**NUCLEAR PHYSICS RESEARCH
QUARTERLY REPORT
JANUARY, FEBRUARY, MARCH
1960**

THE STAFF OF NUCLEAR PHYSICS RESEARCH

APRIL 20, 1960

HANFORD LABORATORIES

HANFORD ATOMIC PRODUCTS OPERATION
RICHLAND, WASHINGTON

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HW-64866

UC-34, Physics and Mathematics
(TID-4500, 15th Ed.)

NUCLEAR PHYSICS RESEARCH QUARTERLY REPORT
JANUARY, FEBRUARY, MARCH, 1960

By

The Staff of Nuclear Physics Research Operation

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H. L. Henry	Supervisor Experimental Reactors
B. R. Leonard, Jr.	Supervisor Experimental Nuclear Physics

Physics and Instrument Research and Development
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April 20, 1960

HANFORD ATOMIC PRODUCTS OPERATION
RICHLAND, WASHINGTON

Work performed under Contract No. AT(45-1)-1350 between the
Atomic Energy Commission and General Electric Company

Printed by/for the U. S. Atomic Energy Commission

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Printed in USA. Price \$2.75. Available from the
Office of Technical Services
U. S. Department of Commerce
Washington 25, D. C.

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HW-61181	April, May, June 1959
HW-62727	July, August, September 1959
HW-63576	October, November, December 1959

EXPERIMENTAL NUCLEAR PHYSICS

Subthreshold Fission - U^{234}

R. H. Odegarden

The fission cross section of U^{234} was measured in the vicinity of the known total cross-section resonance at 5.20 ev. The DR crystal spectrometer was used with a resolution of $1.06 \mu\text{sec/m}$ using the Be $[11\bar{2}0]$ crystal planes. A BF_3 chamber was used to detect the incident flux. It was calibrated against a boron chamber which in turn had been calibrated using a U^{235} sample and assuming a fission cross section of 590.8 barns at 2200 m/sec. The two foil samples of U^{234} were loaded into a multiple plate fission chamber to measure the fissions. The two foils were prepared on 5 mil platinum backings and deposited on 7.92 cm^2 circular deposits. The mass of the sample was determined to be 0.886 mg/cm^2 of U^{234} from the measured alpha activity in a calibrated low geometry alpha counter and the known isotopic content.

The isotopic content of this sample was reported (ORNL) to be:

<u>U^{234} Sample</u>	
U^{234}	93.01%
U^{235}	5.27%
U^{238}	1.72%

The measured resonance in U^{234} can be seen in Figure 1 where $E_0 = 5.20 \pm 0.05 \text{ ev}$. A resonance in U^{235} occurs at 4.84 ev and using a value of 20 barns for the fission cross section from BNL-325⁽¹⁾ and the isotopic content it falls short by a value of four or five of explaining the observed resonance peak height. Thus the observed resonance must be in U^{234} and the correction due to the U^{235} contribution is small.

Figure 1
 ^{234}U Fission Cross Section

TABLE I

<u>E (ev^o)</u>	<u>σ_{fo} obs (barns)</u>	<u>Γ_f obs (mv)</u>	<u>Area (b-ev)</u>
5.20	4.86	450	2.38
± 0.05	± 0.34	± 50	± 0.37

Table I shows the measured resolution and doppler broadened fission cross section. The observed fission width is only the resolution of the spectrometer due to the small fission width. Also shown is the area under a fission cross section vs. energy curve.

The fission cross section can be obtained from $\text{area} = \frac{\pi}{2} \sigma_{fo} \Gamma^{(2)}$ by assuming the total width equal to 30.4 ± 6 mv from total cross-section data given in BNL-325.⁽¹⁾ By this area method of analysis the fission cross section is equal to 50 ± 8 barns.

The resonance parameters were also obtained by shape analysis.⁽³⁾ The doppler width, Δ , was calculated to be 48.16×10^{-3} ev assuming the target temperature to be 300 K. The doppler broadened fission width was obtained from tables of the integral $\psi(x, t)^{(4)}$ and obtained approximately from the equation $\frac{\Gamma \Delta}{\Gamma} = 0.62 + 1.66 \frac{\Delta}{\Gamma}^{(3)}$ with the same results, $99 \pm \text{mv}$, using 30.4 mv for the total cross-section width. The doppler broadened fission cross section was found by correcting for the resolution of the spectrometer. The resolution width of the spectrometer was calculated to be 352 mv and the measured value was 450 mv. The average resolution width of 401 ± 50 mv was used as the best value of the two methods. The doppler broadened fission cross section was found equal to 20 ± 5 barns. The value $\frac{\sigma_{fo} \Delta}{\Gamma} = 0.403$ was obtained from interpolating the tables of the integral $\psi(x, t)^{(4)}$ giving a value of 49 ± 9 barns for the fission cross section. From the two values the fission cross section of 50 ± 9 barns was taken as the best value. The total cross section was calculated to be $(7.35 \pm 2.40) \times 10^4$ barns using the total cross-section data given in BNL-325.⁽¹⁾ The fission width was then obtained from $\frac{\Gamma_f}{\Gamma} = \frac{\sigma_{fo}}{\sigma_{to}}$ to be $20^{+22}_{-10} \mu\text{v}$.

The fast neutron fission cross section exhibits a threshold (energy at which the cross section is one-half of the plateau value) at about 0.6 mev.⁽¹⁾ The level spacing, D , is given to be 12 ± 3 ev⁽⁵⁾ and according to Hill and Wheeler⁽⁶⁾ the average fission width well below threshold is given by

$$\begin{aligned} \Gamma_f &\approx \frac{D}{2\pi} e^{\frac{-2\pi (E_{\Gamma} - E)}{\hbar\omega}} \\ &\approx \frac{12}{2\pi} e^{\frac{-2\pi (0.6)}{0.5 \text{ to } 1.0}} \\ &\approx (1.0 \text{ to } 44.0) \text{ mv} \end{aligned} \tag{1}$$

Obtaining $\hbar\omega$ ⁽⁷⁾ from the slope of the curve at the threshold energy, E_T , the calculated average Γ_f from equation 1 is about 34 mv. The measured fission width (corrected for resolution and doppler effects) is seen to be about two orders of magnitude smaller than the lower expected theoretical limit.

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High Resolution Neutron Reflectivity Measurements
on Single Crystals

D. A. Kottwitz

Introduction

The Hanford three-axis crystal spectrometer was used by McCrary to make measurements relating to the differential (in energy and angle) scattering cross section of water for thermal neutrons.⁽¹⁾ However, in order to obtain sufficient intensity, he was forced to sacrifice resolution considerably by widening the beam collimators. This unsatisfactory situation was judged to be due mainly to the inadequate reflecting power of the pair of diffracting crystals in use as monochromator and analyzer. Thus a series of measurements on these and other crystals on hand was begun by E. J. Seppi and the writer. It was later extended to a number of new aluminum and lead crystals grown in this laboratory by W. J. Friesen.⁽²⁾

The primary purpose of these reflectivity measurements is to select crystals suitable for use as spectrometer monochromators and analyzers. A secondary purpose is to obtain reflectivity data on the finally chosen crystals, so that quantitative predictions of spectrometer performance, i. e., resolution and counting rates, can be more accurately made.

Experimental Method

The principle of the measurements is quite simple. A parallel, monochromatic beam of neutrons impinges on a rotatable single crystal large enough to intercept the whole beam. For an arbitrary angular position of the crystal, the beam is not diffracted. However, when the crystal angle θ satisfies the Bragg condition

$$n\lambda = 2d \sin \theta \quad (1)$$

a diffracted beam is obtained at an angle 2θ from the incident beam. See Figure 1. The theory of crystal diffraction shows that appreciable diffraction from a perfect crystal occurs over a small but finite angular interval of the order of seconds of arc.⁽³⁾ However, if the crystal is imperfect,

Figure 1
Diffraction Geometry

consisting of a number of slightly misoriented regions or having bent or twisted regions formed during growth or subsequent treatment, diffraction can occur over a much larger angular range, e.g., degrees of arc.

In any case, it is possible to define the reflectivity as the ratio of diffracted beam strength to incident beam strength for any incident angle θ . As a function of θ , the reflectivity for a perfect crystal consists of a single narrow spike, while for an imperfect real crystal it may be a series of such spikes or one or more broad peaks or any combination of these.

An ideal crystal for our intended use in neutron spectrometry should have a reflectivity curve consisting of a single peak 50 per cent or more high and about $1/4$ to $1/2$ degree wide.

Apparatus and Procedure

The experimental set-up is shown schematically in plan view in Figure 2. Calcite crystals of good quality are mounted at each of the first two axis positions for diffraction by the cleavage planes. They are set up in the transmission (Laue) mode for double diffraction in "antiparallel" alignment.⁽³⁾ Thus the diffracted beam from the second calcite crystal is a bundle of rays parallel to the same degree as the crystalline perfection of the calcites; "parallel" rocking curves for the calcite pair show this to be $\simeq 0.01^\circ$. The energy resolution of the doubly diffracted beam at the neutron energy (0.086 eV) used in most of the experiments is then $\simeq 0.2$ per cent.

The high resolution probe beam strikes a sample crystal mounted at the third axis. The beam diffracted by the sample is measured with a group of four shielded BF_3 tubes, which can also be rotated to measure the beam incident on the third axis.

A thin boron chamber is used to monitor the beam diffracted by the first calcite crystal. Note that the various collimating openings serve only to define the beam areas and to reduce background, since the probe beam divergence is determined completely by the calcite crystals.

Figure 2
Schematic Plan View of Apparatus

The experimental procedure will now be described. The probe beam is established by setting the first arm angle to correspond to the desired neutron energy and then adjusting the calcite crystals for maximum counting rate. A reflectivity run is begun by setting the BF_3 counter arm to a calculated or previously measured position to receive the beam diffracted from the sample. The sample is mounted and aligned for maximum counting rate of the third diffracted beam. Then this diffracted beam is counted for a number of angular positions of the sample crystal in the neighborhood of the Bragg peak(s). After removing the sample the counter arm is shifted to measure the incident beam. Counting of the diffracted and incident beams is controlled by the monitor count. No additional measurements are made to correct for higher order neutrons, since this is certainly a small effect in the great majority of cases.

An important part of the measurement of each sample is the choice of diffracting planes and orientation of the crystal with respect to the probe beam. Measurements of $\{220\}$ planes predominate for aluminum, for example, since their interplanar spacing of intermediate size makes them a reasonable compromise between intensity and energy resolution; however, $\{200\}$ and $\{111\}$ planes have also been measured. As for orientation, both transmission (Laue) and reflection (Bragg) modes have been used, with transmission predominant, since the probed volume is then more compact. We have also tended to concentrate on the central region of most crystals, although measurements have also been made in several regions of some samples to check homogeneity. To sum up, the measurements constitute a reasonable survey rather than an exhaustive study of each crystal.

Results

Sample reflectivity curves are presented in Figures 3 through 8. The calcite curve in Figure 3 is hardly a true reflectivity curve, since the sample crystal has an angular width of the same magnitude as the probe beam itself. However, it does demonstrate that the resolution of the probe beam is about 0.01° . The remaining figures show curves for a variety of crystals, including beryllium, lead, sodium chloride, and

Figure 3
Reflectivity of Calcite Crystal

Figure 4
Reflectivity of Pb#1 (200)

Figure 5
Reflectivity of Be#1 (0002)

Figure 6
Reflectivity of Be#2 (0002)

Figure 7
Reflectivity of NaCl#1A (200)

Figure 8
Reflectivity of Al#OA (200)

aluminum. The plotted reflectivity values are estimated to have errors of the order of 10 per cent. A number of similar curves for aluminum are given in Friesen's paper.⁽²⁾

In Table I is given a summary of results obtained on a number of crystals under a variety of conditions. All measurements except two on Al No. OA were made at a neutron energy of 0.086 ev. The first column of Table I identifies the crystal, and the second gives the Miller-Bravais indices of the diffracting planes. The third indicates whether the beam was diffracted in transmission, reflection, or an intermediate mode. The fourth column gives R_p , the peak reflectivity value measured; while the fifth gives W , the full width at half maximum if the peak is reasonably "single" or the full width of a group of peaks. For reasonably single peaks the sixth column gives $R_p \times W$, which is a crude measure of the integrated reflectivity.

Discussion of Results

The beryllium and sodium chloride crystals listed in Table I are crystals that have been sawed for use as monochromators and thus have only a limited amount of possible useful volume and a limited number of useful planes. Thus the results are almost definitive in evaluating the crystals for spectrometer use. Be No. 1 is a moderately good crystal, while Be No. 2 is an excellent one; both have been used in Hanford neutron spectrometers. On the other hand, the sodium chloride crystals do not have nearly enough reflecting power.

The situation regarding the lead and aluminum crystals is not nearly so simple. Here the samples are relatively large ingots (cylinders 7 inches long by 2 or 3 inches in diameter, or slabs measuring 6 x 3 x 1 inches) or irregularly shaped pieces cut from these ingots. The measurements have been made under widely different conditions of crystal size, shape and orientation, and so the results can be compared only semi-quantitatively among themselves or with theoretical calculations. Nevertheless, some conclusions may be drawn. The scanty measurements on lead did not locate a suitable crystal; however, they indicate that under

TABLE I
SUMMARY OF REFLECTIVITY MEASUREMENTS

<u>Crystal</u>	<u>Bragg Planes</u>	<u>Mode</u>	<u>R_p</u>	<u>W(deg)</u>	<u>R_p x W(deg)</u>
Be No. 1	10 $\bar{1}$ 0	T	0.15	0.30	0.045
Be No. 1	12 $\bar{3}$ 0	T	0.04	0.33	0.013
Be No. 1	0002	R	0.19	0.78	0.15
Be No. 2	0002	R	0.41	0.26	0.11
Be No. 2	11 $\bar{2}$ 0	T	0.40	0.13	0.052
Be No. 2	10 $\bar{1}$ 0	T	0.25	0.23	0.057
NaCl No. 1A	200	R	0.05	0.28	-----
NaCl No. 2A	200	T	0.11	0.09	0.010
Pb No. 1	200	I	0.17	0.09	0.014
Pb No. 2A	220	R	0.071	0.71	0.050
Al No. 0 A ¹	200	T	0.34	0.16	0.054
Al No. 0 A ²	200	T	0.12	0.14	-----
"	200	T	0.31	0.12	0.037
"	200	T	0.25	0.23	0.058
"	200	I	0.26	0.11	0.029
Al No. 0 B	200	R	0.17	0.27	0.046
"	200	T	0.12	0.29	-----
Al No. 1	220	R	0.056	0.7	-----
"	220	T	0.059	1.0	-----
Al No. 2	200	R	0.022	0.70	0.015
"	200	T	0.080	0.52	0.042
"	200	T	0.084	1.1	0.089
"	220	T	0.065	1.0	0.064
"	200	R	0.11	0.37	0.041
"	220	T	0.11	0.74	0.081
Al No. 3	220	T	0.050	0.88	0.044
"	220	R	0.032	0.90	0.029
Al No. 4	220	R	0.046	0.90	0.041
"	220	T	0.084	0.73	0.061

TABLE I (Contd.)

<u>Crystal</u>	<u>Bragg Planes</u>	<u>Mode</u>	<u>R_p</u>	<u>W(deg)</u>	<u>R_p x W(deg)</u>
Al No. 5	220	T	0.055	0.86	0.047
Al No. 6	220	R	0.060	0.50	0.030
"	220	T	0.075	0.61	0.045
Al No. 6A	220	R	0.082	0.54	0.044
"	220	R	0.066	0.49	0.032
"	220	T	0.077	0.50	0.038
"	220	T	0.065	0.64	0.042
Al No. 6A2	220	R	0.052	0.52	0.027
Al No. 6B	220	T	0.030	0.5	0.015
Al No. 8	220	T	0.058	0.41	0.024
Al No. 9A	220	T	0.15	0.033	0.0050
Al No. 9A1	220	T	0.25	0.040	0.010
"	220	R	0.26	0.047	0.012
Al No. 9B1	220	R	0.28	0.077	0.022
"	220	T	0.096	0.13	0.012
Al No. 9C1	200	T	0.062	0.32	0.020
Al No. 10	220	T	0.12	0.14	-----
"	220	T	0.20	0.05	-----
"	220	T	0.16	0.30	-----
"	111	T	0.11	0.24	0.026
"	220	I	0.15	0.08	0.012
"	220	I	0.096	0.11	0.010
"	220	R	0.22	0.3	-----
Al No. 11	200	T	0.18	1.1	-----
"	200	T	0.14	1.1	-----
"	200	T	0.13	1.1	-----
Al No. 12	111	T	0.24	0.3	-----
"	111	T	0.17	0.34	-----
"	111	T	0.24	0.16	0.038
"	220	R	0.18	0.04	0.007

TABLE I (Contd.)

<u>Crystal</u>	<u>Bragg Planes</u>	<u>Mode</u>	<u>R_p</u>	<u>W(deg)</u>	<u>R_p x W(deg)</u>
Al No. 12	220	R	0.14	0.06	0.008
"	220	T	0.20	0.096	0.019
"	220	T	0.15	0.09	0.013
"	220	T	0.11	0.14	0.015

¹ Measurement at 0.035 ev.

² Measurement at 0.30 ev.

different conditions of crystalline perfection lead would be a suitable material.

The measurements of aluminum fall into four groups. The first are those on two grains (Al No. OA and OB) of moderate size sawed from aluminum ingot cast some years ago. These were the first pieces of aluminum measured, and the results were quite encouraging, since both R_p (20 to 30 per cent) and W (0.1 to 0.3 degree) have values within a factor of 2 of the desired ones.

The second group of measurements was made on a batch of ingots (Al No. 1 through Al No. 8) cast in a graphite crucible with a large temperature gradient in the melt.⁽²⁾ These crystals have surprisingly uniform characteristics, low peak reflectivities of about 6 to 8 per cent and large widths of about 0.5 to 1.0 degree. They are not suitable for monochromators.

The third group was made on the irregular grains (Al No. 9A, 9B, and 9C) cut from an ingot cast in a soft mold.⁽²⁾ These grains have good peak reflectivities (10 to 30 per cent), but are too perfect, having widths of only 0.04 to 0.13 degree.

The fourth group of measurements has been made on a batch of ingots (Al No. 10 through Al No. 12) cast in a soft mold with a low temperature gradient in the melt.⁽²⁾ Al No. 11 is multiple throughout, but the other two contain reasonably large regions of good reflectivity

(15 to 25 per cent) and small widths (0.04 to 0.3 degree). These regions are not suitable for monochromators in their present condition, but they are good material for attempting to improve reflectivity characteristics by artificially and controllably introducing the proper amount of crystalline imperfection or by assembling a composite monochromator from a number of pieces.

Acknowledgments

Much of the credit for designing and testing the experimental method belongs to E. J. Seppi, who also generously shared his knowledge of the spectrometer. Thanks are also due to I. Holtzman for assistance in carrying out some of the later measurements.

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Growth of Aluminum Single Crystals

W. J. Friesen

The purpose of this report is to describe briefly two methods employed recently in attempts to grow aluminum single crystals from the melt, suitable as neutron monochromating crystals for the neutron crystal spectrometers, and some of the results obtained to date.

A good neutron monochromating crystal in general requires a higher degree of perfection over a larger region than is required in other applications of single metal crystals. The dimensions required to fully utilize existing neutron beams are of the order of two to three inches by six inches. Single metal crystals of these dimensions are not readily available commercially and the perfection of crystals which are available are seldom or never stated. The techniques for examining large regions of a crystal may not always be available to commercial manufacturers of single metal crystals.

