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IS-849

Engineering and Equipment (UC-38)
TID 4500, June 1, 1964

UNITED STATES ATOMIC ENERGY COMMISSION

Research and Development Report

A FINITE DIFFERENCE MODEL
OF THE PULSE COLUMN

by

Stanley B. Collins and L. E. Burkhart

July, 1963

Ames Laboratory
at
Iowa State University of Science and Technology
F. H. Spedding, Director
Contract W-7405 eng-82

IS-849

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Printed in USA. Price \$ 1.00 Available from the

Office of Technical Services
U. S. Department of Commerce
Washington 25, D. C.

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IS-822

A FINITE DIFFERENCE MODEL OF THE PULSE COLUMN*

Stanley B. Collins and L. E. Burkhart

ABSTRACT

Equations for a stagewise model of the pulse column were developed using the approach of Burkhart and Fahien (2). The model represents operation of the column in the mixer-settler region at steady state. Linear equilibrium lines, nonequilibrium contact, zero organic recycle and equal holdup in all stages were assumed for the analysis. The resulting set of equations was solved using finite difference calculus. A closed form general solution resulted.

The general solution was programed for digital computer evaluation because of the complex nature of the expressions required for the evaluation of the arbitrary constants in the general solution. The model predicted performances close to those obtained experimentally by Burkhart and Fahien (2).

* This report is based on an M.S. thesis submitted by Stanley B. Collins July, 1963, to Iowa State University, Ames, Iowa.

INTRODUCTION

The application of mechanical agitation to liquid-liquid extraction equipment made a significant improvement in the efficiency of such equipment. This improvement has led to increased application of this versatile separations process.

The pulse column, shown schematically in Figure 1, is one such mechanically agitated extraction device and is the subject of this investigation. The pulse column was first described in a patent (9) by Van Dijck in 1935, but industrial scale application of the pulse column was not made until 1949 when the United States Atomic Energy Commission used the pulse column for uranium processing (6). Interest in the pulse column soon spread.

Sege and Woodfield (6) studied the general flow characteristics of the pulse column. Their observations, shown in Figure 2, indicate five regions of operation:

- A. A region of total flooding due to insufficient pulsation.
- B. A region of complete coalescence known as the mixer-settler region.
- C. A region of incomplete coalescence known as the emulsion region.
- D. A region of instability due to local flooding.
- E. A region of complete flooding due to emulsification.

Figure 1. Diagram of pulse column

Figure 2. Operating characteristics of the pulse column

Operation of the pulse column is usually confined to regions 2 and 3.

Edwards and Beyer (4) derived an equation for predicting conditions of insufficient pulsation and for calculating the volume of liquid recycled for any operating conditions.

A steady-state analog computer model of the pulse column based of empirical correlations of the flooding behavior and the over-all height of a transfer unit (HTU) for operation in the emulsion region was developed by Troutman and Consiglio (7). Tests of the model could not be justified because the correlations employed in the development of the model were only accurate to ± 20 per cent.

Using a stagewise approach, Burkhardt and Fahien (2) mathematically analyzed the pulse column. A steady-state model for mixer-settler operation was developed; it involved two "stage" lines, two "operating" lines, and two stage efficiencies, one each for the upstroke and the down stroke of the pulse generator. Stage concentrations were stepped off in a manner analogous to the McCabe-Thiele Method.

DiLiddo and Walsh (3) studied the dynamic characteristics of a mathematical model of the pulse column. A digital computer was used to analyze a set of difference-differential material balance equations, generalized flooding and efficiency equations, and control system equations.

Biery and Boylan (1) studied the transient start-up behavior of the pulse column. Nine mathematical models of

the pulse column were proposed to describe the transient response of the pulse column operating in the emulsion region. The effect of pulsation on material balance equations was ignored in the derivation of the models. Comparison of experimental measurements with the mathematical models showed that a nonequilibrium, non-uniform mixing model gave the best column simulation. Time to steady state and response curves were estimated reasonably well if thin subdivisions of the column were used.

SCOPE OF THE INVESTIGATION

The intent of this study was to develop a closed solution to a steady state model of the pulse column. The stagewise approach of Burkhart and Fahien (2) was used to formulate the model.

Because the stagewise nature of the pulse column produces a finite interstage jump in liquid concentration, the analysis of the model lends itself to treatment by the methods of finite difference calculus. The general solution obtained by the methods of finite difference calculus was programed for a digital computer. The model was compared with experimental data collected by Burkhart and Fahien (2).

