The Spectrum of Excitations for Real Nuclei*

by

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ABSTRACT

The many-body problem for finite systems is discussed from a point of view which is largely that of the intermediate-coupling and configuration-mixing shell models. Emphasis is placed upon the determination of excitation spectra for two simple systems, the single hole system and the single particle system. It is shown how the spectra for these are obtained by passing from pure independent model to quasi-particle model and thence to representations of the real states. The configuration mixing of the description is generated by allowing for the existence of density correlations in the many-body system. Such correlations manifest themselves as the oscillations of the nuclear surface. Single particle (hole) excitations are altered by the presence of these collective motions. Other, non-collective, configurations serving to modify the single particle spectrum are also discussed.

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DISCLAIMER

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1. Introduction

This paper is largely descriptive. Its purpose is to give as complete an account of the many-body problem, in its application to finite systems, as is currently possible. The emphasis is upon those aspects already familiar from the intermediate coupling and configuration-mixing shell model; a Green's function formalism is used throughout. This method of the test-particle, as it is sometimes called, emphasizes the spectrum associated with single particles and holes. It is after all the central problem of the "shell model" to define in some way a complete set of excitations in the presence of interactions. We will refer to the independent excitations as quasi-particles. In order to be precise, we shall need to define and clarify our usage of certain terms. A shell model basis set will be that set of one-particle functions which is defined in an energy-independent, state-independent, one-body potential U_o. No such potential exists, in nature, for the problem under discussion. The many-body wave function of the shell model is a Slater determinant in the basis set noted.

We may go beyond this concept of the shell model. It is possible and traditional to extend the model in two simple ways. 1) We may introduce the mixing of configurations of shell model, many-body, wave functions. 2) Slater determinants of more sophisticated one-particle functions, e.g., quasi-particles, may be used to represent the many-particle system. Then, the configuration mixing is studied in this

more general situation. Approach 2) has not received a great deal of attention. The early impetus in this direction owes to Brueckner and his collaborators[1,2]. Our only criticism of this effort is that it does not go sufficiently far. It does not contain specific aspects which we believe to be relevant to the specification of finite systems. We shall undertake to make these statements precise at the appropriate points in our discussion. The nuclear many-body problem is unmistakably hard. It is not, however, clear to us that the study of finite systems is necessarily elucidated by an analysis of nuclear matter. Indeed, the two systems are so disparate in, for example, their thermodynamic behavior, that they are most likely unrelated. Further, we do not view the relating of the values of dynamical quantities computed in the infinite system to those for the finite system as specifically useful. It is submitted that a more natural procedure is available. The direction which the approach takes is based upon our traditional ideas of the configuration-mixing shell model. Some elaborations are of course indicated. The approach is formally based upon the expansion of the Martin-Schwinger[3], (M-S), Green's functions in terms of those defined by Brueckner. Moreover, the Hartree-Fock (H-F) method, as developed by Thouless[4], will be seen to play a dominant role in our discussions. While the development of our treatment is quite formal, the results are easily translated into numerical form. A program of numerical evaluation (nuclear properties and single-particle spectra), based upon the notions of this paper, is currently in progress. Hopefully, the results will throw light upon

those aspects which we emphasize here.

It is intrinsic in the shell model, as usually conceived, that a diagonalization of the residual force be carried out. Such a force arises as a reflection of the difference existing between the model and the actual Hamiltonian. This same underlying feature will be present in the extended shell model of this discussion. In order to effect as large a diagonalization of the given (physical) two-particle force as possible, we have to choose a good quasi-particle representation. This first step is to be devoid of considerations arising from perturbation theory. Experience has taught us that it is proper to look for a quasi-particle representation. Several such representations are discussed in section II. The definition of these representations follows from their association with corresponding one-body, energy operators. Such operators are diagonal in the basis of the representation.

At this point, we have to treat the residual force. This is done partially by non-perturbative methods and otherwise by treating the force as small. There is associated with the representation basis, a non-abstract space. This is the space of occupied or unexcited states. The one-body, energy operators will not be diagonal with respect to those configurations of N particles containing a hole in unexcited states and a particle in excited states. These configurations are the next most complicated, when compared with the ground state configuration, having N particles in unexcited states. The residual force is to be partially diagonalized over the space of hole-particle, (h,p), states.

This step is also non-perturbative. It introduces a set of collective states. Such states are discussed in section III.

It is shown there that the positions-in-energy of the states are associated with the poles of the two-particle propagator G_2[†]. In addition, we shall find that the spectrum of (h,p)-excitations is always corrected by its coupling to that of (2h,2p) excitations. A state-energy expansion is introduced for G_2 in order to achieve these general results.

The foregoing statements express the fact that we will take as trial function for the N-particle system, an appropriate linear combination of ground state and (h,p) configurations. One direct consequence of this assumption is that the quasi-particle spectrum is now shifted. This effect comes about, in part, owing to the coupling of quasi-particle and collective states. Evidently, there can be distortions of the spectrum which arise through couplings to states of non-collective, (h,p)-character. The perturbed quasi-particle spectrum is discussed in section IV. Some attention is also given there to the perturbation treatment of the residual force.

We attempt to make some specification of the residual force in section V. It is appreciated that in any consistent use of the new Tamm-Dancoff method[5]) this is formally unnecessary. Nevertheless, we

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[†] The poles actually appear in the function L, defined as $L = G_2 - GG$.

will parametrize the theory whenever the residual force can be treated as a perturbation. This approximation, introduced for simplicity, can be checked numerically. Here, one has to determine from the evaluation of the state-, and energy-dependent difference of given and average interactions, the quality of the approximation of a state-, and energy-independent parametric, potential form.

The pairing interaction is also a part of the residual force. It is one of those contributions, however, which must not be treated from the point of view of standard perturbation theory. At the same time, it is still possible to introduce a sort of perturbation theory for this effect. The details are sketched in section VI. A more complete discussion based upon the Nambu[6] representation will be published elsewhere.

Section VII contains a summary of the principle results and a recapitulation of the methods used. There, also is an enumeration of the data chosen by us for subsequent numerical summary and evaluation.

2. Quasi-Particle Representation

We shall discuss two of the possible quasi-particle representations in this section. Our aim here is not so much to rederive well-known results but rather to describe some procedures for handling these in the many-body problem. These procedures are to be ultimately used in a computational scheme.

First, it is desired to compute the ground state properties of an

N-particle system. To do this we must have the corresponding many-particle wave function, Φ_o, eigenfunction of the energy operator H, with eigenvalue E_o. In the Heisenberg representation we may write

$$H(t) \equiv H_o + V = \frac{1}{2M} \int d\hat{x}\ \nabla \psi^\dagger \cdot \nabla\psi(x)$$

$$+ \frac{1}{2} \int d\hat{x}\ d\hat{x}'\ \psi^\dagger(x)\psi^\dagger(x')\ v(x,x')\ \psi(x)\ \psi(x')\ ; \qquad (1)$$

$$x = (\hat{x},t),\ v(x,x') = \delta(t - t')\ v|\ \hat{x} - \hat{x}'|)\ .$$

The field operators are expanded in terms of some one-particle basis set u_j as

$$\psi^\dagger(x) = \sum a_j^\dagger\ (t)\ u_j^*\ (\hat{x}),\ \psi(x) = \sum a_{j'}\ (t)\ u_{j'}\ (\hat{x})\ .$$

The basis functions are solutions of the one-particle equation

$$\left[\epsilon_j - T_j - U_o\ (j)\right] u_j\ (\hat{x}) = 0 \qquad (1')$$

and the $a_j^\dagger$, a_j are respectively creation and annihilation operators for the state j. One determines the potential U_o,

$$U_o = \int d\hat{x}\ \psi^\dagger\ (x)\ U\ (\hat{x})\ \psi(x),$$

in terms of the given two-body potential v. This is done by writing H as

$$H = (H_o + U_o) + (V - U_o)$$

and forming the variational expression,

$$(\delta \Phi_o | H | \Phi_o) = 0 \ ,$$

for the ground state energy. All of this has been done in a very nice way by Thouless, so we will only quote the results. One finds that

$$U_o(j) = \sum_i \ (v_{ij,ij} - v_{ij,ji}) \tag{2}$$

and that

$$\epsilon_j = \frac{\int d\hat{x} \ u_j^*(p^2/2M) \ u_j(\hat{x}) + \int d\hat{x} \ u_j^* \ v_D u_j(\hat{x}) - \sum\!\!\!\!\!\!\int_k d\hat{x} \ d\hat{x}_1 \ u_j^*(\hat{x}) v_X^{kj}(\hat{x},\hat{x}_1) u_j(\hat{x}_1)}{\int d\hat{x}' \ u_j^* \ (\hat{x}') \ u_j(\hat{x}')} \ ; \tag{3}$$

where

$$v_D \ (\hat{x}) = \sum\!\!\!\!\!\!\int_k d\hat{x}' \ u_k^* \ (\hat{x}') \ v \ (|\hat{x} - \hat{x}'|) \ u_k \ (\hat{x}') \ ,$$

and

$$v_X^{kj} \ (\hat{x},\hat{x}_1) = u_k^* \ (\hat{x}_1) \ v \ (|\hat{x} - \hat{x}_1|) \ \frac{1 + t_{kj}}{2} \ u_k(\hat{x}) \ ;$$

$$t_{kj} = \tau_3 \ (j) \ \tau_3(k) \ .$$

We have written all of this explicitly because we wish to solve

eq. (2), representing the H-F approximation of eq. (1). Let us first, however, make the customary, preliminary remarks. It is necessary to read for $\hat{x}_1$ the set of variables $(\hat{x}_1 \hat{\sigma} \hat{\tau})$; the notation is standard. There is no exchange for non-identical particles. This is assured by the presence of the operator t_{jk}. It is our intention to circumvent the difficulty produced by the presence of a hard core in the given, two-body potential v. This can be done if we take v to be the potential of the modified boundary condition model, (BCM),[7]. That model supposes that at some radius r_o, the logarithmic derivatives $F(\ell jST)$ are specified for every open channel of the relative, two-body system. Outside of the radius r_o, there is a specified potential tail. For the choice of one-, plus, two-pion exchange potential tail (O.P.E.P. plus T.P.E.P.), and a matching radius of 2 pion Compton wavelengths, $r_o = 0.7$ f, the model has enjoyed considerable success. It summarizes the available two-nucleon data with far fewer parameters ($\sim 1/3$ less) than the Brueckner-Gammel-Thaler potential, (BGT). There is also an absence of difficulty in explaining the high-energy behavior of the $_1\delta_2$, singlet-D phase shift, which difficulty is characteristic of BGT. Finally, the choice of potential tail is fully supported by the work of Cottingham and Vinh Mau[8]. These authors, starting from the causal matrix, obtain as the equivalent, static nucleon potential, the O.P.E.P. plus T.P.E.P. tail. This representation is known to be suspect for interparticle separations of 0.5 f or less.

In our many-body computations, we will take the pair suppression

parameter of ref. 7 to be unity and the ladder parameter to be zero. This specifies our potential tail, which we denote as $v^{(1)}$. The effect of the boundary condition is taken into account through the introduction of a pseudo-potential[9]†. It is required that we choose this operator so as to guarantee hermitian, time-reversal-, and, parity-invariant, two-particle matrix elements. Even so, considerable ambiguities arise in the choice of a pseudo-potential. We have tentatively settled upon the <u>radial</u> form for the internuclear force given as

$$v_{BCM} = \frac{\hbar^2}{M}\left(\frac{F}{r_o}\,\delta(r - r_o) + \left[\frac{d^L}{dr}\,\delta(r - r_o^-) - \delta(r - r_o^+)\,\frac{d}{dr}\right]\right)_{j\ell ST} \quad ; \quad (4)$$

$$r_o^- = r_o - 0^-,\ r_o^+ = r_o + 0^+ \ .$$

The left-hand derivative d^L/dx appears in the above form. No one of the derivatives is to act upon the spherical volume element. Finally, we may write the given force as

$$v = v^{(1)} + v_{BCM} \ ;$$

$$v^{(1)} = v(\text{O.P.E.P.}) + v(\text{T.P.E.P.}),\ |\hat{x}_1 - \hat{x}_2| > r_o \quad (5)$$

$$= 0\ ,\quad |\hat{x}_1 - \hat{x}_2| \leq r_o \ .$$

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† This was pointed out to us by S. Kahanna.

This potential form is eminently suited to the computation of matrix elements such as those appearing in (3).

In order to solve eq. (3) we shall have to make a guess for the u_j. This is facilitated by going over to a method of combined variation and iteration. The substance of the approximation is to expand the members of the quasi-particle basis set over the set of shell model functions ψ_s. A convenient shell model basis is provided by the wave functions of the isotropic, harmonic oscillator. Thus we write

$$\psi_s(\hat{x}) = R_{n\ell}(r)\, Y_{\ell m}(\hat{x})\, \chi^{m_s}_{1/2}(\alpha)\, \chi^{t}_{1/2}(\zeta) \quad . \tag{6}$$

Again, the notation is altogether standard. The quasi-particle expansion is given by

$$u_j(\hat{x}) = \sum_s c_{js}\, \psi_s(\hat{x}) \quad . \tag{7}$$

Now, evidently, we are going to treat the c_{js} as variational parameters in the expression of eq. (3). The procedure is to take successively larger numbers of "principal orbitals" n (the principal quantum number) until convergence, if at all possible, obtains. There are some additional technical details involved in the solution of (3) but these will become evident in any practical evaluation. One advantage associated with the use of the ψ_s defined by (6), is the decomposition of product pairs into factors. These factors separately involve the center of mass and the relative coordinate of position. This decomposition is formally expressed

in terms of the familiar Moshinsky-Brody[10] transformation brackets. The ψ_s of (3) are not the only possible shell model basis set. We have suggested[11] that one may use the two-parameter set

$$\psi_s(\hat{x}) = \sum \left[\begin{array}{cc|c} l & 1/2 & j \\ m & m_s & \mu \end{array}\right] f_{n\ell j}(\alpha_j, \beta_j; r)\, Y_{\ell m}(\hat{x})\, \chi^{m_s}_{1/2}(\alpha)\chi^{t}_{1/2}(\zeta). \quad (8)$$

Here, the radial functions $f_{n\ell j}$ have been determined in a non-local potential of the Yamaguchi form. The parameters (α_j, β_j) are fixed by the binding energy of a particle in the orbital and the mean square radius of the orbital[12]. The advantage afforded by the functions of (8) is that they decay exponentially. The radial functions are somewhat more complicated than those of the oscillator. Additionally, one is now forced to give up the Moshinsky-transformation when using the functions of (8). For the sake of completeness, we note that the integral representation pertaining to (8) is

$$f_{n\ell j}(\alpha,\beta;r) = \int \left[\frac{2}{p^2} Q_l \left(1 + \frac{\beta^2}{2p^2}\right)\right]^{1/2} \frac{1}{\alpha^2 + p^2}\, j_l(pr)\, p^2 dp \quad . \quad (9)$$

The form factor in momentum is a product of Yamaguchi and Mitra[13] form factors. In the latter, $Q_l(z)$ is a Legendre function of 2nd kind.

Finally, we should give some additional detail regarding the solution of (3). The procedure is the following. 1) Expand the states $u_j (j \neq k)$ according to (7). This means that $j = (n\ell j)$ or $(n\ell)$ while $s = (n'\ell j')$ or $(n'\ell)$. There are M-1 states k and we want these to be

known at every stage of iteration. 2) Write for these states, k, the shell model functions of (6) or (8). 3) Carry out the variation with respect to the c_{js} to obtain $\epsilon_j^{\,o}$ and $u_j^{\,1}$. 4) Compute v^1, namely $v_D^{\,1}$ and $\left(v_X^{\,kj}\right)^1$. 5) Assume a new set of c_{js} $(j = 1,\ldots,M)$ and repeat step 3) to obtain $\epsilon_j^{\,1}$, $u_j^{\,2}$. 6) Continue until convergent.

We have now determined the H-F quasi-particle basis for bound, unexcited, or occupied, states. In the finite system there will be bound and continuum excited states. To discuss these, we must go from the ground state, N-particle system to the (N + 1)-system. That is, we add one particle to the ground state N-system. The motion of that particle, in each state $(n\ell j)$ or $\hat{k} = (k\ell_k)$, is determined by solving (1'). The average of H-F field is that owing to the N "quiescent" particles in occupied states. This procedure will yield distorted-wave states $u_j = u_{\hat{k}}^{(\pm)}$ for the continuum situation. The representation of (8), and one similar to it for the continuum, is used exclusively in this part of the computation. An iteration procedure is readily available to solve the integro-differential equation (recall that we have exchange) of (1'). This is not discussed here. The H-F quasi-particle basis has been determined for the physical problem. Evidently, we shall be able to compute binding energies and radii for closed shell systems. These systems provide the physical vacuum for the special (N + 1)-configurations discussed above.

All of this labor has not taken us very far. The H-F basis is an approximation to the physical situation and omits much of the physics.

It is nevertheless a useful first step to obtain that basis. We know, for example, that a description involving only passive, unexcited particles[14] will never give us the quasi-particle, finite-lifetime effects which we observe experimentally. These are, however, included if we can establish or determine the BHF-basis. And, evidently, to obtain this basis, which is biorthogonal[15], reflecting a non-hermitian, one-body problem, we shall proceed from the H-F basis. The H-F basis provides then a useful set of trial functions for the new problem of self-consistency.