The perfection of the single crystals grown was studied from the standpoint of their ability to reflect neutrons and their use as neutron monochromators. These two properties of a single crystal depend on more than one type of ordering in the crystal and must be defined in terms of the experimental conditions desired. It may be helpful to state briefly a few concepts and definitions related to single crystals.

The reflectivity will be defined as the percentage of neutrons reflected from a given crystal for a given neutron energy for a given angle between a parallel monoenergetic neutron beam and the crystal lattice.

An ideally perfect crystal is an arrangement of atoms all on perfect lattice sites. According to Bacon⁽¹⁾ the number of lattice planes necessary to achieve 100 per cent reflectivity for a beam of neutrons of

1. Bacon, G. E. "Neutron Diffraction" Oxford University Press, London, Chapter III, p. 53. 1955.

$1 \text{ \AA}$ wavelength satisfying the Bragg condition is of the order of 10,000 planes. For solids this thickness is of the order of 10^{-4} cm. This thickness is a function of the wavelength, the thickness increasing with decrease in wavelength. The reflection occurs over a small angle about the Bragg angle of the order of seconds of arc.

An ideally imperfect crystal is composed of small blocks of ideally perfect crystal slightly disoriented about some average angle with dimensions of the order required for much less than 100 per cent reflectivity. An ideally imperfect crystal is suited to mathematical calculation but would not necessarily be a good monochromating crystal.

The ideal monochromating crystal would have a reflectivity of 100 per cent over and limited to the maximum angular divergence of the collimation desired in a given experiment. The ideal monochromating crystal implies at least two requirements for monoenergetic neutrons. One requirement is a complete extinction of the primary beam. The second requirement is that the projected path through the crystal of every neutron in the incident beam intersect regions oriented at all angles within the desired collimation about the average Bragg angle. Neglecting true absorption and incoherent scattering, such a crystal might in concept, at least, be composed of a series of sheets of ideally perfect crystal of thickness required for 100 per cent reflectivity. These sheets would be uniformly disoriented over the collimation angles about an average sheet by rotations about two perpendicular axes in the plane of the average sheet spaced at angular intervals of the reflection width of an ideally perfect crystal. This crystal should yield about 100 per cent reflectivity for the reflection case from the average plane and 50 per cent reflectivity for the transmission case. It should be noted that the properties of this crystal are not isotropic upon a complete rotation of the crystal with respect to the neutron beam. If a random distribution of blocks, for example, is substituted to avoid this anisotropy, the reflectivity is more difficult to estimate due to secondary extinction.

Imperfections of several types are observed in real crystals. On a scale of small dimensions comparable with the ideally imperfect crystal blocks are imperfections such as vacancies, dislocations and impurities. These types of imperfections lead to broadening of diffraction peaks and reduction in reflectivities. From theoretical considerations the perfect lattice is not stable except at a temperature of absolute zero and will contain some defects at any other temperature.⁽²⁾ Macromosaics are another type of imperfection found in crystals grown from the melt, on a scale of larger dimensions of the order of millimeters. They are regions which differ in orientation from the average orientation of the crystal in the range of 0° to 5° . Other terms often used to describe structures of this type are lineages and striations. They give the crystal a structure similar to a bundle of rods. This type of structure produces separated peaks and broad summation peaks as seen in crystal rocking curves. From the results of the few systematic studies of macromosaics in metal crystals, grown from the melt, reported in literature, one finds that the size and degree of orientation of these structures can be varied considerably with conditions of crystal growth.⁽³⁾ The existence of these structures have been reported for practically all types of metal crystals grown from the melt.⁽⁴⁾

The definition of a single crystal generally used includes the existence of a macromosaic structure. This definition of a single crystal tends to be loose but is realistic and clearly distinguished from the idea of polycrystal with random unrelated orientation of structures.

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Two modes of crystal growth from the melt can be obtained. Since the growth of a crystal from its melt is a nonequilibrium situation, heat must be removed from the system in order for the solid phase to advance. The direction of removal of the latent heat of solidification from the solid-liquid interface defines two modes of growth: heat flow from the interface to the solid defines normal growth; heat flow from the interface to the liquid defines dendritic growth. Since it is possible to supercool the metals both normal growth and dendritic growth are possible.

Methods of Growth and Equipment

To obtain the starting crystal nucleus, the Bridgman technique for nucleation was employed by machining the beginning end of the charges to a conical tip. In the design of the furnace the attempt was made to produce a heat flow pattern such that the liquid-solid interface was concave toward the liquid. This was done to reduce the possibility of ingrowth of stray nucleation at the boundaries of the melt and to minimize the concentration in the liquid of impurities escaping from the interface. With the exception of specimen Al-0, the crystals presented in this report were grown from 99.996 per cent purity aluminum supplied by the Kaiser Aluminum Company. The charges were machined to shape from large ingots to reduce the possibility of contamination in melting and casting shapes, Aluminum Al-0 was grown from aluminum of 99.95 per cent purity and was cast to shape.

Method I

This method consists of maintaining a temperature gradient in the aluminum charge while slowly reducing the temperature through the melting point. The method most closely resembles a combination of the Kapitza⁽⁵⁾ and the Bridgman⁽⁶⁾ method. Eight single crystals were grown by this method.

5. Kapitza, P. Proc. of Royal Soc. A119, 385. 1929.

6. Bridgman, P. W. Proc. Nat. Acad. Sci. 60, 306. 1925.

The details of the furnace construction are shown in Figure 1. The melt is contained in a graphite crucible which is supported by the heat leak. The temperature gradient is established by the flow of heat from a heater surrounding the crucible through the crucible and charge and down the heat leak to the water-cooled plate. The temperature was lowered by reducing the heater power by a moter driven variac. A system pressure between 20μ and 50μ Hg was maintained during growth and cooling of the crystals by means of a two stage mechanical forepump. One difficulty encountered several times was that the aluminum adhered to the graphite crucible.

The growth rate was varied as a parameter as shown in Table I. The perfection of these crystals appeared to vary only slightly as a function of growth rate, and the perfection was less than was desired. In attempting to vary the growth rate as a parameter in this method, it was not clear that the thermal distribution or mode of growth could be separated as variables. Aluminum is characterized, as are most of the metals, by a large thermal conductivity and it should be strongly coupled to the apparatus. In many of the studies reported in the literature of the macromosaic structure as a function of growth rate, for other metals, methods similar to Method I were employed. Since Method I does not represent a simple liquid-solid system proceeding to its lowest energy state, but rather a more complicated system with interrelated constraints, and would not necessarily provide results characteristic of the basic growth mechanisms, further study of the variation of parameters of Method I did not appear fruitful.

T. S. Noggle⁽⁷⁾ demonstrated that levigated alumina powder could be used for the containment of aluminum and grew aluminum single crystals, 1 cm x 1 cm x 6 cm by the Bridgman technique. He also demonstrated that those crystals were of a high degree of crystal perfection, which compared with crystals grown by strain-anneal techniques.⁽⁸⁾

7. Noggle, T. S. Rev. Sci. Instru. 24, 184. 1953.

8. Noggle, T. S. and J. S. Koehler. Acta Met. 3, 260. 1955.

Figure 1

Crystal Growing Furnace, Method I

TABLE I

	Dimensions (inches)		Method	Average Linear Growth Rate (cm/hr)	Atmosphere	Surface Appearance After Acid Etching	Plane Nearest ⊥ to Specimen Axis	Surface Appearance After Growing
	Diameter	Length						
Al-1	2	8	I	6.8	Vacuum	Striations	111	Thin Oxide Coating
Al-2	2	8	I	1.6	20 μ Hg	Small Surface Grains	110	" " "
Al-3	2	8	I	3.8	"	Narrow Striations Grains	100	" " "
Al-4	2	8	I	0.86	"	Small Surface Grains	111	" " "
Al-8	3.375	8	I	0.78	"	-----	110	" " "
Al-10	3	6	II	0.56	Air	No Striations	111	Heavy Oxide Coating
Al-11	3	5.75	II	5.3	Air	Striations	100	Heavy Oxide Coating. Large Gas Holes in Top
Al-12	3	5.75	II	3.1	Vacuum	No Striations	111	Thin Oxide Coating
Al-9	3.375	8	Embedded in Aluminum Oxide	(2.5 to 14)	Air	Large Grains and Dendrites		
Al-0	3	5	Similar to I	3	Air	4 Large Longitudinal Grains, Several Small Grains	100	Heavy Oxide Coating

Estimated Temperature Gradient

Method I 5° c/cm

Method II 0.7° c/cm

To test the method of containment for larger pieces of aluminum, a 3 3/8 -inch diameter by 8-inch long aluminum charge was embedded in ignited aluminum oxide powder in a stainless steel can. This charge was melted in air and allowed to freely cool. The initial dimensions of this charge were well preserved after melting with the exception of the center where a deep pipe formed at the top. Many large grains were formed, some of which indicated high reflectivities and narrow peak widths, see Figure 7, Al 9-Al. This charge is labeled Al-9 even though it is not a single crystal.

Method II

This method is an attempt to approximate the concept of the system of a liquid energetically isolated from its surroundings and coupled only to its solid phase. The energy of the system is lowered by removing heat from the solid phase.

Three single crystals have been grown by this method and further development is in progress.

The details of the furnace construction are shown in Figure 2.

The Bridgman technique of nucleation was used to provide the initial solid phase. Ignited aluminum oxide powder was used to contain the aluminum to approximate mechanical isolation of the aluminum. The approximation to thermal isolation of the liquid was obtained from the thermal insulating properties of aluminum oxide powder and by maintaining the lowest temperature of the inner furnace wall from 5 to 10 C above the melting point. Copper was used for both the inner and outer furnace walls to reduce the temperature gradient vertically along the furnace wall. During the growth of a crystal the temperature observed at the top of the inner copper furnace wall was about 40 C higher than at the bottom of this copper wall. Ideally the temperature of the inner furnace wall would be uniform and would be maintained at the average temperature of the liquid.

The growth rate and the atmosphere were varied for the three crystals grown by this method and are shown in Table I. The first two

Figure 2

Crystal Growing Furnace, Method II

crystals grown by this method were grown in an atmosphere of air. The second crystal was grown at a rate approximately ten times larger than the first crystal grown. The second crystal grown showed large gas holes in the top. Figures 8 and 9 show the appearance of this crystal upon removal from the furnace. For the third crystal grown the system was evacuated to a pressure of approximately 20μ Hg to reduce the effect of gas entrained or dissolved in the melt on the growth of the crystal. This crystal was free of visible gas bubbles on its surface.

Examination of Large Metal Crystals

Neutron Examination of Crystals

The crystals grown by Method I and Method II were examined by D. A. Kottwitz using a double crystal neutron spectrometer.⁽⁹⁾ This apparatus provides a probing beam of neutrons of fairly uniform intensity over the cross sectional area of the beam of dimensions 1 in. x 1 1/4 in. The energy resolution is approximately 0.2 per cent and the angular resolution about the axis of rotation of the specimen is of the order of 0.01 degree. The large area of the beam and high resolution make this instrument ideally suited to the examination of both the mosaic and the macromosaic structures of large metal crystals. Figures 3 through 7 are a portion of the data which have been obtained and show several of the characteristics of the aluminum crystals which have been grown. A neutron energy of 0.086 ev which corresponds to a neutron wave length of 0.98 Å was used in obtaining the data for these figures. These figures represent the cases where the neutron beam is transmitted through the crystal and diffracted from planes nearly perpendicular to the cylindrical axis of the specimens in order to intercept a large volume of the crystal. The maximum value of the reflectivity of an ideally perfect crystal intercepting the entire beam in the transmission case is 50 per cent, neglecting resolution, true absorption, incoherent scattering, and possible

Figure 3

Comparison of Crystal Reflectivity Curves Obtained from Crystals
Grown by Method I and Method II
The Curves are Plotted to the Same Scales for Comparison

Figure 4

AL-10 Crystal Reflectivity Curve Observed
at Positions Along the Direction of Growth
The Neutron Beam was Nearly Perpendicular to Direction of Growth
The Distance Between Observing Positions was 3 Inches

Figure 5

Al-11 Crystal Reflectivity Curve Observed
at Positions Along the Direction of Growth
The Neutron Beam was Nearly Perpendicular to Direction of Growth
The Distance Between Observing Position was 1.5 Inches

Figure 6

Al-12 Crystal Reflectivity Curve Observed
 at Positions Along the Direction of Growth
 The Neutron Beam was Nearly Perpendicular to Direction of Growth
 The Distance Between Observing Position was 1.5 Inches

Figure 7

Crystal Reflectivity Curves of Selected Grains from Two Melts
Al-9A1 was Obtained from a Melt Which was Embedded in Aluminum Oxide
and Allowed to Freely Cool

This Grain Contains Several Large Dendrites
Al-0-A was Obtained from a Position Near the Beginning Tip
of a Crystal Grown by a Method Similar to Method I in an Air Atmosphere

Figure 8

Al-11 Crystal Grown by Method II in Air Atmosphere Upon Removal from Furnace

Figure 9
Al-11 Crystal Grown by Method II in Air Atmosphere Upon Removal from Furnace
Note Large Gas Holes

geometric effects due to the spatial extent of the crystal and detector.

Figure 3 is a comparison of the crystal rocking curves obtained from crystals grown by Method I and by Method II. The crystals grown by Method I are characterized by broad peaks, low reflectivities and total macromosaic widths of the order of 1.5° to 2° . The crystals grown by Method II show considerably higher reflectivities, narrow peak widths, some which begin to approach the resolution width of the spectrometer, and total macromosaic widths of the high reflectivity regions within 0.5 degrees.

The data for Figures 4, 5, 6 of Al-10, Al-11, Al-12 were obtained by translating the crystals along their cylindrical axis, which is also the axis of growth, with the probing beam incident almost perpendicular to this axis. Figures 5 and 6 show the persistence of the structure throughout the length of the crystal along the growth axis. These figures also indicate a decrease in crystal perfection as growth proceeds as indicated by the broadening of the pattern and decrease in the peak reflectivities as one proceeds along the direction of growth.

Figure 7 shows the rocking curves from selected grains in two other large melts. These grains yield the highest reflectivities and narrowest peak widths which were observed for grains in these two melts.

The production of Al-9 was described above. Al-0 was grown several years ago by the author in an air atmosphere by a method similar to Method I with the exception that the crucible was heated only through a heated graphite block resting on top of the crucible. Considerable oxidation of the surface of this specimen occurred. This three-inch diameter specimen was characterized by a division into four grains parallel to the axis with an angular misorientation of the order of two degrees and several small ingrown grains. A single crystal neutron examination of these grains near the top of the specimen indicated a broad macromosaic structure.

Macroscopic and Optical Features

Two etches were used to aid in examination of the crystals grown. Due to the anisotropic chemical reaction rates with respect to the

crystallographic orientation of a solid, it is possible to obtain macroscopic features associated with the structure of a single crystal. The etches used were aqua regia plus HF, which leaves small facets parallel to 100 planes, and NaOH, which leaves facets possibly parallel to 335 planes.⁽¹⁰⁾ The acid etch has been useful in observing low angle grain boundaries and lineages, probably down to orientations differing by about 0.5 degrees. It has also been useful in determining the crystallographic orientation of a specimen by reflecting a beam of light from the 100 facets to determine the orientation of the 100 directions in the sample.

The NaOH etch reveals different interesting macroscopic features of a single crystal specimen. It is difficult to obtain reproducible results with this etch but in general the best results are obtained from a hot saturated solution and a violent reaction interrupted by cooling and washing of the sample in water. In this etch the reaction rate of impurities is much less than for aluminum and at least some impurities can be readily observed.

Figure 10 shows the results of the two etches on longitudinal sections of two crystals. The sample to the left in each case shows the result of acid etching the sanded surface. The striated structure or lineages can be seen. The mating halves of the samples, to the right in each case, were moderately etched in NaOH. No sanding was done after sawing and the saw marks cross the samples. Coring of segregated impurities is seen along the axis and at the top of the samples.

Figures 11 and 12 shows the segregation of impurities in Al-8 near the middle of the crystal and at the top. All crystals grown by Method I showed marked segregation of impurities at the top of the specimens.

A record of the liquid-solid interface can be seen in Figures 13, 14, 15. These faint lines seen in the tips of these crystals are intersections of curved surfaces on a plane parallel to the axis of the specimens. These may be due to interruptions or changes in heater power, since Method I employed a motor driven variac which is stepwise in its action, and Method II

10. Orem, T. H. J. of Res. Nat. Bureau of Standards. 58, No. 3. 157. 1957.

Figure 10

Comparison of Results of Aqua Regia Plus HF and NaOH Etches
From Left to Right Al-2-Acid Etch, Al-2-NaOH Etch, Al-6-Acid Etch, A-6-NaOH Etch
Note Striations Revealed by Acid Etch, and Coring of Segregated Material by NaOH Etch
These Two Crystals were Grown by Method I

Figure 11

One Half of Al-8 after Heavy Etching in NaOH
Note Segregated Material Near Center at Edge and at Top of the Crystal
This Crystal was Grown by Method I

Figure 12

Exterior of Al-8 after Heavy Etching in NaOH
Note Segregated Material Near Center and Top of Crystal
This Crystal was Grown by Method I

Figure 13

Section Through Tip of Al-10 After Heavy Etching in NaOH
Note Narrow Bands Which are Thought to be Due to Interruptions in Growth
of the Crystals Due to an On-off Temperature Controller of Furnace
This Crystal was Grown by Method II

Figure 14

Section Through Tip of Al-2 after Heavy Etching in NaOH
Note Narrow Bands Which are Thought to be Due to Interruptions in the Growth
of the Crystal Due to Stepwise Action of Variac Used for Furnace Power
This Crystal was Grown by Method I

Figure 15

Tracing of Faint Lines Observed on Section Through Al-8
after Heavy Etching in NaOH
This Crystal was Grown by Method I

employed an on-off controller. Bands and lines of this nature have been reported by others and were assumed to be due to variations in heater power.⁽¹¹⁾ These surfaces have only been seen in the more slowly grown crystals. The greater curvature of Al-10, grown by Method II, than Al-2 and Al-8 is expected, due to the higher thermal conductivity of the graphite crucible than the aluminum oxide powder.

Figure 16 shows the result of heavy etching in NaOH of sections of the starting tips of Al-2 grown by Method I and Al-10, 11, and 12 grown by Method II. An interesting feature that is indicated is that the beginning portion of all of these crystals appears to be fairly uniform and free of disturbed regions. Al-2 shows an abrupt change in character about halfway up the tip. Al-10, 11, 12 show a uniform central region surrounded by a shell of different character which contains impurities. This tends to suggest that the central core of the crystals grown by Method II may be of a higher degree of crystal perfection with impurities discharged to the exterior of melt. This might possibly explain the mixed character of the observed diffraction pattern of Al-11, Figure 5. Of the three crystals grown by Method II, striations were observed (after acid etching) on only the surface of Al-12.

Figure 17 is a section from Al-9 which is heavily etched in NaOH. Note the large dendritic structures which indicate supercooling and the dendritic mode of growth. The grain Al-9-A1 (reflectivity curve shown in Figure 7) was located in the outer surface of the melt just below this slice. A dendrite in this grain extends 2.5 in. in length. The main stalk is in the 100 direction, as would be expected for the face centered cubic lattice. This outer region is characterized by isolated large dendrites, surrounded with material fairly uniform in composition, which are not dissolved as rapidly as the surrounding material, indicating the segregation of impurities along the dendrites. A concentration of impurities is indicated in the central region.

