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COMPLEX NEGATIVE SIGNATURE TRAJECTORIES
AND THE POMERANCHUK THEOREM

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ABSTRACT

Within the framework of complex angular momentum methods, it is shown that amplitudes which violate the Pommeranchuk theorem require negative signature trajectories which are of the form $\alpha(t) = 1 \pm \text{const.} \sqrt{t} + O(t)$ near $t = 0$. There must be corresponding positive signature trajectories. The character of the singular surfaces with negative signature is discussed briefly.

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I. INTRODUCTION

In previous papers we have discussed Regge pole-and branch-point trajectories $\alpha(t)$ which are complex for real $t < 0$ due to left hand cuts in the t -plane of these functions.¹⁾ Such trajectories are of interest for the description of diffraction scattering.²⁾³⁾ In particular, pole-cut systems with complex trajectories can be used for the construction of rather general and physically meaningful amplitudes which imply different, constant total cross-sections for particle and antiparticle scattering.⁴⁾⁵⁾

It is the purpose of this note to show that amplitudes which violate the Pomeranchuk theorem⁶⁾ generally require negative signature trajectories with square-root branch points at $t = 0$. They are of the form $\alpha(t) = 1 \pm \text{const.} \sqrt{t} + O(t)$. It then follows that there must be corresponding positive signature trajectories, a fact which can also be proven directly.⁷⁾ The character of these singular surfaces of the continued partial wave amplitude will be discussed briefly.

II. ℓ -PLANE ARGUMENT

We denote the scattering amplitudes for particle and anti-particle scattering by $F(s, t)$ and $\bar{F}(s, t)$ respectively, and we introduce the combinations

$$F_{\pm}(s, t) = F(s, t) \pm \bar{F}(s, t). \quad (1)$$

Assuming constant asymptotic total cross sections given by σ and $\bar{\sigma}$, we have for $s \rightarrow \infty$

$$\text{Im } F_{\pm}(s, 0) \sim s \frac{\sigma \pm \bar{\sigma}}{16\pi}, \quad (2)$$

$$\text{Re } F_{\pm}(s, 0) \sim -\frac{2}{\pi} s (\lg s) \frac{\sigma - \bar{\sigma}}{16\pi}, \quad (3)$$

with $\text{Re } F_{+}(s, 0)$ being of the order $s(\lg s)^{-1}$.²⁾ These properties follow from the familiar dispersion relations.⁸⁾ From Eq. (3) and the general postulates of dispersion theory or of local field theory, we obtain the bounds

$$\frac{(\sigma - \bar{\sigma})^2}{4\pi^3 a} \leq \sigma_{el} \leq \sigma \quad (4)$$

for the elastic cross-section

$$\sigma_{el}(s) \sim \frac{16\pi}{s^2} \int_{-s}^0 dt |F(s, t)|^2. \quad (5)$$

Here $\underline{a}$ is a constant defining the maximal relevant angular momentum $L = \frac{1}{2}\sqrt{as} \lg s$ in the s -channel partial wave expansion of $F(s, t)$ for large values of s .⁴⁾⁹⁾⁶⁾

Let us consider the continued partial wave amplitudes $F_{\pm}(t, \lambda)$. These functions have the usual analytic properties in the complex manifold (t, λ) .²⁾ In particular, they satisfy the continued elastic unitarity condition for $4m^2 \leq t < t_i$ (t_i = first inelastic threshold). This condition forbids certain kinds of singularities. We ask: What are the characteristic features of allowed, isolated singularities of $F_{\pm}(t, \lambda)$ near $(t, \lambda) = (0, 1)$ which are required by the special conditions (2), (3) and (4)? We assume that $F_{\pm}(t, \lambda)$ has a finite number of such singularities at $\lambda = \alpha_k(t)$, $K = 1, 2, \dots$. Irrespective of the character of these singularities, each one can contribute a term to the asymptotic expansion of $F_{\pm}(s, t)$ for $s \rightarrow \infty$ which, except for logarithmic factors, is of the form $s^{\alpha_k(t)}$. Equation (2) requires then that $\alpha_k(0) \leq 1$, with $\alpha_k(0) = 1$ for at least one trajectory of each signature. We assume now that the trajectories $\alpha_k(t)$ are regular functions near $t = 0$, so that the relevant ones are of the form

$$\alpha_k(t) = 1 + \alpha'_k t + O(t^2). \quad (6)$$

They give diffraction peaks which shrink at most like $(\lg s)^{-1}$.

But if we calculate $\sigma_{e1}(s)$, we find that negative signature trajectories of the form (6) give rise to a logarithmically increasing contribution to σ_{e1} . The logarithmic shrinkage is not sufficient for the bound (4).