The BHF basis is obtained by studying the spectrum of excitations of the $N \pm 1$ systems. This information is carried by the one-particle Green's function, G. We shall proceed to the G of Brueckner theory by starting with that defined by Martin and Schwinger. The development which makes this possible has been given elsewhere[16]. However, the relevant equations will be reproduced here. The significance of the expansion and the motivation for it is easily discussed. Brueckner, recognizing that the phase space for particles is much greater than that for holes, uses a scattering operator t defined in terms of particle propagators G^+. Corrections to the self-energy Σ which corresponds to this t will be important for those states near the Fermi level $\epsilon_f(N)$. This was also recognized by Brueckner. Such states, being weakly influenced by the exclusion principle, are filled and depleted a great part of the time. We thus speak of the probability of depletion of an occupied state $s \leq \epsilon_f$. In like manner, the excited states $s > \epsilon_f$, for quasi-

particles, will be filled with some probability even when we discuss the ground state N system. The t-operator of M-S, say, $\tilde{t}$ has hole-particle symmetry as does the corresponding $\tilde{\Sigma}$. When we are far away from ϵ_f we shall not need this symmetry. In the vicinity of ϵ_f it is quite important. It then makes sense to expand $\tilde{\Sigma}$ in terms of Σ in order to recover the symmetry between holes and particles. The corrections arising to Σ may be thought of as density-fluctuation contributions. In fact, one chooses to generate them from the density-density correlation function L, within the scheme of conserving approximations to G_2 (= L + GG), discussed by Baym and Kadanoff[17].

The one-particle propagator G, defined as

$$G(1, 1') = -i \langle T\, \psi(1)\, \psi^{+}(1') \rangle_o \,, \quad (1 = \hat{x}t) \,, \tag{10}$$

the expectation of the time-ordered product of Heisenberg operators over the N-particle ground state, satisfies the equation

$$G_o^{-1}(1, \bar{1}')\, G(\bar{1}', 2) = 1 - i\, v(1, \bar{2})\, G_2(1\,\bar{2}, 2\,\bar{2}^{+}) \,. \tag{11}$$

This is in the matrix notation of ref. 17. The given potential is the 4-dimensional generalization of (5). The unperturbed propagator G_o is defined by

$$G_o^{-1}(1, 2) = \left(i \frac{\partial}{\partial t_1} - \frac{p_1^2}{2M} \right) \delta(1 - 2) \,. \tag{12}$$

The ladder approximation,

$$v(1,2)\, G(1\,2,1'2') = t(1\,2,\bar{1}'\bar{2}') \left[G(\bar{1}',1')\, G(\bar{2}',2') - G(\bar{1}',2')\, G(\bar{2}',1')\right] \quad (13)$$

provides an equation for t, namely,

$$t(1\,2,1'2') = v(1\,2,1'2' - 2'1') + i\, v(1\,2)\, G(1,\bar{3})\, G(2,\bar{4})\, t(\bar{3}\,\bar{4},1'2') \ , \quad (14)$$

when we assume the Bethe-Salpeter equation for G_2,

$$G_2(1\,2,1'2') = G(1,1')\, G(2,2') - G(1,2')\, G(2,1')$$

$$+ G(1,\bar{3})\, G(2,\bar{4})\, \Gamma\,(\bar{3}\,\bar{4},\, \bar{5}\,\bar{6})\, G(\bar{5},1')\, G(\bar{6},2') \ ; \quad (15)$$

and introduce the approximation that the vertex Γ is functionally independent of G.

$$\Gamma\,(1\,2,\, 1'2') = i\, v\,(1\,2) \left[\delta(1,1')\, \delta(2,2') - \delta(1,2')\, \delta(2,1')\right]$$

$$\equiv i\, v\,(1\,2,\, 1'2' - 2'1') \ . \quad (16)$$

The irreducible self-energy Σ is defined as

$$\Sigma\,(1,\bar{1})\, G(\bar{1},1') = -\, i\, v(1,\bar{2})\, G_2(1,\bar{2},1'\bar{2}^{+}) \ , \quad (17)$$

and from the above discussion it follows that we also have

$$\Sigma\,(1,1') = -\, i\, t(\bar{1}\,2,\bar{3},1')\, G(\bar{1},\bar{3}^{+}) \ . \quad (18)$$

Equation (18) is the M-S expression for the nucleon self-energy, previously called $\tilde{\Sigma}$. Now when the Brueckner operator

$$t = v_a + i\, v\, G^{+}\, G^{+}\, t \ ; \ (v_a = v(1\,2,1'2' - 2'1')) \ , \quad (19)$$

is used, the self-energy of (18) becomes the power series (in the Brueckner operator) shown in figs. 1, 2 and 3. We will always draw Goldstone diagrams. Thus, the reader may attach external lines to our diagrams to obtain the appropriate self-energy for particles and holes. Furthermore, only the operator defined by (19) will play a role in our discussions.

We wish now to pass to the BHF-basis, as we have called it, and to deal with the problem of self-consistent determination of that basis. This first of all implies that we shall employ a state-energy representation for all of the operators of the theory. The state representation in terms of our biorthogonal basis ϑ_s is, for G,

$$G(1,1') = \sum\!\!\!\!\!\!\int \; | \; \vartheta_s(\hat{x}_1) > G_s \; (t_1 - t_1') < \tilde{\vartheta}_s \; (\hat{x}_1') \; | \quad . \qquad (20)$$

This involves both continuum and discrete states. We are assuming spatial homogeneity and that the processes under consideration are stationary when we write eq. (20); thus $G(1,1') = G(1 - 1')$. The boundary conditions are implicit in the functions ϑ_s and $\tilde{\vartheta}_s$. These will reflect the characteristic finiteness of the nuclear system. The frequency or energy spectrum is introduced by means of Langer's[18] representation,

$$G_s(t_1 - t_1') = - i \; \theta(t_1 - t_1') \int_{\mu}^{\infty} d\omega \; A_s \; (\omega - \mu) \; e^{i\omega(t_1 - t_1')}$$

$$+ i \; \theta \; (t_1' - t_1) \int_{-\infty}^{\mu} d\omega \; B_s \; (\omega - \mu) \; e^{i\omega(t_1 - t_1')} \quad . \qquad (21)$$

This representation, as written, is only appropriate to the infinite system. The continuous energy variable ω is symmetrically defined about a chemical potential μ. Symmetry between holes and particles is thereby implied. The functions A_s and B_s are respectively the spectral functions for particles and holes. The characteristic asymmetry of the finite system manifests itself through the existence of two quantities μ, namely, $\mu(N + 1) = E_o(N) - E_o(N + 1)$ and $\mu(N - 1) = E_o(N - 1) - E_o(N)$. Even in a large but finite system these quantities are unequal. It is also true that the hole-particle asymmetry is reflected in the definition of the spectral functions.

The spectral functions have the Lorentz form. We have, for example

$$A_s(\omega - \mu) = -2\, I_m \frac{1}{\omega - T_s(\omega) - |B_+| - \Sigma_s(\omega)}$$

$$= \frac{\Gamma_s(\omega)}{\left[\omega - T_s(\omega) - |B_+| - \text{Re}\ \Sigma_s(\omega)\right]^2 + \Gamma_s^2(\omega)/4}\,, \qquad (22)$$

with a similar statement holding for B_s. Actually, the negative of (22) with B_- substituted for B_+ gives B_s. The quantities B_+ and B_- are the binding energies of the last particle in the $(N + 1)$-, and N-systems, respectively. In order to define the spectral functions, we introduce the well of fig. 4, which is typical of a finite system. Next, we proceed to define the spectral functions on a finite set of points for

continuum and bound states. These definitions will be consistent with the existence of L^2-functions, in the Fourier transform sense, on the infinite internal ($-\infty \leq \omega \leq \infty$). Let us proceed to do this. In the limit of zero width, (22) goes over to

$$A_s(\omega - \mu) = 2\pi\delta(\omega - \omega_s)\, f_s \quad ;$$

$$f_s = \left[1 - \frac{\partial}{\partial\omega} \mathrm{Re}\ \Sigma_s(\omega)\right]^{-1}_{\omega = \omega_s} \quad ; \tag{23}$$

$$\omega_s - T_s(\omega_s) - |B_+| - \mathrm{Re}\ \Sigma_s(\omega_s) = 0 \quad .$$

Referring to fig. 4, we note that s is the discrete index, either $(n\ell j)$ or $(n\ell)$ for all of the states of negative total energy ω. We can then write the following.†

$$B_s(\omega - \mu) = 2\pi\delta(\omega - \omega_s)\, f_s \ ; \ \omega_s = \omega_1, \cdots \omega_N \quad ,$$

$$(-|B(N)| - \epsilon_f(N) \leq \omega_s \leq -|B(N)| \quad ;$$

$$= 0 (\omega_s < -|B(N)| - \epsilon_f(N) \quad .$$

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† The spectral functions are given for the ground state configuration. An excited configuration, say, that of one excited particle, i, will have $B_{s_i} \sim \bar{f}_{s_i}$ and $A_{s_i} \sim f_{s_i}$.

$$A_s(\omega - \mu) = 2\pi\delta(\omega - \omega_s)\ \bar{f}_s;\ \omega_s = \omega_{N+1},\cdots,\omega_M\ ,\ \bar{f}_s = 1 - f_s\ ,$$

$$(-\ |B(N)| < \omega_s < 0)\quad . \tag{24}$$

There are precisely N hole-states of the (N-1)-system which can be generated from the N-particle ground state. We can, however, sustain (M-N) bound, excited states of the (N+1)-system within the interval $|B(N)|$. For the positive energy states, s is a mixed, continuous and discrete label, i.e., $(l_k\ k)$. However, recall that the scattering of a particle from an average field of short range shows resonances. It is then meaningful to write

$$\begin{aligned} A_s(\omega - \mu) &= 0(\omega_s > \omega_{MAx})\quad ; \\ &= \frac{\Gamma_s(\omega)/2}{(\omega - \omega_s)^2 + \Gamma_s{}^2(\omega)/4}\ ;\ (\omega_s = \omega_{M+1},\cdots,\omega_{M^*})\ , \\ &\qquad (0 < \omega_s \leq \omega_{MAx})\quad . \end{aligned} \tag{25}$$

The discrete states $\omega_s = \omega(n_k\ \ell_k)$ are the positions of the scattering resonances of the particle in the average field; $\Gamma_s(\omega_s)$ is the associated width of the resonance. The quantity ω_s is just a number, e.g., 1 MeV. We have insisted upon appending a subscript 'k' to the principal quantum number and orbital angular momentum. This is because we wish to indicate that there is a problem of self-consistency associated with the determination of ω_s for continuum states.

The definitions of (24) and (25)† permit us to use the transform of (21). We have cut off the spectrum for the (N-1)-system from below and that for the (N+1)-system from above. This corresponds to the physical situation for the (N-1)-system. The upper cutoff must be employed if we are to have a finite mathematical representation. It is implied by (24) that the bound, excited (N+1)-states do not have a width. This is not true and the statement is an approximation. We shall have to compute the widths by means of perturbation theory. The Green's function method provides an average description. The references to states of the (N±1)-systems must be understood in this sense. Averages are taken over the actual (physical) states of these systems.

We are not very far along in describing the problem of self-consistency which will yield the BHF-basis. It must be kept in mind that the ω_s of (24) and (25) are not known. Nevertheless, sufficient formal methods are now available to us. The self-energy operates Σ, eq. (18), and the scattering operator t, eq. (19), are put into the state-energy representation. To do this, it is necessary to consider that Σ is a homogeneous function of its arguments, $\Sigma(1,1') = \Sigma(1-1')$. The operator t is taken as the retarded part of a more general function. The reader will recall that $< 1\ 2|t|1'2' >$ depends upon just two times, say, t_2 and t_2'. There are delta functions in the definition of t which set t_1 equal to t_2 and

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† The statements of (24) and (25) constitute a heuristic prescription, the basis for which is supported by the work of the Appendix. There, we obtain $G_p(t)$ as a stationary, time series.

t_1' equal to t_2'. Again, we demand homogeneity in the time-dependence. As t_2 can exceed t_2', or be less than t_2', we can formally generate both the retarded ($t_2 \leq t_2'$) and advanced ($t_2 > t_2'$) parts of t. The retarded part of t describes the scattering of pairs of particles. Hole scattering contributions do not exist for the definition of (19). The appropriate results, which are consistent with the foregoing statements, appear below.

$$\Sigma_s(\omega) = \sum_{p \leq \epsilon_p(N)} [< sp|t|sp > - < sp|t|ps >] f_p$$

$$- \sum\!\!\!\!\!\!\int_{\ell_2 p' \ell_4} < sp'|t_a|\ell_4\ell_2 > \frac{1}{\omega - \omega_{\ell_4} + \omega_{p'} - \omega_{\ell_2}} < \ell_2\ell_4|t_a|p's > f_{p'} f_{\ell_4} f_{\ell_2} \quad ; \tag{26}$$

$$t_a|p_1p_2 > = t|p_1p_2 > - t|p_2p_1 > \quad .$$

$$<s'p'|t_a(\omega)|sp > = < s'p'|v_a(\omega)|sp > + \sum\!\!\!\!\!\!\int_{m_1 m_2} < s'p'|v|m_1m_2 >$$

$$\frac{1}{\omega - \omega_{m_1} - \omega_{m_2} + i0^+} <m_1m_2|t_a|sp > \quad . \tag{27}$$

Equation (26) results from the assumption that we will compute the self-energy from the diagrams of fig. 1. This includes the sum of BHF and rearrangement energies. No designations have beeen made for the states $(\ell_2 p' \ell_4)$. We do this now. It is to be understood that $\ell_2, \ell_4 \leq \epsilon_f(N)$

and $p > \epsilon_f(N)$, holds, independently of whether s is less than or greater than $\epsilon_f(N)$. Equation (27) is to be understood as applying when $(s,p) \leq \epsilon_f(N)$ holds and either $(s'p') \leq \epsilon_f(N)$ or $(s'p') > \epsilon_f(N)$ holds. The notation $(s\ p) \gtrless \epsilon_f$ is meant to imply that one or both of the variables in parenthesis satisfies the given inequality. For the states m_1 and m_2 we can mitigate against violations of the exclusion principle by replacing $|m_1 m_2>$ by $|\det|m_1 m_2|>$ in the potential matrix element. These states satisfy the condition $(m_1 m_2) > \epsilon_f(N)$. The restrictions on the states are consistent with those of the discussion of Bethe[19]). Our expressions have an unsymmetrical appearance, the probability-of-occupation factors f_j of (26) being absent in (27). This is tantamount to the approximation that intermediate states m_1, m_2 quite far from the Fermi level are principally involved in the determination of t. In the case of Σ we would say that the presence of the factors f_j shows that we are always computing the interaction energy in a system of interacting particles. Typically, the binding energy $|B(N)|$ will be of the order of 8 MeV or more in the closed shell systems which we discuss. The observed spectra of quasi-particles, namely that of single holes in the (N-1) system and single particles in the (N+1)-system, persist up to excitations of perhaps 3-5 MeV. At higher energies, more complex types of excitations appear. The quasi-particle representation of (23) and (24) then ceases to be valid. For this reason, the sum rule

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \left[A_s(\omega - \mu_+)\ \theta(\omega - \mu_+)\ B_s(\omega - \mu_-)\ \theta(\mu_- - \omega)\right] = 1$$

is certainly exhausted in the interval $\pm|B(N)|$ about $\epsilon_f(N)$. In view of this, we write

$$\frac{1}{2\pi}\left[\int_{-|B(N)|}^{\epsilon_f(N)} d\omega\, B_s(\omega - \mu_-) + \int_{\epsilon_f(N)}^{|B(N)|} d\omega\, A_s(\omega - \mu_+)\right] = 1 \quad .$$

It is proposed then that the negative energy states of the $(N\pm1)$-systems be treated together. The self-consistency of the continuum $(N+1)$-states is discussed as a separate matter and will only be approximate. A similar situation arose in our discussion of the H-F basis. The spectral function for the optical potential states is that given by (25). While we may sometimes refer to a particle in such states as a quasi-particle, this cannot be the case. In reality the optical states are packets of energy ω and width $\Gamma(\omega)$. The quasi-particle states, those of (24), are characterized by the factors f_j and $\bar{f}_j$. As these factors are not always 1 or 0, we have the interpretation that a quasi-particle is a linear combination of shell model hole and particle states.

The self-consistency involves the eqs. (26) and (27), and an eigenvalue statement such as is given in (23). The approximation is made that the occupation factors f_j are unity. Again, restricting our attention to quasi-particle states, we proceed as follows. 1) Make a guess for the energies in (26) and (27). The H-F energies ϵ_i associated with the u_i provide a starting point. 2) Compute the matrix elements of t, diagonal and non-diagonal, from (27), in the u_i-basis. Use the

principal value of (27). 3) Determine the self-energy for each of the quasi-particle states from (26). 4) Determine if the eigenvalue statement of (23) is satisfied for each state s and its assumed ω. 5) If ω does not equal ω_s, readjust the ϵ_i and repeat steps 2) - 5) until ω equals ω_s. 6) If $\omega = \omega_s$ is still not possible, start from 1) and vary all of the u_i's by making the root mean square radius $\sqrt{<R_N>^2}$ larger or smaller. The u_i are functions which one has tabulated and plotted. A linear variation of the r.m.s. radius is straightforward for both bound and scattering states. 7) Repeat 2)-5) with the new u_i's ($\equiv \hat{u}_i$'s) until $\omega = \omega_s$ is satisfied, for each state.