11. Collins, J. A. and C. H. Mathewson. Trans. AIMME. 137:150. 1940.

Figure 16

Sectioned Tips of Crystals after Heavy Etching in NaOH
From Left to Right Al-2 Grown by Method I, Al-10, Al-11, Al-12 Grown by Method II
Note Difference of Appearance of Material Near Surface
of Crystals, Especially, Those Grown by Method II

-51-

Figure 17

Slice Near Middle of A1-9 after Heavy Etching in NaOH
Note Large Dendrites and Increased Segregation of Material Toward Center of Ingot
This Ingot was Embedded in Aluminum Oxide and Melted and Freely Cooled in an Air Atmospher

HW-64866

Figure 18 shows a section along the axis of a portion of Al-9 showing gas bubbles. The pattern of these bubbles indicates the direction of growth as being toward the center of the melt.

Summary and Discussion

Single aluminum crystals grown by Method I in graphite crucibles are characteristic of the properties which, in the past, have generally been associated with metal crystals grown from the melt. Their properties as neutron monochromators are characterized by low reflectivities and broad macromosaic angular distributions.

An exception was noticed in a small region of Al-0, Figure 7. However, the presence of a considerable oxide layer on the surface of the melt could considerably change the thermal distributions in the two systems. Method I produces strong segregation of impurities but the impurities could possibly be associated with impurities in the graphite. This method does not appear to be promising for the production of large aluminum crystals of the perfection desired for neutron spectrometry.

The single aluminum crystals grown by Method II are characterized by large regions of very high crystal perfection for a metal. The macromosaic structure can still be observed but its total angular divergence is greatly reduced over crystals grown by Method I. For the crystals which have been grown by Method II, the crystal perfection appears to decrease as growth proceeds as indicated, for example, in Figure 6. Impurities appear to be dispersed to the boundaries of the melt. The method appears to be very promising for the production of large aluminum single crystals which meet the specification required of a good neutron monochromating crystal as for example the curve to left in Figure 6.

The major differences which contribute to the marked difference in the over-all perfection of crystals grown by Method II as compared to Method I have not been determined.

Figure 18

Section Along Axis of a Portion of Al-9
Note Direction of Gas Bubbles Indicating Solidification Proceeded Inward
from the Surface of the Ingot
The Ingot was Embedded in Aluminum Oxide and Freely Cooled in an Air Atmosphere

Some differences between these methods can be pointed out:

1. Difference in mechanical stresses at the surfaces of the crystals. Method I employed solid graphite crucibles to which several of the crystals adhered. Method II employed aluminum oxide powder for containment which could yield to changes in dimensions of the aluminum.
2. The graphite used in Method I might introduce more impurities into the melt than the aluminum oxide used in Method II.
3. Larger radii of curvature of liquid solid interface in Method I as shown in Figures 14 and 15 than in Method II as shown in Figure 13.
4. Larger temperature gradients in Method I; approximately 5 C/cm as compared to approximately 0.7 C/cm in Method II.
5. Method II may more closely represent an isolated simple liquid metal system as compared to a more complicated system for Method I.

REACTOR PHYSICSMonoenergetic Age Measurement

D. G. Foster, Jr.

An earlier report in this series, HW-61181, carried the abstract of a paper submitted for publication, which described a measurement of Fermi age in water and kerosene using Na-Be sources. The data reported therein have since been reanalyzed.

The new analysis results in a change in some conclusions regarding the distribution of slow neutrons at large distances from the source. The new conclusions are discussed in some detail in an accompanying section⁽¹⁾ of this report, since they have some interesting features which are not particularly relevant to the measurement of age. The revised distributions lead to values of the age which are slightly more than 2 per cent smaller than previously reported. The abstract of the revised version of the paper follows:

"The age to indium resonance of nearly monoenergetic 0.97 Mev neutrons from spherical Na- γ -Be sources has been measured in water and kerosene. The age from a point source is inferred by extrapolation from measurements made with sources 3/4 in. and 3/8 in. in diameter. The flux age is $13.9 \pm 0.2 \text{ cm}^2$ in water and $13.8 \pm 0.2 \text{ cm}^2$ in kerosene. Calculations by the moments method give $13.9 \pm 0.1 \text{ cm}^2$ in each medium, in excellent agreement with the measurements. The thermal migration area measured concurrently is $21.5 \pm 0.4 \text{ cm}^2$ in water and $20.6 \pm 0.4 \text{ cm}^2$ in kerosene. The migration area calculated from the resonance age is $22.2 \pm 0.5 \text{ cm}^2$ in water and $21.8 \pm 0.5 \text{ cm}^2$ in kerosene. Both of these are substantially larger than the measured values."

References

1. "Asymptotic Slowing-Down Flux in Hydrogenous Moderators", in this issue of NPR Quarterly Report, page 56.

Asymptotic Slowing-Down Flux in Hydrogenous Moderators

D. G. Foster, Jr.

In a recent measurement of Fermi age in hydrogenous moderators,⁽¹⁾ the saturated activity of bare and cadmium-covered indium was measured in water (H_2O) and kerosene ($H_2C_{1.014}$). The activities were measured out to 35 cm (nine slowing-down lengths) from a spherical Na^{24} - γ -Be source having a diameter of 1.905 cm. This is believed to be the first high-precision measurement of slowing-down flux obtained with a nearly monoenergetic source of neutrons. It is the purpose of this note to discuss certain features of the flux distributions at large distances from the source.

Neutrons obtained from a Na^{24} - γ -Be source have ideally an average energy of 0.966 Mev. The kinetics of the (γ, n) reaction introduce a spread of ± 0.014 Mev around this mean. In any source of practical size, however, some of the photons will undergo Compton scattering before giving rise to the (γ, n) reaction, and some of the resulting neutrons will collide with other nuclei before escaping from the source. In the source used in the present experiment, 28 per cent of the neutrons had been degraded by at least one such interaction. The calculated spectrum (neglecting later collisions) is shown in the upper part of Figure 1.

The characteristic feature⁽²⁾ of neutron moderation in a hydrogenous medium is the strong first-collision nature of the process. There is a high probability that a neutron will lose a substantial fraction of its initial energy in its first collision with a hydrogen nucleus. Because the scattering cross section of hydrogen increases rapidly with decreasing energy in the region of a few Mev, the mean free path for the second collision will be much shorter than for the first. Thus, a neutron usually is slowed down to a low energy relatively close to the position of its first scattering from hydrogen after leaving the source. The distribution of slow neutrons measured far from the source, therefore, is largely dominated by the distribution of first collisions.

Figure 1

Top: Calculated Energy Spectrum of Neutron Source
The Calculation Includes Only the First Moderating Collision

Bottom: Variation with Energy of the Total Cross Section
of the Moderators Used in this Measurement

If this were strictly true, the slowing-down flux from a mono-energetic source would give a curve of $\log r^2 \phi$ versus r which is asymptotic to a straight line as r tends to infinity. Here r is the distance from the center of the source, and ϕ is the flux measured at some low energy; e. g. at the 1.45 ev resonance in indium.

However, small-angle scattering from hydrogen does not result in a very large energy loss. Furthermore, because of the diffusion of a neutron during the remainder of the moderation process, a neutron which has been scattered through a small angle is not necessarily "lost" from the primary beam, with respect to a point on its original path. Consequently, a neutron captured at 1.45 ev by a foil very far from the source is more likely to have undergone one or more small-angle scatterings before reaching the neighborhood of its ultimate capture, than to have made a long first flight with no collisions at all. The effect of multiple small-angle scattering is, roughly speaking, to degrade the energy of the "primary" beam without actually removing the neutrons altogether. This manifests itself in a gradual increase in the cross section for the first large-angle scattering, and hence in a downward curvature of $\log r^2 \phi$.

In practice, the situation is complicated by two further factors. Since the moderator contains other nuclei besides those of hydrogen, the first collision may result in negligible energy loss even though a large change in angle results. Furthermore, the total scattering cross section of the moderator usually acquires numerous resonances from these other nuclei.

Finally, few experiments actually can be made with monoenergetic neutrons. During deep penetration into the moderator, a complex primary spectrum undergoes extensive "hardening". The less penetrating components are selectively removed from the beam, and give rise to a large flux of low-energy neutrons near the source, while the more penetrating components reach out into the remote region with steadily decreasing average cross section. For scattering from pure hydrogen, the most penetrating neutrons are those of highest energy. In a more complex

medium, on the other hand, the spectrum will reach a transient equilibrium at a point of minimum cross section between resonances. As long as appreciable hardening is taking place, $\log r^2 \phi$ will be concave upward.

After hardening is substantially complete, however, the resulting spectrum is nearly monoenergetic, and thereafter undergoes the same gradual degradation as an initially monoenergetic spectrum. Thus, $\log r^2 \phi$ for the slowing-down distribution from a source with a broad spectrum will have positive curvature at moderate values of r , and negative curvature as r tends to infinity. The transition region may cover several slowing-down lengths, and give an extended section which is indistinguishable from a straight line. Limits on the strength of the source frequently make it impossible to observe the eventual negative curvature, so that many experimenters have concluded that the simple first-collision approximation is an adequate description of the asymptotic distribution.

The early measurements of age to indium resonance in water, performed with a Ra- α -Be source, ⁽²⁾ gave such a straight line between two and four slowing-down lengths from the source. The classic age measurement in water for a fission spectrum, that of Hill, Roberts, and Fitch, ⁽³⁾ was extended to eighteen slowing-down lengths by using the thermal neutron distribution. The last four slowing-down lengths were straight within 1 per cent. The slowing-down flux of neutrons produced by bombarding deuterium with 250 kev deuterons was measured by Spiegel, Oliver, and Caswell ⁽⁴⁾ out to eight slowing-down lengths. The spectrum for this source is narrow enough that the curvature never becomes positive, although the indium resonance flux is very close to linear in its outer reaches. In the measurements covered by this report, using a nominally monoenergetic source, the resonance and thermal fluxes both exhibit asymptotic distributions with negative curvature between four and nine slowing-down lengths from the source.

For the analysis of the present work, the asymptotic distributions were fitted by curves of the form:

$$\phi = \phi_0 r^{n-2} \exp(-r/\lambda). \quad (1)$$

The form of this is suggested by some work of Fano⁽⁵⁾ on the deep penetration of gamma rays. It corresponds rigorously to the asymptotic distribution of monoenergetic photons subject only to Compton scattering. In this expression, λ is the mean free path at the source energy, and n , which may be called the build-up exponent, is calculated from the Klein-Nishina formula for the Compton process. The more complicated process which occurs if pair-production is included (in which case the total cross section passes through a minimum) still leads to the same asymptotic shape (with a different value for n), as long as the source energy is below the energy at which the minimum cross section occurs. If the source energy lies above the minimum, rapid degradation to the energy of the minimum occurs, and the asymptotic distribution acquires the form:

$$\phi = \phi_0 B(r) \exp(-r/\lambda'), \quad (2)$$

where λ' is the mean free path at the minimum and $B(r)$ is a build-up function which is more complicated than r^{n-2} , but qualitatively similar to it over a limited range of r .

The total cross sections of water and kerosene are shown in the bottom of Figure 1, where they may be compared to the source spectrum. The main peak of the source spectrum lies in the low-energy edge of the second resonance in oxygen. In accordance with the arguments just outlined, in water we would predict very rapid degradation into the minimum, which occurs near 0.85 Mev, followed by a distribution somewhat similar to that given by equation (1). In kerosene, where the cross section varies monotonically with energy, the dominant initial process should be hardening of the source spectrum, followed by smooth degradation.

The experimental data are not sufficiently precise to resolve any fine distinctions between these processes. This lack of precision is aggravated by the fact that the distribution of neutrons from the source is

masked, beyond 30 cm from the source, by neutrons generated locally from deuterium in the moderator by photons from the source. Nonetheless, the resonance neutron distributions in both media are statistically in excellent agreement with equation (1). The thermal flux measurements were made with higher counting rates, and hence bear smaller experimental variances. Under the improved precision which is thus available, the fits to equation (1) are decidedly inferior to those found for resonance neutrons, in that the random deviations exceed the internally-measured standard deviations by a factor of nearly three. However, the deviations are still less than 2 per cent out through 26 cm from the source. Much of the extra variance can reasonably be accounted for by known experimental causes.

One consequence of this modified first-collision theory is that the value obtained for λ should be independent of the detector energy in a given moderator. The values of λ required to achieve independent least-squares fits of equation (1) to the resonance and thermal fluxes in each moderator are, indeed, found to be very nearly equal. Requiring them to be the same causes a negligible change in the index of precision for the fit. The parameters resulting from this simultaneous fit to both curves are listed in the following table:

Parameters of Asymptotic Flux Distributions

<u>Moderator</u>	<u>Water</u>	<u>Kerosene</u>
Build-up exponent, n		
Resonance	1.98 ± 0.22	2.21 ± 0.18
Thermal	3.55 ± 0.29	3.41 ± 0.25
Relaxation length, λ	2.37 ± 0.01 cm	2.35 ± 0.01 cm
Cross section per molecule	12.7 ± 0.1 b	12.5 ± 0.1 b

Figures 2 and 3 show the complete fitted curves, together with the data.

The standard deviations listed in the table should be used with caution, since the nature of the least-squares procedure introduces a strong covariance between the adjustable parameters. Thus, in spite of

Figure 2

Flux Distributions in Water
 Points Which Lack Error Flags Bear Standard
 Deviations Smaller than the Plotted Symbol
 The Lines Represent the Analytical Fits Described in the Text

Figure 3

Flux Distributions in Kerosene
 Points Which Lack Error Flags Bear Standard
 Deviations Smaller than the Plotted Symbol
 The Lines Represent the Analytical Fits Described in the Text

the near-equality of the values of n in water and the corresponding values in kerosene, and the large standard deviations assigned to them, the conclusion that these numbers are exactly equal is statistically inadmissible. One can, of course, set corresponding values of n equal in water and kerosene, and obtain an acceptable fit by using different values of λ for resonance and thermal neutrons in the same medium, but this is contrary to the spirit of the model.

The cross sections corresponding to the adopted values of λ are listed in the table also. In water, the cross section agrees with the total cross section at the minimum within 2 per cent. Somewhat surprisingly, the energy at which the total cross section in kerosene is the same as that found from λ is also about 0.85 Mev. It appears from this observation that the lower average energy of the actual spectrum results in an appreciable increase in the cross section over that to be expected from an infinitely thin Na-Be source.

References

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Material Bucklings of Graphite Uranium Lattices

G. W. R. Endres and D. E. Wood

Introduction and Summary

Material buckling measurements have been completed for a series of lattices with a tube-in-tube natural uranium fuel element, for most lattice spacings with tube-and-rod and I and E fuel elements, and for one lattice using a second tube-and-rod fuel elements. The measurements were made in exponential piles four feet long and approximately six feet wide. Bucklings were measured for both air and water coolant. The series of measurements on the second tube-and-rod fuel element was dropped because the results were close to the results obtained from the I and E fuel elements.

Tables I, II, III, and IV show material buckling values and side-to-side extrapolation lengths for the tube-in-tube, tube-and-rod, and I and E fuel elements, respectively. The buckling values are plotted in Figures 1, 2, and 3 as a function of lattice spacing. The dimensions of the fuel elements, cladding and of the process tubes are given in Tables V and VI. Effective densities of uranium are given in Table VII. Volume ratios and volumes per centimeter of length are given in Tables VIII, IX, X, XI, and XII. Some special extrapolation length measurements are discussed and the results shown in Tables XIII, XIV, XV, and XVI. Standard deviations in the material bucklings due to uncertainties in the "least squares" fit and the extrapolation length are given in Table XVII. Tables XVIII, XIX, and XX show the results of all horizontal traverses taken in support of the extrapolation length studies.

The results of the material buckling measurements indicate that the crossover point for the tube-in-tube is at a lattice spacing of about $8 \frac{1}{4}$ in. with a carbon-to-uranium ratio of 20.42. The crossover point for the tube-and-rod fuel element is at a lattice spacing of about $8 \frac{7}{8}$ in. with a carbon-to-uranium ratio of 17.79. The crossover point for the I and E fuel element is at a lattice spacing of about $8 \frac{3}{8}$ inches with a carbon-to-uranium ratio of 21.12. The crossover point is the lattice spacing for which the buckling does not change upon loss of coolant.

TABLE II

MATERIAL BUCKLING VALUES FOR TUBE-AND-ROD FUEL ELEMENTS
2.5 in. x 1.6 in. and 1.17 in. Solid

<u>Lattice Spacing (in.)</u>	<u>Buckling (10^{-6} cm^{-2})</u>	<u>λ (s-s) (inches)</u>	<u>$\sigma(B^2)$ (10^{-6} cm^{-2})</u>
8 3/8 dry **	-111	1.5	7
8 3/8 wet **	-51	1.5	6
10 3/8 dry	+87	1.5 †	7
10 3/8 wet	+7	1.7 †	7
12 3/8 dry	(+110) *	-- *	-
12 3/8 wet	(-10) *	-- *	-
14 9/16 dry	+65	2.2	6
14 9/16 wet	-68	1.5 †	5

† Not taken at equivalent cylindrical radius.

* Estimated from calculated curves. Experimental measurements have not been made.

** The top layer had 1.92-inch fuel elements at 8 3/8-inch spacing. The properties of this region only approximated the properties of the region of interest; therefore, the data were analyzed using multiregion equations. (2)

TABLE III

MATERIAL BUCKLING VALUES FOR TUBE-AND-ROD FUEL ELEMENTS
2.5 x 1.6 with 0.5 Solid

<u>Fuel Element</u>	<u>Lattice Spacing (in.)</u>	<u>Buckling (10^{-6} cm^{-2})</u>	<u>λ (inches)</u>	<u>$\sigma(B^2)$ (10^{-6} cm^{-2})</u>
2.5 x 1.6 with 0.5 in.	14 9/16 dry	+63	1.9	2
2.5 x 1.6 with 0.5 in.	14 9/16 wet	-111	2.0	2

TABLE IV

MATERIAL BUCKLING VALUES FOR I AND E FUEL ELEMENTS
2.5 in. x 1.6 in.

<u>Lattice Spacing (in.)</u>	<u>Buckling (10^{-6} cm^{-2})</u>	<u>λ (s-s) (inches)</u>	<u>σ (B^2) (10^{-6} cm^{-2})</u>
8 3/8 dry	-65	1.5	6
8 3/8 wet	-68	1.5	4
10 3/8 dry	+100	1.6 †	7
10 3/8 wet	-34	1.8 †	7
12 3/8 dry	(+110) *	(Est.) *	-
12 3/8 wet	(-50) *	(Est.) *	-
14 9/16 dry	+70	1.9	7
14 9/16 wet	-129	2.1	7

* Estimated from calculated curves. Experimental measurements have not been made.

** The top layer had 1.92-inch fuel elements at a lattice spacing of 8 3/8 inches. The properties of this region only approximated the properties of the region of interest; therefore, the data were analyzed using multiregion equations. (2)

† Not taken at equivalent cylindrical radius.

Figure 1
Material Buckling for Tube-in-tube Fuel Elements

Figure 2
Material Buckling for Tube-and-Rod Fuel Elements

Figure 3
Material Buckling for I and E Fuel Elements

TABLE V
DIMENSIONS FOR TUBE-IN-TUBE FUEL ELEMENTS
IN INCHES

<u>Uranium Element</u>		<u>Process Tube</u>		<u>Outer Can</u>		<u>Inner Can</u>		<u>Graphite</u>	
<u>OD</u>	<u>ID</u>	<u>OD</u>	<u>ID</u>	<u>OD</u>	<u>ID</u>	<u>OD</u>	<u>ID</u>	<u>Hole Diameter</u>	
2.500	2.000	3.151	3.021	2.580*	2.510	1.990	1.920	3.177±	.005 in.
1.660	1.120	--	--	1.732	1.677	1.110	0.970	---	

* Effective diameter including aluminum runners is 2.583 inches.

Machining tolerances specified for uranium were $\pm .005$ in., but most of the pieces are within $\pm .002$ in.