MATHEMATICAL MODEL

The approach of Burkhart and Fahien (2) was used to derive equations to represent the pulse column. This analysis used a stagewise approach to the problem of simulation. A stage is defined as that volume of the column located between two sieve-plates. The following assumptions were made in this analysis of the column:

1. Mixer-Settler operation
2. Nonequilibrium contact
3. Zero organic recycle
4. Constant fluid properties
5. Linear equilibrium lines
6. Constant and equal holdup in all stages
7. An element of fluid passes through only one stage per half cycle of the pulse generator
8. Perfectly mixed phases

The following nomenclature will be used in the analysis:

E = Organic phase holdup per stage (liters)

R = Aqueous phase holdup per stage (liters)

G = Organic phase interstage flow rate (liters/cycle)

H = Aqueous phase interstage flow rate (liters/cycle)

A = Aqueous phase recycle flow rate (liters/cycle)

x = Aqueous phase concentration (grams/liter)

y = organic phase concentration (grams/liter)

m = distribution coefficient

e_u = upstroke efficiency

e_d = down stroke efficiency

Primed quantities (') refer to conditions immediately after an upstroke before the down stroke begins. Unprimed quantities refer to conditions immediately after the down stroke but before the beginning of the upstroke.

General Stage

Each surge of the liquid was considered to take place in two steps. First the two phases are mixed and the solute transferred from one phase to the other, then the discontinuous phase coalesces and mixes with the holdup remaining in the stage. Letting concentrations with an overbar be transient concentrations and examining Figure 3, the following equations for the n^{th} stage of the column can be derived through material balances:

I. Upstroke

$$A. \quad Ax_{n-1} + (R-A)x_n + Gy_{n-1} = G\bar{y}'_n + Rx'_n \quad (1)$$

$$B. \quad G\bar{y}'_n + (E-G)y_n = Ey'_n \quad (2)$$

II. Down stroke

$$A. \quad (H + A)x'_{n+1} + Ey'_n = (H + A)\bar{x}_n + Ey_n \quad (3)$$

$$B. \quad (H + A)\bar{x}_n + (R-H-A)x'_n = Rx_n \quad (4)$$

Figure 3. Distribution of phases for a pulse column operating in the mixer-settler region

If an asterisk designates equilibrium concentration, the following distribution equations hold:

$$\bar{y}'_n * = mx'_n \quad (5)$$

$$y_n = m\bar{x}^*_n. \quad (6)$$

The deviation from equilibrium may be specified by Murphree plate efficiencies for the upstroke and down stroke respectively:

$$e_u = \frac{\bar{y}'_n - y_{n-1}}{\bar{y}'_n * - y_{n-1}} \quad (7)$$

$$e_d = \frac{\bar{x}_n - x'_{n+1}}{\bar{x}_n * - x'_{n+1}}. \quad (8)$$

Combination of Equations 5 and 7 gives

$$\bar{y}'_n = e_u mx'_n + (1-e_u)y_{n-1}. \quad (9)$$

Similarly Equations 6 and 8 give

$$\bar{x}_n = \frac{e_d}{m} y_n + (1-e_d)x'_{n+1}. \quad (10)$$

If E, R, G, H, A, m, e_u , and e_d are specified, nine variables remain and six independent equations must be written to relate them.

Elimination of x_n , x_{n-1} , $\bar{x}_n$, $\bar{y}'_n$ between Equations 1, 4, 7, and 8 gives

$$\begin{aligned}
& \frac{(R-A)(H+A)(1-e_d)}{R} x'_{n+1} + \left[\frac{A(H+A)(1-e_d)}{R} + \frac{(R-A)(R-H-A)}{R} \right. \\
& \left. Ge_u^{m-R} \right] x'_n + \frac{A(R-H-A)x'_{n-1}}{R} = \frac{(R-A)(H+A)e_d}{Rm} y_n + \left[\frac{-Ge_u}{Rm} \right. \\
& \left. \frac{(R-A)(H+A)e_u}{Rm} \right] y_{n-1} \quad (11)
\end{aligned}$$

