1148

AUGUST 1975

MATT-1148

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AND THE MHD THEORY OF PLASMA
FLOW INTO AN ABSORBING WALL

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The Bohm Stability Criterion and the MHD Theory of
Plasma Flow Into an Absorbing Wall

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ABSTRACT

The consistency of the Bohm stability criterion and the MHD theory of plasma flow into an absorbing wall is shown. This provides strong evidence for the validity of an MHD analysis of the complex problem of plasma flow into a divertor.

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I. INTRODUCTION

When a plasma flows into an absorbing wall, an electrostatic sheath is established to equalize the electron and ion currents. Bohm¹ derived a so-called stability criterion for this electrostatic sheath. He assumed zero ion temperature, conservation of plasma particles, constant electron temperature, no collisions, and one-dimensional flow. Bohm showed that a spatially oscillatory electric field will penetrate into the body of the plasma for $v < (T_e/m)^{1/2}$. T_e is the electron temperature, m the mass associated with a positive charge, and v the flow velocity of the plasma into the sheath. Consequently, to have a proper electrostatic sheath, he required $v \geq (T_e/m)^{1/2}$.

The Bohm stability criterion has been a standard method for estimating the flow velocity of plasma into absorbing walls^{1,2} and has been used as a test of the validity of other flow models.³ However, it has not been recognized that the Bohm criterion, in a generalized form, must be identically satisfied. Oversatisfying the ordinary Bohm criterion,³ $v \geq (T_e/m)^{1/2}$, will be shown to imply a finite ion pressure, that is a finite second moment of the ion distribution function.

If a proper electrostatic sheath forms between impinging plasma and the wall, the plasma flow could be calculated using the MHD equations. These equations have not been used for this problem. There is the question of whether the solution derived from the MHD equations automatically satisfies the Bohm criterion. This paper will demonstrate that the more powerful MHD theory of plasma flow agrees with the predictions of the Bohm stability criterion for the simple problems to which the stability criterion applies. This provides strong evidence for the validity of MHD methods for the more complex problems of plasma flow in a divertor.

II. A GENERALIZED BOHM CRITERION

To show the close relation of the Bohm stability criterion and MHD theory we will rederive the criterion for the general one-dimensional case ignoring momentum transfer with the neutrals.

Bohm pointed out in his paper on sheath stability that the sign of the derivative of the charge density with respect to the electrostatic potential controls the stability of the sheath. The potential is determined by Poisson's equation

$$d^2\phi/dx^2 = -4\pi e(n_+ - n_e) .$$

Near a place where the electron density, n_e , equals the positive charge density, n_+ ,

$$n_+ - n_e \approx [d(n_+ - n_e)/d\phi]\phi$$

with the potential assumed zero at the place where $n_e = n_+$. As $n_e \rightarrow n_+$, the Taylor expanded form of the charge density implies that if

$$\lim_{n_e \rightarrow n_+} [d(n_+ - n_e)/d\phi] > 0 , \phi(x) \text{ is oscillatory}$$

$$\lim_{n_e \rightarrow n_+} [d(n_+ - n_e)/d\phi] < 0 , \phi(x) \text{ converges exponentially} .$$

Actually the case where the derivative vanishes as $n_e \rightarrow n_+$ will be the one of interest. We neither want $\phi(x)$ oscillatory, nor do we want it to exponentially converge to zero in the plasma. Some electric field is required in the plasma to couple the electron and ion motions.

The equation for the ions which is valid for $d(nv)/dx \neq 0$ and holds even if the distribution is non-Maxwellian is

$$d(mn_+ v^2 + P_+)/dx + en_+ d\phi/dx = - R$$

R is the drag and thermal force on the electrons due to the ions⁴ and P_+ is the ion pressure. The electrons obey the equation

$$dP_e/dx - en_e d\phi/dx = + R$$

To evaluate $dn_e/d\phi$, n_e is used as the independent variable.

This gives

$$dn_e/d\phi = en_e/[dP_e/dn_e - R dx/dn_e]$$

Similarly for the ions

$$dn_+/d\phi = - en_+[d(mn_+ v^2 + P_+)/dn_+ + R dx/dn_+]$$

These results imply

$$\lim_{n_e \rightarrow n_+} \frac{d(n_+ - n_e)}{d\phi} = - en \frac{d(mnv^2)/dn + dP/dn}{[d(mnv^2 + P_+)/dn + R dx/dn][dP_e/dn - R dx/dn]}$$

where $P = P_e + P_+$. To look at the term $d(mnv^2)/dn$ further, let

the fluid current $nv = \Gamma$. Then

$$d(mnv^2)/dn = -mv^2 + m(d\Gamma^2/dn)/n.$$

In the MHD theory, $d\Gamma^2/dn$ is negative. This sign will be assumed. We will also assume $dP_{+,e}/dn > 0$ and that $|R dx/dn| < |dP_e/dn|$. The assumption on R can be shown to hold if the standard form is taken.⁴ Then as $n_e \rightarrow n_+$, $d(n_+ - n_e)/d\phi \leq 0$ provided

$$v^2 \geq \frac{1}{m} \frac{dP}{dn} - \frac{1}{n} \left| \frac{d\Gamma^2}{dn} \right|.$$

This is the generalized Bohm criterion. The generalized criterion reduces to Bohm's original statement when his assumptions are made.

III. THE MHD THEORY

The MHD equation is derived by setting $n_e = n_+$ and adding the electron and ion equations. This gives

$$d(mnv^2 + P)/dx = 0.$$

Letting P_0 be the pressure at the place where v is zero, the fluid current $\Gamma = nv$ is

$$\Gamma^2 = n(P_0 - P)/m.$$

If a form for $P(n)$ is assumed, a graph of Γ^2 versus n can be drawn. See Fig. 1. The dashed portion of the curve is not obtainable for $d\Gamma/dx$ monotonic.⁵

The MHD results must be differentiated to put them in a form that resembles the Bohm criterion.

$$d\Gamma^2/dn = [(P_0 - P) - n dP/dn]/m$$

Using $d\Gamma^2/dn = - |d\Gamma^2/dn|$, we find

$$v^2 = \frac{1}{m} \frac{dP}{dn} - \frac{1}{n} \left| \frac{d\Gamma^2}{dn} \right|$$

Consequently, the solution of the MHD equation marginally satisfies the Bohm criterion as required for the existence of finite but non-oscillatory electric fields deep in the plasma.

This work was supported by U.S.AEC Contract AT(11-1)-3073.

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⁵This problem is similar to the fluid mechanics problem of nozzle flow. See p. 347 of L. D. Landau and E. M. Lifshitz, Fluid Mechanics, (Addison-Wesley, Reading, MA., 1959).

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Fig. 1. The Current versus Density. In MHD theory, the fluid current, Γ , is a definite function of the density, n , if the pressure is a known function of density. However, the dashed portions of the Γ versus n curve are not obtainable under usual conditions.

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