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**FAST FLUX TEST FACILITY
BACKUP DESIGN PROJECT
FIFTH QUARTERLY REPORT
JUNE-AUGUST 1968**

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U.S. ATOMIC ENERGY COMMISSION
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BETWEEN PACIFIC NORTHWEST LABORATORY
AND THE GENERAL ELECTRIC COMPANY

ADVANCED PRODUCTS OPERATION

GENERAL ELECTRIC

SUNNYVALE, CALIFORNIA

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FAST FLUX TEST FACILITY BACKUP DESIGN PROJECT

FIFTH QUARTERLY REPORT

June 1, 1968—August 31, 1968

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SECTION I

INTRODUCTION

This quarterly report documents work conducted on the FFTF Backup Design Project on Contract BDR-341 between Pacific Northwest Laboratory and the General Electric Company during the period June 1, 1968 through August 31, 1968.

The objective of the FFTF backup design is to provide the capability of replacing the reference FFTF dispersed core design with an open lattice vertical core and associated handling system, instrumentation, control rod and top head configuration. To do this, use is being made of the reference design vessel, the hot cell configuration, and the balance of plant components and arrangements.

As a result of analysis work done during this quarter, a reference control drive concept has been established utilizing a ten inch offset beam between the control drive extension and the control rods. A dash pot below the control rods decelerates the drive following scram action to a relatively low speed. The remaining energy of the rods is transferred to springs located approximately at the gravity center of the drive beneath the offset beam at the end of the control drive stroke.

A reference backup core clamping mechanism based on the SEFOR torsion bar concept has been selected. Also, a reference backup core support-inlet plenum interface with the PNL vessel concept has been selected.

Good progress has been made on the closed loop and instrumentation layout designs.

The calculated backup-design peak neutron flux at the beginning of an equilibrium fuel cycle remains at approximately 1×10^{16} neutrons/cm²-sec.

The two-detector cross correlation neutron fluctuation analysis technique has been applied to the backup design and shows considerable promise as a means of measuring subcriticality in the range of interest. The incentive to develop gamma compensated (and perhaps gamma shielded) detectors has been clearly demonstrated.

The vessel head design and head analyses for DBA loadings and gamma shielding continues to improve.

Detailed explanation is given on the theoretical model which will be used to predict circumferential sodium velocity profiles around fuel pins adjacent to the fuel subassembly or duct wall.

SECTION II

CONTROL SYSTEM DESIGN ANALYSIS

2.1 INTRODUCTION AND SUMMARY

As reported in the Fourth Quarterly Report, GEAP-5650, the critical parameters requiring evaluation for establishing the allowable magnitude of the offset of the control rod from the control drive line were the loads on the offset during the deceleration phase of the control rod travel. Effort during this quarter was directed at evaluation of the effectiveness of decelerating "dash pot" devices or "cushions" at various locations (bottom of control rod, under the offset beam, and in the drive mechanism). The maximum force at the offset and the terminal velocity were established as a function of clearances and tolerances in the devices. A review was made of the allowable moment which could be safely imposed on the top of the control rod assembly at the offset, and the upper limit was established at 10,000 in.-lb. This was an increase from the 5,000 in.-lb previously established as a reference. This increase was obtained by increasing the clearance between the hexagonal housing around the poison pins and the channel wall, from 0.150 in. to 0.225 in. The reduction in volume fraction of boron carbide is sufficiently small that a small increase in the enrichment of $B_4^{10}C$ in the poison compensates for this change in volume fraction.

The best location of a "dash pot" from bending moment considerations is beneath the offset beam close to the center of gravity of the moving assembly; however, the alignment problems of a piston on the offset beam mating with an orifice on the fixed structure resulted in discarding this idea. A dash pot at the bottom of the control rod was selected since this resulted in smaller moments at the offset than a dash pot in the mechanism. Placing a dash pot at both ends was considered but the matching of forces under tolerance and eccentricity variations to meet the allowable offset moment seemed less desirable than the above selection.

The terminal velocity of the rod, using a flow area which varied with travel in order to keep the deceleration approximately uniform (i.e., to minimize load on the offset), was relatively insensitive to the maximum velocity and was a function primarily of final clearance between piston and orifice. Figure 2-5 shows the sensitivity of this velocity to final clearance. To reduce the final velocity to the 0.5 to 1 ft/sec range where a solid impact could be tolerated, the gap between piston and orifice should be 1 to 2 mils. The feasibility of easily achieving this tolerance appears questionable. The consequence of impacting in the 3 to 4 ft/sec velocity range was evaluated. It was determined that a spring with a travel of 3 to 6 in. could limit the maximum deceleration to 3 to 5 g for impact velocities of 3 to 4 ft/sec, and a rebound, if perfectly elastic, of 2 to 4 in. would result.

An evaluation of the effect of inertia loads on the stability of the drive line showed that with a 300 pound force driving the rod down, an additional inertia load of 6 g on the extension rod could be safely tolerated. The location of the spring was established beneath the offset beam for two reasons: the moments induced in the drive are small (since the spring is near the center of gravity of the moving assembly), and adequate space is available at this location without requiring much additional length in the control rod channel or the mechanism.

The spring at the offset beam should have a spring constant in the range 500 to 1000 lb/in and a load capacity of 3000 to 4000 lb to keep the maximum deceleration in the 3 to 5 g range for impact velocities in the 3 to 4 ft/sec range, as discussed in section 2.8. The specific requirement on the dash pot dimensions and clearances and on the consequent spring characteristics will be established by the development program.

2.2 ALLOWABLE LOAD AT OFFSET FROM STRESS AND DEFLECTION LIMITATIONS

A review was made of the optimum location of bearing pads on the control rod, of the beam stiffness, and of the clearance between rod and channel. The allowable bending moment on the drive at the offset, due to pneumatic scram activation forces in the mechanism, and inertia loads during deceleration established the maximum offset that can be accepted. The following analysis indicates from stress and deflection considerations that 5000 in.-lb per control rod or 10,000 in.-lb. per assembly of two rods is both practical and feasible.

Three locations of F (Figure 2-1; AF = 45 in., 60 in., 75 in.) were considered along with three degrees of taper of FA (2.76 in. diameter at F, to 2.76, 2.26 and 1.76 in. diameter at A). Two wall thicknesses, 0.1 and 0.2 inches, for the wrapper around the poison pins were considered.

Table 2-1 shows the allowable offset moment at the top of one control rod based upon limiting the deflection to the available clearance. In the poison carriage region, JG, an allowance for tolerance and thermal bow of 0.116 inch was taken off the nominal gap. At the top, point A, 0.050 inch allowance for tolerance and alignment was used. Table 2-2 shows the allowable bending moment as a function of section modulus for different configurations.

In addition to the above clearance limitations, the allowable moment from stress limits was considered. The results are shown in Table 2-3. An allowable bending stress of 5,000 psi was selected as a conservative value to permit accommodation of stress risers and thermal stress which had been neglected. The yield strength of Type 304 stainless steel at 1200° F is 15,000 psi and Incoloy is 20,000 psi.

2.3 CONTROL ROD DECELERATION HYDRAULIC ANALYSIS

The scram rate of the control rod assembly is closely related to the deceleration mechanism design. A control rod scram insertion rate of at least 1 g (32.2 ft/sec^2) has been tentatively established to provide an adequate rate of reactor shut down from consideration of potential accidents. To attain a 1 g scram system, an external scram force of ~300 lb is needed. The force provides a dual function: a scram force and a positive holddown force after completion of the scram.

The major problem is to accommodate the forces exerted on the control rod offset during deceleration. The initial analysis was performed assuming an acceptable limit of a total 5,000 in.-lb moment at the offset (i.e., 500 lb on a 10 inch offset). A revised evaluation established an acceptable moment of 10,000 in.-lb (or 1,000 lb on 10 inch offset).

The limitation of moment on the offset was established from the deflection of the control rod coming into contact with the outer hexagonal channel (see Table 2-1).

FIGURE 2-1. CONTROL ELEMENT FLEXURE

TABLE 2-1

ALLOWABLE MOMENT FROM DEFLECTION - CLEARANCE COMPATIBILITY

FJ	t _{Carr}	Gap _{JG}	δ^* _{flex JG}	M ^{**} _{Allowable JG (in.-lb.)}	A _F	Diameter _F	Diameter _A	Clearance _A	δ^{***} _{Allowable A}	M ^{**} _{Allowable FA (in.-lb.)}
0	0.1	0.15	0.034	3550	75	2.76	2.76	0.2	0.15	1300
0	0.2	0.15	0.034	6300	75	2.76	2.76	0.2	0.15	1800
0	0.1	0.15	0.034	3550	75	2.76	2.26	0.45	0.40	2670
0	0.2	0.15	0.034	6300	75	2.76	2.26	0.45	0.4	3250
0	0.1	0.15	0.034	3550	75	2.76	1.76	0.7	0.65	2800
0	0.2	0.15	0.034	6300	75	2.76	1.76	0.7	0.65	3180
15	0.1	0.15	0.034	2520	60	2.76	2.76	0.2	0.15	1920
15	0.2	0.15	0.034	4250	60	2.76	2.76	0.2	0.15	2340
15	0.1	0.15	0.034	2520	60	2.76	2.26	0.45	0.4	3900
15	0.2	0.15	0.034	4250	60	2.76	2.26	0.45	0.4	4650
15	0.1	0.15	0.034	2520	60	2.76	1.26	0.7	0.65	4250
15	0.2	0.15	0.034	4250	60	2.76	1.26	0.7	0.65	4670
30	0.1	0.15	0.034	2020	45	2.76	2.76	0.2	0.15	2580
30	0.2	0.15	0.034	3100	45	2.76	2.76	0.2	0.15	3560
30	0.1	0.15	0.034	2020	45	2.76	2.26	0.45	0.40	6350
30	0.2	0.15	0.034	3100	45	2.76	2.26	0.45	0.40	7000
30	0.1	0.15	0.034	2020	45	2.76	1.76	0.7	0.65	7000
30	0.2	0.15	0.034	3100	45	2.76	1.76	0.7	0.65	7580
15	0.1	0.2	0.084	6200						
15	0.2	0.2	0.084	10500						
30	0.1	0.2	0.084	5000						
30	0.2	0.2	0.084	7700						
30	0.1	0.25	0.134	10000						

* Gap—0.116 in. (tolerance + thermal bow = 0.070 + 0.046 = 0.116 in.)

** Per rod.

*** Use clearance_A = 0.050 in.

TABLE 2-2

SECTION MODULUS AND ALLOWABLE BENDING MOMENT
FOR VARIOUS CONTROL ROD CONFIGURATIONS

Configuration	I/C	M (in.-lb.)
2.76 in. diam shaft	2.06	10,000
2.26 in. diam shaft	1.13	5,700
1.76 in. diam shaft	0.535	2,700
Hexagonal carriage H = 2.76 in., t = 0.1 in.	0.72	3,600
Hexagonal carriage H = 2.76 in., t = 0.2 in.	1.28	6,400

TABLE 2-3

ALLOWABLE MOMENT FROM STRESS LIMITS

FJ (inches)	T _{Carr} (inches)	M _J allowable (in.-lb.)	M _F allowable (= M _A allowable) (in.-lb.)
0	0.1	3600	3600
	0.2	6400	6400
15	0.1	3600	4650
	0.2	6400	8250
30	0.1	3600	5550
	0.2	6400	9900

The following reference configuration is selected:

AF = 45
FJ = 30
Gap around poison pin carriage 0.225
Diameter at F 2.76 inches
Diameter at A 2.26 inches
Carriage thickness 0.1 inch

Moment Capacity = 5,550 in.-lb./rod.
11,100 in.-lb./drive.

Hydraulic decelerating devices in several locations, acting independently and in combination, were investigated. The three locations were at the upper extension rod, at the control rod channel, and at the channel adjacent to the control rod under the offset beam.

The following general assumptions were made for the control rod deceleration schemes (see Appendices A and B for derivation of the equations used to calculate these results):

1.	Acceptable moment on offset	5,000 in.-lb.
2.	Weight of components	
a.	Control rods (two)	550 lb
b.	Extension rod (upper)	150 lb
3.	B ₄ C section	
a.	Pins per subassembly	37
b.	Carriage hexagonal (o.d. across flat)	2.86 in.

c.	Clearance gap (hexagonal channel - carriage)	0.150 in.
d.	B ₄ C clad thickness	0.045 in.
e.	Pin diameter	0.43 in.
f.	Length	36 in.
g.	Heat generation rate	
	(1) B ₄ C	40 W/gm
	(2) Stainless steel	4.7 W/gm
4.	Gas plenum length	15 in.
5.	Nickel follower	
a.	Length	36 in.
b.	Hexagonal (o.d. across flat)	2.76 in.
c.	Clearance (channel hexagonal - nickel)	~0.200 in.
d.	Cooling holes	
	(1) Number	7
	(2) Size	3/8 in. -diam.
6.	Lower extension rod (above poison pin region)	
a.	Length (used for pressure drop calculations)	30 in.
b.	Diameter of rod (mean diameter)	2.555 in.

Initial investigation entailed hydraulic cushioning using the viscous forces from the displaced sodium as the control rod crossed the end of the scram stroke. To be effective, this design required the high pressure plenum supply to be available during all modes of scram. At low reactor power and some accident situations, high pressure may not be available in the inlet plenum.

During normal scram where high pressure is available, the viscous force at the beginning of the cushioning (10 ft/sec for a 1 g system) is 950 lb (two control rods). For the conditions where high pressure plenum is not available, the viscous force decreases to 160 lb (two control rods). The large reduction is due to the large orifice requirement needed to supply the sodium flow for cooling and bypass flow from the large clearances between the outer hexagonal channel and the carriage during normal operation. The diminished viscous forces for the loss of high pressure are not tolerable in decelerating the control rods. Therefore, an alternate design for a closed-end piston device was conceived. The following analyses are the viscous force decelerating designs of the closed-end piston principle.

2.4 SCRAM FORCE REMOVED PRIOR TO START OF DECELERATION

2.4.1 Cushioning Using Adjacent Channel Piston Only

Deceleration of the control rod is accomplished by installing a closed-end piston device above the adjacent channel nickel reflector (adjacent to the control rod positions). The hydraulic device consists of a piston in a closed-end chamber. The decelerating force is obtained by displacing the sodium in the chamber and restricting the quantity of fluid flowing outward by tapering the piston which gives a variable orifice gap with travel. The force is exerted on the offset at the position shown in Figure 2-2.

The effects of hydraulic piston diameter, size, initial velocity of control rod assembly at the beginning of the cushioning stroke, and the moment exerted on the offset were investigated. Piston size was found to have minor effects on the cushioning ability. The clearance gap appeared to have the most sensitive single effect on moments and final velocity. The control rod final velocity was dependent on the final piston gap (clearance at the end of the cushioning stroke). The

FIGURE 2-2. ADJACENT CHANNEL HYDRAULIC DECELERATION SCHEMATIC

moment on the offset was found to be sensitive to the initial gap and gap profile throughout the hydraulic piston stroke.

The design problem of alignment of the piston and orifice precluded the selection of this concept. The alignment problem arises from the lateral deflection of the offset created by external forces.

2.4.2 Upper Extension Rod Deceleration

The design of an upper extension rod deceleration mechanism consists of a pneumatic actuated piston device. The assembly drive furnishes the scram force and maintains a position hold down force. The feasibility of the pneumatic assembly in cushioning the control rod assembly was investigated. The maximum moment on the 10 inch offset was excessive. This concept was accordingly abandoned in favor of dampers at the bottom of the control rods.

2.4.3 Control Rod Channel Hydraulic Piston

The control rod channel hydraulic piston concept and design is analogous to the adjacent channel piston design. The hydraulic piston assembly is located at the bottom of the control rod nickel follower (see Figure 2-3).

The conclusion of the analysis for this concept is the same as for the adjacent channel analysis. The final velocity of the control rod is sensitive to the final gap of the piston. Table 2-4 is the tabulation of piston size, gap profile, and initial cushioning velocity. The hydraulic piston location under the control rods causes deceleration without the potentially large misalignment problem of the piston under the offset.

