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A THEORETICAL EXPLANATION
OF A VERTICAL FLIGHT FREE-FALL
BALLISTIC CHART

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A THEORETICAL EXPLANATION OF A VERTICAL FLIGHT
FREE-FALL BALLISTIC CHART

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ABSTRACT

A theoretical analysis of the translational motion of objects dropped from rest at altitude is performed to explain the inherent characteristics of a ballistic chart that was developed by empirical means.

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Summary

An empirical free-fall ballistic chart that is used very frequently at Sandia Laboratories gives the velocity of an object that was released from rest if the distance fallen and the time of fall are known. This chart, developed using experimental drop test data and some supplementary point mass trajectory calculations, gives accurate results that are independent of altitude and vehicle ballistic coefficient whenever the distance fallen is less than approximately 10,000 feet and the ballistic coefficient is constant. The chart is applicable to freely falling stable vehicles, parachute retarded vehicles, and tumbling bodies that rotate rapidly enough to satisfy the condition of a constant average drag coefficient.

This analysis develops the inherent theoretical basis for the ballistic chart and shows why, through the unique presentation, the chart does not depend explicitly on either altitude or body characteristics.

List of Symbols

- A - Aerodynamic reference area, ft^2 .
 C_D - Drag coefficient.
g - Acceleration of gravity, ft/sec^2 .
m - Vehicle mass, slugs.
S - fall distance, ft.
t - Time, seconds.
V - Velocity, ft/sec .
W - Vehicle weight, lbs.
 β - Vehicle ballistic coefficient, $W/C_D A$ lb/ft^2 .
 γ - Constant defined in Equation 7, $1/\text{sec}$.
 λ - Atmospheric scale height for exponential density, $1/\text{ft}$.
 ρ - Atmospheric density, slugs/ft^3 .

A THEORETICAL EXPLANATION OF A VERTICAL FLIGHT FREE-FALL BALLISTIC CHART

Introduction

Several years ago, Mr. W. V. Hereford of Sandia Laboratories developed a very useful ballistic chart (Figure 1) that gives the velocity of an object falling vertically from rest if the time of flight and the distance fallen are known. The chart was developed using a large amount of experimental and numerical trajectory data for all types of vehicles, with and without parachutes, dropped at altitude with little or no velocity. The chart has been used extensively by the Aerospace Nuclear Safety Department of Sandia Laboratories for the estimation of SNAP capsule terminal velocities, and it is known to give accurate results.

One interesting characteristic of the chart is that altitude (atmospheric density) and vehicle characteristics (weight, area, and drag coefficient) do not appear explicitly. The purpose of this analysis is to develop a simplified theory for the chart and to show why the results do not depend explicitly on the altitude and the vehicle parameters.

Theory

The translational motion of a body with constant mass falling vertically through the atmosphere of a nonrotating planet is governed by the differential equation

$$m \frac{dv}{dt} = W - \frac{\rho A C_D}{2} v^2, \quad (1)$$

or

$$\frac{dV}{dt} = g - \frac{\rho A C_D}{2m} V^2 . \quad (2)$$

The ballistic coefficient of the vehicle is given by

$$\beta \equiv \frac{W}{C_D A} , \quad (3)$$

and Eq. (2) can be written as

$$\frac{dV}{dt} = g \left[1 - \frac{\rho V^2}{2\beta} \right] . \quad (4)$$

Transferring from time to distance travelled as the independent variable in Eq. (4) gives

$$\frac{d(V^2)}{dS} = 2g \left[1 - \frac{\rho V^2}{2\beta} \right] . \quad (5)$$

If the analysis is now restricted to changes in altitude of approximately 10,000 feet or less so that the atmospheric density can be assumed constant, $\rho = \rho_o$, and if the vehicle ballistic coefficient is constant, Eqs. (4) and (5) have the solutions, respectively,

$$V^2 = \frac{2\beta}{\rho_o} \left[\frac{e^{\gamma t} - 1}{e^{\gamma t} + 1} \right]^2 = \frac{2\beta}{\rho_o} \tanh^2 \left(\frac{\gamma t}{2} \right) , \quad (6)$$

where

$$\gamma = g \sqrt{\frac{2\rho_o}{\beta}} , \quad (7)$$

and

$$v^2 = \frac{2\beta}{\rho_0} \left[1 - e^{-\frac{\rho_0 g}{\beta} S} \right] \quad (8)$$

for the initial conditions that $V = 0$ when S and t are zero.

Equations (6) and (8) can be combined to give

$$S = \frac{2\beta}{\rho_0 g} \left[\ln \left(\frac{e^{\gamma t} + 1}{2} \right) - \frac{\gamma t}{2} \right], \quad (9)$$

or alternately

$$t = \frac{1}{g} \sqrt{\frac{\beta}{2\rho_0}} \ln \left[\frac{1 + \sqrt{1 - e^{-\frac{\rho_0 g}{\beta} S}}}{1 - \sqrt{1 - e^{-\frac{\rho_0 g}{\beta} S}}} \right]. \quad (10)$$

The effects of atmospheric and vehicle characteristics on the motion of the body appear in Eqs. (6) through (10) only as the ratio ρ_0/β , and it is this behavior that eliminates the explicit appearance of altitude and ballistic coefficient in the ballistic chart shown in Figure 1. If either Eq. (6) or (8) could be solved explicitly for β/ρ_0 , the resulting equation could be substituted into either Eq. (9) or (10) to give an equation relating distance and time, with velocity as a parameter, which would yield the curves of Figure 1. Although this result cannot be obtained algebraically, it can be achieved without difficulty by either graphical or numerical methods.