Combination of Equations 2, 3, 7, 8 to eliminate x_n , y'_n and $\bar{y}'_n$ gives

$$[(H+A)e_d]x'_{n+1} + [mGe_u]x'_n = \left[\frac{(H+A)e_d + G}{m} \right] y_n + [-G(1-e_u)]y_{n-1} \quad (12)$$

Equations 11 and 12 form a set of linear difference equations with constant coefficients. Equations 11 and 12 may be written in operator notation as follows:

$$[K_1E^2 + K_2E + K_3]x'_n = [K_4E + K_5]y_n \quad (13)$$

$$[K_6E^2 + K_7E]x'_n = [K_8E + K_9]y_n \quad (14)$$

"E" is the difference operator defined as $E(x_n) = x_{n+1}$. The K_i 's are the coefficients of the terms of Equations 11 and 12.

By elimination of (5) y_n between Equations 13 and 14

$$[l_1E^3 + l_2E^2 + l_3E + l_4]x'_n = 0 \quad (15)$$

is obtained.

l_i 's are as follows:

$$l_1 = K_1K_8 - K_4K_6 \quad (16)$$

$$l_2 = K_2K_8 + K_1K_9 - K_4K_7 - K_5K_6 \quad (17)$$

$$l_3 = K_3K_8 + K_2K_9 - K_5K_7 \quad (18)$$

$$l_4 = K_3K_9. \quad (19)$$

Let the roots of characteristic equation of Equation 15 be α , β , and γ . Then x'_n is given by

$$x'_n = c_1\alpha^n + c_2\beta^n + c_3\gamma^n. \quad (20)$$

Similarly, y_n is given by

$$y_n = c_4\alpha^n + c_5\beta^n + c_6\gamma^n. \quad (21)$$

Substitution of Equations 20 and 21 into Equation 14 and equating like powers of α , β , and γ gives the following relationships:

$$c_1 = c_4f(\alpha) \quad (22)$$

$$c_2 = c_5f(\beta) \quad (23)$$

$$c_3 = c_6f(\gamma) \quad (24)$$

Where $f(t)$ is defined as

$$f(t) = \frac{K_8t + K_9}{K_6t^2 + K_7t}. \quad (25)$$

Equation 20 becomes

$$x'_n = c_4f(\alpha)\alpha^n + c_5f(\beta)\beta^n + c_6f(\gamma)\gamma^n. \quad (26)$$

Return to the original Equations 1, 2, 3, 4, 7, and 8 and eliminate x_n , x'_{n+1} , y'_n , and $\bar{y}'_n$ to obtain the following

pair of equations:

$$\begin{aligned}
 & \frac{(H+A)(1-e_d)(R-A)}{mGe_u + R} x_{n+1} + \left[\frac{(R-H-A)(R-A)}{mGe_u + R} - \frac{R+A(H+A)(1-e_d)}{mGe_u + R} \right] x_n \\
 & + \frac{A(R-H-A)}{mGe_u + R} x_{n-1} = - \left[\frac{(H+A)e_d}{m} + \frac{(H+A)(1-e_d)Ge_u}{mGe_u + R} \right] y_n - \\
 & \frac{(R-H-A)Ge_u}{mGe_u + R} y_{n-1}
 \end{aligned} \tag{27}$$

$$\begin{aligned}
 & \frac{(H+A)e_d(R-A)}{mGe_u + R} x_{n+1} + \left[\frac{A(H+A)e_d + mGe_u(R-A)}{mGe_u + R} \right] x_n + \frac{AmGe_u}{mGe_u + R} x_{n-1} \\
 & = \left[\frac{(H+A)e_d}{m} + \frac{G - Ge_u(H+A)e_d}{mGe_u + R} \right] y_n - \left[\frac{G(1-e_u) + mG^2e_u^2}{mGe_u + R} \right] y_{n-1}
 \end{aligned} \tag{28}$$

When written in operator form Equations 27 and 28 become

$$[M_1 E^2 + M_2 E + M_3] x_n = [M_4 E + M_5] y_n \tag{29}$$

$$[M_6 E^2 + M_7 E + M_8] x_n = [M_9 E + M_{10}] y_n \tag{30}$$

The M_i 's are coefficients of Equations 27 and 28. Applying the technique used to solve Equations 13 and 14, the following solutions are obtained for Equations 29 and 30.

