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**BOW-V: A CDC-3600 PROGRAM TO CALCULATE
THE EQUILIBRIUM CONFIGURATIONS
OF A THERMALLY BOWED REACTOR CORE**

D. A. Kucera and D. Mohr

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**EBR-II PROJECT
ARGONNE NATIONAL LABORATORY
Argonne, Illinois - Idaho Falls, Idaho**

January 1970

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NOMENCLATURE

$C(z)$	--	Constant depending on temperature distribution around periphery of subassembly hex can
c_{1j}	--	Change in node LI constraint position for row j necessary to produce load balance
c_{2j}	--	Change in node MI constraint position for row j necessary to produce load balance
c_{kj}	--	Change in constraint position for node k of row j necessary to produce load balance
D	--	Equivalent diameter of hex can
DL_j	--	Nominal distance minus initial clearance (amount of "solid material") between centerlines of rows j and $j+1$ at node MI, for all power levels
DD_j	--	Analog of DL_j at node LI (at power level, P)
DDL_j	--	Zero-power value of DD_j
F_j	--	Compression displacement caused per pound of applied force for row j
ΔF_{kj}	--	Change in required enforcing load at node k of row j
i	--	Node identifier for axial stations of hex cans
j	--	Row number
k	--	Node identifier for hex can contact points (intermediate or top level)
k_{ikj}	--	Constant depending on bending mode of row j
Mode 0	--	Configuration in which Y_1 and Y_2 are undefined (no contact with neighboring rows)
Mode I	--	Configuration in which Y_1 is defined and the beam is pivoted so that the load at X_1 moves to Y_1 while X_N does not contact the lower grid; Y_2 is undefined
Mode I_T	--	Configuration in which Y_2 is defined and the beam is pivoted so that the load at X_2 reaches Y_2 while X_N does not contact the lower grid; Y_1 is undefined
Mode II	--	Configuration in which Y_2 is undefined and free pivoting is followed by loads at X_1 and X_N
Mode II_T	--	Configuration in which Y_1 is undefined and free pivoting is followed by loads at X_2 and X_N

Mode III }
 Mode III_T } -- Configurations in which loads exist at X_1 , X_2 , and X_N
 Mode V }
 Mode IV -- Configuration in which the load at X_1 is counterbalanced
 by a load at X_2 ; no load at X_N
 Node MI -- Top position for subassembly contact
 Node LI -- Intermediate position for subassembly contact
 P -- Fractional power level
 P₀ -- Reference power level (unity corresponds to full power)
 P'_j -- New load at node MI after an iteration
 ΔP_j -- Change in load at node MI for row j
 P^c_j -- Resultant compressive force acting on node MI of row j due
 to compression between pairs (j-1, j) and (j, j+1)
 P_j -- Resultant force due to bending required to enforce the
 row-j constraint at node MI
 Q^c_j -- Compressive load between any two adjacent rows, j and j+1
 (general expression for compressive loads W_j^c and P_j^c)
 R_j -- Total distance adjacent nodes are compressed (amount of po-
 tential interference between rows j and j+1)
 r_j }
 r_{j+1} } -- Individual compression displacements
 r_{kj} -- Constraint point location for node k of row j
 T_j(P) -- Relative hex-can (average) temperature at intermediate
 level for row j at power level P
 ΔT_{ij} -- Input array of temperature difference across hex can at
 node i and row j
 ΔT(z) -- Temperature difference between opposite walls of a hex
 can at elevation z
 W^c_j -- Resultant compressive force acting on node LI of row j
 due to compression between pairs (j-1, j) and (j, j+1)
 W_j -- Resultant force due to bending required to enforce the
 row-j constraint at node LI
 W'_j -- New load at node LI after an iteration
 ΔW_j -- Change in load at node LI for row j

- X_1 -- Location of intermediate (button) node (LI)
- X_2 -- Location of top node (MI)
- X_N -- Nozzle position at lower grid
- Y_1 -- Button constraint position (node LI)
- Y_2 -- Top constraint position (node MI)
- $\left. \begin{matrix} y_A^P \\ y_B^P \\ y_N^P \end{matrix} \right\}$ -- Relative displacements due to loads at points A, B, and N of a beam
- $y(z)$ -- Radial deflection at elevation z
- α -- Coefficient of thermal expansion for hex cans
- α_M -- Mode-dependent values of coefficients, k_{ikj} , relating load changes to deflections
- (α_k, β_k) -- Node MI freely pivoting interval for row k
- $\left. \begin{matrix} \alpha_k \\ \beta_k \end{matrix} \right\}$ -- Free-pivot limits for node MI of row k
- β_M -- Mode-dependent scaling factor relating displacements at nodes LI and MI
- γ_M -- Mode-dependent scaling factor relating loads at nodes LI and MI
- ξ_1 -- Arbitrary constraint point for node LI
- ξ_2 -- Constraint point for node MI at which node LI lies at X_1^*
- ϵ_p -- Specified convergence criterion for load equilibrium at node MI
- ϵ_w -- Specified convergence criterion for load equilibrium at node LI
- κ_j -- Proportionality constant for compressive loading of the pair $(j, j+1)$ at node LI
- κ_j' -- Proportionality constant for compressive loading of the pair $(j, j+1)$ at node MI
- $\left. \begin{matrix} \kappa_{1j} \\ \kappa_{2j} \end{matrix} \right\}$ -- Constants that relate total exerted load to compression displacements for rows j and $j+1$ at LI and MI nodes, respectively
- $\left. \begin{matrix} \lambda_k \\ \mu_k \end{matrix} \right\}$ -- Free-pivot limits for node LI of row k

(λ_k, μ_k) -- Node LI freely pivoting interval for row k

δ_j -- Amount of potential interference between rows j and j+1
at contact node LI

δ'_j -- Amount of potential interference between rows j and j+1
at contact node MI

$\Delta\rho$ -- Total reactivity change (bowing effects only)

ω_j -- Node LI contact indicator for the pair (j, j+1)

ω'_j -- Node MI contact indicator for the pair (j, j+1)

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ABSTRACT

Thermal bowing of subassemblies in the core of a nuclear reactor can cause mechanical interactions between adjacent subassemblies. The BOW-V program calculates the equilibrium configurations of closely spaced, parallel beams (simulating the subassemblies) which are thermally distorted. Each beam has a limited-pivot support near its lower end. The beams can contact and thus mechanically interact with each other at their tops and at an intermediate level. Interference between the beams and a fixed grid below the pivot is also possible. The axial distribution of radial temperature gradients across the beams is arbitrary. The BOW-V model provides for the influence of bowing in one row of subassemblies on any other row in the reactor. All possible modes of bending are included, and each beam can have different moments of inertia above and below the pivot. The contact points have arbitrary compressibility. The model provides for thermal expansion of the lateral dimensions of the subassembly hex can at the intermediate contact point. The program determines the positions of free-pivoting (interacting, but unloaded) beams as well as the deflections of fully loaded beams.

The report contains equation derivations, input/output specifications, and FORTRAN listing. The program is written for the CDC-3600 computer, but may be converted for the IBM-360 computer in 1970. Certain parts of the program, such as the matrix inversion subroutine, have

been modified for double precision. The program is fully operative. A sample problem (EBR-II run 25) is included in this report.

I. INTRODUCTION

A. The Bowing Problem

In a typical EBR-II reactor core, such as the one shown in Fig. 1, the radial temperature gradients across the subassemblies are different at various axial locations. If the axial thermal expansion of the opposite walls of the hexagonal can of the subassembly is non-uniform, the can is deformed, or bowed. Such bowing can cause reactivity changes due to movement of fuel and can strongly influence the dynamic stability of the reactor.

Figure 2 shows the design of a typical core subassembly for a fast reactor. The degree of thermal bowing of such a subassembly, if unrestrained, depends only on the axial and radial temperature profiles in the subassembly, the effective diameter of the subassembly, and the coefficient of thermal expansion. However, because of the proximity of subassemblies within a reactor, the subassemblies usually tend to touch as they bow. After initial contacts are made, any additional deformation (overlap) causes each subassembly to exert a load against its neighboring subassemblies. A subassembly thus is subjected to two distinct influences: a purely thermal bowing effect and a purely mechanical subassembly-interaction effect. It follows that a thermally bowed core, in general, will contain some deformed subassemblies which are unrestrained (unloaded) and some number which are in mutual contact, with each exerted load balanced by an equal and opposite load. Figure 3 shows these effects qualitatively.

B. Outline of the Model

Determination of the equilibrium configuration of such a core is equivalent to specifying a suitable set of constraint points for the centerline of each subassembly. Each subassembly then may be viewed as an isolated beam whose initial shape satisfies the unrestrained thermal-deflection equation (Eq. 1 in Section IV.C) and whose final shape is attained by

LEGEND		NUMBER OF GRID POSITIONS	
SECTORS	A TO F	CORE	61
CONTROL RODS (12)	C	INNER BLANKET	66
SAFETY RODS (2)	S	OUTER BLANKET	510
THERMOCOUPLES (26)	•		
FIXED DUMMY	D	TOTAL	637
NEUTRON SOURCE	S		

Fig. 1. Typical Arrangement of Subassemblies in EBR-II

Fig. 2. EBR-II Core Subassembly

Fig. 3. Bowing and Interaction Effects in Thermally Distorted Subassemblies

forcing its centerline to pass through the adjustable constraint points. This forcing requires a set of loads to be applied at the constraint points. The values of these loads may be calculated with beam formulas, using the mechanical properties of the subassembly. A suitable set of constraint points for any loaded subassembly is one in which each constraint point lies where contact is made with an adjacent subassembly and where the loads at the point are equal but exactly opposite.

BOW-V calculates the suitable constraint points for a specified set of power levels.

II. GENERAL OPERATION AND DEVELOPMENT OF THE BOWING CODE

A. General Operation

BOW-V consists of two major computational blocks, numbered 1 and 2 in the flow chart presented in Fig. 4. In block 1, a given set of constraints is used to calculate the imposed forces required at each row to enforce such constraints. In block 2, a new set of constraints is generated, based on the load imbalances affecting the existing set of constraints. For each power level, these steps are repeated until either a set of constraints is produced in which all loads are balanced or a specified maximum number of iterations has been completed.

B. Evolution of Bowing Models

This general procedure has been used in each of the predecessors of BOW-V. The earliest of these, which used the physical model described in Section III, was BOW-II.¹ This program contained a number of approximations in blocks 1 and 2, which provided speed and simplicity but also led to an unacceptable loss of accuracy.

1. Mode Formulas

A completely general but precise system of equations for block 1, presented in Appendix B, was derived for BOW-III.² This system is developed for each of the qualitatively distinct possible configurations into which the imposed constraints may be partitioned; each such distinct configuration is termed a mode. The specific description of the modes is presented in Appendix B. By means of a unified treatment of mirror-image

Fig. 4. BOW-V Flow Chart

cases in each mode, the program segment for this new block 1 became shorter than the original and decreased the running time. The increased precision of block 1 defined the BOW-III program.

2. Determination of Constraint Positions

BOW-III retained the initial block 2 as given in BOW-II, and results showed that improvement in this block also was necessary. This was accomplished by using a totally new approach, a matrix solution, which resulted in a complete reprogramming of block 2. The use of this new approach defined the program BOW-IV, which determined configurations whose loads were arbitrarily close to complete equilibrium and thereby became the first of the bowing codes to truly reflect the physical bowing model assumed.

Although compressibility was included at the intermediate, or button, level (node LI), BOW-IV contained the assumption that all subassemblies were incompressible at their tops (node MI). This assumption actually required extensive programming, which made this segment much more complicated and time-consuming than the one treating the compressible node LI. Since incompressibility formally can be made equivalent to negligible compressibility, this assumption can be realized simply by inputting zero compressibility at the top node MI. This representation broadened the scope of the program by permitting an arbitrary amount of compressibility at each node (LI and MI). It also permitted a unified treatment of both contact nodes with a considerable reduction in program size and running time. This change in block 2 plus other changes described below defined the BOW-V program.

III. PHYSICAL ASSUMPTIONS IN THE BOW-V MODEL

A. Geometry Selected

BOW-V treats a reactor as a cylindrical system in which each row consists of an annulus of specified inner and outer radii. In the natural $r\theta z$ geometry, all properties of each row are assumed to be independent of the θ coordinate. This permits the reactor to be represented by a single rz plane extending from $r = 0$ along an arbitrary value of θ , as illustrated by Fig. 5.

Fig. 5. BOW-V Geometry

Due to its necessary θ symmetry, the central subassembly (row 1) of the reactor must always have its centerline at $r = 0$. Furthermore, the single subassembly representing each row in this representation must have the mechanical properties of the entire row to which it belongs. With these assumptions, the reactor becomes equivalent to a series of subassemblies whose centerlines lie in a single plane through the core centerline; this viewpoint will be used for its simplicity.

An important advantage of BOW-V is its ability to handle interactions between all rows of subassemblies, i.e., the potential influence of any row on any other row. The model assumes fixed guides at the bottom: the lower grid (nozzle entrance) and upper grid (pivot). The present model does not allow a fixed grid to be imposed above the upper grid; only mutual subassembly contact at two levels (intermediate and top) is allowed.

B. Models for Subassembly and Reactor Row

Each subassembly is assumed to consist of two distinct sections, as depicted in Fig. 6. The nozzle section, lying between the upper and lower grids, consists only of structural material. The remainder of the subassembly, lying above the upper grid, consists of a hexagonal can containing fuel, reflector, or other material, depending on its location in the reactor. Near the core midplane, each face of the subassembly has a protruding spacer button. The seating of a subassembly in the grid plate prevents translational movement but is sufficiently loose to permit some free pivoting.

As a conceptual simplification, the centerline of each row representative alone is sufficient to describe subassembly positions and determine imposed loads. This representation of a row is shown in Fig. 5. The centerline of each row at zero power is exactly vertical unless an initial slope is specified.

C. Axially Varying Quantities

Quantities which vary with axial position, such as the radial temperature differentials, are specified in the input at a number of distinct axial points, termed nodes. Two node identifications are used: node MI is located at the top of the subassembly, and node LI is located at the button

Fig. 6. Typical Undeformed Subassembly

level. MI also equals the total number of nodes. All nodes are equally spaced, and the pivot level corresponds to node number zero.

D. Contact and Compression of Hex Cans

The model permits contact between rows only at nodes LI or MI; thus, while a row might be in contact with an adjacent row at both, either, or neither of nodes LI and MI, it cannot be in contact elsewhere. All sub-assemblies are compressible under imposed loading; a purely elastic relationship (spring constant) exists between compression distance and imposed load. Since loads can be imposed only by contact between rows, the compressibility of each row is fully described by two such spring constants, associated with nodes LI and MI. A distinct pair of constants is required for each row.

E. Sign Convention and Load Orientation

Finally, it is necessary to establish sign conventions to specify the direction of loads and changes in position. Displacements are oriented in the natural way, with radially outward movements defined as positive. Loads are defined, however, to be positive when they are imposed on a row from the outside. Figure 7 illustrates the sign convention for loads.

The loads a row itself exerts against its constraints are completely determined by those constraints. The loads of interest are the "external" loads imposed to "enforce" the chosen constraints upon the row, since it is the existence of such loads which determines the suitability of a given set of constraints. Distinction between loads at the two nodes is made by identifying all loads acting at node LI by the symbol W and all loads acting at node MI by the symbol P. These symbols are superscripted to distinguish between the separate components acting at each node.

IV. METHOD OF SOLUTION

A. General

For each specified power level, BOW-V uses an iterative method to determine the equilibrium set of constraint points. From Fig. 4, one notes

Fig. 7. BOW-V Sign Conventions

that an iteration consists of selection of constraint points, determination of the loads required to enforce such constraints, and examination of such loads for equilibrium. If the existing set of constraint points results in load equilibrium, the iterations end, and calculations for the next power level are begun. If equilibrium is not achieved, iterations continue until either equilibrium is attained or a specified maximum number of iterations has been made for that power level.

B. Concept of Row Units

It is obvious that a row can be physically influenced by another row only when it is in: (1) direct contact with the other row or (2) indirect pairwise contact with the other row through all the intervening rows. This observation motivates the concept of the row unit, or, simply, unit, which is defined as a set of consecutive rows contacting one another at either or both of nodes LI and MI. A unit may consist of all rows in the reactor, as few as two rows, or, in the absence of contacts, may not even exist at all. In general, the rows of subassemblies can be partitioned into a number of units plus a number of individual rows which contact neither neighbor. Each such row or unit constitutes a distinct subset of the reactor core, and is affected by no other subset.

Consider now some unit, consisting of rows $(m_1, m_1 + 1, \dots, m_2 - 1, m_2)$. By definition, neither of the pairs $(m_1 - 1, m_1)$ nor $(m_2, m_2 + 1)$ is in contact, and each row in the unit is totally independent of all rows not in the unit. Each unit thus behaves as a closed system, with each row member constrained at a known position and lying in a known mode with known imposed loads.

C. Thermal-deflection Equations

The radial deformation $y(z)$ at each axial point z is governed by the differential equation

$$\frac{d^2}{dz^2} y(z) = \frac{\alpha \cdot C(z) \Delta T(z)}{D}, \quad (1)$$

where $C(z)$ depends on the temperature distribution around the periphery of the hex can, α is the coefficient of thermal expansion for the

subassembly, D is the equivalent diameter of the hex can, and $\Delta T(z)$ is the radial temperature difference between opposing walls of the subassembly. The derivation of Eq. 1 and the definition of $C(z)$ are given in Appendix G. The equation is integrated twice in the program to yield the desired unrestrained thermal deflection, $y(z)$.

The input temperature array, ΔT_{ij} , which represents the continuous function $\Delta T(z)$ in Eq. 1 for each node i and row j , is associated with an implicit reference power level, P_o . The actual power levels in BOW-V are specified in terms of fractions of P_o by specified initial and final fractions of P_o and a fractional increment. At each fractional power level, P , $\Delta T_{ij}(P)$ is determined from the linear relationship

$$\Delta T_{ij}(P) = P \cdot \Delta T_{ij}(P_o). \quad (2)$$

Similarly, $T_j(P)$, the relative axial temperature at the button level, proceeds from the expression

$$T_j(P) = P \cdot T_j(P_o). \quad (3)$$

If the arbitrary linear relationships between power and temperature are not desired, the use of these relationships for generating temperatures can be bypassed by setting all the power-level variables equal to unity (see Section V.B, Card Set 1). Each BOW-V problem then would correspond to a set of input temperatures, and the power level would be treated as a purely artificial variable. This method allows analysis with time-varying inputs on a quasi-static basis.

D. Load-deflection Equations

1. General Bending Equation

As shown in Appendix C, there corresponds to each mode linear relationships having the form

$$\Delta F_{ij} = \sum_{k=1}^2 k_{ikj} c_{kj}, \quad (4)$$

where j is the row number, i and k are node identifiers ranging between 1 (node LI) and 2 (node MI), k_{ikj} is a constant depending only on the mode of row j , and ΔF_{ij} is the change in the required enforcing load at node k of row j caused by the changes c_{kj} of the row- j constraint positions.

2. Subassembly Contact and Compression

The subassembly cans are assumed to be compressible only at the two contact points under loading. The assumption of perfectly elastic compressibility requires that the amount of compression be a constant multiple of the imposed load, so that loads and compression distances are expressible in terms of one another.

In BOW-V, the constraint points determine the position of nodes LI and MI for each row. It follows that clearances or amounts of compression between pairs of rows are dependent solely on the set of constraint points. Since the constraint points are the independent variables in the problem, imposed loads are computed in terms of the constraint points in the following manner.

As shown in Fig. 8, DL_j is that separation of the centerline positions at which rows j and $j + 1$ contact one another. (The quantity DL_j is the nominal distance minus the initial clearance, i.e., amount of "solid material" between the centerlines of adjacent rows j and $j + 1$ at node MI.) It follows that the pair $(j, j + 1)$ is in compressive contact only if the quantity

$$\delta'_j = DL_j - [y_{j+1}(MI) - y_j(MI)] \quad (5a)$$

is positive, where δ'_j is the interference between rows j and $j + 1$ and y_j and y_{j+1} denote the centerline positions of node MI. This condition is equivalent to

$$\omega'_j \delta'_j > 0, \quad (5b)$$

where ω'_j is the node-MI contact indicator for the pair $(j, j + 1)$.

$$\omega'_j = \begin{cases} 0, & \text{if } \delta'_j < 0 \\ 1, & \text{if } \delta'_j > 0 \end{cases} \quad (5c)$$

The load which each of rows j and $j + 1$ exerts on the other at node MI has the magnitude

$$\kappa'_j \omega'_j \delta'_j, \quad (5d)$$

where κ'_j is the proportionality constant for compression loading of the pair $(j, j + 1)$ at node MI, derived from input quantities by means presented in

Fig. 8. BOW-V Module

Appendix D. As a result of the sign convention for loads, the resultant compressive force acting on node MI of row j , due to the compression of pairs $(j - 1, j)$ and $(j, j + 1)$, is

$$P_j^C = - \kappa_{j-1}' \omega_{j-1}' \delta_{j-1}' + \kappa_j' \omega_j' \delta_j'. \quad (6a)$$

The derivation of compression loads at node LI is similar to that used for node MI, and the analog of Eq. 5c is

$$W_j^C = - \kappa_{j-1} \omega_{j-1} \delta_{j-1} + \kappa_j \omega_j \delta_j, \quad (6b)$$

where W_j^C is the resultant compressive force acting on node LI of row j due to compression of pairs $(j - 1, j)$ and $(j, j + 1)$.

3. Thermal Expansion of the Hex Can across the Flats

At node LI, the analog of Dl_j is DD_j . Although Dl_j is constant, DD_j varies with power level according to the relationship

$$DD_j(P) = DDl_j \left[1 + \alpha \left\{ [T_j(P) + T_{j+1}(P)]/2 \right\} \right], \quad (7)$$

where DDl_j is the distance at zero power, α is the thermal expansion coefficient (constant for all rows), P is the fractional power level, and T_j is the average node-LI temperature at P . Figure 8 compares Dl_j and DD_j .

E. Iteration Technique

1. Initiating the Iteration Process

BOW-V internally provides the first guess of constraint point at the initial power level or initial set of input temperatures. However, the first guess used at each subsequent power level is the final constraint points coincident with the previous power level. Thus, an initial set of constraint points is always defined, and the procedure presented above is always applicable. This process utilizes the equations derived in Appendix B.