Machining tolerances on wall thickness for aluminum tubes are $\pm .003$ in. for cans, and $\pm .005$ in. for process tubes.

TABLE VI
DIMENSIONS FOR TUBE AND ROD FUEL ELEMENT
IN INCHES

<u>Uranium Element</u>		<u>Process Tube</u>		<u>Outer Can</u>		<u>Inner Can</u>		<u>Graphite</u>	
<u>OD</u>	<u>ID</u>	<u>OD</u>	<u>ID</u>	<u>OD</u>	<u>ID</u>	<u>OD</u>	<u>ID</u>	<u>Hole Diameter</u>	
2.500*	1.600	3.151	3.021	2.580**	2.510	1.585	1.515	3.177	
1.175	--	--	--	1.250	1.250	--	--	--	
0.500	--	--	--	0.596	0.504	--	--	--	

* This tube also used for I and E fuel element.

** Effective diameter including aluminum runners is 2.583 inches.

† Rod used in second tube-and-rod fuel element.

TABLE VII
EFFECTIVE DENSITY OF URANIUM SLUGS

<u>Diameter</u>	<u>Density</u>
2.5 x 2.0	18.95 gm/cm ³
1.66 x 1.12	18.60 gm/cm ³
0.500 Solid	18.65
2.5 x 1.6	18.78 gm/cm ³
1.175 Solid	18.95 gm/cm ³

TABLE VIII
TUBE-IN-TUBE
Volume Ratios

<u>Lattice Spacing (in.)</u>	<u>C/U</u>	<u>Al/U</u>	<u>H₂O/U</u>
7 3/16	14.85	0.498	1.097
8 3/8	21.12	0.498	1.097
10 3/8	33.86	0.498	1.097
12 3/8	49.31	0.498	1.097
14 9/16	69.32	0.498	1.097

Atom Ratios

<u>Lattice Spacing (in.)</u>	<u>C/U</u>	<u>Al/U</u>	<u>H₂O/U</u>
7 3/16	25.82	0.630	0.770
8 3/8	36.73	0.630	0.770
10 3/8	58.88	0.630	0.770
12 3/8	85.75	0.630	0.770
14 9/16	120.55	0.630	0.770

TABLE IX
TUBE AND ROD
Volume Ratios

<u>Lattice Spacing (in.)</u>	<u>C/U</u>	<u>Al/U</u>	<u>H₂O/U</u>
8 3/8	15.62	0.309	0.628
10 3/8	25.03	0.309	0.628
12 3/8	36.46	0.309	0.628
14 0/16	51.25	0.309	0.628

<u>Atom Ratios</u>			
<u>Lattice Spacing (in.)</u>	<u>C/U</u>	<u>Al/U</u>	<u>H₂O/U</u>
8 3/8	27.15	0.390	0.441
10 3/8	43.52	0.390	0.441
12 3/8	63.40	0.390	0.441
14 9/16	89.12	0.390	0.441

TABLE X
I AND E
Volume Ratios

<u>Lattice Spacing (in.)</u>	<u>C/U</u>	<u>Al/U</u>	<u>H₂O/U</u>
8 3/8	21.47	0.378	1.286
10 3/8	34.41	0.378	1.286
12 3/8	50.11	0.378	1.286
14 9/16	70.44	0.378	1.286

<u>Atom Ratios</u>			
<u>Lattice Spacing (in.)</u>	<u>C/U</u>	<u>Al/U</u>	<u>H₂O/U</u>
8 3/8	37.60	0.479	0.904
10 3/8	59.94	0.479	0.904
12 3/8	87.29	0.479	0.904
14 9/16	122.71	0.479	0.904

TABLE XI

2.5 x 1.6 WITH 0.5 in. TUBE-AND-ROD FUEL ELEMENT

<u>Volume Ratios</u>			
<u>Lattice Spacing (in.)</u>	<u>C/U</u>	<u>Al/U</u>	<u>H₂O/U</u>
14 9/16	65.98	0.380	1.114
<u>Atom Ratios</u>			
<u>Lattice Spacing (in.)</u>	<u>C/U</u>	<u>Al/U</u>	<u>H₂O/U</u>
14 9/16	114.94	0.481	0.783

TABLE XII
VOLUMES IN CM³/CM LENGTH

<u>Tube-in-Tube Fuel Element</u>	
Uranium	19.001
Aluminum	9.458
H ₂ O	20.853
Polyethylene *	0.072 *
<u>2.5 x 1.6 With 1.17 Tube-and-Rod Fuel Element</u>	
Uranium	25.698
Aluminum	7.929
H ₂ O	16.317
Polyethylene *	0.077 *
<u>2.5 x 1.6 I and E Fuel Element</u>	
Uranium	18.698
Aluminum	7.065
H ₂ O	24.053
<u>2.5 x 1.6 With 0.5 Tube-and-Rod Fuel Element</u>	
Uranium	19.96
Aluminum	7.50
H ₂ O	22.34
Polyethylene *	0.26

* The polyethylene is used to space the center rods and tubes. The volume of polyethylene is included with the H₂O for wet bucklings.

TABLE XIII

λ - WITH AND WITHOUT SHIELD

<u>Fuel Element</u>	<u>Lattice Spacing</u>	<u>λ (side-side) (inches)</u>	<u>Position</u>
2.5 in. x 1.6 in. with 0.5 in.	14 9/16 wet	1.93 ± 0.2	With shields
2.5 in. x 1.6 in. with 0.5 in.	14 9/16 wet	2.05 ± 0.2	Without shields

TABLE XIV

λ - (FRONT-REAR) COUNTER INSERTED FROM SIDE

<u>Fuel Element</u>	<u>Lattice Spacing</u>	<u>λ (inches)</u>	<u>Position</u>
2.5 x 1.6 in. with 1.17	10 3/8 dry	1.13	Front
2.5 x 1.6 in. with 1.17	10 3/8 dry	1.00	Rear

TABLE XV λ - CENTER LINE AND ONE FOOT TO REAR

<u>Fuel Element</u>	<u>Lattice Spacing</u>	<u>λ (side-side) (inches)</u>	<u>Position</u>
2.5 x 1.6 in. with 1.17	14 9/16 wet	1.50 ± 0.2	Center line
2.5 x 1.6 in. with 1.17	14 9/16 wet	1.47 ± 0.2	One foot to rear

TABLE XVI λ - THERMAL AND EPITHERMAL

<u>Fuel Element</u>	<u>Lattice Spacing</u>	<u>λ (side-side) (inches)</u>	<u>Counter</u>
2.5 x 1.6 in.	14 9/16	1.5 ± 0.2	BF ₃ , Bare - Cd
2.5 x 1.6 in. with 1.17	14 9/16	1.3 ± 0.2	BF ₃ , Cadmium covered
2.5 x 1.6 in. with 1.17	8 3/8 dry	1.4 ± 0.1	BF ₃ , Bare - Cd
2.5 x 1.6 in. with 1.17	8 3/8 dry	1.4 ± 0.05	BF ₃ , Cadmium covered

TABLE XVII

STANDARD DEVIATION IN B_n^2 FROM EFFECTIVE SIZES, λ ,
AND VERTICAL RELAXATION LENGTH, B_{11} , IN 10^{-6}cm^{-2}

Fuel Element	λ (side-side)	λ (front-rear)	λ_{total}	B_{11}	B_{11} and λ_{total}	Lattice Spacing (inches)
Tube-in Tube	2	2	3	10	10	7 3/16 dry
	2	2	3	10	10	7 3/16 wet*
	3	2	4	5	6	8 3/8 dry
	3	2	4	4	6	8 3/8 wet*
	5	2	5	5	7	10 2/8 dry*
	3	2	4	4	6	10 3/8 wet*
	2	2	3	4	5	12 3/8 dry*
	2	2	3	1	3	12 3/8 wet*
	4	2	4	2	5	14 9/16 dry*
	4	2	4	1	4	14 9/16 wet*
Tube-and Rod	2	2	3	6	7	8 3/8 dry
	2	2	3	5	6	8 3/8 wet
	6	2	6	3	7	10 3/8 dry
	3	2	4	6	7	10 3/8 wet
	4	2	4	5	6	14 9/16 dry
	4	2	4	3	5	14 0/16 wet
I and E	3	2	4	5	6	8 3/8 dry
	2	2	3	3	4	8 3/8 wet
	6	2	6	4	7	10 3/8 dry
	3	2	4	6	7	10 3/8 wet
	4	2	4	3	5	14 9/16 dry
	4	2	4	6	7	14 9/16 wet

* These results have been previously reported. (1)

TABLE XVIII
HORIZONTAL TRAVERSES

<u>Fuel Element</u>	<u>Lattice Spacing</u>	<u>λ (inches)</u>	<u>Cell Position*</u>
2.5 x 2.0 with 1.6 x 1.1	8 3/16 dry	1.5	4
		1.6	1
	7 3/16 wet	1.4	4
		0.7	2
		0.4	1
	8 3/8 dry	1.13	1 + 2
		1.63	5
		1.50	3
		1.0	F - R
	8 3/8 wet	1.2	3
		1.1	1 + 2
		1.8	1 + 2
		1.3	3
		1.4	2
		1.3	4
	10 3/8 dry	1.7	4
		1.2	4
		1.3	1 + 2
		1.3	1 + 2
		1.5	3
		1.8	1 + 2
	10 3/8 wet	1.4	1 + 2
		1.5	4
		1.6	4
		1.3	2
	12 3/8 dry	1.5	2
		2.5	4
		0.9	F - R
		1.5	1 + 2
		1.0	F - R
		2.3	3
		2.4	5
	12 3/8 wet	1.4	2
		2.1	4
		1.5	1 + 2
		1.4	4
		1.0	F - R

TABLE XVIII (Contd.)

<u>Fuel Element</u>	<u>Lattice Spacing</u>	<u>λ (inches)</u>	<u>Cell Position*</u>
2.5 x 2.0	14 9/16 dry	1.8	3
with 1.6 x 1.1	14 9/16 wet	2.2	3

TABLE XIX

HORIZONTAL TRAVERSES

<u>Fuel Element</u>	<u>Lattice Spacing</u>	<u>λ (inches)</u>	<u>Cell Position*</u>
2.5 x 1.6 with 1.17	8 3/8 dry	1.5	3
	8 3/8 wet	1.5	3
		1.55	4
		1.26	4
		1.13	1 + 2
		1.37	3
		1.38 Cd covered counter	3
	10 3/8 dry	1.1	F - R
		1.5	1 + 2
		1.6	4
		1.4	1 + 2
	10 3/8 wet	1.8	1 + 2
		1.4	5
		1.7	1 + 2
	14 9/16 dry	2.2	5
		1.9	2
		2.2	5
	14 9/16 wet	1.5	2
		1.5	2
		1.5	3
		1.9	5

* Refer to Figure 4 for the actual cell position at which the traverse was taken.

TABLE XX

HORIZONTAL TRAVERSES

<u>Fuel Element</u>	<u>Lattice Spacing</u>	<u>λ (inches)</u>	<u>Cell Position*</u>
2.5 x 1.6 I and E	8 3/8 dry	1.4	4
		1.5	3
		1.0	3
	8 3/8 wet	1.5	4
		1.5	3
		1.2	4
		1.0	1 + 2
		1.2	1 + 2
		1.2 in tubes	-- lucite bar
		1.0	2
		1.1	1
	10 3/8 dry	1.7	1 + 2
		1.5	4
		1.6	1 + 2
	10 3/8 wet	1.8	1 + 2
		1.4	4
		1.7	1 + 2
	14 9/16 dry	1.9	3
		1.5	3
		1.3 CD Head only	3
		1.5 Bare only	3
	14 9/16 dry	1.3	2
		2.1	3
		1.4	2
		2.3	5

* Refer to Figure 4 for the actual cell position at which the traverse was taken.

<u>Position</u>	<u>Description</u>
1	Cell Corner, Between Columns of Tubes.
2	Cell Edge; Under Columns of Tubes.
3	Cell Edge, Radius of Equivalent Cylindrical Cell (Approximate).
4	Between Tubes, Tube Row.
5	Corresponding Positions, Tube Row.

Figure 4
Typical Lattice Cell

Discussion

The λ shown in Tables I, II, and III is the side-to-side extrapolation length and is an average of values measured for each side of the exponential piles. The data used in calculating most of the λ 's were taken as close as possible to the radius of the equivalent cylindrical cell.

The front-to-rear extrapolation length used in all cases was 1.03 inches. A summary of the front-to-rear extrapolation length measurements has been previously reported⁽¹⁾, showing that λ is constant for front-to-rear traverses.

The $\sigma(B^2)$ quoted in Tables I, II, and III is the standard deviation of the buckling as derived from standard deviations in fitting the exponential decay and the transverse cosines, and does not include uncertainties in the extrapolation length due to variations of λ with cell position.

Several special extrapolation length measurements have been made in order to clarify various problems as they were recognized. The extrapolation length has been measured with and without cadmium shields placed at the sides of an exponential pile. This was done to determine what effect backscattered neutrons might have on the extrapolation length. The results shown in Table XI indicate that λ is the same with and without the shields. This pile was constructed in the middle of a large room, with very little scattering material nearby. Thus the conclusion that λ is unaffected by the shields might not hold for piles built near a concrete wall. Also the four sources were clustered in the center of the base, minimizing leakage from the base. Part of the purpose of this measurement was to estimate the effect of epithermal neutrons escaping through the shields and scattering back in, resulting in a higher flux near the edge of the pile, hence an erroneously high extrapolation distance. Since the effect of thermal neutron backscatter is undetectable, the possibility of epithermal backscatter is extremely remote.

The BF-3 neutron counting tubes are usually inserted from the front of the exponential pile. A front-to-rear horizontal traverse has been taken with the counter inserted from the side of the exponential pile. This traverse was taken in order to determine whether or not a possible displacement of the effective counting center of the BF-3 tube from the physical center of the tube had any effect on the measured front-to-rear extrapolation length. The results shown in Table XII gave a front-to-rear λ of 1.06 inches which, within the errors of the measurement, was the same as had been previously measured with the counter inserted from the front.⁽¹⁾

Diffusion theory as applied to exponential piles assumes that the neutron flux distribution is separable into a product of the fluxes in the three dimensions, that is, the flux in the x-direction is a function of x only, etc. To test this assumption experimentally, two horizontal traverses were taken, one on the exponential pile center line, the other one foot to the rear of center. The extrapolation lengths, shown in Table XIII, calculated from the horizontal traverse data, verify the separability of the fluxes for this case.

To determine whether or not the extrapolation length is independent of neutron energy, horizontal traverses were taken with a bare counter and a cadmium-covered counter in two different exponential piles. The results shown in Table XIV indicate that, within the uncertainties of the measurements, λ is the same for thermal and epithermal neutrons.

The independent fit to the thermal and epithermal fluxes is a check of the cadmium ratio criteria on the validity of data. Ordinarily we require that the cadmium ratio be constant in the region where traverses are taken. In the past, the cadmium ratio in the horizontal traverses has been constant, within statistics, all the way out to the last process tubes at each side. However, H. Kouts⁽³⁾ found indications that the comparison of independent fits to thermal and epithermal data is a more sensitive criterion for the existence of an asymptotic energy distribution in the region of measurement. In his work, cases had appeared where the ratio of thermal to epithermal extrapolation distance varied as data points were dropped from the fitting

process, starting with the points nearest the edge. This is possible since the fits to the traverses include considerably more data than the individual cadmium ratio measurements. Thus random deviations are minimized and small trends can be seen which would otherwise appear to be simply a small part of the random fluctuations in the cadmium ratio.

The magnitude of the extrapolation lengths in Table XIV is not necessarily correct since the cadmium shutter method was not used. The differences are valid since the effect of the cadmium shutter method is to raise both λ 's by the same amount, usually a few tenths of an inch.

All special λ 's mentioned above, were measured as close as possible to the radius of the equivalent cylindrical cell.

The front-to-rear extrapolation length has been measured down the center annulus of the tube-in-tube fuel element in the dry 8 3/8 in. lattice. Although several previous front-to-rear traverses in the graphite had given a constant λ , the traverse in the fuel was taken as a further check. The result was 0.96 inches, which is close to 1.03 inches, the value measured in the graphite. Since the traverse in the center annulus shows no significant deviation of λ from the value in the graphite, the front to rear λ has been assumed constant at $1.03 \pm .05$ inches, and front to rear traverses are no longer being taken in four foot piles.

All available evidence on the variation of the side-to-side λ now indicates that the variation is a function of the fine structure flux in the lattice cell. Apparently the total flux is not a simple product of the cell flux and the over-all cosine, as is assumed in the usual technique of homogenizing the cell and analyzing the resulting homogeneous medium for multiplying properties. Thus the small pile flux distribution is not properly reproducing the infinite medium properties, where the flux is rigorously a product of the cell variation and the over-all flux.

One way of looking at the fine structure problem is to consider leakage from the edge in terms of the average position in the cell from which the leakage occurs. For a pile composed of an integral number of cells, the leakage occurs all along the cell edge, so the average is along

the cell edge, closer to the corner than the middle. This location is approximately the point where the flux is the same as in the equivalent cylindrical cell, which is the right place for a horizontal traverse according to the homogenization procedure. However, a traverse at the cell corner would correspond to leakage from a larger cell at the edge and a traverse at the center of the cell edge or even closer to the fuel element would correspond to leakage from a smaller cell at the edge. Thus it should be possible to add or subtract graphite at the edge of the pile, depending on the position of the traverse, and get the same extrapolation distance for all traverse positions. As an example, for the 14 9/16 in. lattice the traverse at the cell corner would correspond to adding 2.1 in. of graphite at the edge of the pile, and the traverse at the middle of the cell edge would correspond to removing 1.6 in. of graphite at the edge of the pile.

The experimental data used in calculating $\sigma(B^2)$ is given in Table XV. Column 2 is the standard deviation in the buckling associated with a standard deviation in the front-to-rear extrapolation length of 0.05 inches. Column 1 is the standard deviation in the buckling due to standard deviations calculated in fitting side-to-side horizontal traverses. Column 3 is the square-root of the sum of the squares of the values listed in columns 1 and 2. Column 4 is the standard deviation in the buckling derived from the "least-squares" fit. Column 5 is the square-root of the sum of the squares of columns 3 and 4 and is the total standard deviation in the buckling.

Densities used were 1.65 gm/cm³ for graphite, 2.70 gm/cm³ for aluminum, and 1.00 gm/cm³ for water.

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Fast Source Theory Harmonic Corrections*

C. Metelmann and D. E. Wood

Introduction

This note is written to clarify the equations and normalizing factors used in fast source theory harmonic corrections for diffusion length measurements. The fast source equations have been used in previous work, ^(1, 2, 3) but the method of using the equations has not been clearly defined.

The fast source theory is an alternative to the usual cadmium shutter technique in correcting for the effect of the fast neutron source. In the shutter technique, a traverse is taken with cadmium between the base and the experimental region (see Figure 1). The shutter traverse is subtracted from the traverse without the shutter to eliminate neutrons which cross the boundary before slowing down below the cadmium cutoff. Then the harmonic corrections are calculated as if the source emitted thermal neutrons only. The fast source theory calculates the thermal neutron distribution from a fast neutron source, so the fast source harmonic corrections are applied to bare counter data, after subtracting the epicadmium neutrons at the counter position. The shutter is not used.

Theory

The thermal flux is determined from the solution to the combined age-diffusion equation as

$$\phi(z) = \frac{K}{C_e C_h} e^{-z/B_{11}} \quad (1)$$

where C_e and C_h are the end and harmonic corrections, and B_{11} is the relaxation length of the fundamental mode.