TABLE 2-4
CONTROL ROD DECELERATION USING
THE CONTROL ROD CHANNEL HYDRAULIC PISTON

Diameter of Piston (in.)	Piston Area (in. ²)	Initial Gap Clearance (in.)	Final Gap Clearance (in.)	Maximum Force (One Piston) (lb)	Maximum Force on Offset (Total) (lb)	Initial Piston Velocity (ft/sec)	Final Piston Velocity (ft/sec)
2.0	3.14	0.050	0.030	660	280	10	9.8
3.0	7.07	0.050	0.030	930	395	10	4.2
2.0	3.14	0.030	0.030	520	220	10	8.4
2.0	3.14	0.025	0.015	860	370	10	4.6
2.5	4.91	0.020	0.005	2640	1130	10	0.73
2.5	4.91	0.030	0.005	1280	550	10	0.79
2.5	4.91	0.030	0.003	1310	560	10	0.45
2.5	4.91	0.035	0.003	1200	510	10	0.46
2.5	4.91	0.035	0.003	700	300	6	0.46
2.5	4.91	0.035	0.003	900	385	8	0.46
2.5	4.91	0.035	0.003	1600	680	12	0.46

FIGURE 2-3. CONTROL ROD CHANNEL HYDRAULIC CUSHION

2.4.4 Combined Adjacent Channel and Control Rod Channel Cushioning

Combined hydraulic piston operation in the adjacent channel and control rod channel was analyzed. Variations of the hydraulic gap profiles were calculated. The final control rod velocity and moment on the offset did not appreciably differ from the individual snubbing cases. So no real incentive existed for this design. The design's disadvantages were the potential misalignment of the adjacent channel piston and the matching of the two hydraulic snubbers.

2.5 HYDRAULIC SNUBBER ON CONTROL ROD AND PNEUMATIC SNUBBER ON THE UPPER EXTENSION ROD

The pneumatic scram force mechanism is intimately related to the viscous damping mechanism performance. The original analysis was expanded to include an evaluation on the interrelationship of the pneumatic and viscous mechanism.

The pneumatic mechanism provides the scram force, counteracts the scram force, actuates the control rod during normal operation, and provides a positive hold down during post scram. In addition, pneumatic cushioning synchronized with the hydraulic snubbing at the control rod channel was investigated.

The sketch of the pneumatic scram cylinder is conceptually shown in Figure 2-4. The cylinder design has a self-centering orifice (radial self-centering) sized to have nominal pressure drop during the scram stroke. Thus the scram force is not impeded. The orifice area is decreased during the decelerating stroke. This provides a pneumatic cushioning force strong enough to counteract the scram force and the decelerating force of the upper extension rod weight.

The operation of the mechanism is based on scram pressure supplied to chamber A equivalent to a total force of 500 pounds. When the decelerating stroke begins (12 inches from the end of the travel), the scram force is opposed by the displaced gas pressure in chamber B. A tapered shaft regulates the pressure in chamber B. During the entire scram operation, the relief line is C. Line C also acts as the supply pressure line for operating the control rod during shim operation.

The orifice area of the pneumatic cylinder was tailored to obtain a minimum force on the offset. Incorporation of manufacturing tolerances and eccentricity of the hydraulic piston revealed that the force on the offset was extremely sensitive to gaps of both the hydraulic and pneumatic orifices. Combined manufacturing tolerances and axial variation from thermal expansion is capable of changing the relative gap so undesirable tolerance conditions might arise. The problem was alleviated by increasing the pneumatic cylinder orifice gap. However, the effectiveness of the pneumatic snubber was then reduced. This concept was not considered promising and was no longer considered.

2.6 CONTROL ROD CHANNEL CUSHIONING WITH AND WITHOUT SCRAM FORCES

The hydraulic piston design located in the control rod channel location was discussed in Section 2.4.3. In that analysis, a nominal gap with a concentric entry of the piston into the orifice hole proved capable of snubbing the control rod. Yet, there is no assurance that concentricity of the piston and orifice would exist through the entire snubbing stroke. Therefore, flow

FIGURE 2-4. UPPER EXTENSION ROD COMPRESSIBLE FLUID PISTON

in concentric and eccentric annuli of small clearance with relative motion of the boundary was investigated. The flow rate through an eccentric annulus increases; therefore, the effectiveness of the hydraulic snubbing is reduced. Table 2-5 illustrates the effect of eccentricity on final rod velocity.

TABLE 2-5
CONTROL ROD CHANNEL HYDRAULIC
CUSHION WITH ECCENTRICITY AND TOLERANCE EFFECTS

Initial Gap (in.)	Final Gap (in.)	Maximum Hydraulic Force on Two Control Rods (lb)	Maximum Force on Offset (lb)	Final Velocity (ft/sec)	Maximum Hydraulic Force on Two Control Rods (lb)	Maximum Force on Offset (lb)	Final Velocity (ft/sec)
Concentric Case					Eccentric Case		
Scram Force = 0							
0.035	0.003	2220	475	0.61	2380	510	0.90
0.035	0.005	2140	460	1.08	2250	485	1.73
0.035	0.007	2060	440	1.59	2140	460	2.64
Scram Force = 300 lb							
0.033	0.003	2550	780	0.82	2940	865	1.19
0.033	0.005	2440	760	1.45	2795	835	2.20
0.033	0.007	2340	740	2.10	2650	805	3.28
0.035	0.003	2610	795	0.84	3100	900	1.22
0.035	0.005	2500	770	1.47	2950	870	2.30
0.035	0.007	2395	750	2.15	2650	845	3.43
0.037	0.003	2685	810	0.84	3260	935	1.26
0.037	0.005	2580	790	1.50	3110	900	2.40
0.037	0.007	2470	765	2.21	3140	910	3.59

Manufacturing tolerance is another factor that influences the hydraulic snubbing. Shown in Table 2-5 are the expected results of tolerance effects. Analysis has been accomplished for both cases of scram force applied (300 lb) and scram force removed.

The initial velocity of the control rod at the beginning of the cushion stroke is shown in Table 2-4. The final velocity is primarily a function of final clearance between the piston and orifice and not initial velocity. Figure 2-5 shows the sensitivity of velocity to the final clearance. It can be seen by the dotted region that in order to get the final velocity down into the 0.5 to 1 ft/sec range, the radial gap tolerances of 1 to 2 mils are required.

The possible impacting velocity is in the 3 to 4 ft/sec velocity range for radial gaps in the 6 to 8 mil range. Energy absorbing impact springs are dictated for this concept.

The design of a single hydraulic piston located in each control rod channel of the closed-end type appears on balance to be the most acceptable design approach. Tests would have to be conducted to verify the acceptable gaps for the piston orifice clearance. If tight clearance of 1 to

FIGURE 2-5. FINAL CONTROL ROD VELOCITY AT THE END OF CUSHIONING STROKE AS A FUNCTION OF FINAL RADIAL GAP OF HYDRAULIC CUSHIONS.

2 mil radial gaps should provide be acceptable, 0.5 to 1 ft/sec final velocities could be expected, and the energy absorbing springs eliminated. Impact velocities of this magnitude are tolerable for solid impact. If, as expected, the tight tolerances are not practical, then an energy absorbing spring device can be developed to take velocities in the 3 to 4 ft/sec range.

2.7 ALLOWABLE TERMINAL VELOCITY OF CONTROL ROD

From the evaluation of dash pots given in section 2.6, the terminal velocity of the control rod is seen to be dependent upon final clearances between piston and orifice.

The effect of hitting a stop at the bottom of the control stroke in the upper mechanism was evaluated. To meet an offset moment limit of 10,000 in.-lb. with a 300 pound scram force, the impact velocity is limited to about 0.5 ft/sec. Since this requires exceedingly small clearances in the dash pot (<2 mils radial gap), a spring mounted under the offset beam (A-A' in Figure 2-6) was evaluated. This is close to the center of gravity of the moving assembly. Therefore, bending moments in the drive are no longer limiting. Buckling of the drive line under inertia loads limits the maximum deceleration of the moving assembly to 6 g (as shown in section 2.7.2). This defines the spring constant and the length of compression required in the spring.

2.7.1 Allowable Impact Velocity at End of Control Rod Travel From Offset Moment Considerations

$$W_{AE} = 200 \text{ lb}$$

$$W_{AK} = 250 \text{ lb} \times 2 \text{ elements} = 500 \text{ lb}$$

Assume following geometry and allowable M at the offset:

$$\begin{aligned} GJ &= 55 \text{ in. (0.1 in. carriage thickness)} \\ JF &= 30 \text{ in. (solid hexagonal 2.76 in. across flats)} \\ FA &= 45 \text{ in. tapered 2.76 in. diameter to 2.26 in. diameter} \\ M_{\text{allowable}} &= 12,000 \text{ in.-lb. (two elements)} \end{aligned}$$

For impact at K:

$$M = \left(F + W_{AE} \frac{\ddot{x}_{AE}}{g} \right) e = Pe$$

To find $\ddot{x}$ assume W_{AE}/g acts as a single degree of freedom mass on a spring A' AG.

Calculation of spring constant "K" of A' AG: due to a unit vertical load at A', the deflection of A' is obtained as the rotation of A x e (assuming deflections due to direct stresses are small compared to bending stresses).

$$\theta_A = \theta_F + \theta_A \text{ relative to tangent at F.}$$

FIGURE 2-6. TERMINAL VELOCITY DRIVE MODEL

$$\theta_F = \frac{Pe}{6E} \frac{55}{\left(1 + \frac{30}{55}\right)^2} \left\{ \frac{2}{1.86} + \frac{2 \left(1 + \frac{30}{55}\right)^3 - 2}{2.83} \right\} = \frac{11.4 Pe}{E}$$

$$\theta_{A-F} = \frac{Pe}{E} \frac{45}{2.86 \times 3 \times 0.18} \left[1 - \frac{1}{0.82^3} \right] = \frac{24 Pe}{E}$$

$$\theta_A = (11.4 + 24) \frac{Pe}{E} = 35.4 \frac{Pe}{E}$$

$$\frac{\delta_{A'}}{P} = \frac{\theta_{A'} e}{P} = \frac{35.4 e^2}{E}$$

$$\text{For } E = 30 \times 10^6 \text{ and } e = 10$$

$$\frac{\delta_A}{P} = \frac{35.4 \times 100}{30 \times 10^6} = 1.18 \times 10^{-4}$$

or

$$K = \frac{P}{\delta_A} = 8480 \text{ lb/in.}$$

$$\text{Natural Frequency } \omega = \sqrt{\frac{kg}{w}} = \sqrt{\frac{8480 \times 386}{200}} = \sqrt{16400}$$

$$\omega = 128 \text{ rad/sec}$$

Maximum acceleration after impact is

$$\frac{\ddot{x}_{AE}}{g} = \left(\frac{\omega \bar{V}}{g} \right) = \frac{128.0}{386} \times \bar{V} = 0.33 \bar{V} - g's$$

$$W_{AE} \frac{\ddot{x}_{AE}}{g} = 200 \times 0.33 \bar{V} = 66.0 \bar{V} \text{ lb}$$

where the relative velocity of A' to A after impact is $\bar{V}$.

For elastic impact $\bar{V} = 2V_0$, where V_0 is the velocity of AK before impact

$$\frac{\ddot{x}_{AE}}{g} = 0.33 \times 2 V_0 = 0.66 V_0$$

$$W_{AE} \frac{\ddot{x}_{AE}}{g} = 200 \times 0.66 V_0 = 132 V_0$$

$$\text{Allowable } P = (F + W_{AE} \ddot{x}_{AE}) = \frac{M_{\text{Allowable}}}{e} = \frac{M_{\text{Allowable}}}{10}$$

$$\text{For } M_{\text{Allowable}} = 10,000 \text{ in.-lb.}, P_{\text{Allowable}} = 1000 \text{ lb}$$

$$F + 132 V_0 = 1000$$

$$\text{For } F = 0 \quad V_0 = 7.6 \text{ in./sec} = 0.63 \text{ fps}$$

$$F = 300 \quad V_0 = 5.3 \text{ in./sec} = 0.45 \text{ fps}$$

$$F = 500 \quad V_0 = 3.8 \text{ in./sec} = 0.32 \text{ fps}$$

For impact at F,

$$M = W_{AK} \frac{\ddot{x}_{AK}}{g} e$$

$$\text{As before } K = 8480$$

$$W = 500$$

$$\omega = \sqrt{\frac{8480 \times 386}{500}} = \sqrt{6550}$$

$$= 81 \text{ rad/sec}$$

$$\ddot{x}_{AK} = \frac{\omega V_0}{g} = \frac{81 V_0}{386} = 0.21 \bar{V} \text{ lb}$$

$$W_{AE} \ddot{x}_{AE} = 500 \times 0.21 \bar{V} = 500 \times 0.42 V_0 = 210 V_0$$

$$M = 10,000 \quad e = 10 \quad P = 1000$$

$$V_0 = \frac{1000}{210} = 4.75 \text{ in./sec} = 0.4 \text{ ft/sec}$$

2.7.2 Stability of Drive Line under Axial Load plus Inertia Loads

Reference (1) gives the required relationship following the schematic shown in Figure 2-7.

$$P' + 0.5 p' L = \frac{\pi^2 EI}{L^2} \text{ (Euler load)}$$

Under deceleration Ng

$$pL = WN$$

$$p' + 0.5 WN' = \left(\pi^2 \frac{EI}{L^2} \right)$$

FIGURE 2-7. AXIAL LOAD FORCE MODEL

In the 4th Quarterly Report the Euler Buckling load was computed for a push rod comprised of one section 150 in. long and a second section 50 in. long.

For 3 in. o.d. 2-1/2 in. i.d. for Section 1
 1.5 in. diam for Section 2 } the Euler load equivalent to $\frac{\pi^2 EI}{L^2} = 4600$

$$\text{Area of 3 in. o.d. section} = \pi (1.5^2 - 1.25^2) = 2.16 \text{ in.}^2$$

$$w = 2.16 \times 0.3 = 0.65 \text{ lb/in.}$$

$$150 \text{ in.} \rightarrow 150 \times 0.65 = 97.5 \text{ lb}$$

$$1.5 \text{ in. diam area} = 1.76 \text{ in.}^2 \times 0.3 = 0.53 \text{ lb/in.}$$

$$50 \text{ in.} \times 0.53 = 26.5 \text{ lb}$$

$$\text{Total} = 97.5 + 26.5 = \underline{124 \text{ lb}}$$

Use 200 lb to include shaft and piston above top support point.

$$P = F_S + 0.5 WN'$$

$$= F_S + 100 N'$$

For $F_S = 300$

$$P = 300 + 100 N'$$

Using a safety factor of 5 for instability

$$P = \frac{4600}{5} = 920$$

$$300 + 100 N' = 920$$

$$N' = \frac{920 - 300}{100} \approx \underline{\underline{6g}}$$

Maximum deceleration of drive should be limited to 6g from stability considerations.

2.8 ALLOWABLE IMPACT VELOCITY ON SPRING UNDER AA'

Neglecting the flexibility of the drive line determines the spring constant K , necessary for a spring to keep the maximum deceleration less than 6g.

$$\ddot{x}_{\max} = \frac{F_k - F_S}{w} = \frac{k\delta - F_S}{w}$$

F_k = maximum spring force at end of travel (lb)

k = spring constant (lb/in.)

δ = spring travel (in.)

F_S = scram force (lb)

w = weight of moving assembly (lb)

$$\text{For: } w = 750 \text{ lb} \quad F_S = 300 \text{ lb} \quad \ddot{x} = 6$$

$$k\delta = w\ddot{x}_{\max} + F_S = 4800 \text{ lb}$$

Impact velocity, V_0 in in./sec

Equating energy of system at impact to energy at end of travel,

$$\frac{1}{2} \frac{w}{g} V_0^2 + F_S \delta + w\delta = \frac{1}{2} k\delta^2$$

$$\text{Substitute } V_0 = 36 \text{ in./sec and } \delta = \frac{4800}{k}$$

$$\frac{1}{2} \frac{4800^2}{k} - \frac{300 \times 4800}{k} - \frac{750 \times 4800}{k} = \frac{1}{2} \frac{750}{386} \times 36^2$$

$$k = 5150 \text{ lb/in.}$$

$$\delta = 0.93 \text{ in.}$$

Height of Rebound:

Assuming a perfectly elastic spring, the height of rebound is $\frac{V^2}{2g}$

For no down force on top of control drive line,

$$h = \frac{V^2}{2g} = 1.7 \text{ in. for } V = 3 \text{ ft/sec}$$

$$= 3 \text{ in. for } V = 4 \text{ ft/sec}$$

These are sufficiently small that reactor safety is not compromised.