2. Load-equilibrium (Convergence) Criteria

As remarked previously, all loads imposed on a row must cause some finite compression. On the other hand, the equilibrium configuration within a unit may be defined as that configuration in which the load required to enforce a constraint point, for the nodes of any row, is imposed by forces

external to that row. Consequently, the conditions for equilibrium (or convergence) can be stated as

$$\begin{aligned} P_j &= P_j^c \text{ and} \\ W_j &= W_j^c, \end{aligned} \quad (8a)$$

where P_j and W_j are the resultant forces required to enforce the row- j constraint at nodes MI and LI, respectively. Since the computer solves the conditions of Eq. 8a by trial and error, they are rewritten in the program as

$$\begin{aligned} |P_i - P_i^c| &< \epsilon_p \text{ and} \\ |W_i - W_i^c| &< \epsilon_w, \end{aligned} \quad (8b)$$

where ϵ_p and ϵ_w are specified convergence criteria. Equilibrium at a given power level results only when both conditions of Eq. 8b are satisfied for all rows.

3. Required Equations

a. Prediction of Required Position Changes

Suppose now that a given set of constraint points, after an iteration at an arbitrary power level, results in loads that do not satisfy the conditions of Eq. 8b at one or more contact nodes in the unit $(m_1, \dots, m_2)$. The constraint point for node k of row j is denoted r_{kj} , and the quantity c_{kj} denotes the change in r_{kj} necessary to produce load balance. For notational clarity, the superscript o will be used to distinguish existing quantities from their next-iteration values. Thus,

$$r_{kj} = r_{kj}^o + c_{kj}. \quad (9)$$

Eq. 4 may be written

$$\Delta W_j = W_j - W_j^o = k_{11j} c_{1j} + k_{12j} c_{2j} \text{ or} \quad (10a)$$

$$\Delta P_j = P_j - P_j^o = k_{21j} c_{1j} + k_{22j} c_{2j}, \quad (10b)$$

and Eq. 5a becomes

$$\begin{aligned} \delta_j &= D1_j - \left\{ \left[y_{j+1}^{(MI)} + c_{2,j+1} \right] - \left[y_j^{(MI)} + c_{2j} \right] \right\} \\ &= \delta_j^o + c_{j2} - c_{2,j+1}. \end{aligned} \quad (11)$$

b. Evaluation of Total Loads

The coefficients in Eqs. 10a and 10b and in Eqs. 6a and 6b are temporarily assumed not to change under the transformation of Eq. 9. Physically, this is equivalent to assuming that each row in the unit remains in its existing mode and retains its contacts with adjacent rows. Cases where this assumption does not hold are described in Section IV.D.

Second, as shown below in Tables V (Appendix B) and VII (Appendix C), the total load on a given row is assumed to be directly proportional to the number of subassemblies in that row. Thus, the total bending load on a row is the product of the individual load on the subassembly and the number of subassemblies in the row.

At this stage, Eqs. 11 and 6a would yield

$$\begin{aligned} P_j^C &= -\kappa_{j-1}'(\delta_{j-1}' + c_{2,j-1} - c_{2j})\omega_{j-1}' + \kappa_j'(\delta_j' + c_{2j} - c_{2,j+1})\omega_j' \\ \therefore P_j^C &= P_j^{CO} - \omega_{j-1}'\kappa_{j-1}'c_{2,j-1} + (\omega_{j-1}'\kappa_{j-1}' + \omega_j'\kappa_j')c_{2j} - \omega_j'\kappa_j'c_{2,j+1}. \end{aligned} \quad (12)$$

which is combined with Eqs. 8a and 10b to give at node MI:

$$\begin{aligned} P_j^{CO} - P_j^C &= \omega_{j-1}'\kappa_{j-1}'c_{2,j-1} + k_{21j}c_{1j} + (k_{22j} - \omega_{j-1}'\kappa_{j-1}' - \omega_j'\kappa_j')c_{2j} \\ &\quad + \omega_j'\kappa_j'c_{2,j+1}. \end{aligned} \quad (13)$$

A similar combination of Eqs. 6b, 9, 10a, and 11 yields the analogous relation for node LI:

$$\begin{aligned} W_j^{CO} - W_j^C &= \omega_{j-1}\kappa_{j-1}c_{1,j-1} + (k_{11j} - \omega_{j-1}\kappa_{j-1} - \omega_j\kappa_j)c_{1j} + k_{12j}c_{2j} \\ &\quad + \omega_j\kappa_jc_{1,j+1}. \end{aligned} \quad (14)$$

c. Nonloaded Rows in Contact

We now analyze a special case, called unloaded free pivoting, which applies to nodes LI and MI. Node LI is described by Eq. 14 which depends on Eqs. 6b and 10a, neither of which is universally applicable. Eqs. 6b and 10a degenerate to triviality for modes I_T and II_T^* in which node LI is not constrained and hence cannot require external load. As a

* See Fig. 15, Appendix B.

result, Eq. 14 degenerates to triviality. In such a case, node LI is completely passive, and its position depends solely on the conditions at node MI. Eq. 14 then must be replaced by an equation of the form

$$c_{1j} = \beta_j c_{2j}, \quad (15a)$$

where β_j is a mode-dependent constant for row j .

In a completely analogous manner, Eq. 13 must be replaced by

$$c_{2j} = \beta_j c_{1j} \quad (15b)$$

for rows in modes I and II whose node MI contacts neither adjacent row. An important consequence of the unit concept is that the simultaneous degeneration of Eqs. 13 and 14 is impossible. Inasmuch as a row belonging to a unit is by definition in contact with at least one of the adjacent rows, simultaneous degeneration of Eqs. 6a and 6b also is impossible.

4. Establishment of Valid Units

a. Convention for Unconstrained Rows

Removing the degeneracy described above requires modifications in the detailed treatment of unit rows. First, the row-1 centerline always is made coincident with $r = 0$ and thus is always excluded from units, although the possibility of its compressive contact with row 2 is included. As a result, the constraint point 0 can have no physical meaning for unit rows. Instead, the value 0 is used to indicate unconstrained rows, i.e., those having no contact with adjacent rows. Thus, a row requiring Eq. 15 in its description would have a node-LI constraint "position" of 0 listed in the output.

b. Treatment of Idle Rows

The remaining modification is best described by a concrete example. Consider the unit (7, 8, 9, 10) in which row 10 lies in mode I_T and thus contacts row 9 only at node MI. Since row 10 is in mode I_T , it will respond to changes in row-9 position with unloaded free pivoting about the upper grid plate; consequently, compressive contact between rows 9 and 10 at node MI is physically impossible, and Eq. 6a is not applicable. Thus, even though row 10 could be considered a member of the unit, its behavior does not affect the remaining rows of the unit. Such a row is termed idle and is excluded from the unit in the program.

In general terms, a valid unit is one whose end rows are active (or loaded)* and whose effect on the unit (in terms of the load-deflection Eq. 13 or 14) is physically real. Within the unit, a row in the "idle" modes I_T or I must be viewed differently. In this case, such a row cannot pivot away from an adjacent row freely because it is contacted on both sides. Thus, compression of the hex can is physically possible without bending of the subassembly.

c. Matrix Solution for Units

Within a given unit $(m_1, \dots, m_2)$, each node LI is associated with an equation of the type of Eq. 14 or 15a, and each node MI is associated with an equation of the type of Eq. 13 or 15b. Since this unit contains $(m_2 - m_1 + 1)$ rows, its contact nodes, numerically equal to $2(m_2 - m_1 + 1)$, are described simultaneously by the same number of linear equations. A matrix method is the natural means of solution, and this method is applied in BOW-V by means of the utility subroutine MATINV, modified for double precision. This operation is performed in block 2 of Fig. 4. The equations defining block 2 are derived in Appendix C.

The foregoing method is used for each unit in the reactor and can result in a new set of constraint points for all unit rows.

d. Changes in Units during Iteration Process

A given unit may gain or lose rows, separate units may merge to form a larger single unit, and bending modes in rows within a unit may change. The assignment of new constraint points is performed in block 1b of Fig. 4 and is equivalent to a change in position of unit rows. It then becomes necessary to reconsider rows which were free (i.e., unconstrained) when the unit rows were at their previous positions. If such a free row is contacted when unit rows take their new positions, it must be assigned a constraint point so that it becomes a member (either active or idle) of the unit which it contacts. Idle unit rows are similarly affected; these may remain idle, become active (i.e., become loaded unit rows), or become idle, depending on the motion of the unit rows.

* Any row is active (or loaded) if either a bending moment or a purely compressive load is imposed on it.

e. Physical Validity of New Constraint Points

As the final step in the determination of next-iteration constraint points, every non-zero constraint point is tested for physical validity. This operation is performed in block 1a of Fig. 4. Since a row can have a non-zero constraint point only through the action of contact with an adjacent row, such contact is the criterion applied. Any constraint point which fails to satisfy this criterion is either moved to a physically valid location or set to zero, as indicated in block 1b. Such a change in constraint point is accompanied by the appropriate changes in load and mode, if any, and the process of constraint validation is continued until every remaining non-zero constraint point in the reactor is associated with contact. This procedure permits splitting of one unit into several units; it also corrects any violations of the assumptions of both mode and contact invariance which are implicit in the solution of Eqs. 13, 14, 15a, and 15b.

The resulting set of constraint points is used in the next iteration trial. The largest number of iterations required to produce convergence in test cases is about six, with two or three iterations per power level being typical.

V. BOW-V INPUT

A. General

The input to BOW-V is specified solely by punched cards that must be prepared on the IBM-026 keypunch. The quantities described below in the card sets are given in typical FORTRAN format. Since the structure of the BOW-V input deck is fixed, the following card formats, in order, apply to all BOW-V problems regardless of the choice of options.

B. Data Location on Cards (Definitions)

Card Set 1 (1 card): MI, MJ, LI, QQ, QD, QT, TL, PEPS, WEPS (316, 6E10.4)

MI Number of equal-length axial nodes subdividing each row (max number = 20).

MJ Number of radial rows considered (max number = 20).

LI Node number specifying intermediate-contact (button) location
 (any number desired from 1 through MI - 1).
QQ Initial fractional power level.
QD Fractional power increment between power levels.
QT Maximum fractional power level.
TL Length of subassemblies from pivot (upper grid) to top.
PEPS Convergence criterion ϵ_p (lb/contact point) (Eq. 8b).
WEPS Convergence criterion ϵ_w (lb/contact point) (Eq. 8b).

Card Set 2 (1 card): MT, NDB, B8, B9, KP, JP1, JP2, IP1, IP2 (2I6, 2E10.4, I4, 4I3)

MT Maximum number of iterations allowed per power level.
NDB Debugging flag. NDB $\neq$ 0 causes the printing of several diagnostically important internal arrays; NDB = 0 yields standard output.
B8 Separation of row centerlines from either nozzle obstruction.
B9 Nozzle length from pivot to bottom.
KP Punching flag. If KP $\neq$ 0, converged positions will be punched for rows JP1 through JP2, nodes IP1 through IP2. If KP = 0, the remainder of card set 2 is ignored, and no punching occurs.
JP1,
JP2 Innermost and outermost rows, respectively; i.e., positions for all rows JP1 through JP2 will be punched.
IP1,
IP2 Lower and upper nodes, respectively; i.e., positions for all nodes IP1 through IP2 will be punched.

Card Set 3 (1 card): ALPHA, D, YR (3E10.4)

ALPHA Coefficient of thermal expansion, all rows.
D Effective diameter of hex can, including C(z) factor (Eq. 1), all rows.
YR That outward displacement of the row-MJ centerline from YP(MJ) at which the outer edge of row MJ contacts the inflexible restraining ring at node MI.

Card Set 4: [DELT(I,J); I = 1, MI; J = 1, MJ] (8E10.4)

DELT(I,J) ΔT values for node I at row J, at reference power level (Eq. 1).

Card Set 5: [YP(J), J = 1, MJ] (8E10.4)

YP(J) Coordinate of centerline for the row-J pivot.

Card Set 6: [Dl(J), J = 1, MJ] (8E10.4)

Dl(J) Nominal distance minus the initial clearance between centerlines of rows J and J + 1 at node MI ("solid material" thickness) (Eq. 5a).

Card Set 7: [B1(J), J = 1, MJ] (8E10.4)

B1(J) Initial-tilt slope of row J (radians).

Card Set 8: [B4(J), J = 1, MJ] (8E10.4)

B4(J) The quantity $I_{A_N}^L / I_{N_A}^L$ for row J, where the subscripts A and N refer to subassembly and nozzle, respectively.

Card Set 9: [B6(J), J = 1, MJ] (8E10.4)

B6(J) The quantity $3EI_A$ for row J. (See Appendix B for definition of terms.)

Card Set 10: [DDL(J), J = 1, MJ] (8E10.4)

DDL(J) Nominal distance minus the initial clearance between centerlines of rows J and J + 1 at zero power and at node LI (Eq. 7).

Card Set 11: [T(J), J = 1, MJ] (8E10.4)

T(J) Average temperature of hex can at node LI and at reference power, relative to temperature of coolant at subassembly inlet (Eq. 7).

Card Set 12: [F1(J), J = 1, MJ] (8E10.4)

F1(J) Effective compressibility per hexagonal face for row J at node I (see Appendix D).

Card Set 13: F2(J), J = 1, MJ (8E10.4)

F2(J) Effective compressibility per hexagonal face for row J at node MI (see Appendix D).

C. Selection and Generation of Input Data

Most of the input quantities are believed to be adequately defined above. Some of them, however, are discussed here.

The power-related quantities on Card Set 1 (QQ, QD, and QT) are referenced to the nominal reactor power level, which is not an input quantity. The values of DELT(I,J) and T(J) given on Card Sets 4 and 11, respectively, correspond to the reference power level of 1.0, which is arbitrarily equivalent to the actual reactor power level.

The convergence criteria, PEPS and WEPS, are non-normalized quantities and represent load in pounds per button. BOW-V has converged successfully with very small values (i.e., $\epsilon_p = \epsilon_w < 0.1$ lb/button).*

The selection of MT may depend on the particular problem studied. Convergence has been obtained, however, within six iterations in all problems studied.

The constant D (equivalent diameter of the hex can) includes the effects of temperature distribution along the periphery of the can because C is at present not an input quantity in BOW-V. Thus, D is a precalculated input quantity which appropriately relates the unrestrained thermal deflection to the product of $\alpha\Delta T(z)$ by incorporating the constant C. (See Eq. 1 and Appendix G.)

The maximum temperature differentials across the hex cans (DELT) are assumed to lie 180° apart on a diameter through the core center. If these conditions are not realized, the actual $\Delta T(\max)$ should be adjusted to establish an effective DELT.

If no outer restraining ring is present, YR is made arbitrarily large. Analogously, if infinitely stiff buttons (or hex faces) are desired, the compressibility, F1 or F2, is made arbitrarily small.

Care must be taken when adjusting input clearances between the faces of hex cans in adjacent rows. Although BOW-V input treats clearances implicitly, they can be expressed as functions of the following input quantities according to Fig. 8. Let

* PEPS = WEPS = 0.1 lb/button in the sample problem in Appendix E.

CM(J) = Clearance between rows J and J + 1 at the top level, and

CL(J) = Clearance between rows J and J + 1 at the button levels.

. . CM(J) = YP(J+1) - YP(J) - D1(J), and

CL(J) = YP(J+1) - YP(J) - DD1(J).

Thus, for a given row spacing, the quantities D1 and DD1 are used to change subassembly clearances at zero power.

VI. BOW-V OUTPUT

A. General

The BOW-V program produces printed output, but has an option parameter (KP) to also provide a limited punched output. A second option (NDB) determines whether standard or diagnostic output is printed.

B. Output Pages

1. Standard Output (NDB = 0)

The diagnostic flag NDB (Card Set 2) usually will be zero. Printed output consists first of a one-page inventory of input quantities. The definitions of these inputs are given in Section V, and a typical listing of them is given in the sample problem, Appendix E.

This page is followed by two more pages of information for each selected power level. The first lists the power-dependent quantities YT(I,J)--the unrestrained thermal deflection--and DD(J)--the thickness of "solid material" between the centerlines of rows J and J + 1 at node LI.

The second page contains the description of the final configuration. First, the power level and final iteration number are given along with the reason for finality at that power level (i.e., either "converged values" or "maximum iteration number"). The remaining quantities describe final configurations quantitatively and are labeled as follows for each power level.

Y(I,J) Node-wise centerline position

MODE Numerical description of load arrangement (see Table V, Appendix B for definitions)

YF, YL Constraint-point positions of node MI and LI, respectively

P, W Net bending load required to enforce YF and YL,
 respectively
PSQ, WSQ Net compressive load at YF and YL, respectively
OM1, OM Contact indicator for nodes MI and LI, respectively

2. Diagnostic Output

A non-zero value of NDB yields a diagnostic output which includes the standard output for use in a detailed study of the iteration steps and matrix solutions for each unit.

C. Punched Output

If the punch option is selected, node positions (converged values only) are printed for each power level, in the format (6E12.5). The positions of selected nodes are given for each of the selected rows, with the first selected node of each row starting a new card. If the maximum number of iterations allowed for a given power level is reached before converged positions are found, punching for that power level is omitted.

These cards form part of the input set for an auxiliary program, BOWREAC, which converts BOW-V output node positions to reactivity changes due to bowing (see Appendix H).

VII. SUMMARY AND CONCLUSIONS

A. Operational Characteristics

The BOW-V program is operationally very stable and rapid. It requires about 1.6 sec per iteration and normally converges in less than 4 iterations. It can perform either purely static or quasi-dynamic computations with arbitrary temperature inputs. In either case, the code actually does a static calculation because it determines a core equilibrium configuration for every set of temperatures. For a transient problem, the successive sets of input temperatures represent a temperature field varying with time.

The program has been used extensively to analyze reactivity effects due to bowing in past EBR-II cores. It also is being used as a design tool to minimize effects of bowing in future cores.

B. Parameter Study

An extensive parameter study has been made while debugging the code. Sensitivity to the following parameters was determined.

1. Magnitude of temperature differentials in the hex can at a given power level
2. Sign of the temperature differential of the hex can (i.e., outward vs inward bowing)
3. Influence of outer rows on deflection of driver S/A's in the core
4. Compressibility of the hex can at S/A buttons
5. Temperature distribution around the periphery of the hex can
6. Bending stiffness of drive-core S/A's
7. Bending stiffness of reflector and blanket S/A's
8. S/A clearances at buttons
9. Axial distribution of temperature gradients

Of the parameters studied, the first five affect core deflections the most. (Item No. 3 was not studied as a parameter per se, but cases were calculated both with and without the outer rows present.)

The sample problem (run 25) in Appendix E illustrates a past real problem in EBR-II. In run 25, the core gradually compacted throughout most of the power range. In this case, the influence of outer rows 7 and 8 was extremely important because they formed a "buggy spring" which compacted the core (rows 1 through 6).

The effect of item No. 5 was studied in a separate analysis (see Appendix G), since the actual (peripheral) temperature distributions are known only approximately. The study showed that $C(z)$ (in Eq. 1) could vary from about 0.65 to about 0.75 depending on the various calculated temperature distributions encountered in the core for run 25. When a representative value ($C=0.70$, for all rows) was used, the BOW-V results matched EBR-II measurements within experimental uncertainty.

C. Future Improvements

Certain sub-models in BOW-V could be improved, and additional effects, such as swelling of the hex can induced by fast flux, could be added. If the peripheral temperature distribution in the hex can differs significantly

from row to row, BOW-V could be altered to accept the quantity $C(z)$ in tabular input analogous to $\Delta T(z)$.

The program may be expanded to include an option for inserting a series of fixed grids above the upper grid level. Effects of nonsymmetry in the core, caused by the presence of different types of subassemblies in a given row, may be investigated. In addition, high-temperature creep deformation of the hex cans may later be included in the BOW-V model.

Finally, the feasibility of converting this program to an IBM-360, Model 50/75 series, will be investigated. A problem could arise, however, because the double-precision computation required on the CDC-3600 will necessitate double or triple precision on the IBM-360.

APPENDIX A

Placement of Freely Pivoting Subassemblies

1. Introduction

A given row in the reactor must find itself in one of three conditions with respect to its neighboring rows: (a) no interaction, (b) loaded interaction, or (c) unloaded interaction. In the first condition (a), the row position is described completely by Eq. 1, Section IV, and the nozzle of the subassembly remains vertical. In the second condition (b), the load-deflection equations describe the deformation and position of the row. In both of these conditions, every axial point of the row lies at a uniquely determined radial position, i.e., there is a one-to-one correspondence between row configuration and temperature distribution.

The third condition (c) is illustrated in Fig. 9a, in which a pair of interacting rows has sufficient clearance to allow free pivoting. Thus, the implied initial overlap must be reduced to a condition of unloaded contact by pivoting about the upper grid. In this condition, there is a range of free pivoting for each row, limited by the lower grid (Fig. 9b illustrates this condition). Without special provision, the basic bowing model cannot choose a unique position within this limited range; so an additional assumption must be added.

Since all rows in general have pivoting freedom, the interaction of a row with its neighbors depends, in part, on the maximum amount of free pivoting available for each row. Therefore, the freedom to pivot causes an important analytical problem, with the particular example described here actually representing only a very special case. The general free-pivoting problem will be explored first, and the special case will be postponed until Section 3 of this appendix.

2. General Problem of Rows with No Bending Loads

In solving this type of problem, some arbitrary power level is specified so that the thermal deformations of all rows have specific values. With these thermal deformations and the resulting interactions, a row is

a. NO NOZZLE CONTACT

b. EXTREME POSITIONS (LIMITED BY NOZZLE CONTACT)

Fig. 9. Interactions of Unloaded Subassemblies

free to pivot only if an interval of physical movement (rotation about the pivot) can be assigned to an axial point of the row. If this is possible, then a corresponding interval may be assigned to any other axial point of the row; in particular, at the button (node LI) and top (node MI). The extreme points of this interval will be termed the pivoting limits for the axial point.

a. Contact at Top Level

As a first step, restrictions at node MI are considered and are incorporated into the node-MI pivot limits, which are labeled α_k and β_k for a general row k . If the thermally deformed positions of row k , independent of other rows, are y_1^k and y_2^k at nodes LI and MI, respectively, then, as Fig. 10 illustrates,

$$\alpha_k \geq y_2^k - B2, \text{ and} \quad (16a)$$

$$\beta_k \leq y_2^k + B2, \quad (16b)$$

where α_k = lefthand restriction for row k , β_k = righthand restriction for row k , and $B2 = (\ell_A/\ell_N) B8$ = free-pivoting extrema at top of subassembly.

