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THE THERMAL AND HYDRAULIC ANALYSIS
OF A PHOENIX FUEL
IN THE MATERIALS TEST REACTOR

R. M. Hiatt

June 1970

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THE THERMAL AND HYDRAULIC ANALYSIS
OF A PHOENIX FUEL IN THE
VATERIALS TEST REACTOR

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June 1970

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ABSTRACT

A Phoenix fuel loading of the Materials Test Reactor at Idaho was investigated to determine safe operating power levels based on thermal and hydraulic considerations. Design points are clearly stated with emphasis placed on flow problems and heat transfer burnout limits.

THE THERMAL AND HYDRAULIC ANALYSIS
OF A PHOENIX FUEL IN THE
MATERIALS TEST REACTOR

INTRODUCTION

The use of plutonium fuels for compact reactor applications is currently under investigation by Battelle-Northwest. One such fuel, a plutonium-aluminum alloy, is being fabricated and prepared for an irradiation experiment, one of the first of its kind, and is to be conducted in the Materials Test Reactor (MTR) at Idaho Falls, Idaho. In addition to evaluating the "Phoenix effect"* of a plutonium fuel,⁽¹⁾ the experiment will serve to improve analytical methods and techniques for predicting plutonium fuel performance in light water moderated reactors, and help determine the marketable value of plutonium in present uranium systems.

The fuel element design adopted for the test is similar in structure to the standard uranium fueled subassembly with the same coolant to metal ratio. However changes in the fuel plate thickness and widening of the coolant channel gaps warrant a complete evaluation of the new design concept. This preliminary analysis is essential to determine safe operating power levels for the experiment. The importance of the study is realized when comparing the nuclear behavior of plutonium fuels to uranium fuels. Plutonium fuels are subjected to

* The Phoenix effect involves the fissile isotope Pu^{239} being converted to Pu^{240} , which upon capturing a neutron is converted to Pu^{241} a fissile isotope.

unusually high local power concentrations due to their high fission cross sections. These localized high power densities are located in areas where there is excess coolant or reflector material that tends to thermalize the neutrons. The duration of these spikes during burnup is not certain but limited evidence indicates that their duration may be lengthy. This problem is accentuated at the bottom of the core due to the top entry control system and irregular fissile material distribution at the lower end of the fuel plates. Both conditions incite power spikes which may be a limiting design factor. Since subcooled incipient nucleate boiling* is to be avoided at all operating power levels, it is essential to have a good description of the coolant properties in the system. A useful analysis must include the effect of the velocity disparity and variation in nuclear behavior from plate to plate on the transfer of heat into the coolant channels. The rest of this report is such a thermal and hydraulic analysis of the Phoenix fuel design concept. Analytical predictions were confirmed by experimentation when possible.

PURPOSE OF THIS STUDY

To effectively delineate reactor core designs within desired performance levels, it is necessary to define heat transfer rates which will preserve the structural integrity of the core. These

* Subcooled incipient nucleate boiling is surface boiling when the fluid bulk temperature is below saturation. Thus the bubbles leaving a hot surface migrate into the fluid stream and collapse distributing their excess heat to the subcooled fluid.

limits are established from maximum fuel and cladding temperatures, or, the point at which a severe deterioration of the heat transfer mechanism occurs which may result in excessive fuel and cladding temperatures. The occurrence of this phenomenon, known as boiling crisis or departure from nucleate boiling, is enhanced in the MTR core, since any bubbles formed could potentially block a coolant flow channel and thus result in damage to the fuel containment wall from an over heated condition. Since the consequences of fuel failures can endanger human life the safeguards for reactor operation are stringent. The reactor system is essentially a constant energy system under normal operation and therefore burnout problems must be predicted and identified to insure safe operation. This effort investigated the thermal hydraulic performance of a stationary Phoenix fuel element described later in the report. The power distribution was taken from experimental data on a plutonium fueled system which was a mockup of the Phoenix fuel experiment. The fuel design limits are defined and an analytical calculation is presented which identifies expected thermal hydraulic problems. Finally a hydraulic test was conducted to confirm the analytical calculations and assumptions. The information presented in the remainder of this report gives detailed data used in determining safe operating powers during the irradiation of a Phoenix fuel in the MTR.

METHOD OF ANALYSIS

The thermal and hydraulic design procedure for the Phoenix core was similar to that established for the standard MTR core. This helped to insure that the thermal and hydraulic characteristics were within

the licensed capability of the MIR facility. An analytical approach was used to predict the coolant flow patterns and map the spatial pressure profiles in the subassembly. The development of the channel flow equations is given in Appendix A. Using these equations and a hot spot analysis, safe operating powers were predicted for the subassembly. Next a dummy Phoenix fuel element was fabricated and tested in the HTF (Hydraulic Test Facility) to confirm the hydraulic performance analytically predicted. These flow data were input into MACABRE-11,⁽²⁾ a steady state thermal and hydraulic digital computer code developed at the test reactor site in Idaho. The results of the computer analysis were used to determine the thermal characteristics in the subassembly. The spatial power distribution used in this investigation had been evaluated earlier from a small scale plutonium fueled model irradiated in the Plutonium Recycle Critical Facility (PRCF) by the Reactor Physics group.

Calculation of film buildup due to corrosive influence was based on a 3000 MWD operation at full power which is considered slightly in excess of the life of the fuel. This assumption was considered conservative. Additionally since burnup data were not available it was assumed that the initial power peaks would exist throughout the experiment, which is also a pessimistic assumption. Commensurate with present "hot channel methods" for predicting safe operating powers, various uncertainty factors unique to the Phoenix core were incorporated into the study. The uncertainty factors constitute the best presently known values and were based upon the results of the PRCF experiment. Where no data was available the uncertainty factors

were developed from the fuel design specifications and it was assumed that the worst possible conditions existed under these standards. Close agreement was found to exist between these parameters and those obtained by Idaho Nuclear Corporation for a standard fuel sub-assembly.

The **MACABRE-II** code was designed to compute the differential heat transfer rate between two adjacent coolant channels. Thus a variation in the thermal resistance in the direction normal to the fuel plate surface is compatible with the code. It does not account for axial and azimuthal heat conduction in the fuel plate,* The worst azimuthal power peaking conditions across each fuel plate were used in this study for the axial power profiles through the core. Calculations were therefore based on "hot channel" conditions without considering intera-channel mixing and simulates the worst possible nominal** conditions. Uncertainty factors were then used to simulate the hottest conditions and therefore the worst possible thermal conditions.

The author was required to add the Keenan and Keyes steam tables,(4) the Bernath burnout correlation⁽⁵⁾ and the Jens and Lottes correlations⁽⁶⁾ in separate subroutines into the **MACABRE-II** code. This made it possible to obtain incipient nucleate boiling data and burnout heat

* This conditions was investigated at the hot spot using the Thermal Analyzer Program, TAP⁽³⁾

** Nominal conditions are defined to be those conditions calculated by standard procedures without considering the influence of core uncertainty factors.

flux comparisons at desired mesh points through the core. The Dittus and Boelter heat transfer correlation⁽⁷⁾ equation was used for convective heat transfer and the energy equations are solved exactly in the MACABRE code as the demanded convergence on pressure loss approaches zero at each axial node. It was also possible to analyse the thermal and hydraulic conditions for a complete fuel element (up to 18 plates and 19 coolant channels) at a time. Thus knowing the inlet and exit head loss coefficient for each channel, a flow balance and velocity distribution across a fuel element was calculated which was compatible with actual flow patterns. This provided more reliability on local heat flux predictions since it improves the accuracy for calculating the heat transferred to adjacent coolant channels. This additional utility establishes a more concise and accurate method of calculating safe operating power limits in the MTR. In summary the method used by the author in this analysis included the following procedures:

- (1) Develop a method for calculating the spatial pressure profile based upon an analytical approach.
- (2) Predict the velocity disparity from analytical considerations.
- (3) Calculate a maximum operating power based upon conditions at the hot spot in the core.
- (4) Confirm the analytical hydraulic predictions by experimentation.

- (5) Determine the maximum nominal reactor power limits at startup and after a 3000 MWD burnup cycle using the worst azimuthal peaking factors for each plate in a fuel element and using the experimental data in the MACABRE code.
- (6) Determine the maximum trip power limits.
- (7) Determine the hot channel conditions based on design uncertainties at design power. The power uncertainties were developed by the Theoretical Physics group.

It was assumed that the element in the core with the largest power density was the critical element and the analysis was based on this element. Further it was assumed that the oxide accumulation due to corrosive influence calculated over a 3000 MWD time period simulated oxidation growth comparable to a 50% fuel burnup. Since the Bohemite corrosion product formed and retained on the aluminum heat transfer surface was a significant resistor to heat transfer, MACABRE-I1 offered an attractive package for analyzing long reactor operating cycles.

SUBASSEMBLY DESCRIPTION

The basic element of the Phoenix fuel subassembly was a flat plate with nominal dimensions of 0.080 inches thick by 2.648 inches wide by 25.125 inches long. The plate consisted of three layers, an aluminum layer of 0.020 inches thick, a fuel sheet composed of a plutonium aluminum alloy of 0.040 inches thick and another 0.020 inch thick aluminum layer. The fuel was placed between the two

FIGURE 1 A Sketch of the Stationary Phoenix Fuel Element
Figures 2 & 3 Illustrates Views A, B, & C

FIGURE 2 The Diffuser Type End Adapters of a Stationary Phoenix Fuel Element. Section A is the entrance adapter and Section B is the exit adapter. Figure 1 illustrates the view for each section

FIGURE 3 A Sketch of the Fuel Plates for a Stationary Phoenix Fuel Element

sheets of aluminum, heated and rolled into one sheet with positive bonding between the layers. Since the fuel section was not as long nor as wide as the two aluminum sheets, the fuel was encased entirely by the aluminum in the rolling process. This flat plate called a fuel plate was next curved in an arc with a 5.5 inch radius about the longitudinal axis. This curvature of the plates gave additional strength during operation at full power. Sixteen fuel plates attached to two non-fueled aluminum side plates formed the basic structure for the subassembly which was the smallest single unit in fueling the MTR. The two side plates were grooved with 16 slots evenly spaced in which the edges of the fuel plates were fixed and anchored by pressing the metal of the side plates into the fuel plates. This process is a roll-swaging process. Figure 3 illustrates the attachment of the fuel plates to the side plates. A spacer comb was fixed at the center of the subassembly at top and bottom to support the plates and maintain proper spacing.