* Work performed as part of Nuclear Engineering R501, "Reactor Physics Laboratory", a course taught by Hanford Laboratories, General Electric Company, in co-operation with the University of Washington Graduate Center at Richland.

The length is $b-2l$ where l is the extrapolation distance.

Figure 1
Simplified Diagram of Hanford Standard Pile

The harmonic correction in the fast source theory is:

$$C_h = \frac{A_{11} B_{11} F_{11} e^{-z/B_{11}}}{\sum_{\substack{m=1 \\ n=1}}^{\infty} A_{nm} B_{nm} F_{nm} J_{nm} e^{-z/B_{nm}}} J_{11}(\infty) \quad (2)$$

The various terms are defined as:

$$A_{nm} = \frac{2}{1 + \coth(z'/B_{mn})} \quad \text{the finite base correction}$$

$$B_{nm}^{-2} = \frac{1}{L^2} + \pi^2 \left(\frac{n^2}{a^2} + \frac{m^2}{b^2} \right) \quad \text{the relaxation length of each mode}$$

$$F_{nm} = \cos \left(\frac{n\pi x'}{a} \right) \cos \left(\frac{m\pi y'}{b} \right) \sum_i \cos \left(\frac{n\pi x_i}{a} \right) \cos \left(\frac{m\pi y_i}{b} \right)$$

the correction for counter (x' , y') and source (x_i , y_i) displacement from the lateral center of the pile.

$$J_{nm} = \sum_{i=1}^3 f_i e^{\tau_i/L^2} \left\{ 1 + \operatorname{erf} \left(\frac{z}{2\sqrt{\tau_i}} - \frac{\sqrt{\tau_i}}{B_{nm}} \right) + e^{2z/B_{nm}} \left[1 - \operatorname{erf} \left(\frac{z}{2\sqrt{\tau_i}} + \frac{\sqrt{\tau_i}}{B_{nm}} \right) \right] \right\} \quad (3)$$

is the fast source correction.

$$J_{11}(\infty) \text{ is } \lim_{z \rightarrow \infty} J_{11}(z)$$

Evaluation yields

$$J_{11}(\infty) = 2 \sum_{i=1}^3 f_i e^{\tau_i/L^2} = 2.2516$$

f_i and τ_i are the fractional yields and ages for the three spectral components of the Ra-Be source.

The thermal source harmonic corrections have the same form with $J_{11}(\infty)/J_{nm} = 1$. The $J_{11}(\infty)$ is essentially a normalizing factor so that the ratio of the fast C_h to the thermal C_h is a measure of the fast source effect which approaches 1 as z increases.

Calculations

Calculations of C_h were made to check equation (2) against the results in References 1 and 2. Reference 1 has the normalizing factor $J_{11}(\infty)$ used, but the equivalent of equation (2) is not quoted. Reference 2 has a misprint in the equivalent of equation (3) and does not use the normalizing factor.

For the calculations, the source and counter were taken at the lateral center so $F_{nm} = 1$. Two counting slots were considered, with $z = 59.77$ cm and 81.04 cm. Harmonics were included only through the third to reduce the complexity of the calculation. The results are compared in Table I with the results of Reference 1. The agreement is satisfactory since fewer harmonics were used in the present calculations, and the newer values of f_i and τ_i from Reference 2 were used. The values of C_h (fast) include the normalizing factor $J_{11}(\infty)$.

TABLE I
COMPARISON OF FAST SOURCE CORRECTIONS

<u>z</u> <u>(cm)</u>	<u>Thermal C_h</u> <u>Calculated</u>	<u>Thermal C_h</u> <u>Reference 1</u>	<u>Fast C_h</u> <u>Calculated</u>	<u>Fast C_h</u> <u>Reference 1</u>
59.77	0.660	0.624	0.728	0.738
81.04	0.745	0.738	0.765	0.792

References

1. Davenport, D. E., G. L. Lynn, and D. C. Pound. Hanford Standard Pile, HW-21793. July 30, 1951.
2. Richey, C. R. and E. Z. Block. Graphite Diffusion Length Measurements at Hanford, HW-45035. September 10, 1956.
3. Matsumoto, D. D. 709 Program for Reduction of Exponential Pile Data, HW-60388. May 20, 1959.

Theory of Control Rod Studies in Exponential Assemblies

D. E. Wood

Introduction

It would be desirable to determine the strength of the control system in a new reactor with methods other than theoretical calculations (which may be inaccurate) or startup tests (which may be too late). Unless a critical mockup of the reactor were available, the only facilities to be considered are the PCTR and an exponential assembly. This paper discusses the analytical methods necessary to interpret measurements of control rod effects in an exponential assembly. Methods used for PCTR experiments are discussed elsewhere in this Quarterly.

The exponential method was developed within the limits of the theory of a bare, homogeneous, subcritical system. Heterogeneous systems are analyzed by using a reasonably large number of cells so that the cells may be approximated by their equivalent homogeneous medium, insofar as the measured flux distributions are concerned. This requires that the total flux be a simple product of the infinite medium cell flux ("microscopic" flux or "fine structure" flux) and the macroscopic over-all flux which is the same as the flux in the equivalent homogeneous system. When control rods are inserted in the assembly, the homogenization concept is valid only for a fairly large number of rods ("supercells"). A single rod measurement can be analyzed only if the placement of the rod is restricted to a location parallel to the axis of relaxation in the exponential, and fully inserted. With these restrictions the effect of the rod can be interpreted in terms of the change in relaxation length produced relative to the assembly without the rod. Then the change can be related to the effect in a full size reactor by assuming that the extrapolation distance into the rod is independent of reactor size, which is reasonable if the rod is far enough from the edge to be unaffected by boundary transients.

The theory of a central rod in a cylindrical exponential pile is developed with 1, 2, and 3 energy groups. Application of the data to a full-size reactor is shown in terms of a single fully inserted rod, with comments on normalizing theory in order to allow for partial or off-center insertion. Extension of the analysis to a rectangular system has not been made, although the transformation for a critical system has been given by Gast.⁽¹⁾

Experiments

Several experiments are needed to fully exploit the potential value of exponential experiments with control rods. The first would be the measurement of the change in inverse relaxation length, $\delta\gamma$, due to insertion of a control rod into pure graphite. This determines the thermal neutron extrapolation distance, d_2 , into the rod, providing this information for use in the lattice experiment.

Second, $\delta\gamma$ is determined for insertion of a control rod into the lattice of interest and an expression derived for $\delta\gamma$ in terms of d_1 and d_2 . Then with d_2 known, this experiment defines d_1 , the epithermal extrapolation distance. Since the epithermal effect of a rod is usually confined to a small range in energy just above thermal, a third extrapolation distance, d_0 , is defined for the remainder of the fast group. Although no experiment is available to measure d_0 , the usual control rod is relatively transparent to fast neutrons, and d_0 may be assumed infinite with little error. Even if d_0 is not infinite, d_0 may be calculated assuming scattering only, with thermal and epithermal absorption expressed through d_1 and d_2 .

The distribution of thermal and epithermal neutrons in the pile near the control rod may be measured to compare with calculated flux distributions using various assumed values of d_1 and d_2 . However, it is doubtful whether the measured flux would be detailed and sensitive enough to d_1 and d_2 to allow any definite conclusions.

1. Gast, P. F. 1955 Geneva Conference, Paper No. 607.

The $\delta\gamma$ due to several rods may be measured just as a single rod. However, the interpretation used is different. The rods are spaced as they would be in a full size reactor and the measured buckling with rods is then the buckling of the reactor with all rods fully inserted. No conversion is needed since the exponential is now composed of control rod supercells.

The effect of two rods can be measured to obtain an approximate measure of the shadowing effect. The interpretation is not too clear in terms of the full size reactor, so this measurement would define the shadowing effect only qualitatively. Two effects are combined here: the effect of an off-center rod, and the shadowing. The theory of scaling up the combination of these effects has not been worked out. The present treatment is limited to a single rod.

Analysis of Single Rod Measurement

I. One Group Theory

Consider a cylindrical exponential assembly, extrapolated radius R_o , driven from one end, with a control rod, radius a , running full length along the axis.

Without the rod, the material buckling is

$$B_m^2 = \left(\frac{2.405}{R_o} \right)^2 - \gamma_o^2 \quad (1)$$

where γ_o is the inverse relaxation length.

With the rod inserted, the material buckling is unchanged. However, the radial leakage, both inward to the rod and through the outer boundary, is increased. The increased leakage reduces the relaxation length, so that

$$B_m^2 = B_o^2 - \gamma^2 \quad (2)$$

where γ and B_o^2 are the new inverse relaxation length and radial geometric buckling. The extrapolation distance, λ , at the outside is assumed unchanged.

The internal boundary is represented through the boundary condition on the linear extrapolation distance

$$d_2 = \frac{\phi(a)}{\phi'(a)} \quad (3)$$

The external boundary is represented by

$$\phi(R_o) = 0 \quad (4)$$

either with or without the rod.

The flux $\phi(r)$ is no longer just $J_o(Br)$ with the rod present, because the diffusion equation is not applied in the rod region. Thus

$$\phi(r) = AJ_o(B_o r) + CY_o(B_o r) \quad (5)$$

where A and C are to be determined by the boundary conditions (3) and (4).

Apply (4) first to find

$$\phi(r) = AJ_o(B_o r) - A \frac{J_o(B_o R_o)}{Y_o(B_o R_o)} Y_o(B_o r) \quad (6)$$

Then apply (3) to get

$$d_2 = \frac{1}{B_o} \frac{Y_o(B_o R_o) J_o(B_o a) - Y_o(B_o a) J_o(B_o R_o)}{Y_1(B_o a) J_o(B_o R_o) - Y_o(B_o R_o) J_1(B_o a)} \quad (7)$$

Measured values of γ_o^2 and γ^2 are used in (1) and (2) to find B_o . Then B_o , R_o , and a are used in (7) to find d_2 .

The result is applied to a full size reactor, radius R, by assuming that d_2 and γ are unchanged. Assuming d_2 unchanged amounts to ignoring the effects of outer boundary transients on the center of the reactor which is reasonable, and γ is trivial in the full size reactor, and measured in the exponential without rod.

Now apply the derivation to the reactor to find

$$d_2 = \frac{1}{B} \frac{Y_o(BR) J_o(Ba) - Y_o(Ba) J_o(BR)}{Y_1(Ba) J_o(BR) - Y_o(BR) J_1(Ba)} \quad (8)$$

where B is the radial geometric buckling of the reactor with the rod inserted, and (8) is to be solved by iteration for B . In practice a graph of B^2 versus R would be generated (for a fixed d_2) to allow application to any size reactor.

The clean reactor buckling is just $(2.405/R)^2$, so the change in buckling for the reactor due to rod insertion is

$$\delta B^2 = \left(\frac{2.405}{R} \right)^2 - B^2 \quad (9)$$

If the reactor is of finite length, the bucklings are modified by $(\pi/H)^2$, but δB^2 is unchanged.

The calculation of d_2 was used as an aid to understanding the effect and was not actually necessary to the calculation of δB^2 . If (7) and (8) are combined, eliminating d_2 , then $\delta \lambda^2$ yields δB^2 directly, and the precision of the determination of d_2 becomes unimportant.

This feature makes the experimental result more valid than the diffusion theory analysis used on the system. Generally one would expect a transport correction to be necessary near a strong absorber like a control rod. However, the correction should be the same for both the exponential and the reactor, so the application of δB^2 directly to the reactor bypasses this weakness of diffusion theory.

The preceding analysis is inaccurate if applied to a multiplying system since the rod absorption is a strong function of energy. However, if a diffusing medium with a thermal source is used, then a value for d_2 may be obtained which applies to thermal neutrons only, since no fast neutrons are present.

Thus the first experiment should be performed in a graphite stack, with and without the rod, to determine d_2 for the rod. Then d_2 is to be used in a 2 or 3 group analysis of a measurement in a multiplying medium, as discussed in the following sections.

The restriction to cylindrical piles has been made to avoid treatment of the angular harmonics generated by a cylindrical rod in a square pile.

II. Two-Group Theory

The two-group analysis is developed to allow more accurate treatment of the energy dependence of the absorption of the rod. The thermal neutron extrapolation distance, d_2 , is assumed known from a measurement in pure graphite, and the exponential experiment on the multiplying medium is then analyzed to determine the fast extrapolation distance, d_1 .

Conceptually, the problem is not as easily visualized in the two-group treatment, since the relationship between material and geometric bucklings is not simple. Mathematically, the problem merely amounts to deriving an equation similar to (6), but now there are four equations in four unknowns instead of only two.

The two-group diffusion equations are

$$\nabla^2 \phi_f - \frac{\phi_f}{\tau} + \frac{k}{\beta} \frac{\sum_s}{D_f} \phi_s = 0 \quad (10)$$

$$\nabla^2 \phi_s - \frac{\phi_s}{L^2} + \frac{\sum_f}{D_s} \phi_f = 0 \quad (11)$$

Solutions are sought for the homogeneous parts of these:

$$\nabla^2 \phi_f + B^2 \phi_f = 0 \quad (12)$$

$$\nabla^2 \phi_s + B^2 \phi_s = 0 \quad (13)$$

Where B^2 is taken as the same for both, and substitution in the inhomogeneous equations (10) and (11) determines the form of B^2 . Two solutions are found for B^2 :

$$\mu^2 = \frac{1}{2} \left[-\frac{1}{\tau} - \frac{1}{L^2} + \sqrt{\left(\frac{1}{\tau} + \frac{1}{L^2}\right)^2 + \frac{4(k-1)}{L^2}} \right] \quad (14)$$

$$-\nu^2 = \frac{1}{2} \left[-\frac{1}{\tau} - \frac{1}{L^2} - \sqrt{\left(\frac{1}{\tau} + \frac{1}{L^2}\right)^2 + \frac{4(k-1)}{\tau L^2}} \right] \quad (15)$$

The wave equations to be satisfied are:

$$\nabla_r^2 \phi_1 + \frac{\partial^2 \phi_1}{\partial Z^2} + \mu^2 \phi_1 = 0 \quad (16)$$

$$\nabla_r^2 \phi_2 + \frac{\partial^2 \phi_2}{\partial Z^2} - \nu^2 \phi_2 = 0 \quad (17)$$

Now the usual separation of variables is made, letting $\phi(r, z) = R(r)Z(z)$, and the separated terms set equal to constants, yielding

$$-\alpha_1^2 + \gamma_1^2 + \mu^2 = 0 \quad (18)$$

$$-\alpha_2^2 + \gamma_2^2 - \nu^2 = 0 \quad (19)$$

The α 's may be interpreted as radial geometric bucklings, and are related to the material bucklings μ^2 and $-\nu^2$ through γ , the inverse relaxation length in the axial direction. At this point, it should be recognized that $\gamma_1^2 = \gamma_2^2 = \gamma^2$ for a system with no internal axial boundaries. With no boundaries to introduce axial transients, there is only a single relaxation length axially corresponding to the eigenvalue γ . This is analogous to the radial solution for a single region where the term containing ν goes to zero.

Also the material bucklings are related by (from 14 and 15)

$$\mu^2 = \nu^2 - \frac{1}{\tau} - \frac{1}{L^2} \quad (20)$$

The radial fluxes have the form

$$R_f = R_1 + R_2 \quad (21)$$

$$R_s = S_1 R_1 + S_2 R_2 \quad (22)$$

with $R_1 = AJ_o(\alpha_1 r) + CY_o(\alpha_1 r)$ (23)

$$R^2 = EI_o(\alpha_2 r) + FK_o(\alpha_2 r) \quad (24)$$

where the Y_o and K_o terms are retained because the origin is not included. The equations are applied only to the region outside of the rod.

The boundary conditions applied to the radial fluxes are:

$$R_1(R_o) = 0 \quad (25)$$

$$R_2(R_o) = 0 \quad (26)$$

$$R_f(a)/R'_f(a) = d_1 \quad (27)$$

$$R_s(a)/R'_s(a) = d_2 \quad (28)$$

$$\text{Define } X = J_o(\alpha_1 a) - \frac{J_o(\alpha_1 R_o)}{K_o(\alpha_1 R_o)} Y_o(\alpha_1 a) \quad (29)$$

$$Y = I_o(\alpha_2 a) - \frac{I_o(\alpha_2 R_o)}{K_o(\alpha_2 R_o)} K_o(\alpha_2 a) \quad (30)$$

The critical condition is obtained from Equations 25-28, setting the determinant of the coefficients of A, C, E, F, to 0.

$$S_1 \frac{X/X' - d_2}{X/X' - d_1} = S_2 \frac{Y/Y' - d_2}{Y/Y' - d_1} \quad (31)$$

where the prime denotes d/da .

The major buckling in the exponential without the rod is defined by

$$-\alpha_{10}^2 + \gamma_0^2 + \mu^2 = 0 \quad (18')$$

where $\alpha_{10}^2 = \left(\frac{2.405}{R_o} \right)^2$ without the rod, and γ_o is the relaxation length.

With the rod in place, a new γ is measured to determine α_{10} and α_{20} :

$$-\alpha_{10}^2 + \gamma^2 + \mu^2 = 0 \quad (18')$$

$$-\alpha_{20}^2 + \gamma^2 - v^2 = 0 \quad (19')$$

with v^2 from Equation (20).

The α_{10} , α_{20} , and the d_2 from the pure graphite experiment are inserted in Equation (31) to find d_1 , and now all parameters are available to apply to a full size reactor, radius R . The bucklings α_1 and α_2 for the reactor are found from Equation (31) using known d_1 , d_2 , μ , v , and R instead of R_0 . The change in the major buckling of the reactor due to rod insertion is

$$\alpha_1^2 = \alpha_{10}^2 - \left(\frac{2.405}{R} \right)^2 \quad (32)$$

The change in the minor buckling can be found correspondingly, although values for τ and L^2 must be calculated.

III. Three-Group Theory

The two-group analysis is adequate for a rod absorbing thermal neutrons. For a resonance absorber, the absorption is limited to the lower end of the fast group, and the two-group method suffers the same weakness as the one-group method applied to a thermal absorber. A three-group method is proposed with epithermal absorption in the middle group, and a fast group with slowing down, but no true absorption. In this way the fast extrapolation distance, d_0 , may be defined from scattering properties only, and all true absorption represented in d_1 and d_2 . This division is particularly important for internally cooled rods where slowing down in the rod is quite different from slowing down in the external graphite moderator.

The equations are too complex for hand calculations, so the analysis will only be outlined with a minimum of detail. A computer program should be written, or imported, to handle the three-group, two-region problem with the appropriate logarithmic derivative boundary conditions at the rod. The albedo boundary condition used in $F_3^{(2)}$ would probably be suitable.

(2) Keppler, J. G. and W. L. Orr. Three-Group Neutron Diffusion Calculation (Program F₃ - IBM-704), XDC-58-7-18.

The diffusion equations are:

$$\text{fast:} \quad D_0 \nabla^2 \phi_0 - \phi_0 \Sigma_0 + \Sigma_{2,p}^k \phi_2 = 0 \quad (33)$$

$$\text{epithermal:} \quad D_1 \nabla^2 \phi_1 - \phi_1 (\Sigma_1^a + \Sigma_1^{sl}) + p \Sigma_0 \phi_0 = 0 \quad (34)$$

$$\text{thermal:} \quad D_2 \nabla^2 \phi_2 - \phi_2 \Sigma_2 + \phi_1 \Sigma_1^{sl} = 0 \quad (35)$$

with resonance capture between groups 0 and 1, and slowing down in the epithermal group represented by Σ_1^{sl} .

