NUCLEAR STRUCTURE THEORY

Annual Technical Progress Report

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Abstract

This report summarizes progress during the past year in the following areas of nuclear structure and reaction theory:

Meson interactions with nucleons and nuclei, including inelastic scattering of pions, three-body theories of scattering and absorption of pions by deuterons, and π-ρ bremsstrahlung.

Theory of the effective interaction, including behavior of the expansion in orders of the reaction matrix.

Statistical spectroscopy including fluctuations in energy levels and excitation strengths, and sum rules and strength distributions for various excitation processes, including single-nucleon transfer, β decay and multipole giant resonances.

Studies of the inverse scattering problem.

Studies of nuclear symmetries, of nuclear clustering, and of general nuclear structure by α-transfer reactions, and of nuclear shapes by (d, 3He) reactions.
The study of inelastic scattering of pions by nuclei begun last year, has been extended. In the earlier approach, based on the impulse approximation, it was shown that the amplitude for exciting the Giant Dipole resonance is particularly sensitive to the velocity-dependence (or non-locality) of the pion-nucleon interaction. More recently, we have performed full distorted-wave calculations to investigate the role of the (elastic) optical distortion of the pion wave. We find considerable sensitivity to the degree of non-locality of the optical potential (i.e. Kisslinger vs. Laplacian) as well as to that of the impulse amplitude. As in the earlier study, the different interactions show rather different behavior at small angles. This is in contrast to elastic scattering, for which most theories give similar predictions for small angles. We have extended our study to a number of nuclear targets, and to excitations besides the electric dipole, including, for example, electric quadrupole.

The conventional multiple scattering theory, as applied to pion-nucleus reactions, is consistent only if one ignores the role of pions as a major component in the potential interactions among the target nucleons. Particularly glaring examples of inconsistency show up in applications to the absorption of pions by nuclei. We have been able to formulate a more satisfactory theoretical approach, which puts pion absorption and scattering on the same footing, and keeps proper track of pion contributions to scattering and to nuclear forces. This theory is cast in both relativistic (Green-function) and non-relativistic (Schrodinger equation) forms. The latter leads to a more practical system of equations for low-energy pion reactions. For the p-d system, we obtain an extension of the usual Faddeev equations; for large nuclei,
we generalize the optical-model theory. Detailed calculations using these methods are under way for the low-energy scattering and absorption of pions by deuterons, and should be completed shortly.

In connection with this, we have been reanalyzing the theory of zero-energy pion-nucleus scattering, which is used to interpret the energy-level shifts in pionic atoms.

A study of the expansion of the effective interaction between nucleons in light nuclei, in the linked-cluster theory, has been completed and published. We have found rather slow convergence of the G-matrix expansion through fourth-order. A small number of terms associated with a particular perturbation-diagrammatic structure appear to dominate the behavior. This points to possible methods (partial summations) for producing a more convergent expansion of the effective interaction.

The work on eigenvalue distributions for ensembles of Hamiltonians has been given new life by the recognition that the moment covariances, whose asymptotic values determine that there is ergodicity for the eigenvalue density, also fix all the two-point fluctuations. Thus we find easy methods for the evaluation of many of the GOE energy level-fluctuation measures (spacing-distribution widths and so forth) which are very difficult to deal with by the classical methods of Wigner, Dyson and Mehta. This method moreover extends readily to the ensembles of k-body interactions which are physically better motivated than GOE. We are now able to "see" why it is that essentially all ensembles give the same fluctuations, thereby resolving a problem discussed first and at length at the 1971 Albany Conference (there is still however a formal problem to be solved for a complete derivation of this result). We are able to show also that
the pattern of the energy-level fluctuations which is observed in the slow-neutron reactions should extend over the entire spectrum. This is in agreement with experiment and appears to be of real consequence with regard to the basic question of the information content of complex spectra. The covariance method extends also to give a theory for fluctuations in expectation values and strengths. A review is being prepared, in collaboration with S. S. M. Wong of the University of Toronto, and T. Brody, J. Flores and P. A. Mello of the National University of Mexico, on energy-level and related fluctuations.