An adapter, formed to fit the end of the subassembly and tapering to a 2.0 inch radius (refer to Figure 2, Section A) was welded to the top of the subassembly. The bottom adapter tapers to approximately a 2 inch square as illustrated in Figure 2, Section B. Figure 1 illustrates a front view of the assembly. A detailed print is shown in Figure A-4 of Appendix A. All units are in inches.

POWER DISTRIBUTION

The design power distribution⁽⁸⁾ is somewhat dependent upon the fuel arrangement in the MTR core. Figure 4 illustrates the proposed position for the Phoenix load. It will require some effort to adjust the results of the PRCF experiment to be applicable to incore conditions. However using presently available data from the PRCF experiment, it was found that maximum power peaks occur at approximately 18.5 inches below the top of the core in MTR elements number L-25 and L-45* (refer to Figure 4). These data were taken with the control rods withdrawn approximately 12.5 inches and represents startup conditions. The extrapolated peaking factor at these locations was found to be 2.5 and represents the ratio of the local power to the core average. L-24, L-26, L-44, and L-46 are shim rod locations and were measured to have maximum to core average power ratios of 2.4 occurring approximately 12.5 inches below the top of the fuel in the shim rod. Should it be found in later investigations that this factor increases, the shim rods could possibly be the critical elements and should be used to set the safe operating power for the reactor.

Figure 5 illustrates the power profile through the core of the stationary fuel element L-25. Two profiles are shown,

* It is assumed that symmetry exists in the core and therefore L-25 and L-45 behave as identical elements. Future discussion will reference L-25 but will also apply to L-45.

FIGURE 4 Proposed Arrangement of MTR Active Core with Phoenix Fuel Load

FIGURE 5 Axial Power Profiles in Plates of Stationary Fuel

one for the center plate of the element and one for the exterior fuel plate. Figure 6 illustrates the azimuthal or lateral traverse power profile parallel to the plate surface in L-25 and one half the distance across L-35 at approximately 18.5 inches below the top of the core. It was assumed that this lateral power shape was constant at all axial positions but its integrated value varied according to location. Integrating these data and performing an energy balance it was found that element L-25 generated 4.77 percent of the total reactor power. An energy balance based on predictions using the MACABRE-I1 code showed 4.89 percent of the reactor power was produced in element L-25, which is a conservative analysis. To further consider safe design parameters for this element it is necessary to review the MTR operating limits.

MTR OPERATING LIMITS

The major thermal and hydraulic limits placed on MTR operation and fuel design are summarized as follows:

1. The nominal heat flux at the hot spot in the hot channel should not exceed 1×10^6 Btu/hr-ft².
2. There should be a 95% confidence level that subcooled incipient nucleate boiling does not occur in the reactor under nominal operating conditions. This was defined to be two standard deviations below the predictions obtained using the Jens' and Lottes' correlation, For the purpose of this report a standard deviation was assumed to be 18°F. This was 2 degrees higher than that reported by Little in his document. (9)

FIGURE 6 Power Profile for a Lateral Traverse in East West Direction at 18.5 inches Below Top of Fuel Elements in Position L-25 and L-35

3. The maximum reactor heat flux should be at least 3 standard deviations less than the burnout heat flux as predicted from the Bernath correlation and calculated at the reactor trip power. A standard deviation was assumed to be 10.05 percent of the maximum nominal heat flux, which was in accordance with the Little document.
4. Temperatures should be maintained at levels that will safeguard the structural integrity of the core. This proved not to be a limiting factor.
5. The maximum nominal power level for MIR operation was 40 MW_t .
6. Inlet pressure at the reactor vessel was not to exceed 75 psia.
7. The differential pressure across the reactor core will not exceed 50 psi nor will it be less than 30 psi.
8. The inlet coolant temperature should not exceed 130°F . The minimum temperature may be limited from environmental considerations since the heat sink is the atmosphere. However, the reactor currently operates at an inlet temperature of $105^\circ\text{F}^{(9)}$ at a power level of 40 MW_t .

The thermal and hydraulic limits listed above were consistent with operating limits established by Idaho Nuclear Corporation (INC).⁽¹⁰⁾

It was necessary to establish that the Phoenix fuel load in the MTR would perform within these operating limits. To present such a proof required a knowledge of the flow distribution in a subassembly upon which the calculations of the thermal and hydraulic characteristics of the core could be based. An analytical flow analysis is presented in Appendix A and a hydraulic experiment, discussed later in this report, was performed to confirm the analytical assumptions and results. A summary of major results is presented next for easy reference to the design limits established above.

SUMMARY OF MAJOR RESULTS

A comparison of the hydraulic characteristics for a Phoenix fuel load and a standard fuel load in the MTR is shown in Table I. Table II illustrates a comparison of the heat transfer conditions for the two cores. Table III shows a comparison of the uncertainty factors used for generating hot channel conditions for the two fuels. A summary of the major results based on presently available data were found to be as follows. A Wang desk calculator and computer results are shown to a 4 figure accuracy.

1. The reactor operating power for the Phoenix core was found to be 33 MW_t , which represents a 17.5% power decrease from the standard 40 MW_t operating power limit. Adapting the PRCF experiment results to incore conditions will require an adjustment in this prediction. Further, should the control rod purport the limiting condition, a reassessment of this operating power will be required.
2. The trip power for the Phoenix core can be set at 49.5

TABLE I

Comparison of Core Characteristics for a Standard
MTR Fueled Core vs an MTR-Phoenix Fueled Core

	<u>Standard</u>	<u>Phoenix</u>
Inlet Coolant Temperature, °F	105	105
Inlet Reactor Pressure, psia	60	60
Core Pressure Differential, psi	40	40
Reactor Tank Pressure Differential, psi	43	43
Average Reactor Coolant Flow per Stationary Fuel Element, gpm	620	470
Average Reactor Coolant Flow per Cd-U Shim Rod, gpm	550	445
Total Flow Through Reactor, gpm	23000	19300
Average Coolant Flow Rate in Fuel Element Channel, ft/sec	31.4	30.04
Average Coolant Flow Rate in Shim Rod Fuel Channel, ft/sec	36	34.4
Hydraulic Diameter of Fuel Element Channel, inches	0.2223	0.228
Hydraulic Diameter of Shim Rod Fuel Channel, inches	0.2236	0.1924 (outside) 0.2274 (inside)
Fuel Element Flow Area, ft^2	0.0406	0.035
Shim Rod Fuel Section Flow Area, ft^2	0.0304	0.029
Heat Transfer Area Per Element, ft^2	15.49	13.31
Heat Transfer Area Per Shim Rod Fuel Section, ft^2	10.65	10.39
Total Core Heat Transfer Area, ft^2	398.87	335.9
Number of Fuel Plates in Fuel Element	19	16
Element Fuel Plate Thickness, inches	0.05 (except two outside 0.065")	0.08
Coolant Channel Thickness, inches	0.116	0.119
Number of Fuel Plates in Shim Rod Fuel Section	14	13
Shim Rod Fuel Plate Thickness, inches	0.06	0.08
Shim Rod Coolant Channel Thickness, inches	0.117	0.119 (inside) 0.100 (outside)
Core Power	40	33
Number of Elements in Core	23	19
Number of Shim Rods in Core	4	8
Position of Core in Reactor	North Side	Center

TABLE II

Heat Transfer Predictions and Comparisons for a MIR Core Fueled
With Standard Fuel and a Phoenix Load

<u>Reactor Power</u>	<u>Standard</u>	<u>Phoenix</u>		
Nominal power level	40 MW _t	33 MW _t		
Uncertainty hot channel factor*	1.27	1.33		
Uncertainty heat flux factor*	1.38	1.40		
Trip power level**	60 MW _t	49.5 MW _t		
	<u>Fuel Element</u>		<u>Shim Rod</u>	
	<u>Standard</u>	<u>Phoenix</u>	<u>Standard</u>	<u>Phoenix</u>
<u>Coolant Temperature, °F</u>				
Entering reactor	105	105	105	105
Average exit, hottest element	128	128	121	125
Average exit, hottest channel	137	137	125	133
Maximum exit, hottest channel*	148	151	133	142
Saturation, hot spot	248	252	273	250
<u>Heat Fluxes, BTU/Hr-Ft²</u>				
Average, all elements	3.1+5	3.06+5	2.9+5	3.57+5
Average, hottest element	3.8+5	3.84+5	3.2+5	4.07+5
Average, hottest channel	4.4+5	4.26+5	3.6+5	4.75+5
Hot spot, hottest channel	8.2+5	8.38+5	7.0+5	7.84+5
Maximum hot spot, hottest channel*	1.13+6	1.17+5	9.6+5	1.13+6
Standard deviation	+8.2+4	+8.38+4	+8.2+4	+8.38+4
Burnout, trip power**	1.56+6	1.52+6		1.90+6
Maximum allowed, trip power**	1.31+6	1.27+6		1.66+6
Maximum, trip power**	1.23+6	1.26+6	1.05+6	1.18+6
<u>Fuel Plate Surface Temperature, °F</u>				
Average, hottest element	168	169	153	165
Average, hottest channel	188	189	163	183
Maximum hot spot, hottest channel*	322	308	282	278
Standard deviation	+16	+18	+16	+18
Incipient nucleate boiling	305	303		306
Maximum allowed, hot spot	273	267		270
Hot spot, hottest channel	243	255	208	223

* These calculations were based on nominal conditions multiplied by an uncertainty factor (refer to Table III). Thus they represent the worst possible conditions.

** These conditions were calculated at trip power. Trip power = 1.5x the nominal power = 49.5 MW_t.

TABLE III

Hot Spot - Hot Channel Factors and Uncertainties for
A Standard Fueling and a Phoenix Load in the MTR

<u>Hot Channel Factors</u>	<u>Standard</u>	<u>Phoenix</u>
Variation in velocity from channel to channel	1.25	1.134
Fraction of total power generated in fuel and coolant	0.95	0.985
<u>Heat Flux Factors</u>		
Fraction of reactor power generated in fuel	0.90	0.95
<u>Heat Transfer Coefficient Factors</u>		
Variation in velocity from channel to channel	0.84	0.91
Variation in velocity within the channel	0.96	0.96
<u>Hot Channel Uncertainties</u>		
Reactor power measurement error	1.05	1.05
Plat to plate fuel variation	1.02	1.05
Dimensional effects on mass flow	1.08	1.08
Fuel element power calculation error	1.10	1.12
Total	<u>1.27</u>	<u>1.33</u>
<u>Heat Flux Uncertainties</u>		
Vertical maximum-to-average flux ratio measurement	1.05	1.05
Reactor power measurement error	1.05	1.05
Fuel element power calculation error	1.10	1.12
Fuel variation plate to plate	1.02	1.05
Fuel core alloy thickness variation	1.05	1.04
Fuel core alloy area variation	1.06	1.04
Total	<u>1.38</u>	<u>1.40</u>
<u>Heat Transfer Coefficient Uncertainties</u>		
Correlation equation	0.80	0.80
Local dimensional effects	0.96	0.96
Total	<u>0.77</u>	<u>0.77</u>

MW_t which is within the three standard deviations required by burnout design limits.