The eigenfunctions and energies are then $(\omega_s, \vartheta_s = \hat{u}_s)$. These functions constitute the hermitian approximation to the BHF quasi-particle basis. The self-energies $\Sigma_s(\omega_s)$ are known for those states within the interval $2|B(N)|$ about $\epsilon_f(N)$. It is then possible to construct the factors f_s and $\bar{f}_s$, for these states, by numerical differentiation. A more reliable procedure involves the direct differention of (26). This is straightforward if $t(\omega)$ is not too strongly energy-dependent within the region of interest. The deeply bound states, $\omega_s < -|B(N)|$, are pure hole excitations of the (N-1)-system. We have yet to discuss the quasi-particle damping which is associated with the excitations of the (N+1)-system. This will be done in a later section.

We turn now to the optical model states. Without disturbing the previously obtained basis, one computes the hermitian interaction energy of (26), written now as Re $\Sigma_k(\omega)$. The anti-hermitian part $\Gamma_k(\omega)$ is com-

puted in the approximation of fig. 5. As yet we have no wave functions to complete our discussion of the basis. The H-F basis functions $u^{(\pm)}_{\hat{k}}(\hat{r})$ comprise a complete set. We define the quantity

$$U_o(\hat{r},\hat{r}') = \sum_{\hat{k}} |u^{(+)}_{\hat{k}}(\hat{r}) > \Sigma_k (\omega) < u^{(+)}_{k}(\hat{r}')| \qquad (28)$$

as the complex non-local potential for the particle in continuum states. It is of course true that $\omega = T_k + |B(N)|$, where T_k is the asymptotic kinetic energy of the incident particle. One can then solve the equation

$$(\omega - T)\, \vartheta^{(\pm)}(\hat{r}) = \int d\hat{r}'\, \vartheta^{(\pm)}(\hat{r}')\, U_o(\hat{r},\hat{r}') \qquad (29)$$

in order to obtain the basis functions for energy ω. As is the case with all statements made "in principle", it is not suggested that one perform the computation of (29). What is intended has to do with the demonstration that the H-F basis functions provide a means of determining the BHF basis. We use the set $u^{(\pm)}_{\hat{k}}$ in (28) instead of plane waves simply because the former more nearly pertain to the physical situation under discussion.

3. Collective States

We shall obtain the collective states of the N-spectrum. These have significance for the ultimate determination of the (N±1)-states. Collective oscillations are described by us as density fluctuations. Mainly,

our concern is with those motions of particle-hole structure. These states will occur at finite frequencies. This is in contrast to the zero-frequency collective state, that leading to uniform dilatation of Fermi sphere, which characterizes pairing.

It has been explained that hole scattering corrections to Brueckner theory are of some importance. These play a dominant role in the vicinity of the Fermi level. It is especially relevant to discuss these corrections in finite systems. For these systems we seldom get pure quasi-particle behavior, even at low excitations, once the medium weight ($A \sim 60$) nuclei are reached.

Hole corrections to particle-particle interactions take their simplest form in the symmetric (ph)-scattering graphs. These may be included in the formalism already given without the occurence of double countings. The one-body, non-hermitian operators, defining the quasi-particle basis, are not diagonal with respect to configurations having a hole in unexcited states and a particle in excited states. We restrict ourselves to configurations having just N-particles. The aim then is to incorporate the zero-point oscillations and the correlations associated with these into our description of what has been called the non-degenerate, N-particle, ground state, $|0>$. The new state $|\Psi(N)>$, upon which the quasi-particle operators $a_j^\dagger$ and a_j act, will be an admixture of excited and ground states.

The excited configurations with a particle and a hole present are most simply discussed in terms of the operator $L(1\ 2,\ 1'2')$. This is

the operator for the density correlations. In particular, we have the defining statements

$$L(1\ 2,\ 1'2') = G_2\ (1\ 2,\ 1'2') - G(1,1')\ G(2,2'), \tag{30}$$

$$< n(1),\ n(2) >_o = -\ 2i\ \mathrm{Im}\ L(1\ 2,\ 1^+2^+)\ ;\ n = \psi^+\psi\ , \tag{31}$$

which suggest the interpretations just given. In addition to the possibility of having the independent propagation of (p,h)-pair (fig. 6), we can have correlations within the pair. This is described by saying that the particle and hole can scatter themselves into a bound state. Both the scattering and the bound state configurations are contained in the propagator L. We shall extract the bound states from the solution of the integral equation for L, given in ref. 17. Our treatment is very similar to that of Gottfried and Picman[20] and a preliminary discussion has appeared elsewhere[21].

The integral equation for L, defined as the negative of the functional derivative, $[\delta G(1,1';U)/\delta U(2',2)]_{U=0}$, U being a scalar external field, is found to be

$$L(1\ 2,1'2') = -\ G(1,2')\ G(2,1') + \int G(1,\bar{3})\ G(\bar{4},1')\ \Xi\ (\bar{3}\ \bar{5},\ \bar{4}\ \bar{6})\ L(\bar{6}\ 2,\bar{5}\ 2'). \tag{32}$$

The vertex operator Ξ, is obtained for the ladder approximation, as

$$\Xi\ (\ 12,\ 1'2') = [\delta\Sigma)1,2)/\delta G(2',1')\ ]_{U=0}$$

$$= -\ i < 1\ 2|t|1'2' > + \int < 1\ \bar{3}|t|2'\bar{4} > G(\bar{4},\bar{5})\ G(\bar{6},\bar{3})\ < 2\ \bar{5}|t|1'\bar{6} > \ .$$

One proceeds by putting (32) into the state-energy representation, the basis for which has already been discussed. The first step of the procedure yields

$$L(\alpha t_1,\ \beta t_2;\ \mu t_1',\ \nu t_2') = -\delta_{\alpha\nu}\,\delta_{\beta\mu}\,G_\alpha(t_1 - t_2')\,G_\beta(t_2 - t_1')$$

$$+\sum\!\!\!\!\!\!\int G_\alpha(t_1-\bar{t}_3)\,G_\mu(t_4-t_1')<m\bar{t}_5,\alpha t_3|\Xi|\mu\bar{t}_4,k\bar{t}_6>L(k\bar{t}_6,\beta t_2;\ m\bar{t}_5,\nu t_2'). \quad (33)$$

One is to apply the Fourier transformation to this expression. The two contributions to the vertex are

$$\delta(\bar{t}_5 - \bar{t}_3)<m\alpha\ ;\ \bar{t}_3|t|\ \mu k\ ;\ \bar{t}_4>\delta(\bar{t}_4 - \bar{t}_6)$$

$$+\sum_{\sigma p}<mp\ ;\ \bar{t}_5|t|\ \mu\sigma\ ;\ \bar{t}_4>G_p(\bar{t}_3 - \bar{t}_5)\,G_\sigma(\bar{t}_4 - \bar{t}_6)<p\alpha\ ;\ \bar{t}_3|t|\sigma k\ ;\ \bar{t}_6>.$$

And, the propagator is to be specialized as

$$L(\alpha\ t_1,\ \beta\ t_2;\ \mu\ t_1',\ \nu\ t_2') = L_{\alpha\nu;\beta\mu}\ (t_1 - t_2';\ t_2 - t_1') \quad . \quad (34)$$

This says that a hole in state μ at time t_1' propagates to t_2, state β, while a particle in ν, at time t_2', propagates to t_1, state α. The Fourier transform of (33), with restrictions to be discussed shortly, takes the form

$$L_{\alpha\nu,\beta\mu}(E_1,E_2) = -\delta_{\alpha\nu}\delta_{\beta\mu} + (2\pi)^2 \sum_{km} \int d\omega \, \langle m\alpha|t(\omega)|\mu k\rangle \, L_{k\nu,\beta m}(\omega - E_{12}^{(-)})$$

$$-(2\pi)^3 \left[E_{12}^{(-)}\right]^{-2} \sum_{kmp\sigma} \langle mp|t(E-E_{12}^{(-)})|\mu\sigma\rangle$$

$$\cdot \langle p\alpha|t(E_{12}^{(-)})|\sigma k\rangle \, L_{k\nu,pm}(E_{12}^{(-)})$$

$$= -\delta_{\alpha\nu}\delta_{\beta\mu} + (2\pi)^2 \sum_{km} (\omega_{km} - E_{12}^{(-)})^{-1} \langle m\alpha|t|\mu k\rangle \, L_{k\nu,\beta m}$$

$$-(2\pi)^3 \left[E_{12}^{(-)}\right]^{-2} \sum_{kmp\sigma} \langle mp|t(-E_{12}^{(-)})|\mu\sigma\rangle$$

$$\cdot \langle p\alpha|t(E_{12}^{(-)})|\sigma k\rangle \, L_{k\nu,\beta m} \quad . \qquad (35)$$

The expressions on the right-hand side of (35) have been arranged so as to depend only upon the energy difference $E_1 - E_2 \equiv E_{12}^{(-)}$. This is the energy of the (p,h)-pair (ν,β). Then, $L_{\alpha\nu,\beta\mu}$ no longer depends separately upon the variables E_1 and E_2. The first term of (35) is the expected convolution in energy. It describes the transition of particle-hole pairs (ν,β) into those (k,m). The latter are at the relative excitation energies $(\omega_{km} - E_{12}^{(-)})$. It is possible to have excited configu-

rations of two particles, (αk), and two holes, $(m\mu)$, in the N-particle system. These configurations are coupled to those in which a single (h,p)-pair, (p,σ), is excited. This is the situation described by the second term of (35). The more general energy dependence of this term has been restricted so as to yield the homogeneous variable $E_{12}^{(-)}$. The formal over-simplification introduced is that of having 2-particle, 2-hole excitations at $E_{12}^{(-)} - (-E_{12}^{(-)}) = 2E_{12}^{(-)}$, or twice the excitation energy of the (p,h)-pair (ν,β).

Eq. (35) is to be regarded as a set of algebraic equations for an abstract particle-hole vector $\hat{X}^{(k)}$, $k = (\nu,\beta)$, whose components are $X_s^{(k)}$, $s = (\alpha,\mu)$; $(s,k = 1, 2,...)$. The product of delta functions in (35) is written as that δ_{sk} for the abstract, particle-hole space. Then, we write the algebraic statement

$$X_s^{(k)} = \delta_{sk} + \sum a_{\mu'}^{s} X_{\mu'}^{(k)} \quad . \tag{36}$$

This has the solution

$$X_s^{(k)} = \frac{-1}{\det \underline{M}_{(k)}} \tilde{M}^{sj} \delta_{jk} = \frac{-1}{\det \underline{M}_{(k)}} \tilde{M}^{sk} \quad ;$$

$$\underline{M}(k) = [\underline{1} - \underline{a}]$$

$$= \begin{vmatrix} 1 - a_1^1 & - a_2^1 & - a_3^1 \cdots \\ - a_1^2 & 1 - a_2^2 & - a_3^2 \cdots \\ - a_1^3 & - a_2^3 & 1 - a_3^3 \cdots \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{vmatrix} \quad ;$$

$$a^s_{\mu'} = (2\pi)^2 \, (\omega_{\mu'} - E^{(-)}_{12})^{-1} \; \langle m'\alpha|t|\mu k' \rangle$$

$$- (2\pi)^3 \, [E^{(-)}_{12}]^{-2} \sum_{p\sigma} \langle m'p|t|\mu\sigma \rangle \langle p\sigma|t|\sigma k' \rangle \; ; \; \mu' = (k'm') \quad .$$

Collective states appear at the zeros of det $\underline{M}_{(k)} = 0$. This is an eigenvalue problem for the unknown energies $E^{(-)}_{12}(k)$. The quadratic dependence of the coefficients $a^s_{\mu'}$ upon these energies has its origin in the (2p,2h) correction terms. Otherwise, the eigenvalue problem is that first described by Brown and his collaborators. The correction terms are associated with ground state correlations and are troublesome to compute. They are the more important the farther the $E^{(-)}_{12}$ states move away from the unperturbed quasi-particle values ω_k, and toward zero energy. The ground state correlations have been discussed by Brown et al., and also by Kerman and Klein[22]), and Lewis-Walecka[5]). We would note that the formalism presented here does contain the correlation effects in a way which is unrelated to perturbation theory. This

will mean that low-lying collective states, e. g., 3^-, which move down by large amounts and those such as 1^+, 2^+, 3^+ built upon (p,h)-states of low excitation, within a sub-shell, e. g., Ni^{58}, can be properly determined.

The (p,h)-collective states are established by the residual, two-nucleon interaction. The actual (observed in free scattering) interaction is used to build the quasi-particle spectrum. This spectrum is sufficient to determine the positions of the unperturbed (p,h)-states. The residual force is non-singular and not too energy-dependent for states at the Fermi sea, once saturation has been achieved. The tremendous success of the descriptions of the residual force in terms of, for example, Rosenfeld and Soper mixtures, is indeed meaningful, in the conventional shell model theory, that without configuration mixing. The latter is an essential part of what is termed the intermediate coupling shell model. The intermediate coupling shell model has been extensively utilized by Gillet[23]. On the other hand, the residual force is the difference between the 2-body operator corresponding to the given force and the 1-body operator of the average field. The non-diagonal (p,h)-transitions in the secular matrix for the collective state can only be induced by the given force. The unperturbed (p,h)-energies are, to good approximation, given by the average field, either H-F or BHF. Thus, the average field drops out of transition matrix elements, as it should. The collective state is established from both average and given interactions. But, the force parametrization of conventional shell model

theory is inappropriate. One encounters precisely this situation in applications of the configuration mixing shell model. It is to be seen that the theory presented here does imply the configuration-mixing shell model.

It is, finally, indicated that once the basis vectors $\hat{X}^{(k)}$ have been determined the L-propagator for the bound states (collective states) can be written, in state-energy representation, as

$$L^{(k)}(1\ 2,1'2';\omega_k) = \sum_s \frac{|X_s^{(k)} > < X_s^{(k)}|}{\omega_{(k)} - \omega_s}\ ,\ \omega_{(k)} = E_{12}^{(-)}(k)\quad ; \qquad (37)$$

$$L(1\ 2,1'2') = \sum_k L^{(k)}\ (1\ 2,1'2';\omega_k)\quad .$$

There also exist scattering contributions to the propagator

$$L'(1\ 2,1'2';\omega) = \sum \frac{|\psi_\alpha > < \psi_\alpha|}{\omega - \omega_\alpha}\ ;\ \int d\omega\ L'(1\ 2,1'2';\omega) = L'(1\ 2,1'2').$$

However, we have no interest in these as their spectrum is just the same as that which is implied by the (N+1) and (N-1) representations.

4. Single Particle Spectrum

We shall discuss some very simple excitation spectra and how these may be computed. Our results apply mainly to the (N+1)-spectrum as it

is influenced by states of the N-system. One example, that of the 2-particle bound state, is specifically relevant to the coupling of the (N+2)-, and N-states. The selection of topics and their order of discussion is somewhat arbitrary. First, a non-perturbative coupling of quasi-particles to vibrations is discussed. This is an old problem which was treated quite early on by Kisslinger and Sorensen[24]. Our remarks are therefore cursory. Second, we examine the second order perturbation theory for coupling of quasi-particle and vibration. The difference between the first and second examples has to do with relative quasi-particle purity of the excitation spectrum. The second example has been discussed from another point of view by Schrieffer[25]. The final and third discussion summarizes some of the aspects arising in the analysis of quasi-particle couplings to non-collective (p,h)-states[26] and (2p,1h)-states[27]. As our interest is primarily with single particle spectra, we do not deal with the possible many-particle spectra. Mottelson[28] has, however, given such a discussion in some detail.

We distinguish here between the quasi-particle spectrum and the single-particle spectrum, namely, that for a real particle. It is necessary to approximate to the latter in some reasonable way. At low excitations in the (N+1)-system, the spectrum may very well look like that for a quasi-particle. At somewhat higher energies, excitations of the core become important. The most obvious way to generate the single-particle spectrum in this regime is to couple the quasi-particle and core states. This must be done in the strong-coupling limit. A

diagonalization of the particle-core interaction over the basis noted is carried out. The result of the diagonalization is to give a single-particle spectrum which cannot be characterized as being that for a quasi-particle. The diagonalization has also taken into account that an extra nucleon has been added to a real system, one undergoing vacuum fluctuations. In general, the low-lying core state is the 2^+ vibration. However, in many isotopes, e.g., those of Ni, the odd parity 3^- vibration must also be taken into account.

The calculation proceeds along the lines of the Tamm-Dancoff approximation. One, of course, is instructed to form states of good J^{II}, etc. In view of the nature of this approximation, the quasi-particle basis states are employed at their unperturbed energies. Since the calculation fixes the single particle spectrum, no attempt is made to compute the shifts of the quasi-particle spectrum. In fact, for a sufficiently strong interaction between quasi-particle and vibration, the resulting single-particle spectrum will achieve the Paschen-Back limit.

These points are mentioned owing to the fact that some attempts to compute quasi-particle energy shifts have been made in the past. These have usually been done in the weak-coupling limit, and involve difficult questions concerning the self-consistency of the quasi-particle basis energies. An overall question in the discussion of the quasi-particle coupling to a vibration is that of the coupling potential. Again, as with our earlier discussions, the interaction is that remaining after subtraction of the self-consistent field from the given two-body inter-

action. We have noted that in Brueckner theory this statement gets translated into $t_{ij} - \langle t_i \rangle_o - \langle t_j \rangle_o$.