2.8.1 Additional Energy Absorbing Spring Results

The analysis of section 2.7.2 indicated that a 6g deceleration is the upper limit for the final spring cushion. In considering the spring requirements for the final spring cushion, Figure 2-8 was developed. It shows the spring deflection (travel after impact) and the spring constant at various final dash pot velocities for selected deceleration rates. The basis for this curve is to equate the compression work on the spring to the kinetic energy of the control rod assembly at spring contact, plus the potential energy obtained from the control rod assembly weight and the additional scram force traveling through the spring stroke. The control rod assembly weight and additional scram force were taken at 750 pounds and 300 pounds respectively. A good location to place the final cushion spring is in the hexagonal reflector channel adjacent to the paired control rods and on the side away from the core. To provide the final stop, the largest helical coil spring which can be fitted into a single hexagonal channel location, is limited by stress to a deceleration rate of approximately 3-1/4 g. Belleville spring washers have been considered. They are fairly stiff in practical stacked heights and allow deceleration of approximately 6 g.

To provide the flexibility for covering the range of decelerations that may be required for the final design selection, more than a single reflector hexagonal channel may need to be used for spring locations. A cluster of three reflector channels are available for this purpose. Their location is directly under the offset, from control rod to drive extension shaft.

FIGURE 2-8. SPRING CUSHION CHARACTERISTICS

SECTION III

CORE MECHANICAL DESIGN

3.1 CORE SUPPORT AND INLET PLENUM

Design of the inlet plenum and core support plate which provides support for the reactor internals was continued during the quarter. A proposed design of the interface between the core support structure for the backup design and the reference pressure vessel has been established and is shown in Figure 3-1. The inlet plenum for the backup design consists of two perforated plates which would be bolted to the reference design core structure replacing the two perforated plates that form the inlet plenum for the reference design. Core by-pass leakage that might occur at the junction of the perforated plates and the reference structure will be controlled by metal o-ring seals at each of the junctions. The o-ring seals will be compressed by the plate hold-down bolts. The upper plate is bolted directly to the reference structure while the lower plate is held down by a set of special fasteners located just inside the circle of bolts that fasten the upper plate to the reference structure.

The upper plate has an outside diameter of 70 inches, a thickness of 6 inches, and is perforated with 127-2.390 inch diameter holes on a 3.39 inch equilateral triangular spacing. The lower plate has an outside diameter of 59 inches, a thickness of 6 inches and is perforated with 127-2.265 inch diameter holes on the same pitch and pattern as the upper plate. The alignment of the hole patterns between the two plates is very important since the nose piece of the fuel assemblies must penetrate both plates to be properly installed. The hole pattern alignment will be controlled by machining the holes in both plates at the same time while they are clamped together. During reactor shutdown conditions the upper grid plate will support the dead weight of the entire core which includes the driver fuel bundles, closed loops, open loops, and control rod shrouds. During operating conditions the core components will be held down to the upper grid plate against the hydraulic forces by a combination of their weight and utilization of the hydraulic hold-down effect from the pressure of the inlet coolant. As the coolant pressure within the inlet plenum increases, the load on the upper grid plate will change from a downward force to a net upward force at a point when the hydrostatic force from the inlet plenum pressure exceeds the weight of the core components. The pressure difference across the upper grid plate during operating conditions will produce a load on the plate which will exceed the dead weight load on the plate. This is the most important factor in the determination of the thickness of the plate. At installation the hole patterns of the two plates will be kept in alignment by several specially sized keys which will extend through the upper plate and into the holes of the lower plate, holding them in proper position until the bolts are fastened, after which the keys would be removed.

The lower end of the fuel assemblies and other core components will extend through the upper plate into and through the lower plate forming a seal at each of the plates. A piston ring type of seal will be used to reduce the amount of core by-pass flow that will occur at the penetration into the high pressure plenum. The coolant pressure difference between the inside and outside of the lower end of the fuel assembly acting over the cross-sectional area of the lower end will produce a force which in addition to the weight of the fuel assembly will be sufficient to keep the fuel assembly seated on the upper grid plate. The coolant pressure in the

region just beneath the lower plate of the inlet plenum is maintained at a low value relative to the pressure within the inlet plenum by means of tubes connecting this region to the area above the inlet plenum. These tubes pass through the outer edge of the inlet plenum as shown on Figure 3-1.

The bolted in grid plate in the reference design would have the advantage of providing a relatively simple means of replacing the upper grid plate at any time over the life of the plant should this become necessary due to radiation damage or other reasons. Actual removal of installation of the grid plates could be accomplished by remote means and would require only the rotation of the two sets of screws and a means to grapple the two plates. The proposed joint between the reference and vertical core structure would be located at a radius great enough to allow for a change in core size of the reference core and vertical core.

3.2 CORE CLAMPING

A core clamping system for the FFTF back-up design has been established. The concept utilizes a torsion rod type of clamping mechanism similar to the type installed in the SEFOR reactor. A drawing of the clamping device is shown in Figure 3-2. In the proposed system, a clamping device would be installed within each set of reflector assemblies which form a complete ring around the fuel assemblies. The ring of clamping mechanisms would be located radially beyond the outside diameter of the reflector plug. The clamping mechanisms are actuated by a special tool which is inserted through access holes located in the reactor outer head.

The clamping mechanisms are connected by roller chains, as shown in Figure 3-2, in groups of three so that 12 access holes are necessary to actuate the 36 clamping mechanisms required to form the complete ring of clamping force around the reactor core. The actuating tool rotates the torsion rods of the clamping mechanisms to some determined torque value. The torque is then locked in the torsion rods by means of a splined connection. The reaction to the torque stored within the torsion rods is used to force the hexagonal channel, in which the clamping mechanism is installed, radially inward. The tightening process, if repeated at 12 locations around the periphery of the core, will create a complete ring of radially inward directed forces which will hold the fuel assemblies in a compact array. Differential thermal expansion between the core and surrounding reflector region is accommodated by more or less rotation of the torsion rod of the clamps which functions as a torsional spring.

The above method of clamping the FFTF core has the major advantages of not interfering with refueling or experiment handling since the clamping mechanisms are located outside of the core. Also the core could be tightened and verified before startup since no core temperature changes or hydraulic pressure are required for clamp actuation.

3.3 CLOSED LOOP

The closed loops, which are designed to provide a test region within the reactor core, complete with an isolated independent coolant system, consist of two different types. There will be a total of six closed loops installed within the FFTF core. Five of the six loops are of the re-entrant, or counterflow, type, as shown in Figure 3-3, and are located at the same

FIGURE 3-1. CORE SUPPORT AND CLAMPING

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FIGURE 3-2. CORE CLAMPING MECHANISM

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FIGURE 3-3. SINGLE CLOSED LOOP

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radial position within the core, approximately 13.5 inches from the core center. The test sections of the closed loops are suspended within the core by means of a hanger rod or handle which extends to the top of the closed loop above the refueling plug. The test sections may be removed from the closed loops by means of the handle which also serves to seal the closed loop inlet flow from the outlet flow, thus providing a coolant flow circuit through the closed loop. In the re-entrant type loop, the inlet coolant flows downward through an outer annular path and then flows upward through the test region to the outlet pipe. The sixth closed loop, shown in Figure 3-4, to be installed at the center of the core, differs from the other five closed loops in occupying two hexagonal channel positions in the core. One of the hexagonal channels is used for the inlet flow and the other is used to house the test region, allowing for a larger diameter test assembly and more coolant flow than can be accommodated in the five re-entrant type closed loops. The upper end of both types of closed loops is essentially identical above the elevation of the point of attachment of the closed loops to the inlet and outlet piping crossing over the core. Shielding will be provided as a part of the test section handle over the length of closed loop that passes through the refueling plug. The lower end of the closed loops will be similar to the nose piece of the driver fuel elements and will extend through the support plates which form the inlet plenum, thereby taking advantage of the hydraulic hold down effect. Differential expansion of the closed loops with respect to the refueling plug will be accommodated by having the closed loops connected to the refueling plug by means of a bellows which will allow the required axial motion of the closed loop. Designing the closed loops with a lower end similar to that of a driver fuel element will allow a driver fuel element to be installed in a test loop position if it is desired to operate the reactor with less than six closed loops installed.

All of the closed loops are connected to the coolant inlet and outlet lines by means of remotely operable couplings. The couplings are located at an elevation just below the refueling plug and at a radial position above the reflector-control channels which surround the fuel assemblies. The closed loop couplings are accessible when the refueling plug has been removed since they are located within the refueling plug opening. At the present time the couplings are operated by rotating two screws that hold the two halves of each coupling together. Rotation of the two screws also causes the joint to separate in a horizontal direction through a cam and follower to free the closed loop from the piping to allow removal. Contacts with a vendor have produced information on a new type of coupling which is remotely operable by special tools, which requires no threaded fasteners, and which has smaller envelope dimensions. This new type, presently being tested by the vendors, appears to have some desirable features for use on FFTF closed loops.

Figure 3-5 shows a typical re-entrant type closed loop. It is connected to the external coolant lines with the reactor refueling plug in place and in a disconnected condition, ready for removal, with the refueling plug already removed.

3.4 FUEL PIN STRESS ANALYSIS

A preliminary evaluation was made of the stresses at the end-plug to cladding weld joint. The effects considered were of internal and external pressures and temperature differences between end plug and cladding.

The following conditions were evaluated:

1. Steady State
2. Scram with primary pumps at rated flow
3. Cold fuel (100° F) insertion into sodium (400° F) during refueling.

Temperature distributions in the end plug region during transients were computed using the TIGER V digital computer code. The maximum temperature differences between the end plug and the thin cladding were 56° F for the scram condition without pump coast down and 225° F for a rapid (> 5 ft/min) refueling insertion rate. The discontinuity bending stresses for these temperature differences are 32,000 psi and approximately 190,000 psi respectively. A stress concentration factor of 1.61 $\left[k = 0.293 \sqrt{t/\delta} \right]$ was incorporated. Pressure induced stresses were neglected since the net effect was to reduce the above values. The joint can probably withstand the scram transient since the peak stress is less than twice the yield stress. But, further evaluation is needed to determine to what degree, if necessary, fuel elements should be pre-heated during refueling.

3.5 INSTRUMENT SUPPORT STRUCTURE

Instrumentation for the driver fuel elements is supported by the structure hanging from the refueling plug (Figure 3-6). With the plug in place on the vessel, instrument packages extend into the handle section of each fuel element. The refueling plug and instrumentation are removed to gain access to the fuel region. To avoid interference with the closed loop piping, provisions have been made to move the instrumentation from beneath the pipes before the refueling plug is raised.

Figure 3-6 shows the refueling plug and structure designed to accommodate six closed loops and twelve open loops.

The instrumentation is fabricated in modules which are attached to the refueling plug. Each module supplies instrumentation for 10 to 12 fuel elements. The replacement of a defective instrument sensor will require the replacement of an entire module. Changing the instrument modules will occur with the refueling plug in its refueling cell storage position. Each instrument module is attached to the support structure at both ends. Exchanging the modules can be done with conventional remote handling equipment. The top connection supports the weight of the module and provides the clamping force for the seal between the instrument module and refueling plug. This connection can be tightened and loosened with a rotary motion, possibly with an impact wrench. The lower connection is shown as a band clamp although other methods are being considered. The lower connection is complicated by the requirement to capture and positively secure movable parts inside the vessel.

Figure 3-6 shows the arrangement of the sodium sample lines in the instrument support structure. Although these lines are intended to be a permanent part of the structure, it would not be impossible to replace one if needed. The sample lines are attached to the support structure at frequent intervals to avoid excessive vibration.

The sample lines extend from the fuel elements to midway through the refueling plug and terminate around its periphery. No mechanical connection is made between these sample lines

FIGURE 3-4. CLOSED LOOP - CORE CENTER

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FIGURE 3-5. DETAIL ON CLOSED LOOP BRANCH PIPING DISCONNECT FEATURES

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FIGURE 3-6. REFUELING PLUG AND INSTRUMENTATION

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and the corresponding lines in the annular plug. The sodium is forced through the tubes with sufficient velocity to jump the gap between the two plugs.

In Figure 3-6, section F-F shows how the sample lines are connected to positions directly below the instrument module support tube. For these positions a portion of the support tube is included on the instrument module. A permanent sample line to these positions would interfere with the module when it is being installed or removed.

SECTION IV

PHYSICS

4.1 CROSS SECTIONS

New 60 group fuel cross sections have been produced for the two enrichment zones in the FFTF Backup Design. These cross sections were generated from the General Electric BASO3 cross section data.

The 60 group, cross sections were lessened to 16 energy groups for use in 2-D synthesis calculations. The condensation was performed in several reactor regions and several reactor conditions (control in, control out, and sodium voided) to include spatial variations in the neutron energy spectrum. The energy structure of the 16 group, cross-section set (see Table 4-1) was established using a flux, adjoint flux, and $\Delta \sigma_{\text{cap}}$ in U-238 (due to Doppler) weighting as described in Reference (2).

TABLE 4-1

ENERGY STRUCTURE OF FFTF BACKUP DESIGN
16 GROUP CROSS SECTION FILE

<u>Group</u>	<u>E_L</u>	<u>U_L</u>	<u>ΔU</u>
1	2.2 MeV	1.5	1.5
2	0.825	2.5	1.0
3	0.5	3.0	0.5
4	0.3	3.5	0.5
5	0.18	4.0	0.5
6	0.11	4.5	0.5
7	61.0 keV	5.1	0.6
8	25.0	6.0	0.9
9	5.5	7.50	1.50
10	2.0	8.50	1.00
11	1.3	8.97	0.47
12	1.0	9.20	0.23
13	630.0 eV	9.67	0.47
14	502.0	9.90	0.23
15	150.0	11.10	1.200
16	Thermal	----	----

4.2 DESIGN PARAMETERS

4.2.1 Power Distribution

The radial power distribution at the beginning and end of an equilibrium burnup cycle are shown in Figure 4-1. The plutonium enrichment is distributed between the two enrichment zones so the peak linear power density in a fresh, newly loaded fuel bundle is the same in each zone at the beginning of an equilibrium fuel cycle. This is the limiting power density condition. The power peaking factors for the beginning of cycle condition are listed in Table 4-2.

TABLE 4-2

POWER PEAKING FACTORS AT BEGINNING OF CYCLE

	<u>ZONE 1</u>	<u>ZONE 2</u>
Radial	1.30	1.28
Axial	1.21	1.21
New to Average	1.06	1.08
Fabrication	1.035	1.035
Experiment Compensation	<u>1.025</u>	~1.00
Total	1.77	
Average Linear Power Density	8.16* kW/ft	
Peak Linear Power Density	14.4 kW/ft	
Over Power Factor	1.2	
Peak Linear Power Density Including		
Over Power Factor	17.3 kW/ft	
Peak Linear Power Density Allowable	18.0 kW/ft	

*Value Based on 91% of total energy absorbed in fuel.

Zone one and zone two fuel bundles can be interchanged at the boundary between the two zones to compensate for experimental reactivity changes. Interchanging 12 fuel bundles causes a reactivity change of $\sim 1\% \Delta k/k$. When 12 zone two fuel bundles are replaced with zone one fuel, the power density near the core center increases about 2.5%, hence the 1.025 experimental compensation power peaking factor.

The peak linear power density, including the 1.2 overpower factor, calculates to be 17.3 kW/ft while the maximum allowable value is 18.0 kW/ft. Since the calculated peak linear power density is not yet limiting, possible uncertainties in analytical techniques can be allowed.