Setting the determinant of the coefficients of the fluxes to 0 and introducing the buckling for the homogeneous parts of the equations yields:

$$k = (1 + L^2 B^2)(1 + \tau_1' B^2)(1 + \tau_0 B^2) \quad (36)$$

$$\text{where} \quad L^2 = D_2 / \Sigma_2$$

$$\tau_0 = D_0 / \Sigma_0$$

$$\tau_1' = (\Sigma_1^a + \Sigma_1^{sl}) / D_1 = \Sigma_1^a / D_1 + \tau_1$$

$$\tau = \tau_0 + \tau_1$$

The solution of the cubic equation in B^2 has three bucklings, μ^2 , $-\nu^2$, and $-\theta^2$. Just as in the two-group case one of these is the major buckling, μ^2 , and the others are minor bucklings which are related to μ^2 through L^2 , τ_1 , and τ_0 .

The fast flux is

$$\begin{aligned} \phi_0 = & AJ_0(\alpha_1 r) + CY_0(\alpha_1 r) + EI_0(\alpha_2 r) + FK_0(\alpha_2 r) + GI_0(\alpha_3 r) \\ & + HK_0(\alpha_e r) = 0 \end{aligned} \quad (37)$$

and the ϕ_1 and ϕ_2 have the same form with appropriate coupling coefficients.

The α 's are again related to the material bucklings:

$$\alpha_1^2 = \gamma^2 + \mu^2 \quad (38)$$

$$\alpha_2^2 = \gamma^2 - \nu^2 \quad (39)$$

$$\alpha_3^2 = \gamma^2 - \theta^2 \quad (40)$$

with γ^2 replaced by $(\pi/H)^2$ for the reactor case.

The same method is applied as in the two-group system, except that d_0 must be calculated. The six constants require six boundary conditions which are

$$\phi_i(R) = 0 \quad (41)$$

$$\phi_i(a)/\phi'_i(a) = d_i \quad i = 0, 1, 2 \quad (42)$$

The measured $\delta\gamma^2$ is used to determine d_1 and then d_1 is used in the reactor critical condition to find $\delta\mu^2$.

Since the rod is not an absorber for the fast group, diffusion theory inside the rod can be used to find d_0 . ϕ_0 in the rod is proportional to $I_0(\beta r)$, where

$$\beta^2 = \frac{1}{\tau_{or}} + \gamma^2 \quad \text{for the exponential} \quad (43)$$

$$= \frac{1}{\tau_{or}} + \frac{\pi^2}{H^2} \quad \text{for the reactor}$$

Then application of continuity of fast flux and current at the core-rod interface yields:

$$\frac{\phi_0(a)}{\phi'_0(a)} = d_0 = \frac{D_0 I_0(\beta a)}{D_{or} \beta I_1(\beta a)} \quad (44)$$

τ_{or} and D_{or} are the age and diffusion coefficient of the rod for the fast group.

The solution of the above equations is left as a digital computer problem, since many multigroup diffusion codes exist. The main problem is to use the proper boundary condition on the fluxes outside the rod.

Although the present treatment assumes the need to calculate d_0 , with d_1 and d_2 determined by experiment, there remains the possibility that d_0 can be experimentally determined, at least under special conditions. If the resonance absorption is primarily due to a material with a fairly narrow resonance, and the slowing down is primarily due to a weakly absorbing moderator (water), then d_1 and d_0 might be separately determined by measuring the rod effect with and without the water moderator. The measurement would be definitive if slowing down in the rod materials other than water could be neglected.

Reflected Reactors

The bare reactor treatment can be directly extended to reflected systems by treating the reflector as a simple extension of the core whose size is measured by the usual reflector savings. This is valid only if the core is large enough so that the minor buckling transients at the two interfaces (core-rod and core-reflector) do not interact. If the system is too small to make that assumption, then a three-region treatment is necessary, and the only reasonable solution is a multigroup, multiregion computer code. This does not affect the value of the experiment, but only complicates application of the data to specific systems.

Partially Inserted Rods

The exponential pile is basically unsuited to the measurement of the effect of partially inserted rods. However, the experiments would provide normalization points (zero and full insertion) for any proposed theoretical treatment. For example, application of one-group perturbation theory to a partially inserted rod yields

$$\frac{\delta k}{(\delta k)_0} = \frac{z}{H} - \frac{\sin(2\pi z/H)}{2\pi} \quad (45)$$

where z is the distance inserted and $(\delta k)_0$ is the fully inserted rod effect. Even though this treatment might not predict $(\delta k)_0$ correctly, it should be

fairly good to predict the variation of δk with z , so $(\delta k)_0$ would be normalized to the exponential measurement.

Off-Center Single Rods

A measurement could be done in an exponential with an off-center rod, but the interpretation in terms of scaling up the effect to the full size reactor is not clear. Fortunately the experiment has been done previously, ⁽³⁾ and the result was interpreted in a fairly simple manner. The buckling change in an exponential was measured as a function of rod position radially, and the result was fitted by the expression

$$\delta\gamma^2 = J_0^{2.2}(Br) \quad (46)$$

which is normalized to one at the center. Now one-group perturbation theory would predict

$$\delta\gamma^2 = J_0^2(Br) \quad (47)$$

which is very close to the experimental result. The difference in the exponent (2.2 versus 2.0) would become smaller for either a larger reactor or a weaker rod, since the rod effect becomes smaller and perturbation theory could be expected to give a more accurate answer.

Since this experiment is already available, there is no need here for further studies at this time.

(3) Persson, R., et al. Exponential Experiments on Heavy Water - Natural Uranium Metal and Oxide Lattice. 1958 Geneva Conference Paper 160.

PCTR Measurements of Control Rod Strength

P. F. Nichols and R. E. Heineman

Introduction

This paper describes the measurement of control rod strength in the PCTR. The first section outlines the theory and method of such a measurement, and the second section presents the results of an actual measurement of the Experimental Gas Cooled Reactor (EGCR) control rod strength. The reactor parameter actually inferred is the difference in the infinite multiplication factor for a control rod supercell with and without the control rod in place in the supercell. These measurements are, therefore, very closely related to the measurement of k_{∞} for an ordinary lattice cell. ^(1, 2)

The first section considers the problem first in the most simple manner possible, i.e., a one-group treatment of the measurement on an infinitely long control rod. The complications and refinements involved in a two-group treatment are then discussed first for an infinitely long control rod, and finally for one of finite length.

In the second section, a measurement of the EGCR control rod strength is interpreted with the use of the two-group method for a control rod of finite length. The resulting control rod worth in a 4 x 4 array of EGCR cells 21.875 in. in length is $\Delta k_{\infty}^{cr} = 0.158 \pm 0.012$.

Theory of the Measurements

Case 1 - Infinitely Long, Cylindrical PCTR - One Group Theory

If a uniform array of cells is poisoned to an infinite multiplication factor of unity, no change in the reactivity, ρ , of the PCTR is observed when the array is inserted into the cavity of the PCTR as long as the neutron or adjoint flux spectrum has been properly matched to that of the cells. In the one-group approximation no such matching problem exists, however, and the null reactivity condition may be expressed in terms of one-group

perturbation theory as

$$\Delta\rho = \Sigma_a^f \phi^f m (\eta-1) - \Sigma_a^i \phi^i m - \Sigma_a^p \phi^p m = 0 \quad (1)$$

where the superscripts f, i, and p refer to the fuel, other cell components, and the poison, respectively. Equation (1) may be written as

$$k_\infty = \eta f = 1 + \frac{\Sigma_a^p \phi^p}{\Sigma_a^f \phi^f + \Sigma_a^i \phi^i} = 1 + \frac{A^p}{A^{\text{cell}}} \quad (2)$$

where A^p is the neutron absorption rate in the poison, and A^{cell} is the absorption rate in the remainder of the cell.

One now simultaneously inserts the control rod and removes sufficient poison such that no change in reactivity occurs. Proceeding as above, we now have

$$k'_\infty = \eta f' = 1 + \frac{\Sigma_a^{p'} \phi^{p'}}{\Sigma_a^f \phi^f + \Sigma_a^i \phi^i + \Sigma_a^{\text{cr}} \phi^{\text{cr}}} = 1 + \frac{A^{p'}}{A^{\text{cell}'}} \quad (3)$$

where the primes refer to the case in which the control rod is in the supercell. We now define

$$\Delta k^{\text{cr}} = k_\infty - k'_\infty = \frac{A^p}{A^{\text{cell}}} - \frac{A^{p'}}{A^{\text{cell}'}} \quad (4)$$

If we wish to account for the possibility that the relative flux distribution over the supercell components may change during the addition or removal of poison, we use the following expression

$$\Delta k^{\text{cr}} = \frac{f}{f^p} - \frac{f'}{f^{p'}} \quad (5)$$

It should be noted that Equation (5) reduces to Equation (4) for the special case of unchanged flux distributions.

Thus, the control rod strength is given in terms of known macroscopic cross sections and measured relative fluxes in the supercell. In the one group approximation, the errors in the measurement of k_{∞} and k'_{∞} are caused only by the errors in the cross sections and relative fluxes, and are denoted by $\delta(\Delta k_{\infty})_{rel}$ and $\delta(\Delta k'_{\infty})_{rel}$. These errors are in general less than two per cent of $(k_{\infty}-1)$ and $(k'_{\infty}-1)$ respectively, and are not independent of one another.

Case 2 - Infinitely Long, Cylindrical PCTR - Two Group Theory

We now take into account the difference in worth of fast and thermal neutrons. The control rod worth is still calculated from Equation (4) or (5), but we now have an additional source of error, namely the flux and adjoint mismatches. It thus becomes necessary to measure not only the quantities in Equation (4) or (5), but also the radial flux and adjoint ratio mismatches. The resulting error for the measurement of k_{∞} or k'_{∞} is given by

$$\delta(\Delta k_{\infty})_{mis} = -k_{\infty} \left[\frac{\left(\frac{\Delta \phi_1}{\phi_2} \right)}{\left(\frac{\phi_1}{\phi_2} \right)} \cdot \frac{\left(\frac{\Delta m_1}{m_2} \right)}{\left(\frac{m_1}{m_2} \right)} \right] \quad \text{area ave.} \quad (6)$$

or

$$\delta(\Delta k'_{\infty})_{mis} = k'_{\infty} \left[\frac{\left(\frac{\Delta \phi_1'}{\phi_2'} \right)}{\left(\frac{\phi_1'}{\phi_2'} \right)} \cdot \frac{\left(\frac{\Delta m_1'}{m_2'} \right)}{\left(\frac{m_1'}{m_2'} \right)} \right] \quad \text{area ave.} \quad (7)$$

where the quantities in the brackets are area averaged mismatches.

If the control rod is very weak, the errors in the measurements are caused primarily by the errors in the cross sections and relative fluxes, $\delta(\Delta k_{\infty})_{rel}$ and $\delta(\Delta k'_{\infty})_{rel}$, as in the one group case. However, if the control rod is strong, the errors from the flux and adjoint mismatches can be expected to predominate.

Case 3 - Finite Length Cylindrical PCTR - Two Group Theory

In this case, we have flux and adjoint mismatches in the axial direction as well as the radial direction. The errors resulting from the mismatches are given by

$$\delta(\Delta k_{\infty}) = -k_{\infty} \left[\frac{\left(\frac{\Delta \phi_1}{\phi_2} \right)}{\left(\frac{\phi_1}{\phi_2} \right)} \cdot \frac{\left(\frac{\Delta m_1}{m_2} \right)}{\left(\frac{m_1}{m_2} \right)} \right] \quad \text{volume ave.} \quad (8)$$

and

$$\delta(\Delta k'_{\infty}) = -k'_{\infty} \left[\frac{\left(\frac{\Delta \phi_1}{\phi_2} \right)'}{\left(\frac{\phi_1}{\phi_2} \right)'} \cdot \frac{\left(\frac{\Delta m_1}{m_2} \right)'}{\left(\frac{m_1}{m_2} \right)'} \right] \quad \text{volume ave.} \quad (9)$$

where the quantities in the brackets are now volume weighted averages.

Experimental Measurement of EGCR Control Rod Worth

The worth of a boron carbide control rod in a 4 x 4 array of EGCR lattice cells 21.875 in. long was measured in the PCTR. The control rod consisted of B₄C pellets with an OD of 3.257 in. and ID of 2.730 in., and a density of 1.86 gm/cm³. It had an exterior stainless steel cladding with an OD of 3.432 in. and a wall thickness of 0.088 ± 0.007, and an interior stainless steel liner with an OD of 2.619 in., and a wall thickness of 0.044 ± 0.001. See Figure 1. The EGCR fuel assemblies are described elsewhere in References 3 and 4. In this case, k_∞ had been measured previously⁽³⁾ for the uniform lattice without the control rod in position, so the problem was reduced to a measurement of k'_∞ with the control rod in place. k'_∞ was obtained from the expression

$$k'_{\infty} = 1 + \frac{A_p'}{A_{\text{cell}}'} \quad (2a)$$

This was used instead of

$$k'_{\infty} = \frac{f'}{f^p} \quad (5a)$$

Figure 1
PCTR Measurement of EGCR Control Rod Strength

because the quantity of poison present was so small (actually negative) that no significant change in the flux distributions occurs. As a result it was necessary to obtain only one flux traverse. The flux traverse in the moderator is shown in Figure 2.

Absorption rates in cell components other than the control rod were calculated from

$$A_i = (\Sigma_a \phi_{1/v})_i \quad (10)$$

In order to obtain the absorption rate in the copper, A^P , it was necessary to measure the mass of copper required to poison the 4 x 4 array with the control rod in place to an infinite multiplication factor of unity. This was done by making reactivity measurements, using the array without the control rod, but poisoned to a k_∞ of unity as the standard. In this particular experiment, the 4 x 4 array with the control rod had a k'_∞ of slightly less than unity, even with all the poison removed. Thus the mass of copper required to poison the 4 x 4 array with the control rod to an infinite multiplication factor of unity was "negative", but could be estimated from the worth of copper and the $\Delta\rho$ reactivity measurements.

The absorption rate in the control rod (included in A^{cell}) was calculated from

$$A^{cr} = -L \frac{\overline{Z(z)}}{\overline{Z(0)}} 2\pi R D \left. \frac{d\phi}{dr} \right|_{\substack{r=R \\ z=0}} \quad (11)$$

where L is the length of the control rod, $\overline{Z(z)}/\overline{Z(0)}$ is the ratio of the average flux along the control rod to the flux at $z = 0$, R is the radius of the control rod, D is the diffusion coefficient, and $d\phi/dr$ is measured at $z = 0$. In calculating a value for A^{cr} , a radius ($R = 3$ in.) somewhat larger than the radius of the control rod was chosen to make the diffusion theory approximation for the current more valid. (5)

Figure 2
Flux Traverse in Moderator

The relative absorption rates in the various cell components are listed in Table I.

The value of k_{∞}' obtained from Equation (2a) is 0.989 ± 0.011 . Now since the uniform array without the control rod has a k_{∞} of 1.147 ± 0.004 , we have

$$\Delta k^{cr} = (1.147 \pm 0.004) - (0.989 \pm 0.011) = 0.158 \pm 0.012$$

TABLE I
RELATIVE ABSORPTION RATES IN CELL COMPONENTS

<u>Component</u>	<u>Mass (kg)</u>	<u>$\Sigma_a \phi_1 / v$ (arbitrary unit)</u>
Fuel	114.0	4787
Control Rod	1.6	361
Cladding	14.2	581
Spiders and End Caps	7.1	322
Graphite	529.4	219
MgO Pellets (adjacent to end caps)	0.7	0.5

The error of 0.011 in k_{∞}' results almost entirely from the radial flux and adjoint mismatches.

It is not possible to measure the individual mismatches directly for a supercell, because only one complete supercell can be accommodated in the PCTR cavity. The mismatches were estimated in the following manner. The changes in the flux and adjoint ratio were measured which occur between the poisoned uniform array without the control rod, and the array with the control rod in place. It was then assumed that these changes in the mismatches obtained in this manner approximate the actual mismatch which occurs between the PCTR loading with the control rod in place, and an infinite array of such cells. This approximation probably overestimates the actual mismatch close to the control rod. A volume weighted average was used in the calculation.

The results of this experiment were extrapolated to a 4 x 4 array 29.000 in. long, and 3 x 3 array 21.875 in. long and 29.000 in. long. The results are listed below, but the details of the calculation are cumbersome, and will not be included here.

$$\Delta k_{\infty}^{cr} = 0.289 \pm 0.020 \quad 3 \times 3 - 21.875 \text{ in.}$$

$$\Delta k_{\infty}^{cr} = 0.163 \pm 0.012 \quad 4 \times 4 - 29.000 \text{ in.}$$

$$\Delta k_{\infty}^{cr} = 0.294 \pm 0.020 \quad 3 \times 3 - 29.000 \text{ in.}$$

Acknowledgments

The work done by F. C. Engesser, T. J. Oakes and the personnel of Experimental Reactors Operation is gratefully acknowledged.

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Neutron Rethermalization in Graphite in Cylindrical Geometry

R. A. Bennett

Introduction

This is a status report on a second series of experiments performed in the PCTR in the study of the space-energy distribution of neutrons in graphite-moderating media with nonuniform physical temperatures. The purposes of this work are to determine rethermalization cross sections of graphite as a function of the temperature of the media in which the neutrons are being moderated and to test the validity of the two-maxwellian group model by comparison of calculated and measured traverses of the activity of a non- $1/v$ detector like lutetium. Secondary objectives of this work are to establish analytical methods which treat the entire PCTR rather than just the experimental region of the reactor core, and which can evaluate simultaneously the different rethermalization cross sections of dissimilar media or of a homogeneous medium with two temperature regions.

The experimental curves and data are included in this report together with a description of the experiments, methods of analysis, and results obtained thus far.

Experimental Geometry and Theoretical Model

In the analysis of these experiments the entire PCTR is treated and is assumed to be an infinite cylindrical assembly with two temperature regions and five geometrical regions. The geometry and temperatures are shown in Figure 1. Region one is a homogeneous cylinder of graphite; region two is an annulus of insulation of lampblack contained between two 1/16-inch thick cans of 2S aluminum; region three is a homogeneous annulus of graphite; region four is an homogeneous annulus of Lucite, U-235, and lead; and region five is a homogeneous annulus of graphite.

Region one only is at the physical temperature T_1 . Regions three, four, five are all at the physical temperature T_2 . Region two is at a mean

Figure 1
PCTR - Equivalent for Analysis

temperature between T_1 and T_2 which is defined by the characteristic logarithmic variation of temperature in cylindrical geometry.

The control rods and end leveling slugs of the reactor are neglected. The neglect of these is justified on the basis that the axial distribution of the flux in the experimental region is flat to within a few percent.

The activity data were taken in radial traverses through regions one and three only, in most cases. Where possible both bare and cadmium covered activity traverses were made simultaneously. Thermal activity and epithermal activity traverses were calculated from these data.

Theory

The theoretical model for the analysis of these experiments is essentially that reported in the earlier experiments.^(1, 2) As a first approximation, the neutron spectrum at all points in the reactor is assumed to be a single $1/E$ epithermal spectrum plus a weighted sum of two maxwellian energy groups $M(E, T_1)$ and $M(E, T_2)$, which are characterized by the physical temperatures, T_1 and T_2 .