An example of the weak coupling of a quasi-particle to a vibration will be considered. Here one wants to obtain the quasi-particle energy shift. Fortunately, one can see where the self-consistency is likely to go wrong, if we use the Green's function methods. This is to be done. In addition, the answer to the problem is known from other considerations[29]. We attempt to compute the self-energy Σ owing to a nucleon being in a bound state of (p,h)-type. The bound state propagation is described by L' of eq. (3), which we call L_B. The first assumption is summarized by writing

$$G_o^{-1} G = 1 - i v G_2 \rightarrow 1 - i v L_B \quad .$$

This removes the independent particle-pair propagation in G_2. The next assumption considers that it is possible to define the scattering operator, t, as

$$v L_B = - t G G \Big|_X$$

with, moreover,

$$t = v + i t G G v.$$

The exchange part $G G\big|_X$ of independent particle propagation, G G, is that identified as the propagation of independent particle and hole pair. Now writing

$$t = [1 + i\, t\, G\, G]\, v$$

so that

$$[1 + i\, t\, G\, G] v\, L_B = - [1 + i\, t\, G\, G]\, t\, G\, G\, |_X \,,$$

we obtain

$$t\, L_B = - \Xi\, G\, G|_X \quad .$$

If we assume that

$$L_B = - G\, G|_X + L_B\, \Xi\, G\, G$$

holds, then

$$t\, G\, G|_X = - \Xi\, G\, G + t\, L_B\, \Xi\, G\, G \quad .$$

For the self-energy, defined as

$$\Sigma\, G = - i\, v\, L_B \quad ,$$

there follows

$$\Sigma = - i\, [\Xi\, G - t\, L_B\, \Xi\, G] \quad . \tag{38}$$

Suppose now that we retain the lowest-order bound-state term

$$\Sigma_B^{(2)} = i\, t\, L_B\, t\, G \quad . \tag{39}$$

This self-energy may be characterized as the second-order, weak-coupling contribution owing to the fact the nucleon is sometimes in a correlated state. The nucleon is supposed to have the additional self-energy $\Sigma' = - itG + tGGtG \equiv - i \Xi G$, which is that defining the BHF quasi-particle basis. On the face of things, the approximate self-energy $\Sigma' + \Sigma_B^{(2)}$ describes the combined bound state and independent motion of a nucleon. However, it need scarcely be pointed out that Σ, defined as above, with t being a given, instead of derived, operator, is, only artifically consistent with the results of section II. The self-energy defined previously as $\Sigma G = ivG_2$ has, in addition to the terms appearing in (38), contributions which may be identified as mass renormalizations of hole lines. Even if these are ignored the two self-energy expressions are not to be formally equated. We have only found a formal device for reproducing the quasi-particle contribution to the self-energy, in the presence of correlated nucleon motions. The motivation for this sort of description is provided by one's view of the physical situation. The mathematical operation is that of projecting on to a transpose space all of the operators, and relations between these, defined in sec. II. We note that the weak-coupling result,

$$\Sigma = - i [\Xi G - t L_B t G] , \qquad (40)$$

fails to correct the quasi-particle motion for the presence of the correlations. This feature defines the approximation.

In discussing $\Sigma_B^{(2)}$ of eq. (40) we encounter an interesting feature.

The contribution will become large for states in the quasi-particle spectrum whose energy separation is just equal to the excitation of the collective state. There is not, however, a pole in the $\Sigma_B^{(2)}$-term when the frequency condition is satisfied. We will explain what occurs shortly. The behavior noted is similar to that of the continuum, two-particle bound state discussed by Fonda[30]. At the positions of these states, the S-matrix will not exhibit poles. We describe the phenomenon under discussion as a "2-particle bound state". The frequency condition involved just two states of the quasi-particle spectrum. The nomenclature is not misleading even though one of the states is virtual. In the (N+1)-system, the correlation between the two such states could produce significant energy shifts in the energy of a real (as against virtual), bound quasi-particle. The scattering configurations of the (N+1)-system will be similarly influenced, for sufficiently low energies. Here, the correlation of states, induced by the virtual phonon for a nuclear surface oscillation, will modify the real part of the average, one-body potential. Additionally, it is expected that the imaginary part of that potential will also change. The correlation increases the average lifetime of the one-body, wave-packet state. The effect is to change the energy dependence of the imaginary part of the optical potential in the vicinity of zero energy.

The two-particle bound state persists when we consider two real quasi-particles, states of the (N+2)-system and also two virtual quasi-particles, again in the (N±2)-system. These conclusions are supported

by the analysis of Schrieffer. In the (N±2) systems the two-particle bound state gives an extended description of pairing. Two quasi-particles in different orbitals correlate over large distances by the exchange of a virtual phonon. The manifestation of the bound state is likely to be stronger in the (N+1)-, and (N-2)-systems than in the (N+2)-system. This is predicated upon assumptions regarding the extent to which the core nucleons will be excited.

As is customary, we go over to the state-energy representation in order to discuss $\Sigma_B^{(2)}$. It is probably helpful to note that the operator multiplication in eq. (30) is that specified by the diagram of fig. 7. We have, according to this diagram,

$$\Sigma_B^{(2)}(1',4') = i < 1'\bar{2}'|t|\bar{1}\,\bar{2} > L_B(\bar{2}'\bar{3},\ \bar{3}'\bar{2}) < \bar{3}\,\bar{4}|t|\bar{3}'4' > G(\bar{1},\bar{4}) \quad .$$

The propagator is expressed as

$$L_B(2'3;3'2) = \delta(t_2' - t_3')\,\delta(t_2 - t_3) \sum_s |\tilde{\psi}_s(x_2x_3) > L_s(t_2' - t_2)$$

$$< \psi_s(s_2'x_3)| \quad . \tag{41}$$

The frequencies of the bound states, ψ_s, are ω_s. Furthermore, these states may be expressed as

$$\psi_s(x_2,x_2') = \sum c_{sr} |\vartheta_k(x_2') > < k|\Omega_q^{\kappa}|\ \tilde{\ell} > < \tilde{\vartheta}_\ell\ (x_3')| \quad , \tag{41}$$

in terms of the biorthogonal BHF basis set, where time-reversal, $K\vartheta$, is denoted by $\tilde{\vartheta}$. The remaining, undesignated quantities are: c_{sr}-expansion amplitude or direction cosine for the projection of the bound state vector along the axes of the abstract (p,h)-basis system; $s - (q,\kappa)$, $-q \leq \kappa \leq q$; $r = (k\ \tilde{\ell})$, $k = (n_1\ \ell_1\ j_1;\ m_1)$, $\tilde{\ell} = (n_2'\ \ell_2'\ j_2'\ ;\ -m_2')$; and

$$< k|\Omega_q^{\kappa}|\tilde{\ell} > = \begin{bmatrix} j_1 & j_2' \\ m_1 & -m_2' \end{bmatrix} \begin{vmatrix} q \\ \kappa \end{vmatrix} \epsilon^{i\Phi(k,\tilde{\ell})}$$

is the product of vector coupling coefficient and the phase utilized in the definition of the (p,h)-basis states. Finally, if

$$L_s(t_1 - t_1') = \frac{1}{2\pi} \int d\omega\ \epsilon^{-i\omega(t_1 - t_1')}\ L_s(\omega) \quad ,$$

the spectral function is

$$L_s(\omega) = \frac{\bar{\rho}_s(\omega)}{\omega - \omega_s} \quad ,$$

where the weight $\bar{\rho}_s(\omega_s)$ will in general be approximated as unity by us. The scattering operators in (39) are no longer assumed to have the homogeneous time dependence of our earlier discussions. This is to say that the spectral forms for these operators will, in general, be off of the energy shell. Our final result, incorporating all of the foregoing, is

$$\Sigma_B \equiv \Sigma_{k_i}^{(2)}(E) = i \sum c_{sg} < k|\Omega_q^{\kappa}|\tilde{\ell} > c^*_{sp} < r|\Omega_q^{-\kappa}|\tilde{j} > \rho_m(\epsilon_m)$$

$$< k_1 k;\ E - \omega_s |t| mj;\ \epsilon_m > < rm;\ \epsilon_m |t| \ell k_1;\ E > \quad ; \tag{42}$$

where $g = (k\ \tilde{\ell})$, $c^*_{sp} = (-1)^{\kappa} c_{sp}$, $p = (r\ \tilde{j})$; $\rho_m(\epsilon_m)$ is the density-in-energy of quasi-particle states of type m, at the energy ϵ_m.

The expression of eq. (42) yields the width $\Gamma_{k_1}(E)$ for the process under discussion. The particle in state k_1 at the energy E experiences a complex self-energy, Re $\Sigma_{k_1}(E)$ + i Im $\Sigma_{k_1}(E)$; - 2 Im $\Sigma_{k_1}(E + i0^+) = \Gamma_{k_1}(E)$. Eq. (42) pertains to the scattering configurations of the (N+1)-system. Therefore, the state label k_1 and the energy E corresponding to that state are to be understood as continuous variables. The contribution to the optical potential supplied by (39) would be determined by computing Re $\Sigma_{k_1}(E)$. One can show in general that

$$\text{Re}\ \Sigma_s(z) = \frac{P}{2\pi} \int_{-\infty}^{\infty} dx\ \frac{\Gamma_x(x)}{x - z}\ ; \quad (z = E + iy\ ;\ x = E'). \tag{43}$$

It is then possible to compute the real part of Σ being given its discontinuity across the real axis. This evaluation is left for the reader. The principal value is implied in (43). It is also clear from this form that Re $\Sigma_{k_1}(E)$ will not have poles.

Eq. (42) suggests the interpretation that the particle in the wave packet state (k_1,E) is coupled to the bound, quasi-particle state (m,ϵ_m) through the vibration $(q, \kappa; \omega_s)$. Inspection of the matrix elements shows that the optical-state particle excites the target with which it interacts. A schematic representation of the excitation process is indicated in fig. 8. As the optical particle must scatter back into the entrance channel from the intermediate state in order that $\Sigma_{k_1}(E)$ be calculable, we are confronted by the need to introduce approximations. If $\omega_s \ll E$ is satisfied then eqs. (42) and (43) give a good approximation to a physical process (elastic scattering). By $\omega_s \ll E$ we shall mean $\omega_s \lesssim 0.3\ E$. Even so, the separate matrix elements of (42) involve off-energy-shell operators. The resulting self-energy may indeed be quite small owing to this aspect. On the other hand, the singling out of an ϵ_m, satisfying $\epsilon_m = E - \omega_s$, together with our inequality, produces on-shell matrix elements. The corresponding self-energy is significantly larger. We rely here upon the "rapid" drop-off of matrix elements as one goes off of the energy shell. This is a rather tenuous proposition. However, the nature of two-particle bound state has been disclosed. This is as far as we need to go.

Some brief remarks can be made concerning the coupling of the continuum (N+1)-spectrum for quasi-particles into non-collective states. The coupling to (p,h)-states has been studied by Shakin and Lemmer who investigate neutron scattering from N^{15}. The presence of the intermediate (p,h)-states leads to resonances in the neutron scattering

cross section. In the energy range $0 < E_n \leq 10$ MeV, the resonance widths vary between 0.6 to 800 keV. The mean spacing of the states is roughly $1 \cdot \text{MeV}^{-1}$. The reader is referred to the cited reference for additional details. Some consideration has also been given to the couplings to (2p,1h)-states. The effect of such states is also seen as resonances in the total neutron scattering cross sections. The experimental evidence is beautifully summarized by the data of Glasgow and Foster[31]. For $2 \leq E_n \leq 6$ MeV, and $20 \leq A \leq 208$, the resonances appear. The widths are typically of the order of 100 keV and the mean spacings are $2\text{-}4 \cdot \text{MeV}^{-1}$. Both the resonances arising from (p,h)-intermediate states and those from (2p,1h)-intermediate states are described as "the resonances of intermediate structure".

5. Residual Forces

We want to make some general remarks concerning the appearance of residual forces and their treatment. The problem arises only if we decide to parametrize the effect of these forces in terms of some selected functional form. Traditional shell model computations have followed this approach. The reason for this has been to make a complicated problem tractable.

The many-body descriptions of the fermion systems seek to produce a self-binding system, at the given density and energy. We will consider finite systems as has been done throughout these discussions. The self-binding is produced by the delicate balance between the attractive forces, principally that in the ^1S_o state, and the kinetic energy. The

finite size of the system is carried by the representation basis. This basis has been generated in this analysis from the diagrams of fig. 1. The procedure has been to generate the Brueckner operator, t, from the given force v. Let us now designate this t as the given force. Now the diagrams of fig. 1 generate the average potential $< t_j >_o \equiv U_o(j,\epsilon_j)$ which defines the basis. At each vertex in the diagrams, the given force, t, acts. For <u>all</u> other diagrams, we describe the vertex operator by $t' = t - < t >_o$, the residual interaction.

The average potential is a set of numbers, $U_o\ (j,\epsilon_j)$, for any many-body computation. We were given a coordinate representation for v. Corresponding to this v there is a functional, coordinate-representation for the given force, t. On the other hand, the residual force, t', is the state-, and energy-dependent set of numbers $t'(j,\epsilon_j)$. As long as we are doing numerical computations on fast computers, as part of a systematic program of evaluation, the t'-interaction is just as easily handled as that t. The problem of what to do with the residual force never arises. It is only when the evaluations involving t' are not part of such an established program, and we utilize a functional form for t', that difficulties arise. In general, it sill not be easy to find simple, tractable forms which replace to good approximation the non-local, velocity-dependent, interaction implied by the many-body theory. Very often, if we work within a given shell, e.g. p-, or (sd)-, the form parametrization can be achieved. Thus we encounter force mixtures of Rosenfeld and Soper types, for example.

In an earlier section, while discussing (p,h)-states, we noted that the transition matrix elements of t' really reduced to those of t. One was then forced to remark that only the given force, t, was relevant to the specific computation. The use of an equivalent functional form, which we will now describe as the effective force, for t', the residual force, simplifies the computation of transition matrix elements. The effective force is to be evaluated within excited configurations, or within configurations other than the chosen configuration. This is its role. The following considerations serve to illustrate this.

We stated earlier that a trial function, $\Psi(N)$, would be used for the N-system consisting of the usual Slater determinant in occupied states, plus additional determinants of (h,p)-type. The physical vacuum $\Psi(N)$ is then expressed as

$$\Psi(N) = c_o \Phi_o + \sum c_{ml} \alpha_k^+ \alpha_l \Phi_o \ , \tag{44}$$

the c's being expansion coefficients and $\Phi_o(N)$ the true vacuum. The operators (α,α^+) for the quasi-particles satisfy anticommutation rules. In particular, we also have the relations for these operators, symbolizing dressed particles,

$$\alpha_k \Psi(N) \neq 0, \ (k > k_f; \ k \leq k_f) \ .$$

We then introduce the customary definitions,

$$\alpha_k = \begin{cases} a_k\ , & (k > k_f) \\ b_k\dagger, & (k \leq k_f) \end{cases} \quad ; \quad a_k^\dagger = \begin{cases} a_k\dagger, & (k > k_f) \\ b_k\ , & (k \leq k_f) \end{cases} \quad , \tag{45}$$

of the quasi-particle operators in terms of those for bare or shell model states. For the latter operators, the following relations hold

$$a_k \Phi_o = 0, \ (k > k_f) \ ; \quad b_k \Phi_o = 0, \ (k \leq k_f) \quad .$$

The vacuum state is rewritten as

$$\Psi(N) = c_o \Phi_o + \sum c_{ml}\ a_m^\dagger\ b_l^\dagger\ \Phi_o \quad . \tag{46}$$

The states of the (N±1)-systems have the representations

$$\Psi(N-1) \equiv \alpha_k\ \Psi(N) = \begin{cases} 0 \quad + \sum c_{kl}\ b_l^\dagger\ \Phi_o & , \ (k > k_f) \\ c_o\ b_k^\dagger\ \Phi_o \ + \sum c_{ml}\ a_m^\dagger\ b_l^\dagger\ b_k^\dagger\ \Phi_o & , \ (k \leq k_f) \end{cases} \ ; \tag{47}$$

and

$$\Psi(N+1) \equiv \alpha_k^\dagger\ \Psi(N) = \begin{cases} c_o\ a_k^\dagger\ \Phi_o \ + \sum c_{ml}\ a_m^\dagger\ b_l^\dagger\ a_k^\dagger\ \Phi_o & , \ (k > k_f) \\ 0 \quad + \sum c_{mk}\ a_m^\dagger\ \Phi_o & , \ (k \leq k_f) \end{cases} \ . \tag{48}$$

These lowest states of the (N±1) systems are linear combinations of hole

and (2h,1p)-states, particle and (2p,1h)-states.

The chosen configuration, that described by (46), determines the configurations of the (N±1)-system within which the evaluations of the many-particle matrix elements occur. In particular, we shall not attempt to determine the excited configurations of (47) and (48) in any self-consistent way. Such configurations are computed from perturbation theory. We deal with the states $\Psi(N-1)$ first. The diagrams of fig. 1 are given again in fig. 9, and labelled so as to pertain to these states. The interaction operators of this diagram must be described.

