in Ref. [2].)

The principal effect of impurities in a two-component reactor is the additional Coulomb drag on the energetic ions by the impurity ions and their neutralizing electrons. Alternatively, if the total plasma pressure is limited, addition of impurities requires a depletion of the triton population, thus decreasing the fusion reaction rate. Another impurity effect is enhanced pitch-angle scattering, which results in appreciable transverse energy for a tangentially injected beam. In the case of major-radius compression, the effectiveness of the available compression ratio is reduced, since $\Delta W_{\perp} \propto C$, while $\Delta W_{\parallel} \propto C^2$. For the illustrative example discussed above, if $Z_{\text{eff}} = 3$ the overall Q is reduced to 1.06 for iron impurity or 0.86 for oxygen. The Fokker-Planck solutions demonstrate that pitch-angle scattering aggravates the energy dispersion that occurs during compression and other forms of clamping — but this dispersion can grow to several tens of percent before substantially affecting Q .

2 POWER PRODUCTION

A two-component reactor with a 100% tritium bulk plasma can achieve a much larger power density than a one-component reactor of the same β -value [5], using a 50-50 D-T mixture. On the other hand the one-component reactor can achieve very high Q -values, while the two-component reactor is limited ideally to $Q \lesssim 5$, and for practical cases probably to $Q \sim 1-2$. In Figs. 4 and 5 we consider the entire family of possible two-component operating regimes, ranging from the pure two-component case to the pure one-component case, as a function of increasing $n\tau_E$ -value, with the fractional admixture of tritium optimized for maximum Q in each case. The plasma temperature is assumed in all cases to be maintained by a 200-keV reacting deuteron beam, since this always yields an improvement, but for very large $n\tau_E$ -values the required injection power becomes so small that the two-component contribution to the total fusion power density P_f is unimportant. The system Q -value is now defined by

$$Q_f = \frac{n_D n_T \overline{\sigma v E_f} + Q_b P_b}{P_b} = \frac{P_f}{P_b} \quad (7)$$

where Q_b is given by Eq. (1) and P_b by Eq. (2). (The power deposition by fusion alpha particles is neglected here.) Figure 4 shows the trade-off between high Q_f and high P_f for a given bulk plasma temperature $T_e = T_i = 8$ keV, and a fixed beam energy of 200 keV. Figure 5 illustrates the temperature dependence of the Q_f - P_f trade-off.

There are practical applications, such as the production of neutrons for the breeding of fissile fuel, where the economics favors high P_f (so as to reduce capital investment), rather than high Q_f . In that case, the regimes on the right-hand side of Fig. 5 are most attractive [5]. On the other hand, for production of economical power from pure fusion systems, very high Q -values may be necessary, and operation would then shift to the left side of Fig. 5. It is interesting to note that even in pulsed ignition devices (instantaneous $Q \rightarrow \infty$) the use of reacting-beam heating continues to make some useful contribution, by reducing the start-up power requirements [4].

3 GROSS EQUILIBRIUM

To a good approximation, the injected energetic-ion component in a large tokamak produces the same effects as an equal increment in the pressure of the bulk plasma. If a system of parallel and antiparallel injection is used, there will result a pressure anisotropy, but the modification of the MHD equilibrium solution is very small for the usual range of tokamak parameters.

On injection, the electric current of the energetic ions is cancelled identically by induced electron currents. These counter-currents relax with a very long time-constant, equal to the bulk-plasma skin time (and even in steady state the ion current tends to be partly shielded by electron counter-currents). To investigate the possible effect of unshielded injected-ion currents, we have carried out a spatially two-dimensional computer simulation, where a monoenergetic group of 2500 superparticles (D) is injected instantaneously, and the electron counter-current is removed in 100 μ sec. The (collisionless) guiding-center motions of the superparticles are followed in time and used to compute contributions to the toroidal current and plasma pressure. The background plasma (T) is modeled as a fluid. For various beam energies and peaked deposition profiles, the equilibrium vertical magnetic field and the temporal evolution of the flux-surface configuration and radial current-density and pressure profiles are calculated.