The work on sum rules and strength distributions, described briefly in last year's report, has been greatly extended. The essential method here is that of evaluating statistically the response of the system when the Hamiltonian is perturbed in an appropriate way. The results for expectation values (and hence for sum rules) are expressed as an expansion in orthogonal polynomials for which the density is the weight function; because of the operation of the central limit theorem the expansion is very rapidly convergent, being usually dominated by the first two terms (the only ones not inhibited by the CLT) so that expectation values are basically linear in the energy. Similarly the microscopic strength function (spectroscopic factor, B(E2), etc) has a double-polynomial expansion and is basically bilinear in the energy (linear in the initial and final energies). A short account of the general theory has been published, as well as a detailed comparison with a (ds)-shell example for sum rules and strengths, in which the agreements for total strengths, centroids and widths are remarkably good. Two further papers show how single-state occupancies, calculable by these methods and measurable via nucleon transfer, can determine significant parameters of the effective
interaction, and, for example, how, by dealing with energy-weighted sum-rules, one can sometimes identify weak-coupling particle and hole states. General studies of giant multipole resonances and of the $\beta$-decay giant resonance are now under way.

Because the nucleon-nucleon interaction plays a fundamental role in nuclear physics, it is useful to determine which of its features are firmly fixed by elastic scattering data. The inverse scattering theory of Gel'fand-Levitan provides a formalism for going directly to a potential from the phase-shift information. Hence, an arbitrary form for the potential is avoided. This inverse approach has not, however, been exploited, because it is computationally involved. We have used several realistic phase-shift models of the $^1S_0$ partial-wave in order to study the numerical properties of the inverse-problem formalism. A paper is in preparation. Alternative procedures are being developed.

The bremsstrahlung reactions $\pi^\pm + p \rightarrow \pi^\pm + p + \gamma$, observed several years ago at the Lawrence Berkeley Laboratory 184" synchrocyclotron, failed to show the expected $\Delta(1236)$ resonance in the final state. An explanation for this probably involves the off-shell behavior of the pion-nucleon reaction, and calculations, in collaboration with Quang Ho-Kim of Laval University, Quebec, are underway to study this. The Chew-Low pion-nucleon model with crossing is being used as the preliminary model for $\pi^\pm p$ scattering. Numerical investigations of the structure and stability of the solutions of the Chew-Low non-linear integral equations are in progress. Accurate solutions of these equations will of course be of general use in low-energy pion studies.

Heavy-ion-induced multiparticle transfer processes probe selectively multinucleon correlations in nuclei. Motivated by: (a) the intriguing interplay between rotational and clustering degrees of freedom found among
the sd-shell nuclei; b) the capability of using the MP Tandem accelerator of the Rochester Nuclear Structure Research Laboratory to obtain high resolution (\(^6\text{Li},d\)) data; c) the availability of the exact-finite-range (EFR) distorted-wave-Born-approximation (DWBA) code LOLA for analyzing the data; d) a familiarity with group-theoretical techniques for carrying out suitable calculations, we are comparing the experimentally determined alpha-particle stripping strengths on light sd-shell targets with the corresponding shell-model predictions. To begin with, relying on known successes of the SU\(_3\)-SU\(_4\) model and the selectivity of alpha transfer processes within such a picture, simple theoretical predictions for alpha-particle stripping and pickup strengths among the stable sd-shell nuclei with A \(\leq 28\) were made. Detailed shell-model calculations which substantiate the simple predictions have since been carried out, in collaboration with N. Anantaraman, for \(^{21}\text{Ne} + \alpha \rightarrow ^{25}\text{Mg}\), and with P. Nanakos and M. Conze from Darmstadt, for \(^{18}\alpha + \alpha \rightarrow ^{22}\text{Ne}, ^{20}\text{Ne} + \alpha \rightarrow ^{24}\text{Mg}\). Systematic effects, such as a drop in stripping strength with increasing spin of the residual nucleus, are expected. Definite predictions, for example, of a weak transfer to the lowest \(J^\pi=4^+\) in the \((^{20}\text{Ne} + \alpha \rightarrow ^{24}\text{Mg})\) reaction, with the second \(J^\pi=4^+\) at only 1.5 MeV higher excitation expected to be strongly populated, suggest crucial experimental tests of the theory.