However, it is necessary to consider the effects of other rows on row k . If we assume temporarily that the quantities α_{k-1} and β_{k+1} are known, it is clear from Fig. 11 that the conditions

$$\alpha_k \geq \alpha_{k-1} + D1_{k-1} \text{ and} \quad (17a)$$

$$\beta_k \leq \beta_{k+1} - D1_k \quad (17b)$$

are necessary for row k to be physically pivoted. Thus, the combinations of Eq. 16a with Eq. 17a and Eq. 16b with Eq. 17b yield finally the desired definitions

$$\left. \begin{aligned} \alpha_k &\equiv \max [y_2^k - B2, \alpha_{k-1} + D1_{k-1}] \\ \beta_k &\equiv \min [y_2^k + B2, \beta_{k+1} - D1_k] \end{aligned} \right\} \quad (18)$$

The form of Eq. 18 shows that the pivoting limits of each row are obtained from its two immediate neighbors by an iteration on row number. The above values of α_k and β_k describe the largest possible interval of node-MI pivoting limits for row k . These limits are determined by the positions of some

Fig. 10. Row-k Free-pivoting Extrema, Node MI

Fig. 11. Restrictions of Row-k Pivoting, Node MI

minimum and maximum rows which cannot pivot because of constraints. For example, a row in loaded interaction would provide such a fixed-point constraint, as would fixed row 1 or the outer restraining ring.

Since the conditions of Eq. 18 assign a unique interval to each non-fixed row k , the interval (α_k, β_k) exists, in principle, for all nonfixed rows. We now have to deal only with those rows for which $\alpha_k > \beta_k$.

First, suppose that either

$$\alpha_k > y_2^k + B2 \text{ or} \quad (19a)$$

$$\beta_k < y_2^k - B2 \quad (19b)$$

is satisfied. The physical interpretation of these conditions is presented in Figs. 12a and 12b. The condition shown in Fig. 12a suggests that the existing thermal deformations are so great that the subassembly must bend (i.e., experience loaded interaction due to nozzle contact). Hence, Eq. 19a requires specifically that rows $k-1$ and k be in loaded interaction, and Eq. 19b requires that rows k and $k+1$ be in loaded interaction. If either Eq. 19a or Eq. 19b is satisfied, it becomes necessary to define, by Eq. 18, a new set of pivot limits for each of the remaining rows in the original subdivided set of rows.

In general, a number of sets of rows remain in which no member satisfies Eq. 19a or Eq. 19b. If any such row, however, does satisfy $\alpha_k > \beta_k$ (hex-can compression), then

$$y_2^k \leq B2 \leq \beta_k < \alpha_k \leq y_2^k + B2 \quad (20)$$

is satisfied. Comparison with Eq. 18 then gives

$$\alpha_k = \alpha_{k-1} + D1_{k-1}, \text{ and} \quad (21a)$$

$$\beta_k = \beta_{k+1} - D1_k, \quad (21b)$$

so that

$$\alpha_{k-1} = \alpha_k - D1_{k-1}, \text{ and} \quad (22a)$$

$$\beta_{k+1} = \beta_k + D1_k. \quad (22b)$$

But then Eq. 18 yields, in combination with Eqs. 22a and 22b,

a. CONDITION FOR LEFT SIDE OF EQUATION 19a

b. CONDITION FOR RIGHT SIDE OF EQUATION 19b

Fig. 12. Conditions Requiring Bending of Row k

$$\alpha_{k-1} = \alpha_k - D1_{k-1} > \beta_k - D1_{k-1} \geq \beta_{k-1}, \text{ and} \quad (23a)$$

$$\beta_{k+1} = \beta_k + D1_k < \alpha_k + D1_k \leq \alpha_{k+1}, \quad (23b)$$

so that $\alpha_{k-1} > \beta_{k-1}$ and $\alpha_{k+1} > \beta_{k+1}$. Hence, if one row in a subset satisfies Eq. 20, all other rows in that subset also satisfy Eq. 20. However, since a row that satisfies Eq. 20 cannot satisfy either Eq. 19a or Eq. 19b, none of these rows (that satisfy Eq. 20) will require bending loads resulting from nozzle obstruction. It follows that such a row is prevented from pivoting freely only by the fact that compressive contacts on both sides are caused by the proximity of the fixed rows governing Eq. 18. Because of the convention in BOW-V, these rows then become members of a unit containing the fixed rows since they are not free pivoting.

At this point, the node-MI thermal deformations and centerline clearances have been considered, and free pivoting is possible only for those rows which satisfy $\alpha_k < \beta_k$.

b. Contact at Button Level

Here it becomes necessary to consider thermal deformations and centerline clearances at node LI. The node-LI free-pivoting interval is denoted (λ_k, μ_k) for row k. For those rows, k, that satisfy $\alpha_k < \beta_k$, it is clear from Fig. 13 that

$$\lambda_k = y_1^k + \frac{\ell_B}{\ell_A} (\alpha_k - y_2^k), \text{ and} \quad (24a)$$

$$\mu_k = y_1^k + \frac{\ell_B}{\ell_A} (\beta_k - y_2^k). \quad (24b)$$

However, for free pivoting it is necessary that:

$$\lambda_k \geq \lambda_{k-1} + DD_{k-1}, \text{ and} \quad (25a)$$

$$\mu_k \leq \mu_{k+1} - DD_k. \quad (25b)$$

These conditions are analogous to those of Eqs. 17a and 17b at node MI. As a result, the analog of Eq. 18 is

Fig. 13. Node-LI Positions for Node-MI Pivot Limits

$$\lambda_k = \max \left[y_1^k + \frac{l_B}{l_A} (\alpha_k - y_2^k), \lambda_{k-1} + DD_{k-1} \right], \text{ and} \quad (26)$$

$$\mu_k = \min \left[y_1^k + \frac{l_B}{l_A} (\beta_k - y_2^k), \mu_{k+1} - DD_k \right],$$

where α_k and β_k retain the values derived from Eq. 18.

Treatment of the intervals (λ_k, μ_k) obtained from Eq. 26 is completely analogous to that of the intervals (α_k, β_k) from Eq. 18. The condition $\lambda_k > \mu_k$ does not allow row k to be freely pivoted, and row k becomes one of a pair of loaded rows or part of a larger unit. If row k becomes loaded, it then becomes a fixed row from the viewpoint of free pivoting, and the process of determining pivot limits for its neighboring rows must begin again from Eq. 18.

All interactions caused by thermal deformation have now been considered, and those rows which satisfy $(\lambda_k < \mu_k)$ or $(\alpha_k < \beta_k)$ are the only rows that may be pivoted freely. Testing for both the (α_k, β_k) and (λ_k, μ_k) intervals is necessary; together they comprise the criteria for a free-pivoting capability.

3. Assignment of Positions to Freely Pivoting Rows

a. General Assumptions

Every row in free pivoting has now been identified, and it is necessary to assign a unique position to each of these rows. As a first approximation, each row is positioned so that its nozzle lies at its previous position. At the very beginning of the problem, however, the nozzle position corresponded to the given initial lean of the row. If the row in this position did not interact with its neighboring rows, then this position was accepted as final. If interaction occurs, the assignment of positions to the interacting rows is made by using the procedure described below.

The physical bowing model described previously cannot assign unique positions to freely pivoting rows that interact. To accomplish this assignment, we first assume that every subassembly (or row) contains a finite frictional resistance to pivot rotation. Second, we assume this resistance to be identical for every subassembly. Third, we assume that the effect of frictional resistance to pivot rotation is negligible compared to bending effects, and that it affects only those rows that do not

interact with loaded rows, i.e., precisely those rows that can pivot freely.

b. Analysis

Since we assume that row 1 is immovable, free pivoting involves the simultaneous free pivoting of 6 (k-1) subassemblies for a given row k (any row other than row 1). Hence, each freely pivoting row k can move from its given position only under a resultant force of at least 6 (k-1) friction units. Suppose that all members of a group of rows (M, M + 1, ..., N - 1, N) can pivot freely and are mutually interacting. Then, as shown below, there will be one row of this group that will remain fixed, i.e., its slope at the pivot will remain fixed. This row, k_o , thus satisfies the relationship:

$$\left| \sum_{j=M}^{k_o-1} 6(j-1) - \sum_{j=k_o+1}^N 6(j-1) \right| < 6(k_o-1), \quad (27a)$$

where: $\sum_{j=M}^{k_o-1} 6(j-1)$ = resultant force on row k_o from the inside, and

$\sum_{j=k_o+1}^N 6(j-1)$ = resultant force on row k_o from the outside.

Equivalently, we can write

$$\left| \sum_{j=M}^{k_o-1} 6(j-1) - \sum_{j=k_o+1}^N 6(j-1) \right| < 6(k_o-1), \quad (27b)$$

and we also know that

$$\sum_{j=1}^r j = \frac{r(r+1)}{2}, \quad (28)$$

because

$$\sum_{j=r}^s j = \sum_{j=1}^s j - \sum_{j=1}^{r-1} j.$$

Therefore, where s and r are positive integers,

$$\sum_{j=r}^s j = \frac{s(s+1)}{2} - \frac{r(r-1)}{2} = 1/2 (s^2 + s - r^2 + r) = 1/2 (s^2 - r^2 + s + r) .$$

Substituting the appropriate limits for s and $r-1$, we can write

$$|k_o^2 - 2k_o - C_1| < k_o - 1 , \quad (29)$$

where $C_1 = 1/2 [N(N-1) + M(M-3)]$.

$$\text{Let } \ell_o = k_o - 1 ,$$

$$\text{then } |(\ell_o + 1)^2 - 2(\ell_o + 1) - C_1| < \ell_o = |\ell_o^2 - C_1 - 1| < \ell_o |\ell_o^2 - C_2| < \ell_o . \quad (30)$$

$$\text{Let } C_2 = C_1 + 1 , \quad (31)$$

$$\text{then } |\ell_o - C_2/\ell_o| < 1 ,$$

$$\text{or } -1 < \ell_o - C_2/\ell_o < 1 . \quad (32)$$

Taking the first part of the inequality, we see that

$$\ell_o - C_2/\ell_o > -1 \text{ leads to } (2\ell_o + 1)^2 > 4C_2 + 1 ; \quad (33)$$

and taking the second part, we see that

$$\ell_o - C_2/\ell_o < 1 \text{ leads to } (2\ell_o - 1)^2 < 4C_2 + 1 . \quad (34)$$

Thus, we can rewrite Eq. 34:

$$-1 + \sqrt{4C_2 + 1} < 2\ell_o < 1 + \sqrt{4C_2 + 1} ,$$

$$\text{or } 1 + \sqrt{4C_2 + 1} < 2k_o < 3 + \sqrt{4C_2 + 1} . \quad (35)$$

By inspection, the interval $(1 + \sqrt{4C_2 + 1}, 3 + \sqrt{4C_2 + 1})$ can contain at most one even integer; but suppose there is no even integer, i.e., suppose

$$\sqrt{4C_2 + 1} = 2r + 1 .$$

$$\begin{aligned} \text{Then, } 4C_2 + 1 &= 4r^2 + 4r + 1 = 1 + 4 \left\{ 1 + 1/2 [N(N-1) + M(M-3)] \right\} \\ &= 5 + 2N(N-1) + 2M(M-3) . \end{aligned}$$

Let $N = M + I$, $I \geq 1$ (I is an even or odd integer).

$$\text{Then } 4r^2 + 4r + 1 = 5 + 2(M + I)(M + I - 1) + 2M(M - 3) .$$

Thus,

$$4C_2 + 1 = 4M^2 + 4(I - 2) M + (2I^2 - 2I + 5) ,$$

or

$$4C_2 + 1 = (2M + I - 2)^2 + (I + 1)^2 . \quad (36)$$

We note that the terms in Eq. 36 are pythagorean squares of odd/even pairs, i.e., $a^2 + b^2 = c^2$ (c^2 is odd), and

$$a^2 = c^2 - b^2 = (c + b) (c - b) .$$

Since N in BOW-V is limited to 20 rows,

$$c = \sqrt{4C_2 + 1} < 29 .$$

$$\therefore a < 29, \text{ and}$$

$$b < 29 .$$

Allowing a to be odd, we show all the required examples where a, b, and c are always integers:

$$a = 3, a^2 = 9; 1(9) \Rightarrow 3, 4, 5$$

$$a = 5, a^2 = 25; 1(25) \Rightarrow 5, 12, 13$$

$$a = 7, a^2 = 49; 1(49) \Rightarrow 7, 24, 25$$

$$a = 9, a^2 = 81; 1(81) \Rightarrow 9, 40, 41; 3(27) \Rightarrow 9, 12, 15$$

$$a = 11, a^2 = 121; 1(121) \Rightarrow 11, 60, 61$$

$$a = 13, a^2 = 169; 1(169) \Rightarrow 13, 84, 85$$

$$a = 15, a^2 = 225; 1(225) \Rightarrow 15, 112, 113; 3(75) \Rightarrow 15, 36, 39;$$

$$5(45) \Rightarrow 15, 20, 25; 9(25) \Rightarrow 15, 8, 17$$

$$a = 17, a^2 = 289; 1(289) \Rightarrow 17, 144, 145$$

$$a = 19 \text{ (prime no.)}, a^2 = 361; 1(361) \Rightarrow 19, 180, 181$$

$$a = 21, a^2 = 441; 1(441); 3(147); 7(63) \Rightarrow 21, 28, 35; 9(49) \Rightarrow 21, 20, 29$$

$$a = 23 \text{ (prime no.)}, a^2 = 529; 1(529) \Rightarrow 23, 264, 265$$

$$a = 25, a^2 = 625; 1(625); 5(125) \Rightarrow 25, 60, 65$$

$$a = 27, a^2 = 729; 3(243); 9(81) \Rightarrow 27, 36, 45$$

Thus, we must check the following combinations of cases a, b, and c (tabulated below) that are mathematically possible:

<u>Case</u>	<u>i</u>	<u>ii</u>	<u>iii</u>	<u>iv</u>	<u>v</u>	<u>vi</u>
a	3	5	7	9	15	15
b	4	12	24	12	20	8
c	5	13	25	15	25	17

We can write the following two choices for a and b:

$$\begin{aligned} (1) \quad 2M + I - 2 = a, \text{ or } (2) \quad 2M + I - 2 = b \\ I + 1 = a \qquad \qquad \qquad I + 1 = b \end{aligned} \quad (37)$$

for chosen values of a and b above.

In general, let $2M + I - 2 = \alpha$, and
 $I + 1 = \beta$.

Then we have

$$I = \beta - 1, M = 1/2 (\alpha - \beta + 3) .$$

Table I lists possible solutions of Eq. 37 corresponding to potentially stationary rows under free pivoting in EBR-II.

TABLE I. Solutions of Eq. 37 Corresponding to Potentially Stationary Rows under Free Pivoting in EBR-II

<u>Basic Case</u>		<u>Choice 1</u>		<u>Choice 2</u>	
i	3, 4	$\alpha = 3$ $\beta = 4$	$I = 3, M = 1$	$\alpha = 4$ $\beta = 3$	$I = 2, M = 2$
ii	5, 12	$\alpha = 5$ $\beta = 12$	$I = 11, M = -2$	$\alpha = 12$ $\beta = 5$	$I = 4, M = 5$
iii	7, 24	$\alpha = 7$ $\beta = 24$		$\alpha = 24$ $\beta = 7$	$I = 6, M = 10$
iv	9, 12	$\alpha = 9$ $\beta = 12$	$I = 11, M = 0$	$\alpha = 12$ $\beta = 9$	$I = 8, M = 3$
v	15, 20	$\alpha = 15$ $\beta = 20$		$\alpha = 20$ $\beta = 15$	$I = 14, M = 4$
vi	8, 15	$\alpha = 8$ $\beta = 15$		$\alpha = 15$ $\beta = 8$	$I = 7, M = 5$

Only cases i(2), ii(2), iv(2), and vi(2) are physically possible in EBR-II. Its row-index restrictions can be expressed: $2 < M < 15$ and $M + I < 15$; Table II lists the indeterminate cases. (Those cases that allow no unique solutions for k_o).

TABLE II. Indeterminate Cases Shown in Table I

<u>Case</u>	<u>M</u>	<u>N</u>	<u>I</u>	<u>$1/2(1 + \sqrt{4C_2 + 1})$</u>	<u>c</u>
i(2)	2	4	2	3	5
ii(2)	3	11	8	8	15
iv(2)	5	9	4	7	13
vi(2)	5	12	7	9	17

From Eq. 35, we attempt to solve for k_o as follows in these physically indeterminate cases:

$$0.5 (1 + \sqrt{4C_2 + 1}) < k_o < 0.5 (3 + \sqrt{4C_2 + 1}). \quad (38)$$

We have:

$$\sqrt{4C_2 + 1} = 5 + 2N(N - 1) + 2M(M - 3).$$

Table III lists those freely pivoting cases that allow no direct solution for the stationary row, k_o , of EBR-II.

TABLE III. Freely Pivoting Cases Allowing No Direct Solution for the Stationary Row, k_o , of EBR-II

<u>Case</u>	<u>k_o</u>
i(2)	$3 < k_o < 4$
ii(2)	$8 < k_o < 9$
iv(2)	$7 < k_o < 8$
vi(2)	$9 < k_o < 10$

Because the above cases allow no solution for k_o (k_o must be an integer), we arbitrarily allow the existence of k_o in all cases by re-writing Eq. 38 as follows:

$$0.5 (1 + \sqrt{4C_2 + 1}) < k_o \leq 0.5 (3 + \sqrt{4C_2 + 1}) . \quad (39)$$

Thus, the freely pivoting row, k_o , whose slope (at the pivot) remains fixed will correspond to the nearest integer larger than the left side of Eq. 39.

APPENDIX B

Derivation of Mode Formulas

1. General Assumptions

This appendix describes the contents of blocks 1 and 1b in Fig. 4.

BOW-V treats each row representative as a pivoted vertical beam composed of uniform, but distinct, regions above and below the pivot, i.e., all rows can have different mechanical properties above and below the pivot. The centerline of the nozzle extending below the pivot initially is a straight line whose end lies midway between a pair of separated nozzle supports (lower grid). Above the pivot, the temperature difference across the beam describes a curve that satisfies the thermal-deformation Eq. 1. Because this curvature is small compared to the length of span, the beam is assumed to have the deformation properties of a perfectly straight beam.

2. Potential Load Locations

Referring to Figs. 14 and 15, two axial locations above the pivot may be constrained to lie at specified positions. These locations correspond to nodes MI and LI (top and button positions in EBR-II), and are designated X_2 and X_1 , respectively. The corresponding constraint positions at these nodes are designated Y_2 and Y_1 . As the beam is forced to pass (in a radial direction) through positions Y_2 and Y_1 , the nozzle tip may swing into contact with one of the restraining supports at the nozzle position X_N . If further deformation at X_2 and X_1 is required, however, the nozzle section will be deformed, because the nozzle supports are assumed to be immovable. In general, three loads (at X_2 , X_1 and X_N) are necessary to locate the beam at Y_2 and Y_1 . Since the radial position at X_N is fully determined by the choices for Y_2 and Y_1 , the position of X_N (and the load there, if any) and the load at the pivot are not treated explicitly in the equations to be developed here. Nevertheless, the conditions at X_N are necessary for the physical description of modes in this report.

The scope of BOW-V requires that the loads be computed by a completely arbitrary choice of Y_2 and Y_1 . As explained under

Fig. 14. Subassembly as a Loaded Beam

Fig. 15. Mechanical Deformation Modes

Section 3, such flexibility gives rise to a number of distinct possible combinations of loads at the points X_2 , X_1 , and X_N . Each such combination is termed a mode, and each mode is assigned a unique mode number.

3. Mode Classification

The placement of the beam at its selected constraint points is most easily understood as occurring in two steps. In the first step (contact at subassembly top and nozzle), the constraint Y_1 (button) is ignored, and the beam is moved from its initial configuration so that X_2 lies at Y_2 . The second step (contact at button level and nozzle) then consists of moving X_1 to Y_1 while holding X_2 at Y_2 . If either constraint position (Y_2 or Y_1) is unnecessary, the corresponding step is omitted.

a. Step One: Contact at Subassembly Top and Nozzle

Step one may be divided into three cases, depending on the choice of Y_2 . The first case results in no contact of a row with its neighbors and therefore is trivial. In this case, Y_2 is undefined and therefore is assigned mode number zero (0). In the second case, Y_2 is defined, and the beam is pivoted so that X_2 reaches Y_2 while X_N remains separated from the lower grid; this case is assigned mode number I_T . In the third case, the clearance of X_N is insufficient for X_2 to reach Y_2 through free pivoting. Thus, X_2 can reach Y_2 only by means of bending caused by loads at X_2 and X_N ; this case is assigned mode number II_T . Modes 0, I_T , and II_T describe the behavior of those rows for which Y_1 is undefined. For rows in which Y_1 is defined, however, these three modes are only the first step in the determination of the final mode.

b. Step Two: Contact at Button Level and Nozzle

Step two is most easily described in terms of the effects of Y_1 on the three cases of step one. If Y_2 is undefined, the movement of X_1 to Y_1 separates into cases in which either: (a) free pivoting is not followed by any loads at X_1 and X_N (mode I); or (b) free pivoting is followed by loads at X_1 and X_N (mode II).

(1) Contact at Top, Button, and Nozzle. If X_2 reaches Y_2 with clearance at the nozzle but with interference at X_1 , movement of X_1 towards Y_1 requires a load at X_1 that will be counterbalanced by a load at X_2 . Thus, X_N will be pivoted in this configuration which is defined as mode IV.

If the displacement of X_1 to Y_1 is so large that X_N contacts a nozzle support, a third load then arises at X_N . This new mode actually is separated into three cases (III, III_T and V); each has loads at X_1 , X_2 , and X_N .

Suppose now that X_2 reaches Y_2 only after loads are applied to X_2 and X_N . Initially, the movement of X_1 to Y_1 must produce an increase in one of the loads at X_2 or X_N and a decrease in the other as a result of net subassembly rotation about the pivot. An increase in the load at X_N is characteristic of mode III, but an increase in the load at X_2 is characteristic of mode III_T.

If the movement of X_1 to Y_1 is large, modes III and III_T could yield to other modes. Thus, mode III is applicable until X_1 reaches a point at which the load at X_2 is reduced to zero; a row in this configuration reverts to mode II. Continued movement of X_1 eventually requires a load at X_2 in the opposite direction of the load originally required by mode II_T. The occurrence of this load at X_2 determines mode V in which loads again exist at X_1 , X_2 , and X_N . If the load at X_N is fully relieved because of further movement of X_1 , mode III_T would revert to mode IV with the nozzle pivoting towards its opposite restraint. If X_N ultimately contacts this restraint, a load is introduced at X_N , and mode V again occurs. It follows that the most complex modes that can arise are III, III_T, or V. Each mode is sketched in Fig. 15, and Fig. 16a presents a complete schematic diagram of possible sequences of modes. Figure 16b shows the detailed steps leading to mode V.

c. Mode Polarity

A row may be physically in a given mode in one of two distinct, mirror-image configurations, i.e., of plus or minus polarity. Although the sign convention for loads in BOW-V requires that a distinction be made between these two cases, the descriptive equations are identical in each case, because they are written in incremental form. For completeness and clarity, mode V in Fig. 15 is sketched for both polarities.