We adopt the operator point of view developed by Watson[32] whereby the one-, and three-quasi-particle configurations are projected out by operators P_1 and P_3. The average values of the given force, t, computed in these configurations are t_1 and t_3. The interaction operator is then written as

$$t = (1 - P)\, t + Pt \equiv I + Pt = I + P_1 t_1 + P_3 t_3 \quad . \tag{49}$$

The Watson, I-operator is the residual force, t', of this discussion. The quasi-particle basis is defined in terms of the one-body operators deriving from the averages

$$[\epsilon_k - T_k - U_k(\epsilon_k)]\, \vartheta_k = 0 \quad ; \tag{50}$$

$$U_k(\epsilon_k) = \sum_s \langle ks|t_1|ks\rangle - \sum_{p\ell m} \langle km|t_3|p\ell\rangle \frac{1}{e(p\ell m;\ell)} \langle p\ell|t_3|km\rangle \tag{51}$$

$$= \langle t_1 \rangle_k - \langle t_3 \frac{1}{e_3} t_3 \rangle_k \; ; \quad (s \leq k_f \; ; \; p, \, m \leq k_f, \, \ell > k_f) \quad .$$

The expression for the average field is determined by the vanishing of the sum of diagrams in fig. 9. We have previously discussed the self-consistent determination of the states $k \leq k_f$. Note that in (51), e_3 is the excitation of the 3 quasi-particle configurations relative to that for a single quasi-particle.

Next, we examine the states of the (N+1)-system. It is considered that no more hole graphs are to be accounted for in the self-consistency. The five graphs of fig. 1, 2, and 3 are repeated in figs. 10, 11, 12, and 13 with, however, a different grouping.† Excited configurations, $\Psi^*(N+1)$, of the (N+1)-system are produced by the conventional operation upon (48). These consist of linear combinations of (1p)-, (2p,1h)-, and (3p,2h)-states. Again, we should only like to account for these configurations in an approximate way. The graphs of figs. 11 and 13 are discarded, on this basis, in the self-consistent determinations. That of fig. 12, (a), could indeed belong to $\Psi(N+1)$. It is of 3rd-order and should be a perturbation on the interaction energy. We shall also discard this graph in the self-consistency. If we were to retain any 3rd-order process, it would be that represented by fig. 12, (a). However, in order to take full account of (p,h)-correlations, we would sum the

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† The arrangement of graphs is governed by the configuration assignments.

ladder of repeated (p,h)-interactions. This would define the J-vertex of Shaw[33], fig. 14. Let us follow this procedure, observing that the new operator has the average J_3 in the (2p,1h)-states. We can now form the representation for the self-consistent potential as in fig. 15. The 3rd-order term, involving J,

$$J = t + i\, t\, G^{+}\, G^{-}\, J \quad , \tag{52}$$

the operator for (p,h)-interactions, is still in general smaller than the 1st-order term. Particle self-consistency would then be determined by the graph of fig. 10. That the graph should vanish, yields the self-consistent potential defined on occupied states. The role of the (2p,1h)-states is to determine the anti-hermitian part of t as indicated by fig. 5.

The omission of the (3p,2h)-graph of fig. 11 is known to lead to a violation of the equilibrium thermodynamics of the <u>infinite</u> system[34]. The corresponding restriction does not apply to the finite system, in any event. The determinations we have made, leading to different statement of self-consistency in the (N±1)-systems, are reflections of the relations in (47) and (48).

The question of residual forces can be raised again. Referring to our diagrams, the choice of residual interaction, either $(t-t_1)$ or $(t-t_3)$, follows from the definition $I = t - P_1 t_1 - P_3 t_3$. The one-body operators t_1 and t_3 describe the interaction of the quasi-particle, namely, the excitation k of the (N±1) systems, with the background of occupied states, and with the excited configurations of two particles

and a hole (two holes and one particle). In computing the scattering of (h,p)-pairs, say, (ℓ,m_1) and $\ell_2,m_2)$, as in fig. 16 (a), we obtain in the matrix element

$$< \ell_2 m_1 | t | m_2 \ell_1 - \ell_1 m_2 > - \delta_{m_2 m_1} < \ell_2 | t_3 | \ell_1 > - \delta_{\ell_2 \ell_1} < m_1 | t_3 | m_2 > \quad .$$

For the hole states, t_3 projects off the (2h,1p)-background interactions. The (2p,1h)-background is to be projected off for particles. However, we have seen that these states play a rather definite role in determining particle lifetimes. There is as noted earlier no t_3-projection operator for particle states. We can maintain the formal structure of the matrix elements by replacing t_3 by t_5 for particle states. This has additional implications. An excited state of the (N+1)-system was noted as having (1p)-, (2p,1h),- and (3p,2h)-contributions. It is immediately clear that unless we add one particle, at the Fermi level, to a quiescent N-particle system, the excited (N+1)-system is always under discussion. Having already accounted for the (1p)-and (2p,1h)- contributions, we need then to incorporate the remaining contribution, fig. 11. This is done by defining an interaction operator $I = t - P_1 t_1 - P_3^- t_3^- - P_5^+ t_5^+$ with the additional projections (±) on quasi-particle and quasi-hole states. The self-consistency for particles is determined by the operator $U_k(\epsilon_k)$ of (51) with $t_3 \equiv t_3^-$ replaced by t_5^+. Order arguments are then used to eliminate all diagrams except those of fig. 16. These together with that of fig. 5 determine the (N+1)-excitations. We could have arrived at these results in a direct way. However, the chain of argument is

instructive. The (3p,2p)-configurations actually involve (2h,1p)-intermediate states, thus $t_3 = P_3^- t_3^- + P_3^+ t_3^+ = P_3^- t_3^- + P_5^+ t_5^+$. What has been indicated is the very basic asymmetry in the discussion of holes and particles.

The (p,h)-matrix element which we introduced above is now written as

$$< \ell_2 m_1 | t | m_2 \ell_1 - \ell_1 m_2 > - \delta_{m_1 m_2} < \ell_2 | t_3 | \ell_2 > - \delta_{\ell_1 \ell_2} < m_1 | t_5 | m_2 > \quad .$$

This is just the matrix element $< \ell_2 m_1 | I | m_2 \ell_1 - \ell_1 m_2 >$ of the residual interaction. Evidently, the non-diagonal elements of I are equal to those of t, the given interaction, in this case. The situation is no different for a conventional H-F computation with the diagram of fig. 16 (b). One simply replaces t_3 and t_5 by v_{HF}.

When we employ effective forces to rewrite

$$t = (1 - P_o)\, t + P_o\, t$$

as

$$t = t' + P_o\, t \quad ,$$

with a functional form t' replacing what was I, a great deal of the descriptive simplicity is lost.

It is supposed that the function t' is equal to the actual force t, but evaluated in the medium. The qualification states that the presence of other particles is important for the pair-interaction under scrutiny.

This renormalization of the interaction can be quite important. One would find this to be the case in computing two-body interactions at the middle of a shell. One can say this in another way. There are appreciable components of long-range multipoles which contribute to the average field, P_o t. The situation is well-known in, for example, the discussion of vibrations about the deformed shape. If we discuss vibrations about the spherical shape, there can still be important polarization effects. Two particles, being part of a vibration, may chiefly interact through virtual emission and absorption of a phonon.

There is, as noted, some reason to believe that the dynamical polarization effects may well be important. Chiefly, we expect these to manifest themselves in the (N±2) systems. It is assumed that the N-system has a spectrum of comparatively low-frequency, vibrational states. This spectrum is established with little or no renormalization of the given force. One relies upon coherent momentum transfers, of nearly <u>equal</u> and <u>small</u> amounts, to build the collective states. This is physically the meaning of the formal summation of the "ring diagrams". As long, then, as we work in the N-system, almost any functional approximation, t', to the near-zero-momentum transfer components of t is likely to be adequate.

Now one can argue indefinitely that the hard core, or high-momentum components of the force, drive the diagrams of figs. 11 and 13 (b). This has no relevance for the finite system. Again, the renormalization encountered in going from v to t can never be important at low momentum transfers.

The given force is still evaluated as a diagonal operator, symbolized by P_o t. However, the parameters in t' are not well-determined. The recourse to empirical data as a means of fixing the matrix elements of P_o t does not necessarily work as satisfactorily for t'. In using t' we will of course compute transition matirx elements. At the same time, we have the rule that these matrix elements are formally equal to those of the given force. The practical difficulty is largely with the difference in the momentum spectra of t' and t. In most applications we choose a t' which reproduces t over a part of its momentum-transfer spectrum. Forces, t', with predominately low momentum components are used to summarize the scattering data up to perhaps 20 MeV. Alternatively, they do this in a quite approximate way and give the level spectra for some class of nuclei, e.g., p-shell systems. It is also possible to use forces t' with appreciable high momentum components which summarize the two-body scattering data up to 100 and sometimes up to 310 MeV. Most of the nuclear phenomena we are trying to describe, in an intermediate-coupling or a configuration-mixing shell model calculation, are insensitive to the existence of the very high momentum components of the force. We believe then that we are not forced to reproduce t beyond perhaps 45 MeV, or at the most 100 MeV.

Owing to the absence of suitable H-F and BHF calculations for finite systems we never establish a quasi-particle basis. Instead, one assumes a shell model basis with the states being fixed by empirical data. The shell model basis, to date, is that of the isotropic, harmonic oscillator.

Despite the overwhelming success of this description, one is still troubled in any computation. The higher oscillator levels, Λ, $\Lambda = 2n + \ell - 2$, are characterized as having wave functions with too much kinetic energy. This, in effect, simulates the behavior of a very hard force.

Often, a two-body force, $t' \simeq \lambda\, \delta\, (\hat{r}_1 - \hat{r}_2)$, of zero-range is employed in the computations. This force fills the entire momentum spectrum with uniform weight. It is conventionally argued that t' represents the fluctuations about the average field $P_o t$. The effective force, given above, is supposed to be a reasonable approximation to the short range, fluctuating field. In many situations, however, t' contains additional long range correlations to which the nucleon motions are quite sensitive.† One uses the given effective force, in most cases, owing to its simplicity. The volume strength parameter, $(\lambda/4\pi)$, should be approximately 430 MeV $\cdot$ f^3, corresponding to the observations in the two-body system.

The shell model computations, diagonalizing the effective force, are very often sensitive to the functional form chosen for t'. This sensitivity seems to be dependent upon the type of nuclear state being computed. Discussions of this effect are abundant in the literature.

Before the advent and adoption of hard core potentials, the shell model had the burden of explaining saturation. This was manifest in the use of exchange mixtures satisfying the saturation conditions, in

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† This point has been especially stressed by Mottelson.

full or partially. Brueckner has demonstrated that nuclear matter is stable against collapse about the point of normal density, as given by the empirical mass formula. In the computations pertaining to finite systems one can take the view that the model operator defining the BHF basis is not required to saturate. This aspect is not altogether clear[35]). Independently of whether this is so, we are entitled to feel uncomfortable if the result of an empirical search for a best effective force, simulating t', produces an exchange mixture implying saturation.

Most of the effective forces in use utilize the relative coordinate dependence $v(r) = v_o(\mu/r) \exp(-r/\mu)$. The range is taken to be that given by one pion exchange, roughly $\mu = 1.4$ f. This emphasizes again the long range nature of the force, its absence of high momentum components. The hard core is believed to be about 0.35 f. This is seen, with any probability, in collisions of about 85 MeV in the relative system. We are talking about potentials, here, whose momentum components are exhausted by the time one reaches 35 MeV of momentum transfer in the relative system.

It is customary to neglect the tensor force in the description of the effective force. The necessity of having such a contribution is well-known in connection with the average field giving the binding of nuclear species. There is also the possibility that the correct two-body tensor force will generate the average, one-body spin-orbit field. This was investigated extensively by Kisslinger[36]) within the framework of Brueckner theory. Relative to the computation of certain transition

matrix elements, the tensor force has been required for the description of the β-decay in light elements[37].

Finally, we note that the effective force may have matrix elements which are energy-, and, momentum-transfer-dependent. The many-body theories extant would prefer this. The sort of force implied would have high momentum components. A partial list of some commonly used effective forces is given in Table 1, and some of their properties are noted. The reader must note that none of the forces listed can establish the single particle basis or spectrum which is summarized by the nuclear shell model. The forces are effective interactions, then, in this specific sense.

6. Pairing Force

Up to this point, no account has been given of the pairing interaction. The shell model summaries of nuclear properties indicate the importance of this interaction for systems having $Z \geq 28$ [39]. We shall give a relatively brief discussion of how the pairing is to be taken into account in the presence of the self-consistency of the previous discussions.

Our point of view, here, is the same as that of the earlier sections. Pairing is described through the introduction of the so-called gap-function, F, into the one-particle propagator[40]. The result is to generalize this propagator to a matrix function. Again, we observe that the addition of a particle or a hole to the N-particle system does not take place at the equilibrium density $\rho_o(N)$ of that system. It is

however assumed that, the relevant density, ρ, departs by a small amount from ρ_o. The density fluctuations are governed by the quantity $\overline{(\rho - \rho_o)^2}$, the average being performed over the chosen configurations of the N-system. Density fluctuations are introduced by computing the system-response to a scalar, external field, this being ultimately set equal to zero. The information concerning the fluctuations is contained in the correlation function, L. The external field induces t^2-terms in the equation of motion for L. These terms describe the re-establishment of equilibrium through the mechanism of collisions. Single particle (quasi-particle) self-energies, Σ, are modified by their coupling to the density fluctuations. The new self-energies depend upon the Brueckner scattering operator, t, to 4th-order. This result follows from a perturbation expansion of the Martin-Schwinger operator in terms of that of Brueckner.

It is believed that the pairing force is a short range, fluctuating residual interaction, in the sense of the shell model discussion of the previous section. In view of this, we can represent the interaction as a delta function in coordinate space. Many other alternative possibilities exist[41]. For the present, we shall confine our remarks to infinite systems and the spectral functions associated with these. The discussion of the pairing in finite systems and the corresponding spectral functions, represented as stationary time series, is deferred to a subsequent publication. The transition from infinite to finite system is only a matter of detail.

We replace the Heisenberg operators of our earlier discussions by

two-component spinors. These are defined as

$$\Psi = \begin{bmatrix} \psi_{\uparrow} \\ a\psi_{\downarrow}^{\dagger} \end{bmatrix} \quad \text{and} \quad \Psi^{\dagger} = \begin{bmatrix} \psi_{\uparrow}^{\dagger} & a^{\dagger}\psi_{\downarrow} \end{bmatrix} . \tag{52}$$

The operator a annihilates two particles of opposite spin (total angular momentum) projection. The projection is symbolized by the arrows in (52). The operator a,

$$a = c \int d\hat{r}\, \psi_{\downarrow}(\hat{r}t)\, \psi_{\uparrow}(\hat{r}t) \; ; \; c \text{ - constant}, \tag{53}$$

keeps the mean number of pairs constant in the N and (N±1)-systems. Our motivation for the representation of (52) and the dependent relations,

$$\mathcal{G}(1,1') = -i < T\,[\Psi(1)\, \Psi^{\dagger}(1')] >_o$$

$$= -i < \begin{bmatrix} \psi_{\uparrow}(1)\, \psi_{\uparrow}^{\dagger}(1') & a\, \psi_{\downarrow}^{\dagger}(1)\, \psi_{\uparrow}^{\dagger}(1') \\ a^{\dagger}\psi_{\uparrow}(1)\, \psi_{\downarrow}(1') & a^{\dagger} a\, \psi_{\downarrow}^{\dagger}(1)\, \psi_{\uparrow}(1') \end{bmatrix} >_o \; ;$$

$$\mathcal{G}(1,1') = \begin{bmatrix} G(1,1') & F(1,1') \\ F(1',1) & -G(1',1) \end{bmatrix} , \tag{54}$$

is quite straightfoward. It has been tacitly assumed that the N-particle,

physical vacuum is non-degenerate. This is not the case. There exist a set of gauge and spin transformations upon Ψ which leave the many-body Hamiltonian unchanged, (see ref. 6). In general, the existence of such invariances, independent of the existence of inter-particle interactions, will lead to the presence of a zero-mass boson[42]). Here, this boson is manifest as the zero-frequency, pairing state. Eq. (54) tells us that a particle in the many-body system not only undergoes independent propagation, G, but also has a finite probability for being in a projection-correlated state, F.