Together with H. E. Gove and N. Anantaraman of NSRL, we have studied the \((^{6}\text{Li},d)\) reaction on targets of \(^{16,18}\text{O}, ^{20,21,22}\text{Ne}, ^{24,25}\text{Mg}\). Typical distributions were taken in the angular range \(\theta_{\text{LAB}}\) of 5° to 60°, with resolution <70 keV. Since unnatural parity states were observed to be down in strength by a factor of 5–10 from the dominant ground-state-band transitions, we analyzed the data using the EFR-DWBA code LOLA assuming a direct cluster-transfer-mechanism. Characteristic L-shapes were obtained for L=0,2,4 transfers and could therefore be used to confirm and, in
favorable cases, assign spin values. For low-lying states the experimentally observed alpha-particle spectroscopic strengths agree well with predicted values. Absolute strengths relative to $^{16}O + \alpha + ^{20}Ne$ all come within a factor of two of the predicted value which, since the values themselves vary over two orders of magnitude, is considered quite acceptable. In $^{16,18}O + \alpha + ^{20,22}Ne$ the experimentally observed strengths to $J^T=0^+$ members of the ground state band are less than expected, which may reflect competition between shell closure and clustering correlations in these nuclei.

This work will continue; it is proposed to write a review article which, among other things, will include a systematic comparison of the experimentally measured and theoretically predicted relative strengths to low-lying ($\leq 5$ MeV) natural-parity states of the $A \leq 28$ sd shell nuclei. Absolute strengths (relative to $^{16}O + \alpha + ^{20}Ne$) will be given and systematics correlated with available ($^6Li,d$) data on targets with $28 \leq A \leq 90$.

To explore the effect of channel couplings and to fix in yet another way the intrinsic shapes of $^{27}Al$ and $^{28}Si$, we are, with R. N. Boyd, exploring structure effects which might explain the observed cross sections in the $^{28}Si(d,^3He)^{27}Al$ pickup reaction. The prolate $(\lambda\mu)=(1,0)$ shape in $^{28}Si$ for example, has no direct single-particle parentage with a $K^T=5/2^+$ band in $^{27}Al$, the ground-state configuration. The oblate $(0,12)$ shape in $^{28}Si$ on the other hand furnishes the appropriate link. It follows that the $7/2^+$ and $9/2^+$ members of the ground state band which cannot be populated directly via $d_{5/2}$, $s_{1/2}$ or $d_{3/2}$ pickup should therefore be describable via a two-step mechanism through inelastic excitations of the corresponding oblate shapes. Similar arguments apply to the two low-lying $K^T=1/2^+$ bands; one has direct single-particle coupling with only the prolate shape and the other with the oblate shape. Initial results indicate that both magnitudes and shapes of the observed angular distributions are consistent
with assigning oblate shapes to the ground state band of $^{28}\text{Si}$ and the $K_{\pi}=5/2^+$ and $1/2^+$ bands in $^{27}\text{Al}$.

The work reported here includes that of J.P. Drayer and J. P. Lavine and three graduate students, K. Kar, T. Mizutani, V. Potbhare, all at Rochester. There has been an active collaboration with persons in many laboratories including P. Goode at Rutgers University; O. Nalcioglu at the University of Wisconsin; P. Myhrer at SIN, Switzerland; S. S. M. Wong at the University of Toronto; T. Brody, J. G. Flores and P. A. Mello at the National University of Mexico; K. K. Mon at Cornell University; S. P. Pandya at Physical Research Laboratory, Ahmedabad, India; M. Bawin at the University of Liege; Quang Ho-Kim at Laval University; P. Manakos, M. Conze and H. Feldmeier at the Technische Hochschule, Darmstadt. It is clear also from the report that there has been a very fruitful collaboration with experimentalists at the Nuclear Structure Research Laboratory here.

During the year we have had a number of visitors for periods of one to several weeks, in some cases supported in part by the contract. These have included S. P. Pandya, Physical Research Laboratory, Ahmedabad; S. S. M. Wong, University of Toronto; A. Reitan of the University of Trondheim; J. G. Flores and P. A. Mello of the National University of Mexico.

Through our research in collaboration with S. S. M. Wong at the University of Toronto, and P. Goode at Rutgers University, we have effectively been aided by large grants of computer time by these institutions.

Dr. J. P. Draayer will probably leave the Department at the end of the summer to take up a faculty position at Louisiana State University. Dr. B. D. Chang of the University of Chicago will join the nuclear theory group in September.

