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EFFECTIVE CADMIUM CUTOFF ENERGIES

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OAK RIDGE NATIONAL LABORATORY

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Effective Cadmium Cutoff Energies

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Abstract

Effective cutoff energies for point $1/v$ absorbers inside of spherical and cylindrical cadmium filters have been calculated for thermal reactor neutrons. The neutron spectrum was assumed to consist of a Maxwellian plus a $1/E$ component, and the parameters varied were the thickness of filter, the Maxwellian temperature and the Maxwellian to $1/E$ flux ratio. Because of the sensitivity of the effective cutoff to Maxwellian flux parameters for thin filters it is recommended that filter thicknesses of about 40 mils be used. Forty mil filters show effective cutoffs at about 0.50 to 0.55 ev. for temperatures up to about 500°A (or about 0.045 ev). Effective cutoff energies for boron filters were also calculated for purposes of comparison. The cutoffs for cylindrical cadmium filters should be applicable to a properly designed experimental facility.

Introduction

For many years cadmium has been used as a filter for thermal neutrons in measuring thermal vs epithermal reaction rates for reactor spectrum neutrons. The effective Cd cutoff energy has been estimated as several tenths of an ev depending on the thickness of the filter. Hughes (1) estimated about 0.40 and 0.57 ev for 10 and 30 mil cadmium thicknesses, respectively, from a plot of the calculated relative number of neutrons transmitted vs energy. More recently Dayton and Pettus (2) calculated effective Cd cutoff energies for flat filters of large area for both a monodirectional and an isotropic flux. They assumed only a $1/E$ flux, with lower and upper limits of 0.025 ev and 10 mev (although the upper limit was not important) in their numerical integrations over the angle and energy. They defined the effective cutoff energy as the cutoff of a perfect (infinitely sharp) filter that allows the same total number of absorptions by the filtered sample as does the given cadmium filter. In their calculations they assumed that the filtered sample had a $1/v$ cross section.

The purpose of this paper is to calculate effective Cd cutoff energies, as defined by Dayton and Pettus, for a hypothetical $1/v$ absorber and for an idealized experimental configuration which approximates our actual irradiation conditions. Two geometrical configurations have been considered: spherical and cylindrical filters with point absorbers at the centers. The authors have used cylindrical filters (one cm diam. by two cm high) in reactor irradiations. While the samples and monitors were not points, they were very thin (so that negligible self-shadowing occurred) and rather small in area. It can be shown that if a point sample is moved a significant distance from the center of the cylindrical shell its corresponding effective Cd thickness is changed very little. For example, if the point is moved along the axis half way to one of the faces (i.e., to one quarter of the height from the nearer face) the effective Cd thickness is changed about 0.2%. Hence a thin

finite sample which approximates an integral of such points should show very nearly the same effective cutoff as does the central point. Actually an effective thickness could be calculated and a correction made, if desired. In the case of the spherical filters the correction would be larger. From an experimental point of view, however, the cylindrical filters are easier to make and to use, in addition to allowing an effective cutoff which is relatively insensitive to position of sample.

Effective cutoff energies were also calculated for boron filters in order to compare the variation of effective cutoffs with filter thickness and spectral parameters to that of cadmium.

In general real filters such as cadmium and boron only approximate our defined perfect filter. An additional factor that affects the usefulness of a filter is the sharpness with which the transition from opacity to transparency is made at the cutoff. A simple measure of this efficiency was made here by calculating the fraction of the total number of reactions occurring in the $1/v$ test sample at energies below the computed cutoff energies (FRACT). Thus the hypothetical perfect filter would yield a FRACT = 0. A comparison of this efficiency for cadmium and boron filters has been made in this paper. In general much lower values of this parameter FRACT were found for cadmium than for boron. This is of course consistent with the large resonance in cadmium giving rise to a sharp cutoff in contrast to the $1/v$ cross section dependence of boron.

Model, Assumptions and Calculations

The flux was assumed to be isotropic with respect to the point $1/v$ absorber and to consist of a Maxwellian thermal plus a $1/E$ epithermal spectrum. Following Westcott (3) the $1/E$ flux was assumed to go to zero (sharply) at five times the Maxwellian characteristic energy E_m (where $E_m = kT$). For a Maxwellian spectrum the derivative of the neutron density n

with respect to the energy $\underline{E}$ is given by the equation

$$\frac{dn}{dE} = \frac{2n}{\sqrt{\pi}} \frac{E^{1/2}}{E_m^{3/2}} e^{-E/E_m} \quad (1)$$