4. Final Equations

a. General

Modes zero, I, and I_T involve no deformations caused by bending and are governed by the unloaded conditions of unrestrained thermal deflection and free pivoting. Modes II, II_T, and IV require mechanical

STEP ONE:

STEP TWO:

a. MODE PROGRESSION

b. TYPICAL MODE SEQUENCE FROM MODE II_T TO MODE V

Fig. 16. Mode Relationships and Typical Mode Sequence

deformations arising from loads at two points of the pivoted beam, and these modes are governed by formulas available in standard tables.³ In modes III, III_T, and V, deformations arising from three loads acting on a pivoted beam are required. In these modes, the beam is redundantly loaded; formulas governing these modes are not available in standard tables. The derivations of equations corresponding to modes II through V are outlined in the following section.

b. Derivation of Equations

As Tables IV and V indicate, the final expressions relating load and displacement are identical for modes III, III_T, and V. Because modes II, II_T, and IV are less complex, the derivation is shown only for mode V. The derivations of the auxiliary expressions in Table V are omitted, because the procedure is similar to that used in Table IV.

The derivation of the mode-V equations is shown using the Second Area Moment Proposition⁴ which states:

"If A & P are points on a deflection curve, the vertical distance of A from the tangent to the curve at P is equal to the moment with respect to A of the area of the bending moment diagram between A & P, divided by the flexural rigidity, EI."

In equation form, this statement reduces to:

$$y_A^P = \int_P^A \frac{M_P^A dx}{EI} . \quad (40)$$

Referring to Fig. 17, we write the remaining two necessary relationships:

$$F_P = F_A + F_B + F_N, \text{ and} \quad (41)$$

$$F_N = \frac{1}{\ell_N} (F_A \ell_A + F_B \ell_B) . \quad (42)$$

Using Eq. 40, we write the relative deflections for points A, B, and N of the deflected beam, noting that the beam stiffness is assumed to differ in lengths AP and PN:

TABLE IV. Summary of Mode Definitions

Mode	No.	Load Configuration			
		Top Load	Button Load	Nozzle Load	Relative Load Polarity
0	0	No contact	(No contact)	(No contact)	—
I	1	No contact	(No load; contact only)	(No contact)	—
I _T	10	(No load; contact only)	(No contact)	(No contact)	—
II	2	Absent	Present	Present	Both loads same direction
II _T	20	Present	Absent	Present	Both loads same direction
III	3	Present	Present	Present	All loads same direction
III _T	30	Present	Present	Present	Button load opposite other two
IV	4	Present	Present	Absent	Both loads are opposite
V	5	Present	Present	Present	Top load opposite other two

TABLE V. Summary of Possible Bending Modes and Resulting Equations Used in Blocks 1 and 1b^a

<u>Mode</u>	<u>Governing Displacement</u>	<u>Resulting Increments</u>
I _T	Δy_A	$\Delta y_B = \frac{l_B}{l_A} \Delta y_A$ $\Delta W = \Delta P = 0$
II _T	Δy_A	$\Delta y_B = \frac{l_B}{l_A} K \Delta y_A$ $\Delta W = 0$ $\Delta P = - \frac{3EI_A \Delta y_A}{l_A^3 (1 + \rho)}$
I	Δy_B	$\Delta y_A = \frac{l_A}{l_B} \Delta y_B$ $\Delta W = \Delta P = 0$
II	Δy_B	$\Delta y_A = \frac{l_B (3l_A - l_B) + 2 l_A^2 \rho}{2 l_B (l_B + l_A \rho)} \Delta y_B$ $\Delta P = 0$ $\Delta W = - \frac{3EI_A}{l_B^2 (l_B + l_A \rho)^2} \Delta y_B$

^aSee Fig. 4, BOW-V Flow Chart.

TABLE V (contd)

<u>Mode</u>	<u>Governing Displacement</u>	<u>Resulting Increments</u>
IV	Δy_B	$\Delta y_A = 0$ $\Delta W = \frac{-3EI_A \ell_A}{\ell_B^2 (\ell_A - \ell_B)^2} \Delta y_B$ $\Delta P = -\frac{\ell_A}{\ell_B} \Delta W$
III, III _T , V	Δy_B	$\Delta y_A = 0$ $\Delta W = \frac{6EI_A \ell_A}{\ell_B^2 (\ell_A - \ell_B) [\ell_B + (2\ell_A - \ell_B)K]} \Delta y_B$ $\Delta P = \frac{\ell_B}{\ell_A} K \Delta W$

NOTES: (1) $\rho = \frac{I_A \ell_N}{I_N \ell_A}$.

$$(2) K = -\frac{\ell_B (3\ell_A - \ell_B) + 2\ell_A^2 \rho}{2\ell_A^2 (1 + \rho)}.$$

(3) All overall constants relating deflection to load are actually row dependent. Thus, in the BOW-V program, individual subassembly loads (shown in the table) are multiplied by the constant KK for hex-can geometry, where KK = number of subassemblies per row = 6(k - 1) for k > 1 (where k = general row number).

$$y_A^P = \frac{1}{(EI)_A} \left[\frac{F_A \ell_A^3}{3} + \frac{F_B \ell_B^2}{2} (\ell_A - \frac{\ell_B}{3}) \right], \quad (43)$$

$$y_B^P = \frac{1}{(EI)_A} \left[F_A \frac{(\ell_A - \ell_B)}{2} \ell_B^2 + \frac{F_A \ell_B^3}{3} + \frac{F_B \ell_B^3}{3} \right], \text{ and} \quad (44)$$

$$y_N^P = \frac{1}{(EI)_N} \left[\frac{F_A \ell_A + F_B \ell_B}{3} \right] \ell_N^2 = \frac{F_N}{(EI)_N} \frac{\ell_N^3}{3}, \quad (45)$$

where: E = Young's modulus, and
 I = moment of inertia with respect to vertical
 (z) axis.

To establish the relationship between F_A and F_B , assume that
 (a) displacements and forces are incremental quantities and (b) y_A (total
 displacement) = 0. Condition (b) is expressed as follows:

$$y_A = 0 = y_A^P + (\ell_A / \ell_N) y_N.$$

$$\therefore 0 = \frac{1}{(EI)_A} \left[\frac{F_A \ell_A^3}{3} + \frac{F_B \ell_B^2}{2} (\ell_A - \frac{\ell_B}{3}) \right] + \frac{\ell_A}{\ell_N (EI)_N} \left[(F_A \ell_A + F_B \ell_B) \frac{\ell_N^2}{3} \right].$$

Thus

$$F_A = \frac{\ell_B}{\ell_A} K F_B,$$

where

$$K = - \frac{\ell_B (3\ell_A - \ell_B) + 2\ell_A^2 \rho}{2\ell_A^2 (1 + \rho)} \quad \text{and}$$

$$\rho = \frac{I_A \ell_N}{I_N \ell_A}.$$

a. RADIAL DISPLACEMENT DIAGRAM

b. BENDING MOMENT DIAGRAM

Fig. 17. Subassembly Displacement and Bending-moment Diagrams

To establish the relationship between F_B and y_B , assume that:

(a) displacement and forces are incremental quantities and (b) y_B (total displacement) = $y_B^P - \frac{\ell_B}{\ell_A} y_A^P$. Applying condition (b), we have:

$$y_B = \frac{1}{(EI)_A} \left[\frac{F_A(\ell_A - \ell_B)\ell_B^2}{2} + \frac{F_A\ell_B^3}{3} + \frac{F_B\ell_B^3}{3} \right] - \frac{\ell_B}{\ell_A(EI)_A} \cdot \left[\frac{F_A\ell_A^3}{3} + \frac{F_B\ell_B^2}{2} (\ell_A - \frac{\ell_B}{3}) \right]. \quad (47)$$

Thus

$$F_B = \frac{-6(EI)_A\ell_A}{\ell_B^2(\ell_A - \ell_B) \underbrace{[\ell_B + K(2\ell_A - \ell_B)]}_x} y_B. \quad (47)$$

We note that $x \leq 0$ for all values of ℓ_B and ρ because:

$$\ell_B \leq \ell_A \quad \text{and}$$

$$\rho \geq 0.$$

The necessary relationships for modes II, II_T, and IV are derived analogously. Table IV lists the formulas governing each mode. Because the load and displacement variables are all incremental, the final equilibrium values are determined by summing the successive increments. For example, the displacement y_B in mode V is that displacement required to move the load at X_1 to Y_1 from the point at which further movement of X_1 is governed by mode V.

Table VI lists the corresponding node-dependent displacements at arbitrary node points. These additional quantities are not necessary for the basic BOW-V solution, but they are helpful in using the BOW-V solution for determining such things as the power-dependent reactivity effects caused by bowing.

TABLE VI. Displacement of Any Node (i) for Any Bending Mode

Mode	Displacement of Node i
I_T	$\Delta y_i = \frac{\ell_i}{\ell_A} \Delta y_A$
II_T	$\Delta y_i = \Delta y_A \frac{\ell_i [2\ell_A^2 \rho + \ell_i (3\ell_A - \ell_i)]}{2\ell_A^3 (\rho + 1)}$
I	$\Delta y_i = \frac{\ell_i}{\ell_B} \Delta y_B$
II	$\Delta y_i = \Delta y_B \frac{\ell_i [\ell_B (3\ell_i + 2\ell_A \rho) - \ell_i^2]}{2\ell_B^2 (\ell_A \rho + \ell_B)}, \text{ when } i \leq LI$ $= \Delta y_B \frac{\ell_i (2\ell_A \rho + 3\ell_B) - \ell_B^2}{2\ell_B (\ell_A \rho + \ell_B)}, \text{ when } LI < i \leq MI$
IV	$\Delta y_i = \Delta y_B \frac{\ell_i [\ell_B (2\ell_A - \ell_B) - \ell_i^2]}{2\ell_B^2 (\ell_A - \ell_B)}, \text{ when } i \leq LI$ $= \Delta y_B \frac{(\ell_A - \ell_i) [\ell_i (2\ell_A - \ell_i) - \ell_B^2]}{2\ell_B (\ell_A - \ell_B)^2}, \text{ when } LI < i \leq MI$

TABLE VI (contd)

Mode

Displacement of Node i

$$\begin{aligned}
 \Delta y_i &= \Delta y_B \frac{\ell_i [\ell_B (\ell_A - \ell_i) (2\ell_A - \ell_i) K + \ell_A \ell_i^2 - 3\ell_A \ell_B \ell_i + 3\ell_A \ell_B^2 - \ell_B^3]}{\ell_B^2 (\ell_A - \ell_B) [\ell_B + (2\ell_A - \ell_B) K]}, \text{ when } i \leq LI \\
 &= \Delta y_B \frac{(\ell_A - \ell_i) [\ell_i (2\ell_A - \ell_i) K + \ell_B^2]}{\ell_B (\ell_A - \ell_B) [\ell_B + (2\ell_A - \ell_B) K]}, \text{ when } LI < i \leq MI
 \end{aligned}$$

APPENDIX C

Mode-dependent Constraint-change Equations

1. General

Treatment of changes in constraint point used in the inverse matrix solution in block 2 of Fig. 4 requires the complete set of relations implied by Eq. 4, which may be written equivalently as:

$$\begin{aligned}\Delta W_j &= W_j^0 + k_{11j} c_{1j} + k_{12j} c_{2j}, \text{ and} \\ \Delta P_j &= P_j^0 + k_{21j} c_{1j} + k_{22j} c_{2j},\end{aligned}\tag{48}$$

where all variables are incremental quantities, and c_{1j} and c_{2j} are iterated values of Δy_B and Δy_A , respectively. The constants k_{ikj} are now determined under the assumption that the displacements of constraint point c_{ij} leave row j in its initial mode.

The procedure is similar for all modes and will be presented here for mode V only. Because mode V is the most complicated, it requires the largest use of the necessary techniques. Each mode consists of two mirror-image cases; vector notation is used to denote incremental loads and displacements, to permit the choice of sign to be implicit, and thereby to collapse two cases into one.

2. Description of Mode Sequence

Mode V is most easily viewed as the result of the sequence: mode I--mode II--mode V. The following notation will be used consistent with Fig. 18: nodes LI and MI lie at radial positions X_1 and X_2 , respectively, at the instant that free pivoting ends because of nozzle contact. Mode-II movement then occurs until node MI has arrived at the constraint point ξ_2 , at which node LI lies at point X_1^* (the new position of the button node). At this point, a load $\underline{P}$ is introduced at ξ_2 to restrain node MI there, while node LI is further displaced from X_1^* to the given constraint point ξ_1 . The values of X_1^* , ξ_2 , and ξ_1 determine the applicable positions and loads. The corresponding procedure is then applied for new constraints,

Fig 18. Vectorial Description of Derivations in the Constraint-change Equation

and the corresponding configuration defined by X_1' , ξ_2' , and ξ_1' is compared with the original to provide the desired relationships. As a notational simplification, a subscripted quantity of α_M will be used to represent the mode-dependent values of constant k_{ikj} relating load changes to deflections. The quantity β_M will denote the mode-dependent displacement-scaling factor, and γ_M will denote the mode-dependent load-scaling factor. (The subscript M is the mode number.)

3. Derivation of Equations by Vectorial Superposition

The steps in the calculation are summarized in Fig. 18. As a first step, observe that

$$\begin{aligned} \underline{\Delta W}' - \underline{\Delta W} &= -\alpha_V (\overrightarrow{X_1' \xi_1'} - \overrightarrow{X_1^* \xi_1}) = -\alpha_V [(\overrightarrow{X_1' X_1} + \overrightarrow{X_1 \xi_1'}) - (\overrightarrow{X_1^* X_1} + \overrightarrow{X_1 \xi_1})] \\ &= -\alpha_V [\overrightarrow{X_1 \xi_1'} - \overrightarrow{X_1 \xi_1} + (\overrightarrow{X_1' X_1} - \overrightarrow{X_1^* X_1})] \\ &= -\alpha_V [\overrightarrow{\xi_1 \xi_1'} + (-\overrightarrow{X_1 X_1'} + \overrightarrow{X_1 X_1^*})], \end{aligned} \quad (49)$$

which becomes, upon insertion of (1a) and (1a') from Fig. 18,

$$\underline{\Delta W}' - \underline{\Delta W} = -\alpha_V [\overrightarrow{\xi_1 \xi_1'} + \beta_{II}^{-1} (-\overrightarrow{X_2 \xi_2'} + \overrightarrow{X_2 \xi_2})] = -\alpha_V \overrightarrow{\xi_1 \xi_1'} + \alpha_V \beta_{II}^{-1} \overrightarrow{\xi_2 \xi_2'}. \quad (50)$$

As a result, Eqs. 2a, 2b, 2a', and 2b' from Fig. 18 yield

$$\underline{P}' - \underline{P} = -\gamma_V (\underline{\Delta W}' - \underline{\Delta W}) = \gamma_V \alpha_V \overrightarrow{\xi_1 \xi_1'} - \gamma_V \alpha_V \beta_{II}^{-1} \overrightarrow{\xi_2 \xi_2'}. \quad (51)$$

By definition,

$$\begin{aligned}
 \underline{W}' - \underline{W} &= (\underline{W}'_0 + \underline{\Delta W}') - (\underline{W}_0 + \underline{\Delta W}) = (\underline{W}'_0 - \underline{W}_0) + (\underline{\Delta W}' - \underline{\Delta W}) \\
 &= -\alpha_{II}(\overrightarrow{X_1 X_1'} - \overrightarrow{X_1 X_1^*}) + (-\alpha_V \overrightarrow{\xi_1 \xi_1'} + \alpha_V \beta_{II}^{-1} \overrightarrow{\xi_1 \xi_1'}) \\
 &= -\alpha_{II}(\beta_{II}^{-1} \overrightarrow{X_2 \xi_2'} - \beta_{II}^{-1} \overrightarrow{X_2 \xi_2}) - \alpha_V \overrightarrow{\xi_1 \xi_1'} + \alpha_V \beta_{II}^{-1} \overrightarrow{\xi_2 \xi_2'} \\
 &= -\alpha_V \overrightarrow{\xi_1 \xi_1'} + \beta_{II}^{-1} (\alpha_V - \alpha_{II}) \overrightarrow{\xi_2 \xi_2'} ,
 \end{aligned} \tag{52}$$

and

$$c_{1j} = \overrightarrow{\xi_1 \xi_1'}, \text{ and } c_{2j} = \overrightarrow{\xi_2 \xi_2'} . \tag{53}$$

Thus, Eqs. 51 and 52 become

$$\begin{aligned}
 W_j' &= W_j - \alpha_V c_{1j} + \beta_{II}^{-1} (\alpha_V - \alpha_{II}) c_{2j}, \text{ and} \\
 P_j' &= P_j + \gamma_V \alpha_V c_{1j} - \gamma_V \alpha_V \beta_{II}^{-1} c_{2j} .
 \end{aligned} \tag{54}$$

Table V contains a complete list of the coefficients k_{ikj} in Eq. 48 for all modes.

4. Relationship of Constraint-change Equations to Mode Formulas

The set of equations given in Table VII is the complete set that comprises all the necessary relationships between forces on and deflections of any beam in any mode. Although the block-1 equations were derived differently than the block-2 equations, the former could actually have been extracted from the latter. Because blocks 1 and 1b were derived separately by another author,² the two methods of derivation were given above.

TABLE VII. Summary of Equations Used in Block 2^a

Mode	Equations
I _T	$c_{1j} = \beta_{I_T}^j c_{2j}$
II _T	$c_{1j} = \beta_{II_T}^j c_{2j}; P_j' = P_j - \alpha_{II_T}^j c_{2j}$
I	$c_{2j} = \beta_I^j c_{1j}$
II	$c_{2j} = \beta_{II}^j c_{1j}; W_j' = W_j - \alpha_{II}^j c_{1j}$
IV	$\begin{cases} W_j' = W_j - \alpha_{IV}^j c_{1j} + \alpha_{IV}^j (\beta_I^j)^{-1} c_{2j} \\ P_j' = P_j + \gamma_{IV}^j \alpha_{IV}^j c_{1j} - \gamma_{IV}^j \alpha_{IV}^j (\beta_I^j)^{-1} c_{2j} \end{cases}$
III, III _T , V	$\begin{cases} W_j' = W_j - \alpha_{III}^j c_{1j} + \alpha_{III}^j \beta_{II_T}^j c_{2j} \\ P_j' = P_j + \gamma_{III}^j \alpha_{III}^j c_{1j} - (\gamma_{III}^j \alpha_{III}^j \beta_{II_T}^j + \alpha_{II_T}^j) c_{2j} \end{cases}$

Mode	α_M	β_M	γ_M
I _T	---	$\frac{l_B}{l_A}$	---
II _T	$\frac{3EI_A}{l_A^3(1+\rho)}$	$-\frac{l_B}{l_A} K$	---
I	---	$\frac{l_A}{l_B}$	---

^aSee Fig. 4, BOW-V Flow Chart.

TABLE VII (contd)

Mode	α_M	β_M	γ_M
II	$\frac{3EI_A}{\ell_B^3(1 + \frac{\ell_A}{\ell_B} \rho)}$	$\frac{(1/2)\ell_B(3\ell_A - \ell_B) + \ell_A^2 \rho}{\ell_B^2 + \ell_A \ell_B \rho}$	---
IV	$\frac{3EI_A \ell_A}{\ell_B^2(\ell_A - \ell_B)^2}$	---	$\frac{\ell_B}{\ell_A}$
III, III _T , V	$\frac{-6EI_A \ell_A}{\ell_B^2(\ell_A - \ell_B)[\ell_B + (2\ell_A - \ell_B)K]}$	---	$-\frac{\ell_B}{\ell_A} K$

NOTES: (1) $\rho = \frac{I_A \ell_N}{I_N \ell_A}$

$$(2) \quad K = - \frac{\ell_B(3\ell_A - \ell_B) + 2\ell_A^2 \rho}{2\ell_A^2(1 + \rho)}$$

(3) All overall constants relating deflection to load are actually row dependent. Thus, in the BOW-V program, individual subassembly loads (shown in the above table) are multiplied by the constant KK for hex-can geometry, where

KK = number of subassemblies per row

$$= 6(k - 1) \quad \text{for } k > 1.$$

k = general row number.

The set of relationships in Appendix B is obviously limited and describes only specific loads at arbitrary constraint-point displacements. Once these initial loads are determined, block 2 terminates the iteration process by computing: (a) new constraint displacements, if necessary, and (b) the necessary loads with a matrix inversion solution. (See program listing, Appendix F.)

APPENDIX D

Compression-spring Constants for Hex Cans

1. Assumptions

Under the assumption of θ symmetry, it can be shown that contact between rows j and $j + 1$ is physically equivalent to the mutual, simultaneous contact of $(12j - 6)$ outward faces of row- j hexagons with the same number of inward faces of row $(j + 1)$ hexagons. (At node LI, this simultaneous contact is accomplished in the EBR-II core by the contacting of buttons formed on the faces rather than contacting of the subassembly faces themselves.)

For simplification, BOW-V also assumes that each face or button of a given hex can is loaded equally, as shown in Fig. 19. Hence, all faces of any given hexagon are equally compressed, and such compression is assumed to be perfectly elastic.

2. Derivation of Individual Hex-face Rigidity

The hex-face rigidity of a typical EBR-II hex can was determined theoretically by making the following assumptions:

- (a) The sides of a given hex can are always loaded equally, regardless of position in the reactor
- (b) The length of the faces is infinitely long as compared to the width
- (c) The net clearance between fuel pins and spacers inside the hex can is sufficient to prevent any contribution to the total hex-face rigidity from these components
- (d) The button itself, if present, has no effect on the hex-face rigidity
- (e) The loads are concentrated over a small area on each face

The first assumption allows each face of the hex can to be considered individually. Since the hex can is loaded symmetrically, the corners do not rotate and the edges may be considered clamped. Since the length-to-width ratio of the flat plate is essentially infinite, we have the following expression⁴ for button compressibility:

Fig. 19. Hex-can Compression-loading Model

$$\frac{\Delta y}{\Delta Q} = \frac{(0.0791)b^2}{Et^3} . \quad (55)$$

For convenience, we redefine button compressibility, $\frac{\Delta y}{\Delta Q}$, as F_{1k} .

The sample problem discussed in Appendix E assumes a rigidity at the intermediate (button) level different from that at the top level. Because EBR-II subassemblies have raised buttons at the intermediate level, assumption e and F_{1k} above should be valid. The top level, however, has no buttons, and an arbitrary, small value ($F_{2k} = 10^{-8}$ in./lb) was chosen to approximate zero compressibility.