3. A reduction in coolant flow area through a stationary fuel element reduces the flow from 620 gpm to 470 gpm at a 40 psi pressure drop.
4. A 13% velocity disparity was calculated for the Phoenix stationary fuel element compared to a 25% velocity disparity reported for the standard element. The difference was attributed to the reduction in flow area for the Phoenix subassembly.
5. The insertion of a flux monitoring wand into channel 14 affected local flow patterns within acceptable limits.
6. The heat transfer area for the Phoenix core was approximately 18% less than a standard fueled core.
7. The maximum nominal heat flux for a stationary Phoenix fuel element at a 33 MW_t reactor operating power level was 8.4×10^5 Btu/hr-ft² compared to 8.2×10^5 Btu/hr-ft² for standard operation at 40 MW_t .
8. The calculated burnout heat flux was 1.52×10^6 . This was evaluated at 60% of predictions made using the Bernath correlation.
9. The maximum value for the corrosive film buildup for a 3000 MWD reactor operating cycle was calculated to be 1.6×10^{-3} inches. This corresponds with 0.3×10^{-3} obtained from experimental results.(11)

HYDRAULIC EXPERIMENT

A dummy stationary Phoenix fuel element was hydraulically tested at Idaho to confirm the analytically determined flow distribution presented in Appendix A. The test loop was a closed cycle low temperature system consisting primarily of a centrifugal pump, counter flow heat exchanger, an orifice type metering device and a test section. These components were connected by 8 inch OD stainless steel pipe. Figure 7 illustrates the system. The pump work energy was used to bring the water to the desired temperature level and was maintained at this level by removing the excess heat in a counterflow heat exchanger. A Wheeler economy pump, operating at approximately 1000 gpm was used to cycle the water through the system. The flow through the test section was measured using a calibrated orifice and was regulated by the excess flow being valved through a bypass loop. A system pressure tap was located on the high side of the pump. The pressure drop across the test section was measured using two pressure taps, one positioned 8 feet upstream and one located 2 feet downstream from the test section. Six differential pressure indicators (DPI) were used to monitor the pressure differential between any two points in the test element. An effort was made to keep the DPI dial near the central increments registered on the face of the gauge. The capacities of the gauges used are shown in Table IV. The gauges were mounted on a board and were equipped with quick couplers for ease of operation. This is illustrated in Figures 8A and 8B.

FIGURE 7 A Simplified Sketch of the Hydraulic Test Facility Loop at Idaho

TABLE IV

Type and Range of Differential Pressure Gauges
Used in the Hydraulic Experiment

<u>Model No.</u>	<u>Brand Name</u>	<u>Range</u>	<u>Increment</u>
720-13949	Barton Instrument Corp.	0.10 psi	0.05 psi
720-13948	Barton Instrument Corp.	0-25 psi	0.10 psi
720-13950	Barton Instrument Corp.	0-50 psi	0.25 psi
MR	Foxboro	0.100 in H ₂ O	1.0 inch H ₂ O
720-13947	Barton Instrument Corp.	0-150 psi	1.0 psi
H43245	Heise	0-400 psi	1.0 psi

TEST SECTION

The test section was designed to simulate as closely as possible the wetted perimeter and hydraulic diameter descriptive of the peripheral channels around the fuel assembly during incore operation. Thus the inside walls of the test section represented outside surfaces of adjacent fuel elements. Additionally the fuel element was positioned in the test section using spacers representative of the upper and lower grid plates in the MTR core. However the test was conducted with the test section in a horizontal position and the in-pyle dynamic head monitoring device was not in position during the experiment. The influence of these two factors was accounted for by reducing the rated flow measured by 10 percent which is in agreement with MTR standard practices and procedures. (9) The 10 percent correction was deduced from a

FIGURE 8A Differential Pressure Gauge Mounting
For the Phoenix Fuel Experiment

FIGURE 8B Rear View of the
Pressure Gauge Mounting
Board Showing The Quick
Coupler Arrangement

statistical analysis based on a number of standard fuel elements tested in the Hydraulic Test Facility (HTF) at Idaho. Figure 9 illustrates the test section positioned in the test loop.

INSTRUMENTATION OF THE TEST ELEMENT

The Phoenix fuel model was a full scale mockup and was instrumented with static pressure taps as illustrated in Figure 10. One of the primary purposes of the experiment was to determine the velocity disparity across the element. Since all the channels could not be instrumented without greatly disrupting the flow patterns, the channels selected for instrumentation were channels containing the maximum and minimum flow. Emphasis was placed on the outside channels for three reasons:

- ° They represented the minimum flow regions
- ° The influence of the flux monitoring wand on the flow in surrounding channels should be determined. The wand was located in channel 14 next to an outside channel.
- ° The most rapid change in velocity (highest change in slope) occurred in the outside channels.

The outside channels selected were 2, 3, 14, 15 and 16 shown in Figure 10. Note that channel 14 was instrumented with three pressure taps equally spaced from the side plate to the center of the channel. This arrangement was used to determine potential interchannel mixing resulting from a pressure disparity in the channel and also indicate the flows which result in a breakdown of the boundary layer.

FIGURE 9 A View of the Test Section Positioned in the Hydraulic Test Facility Low Pressure Loop. The Pressure tap lead mounting device and the location of the leads as they exit from the test section are shown. The direction of flow was from left to right.

- Static differential pressure lead locations. The ΔP was measured from Tap A to Tap B.

FIGURE 10 Location of the Leads and Differential Pressure Taps for the Phoenix Fuel Experiment

Channel 9, a central channel, was selected as representative of the maximum flow region. Data obtained from these channels were used to scribe a smooth pressure drop versus channel curve which represented the flow distribution in the test element. Figure 11 illustrates the inlet and exit head loss coefficients obtained from the data and used in the MACABRE code.

The instrumentation consisted of 16 static pressure taps and lead wires (wire diameters were 0.0425 OD by 0.0275 inch ID) connected externally to six differential pressure indicators. Each pressure tap located above the horizontal midplane of the element was designated position A and corresponding pressure taps below the horizontal midplane were designated position B.* Each lead was equipped with a quick coupler to facilitate any combination of differential pressure measurements desired. Pressure taps 1A, 1B, 2A, 2B, 8A, 8B, 13A, 13B, 13'A, 13'A, 13A, 14A, 14B, 15A and 15B were positioned 3 inches below the top of the fuel plates and 3 inches above the bottom of the fuel plates in their respective channels (See Figure 10). Pressure taps 9A and 9B were located in the upper and lower plenum just above and below the fuel plates respectively. Two additional pressure taps, 16A and 16B which are not shown in Figure 10, were external

* In operation the element is in a vertical position with the flow entering through the top. The test element was in a horizontal position for the test and should not be confused by the above notation. The entering section is the upper and exiting section is referred to as lower.

FIGURE 11 Entrance and Exit Pressure Drop Coefficient for a Stationary Phoenix Fuel Element (Including end fittings and channel entrance and exit effects.)

to the test section and were permanent fixtures of the loop (Figure 7). 16A was positioned 8 feet upstream and 16B was positioned 2 feet downstream from the test section. These two pressure taps were used to give the overall static pressure drop across the test section.

The leads internal to the test element were clustered in the lower plenum and exited through an opening in the extension of the outside fuel plate near the lower spacer comb (Figure 1). Provision was made for each lead to exit from the test section separately as illustrated in Figure 9 and no leakage was detected. This arrangement did alter the flow patterns slightly in the lower plenum which will be discussed later. An effort was made to keep the lead wires positioned next to the side plate when possible. An example differential pressure combination of interest is shown in Table V.

TABLE V

Illustration of Differential Pressure Measurements Made
In the Hydraulic Experiment of a Phoenix Fuel Element

<u>Pressure Tap Combination,</u>	<u>Location of the Differential Pressure Drop Measured</u>
16A to 9A	Through the inlet adapter into the upper plenum
9A to 8A	From upper plenum into coolant channel 8
8A to 8B	Through channel 8 (frictional losses)
8B to 9B	From channel 8 to lower plenum
9B to 16B	From lower plenum through lower end adapter
16A to 16B	Through the test section and should equal the sum of the above losses

PROCEDURES

The test section was positioned into the loop using two flanges fixed at the ends for that purpose. The loop was filled with water and brought to the desired flow rate, temperature (105°F) and pressure level. Sufficient time was allowed at the various flow rates to insure that the data were obtained at steady state conditions. It was assumed that steady state was achieved when no measurable changes occurred during three consecutive time intervals. Differential pressure drop data were gathered for flow rates of 50, 75, 100, 125, and 150* percent of rated flow. Rated flow was defined as the flow obtained with a 40 psi pressure drop across the element. A duplicate effort was made for each flow rate to check the testing procedure. Duplicate differential pressure data were recorded within ± 0.05 percent which was within instrumentation error.

After all data was obtained for the instrumented model, the test section was removed from the test loop and the instrumentation removed from the element. The noninstrumented assembly was reinstated in the test loop and the pressure drop across the element was determined for similar flow rates. This procedure was designed to determine the influence of the instrumentation on pressure drop. It was assumed that nearly all of this loss (the loss attributed to

* In some cases ~~150%~~ of rated flow exceeded the loop capacity. In this instance data for the maximum capacity were recorded.

the instrumentation) occurred in the lower plenum, since the leads were clustered at this location resulting in a large reduction in flow area, and the head loss through the end adapter was adjusted accordingly.

It was necessary to determine the net flow or the flow through the test element to adequately predict the flow patterns in the test element. It was easier to determine the flow external to the test element and ~~subtract this~~ from the total flow. This was done by removing the test element from the test section and plugging the element. The element was reinserted into the test loop and the external flow was recorded for various pressure drops. Again duplicate readings were obtained to verify the testing procedure. The net flow was obtained by plotting these data points as illustrated in Figure 12. The 10% adjustment made in the net flow for MIR operation was due to the gravitational head loss with the element in a vertical position and additional losses encountered in-pipe due to instrumentation.

RESULTS

The results of the hydraulic experiment and analytical predictions are illustrated in Figures 13 through 20. Figure 13 shows the coolant flow rate through an entrance adapter of a stationary Phoenix fuel element in **lb/sec** and ~~gm~~ at a coolant density of 61.92 lb/ft^3 . This corresponds to an inlet temperature of **105°F**.

