MASTER

TEMP-2 A ONE DIMENSIONAL TRANSIENT THERMAL STRESS CODE FOR THE IBM-704

April 1960

CONTRACT AT-11-1-GEN-14

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PITTSBURGH, PENNSYLVANIA**

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TEMP-2 A ONE DIMENSIONAL TRANSIENT
THERMAL STRESS PROGRAM FOR THE IBM 704

L. M. Culpepper and D. Jortner

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April 1960

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This report describes a computer program to solve transient thermal stress problems in one dimension. For given boundary and initial conditions this program computes the transient temperature distribution then solves for stresses. Spherical, cylindrical, and slab geometries can be treated by this program

TEMP-2 -- A ONE DIMENSIONAL TRANSIENT
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I. INTRODUCTION

There are a number of one-space dimension thermal stress problems of practical importance in which the temperature varies with time. Three of these, which may be treated by this program, are

- A. A sphere, with temperature distribution symmetrical about the center.
- B. A long cylinder, with temperature distribution symmetrical about the axis.
- C. A plate or slab, with temperature distribution across the thickness and surfaces free or restrained.

Bodies with non-homogeneous material composition are contemplated.

The TEMP-2 program solves the difference form of the one-dimensional transient heat-conduction equation for a body with an arbitrary initial temperature distribution and either the temperature, its normal gradient, or a combination of the two specified on the boundaries. The thermal stresses resulting from the temperature distribution are then obtained by a regionwise application of the analytical stress

expressions of Ref (1). The solution of a 41-point problem requires about five seconds of computer time per time step.

II. NOMENCLATURE

A region is characterized by spatially-constant elastic and thermal properties and is represented by a set of mesh points separated by a constant distance or mesh spacing. If the total number of mesh points representing the body is numbered starting with zero, then the boundaries of a region must fall on even-numbered mesh points. Each region must contain at least one interior mesh point or two mesh intervals. The symbols used are:

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
E	modulus of elasticity	(F/L ²)
ν	Poisson's ratio	(-)
α	coefficient of thermal expansion	(1/T)
σ	normal stress	(F/L ²)
ϵ	strain	(-)
F	force	(F)
h	slab thickness	(L)
T	temperature	(T)
t	time	(θ)
ρ	density (weight or lb mass)	(F/L ³)
C	specific heat	(H/TF)
G	conductivity	(H/LT θ)
Q	internal heat generation	(H/I. ³ θ)
N	number of regions	(-)
k	the index of a region which has its outer boundary at the h th interface. At the inner boundary k = 0,	(-)

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
	at the outer boundary $k = n$	
$\bar{T}_k$	average temperature of region $k = \bar{T}_k = \frac{\int_{r_{k-1}}^{r_k} T dV}{\int_{r_{k-1}}^{r_k} dV}$	(T)
r_0	coordinate of inner boundary	(L)
r_n	coordinate of outer boundary	(L)

A time step with length Δt is a period of time during which the thermal and elastic properties and the temperature remain constant.

III. RESTRICTIONS

The program provides for a minimum of 7 and a maximum of 251 mesh points which may be distributed over a minimum of 3 and a maximum of 25 regions. Although the program requires that at least 3 regions be specified, these regions may be identical so that homogeneous bodies may be represented. The program also requires that a constant time increment be used and, in order to reduce the amount of time required to solve the problem, it is suggested that, if a slow transient is to be studied, the following method be used:

If, for example, an hour-long transient is to be studied, the problem could be solved using a coarse time increment of 60 seconds. If a particular part of the transient is of interest then the temperature distribution at that time could be used as the initial temperature distribution for another problem with a smaller time increment.

Since an implicit method is used to solve the transient heat conduction equation, the finite difference solution converges to the true solution in a manner

which is independent of the relationship between Δt and Δx . The accuracy of the solution is, however, a function of the time and mesh increments, the error being $E = O(\Delta t) + O[(\Delta x)^2]$.

IV. SOLUTION OF HEAT EQUATION

The one dimensional heat-conduction equation

$$\rho C(r, T) \frac{\partial T}{\partial t}(r, t) = \nabla \cdot G(r, T) \nabla T(r, t) + Q(r, t)$$

is solved by replacing the time derivative by a backward difference quotient. The resulting equation

$$\rho C(r, T^i) \left[\frac{T^i(r) - T^{i-1}(r)}{\Delta t} \right] = \nabla \cdot G(r, T^i) \nabla T^i(r) + Q^i(r)$$

is then linearized by requiring that the coefficients ρC and G be constant within a region and functions only of the average temperature in that region at the beginning of a time step.