$$x_n = d_1 \delta^n + d_2 \epsilon^n + d_3 \lambda^n \tag{31}$$

$$y_n = d_4 \delta^n + d_5 \epsilon^n + d_6 \lambda^n \tag{32}$$

Substituting 31 and 32 into 30 and equating like powers of δ , ϵ , and λ gives

$$d_1 = d_4 g(\delta) \quad (33)$$

$$d_2 = d_5 g(\epsilon) \quad (34)$$

$$d_3 = d_6 g(\lambda) \quad (35)$$

Where $g(t)$ is defined as

$$g(t) = \frac{M_9 t + M_{10}}{M_6 t^2 + M_7 t + M_8} . \quad (36)$$

Equation 31 becomes

$$x_n = d_4 g(\delta) \delta^n + d_5 g(\epsilon) \epsilon^n + d_6 g(\lambda) \lambda^n. \quad (37)$$

Since y_n is a unique concentration, Equations 21 and 32 give

$$c_4 \alpha^n + c_5 \beta^n + c_6 \gamma^n = d_4 \delta^n + d_5 \epsilon^n + d_6 \lambda^n. \quad (38)$$

Thus, equations relating concentration to stage number become

$$y_n = c_4 \alpha^n + c_5 \beta^n + c_6 \gamma^n \quad (39)$$

$$x'_n = c_4 f(\alpha) \alpha^n + c_5 f(\beta) \beta^n + c_6 f(\gamma) \gamma^n \quad (40)$$

$$x_n = c_4 g(\alpha) \alpha^n + c_5 g(\beta) \beta^n + c_6 g(\gamma) \gamma^n. \quad (41)$$

The constants c_4 , c_5 , and c_6 can be evaluated if three concentrations are specified.

End-Sections

Dispersed phase can enter the bottom end-section from two distinct sources, the sieve-plate and the light phase inlet nozzle. On the down stroke, a transient concentration would be generated by the light phase leaving the inlet nozzle and passing up through the continuous heavy phase. Another transient concentration would be generated by the dispersed heavy phase passing down through the light phase holdup. The streams with transient concentrations then mix to give steady state concentrations. The following material balance equations are obtained as a result of this analysis:

$$(H+A)x'_2 + (E-G/2)y'_1 = (H+A)\bar{x}_1 + (E-G/2)\bar{y}_1 \quad (42)$$

$$(G/2)y_s + (R-H-A)x'_1 = (R-H-A)\bar{x}_1 + (G/2)\bar{y}_1 \quad (43)$$

$$(E-G/2)\bar{y}_1 + (G/2)\bar{y}_1 = Ey_1 \quad (44)$$

$$(H+A)\bar{x}_1 + (R-H-A)\bar{x}_1 = Rx_1 \quad (45)$$

The bottom end-section is stage one.

During the upstroke, the only source of dispersed phase is the light phase inlet nozzle. The following equations apply:

$$(G/2)y_s + (R-A)x'_1 + Ax'_1 = (G/2)\bar{y}'_1 + Rx'_1 \quad (46)$$

$$(G/2)\bar{y}'_1 (E-G)y_1 = (E-G/2)y'_1. \quad (47)$$

Equations 9 and 10 apply in the following form:

$$\bar{y}'_1 = e_u mx'_1 + (1-e_u)y_s \quad (48)$$

$$\bar{x}_1 = (e_d/m)\bar{y}_1 + (1-e_d)x'_2. \quad (49)$$

$$\bar{x}_1 = (e_d/m)\bar{y}_1 + (1-e_d)x'_2 \quad (50)$$

The system of 9 equations contains 11 independent variable concentrations. Assume that y_s , the concentration of the entering light phase, and x'_1 , the concentration of the leaving heavy phase, are known; the following relationships may be derived from Equations 42 through 50:

$$x_1 = \frac{(R+Ge_um/2-A)x'_1 - (e_uG/2)y_s}{R - A} \quad (51)$$

$$y_1 = (G/2)(1-e_u)y_s + (G/2)e_umx'_1 - (S_4/S_1)[(G/2)y_s + (R-H-A)x'_1 - S_3x_1] + S_6x_1 / (S_5-S_2S_4/S_1) \quad (52)$$

$$x'_2 = [(G/2)y_s + (R-H-A)x'_1 - S_2y_1 - S_3x_1]/S_1 \quad (53)$$

The S_i 's are defined as follows:

$$S_1 = \frac{(R-H-A)(1-e_d) - 2(R-H-A)e_d(E-G/2)R(1-e_d)}{(m+G/2)GT} \quad (54)$$