The differential of the total flux $\underline{F}_{\text{tot}}$ in any interval $\underline{dE}$ then becomes

$$dF_{\text{tot}} = v \frac{dn}{dE} dE = \frac{2nv}{\sqrt{\pi}} \frac{E^{1/2}}{E_m^{3/2}} e^{-E/E_m} dE + \phi_r dE/E, \quad 5E_m \leq E < \infty \quad (2)$$

$$= \frac{2nv}{\sqrt{\pi}} \frac{E^{1/2}}{E_m^{3/2}} e^{-E/E_m} dE, \quad 0 \leq E \leq 5E_m \quad (2a)$$

where ϕ_r is the epithermal or $1/E$ flux per unit $\ln E$. On changing the variable to $x \equiv E/E_m$, the differential of the total reaction rate $\underline{R}$ per atom for a $1/v$ absorber becomes

$$dR = \sigma_a dF_{\text{tot}} = \frac{\sigma_o v}{v} v \frac{dn}{dx} dx = \frac{2nv \sigma_o}{\sqrt{\pi}} x^{1/2} e^{-x} dx + \sigma_o x_o^{1/2} \phi_r dx/x^{3/2}, \quad (3)$$

where

$$\phi_r = 0 \text{ for } x < 5 \text{ (i.e., } E < 5E_m \text{)}.$$

Here the subscript zero refers to a standard energy or velocity (here taken as 2200 m/s).

On normalizing to unit ϕ_r and σ_o ,

$$dR' = \frac{dR}{\phi_r \sigma_o} = \frac{2r}{\sqrt{\pi}} x^{1/2} e^{-x} dx + x_o^{1/2} dx/x^{3/2}, \text{ for } 5 \leq x < \infty \quad (4)$$

$$= \frac{2r}{\sqrt{\pi}} x^{1/2} e^{-x} dx, \quad \text{for } 0 \leq x \leq 5 \quad (4a)$$

where $\underline{r} \equiv nv_o/\phi_r$, i.e., the ratio of the "conventional" thermal flux (3) to the resonance flux per unit $\ln E$.

Two geometrical configurations will be considered: spherical and cylindrical cadmium shells with point $1/v$ absorbers at their centers. As seen in Fig. 1 the only neutrons going through the spherical shells and also going through the point samples at the centers go along a diameter, and hence the thickness of cadmium traversed before reaching the samples is just the actual thickness of the spheres l_o . Effects of scattering* of neutrons by the shells would not in general be important and are not considered in these calculations. Using the same definition of effective Cd cutoff energy E_c as that used by Dayton and Pettus (2) the equation for the spheres becomes

$$\int_{E_c}^{\infty} dR^* = \int_0^{\infty} e^{-\sum_{Cd} l_o} dR', \quad (5)$$

where $\sum_{Cd} l_o$ is the macroscopic cross section of cadmium times the thickness of the spheres. On substituting equations (4) and (4a) into (5),

$$\frac{2r}{\sqrt{\pi}} \int_{x_c}^{\infty} x^{1/2} e^{-x} dx + \int_{x_c'}^{\infty} \frac{x_o^{1/2} dx}{x^{3/2}} = \frac{2r}{\sqrt{\pi}} \int_0^{\infty} x^{1/2} e^{-x} e^{-\sum_{Cd} l_o} dx + \int_5^{\infty} \frac{x_o^{1/2} e^{-\sum_{Cd} l_o} dx}{x^{3/2}} \quad (6)$$

where

$$x_c' = x_c \text{ if } x_c > 5$$

$$x_c' = 5 \text{ if } x_c \leq 5.$$

For values of x significantly greater than unity the first term on the left can be integrated analytically in series form, the indefinite integral becomes

* Scattering effects should cancel to a good approximation and show little net effect.

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$$\theta_L = \tan^{-1} \frac{D}{H}$$

$$\frac{l_0}{l_1} = \sin \theta$$

$$\frac{l_0}{l_2} = \cos \theta$$

Fig. 1. Spherical and Cylindrical Cadmium Filters.

$$\int x^{1/2} e^{-x} dx = -x^{1/2} e^{-x} \left(1 + \frac{1}{2x} - \frac{1}{(2x)^2} + \frac{3}{(2x)^3} \dots \right). \quad (7)$$

Equation (7) was used to evaluate this integral for values of x_c greater than 5. For values of x_c less than 5, the first integral on the left was divided into two parts: that from x_c to 5 and that above 5; the first part was evaluated numerically using Gaussian quadrature and equation (7) was used for the remainder of the integral.

The magnitude of the error made in using equation (7) is less than the last term in parentheses. For values of x_c greater than 5 this error is less than 0.3%. It is not recommended that this approximation be used for x_c values less than about 5.