Then

$$\rho C^i(r) \frac{T^i(r) - T^{i-1}(r)}{\Delta t} = \nabla \cdot G^i(r) \nabla T^i(r) + Q^i(r) \quad (1)$$

with $T^0(r)$ supplied as input is solved for each time step with boundary conditions

$$T^i(r_0) + A^i \left. \frac{dT^i}{dr}(r) \right|_{r=r_0} = B^i \text{ and } T^i(r_n) + C^i \left. \frac{dT^i}{dt}(r) \right|_{r=r_n} = D^i.$$

Equation 1 is solved by a standard 3-point central-difference method described, for example, in Ref 2.

V. SOLUTION OF STRESS EQUATIONS

The thermal stress equations for the three types of problems considered are given below.

A. Sphere

For this case, only the normal stresses σ_r , σ_θ and σ_ϕ (the last two are

equal) exist. The shear stresses are zero due to symmetry.

The stresses are given by the expressions

$$\sigma_r = \frac{-2\alpha_k E_k}{(1-\nu_k)} \frac{1}{r^3} \int_{r_{k-1}}^r Tr^2 dr + \frac{E_k A_k}{(1-2\nu_k)} - \frac{2E_k B_k}{(1+\nu_k)} \frac{1}{r^3} \quad (1)$$

$$\sigma_\theta = \frac{\alpha_k E_k}{(1-\nu_k)} \frac{1}{r^3} \int_{r_{k-1}}^r Tr^2 dr + \frac{E_k A_k}{(1-2\nu_k)} + \frac{E_k B_k}{(1+\nu_k)} \frac{1}{r^3} - \frac{\alpha_k E_k T}{(1-\nu_k)} \quad (2)$$

k is the index of the regions. The values of k go from 1 to n . The radius r_k is located at the outer surface of region k .

The constants A_k and B_k (which total $2n$ in number) are determined by the fact that radial deformations and stresses are continuous at the $n-1$ internal interfaces and by the use of boundary conditions at the inner and outer radii of the sphere. The quantities α_k , E_k , and ν_k are functions of material composition and temperature which are known functions of the radius r .

Therefore, in order to obtain the A_k and B_k constants the following simultaneous equations must be solved:

a. Internal Interfaces:

(1) Continuity of radial deformation.

$$\frac{(1+\nu_k)}{(1-\nu_k)} \alpha_k \frac{1}{r_k^2} \int_{r_{k-1}}^{r_k} Tr^2 dr + A_k r_k + B_k \frac{1}{r_k^2} = A_{k+1} r_k + B_{k+1} \frac{1}{r_k^2} \quad (3)$$

This is applied at $k = 1, 2, \dots, n-1$.

(2) Continuity of radial stress.

$$\begin{aligned} & -\frac{2A_k E_k}{(1-\nu_k)} \frac{1}{r_k^3} \int_{r_{k-1}}^{r_k} Tr^2 dr + \frac{E_k A_k}{(1-2\nu_k)} - \frac{2E_k B_k}{(1+\nu_k)} \frac{1}{r_k^3} \\ & = \frac{E_{k+1} A_{k+1}}{(1-2\nu_{k+1})} - \frac{2E_{k+1} B_{k+1}}{(1+\nu_{k+1}) r_k^3} \end{aligned} \quad (4)$$

This is applied at $k = 1, 2, \dots, n-1$.

b. Boundary Conditions:

(1) Outer boundary - the external radial stress σ_{r_n} is specified.

$$-\frac{2\alpha_n E_n}{(1-\nu_n)} \frac{1}{r_n^3} \int_{r_{n-1}}^{r_n} Tr^2 dr + \frac{E_n A_n}{(1-2\nu_n)} - \frac{2E_n B_n}{(1-\nu_n)} = \sigma_{r_n} \quad (5)$$

(2) Inner boundary.

(a) If sphere is hollow, σ_{r_0} is specified and:

$$\frac{E_1 A_1}{(1-2\nu_1)} - \frac{2E_1 B_1}{(1+\nu_1)} \frac{1}{r_0^3} = \sigma_{r_0} \quad (6)$$

(b) If the sphere is solid, radial deformation is zero at $r = 0$ and this requires that

$$B_1 = 0 \quad (7)$$

Thus by using the $2n-2$ internal continuity relations and the 2 boundary conditions, the $2n$ quantities A_k and B_k are determined.

