Intrabeam Scattering with Coupling

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Intra beam Scattering with Coupling

At high γ , vertical growth ^{rate} is almost zero.
Coupling might reduce growth rates
by as much as a factor of 2.

However, smaller growth rates
need not cause large changes in
final state of beam after 10 hours.

~~It~~
Growth rate $\sim \frac{1}{\sigma} \frac{d\sigma}{dt} \sim N_b / 6\text{-dimensional phase space}$

$$\frac{1}{\sigma} \frac{d\sigma}{dt} \sim N_b / \sigma^6$$

Factor 2 in growth rate can be
compensated for by changing σ by $2^{1/6}$
which is a 12% change in σ .

~~It~~ ^{It} was observed previously, that
when N_b is changed by factor 2, σ
only changes by about 12%.

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How to do Intra beam Scattering with Coupling

Review of IBS theory

It is assumed that, in the absence of Intra beam scattering ~~scattering~~

$$P(x, x', y, y') \cong \exp \left[- \left(\frac{x^2}{2\sigma_x^2} + \frac{x'^2}{2\sigma_{x'}^2} + \frac{y^2}{2\sigma_y^2} + \frac{y'^2}{2\sigma_{y'}^2} \right) \right]$$

at elements where $\alpha=0$, and

$$P(x, x', y, y') \cong \exp \left[- \left(E_x(x, x') / \bar{E}_x + E_y(y, y') / \bar{E}_y \right) \right]$$

$$\bar{E}_x = 2\sigma_x^2 / \beta_x \quad \bar{E}_y = 2\sigma_y^2 / \beta_y$$

Note that

$$\frac{dP}{dt} = 0 \quad \left(\text{if no linear coupling} \right)$$

in the absence of Intra beam Scattering.

Review of IBS Theory (Continued)

In the presence of Intrabeam Scattering it is assumed that $P(x, x', y, y')$ keeps the same form

$$P(x, x', y, y') \simeq \exp \left[- \left(\epsilon_x(x, x') / \bar{\epsilon}_x + \epsilon_y(y, y') / \bar{\epsilon}_y \right) \right]$$

but now $\bar{\epsilon}_x = \bar{\epsilon}_x(t)$, $\bar{\epsilon}_y = \bar{\epsilon}_y(t)$ and grow slowly with time.

Using the result ~~that~~

$$\bar{\epsilon}_x = \langle \epsilon_x \rangle = \int dx dx' dy dy' P(x, x', y, y') \epsilon_x(x, x'),$$

and the collision cross-section $\sigma(\theta)$, one can then find expressions for

$$\frac{1}{\bar{\epsilon}_x} \frac{d \bar{\epsilon}_x}{dt}, \quad \frac{1}{\bar{\epsilon}_y} \frac{d \bar{\epsilon}_y}{dt} \quad \text{etc}$$

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Intra beam Scattering with Linear Coupling

The following procedure could be used

There are now two invariants

$$I_1(x, x', y, y') \quad I_2(x, x', y, y')$$

Assume for $P(x, x', y, y')$, in the absence of Intra beam scattering,

$$P(x, x', y, y') \cong \exp \left[- \left(I_1 / \bar{I}_1 + I_2 / \bar{I}_2 \right) \right]$$

$\bar{I}_1$ and $\bar{I}_2$ are constants and can probably be related to $\langle I_1 \rangle$ and $\langle I_2 \rangle$.

Note

$$\frac{dP}{dt} = 0$$

in the absence of Intra beam Scattering

~~In the~~

In the presence of Intrabeam Scattering,
it is assumed that $\bar{I}_1 = \bar{I}_1(t)$, $\bar{I}_2 = \bar{I}_2(t)$
and grow slowly with time.

Using the relationship between
 $\bar{I}_1$, $\bar{I}_2$ and $\langle I_1 \rangle$ and $\langle I_2 \rangle$ one
can then find expressions for

$$\frac{1}{\bar{I}_1} \frac{d\bar{I}_1}{dt} \quad \text{and} \quad \frac{1}{\bar{I}_2} \frac{d\bar{I}_2}{dt}$$

Above procedure would be difficult
to carry out. ^{The} Expressions for $\bar{I}_1$ and $\bar{I}_2$
are complicated.

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Intrabeam Scattering ~~classical intrabeam coupling~~ with Complete Coupling

It is assumed that

$$\mathcal{E}_t = \mathcal{E}_x(x, x') + \mathcal{E}_y(y, y')$$

a constant of the motion. One assumes
for $P(x, x', y, y')$

$$P(x, x', y, y') \approx \exp[-\mathcal{E}_t(x, x', y, y')/\bar{\mathcal{E}}],$$

where ~~the energy is~~ $\bar{\mathcal{E}} = \bar{\mathcal{E}}(t)$ and grows slowly with time

$$\bar{\mathcal{E}} = \frac{1}{2} \langle \mathcal{E}_t \rangle = \frac{1}{2} \int dx dx' dy dy' P(x, x', y, y') \mathcal{E}_t(x, x', y, y')$$

One can then find an expression for

$$\frac{1}{\langle \mathcal{E}_t \rangle} \frac{d \langle \mathcal{E}_t \rangle}{dt}$$

At high γ , starting at $t=0$
 with equal E_x and E_y , the
 growth rate for E_z is $1/2$
 the growth rate for E_x
 in the absence of linear coupling

The present program
 for computing Intrabeam Scattering
 can be modified to compute
 the beam growth with the above
 assumption for $p(x, x', y, y')$
 The program then computes
 the growth of E_z

σ_x, δ versus γ
after 10 hours
showing
Effect of Coupling. (Gold)

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Why does δ grow?

In the old E_x, E_y theory, at high δ ,
there exists the time invariant

$$\sigma_E^2 - \sigma_x^2 = \text{const}$$

$$\sigma_E = X_p \delta$$

and $\sigma_E \sim \sigma_x$ at large t

In the new E_t theory, the
invariant becomes

$$\sigma_E^2 - 2\sigma_x^2 = \text{const}.$$

and $\sigma_E \rightarrow 1.4 \sigma_x$ at large t

σ_E^2 versus σ_x^2

over 10 hours

at $\delta = 100$ for Gold

$$\sigma_E = X_p \delta$$

New ϵ_t Result