FIGURE 12 Pressure Drop Versus Flow Rate for a Stationary Phoenix Fuel Element

FIGURE 13 Flow Through an Entrance Adapter of a Stationary Phoenix Fuel Element (Coolant Density = 61.92 lb/ft³)

FIGURE 14 Velocity Versus Flow Rate Through an End Adapter of a Stationary Phoenix Fuel Element (Area = $\pi/144$)

FIGURE 15 Entrance and Exit Pressure Drop for Channel #2
in a Stationary Phoenix Fuel Element

FIGURE 16 Entrance and Exit Pressure Drop for Channel #3
in a Stationary Phoenix Fuel Element

FIGURE 17 Entrance and Exit Pressure Drop for Channel #9
in a Stationary Phoenix Fuel Element

FIGURE 18 Entrance and Exit Pressure Drop for Channel #14
in a Stationary Phoenix Fuel Element

FIGURE 19 Entrance and Exit Pressure Drop for Channel #15 in a Stationary Phoenix Fuel Element. A Flux Monitoring Wand was Inserted into Channel 15 next to the Side Plate

FIGURE 20 Entrance and Exit Pressure Drop for Channel #16 in a Stationary Phoenix Fuel Element. A Flux Monitoring Wand was Inserted in Channel #15, and its influence on Pressure Drop in Channel 16 is illustrated

The rated flow at a pressure drop of 40 psi across the element is also indicated. Figure 14 illustrates the same condition in velocity units of ft/sec and gpm. Figure 15 through 20 illustrates the pressure drop versus flow for channels 2, 3, 9, 14, 15, and 16, respectively. For purposes of input to the MACABRE code, the remaining channels 4, 5, 6, 7, 8, 10, 11, 12, and 13 were considered to have the same flow as channel 9. A flow balance showed that this assumption was within acceptable limits since the total flow calculated agreed well with experimental values.

It was found that the flux monitoring wand inserted into channel 15 had little effect on the flow patterns. Consequently, only one or two data points were obtained with the wand inserted. Figure 20 illustrated the small variation in flow due to inserting the wand.

It was also found that the pressures in channel 9 were uniform azimuthally for all practical flows. However for very high flows, there were small differences in static pressure in channel 13 as measured in locations 13"A, 13'A, and 13A (refer to Figure 10 for location definition). This indicated that there was a destruction of the boundary layer at high flows. The flow at which this occurred was 1.5 times the flow that will be experienced in the reactor. However, at high flows the fuel plate stress is increased considerably with the breakdown of the boundary layer since the pressure differential between adjacent channels may increase and thus is somewhat a measure of the plate strength.

The fuel assembly dimensions were taken before and after the experiment to determine the plate integrity. It was found that some of the plates in the central region had moved sufficiently to cause some concern. Further investigation showed the upper spacer comb had come loose allowing the plate movement. It was believed the spacer pin had not been assembled properly, and thus the plate movement was not a measure of the integrity of the Phoenix fuel element. All the fuel elements will be subjected to testing in the HTF, and only those elements that meet the MTR requirements will be loaded.

The analytical approach is illustrated in Appendix A. The results of that analysis agreed well with the experimental data. The final equations derived were:

$$H_1 = [9.84 + 27.96 (\mu/\dot{\omega})^{.437}] (\dot{\omega}/\rho)^2$$

$$H_2 = 25.33 (\dot{\omega}/\rho)^2$$

$$H_3 = 13.953 + 1593.2 (\mu/\dot{\omega})^{.437}] (\dot{\omega}/\rho)^2$$

$$H_4 = 8.273 + 32.265 (\mu/\dot{\omega})^{.437}] (\dot{\omega}/\rho)^2$$

$$H_t = [57.396 + 1653.425 (\mu/\dot{\omega})^{.437}] (\dot{\omega}/\rho)^2$$

Where:

H_1 = The head loss at the inlet section psi

H_2 = The head loss in the diffuser section psi

H_3 = The head loss in the core section psi

H_4 = The exiting head loss from the core section psi

H_t = The total head loss psi

ω = The mass flow rate in lb/sec

ρ = The density of the flowing fluid lb/ft³

μ = The dynamic viscosity of the flowing fluid

The flow distribution equations derived were:

$$\dot{\omega}_{s-9} = (.54111[1-.92474] + .04111[.87111-1]) 2\dot{\omega} = .07085 \dot{\omega}$$

$$\dot{\omega}_{s-8,10} = (.54111[.92474-.77643] + .04111[.6400-.87111])\dot{\omega} = .07075 \dot{\omega}$$

$$\dot{\omega}_{s-7,11} = (.54111[.77643-.63141] + .04111[.44444-.64000])\dot{\omega} = .07043 \dot{\omega}$$

$$\dot{\omega}_{s-6,12} = (.54111[.63141-.49025] + .04111[.28444-.44444])\dot{\omega} = .06981 \dot{\omega}$$

$$\dot{\omega}_{s-5,13} = (.54111[.49025-.35378] + .04111[.160-.28444]) \dot{\omega} = .06873 \dot{\omega}$$

$$\dot{\omega}_{s-4,14} = (.54111[.35378-.22338] + .04111[.07111-.16000])\dot{\omega} = .06691 \dot{\omega}$$

$$\dot{\omega}_{s-3,15} = (.54111[.22338-.10178] + .04111[.01778-.07111])\dot{\omega} = .06361 \dot{\omega}$$

$$\dot{\omega}_{s-2,16} = (.54111[.10178-0] + .04111[0-.1778])\dot{\omega} = .05434 \dot{\omega}$$

Where the subscript S- X_1, X_2 refers to subchannel X_1 and subchannel X_2 .

The assumption of flow symmetry in the derivation required that two channels equidistant from the center of the element have the **same** quantity of flow and hence the designation of two subchannels indicated by the above notation.

SUMMARY AND CONCLUSIONS

A Phoenix fuel loading of the MTR **was** investigated to determine possible problem areas based on thermal and hydraulic considerations. Since the reactor system is essentially a constant energy system

under normal operation, and since safety requirements are stringent to safeguard human life, it was necessary to predict burnout conditions and identify safe operating power levels. Two assumptions that formed the basis for the study were the power distribution and the coolant flow disparity existing in the fuel element. Both the power distribution and the flow assumptions were verified as much as possible from mockup experiments.

The power distribution was obtained from a small scale irradiation experiment in the Plutonium Recycle Facility. The experiment was conducted by the Experimental Physics group. The results of the experiment were used to identify a critical subassembly (a subassembly operating at the hot spot in the core) on which this analysis was made. The ratio of the subassembly power to the core average, its power distribution, and the location of the hot element in the core were determined.

The second problem, the hydraulic performance of a Phoenix fuel subassembly, was solved using an analytical approach and the results were confirmed by experimentation. A dummy full scale mockup of the Phoenix fuel subassembly was used in the hydraulic experiment which was conducted in the Hydraulic Test Facility at Idaho. The flow disparity determined in this manner formed the basis on which the hydraulic characteristics of the fuel were determined. Although the power and flow descriptions of the fuel element were obtained out of core, in core conditions were simulated as closely as possible.

The design points were clearly specified and were based on the Material Test Reactor operating license.

The results of the analysis illustrate that the performance of the Phoenix fuel load compares favorably with a standard uranium fuel load at a power level of 33 MW_t . Which represents approximately an 18 percent reduction in power. This results for two reasons. The first is that although the coolant to fuel ratio (on a volume basis) is the same for both the standard fuel and the Phoenix fuel, there was an 18% reduction in heat transfer surface area in the Phoenix core. The analysis showed that the hot spot heat flux approached the maximum allowed at a 49.5 MW_t reactor power. The 49.5 MW_t represents an overpower of 1.5 times the nominal operating power and is the power level at which the automatic shutdown device is activated in the MTR.

The second reason was due to the reduced flow rate in the Phoenix subassembly. Commensurate with the reduction in heat transfer surface area was an 18% reduction in flow area which resulted in a 24 percent reduction in flow. At the hot spot in the hottest channel the plate surface temperature was within 12°F of that allowed by the incipient nucleate boiling criteria at a nominal operating power of 33 MW_t . The total margin was 30°F since the design limit contained a built in margin of two standard deviations (18°F) from subcooled boiling conditions.

It was desired to obtain a 50% fuel burnup to insure proper evaluation of the Phoenix effect. A 50% burnup was equivalent to a

3000 MWD reactor operating cycle. The corrosive film buildup for this operation at full power proved not to be a limiting condition.

It was concluded that the performance of the Phoenix fuel assembly at a reactor power level of 33 MW_t was within the safety limits specified by MTR operations. However, this analysis was based on data from the PROCF critical experiment and a stationary fuel subassembly positioned in the hot spot of the core. Should the calculated power shapes change and the power densities in the control rod increase above that now believed practical, a reassessment of these results should be made.

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APPENDIX A

APPENDIX A

The prediction of the head loss through the Phoenix fuel element is not a straight forward calculation due to the geometry of the inlet and exit adapters. The frictional head loss may be predicted using conventional methods but the dynamic head loss is more complicated. It was necessary to subdivide the adapters into sections illustrated in Figure A-1* to calculate the minor head loss with some degree of accuracy. For the entrance adapter there is a sudden contraction to a round pipe-like flow section, (Section 1), a conical shaped diffuser (Section 2), and a sudden contraction to 15 rectangular-like flow channels (Section 3). The exit adapter illustrates a convergent channel (Section 4), a rectangular flow channel (Section 5), and a sudden expansion into a large tank (Section 6). The equations for the head loss through the 6 sections was developed beginning with Section 1 of the entrance adapter.

A-1 Minor Head Loss for a Sudden Contraction

The process of converting pressure head into velocity head is very efficient;''') hence the head loss from the large tank into Section 1 is small compared with the head loss from the vena contraction area to the location where the flow fills the channel (refer to Figure A-1). This results since the velocity head through the vena

* Refer to Figure A-4 for a more detailed illustration of the Phoenix fuel element.