Now the stresses may be determined from Eqs 1 and 2.

B. Long Cylinder

For this case the normal stresses σ_r , σ_θ , and σ_z exist. The three shear stresses are zero due to angular and longitudinal symmetry. These stresses are given by the expressions

$$\sigma_r = \frac{-E_k \alpha_k}{(1-\nu_k) r^2} \int_{r_{k-1}}^r Tr dr + \frac{E_k A_k}{(1+\nu_k)(1-2\nu_k)} - \frac{E_k B_k}{(1+\nu_k) r^2} \quad (1)$$

$$\sigma_\theta = \frac{-E_k \alpha_k T}{(1-\nu_k)} + \frac{E_k \alpha_k}{(1-\nu_k) r^2} \int_{r_{k-1}}^r Tr dr + \frac{E_k A_k}{(1+\nu_k)(1-2\nu_k)} + \frac{E_k B_k}{(1+\nu_k) r^2} \quad (2)$$

$$\sigma_z = \frac{-E_k \alpha_k T}{(1-\nu_k)} + \frac{2\nu_k E_k A_k}{(1+\nu_k)(1-2\nu_k)} + E_k \epsilon_z \quad (3)$$

As in the spherical case, A_k and B_k are the $2n$ unknown constants of the system. E_k , α_k , and ν_k are the known functions of radius as is the temperature T . One additional constant exists here, ϵ_z . This is the constant axial strain. (This is constant due to the one-dimensional nature of the problem.)

Therefore, in addition to the $(2n-2)$ continuity relations and the 2 radial boundary conditions, an added piece of information must be supplied which determines ϵ_z .

The equations used to find A_k , B_k , and ϵ_z are:

a. Internal interfaces:

(1) Continuity of radial deformation.

$$\frac{(1+\nu_k)}{(1-\nu_k)} \frac{\alpha_k}{r_k} \int_{r_{k-1}}^{r_k} T r dr + A_k r_k + \frac{B_k}{r_k} = A_{k+1} r_k + \frac{B_{k+1}}{r_k} \quad (4)$$

This is applied at $k = 1, 2, \dots, n-1$.

(2) Continuity of radial stress:

$$\begin{aligned} \frac{-E_k \alpha_k}{(1-\nu_k) r_k^2} \int_{r_{k-1}}^{r_k} T r dr + \frac{E_k A_k}{(1+\nu_k)(1-2\nu_k)} - \frac{E_k B_k}{(1+\nu_k) r_k^2} \\ = \frac{E_{k+1} A_{k+1}}{(1+\nu_{k+1})(1-2\nu_{k+1})} - \frac{E_{k+1} B_{k+1}}{(1+\nu_{k+1}) r_k^2} \end{aligned} \quad (5)$$

This is applied at $k = 1, 2, \dots, n-1$.

b. Boundary Conditions:

(1) Outer boundary - the external radial stress is specified.

$$\frac{-E_n \alpha_n}{(1-\nu_n) r_n^2} \int_{r_{n-1}}^{r_n} T r dr + \frac{E_n A_n}{(1+\nu_n)(1-2\nu_n)} - \frac{E_n B_n}{(1+\nu_n) r_n^2} = \sigma_{r_n} \quad (6)$$

(2) Inner boundary.

(a) If cylinder is hollow, σ_{r_0} is specified.

$$\frac{E_1 A_1}{(1+\nu_1)(1-2\nu_1)} - \frac{E_1 B_1}{(1+\nu_1)r_0^2} = \sigma_{r_0} \quad (7)$$

(b) If cylinder is solid, radial deformation is zero at $r = 0$ and this requires that

$$B_1 = 0 \quad (8)$$

(c) Determining ϵ_z :

To determine ϵ_z , a choice is allowed. Either the requestor can specify ϵ_z directly as input, or the requestor can specify the axial force, F_z , acting on the cylinder. In this case, ϵ_z can be determined by

$$2\pi \int_{r_0}^{r_n} r \sigma_z dr = F_z$$

which results in

$$\epsilon_z = \frac{\frac{F_z}{\pi} + 2 \sum_{k=1}^n \left[\frac{E_k \alpha_k}{(1-\nu_k)} \int_{r_{k-1}}^{r_k} r dr - \frac{E_k \nu_k A_k (r_k^2 - r_{k-1}^2)}{(1+\nu_k)(1-2\nu_k)} \right]}{\sum_{k=1}^n E_k (r_k^2 - r_{k-1}^2)} \quad (9)$$

Now the stresses may be determined from Eqs 1, 2, and 3.