FIGURE 4-1. RADIAL POWER DISTRIBUTION

4.2.2 Flux Distribution

The radial neutron flux distribution in the core is shown in Figure 4-2 for the beginning and end of cycle conditions. The peak neutron flux is 0.95×10^{16} at beginning of cycle and 0.92×10^{16} at the end of cycle. These values assume that no fuel shuffling occurs.

4.2.3 Reflector Control Strength

Calculations for reflector control strength are based on a control system with the material composition described in Table 4-3. Of the 42 reflector locations adjacent to the radial core boundary, 38 contain reflector control. The other 4 are located in the area of the refueling cask and are not available as reflector control locations. The 2-D synthesis calculations assume all 42 locations are used and the calculated control strength is reduced by 38/42 to obtain the worth of the control system. The worths of a natural B_4C and $B_4^{10}C$ control system are 5.89% $\Delta k/k$ and 9.87% $\Delta k/k$, respectively. The influence of B^{10} enrichment on the reflector control strength is seen in Figure 4-3.

4.2.4 Delayed Neutron Fraction for two-Zone Core

The effective delayed neutron fraction, β_{eff} , in the FFTF Backup Design is 0.0028. The two enrichment zone FFTF Backup Design has a β_{eff} substantially lower than the 0.003 previously quoted for the one zone design. This reduction in β_{eff} is primarily due to the increased plutonium enrichment and hence higher fraction of fissions in plutonium for the two-zone design. β_{eff} was calculated for both the beginning and end of an equilibrium fuel cycle. Calculations indicate that β_{eff} is essentially constant during the fuel cycle.

TABLE 4-3

MATERIAL COMPOSITION OF REFLECTOR CONTROL SYSTEM

	<u>Nickel In</u>	<u>B₄C In</u>
v/o B ₄ C*	--	35.1
v/o Sodium	29.9	28.0
v/o Steel	--	25.5
v/o Nickel	58.7	--
v/o In-800	11.4	11.4

*This assumes a 0.180 inch gap between the carriage and the channel. The reference control drive analysis of Section II was for a 0.225 inch gap. The physics results were calculated prior to this change and will be updated in subsequent reports.

FIGURE 4-2. RADIAL FLUX DISTRIBUTION

FIGURE 4-3. INFLUENCE OF B¹⁰ ENRICHMENT ON REFLECTOR CONTROL STRENGTH

Calculations were performed with the BISYN computer code. The delayed neutron fraction, β_k , was calculated for each core region using the one group cross sections and material balance from BISYN output and delayed neutron fractions from Keepin⁽³⁾. A delayed neutron spectrum was obtained from Keepin and delayed neutron chi values, $X_{d,i}$, determined for the FFTF 16-energy group cross section set where $\sum_i X_{d,i} = 1$. A BISYN perturbation calculation was made where X was perturbed in each region k and energy group i by the amount $\delta X_{k,i} = \beta_k X_{d,i}$, yielding the value of β_{eff} .

4.3 TEMPERATURE REACTIVITY EFFECTS

The U-238 Doppler coefficient, $T \frac{dk}{dT}$, has been calculated at beginning and end of cycle conditions. The calculations used the 16 group, cross-section file described in Section 4.1. The end of cycle value, U-238 $T \frac{dk}{dT} = -0.0054$, agrees with that calculated with the one zone cross section file. The beginning of cycle value, U-238 $T \frac{dk}{dT} = -0.0047$, is slightly lower than previously calculated. This is probably due to the new energy structure of the 16 group, cross-section file and the presence of slightly more boron at the beginning of cycle condition.

Calculations have been performed for the FFTF zoned core design to evaluate the reactivity effect of changing the reactor condition from cold refueling temperature (400° F isothermal) to full power (400 MWt). The total temperature reactivity effect was calculated to be -1.35% $\Delta k/k$, cold refueling to full power operation.

In addition, the reactivity effect from 700° F isothermal to full power was calculated. The results of these calculations are shown in Table 4-4. The assumed material temperatures and the corresponding average linear expansions at each temperature state are given in Table 4-5.

TABLE 4 4

TEMPERATURE REACTIVITY EFFECT FOR THE FFTF BACKUP ZONED CORE AT THE END OF AN EQUILIBRIUM BURNUP CYCLE ASSUMING NO CONTROL INSERTED

<u>Case</u>	<u>Expansion and Density Effects</u>	<u>Doppler Effect</u>	<u>Total</u>
400° F Isothermal - 400 MWt	-0.77% $\Delta k/k$	-0.58% $\Delta k/k$	-1.35% $\Delta k/k$
700° F Isothermal - 400 MWt	-0.56% $\Delta k/k$	-0.41% $\Delta k/k$	-0.97% $\Delta k/k$
400° F Isothermal - 700° F Isothermal	-0.21% $\Delta k/k$	-0.17% $\Delta k/k$	-0.38% $\Delta k/k$

TABLE 4-5

ASSUMED MATERIAL TEMPERATURES AND AVERAGE LINEAR EXPANSION $(1 + \alpha \Delta T \text{ from Room Temperature})$

	400° F Isothermal		700° F Isothermal		400 MWt	
	<u>Temperature</u>	<u>$(1 + \alpha \Delta T)$</u>	<u>Temperature</u>	<u>$(1 + \alpha \Delta T)$</u>	<u>Temperature</u>	<u>$(1 + \alpha \Delta T)$</u>
Fuel:						
Average centerline temp. (for axial fuel expansion)	400° F	1.0010	700° F	1.0022	3000° F	1.0200
Average fuel temp. (for Doppler effect)	400° F		700° F		2000° F	
Fuel cladding, average temp.	400° F	1.00316	700° F	1.00626	950° F	1.00893
Channels, average temp.	400° F	1.00290	700° F	1.00573	800° F	1.00672
Sodium, average temp.	400° F	$\rho @ 400^\circ \text{F}$ $= 0.906 \text{ gm/cc}$	700° F	$\rho @ 700^\circ \text{F}$ $= 0.863 \text{ gm/cc}$	850° F	$\rho @ 850^\circ \text{F}$ $= 0.841 \text{ gm/cc}$

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The calculations were performed assuming fuel compositions for the end of an equilibrium burnup cycle. The total temperature reactivity effect (expansion and density changes plus Doppler) at the beginning of the burnup cycle is nearly equal to the end of cycle temperature effect.

The axial fuel expansion reactivity effect at the hot operating condition was found to be $\frac{\Delta k/k}{\Delta h/h} = -0.29$, where h is fuel height. Other individual temperature reactivity effects were not calculated.

4.4 AXIAL REFLECTOR THICKNESS

Calculations indicate that making the axial reflector section of the fuel rods greater than six inches does not significantly improve either k_{eff} or the axial power peaking factor. Reducing the axial reflector thickness to less than six inches will require more detailed examination.

4.5 ENERGY PRODUCED PER FISSION

The energy produced per fission has been calculated for the two zone FFTF Backup Design. The calculated value of 216 MeV per fission agrees with the previously calculated value⁽⁴⁾ of 215 MeV per fission for the one zone core. The fraction of energy absorbed in the fuel was calculated to be 90% as compared with 91% reported in reference (4). The differences listed are due to slightly different assumptions in the analytical methods rather than actual differences. To be consistent, the one zone calculated values will also be used for the two zone core.

4.6 MAXIMUM FUEL BUNDLE WORTH IN FFTF BACKUP DESIGN

The maximum worth of a single fuel bundle has been calculated for the FFTF Backup Design. Assuming a fresh fuel bundle is inserted into a sodium filled hole at the center of the core for beginning cycle conditions, the worth of a Zone 1 fuel bundle is \$2.3, and a Zone 2 fuel bundle is \$3.7 at this location for $\beta_{\text{eff}} = 0.0028$.

SECTION V

SHIELDING

The major emphasis in nuclear shield design during the past quarter was radiation streaming effects through specific penetrations in the top head. The computer program QAD⁽⁵⁾ is being used together with hand calculations to evaluate radiation attenuation in the top head region.

Included is a discussion calling attention to the decision not to use B₄C in the radial shield. The effect is noted on the gamma heating load outside of the reactor vessel.

5.1 SHIELDING REQUIREMENTS FOR THE TOP SHIELD PLUG

The principal source of radiation to be attenuated by the top shield plug comes from the Na²⁴ gamma activity in the primary coolant pool below the plug. The dose rate at the surface of this pool is 2.5×10^8 mR/h. The presently conceived top head design will attenuate this dose rate to 2.4 mR/h, considering mass attenuation only. The contribution, however, of gamma streaming through the many penetrations in the top plug can increase the dose rate considerably in some areas. The results of a preliminary study to determine the dose rate at the top of the plug due to gamma streaming is presented below.

5.1.1 Calculational Model

A simple geometry model was selected for this study with primary emphasis on the annular gaps between the refueling head, reflector head, main head, and vessel wall as shown in Figure 5-1. Results were obtained by a combination of hand calculations and computer runs using QADP5A⁽⁵⁾.

5.1.2 Sodium Pool Gamma Source

During operation, gamma radiation to the top shield plug comes principally from Na²⁴ activity in the sodium pool below the plug. After shutdown, the radiation due to the Na²⁴ activity will decrease with a 15 hour half-life, until in ~9 days the Na²² activity (with a 2.6 year half-life) will exceed that of Na²⁴. The saturated Na²⁴ activity will be used to evaluate the top plug shielding effectiveness. The saturated specific activity of Na²⁴ in the primary coolant sodium has been previously calculated to be 0.071 Ci/cm³. Each disintegration of Na²⁴ produces one 2.8 MeV γ and one 1.4 MeV γ .

5.1.3 Mass Attenuation

The mass attenuation through the top shield plug neglecting all penetrations was calculated using the following formula⁽⁶⁾ and checked with the ray analysis computer code QADP5A:

$$D = k \phi = \frac{k B S_v}{2\mu_s} E_2(b_1) - \frac{E_2(b_1 \sec \theta_1)}{\sec \theta_1}$$

FIGURE 5-1. CALCULATIONAL MODEL (APPROXIMATE DIMENSIONS)

where: K = conversion from γ flux to dose rate

B = build-up factor

S_v = source strength of volume source ($\text{cm}^{-3} \text{sec}^{-1}$)

μ_s = macroscopic cross section of source material (cm^{-1})

$b_1 = \sum_i^n \mu_i t_i$

μ_i = macroscopic cross section of shield material i (cm^{-1})

t_i = thickness of ith shield (cm)

θ_1 = angle subtended by centerline through cylindrical source and line from outer radius of cylindrical source to receiver point (must be on center line).

$$E_n(b) = b^{n-1} \int_b^\infty \frac{e^{-t}}{t^n} dt \quad \text{for } n \geq 0$$

Comparison of hand calculations and QADP5A results showed excellent agreement (2.4 mR/h from QAD and 2.8 mR/h by hand) with the difference due mainly to hand-interpolated cross-sections.

5.1.4 Attenuation through Annular Gaps

Both hand calculations and QADP5A calculations were done for the annular gaps at the following interfaces:

- 1) Refueling Head - Reflector Head
- 2) Reflector Head - Main Head
- 3) Main Head - Vessel Wall

Hand calculations assumed a cosine distributed gamma source at the top of the sodium pool of 2.5×10^8 mR/h. Attenuation through the annular slot was calculated using the following expression:

$$\frac{\phi}{\phi_0} = \frac{1}{\pi L^2} \left[\left(\cos^{-1} \frac{r}{R} \right) (2R^2 - r^2) - r \sqrt{R^2 - r^2} \right]$$

where:

r = inner radius of annular slot, cm

R = outer radius of annular slot, cm

L = length of slot, cm

Since the refueling head and reflector head both have two offsets, the gamma flux at the bottom of each slot section was the same as the mass flux at that particular height. The flux at the top of the slot section was then attenuated through the steel wall surrounding the slot until reaching the top of the shield plug. The results of these calculations are summarized in Table 5-1.

TABLE 5-1
DOSE RATES AT THE TOP OF THE HEAD

Streaming Path	Dimensions	Dose Rate mR/h	
		Hand Calculation	QADP5A
Refueling Head - Reflector Head Annular Duct	50 in. Diameter *0.1 in. thick annular slot (2 steps)	13	
Reflector Head - Main Head Annular Duct	84 in. Diameter *0.15 in. thick annular slot (2 steps)	24	
Main Head - Vessel Wall Annular Duct	190 in. Diameter *1.0 in. thick annular slot (1 main step)	~107	63
Typical 4 in. Diameter Cylindrical Duct Voided	4 in. Diameter Cylindrical duct total length of top shield plug	3.3×10^5	3.4×10^5

* Since the plug could be offset to one side, the slot thickness was assumed to be two times the given thickness for calculations.

The design dose rate at the top of the shield plug is ≤ 50 mR/h. As can be seen the analysis shows no unusual problems for the refueling head - reflector head and the reflector head - main head interfaces; although other penetrations through these areas may further increase the dose rate. Work will continue in this area. The gap between the main head and the vessel wall will be the subject of future detailed analysis.

5.2 FFTF BACKUP DESIGN RADIAL SHIELD

The second quarterly, GEAP-5614, reported on a radial shield optimization study. The specific goals of the analysis were to minimize the fast neutron fluence at the reactor vessel and to minimize gamma heating external to the reactor vessel. When the B_4C content of the 18-inch thickness radial neutron shield varied from 60% down to 0%, the fast neutron fluence ($E_n > 0.1$ MeV) remained well below the tentative limit (1×10^{20} nvt) established by PNL.

However, this study indicated that the gamma heat generation rate varied from 0.278 watts/cm² (0% B_4C) down to 0.042 watts/cm² (60% B_4C). This is in excess of the 0.035 watts/cm² recommended by PNL. Yet, the cooling system for components located externally and adjacent to the reactor vessel has been tentatively sized by PNL in the range of 100 to 500 kW. If this heating rate occurs over a 15-foot height, the total gamma heating rates illustrated by Table 5-2 are generated. Table 5-2 shows the total gamma heat load varying from a low of 26 kW

to a high of 201 kW. Thus, in all cases, the total heat load is well under the tentative limit of 500 kW. Therefore, considering the cost of B_4C , the decision has been made to recommend elimination of B_4C from the core radial shielding. This decision, following the study reported in the third Quarterly, GEAP-5614, showing it both feasible and acceptable to remove all B_4C from the upper axial neutron shield, resulted in the removal of all B_4C from the backup core shielding which is located inside the reactor vessel.

TABLE 5-2

FFTF BACKUP DESIGN 400 MW THERMAL, GAMMA HEAT GENERATION
RATE IN THE COMPONENTS LOCATED EXTERNALLY ADJACENT TO THE REACTOR VESSEL

Neutron Shield Composition			Gamma Heat Generation Rate		
Volume Percent			MAC $\propto$ Slab*	1/r Correction*	Total Heat Over 15-Foot Height
B_4C	SS-304	Na	Watts/cm ²	Watts/cm ²	kW
0	70	30	1.169	0.278	201
10	60	30	0.168	0.057	41
20	50	30	0.104	0.043	31
40	30	30	0.072	0.036	26
60	10	30	0.097	0.042	30

* Includes 0.026 watts/cm² due to Na^{24} decay activity (absorption in the reactor vessel was neglected).

SECTION VI

SUBCRITICALITY ANALYSIS - SAFEGUARDS

6.1 INTRODUCTION

The purpose of this study was to evaluate the feasibility of using neutron noise analysis techniques to measure subcritical reactivity in the FFTF backup design. Accurate knowledge of the system reactivity is desirable during shutdown and refueling procedures. Traditional reactivity measuring techniques are restricted in the desired application by accuracy limits, perturbations that must be applied to the system, or the length of time required to acquire sufficient data. Sheff has summarized the arguments for a continuous indication of shutdown reactivity.⁽⁷⁾ Neutron noise analysis techniques offer the possibility of nearly continuous reactivity measurement in far subcritical systems with no perturbation to the system. A "noise" system is not a prerequisite for FFTF licensing, normal procedural controls being considered totally adequate to eliminate consideration of a refueling accident. However, the successful development of this technique would be a valuable backup to procedural controls.