The two integrals on the right were evaluated numerically using Gaussian quadrature. In the numerical integration finite upper limits had to be set, so all the four upper limits in equation (5) were set at $x = 1000$. It can be shown that the error made in setting these limits at 1000 is less than about 0.1% in the reaction rate with even the thicker cadmium shells, by expanding the exponential in the last integral, dropping all terms after the second and integrating analytically. It should be noted that the error is not as great as the values of the integrals above $x = 1000$, since these approximately cancel on the two sides of equation (6).

After evaluating the right hand side of (6), x_c (and hence E_c) was evaluated by successive approximations. The ORACLE (the Oak Ridge National Laboratory digital computer) and the IBM-704 at the Oak Ridge Gaseous Diffusion Plant were used for these calculations.

Also calculated in many cases was the fraction FRACT of total reactions of the central 1/v sample which occurred below the effective cutoff.

The form of the expression used for the cross section of Cd was a one level Breit-Wigner two parameter equation in which the parameters were adjusted to fit the absorption curve given in ENL-325 (4). Explicitly the equation was

$$\Sigma_{Cd}^{10} = \frac{5.076 \times 10^{-3} l_0}{\sqrt{E} [4(E-0.178)^2 + 0.013112]} \quad (8)$$

where l_0 is in mils of metallic Cd and E is in ev. The total absorption cross section of cadmium is essentially due to the 0.178 ev resonance of Cd^{113} , at least out to an energy of many ev. Hence the parameters in equation (8) are those for this nuclide at its natural isotopic abundance.

In the case of the cylindrical shells the transmission of neutrons, as seen in Fig. 1, depends upon the angle between the direction of the neutron and the axis of the cylinder as well as upon the neutron energy. In equation (8) l_0 must be replaced by $l_0/\cos\theta$ and $l_0/\sin\theta$ for values of θ between zero and $\theta_L = \tan^{-1}(D/H)$ and between the latter value and $\pi/2$, respectively. Here H and D are the height and diameter, respectively, of the cylinders. After changing the variable to $\mu = \cos\theta$, using approximation (7) and adding

$2x_0^{1/2} / \sqrt{1000}$ to both sides, the equation analogous to (6) becomes

$$\begin{aligned}
 & \frac{2x_0^{1/2}}{x_c^{1/2}} + \frac{2r}{\sqrt{\pi}} x_c^{1/2} e^{-x_c} \left(1 + \frac{1}{2x_c} - \frac{1}{(2x_c)^2} + \frac{3}{(2x_c)^3} \dots \right) = \frac{2x_0^{1/2}}{\sqrt{1000}} + \\
 & + \frac{2r}{\sqrt{\pi}} \int_{\mu(\pi/2)}^{\mu(\theta_L)} \int_0^5 \left(e^{-\sum Cd^{1/2}_0 / \sqrt{1-\mu^2}} \right) x^{1/2} e^{-x} dx d\mu \\
 & + \int_{\mu(\pi/2)}^{\mu(\theta_L)} \int_5^{1000} \left(e^{-\sum Cd^{1/2}_0 / \sqrt{1-\mu^2}} \right) \left(\frac{2r}{\sqrt{\pi}} x^{1/2} e^{-x} + \frac{x_0^{1/2}}{x^{3/2}} \right) dx d\mu \\
 & + \frac{2r}{\sqrt{\pi}} \int_{\mu(\theta_L)}^{\mu(0)} \int_0^5 \left(e^{-\sum Cd^{1/2}_0 / \mu} \right) x^{1/2} e^{-x} dx d\mu \\
 & + \int_{\mu(\theta_L)}^{\mu(0)} \int_5^{1000} \left(e^{-\sum Cd^{1/2}_0 / \mu} \right) \left(\frac{2r}{\sqrt{\pi}} x^{1/2} e^{-x} + \frac{x_0^{1/2}}{x^{3/2}} \right) dx d\mu \quad (9)
 \end{aligned}$$

Here the upper limit of energy in the numerical integration was taken as $x = 1000$ with the energy range being divided into 5 intervals* (i.e., 0-2, 2-5, 5-15, 15-100, 100-1000) and the range of θ into 2 (i.e., $0 - \tan^{-1}(D/H)$ and $\tan^{-1}(D/H) - \pi/2$). The integration was carried out using Gaussian quadrature

* Some calculations were also carried out using 10 energy intervals and these results agreed to within 0.1% with those using 5 intervals.

with 8 points per interval (of either x or μ) on the IBM-704 computer at the Oak Ridge Gaseous Diffusion Plant.

Also calculated was the fraction FRACT of the total reactions of the $1/v$ sample which occurred below the effective cutoff.