The equation of motion for $\mathcal{G}$ is derived in the conventional manner. One must first consider that obtaining for Ψ. To do this, it is necessary to introduce the matrix set $(\underline{1}, \tau_1, \tau_2, \tau_3)$. The equal-time, anti-commutator of Ψ and $\Psi^\dagger$ generates the unit matrix $\underline{1}$. It is common knowledge that the general 2 x 2 matrix may be expressed in terms of $(\underline{1},\hat{\tau})$. The matrix equation of motion decomposed over this basis is found to be

$$\mathcal{G}_o^{-1}(1,\bar{3})\,\mathcal{G}(\bar{3},1') = \delta(1-1') + i\tau_3^{(1)} \int tr^{(2)}\, v\,(1-\bar{2})\tau_3^{(2)}\mathcal{G}_2(1\,2,1'2^\dagger), \qquad (55)$$

$$\mathcal{G}_o^{-1}(1,1') = \left[i\frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2M}\ \tau_3^{(1)} \right] \delta(1-1') \quad .$$

As we have no special interest in the gauge-invariant formulation of the problem, the discussion proceeds as before. The 2-particle propagator is considered as satisfying the Bethe-Salpeter equation. The laddar approximation generates the matrix scattering operator. The density

correlation function is defined as a variational derivative with respect to a scalar external field. This procedure also generates the equation of motion for the correlation function. Leaving aside the matrix representation for the moment, we carry out a set of manipulations which can then be directly generalized to the matrix functions. One has

$$\Sigma\, G = i\, v\, G_2 = t\, G\, G\ ;\ \Sigma = t\, G\ ,$$

and

$$\Sigma\, G = i\, v\, (\, G\, G + L\,) = t\, G\, G\ .$$

We introduce the operation

$$(1 + i\, t\, G\, G)\ \Sigma = (\, 1 + i\, t\, G\, G)\, v(G\, G + L)\ ,$$

to obtain

$$(1 + i\, t\, G\, G)\ \Sigma = t(\, G\, G + L)\ .$$

The next step is to consider the perturbation expansion generated by $t = \lambda t$, $\Sigma = \lambda\Sigma_1 + \lambda^2\Sigma_2 + \ldots$, $L = L^{(o)} + \lambda L^{(1)} + \lambda^2 L^{(2)} + \ldots$. Of course, we also have $G = G_o + G_o\, \Sigma G = G^{(o)} + \lambda\, G_o\, \Sigma_1\, G^{(o)} + \ldots = G^{(o)} + \lambda G^{(1)} + \ldots$, but it is not required that we carry this explicitly. The perturbation expansion leads to the self-energies

$$\Sigma_1 G = t\, G\, G + t\, L^{(o)}$$

$$\Sigma_2 G = - i\, t\, G\, G\, \Sigma_1\, G + t\, L^{(1)}$$

$$\Sigma_3 G = - i\, t\, G\, G\, \Sigma_2\, G + t\, L^{(2)}$$

$$\begin{array}{cc} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{array}$$

These expressions, together with

$$L = - G\, G + G\, G\, \Xi\, L \quad ,$$

$$\Xi = t + i\, t\, G\, G\, t \quad ,$$

define a perturbation expansion which is reminiscent of that which appears in section II. There, however, we did not expand the correlation function, L. Indeed, in order to discuss the bound states of L, including that owing to pairing, the expansion on L is not permissible. As the only question at issue is one of counting, we may formally sum the L-contributions, when there is a bound state present, and rearrange the perturbation expansion as

$$\Sigma_1 G = t\, G\, G + t\, L$$

$$\Sigma_2 G = -\, i\, t\, G\, G \quad \Sigma_1 G$$

. .
. .
. .

We argue then that the bound states of L appear in the "first-order" of the perturbation expansion. The remaining terms of Σ, i.e., Σ_N contain the higher order, weak-coupling ocrrections to Σ_1. It is easily seen that the computation of Σ_1 G accounts for those terms, up to t^3, which were enumerated in section II.

In keeping with the discussion just given, we write the following matrix equations

$$\sum_1 \mathcal{G} = T\, \mathcal{G}\, \mathcal{G} + T\, \mathcal{L} \quad , \tag{56}$$

$$\mathcal{L} = -\, \mathcal{G}\, \mathcal{G} + \mathcal{G}\, \mathcal{G}\, T\, \mathcal{L} \quad , \tag{57}$$

$$T = v\, \underline{1} + i\, v\, \mathcal{G}\, \mathcal{G}\, T \quad , \tag{58}$$

$$\mathcal{G} = \mathcal{G}_o + \mathcal{G}_o \sum_1 \mathcal{G} \quad . \tag{59}$$

The correlation equation, (57), has been approximated. This approximation, neglecting the T^2-terms, ignores the hole-hole and particle-particle

correction to $\mathcal{L}$. These terms are not necessarily small corrections to the pairing interaction, but their omission will not invalidate the discussion of the lowest-order pairing situation. It is required that eqs. (56) - (59) be solved self-consistently. The eqs. of (58) are solved by first carrying out the projections in $\hat{\tau}$-space, by means of suitable trace-operations. At the same time, we must introduce the spectral representation

$$\mathcal{G}(\omega,\epsilon) = (\omega - \epsilon\tau_3 - \Delta\,\tau_1)^{-1}$$

where Δ is the undetermined gap parameter. The spectral representation of T generates that for $\mathcal{L}$, (57). Both representations determine that of Σ_1, (56), and finally that of $\mathcal{G}$, (59). The gap parameter appears as an eigenvalue in the sum-rule expression for the spectral representation of the gap-function F. It is completely obvious that in solving the self-consistent set of equations we will approximate G by that of the BHF-basis. The formal manipulations are straightforward, but tedious. We will find that the quasi-particle energies, $\mathcal{E}$, are defined now as $([\mathcal{E} - \mu(N)]^2 + \Delta^2)^{1/2}$, and, as well, an integral equation for Δ. These results are all well-known. Our only purpose in re-iterating such statements, is to indicate that the pairing interaction may be taken into account in the presence of the previously obtained, many-body self-consistency.

In any determination of the pairing, we will be guided by the empirical classifications furnished by the shell model. We shall be

restricted to (j-j)-coupling. It is possible to summarize any representations (state-energy) of dynamical quantities in terms of their invariances under the operations of the symplectic group. In this connection, it will be helpful to recall that the quantum numbers referring to reduced isotopic spin and seniority have special relevance for the interaction of identical particles in the same j-shell.

7. Summary and Conclusions

A discussion of the many-body problem has been given from the point of view of the configuration-mixing and intermediate-coupling shell models. It has been emphasized that the configuration-mixing density correlations can be taken account of in a formally consistent manner. These correlations determine the many-body configurations within which we carry out our evaluations. They moreover specifically influence the nature of the quasi-particle spectrum which one can obtain.

All of our remarks have been, and are, restricted to systems having spherical symmetry. The quasi-particle basis which pertains to such systems has been described by us as that of Brueckner-Hartree-Fock (BHF). We were able to show that the non-hermitian, one-body, energy-operator, which defines the basis, led to quite different interpretations of the physical configurations appearing in the excitation spectra for the (N+1)-, and (N-1)-systems. The absence of a simple symmetry operation, such as that of particle-hole conjugation[43], to relate the excited (N-1)-configurations to those of the (N+1)-system is completely evident.

The presence of density correlations in the physical system was shown to lead to collective states, the oscillations of the nuclear surface. Our discussion was entirely restricted to the collective states of particle-hole character. The study of more complex states, those of higher quasi-particle number†, would not have been consistent with our configuration assignments. It was found that the (p,h)-matrix elements, determining the positions of the collective states, were to be corrected by matrix elements involving the states of two particles and two holes. The existence of such matrix elements reflected the symmetric description of particles and holes for states near the Fermi level.

The quasi-particle spectrum was replaced by a real spectrum. We have described the latter as that for a single particle. The single particle spectrum includes the couplings between quasi-particles and collective states. Several of the renormalizations which are brought about in this way were described. A combination of perturbation theory and matrix-diagonalization procedures was used in obtaining the descriptions.

Some attention was given the problem of residual forces and the description of these. The enumeration of such forces is dependent upon how we choose and restrict our configurations. Our discussion was patterned after those of the projection method introduced by Watson.

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† The low frequency 2^+ state is one of higher quasi-particle number, i.e., 4, in our nomenclature.

We attempted to outline how one is to incorporate the description of pairing into the computations. In effect one seeks to generalize the BHF basis so that it is defined in the pairing field. It was necessary to use the Nambu, spinor representation of the Heisenberg operators in order to begin the generalization. The many-body self-consistency becomes rather complicated. It is summarized by four, simultaneous matrix equations in the operator functions of the theory.

The differences between the Green's function descriptions for finite systems and infinite systems have been stressed. These are summarized largely in the basis sets and spectral representations which apply in the two situations. There can also be considerable differences in the modes of description. These however are dictated by the physics in every case.

A large part of the current experimental effort is oriented toward measurement of the properties of nuclear excited states. The formal apparatus of the Green's functions is especially suited to the descriptions of such states, if they are not too complicated. Our emphasis has been placed upon the single particle states. Such are to be seen in the low-resolution (d,p)-data[44-46], for example. At least in the region of the Ni-isotopes, we have reason to expect some level shifts of the neutron spectra, owing to the collective couplings, especially to the 2^+ and 3^-. The γ-spectra from fast neutron capture also show fluctuations in this region[47]. It may well be that this selection-rule-restricted process has some appreciable contribution coming from the 2-particle bound state, as we have described it.

Fluctuations in the apparent density of states in the single particle spectrum can arise from quasi-particle couplings to both collective and non-collective states. As to which of these is the more important will depend upon the particular class of experiments. As a general guide we always ask ourselves where the strength for the particular impurity-excitation lies in energy. The total, elastic and inelastic γ-cross sections bear upon this question. Fluctuations of the photon transition strength function have been seen in high-energy, bremsstrahlung monochromator measurements employing small band-widths[48]. Similar to the situation obtaining in (n,γ), the strong operation of selection rules ought to be vastly helpful in restricting the possible formal descriptions.

The analysis of n-O^{15}-scattering by Lemmer and Shakin[26], for neutron energies up to 10 MeV, describe the quasi-particle couplings to non-collective states of (p,h)-character. Such states are also employed in the discussion of the fluctuations in S-wave strength functions given by Block and Feshbach[49]. The 3-300 keV data[50], especially that pertaining to the behavior of P-wave strength function, has some interesting fluctuations in the vicinity of A = 60. We have examined this aspect from the point of view of collective couplings. However, no numerical results are available at this time.

Quasi-particle couplings to (2p,1h)-states have been discussed theoretically[27]. The impetus for this work was provided by the total neutron cross section data of Foster and Glasgow[31], and the reaction

data, $n + Si^{28} \rightarrow \alpha + Mg^{25}$, $12.5 \leq E_n \leq$ MeV, obtained by the CISE group[51]). Numerical determinations are in progress for the two situations.

In explaining, or attempting to explain, those features of physical data which make manifest the continuum intermediate-coupling and configuration-mixing shell model descriptions, certain problems arise. What we look for in the data are certain correlation structures, built upon energy-angle dependences of the cross sections. These structures of the system must be distinguished from the statistical fluctuations[51]) in the level densities of the true states of the system. This is not altogether simple to do. While some general rules might be given the experimentalist, the subject is somewhat young and still under development. Subsequent experience with the models will enable us to state what are, very likely, the most profitable ways in which to collect and analyze the data.

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TABLE 1

Effective Forces

Force	Character	Remarks
Yamaguchi	$(\hat{k}'\|v\|\hat{k})=\lambda\Sigma v_l(k,k')P_l(\hat{n}_k\cdot\hat{n}_{k'});\ v_l(k,k')=v_l(k)v_l(k')$ $v_l(k)=T_l(\hat{S},\hat{k})(\beta_l^2+k^2)^{-2l-1}$, T_l-tensor of rank l formed from total spin $\hat{S}=\hat{\sigma}_1+\hat{\sigma}_2$ and relative momentum $\hat{k}$; λ and β_l are parameters.	Factorization occurs in momentum space of the relative 2-body system. Parameters λ,β_l are determined by fit to scattering and bound state data. The corresponding scattering operator $(\hat{k}\|t\|\hat{k})$ is factorable, and energy-, momentum-transfer-dependent. Owing to the Moshinsky transformation, $(\hat{k}'\|t\|\hat{k})$ can be used with oscillator functions.
Rosenfeld	$\frac{\hat{\tau}_1\cdot\hat{\tau}_2}{3}\,v(\|\hat{r}_1-\hat{r}\|)\ (0.3+0.7\,\hat{\sigma}_1\cdot\hat{\sigma}_2)$; one chooses as a rule either $v(r)=v_o(\mu/r)\exp(-r/\mu)$, or $v(r)\rightarrow\delta(\hat{r}_1-\hat{r}_2)$.	The exchange mixture satisfies the full saturation conditions. It assumes singlet-to-triplet ratio of deutron.
Soper	$v(\|\hat{r}_1-\hat{r}_2\|)(0.865+0.135\,\hat{\sigma}_1\cdot\hat{\sigma}_2)$.	This exchange mixture is a fit to p-shell data, that in Li-isotopes.
Hulthen and Sugawara†	$\left[v_1(r)P_\sigma^{\,1}+v_3P_\sigma^{\,3}\right]\frac{1+P_M}{2}$; $v_i=v_o^{\,i}(\mu_i r)^{-1}\exp(-\mu_i r)$; $v_o^{\,1}=46.87$ MeV, $v_o^{\,3}=52.13$ MeV, $\mu_1=0.8547\ f^{-1}$. $\mu_3=0.7261\ f^{-1}$.	The parameters are adjusted to fit the low energy (n,p) data.
Intermediate Coupling (typical)	$\zeta\,\hat{\sigma}\cdot\hat{l}$ + Rosenfeld 2-body force; Yukawa form.	The parameter ζ is fixed upon diagonalization of the energy matrix. The term $\zeta\,\hat{\sigma}\cdot\hat{l}$ is a one-body force.

† See ref. 38.

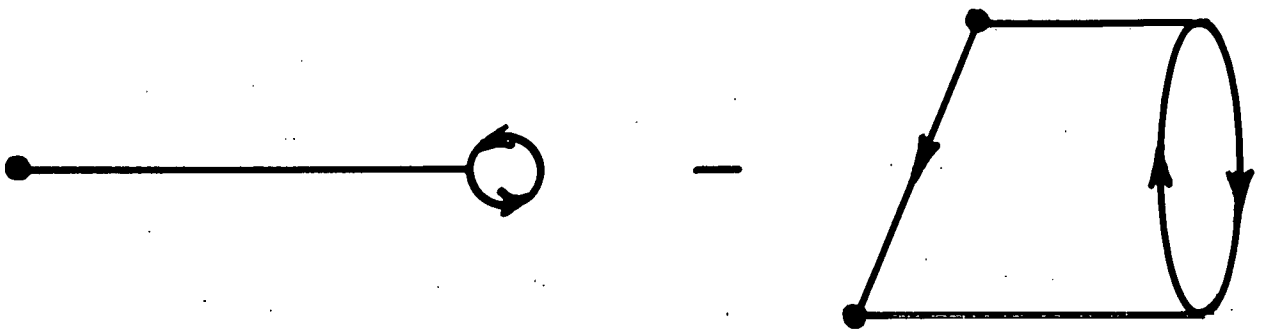

Fig. 1. Goldstone-type graphs determining matrix elements of one-body interaction operator in the quasi-particle basis. Horizontal lines are Brueckner t-operators. External lines are attached to heavy dots.

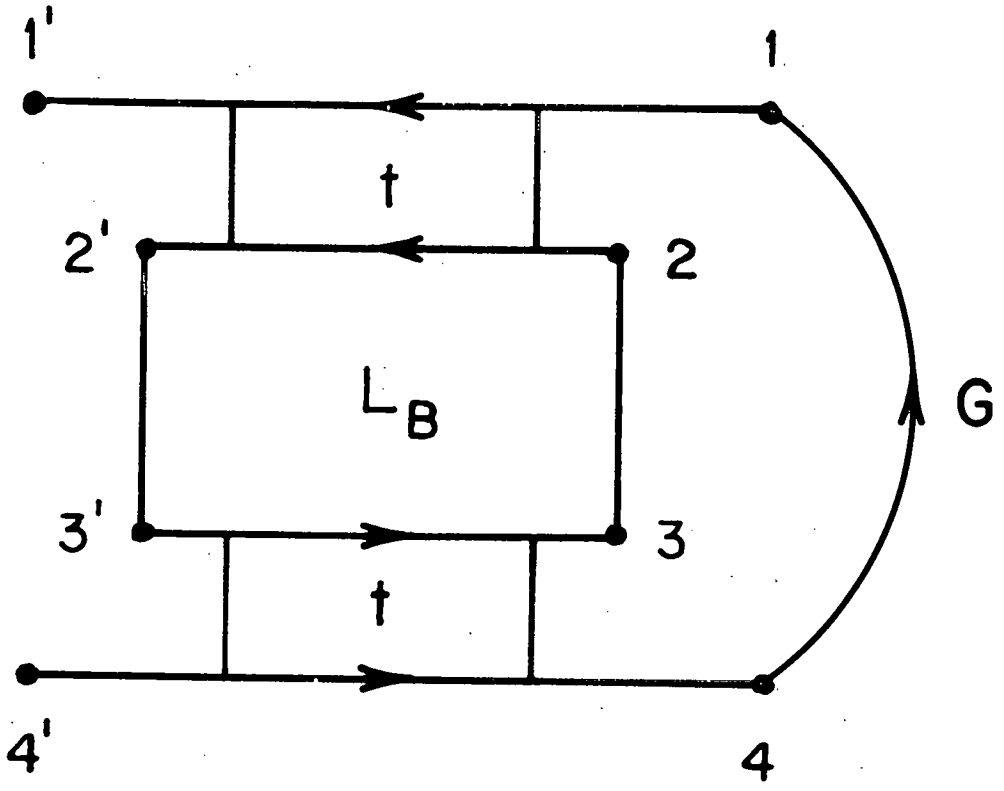

Fig. 2. Third-order, single particle self-energy, with mass renormalization δm on hole lines. This graph contributes to the polarization operator.