This section summarizes the preliminary evaluation of the feasibility of using noise analysis for the FFTF backup design. This section also outlines a simplified theory of the most promising noise analysis technique, indicates the areas that must be studied to evaluate this technique, and quantitatively discusses the application to the FFTF backup design.

6.2 BASIC THEORY

The basis idea of reactivity measurement by neutron noise analysis is as follows: the neutrons released in fission events provide a noise source of known frequency distribution (Poisson) that is modified by the reactor transfer function to provide a neutron flux noise spectrum related to the reactivity state of the system. Spectral analysis of the output signal from conventional neutron detectors provides the basis for measurement of the reactivity. The derivation [based on Reference (8)] of the relevant equations for the most promising method, two-detector two-channel cross correlation, is outlined below:

The generating function for the detector outputs, γ_1 and γ_2 , is defined as

$$f(U_1, U_2) \equiv \int_0^\infty \int_0^\infty p(\gamma_1, \gamma_2) \exp(U_1 \gamma_1 + U_2 \gamma_2) d\gamma_1 d\gamma_2 \quad (6-1)$$

where $p(\gamma_1, \gamma_2) d\gamma_1 d\gamma_2$ is the probability that the signal from detector 1 is within $d\gamma_1$ about the value γ_1 and the signal from detector 2 is within $d\gamma_2$ about the value γ_2 . This generating function is useful since the time averaged signals from the detectors and the time averaged product of the signals may be expressed as simple derivatives of the generating function, evaluated at $U_1 = U_2 = 0$:

$$\gamma_1(t) = \left. \frac{\partial f(U_1, U_2)}{\partial U_1} \right|_{\substack{U_1=0 \\ U_2=0}}, \quad \gamma_2(t) = \left. \frac{\partial f(U_1, U_2)}{\partial U_2} \right|_{\substack{U_1=0 \\ U_2=0}} \quad (6-2)$$

and,

$$\gamma_1(t) \gamma_2(t) = \left. \frac{\partial^2 f(U_1, U_2)}{\partial U_1 \partial U_2} \right|_{\substack{U_1=0 \\ U_2=0}}$$

The notation indicates that the derivatives are evaluated at the point $U_1 = U_2 = 0$. As shown below, the derivatives of the generating function, $f(U_1, U_2)$, are directly related to the system reactivity. Thus by measuring the time averaged signals, the reactivity may be measured. The generating function for the distribution of detector signals due to all nuclear events, $f(U_1, U_2)$, may be related to the generating function for signals due to the introduction of a single particle, $a(t, U_1, U_2)$, to provide the following relations:

$$\overline{\gamma_1} = \lim_{T \rightarrow \infty} S_0 \int_0^T \left. \frac{\partial a(t, U_1, U_2)}{\partial U_1} \right|_{\substack{U_1=0 \\ U_2=0}} dt \quad (6-3)$$

$$\overline{\gamma_1 \gamma_2} = \overline{\gamma_1} \overline{\gamma_2} + 2 F \alpha_2 \lim_{T \rightarrow \infty} \int_0^T \left. \frac{\partial a(t, U_1, U_2)}{\partial U_1} \right|_{\substack{U_1=0 \\ U_2=0}} \left. \frac{\partial a(t, U_1, U_2)}{\partial U_2} \right|_{\substack{U_1=0 \\ U_2=0}} dt$$

where

- S_0 = the neutron source rate due spontaneous fission,
- F = the fission rate,
- $\alpha_2(t, U_1, U_2)$ = the generating function for signals at time t due to the introduction of one particle at time zero.
- α_2 = is the average number of neutron pairs per fission

These expressions are simplified by noting that

$$\left. \frac{\partial a(t, U_1, U_2)}{\partial U_1} \right|_{\substack{U_1=0 \\ U_2=0}}$$

is the expected signal from detector 1 at time t due to a unit neutron source at time zero. Accordingly,

$$\left. \frac{\partial a(t, U_1, U_2)}{\partial U_1} \right|_{\substack{U_1 = 0 \\ U_2 = 0}} = \frac{W_1 k}{\ell \nu} \int_0^t n(\tau) g(t-\tau) d\tau \quad (6-4)$$

where:

- w = the detector efficiency in detections per fission
- k = the system multiplication constant
- ℓ = the prompt neutron lifetime
- ν = the average number of neutrons emitted per fission
- $n(\tau)$ = the neutron population at time τ due to a unit source at time zero, i.e., the solution to the point kinetics equation with a delta source at time zero.
- $g(t-\tau)$ = the response of the detector and associated electronics at time $t-\tau$ due to a detection event at time τ .

Equations 6-3 and 6-4 are manipulated to obtain more useful forms:

$$\begin{aligned} \overline{\gamma_1} &= W_1 F \lim_{T \rightarrow \infty} \int_0^T g_1(t) dt \\ \overline{\gamma_2} &= W_2 F \lim_{T \rightarrow \infty} \int_0^T g_2(t) dt \\ \overline{\gamma_1 \gamma_2} &= \overline{\gamma_1} \overline{\gamma_2} + \left(F W_1 W_2 \frac{\nu(\nu-1) k^2}{(\ell \nu)^2} \right) \times \\ &\quad \left(\lim_{T \rightarrow \infty} \int_0^T \int_0^t n(\tau) g_1(t-\tau) dt \int_0^t n(\tau) g_2(t-\tau) (d\tau dt) \right) \end{aligned} \quad (6-5)$$

The Fourier transform forms of these equations are

$$\begin{aligned} \overline{\gamma_1} &= W_1 F G_1(0) \\ \overline{\gamma_2} &= W_2 F G_2(0) \\ \overline{\gamma_1 \gamma_2} &= \overline{\gamma_1} \overline{\gamma_2} + F W_1 W_2 \frac{\nu(\nu-1) k^2}{(\ell \nu)^2 2\pi} \int_0^\infty G_1(\omega) G_2(-\omega) |H(\omega)|^2 d\omega \end{aligned} \quad (6-6)$$

where

$$G_1(\omega) = \int_0^{\infty} e^{i\omega t} g_1(t) dt$$

$$H(\omega) = \int_0^{\infty} e^{i\omega t} n(t) dt$$

From equation 6-6 we note that spectral analysis of the product signal $\gamma_1 \gamma_2$ combined with knowledge of the transfer function of the detector and associated electronics, $G(\omega)$, will provide a measurement of the square modulus of the reactor transfer function, $H(\omega)$.

In the two detector two channel cross correlation technique the signal from each detector is passed through a wide band amplifier and a tuned bandpass filter with the transfer function $B(\omega)$. The signals are then multiplied and integrated as shown in Figure 6-1.

FIGURE 6-1. SCHEMATIC DIAGRAM OF TWO DETECTOR TWO CHANNEL CROSS CORRELATION MEASUREMENT

If the average charge at the amplifier output per detection event is q and the detectors and amplifiers have essentially flat responses in the frequency range of interest then $G_1(\omega) = q_1 B(\omega)$ and $G_2(\omega) = q_2 B(\omega)$. The output signal from this system will be

$$S_1 = q_1 q_2 W_1 W_2 F^2 B_1(0) B_2(0) + q_1 q_2 W_1 W_2 F \frac{\nu(\nu-1) k^2}{(\ell \nu)^2 2\pi} \times \operatorname{Re} \int_0^{\infty} B_1(\omega') B_2(\omega') |H(\omega')|^2 d\omega' \quad (6-7)$$

The bandpass filters are tuned so that only signals in a narrow frequency range are transmitted, that is

$$B_1(\omega') B_1(-\omega') = 0, \text{ if } \omega' \text{ is not within } \Delta\omega_1 \text{ about } \omega_1. \quad (6-8)$$

The band width, $\Delta\omega$, is chosen so that $|H(\omega)|^2$ is essentially constant over $\Delta\omega$ and therefore may be taken outside the integral in equation 6-7. The output signal may now be expressed as

$$S_1(\omega_1, \Delta\omega_1) = q_1 q_2 F W_1 W_2 \frac{\overline{\nu(\nu-1)k^2}}{(\nu\ell)^2} |H(\omega)|^2 \Gamma_1(\omega_1, \Delta\omega_1)$$

where

$$\Gamma_1(\omega_1, \Delta\omega_1) = \text{Re} \left(\frac{1}{2\pi} \int_0^\infty B_1(\omega') B_2(-\omega') d\omega' \right) \quad (6-9)$$

To relate the output $S(\omega, \Delta\omega)$ to the system parameters we note that at sufficiently high frequencies, the effect of delayed neutrons may be ignored, and the zero power reactor transfer function is given by

$$H(\omega) = \frac{1}{\alpha + i\omega}, \quad |H(\omega)|^2 = \frac{1}{\alpha^2 + \omega^2}$$

where

$$\alpha = \frac{1 - k(1 - \beta)}{\ell}$$

The ratio of the compensated signals from the two channels is thus

$$\frac{S_1(\omega_1, \Delta\omega_1) / \Gamma_1(\omega_1, \Delta\omega_1)}{S_2(\omega_2, \Delta\omega_2) / \Gamma_2(\omega_2, \Delta\omega_2)} = \frac{\alpha^2 + \omega_1^2}{\alpha^2 + \omega_2^2} \quad (6-10)$$

If the ω 's are chosen so that $\omega_1^2 \gg \alpha^2$ and $\omega_2^2 \ll \alpha^2$, then the ratio of the channel signals is

$$\frac{S_1(\omega_1, \Delta\omega_1) / \Gamma_1(\omega_1, \Delta\omega_1)}{S_2(\omega_2, \Delta\omega_2) / \Gamma_2(\omega_2, \Delta\omega_2)} = \left[\frac{1 - k(1 - \beta)}{\omega_1 \ell} \right]^2 \quad (6-11)$$

The restrictions on the frequencies ω_1 and ω_2 are simply that ω_1 is well above the break frequency of the reactor transfer function and ω_2 is well below this frequency. The break frequency,

$$\omega_0 = \frac{\beta}{\ell} \left(1 - \frac{k-1}{\beta} \right),$$

and the dependence on reactivity of this frequency are shown in Figure 6-2. The following data were used to construct Figure 6-2: $\beta = 0.0032^*$, $\ell = 3 \times 10^{-7}$ second. Introducing the definition

* This value was chosen before the work reported in Section IV for the 2 zone core was completed, giving $\beta = 0.0028$.

FIGURE 6-2. MODULUS OF ZERO POWER TRANSFER FUNCTION FOR FFTF AS A FUNCTION OF SUBCRITICALITY

of reactivity in dollars $\rho \triangleq \frac{k-1}{\beta}$ and noting that $\beta \ll 1$, we obtain the desired equation for evaluating the two detector two channel cross correlation technique:*

$$\text{Reactivity in Dollars} = \rho = \frac{k-1}{\beta} = 1 - \frac{\ell \omega_1}{\beta} \sqrt{\frac{S_1(\omega_1, \Delta\omega_1) \Gamma_2(\omega_2, \Delta\omega_2)}{S_2(\omega_2, \Delta\omega_2) \Gamma_1(\omega_1, \Delta\omega_1)}} \quad (6-12)$$

Equation 6-12 provides a means for determining reactivity by a simple analog processing of the output of the two frequency channels shown in Figure 6-1.

6.3 ACCURACY LIMITATIONS OF MEASURED REACTIVITY

The error in the measured reactivity obtained using the two detector two channel cross correlation method stems from several sources:

1. Statistical uncertainty due to finite sampling time
2. Error in measured constants $\left(\Gamma_1, \Gamma_2, \frac{\ell}{\beta}\right)$.
3. Errors due to approximations made in theoretical derivation such as the use of the point kinetics model
4. Errors due to hardware such as amplifier drift or detector burnout.

The accuracy of this technique does not critically depend on the following data as do other familiar techniques:

1. Accurate knowledge of source strength and distribution
2. Spatial flux distribution
3. Absolute value of detector output
4. Accurate knowledge of material (i.e., control rod) worth far from operating conditions.

Only the accuracy limitations due to the finite sampling time and the stochastic nature of the process have been considered here.

The statistical error in the measured reactivity is dependent on the ratio of the correlated to uncorrelated noise. It has been shown in References 9 and 10 that this ratio is given by the expression,

$$Q(\omega) = \frac{W \overline{\nu(\nu-1)}}{(\nu \beta)^2 \text{RD}} \left[\frac{1}{(1-\rho)^2 + \left(\frac{\omega \ell}{\beta}\right)^2} \right] \quad (6-13)$$

* Reactivity in dollars is usually defined as $\rho' = \frac{k-1}{k\beta}$, thus $\rho' = \frac{\rho}{1+\beta\rho}$, and for a subcritical system, $|\rho'| > |\rho|$. The use of ρ' rather than ρ would slightly modify the error analysis in the following sections but would not change the qualitative conclusions.

where:

$$R = \overline{q^2} / \bar{q}^2$$

$$D = 1 + \frac{\phi_\gamma}{\phi_n} Z$$

$$\phi_\gamma = \text{gamma flux at detector} \left(\frac{R}{h} \right)$$

$$\phi_n = \text{neutron flux at detector} \left(\frac{\text{neutron}}{\text{cm}^2 - \text{sec}} \right)$$

$$Z = \text{relative gamma effectiveness in detector} \left(\frac{\text{current}/R/h}{\text{current}/\text{neutron}/\text{cm}^2 - \text{sec}} \right)$$

The factor R in Equation 6-13 accounts for the signal fluctuations due to randomness in the detection process. The factor D accounts for the uncorrelated noise due to gamma ray interaction with the detector. The relative standard deviation in the cross spectral density, assuming independent errors⁽⁹⁾, is

$$\epsilon_{\text{CSD}}(\omega, \Delta\omega) = \frac{\sigma_{\text{CSD}}(\omega, \Delta\omega)}{S(\omega, \Delta\omega)} \approx \left\{ \frac{1.5 + 2 [Q(\omega)]^{-1} + [Q(\omega)]^{-2}}{\frac{\Delta\omega}{2} T} \right\}^{1/2} \quad (6-14)$$

where T is the measuring time in seconds. The relative standard deviation of the reactivity is found from Equation (6-12), again assuming Gaussian error propagation,

$$\frac{\sigma_\rho}{\rho} = \frac{1 - \rho}{2|\rho|} \left[\epsilon_{\text{CSD}}^2(\omega_1) + \epsilon_{\text{CSD}}^2(\omega_2) \right]^{1/2} \quad (6-15)$$

where ω_1 and ω_2 are the center frequencies of the two bandpass filters shown in Figure 6-1.

Equations (6-14) and (6-15) have been used to determine the achievable reactivity measurement accuracy under reasonable conditions in the FFTF backup design and to investigate the dependence of the accuracy limitation on the measurement system parameters.

6.4 ANALYSIS OF STATISTICAL UNCERTAINTY IN MEASURED REACTIVITY FOR FFTF BACKUP DESIGN, NO GAMMA BACKGROUND

A simple FFTF backup design schematic core arrangement and nominal dimensions are shown in Figure 6-3. The statistical uncertainty in the measured reactivity was determined for detectors moved along the axial and radial lines shown in Figure 6-3. The following assumptions and data were used in the analysis:

Assumptions

1. The two detectors are identical and are placed next to each other so that the efficiencies are assumed to be equal.

FIGURE 6-3. FFTF BACKUP DESIGN CORE AND REFLECTOR ARRANGEMENT (cm)

- 2.. Each detector contains 1 gram of B^{10} .
3. The gamma ray interaction with the detectors is negligible, i. e., $D = 1.0$. (The effect of gamma ray interaction is shown later in Figure 6-8.)
4. Measuring time = 30 minutes

$$\frac{\nu(\nu - 1)}{\nu^2} = 0.815$$

$$R = 1.20$$

$$\beta = 0.0032 \text{ (This value was chosen before the work reported in Section IV for the 2 zone core was completed, giving } \beta = 0.0028.)$$

$$\ell = 3 \times 10^{-7} \text{ sec}$$

$$\omega_1 = 3\omega_0 \text{ where } \omega_0(\rho) = \frac{\beta}{\ell} (1 - \rho)$$

$$\omega_2 = 1/3 \omega_0$$

$$\Delta\omega_1/\omega_1 = \Delta\omega_2/\omega_2 = 0.5.$$

The detectors assumed for this analysis could be multiple plate Boron coated detectors ~1 inch in diameter and ~18 inches long. The type of detectors recommended for actual use will depend on the particular size and environment limitations and sensitivity to gamma flux that would be determined during a development program. A development program would also consider the use of pulse mode detectors and multiple detectors to determine the type of detection system that best meets the overall requirements in the specified environment.