It should be noted that if the value of x_c is less than 5 the second expression on the left of equation (9) should not be given in the series expansion shown. Rather it should be evaluated, as in the sphere case, by using the following expression

$$\frac{2r}{\sqrt{\pi}} \left[\int_{x_c}^5 x^{1/2} e^{-x} dx + \sqrt{5} e^{-5} \left(1 + \frac{1}{10} - \frac{1}{(10)^2} + \frac{3}{(10)^3} \right) \right]$$

The value of x_c may then be obtained by successive approximations using numerical integration. This procedure was necessary in the calculations on both the thinner cadmium and boron filters.

In the case of boron filters the energy range (values of x) was divided into 8 intervals (0-2; 2-5; 5-15; 15-100; 100-1000; 1000-10,000; 10,000-100,000; 100,000-1,000,000) because of the slow decrease of the boron cross section with energy. The upper limits on all integrals here becomes 1,000,000 and the term added to both sides becomes $x_o^{1/2}/500$. For the boron computations the expression $\sum_{Cd}^1$ was replaced by

$$\sum_{B} M_B = 0.042039 M_B x_o^{1/2} / x^{1/2}, \quad (10)$$

where $\sum_{B} M_B$ is the macroscopic cross section of boron times the thickness, M_B being in the units of mg B/cm². The 2200 m/s cross section used here was 755 b.

Results and Discussion

Cd Spheres. Effective cutoff energies for spherical cadmium shells are presented in tabular form in Appendix A and are shown for three values of the Maxwellian to epithermal flux ratio $\underline{r}$ as a function of the characteristic Maxwellian energy $\underline{E}_m = kT$ in Fig. 2. Values are also given for 50% transmission (i.e., the energy at which the exponential transmission factor is equal to one-half) and for $\underline{r} = 0$ ($\underline{E}_m = 0.0253$ ev). It can be seen that the cutoff energies at 20 mils thickness and below are sensitive to the values of $\underline{r}$ as well as $\underline{E}_m$, even at lower values of the latter. For 30 mil and thicker filters the cutoffs are nearly independent of $\underline{r}$ at the lower Maxwellian temperatures. The values for 50% transmission are rather close to calculated cutoffs for the thicker filters and lower temperatures.

It would be desirable to have a cutoff near the assumed lower limit of the $1/E$ flux, i.e., $5\underline{E}_m$. However, this would require thin filters, which are very sensitive to the flux ratio $\underline{r}$. Hence the authors suggest that the best compromise would be achieved with filters of about 40 mil thickness.

Approximate calculations for cylindrical and cubic filters were made, based on the sphere calculations and the assumption that the transmission could be calculated simply by using effective thicknesses seen by the neutrons in the transmission exponential factor. The effective thickness for cylinders can easily be shown to be 1.13 and 1.22 times the actual thickness for an $\underline{H/D}$ (height/diameter) of 1 and 2, respectively. Then the approximate effective cutoffs for a cylinder would be given by the results for a sphere with a thickness of 1.13 and 1.22, respectively, times the actual thickness of the cylinders. Such approximate and accurate cutoffs for a cylinder ($\underline{H/D} = 2$)

Fig. 2. Effective Cadmium Cutoffs for Spherical Shells.

are given in Table I for two values of E_m (0.0253 and 0.05 ev). Here the first column gives the actual thickness, the second gives the effective thickness and the last column gives the energy at which 50% of the neutrons are transmitted. The corresponding thickness ratio (effective/actual) for a cubic filter is about 1.20.

Some calculations were also carried out under conditions which should have given results in agreement with those of Dayton and Pettus (2). Using the sphere model, described above, with no Maxwellian (i.e., $r = 0$) and with a lower limit of the $1/E$ flux at 0.025 ev the results should have agreed with their monodirectional flux calculations for very thin samples (see the third and fourth columns of Table II). The data of Dayton and Pettus were read from a graph but should be accurate enough to indicate a distinct difference*. Their resonance parameters for Cd were slightly different from those used in this paper, but hardly enough to account for the difference in results.

Reaction rates per unit energy vs E and total reactions below E have been calculated for the $1/v$ absorber as a function of energy for 10 and 40 mil cadmium filters, $E_m = 0.0253$ and $r = 20$. The solid curves in Figs. 3 and 3a show the fractions of the total number of reactions occurring at energy E per unit energy; the dotted curves show the fractions of the total reactions which occur between energies zero and E . The total number of reactions occurring

* A recent communication (2 June 1959) from W. G. Pettus contained some recalculated values and these (in the case of the beam flux) were much closer to the results obtained in this work than were Dayton's and Pettus' original values. (See Table II and ref. (2)).