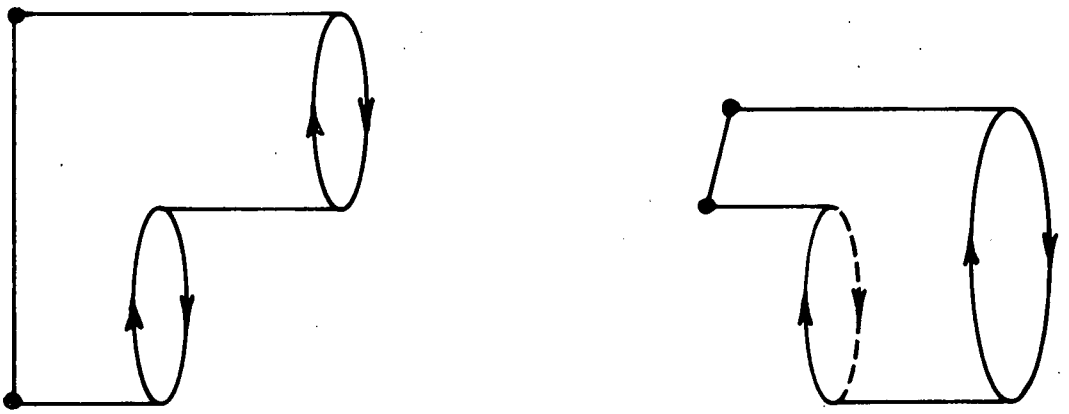

Fig. 3. Third-order, single particle self-energies which contribute to the mass operator.

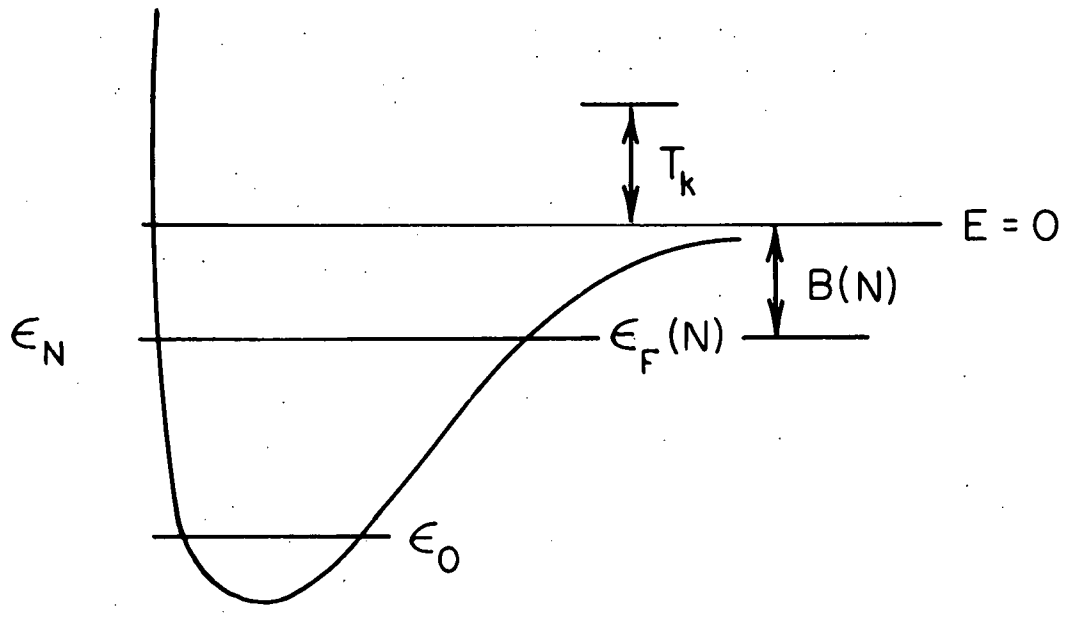

Fig. 4. Idealized, single-particle well generated by the ground state system of N particles. The last nucleon of that system is bound by an amount B(N). A nucleon added to this system in continuum states has asymptotic total energy T_k. We define $\epsilon_N - \epsilon_o$ as $\epsilon_f(N)$.

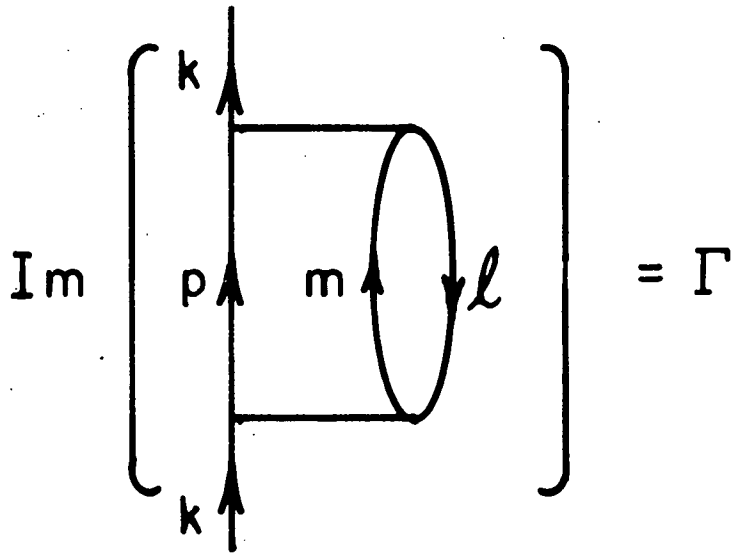

Fig. 5. The lifetime Γ of a single-particle state, k, as determined in the small-width approximation.

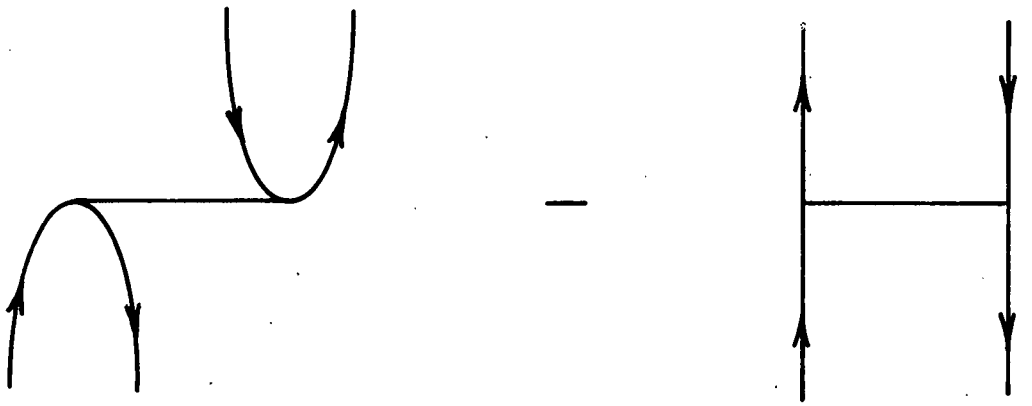

Fig. 6. Direct minus exchange scattering of independent, particle-hole pairs.

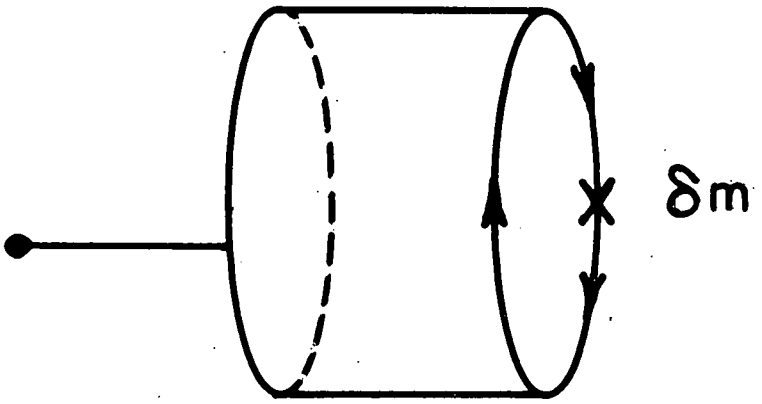

Fig. 7. The self-energy $\Sigma_B^{(2)}(1',4')$ which yields the two-particle bound state. This graph is one of Baym-Kadanoff type.

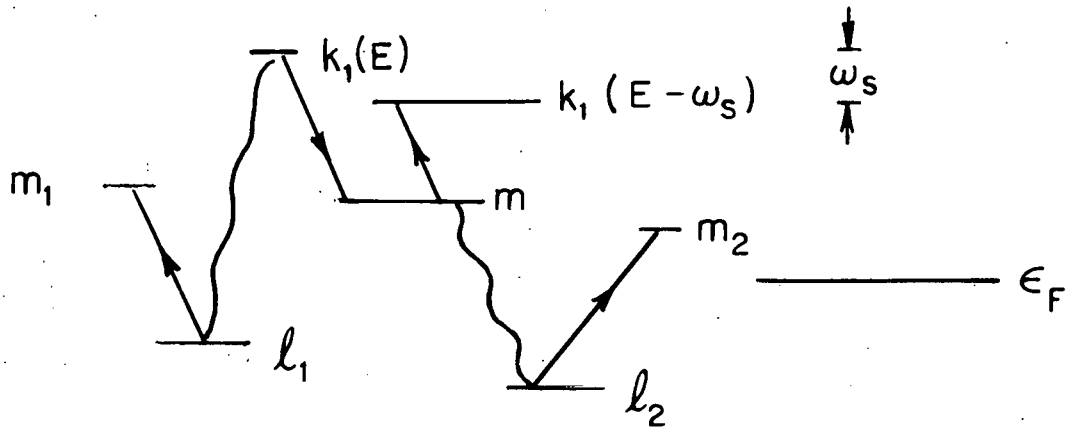

Fig. 8. Excitation sequence describing the formation of a two-particle bound state. Wavy lines connect interacting particles which then make the transitions shown by arrows. The frequency of the collective state, correlating the states k, and m, is ω_s.

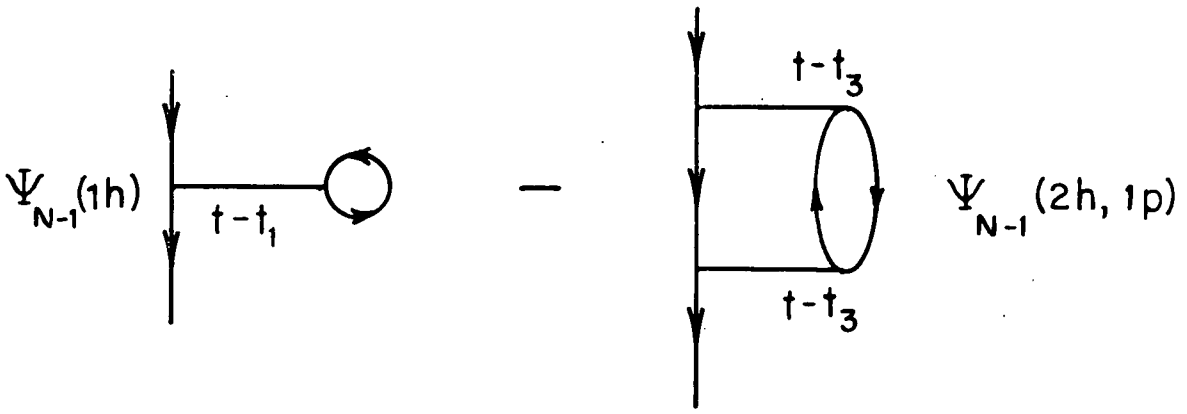

Fig. 9. Hole self-energies, i.e., those of the (N-1)-system which define the quasi-particle basis. Configuration labels Ψ are indicated. The operators t_1 and t_3 are projected on to 1 and 3 quasi-particle states, thus $(t-t_1)$, $(t-t_3)$ are residual interactions. The lowest configurations appear here.

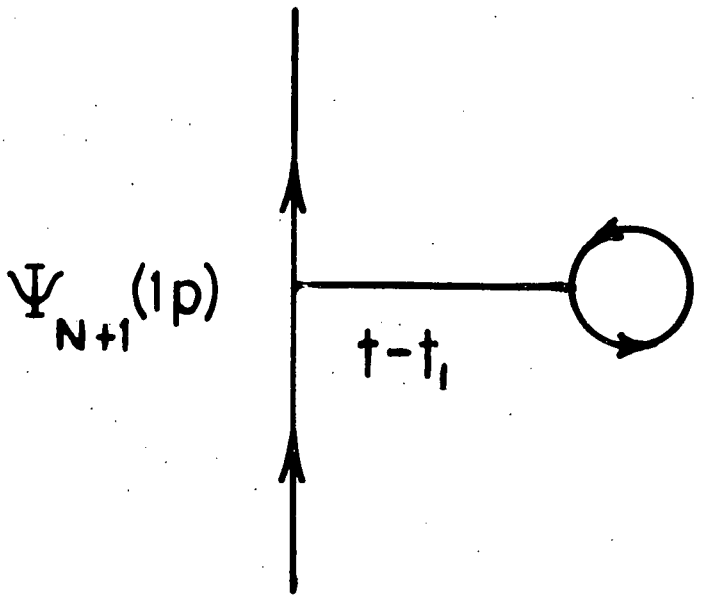

Fig. 10. Particle self-energy, first-order, belonging to lowest, (N+1)-configuration.

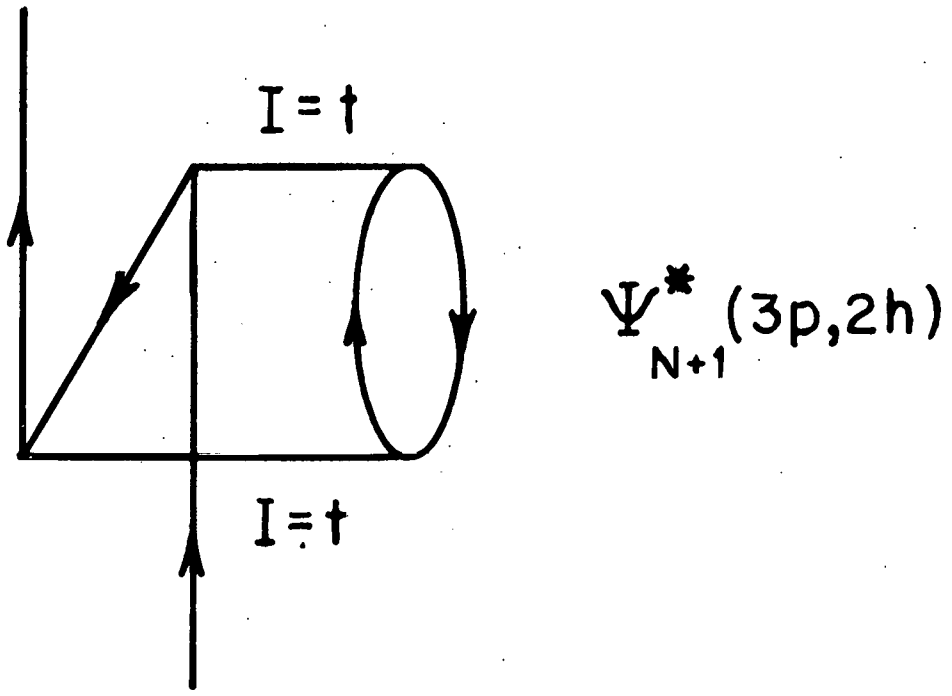

Fig. 11. Second order, particle, self-energy coming from the excited (next-lowest), (N+1)-configuration. The Watson, I-operator is defined in the text.

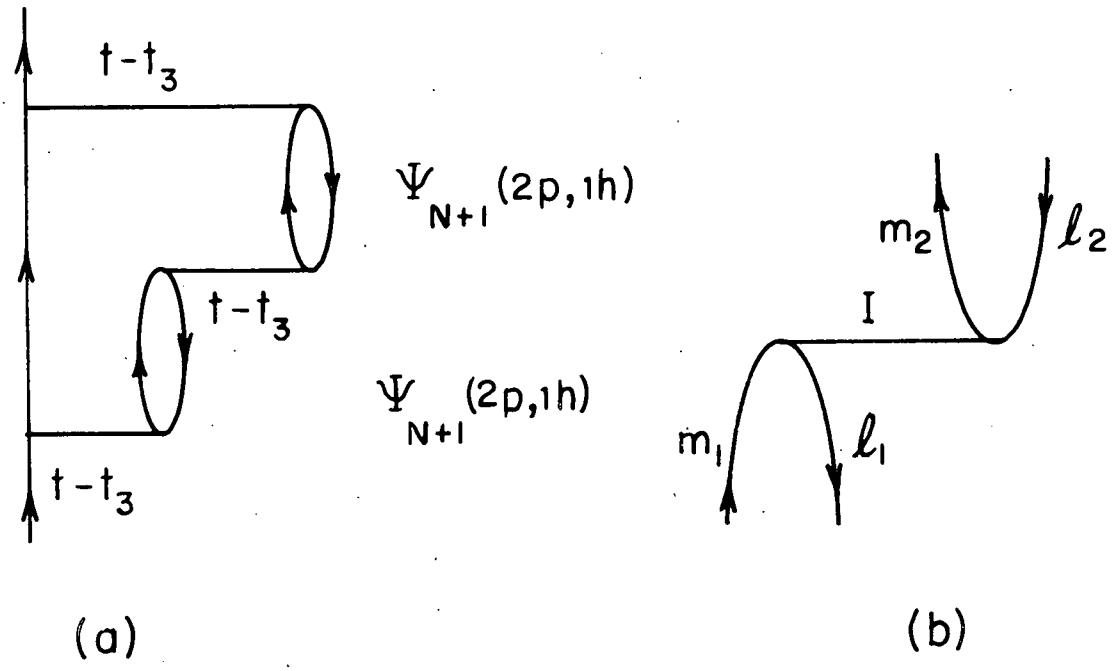

Fig. 12. (a) Same as fig. 10, except for order; (b) The elementary particle-hole scattering generated by (a).