C. Slab or Plate

For this case, the only stresses existing are σ_x , σ_y , σ_z where σ_x is a known constant as a consequence of the equations of equilibrium and the boundary conditions. σ_x is inserted as input to the problem.

If the restraints in the y and z directions are identical, the other stresses are

$$\sigma_y = \sigma_z = \frac{E_k}{(1-\nu_k)} \left[\epsilon_y - \alpha_k T + \frac{\nu_k \sigma_x}{E_k} \right] \quad (1)$$

In the most general form, $\epsilon_y = \epsilon_z$ may be in the form $(L + M \frac{x}{h})$ where L is the strain at any convenient point, say the left surface, and h is the slab thickness. However, in the interest of simplicity not all possible combinations of L and M are considered.

The cases which are considered are:

- a. Specify L and M.
- b. Specify the force existing in the y direction and M.
- c. Specify complete lack of restraint, i.e., force and moment = 0.

In case a, since σ_x is specified, the stresses are automatically determined when L and M are specified. This case includes the completely restrained situation where L and M are equal to zero.

In case b, the only unknown constant is L. This is determined from

$$\int_{X_0}^{X_h} \sigma_y dx = F_y \quad (2)$$

where F_y is the force in the y direction per unit length of slab.

This results in

$$\sum_{k=1}^n \frac{E_k}{(1-\nu_k)} \int_{X_{k-1}}^{X_k} \left[L + \frac{M}{h} x - \alpha_k T + \frac{\nu_k \sigma_x}{E_k} \right] dx = F_y \quad (3)$$

or

$$\begin{aligned}
L \sum_{k=1}^n \frac{E_k (X_k - X_{k-1})}{(1-\nu_k)} + \frac{M}{h} \sum_{k=1}^n \frac{E_k (X_k^2 - X_{k-1}^2)}{2(1-\nu_k)} - \sum_{k=1}^n \frac{E_k \alpha_k \int_{X_{k-1}}^{X_k} T dx}{(1-\nu_k)} \\
+ \sigma_x \sum_{k=1}^n \frac{\nu_k (X_k - X_{k-1})}{(1-\nu_k)} = F_y
\end{aligned} \quad (3')$$

L is found from this equation.

In case c, L and M are both unknowns. They are determined from Eq 3' with F_y equal to zero and from the fact that there is also no moment due to lack of restraint. This condition is expressed by

$$\int_0^h \sigma_y x dx = 0. \quad (4)$$

This results in

$$\sum_{k=1}^n \frac{E_k}{(1-\nu_k)} \int_{X_{k-1}}^{X_k} \left[Lx + \frac{M}{h} x^2 - \alpha_k T x + \frac{\nu_k \sigma_x x}{E_k} \right] dx = 0 \quad (5)$$

or

$$\begin{aligned}
L \sum_{k=1}^n \frac{E_k (X_k^2 - X_{k-1}^2)}{2(1-\nu_k)} + \frac{M}{h} \sum_{k=1}^n \frac{E_k (X_k^3 - X_{k-1}^3)}{3(1-\nu_k)} - \sum_{k=1}^n \frac{\alpha_k E_k}{(1-\nu_k)} \int_{X_{k-1}}^{X_k} T x dx \\
+ \sigma_x \sum_{k=1}^n \frac{\nu_k (X_k^2 - X_{k-1}^2)}{2(1-\nu_k)} = 0.
\end{aligned} \quad (5')$$

Eq 3' with $F_y = 0$ and Eq 5' are sufficient to determine L and M. Thus the stresses for all the cases to be considered are defined.

VI. INPUT FORMAT

A. Data Deck

Each data deck must contain a title card, data cards, end of data card, and

a blank card.

B. Title Card

This card must be punched as follows: Columns 1 and 67 are to be blank. Columns 2-9 contain the problem number. Columns 64-66 contain control information. Columns 68-72 must contain the characters TEMP2. The remaining columns may be punched in any manner and will be reproduced on each page of output.