FIGURE A-1 A Simplified Sketch of the End Adapters to a Stationary Phoenix Fuel Element

contracta area is being reconverted into pressure head, an inefficient process. For convenience, define the following terms:

$F_{(x)}$ the force in the X direction

$\Sigma F_{(x)}$ the summation of the forces in the X direction

$\dot{W}$ the mass flow rate (lb/sec)

U the free stream velocity where $dU/dr = 0$

u the variable velocity across the channel ($du/dr \neq 0$
except where $u = U$)

variables with a bar ($\bar{U}$) represent average conditions

variables with a subscript v represents conditions at a vena
contracta

variables with a prime (U') represent inlet conditions

variables without a prime but with a subscript (U_1) represent
exit conditions for Section 1

variables without a subscript or a prime (U) represent general
conditions

F_f is the friction force acting on the flow boundaries (lb)

P is the pressure (psi)

A the cross section area (ft^2)

ρ the mass density (lb/ft^3)

g_c the gravitational constant (ft/sec^2)

H_T the total head loss (ft)

H_{KE} the kinetic head loss (ft)

- C_c the contraction coefficient
 L the length (ft)
 R the radius (a constant in ft)
 r the variable radius along R (ft)
 f the Darcy Weisbach friction coefficient
 D_h the hydraulic diameter (ft)

In general the kinetic head loss for a sudden contraction can be developed as follows. Equating the summation of the forces acting on the boundaries of Section 1 in Figure A-1 to the rate change of momentum for Section 1 gives:

$$\sum F_{(x)} = \dot{W} (\bar{U}_{v1} - \bar{U}_1)$$

the forces acting are:

- $P_1 A_1$ the pressure force at the vena contracta
 $-P_1 A_1$ the pressure force acting at the exit
 $-F_{f1}$ the frictional force acting along the walls

Substitution into the above equation gives:

$$P_{v1} A_{v1} - P_1 A_1 - F_{f1} = \dot{W} (\bar{U}_{v1} - \bar{U}_1)$$

but $A_{v1} = A_1$

$$(P_{v1} - P_1) A_1 - F_{f1} = \dot{W} (\bar{U}_{v1} - \bar{U}_1)$$

For an incompressible fluid, the continuity equation shows

$$\rho_1 A_{v1} \bar{U}_{v1} = \rho A_1 \bar{U}_1 = \dot{W}$$

or

$$\frac{A_{v1}}{A_1} = \frac{\bar{U}_1}{\bar{U}_{v1}}$$

Substitution and multiplication by $\frac{1}{\rho g_c}$ gives:

$$\begin{aligned} \frac{1}{\rho g_c} (P_{v1} - P_1) - \frac{F_{f1}}{\rho g_c A_1} &= \frac{\rho A_1 \bar{U}_1}{\rho g_c A_1} (\bar{U}_{v1} - \bar{U}_1) \\ \frac{1}{g_c} (P_{v1} - P_1) - \frac{F_{f1}}{\rho g_c A_1} &= \frac{2\bar{U}_1}{2g_c} (\bar{U}_{v1} - \bar{U}_1) \end{aligned} \quad A-1$$

The Bernoulli equation can also be applied to Section 1.

the body forces we have:

$$\frac{1}{\rho g_c} (P_{v1} - P_1) = H_1 - \frac{1}{2g_c} (\bar{U}_{v1}^2 - \bar{U}_1^2) \quad A-2$$

Since the Bernoulli equation is derived from energy considerations' and equation 1 is based on the rate change of momentum principle, the two equations are linearly independent and can be used to eliminate the velocities and pressures. Subtracting equation 2 from equation 1 we have:

$$H_1 - \frac{F_{f1}}{\rho A_1 g_c} = \frac{1}{2g_c} (\bar{U}_{v1}^2 - 2\bar{U}_{v1}\bar{U}_1 + 2\bar{U}_1^2 - \bar{U}_1^2)$$

$$H_1 - \frac{F_{f1}}{\rho A_1 g_c} = \frac{1}{2g_c} (\bar{U}_{v1} - \bar{U}_1)^2$$

and multiplying the right hand side by $\bar{U}_1^2/\bar{U}_1^2$ gives

$$H_1 - \frac{F_{f1}}{\rho A_1 g_c} = \frac{\bar{U}_1^2}{2g_c} \left(\frac{\bar{U}_{v1}}{\bar{U}_1} - 1 \right)^2$$

But from above $\frac{\bar{U}_{v1}}{\bar{U}_1} = \frac{A_1}{A'_{v1}}$, also $H_1 - \frac{F_{f1}}{\rho A_1 g_c} = H_{KE1}$

Substitution into the above

$$H_{KE1} = \frac{\bar{U}_1^2}{2g_c} \left(\frac{A_1}{A'_{v1}} - 1 \right)^2 \quad \text{A-3}$$

The area A'_{v1} is not easily determined since it is the flow area at the vena contracta. Define a contraction coefficient (C_c) as:

$$\frac{1}{C_c} = \frac{A_1}{A'_{v1}}$$

and equation 3 becomes

$$H_{KE1} = \frac{\bar{U}_1^2}{2g_c} \left(\frac{1}{C_c} - 1 \right)^2 \quad \text{A-4-1}$$

or in terms of the flow rate (W)

$$H_{KE1} = \frac{1}{2g_c} \left(\frac{\dot{W}}{\rho A_1} \right)^2 \left(\frac{1}{C_c} - 1 \right)^2 \quad \text{A-4-2}$$

which is the general equation for the kinetic head loss for a sudden contraction. C_c has been determined experimentally⁽¹³⁾ for various values of (A_1/A'_1) and some of the values are illustrated in Table A-1.

TABLE A-1

Experimental Determined Head Loss Coefficients
Due to Contraction and Defined by $1/cc = A_1/A'_1$

$\underline{A_1/A'_1}$	$\underline{C_c}$	$\underline{A_1/A'_1}$	$\underline{C_c}$
0.0	0.585	0.6	0.712
0.1	0.624	0.7	0.755
0.2	0.632	0.8	0.813
0.3	0.643	0.9	0.892
0.4	0.659	1.0	1.0
0.5	0.681		

The flow into Section 1 is from a large tank. The tank feeds a number of fuel elements that are spaced on 3.189 inch centers. There is also flow around the outside of the fuel element. Therefore the area (A'_1) to be used is taken as the cross section area of the fuel element. The outside dimensions of the element are approximately 3 inches by 3 inches. The inside radius of the inlet adapter is 1 inch. Therefore (A_1/A'_1) becomes:

$$(A_1/A'_1) = \frac{3.14}{9} = .3489$$

from Table A-1 and $(A_1/A'_1) = .3489$, $C_c = .651$. Substitution for C_c , A_1 , and $g_c = 32.2 \text{ ft/sec}^2$ into equation 4-2 gives the kinetic head loss for Section 1.

$$H_{KE1} = 9.3857 \left(\frac{\dot{W}}{\rho} \right)^2 \quad \text{A4-3}$$

which is a function of the mass flow rate ($\dot{W}$) in lb/sec and the density (ρ) in (lb/ft^3) . The frictional head loss (H_{f1}) equation for Section 1 is:

$$H_{f1} = \frac{f_1 L_1}{2R_1} \frac{\bar{U}_1^2}{2g_c} = \frac{f_1 L_1}{4R_1 g_c} \left(\frac{\dot{W}}{\rho A_1} \right)^2 \quad \text{A5-1}$$

From Figure A-1 we find

$$L_1 = .4844 \text{ ft}$$

$$R_1 = .08333 \text{ ft}$$

$$A_1 = .0218 \text{ ft}^2$$

Then

$$H_{f1} = 94.963 f_1 (\dot{W}/\rho)^2$$

and the total head loss (H_1) for Section 1 becomes

$$H_1 = H_{KE1} + H_{f1}$$

$$H_1 = 9.3857 (\dot{W}/\rho)^2 + 94.963 f_1 (\dot{W}/\rho)^2$$

$$H_1 = (9.3857 + 94.963 f_1) (\dot{W}/\rho)^2 \quad \text{A-6}$$

Equation A-6 can now be evaluated if the friction factor can be determined. Experimental effort at Idaho Nuclear illustrated that the equation

$$f = .0048 + .716 \text{ Re}^{-0.437}$$

gave excellent results for the Darcy Weisbach friction coefficient applicable to a standard MTR element. This expression will be used throughout this analysis. Converting the Reynolds number to flow rate and substitution into equation A-6 gives:

$$\text{Re} = \frac{4\dot{W}}{\mu C}$$

where

$\dot{W}$ = the mass flow rate in lb/sec

μ = the dynamic viscosity

C = the wetted perimeter

for the Phoenix core

$$C = 2(3.14)/12 = 0.52333 \text{ ft}$$

$$f_1 = .0048 + .716 \left(\frac{4}{.52333} \right)^{-0.437} \left(\frac{\dot{W}}{\mu} \right)^{-0.437} = 0.0048 + 0.2944 \left(\frac{\mu}{\dot{W}} \right)^{.037}$$

$$H_1 = \left[9.84 + 27.96 \left(\frac{\mu}{\dot{W}} \right)^{0.4377} \right] \left(\frac{\dot{W}}{\rho} \right)^2 \quad \text{A6-1}$$

A-2 Head Loss for a Diffuser

Fluid flow through a diffuser is one of the most common, and most troublesome problems in practice. The flow involves a conversion

of velocity head to pressure head. The process is much more difficult to analyze than the reverse process of flow in a converging nozzle. In the diverging channel, the viscous effects in the growing boundary layer and the inertial forces dissipate energy which becomes unavailable to the system as adjacent layers of fluid slide over each other. Additional energy is lost if an adverse pressure gradient should develop resulting in the fluid separating from the wall. This flow process becomes unstable and inefficient resulting in large kinetic head losses. We will proceed to analyze the process using the principle of conservation of energy and the continuity equation?

The first law of thermodynamics states that the difference between the heat transferred across the boundaries of a control volume and the work done must appear as a change in internal energy of the substance within the system. Thus

$$dq - \frac{1}{J} dW = de \quad \text{A-7}$$

where

dq = an infinitesimal amount of heat flow per unit mass
between the confining volume and the surroundings

dW = the work done by the system

de = the change in internal energy

and

J = is a constant representing the mechanical equivalent
of heat

* The following analysis is the development of the energy equation.
One may skip to equation A-12 page A-15 without loss of continuity.

For a control volume the work performed on the boundaries of the system is called the shaft work and may be expressed in differential form as

$$\text{shaft work} = dw$$

$$\text{shaft power} = \dot{W} dw$$

A-8

Three other types of work which must be considered are:

The kinetic work involved by the fluid as it moves across the boundaries of the control volume.

Gravitational work of the flowing fluid as it changes elevation

Flow work or displacement work which is the work performed by the fluid as it moves against a pressure head.