Studies have been made for 80-keV and 150-keV beams ($\bar{\Gamma} \lesssim 0.5$) injected into plasmas of $I \sim 1$ MA, $n_e \sim 5 \times 10^{13} \text{cm}^{-3}$, $T_e \sim 1-6$ keV. For the case of beams injected parallel to the toroidal current (co-injection), the plasma surface expands somewhat, but the flux surfaces are undistorted, even when the beam-ion current is comparable to the (ohmic) plasma current. The spatial distribution of the beam particles after 100 μ sec is essentially the same as the injected distribution.

Computations with counter-injected beams alone indicate serious deformation of the magnetic surfaces, with appreciable outward drift of the injected particles. This behavior may be thought of as due to repulsion between the beam-induced current and the ohmic plasma current. When both co- and counter-injected beams are introduced in equal numbers, the

undesirable effects of the counter-injected beam alone are markedly reduced. Under realistic experimental conditions, the combination of fast-ion scattering and persistent shielding by the electron current would tend to suppress the irregularities produced in the present computational model.

4 VELOCITY-SPACE STABILITY

4.1 Infinite-Medium Modes

In the ideal case of isotropic energetic-ion injection from steady-state sources, the slowing-down distribution takes the well-known form $f_b(v) \sim (v_c^3 + v^3)^{-1}$ for $v \leq v_{bo}$, where v_{bo} is the injection velocity, and we define $v_c^3 = 3\pi^{1/2} T_e^{3/2} / (2m_e)^{1/2} m_i$. Since this distribution is monotonically decreasing (and isotropic), there is absolute stability against infinite-medium modes. The ideal isotropic distribution would be entirely suitable for two-component reactor purposes. It is of interest, however, to examine the stability of some other distributions that have potential practical advantages: for example, the case of injection parallel and antiparallel to the magnetic axis, and the case of energy-clamping. The case of injection perpendicular to the magnetic axis, which was examined in Ref. [6], is of somewhat lesser practical interest for two-component tokamak purposes.

The steady-state distribution for slowing-down of a parallel-antiparallel-injected deuteron beam in a tritium bulk plasma is illustrated in Fig. 6a. In this case there exist velocity anisotropies, as well as local nonmonotonicities $\partial f_b / \partial v_{||}^2 > 0$ and $\partial f_b / \partial v_{\perp}^2 > 0$. (Note, however, that while $\partial f_b / \partial v_{||}^2 > 0$, for example, occurs locally, the averages that appear in determination of stability criteria, such as $\partial / \partial v_{||}^2 \int f dv_{\perp}^2$, are generally negative.) The resultant instability-driving sources are difficult to tap unless v_{bo} exceeds the Alfvén velocity v_A . Since we have identically $v_{bo}/v_A = (3\beta W_0/4T)^{1/2}$, the regime where $v_{bo} \geq v_A$ is not normally accessible for tokamak parameters; however, it may become accessible in the future if specially shaped tokamaks succeed in raising the MHD-stable β -limit to ≥ 0.1 . Instability can also be enhanced by the presence of high-Z impurities, which increase pitch-angle scattering relative to diffusion in energy, so that the distribution becomes "crescent-shaped" in velocity space and the $\partial f_b / \partial v_{\perp}^2$ term is strengthened (Fig. 6b). A similar effect is produced by energy-clamping (cf. Section 1).

We will consider first the possible excitation of the three standard parallel-propagating plasma modes ($k_{||} \gg k_{\perp}$), namely the longitudinal ion sound wave, and the slow and fast Alfvén waves [7]. The sound wave is driven by the $\partial f_b / \partial v_{||}^2$ term at the resonance $v_{||} = v_s$, but is very strongly stabilized — first of all, by bulk-plasma ion Landau damping. Even in the case of cold plasma ions and with the assumption of an energetically narrow injected population of velocity spread $\Delta v_{||b}/v_{||b}$, electron Landau damping gives the stability condition

$$\bar{\Gamma} < \left(\frac{W}{T_e}\right)^2 \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{\Delta v_{||b}}{v_{||b}}\right)^2. \quad (8)$$

This condition would be difficult to violate, except with very narrowly peaked beams.