The co-principle investigators, Prof. J.B. French and Prof. D.S. Koltun have devoted approximately two thirds of their effort to the research project, during the academic year.
Reports and Publications  
July 1, 1974 - June 30, 1975

1. F. Myhrer and D. S. Koltun  
**Elastic Pion-Deuteron Scattering in the Resonance Region as a Three-Body Problem**  

2. P. Goode and D. S. Koltun  
**The Average Effective Interaction in Linked Cluster Theory**  

**Shapes of (^6Li,d)** Angular Distributions as Signatures of Rotational Bands in ^29Si  

**Spectroscopic Strengths for ^6Li Induced Alpha Transfer on ^24Mg**  

5. J. P. Draayer  
**Alpha-Particle Spectroscopic Amplitudes for sd-Shell Nuclei**  

6. V. Potbhare and S. P. Pandya  
**Ground-state Occupancies and Effective Interactions in (sd)-Shell Nuclei**  
Submitted to Nuclear Physics (COO-2171-48).

7. K. K. Mon and J. B. French  
**Statistical Properties of Many-Particle Spectra**  
Submitted to Annals of Physics (COO-2171-46).

8. J. B. French  
**Aspects of Statistical Spectroscopy Relevant to Effective-Interaction Theory**  

**Polynomial Expansions for Excitation Strengths**  

**E2 Polynomial Strengths: A comparison with Shell-Model Results**  

11. J. P. Draayer, J. B. French, V. Potbhare, M. Prasad and S. S. M. Wong  
**Spectral Averaging for Single-Nucleon-Transfer Sum Rules**  

**The ^24Mg(^6Li,d)^28Si Reaction: Theory and Experiment**  
13. J. P. Draayer
Theoretical Predictions for Alpha Particle Spectroscopic Strengths
COO-2171-45 (not published).

14. J. P. Draayer and N. Anantaraman
The Nuclear SU3 Model
Based on lectures by J.P.Draayer (report NSRL-102).

15. M. Bawin and J. P. Lavine
Comment on Bose Condensation in Supercritical External Fields

16. Annual Technical Progress Report for Period October 1, 1973 -

Papers Contributed to Conferences and Meetings

1. O. Nalcioglu and D. S. Koltun
The Effect of Fermi Motion on Inelastic Pion Scattering
VI International Conference on High Energy Physics and Nuclear Structure,
Santa Fe, June 9-13, 1975 (selected for oral presentation).

2. P. Goode
Behavior of the G-matrix Expansion for the Average Effective Interactions
(Invited paper - collaboration with D. S. Koltun)
International Topical Conference on Effective Interactions and
Operators in Nuclei, Tucson, June 2-6, 1975.

3. J. P. Draayer and N. Anantaraman
Spectroscopic Amplitudes for Alpha Particle Stripping and Pickup Between
25Mg and 21Ne

4. N. Anantaraman, R. M. DeVries, J. P. Draayer, H. E. Gove and
J. P. Trentelman
A Study of the 16O(6Li,d)22Ne Reaction at 32 MeV

5. N. Anantaraman, R. M. DeVries, J. P. Draayer, H. E. Gove and
J. P. Trentelman
A Comparison Between 16O(6Li,d)20Ne and 16O(6Li,d)22Ne
Proceedings of the Conference on Reactions Between Complex Nuclei,
Shapes of $^{6}$Li,d Angular Distributions as Signatures of Rotational Bands in $^{29}$Si

7. N. Anantaraman, J. P. Draayer and H. E. Gove
Energy Dependence of $^{16}$O($^{6}$Li,d) Spectroscopic Factor Ratios

8. N. Anantaraman, J. P. Draayer, H. E. Gove and J. Toke
A Study of the $^{20}$Ne($^{6}$Li,d)$^{24}$Mg Reaction at 32 MeV

L-Admixtures in the $^{21}$Ne($^{6}$Li,d)$^{25}$Mg Reaction

10. N. Anantaraman, J. P. Draayer and H. E. Gove
Systematics of Alpha-Particle Strengths of Low-Lying Levels in the Light sd-Shell Nuclei
Proceedings of the Second International Conference on Clustering Phenomena in Nuclei, University of Maryland.

11. J. P. Draayer
Theoretical Predictions for Alpha Particle Spectroscopic Strengths

12. J. B. French
Spectral and Strength Distributions in Many-Particle Spectroscopy

13. J. B. French
Sum Rules and Strength Distributions
Invited Talk at Nuclear Physics Symposium (Mexico City, August 1974).

14. J. B. French (see publication #8 above)

15. J. P. Draayer (see publication #12 above)
International Conference on Nuclear Structure and Spectroscopy, Amsterdam (selected for oral presentation).