$$S_2 = \left[\frac{(R-H-A)e_d + G/2}{m} \right] \left[\frac{2E - 4E(E-G/2)(R-H-A)e_d}{G} \right] \frac{1}{mG^2T} \quad (55)$$

$$S_3 = \left[\frac{E-G/2 + (H+A)e_d}{m} \right] \frac{2(E-G/2)R}{GT} \quad (56)$$

$$S_4 = \left[\frac{E-G/2 + (H+A)e_d}{m} \right] \left[\frac{R(1-e_d) - e_d(H+A)}{1} \right] \quad (57)$$

$$S_5 = \left[\frac{E-G/2 + (H+A)e_d}{m} \right] \left[2E(R-H-A)e_d \right] + G-E \quad (58)$$

$$S_6 = \left[\frac{E-G/2 + (H+A)e_d}{m} \right] R/T \quad (59)$$

$$T = \frac{2(R-H-A)e_d(E-G/2)}{mG} - \frac{(H+A)e_d}{m} \quad (60)$$

Evaluation of x_1 , y_1 , and x'_2 allows c_4 , c_5 , and c_6 to be calculated from Equations 39, 40, and 41.

The top end-section analysis is similar to the bottom end-section analysis. Material balance equations over the top end-section give:

I. Upstroke

$$A. \quad R\bar{x}_N + Gx_{N-1} = R\bar{x}'_N + Gy_N \quad (61)$$

$$B. \quad R\bar{x}'_N + Ax_{N-1} + (H/2)x'_A = (R+H/2+A)x'_N \quad (62)$$

II. Down stroke

$$A. \quad (R-H/2)x'_N + (H/2)x'_A = R\bar{x}_N \quad (63)$$

Equilibrium and efficiency equations combine to give

$$y'_N = me_u \bar{x}'_N + (1-e_u)y_{N-1} \quad (64)$$

Stage N is the top end-section.

If x'_N , y_{N-1} , and x_{N-1} are assumed known, combination of the set of Equation 61 through 64 gives

$$y_N = \frac{(R+A/2)x'_N - (A/2)x_{N-1} + \left[\frac{G/2 + R(1-e_u)}{me_u} \right] y_{N-1}}{(G/2 + R/me_u)} \quad (65)$$

$$x'_A = \frac{(H+A)x'_N + Gy_N - Gy_{N-1} - Ax_{N-1}}{H} \quad (66)$$

Flow Characteristics

Edwards and Beyer (4) developed an equation relating operating variables to the volume of liquid recycled. Their equation, modified to allow for continuous removal of the aqueous phase from the bottom end-section and the following equation is

$$A = \frac{a\pi d^2}{4} \cosh \theta - G/2 - H/2 \quad (67)$$

where

a = pulse amplitude (cm)

d = column diameter (cm)

$$\theta = \frac{4G}{a\pi^2 d^2} \quad (68)$$

DIGITAL COMPUTER PROGRAM

The use of a digital computer to perform calculations is justified because of the extreme complexity of the algebraic expressions needed to evaluate the constants in the general solution. Figure 4 shows a block diagram of the digital computer program. In the following discussion the sub-routines will be referred to by their call names as shown in Figure 4.

FLOW calculates the recycle flow rate using Equation 67. The aqueous holdup is also calculated by subtracting the organic phase holdup from the volume of the stage.

COEFNT calculates the constants associated with Equations 13, 14, 15 and 30. The constants needed for the evaluation of the roots of Equation 15 by Cardan's Formulas (8) are calculated from constants associated with Equation 15.

At this point a test is made to insure that the roots of Equation 15 will not be complex. Complex roots do not correspond to any real model.

CUBIC computes the roots of Equation 15. A trigonometric form of Cardan's Formulas was used to evaluate the roots.

All powers of the roots from one to N are computed and

Figure 4. Block diagram of computer program

and stored by POWER. N is the number of stages in the column including the bottom and top end-sections.

Bottom end-section calculations are made using Equations 51 through 60 by BOUND. Concentrations from the bottom end-section and Equations 39, 40, and 41 are used to evaluate c_4 , c_5 , and c_6 .

COLUMN calculates the compositions throughout the remainder of the column including the top end-section.

MATBAL check the material balance around each stage and over the whole column.