The selection of channel frequencies and filter band widths indicated above does not represent an optimum choice but rather a reasonable estimate that will be modified as an optimum strategy for use of this technique is developed.

The detector efficiency was determined using the assumed detector composition (1 gm of B^{10} in each detector) and the flux and fission distributions from the FFTF backup nuclear design and shielding analyses. The average detector efficiency as a function of radial and axial position is shown in Figures 6-4 and 6-5. Since B^{10} is a $\frac{1}{V}$ absorber the detector efficiency curves in Figure 6-4 and 6-5 do not precisely follow the total flux distributions due to the spatial variation in neutron energy spectrum.

The statistical uncertainty in the measured reactivity obtained by using the data and assumptions stated above is shown in Figures 6-6 and 6-7 to be a function of detector location and reactivity. The uncertainties shown correspond to 2σ variations which represents a 95% confidence level. A measuring time of 30 minutes has been used in these calculations. This measuring time was chosen as a nominal estimate of the minimum time available to measure reactivity while a fuel bundle is being removed from the core area to storage and a new fuel bundle is being transferred to the core area. The system reactivity during refueling will be between 15\$ and 25\$, assuming most control rods are inserted, and the worth of a bundle will be less than 4\$ as reported in Section IV. Thus, the information in Figures 6-6 and 6-7 indicates, with respect to the statistical limitations of the method, subcriticality may be adequately (i.e., within much less than a fuel bundle worth) demonstrated during refueling with detectors located in the core, in radial reflector and support area, or in axial reflector and gas space. The actual uncertainty requirements will be determined by the strategy used and the additional information required in addition to the assurance of adequate subcriticality during refueling. This requirement when combined with the equipment uncertainty and the statistical uncertainty will define further limitations of the areas acceptable for detector location.

6.5 EFFECT OF GAMMA INTERFERENCE ON ACCURACY

The calculations described in section 6.4 assumed that the detection system is essentially insensitive to the gamma flux due to the detector design, compensating chambers, or shielding of the detector (e.g. many inches of lead). The results are therefore representative of the best results that could be achieved with this approach. From practical considerations such a perfect system has not been developed. In actual practice the gamma flux interference will degrade the statistical accuracy of the reactivity measurement. This occurs, because in a substantially subcritical reactor, the gamma flux near the core results from fission product decay and is completely uncorrelated with the neutron flux. Accordingly, the gamma flux contributes an uncorrelated noise signal to the detector output. The effect of this additional uncorrelated signal is shown in Figure 6-8. The gamma and neutron fluxes at about 10 hours after shutdown have been estimated from operating condition calculations. The increase in uncertainty becomes more

FIGURE 6-4. DETECTOR EFFICIENCY vs RADIAL DISTANCE

FIGURE 6-5. DETECTOR EFFICIENCY vs HEIGHT

FIGURE 6-6. REACTIVITY UNCERTAINTY vs RADIUS (1 gm B^{10} DETECTOR, FFTF BACKUP DESIGN)

FIGURE 6-7. REACTIVITY UNCERTAINTY vs HEIGHT (1 gm B^{10} DETECTOR, FFTF BACKUP DESIGN)

FIGURE 6-8. REACTIVITY UNCERTAINTY VERSUS REACTIVITY FOR SEVERAL DETECTORS
 (DETECTORS WITH 1 gm B¹⁰ AT RADIAL REFLECTOR-SUPPORT
 STRUCTURE BOUNDARY)

pronounced as the system becomes more subcritical since the neutron flux is approximately inversely proportional to reactivity. For this example it has been assumed that the two detectors, each containing one gram of B^{10} , are located at the radial reflector-support structure boundary ($r \approx 70$ cm). The detectors (and required associated shielding) may be enclosed in a stainless steel capsule to prevent reactions with the sodium. This capsule could be inserted into a vacant channel of the outer row of reflector bundles at the start of refueling operations. The gamma flux from the core at the detector location is approximately 5×10^5 R/h with no detector shielding. The gamma flux from the radioactive sodium (assuming 50 volume percent of the reflector and radial core shielding region is sodium) is approximately 1.25×10^5 R/h. With no detector shielding the total gamma flux at the detector is 6.5×10^5 R/h. Each inch of lead shielding around the detector reduces the gamma flux by a factor of ten. Thus, the gamma flux at the detector is approximately 6.25×10^5 R/h with no shielding, 6.25×10^4 R/h with one inch of lead, and 6.25×10^3 R/h with two inches of lead.

Two types of detectors have been considered, an "optimized"⁽¹⁰⁾ detector with a relative gamma effectiveness in the detector, $Z \approx 2 \frac{\text{neutron/cm}^2\text{-sec}}{\text{R/h}}$ (see Equation 6-13) and a "conventional" detector with the same gamma to neutron sensitivity, $Z \approx 20$, as the Reuter Stokes RSN-5-S Boron Trifluoride detector. For each detector, a shielded and an unshielded condition have been calculated. The shielded condition uses the equivalent of one inch of solid lead shielding around each detector. Note that two inches of lead shielding on the conventional detector is equivalent to the optimized detector with one inch of lead. The detector design would consider the ambient sodium temperature during refueling, 400–500 °F, the gamma induced heating of lead, the melting point of lead (if lead were used), and the allowable detector temperature (300°C). Figure 6-8 illustrates that the gamma interference severely affects the measurement uncertainty. In fact it renders the unshielded "conventional" detector useless under the assumed conditions.

The shielded conventional detector and unshielded optimized detector lead to uncertainties much larger than those with no gamma interference. However, these two systems appear to reliably indicate whether or not the system is subcritical by at least the maximum bundle worth. Although these systems do not provide all the possible benefits of noise analysis, they do perform the basic safeguards function, namely to assure sufficient subcriticality during refueling.

The shielded optimized detector provides more information but is still considerably less certain than a gamma insensitive system.

SECTION VII

VESSEL AND HEAD DESIGN

7.1 INTRODUCTION

Consistent with program emphasis on mechanical design areas agreed to be of prime importance, work has continued to result in a more detailed conceptual design of the FFTF backup design head and vessel interface. The components included in this work are the vessel flange area including the vessel support and DBA hold-down facilities, and the vessel head, the refueling plug, and other head and plug penetrations.

Since the backup reactor and head design are to be compatible with the Reference FFTF vessel, the work related to the vessel is directed toward the head-vessel interface area. In section 7.3, upper vessel and flange preliminary temperature and stress analyses are described. The general concept of the backup head design is to provide an upper structural region and a lower thermal gradient and "sodium vapor trap" type layer for each of the head components. The structural region is maintained near ambient temperature ($<200^{\circ}\text{F}$) by cooling coils located near the lower edge of the structure. The temperature gradient between the 1200°F design temperature sodium-free surface and the structure is provided in the vessel and head components by the insulating properties of the lower structure.

The Reference plant designers are considering both "cold" and "hot" (structure) head designs. The Backup Designers prefer the "cold" head concept, because it minimizes the possibility of radioactive sodium entering the head shielding (structure) region, requires no extra plug cool-down time for rapid core access, and because the improved lower temperature properties of steel require less bulky head structures. As reported previously, the major part of the structural requirements in the head components is due to maintaining integrity during the effects of a Design Basis Accident (DBA).

7.2 VESSEL HEAD DESIGN FEATURES

The vessel head functional requirements include:

1. Containment of primary sodium and vapor (except during DBA).
2. Adequate nuclear shielding for on-line access to the head.
3. Rapid access to driver fuel and in-core experiments.
4. Access to reactor components including: control, reflector, core clamping, and shield rods; closed loop branch piping; fuel leak location detection tubes and in-vessel irradiation facilities.
5. Heat removal capability from head, vessel, and penetrations.

6. Adjustment for lining up center-line of head and core after plant operation.
7. Adequate support for the control rod drives, access, support, and viewing port gear, and the refueling plug. The refueling plug supports the driver fuel instrumentation and rack, the backup fuel holddown plate, the closed loop reactor tube protrusion and cover, and the backup scram control drives.
8. Minimum DBA effects by providing: energy absorption below the head and in stretch head-holddown bolts; equipment designed to minimize sodium (water) hammer on the head and to maintain integrity under all conditions.

The following paragraphs, numbered and related to the respective items in the functional requirements, indicate the work accomplished, the observations and conclusions generated, and the work to be done to complete this conceptual design.

1. The head structure contains the sodium and vapor in the vessel. The DBA loading of the head appears to be the most significant loading for the structure as discussed in this section and in Appendix C. A multiple head planned for this closure is described later in this section. The governing structural loading is the inertia load due to slowing of the head velocity by the holddown bolts following the DBA impulse. The closure geometry may result in some local yielding under this accident loading, but gross yielding or failure is not to be expected. The shielding mass for the closed loop test piping has not yet been factored into the head analysis. The central refueling plug and the annular plug are analyzed as solid pierced plates (the thickness required for shielding rather than strength, governs). These elements are simply supported to each other and to the outer portion of the head by means of shear keys. The shear ties are designed either to support different impulses to the components from the DBA or to support inertia loads by using the holddown bolts to retard initial velocity. The outermost ring section of the head (relatively permanent in location) is a box girder ring built up of plates to form a torsion box, a beam section with bulkheads for shear distribution. The tiedown is by perimeter bolts to the building structure as reported in the Third Quarterly (GEAP-5614) and Fourth Quarterly (GEAP-5650). The loading employed in the analysis was 850 psi, equivalent to the maximum inertial load imposed by the restraint of the holddown bolts. The inner plug and ring were analyzed by conventional flat plate formulas with equivalent properties of pierced plate. Moments and shears in the outer structure were by computer solution of a variable thickness; solid plate equilibrated to the double plate geometry. The stresses in the inner plug and ring are much below the ASME Boiler and Pressure Vessel (Section III), the limits with the thicknesses determined by shielding requirements. The outer ring is sized for yield strength at 200 °F.
2. Biological shielding analyses and requirements for the head were presented in Section V of this report and were discussed in the last quarterly report. The shield design criteria for the backup head continues to provide adequate protection for short time personnel having access to the head during or shortly after operating periods.
3. As reported previously, rapid access to the driver fuel and in-core experimental facilities is provided by a 4-foot diameter refueling plug. Considerable attention has

been and will continue to be devoted to accommodating the various refueling plug penetrations for experimental facilities (open and closed loops), control safety rod drives, sodium fuel leak detection location lines, and driver fuel instrumentation. As described in the figures, the present plug structure consists of a solid plate with the thickness determined by shielding requirements and perforated as required for the many test and driver fuel instrumentation lead connections.

4. Similar to the refueling plug, the annular reflector plug structure has been changed to solid construction. The shielding and structural requirements and penetrations have not changed this quarter.
5. There were no changes in the head cooling concepts this quarter. A review of system requirements and updated heat loads is scheduled for the next quarter.
6. Head position adjustment to line up the center line of the head and core assembly after operation remains unchanged.
7. Structural analysis of the head under DBA conditions was described in item 1. Normal structural requirements and conditions were reported last quarter and have not changed.
8. See Item 1 and Appendix C. The head hold-down stretch bolt requirements have not changed but the head structure has been modified to provide more adequate restraint during the DBA and to incorporate the unwieldy gusset structure within the box type head structure. More details are described in the next section.

Figures 7-1, 7-2, 7-3, and 7-4 describe the recent vessel head configurations and implement the text in Section VII. Significant items in each figure are listed below:

Figure 7-1: Plan View of the Top of the Head

- Note the biological shield covering the six groups of branch piping and sodium fuel leak detector lines. The shields weigh almost as much as the head structure.
- Dotted lines indicate typical locations of the closed loop branch piping and the sodium sample lines. The branch piping is separated below the reflector plug to implement pipe support when the reactor tube is disconnected. Shield holddown structures are not shown.
- Note the number and diversity of mechanical operators and information lead connectors on the central refueling plug (~4 ft diameter) and the annular reflector plug (~7 ft diameter).
- Twelve plug access holes in the reflector plug are used for core tightening and general access.
- Six large access holes in the head are for core tightening and auxiliary purposes such as lighting and viewing.

FOR 15" 6" OD VESSEL
54" OD. CTR. HEAD
52" DIA. REFUELING ACCESS

FIGURE 7-1. VESSEL HEADS

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FIGURE 7-2. ELEVATION - VESSEL HEADS

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FIGURE 7-3. CONCEPTUAL - VESSEL HEADS

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PRIMARY & SECONDARY NaK
COOLANT LOOPS

FIGURE 7-4. CONCEPTUAL - VESSEL HEADS

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- The twelve largest circles near the branch piping shields are spring hangers for the piping and do not penetrate the head.
- The holddown bolt locations are near the periphery of the head.
- The circumferential broken line at about 3/4 of the diameter indicates the vessel diameter.

Figure 7-2: Elevation A-A of the Head—Typical Penetrations

- This head structure does not show the gussets required between the holddown bolts and the head. See the right side of Figure 7-3 for a box structure alternate for the gussets. Apparent void areas in the structure are filled with appropriate density shot.
- Four (4) inches of insulation and eleven (11) inches of lead shielding are shown around the branch piping above the head. The branch piping penetration has not been completed.
- Plug penetrations include control drive rods, open loop stalks, in-core control drives, closed loops and driver fuel instrumentation leads. The core tightening and access penetration is shown on the right side of the head.
- Below the head structure are four layers of energy absorbing insulation, and below this, on the head, is the water-hammer attenuator. The coolant to structure temperature gradient occurs across the insulation layers.
- The shear rings that transmit DBA impact energy from the plugs to the head are shown on either side of the control drive rod penetration near the upper plate. They are located as high as possible for convenient access or maintenance.

Figure 7-3: Elevation B-B of the Head—Sodium Sample Lines

- Note location and route of sodium sample lines from the driver fuel elements. Fluid jump connectors are utilized in both plug interfaces. All lines are disconnected when the head is removed.
- The structure on the right side has been modified to show a box type structure. No gussets are required with this structure. The space within the structure is filled with lower density shot than the left-side design concept.
- The refueling and reflector plug structures are shown as solid steel. The cooling ducts near the bottom of the structure are recessed in the structure. The 18-inch thickness is required for bulk shielding requirements.

Figure 7-4: Head Plan View at -18 inch Elevation

- This section shows a conceptual layout of the plug cooling coils. A redundant system is shown in each plug. Head cooling is not shown.

- Plug penetration sizes are apparent. Component identification is the same as shown in Figure 7-1.
- Location of the two branch pipes for each closed loop and the sample lines are shown straddling the $2/3$ diameter circumference line. Note proximity of vessel and head circumferences.

7.3 VESSEL TEMPERATURE AND STRESS DISTRIBUTIONS NEAR THE HEAD

Steady state and transient thermal stresses in the upper section of the vessel have been re-examined. These analyses are based on the cool head structure concept which requires the temperature gradient ($\sim 1000^\circ\text{F}$) between the sodium pool and the ambient temperature vessel flange to be distributed along the upper 30 inches of the vessel. Prior stress analysis has indicated that with about a one-inch thickness of vessel wall and a reasonable distance between sodium surface and the head structure (~ 30 inches), the steady state gradient between a cool closure and the hot sodium pool will not excessively load the wall. Thermal analysis of the wall temperature and subsequent stress analysis has indicated the need for additional analysis of this region.