3. Derivation of Compression Constants by Row

Consider a node to be compressed by a total distance R_j at each pair of buttons for rows j and $j + 1$. For any row k , the quantity F_k specifies the compression displacement caused per pound of applied force on one button of row k . Further, the individual compression displacements r_j and r_{j+1} must result in equal forces upon the buttons of rows j and $j + 1$. Hence, we can write:

$$\begin{aligned} R_j &= \text{amount of interference between rows } j \text{ and } j + 1. \\ \therefore R_j &= r_j + r_{j+1}, \text{ and} \\ \frac{1}{F_j} \cdot r_j &= \frac{1}{F_{j+1}} \cdot r_{j+1} , \end{aligned} \quad (56)$$

which yields

$$r_j = R_j \left(\frac{F_j}{F_j + F_{j+1}} \right) , \text{ and } r_{j+1} = R_j \left(\frac{F_{j+1}}{F_j + F_{j+1}} \right) . \quad (57)$$

Thus, the compressive load Q_j^c between any two rows j and $j + 1$ can be written in general:

$$Q_j^c = \left(\frac{12j-6}{F_j} \right) \left(\frac{R_j F_j}{F_j + F_{j+1}} \right) = \kappa_j R_j . \quad (58)$$

The resultant (total) compressive load Q_j^c on a given row j can then be expressed:

$$Q_j^c = Q_j^c - Q_{j-1}^c . \quad (59)$$

The constant κ_j that relates total exerted load to compression displacement for rows j and $j + 1$ is defined more specifically as follows:

$$\kappa_{1j} = \frac{12j - 6}{F1_j + F1_{j+1}} , \text{ Intermediate level (Node LI) } , \quad (60a)$$

and

$$\kappa_{2j} = \frac{12j - 6}{F1_j + F2_{j+1}} , \text{ Top level (Node MI) } . \quad (60b)$$

$F1_k$ and $F2_k$ are the compression per pound of each subassembly face at nodes LI and MI, respectively, of a general row k .

APPENDIX E

BOW-V Sample Problem

1. Introduction

The bowing problem described here was first studied extensively on EBR-II for the reactor core configuration⁵ in run 25. The EBR-II reactor has been described elsewhere.^{6,7} Although many core changes have been made since 1967, the bowing effects associated with the run-25 core have provided a practical problem to be solved. Moreover, the subject continues to be of interest today in EBR-II, because some bowing effects are normally present and cause significant reactivity feedback effects.

2. Input

The BOW-V program uses any consistent set of units, and the sample problem is described in the inch-pound-°F system. The reference power level of the reactor was 45 MWt, which corresponds to a P_0 (normalized) of 1.0. The basic input is printed out as page 1 of the output.

For EBR-II run 25,

$$b = 1.322 \text{ in.}$$

$$E = 25 \times 10^6 \text{ lb/in.}^2$$

$$t = 0.040 \text{ in.}$$

$$\text{Thus, } F_{1k} = 0.865 \times 10^{-4} \text{ in./lb (per button).}$$

3. Output

Typical output quantities are printed out for reactor power levels (P_0) from 0.5 to 1.0, in increments of 0.5. The converged configurations of row centerlines are shown also in Fig. 20. In this sample problem, rows 1 through 6 and nodes 4 through 6 define the boundaries of the driver core. The intermediate button level corresponds to node 5.

The output for EBR-II run 25 is reproduced below.

BCW-V INPLT INVENTORY

MJ=15 MI=10 LI= 5 MT=20 NCB= 0

P0= 0.500 PMAX= 1.000 PDEL= 0.500 WEPS= 1.00000-001 PEPS= 1.00000-001

TL= 65.0000 D= 3.5200 YR= 0.0300 H8= 0.0040 H9= 10.0000 ALPHA= 1.00000-005

DELT

ROW/NODE	1	2	3	4	5	6	7	8	9	10
1	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000
2	-0.000+000	-0.000+000	-0.000+000	-0.000+000	9.000+000	2.300+001	1.700+001	-0.000+000	-0.000+000	-0.000+000
3	-0.000+000	-0.000+000	-0.000+000	-0.000+000	1.000+001	2.500+001	1.900+001	-0.000+000	-0.000+000	-0.000+000
4	-0.000+000	-0.000+000	-0.000+000	-0.000+000	1.200+001	2.900+001	2.200+001	-0.000+000	-0.000+000	-0.000+000
5	-0.000+000	-0.000+000	-0.000+000	-0.000+000	1.200+001	2.900+001	2.200+001	-0.000+000	-0.000+000	-0.000+000
6	-0.000+000	-0.000+000	-0.000+000	-0.000+000	1.000+001	2.400+001	1.800+001	-0.000+000	-0.000+000	-0.000+000
7	-0.000+000	-0.000+000	-0.000+000	4.000+000	2.000+001	4.800+001	7.400+001	9.000+001	9.200+001	9.500+001
8	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-1.500+001	-3.500+001	-5.300+001	-6.600+001	-6.600+001	-6.600+001
9	-0.000+000	-0.000+000	3.000+000	9.000+000	1.000+001	7.000+000	-3.000+000	-1.400+001	-1.800+001	-1.800+001
10	-0.000+000	-0.000+000	4.000+000	9.000+000	3.000+001	5.800+001	7.200+001	7.400+001	7.600+001	7.800+001
11	-0.000+000	-0.000+000	3.000+000	5.000+000	2.000+001	4.400+001	5.200+001	4.500+001	3.800+001	3.200+001
12	-0.000+000	-0.000+000	-0.000+000	-0.000+000	8.000+000	2.000+001	2.500+001	2.300+001	2.000+001	1.800+001
13	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.300+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000
14	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000
15	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000	-0.000+000

ROW	YP	D1	DD1	T	B4	B6	F1	F2
1	-0.000000	2.191000	2.218000	7.9000+001	2.4000-001	1.6650+007	7.2000-005	1.0000-008
2	2.220000	2.014000	2.038000	7.9000+001	2.4000-001	1.6650+007	7.2000-005	1.0000-008
3	4.260000	2.073000	2.098000	8.1000+001	2.4000-001	1.6650+007	7.2000-005	1.0000-008
4	6.360000	2.073000	2.098000	9.5000+001	2.4000-001	1.6650+007	7.2000-005	1.0000-008
5	8.460000	2.083000	2.108000	9.4000+001	2.4000-001	1.6650+007	7.2000-005	1.0000-008
6	10.570000	2.083000	2.108000	7.9000+001	2.4000-001	1.6650+007	7.2000-005	1.0000-008
7	12.680000	2.073000	2.098000	6.2000+001	2.4000-001	1.6650+007	7.2000-005	1.0000-008
8	14.780000	2.083000	2.108000	3.7000+001	2.4000-001	1.6650+007	7.2000-005	1.0000-008
9	16.890000	2.073000	2.098000	6.7000+001	2.4000-001	1.6650+007	7.2000-005	1.0000-008
10	18.990000	2.083000	2.108000	5.0000+001	2.4000-001	1.6650+007	7.2000-005	1.0000-008
11	21.100000	2.093000	2.118000	-0.0000+000	2.4000-001	1.6650+007	7.2000-005	1.0000-008
12	23.220000	2.093000	2.118000	-0.0000+000	2.4000-001	1.6650+007	7.2000-005	1.0000-008
13	25.340000	2.083000	2.108000	-0.0000+000	2.4000-001	1.6650+007	7.2000-005	1.0000-008
14	27.450000	2.073000	2.098000	-0.0000+000	2.4000-001	1.6650+007	7.2000-005	1.0000-008
15	29.550000	-0.000000	-0.000000	-0.0000+000	2.4000-001	1.6650+007	7.2000-005	1.0000-008

FRACTIONAL POWER= 0,50000

ROW--	1	2	3	4	5	6	7	8	9	10
DD	2,218876	2,036815	2,098923	2,098991	2,108912	2,108743	2,098519	2,108548	2,098614	2,108264
YT--										
I=10	0,000+000	1,128-002	1,242-002	1,452-002	1,452-002	1,200-002	4,957-002	-3,499-002	5,701+003	5,504-002
I= 9	0,000+000	8,342-003	9,182-003	1,074-002	1,074-002	8,882-003	2,985-002	-2,088-002	6,051+003	3,563-002
I= 8	0,000+000	5,359-003	5,894-003	6,907-003	6,907-003	5,716-003	1,570-002	-1,078-002	5,354+003	2,079-002
I= 7	0,000+000	2,616-003	2,876-003	3,383-003	3,383-003	2,806-003	6,807-003	-4,476-003	3,906+003	1,042-002
I= 6	0,000+000	8,052-004	8,877-004	1,053-003	1,053-003	8,752-004	2,231-003	-1,300-003	2,283+003	4,251-003
I= 5	0,000+000	1,125-004	1,250-004	1,500-004	1,500-004	1,250-004	4,801-004	-1,875-004	1,003+003	1,373-003
I= 4	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	5,001+005	0,000+000	2,851+004	3,426-004
I= 3	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	3,751+005	5,001-005
I= 2	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000
I= 1	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000

ROW--	11	12	13	14	15
DD	2,116000	2,116000	2,108000	2,098000	-0,000000
YT--					
I=10	3,697-002	1,584-002	0,000+000	0,000+000	0,000+000
I= 9	2,461-002	1,009-002	0,000+000	0,000+000	0,000+000
I= 8	1,454-002	5,566-003	0,000+000	0,000+000	0,000+000
I= 7	7,202-003	2,423-003	0,000+000	0,000+000	0,000+000
I= 6	2,841-003	7,102-004	0,000+000	0,000+000	0,000+000
I= 5	8,977-004	1,000-004	0,000+000	0,000+000	0,000+000
I= 4	2,351-004	0,000+000	0,000+000	0,000+000	0,000+000
I= 3	3,751-005	0,000+000	0,000+000	0,000+000	0,000+000
I= 2	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000
I= 1	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000

CONVERGED VALUFS, FRACTIONAL POWER= 0.50000

ITERATION NUMBER 6

Row--	1	2	3	4	5	6	7	8	9	10
Y--										
I=10	0.000000	2.226485	4.264456	6.363888	8.461539	10.566633	12.707991	14.780991	16.923287	19.068882
I= 9	0.000000	2.224024	4.262012	6.361171	8.459056	10.565049	12.690842	14.791052	16.920856	19.047093
I= 8	0.000000	2.221521	4.259520	6.358398	8.456519	10.563421	12.679229	14.797148	16.917376	19.029865
I= 7	0.000000	2.219257	4.257299	6.355939	8.454294	10.562047	12.672800	14.799490	16.913147	19.017108
I= 6	0.000000	2.217927	4.256108	6.354671	8.453262	10.561653	12.670592	14.798794	16.908743	19.008557
I= 5	0.000000	2.217714	4.256142	6.354832	8.453658	10.562440	12.671078	14.796150	16.904681	19.003295
I= 4	0.000000	2.218081	4.256813	6.355746	8.454806	10.563852	12.672728	14.792721	16.901184	18.999880
I= 3	0.000000	2.218561	4.257610	6.356809	8.456105	10.565389	12.674625	14.789263	16.898167	18.997203
I= 2	0.000000	2.219041	4.258407	6.357873	8.457403	10.566926	12.676472	14.785979	16.895383	18.994769
I= 1	0.000000	2.219520	4.259203	6.358936	8.458702	10.568463	12.678253	14.782886	16.892670	18.992384

MODE	0	-1	-1	-1	-1	-1	4	-30	2	1
YF	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	12.707991	14.780991	0.000000	0.000000
YL	0.000000	2.217714	4.256142	6.354832	8.453658	10.562440	12.671078	14.796150	16.904681	19.003295
P	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	2.4213+001	-2.4213+001	0.0000+000	0.0000+000
PSQ	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	2.4213+001	-2.4213+001	0.0000+000	0.0000+000
OM1	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
W	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	-4.8427+001	1.0669+001	-1.0669+001	0.0000+000
WSQ	0.0000+000	-3.3809-020	8.6579-020	-1.9380-019	2.8982-019	-2.6195-020	-4.8427+001	1.0669+001	1.0669+001	0.0000+000
OM	1.0	1.0	1.0	1.0	1.0	1.0	0.0	1.0	1.0	1.0

Row--	11	12	13	14	15
Y--					
I=10	21.158295	23.254755	25.355116	27.461116	29.557116
I= 9	21.143797	23.247119	25.353605	27.460005	29.556405
I= 8	21.131593	23.240699	25.352093	27.458893	29.555693
I= 7	21.122126	23.235664	25.350581	27.457781	29.554981
I= 6	21.115633	23.232060	25.349070	27.456670	29.554270
I= 5	21.111558	23.229558	25.347558	27.455558	29.553558
I= 4	21.108763	23.227566	25.346047	27.454447	29.552847
I= 3	21.106434	23.225675	25.344535	27.453335	29.552135
I= 2	21.104264	23.223783	25.343023	27.452223	29.551423
I= 1	21.102132	23.221892	25.341512	27.451112	29.550712

MODE	1	1	1	1	1
YF	0.000000	0.000000	0.000000	0.000000	0.000000
YL	21.111558	23.229558	25.347558	27.455558	29.553558
P	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000
PSQ	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000
OM1	0.0	0.0	0.0	0.0	0.0
W	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000
WSQ	0.0000+000	0.0000+000	0.0000+000	0.0000+000	-0.0000+000
OM	1.0	1.0	1.0	1.0	0.0

FRACTIONAL POWER= 1.00000

ROW--	1	2	3	4	5	6	7	8	9	10
DD	2,219752	2,039630	2,099846	2,099983	2,109823	2,109486	2,099039	2,109096	2,099227	2,108527
YT--										
I=10	0,000+000	2,257-002	2,485-002	2,905-002	2,905-002	2,401-002	9,914-002	-6,998-002	1,140+002	1,101-001
I= 9	0,000+000	1,668-002	1,836-002	2,149-002	2,149-002	1,776-002	5,970-002	-4,177-002	1,210+002	7,127-002
I= 8	0,000+000	1,072-002	1,179-002	1,381-002	1,381-002	1,143-002	3,141-002	-2,155-002	1,071+002	4,158-002
I= 7	0,000+000	5,231-003	5,751-003	6,767-003	6,767-003	5,611-003	1,361+002	-8,952-003	7,812-003	2,083-002
I= 6	0,000+000	1,610-003	1,775-003	2,105-003	2,105-003	1,750-003	4,461+003	-2,601-003	4,566-003	8,502-003
I= 5	0,000+000	2,251-004	2,501-004	3,001-004	3,001-004	2,501-004	9,602-004	-3,751-004	2,005+003	2,746-003
I= 4	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	1,000-004	0,000+000	5,701-004	6,852-004
I= 3	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	7,502+005	1,000-004
I= 2	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000
I= 1	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000
ROW--	11	12	13	14	15					
DD	2,118000	2,116000	2,108000	2,098000	-0,000000					
YT--										
I=10	7,395-002	3,168-002	0,000+000	0,000+000	0,000+000					
I= 9	4,922-002	2,019-002	0,000+000	0,000+000	0,000+000					
I= 8	2,907-002	1,113-002	0,000+000	0,000+000	0,000+000					
I= 7	1,440-002	4,846-003	0,000+000	0,000+000	0,000+000					
I= 6	5,681-003	1,420-003	0,000+000	0,000+000	0,000+000					
I= 5	1,795-003	2,000-004	0,000+000	0,000+000	0,000+000					
I= 4	4,701-004	0,000+000	0,000+000	0,000+000	0,000+000					
I= 3	7,502-005	0,000+000	0,000+000	0,000+000	0,000+000					
I= 2	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000					
I= 1	0,000+000	0,000+000	0,000+000	0,000+000	0,000+000					

CONVERGED VALUES, FRACTIONAL POWER= 1.00000

ITERATION NUMBER 3

ROW--	1	2	3	4	5	6	7	8	9	10
Y--										
I=10	0.000000	2.221938	4.256535	6.355615	8.452106	10.553843	12.705220	14.778220	16.935799	19.117852
I= 9	0.000000	2.218120	4.252918	6.351504	8.448397	10.551822	12.677760	14.795663	16.932938	19.078977
I= 8	0.000000	2.214216	4.249206	6.347283	8.444577	10.549712	12.661069	14.805435	16.927982	19.049073
I= 7	0.000000	2.210792	4.246034	6.343687	8.441383	10.548111	12.654140	14.808219	16.921525	19.027809
I= 6	0.000000	2.209234	4.244923	6.342476	8.440574	10.548471	12.654745	14.805707	16.914719	19.014502
I= 5	0.000000	2.209912	4.246262	6.344121	8.442620	10.551192	12.659524	14.800337	16.908597	19.007167
I= 4	0.000000	2.211749	4.248873	6.347263	8.446160	10.555146	12.665216	14.794597	16.903610	19.002833
I= 3	0.000000	2.213812	4.251719	6.350654	8.449923	10.559253	12.670178	14.789767	16.899622	18.999444
I= 2	0.000000	2.215875	4.254533	6.353941	8.453535	10.563163	12.674108	14.785923	16.896171	18.996234
I= 1	0.000000	2.217937	4.257298	6.357074	8.456919	10.566778	12.677263	14.782767	16.892969	18.993044
MODE	0	-1	-2	-2	-2	-2	30	-30	2	5
YF	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	12.705220	14.778220	0.000000	19.117852
YL	0.000000	2.209912	4.246262	6.344121	8.442620	10.551192	12.659524	14.800337	16.908597	19.007167
P	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	2.6887+002	-2.6887+002	0.0000+000	1.6515+002
PSQ	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	2.6887+002	-2.6887+002	0.0000+000	1.6515+002
OM1	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0
W	0.0000+000	0.0000+000	3.8867+000	1.8752+001	3.6807+001	5.9562+001	-5.2904+002	5.2242+002	-5.6496+001	4.1123+002
WSQ	0.0000+000	-2.2473+020	3.8867+000	1.8752+001	3.6807+001	5.9562+001	-5.2904+002	5.2242+002	-5.6496+001	4.1123+002
OM	1.0	1.0	1.0	1.0	1.0	1.0	0.0	-1.0	1.0	1.0

ROW--	11	12	13	14	15
Y--					
I=10	21.200852	23.293852	25.386852	27.485033	29.580000
I= 9	21.173560	23.277501	25.380930	27.481399	29.576974
I= 8	21.150841	23.263620	25.375096	27.477765	29.573945
I= 7	21.133571	23.252588	25.369440	27.474131	29.570909
I= 6	21.122207	23.244534	25.364051	27.470498	29.567865
I= 5	21.115624	23.236842	25.359018	27.466864	29.564809
I= 4	21.111542	23.234365	25.354411	27.463240	29.561744
I= 3	21.108355	23.233324	25.350227	27.459680	29.558697
I= 2	21.105482	23.226557	25.346444	27.456244	29.555702
I= 1	21.102712	23.223102	25.343043	27.452997	29.552792
MODE	5	2	-30	2	5
YF	21.200852	23.293852	25.386852	0.000000	29.580000
YL	21.115624	0.000000	25.359018	27.466864	29.564809
P	1.6424+001	-5.2195+001	-1.2937+002	0.0000+000	5.0915+000
PSQ	1.6424+001	-5.2195+001	-1.2937+002	0.0000+000	5.0915+000
OM1	1.0	1.0	0.0	0.0	1.0
W	-5.4697+001	0.0000+000	1.6046+002	-9.8765+001	-6.1698+001
WSQ	-5.4697+001	0.0000+000	1.6046+002	-9.8765+001	-6.1698+001
OM	0.0	0.0	1.0	1.0	0.0

Fig. 20. Equilibrium Configuration of Subassemblies Distorted by Radial Temperature Gradients in an EBR-II Stainless-steel-reflected Core

APPENDIX F

BOW-V Program Listing

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PROGRAM BOW V
  DIMENSION A6(20),A7(20),AY(20,20),B1(20),B3(20),B35(20),
1 B35L(20),B3L(20),B36(20),B37(20),B38(20),B39(20),B4(20),B40(20),
2 B4L(20),B5(20),B6(20),BC1(20),BC2(20),BC3(20),BC4(20),CAP(20),
3 D0(20),D1(20),DD(20),DD1(20),DELT(20,20),F1(20),
4 GL(20),HL(20),LU1(10),LU2(10),MODE(20),NF(20),
5 OM(20),OM1(20),P0(20),PL(20),PLP(20),PR(20),PRP(20),Q(30,30),
6 T(20),V(30),W0(20),WL(20),WR(20),XDLT(43,20),XX(20),YF(20),
7 YI(20,20),YL(20),YP(20),YPT(20,20),YT(20,20),BII(20),BIIT(20),
8 WSQ(20),P(20),D01(20),YN(20),YN0(20),PSQ(20),F2(20),CAP1(20)
  COMMON/402/INDEX(100)
  TYPE DOUBLE A6,A7,AY,B35,B35L,B36,B37,B38,B39,B3L,B40,B4L,B6,
1 BC1,BC2,BC3,BC4,BII,BIIT,C1,C2,C3,C3M,C4,C4M,CAP,CAP1,CL,D0,D01,
2 DF1,DF2,P,P0,PL,PR,PSQ,Q,V,W0,WA,WB,WL,WR,WSQ,YF,YL,YPT,YW,DD,D1,
3 DF0,R,YT,DL,DL1,DR,DR1,C,D1X,DF3,YN,YN1
  READ1000,M1,MJ,LI,QQ,QD,QT,TL,PEPS,WEPS
  READ 5041,MT,NDB,B8,B9,KP,JP1,JP2,IP1,IP2
5041 FORMAT(2I6,2E10,4,I4,4I3)
  READ 2000,ALPHA,D,YR
  READ2000,((DELT(I,J),I=1,MJ),J=1,MJ)
  READ2000,(YP(J),J=1,MJ)
  READ2000,(D1(J),J=1,MJ)
  READ2000,(B1(J),J=1,MJ)
  READ2000,(B4(J),J=1,MJ)
  READ2000,(B6(J),J=1,MJ)
  READ2000,(DD1(J),J=1,MJ)
  READ2000,(T(J),J=1,MJ)
  READ2000,(F1(J),J=1,MJ)
  READ 2000,(F2(J),J=1,MJ)
1000 FORMAT(3I6,6E10,4)
2000 FORMAT(8E10,4)
  PRINT 6000
6000 FORMAT(1H148X,*BOW-V INPUT INVENTORY*//)
  PRINT 6010,MJ,M1,LI,MT,NDB
6010 FORMAT(* MJ=*I2,5X,*M1=*I2,5X,*LI=*I2,5X,*MT=*I2,5X,*NDB=*I2/)
  PRINT 6015,QQ,QT,QD,WEPS,PEPS
6015 FORMAT(* P0=*F7,3,10X,*PMA=*F7,3,8X,*PDEL=*F7,3,8X,*WEPS=*
1 E12,5,3X,*PEPS=*E12,5/)
  PRINT 6020,TL,D,YR,B8,B9,ALPHA
6020 FORMAT(* TL=*F8,4,9X,*D=*F8,4,10X,*YR=*F8,4,9X,*B8=*F8,4,9X,
1 *B9=*F8,4,9X,*ALPHA=*E12,5/)
  DO 6025 I=1,M1,10
  I1=I+9 $ IF(I1.GT.M1) I1=M1
  PRINT 6030,(J,J=1,I1)
6030 FORMAT(/5H DELT//9H ROW/NODE1X,10(5X,I2,4X))
  DO 6025 J=1,MJ
6025 PRINT 6040,J,(DELT(K,J),K=1,I1)
6040 FORMAT(I3,9X,10(E10,3,1X))
  PRINT 6050
6050 FORMAT(/4X,3HPOW8X,2HYP10X,2HD1,10X,3HDD110X,1HT11X,2HB410X,
1 2HB610X,2HF110X,2HF2/)
  DO 6055 I=1,MJ
6055 PRINT 6060,I,YP(I),D1(I),DD1(I),T(I),B4(I),B6(I),F1(I),F2(I)
6060 FORMAT(I6,4X,3F12,6,6(1X,E11,4))
  QQ=QQ
  NU=0 $ MK=MJ-1 $ XM1=M1 $ XLI=LI