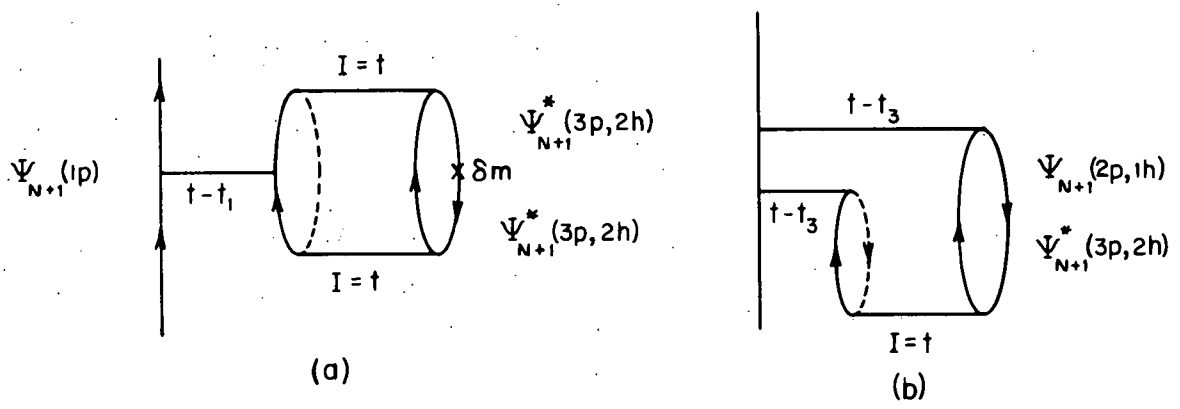

Fig. 13. Particle, self-energies of 3rd-order mixing the lowest and next lowest, (N+1)-configurations.

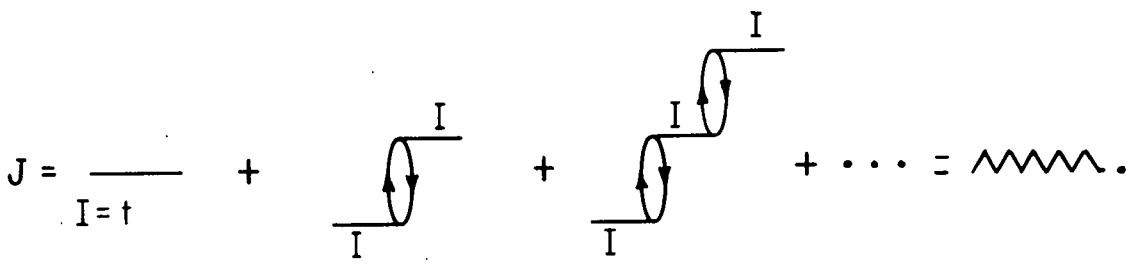

Fig. 14. Sequence of interactions which defines the Shaw-operator, J. This is the operator for hole-particle interactions.

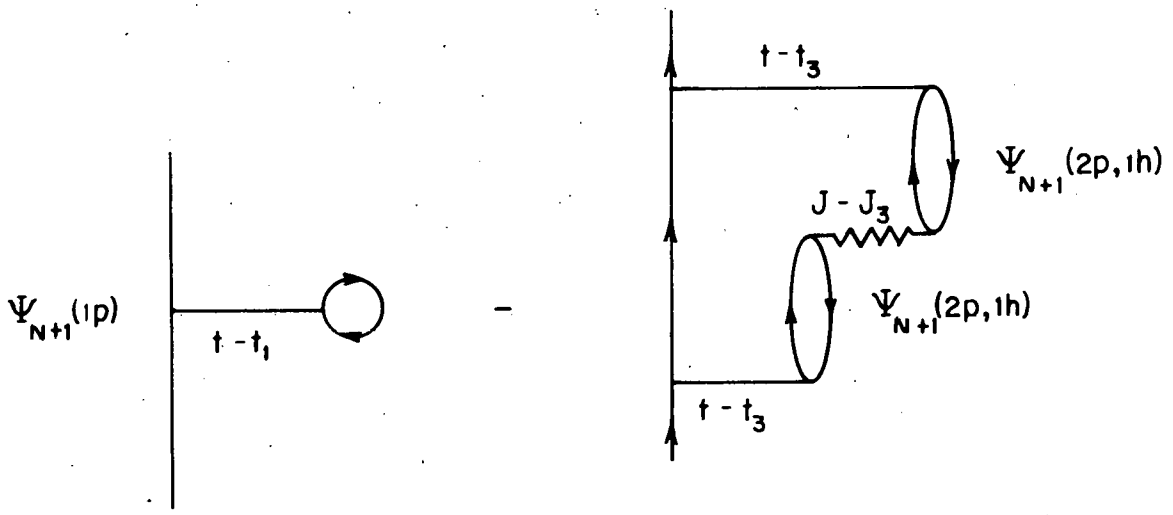

Fig. 15. Single particle energies of the lowest, (N+1)-configuration.

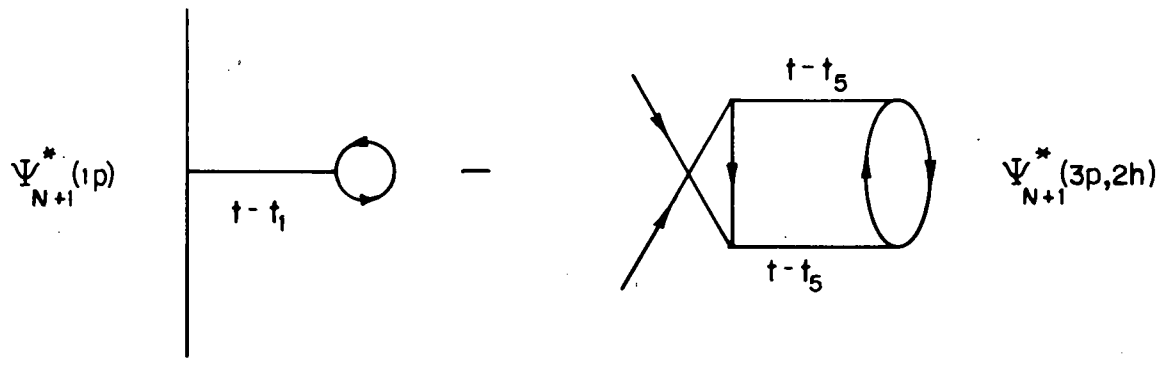

Fig. 16. Particle self energies which define the quasi-particle basis. The next-lowest (N+1)-configuration is the chosen configuration in the sense of Bethe.

Appendix

It is necessary to have a spectral representation of the Green's function G, which is appropriate to the discussion of finite systems. The necessary generalization consists of the incorporation of a stationary time series in the usual definition of the spectral function.

For purposes of orientation we consider the finite, one-body well associated with the ground state, $E_o(N)$, N-body system as that shown in fig. 1A. When we add or remove a particle from this system, the excited state energies are given by

$$E(N+1) - E_o(N) = \epsilon_X(N+1) - \mu(N+1) \; ; \quad \mu(N+1) = E_o(N) - E_o(N+1) \; ;$$

$$E_o(N) - E(N-1) = \mu'(N-1) - \epsilon_X'(N-1); \; \mu'(N-1) = E_o(N-1) - E_o(N) \; .$$

The excitation energies in the (N=1)-systems are shown in fig. 2A. Note that $\mu(N+1) = |B(N+1)|$ and $\mu'(N-1) = |B(N)|$ hold, where B is the binding of the last nucleon.

The Green's function is defined in the time-domain as

$$G_p(t_1-t_2)=(-i)T < \alpha_p(t_1)\alpha_p^\dagger(t_2) >_o=(-i)T < \epsilon^{iE_o^N t_1} \alpha_p \epsilon^{-iH(t_1-t_2)} \alpha_p^\dagger \epsilon^{-iE_o^N t_2} >_o$$

$$=(-i)\,\theta\,(t_1-t_2) < a_p \epsilon^{-i(H-E_o^N)(t_1-t_2)} a_p^\dagger >_o$$

$$+ i\theta(t_2-t_1) < a_p^\dagger \epsilon^{i(H-E_o^N)(t_1-t_2)} a_p >_o \quad . \qquad (1A)$$

We require the Fourier transform

$$G_p(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \epsilon^{-i\omega t - |\alpha| t} \; G_p(t); \; \alpha \to 0^+, \; t = t_1 - t_2 \quad , \tag{2A}$$

which defines the spectral function.

The transform of (1A) leads to

$$G_p(\omega) = \sum | < s|a_p^\dagger|0 > |^2 \, \delta(E_o^N - E_s^{N+1} - \omega)$$

$$- \sum | <s'|a_p|0 > |^2 \, \delta(E_{s'}^{N-1} - E_o^N + \omega)$$

$$= A_p(\omega) + G_p(\omega) \quad . \tag{3A}$$

We introduce the retarded function

$$G_p^-(t) = i\theta(-t) \sum \mathcal{B}_p(\omega_N' - \mu') \, \epsilon^{i\omega_N' t} \quad (\omega_N' = - N\pi\zeta'\Delta) \tag{4A}$$

where ζ' is an, as yet, arbitrary, irrational number. This expression has a transform which leads to the B_p of (3A), if we make the identification

$$| < s'|a_p|0 >|^2 = \mathcal{B}_p(\omega_s' - \mu'); \; \omega_s' = \epsilon_{Xs'}(N-1) \quad . \tag{5A}$$

We write, then,

$$G_p^- (\omega) = \sum \beta_p \, (\omega_N' - \mu') \, \delta(\omega - \omega_N') \quad . \tag{6A}$$

The representation for the advanced function is taken to be

$$G_p^+ (t) = -i\theta(t) \left\{ \sum G_p(\omega_N - \mu) \, e^{i\omega_N t} + \int_\mu^\infty d\epsilon \, A_p' \, (\epsilon - \mu) \, e^{i\epsilon t} \right\} \tag{7A}$$

with $(\omega_N - \mu) = - N\pi\zeta\mu$. The corresponding transform is

$$\begin{aligned} G_p^+ (\omega) &= \sum G_p(\omega_N - \mu) \, \delta(\omega - \omega_N) + A_p' \, (\omega - \mu) \\ &= \sum G_p(\omega_N - \mu) \, \delta(\omega - \omega_N) + \frac{1}{2\pi i} \int_\mu^\infty dx \, \frac{A_p'(x-\mu)}{x-\omega-i\epsilon} \quad . \end{aligned} \tag{8A}$$

The function $G_p^- (\omega)$ is defined on the real axis as the limit from below, $G_p^- (\omega - i0^+)$. It is analytic in the lower-half, complex, energy plane. The complex variable representation for the function is

$$G_p^- (z) = \frac{1}{2\pi i} \, \pi \mathrm{ctn}\pi z \, \hat{\beta}_p \, (z - \mu'); \; z = \omega - iy \quad , \tag{9A}$$

as is known from the general theorems concerning the summability of series.

The advanced function of (8A) has a branch cut. The discontinuity across the cut is just the spectral function A_p'. We observe that the

advanced function will be analytic in the lower half-plane. The analytic continuation of (8A) is

$$G_p^+(z) = \frac{1}{2\pi i}\left\{\pi \operatorname{ctn}\pi z\, \hat{G}_p(z) + \int_\mu^\infty dx\ \frac{A_p'(x-\mu)}{x - z}\right\}. \tag{10A}$$

It is required that the inverse transforms of (9A) and (10A), namely,

$$\int_c e^{izt}\, G_p\,(z)\, dz \tag{11A}$$

return us to (3A). In (3A) we have not taken the usual infinite state limit in order to obtain a continuum representation. The relevant contours for the evaluation, $(t < 0; t > 0)$, of (11A) are shown in fig. 3A. The reader can verify that the representations of (4A) and (7A), $G_p(\omega_s - \mu) = |\langle s|a_p^\dagger|0\rangle|^2$, $A_p' = A_p$, do indeed reproduce the mixed, spectral representation of (3A). The single remaining detail is a common one. In expressions such as (6A), the density-in-energy of the actual compound states (s,s') is to be identified with the energy-conserving delta function. Our results are then summarized by writing

$$G_p^-(\omega) = \sum \mathcal{B}_p(\omega_N' - \mu')\ \rho_\omega\ (\omega_{N'})$$

$$G_p^+(\omega) = \sum G_p(\omega_N - \mu)\ \rho_\omega(\omega_N) + \frac{1}{2\pi i}\int_\mu^\infty dx\ \frac{A(x-\mu)}{x-\omega-i\epsilon}$$

The relevant point which arises, here, has to do with the execution of the integration of (11A). The insertion of factors $\rho_p(z)$ in the expression of (9A) and (10A), i.e., $\hat{\beta}(z) = \beta_p(z)\ \rho_p(z)$ and $G_\rho(z) = G_p(z)\rho_p(z)$, takes care of what would otherwise appear to be an ambiguity.

The undefined, irrational numbers ζ and ζ' of (4A) and (7A) permit us to adjust the formal spectra into a rough correspondence with the physical spectra. We will know the number of states $N = N_o$ in a practical computation. Then, ζ and ζ' are adjusted to fit the end-points of the corresponding spectra.

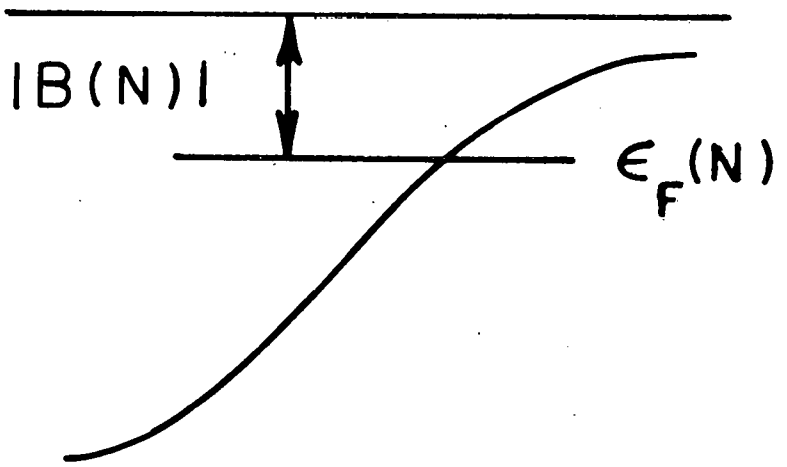

Fig. 1A. Typical finite, one-body well generated by N-system. The binding of the last nucleon is B(N).

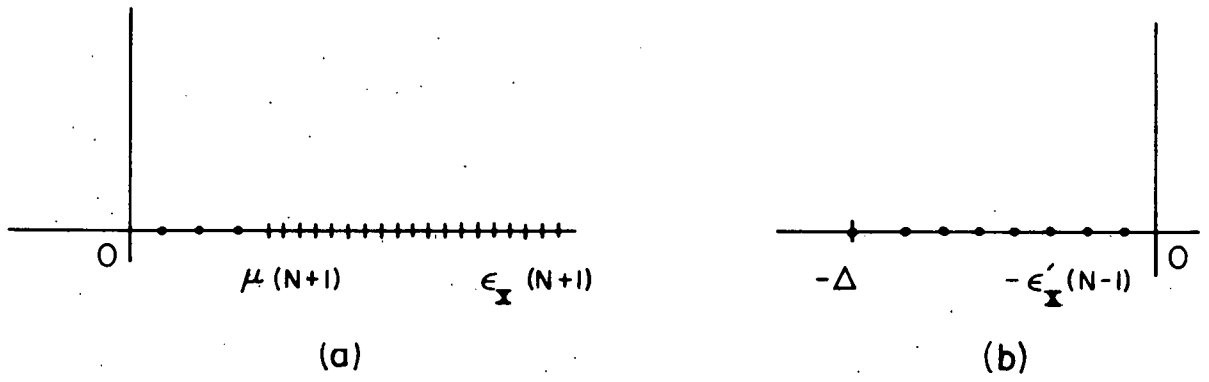

Fig. 2A. Excitation spectra for (a) the (N+1)-system and, (b) the (N-1)-system. The interval Δ is that separating the Fermi- and $1s_{1/2}$-levels.

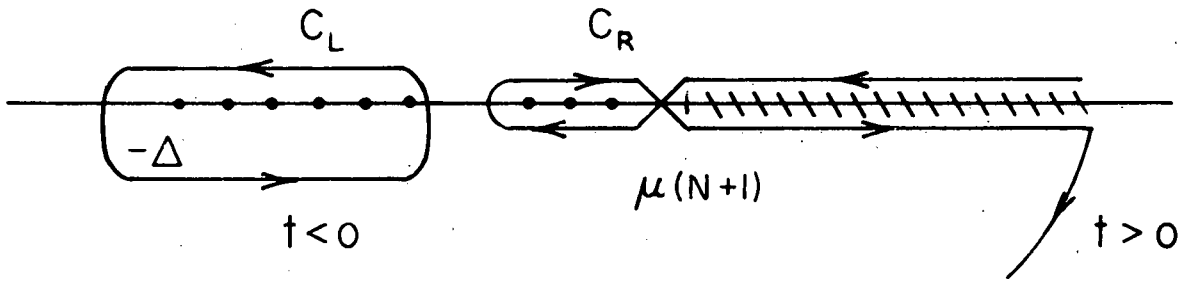

Fig. 3A. The integration contours c_L and c_R for the inversion of (11A).