Table I

Approximate vs Accurate Effective Cadmium Cutoffs for Cylinders
($H/D = 2$) for $\underline{r} = 20$

l_0	Eff. l cyl.	$E_m = .0253$			$E_m = .05 \text{ ev}$			$E_{1/2}$
		sphere	cylinder		sphere	cylinder		
			calc.	approx.		calc.	approx.	
10	12.2	.126	.148	(.160)	.224	.250	(.262)	.354
20	24.4	.375	.419	.430	.375	.401	.410	.405
30	36.6	.476	.502	.503	.448	.472	.481	.455
40	48.8	.518	.547	.550	.491	.522	.529	.506
60	73.2	.583	.615	.617	.567	.600	.606	.56
100	122.0	.680	.718	.723	.673	.712	.718	.66

Table II
Effective Cd Cutoff Energies for Monodirectional $1/E$ Flux

Cd Thickness (mils)	Flux Lower Limit in ev		
	This Paper <u>5×0.0253</u>	<u>0.025</u>	Dayton and Pettus ^a <u>0.025</u>
10	0.362	0.237	0.26
20	.431	.429	.34
40	.520	.520	.42
60	.584	.583	.47
100	.679	.679	.56

^a Values read from a graph (2).

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Fig. 3. Reaction Rates and Fractions of Total Reactions Below Energy E for Spherical Cadmium Shells.

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Fig. 3a. Reaction Rates and Fractions of Total Reactions Below Energy E for Spherical Cadmium Shells.

at all energies was normalized to unity. Actually of course there are more reactions in the 10 than in the 40 mil case. As seen in Fig. 3 about 60% of the total reactions occur below the cutoff with Maxwellian neutrons in the case of the 10 mil thickness. In the 40 mil case very few reactions involve Maxwellian neutrons and only a few percent of the reactions occur below E_c .

Boron Spheres. The calculated results for boron spheres are presented in Appendix B.

Cd Cylinders. The results for the cylindrical shells are shown in Fig. 4. The cutoffs are seen to be similar to but a little higher than those for the spherical filters, as expected. Again a thickness of about 40 mils is indicated in order that the cutoffs be insensitive to the flux ratio at the lower Maxwellian temperatures. Since the melting point of cadmium metal is 321°C (594°A) the higher temperatures may be unrealistic; however cadmium in some chemical form could be used as a filter at higher temperatures by cladding it with some high melting material. Values of E_c vs. Cd thickness are shown in Fig. 5 for $r = 0, 12$ and 30 for $E_m = 0.0253$ ev.

The fractions of the total reactions by the $1/v$ sample below the effective cutoff are shown in Fig. 6. For $r \neq 0$, this fraction is increased by increasing E_m at any thickness. At low E_m (i.e., around 0.0253 ev), the fraction increases with r only for thin filters; at the higher E_m values the fraction increases with r for all thicknesses. At $E_m = 0.075$ ev a distinct minimum occurs in the fraction curve at about 15 mils Cd. At $E_m = 0.1$ ev the minimum is broader and less distinct.

See Appendix C for calculated values for cylindrical Cd filters.

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Fig. 4. Effective Cadmium Cutoffs for Cylindrical Filters (Height/Diameter = 2).

Fig. 5. Effective Cadmium Cutoffs for Cylindrical Shells (Height/Diameter = 2) for $E_m = 0.0253$ ev.

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Fig. 6. Fractions of Reactions Below Cadmium Cutoff for Cylindrical Filters (Height/Diameter = 2).

Boron Cylinders. Figs. 7a and 7b show typical curves for E_c and FRACT vs. thickness, respectively, for boron and cadmium. As expected, there is no leveling off of E_c with increasing thickness in the case of Boron. Also as expected the boron shows a much larger FRACT than cadmium and shows the effects of the Maxwellian neutrons over a wider range of thicknesses. Also see Appendix D.

Conclusions

Effective cadmium cutoff energies have been calculated for a wide variety of Maxwellian neutron temperatures, ratios of Maxwellian to epithermal flux, and thicknesses of Cd filters. Because of the sensitivity to Maxwellian flux parameters for thin filters (less than about 30 mils) it is recommended that filters of about 40 mils be used in Cd ratio work. Depending on the exact conditions the effective cutoffs for 40 mil Cd lie between 0.50 and 0.55 ev for Maxwellian temperatures of 300 to 500^oA. The values presented here should hold to a good approximation for small samples centrally located inside cylindrical cadmium filters.

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Fig. 7a. Effective Cutoffs for Cadmium and Boron Filters for $E_m = 0.0253$ ev and $r = 20$.

Fig. 7b. Fractions of Reactions Below Effective Cutoffs for Boron and Cadmium Filters for $E_m = 0.0253$ ev and $r = 20$.