The slow Alfvén wave is excited at low frequencies ($\omega \ll \Omega_b =$ beam-ion cyclotron frequency) by the $\partial f_b / \partial v_{||}^2$ term, provided that the resonance $v_{||b} = v_A$ can be reached. Instability requires

$$\frac{\partial}{\partial v_{||}^2} \int f_b v_{||}^4 d^2 v_{\perp} > 0 \quad (9)$$

which is not generally satisfied by steady-state slowing-down distributions, but could be satisfied for transient (or clamped) injection. In the latter case, if we have a peaking characterized by $\Delta v_{||b} \sim v_{||b} \sim v_{\perp b}$, the condition for stabilization by electron Landau damping is

$$\bar{\Gamma} < C \frac{v_{||b}}{v_e} \quad (10)$$

The constant C depends on details of f_b , and is likely to exceed 4; nonetheless, it appears possible to excite instability for $\bar{\Gamma}$ -values of order unity. The principal reason for the unimportance of this mode in practice is the tendency of v_{bo} to lie well below v_A .

The fast Alfvén wave is excited by the anisotropy of the parallel-injected ion distribution at the resonance $v_{||b} = v_{\text{wave}} + \Omega_b / k_{||}$, provided that $\partial f_b / \partial v_{||}^2 < 0$ and also $|\partial f_b / \partial v_{||}^2| > |\partial f_b / \partial v_{||}^2|$. These conditions are generally met by the steady-state slowing-down distribution. Stabilization by electron Landau damping is unimportant. The main limitation on the instability is given by the value $v_{||b}$ below which the resonance cannot be achieved. For $\Omega_b = \Omega_i$, the instability criterion is thus

$$v_{||b} < 2.6 v_A \quad (11)$$

This criterion is evidently even more difficult to violate than that for the slow Alfvén wave.

Lastly, we must consider transverse waves ($k_{\perp} \gg k_{||}$), which are driven by the $\partial f_b / \partial v_{||}^2$ and $\partial f_b / \partial v_{\perp}^2$ terms, and opposed by electron Landau damping. The most significant instabilities obtainable in this range of $k_{\perp} / k_{||}$ arise as the slow Alfvén wave goes over into the shear Alfvén wave, and is excited at resonances of the beam ion-cyclotron frequency Ω_b and its higher harmonics [8]. These resonances are accessible for $v_{bo} < v_A$, the normal case of interest. For perpendicular injection, these modes have been considered in Ref. [6]. For steady-state parallel injection, the nonmonotonicities of f_b are too weak to give instability. Distributions resulting from enhanced pitch-angle scattering (Fig. 6b) tend to be more susceptible to instability, but the steady-state slowing-down distributions obtained even with very high Z_{eff} appear to be stable. (Furthermore, since very small values of $k_{||}$ are typically required for these modes, the finite-medium solution is expected to be sensitive to shear-stabilization.) The ion-cyclotron or "loss cone" modes should, however, be of significant practical interest in limiting the permissible degree of peaking in short-pulse operation or in energy-clamped distributions.

4.2 Finite-Medium Modes

We consider next the effects of finite-medium modes, such as drift waves, trapped-particle modes, and kink-like MHD modes, on the energetic-ion component. Since anomalous transport is dominant in all present-day tokamak experiments, we must ask, first of all, what effect even the bulk-plasma-driven modes might have on the confinement of the energetic ions, in the absence of special beam-driven modes. The bulk-plasma modes are characterized by $\omega \sim \omega_{*e} \ll \omega_{*b}$, (where ω_{*e} and ω_{*b} are the diamagnetic drift frequencies of the electrons and the injected ions, respectively). In view of this frequency mismatch, we expect the primary effect on the energetic ions to be enhanced cross-field transport, rather than anomalous energy loss. The ratio τ_b/τ_p of the energetic-ion confinement time to the bulk-plasma confinement time may be estimated as follows: $\tau_j^{-1} \propto \langle (E/B)^2 \rangle t_j$, where $\langle (E/B)^2 \rangle$ is the average fluctuating drift velocity due to the instability, and t_j is the correlation time for the j^{th} -species, given roughly for ions by $t_j = \gamma/(\omega^2 + k_{\perp}^2 v_j^2)$ (with γ the growth rate, and ω the frequency, of the mode). For typically expected modes in a sheared field, $\omega \sim k_{\perp} v_i$. Hence we conclude:

$$\frac{\tau_b}{\tau_p} \sim \frac{\bar{W}_b}{T} \quad (12)$$

For typical two-component tokamak parameters, the confinement time τ_b is thus substantially greater than τ_p . Since we generally assume $\tau_s \lesssim \tau_p$ ($\sim \tau_E$), in order that the bulk plasma temperature may be maintained by the injected ions for $\bar{\Gamma} \lesssim 1$, it follows that the effect of the plasma-driven drift waves and trapped-particle modes on the energetic ions is not important. The presence of MHD-activity in the tokamak plasma might lead to loss of high- $v_{\perp}$ ions by way of faulty magnetic surfaces. An empirical estimate of the importance of this effect can be derived from measurements on high- $v_{\perp}$ electrons [9]: for normally quiescent tokamak conditions, the loss of energetic ions estimated on this basis would not be significant.