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INPUT

DEFINITIONS OF CONSTANTS
(INPUT AND POWER INDEPENDENT)

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      DELX=TL/XMI $ XLM=XLI/XMI
      A4=XLI*DELX
      A42=A4**2
      A2=TL-A4
      A5=A2**2
      B2=B8*TL/B9
      E2=TL/(A42*A5)
      A15=XMI-XLI
      A20=2,*XMI-XLI
      DO240 I=1,MI
      XI=I
      XX(I)=(1.5*(XMI/XI)-.5)*((XI/XMI)**2)
      A1=XI*DELX
      A3=TL-A1
      A32=A3**2
      A8=2,*TL
      A6(I)= A1*(A8*A3-A5-A32)/(2,*A42*A2)
      GL(I)=((XI/XLI)**2)*(1.5*(XLI/XI)-.5)
      HL(I)=(XI-XLI)/XLI
      B36(I)=XI*(XMI-XI)*(2,*XMI*XI)
      IF(I,GT,LI) GO TO 239
      B39(I)=XI*(XI**2-3,*XLI*XI+XLI**2*(3,-XLM))
      GO TO 240
239  B39(I)=XLI**3*(XMI-XI)/XMI
240  A7(I)= A3*(A8*A2-A5-A32)/(2,*A4*A5)
      DO74 J=1,MJ
      YN0(J)=0.
      CAP(J)=(12.*J-6,)/(F1(J)+F1(J+1))
      CAP1(J)=(12,*J-6,)/(F2(J)+F2(J+1))
      B5(J)=B6(J)*6,*(J=1)/TL**3
      B6(J)=6,*(J-1)*B6(J)*E2
      B3(J)=B4(J)+1.
      B35(J)=B5(J)/B3(J)
      B4L(J)=B4(J)/XLM
      B3L(J)=B4L(J)+1.
      B5L=B5(J)/XLM**3
      B35L(J)=B5L/B3L(J)
      B11T(J)=(B4(J)+XX(LI))*XLM/B3(J)
      B11(J)=1./(1.+(B4L(J)+1.5)*HL(MI)/B3L(J))
      B37(J)=A20*A15*.5/(B3(J)*XMI**2) -1,
      B36(J)=2.*B6(J)*A15/(A20*B37(J)*XLI)
      B37(J)=B37(J)*XLM
74  B40(J)=XLI*A15*(B37(J)*A20*XLI*XLM)
      CAP1(MJ)=(12.*MJ-6,)/F2(MJ)
      DO80 J=1,MJ
      DO80 I=1,MI
      XI=I
80  YI(I,J)=YP(J)+B1(J)*DELX*XI
      GO TO 60
304  QQ=QQ+QN
      Q=(QQ-Q0)/(QT-Q0)
      IF(Q,LE.1,) GO TO 60
61  STOP
60  T(NJ+1)=0.
      DO 50 J=1,MJ
      DD(J)=DD1(J)*(1.+0.5*ALPHA*(T(J)+T(J+1))*QQ)

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INITIALIZATION OF POWER-DEPENDENT CONSTANTS

```

      XDLT(1,J)=XDLT(3,J)=0.
      DO50 I=1,M1
      XDLT(2*I+3,J)=QQ*DELT(I,J)
50   XDLT(2*I+2,J)=-((1./6,)*XDLT(2*I+1,J)+(3./4,)*XDLT(2*I+1,J)+(3./8,)*
      1 *XDLT(2*I+3,J)
      M=0
      IF(QQ.EQ.Q0) GO TO 14
C     DEFINE NOZZLE POSITIONS
      DO 17 I=2,MJ
      KU=XABSF(MODE(I)) $ S=1,
      IF(KU.GE.2) GO TO 19
      IF(P0(I).EQ.0,) GO TO 18
      IF(P0(I).LT.0,) S=-S $ GO TO 552
18   YN0(I)=B9*(YP(I)+YT(MI,I)-AY(MI,I))/TL
      GO TO 17
19   IF(KU.NE.4) GO TO 20
      DF1=YF(I)-YP(I) $ DF2=A20*(YP(I)-YL(I))
      YN0(I)=.5*B9*(XLM*DF1+DF2/XLI)/A2 $ GO TO 17
20   IF(KU.EQ.30) S=-S
      IF(W0(I).LT.0,) S=-S
552  YN0(I)=S*B8
17   CONTINUE
14   DO 15 J=1,MJ
      SUM1=SUM2=0.
      DO 15 I=1,M1
      TERM1=(ALPHA/D)*(DELX/6,)*(XDLT(2*I+1,J)+4,*XDLT(2*I+2,J)+XDLT(2*I
      1 +3,J))
      SUM1=SUM1+TERM1
      TERM2=DELX*(SUM1-TERM1/2,)
      SUM2=SUM2+TERM2
15   YT(I,J)=SUM2
      DO635 J=1,MJ
      DO635 I=1,M1
635  YPT(I,J)=YI(I,J)+YT(I,J)
      PRINT 6100,Q0
6100  FORMAT(1H146X*FRACTIONAL POWER=*F8,5//)
      DO 6105 I=1,MJ,10
      I1=I+9 $ IF(I1.GT,MJ) I1=MJ
      PRINT 6110,(J,J=I,I1)
6110  FORMAT(/6X,*ROW--*4X,10(I2,9X))
      PRINT 6120,(DD(J),J=I,I1)
6120  FORMAT(/6X,2HDD,2X,10F11,6)
      PRINT 6130
6130  FORMAT(5H YT--)
      DO 6105 J=1,M1
      J1=MI+1-J
6105  PRINT 6140,J1,(YT(J1,K),K=1,I1)
6140  FORMAT(6X,*I=*I2,10(1X,E10,3))
      GO TO 810
125  M=M+1
      IF(NDB.EQ.0) GO TO 103
      PRINT 6275,(YF(I),I=1,MJ)
      PRINT 6280,(YL(I),I=1,MJ)
C
C     FIT TOP-LEVEL CONSTRAINTS
C

```

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START OF NEXT ITERATION: RELATIONSHIP OF LOADS TO CONSTRAINTS
(Estimate is defined for first power level and for succeeding power levels causing no interaction of rows)

```

103 DO 607 J=1,MJ
    PR(J)=PL(J)=0.
    BC1(J)=BC2(J)=BC3(J)=BC4(J)=0.
    IF(YF(J),EQ,0.) GO TO 609
    IF(YPT(MI,J),NE,YF(J)) GO TO 611
609 DO 610 I=1,MI
    AY(I,J)=YPT(I,J)
610 GO TO 616
611 CL=YF(J)-YPT(MI,J)
    SGN=1.
    IF(CL,LT,0.) SGN=-1.
    B2S=62*SGN*TL*B1(J)

C
C
C
    MODE 1

    C1=CL $ ASSIGN 618 TO M1 $ V(J)=CL*B9/TL
    IF(ABSF(CL),LE,B2S) GO TO 614
    C1=SGN*B2S $ ASSIGN 616 TO M1
614 DO 615 I=1,MI
    XI=I
615 AY(I,J)=YPT(I,J)+C1*XI/XMI
    C2=CL-C1
    GO TO M1

C
C
C
    MODE 2

616 BC4(J)=B35(J)
    DO 617 I=1,MI
    XI=I
617 AY(I,J)=AY(I,J)+C2*(B4(J)+XX(I))*XI/(XMI*B3(J))
    P=C2*B35(J)
    IF(SGN,LT,0.) PR(J)=P
    IF(SGN,GT,0.) PL(J)=P
618 PLP(J)=PL(J)
    PRP(J)=PR(J)
607 CONTINUE
    DO241 J=1,MJ
    WL(J)=0.
241 WR(J)=0.

C
C
C
    SUPERIMPOSE BUTTON-LEVEL CONSTRAINTS

    DO 900 J=1,MJ
    MODE(J)=0
    IF(YL(J),EQ,0.) GO TO 900
    IF(AY(LI,J),EQ,YL(J)) GO TO 900
    CL=YL(J)-AY(LI,J) $ SGN=1. $ IF(CL,LE,0.) SGN=-1.
    CHOOSE INITIAL MODE
C
904 IF(YF(J),NE,0.) GO TO 908
C
C
C
    MODE 1

    MODE(J)=1
    BC3(J)=BC4(J)=0.
    DS=44/B9*(SGN*B8+B9*(4,*AY(1,J)-AY(2,J)-3,*YP(J))/(2,*DELX))
    C1=DS

```

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```

        ASSIGN 916 TO M1
        IF(ABSF(C1).LT.ABSF(CL)) GO TO 912
        C1=CL $ ASSIGN 900 TO M1
912     DO 907 I=1,M1
        XI=I
907     AY(I,J)=AY(I,J)+C1*XI/XLI
        IF(C1.LT.0.) MODE(J)=-1
        CL=CL-C1
        GO TO M1

C
C       MODES 3 AND 3A
C
908     BC1(J)=-B36(J)
        BC2(J)=-B36(J)*BIIT(J)
        BC3(J)= B36(J)*B37(J)
        BC4(J)= B35(J)+BC3(J)*BIIT(J)
        IF(PLP(J).EQ.0.) GO TO 909
        PP=PLP(J)
        IF(CL) 911,900,910
909     PP=PRP(J)
        IF(PP.NE.0.) GO TO 906
        DNOZ=V(J)-SGN*B8 $ GO TO 922
906     IF(CL) 910,900,911
C       MODE 3
910     MODE(J)=3
        C3M=-PP/(B36(J)*B37(J))
        IF(ABSF(C3M).GE.ABSF(CL)) GO TO 914
        C3=C3M
        ASSIGN 929 TO M3
        GO TO 915
C       MODE 3A
911     MODE(J)=30
        C3M=-PP/(B36(J)*(B37(J)+XLM))
        IF(ABSF(C3M).GE.ABSF(CL)) GO TO 914
        C3=C3M
        ASSIGN 922 TO M3
        DNOZ=-SGN*2.*B8
        GO TO 915
C       MODES 3,3A
914     C3=CL
        ASSIGN 900 TO M3
915     DO 932 I=1,M1
932     AY(I,J)=AY(I,J)+C3*(B37(J)+B38(I)+B39(I))/B40(J)
        WB=C3*B36(J)
        WA=WB*B37(J)
        IF(CL.LT.0.) WR(J)=WB
        IF(CL.GT.0.) WL(J)=WB
        IF(PP.LT.0.) PL(J)=PL(J)+WA
        IF(PP.GT.0.) PR(J)=PR(J)+WA
        IF(C3.LT.0.) MODE(J)=-MODE(J)
        CL=CL-C3
        GO TO M3

C
C       MODE 2
C
916     BC1(J)=B35L(J)

```

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```

      BC2(J)=0,
      MODE(J)=2
      C2=CL
      DO 919 I=1,MI
      XI=I
      IF(I.GT.LI) GO TO 920
      YW=C2*(B4L(J)+GL(I))*XI/(XLI*B3L(J))
      GO TO 919
920  YW=C2*(1.+(1.5+B4L(J))*HL(I)/B3L(J))
919  AY(I,J)=AY(I,J)+YW
      WB=B35L(J)*C2
      IF(CL,LT,0.) WR(J)=WR(J)+WB
      IF(CL,GT,0.) WL(J)=WL(J)+WB
      IF(C2,LT,0.) MODE(J)=-2
      GO TO 900

C
C      MODE 4
C
922  BC1(J)=B6(J)
      BC2(J)=XLM*B6(J)
      BC3(J)= XLM*BC1(J)
      BC4(J)= XLM*BC2(J)
      MODE(J)=4
      C4M=-DNOZ*2. *XLI*A2/(B9*A20)
      IF(ABSF(C4M),GE,ABSF(CL)) GO TO 923
      C4=C4M
      ASSIGN 929 TO M4
      GO TO 924
923  C4=CL
      ASSIGN 900 TO M4
924  DO 926 I=1,MI
      IF(I,GT,LI) GO TO 925
      YW=C4*A6(I)
      GO TO 926
925  YW=C4*A7(I)
926  AY(I,J)=AY(I,J)+YW
      WB=C4*B6(J)
      WA=WB*XLM
      IF(CL,LT,0.) GO TO 927
      WL(J)=WL(J)+WB
      PR(J)=PR(J)+WA
      GO TO 928
927  WR(J)=WR(J)+WP
      PL(J)=PL(J)+WA
928  CL=CL-C4
      IF(C4,LT,0.) MODE(J)=-4
      GO TO M4

C
C      MODE 5
C
929  BC1(J)=-B36(J)
      BC2(J)=- (B36(J)+B35L(J))*B1I(J)
      BC3(J)= B36(J)*B37(J)
      BC4(J)=BC3(J)*B1I(J)
      MODF(J)=5
      WB=CL*B36(J)

```

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```

      WA=WB*B37(J)
      DO 933 I=1,M1
933  AY(I,J)=AY(I,J)+CL*(B37(J)*B38(I)+B39(I))/B40(J)
      IF(CL.LT.0.) GO TO 930
      WL(J)=WL(J)+WA
      PR(J)=PR(J)+WA
      GO TO 900
930  WR(J)=WR(J)+WB
      PL(J)=PL(J)+WA
      MODE(J)=-5
900  CONTINUE
      DO 901 J=1,MJ
      PO(J)=PL(J)+PR(J)
      WO(J)=WL(J)+WP(J)
      IF(YL(J).NE.0.) AY(LI,J)=YL(J)
      IF(YF(J).NE.0.) AY(MI,J)=YF(J)
901  CONTINUE
C      QUALITATIVE CONSTRAINT JUSTIFICATION
      K1=0
      DO 935 I=2,MJ
C      TOP=CONSTRAINT
      KU=XAHSF(MODE(I))
      DL=AY(MI,I)-AY(MI,I-1)-D1(I-1)
      DR=AY(MI,I+1)-AY(MI,I)-D1(I)
      IF(I.EQ.MJ) DR=YF(MJ)+YR-AY(MI,MJ)
      DL1=AY(LI,I)-AY(LI,I-1)-DD(I-1)
      DR1=AY(LI,I+1)-AY(LI,I)-DD(I)
      IF(I.EQ.MJ) DR1=YF(MJ)+YR-AY(LI,MJ)
      IF(DL.LT.1.E-6) DL=0.
      IF(DR.LT.1.E-6) DR=0.
      IF(DL1.LT.1.E-6) DL1=0.
      IF(DR1.LT.1.E-6) DR1=0.
      IF(YF(I).EQ.0.) GO TO 939
      R=YF(I)+YT(MI,I)
      IF(ABSF(PO(I)).GT.1.E-3) GO TO 936
      IF(DL*DR.EQ.0.) GO TO 939
      K1=1
      IF(KU.EQ.0) GO TO 946
      YF(I)=0, $ GO TO 935
      IF MODE (i) ≠ IT, YF(i) ≠ 0 IS IMPOSSIBLE
946  C=DP $ C1=DR1
      IF(R.GT.AY(MI,I)) GO TO 952
      C=-DL $ C1=-DL1
952  IF(C1.EQ.0.) GO TO 954
      C1=C1/XLM $ R=R-AY(MI,I)
      IF(C1/C.LT.1.) C=C1
      IF(R/C.GT.1.) GO TO 953
      AY(MI,I)=YPT(MI,I) $ AY(LI,I)=YPT(LI,I)
      YF(I)=0, $ GO TO 935
      IF(C1.EQ.0.) GO TO 954
      AY(MI,I)=YF(I)+AY(MI,I)+C
      AY(LI,I)=AY(LI,I)+C*XLM
      IF(C/C1.LT.1.) GO TO 935
954  YF(I)=0, $ YL(I)=AY(LI,I)
      GO TO 935
936  C=DR $ IF(PO(I).LT.0.) C=-DL
      IF(C.EQ.0.) GO TO 939
      C1=PO(I)/RC4(I)

```

ROW-1 CONTACTS

MOVE ROW i TO POSITION OF CONTACT
OR TO VERTICAL POSITION. IF CONTACT
POINTS ARE CLOSER FOR NODE LI,
CHANGE MODE (i) TO 1.

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↑
ASSURANCE THAT IMPOSED (NON-ZERO) CONSTRAINTS ARE ASSOCIATED WITH ACTUAL CONTACT
↓

```

      IF(KU,NE,0) GO TO 948
      C2=DR1 $ IF(P0(I),LT,0.) C2=-DL1
      IF(C2,NE,0.) GO TO 957
      YL(I)=AY(LI,I) $ GO TO 948
957  C2=C2/BIIT(I) $ IF(C2/C,LT,1.) C=C2
      IF(C1/C,GT,1.) GO TO 951
      AY(MI,I)=YF(I)=AY(MI,I)+C1
      AY(LI,I)=AY(LI,I)+C1*BIIT(I)
      GO TO 935
951  AY(MI,I)=YF(I)=AY(MI,I)+C
      AY(LI,I)=AY(LI,I)+C*BIIT(I)
      IF(C2,EQ,C) YL(I)=AY(LI,I)
      GO TO 935
948  IF(C1/C,GT,1.) GO TO 937 ← MODES III, IIIT, IV. AND V
      YF(I)=0. $ GO TO 938
937  C1=C $ YF(I)=YF(I)+C1
938  AY(MI,I)=AY(MI,I)+C1
      K1=1 $ GO TO 935
C    BUTTON-CONSTRAINT
939  IF(YL(I)-EQ,0.) GO TO 935
      R=YF(I)+YT(LI,I)
      IF(ABSF(W0(I)),GT,1,E-3) GO TO 940
      IF(DL1*DR1,EQ,0.) GO TO 935
      K1=1
      IF(KU,LE,2) GO TO 947
      YL(I)=0. $ GO TO 935
947  C=DR1 $ C1=DR
      IF(R,GT,AY(LI,I)) GO TO 950
      C=-DL1 $ C1=-DL ← MODE I
950  R=R-AY(LI,I) $ C1=C1-XLM
      IF(C/C1,GT,1.) C=C1
      IF(C/R,LT,1.) GO TO 955
      AY(LI,I)=YPT(LI,I) $ AY(MI,I)=YPT(MI,I)
      YL(I)=0. $ GO TO 935
955  YL(I)=AY(LI,I)=AY(LI,I)+C
      AY(MI,I)=AY(MI,I)+C/XLM
      IF(C/C1,LT,1.) GO TO 935
      YL(I)=0. $ YF(I)=AY(MI,I)
      GO TO 935
940  C1=DR1 $ C2=DR
      IF(W0(I),GT,0.) GO TO 941
      C1=-DL1 $ C2=-DL
941  IF(C1/W0(I),LE,1,E-8) GO TO 935
      C=W0(I)/BC1(I) $ K1=1
      IF(KU,GT,2) GO TO 942
      C4=C2*BIIT(I) $ C3=C1
      IF(C4/C3,LT,1.) C3=C4 ← MODE II
      IF(C/C3,GT,1.) GO TO 956
      YL(I)=AY(LI,I)=AY(LI,I)+C
      AY(MI,I)=AY(MI,I)+C/BIIT(I)
      GO TO 935
956  AY(LI,I)=YL(I)=AY(LI,I)+C3
      AY(MI,I)=AY(MI,I)+C3/BIIT(I)
      IF(C3/C1,EQ,1.) GO TO 935
      YL(I)=0. $ YF(I)=AY(MI,I)
      GO TO 935

```

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MODES III, III_T, IV, V

```

942 C2=C $ IF(C/C1.GT,1,) C2=C1
    AY(LI,I)=YL(I)=AY(LI,I)+C2
    IF(C2.EQ,C) YL(I)=C,
935 CONTINUE
    IF(K1.EQ,0) GO TO 945
    IF(NDB,LT,1) GO TO 103
    PRINT 5121,(YF(J),J=1,MJ)
5121 FORMAT(4H YF*6X,10F11,5)
    PRINT 5122,(YL(J),J=1,MJ)
5122 FORMAT(4H YL*6X,10F11,5)
    GO TO 103
945 DO 10 I=1,MK
    OM(I)=0, $ OM1(I)=0,
    IF(AY(LI,I+1)-AY(LI,I) ,LE,DD(I)) OM(I)=1,
    IF(AY(MI,I+1)-AY(MI,I) ,LE,D1(I)) OM1(I)=1,
    D01(I)=(D1(I)-AY(MI,I+1)+AY(MI,I))*OM1(I)
    D0(I)=(DD(I)-AY(LI,I+1)+AY(LI,I))*OM(I)
10 CONTINUE
    OM1(MJ)=0,
    IF(AY(MI,MJ).GE,YP(MJ)+YR) OM1(MJ)=1,
    D01(MJ)=(AY(MI,MJ)-YP(MJ)-YR)*OM1(MJ)
    DO 902 I=2,MK
    PSQ(I)=D01(I)*CAP1(I)-D01(I-1)*CAP1(I-1)
902 WSQ(I)=D0(I)*CAP(I)-D0(I-1)*CAP(I-1)
    PSQ(MJ)=-D01(MJ-1)*CAP1(MJ-1)+D01(MJ)*CAP1(MJ)
    WSQ(MJ)=-D0(MJ-1)*CAP(MJ-1)