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Appendix A

Effective Cadmium Cutoffs E_c (in ev) for Spherical Filters
or Unidirectional Beams

$$r = nv_o/\phi_r =$$

E_m (ev)	l_o (mils Cd)	12		20		30	
		E_c	FRACT	E_c	FRACT	E_c	FRACT
.0253	10	0.141	0.459	0.126	0.585	0.118	0.678
	15	0.283	0.163	0.235	0.241	0.202	0.323
	20	0.396	0.082	0.375	0.096	0.351	0.115
	30	0.478	0.067	0.477	0.068	0.475	0.069
	40	0.519	0.068	0.518	0.069	0.518	
	50	0.552	0.071	0.552		0.552	
	60	0.583	0.073	0.583		0.583	
	80	0.635		0.635		0.635	
	100	0.680	0.074	0.680		0.680	
	120	0.719		0.719		0.719	
.0300	10	0.158	0.455	0.144	0.581	0.137	0.675
	15	0.285	0.162	0.245	0.239	0.220	0.319
	20	0.395	0.083	0.374	0.097	0.351	0.117
	30	0.477	0.068	0.476	0.069	0.474	0.070
	40	0.521		0.520		0.520	
	50	0.555		0.555		0.554	
	60	0.584		0.584		0.584	
	80	0.635		0.635		0.635	
	100	0.679		0.679		0.679	
	120	0.719		0.719		0.719	
.0400	10	0.196		0.186		0.181	
	15	0.302		0.278		0.265	
	20	0.391		0.372		0.357	
	30	0.474		0.469		0.462	
	40	0.516		0.514		0.511	
	50	0.551		0.549		0.547	
	60	0.581		0.578		0.578	
	80	0.634		0.633		0.632	
	100	0.680		0.680		0.679	
	120	0.720		0.720		0.719	

Appendix A (Cont'd)

$$r = nv_o / \phi_r$$

E_m (ev)	l_o (mils Cd)	12		20		30	
		E_c	FRACT	E_c	FRACT	E_c	FRACT
.0500	10	0.230	0.	0.222	0.492	0.219	0.572
	15	0.321	0.180	0.308	0.239	0.301	0.298
	20	0.387	0.131	0.373	0.167	0.363	0.206
	30	0.457	0.105	0.446	0.128	0.436	0.154
	40	0.502		0.493	0.111	0.481	
	50	0.540		0.532		0.521	
	60	0.574		0.566	0.094	0.559	
	80	0.630		0.625		0.620	
	100	0.676		0.672	0.083	0.670	
	120	0.716		0.714		0.711	
.0600	10					0.256	
	15			0.331		0.324	
	20			0.376		0.373	
	30			0.432		0.426	
	40			0.467		0.462	
	60			0.536		0.524	
	100			0.650		0.639	
.0750	10	0.288	0.286	0.286	0.343	0.286	0.380
	15	0.353	0.188	0.349	0.237	0.346	0.275
	20	0.392	0.185	0.386	0.227	0.383	0.261
	30	0.435	0.206	0.428	0.264	0.424	0.316
	40	0.471		0.460	0.267	0.457	
	60	0.529		0.514	0.246	0.508	
	100	0.623		0.606	0.192	0.592	
.1000	10	0.317	0.245	0.317	0.275	0.316	0.292
	15	0.365	0.211	0.364	0.245	0.364	0.266
	20	0.395	0.202	0.394	0.239	0.393	0.265
	30	0.438	0.197	0.434	0.239	0.433	0.270
	40	0.477		0.467		0.469	
	60	0.526		0.517		0.514	
	100	0.597		0.593		0.584	

Appendix B

Effective Boron Cutoffs E_c (in ev) for Spherical Filters
or Unidirectional Beams