Consider the control volume shown in Figure A-2. If a unit of fluid is accelerated or decelerated, according to Newton's second law of inertia a force is involved. In a time (dt) if the element velocity changes from U to $U + dU$, it has accelerated by an amount dU/dt . The mass of the fluid is

$$M = \rho \frac{[(A+dA) + A]}{2} dx$$

Expanding this expression and neglecting second and third order terms gives:

$$M = \rho A dx$$

FIGURE A-2 Displacement of a Fluid Element Against Pressure

The inertial force (dF_K) acting on the element is therefore

$$\begin{aligned} dF_K &= \rho A dx \frac{dU}{dt} \\ &= \rho A \frac{dx}{dt} dU \\ &= \rho A U dU \\ dF_K &= W dU \end{aligned}$$

The kinetic power becomes

$$\begin{aligned} dP_K &= W U dU \\ &= \frac{W}{2} d(U^2) \end{aligned} \quad A-9$$

Should the fluid element change elevation as it moves along the diffuser, there is a change in potential energy. In differential form the force in the potential field is

$$dF_g = \rho g A dx$$

This force is along z and while the element moves along the diffuser, in a unit time dt , its elevation changes from z to $z+dz$. The potential work performed on the incremental element considered in Figure 2 becomes:

$$dW_g = \rho g A dx dz$$

but from the relation $\frac{dx}{dt} = U$ or $dx = U dt$ and substitution into the above gives the potential work as

$$\begin{aligned} dW_g &= \rho g A U dz dt \\ &= \dot{W} g dz dt \end{aligned}$$

and the gravitational power becomes

$$dP_g = Wg dz \quad \text{A-10}$$

The flow work is the work required by the fluid to move against the pressure. In Figure A-2 the element BDCE will be pushed on the left hand side by the pressure p . This indicates that the left hand side surroundings will do work on the elemental volume. The velocity at the face BC is U and in time dt , the face will move a distance Udt . Therefore the work done by the left hand side surroundings on the face BC is

$$pAUdt$$

Using the same arguments the element will do work on the right hand side surroundings. The pressure there is $(p+dp)$, the cross section area is $(A+dA)$ and the velocity is $(U+dU)$. Thus the work done by the element on the right hand side is

$$(p+dp)(A+dA)(U+dU) dt$$

and the net flow work becomes

$$dW_f = (p+dp)(A+dA)(U+dU)dt - pAUdt$$

expanding the equation and neglecting second and third order terms gives

$$\begin{aligned} dW_f &= [pAdU + AUdp + pUdA]dt \\ &= d(pAU)dt \\ &= d\left(p \frac{\dot{W}}{\rho}\right) dt \end{aligned}$$

Consequently the flow power is:

$$dP_f = d(p \frac{\dot{W}}{\rho})$$

or for steady state ($\dot{W} = \text{constant}$)

$$dP_f = \dot{W} d(\frac{P}{\rho}) \quad \text{A-11}$$

The total work per unit mass, as represented in Equation A-7, is the sum of all the powers divided by $\dot{W}$. Substitution into equation A-7 gives

$$dq = \frac{1}{J} [dW + \frac{1}{2} d(U^2) + gdz + d(\frac{P}{\rho})] = de$$

or

$$dq = \frac{1}{J} dw = de + [\frac{1}{2} d(U^2) + gdz + d(\frac{P}{\rho})] \frac{1}{J} \quad \text{A-12}$$

which is the differential form of the energy equation for a control volume. The last three terms are those that distinguish flow through a control volume from the energy balance of the fluid contained in a system. For the purpose of this effort it is assumed

$$dq = dW = de = dx = 0$$

Equation A-12 reduces to

$$\frac{(U^2 - U'^2)}{2g} = - \left(\frac{P}{\rho} - \frac{P'}{\rho} \right)$$

and the head loss for the diffuser becomes

$$H_2 = \frac{U^2 - U'^2}{2g} \quad \text{A-13}$$

In terms of the flow rate $\dot{W}$

$$H_2 = \frac{1}{2g} \left(\frac{1}{A_1^2} - \frac{1}{A^2} \right) \left(\frac{\dot{W}}{\rho} \right)^2$$

for the Phoenix fuel

$$A' = .0218 \text{ ft}^2$$

$$A = .04597 \text{ ft}^2$$

Therefore H_2 becomes

$$H_2 = \frac{1}{64.4} \left[\frac{1}{(.0218)^2} - \frac{1}{(.04597)^2} \right] \left(\frac{\dot{W}}{\rho} \right)^2$$

$$H_2 = 25.33 \left(\frac{\dot{W}}{\rho} \right)^2$$

A-14

A-3 Head Loss for Section 3

The head loss for Section 3 consists of a combination of three flow problems, frictional head loss, head loss for a sudden contraction and head loss for a sudden expansion. The frictional losses can be determined using conventional methods. The head loss equation for a sudden expansion was derived in Section A-1 equation A4-1. The equation for the head loss in a sudden expansion may be developed using the same method described in Section A-1 for a sudden contraction. Summing the forces in the x direction as shown in Figure A-2 and equating them to the rate change of momentum gives:

$$P_1 A - P_2 A - F_v = \dot{W} (\bar{U} - \bar{U}') \quad A-15$$

A-17

BNWL

FIGURE A-3 A Simplified Sketch of the Exit Geometry of a Phoenix Fueled Core. The Core is Considered Symmetric and Only 1/4 the Core is Shown

Where it is assumed that the pressure P' acting at the exit of the fuel-plate channel also acts at the beginning of the larger area A , and F_v is the viscous forces acting as the coolant streams emerge. Using the incompressible Bernoulli energy equation and neglecting body forces gives a second relationship of the pressures, and head loss as:

$$\frac{P' - P}{\rho g_c} = H = \frac{\bar{U}'^2 - \bar{U}^2}{2g_c} \quad \text{A-16}$$

Multiplying equation A-15 by $\frac{1}{\rho g_c A}$ and subtracting equation A-16 gives:

$$\begin{aligned} \frac{(P' - P)}{\rho g_c} - \frac{F_v}{\rho g_c A} &= \frac{2\rho A \bar{U}}{2\rho g_c A} (\bar{U} - \bar{U}') \\ - \frac{(P' - P)}{\rho g_c} + H &= \frac{(\bar{U}'^2 - \bar{U}^2)}{2g_c} \\ H - \frac{F_v}{\rho g_c A} &= \frac{1}{2g_c} (\bar{U}'^2 - 2\bar{U}'\bar{U} + 2\bar{U}^2 - \bar{U}^2) \end{aligned}$$

Where from the continuity equation

$$W = \rho A' \bar{U}' = \rho A \bar{U}$$

and

$$A'/A = \frac{\bar{U}}{\bar{U}'}$$

Thus the kinetic head loss for the sudden expansion becomes:

$$H_{KE} = \frac{1}{2g_c} (\bar{U}' - U)^2 = H - \frac{F_v}{\rho g_c A}$$

now multiply through by $\bar{U}'^2/\bar{U}'^2$ and substitute $A'/A = \bar{U}/\bar{U}'$ gives:

$$H_{KE} = \frac{\bar{U}'^2}{2g_c} \left(1 - \frac{A'}{A}\right)^2 \quad A17-1$$

or by substitution $\bar{U}'_s = \frac{\dot{W}_s}{\rho A'_s}$ the equation becomes

$$H_{KE} = \left(1 - \frac{A_s}{A}\right)^2 \left(\frac{1}{2g_c A_s^2}\right) \left(\frac{\dot{W}_s}{\rho}\right)^2 \quad A17-2$$

where the subscript (s) signifies conditions in the subchannel, $\dot{W}_s$ is the mass flow rate in (lb/sec) exiting from each subchannel and ρ is the fluid density in (lb/ft³). For the Phoenix core and Section 3

$$A_s = .002188 \text{ ft}^2$$

$$A = .003659 \text{ ft}^2$$

Then

$$H_{KE_3} = 523.037 \left(\frac{\dot{W}_s}{\rho}\right)^2 \quad A-18$$

from equation A4-2, the kinetic head loss in entering the subchannels of Section 3 is:

$$H_{KE_3} = \frac{1}{2g_c A_s^2} \left(\frac{\dot{W}_s}{\rho}\right)^2 \left(\frac{1}{C_c} - 1\right)^2$$

The value of (C_c) as determined from Table A-1 and the area ratio

$(A_s/A') = (.002188/.003659) = .598$ as $(C_c = .7114)$ and H'_{KE3} becomes

$$H'_{KE3} = 533.32 \left(\dot{W}_s / \rho \right)^2 \quad A-19$$

The frictional head loss for Section 3 (refer to Figure A-1) can be written

$$H_{f3} = \frac{f'_3 L'_3}{2D'_{hc} A'^3_3} \left(\frac{\dot{W}}{\rho} \right)^2 + \frac{f_{s3} L_{s3}}{2D_{hs3} g_c A^2_{s3}} \left(\frac{\dot{W}_s}{\rho} \right)^2$$

Where

$$L'_3 = .2552 \text{ ft}$$

$$L_{s3} = 2.09375 \text{ ft}$$

$$D'_{h3} = .23084 \text{ ft}$$

$$A'_3 = .0534 \text{ ft}^2$$

$$D_{hs3} = .018980 \text{ ft}$$

$$A_{s3} = .002188 \text{ ft}^2$$

$$H_{f3} = 6.02 f'_3 \left(\frac{\dot{W}}{\rho} \right)^2 + 357810 f_{s3} \left(\frac{\dot{W}_s}{\rho} \right)^2 \quad A-20$$

and H_3 becomes

$$\begin{aligned} H_3 &= H_{f3} + H'_{KE3} + H_{KE3} \\ &= 6.02 f'_3 \left(\frac{\dot{W}}{\rho} \right)^2 + 357810 f_{s3} \left(\frac{\dot{W}_s}{\rho} \right)^2 + 533.32 \left(\frac{\dot{W}_s}{\rho} \right)^2 + \\ &\quad + 523.037 \left(\frac{\dot{W}_s}{\rho} \right)^2 \end{aligned} \quad A-21$$

As before in Section A-1 the friction factor is defined by

$$f = .0048 + .716 \left(\frac{4\dot{W}}{\mu C} \right)^{-.437}$$

Therefore for the Phoenix core

$$f_3 = .0048 + .716 \left(\frac{4}{.92533} \frac{\dot{W}}{\mu} \right)^{-.437} = .0048 + 3.776 \left(\frac{\mu}{\dot{W}} \right)^{.437}$$

$$f_{s3} = .0048 + .716 \left(\frac{4}{.46117} \frac{\dot{W}_s}{\mu} \right)^{-.437} = .0048 + .27856 \left(\frac{\mu}{\dot{W}_s} \right)^{.437}$$

substitution into A-21 gives

$$H_3 = [6.02(.0048 + .3776 \left(\frac{\mu}{\dot{W}} \right)^{.437})] \left(\frac{\dot{W}}{\rho} \right)^2 + 1056.4 \left(\frac{\dot{W}_s}{\rho} \right)^2 +$$

$$357810 \left[.0048 + 0.27856 \left(\frac{\mu}{\dot{W}_s} \right)^{.437} \right] \left(\frac{\dot{W}_s}{\rho} \right)^2$$

$$H_3 = [.0289 + 2.273 \left(\frac{\mu}{\dot{W}} \right)^{.437}] \left(\frac{\dot{W}}{\rho} \right)^2 + \left[2773.9 + 99671.6 \left(\frac{\mu}{\dot{W}_s} \right)^{.437} \right] \left(\frac{\dot{W}_s}{\rho} \right)^2$$

A21-1

Equation A21-1 can be evaluated if $\dot{W}_s$ can be determined. This parameter requires some knowledge of the velocity profile in the plenum. The boundary conditions assumed descriptive and appropriate are:

- (1) The velocity is a maximum at the center of the plenum
- (2) The velocity profile along (y) at $X=0$ is similar to the velocity profile along x at $Y=0$ where x and y is the coordinate system originating at the center of the plenum. X is perpendicular to the flow channels.