We are led to consider negative signature trajectories which are not regular at $t = 0$, but have a branch point there. A priori, singular surfaces $\alpha(t)$ of the partial wave amplitudes have only branch points related to certain physical thresholds in the t -channel. However, there may be additional branch points due to the cross over of two or more trajectories of the same character.¹⁾²⁾ Under these circumstances it is possible to prevent these branch points from being inherited by the continued partial wave amplitude itself, where they should not occur. In general, a singular surface near $t = 0$ can be of the form¹⁾

$$\alpha(t) = \alpha(0) + \sum_{i=0}^{\infty} \beta_i t^{i/n}, \quad (7)$$

where n is an integer and the β_i are constants. It corresponds to n trajectories $\alpha_1(t) \dots \alpha_n(t)$ which cross at $t=0$ and form the branches of the multivalued function (7).

Suppose that the negative signature amplitude $F_-(t, \lambda)$ has singular surfaces of the type (7) with $\alpha(0) = 1$. These then give rise to terms in the asymptotic expansion of $F_-(s, t)$ for $s \rightarrow \infty$ which, as far as the power law is concerned, are of the form

$$s^{1+ct^{1/n}} + \dots \quad (8)$$

with $n \geq 2$ and $c > 0$. But since we have assumed constant total cross sections, the amplitude $F_-(s, t)$ has the bound¹⁰⁾

$$|F_-(s,t)| < O(s^{1+\sqrt{at}}) \quad (9)$$

for $s \rightarrow \infty$ and $0 \leq t < 4m^2$. Consequently only the case $n = 2$ is allowed, because otherwise the bound (9) would be exceeded for small, positive values of t . With $n = 2$, we have pairs of singular surfaces

$$\alpha_{1,2}(t) = 1 \pm \text{const} \sqrt{t} + O(t) \quad (10)$$

as a characteristic feature of negative signature amplitudes which violate the Pomeranchuk theorem.

III. S-CHANNEL ARGUMENT

It may be of interest to indicate an alternate but related argument for the presence of singular surfaces of the type (10) in the negative signature amplitude. For large values of s , we can write $F(s,t)$ in the form of a Bessel transform:^{4,5,11)}

$$F(s,t) \sim s \int_0^1 d\xi \psi(\xi, \lg s) J_0(\xi \sqrt{-at} \lg s), \quad (11)$$

where $\underline{a}$ is again determined by the maximal angular momentum $L = \frac{1}{2} \times \sqrt{as} \lg s$. There are corresponding expressions for $\bar{F}$ and for $F_{\pm}$. If we can show that the function $\psi_{-}(\xi, \lg s) = \psi(\xi, \lg s) - \bar{\psi}(\xi, \lg s)$ has support for non-zero values of ξ in the interval $0 \leq \xi \leq 1$ and in the limit $s \rightarrow \infty$, then it follows that the asymptotic expansion of $F_{-}(s,t)$ contains terms of the form

$$s^{1 \pm i \xi \sqrt{-at}} \quad (12)$$

as far as the power law is concerned. In the complex angular momentum plane, these terms correspond again to negative signature trajectories of the type (10). As a contrasting example, we mention that a function like

$$\psi(\xi, \lg s) \sim \xi \lg s e^{-\xi^2 \lg s}$$

would give rise to an asymptotic term with the power behavior $\sim s^{1+at}$

corresponding to a trajectory which is regular at $t = 0$.

From Eqs. (5) and (11) we obtain

$$\sigma_{e1}(s) \sim \frac{32\pi}{a} \int_0^1 d\xi \xi^{-1} \rho(\xi, \lg s), \quad (13)$$

where

$$\rho(\xi, \lg s) \equiv (\lg s)^{-2} |\psi(\xi, \lg s)|^2. \quad (14)$$

using the inequality (4) we conclude then that

- 1) the lower bound implies that the function $\rho(\xi, \lg s)$ must be positive for some values of ξ in the interval $0 \leq \xi \leq 1$.
- 2) the upper bound requires that the support of ρ is not restricted to the point $\xi = 0$ for $\lg s \rightarrow \infty$. Because the integral (5) must converge, we find that ρ has to vanish for $\xi \rightarrow 0$, and hence it must be positive for some nonzero values of $\xi \leq 1$.