E_m (ev)	I_o (mg./cm ²)	$r = nv_o/\phi_r =$					
		12		20		30	
		E_c	FRACT	E_c	FRACT	E_c	FRACT
.0253	20	0.0353	0.384	0.0349	0.399	0.0347	0.407
	50	0.0715	0.545	0.0692	0.587	0.0682	0.611
	100	0.1672	0.524	0.1372	0.599	0.1258	0.655
	150	0.5936	0.441	0.4227	0.490	0.2970	0.543
	200	1.422	0.399	1.207	0.422	0.9990	0.449
	300	3.864	0.373	3.743	0.377	3.600	0.383
	500	11.16	0.368	11.14	0.368	11.13	0.368
.0350	20	0.0432	0.356	0.0428	0.367	0.0427	0.373
	50	0.0858	0.538	0.0841	0.569	0.0832	0.586
	100	0.1681	0.571	0.1507	0.638	0.1442	0.682
	150	0.4192	0.492	0.2781	0.563	0.2295	0.627
	200	1.139	0.430	0.8512	0.472	0.6227	0.518
	300	3.615	0.382	3.344	0.393	3.046	0.406
	500	11.11	0.369	11.06	0.369	11.00	0.370
.0500	20	0.0541	0.324	0.0537	0.331	0.0536	0.336
	50	0.1060	0.517	0.1046	0.538	0.1039	0.550
	100	0.1879	0.603	0.1794	0.655	0.1754	0.685
	150	0.3177	0.561	0.2711	0.637	0.2531	0.690
	200	0.7782	0.485	0.4997	0.553	0.3777	0.618
	300	3.088	0.404	2.586	0.429	2.112	0.458
	500	10.92	0.371	10.74	0.373	10.50	0.376
.0750	20	0.0699	0.288	0.0696	0.293	0.0694	0.296
	50	0.1360	0.483	0.1348	0.497	0.1342	0.505
	100	0.2318	0.610	0.2246	0.641	0.2223	0.662
	150	0.3389	0.621	0.3168	0.680	0.3077	0.716
	200	0.5126	0.571	0.4321	0.648	0.4030	0.703
	300	2.174	0.454	1.520	0.506	1.049	0.561
	500	10.23	0.380	9.613	0.388	8.925	0.398
.1000	20	0.0840	0.263	0.0837	0.267	0.0835	0.269
	50	0.1626	0.455	0.1615	0.465	0.1610	0.471
	100	0.2728	0.599	0.2685	0.625	0.2664	0.639
	150	0.3826	0.641	0.3691	0.687	0.3629	0.713
	200	0.5230	0.623	0.4796	0.689	0.4606	0.730
	300	1.483	0.510	0.9321	0.584	0.7403	0.651
	500	9.155	0.395	8.027	0.413	6.881	0.435

Appendix C

Effective Cd Cutoffs E_c (in ev) for Cylinders (Height/Diam. = 2)
and Fractions of Reactions Below Cutoffs FRACT for Various
Values of the Characteristic Maxwellian Energy E_m

$$r = nv_0/\phi_r =$$

mils Cd	0		12		20		30	
	E_c	FRACT	E_c	FRACT	E_c	FRACT	E_c	FRACT
<u>$E_m = 0.0253$</u>								
10	.378	.0612	.173	.358	.148	.482	.137	.582
15	.419	.0640	.339	.116	.298	.161	.259	.218
20	.452	.0661	.431	.0751	.419	.0819	.404	.0907
30	.504	.0684	.503	.0692	.502	.0695	.501	.0698
40	.547	.0705	.547	.0705	.547	.0706	.547	.0708
60	.615	.0730	.615	.0729	.615	.0730	.615	.0728
100	.718	.0756	.718	.0757	.718	.0756	.718	.0755
<u>$E_m = 0.035$</u>								
10			.206	.346	.189	.466	.181	.565
15			.342	.1190	.308	.161	.283	.213
20			.429	.0789	.416	.0877	.401	.0997
30			.501	.0711	.499	.0726	.497	.0750
40			.545	.0715	.545	.0723	.544	.0733
60			.615	.0734	.614	.0736	.614	.0741
100			.719	.0760	.718	.0760	.718	.0762
<u>$E_m = 0.05$</u>								
10			.260	.301	.250	.398	.245	.479
15			.353	.1552	.338	.202	.329	.252
20			.415	.1218	.401	.153	.390	.188
30			.483	.1003	.472	.120	.462	.143
40			.531	.0915	.522	.105	.513	.122
60			.606	.0838	.599	.0910	.592	.100
100			.714	.0799	.712	.0826	.708	.0860

(Appendix C continued)

$$\underline{E_m = 0.075}$$

10	.427	.0419	.311	.249	.309	.304	.308	.344
15	.451	.0515	.375	.175	.370	.225	.366	.262
20	.473	.0572	.409	.198	.403	.248	.399	.289
30	.515	.0648	.454	.210	.446	.271	.441	.327
40	.552	.0684	.491	.203	.480	.266	.474	.326
60	.617	.0725	.555	.179	.540	.234	.529	.291
100	.719	.0758	.665	.140	.643	.177	.625	.219

$$\underline{E_m = 0.100}$$

10			.336	.234	.336	.267	.335	.285
15			.382	.211	.381	.247	.380	.270
20			.412	.204	.410	.244	.409	.271
30			.456	.197	.452	.241	.450	.274
40			.495	.191	.488	.239	.483	.274
60			.547	.225	.539	.279	.534	.325
100			.631	.236	.617	.304	.610	.370