- (3) The velocity is equal to zero at the face of the plenum.
- (4) The velocity profile flattens in the (y) direction with respect to distance from the center of the plenum along the x axis.

It was assumed that the flow was fully developed after entering Section 3 just before it flows into the subchannels. The channel geometry was assumed rectangular and the flow pattern symmetric. The universal velocity profile law was modified to apply in this Section in the form

$$\frac{u_x}{U} = b\left(1 - \frac{x}{X}\right)^a + cf(x) \quad \text{A-22}$$

$$\frac{u_y}{u_x} = e\left(1 - \frac{y}{Y}\right)^a + gf(y) \quad \text{A-23}$$

where

- u_x = the velocity at x measured from the center of the plenum perpendicular to the subchannels
- U = the velocity at the center of the plenum just prior to entering the subchannel
- u_y = the velocity measured from the center line of the plenum parallel to the subchannels. (The center line is the x axis).
- X = the distance from the center of the plenum to the surface of the plenum measured perpendicular to the subchannels.

x = the variable distance along X

Y = the (y) coordinate associated with X . It is the distance from a plane perpendicular to the subchannels, and through the center of the plenum, and extends to the surface of the plenum parallel to the subchannels.

y = the variable distance along Y .

$f(x)$ = same function of x

$f(y)$ = same function of y

There are six conditions that equations A-22 and A-23 must satisfy.

at $x = y = 0$ $u_x = u_y = U$ (the center velocity)

at $x = y = 0$ $\frac{du_x}{dx} = \frac{du_y}{dy} = 0$

at $x = X$ $u_x = 0$

at $y = Y$ $u_y = 0$

Consider the derivative of equation A-22 and A-23

$$\frac{du_x}{dx} = -\frac{ba}{X} \left(1 - \frac{x}{X}\right)^{a-1} + c \frac{df(x)}{dx}$$

$$\frac{du_y}{dy} = -\frac{ea}{Y} \left(1 - \frac{y}{Y}\right)^{a-1} + g \frac{df(y)}{dy}$$

$$\text{at } x = y = 0, \quad \frac{du_y}{dy} = \frac{du_x}{dx} = 0$$

$$\frac{ba}{X} = c \frac{df(x)}{dx} \quad \text{and} \quad \frac{ea}{Y} = g \frac{df(y)}{dy}$$

Integration gives for $f(x)$ and $f(y)$

$$f(x) = \frac{ba}{cX} (x + d) \quad \text{and} \quad f(y) = \frac{ea}{gY} (y + h)$$

substitution into equations A-22 and A-23 gives:

$$\frac{u_x}{U} = b(1 - \frac{x}{X})^a + \frac{ba}{X} (x+d) \quad \text{or} \quad \frac{u_x}{U} = b \left[(1 - \frac{x}{X})^a + a(d' + \frac{x}{X}) \right]$$

$$\frac{u_y}{U} = e(1 - \frac{y}{Y})^a + \frac{ea}{Y} (y+h) \quad \text{or} \quad \frac{u_y}{U} = e \left[(1 - \frac{y}{Y})^a + a(h' + \frac{y}{Y}) \right]$$

Now applying condition one to these expressions

$$\text{at } x = y = 0 \quad u_x = u_y = U$$

$$u_x = bU \left[(1 - \frac{0}{X})^a + a(d' + \frac{0}{X}) \right] = bU(1 + ad')$$

$$u_y = eU \left[(1 - \frac{0}{Y})^a + a(h' + \frac{0}{Y}) \right] = eU(1 + ah')$$

The last condition gives

$$\text{at } x = X \text{ and } y = Y, \quad u_x = u_y = 0$$

$$u_x = 0 = (1 - \frac{X}{X})^a + a(d' + \frac{X}{X}) \quad \text{or} \quad d' = -1$$

$$u_y = 0 = (1 - \frac{Y}{Y})^a + a(h' + \frac{Y}{Y}) \quad \text{or} \quad h' = -1$$

Therefore from condition 1

$$u_x = bU(1-a) = U \text{ or } b = \frac{1}{(1-a)}$$

$$u_y = eU(1-a) = U \text{ or } c = \frac{1}{(1-a)}$$

substitution gives for the velocity distribution equations

$$u_x = \frac{U}{(1-a)} \left[\left(1 - \frac{x}{X}\right)^a - a\left(1 - \frac{x}{X}\right) \right] \quad \text{A-24}$$

$$u_y = \frac{u_x}{(1-a)} \left[\left(1 - \frac{y}{Y}\right)^a - a\left(1 - \frac{y}{Y}\right) \right] \quad \text{A-25}$$

The next step is to evaluate U in terms of the flow rate $\dot{W}$. This can be done by integrating Equation A-24 and A-25 and solving for the average velocity $\bar{U}$.

$$\bar{U}_x = \frac{U}{(1-a)X} \int_0^X \left(1 - \frac{x}{X}\right)^a - a\left(1 - \frac{x}{X}\right) dx$$

$$\bar{U}_x = \frac{U}{(1-a)X} \left[\frac{-X}{(a+1)} \left(1 - \frac{x}{X}\right)^{a+1} + \frac{aX}{2} \left(1 - \frac{x}{X}\right)^2 \right]_0^X$$

$$\bar{U}_x = \frac{U}{(1-a)} \left[\frac{1}{(a+1)} - \frac{a}{2} \right]$$

and for $\bar{U}_y$

$$\bar{U}_y = \frac{u_x}{(1-a)Y} \int_0^Y \left(1 - \frac{y}{Y}\right)^a - a\left(1 - \frac{y}{Y}\right) dy$$

$$\bar{U}_y = \frac{u_x}{(1-a)} \left[\frac{1}{a+1} - \frac{a}{2} \right]$$

and the average velocity in the plenum becomes:

$$\bar{U} = \frac{\bar{u}_x}{(1-a)} \left[\frac{1}{a+1} - \frac{a}{2} \right]^2 \quad \text{A-26}$$

$$\bar{U} = \frac{U}{(1-a)^2} \left[\frac{1}{a+1} - \frac{a}{2} \right]^2$$

for the continuity equation

$$\rho a \bar{U} = \dot{W} \frac{(2X)(2Y) U}{[1-a]^2} \left[\frac{1}{a+1} - \frac{a}{2} \right]^2$$

or (U) the maximum velocity in the plenum prior to entering the subchannels becomes:

$$U = \frac{\dot{W}(1-a)^2}{\rho(2X)(2Y) \left[\frac{1}{a+1} - \frac{a}{2} \right]^2} = \frac{(1-a)^2 \dot{W}}{4\rho XY \left[\frac{1}{a+1} - \frac{a}{2} \right]^2} \quad \text{A-27}$$

In the integration process u_x is the centerline velocity in the plenum perpendicular to the subchannels and just prior to entering the subchannels. If it is assumed that the area symmetric to the subchannels contain the mass rate of flow associated with each corresponding subchannel, then the flow and average velocity in the subchannels may be determined. To obtain the descriptive velocities within the subchannels, the equation A-24 (equation for u) needs to be modified by the area ratio A'_s/A_s . Substitution of A-27 into A-24 and including the area modification to obtain subchannel velocities gives:

$$u_s = \frac{(1-a) \dot{W} A_s}{4\rho XY \left[\frac{1}{a+1} - \frac{a}{2} \right]^2 A_s} \left[\left(1 - \frac{x}{X}\right)^a - a \left(1 - \frac{x}{X}\right) \right]$$

or the average velocity for each channel along the centerline is:

$$\bar{U}_{xs} = \frac{(1-a) \dot{W} A_s}{4\rho XY (X_o - X_i) \left[\frac{1}{a+1} - \frac{a}{2} \right]^2 A_s} \int_{X_i}^{X_o} \left(1 - \frac{x}{X}\right)^a - a \left(1 - \frac{x}{X}\right) dx$$

$$\bar{U}_{xs} = \frac{(1-a) \dot{W} A_s X}{4\rho XY (X_o - X_i) \left[\frac{1}{a+1} - \frac{a}{2} \right]^2 A_s} \left[-\frac{1}{(a+1)} \left(1 - \frac{x}{X}\right)^{a+1} + \frac{a}{2} \left(1 - \frac{x}{X}\right)^2 \right]_{X_i}^{X_o}$$

Note that $2Y(X_o - X_i) = A$

$$\bar{U}_{xs} = \frac{(1-a) \dot{W}}{2\rho \left[\frac{1}{a+1} - \frac{a}{2} \right]^2 A_s} \left\{ \frac{1}{a+1} \left[\left(1 - \frac{X_i}{X}\right)^{a+1} - \left(1 - \frac{X_o}{X}\right)^{a+1} \right] + \right.$$

$$\left. \frac{a}{2} \left[\left(1 - \frac{X_o}{X}\right)^2 - \left(1 - \frac{X_i}{X}\right)^2 \right] \right\}$$

substitution into equation A-26 gives the average velocity for each channel as:

$$\bar{u}_s = \frac{\dot{W}}{2\rho A_s \left[\frac{1}{a+1} - \frac{a}{2} \right]} \left\{ \frac{1}{a+1} \left[\left(1 - \frac{X_i}{X}\right)^{a+1} - \left(1 - \frac{X_o}{X}\right)^{a+1} \right] + \frac{a}{2} \right.$$

$$\left. \left[\left(1 - \frac{X_o}{X}\right)^2 - \left(1 - \frac{X_i}{X}\right)^2 \right] \right\}$$