C
C
C CONVERGENCE TEST

    ASSIGN 304 TO MM
    DO 11 I=2,MJ
    IF(ABSF(W0(I)-WSQ(I)),GE,WEPS) GO TO 73
    IF(ABSF(P0(I)-PSQ(I)),GE,PEPS) GO TO 73
11 CONTINUE
    PRINT 6200,QQ
6200 FORMAT(1H137X,*CONVERGED VALUES, FRACTIONAL POWER=*F8,5/)
    IF(KP.EQ,0) GO TO 305
    DO 12 I=JP1,JP2
12 PUNCH 7000,(AY(J,I),J=IP1,IP2)
7000 FORMAT(6E12,5)
    GO TO 305
73 IF(M,LT,MT) GO TO 72
    PRINT 6210,QQ
6210 FORMAT(1H133X,*MAXIMUM ITERATION NUMBER, FRACTIONAL POWER=*F8,5/)
    GO TO 305
72 IF(NDR.EQ,0) GO TO 75
    ASSIGN 75 TO MM
    PRINT 6220,QQ
6220 FORMAT(1H141X,*DIAGNOSTIC PRINTOUT, POWER=*F8,5/)
305 PRINT 6240,M
6240 FORMAT(50X*ITERATION NUMBER*I3//)
    DO 6245 I=1,MJ,10
    I1=I+9 $ IF(I1.GT,MJ) I1=MJ
    PRINT 6110,(J,J=I,I1)
    PRINT 6250
6250 FORMAT(/* Y--*)

```

SUBASSEMBLY-
COMPRESSION
QUANTITIES

CONVERGENCE CHECK AND ITERATION PRINTOUT

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```

DO 6246 K=1,MI
J=MJ+1-K
6246 PRINT 6260,J,(AY(J,L),L=1,I1)
6260 FORMAT(3X,*I=*I2,3X,10F11,6)
DO 6265 L=1,I1
IF(XABSF(MODE(L)),GT,0) GO TO 6265
IF(YL(L),NE,0,) GO TO 6265
IF(YF(L),EQ,0,) GO TO 6265
IF(P0(L)) 6261,6263,6262
6261 MODE(L)=20 $ GO TO 6265
6262 MODE(L)=-20 $ GO TO 6265
6263 MODE(L)=-10
IF(YF(L),GT,YP(L)+YT(MJ,L)) MODE(L)=10
6265 CONTINUE
6269 PRINT 6270,(MODE(K),K=1,I1)
6270 FORMAT(/3X,*MODE*3X,10(4X,13,4X))
PRINT 6275,(YF(K),K=1,I1)
6275 FORMAT(4X*YF*4X,10F11,6)
PRINT 6280,(YL(K),K=1,I1)
6280 FORMAT(4X*YL*4X,10F11,6)
PRINT 6290,(P0(K),K=1,I1)
6290 FORMAT(5X,*P*4X,10E11,4)
PRINT 6291,(PSQ(K),K=1,I1)
6291 FORMAT(4X,*PSQ*3X,10E11,4)
PRINT 6292,(OM1(K),K=1,I1)
6292 FORMAT(4X,*OM1*3X,10(4X,F3,1,4X))
PRINT 6295,(W0(K),K=1,I1)
6295 FORMAT(5X,*W*4X,10E11,4)
PRINT 6296,(WSQ(K),K=1,I1)
6296 FORMAT(4X,*WSQ*3X,10E11,4)
6245 PRINT 6297,(OM(K),K=1,I1)
6297 FORMAT(5X,*OM*3X,10(4X,F3,1,4X))
DO 6298 I=2,MJ
IF(XABSF(MODE(I)),EQ,10) MODE(I)=0
IF(XABSF(MODE(I)),EQ,20) MODE(I)=0
6298 CONTINUE
GOTO MM

```

```

C
C
C      DETERMINE UNITS
75  NU=0
DO 7 I=3,MJ
IF(OM(I-1),GT,0,) GO TO 53
IF(OM1(I-1),EQ,0,) GO TO 7
53  IF(NU,EQ,0) GO TO 51
IF(LU2(NU),EQ,I-1) GO TO 52
51  NU=NU+1
LU1(NU)=I-1
52  LU2(NU)=I
7   CONTINUE
C      REMOVE TYPE-A IDLE ROWS
DO 54 I=1,NU
I1=LU1(I) $ I2=LU2(I)
DO 55 J=I1,I2
KU=XABSF(MODE(J))
IF(KU,GE,2) GO TO 56

```

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DEFINITION PHYSICALLY MEANINGFUL UNITS
(LOAD-GOVERNED ROWS)

```

      IF(P0(J).NE.0.) GO TO 56
      IF(J.GT.2) GO TO 48
      IF(YF(2)+YL(2).EQ.0.) GO TO 48
      IF(D0(1)+D01(1).GT.0.) GO TO 56
48    LU1(I)=LU1(I)+1
55    CONTINUE
56    DO 57 J=1,12
      L=I1+I2-J
      KU=XABSF(MODE(L))
      IF(KU.GE.2) GO TO 54
      IF(P0(L).NE.0.) GO TO 54
      LU2(I)=LU2(I)-1
57    CONTINUE
      IF(I2.NE.MJ) GO TO 54
      IF(LU1(I).LT.I2) GO TO 54
      IF(PSG(MJ).EQ.0.) GO TO 54
      LU1(I)=MJ-1 $ LU2(I)=MJ
54    CONTINUE
      I=0
58    I=I+1
49    IF(I.GT.NU) GO TO 581
      IF(LU1(I).LT.LU2(I)) GO TO 58
      NU=NU-1
      DO 59 J=1,NU
        LU1(J)=LU1(J+1)
        LU2(J)=LU2(J+1)
        GO TO 49
59    CONTINUE
      REMEMBER UNITS, DROPPING UNUSABLE UNITS

```

EXCEPTION: RETAIN ROW 2 IF ROWS 1 AND 2
ARE COMPRESSED

EXCEPTION: RETAIN MJ-1, MJ IF MJ IS
COMPRESSED BY RESTRAINING RING

MATRIX TREATMENT OF UNITS

```

C
C
C      CONSTRUCT AND SOLVE G+MATRICES
581  IF(NU.EQ.0) GO TO 810
      IF(NDR.LT.1) GO TO 43
      PRINT 5100,NU
5100  FORMAT(4H NU=I6)
      PRINT 5101,(LU1(I),I=1,NU)
5101  FORMAT(4H LU16X,10I3)
      PRINT 5102,(LU2(I),I=1,NU)
5102  FORMAT(4H LU26X,10I3)
43    DO 3 I=1,NU
      NU0=LU1(I)-1 $ NU1=LU2(I)-NU0
      NU2=2*NU1 $ NF1=0
      DO 8 J=1,NU2
        V(J)=0.
        DO 8 K=1,NU2
          Q(K,J)=0.
          DO 4 J=1,NU1
            NR=NU0+J $ KU=XABSF(MODE(NR))
            NN=2*J-1
            IF(NR.EQ.2) GO TO 40
            IF(J.EQ.1) GO TO 41
            IF(J.LT.NU1) GO TO 40
            IF(NR.EQ.MJ) GO TO 40
            PSQ(NR)=-D01(NR-1)*CAP1(NR-1)
            WSQ(NR)=-D0(NR-1)*CAP(NR-1)
            GO TO 40
          41  PSQ(NR)=D01(NR)*CAP1(NR)

```

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```

      WSQ(NR)=D0(NR)*CAP(NR)
C      EFFECTIVE-MODE CHANGES
40  IF(KU,GE,3) GO TO 6
      C=C1=YF(NR-1)+YL(NR-1)
      IF(NR,EQ,2) C=C1=1,
      C2=YF(NR+1)+YL(NR+1)
      IF(NR,EQ,MJ) C2=1,
      C=OM(NR-1)*C+OM(NR)*C2
      C1=OM1(NR-1)*C1+OM1(NR)*C2
      IF(P0(NR),EQ,0,) GO TO 44
      IF(C,NE,0,) GO TO 46
      Q(NN,NN)=-1, $ Q(NN,2*J)=BIIIT(NR)
      GO TO 66
46  KU=MODE(NR)=3
      BC1(NR)=-B36(NR) $ BC2(NR)=BC1(NR)*BIIIT(NR)
      BC3(NR)=B36(NR)*B37(NR) $ BC4(NR)=B35(NR)+BC3(NR)*BIIIT(NR)
      GO TO 6
44  IF(W0(NR),EQ,0,) GO TO 47
      IF(C1,NE,0,) GO TO 46
      Q(2*J,NN)=-1, $ Q(2*J,2*J)=BII(NR)
      GO TO 6
47  IF(YF(NR),EQ,0,) GO TO 64
      IF(C,NE,0,) GO TO 62
      Q(NN,NN)=XLM $ Q(NN,2*J)=-1,
      GO TO 66
62  KU=MODE(NR)=4
      BC1(NR)=B6(NR) $ BC2(NR)=XLM*B6(NR)
      BC3(NR)=XLM*BC1(NR) $ BC4(NR)=XLM*BC2(NR)
      GO TO 6
64  IF(YL(NR),EQ,0,) GO TO 65
      IF(C1,NE,0,) GO TO 62
65  Q(2*J,NN)=XLM $ Q(2*J,2*J)=1,
C
C      CONSTRUCT COEFFICIENTS
C
C      A) BUTTON
6  DF1=DF2=0,
   IF(J,GT,1) Q(NN,2*J-3)=DF1=OM(NR-1)*CAP(NR-1)
   IF(NR,EQ,2) DF1=OM(1)*CAP(1)
   IF(J,LT,NU1) Q(NN,2*J+1)=DF2=OM(NR)*CAP(NR)
   Q(NN,2*J-1)=-(BC1(NR)+DF1+DF2)
   Q(NN,2*J)=BC2(NR)
   V(NN)=WSQ(NR)-W0(NR)
   IF(Q(2*J,NN),NE,0,) GO TO 4
C      B) TOP
66 DF1=DF2=0, $ NN=NN+1
   IF(J,GT,1) Q(NN,2*J-2)=DF1=OM1(NR-1)*CAP1(NR-1)
   IF(NR,EQ,2) DF1=OM1(1)*CAP1(1)
   IF(J,LT,NU1) Q(NN,2*J+2)=DF2=OM1(NR)*CAP1(NR)
   IF(NR,EQ,MJ) DF2=OM1(MJ)*CAP1(MJ)
   Q(NN,2*J-1)=BC3(NR)
   Q(NN,2*J)=-(BC4(NR)+DF1+DF2)
   V(NN)=PSQ(NR)-P0(NR)
4  CONTINUE
   IF(NDB,LT,1) GO TO 63
   PRINT 5110,((Q(K,J),J=1,NU2),K=1,NU2)

```

IN A UNIT, MODES O, I, I_T, II,
AND II_T BEHAVE AS ASYMPTOTIC
CASES OF OTHER MODES

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```

5110  FORMAT(2H Q8X,10E10,3)
      PRINT 5111,(V(J),J=1,NU2)
5111  FORMAT(3H V07X,10E10,3)
C
C      SOLVE UNIT MATRIX
C
63    DO 67 J=2,NU2
      NN=(J-1)*NU2
      DO 67 K=1,NU2
67    Q(NN+K)=Q(K,J)
      CALL MATINV(Q,NU2,V,1,DETERM,NU2)
      IF(NDB,LT,1) GO TO 741
      PRINT 5115,(V(J),J=1,NU2)
5115  FORMAT(3H V 7X,10E10,3)
741   DO 68 J=1,NU1
      NR=NU0+J
      AY(LI,NR)=AY(LI,NR)+V(2+J-1)
      AY(MI,NR)=AY(MI,NR)+V(2+J)
      KU=XABSF(MODE(NR))
      IF(KU,GT,0) GO TO 742
      YF(NR)=AY(MI,NR) $ GO TO 68
742   IF(KU,LE,2) GO TO 743
      YF(NR)=AY(MI,NR)
743   YL(NR)=AY(LI,NR)
68    CONTINUE
3     CONTINUE

C
C      ADJUST NON-UNIT ROWS
C
810   IF(NDB,LT,1) GO TO 813
      PRINT 6275,(YF(J),J=1,MJ)
      PRINT 6280,(YL(J),J=1,MJ)
813   NU1=NU+1 $ YZ=B2+XLM
      NF(1)=1 $ Q(1)=Q(2)=0,
      Q(2+MJ+2)=YP(MJ)+YR $ Q(2+MJ+1)=2,*YP(MJ)
      DO 400 I=2,MJ
400   NF(I)=0
      IF(NU,EQ,0) GO TO 402
      DO 401 I=1,NU
      I1=LU1(I) $ I2=LU2(I)
      DO 401 J=I1,I2
      NF(J)=1
      Q(2+J-1)=AY(LI,J)
401   Q(2+J)=AY(MI,J)
402   DO 405 I=1,NU1
      I1=2 $ IF(I,GT,1) I1=LU2(I=1)+1
      I2=MJ $ IF(I,LT,NU1) I2=LU1(I)=1
      IF(I1,GT,I2) GO TO 405
      I3=I1 $ I4=I2
406   V(1)=Q(2+I3-2) $ V(2)=Q(2+I3-3)
      V(3)=Q(2+I4-2) $ V(4)=Q(2+I4-1)

C
C      FIND AND RESOLVE BENT END=ROWS
C
      LEFT END
      DF1=V(1)+D1(I3-1)-YP(I3)-YT(MI,I3)-B2

```

LOOP 405: FIND ROWS CAPABLE OF FREE
PIVOTING. DENOTE THEM BY
NF(I) = 0; IF A ROW IS INCAPABLE
OF PIVOTING, DENOTE IT BY
NF(I) > 0

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PLACEMENT OF FREE-PIVOTING (NONLOADED) ROWS

```

      IF (DF1,LE,0.) GO TO 408
      R=DF1*CAP1(I3-1)/(B35(I3)+CAP1(I3-1)) $ NF(I3)=2
      V(1)=Q(2*I3)=YF(I3)=YP(I3)+YT(MI,I3)+B2+R
      V(2)=Q(2*I3-1)=YP(I3)+YT(LI,I3)+YZ-B37(I3)*R
407  IF (I1,EQ,I3) I1=I1+1
      I3=I3+1 $ IF (I3,GT,I4) GO TO 424
      GO TO 406
408  DF1=V(2)+DD(I3-1)-YP(I3)-YT(LI,I3)+YZ
      IF (DF1,LE,0.) GO TO 409
      R=DF1*CAP(I3-1)/(B35L(I3)+CAP(I3-1)) $ NF(I3)=2
      V(2)=Q(2*I3-1)=YL(I3)=YP(I3)+YT(LI,I3)+YZ+R
      V(1)=Q(2*I3)=YP(I3)+YT(MI,I3)+B2+R*(1.+HL(MI)*(1.5+B4L(I4))
1  /B3L(I4))
      GO TO 407
C      PIGHT END
409  DF1=YP(I4)+YT(MI,I4)-B2-V(3)+D1(I4)
      IF (DF1,LE,0.) GO TO 411
      R=DF1*CAP1(I4)/(B35(I4)+CAP1(I4)) $ NF(I4)=2
      V(3)=Q(2*I4)=YF(I4)=YP(I4)+YT(MI,I4)-B2+R
      V(4)=Q(2*I4-1)=YP(I4)+YT(LI,I4)-YZ+B37(I4)*R
410  IF (I2,EQ,I4) I2=I2+1
      I4=I4+1 $ IF (I4,LT,I3) GO TO 424
      GO TO 409
411  DF1=YP(I4)+YT(LI,I4)-YZ-V(4)+DD(I4)
      IF (DF1,LE,0.) GO TO 412
      R=DF1*CAP(I4)/(B35L(I4)+CAP(I4)) $ NF(I4)=2
      V(4)=Q(2*I4-1)=YL(I4)=YP(I4)+YT(LI,I4)-YZ+R
      V(3)=Q(2*I4)=YP(I4)+YT(MI,I4)-B2-R*(1.+HL(MI)*(1.5+B4L(I4))
1  /B3L(I4))
      GO TO 410
C
C      COMPUTE TOP PIVOTING-LIMITS
C
412  BC1(I3)=YP(I3)+YT(MI,I3)-B2
      IF (BC1(I3),LT,V(1)+D1(I3-1)) BC1(I3)=V(1)+D1(I3-1)
      BC2(I4)=YP(I4)+YT(MI,I4)+B2
      IF (BC2(I4),GT,V(3)-D1(I4)) BC2(I4)=V(3)-D1(I4)
      I30=I3+1 $ I40=I4-1 $ I5=I6=0
      DO 413 J=I30,I4
      BC1(J)=YP(J)+YT(MI,J)-B2
      IF (BC1(J),LT,BC1(J-1)+D1(J-1)) BC1(J)=BC1(J-1)+D1(J-1)
      IF (BC1(J),LE,YP(J)+YT(MI,J)+B2) GO TO 413
      I5=J $ GO TO 414
413  CONTINUE
414  DO 415 J=I3,I40
      K=I40+I3-J
      BC2(K)=YP(K)+YT(MI,K)+B2
      IF (BC2(K),GT,BC2(K+1)-D1(K)) BC2(K)=BC2(K+1)-D1(K)
      IF (BC2(K),GT,YP(K)+YT(MI,K)-B2) GO TO 415
      I6=K $ GO TO 416
415  CONTINUE
416  K1=I5+I6
      IF (I5,EQ,0) GO TO 418
      K=I5 $ DF0=BC1(K)-YP(K)-YT(MI,K)-B2
      C2=BC1(K-1) $ C3=YP(K)+YT(MI,K)+B2
      C4=XLM*(BC1(K-1)-YP(K-1)-YT(MI,K-1)) $ C5=YZ

```

(I1,I2): BLOCK OF ROWS BETWEEN
EXISTING UNITS; FREE-
PIVOTING CANDIDATES,
BARRING A LOADED
INTERACTION

(I3,I4): BLOCK OF ROWS [IN (I1,I2)]
NOW BEING CONSIDERED
AND WHICH HAVE NOT YET
CONTRADICTED FREE-
PIVOTING POSSIBILITIES

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417 C1=CAP1(K-1)/(B35(K-1)*B35(K)+CAP1(K-1)*(B35(K-1)+B35(K)))
    C1=C1*DF0
    DF1=C1*B35(K-1) $ DF2=C1*B35(K) $ NF(K)=NF(K-1)=3
    Q(2*K)=C3+DF1
    Q(2*K-1)=YP(K)+YT(LI,K)+C5*B37(K)+DF1
    Q(2*K-2)=C2-DF2
    Q(2*K-3)=YP(K-1)+YT(LI,K-1)+C4*B37(K-1)+DF2
    YF(K-1)=Q(2*K-2) $ YF(K)=Q(2*K)
418 IF(I6.EQ.0) GO TO 419
    IF(NF(I6)+NF(I6+1).GT.0) GO TO 419
    K=I6+1 $ I6=0 $ DF0=YP(K)+YT(MI,K)-B2-BC2(K)
    C4=-YZ $ C5=XLM*(BC2(K)-YP(K)-YT(MI,K))
    C2=YP(K-1)+YT(MI,K-1)-B2 $ C3=BC2(K) $ GO TO 417
419 IF(K1.GT.0) GO TO 420

C
C      COMPUTE BUTTON PIVOTING-LIMITS
C
R=1,-XLM
BC3(I3)=YP(I3)*R+YT(LI,I3)-XLM*(YT(MI,I3)-BC1(I3))
IF(BC3(I3).LT.V(2)+DD(I3-1)) BC3(I3)=V(2)+DD(I3-1)
BC4(I4)=YP(I4)*R+YT(LI,I4)+XLM*(BC2(I4)-YT(MI,I4))
IF(BC4(I4).GT.V(4)-DD(I4)) BC4(I4)=V(4)-DD(I4)
DO 425 J=I3,I4
    DF0=YP(J)*R+YT(LI,J)-XLM*YT(MI,J)
    BC3(J)=DF0+XLM*BC1(J)
    IF(BC3(J).LT.BC3(J-1)+DD(J-1)) BC3(J)=BC3(J-1)+DD(J-1)
    IF(BC3(J).LE.DF0+XLM*BC2(J)) GO TO 425
    I5=J $ GO TO 426
425 CONTINUE
426 DO 427 J=I3,I40
    K=I40+I3-J
    DF0=YP(K)*R+YT(LI,K)-XLM*YT(MI,K)
    BC4(K)=DF0+XLM*BC2(K)
    IF(BC4(K).GT.BC4(K+1)-DD(K)) BC4(K)=BC4(K+1)-DD(K)
    IF(BC4(K).GE.DF0+XLM*BC1(K)) GO TO 427
    I6=K $ GO TO 428
427 CONTINUE
428 K1=I5+I6
    IF(I5.EQ.0) GO TO 430
    K=I5 $ C3=YP(K)*R+YT(LI,K)+XLM*(BC2(K)-YT(MI,K))
    C2=BC3(K-1) $ DF0=BC3(K)-C3
    C4=(BC3(K-1)-YP(K-1)-YT(LI,K-1))/XLM $ C5=B2
429 C1=CAP(K-1)/(B35L(K-1)*B35L(K)+CAP(K-1)*(B35L(K-1)+B35L(K)))
    C1=C1*DF0
    DF1=C1*B35L(K-1) $ DF2=C1*B35L(K) $ NF(K)=NF(K-1)=4
    Q(2*K-1)=C3+DF1
    Q(2*K)=YP(K)+YT(MI,K)+C5+(1.+HL(MI)*(1.5+B4L(K))/B3L(K))*DF1
    Q(2*K-3)=C2-DF2
    Q(2*K-2)=YP(K-1)+YT(MI,K-1)+C4*(1.+HL(MI)*(1.5+B4L(K-1))/
1 B3L(K-1))*DF2
    YL(K-1)=Q(2*K-3) $ YL(K)=Q(2*K-1)
430 IF(I6.EQ.0) GO TO 431
    IF(NF(I6)+NF(I6+1).GT.0) GO TO 431
    K=I6+1 $ I6=0 $ C2=YP(K)*R+YT(LI,K)+XLM*(BC1(K)-YT(MI,K))
    C4=-B2 $ C5=(BC4(K)-YP(K)-YT(LI,K))/XLM
    DF0=C2-BC4(K) $ C3=BC4(K) $ GO TO 429