Equations (6) and (8) were used to calculate the typical variations of velocity with distance fallen and time of fall, shown in Figures 2 and 3, respectively, using representative values of β/ρ_0 corresponding to altitudes from 5000 to 45,000 feet and ballistic coefficients ranging from 10 to 200 lb/ft². Equation (9) was used to calculate the variation of distance fallen with time shown in Figure 4. Figures 2, 3, and 4 are typical of common ballistic charts having altitude and ballistic coefficients as parameters.

In the pretest planning of free-fall drop tests, it is often necessary to know the minimum drop altitude that will allow the vehicle to reach terminal velocity. This information can be obtained by writing Eq. (8) in the form

$$S = -\frac{\beta}{\rho_0 g} \ln \left[1 - \frac{V^2}{V_T^2} \right], \quad (11)$$

where

$$V_T^2 = \frac{2\beta}{\rho_0}. \quad (12)$$

Equation (11) was used to calculate the altitude necessary to allow the vehicle to reach 99 percent of V_T shown in Figure 5.

The data shown in Figures 2 and 3 were used to construct the theoretical free-fall ballistic chart shown in Figure 6. Several points from the empirical chart are plotted for comparison. It can be seen that the agreement is good. For large fall distances, the empirical data are slightly above the theoretical curves; this demonstrates the effect of the actual nonconstant density variation with altitude in the empirical method.

In general, efforts to include variable atmospheric density in the analysis encounter difficulties in the integration of the equations. For variable density, Eq. (5) generally can be integrated without difficulty to give the square of the velocity in terms of either atmospheric density or distance travelled, but the integration for distance in terms of time is generally very difficult, if not impossible, to obtain analytically. For example, with the exponential density variation

$$\rho = \rho_0 e^{\lambda S}, \quad (13)$$

Eq. (5) has the series solution

$$v^2 = \frac{2g}{\lambda} e^{\frac{g\rho_0}{\lambda\beta} \frac{\rho}{\rho_0}} \left\{ \ln \frac{\rho}{\rho_0} - \frac{g\rho_0}{\lambda\beta} \left(\frac{\rho}{\rho_0} - 1 \right) + \left(\frac{g\rho_0}{\lambda\beta} \right)^2 \frac{1}{2 \cdot 2!} \left(\frac{\rho^2}{\rho_0^2} - 1 \right) - \right. \\ \left. \left(\frac{g\rho_0}{\lambda\beta} \right)^3 \frac{1}{3 \cdot 3!} \left(\frac{\rho^3}{\rho_0^3} - 1 \right) + \dots \right\} \quad (14)$$

with the initial conditions $V = 0$ when $S = t = 0$, ($\rho = \rho_0$). It can be seen from Eq. (14) that, for a given fall distance (i.e., density ratio), the velocity of a vertically falling vehicle depends only upon the ratio of the initial atmospheric density to the vehicle ballistic coefficient.* As a result, the chart presented in Figure 1 still would not depend on either altitude or vehicle parameters and the chart can be extended to larger fall distances by using experimental data or numerical integration of the equations of motion with the one restriction of a constant vehicle ballistic coefficient.

Conclusions

A theoretical analysis of the translational motion of vertically falling objects has been performed to explain the inherent characteristics of an empirical free-fall ballistic chart. It has been shown that, for a given planetary model, the motion of an object falling from rest with a constant ballistic coefficient depends on the single parameter ρ_0/β . Because of this behavior, the unique presentation of the ballistic chart does not contain an explicit dependence on release altitude or vehicle size, weight, or shape.

*This characteristic of vertically falling bodies is valid for any density variation that can be written as $\rho = \rho_0 f(S)$ where $f(S)$ is a function of S only that is applicable throughout the atmosphere. If $f(S)$ is applicable only to one zone of the atmosphere, the dependence of the trajectory on a single parameter would not hold for vehicles dropped in different altitude zones.

Figure 1. Empirical free-fall ballistic chart that gives the velocity of an object released from rest if the distance fallen and time of fall are known

Figure 2. The theoretical variation of fall velocity with distance from release for constant atmospheric density

Figure 3. The theoretical variation of fall velocity with time from release for constant atmospheric density

Figure 4.
The theoretical variation of distance travelled with time from release for constant atmospheric density

Figure 5. The variation of the fall distance necessary to reach $0.99 V_T$ with the density ratio ρ_0/β for constant atmospheric density

Figure 6. Comparison between the theoretical and empirical results for the free-fall ballistic chart

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