To obtain a solution to the problem it was necessary to know the concentrations of the entering solvent and the leaving raffinate streams. APPROX, based on linear interpolation, allows calculations to be made for any pair of inlet concentrations. Three or four iterations were usually sufficient to reach the specified inlet concentrations.

CONCLUSIONS AND RECOMMENDATIONS

The results of three digital computer calculations are shown in Figures 5, 6, and 7. The computer output is shown in Tables 1, 2, and 3, Appendix A. Input data for each of the calculations was chosen so that the data would coincide as closely as possible with the experimental data of Burkhart and Fahien (2). Because of the nonlinear equilibrium line, and approximate stage efficiencies inherent with the experimental data, close agreement of the experimental and mathematical model was not expected. However, Table 4, Appendix A shows that the model predicts separations in the correct range even though a linear equilibrium line was assumed for calculation purposes.

A better fit of a nonlinear equilibrium line might be obtained by dividing the column into a small number of parts, each with a different distribution constant and calculating each division separately so that the streams entering and leaving the top of a division correspond to those leaving and entering the bottom of the division above it. Thus the assumption of a constant distribution coefficient in the analysis does not restrict the model to simulation of systems involving linear equilibrium lines.

The usefulness of finite difference calculus for the analysis of the pulse column is apparent. The closed form solution resulting from this analysis is particularly

Figure 5. Plot of calculated concentrations for a pulse frequency of 16.8 cycles/min.

Figure 6. Plot of calculated concentrations for a pulse frequency of 24.88 cycles/min.

Figure 7. Plot of calculated concentrations for a pulse frequency of 30.3 cycles/min.

useful for obtaining estimates of the performance of pulse columns having a large number of stages. Once the constants in the general solution have been calculated, the concentrations at the top of the column may be calculated without the necessity to calculate intermediate column concentrations.

The usefulness of the model for digital computer control is limited by the assumption of constant holdup throughout the column. Any upset of flow rates would cause the local holdup to fluctuate within the column. Analysis of the case involving variable holdup using finite difference calculus would lead to difference equations with variable coefficients. Solution of such equations would require numerical analysis.

Stage efficiencies need further investigation. Burkhart and Fahien (2) made the only experimental measurements which differentiated between upstroke and down stroke efficiencies. No methods have been developed for predicting upstroke and down stroke efficiencies.

NOMENCLATURE

- A = aqueous recycle per cycle, liters/cycle.
 E = organic phase holdup per stage, liters.
 G = organic phase interstage flow rate, liters/cycle.
 H = aqueous phase interstage flow rate, liters/cycle.
 N = top end-section of column.
 R = aqueous phase holdup per stage, liters.

 a = pulse amplitude (displacement from one extreme position to the other), cm.
 f = pulse frequency, cycles/min.
 m = distribution coefficient, no units.
 x = aqueous phase concentration, gm/liter.
 y = organic phase concentration, gm/liter.
 e = Murphree stage efficiency.

Subscripts:

- A = flow stream at the top of the column.
 N = top end-section of column.
 n = typical stage of the column.
 $1, 2, \dots$ = stage numbered from the bottom end-section.

Superscripts:

- $(\prime)$ = prime, refers to conditions at the end of an upstroke, but before the down stroke begins.
 $()$ = no prime, refers to conditions at the end of a down stroke but before the upstroke begins.

(-) = transient composition.

(*) = equilibrium composition.

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APPENDIX A

Tables of Calculated Data

Table 1. Comparison of calculated and experimental concentrations in gm./liter

Pulse Frequency	Experimental y_N	Calculated y_N	Experimental x'_1	Calculated x'_1
24.88	84.1	79.7	69.6	73.8
30.30	80.5	80.0	72.5	73.0
16.80	66.2	66.8	17.5	26.1

Table 2. Calculated data for a pulse column operating at a pulse frequency of 16.80 cycles/min.; concentrations in gm./liter