It remains to determine the signature of the positive contributions to ρ . From Eqs. (3) and (11) we find

$$\int_0^1 d\xi \operatorname{Re} \psi_-(\xi, \lg s) = - \frac{2}{\pi} \frac{\sigma - \bar{\sigma}}{16\pi} \lg s, \quad (15)$$

and hence

$$(\lg s)^{-2} |\operatorname{Re} \psi_-(\xi, \lg s)|^2$$

must make a finite contribution to $\rho(\xi, \lg s)$ for $\lg s \rightarrow \infty$. But then it follows from the previous arguments that $(\lg s)^{-1} \text{Re} \psi_-$ has support for $\lg s \rightarrow \infty$ and nonzero values of $\xi \leq 1$. This is what we wanted to show.

With the dispersive part of $F_-(s, t)$ having terms of the form (12) in the asymptotic expansion, it follows from the dispersion relations that the same must be true for the absorptive part. Hence also $\text{Im} \psi_-(\xi, \lg s)$ has support for positive $\xi \leq 1$. We can use the presence of asymptotic terms like

$$s^{1+\text{const.}\sqrt{t}} \quad (16)$$

in the high energy limit of $\text{Im} F_-(s, t)$ in order to show that also the positive signature amplitude must have trajectories of the type (10): It can be seen from the partial wave expansions that $\text{Im} F(s, t)$ and $\text{Im} \bar{F}(s, t)$ are positive for $0 \leq t \leq 4m^2$. Because $\text{Im} F_-(s, t)$ has asymptotic terms like (16) for $s \rightarrow \infty$, these positivity constraints require that corresponding terms are present in $\text{Im} F_+(s, t)$.

IV. DISCUSSION

We see that amplitudes which violate the Pomeranchuk theorem and which instead satisfy Eqs. (2) and (3), must have complex trajectories of the form $\alpha(t) = l \pm \text{const.} \sqrt{t} + O(t)$ in the negative and the positive signature partial wave amplitudes. Although our arguments are not completely rigorous from the mathematical point of view, they are sufficient as far as the usual complex angular momentum methods are concerned.

There remains the question of the character of the trajectories (10) as singular surfaces of $F_{\pm}(t, \lambda)$. We restrict ourselves here to a few remarks. It is well known that simple pole surfaces in F_{-} lead to an increasing elastic cross-section ($\sim \lg \lg s$), and hence violate the bound (4). Logarithmic branch points are possible near $t = 0$. But in order to be effective, they generally must appear in forms like

$$\lg[(\lambda - \alpha_1(t))(\lambda - \alpha_2(t))]$$

so that $F_{-} \rightarrow \infty$ for $\lambda \rightarrow \alpha_{1,2}$. Terms of this type are incompatible with the continued elastic unitarity condition for $4m^2 \leq t < t_i$, because branch point trajectories usually do not have thresholds at $t = 4m^2$ as do pole trajectories. (2,5)

We had found earlier, that rather natural pole-cut systems with square-root branch points can give rise to acceptable amplitudes¹⁾⁵⁾; these branch points are given in terms of the poles $\alpha_{1,2} = 1 \pm \text{const} \sqrt{t} + 0(t)$ by

$$\alpha_{c1,2}(t) = n\alpha_{1,2}\left(\frac{t}{n^2}\right) - n + 1 = \alpha_{1,2}(t) + 0(t) . \quad (17)$$

In the neighborhood of $(t, \lambda) = (0, 1)$, they degenerate to forms like

$$F_-(t, \lambda) \propto \int_0^1 d\xi \frac{\rho_-(\xi)}{[(\lambda-1)^2 - a\xi^2 t]^{\frac{1}{2}}} \quad (18)$$

We must impose the subsidiary condition

$$\int_0^1 d\xi \frac{\rho_-(\xi)}{\xi} = 0 \quad (19)$$

in order to prevent a singularity at $t = 0$. The terms (18) should be considered as a superposition of Regge poles and associated square root branch points which only coincide near $t = 0$ as indicated in Eq. (17). At $t = 4m^2$, the pole trajectories must develop a threshold, while the branch points do this only at $t = t_i$. The pole-cut relationship can be such that the unwanted branches of the pole trajectories are removed from the physical sheet for $t > 0$.¹⁾⁵⁾ In Eq. (18) we assume that the weight function $\rho_-(\xi)$ is sufficiently well behaved so as not to alter the character of the singularities.

In the s-channel, the amplitude (18) gives rise to expressions like Eq. (11) with

$$\text{Im } \psi_-(\xi, \lg s) = \rho_-(\xi),$$

(20)

$$\text{Re } \psi_-(\xi, \lg s) = \frac{2}{\pi} \lg s \left[\int_0^\xi dx \frac{\rho_-(x)}{x} \right]$$

Of special interest is the question of oscillations in the differential cross section, which can be present with amplitudes of the type discussed here.¹³⁾ We hope to come back to this problem elsewhere.

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