It remains to consider whether the energetic ions themselves will drive important new finite-medium modes. For the electron branch of the drift wave [10], including the trapped-electron modes, the resonant interaction is generally stabilizing, even for nonmonotonic and anisotropic f_b : owing to $\omega \ll \omega_{*b}$, the $\partial f_b / \partial x$ term will dominate the term in $\partial f_b / \partial v_{\perp}^2$. If the sign of $\partial f_b / \partial x$ were to be reversed relative to $\partial n_b / \partial x$ (corresponding to a hollow injected-ion shell), the effect of the beam would be destabilizing — but the interaction would still be weak relative to the ordinary driving and damping mechanisms of the bulk plasma. The excitation of the trapped-ion mode has also been considered [11,12]; it is shown in Ref. [12] to be stabilized by very small spreads of the energetic-ion ∇B - and curvature-drift frequency ω_{Db} . The stability condition for $T_e = T_i$ is

$$\Delta\omega_{Db}/\omega_{Db} \geq \bar{\Gamma} (T/\bar{W}_b)^2 \quad (13)$$

It would be difficult to violate this condition in a two-component tokamak experiment, even with instantaneous mono-energetic injection or perfect energy clamping, since the geometric factors that arise from the spread in

beam-deposition points will generally create a sufficient spread of $\Delta\omega_{Db}$ to satisfy condition (13).

In order to find a finite-medium mode that can be excited significantly by the energetic ion component, we must look for a weakly damped bulk-plasma wave travelling in the direction of the ion diamagnetic drift. The only mode that fits this description is the ion branch of the drift-Alfvén wave; this has indeed turned out to be the most dangerous of the finite-medium instabilities [7,13]. Since we have $\omega \geq k_{\parallel}v_A$ for the ion branch of the drift-Alfvén wave, the beam resonance at $\omega = k_{\parallel}v_{b\parallel}$ cannot normally be reached in two-component tokamak plasmas (cf. Section 4.1). However the magnetic drifts of ions passing through regions of different ω_{Db} as a function of azimuthal angle θ introduce oscillating components into the beam motion, and these higher-frequency harmonics are able to resonate with the wave frequency. The instability is then driven by the spatial displacement of the energetic ions in the direction of $-\nabla B$. (Alternatively, if $\partial f_b / \partial v_{\parallel}^2 > 0$, the mode could be driven by the nonmonotonicity of the energetic ions.)

The drift-Alfvén mode of interest here resembles the kink-like oscillations that arise spontaneously in MHD-unstable tokamak configurations, but is, of course, stable in the absence of excitation by the energetic-ion component. The wave amplitude becomes large at the Alfvén singularity $\omega = k_{\parallel}(r_A)v_A$, so that non-MHD effects become important there. The primary damping effects that oppose the instability arise from the collisions of trapped electrons in the regions $\omega \geq k_{\parallel}v_A$. Hence, to avoid strong electron damping, modes with only a small region of $\omega \geq k_{\parallel}v_A$, near the axis, are favored. Strong damping effects would also arise at the point $k_{\parallel}(r) = 0$, where the safety factor $q = rB_t/RB_p$ equals the pitch m/n of the Alfvén perturbation $\exp[i(n\phi + m\theta)]$; accordingly, the most dangerous modes are obtained by choosing m and n so as to avoid the presence of such a point in the plasma profile. For centrally peaked current densities (the normal tokamak case), where the q -value increases outward from the axis, the case of interest is $q_A > q(0) > m/n$ where q_A is the q -value at the Alfvén singularity. In order to achieve the unstable resonance with the drift harmonics of the beam motion, however, it is necessary to limit the wave frequency by requiring $(q_A - m/n)v_A \leq (q_A - m/n + 1)v_b$. There is a further limiting condition on the radius r at the Alfvén singularity, in order for the beam destabilization to overcome the electron damping, yielding the stability criterion [13]

$$\frac{r_A^2}{r_b^2} > \beta_b \frac{v_b}{v_{ei}R} \left(\frac{r}{R}\right)^{1/2} \frac{(\partial \ln q / r^2)^{-1} R \rho_b}{r_b^4} \frac{(m/n)}{n - (n/q_A)}. \quad (14)$$