$$r = nv_o / \phi_r = 20$$

mils Cd	E_c	FRACT	E_c	FRACT	E_c	FRACT	E_c	FRACT
	$E_m = 0.20$		$E_m = 0.40$		$E_m = 0.70$		$E_m = 1.0$	
10	.371	.170	.390	.087	.403	.048	.411	.033
15	.410	.185	.434	.102	.456	.061	.450	.036
20	.439	.198	.469	.115	.494	.071	.485	.040
30	.484	.220	.520	.133	.550	.082	.546	.049
40	.520	.236	.560	.145	.591	.089	.596	.057
60	.575	.259	.622	.163	.657	.100	.678	.071
100	.657	.286	.715	.192	.757	.119	.792	.090

mils Cd	$E_m = 3.0$		$E_m = 10.0$	
10	.387	.0051	.517	.0031
15	.429	.0060	.631	.0049
20	.467	.0071	.716	.0063
30	.537	.0098	.834	.0083
40	.598	.0126	.910	.0095
60	.702	.0181	.995	.0104
100	.859	.0273	1.073	.0103

Appendix C (Cont'd)

Miscellaneous Results

$$r = nv_o / \phi_r =$$

mils Cd	12		30		12		30	
	E_c	FRACT	E_c	FRACT	E_c	FRACT	E_c	FRACT
$E_m = 0.0253$					$E_m = 0.035$			
25	0.475	0.069	0.468	0.072	0.473	0.072	0.464	0.080
35	0.526	0.070	0.525	0.070	0.524	0.071	0.523	0.074
80	0.670	0.074	0.670	0.074	0.670	0.075	0.670	0.075
$E_m = 0.050$					$E_m = 0.075$			
25	0.454	0.108	0.432	0.161	0.433	0.208	0.422	0.316
35	0.508	0.095	0.488	0.131	0.472	0.207	0.458	0.329
80	0.665	0.081	0.656	0.091	0.612	0.157	0.579	0.252
$E_m = 0.100$								
25	0.436	0.200	0.431	0.274				
35	0.476	0.194	0.467	0.274				
80	0.592	0.238	0.575	0.361				

Appendix D

Effective Boron Cutoffs E_c (in ev) for Cylinders (Height/Diam. = 2)
and Fractions of Reactions below Cutoffs FRACT for Various
Values of the Characteristic Maxwellian Energy E_m

mg. B/cm ²	$r = nv_o/\phi_r$					
	12		20		30	
	E_c	FRACT	E_c	FRACT	E_c	FRACT
$E_m = 0.0253$						
20	.040	.422	.040	.440	.039	.450
30	.054	.500	.053	.528	.053	.544
50	.082	.564	.079	.613	.077	.642
80	.146	.567	.125	.618	.115	.669
100	.246	.549	.173	.623	.149	.673
150	.932	.463	.715	.517	.535	.575
200	2.098	.416	1.869	.444	1.631	.477
300	5.384	.383	5.283	.389	5.160	.397
500	15.322	.375	15.307	.375	15.292	.375
1000	61.227	.374	61.227	.374	61.227	.374
$E_m = 0.050$						
20	.062	.362	.061	.372	.061	.377
30	.082	.449	.082	.464	.081	.471
50	.120	.552	.118	.578	.117	.593
80	.176	.607	.169	.654	.166	.682
100	.221	.605	.205	.665	.198	.703
150	.454	.588	.332	.663	.295	.713
200	1.297	.511	.892	.584	.607	.657
300	4.620	.423	4.083	.454	3.532	.489
500	15.126	.380	14.973	.383	14.790	.388
1000	61.310	.374	61.310	.374	61.310	.374

Appendix D (Continued)

$$r = nv_o/\phi_r$$

mg. B/cm²

12		20		30	
E _c	FRACT	E _c	FRACT	E _c	FRACT

$$E_m = 0.075$$

20	.080	.324	.079	.331	.079	.334
30	.106	.410	.106	.420	.105	.425
50	.154	.522	.152	.540	.152	.550
80	.220	.607	.215	.639	.213	.656
100	.264	.628	.255	.671	.251	.695
150	.406	.627	.367	.679	.349	.723
200	.770	.600	.538	.681	.472	.736
300	3.532	.479	2.682	.539	1.994	.601
500	14.518	.392	13.968	.404	13.327	.419
1000	61.272	.374	61.272	.374	61.272	.375

$$E_m = 0.100$$

20	.096	.298	.095	.303	.095	.306
30	.128	.381	.127	.389	.127	.392
50	.184	.496	.183	.510	.182	.516
80	.260	.594	.257	.617	.255	.630
100	.309	.628	.303	.659	.300	.677
150	.442	.645	.418	.700	.408	.732
200	.638	.645	.557	.704	.526	.742
300	2.570	.539	1.698	.620	1.138	.696
500	13.494	.412	12.364	.437	11.159	.465
1000	61.189	.375	61.189	.376	61.067	.376

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