The weighting factors are the spatial distributions of the integrated thermal fluxes $\phi_1(x)$ and $\phi_2(x)$. The total space-energy distribution may then be written:

$$\Phi(E, T_1, T_2, r) = \phi_2(r) M(E, T_1) + \phi_3(r) M(E, T_2) + \frac{\phi_1(r)}{E} \quad (1)$$

The group diffusion equations are the normal form with a rethermalization term added. In any region i , the differential diffusion equations are:

$$\begin{aligned} D_{1i} \nabla^2 \phi_{1i} - \Sigma_{a1i} \phi_{1i} - \Sigma_{1 \rightarrow 2, i} \phi_{1i} - \Sigma_{1 \rightarrow 3, i} \phi_{1i} &= \nu \Sigma_{f2i} \phi_{2i} + \nu \Sigma_{f3i} \phi_{3i} \\ D_{2i} \nabla^2 \phi_{2i} - \Sigma_{a2i} \phi_{2i} + \Sigma_{1 \rightarrow 2, i} \phi_{1i} - \Sigma_{2 \rightarrow 3, i} \phi_{2i} &= 0 \\ D_{3i} \nabla^2 \phi_{3i} - \Sigma_{a3i} \phi_{3i} + \Sigma_{1 \rightarrow 3, i} \phi_{1i} - \Sigma_{3 \rightarrow 2, i} \phi_{3i} &= 0 \end{aligned} \quad (2)$$

All of the indicated parameters do not exist in all regions, of course, and in general they are based on the physical characteristics of the region of concern. The exceptions are (1) the fuel is contained in region four only; (2) $\Sigma_{1 \rightarrow 2}$ and $\Sigma_{3 \rightarrow 2}$ are defined in regions one and two only; (3) $\Sigma_{1 \rightarrow 3}$ and $\Sigma_{2 \rightarrow 3}$ are defined in all except region one. The latter two definitions are simply described by the statement that neutrons in a given region transfer only into the characteristic distribution of thermal neutrons of that region.

The unknown parameters of Equation 2 are the rethermalization cross sections $\Sigma_{2 \rightarrow 3}$ and $\Sigma_{3 \rightarrow 2}$. These quantities are determined by an iteration procedure on $\Sigma_{2 \rightarrow 3}$ and $\Sigma_{3 \rightarrow 2}$ which fits a theoretical curve of the thermal activity of a $1/v$ detector to an experimental curve. The theoretical activity expression is obtained from Equation (1) by multiplying the thermal fluxes by the activation cross sections and integrating over-all thermal energies. The resulting expression is

$$A_{th}(r) \sim \phi_2(r) + \sqrt{\frac{T_1}{T_2}} \phi_3(r) \quad (3)$$

for all r . A more detailed description of the analytical procedures is given later in this report.

The thermal fluxes $\phi_2(r)$ and $\phi_3(r)$ may also be used to determine activation traverses of detectors with non- $1/v$ response, which may then be used to evaluate validity of the spectral model employed in the analysis. It can be shown by using the thermal components of Equation (1) and cross sections of Lu^{175} and Lu^{176} that the observed ratio of counting rates of the isotopes of Lu at rod drop at position r is

$$\frac{A_{th}^{177}}{A_{th}^{176}}(r) = \alpha \left[\frac{1 + \frac{g^{176}(T_2)}{g^{176}(T_1)} \sqrt{\frac{T_1}{T_2}} \frac{\phi_3(r)}{\phi_2(r)}}{1 + \frac{T_1}{T_2} \frac{\phi_3(r)}{\phi_2(r)}} \right] g^{176}(T_1)$$

where g^{176} is the Westcott non- $1/v$ factor for Lu^{176} , $g^{175} = 1$ and α is a

calibration constant for the counting system used. The constant α is obtained from a single irradiation in a well-thermalized neutron spectrum of a thermal column ($Cd R_0 \geq 10^4$). Although the precisions of the experimental quantities entering in this expression are expected to be good, the sensitivity of this test of the model of the energy spectrum of thermal neutrons is difficult to predict at this time.

Experimental Details

This series of experiments was conducted in essentially the manner described in the first series of experiments. The variations from the first series of experiments are (1) the temperature discontinuity was changed from plane to cylindrical geometry; (2) the insulation was changed from a vacuum to lampblack; (3) the cylindrical geometry allowed traverses to be made in any radial direction rather than perpendicular to the plane temperature discontinuity; this in turn allowed the activation of the Au, Cu, and Lu detectors, both bare and cadmium covered, to be made simultaneously; the latter was accomplished by making the traverses at symmetric locations with respect to the vertical midplane of the PCTR and at a separation of six inches; (4) the temperature range was extended to 800 K where the previous high temperature was 666 K; (5) Lucite rods were inserted in the driver fuel columns and leveling slugs to reduce the thermal source in regions one and three; (6) counting techniques were improved to insure precise normalization of traverse in regions one and three; (7) lutetium was employed to measure neutron temperatures and test the validity of the model of the neutron spectrum.

Experimental Results

Radial traverses of Cu, Au, and Lu, both bare and cadmium covered, were made for the temperature conditions 144 K, 290 K, 523 K, 690 K, 803 K in region one and corresponding values of 283 K, 290 K, 299 K, 308 K, 326 K in region three. Within a few percent the epicadmium traverses were the same for all temperature conditions. The bare and cadmium traverses of copper activity are shown in Figure 2 for

$T_1 = T_2 = 293$ K. The bare and cadmium covered traverses are used to

calculate the thermal traverse of the activity of a $1/v$ detector. The calculated thermal traverse is also shown in Figure 2. The thermal traverses for all five experiments are shown in Figure 3. The traverses of episcadmium activities of Au are a fair measure of the slowing distribution at thermal or the source of thermal neutrons. The spatial variation of these curves with temperature was not significant so that a single curve serves for all experiments. This curve together with the Cu episcadmium curve are shown in Figure 4. The difference in shape between the two curves is indicative of the variation of the energy dependence of the epithermal spectrum with position. The significance of this variation is apparent when one attempts to fit the episcadmium activity of Au with the fast diffusion Equation (1). This problem is discussed in detail under the "Analytical Methods and Results" section of this report.

Under the assumption of no absorption in the graphite and no thermal neutron source one may make fairly good estimates of the relaxation lengths for rethermalization, which are proportional to the rethermalization cross sections. The estimates are determined from the ratio of activities at the center of the heated or cooled region to the activity at the same point at room temperature. This technique is described for plane geometry in the earlier work. These results are shown in Table I together with earlier results obtained in the plane geometry.

TABLE I
RELAXATION LENGTHS FOR RETHERMALIZATION

Graphite Temp. K	Plane Geometry			Cylindrical Geometry
	Preliminary Estimate	Final Result	From Curve	Preliminary Estimate
	cm.	cm.	cm.	cm.
108	35.8	21.0		
144			20.0	17.5
491	12.7	11.4		
523			10.8	8.1
666	6.6	7.5		
690			7.2	5.1
803			6.0	3.3

Gaseous graphite - theoretical result applicable at high temperatures
= 3.5 centimeters.

Figure 2
Radial Activity Traverses of Copper
for
 $T_1 = T_2 = 290$ K

Figure 3
 Radial Traverses of the Thermal Activity
 of Copper for Various Temperature Conditions

Figure 4
 Epicadmium Traverses of the Activity
 of Au and Cu for $T_1 = T_2 = 290 \text{ K}$

The preliminary estimates and final results for plane geometry yield some measure of the worth of the estimating scheme. A comparison of values from the curve for plane geometry and the preliminary estimates for cylindrical geometry shows a rather consistent difference. It is expected that this difference is a geometry effect, which could not be corrected for in the plane geometry. Final conclusions regarding this point must await the completion of the more exact analysis.

Activity traverses of the isotopes of lutetium were obtained for each set of temperature conditions. The radial distributions of the effective temperatures of thermal neutrons determined from the ratio of these activities is shown in Figure 5. These temperatures are calculated under the assumption that the energy distribution of thermal neutrons may be represented by a single maxwellian characterized by a single temperature. In the test of the validity of spectral model used to obtain the rethermalization cross sections the spectrum is assumed to be a weighted sum of two maxwellians (See Equation 1). The test of the compatibility of these models requires the fluxes ϕ_2 and ϕ_3 . The determination of ϕ_2 and ϕ_3 is concurrent with the determination of $\Sigma_{2 \rightarrow 3}$ and $\Sigma_{3 \rightarrow 2}$. Work on the latter is in progress.

Analytical Methods and Results

The evaluation of the rethermalization cross sections from the traverses of the thermal activity of a $1/v$ detector is being done with a new and as yet unreported IBM 709 code called FIT-1. FIT-1 has been coded by J. Lilley who is working closely with the author in the analysis of these data and has provided considerable assistance in the form of program modification and interpretation of the results of the computer analysis.

FIT-1 is a data fitting code which minimized the residual sum of squares between an experimental activity curve and a computed curve by iteration on a single parameter or a number of parameters singly. The computed curves are obtained in FIT-1 with a modified version of F-3⁽³⁾, which is a multiregion diffusion code. This modified version of F-3 includes the "upscattering" or the rethermalization processes described earlier. The known parameters of the diffusion differential equation and

Figure 5
Effective Neutron Temperature
vs
Radius

boundary conditions are specified in the program input. The estimated values and limits of the unknown parameters and the sequence of iteration upon them are specified separately. The program automatically selects the best value of each parameter, one at a time, and proceeds to the next.

Thus far, FIT-1 has been used successfully to obtain the fast diffusion parameters which yield a best fit to the observed traverse of the epicadmium activity of Au, which is the best thermal source distribution found in these experiments. This was done for $T_1 = T_2$, with equal diffusion coefficients in regions one and three, with $\Sigma_{1 \rightarrow 2, 1} = \Sigma_{1 \rightarrow 2, 3}$, and with $\Sigma_{ali} = 0$. The fitted curve is shown in Figure 4. The adjustment of the fast diffusion parameters to yield the correct cadmium ratios and hence the best thermal flux is the current problem of this analysis.

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Lutetium as a Spectral Index Detector

L. C. Schmid and W. P. Stinson

Experimental Technique

The ratio of the activities induced in the element lutetium by neutron capture has been shown⁽¹⁾ to be a sensitive indicator of a spectral index which characterizes the Maxwellian distribution of neutron energies in a thermal column. A recipe for making ceramic foils which are suitable for such measurements at high temperatures has been presented⁽²⁾ also. The interest which has been shown for the use of lutetium as a spectral indicator has made evident the desirability of describing the experimental techniques involved in a measurement.

Neutron capture in the element lutetium produces two radioactive nuclides, Lu-176m and Lu-177. The "m" indicates that the decay of interest proceeds from a metastable level or excited state of the Lu-176 nucleus rather than from the ground state. Both nuclides decay by beta-ray emission to excited states of Hf which in turn decays to its ground state by gamma-ray emission. Thus a system which counts either of these radiations can be used as a detector. The systems used by Reactor Lattice Physics are mainly gamma-ray spectrometers and, for this reason, the discussion which follows is concerned with gamma-ray counting of the foils.

The gamma-ray spectra from a lutetium foil are shown in Figure 1. The curve obtained with a differential pulse-height analyzer is shown as a solid line and that obtained with an integral analyzer is shown as a dashed line. Curves A were obtained one hour after the foil was irradiated. Peaks are observed in the differential spectrum at energies of 27, 55, 88, and 208 kev. The 208-kev peak is caused by radiation emitted in the decay of Lu-177. The other peaks are from radiation that is characteristic of Lu-176m. The 88-kev peak is caused by gamma rays from a transition in Hf-176. The 55-kev peak is caused by K x-rays of Hf which are produced after "internal conversion" of the 88-kev transition. The 27-kev peak is a secondary peak caused by the Hf K x-rays. When an Hf

Figure 1

Gamma-ray Spectra of Lutetium. The dashed and solid curves were obtained with an integral and a differential pulse-height analyzer, respectively. Curves A were obtained one hour after irradiation when the radiation is predominantly from Lu-176m. Curves B were obtained five days after the irradiation. At this time the Lu-176m activity has decayed and the radiation from Lu-177 is observed.

x-ray is absorbed in a NaI(Tl) crystal, I x-rays (28 kev) are produced. Some of these I x-rays are reabsorbed, but some also escape from the crystal and their energy is lost from the system. The 27-kev peak contains counts corresponding to the difference in energy between the Hf and I x-ray energies. A certain percentage of the counts in this peak is due also to absorption of the I x-ray (28-kev) and the escape of radiation which removes the "difference energy".

Curves B of Figure 1 are curves of the gamma-ray spectrum obtained from the same foil as that used for Curves A but counted five days after the irradiation. The half lives of Lu-176m and Lu-177 are 3.68 ± 0.01 hr and 6.72 ± 0.01 d, respectively. Therefore, the intensities of Lu-176m and Lu-177 gamma rays in Curves B are 2×10^{-10} and 0.6 of what they were in Curves A. The short-lived activity has decayed to a negligible amount and the gamma-ray spectrum obtained from the foil now is that characteristic of Lu-177. Photo peaks from gamma rays with energies of 113 and 208 kev are observed. An Hf K x-ray peak and its secondary "escape" peak are again seen at 55 and 27-kev.

There are several ways in which the count rates from the two isotopes can be separated in order to obtain a ratio of the Lu-177 activity to the Lu-176m activity. We have chosen to count the foils with an integral analyzer biased at 37 kev. A particular foil is counted 30-45 minutes after irradiation and again in three or four days. If a foil is counted earlier than 30 minutes after irradiation the aluminum activity from the aluminum oxide matrix of the foil will be counted with the lutetium activity. After three or four days only the long-lived activity remains. The count from this long-lived activity is corrected back to the time the foil was counted the first time and the difference (short-lived activity) is calculated. The two activities (short-lived and long-lived) are then decay corrected back to the end of the irradiation and the ratio $R_m = A^{177}/A^{176m}$ is obtained. The counts for a natural activity (11.8 c/m per 10 mg of lutetium) are subtracted before the counts are decay corrected.

The measured ratio R_m can be represented by

$$R = \frac{A^{177}}{A^{176m}} = \frac{N^{176}(\sigma_O g)^{176} (1 + \frac{rs}{g})^{176} \psi^{177} \epsilon_s^{177} \epsilon_c^{177}}{N^{175} \sigma_O^{175} (1 + rs)^{175} \psi^{176m} \epsilon_s^{176m} \epsilon_c^{176m}} \quad (1)$$

For simplicity the shorthand notation of g , s , and r suggested by Westcott⁽³⁾ has been used. However, the s and r are defined in a more rigorous manner. Some quantities such as the mass of lutetium and geometrical efficiencies have not been included in (1) since they are identical for the two isotopes and cancel. Also g^{175} has not been included since its value is ~ 1 . The quantity N is the number of atoms of a particular isotope and the σ_O are 2200 m/sec cross sections. The ϵ_s are source efficiencies, i.e., what fraction of the total disintegrations are in a form which can be counted by a gamma-ray spectrometer. The ϵ_c are counter efficiencies, i.e., what fraction of the countable radiations are actually counted by the detector. The ψ^{177} and ψ^{176m} are parameters which take into consideration the effect of a non-constant flux during irradiation of the foil. For many experimental situations, the flux increases exponentially from some source level to a constant flux level and then the flux decreases exponentially toward the source level at the end of the irradiation.

If the differential equations which represent the build up and decay of a nuclide during an irradiation are solved with the following conditions on the flux,

$$\begin{array}{ll} \phi = \phi_s & t = 0 \\ \phi = \phi_s e^{\alpha_1 t} & 0 \leq t \leq T_1 \\ \phi = \phi_m & T_1 \leq t \leq T_2 \\ \phi = \phi_m e^{-\alpha_2(t-T_2)} & T_2 \leq t \leq T_3 \\ \phi = \phi_s & t \geq T_3 \end{array} \quad (2)$$

where α_1 and α_2 are inverse periods, and ϕ_s and ϕ_m are source and maximum fluxes, respectively, then

$$\psi = \phi_m e^{-\lambda(T_3 - T_2)} \left\{ \frac{1}{(1 - \alpha_2/\lambda)} \left[e^{-(\alpha_2 - \lambda)(T_3 - T_2)} - 1 \right] + \left[1 - e^{-\lambda(T_2 - T_1)} \right] + \frac{e^{-\lambda(T_2 - T_1)}}{(1 + \alpha_1/\lambda)} \left[1 - e^{-(\lambda + \alpha_1)T_1} \right] \right\} \quad (3)$$

Equation (3) reduces for very fast periods (i.e., α_1 and α_2 are large and $(T_3 - T_2) = T_1 = 0$) to the usual form for a constant flux,

$$\psi_c = \phi_m \left[1 - e^{-\lambda(T_2 - T_1)} \right] \quad (4)$$

The effect of ignoring the periods can be observed by dividing Equation (3) by Equation (4). Thus,

$$E = e^{-\lambda(T_3 - T_2)} \left\{ 1 - \frac{1}{(1 - \alpha_2/\lambda)} \left[\frac{1 - e^{-(\alpha_2 - \lambda)(T_3 - T_2)}}{1 - e^{-\lambda(T_2 - T_1)}} \right] + \frac{e^{-\lambda(T_2 - T_1)}}{1 + \alpha_1/\lambda} \left[\frac{1 - e^{-(\lambda + \alpha_1)T_1}}{1 - e^{-\lambda(T_2 - T_1)}} \right] \right\} \quad (5)$$

Equation 5 can be simplified by using the boundary conditions which the flux must satisfy at $t = T_1$ and T_2 . These are $\phi_m = \phi_s e^{\alpha_1 T_1}$ and $\phi_s = \phi_m e^{-\alpha_2(T_3 - T_2)}$. Since the ratio $\frac{\phi_s}{\phi_m}$ is usually $\sim 10^{-7}$, the terms $e^{-\alpha_1 T_1}$ and $e^{-\alpha_2(T_3 - T_2)}$ can be assumed zero and

$$E = e^{-\lambda(T_3 - T_2)} \left\{ 1 + \frac{\lambda}{1 - e^{-\lambda(T_2 - T_1)}} \left[\frac{1}{\alpha_2 - \lambda} + \frac{e^{-\lambda(T_2 - T_1)}}{\alpha_1 + \lambda} \right] \right\} \quad (6)$$

Usually an irradiation is terminated by "scramming" the reactor so that the flux decreases to the source level with a fast period (large α_2) and $(T_3 - T_2)$ becomes small. In the limit as $\alpha_2 \rightarrow \infty$ and $(T_3 - T_2) \rightarrow 0$, Equation (6) becomes

$$E = \left\{ 1 + \frac{\lambda e^{-\lambda(T_2 - T_1)}}{(\lambda + \alpha_1) \left[1 - e^{-\lambda(T_2 - T_1)} \right]} \right\} \quad (6A)$$

The quantity E becomes larger the shorter the irradiation time $(T_2 - T_1)$ and the longer the rising period (smaller α_1) which is used to reach ϕ_m . Some values for E as determined by Equation (6A) are listed in Table I.

TABLE I
VALUES OF E

The difference (E-1) is the ratio of the number of Lu-176m nuclei, at the end of the irradiation, which were formed during $(T_2 - T_1)$ to those which were formed during the rise to power with a period $(1/\alpha_1)$.