C. Data Cards

Each of these cards must contain the characters DEC in columns 8-10 and blanks in columns 1-7 and 11. In addition, each card in the data deck must contain a card number starting in column 12. Successive pieces of data on the card must be separated by commas. The last piece of data on a card must not be followed by a comma. The first blank column after column 12 signals the end of the data on that card providing all 72 columns are not used. In the discussion below, lower case letters will indicate fixed-point numbers and upper case letters will indicate floating-point numbers. A floating-point number is distinguished by the presence of a decimal point, an "E", or both. For example, -17.03 may be represented as -17.03 or by -1703E-2 or -.1703E2. For positive floating-point numbers, the + sign is not needed. In many cases, the data for each of the card series described below may be punched on 1 card. If, however, more than 1 card is needed, the succeeding cards should be numbered sequentially. All problems not fulfilling this requirement will be rejected.

D. Card Numbers

Again, each data card must be numbered and, for ease in input preparation, the card numbers are separated into the following series.

<u>Card Series</u>	<u>Data</u>
1000	General information.
2000	Mesh spacing and region description.
3000	Time increments.
4000	Initial temperature distribution.
10000	Values of ρC .
20000	Values of thermal conductivity.
30000	Internal heat generation.
40000	Coefficient of thermal expansion.
50000	Modulus of elasticity.
60000	Poisson's ratio
70000	Boundary conditions
80000	End of data.

A more complete description of each type of data card follows.

A. Title Card

Columns 64-66 contain control information as follows.

Column 64 = 0 for cartesian coordinates,
 = 1 for cylindrical coordinates,
 = 2 for spherical coordinates.

Column 65 is used only for cylindrical geometry.

If column 65 = 0 the axial strain ϵ_z is specified,
 = 1 the axial force F_z is specified and ϵ_z is calculated.

Column 66 is used only for cartesian geometry to indicate the
 method of calculating the strain $\epsilon_y = L + M \frac{x}{h}$.

If column 66 = 0 L and M are specified,
 = 1 L is calculated and axial force is specified,
 = 2 L and M are calculated assuming free expansion of the body.

b. Card 1001 contains j numbers depending on the choice of geometry and stress boundary conditions. The general format is DEC 1001, $A_1, A_2, \dots, A_j$.

For spherical coordinates, $A_1 = r_0, A_2 = \sigma_r(r_0); A_3 = \sigma_r(r_n)$

For cylindrical coordinates, $A_1 = r_0, A_2 = \sigma_r(r_0), A_3 = \sigma_r(r_n),$

$A_4 = F_z$ or ϵ_z .

For cartesian coordinates, $A_1 = r_0, A_4 = \sigma_x, A_5 = h.$

A_2 and $A_3 = L$ and M

or F_y and M

or $= 0$ and 0 for complete lack of restraint.

c. The 2000 series contains mesh spacing and region specification data in the form:

DEC 20001, $\Delta r_1, m_1, \Delta r_2, m_2, \dots, \Delta r_n, m_n$

where Δr_1 is the spacing between the points $0, 1, 2, 3, \dots, m_1,$

Δr_2 is the spacing between the points $m_1, m_1 + 1, \dots, m_2,$ etc.

n , the number of regions, is determined by the number of pairs $\Delta r_i, m_i$ and must satisfy $3 \leq n \leq 25$. m_n must satisfy $6 \leq m_n \leq 250$.

E. Region Data

The thermal and elastic properties of a body are described to the TEMP2 code in tabular form. The quantities $\rho C, G, \alpha, E,$ and ν are tabulated as functions of temperature while the quantity Q is tabulated as a function of time. Since the body may be composed of several materials, each table must be identified by the numbers of the regions for which the table applies. Each property of the body is described in a separate card series as follows:

Let h be a number between one and six denoting one of the properties

DEC h0100, $j_1, j_2, \dots, j_i$

DEC h0101, $y_1, x_1, y_2, x_2, \dots, y_{\ell}, x_{\ell}$

DEC h0102, $y_{i+1}, x_{i+1}, \dots, y_m, x_m$

DEC h0200, $j_{i+1}, j_{i+2}, \dots$

DEC h0201, $y_1, x_1, y_2, x_2, \dots, y_n, x_n$

DEC h0202, $y_{n+1}, x_{n+1}, \dots, y_p, x_p$

Thus, if $h = 5$, the property being described is E, Young's modulus. The series of cards shows that there are two materials, one material comprising regions $j_1, j_2, \dots, j_i$ and the other comprising regions $j_{i+1}, j_{i+2}, \dots$. The table for the first material contains m entries of the property y and the independent variable x , while the table for material 2 contains p entries. Each table must contain at least 2 but fewer than 50 entries and may be assigned to as many regions as described.