<div> <div>G= 0.05430 H= 0.03210 E= 0.05430</div> <div>A= 0.00433 R= 0.22767 ED= 0.07000</div> <div>EU= 0.2200 DIA= 0.68640 DIST= 0.76200</div> <div>AMP= 0.12700 FREQ= 16.80000 N= 16</div> <div>SLOPE= 0.60000</div> </div>				
<div> <div>ALPHA= 1.00435 BETA= 0.99796 GAMMA= 0.07835</div> <div>C4= 959.18245 C5=-960.76074 C6= -1.08073</div> </div>				
N	X(N)	Y(N)	XPRM(N+1)	DEL(N)
		1.00000	26.07779	
1	26.46950	2.54885	28.43305	-0.00000
2	29.24725	6.85688	35.18481	0.00000
3	36.06215	11.11117	42.33596	-0.00000
4	43.22187	15.37816	49.54711	0.00000
5	50.43708	19.66312	56.79152	-0.00000
6	57.68531	23.96644	64.06740	0.00001
7	64.96510	28.28840	71.37470	0.00000
8	72.27630	32.62903	78.71350	-0.00000
9	79.61900	36.98838	86.08390	-0.00000
10	86.99330	41.36639	93.48620	0.00001
11	94.39960	45.76342	100.92040	-0.00000
12	101.83770	50.17935	108.38670	0.00001
13	109.30790	54.61439	115.88510	-0.00000
14	116.81020	59.06851	123.41570	-0.00000
15	124.34490	63.54175	130.97920	0.00000
		66.80871	137.39984	-0.00000

Table 3. Calculated data for a pulse column operating at a pulse frequency of 24.88 cycles/min.; concentrations in gm./liter

<div> <div> G= 0.03979 A= 0.00607 EU= 0.0675 AMP= 0.12700 SLOPE= 0.60000 </div> <div> H= 0.04208 R= 0.24218 DIA= 0.68640 FREQ= 24.88000 </div> <div> F= 0.03979 ED= 0.08600 DIST= 0.76200 N= 16 </div> </div>				
<div> ALPHA= 1.00000 C4= 102.16420 </div> <div> BETA= 0.90472 C5= -104.02474 </div> <div> GAMMA= 0.09813 C6= -0.20627 </div>				
N	X(N)	Y(N)	XPRM(N+1)	DEL(N)
		0.10000	73.76482	
1	74.01597	8.03108	80.35115	0.00000
2	80.97097	17.01716	88.65395	0.00000
3	89.26283	25.13212	96.40526	0.00000
4	96.96084	32.47239	103.44149	0.00000
5	103.94460	39.11311	109.80961	0.00000
6	110.26482	45.12108	115.57119	0.00000
7	115.98304	50.55657	120.78381	0.00000
8	121.15641	55.47415	125.49975	0.00000
9	125.83686	59.92317	129.76637	0.00000
10	130.07136	63.94828	133.62644	0.00000
11	133.90237	67.58987	137.11873	0.00000
12	137.36836	70.88447	140.27827	0.00000
13	140.50412	73.86515	143.13676	0.00000
14	143.34110	76.56183	145.72290	0.00000
15	145.90777	79.00156	148.06264	0.00000
		79.66437	148.99996	-0.00000

Table 4. Calculated data for a pulse column operating at a pulse frequency of 30.30 cycles/min.; concentrations in gm./liter

G=	0.03267	H=	0.03455	E=	0.03267
A=	0.01339	R=	0.24930	ED=	0.10000
EU=	0.0200	DIA=	0.68640	DIST=	0.76200
AMP=	0.12700	FREQ=	30.30000	N=	16
SLOPE=	0.60000				
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ALPHA=	1.00000	BETA=	0.00154	GAMMA=	0.22640
C4=	101.05833	C5=	-100.61836	C6=	-0.22586
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N	X(N)	Y(N)	XPRM(N+1)	DEL(N)	
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		0.10000	72.96777		
				-0.00000	
1	73.02825	10.29507	79.93252		0.00000
2	80.42871	19.26603	88.11305		0.00000
3	88.64224	27.32703	95.90055		0.00000
4	96.39617	34.53833	103.01467		0.00000
5	103.46570	41.13335	109.44954		0.00000
6	109.85712	47.03368	115.25566		0.00000
7	115.62333	52.35303	120.49123		0.00000
8	120.82275	57.14864	125.21160		0.00000
9	125.51050	61.47210	129.46728		0.00000
10	129.73676	65.36990	133.30402		0.00000
11	133.54696	68.83395	136.76301		0.00000
12	136.98204	72.05203	139.88148		0.00000
13	140.07894	74.90821	142.69293		0.00000
14	142.87096	77.48320	145.22759		0.00000
15	145.38810	79.80466	147.51273		0.00000
				-0.00000	
		79.97927	148.50115		