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431 IF(K1,NE,0) GO TO 420
C
C      ADJUST TOP PIVOTING LIMITS
C
      R=1,-1,/XLM
      DO 4310 J=I3,I4
      DF0=YP(J)*R+YT(MI,J)-YT(LI,J)/XLM
      BC1(J)=DF0+BC3(J)/XLM
4310  BC2(J)=DF0+BC4(J)/XLM
      DO 4311 J=I3,I40
      IF(BC1(J)+D1(J),LE,BC2(J+1)) GO TO 4311
      I5=J+1 $ GO TO 4312
4311 CONTINUE
      GO TO 424
4312 C2=BC1(I5-1) $ C3=BC2(I5) $ C4=BC3(I5-1) $ C5=BC4(I5)
      DF0=C2+D1(I5-1)-C3 $ K1=I5 $ I6=0
      GO TO 417
C
C      DEFINE NEXT SUBSET
C
420 IF(NF(I3),EQ,0) GO TO 422
      I30=I3+1
      DO 421 J=I30,I4
      IF(NF(J),GT,0) GO TO 421
      I3=J $ GO TO 422
421 CONTINUE
424 IF(I4,EQ,I2) GO TO 405
      I3=I4+1 $ I4=I2 $ GO TO 420
422 DO 423 J=I3,I2
      IF(NF(J),EQ,0) GO TO 423
      I4=J-1 $ GO TO 406
423 CONTINUE
      I4=I2 $ GO TO 406
405 CONTINUE ← NF(i) NOW KNOWN FOR ALL ROWS, 1 THROUGH MJ
C
C      RESOLVE NON-BENT PIVOTING SUBSETS
C
      I1=1
      IF(NDB,EQ,0) GO TO 435
      PRINT 5124,(NF(I),I=1,MJ)
5124 FORMAT(* NF*7X,20I3)
      PRINT 5126,(BC1(I),I=1,MJ)
5126 FORMAT(* BC1*6X,10F12,6/10X,10F12,6)
      PRINT 5127,(BC2(I),I=1,MJ)
5127 FORMAT(* BC2*6X,10F12,6/10X,10F12,6)
      PRINT 5128,(BC3(I),I=1,MJ)
5128 FORMAT(* BC3*6X,10F12,6/10X,10F12,6)
      PRINT 5129,(BC4(I),I=1,MJ)
5129 FORMAT(* BC4*6X,10F12,6/10X,10F12,6)
435 IF(NF(I1),EQ,0) GO TO 436
434 I2=I1+1 $ IF(I1,GT,MJ) GO TO 125
      GO TO 435
436 I2=I2+1 $ IF(I2,GT,MJ) GO TO 437
      IF(NF(I2),EQ,0) GO TO 436
437 I2=I2-1
C      NON-BENT COMPRESSION

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POSITIONING OF ROWS SATISFYING NF(i) = 0

```

      IF(RC1(I1).LE,BC2(I1)) GO TO 439
      DO 438 I=I1,I2
      YF(I)=.5*(BC1(I)+BC2(I)) $ YL(I)=0,
438  NF(I)=5
      I1=I2 $ GO TO 434
439  IF(RC3(I1).LE,BC4(I1)) GO TO 441
      DO 440 I=I1,I2
      YL(I)=.5*(BC3(I)+BC4(I)) $ YF(I)=0,
440  NF(I)=5
      I1=I2 $ GO TO 434
C     FREE-PIVOTING SUBSET
441  DO 442 I=I1,I2
      R=YP(I)+YT(LI,I) $ YN(I)=YN0(I)
      BC1(I)=B9*(R-RC4(I)).*4
      BC2(I)=B9*(R-RC3(I))/A4
      IF(YN(I).LT,BC1(I)) YN(I)=BC1(I)
      IF(YN(I).GT,BC2(I)) YN(I)=BC2(I)
      AY(MI,I)=YP(I)+YT(MI,I)-TL*YN(I)/B9
      AY(LI,I)=R-A4*YN(I)/B9
      YF(I)=YL(I)=0,
      IF(OM(I-1)+OM(I).GT,0.) YL(I)=AY(LI,I)
      IF(OM1(I-1)+OM1(I).GT,0.) YF(I)=AY(MI,I)
442  CONTINUE
      I3=I4=I1
      IF(MDH.EQ,0) GO TO 433
      PRINT 5123,I1,I2,(YN(I),I=I1,I2)
5123  FORMAT(10X,*I1=*I2,*, I2=*I2,*, YN=--*7E12,5,(/28X,7E12,5))
433  IF(I1.EQ,MJ) GO TO 445
443  IF(AY(MI,I3)+D1(I3).GE,AY(MI,I3+1)) GO TO 444
      IF(AY(LI,I3)+DD(I3).GE,AY(LI,I3+1)) GO TO 444
      I4=I3+1 $ IF(I3.LT,I2) GO TO 443
      GO TO 469
444  IF(I4.EQ,I2) GO TO 445 $ I4=I4+1
      IF(AY(MI,I4)+D1(I4).GE,AY(MI,I4+1)) GO TO 444
      IF(AY(LI,I4)+DD(I4).GE,AY(LI,I4+1)) GO TO 444
445  K0=I4*(I4-1)+I3*(I3-3)
      C=2.*K0+5, $ C=.5*(SQRT(C)+1.001)
      K0=C+1 $ YN1=YN(K0)
C     SFT (I3,I4) BY K0-CHOICE
452  AY(LI,K0)=YP(K0)+YT(LI,K0)-A4*YN1 /B9
      AY(MI,K0)=YP(K0)+YT(MI,K0)-TL*YN1 /B9
      IF(MDH.EQ,1) PRINT 5125,I3,I4,C,K0
5125  FORMAT(*I3=*I2,*, I4=*I2,*,C=*E12,5,*,K0=*I2)
      J1=I3 $ J2=K0-1 $ J3=K0+1 $ J4=I4
450  DO 446 I=J1,J2
      J=J1+J2-1
      AY(LI,J)=YP(J)+YT(LI,J)-A4*YN(J)/B9
      AY(MI,J)=YP(J)+YT(MI,J)-TL*YN(J)/B9
      DFO=AY(MI,J)+D1(J)-AY(MI,J+1)
      IF(DFO.LE,0.) GO TO 447
      AY(MI,J)=AY(MI,J+1)-D1(J)
      AY(LI,J)=AY(LI,J)-XLM*DFO
447  DFO=AY(LI,J)+DD(J)-AY(LI,J+1)
      IF(DFO.LE,0.) GO TO 446
      AY(MI,J)=AY(MI,J)-DFO/XLM
      AY(LI,J)=AY(LI,J+1)-DD(J)

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446 CONTINUE
DO 448 J=J3,J4
AY(LI,J)=YP(J)+YT(LI,J)-A4*YN(J)/B9
AY(MI,J)=YP(J)+YT(MI,J)-TL*YN(J)/B9
DF0=AY(MI,J-1)+D1(J-1)-AY(MI,J)
IF(DF0,LE,0.) GO TO 449
AY(MI,J)=AY(MI,J-1)+D1(J-1)
AY(LI,J)=AY(LI,J)+XLM*DF0
449 DF0=AY(LI,J-1)+DD(J-1)-AY(LI,J)
IF(DF0,LE,0.) GO TO 448
AY(MI,J)=AY(MI,J)+DF0/XLM
AY(LI,J)=AY(LI,J-1)+DD(J-1)
448 CONTINUE
C GROWTH OF (I3,I4)
460 K1=0
IF(I3,EQ,I1) GO TO 461
DF1=AY(MI,I3)-AY(MI,I3-1)-D1(I3-1)
DF0=AY(LI,I3)-AY(LI,I3-1)-DD(I3-1)
IF(DF0/XLM,LT,DF1) DF1=DF0/XLM
IF(DF1,GT,0.) GO TO 461
K1=K1+1
461 IF(I4,EQ,I2) GO TO 462
DF2=AY(MI,I4)+D1(I4)-AY(MI,I4+1)
DF0=AY(LI,I4)+DD(I4)-AY(LI,I4+1)
IF(DF0/XLM,GT,DF2) DF2=DF0/XLM
IF(DF2,LT,0.) GO TO 462
K1=K1+2
462 IF(K1,EQ,0) GO TO 471
IF(K1-2) 464,465,463
463 IF(DF1+DF2,GT,0.) GO TO 465
464 I3=I3-1 $ J1=J2=I3 $ J4=J3+1 $ DF0=DF1 $ GO TO 466
465 I4=I4+1 $ J2=J1-1 $ J3=J4=I4 $ DF0=DF2
466 K=I4*(I4-1)+I3*(I3-1)
C=2,*K+5, $ C=.5*(SQRTF(C)+1,001)
K=C+1 $ IF(K,EQ,K0) GO TO 452
YN1=B9*(DF0+YP(K)+YT(MI,K)-AY(MI,K))/TL
K0=K $ GO TO 452
471 DO 472 K=I3,I4
IF(AY(LI,K),LT,BC3(K)) GO TO 473
IF(AY(LI,K),GT,BC4(K)) GO TO 473
472 CONTINUE
GO TO 480
473 J1=K-1 $ J2=K+1
IF(NDH,NE,0) PRINT 5130,K
5130 FORMAT(4H *K=I3)
IF(AY(LI,K),GE,BC3(K)) GO TO 474
AY(MI,K)=AY(MI,K)+(BC3(K)-AY(LI,K))/XLM
AY(LI,K)=BC3(K) $ GO TO 475
474 AY(MI,K)=AY(MI,K)+(BC4(K)-AY(LI,K))/XLM $ AY(LI,K)=BC4(K)
475 DO 476 K=I3,J1
L=I3+J1-K
DF0=AY(MI,L)+D1(L)-AY(MI,L+1) $ IF(DF0,LE,0.) GO TO 477
AY(MI,L)=AY(MI,L+1)-D1(L) $ AY(LI,L)=AY(LI,L)-XLM*DF0
477 DF0=AY(LI,L)+DD(L)-AY(LI,L+1) $ IF(DF0,LE,0.) GO TO 476
AY(LI,L)=AY(LI,L+1)-DD(L) $ AY(MI,L)=AY(MI,L)-DF0/XLM
476 CONTINUE

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DO 478 K=J2,I4
  DF0=AY(MI,K-1)+D1(K-1)-AY(MI,K) $ IF(DF0,LE,0) GO TO 479
  AY(MI,K)=AY(MI,K-1)+D1(K-1) $ AY(LI,K)=AY(LI,K)+XLM*DF0
479  DF0=AY(LI,K-1)+DD(K-1)-AY(LI,K) $ IF(DF0,LE,0) GO TO 478
  AY(LI,K)=AY(LI,K-1)+DD(K-1) $ AY(MI,K)=AY(MI,K)+DF0/XLM
478  CONTINUE
  GO TO 460
480  I3=I4+1 $ IF(I3,LE,I2) GO TO 443
C    ASSIGN CONSTRAINTS TO PIVOTED SUBSET
469  IF(AY(MI,I1-1)+D1(I1-1),GE,AY(MI,I1)) YF(I1)=AY(MI,I1)
  IF(AY(LI,I1-1)+DD(I1-1),GE,AY(LI,I1)) YL(I1)=AY(LI,I1)
  I1=I1+1
  DO 468 I=I1,I2
    IF(AY(MI,I-1)+D1(I-1),LT,AY(MI,I)) GO TO 467
    YF(I-1)=AY(MI,I-1) $ YF(I)=AY(MI,I)
467  IF(AY(LI,I-1)+DD(I-1),LT,AY(LI,I)) GO TO 468
    YL(I-1)=AY(LI,I-1) $ YL(I)=AY(LI,I)
468  CONTINUE
  IF(I2,EQ,MJ) GO TO 470
  IF(AY(MI,I2)+D1(I2),GE,AY(MI,I2+1)) YF(I2)=AY(MI,I2)
  IF(AY(LI,I2)+DD(I2),GE,AY(LI,I2+1)) YL(I2)=AY(LI,I2)
470  I1=I2 $ GO TO 434
END

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      SUBROUTINE MATINV (A,NSUB,B,MSUB,DET,NMAX)
C
C   MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS
      DIMENSION A(NMAX,NSUB), B(NMAX,MSUB)
      COMMON /402/ PIVOT(100), INDEX(100), IPIVOT(100)
      EQUIVALENCE (PIVOT,INDEX,IPIVOT), (AMAX,T), (IROW,I1,IRC)
      TYPE DOUBLE A,B,AMAX,T,SWAP
      DATA (ISHIFT=4096), (MASK=0000000077777777B)
C
      N=NSUB
      M=MSUB
C
C   INITIALIZATION
C
      DETERM=1.0
      DO 20 I=1,N
        IPIVOT(I)=0
      20 CONTINUE
      DO 550 I=1,N
C
C   SEARCH FOR PIVOT ELEMENT
C
        AMAX=0.0
        DO 105 J=1,N
          TEMP=IPIVOT(J).AND..NOT.MASK
          IF (TEMP) 105, 60
        60 DO 100 K=1,N
          TEMP=IPIVOT(K).AND..NOT.MASK
          IF (TEMP) 100, 80
        80 TEMP=ABS(A(J,K))
          IF (TEMP-AMAX) 100, 85, 85
        85 IROW=J
          ICOLUM=K
          AMAX=TEMP
        100 CONTINUE
        105 CONTINUE
        INDEX(I)=INDEX(I) + (ISHIFT+IROW+ICOLUM)
        J=IROW
        BMAX=AMAX=A(J,ICOLUM)
        DETERM=AMAX*DETERM
C
C   RETURN IF MATRIX IS SINGULAR
C
        IF (DETERM) 110, 600
C
        110 PIVOT(ICOLUM)=INDEX(ICOLUM).OR.BMAX.AND..NOT.MASK
C
C   INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
        IF (IROW-ICOLUM) 140, 260
        140 DETERM=-DETERM
        DO 200 K=1,N
          SWAP=A(J,K)
          A(J,K)=A(ICOLUM,K)
          A(ICOLUM,K)=SWAP
        200 CONTINUE

```

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```
      DO 250 K=1,M
      SWAP=B(J,K)
      B(J,K)=B(ICOLUM,K)
      B(ICOLUM,K)=SWAP
250  CONTINUE
C
C      DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
260  K=ICOLUM
      A(ICOLUM,K)=1./
      DO 350 K=1,N
      A(ICOLUM,K)=A(ICOLUM,K)/AMAX
350  CONTINUE
      DO 370 K=1,M
      B(ICOLUM,K)=B(ICOLUM,K)/AMAX
370  CONTINUE
C
C      REDUCE NON-PIVOT ROWS
C
      DO 550 J=1,N
      IF( J-ICOLUM) 400, 550
400  T=A( J, ICOLUM)
      A( J, ICOLUM)=0.0
      DO 450 K=1,N
      A( J,K)=A( J,K)-A(ICOLUM,K)*T
450  CONTINUE
      DO 500 K=1,M
      B( J,K)=B( J,K)-B(ICOLUM,K)*T
500  CONTINUE
550  CONTINUE
C
C      INTERCHANGE COLUMNS
C
600  DO 710 I=1,N
      I1=N+1-I
      IRC=INDEX(I1),AND,MASK
      K=IRC/ISHIFT
      ICOLUM=IRC-K*ISHIFT
      IF (K-ICOLUM) 650, 710
650  DO 705 J=1,N
      SWAP=A(J,K)
      A(J,K)=A(J, ICOLUM)
      A(J, ICOLUM)=SWAP
705  CONTINUE
710  CONTINUE
C
740  DET=DETERM
      RETURN
C
      ENTRY QMATINV
      RETURN
C
      END
```

APPENDIX G

Derivation of Equation for Unrestrained Thermal Deflection

1. Assumptions

The differential equation for unrestrained thermal deflection can be written in general as:

$$\frac{d^2 y}{dz^2} \equiv - \frac{M_T}{I_z E} , \quad (61)$$

where

- M_T = thermal moment,
- I_z = moment of inertia,
- E = Young's modulus,
- z = axial location, and
- y = deflection in radial direction.

Using the expressions developed in Eqs. 65 and 68, we can write:

$$\frac{d^2 y}{dz^2} = \frac{\alpha C(z) \Delta T(z)}{D} , \quad (62)$$

where

- α = coefficient of linear thermal expansion,
- D = effective can diameter,
- $C(z)$ = dimensionless parameter depending on hex-can peripheral temperature distribution, and
- ΔT = maximum temperature difference across hex-can flats.

The expressions for M_T and I_z are now determined explicitly for hex-can geometry, and the constant $C(z)$ is evaluated.

2. Derivation of Thermal Moment (M_T)

Referring to Fig. 21, we can write the general expression:

$$M_T = E\alpha \int T(x,y) x dA. \quad (63)$$

For convenience we have considered the hex-can temperature distribution, $T(x,y)$, as a function of distance along the periphery S . We will now derive M_T for two arbitrary temperature distributions: (a) linear variation with perimeter, and (b) stepwise variation with perimeter.

The hex can is divided into two symmetrical halves along the x axis, so Eq. 63 need be evaluated for only one half because of the symmetry. The total perimeter is logically divided into 6 equal parts, where:

$$P(\text{total}) = 6b,$$

$$b = \frac{d}{2(\cos 60^\circ)} = 0.577d = \text{face width, and}$$

$$d = \text{outside distance across flats.}$$

We assume: $t \ll b$.

a. Linear Temperature Variation

Referring to Fig. 21, we evaluate the integral in Eq. 62 piecewise along S as follows:

$$\begin{aligned} M_T = & 2E\alpha \int_0^{0.288d} \left[T_1 - \frac{(T_1 - T_2)y}{1.73d} \left(\frac{-dt}{2} \right) \right] dy \\ & + 2E\alpha \int_{-\frac{d}{2}}^0 \left[T_1 - \frac{(T_1 - T_2)(0.288d)}{1.73d} - \frac{(T_1 - T_2)(\frac{d}{2} + x)}{1.73d \cos 60^\circ} \right] \frac{txdx}{\cos 60^\circ} \\ & + 2E\alpha \int_0^{\frac{d}{2}} \left[T_1 - \frac{(T_1 - T_2)}{2} - \frac{(T_1 - T_2)x}{1.73d \cos 60^\circ} \right] \frac{txdx}{\cos 60^\circ} \\ & + 2E\alpha \int_0^{0.288d} \left[T_2 + \frac{(T_1 - T_2)y}{1.73d} \right] \frac{td}{2} dy. \end{aligned} \quad (64)$$

CROSS-SECTION OF HEX CAN

Fig. 21. Linear Temperature Variation along Periphery of Hex Can

Evaluating the above,

$$M_T = - (0.368) E \alpha t d^2 (T_1 - T_2), \quad (65)$$

where T_1 and T_2 are the diametrically opposite, maximum and minimum can temperatures, respectively.

b. Stepwise Temperature Variation

The temperature variation chosen is shown in Fig. 22, where the temperature changes from T_1 to T_2 at point A. We thus have the following integral to evaluate for M_T :

$$\begin{aligned} M_T = & 2E\alpha \int_0^{0.288d} T_1 \left(\frac{-td}{2} \right) dy + 2E\alpha \int_{-\frac{d}{2}}^0 \frac{T_2 t x dx}{\cos 60^\circ} \\ & + 2E\alpha \int_0^{\frac{d}{2}} \frac{T_2 t x dx}{\cos 60^\circ} + 2E\alpha \int_0^{0.288d} T_2 \left(\frac{td}{2} \right) dy. \end{aligned}$$

Thus

$$M_T = - (0.288) E \alpha t d^2 (T_1 - T_2). \quad (66)$$

We note that M_T has the same value whether the step in temperature occurs at point A or B.

3. Moment of Inertia (I_z)

For a hollow hexagon, the moment of inertia about the vertical z axis can be written, for $t \ll b$:

$$I_z = t \left(\frac{5}{8} b d^2 + \frac{\sqrt{3}}{12} d^3 \right).$$

Since $b = 0.577d$,

$$I_z = (0.505) t d^3. \quad (67)$$

CROSS-SECTION OF HEX CAN

Fig. 22. Stepwise Temperature Variation along Periphery of Hex Can

4. Final Thermal-deflection Equations

Since the peripheral temperature distribution can change, in general, with axial location, the temperature-distribution "constant" is written:

$$C = C(z).$$

We thus have the following results for C :

C = 0.73 (linear temperature distribution)

C = 0.62 (stepwise temperature distribution; step at point A or B)

Analyses indicate that the value of C in EBR-II ranges in dimensionless units from about 0.65 to 0.75, depending upon row number and elevation.

APPENDIX H

Calculation of Reactivity from Row Displacements

1. General

Reactivity changes caused by bowing effects in EBR-II have been calculated by summing the products of row displacements and row reactivity coefficients according to the equation:

$$\Delta\rho = \sum_{k=k_1}^{k_2} \left(\frac{\partial\rho}{\partial r}\right)_k \cdot \overline{\Delta y}_k, \quad (68)$$

where

$\overline{\Delta y}_k$ = row centerline displacements (effective),

$\Delta\rho$ = total reactivity change (bowing effects only),

$\left(\frac{\partial\rho}{\partial r}\right)_k$ = reactivity coefficient by row, and

k_1, k_2 = inner and outer rows, respectively, of the group being considered.

2. Effective Row Displacements

Two methods have been used to determine effective row displacements for EBR-II from BOW-V output:

(a) row displacement assumed equivalent to displacement of center node (at horizontal midplane of core), and

(b) row displacement assumed equivalent to parabolically weighted displacement of all nodes within the core region.

These two methods show small differences in the calculated reactivity values. Thus, the small, but finite, distortion and inclination of the subassemblies apparently were unimportant. Only method 2b (above) will be considered here.

$$\overline{\Delta y}_k = \frac{1}{6} [Y_k(4) + 4Y_k(5) + Y_k(6)] - YP_k, \quad (69)$$

where

- k = row number;
 $Y_k(4), Y_k(5), Y_k(6)$ = radial positions of lower, intermediate and upper nodes within core, respectively; and
 YP_k = initial centerline position of row k (same for all node points if initial lean = 0).

3. Reactivity Coefficients

Values of the reactivity coefficients (for EBR-II) have been calculated by two means devised by: (a) Persiani and Kucera⁸ and (b) Madell and Jarka.⁹ The assumptions made in these two calculational models are described in the above references, and only the results are given in Table VIII.

TABLE VIII. Comparison of Row-displacement Reactivity Coefficients for EBR-II, Run 25.

Row No.	$(\partial \rho / \partial r)_k, \text{ Ih/mil}$	
	<u>Persiani-Kucera Model, Set a</u>	<u>Madell-Jarka Model, Set b</u>
6	-1.81	-2.60
7	-0.99	-0.27
8	-0.50	-0.13
9	-0.29	-0.03

^aThese values have been adjusted from those originally calculated.¹⁰

The calculated reactivity changes are smaller with coefficient set a than with set b. Set b will be used to relate BOW-V results to reactor experimental results until improved coefficients are obtained. Figure 23 shows both sets of reactivity coefficients and compares the calculated and measured bowing reactivity decrement in EBR-II, run 25.

Fig. 23. Measured vs Calculated Bowing Reactivity on EBR-II

4. BOWREAC Computer Code

A small CDC-1604 program has been written to convert the row displacements (punched output) of BOW-V to reactivity changes. This program sums the reactivity contributions for up to 10 rows and can combine the displacements of the core top, center, and bottom levels for each row into a weighted average displacement. These displacements are multiplied by input values of individual-row reactivity coefficients.

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