In addition, the transient response due to scram or decrease in outlet temperature has been examined. The vessel outlet nozzles and the fuel assembly outlets are about 10 feet below the sodium surface; therefore, for this analysis it was assumed that the core and vessel outlet sodium temperature drops to inlet temperature quickly and then the conductive transfer of heat continues to decrease the upper plenum sodium and vessel temperatures. The initial stratification of the sodium pool would rule out natural convection, and the vertical component of forced convection would be quickly suppressed. Sharp slope change of the gradient is evident near the outlet nozzle, as would be expected, and in this area the transient will be a constraint on the vessel design and thermal shielding whether the head structure is hot or cool. However, the initial analysis indicates that it requires more than two hours to affect the temperature at the top of the pool and the effect then is to moderate the change of slope of the temperature gradients at the pool surface. It appears that the transient response due to scram will not be significant at or above the sodium level in the vessel. Study of the start-up transient response has been initiated, and it is expected that natural convection will produce much quicker changes in temperature at the pool surface and may produce significant change of slope of the gradients in the upper level of the vessel, dependent upon heat-up rate. Flange design and steady state and scram transient temperatures for the vessel wall are plotted in Figures 7-5, 7-6 and 7-7. The combined stress intensities of critical points are superimposed on the temperature plots to indicate the severity of the gradient loading. From evidence available at this time, it seems feasible to cool the vessel closure and provide space above the pool surface for the temperature transition.

7.4 VESSEL HEIGHT CONSIDERATIONS

Vessel and Head interface descriptions were updated and issued in Volume I of the Preliminary Draft for the FFTF Backup Design Interface Definition report. The vessel definition emphasized the design requirements for the vessel support, flange and coolant service nozzles as well as the length (or depth) of the major components within the vessel. The vessel could have been over eight feet shorter if refueling or driver assemblies under the closed-loop-branch

FIGURE 7-5. VESSEL FLANGE DETAILS

FIGURE 7-6. STEADY STATE TEMPERATURE GRADIENT IN THE VESSEL NEAR THE FLANGE

FIGURE 7-7. TRANSIENT TEMPERATURE GRADIENT IN VESSEL BETWEEN SODIUM OUTLET AND FLANGE AFTER SCRAM

piping had been provided when the involved closed loop reactor tube was removed from the vessel. This 20% saving on vessel length appeared warranted. To fully utilize the advantages of the open head refueling (and top of the reactor viewing) concept of the backup design, facilities for safely lowering the sodium level in the vessel are required.

As both the reference and backup reactor designs are being developed it appears that the backup design has the same or slightly smaller vessel diameter requirements; thus diametrical inconsistencies can be filled when the backup design is installed. At this stage in conceptual development, the depth of the vessel (or elevation of the core support plate) and other pertinent component elevations should be noted.

Figures 7-8 and 7-9 indicate the maximum (deepest) and minimum length vessels, respectively, for the backup design. On these drawings the vessel diameter, bottom head, molten fuel containment high pressure plenum (except the grid plates), and the inlet and outlet coolant nozzles are identical to PNL drawing SK-3-14332, Revision 0. The "maximum" length vessel is 8 feet, which is 9 inches deeper than the minimum depth vessel. This additional depth is used to permit driver fuel (and grapple) movement beneath the closed loop branch piping and to store more sodium above the core in case of a casualty to the primary coolant system. The "minimum" length vessel provides only two feet of fuel submergence during refueling of the driver fuel. The shorter vessel, in addition to being smaller and probably requiring a smaller containment building, requires shorter:

- Extension rods on the control rod drives,
- Closed loop lead stalks and coolant system,
- Driver fuel instrument leads and structure,
- Operators for core clamping devices,
- Or less seismic restraint,
- Or improved visibility of reactor components.

For these reasons a short vessel design will be pursued in the continuing conceptual design periods.

The following tentative reference elevations are noted on both drawings:

- Vessel flange face (elevation datum)
- Vessel flange connection to vessel (-1 foot 6 inches) (limit of structure temperature)
- Cover gas and sodium overflow pipes*
- Sodium level (maximum height of outlet temperature)
- Inspection drain**
- Maximum coolant outlet nozzle height (2 foot submergence during inspection)
- Bottom of core elevation (reference design datum)
- Reactor support plate level (core datum)

* A two-duct multiple nozzle gravity overflow and extended cover gas reservoir system is recommended to provide constant sodium level in the vessel. The overflow sodium will flow to the drain tank. A service refill pump is required to return some sodium to the vessel. Actually the nozzles should be located at a slightly lower level than shown with the sodium overflowing into a wier box.

** The inspection drain can be provided as a dip tube with the actual nozzle near the top of the vessel. If these lines penetrate the head, additional shielding and space will be required above and near the head. This drain is sized to lower the sodium level in the vessel to the noted level in about one hour. Inspection draining and observation may be required each time before the refueling plug is replaced.

GEAP-5604

FIGURE 7-8. MAXIMUM LENGTH VESSEL

FIGURE 7-9. MINIMUM LENGTH VESSEL

SECTION VIII

REFUELING AND REACTOR SERVICING

Conceptual design of the refueling grapple was started. Major design requirements of this grapple and the associated tube hoist include the following: 1) Up to 1000 pounds lift force capability, 2) Holddown of adjacent core elements during initial withdrawal motion, 3) Rotational positioning of grappled component, 4) Position readout for all motions, and 5) Force readout for lift and holddown operations.

The amount of offset to be provided has not yet been finalized. The minimum offset is that required to handle driver fuel elements adjacent to the experimental facility stalks; whereas the maximum offset to be considered would enable grappling those driver fuel elements beneath the experimental facility branch piping. Minimizing the offset enhances maneuverability during handling operations. Providing adequate offset for under-branch-pipe handling operations results in greater versatility. However, vertical clear space between branch piping and driver fuel handles must also be provided for under-branch-pipe handling operations. Therefore a trade-off study is required to finalize the offset requirements.

SECTION IX

CIRCUMFERENTIAL TEMPERATURE
DISTRIBUTION FOR CORNER AND SIDE FUEL PINS

9.1 SUMMARY

The theoretical background and numerical methods being used in a digital computer code (now in the debugging stage) to be used in the determination of the circumferential temperature distribution for the outer row of fuel pins in the FFTF fuel subassembly are described.

Emphasis is placed on the development of the velocity profile around a corner pin. The computer code being developed to perform this analysis will with slight modifications be applicable for analysis of the fuel pins along the channel wall. The geometry allowed in the computer model is restricted to a multiple fuel-pin triangular array.

9.2 INTRODUCTION

The FFTF Backup Design and the reference design call for multiple fuel pins in triangular arrays within hexagonal channels. Figure 9-1 shows a schematic plan view of a Backup Design bundle.

A considerable amount of thermal hydraulic analysis has been performed on interior pins by considering them to be in an infinite array, notably the analysis done by Bender, Switick and Field.⁽¹¹⁾ The circumferential velocity distribution and associated eddy diffusivities have been analyzed by Nijssing, Gargantini, and Eifler.⁽¹²⁾

In comparison, very little analytical or experimental work has been performed on the outer row of fuel pins. The outer row of fuel pins is of particular interest due to the nonsymmetrical cooling which exists. The presence of the channel on one side of the pins will disturb the velocity profile and will change the heat transfer through circumferential heat transfer in the wall. Nonsymmetrical cooling also results because, the flow area between the fuel pin and channel wall is normally larger than the flow area on the interior side of the fuel pins.

The nonsymmetrical cooling will cause a circumferential temperature distribution in the fuel pin. This nonuniform temperature distribution introduces the following possibilities:

1. Thermal bowing can occur if the fuel pins are loosely supported.
2. High stresses can occur at the points of support if the fuel pins are rigidly supported.
3. High circumferential thermal stresses may develop.

The purposes for developing this digital code is to determine the temperature distribution for a corner fuel pin in a triangular array. The corner fuel pin was chosen since its temperature distribution should be more severe than a fuel pin along one of the hexagonal flats. The computer code which is being developed will also be applicable for analyzing a side rod by replacing the geometry subroutines.

PITCH-DIAMETER RATIO OF 1.20
PIN DIAMETER OF 0.250 in.

FIGURE 9-1. TYPICAL 127-PIN FUEL BUNDLE

The temperature distribution analysis can be done in two steps. First, it will be assumed that momentum and energy equations are separable (coolant properties will be evaluated at the coolant bulk temperature). The resulting velocity distribution will be based on bulk fluid properties. Secondly these results will be used in a general heat transfer code, such as THTD, ⁽¹³⁾ to generate the circumferential pin temperature profile.

The circumferential variation of the velocity and circumferential eddy diffusivity for momentum will be determined using a modification of the procedure of Nijssing, Gangantini, and Eifler. This procedure was developed for the case of a rod in an infinite array, but by modifying the equations to include the channel wall, it can be used for corner or side rods. This procedure will yield values for the local mean velocity and average circumferential eddy diffusivity for momentum as a function of circumferential position.

The radial variation of the velocity and radial eddy diffusivity for momentum will be based upon an annular model proposed by S. Levy. ⁽¹⁴⁾ By using this model with the average velocity

equal to the local mean velocity (from circumferential analysis) and considering each circumferential segment as an equivalent annulus, the radial variations can be determined.

The superposition of the results of these two procedures will yield a complete nodal velocity field and each nodal position will have circumferential and radial components of the eddy diffusivity for momentum associated with it.

The accuracy of this approach will depend upon several factors:

1. Number of circumferential segments: This will directly effect the accuracy of the local mean velocities distribution and average circumferential diffusivities. It will also determine the amount of deviation from an annular section, effecting the radial distributions.
2. Number of radial segments: Since the annular solution is not a nodal solution but is a continuous function, the number of radial segments will not have a large bearing on the results. It will, however, effect the heat transfer portion significantly.

Therefore, from an accuracy standpoint a large number of radial and circumferential segments is desirable.

The use of an average value for the circumferential eddy diffusivity (for each radial position) is not expected to cause significant errors. Since the pitch-to-diameter ratios under consideration are in the range of 1.20 - 1.30, the use of an annular model at each circumferential position is not expected to cause large errors.

The assumption of superposition of the radial and circumferential solutions has been examined. Although some inaccuracy is expected, the radial distribution is of secondary importance to the heat transfer analysis. The ratio of eddy diffusivities (circumferential/radial) should be greater than unity. Thus, heat transfer in the circumferential direction should predominate and the radial distribution will have only a second order effect on the circumferential transfer. Since the circumferential temperature distribution in the cladding is the desired result, small errors in the radial distribution can be tolerated.

The errors which result from assuming no circumferential or radial velocity components (unidirectional velocity) and neglecting mixing from the spiral wire spacers should be small. For wire wrap pitches, of the order of twelve inches, the flow direction can be considered vertical without introducing significant error. The amount of gross mixing should also be small. Nomenclature for the derivations appears at the end of this Section.

9.3 VELOCITY DISTRIBUTION DERIVATIONS

9.3.1 Assumptions

The major assumptions involved in the derivation of the velocity distribution are as follows:

1. Fully developed, single phase, steady-state turbulent flow.
2. Coolant is incompressible.

3. No momentum flux across the boundaries of the channel.
4. Temperature differentials are not severe so that material properties may be evaluated at the bulk fluid temperature and considered constant across the flow cross section.

Additional assumptions will be required in the analysis and will be discussed as they appear.

9.3.2 Equation of Motion

Since there is no momentum flux across the boundaries and the flow is incompressible, the radial and circumferential velocities must be zero,

$$u_r = u_\phi = 0 \quad (9-1)$$

where u is the time averaged velocity.

The equation of continuity in cylindrical coordinates is:

$$\frac{1}{r} \frac{\partial}{\partial r} (r u_r) + \frac{1}{r} \frac{\partial u_\phi}{\partial \phi} + \frac{\partial u_z}{\partial z} = 0. \quad (9-2)$$

Application of equation (9-1) reduces equation (9-2) to,

$$\frac{\partial u_z}{\partial z} = 0. \quad (9-3)$$

which implies that u_z is constant in the axial direction:

$$u_z = u_z(r, \phi) \quad (9-4)$$

From the r, ϕ components of the momentum equation,

$$P \neq P(r, \phi). \quad (9-5)$$

Therefore, the axial pressure gradient, $\partial P / \partial z$, is constant across the flow section.

The governing flow equation thus becomes the z component of the momentum equation. If one assumes that the molecular and turbulent momentum transport effects are additive, the momentum equation becomes

$$(\nu + \epsilon_{mr}) \frac{\partial^2 u}{\partial r^2} + \frac{(\nu + \epsilon_{mr})}{r} \frac{\partial u}{\partial r} + \frac{(\nu + \epsilon_{m\phi})}{r^2} \frac{\partial^2 u}{\partial \phi^2} = \frac{g_c}{\rho} \frac{\partial P}{\partial z} \quad (9-6)$$

where

$$u = u_z(r, \phi)$$

The quantities ϵ_{mr} and $\epsilon_{m\phi}$ are the eddy diffusivities of momentum in the radial and circumferential directions, respectively.

The solution of Equation (9-6) for a nonsymmetrical geometry is very difficult and no methods have been developed at this time. The method used by Bender, Switick and Field⁽¹¹⁾ cannot be applied for the corner rod geometry (Figure 9-2) due to the presence of the channel wall. It is therefore necessary to look for a simpler approach to the problem.

The first and second terms in Equation (9-6) account for the transfer of momentum normal to the rod surface. Near the surface the transfer mechanism is largely molecular in nature and therefore the molecular coefficient ν is much larger than the eddy diffusivity. As the distance from the wall increases, the eddy diffusivity predominates. Therefore, the radial transfer rate is determined primarily by the conditions in the immediate vicinity of the surface.

FIGURE 9-2. TYPICAL CORNER FUEL PIN GEOMETRY

The third term in Equation (9-6) accounts for transfer of momentum around the rod periphery. Peripheral transfer will occur due to the variation in flow area (and thus velocity gradients) around the rod. Peripheral transfer is determined by the conditions in the central portion of the flow where molecular diffusivities are small. Experimental observations and analytical predictions have shown that $\epsilon_{m\phi}$ is larger than ϵ_{mr} .

Considering the foregoing characteristics of the transfer phenomena, the radial and circumferential transfer of momentum may be considered independently of each other.

9.3.3 Circumferential Velocity Distribution

The model to be considered is shown in Figure 9-3. This model is obtained by taking advantage of the symmetry in the entire geometry (Figure 9-2).

Consider a volume element $dA \cdot dz$ such as that shown in Figure 9-4. In this Figure, dA is the flow area for the region between the rod surface and the outer boundary for a differential segment $d\phi$. Let y_ϕ be the distance from the rod surface to the outer boundary at a given angle ϕ .

The momentum balance can be written in the form:

$$\left\{ \begin{array}{c} \text{rate of} \\ \text{momentum} \\ \text{accumulation} \end{array} \right\} = \left\{ \begin{array}{c} \text{rate of} \\ \text{momentum} \\ \text{in} \end{array} \right\} - \left\{ \begin{array}{c} \text{rate of} \\ \text{momentum} \\ \text{out} \end{array} \right\} + \left\{ \begin{array}{c} \text{sum of forces} \\ \text{acting on} \\ \text{system} \end{array} \right\}$$

The rate of momentum accumulation for a steady-state system is zero. Since the radial and circumferential velocities are zero and the flow is fully developed, the net rate of momentum by convection will also be zero. Therefore, the net change of momentum is due to molecular and eddy transport by the stresses on the bounding surfaces of the element.

Neglecting gravity effects, the forces acting on the system are the pressure forces C and D in Figure 9-4. Using the notation of Figure 9-4, the momentum balance equation becomes:

$$A + B - C + D + E - F = 0 \quad (9-7)$$

The momentum flux A can be expressed in terms of the shear stress along the rod surface by

$$A = \tau_{\phi 1} R d\phi dz \quad (9-8)$$

where:

$\tau_{\phi 1}$ = Shear stress at rod surface at position ϕ .

R = Radius of fuel rod.