Since $\rho A_s \bar{u}_s = \dot{W}_s$ or $\bar{u}_s = \dot{W}_s / \rho A_s$

we have

$$\dot{W}_s = \frac{\dot{W}}{2 \left[\frac{1}{a+1} - \frac{a}{2} \right]} \left\{ \frac{1}{a+1} \left[\left(1 - \frac{x_i}{X}\right)^{a+1} - \left(1 - \frac{x_o}{X}\right)^{a+1} \right] + \frac{a}{2} \left[\left(1 - \frac{x_o}{X}\right)^2 - \left(1 - \frac{x_i}{X}\right)^2 \right] \right\} \quad \text{A-29}$$

Equations A-28 and A-29 are the general equations for the average velocity and the mass flow rate in each channel respectively. The parameters x_i and x_o represent the width of the plenum area that is to be associated with each channel. The next step is to evaluate the constant (a). To do this we reference the standard MIR fuel element geometry for which there is available data.⁽⁹⁾ A velocity disparity of 25% has been measured experimentally in the core. The characteristic dimensions are:

$$X = 1.494 \text{ inches}$$

$$x_i = 0 \quad \text{for central channel}$$

$$x_o = .166 \quad \text{for central channel}$$

$$x_i = 1.328 \quad \text{for outside channel}$$

$$x_o = 1.494 \quad \text{for outside channel}$$

Then from equation A-28 and the relationship

$$\frac{u_{s-o}}{u_{si}} = .75$$

$$.75 = \frac{\frac{1}{a+1} \left[\left(1 - \frac{1.328}{1.494}\right)^{a+1} - \left(1 - \frac{1.494}{1.494}\right)^{a+1} \right] + \frac{a}{2} \left[\left(1 - \frac{1.494}{1.494}\right)^2 - \left(1 - \frac{1.328}{1.494}\right)^2 \right]}{\frac{1}{a+1} \left[\left(1 - \frac{0}{1.494}\right)^{a+1} - \left(1 - \frac{.166}{1.494}\right)^{a+1} \right] + \frac{a}{2} \left[\left(1 - \frac{.166}{1.494}\right)^2 - \left(1 - \frac{0}{1.494}\right)^2 \right]}$$

$$.75 = \frac{\frac{1}{a+1} \left[(.11111)^{a+1} - 0 \right] - .006173 a}{\frac{1}{a+1} \left[(1-.88889)^{a+1} \right] - .10494 a}$$

when $a = .134$ we have $.75 = .7502$ which is sufficient for this effort.

Thus a value of $.134$ is assumed for the Phoenix core since the flow enters through similar end adapters. Equation A-28 and A-29 need to be multiplied by 2 when calculating the flow through the central coolant channel since the x and y axis split the channel into equal parts. The characteristic parameters for the Phoenix core are listed in Table A-2. The flow rate in the channels becomes:

TABLE A-2

Characteristics Parameters for the Phoenix Core
Used in the Analytical Analysis

(X = 1.4925 inches)

Channel	x_i	x_o	$(1-x_i/X)^{1.17}$	$(1-x_o/X)^{1.17}$	$(1-x_i/X)^2$	$(1-x_o/X)^2$
9	.0	.0995	1	.92474	1	.87111
10	.0995	.2985	.92474	.77643	.87111	.64000
11	.2985	.4975	.77643	.63141	.64000	.44444
12	.4975	.6965	.63141	.49025	.44444	.28444
13	.6965	.8955	.49025	.35378	.28444	.16000
14	.8955	1.0945	.35378	.22338	.16000	.07111
15	1.0945	1.2935	.22338	.10178	.07111	.01778
16	1.2935	1.4925	.10178	0	.01778	0

$$\begin{aligned}
\dot{W}_{S-9} &= (.54111 [1-.92474] + .04111[.8711-1]) \dot{W} = .07085 \dot{W} \\
\dot{W}_{S-8,10} &= (.54111 [.92474-.77643] + .04111[.6400-.87111]) \dot{W} = .07075 \dot{W} \\
\dot{W}_{S-7,11} &= (.54111 [.77643-.63141] + .04111[.44444-.64000]) \dot{W} = .07043 \dot{W} \\
\dot{W}_{S-6,12} &= (.54111 [.63141-.49025] + .04111[.28444-.44444]) \dot{W} = .06981 \dot{W} \\
\dot{W}_{S-5,13} &= (.54111 [.49025-.35378] + .04111[.1600-.28444]) \dot{W} = .06873 \dot{W} \\
\dot{W}_{S-4,14} &= (.54111 [.35378-.22338] + .04111[.07111-.16000]) \dot{W} = .06691 \dot{W} \\
\dot{W}_{S-3,15} &= (.54111 [.22338-.10178] + .04111[.01778-.07111]) \dot{W} = .06361 \dot{W} \\
\dot{W}_{S-2,16} &= (.54111 [.10178-0] + .04111[0-.1778]) \dot{W} = .05434 \dot{W}
\end{aligned}$$

For a check we have:

$$\dot{W} = \sum_{i=1}^{15} \dot{W}_{S-i} = 1.00001 \dot{W}$$

Thus for the central channel equation A-29 becomes:

$$\frac{W_S}{\rho} = .07085 \frac{\dot{W}}{\rho}$$

the head loss equation A21-1 can now be written in terms of (W/ρ) as

$$\begin{aligned}
H_3 &= [.0289 + 2.273 \left(\frac{\mu}{W} \right)^{.437}] \left(\frac{\dot{W}}{\rho} \right)^2 + [2773.9 + 99671.6 \left(\frac{\mu}{.07085 \dot{W}} \right)^{.437}] \\
&\quad \left(\frac{.07085 \dot{W}}{\rho} \right)^2 \\
H_3 &= [13.953 + 1593.2 \left(\frac{\mu}{W} \right)^{.437}] \left(\frac{\dot{W}}{\rho} \right)^2
\end{aligned}$$

The head loss equation A-30 was based on the central subchannel in the core since the boundary layer influence was least at this location.

A-4 Head Loss for Sections 4 and 5

The kinetic head loss for a convergent channel is small compared to the frictional effects and the kinetic loss experienced in the sudden expansion of the fluid as it exits from Section 5 (refer to Figure A-1). The general equation for this type of kinetic head loss was developed in Section A-3 equation A17-1.

$$H_{KE} = \frac{\bar{U}^2}{2g_c} \left(1 - \frac{A'}{A}\right)^2$$

Applying this equation for the exit at Section 5 gives:

$$H_{KE5} = \frac{\bar{U}_5^2}{2g_c} \left(1 - \frac{A'_5}{A}\right)^2$$

where

$\bar{U}_5$ = the average velocity exiting from the fuel element

A'_5 = the cross section area at the exit of the fuel element

A = the equivalent expansion area

for the Phoenix core:

$$A'_5 = .02604 \text{ ft}^2$$

$$A = .0625 \text{ ft}^2$$

$$\bar{U}_5^2 = \left(\frac{\dot{W}}{\rho A} \right)^2$$

Therefore

$$H_{KE5} = \frac{1}{2g_c A^2} (1 - .41664)^2 \left(\frac{\dot{W}}{\rho} \right)^2$$

$$H_{KE5} = 7.793 \left(\frac{\dot{W}}{\rho} \right)^2$$

A-31

The frictional losses are

$$H_{f4,5} = \frac{f_4 L_4}{2\bar{D}_{h4} g_c \bar{A}_4^2} \left(\frac{\dot{W}}{\rho} \right)^2 + \frac{f_5 L_5}{2D_{h5} g_c A_5^2} \left(\frac{\dot{W}}{\rho} \right)^2$$

For the Phoenix core

$$L_4 = .5 \text{ ft}$$

$$L_5 = .46354 \text{ ft}$$

$$\bar{A}_4 = \frac{2A_4 A_4}{A_4 + A_4} = .03325 \text{ ft}^2$$

$$A_5 = .02604 \text{ ft}^2$$

$$\bar{D}_{h4} = .2058 \text{ ft}$$

$$D_{h5} = .16128 \text{ ft}$$

$$H_{f4,5} = (34.124 f_4 + 65.83 f_5) \left(\frac{\dot{W}}{\rho} \right)^2$$

A-32

The total head loss for Section 4 and 5 becomes:

$$H_{4,5} = H_{KE5} + H_{f4,5}$$

$$H_{4,5} = (7.793 + 34.124 f_4 + 65.83 f_5) \left(\frac{\dot{W}}{\rho} \right)^2$$

A-33

f_4 and f_5 are calculated with the relationship

$$f_i = .0048 + .716 \left(\frac{4\dot{W}}{\mu C} \right)^{.437}$$

therefore

$$f_4 = f_5 = .0048 + .3228 \left(\frac{\mu}{\dot{W}} \right)^{.437}$$

Equation A-33 becomes

$$\begin{aligned} H_{4,5} &= (7.793 + 99.954 [.0048 + .3228 \left(\frac{\mu}{\dot{W}} \right)^{.437}]) \left(\frac{\dot{W}}{\rho} \right)^2 \\ H_{4,5} &= [8.273 + 32.265 \left(\frac{\mu}{\dot{W}} \right)^{.437}] \left(\frac{\dot{W}}{\rho} \right)^2 \end{aligned} \quad \text{A-34}$$

A-5 Summary of Equations

The total head loss for the Phoenix fuel element can be predicted by summing the head loss across each section

$$H_T = H_1 + H_2 + H_3 + H_{4,5}$$

from equations

$$H_1 = [9.84 + 27.96 \left(\frac{\mu}{\dot{W}} \right)^{.437}] \left(\frac{\dot{W}}{\rho} \right)^2 \quad \text{inlet section} \quad \text{A6-1}$$

$$H_2 = 25.3 \left(\frac{\dot{W}}{\rho} \right)^2 \quad \text{diffuser section} \quad \text{A-14}$$

$$H_3 = [13.953 + 1593.2 \left(\frac{\mu}{\dot{W}} \right)^{.437}] \left(\frac{\dot{W}}{\rho} \right)^2 \quad \text{core section} \quad \text{A-30}$$

$$H_{4,5} = \left[8.273 + 32.265 \left(\frac{\mu}{\dot{W}} \right)^{.437} \right] \left(\frac{\dot{W}}{\rho} \right)^2 \quad \text{exit section} \quad \text{A-34}$$

the total head loss equation becomes:

$$H_T = \left[57.396 + 1653.425 \left(\frac{\mu}{\dot{W}} \right)^{.437} \right] \left(\frac{\dot{W}}{\rho} \right)^2 \quad \text{A-35}$$

Where

H_T = the total head loss in feet of flowing fluid

$\dot{W}$ = the mass flow rate in lb/sec

ρ = the density of the flowing fluid lb/ft³

μ = the dynamic viscosity of the flowing fluid

FIGURE A-4 Detail of Phoenix Fuel Assembly

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