Here we have not invoked $\partial f_b / \partial v_{\parallel}^2 > 0$, so that the condition $\omega < \omega_{*b}$ is also necessary for instability, and was used in deriving (14). The quantity r_b^2 is defined by the energetic-ion density profile $n_b = [1 - (1/2)(r^2/r_b^2)]n_{b0}$, β_b and ρ_b are the β -value and mean gyroradius of the energetic ions, and v_{ei} is the electron collision frequency with the bulk ions. In deriving Eq. (14), both shear-stabilization and the averaging of ω_{Db} over regions of compensating curvature have been taken into account. (In an unsheared field of uniformly "unfavorable" curvature, the instability condition is far more severe [7].) For two-component tokamak experiments of practical interest ($n \sim 10^{14} \text{ cm}^{-3}$, $T_e \sim 5 \text{ keV}$, $B_t \sim 50 \text{ kG}$, $r_b \sim 50 \text{ cm}$, $W_b \sim 100 \text{ keV}$) we can reduce the

stability condition to $r_A/r_b \geq 2(\beta_p)^{1/2}$. For contemplated β_p -values of ~ 0.01 , there will result a modest flattening of the beam density over its central region. If we have $\partial f_b / \partial v_{||}^2 > 0$, so that $\omega < \omega_{*b}$ is not necessary, the quantity r_A/r_b will be increased, making the mode more dangerous to high- β_p operation.

The persistence of the drift-Alfvén mode instability, even for monotonic energetic-ion distributions, may be of particular interest in relation to its possible excitation by the alpha particles of the D-T reaction, even in the absence of neutral-beam-injected ions. The 3.5-MeV alphas could easily satisfy the condition $v_A < v_b$; in this case the mode could be driven by trapped alpha particles with "unfavorable" bounce-averaged ω_{Db} (as well as by the mechanism described here), but this case has not yet been examined in detail.

5 TIME-DEPENDENT TRANSPORT STUDIES

A tokamak radial transport code has been modified to include the heating of the plasma by injected superthermal ions. This code [14] employs local transport coefficients that fall into six regimes, in which the plasma transport is dominated by collisional drift waves, trapped-electron modes, or the trapped-ion instability, in order of decreasing plasma collisionality. Neutral gas and impurity effects are included. The energetic deuterons are deposited into the tritium plasma with a prescribed radial distribution, and are assumed to slow down classically. The code can also make an instantaneous major-radius compression.

Calculations for various heating and compression scenarios have been carried out for plasmas with $I \sim 1-2$ MA, $a \sim 60-90$ cm, $B_t \sim 45$ kG, $\bar{n}_e \sim 10^{14} \text{cm}^{-3}$, $\bar{T}_i \sim \bar{T}_e \sim 5-10$ keV. Figure 7 shows the time-dependent plasma energetics for an illustrative case where a 30-msec, 240-A pulse of 100-keV deuterons is injected into a preheated tritium plasma of central parameters $n = 6 \times 10^{13} \text{cm}^{-3}$, $T_e = T_i = 6.0$ keV. At $t = 55$ msec, an instantaneous major-radius compression of 1.44 increases the beam energy by a factor of 2. The radial profiles of n , T_e , and T_i 5 msec after compression are shown in Fig. 8. At this time β_p , including the beam pressure, attains its maximum value of 2.0. The central neutral density is $4 \times 10^6 \text{cm}^{-3}$. Figure 7 shows that the total fusion energy output in the compressed plasma (1.42 MJ) is considerably larger than the energy residing in the beam ions immediately after compression (1.02 MJ); that is, $Q > 1$ is obtained in the postcompression phase.

ACKNOWLEDGMENTS

We are grateful to Yung-Chiun Sun and Fred Seidl for aid in carrying out computations.

*This work was supported by the U. S. Atomic Energy Commission Contract AT(11-1)-3073.

⁺Present Address: Lawrence Livermore Laboratory, Livermore, California 94450.