F. Boundary Conditions

The boundary condition at the inner boundary is

$$T(r_0) + A_i \left. \frac{dT(r)}{dr} \right|_{r_0} = B_i$$

and at the outer

$$T(r_n) + C_i \left. \frac{dT(r)}{dr} \right|_{r_n} = D_i$$

and are specified in the following manner:

DEC 70100, $A_1, B_1, C_1, D_1, t_1, A_2, B_2, C_2, D_2, t_2, \dots, A_i, B_i, C_i, D_i, t_i$

DEC 70101, $A_{i+1}, B_{i+1}, C_{i+1}, D_{i+1}, t_{i+1}, \dots$

where the A_i, B_i, C_i , and D_i are used when the time t satisfies $t_i \leq t < t_{i+1}$.

In order to obtain a zero derivative at either boundary, set the coefficient of its derivative to 10^{25} .

a. Time Increments

DEC 3001, $\Delta t, n$

b. Initial Temperature Distribution

DEC 4001, $T_0, T_1, T_2, \dots, T_{m_n}$

The initial temperature must be specified at each point.

C. End of Data

DEC 80000

This card followed by a blank card signals the end of the input data for a problem.

VII. OUTPUT FORMAT

Each page of output is numbered and identified by the title card. The temperature and stresses at each point are printed in columnar form together with the point number and coordinate. Each column is identified by a title. The output numbers themselves are signed 8-digit fractions with a signed 2-digit exponent. Leading zeros in the exponent are not printed. Examples are:

$1/2 = 50000000$

$-1/3 = -1-33333333$

$10/3 = 1\ 33333333$

32000001-SAMPLE PROBLEM

11 TEMP2 TMP35001

DEC 1001,2.0,0	TMP35001
DEC 2001,.01,4,.05,12,.01,18,.005,30,.015,36	TMP35001
DEC 3001,.01	TMP35001
DEC 4001,200.0,201.0,202.0,203.0,204.0,209.0,214.0,219.0,224.0	TMP35001
DEC 4002,229.0,234.0,239.0,244.0,245.0,246.0,247.0,248.0,249.0	TMP35001
DEC 4003,250.0,250.5,251.0,251.5,252.0,252.5,253.0,253.5,254.0	TMP35001
DEC 4004,254.5,255.0,255.5,256.0,257.5,259.0,260.5,262.0,263.5	TMP35001
DEC 4005,265.0	TMP35001
DEC 10100,1,2	TMP35001
DEC 10101,60.0,10.00,64.0,15.0,70.0,20.0,72.0,60.0,80.0,100.0	TMP35001
DEC 10102,85.0,150.0,83.0,200.0,30.0,300.0	TMP35001
DEC 10200,3,4,5	TMP35001
DEC 10201,40.0,10.0,50.0,100.0,60.0,300.0	TMP35001
DEC 20100,1,2,3,4,5	TMP35001
DEC 20101,12.0,10.0,15.0,100.0,17.0,300.0	TMP35001
DEC 30100,1,2	TMP35001
DEC 30101,101.0,0,136.0,.025,151.0,.055	TMP35001
DEC 30200,3,4,5	TMP35001
DEC 30201,0,0,0,.055	TMP35001
DEC 40100,1,2	TMP35001
DEC 40101,6.1E-8,10.0,6.5E-8,100.0,6.8E-8,-400.0	TMP35001
DEC 40200,3,4,5	TMP35001
DEC 40201,4.0E-8,15.0,4.9E-8,200.0,5.3E-8,350.0	TMP35001
DEC 50100,1,2,3,4,5	TMP35001
DEC 50101,30.0E6,10.0,55.0E6,400.0	TMP35001
DEC 60100,1,2,3,4,5	TMP35001
DEC 60101,.15,10.0,.15,400.0	TMP35001
DEC 70100,1.E25,0,0,300.0,0,1.E25,0,0,280.0,.01,1.E25,0,0,200.0,.6	TMP35001
DEC 80000	TMP35001

REFERENCE

- [1] S. Timoshenko and J. N. Goodier, "Theory of Elasticity," 2nd Ed. McGraw-Hill, New York, 1951 p 399.