Period or (1/ α_1)	Irradiation Time $(T_2 - T_1)$		
	10 min	30 min	100 min
30 sec	1.049	1.016	1.004
1 min	1.098	1.032	0.008
3 min	1.294	1.095	1.025

Values of 15 min^{-1} and 1 min for α_2 and $(T_3 - T_2)$ respectively were substituted into Equation (6) in order to evaluate the assumptions used to obtain Equation (6A). Of the cases in Table I, the approximation is worst for a 10 min irradiation and $1/\alpha_1 = 30 \text{ sec}$. The value of E as calculated from Equation (6) is 1.054 for this worst case. Thus, for most irradiations Equation (6A) can be used to determine ψ in Equation (1) by

$$\psi^{177} = \psi_c^{177} E^{177} \text{ and } \psi^{176m} = \psi_c^{176m} E^{176m} \quad (7)$$

While the contribution to each ψ because of the rising period is important, it should be pointed out that when the ratio of ψ^{177}/ψ^{176m} is calculated most of the contribution because of the period cancel and

$$\frac{\psi^{177}}{\psi^{176m}} \sim \frac{\psi_c^{177}}{\psi_c^{176m}}$$

In Equation (1) the g and s are quantities which are a function of the spectral index (i. e., neutron temperature). Therefore, when s is known, it should be possible to deduce the spectral index from a measurement of R_m by solving Equation (1) for g^{176} . Because of the lack of sufficient nuclear data the σ_o , and ϵ_s , and ϵ_c cannot be accurately determined independently. However, it is possible to determine them collectively. Equation (1) can be written as

$$R_m = a \frac{(1 + \frac{rs}{g})^{176} \psi^{177}}{(1 + rs)^{175} \psi^{176m}} g^{176} \quad (8)$$

where

$$a = \frac{N^{176} \sigma_o^{176} \epsilon_s^{177} \epsilon_c^{177}}{N^{175} \sigma_o^{175} \epsilon_s^{176m} \epsilon_c^{176m}} \quad (9)$$

The parameter a can be determined by

1. normalization with a foil irradiated in a thermal column,
2. normalization within an experiment,
3. normalization with a secondary standard, or
4. by a determination of $\epsilon_c^{177} / \epsilon_c^{176m}$ when sufficient nuclear data is available to determine the other quantities which define "a". Since this latter method is not now available for use, it will not be discussed.

For an irradiation in a thermal column, Equation (8) becomes

$$R_m = a g^{176} = R_{th}$$

since $s = 0$. Thus $a = R_m / g^{176}$ and g has been calculated⁽¹⁾ so "a" can be determined.

In order to normalize within an experiment it is necessary to know the conditions at some point where a measurement is made. For example, in an experiment where a traverse is being made in a reactor the traverse may contain a measurement in a region for which the spectral index is

is already known. The region could be a reflector where the neutron distribution is highly thermalized or it could be some other region for which the neutron distribution is known.

A counter for which an "a" has been determined can serve as a secondary standard. By counting a foil first in the standard counter and then in a second counter, the "a" for the second counter can be determined. Actually in this method, the relative counter efficiencies are being determined since all other quantities which define "a" are constants that characterize the lutetium source.

The quantities $(1 + \frac{rs}{g})$ for Lu-176 or $(1 + rs)$ for Lu-175 take into consideration the effect of the epithermal neutrons on the measurement. The g is the usual Westcott g factor and r is determined by

$$CdR = \frac{g + rs_{et}}{r \left\{ s_{ec} + (g/K) \sqrt{T/T_o} \right\}} \quad (10)$$

The CdR is a cadmium ratio, T_o is the neutron temperature corresponding to 2200 m/sec and T is the neutron temperature which is to be deduced. The value of K depends on the thickness of cadmium used and is tabulated in Reference 3. The s values are defined by

$$s_i = \sqrt{\frac{4T}{\pi T_o}} \left[\frac{RI_i}{\sigma_o} = \frac{RI_i(1/v)}{\sigma_o} \right] \quad (11)$$

where the quantity in brackets is a resonance integral in excess of its "1/v" component. The RI_i are defined by

$$RI_i = \int_i^{\infty} \sigma(E) \frac{\Delta(E)}{E} dE \quad (12)$$

The values of i which are of interest are the epithermal and epicadmium cutoff energies. These have been denoted in Equation (10) by subscripts et and ec, respectively. The function $\Delta(E)$ is a function which joins the slowing down distribution of neutrons to the Maxwellian distribution. Several forms of Δ are available for consideration. Three which are being considered are a function which goes to zero very rapidly at E_{et} , the

joining function used by Westcott,⁽³⁾ and the joining function calculated by Hurwitz.⁽⁴⁾ The spectral-index value, as deduced from a lutetium activity ratio, will depend on which model is used. In particular, for an index of about 340 K, the index calculated using the Hurwitz model gives a value 17° lower than when the sharp-cutoff model is assumed.

It is desirable, but not necessary, to determine the epithermal effect with a lutetium cadmium-ratio measurement. When this is done, it is more convenient to make the substitution

$$\frac{(1 + \frac{rs}{g})^{176}}{(1 + rs)^{175}} = \frac{\left[\text{CdR} - \frac{\text{RI}_{\text{et}}}{\text{RI}_{\text{ec}}} \right]^{175}}{\left[\text{CdR} - \frac{\text{RI}_{\text{et}}}{\text{RI}_{\text{ec}}} \right]^{176}} \frac{\text{CdR}^{176}}{\text{CdR}^{175}} \quad (13)$$

The cadmium ratios can be substituted directly into Equation (13) and all that must be calculated are ratios of the resonance integrals $\text{RI}_{\text{et}}/\text{RI}_{\text{ec}}$ as a function of neutron temperature. Values for these ratios are being prepared for the three different methods of joining the Maxwellian distribution to the slowing-down distribution and will appear as a subsequent report.

The Equations 4, 6A, 7, 9, and 13 when combined with a measured ratio of $A^{177}/A^{176\text{m}}$ determine g^{176} according to Equation (8). The value for the spectral index can then be found from the calibration curve⁽¹⁾ which has been reproduced in Figure 2.

Figure 2

Plot of g^{176} as a Function of the Spectral Index

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PHYSICS OF NUCLEAR SAFETYNeutron Multiplication Measurements with Pu-Al Alloy Rods in Light Water

V. I. Neeley, R. C. Lloyd and E. D. Clayton

Critical mass data were needed for Pu-Al alloy fuel elements in support of the Plutonium Recycle Program. The data would be used in the preparation of nuclear safety specifications concerning the handling and storage of this kind of fissile material. The critical mass data would also provide desirable check points for correlating critical mass calculations with experiments for heterogeneous Pu systems.

To obtain these data, neutron multiplication and exponential experiments were conducted with Pu-Al alloy rods having 5 wt per cent Pu enrichment. These rods were 24 in. long and 0.506 in. in diameter and were clad in 0.030 in. Zircaloy-2 with 0.020 in. and 0.125 in. thick end caps. The average rod contained 11.01 grams of Pu. This gave a Zr/Pu atomic ratio of 31.92 and an Al/Pu atomic ratio of 168.20. The plutonium contained 5 wt per cent Pu-240.

These experiments were conducted in the TTR reactor room using the vessels and safety system originally set up for the heterogeneous 3 per cent enriched uranium experiments.⁽¹⁾ Figure 1 shows the experimental equipment. The lattices were hexagonal in shape and placed so as to provide effectively infinite water reflection on all sides. The lattice plates were constructed from Lucite.

Experiments were conducted using critical approach, "back-off" and exponential measurement techniques. The "back-off" consisted of removing rods from the system after completing the critical approach, in general it was a more accurate measurement than the critical approach since no safety control channels were necessary when rods were being removed. An external source was used during the critical approach and removed during the "back-off" measurements since the neutron emission from (α , n) and spontaneous fission reactions at high multiplications

Figure 1
Critical Approach Assembly for 5 wt/% Pu in Pu-Al Rods

provided sufficient flux for the measurements. The critical approach was carried to 96 per cent of the critical mass as determined by inverse multiplication plots. The critical number of rods, N_c , was obtained by a least squares fit to the N/ϕ -vs- N plot in the region 85-96 per cent of the critical mass, where N was the number of rods in the system and ϕ was the flux as measured by a BF_3 chamber. Fuel rods were removed in order to make the exponential measurements, i.e., the reactor radius was reduced so that the reactor would remain subcritical for an assumed infinite height.

Figure 2 shows the experimental results of the multiplication and exponential measurements in addition to the critical mass for spherical geometry. The extrapolation lengths were determined by equating the buckling expressions for the critical size as determined from the multiplication measurements and the exponential measurements.

The main difference between these experiments and the multiplication experiments conducted on the heterogeneous 3 per cent enriched uranium rods was the effect of the larger "built-in source" (α, n and spontaneous fission reactions) on the critical approach curves and extrapolations.

It can be shown that the critical approach curve is independent of a homogeneous source. This is evident from the following equations:

It is assumed that the cylindrical reactor is homogenous and that the source is homogenous. Then it can be shown that the flux is represented by⁽²⁾

$$(1) \phi(r, t) \approx \sum_{n=1}^{\infty} \left[A_n e^{(k_n - 1) \frac{t}{T_n}} + \frac{S_n k_{\infty} e^{-B_n^2 \tau}}{(1 + L^2 B_n^2)(1 - k_n)} \right] J_0 \left(\frac{\alpha_n r}{a} \right)$$

where S_n is the homogenous source.

Then assuming long enough periods of time between loadings so that the time variation damps out and that the reactor is close enough to critical so that only the first mode of the buckling is present and that the flux is

-140-

HW-64866

Figure 2
Experimental Multiplication and Exponential Measurements vs
H/Pu Atomic Ratio

being measured at the center of the pile ($r = 0$) then Equation (1) reduces to

$$(2) \phi = \frac{C e^{-B^2 \tau}}{1 + L^2 B^2 - k_{\infty} e^{-B^2 \tau}}$$

Figure 3 is a plot comparing this theoretical expression and the actual critical approach normalized to the experimentally predicted critical size.

In two cases, plutonium contamination problems were encountered due to faulty or ruptured cladding on the rods. In both cases the potentially serious contamination problems were kept to a minimum by careful monitoring procedures.

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Figure 3
 Comparison of Experimental and Theoretical Critical Approach Curves

Critical Approach and Exponential Measurements
with 3.1 Per Cent Enriched Uranium Rods in Light Water

R. C. Lloyd, E. D. Clayton and R. B. Smith

A series of exponential and critical approach measurements have previously been made with 3.063 per cent enriched uranium rods in water. (1-6) Fuel rod sizes were 0.925, 0.600, 0.300 and 0.175 inches in diameter. The uranium was encased in thin walled acrylic resin tubes; a hexagonal pattern was used for the lattice assemblies. The tubes were assumed equivalent to water since one measurement with increased amounts of acrylic resin material showed a very small change in the measured critical mass (-.029%/1% acrylic resin in moderator). Loadings were taken to 96 per cent of the predicted critical mass. The critical masses were determined from a least square fit of $\frac{\sqrt{N}}{CR}$ vs N for the region of N between 85 to 96 per cent of critical. N is the number of tubes and CR the count rate of a BF_3 proportional counter at approximately the center of the loading.

A summary listing of the results is given in Table I. The extrapolation length, λ , was calculated by setting the buckling values equal from the exponential and the critical approach measurements. In the 0.925 inch diameter fuel rods the λ value was extrapolated from data from the other rod sizes since exponential measurements have not been made with these rods. The values for the critical number of tubes, N_c , have been corrected, for the effect of the safety rod guide tubes, source stringer, and BF_3 counter.

It is estimated that the uncertainty in the measured critical masses is one per cent. The critical mass for spherical geometry has an uncertainty of about one per cent also, since changes due to errors in λ tend to cancel out. In the two measurements where exponential measurements only were made the uncertainty in the critical mass is about four per cent.

TABLE I
SUMMARY OF CRITICAL APPROACH AND EXPONENTIAL MEASUREMENTS
WITH 3.063 PER CENT ENRICHED URANIUM

Fuel Rod Diameter (inches)	Lattice Spacing (inches)	h (cm)	H ₂ O/U (vol)	N _c (rods)	R _c (cm)	C. M. Cylinder (lb)	λ ⁽²⁾ (cm)	Buckling 10 ⁻⁶ cm ⁻²	C. M. Sphere (lb)
0.925	1.500	40.64	1.89	72.1	16.98	530.7	6.37	14,068	491
		60.96	1.89	60.0	15.50	662.6			
	1.600	40.64	2.29	63.5	17.01	467.6	6.23	14,212	432
		60.96	2.29	52.4	15.45	578.8			
	1.700	40.64	2.72	58.6	17.35	431.3	6.12	14,027	399
		60.96	2.72	49.2	15.90	543.1			
	1.800	40.64	3.17	56.7	18.08	417.6	6.00	13,536	388
		60.96	3.17	46.9	16.44	518.1			
	1.950	40.64	3.89	60.4	20.21	444.7	5.86	12,108	417
		60.96	3.89	48.6	18.13	536.8			
0.600	1.000	40.64	2.06	152.1	16.45	471.2	6.16	14,834	433
		81.28	2.06	114.9	14.30	711.9			
	1.100	40.64	2.71	118.6	15.97	367.4	6.52	14,855	337
		81.28	2.71	91.2	14.01	564.8			
	1.200	40.64	3.41	104.5	16.36	323.7	5.77	15,435	295
		81.28	3.41	79.9	14.30	494.9			
	1.300	40.64	4.18	98.9	17.24	306.4	6.13	14,115	283
		81.28	4.18	74.1	14.92	458.8			
	1.420	40.64	5.18	100.3	18.97	310.7	5.86	12,984	290
		81.28	5.18	73.8	16.27	457.1			
	1.600	40.64	6.84	122.0	23.57	378.0	5.46	10,576	353
		81.28	6.84	82.4	19.37	510.4			
0.300	0.500 ⁽¹⁾	81.28	2.06	624.0	16.7	964.0	6.80	13,607	465
	0.600	40.64	3.41	387.5	15.75	299.2	6.51	15,098	274
		81.28	3.41	298.1	13.82	460.3			
	0.700	40.64	5.00	296.9	16.09	229.2	6.05	15,352	209
		81.28	5.00	230.0	14.16	355.3			
	0.800	40.64	6.84	271.9	17.59	209.2	6.17	13,758	195
		81.28	6.84	199.9	15.09	308.7			
	0.900	40.64	8.92	285.7	20.79	220.6	5.94	11,986	207
		81.28	8.92	203.8	17.14	314.7			
	0.300 ⁽¹⁾	59.69	2.24	1786.0	16.9	689.0	7.5	11,477	560
0.175	0.375	59.69	4.06	873.3	14.78	337.1	6.91	14,121	257
	0.450	59.69	6.29	628.3	15.04	242.5	6.53	14,292	184
	0.500	59.69	8.00	569.7	15.92	219.9	6.43	13,458	171
	0.550	59.69	9.89	554.2	17.27	213.9	6.30	12,301	171
	0.600	59.69	11.96	572.5	19.15	221.0	6.31	10,809	184

1. Exponential measurement only - λ from extrapolating measured λ curve.
2. λ for 0.925-inch diameter rods from the extrapolated curve, λ for other sizes calculated by setting values of buckling equal; when more than two measurements were made a weighted average was taken.

Figure 1 shows a plot of maximum buckling, and minimum critical mass in spherical geometry, vs rod diameter for the 3.063 per cent enriched uranium. The maximum buckling for the 3.063 per cent enriched uranium is $15,570 \times 10^{-6} \text{ cm}^{-2}$. This value would be obtained for a uranium rod of 0.4 inch diameter at an $\text{H}_2\text{O}/\text{U}$ volume ratio of ~ 3.9 . The minimum critical mass is estimated to be 165 lbs of uranium (2.3 kg U-235); this value would be obtained for uranium rods of about 0.1 inch diameter at an $\text{H}_2\text{O}/\text{U}$ volume ratio of ~ 10.5 .

Figure 2 shows plots of the extrapolation length vs $\text{H}_2\text{O}/\text{U}$ volume ratio. The extrapolation length was measured for the three smaller rod diameters; for the 0.925 inch diameter the values of λ were extrapolated from curves plotted for the small rod diameters. The values of the extrapolation lengths are observed to decrease with increasing rod diameters and $\text{H}_2\text{O}/\text{U}$ volume ratios.

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MAXIMUM BUCKLING AND MINIMUM MASS VS. ROD DIAMETER (3.06 Per cent Enriched Uranium)

Figure 1

Maximum Buckling and Minimum Mass Vs. Rod Diameter
(3.06 Per Cent Enriched Uranium)

Figure 2

Extrapolation Length Vs. H₂O/u Volume Ratio for 3.063 Per Cent Enriched Uranium

Exponential Experiments with Poisoned Moderator

R. C. Lloyd

A series of measurements were completed for determining the amount of neutron absorber to "safe" a heterogeneous system of 1.007 per cent enriched uranium rods. This information can be used for determining the amount of poison needed in dissolver batches as a secondary safety control against criticality.

The fuel rods were 0.925 in. in diameter, and 44 in. in length. They were encased in aluminum tubes having a wall thickness of 0.032 in. Eighty-five of the fuel rods were loaded into a stainless steel tank having a 1/16 in. wall thickness. The lattice and moderator which contained the poison was separated from the light water reflector by this stainless steel tank. Increments of boric acid (H_3BO_3) were added to the moderator until the buckling became negative.

Some difficulty was encountered in measuring the thermal flux when boric acid was added to the moderator since the source neutron rate dropped off rapidly making it difficult to obtain good statistics. Also, some precipitation of the poison was noticed. To counteract this, a continuous mixing pump was installed. The results of the measurements are summarized in Table I. In the calculation of the buckling an extrapolation length, λ , was determined by multiplying values from BNL data⁽¹⁾ by 0.916 to correct for the effect of the stainless steel tank on the extrapolation length. This correction factor was obtained from measurements by R. A. Bennett.⁽²⁾

TABLE I

BUCKLINGS OF 1.007 ENRICHED URANIUM WITH POISONED MODERATOR

<u>Lattice Spacing (inches)</u>	<u>H₂O/U (volume)</u>	<u>H₃BO₃ gm/1</u>	<u>Boron gm/1</u>	<u>λ (cm)</u>	<u>Buckling (10⁻⁶cm⁻²)</u>	<u>H₃BO₃ at B² = 0 (gm/1)</u>
1.40	1.37	0	0	6.92	2794	4.0 ± 0.3
		1.5	0.26		1643	
		3.0	0.53		663	
		4.5	0.79		-269	
1.50 ⁽³⁾	1.74	0	0	6.65	3294	3.4 ± 0.5
		1.5	0.26		1502	
		3.0	0.53		97	
		6.0	1.06		-2303	
1.60	2.15	0	0	6.43	3341	3.2 ± 0.5
		1.5	0.26		2061	
		3.0	0.53		-313	
		4.5	0.79		-1162	

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Exponential Measurements of Natural Uranium - Water Systems

R. C. Lloyd

A series of exponential measurements was started with 0.925 in. diameter natural uranium fuel elements in water. These measurements were being done to obtain further data on the critical mass of natural uranium in light water and to obtain buckling values for the interpretation of planned experiments involving a three per cent enriched uranium lattice reflected by a natural uranium tamper. This size of natural uranium rods, which was readily available, was of interest since the 0.925 in. rod would provide buckling values close to the maximum buckling as indicated by calculations.⁽¹⁾ The maximum buckling was estimated to occur at a diameter between 0.8 and 0.9 in. Other measurements have been made at rod diameters of about 0.5 and 1 in.^(2, 3) Also, measurements have been made with a 0.787-in. diameter rod with an annular air gap and Al tubes.⁽⁴⁾

The fuel elements were encased in thin walled (1/32 in.) Lucite tubes. Eighty-five fuel elements, 44 in. long, were loaded into hexagonal lattice frameworks for these measurements. The results of the two lattices measured are summarized below.

Buckling of Natural Uranium - Water Lattices

<u>Lattice Spacing</u>	<u>H₂O/U Volume Ratio</u>	<u>λ^* (cm)</u>	<u>$(10^{-6}B^2 - 2)$ cm</u>
1.5 in	1.89	7.16	-24
1.6 in	2.29	6.96	-367

The error limits on the buckling are estimated to be about $\pm 250 \times 10^{-6} \text{ cm}^{-2}$.

* The buckling was calculated with values of λ obtained from BNL data.⁽⁵⁾

An exponential measurement is planned with a lattice spacing of 1.40 in. It is estimated from calculations that this spacing will be below the maximum buckling and a curve can then be drawn to better estimate the maximum buckling. This value will be for a clean lattice of 0.925 in. diameter rods of natural uranium since the Lucite tubes are essentially equivalent to water.

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