REFERENCES

- [1] DAWSON, J. M., FURTH, H. P., TENNEY, F. H., Phys. Rev. Lett. 26 (1971) 1156.
- [2] BOK, K., et al., Phys. Rev. Lett. 32 (1974) 661.
- [3] FRIEMAN, E. A., TENNEY, F. H., Bull. Am. Phys. Soc. 18 (1974) 878.
- [4] FURTH, H. P., JASSBY, D. L., Phys. Rev. Lett. 32 (1974) 1176.
- [5] JASSBY, D. L., Princeton Plasma Physics Laboratory Report MATT-1072 (1974); to be published.
- [6] CORDEY, J. G., HOUGHTON, M. J., Nuc. Fus. 13 (1973) 215.
- [7] BERK, H. L., HORTON, W., Jr., ROSENBLUTH, M. N., RUTHERFORD, P. H., in Symposium on Plasma Heating in Toroidal Devices (Varenna, 1974), to be published.
- [8] BERK, H. L., HORTON, W., Jr., ROSENBLUTH, M. N., RUTHERFORD, P. H., to be published.
- [9] VON GOELER, S., et al., in Symposium on Toroidal Plasma Confinement (Proc. 3rd Int. Symp. Garching, 1973) B-25.
- [10] STIX, T. H., Phys. Fluids 16 (1973) 1922.
- [11] OHKAWA, T., BHADRA, D. K., Physics Letters 48A (1974) 140.
- [12] BERK, H. L., to be published in Physics Letters.
- [13] ROSENBLUTH, M. N., RUTHERFORD, P. H., to be published.
- [14] MEADE, D. M., et al., in Plasma Physics and Controlled Nuclear Fusion Research (Proc. 5th Int. Conf. Japan, 1974) Paper A15-4 (this conference).

Fig. 1. Effect of finite bulk-plasma temperature on the fusion energy multiplication factor of fast deuterons injected into a tritium target plasma. (Solid curves are for simple slowing-down, neglecting all energy diffusion effects.) (PPPL 743837)

Fig. 2. (a) Optimal beam energies for clamped (W_c) and unclamped (W) deuteron beams in a 100% tritium target plasma. (b) Maximum energy gains (17.6 MeV per reaction) for clamped (Q_c) and unclamped (Q) beams. (PPPL 743740)

Fig. 3. (a) Time sequence for injection and compression cycle. 60-keV deuterons are injected for 30 msec into a tritium plasma, followed by a "fast compression, a slow "clamping" compression, and a fast decompression. (b) Mean energy of the beam ions. (c) Electron temperature. (Computation includes electron heat loss with $\tau_E = 200$ msec.) (d) $Q = (\text{Fusion energy output})/(\text{Net energy delivered to beam ions by injection and compression})$. (PPPL 743737)

Fig. 4. Dependence of P_f and Q_f on $n_e \tau_E$ for an 8-keV D-T plasma heated by 200-keV D beams [5]. Plasma composition, given by n_T/n_i , is optimized for maximum Q_f . Total plasma pressure in this example is 0.655 J/cm³. Radiation loss rates are included in $1/\tau_E$. (PPPL 743656)

Fig. 5. Q_f versus P_f for plasma of Fig. 4 with various values of $T_e = T_i$ [5]. The dashed curves are contours of constant $n_e T_e$ (cm^{-3}sec). (PPPL 743741)

Fig. 6. Steady-state slowing-down distribution f_b for 150-keV parallel-injected deuterons in a tritium plasma of $T_e = 4$ keV, $T_i = 1$ keV, with effective Z -values of 1 (a) and 4 (b). (PPPL 743738)

Fig. 7. Energetics of fast compression case (1.2 MA post-compression). The total energies in plasma, poloidal field within the limiter, and beams are shown as functions of time, together with the time-integrated energy losses up to that time. The nuclear energy includes both the alpha-particle and neutron components. On the vertical scale, 1.0 equals 1.29 MJ. (PPPL 742059)

Fig. 8. Fast compression case with precompression injection at 100 keV for a 30 msec pulse, and a postcompression current $I = 1.2$ MA. Compression occurs at 55 msec. Radial profiles are given at 60 msec. Each profile is scaled to its maximum value, which is stated; τ is the rotational transform $2\pi R B_p / r B_t$. (PPPL 742053)

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, express or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission to the extent that such employee or contractor prepares, handles or distributes, or provides access to, any information pursuant to his employment or contract with the Commission.