In a similar manner

$$B = \alpha \tau_{\phi 2} d\ell dz \quad (9-9)$$

FIGURE 9-3. MODEL FOR VELOCITY DISTRIBUTION CALCULATIONS

FIGURE 9-4. MOMENTUM BALANCE ON ELEMENTAL VOLUME $dA \cdot dz$

where:

$$\begin{aligned}\tau_{\phi 2} &= \text{Shear stress at channel wall at position } \phi . \\ dl &= \text{Length of channel for differential angle } d\phi .\end{aligned}$$

The variable α serves the following purpose: When the outer boundary is the channel wall, (Regions I and II of Figure 9-3), α has a value of unity. When the outer boundary is a line of symmetry (Regions III-V in Figure 9-3), it carries the value of zero. Therefore, the term may be included in the general formulation and is valid for any circumferential position.

The pressure force C is simply

$$C = PdA \quad (9-10)$$

where:

P = Pressure.

In a similar manner

$$D = \left[P + \frac{dP}{dz} dz \right] dA . \quad (9-11)$$

The momentum flux E can be expressed in terms of the circumferential shear stress by

$$E = \tau_{\phi\phi} y_o dz \quad (9-12)$$

where:

$\tau_{\phi\phi}$ = Circumferential shear stress at position ϕ .

Assuming the molecular and eddy effects are additive:

$$\tau_{\phi\phi} = \frac{\rho}{g_c} (\nu + \epsilon_{m\phi}) \frac{1}{r} \frac{du}{d\phi} . \quad (9-13)$$

Since the circumferential transfer occurs predominately in the central region where molecular effects are small, one may assume an average value of $\epsilon_{m\phi}$ acting at a mean radius. One may also neglect the molecular diffusivity ν . This also requires that a mean velocity be considered for the differential segment.

Then:

$$\tau_{\phi\phi} = \frac{\rho}{g_c} \bar{\epsilon}_{m\phi} \frac{1}{r} \frac{du}{d\phi} . \quad (9-14)$$

Let the mean radius be defined by:

$$\bar{r} = R + y_o/2 \quad (9-15)$$

Therefore, Equation (9-12) can be expressed by:

$$E = \frac{\rho}{g_c} (\bar{\epsilon}_{m\phi}) \frac{y_o dz}{R + y_o/2} \frac{d\bar{u}}{d\phi} \quad (9-16)$$

Then in a similar manner:

$$F = \frac{\rho}{g_c} (\bar{\epsilon}_{m\phi}) \left(\frac{y_o dz}{R + y_o/2} \right) \frac{d\bar{u}}{d\phi} + \frac{\rho}{g_c} (\bar{\epsilon}_{m\phi}) dz \frac{d}{d\phi} \left(\frac{y_o}{R + y_o/2} \frac{d\bar{u}}{d\phi} \right) d\phi \quad (9-17)$$

Substitution of the preceding equations into equation (9-7) yields:

$$\begin{aligned} \tau_{\phi_1} R d\phi dz + \alpha \tau_{\phi_2} dl dz - P dA + \left[P + \frac{dP}{dz} dz \right] dA + \frac{\rho}{g_c} \bar{\epsilon}_{m\phi} \left(\frac{dz y_o}{R + y_o/2} \right) \frac{d\bar{u}}{d\phi} \\ - \left[\frac{\rho}{g_c} (\bar{\epsilon}_{m\phi}) \left(\frac{dz y_o}{R + y_o/2} \right) \frac{d\bar{u}}{d\phi} + \frac{\rho}{g_c} (\bar{\epsilon}_{m\phi}) dz \frac{d}{d\phi} \left(\frac{y_o}{R + y_o/2} \frac{d\bar{u}}{d\phi} \right) d\phi \right] = 0 \quad (9-18) \end{aligned}$$

Equation (9-18) can be reduced directly to:

$$\tau_{\phi_1} R d\phi + \alpha \tau_{\phi_2} dl + \frac{dP}{dz} dA - \frac{\rho}{g_c} \bar{\epsilon}_{m\phi} \frac{d}{d\phi} \left(\frac{y_o}{R + y_o/2} \frac{d\bar{u}}{d\phi} \right) d\phi = 0 \quad (9-19)$$

Dividing by $d\phi$:

$$\tau_{\phi_1} R + \alpha \tau_{\phi_2} \frac{dl}{d\phi} + \frac{dP}{dz} \frac{dA}{d\phi} - \frac{\rho}{g_c} \bar{\epsilon}_{m\phi} \frac{d}{d\phi} \left(\frac{y_o}{R + y_o/2} \frac{d\bar{u}}{d\phi} \right) = 0 \quad (9-20)$$

Consider a momentum balance on an elemental volume $\Delta A \cdot dz$ as shown in Figure 9-5. As in the elemental volume $dA \cdot dz$ the rate of accumulation of momentum and net rate of momentum by convection will be zero. Then the momentum balance becomes:

$$A' - B' + C' + D' = 0 \quad (9-21)$$

The pressure forces A' and B' are:

$$A' = -P \Delta A \quad (9-22)$$

FIGURE 9-5. FORCES ON ELEMENTAL VOLUME ΔA df

$$B' = - \left[P + \frac{dP}{dz} dz \right] \Delta A \quad (9-23)$$

The momentum flux C' can be expressed in terms of the average shear stress along the rod surface by

$$C' = \tau_{av1} \pi R dz \quad (9-24)$$

where:

τ_{av1} = Average shear stress at rod surface.

In a similar manner

$$D' = \tau_{av2} \ell_o dz \quad (9-25)$$

where:

τ_{av2} = Average shear stress at channel wall.

ℓ_o = Length of channel wall.

The momentum balance of Equation (9-21) is therefore;

$$-P\Delta A + \left[P + \frac{dP}{dz} dz \right] \Delta A + \tau_{av1} \pi R dz + \tau_{av2} \ell_o dz = 0 \quad (9-26)$$

which reduces to:

$$-\frac{dP}{dz} = \tau_{av1} \frac{\pi R}{\Delta A} + \tau_{av2} \frac{\ell_o}{\Delta A} \quad (9-27)$$

Upon substitution of Equation (9-27) for the pressure term in Equation (9-20), one obtains:

$$\tau_{\phi 1} R + \alpha \tau_{\phi 2} \frac{d\ell}{d\phi} - \left[\tau_{av1} \frac{\pi R}{\Delta A} + \tau_{av2} \frac{\ell_o}{\Delta A} \right] \frac{dA}{d\phi} = \frac{\rho}{g_c} \bar{\epsilon}_{m\phi} \frac{d}{d\phi} \left(\frac{y_o}{R + y_o/2} \frac{d\bar{u}}{d\phi} \right) \quad (9-28)$$

The continuity equation corresponding to the differential equation above is

$$u_b = \frac{1}{\Delta A} \int_0^{\Delta A} \bar{u} dA \quad (9-29)$$

The boundary conditions are that the momentum crossing the lines of symmetry at $\phi = 0$ and $\phi = \pi$ is zero:

$$\left. \frac{d\bar{u}}{d\phi} \right|_{\phi=0} = 0 \quad (9-30)$$

$$\left. \frac{d\bar{u}}{d\phi} \right|_{\phi=\pi} = 0 \quad (9-31)$$

The overall-effective diffusivity for lateral mass transfer in plane flow has been shown by Elder⁽¹⁵⁾ to be given by:

$$\frac{\epsilon_m}{\nu} = 0.0115 R_e^{7/8} \quad (9-32)$$

where:

ν = Kinematic viscosity of coolant.

The corresponding expression for $\bar{\epsilon}_{m\phi}$ in the nonuniform geometry being considered here is assumed to have the same form. The peripheral variation of $\bar{\epsilon}_{m\phi}$ is accounted for by the local mean Reynolds number. Then,

$$\frac{\bar{\epsilon}_{m\phi}}{\nu} = 0.0115 \left[\frac{\bar{u}}{u_b} \frac{y_e}{d_e} R_e \right]^{7/8} \quad (9-33)$$

where:

y_e = Hydraulic diameter of flow area dA .

d_e = Hydraulic diameter of flow area ΔA .

R_e = Reynolds number based on u_b and d_e .

For a uniform geometry (annulus), an effective shear stress may be defined using the Blasius formula for the friction factor. If this effective stress is τ_u , then

$$\tau_u = 0.1582 \frac{\rho}{g_c} u_b^{1.75} \frac{\nu^{0.23}}{d_e^{1.25}} \quad (9-34)$$

where τ_u is defined by

$$\tau_u = \frac{\tau_i A_i + \tau_o A_o}{A_T dx} \quad (9-35)$$

and where:

- τ_u = Effective shear stress for annulus.
- τ_i = Shear stress on inner surface on annulus.
- τ_o = Shear stress on outer surface of annulus.
- A_i = Area on which τ_i acts.
- A_o = Area on which τ_o acts.
- A = Total flow area.
- dx = Axial length of segment.

Assuming the local shear stresses for a nonuniform geometry are related to the local flow conditions in the same manner as in a uniform geometry,

$$\tau_\phi = 0.1582 \frac{\rho}{g_c} \bar{u}^{1.75} \frac{\nu^{0.25}}{y_e^{1.25}} \quad (9-36)$$

where

$$\tau_\phi = \frac{\tau_{\phi 1} A^*_1 + \tau_{\phi 2} A^*_2}{dA dx} \quad (9-37)$$

and

$$A^*_1 = R d\phi dx \quad (9-38)$$

$$A^*_2 = \alpha d\ell dx. \quad (9-39)$$

Therefore, Equation (9-37) reduces to:

$$\tau_\phi = \left[\tau_{\phi 1} R + \alpha \tau_{\phi 2} \right] \frac{d\phi}{dA} \quad (9-40)$$

Combining Equations (9-36) and (9-40) one obtains:

$$\left[\tau_{\phi 1} R + \alpha \tau_{\phi 2} \frac{d\ell}{d\phi} \right] = 0.1582 \frac{\rho}{g_c} \frac{dA}{d\phi} \bar{u}^{1.75} \frac{\nu^{0.25}}{y_e^{1.25}} \quad (9-41)$$

One may also define an effective stress for the entire flow section by

$$\tau_{av} = \frac{\tau_{av1} \Delta A^*_1 + \tau_{av2} \Delta A^*_2}{\Delta A \cdot dx} \quad (9-42)$$

where:

$$\Delta A^*_1 = \frac{1}{\pi} \int_0^{\pi} A^*_1 d\phi \quad (9-43)$$

$$\Delta A^*_2 = \frac{1}{\phi^*} \int_0^{\phi^*} A^*_2 d\phi \quad (9-44)$$

The average shear stress is also given by:

$$\tau_{av_1} = \frac{1}{\pi} \int_0^{\pi} \tau_{\phi_1} d\phi \quad (9-45)$$

$$\tau_{av_2} = \frac{1}{\phi^*} \int_0^{\phi^*} \tau_{\phi_2} d\phi \quad (9-46)$$

Equation (9-42) thus becomes:

$$\tau_{av} = \frac{1}{\Delta A dx} \left[\frac{1}{\pi} \int_0^{\pi} \tau_{\phi_1} A^*_1 d\phi + \frac{1}{\phi^*} \int_0^{\phi^*} \tau_{\phi_2} A^*_2 d\phi \right] \quad (9-47)$$

Using the symbol α to allow integration of second integral over entire range:

$$\tau_{av} = \frac{1}{\pi \Delta A dx} \int_0^{\pi} \left(\tau_{\phi_1} A^*_1 + \alpha \tau_{\phi_2} A^*_2 \right) d\phi \quad (9-48)$$

Substituting Equation (9-37)

$$\tau_{av} = \frac{1}{\pi \Delta A} \int_0^{\pi} \tau_{\phi} dA d\phi \quad (9-49)$$

and using Equation (9-36):

$$\tau_{av} = \frac{0.1582}{\pi \Delta A} \frac{\rho}{g_c} \int_0^{\pi} \bar{u}^{1.75} \frac{\nu^{0.25}}{y_e^{1.25}} dA d\phi \quad (9-50)$$

Where τ_{av} as given by Equation (9-42) can also be expressed by

$$\tau_{av} = \tau_{av1} \frac{\pi R}{\Delta A} + \tau_{av2} \frac{\ell_o}{\Delta A} \quad (9-51)$$

substitution for ΔA_1^* and ΔA_2^* . Therefore, by combining the two expressions, Equations (9-50 and 9-51), one obtains:

$$\tau_{av1} \frac{\pi R}{\Delta A} + \tau_{av2} \frac{\ell_o}{\Delta A} = \frac{0.1582}{\pi \Delta A} \frac{\rho}{g_c} \int_0^\pi \frac{\bar{u}^{1.75}}{y_e^{1.25}} \frac{\nu^{0.25}}{y_e^{1.25}} dA d\phi \quad (9-52)$$

Upon substitution of Equations (9-33, 9-41, and 9-52) into Equation (9-28) yields:

$$0.1582 \frac{\rho}{g_c} \frac{dA}{d\phi} \frac{\bar{u}^{1.75}}{y_e^{1.25}} \frac{\nu^{0.25}}{y_e^{1.25}} - \left[\frac{0.1582}{\pi \Delta A} \frac{\rho}{g_c} \int_0^\pi \frac{\bar{u}^{1.75}}{y_e^{1.25}} \frac{\nu^{0.25}}{y_e^{1.25}} dA d\phi \right] \\ \frac{dA}{d\phi} = 0.0115 \frac{\rho \nu}{g_c} \left[\frac{\bar{u}}{u_b} \frac{y_e}{d_e} R_e \right]^{7/8} \frac{d}{d\phi} \left(\frac{y_o}{R + y_o/2} \frac{d\bar{u}}{d\phi} \right) \quad (9-53)$$

Multiplying both sides by $\frac{g_c}{\rho} \frac{d_e^{1.25}}{\nu^{0.25} u_b^{1.75}}$, which are all constants:

$$0.1582 \frac{dA}{d\phi} \left[\left(\frac{\bar{u}}{u_b} \right)^{1.75} \left(\frac{d_e}{y_e} \right)^{1.25} - \frac{1}{\pi \Delta A} \int_0^\pi \left(\frac{\bar{u}}{u_b} \right)^{1.75} \left(\frac{d_e}{y_e} \right)^{1.25} dA d\phi \right] \\ = 0.0115 \frac{\nu^{0.75} d_e^{1.25}}{u_b^{0.75}} \left[\frac{\bar{u}}{u_b} \frac{y_e}{d_e} R_e \right]^{7/8} \frac{d}{d\phi} \left[\frac{y_o}{R + y_o/2} \frac{d}{d\phi} \left(\frac{\bar{u}}{u_b} \right) \right] \quad (9-54)$$

Let η be a dimensionless velocity defined by:

$$\eta = \frac{\bar{u}}{u_b} \quad (9-55)$$

Then Equation (9-53) becomes:

$$\begin{aligned}
& 0.1582 \frac{dA}{d\phi} \left[\eta^{1.75} \left(\frac{d_e}{y_e} \right)^{1.75} - \frac{1}{\pi \Delta A} \int_0^\pi \eta^{1.75} \left(\frac{d_e}{y_e} \right)^{1.25} dA d\phi \right] \\
& = 0.0115 \frac{\nu^{0.75} d_e^{1.25}}{u_b^{0.75}} \left(\eta \frac{y_e}{d_e} R_e \right)^{7/8} \frac{d}{d\phi} \left(\frac{y_o}{R + y_o/2} \frac{d\eta}{d\phi} \right) \quad (9-56)
\end{aligned}$$

Examining the right hand side:

$$\begin{aligned}
& \frac{\nu^{0.75} d_e^{1.25}}{u_b^{0.75}} \left(\eta \frac{y_e}{d_e} R_e \right)^{7/8} = \left(\frac{\nu}{u_b d_e} \right)^{0.75} d_e^2 \eta^{0.875} \frac{y_e^{0.875}}{d_e^{0.875}} R_e^{0.875} \quad (9-57) \\
& = R_e^{-0.75} \frac{d_e^2}{d_e^{0.875}} \eta^{0.875} y_e^{0.875} R_e^{0.875} \\
& = \eta^{0.875} d_e^2 \left(\frac{y_e}{d_e} \right)^{0.875} R_e^{0.125}
\end{aligned}$$

Therefore, Equation (9-56) can be reduced to:

$$\begin{aligned}
& \frac{dA}{d\phi} \left[\eta^{1.75} \left(\frac{d_e}{y_e} \right)^{1.25} - \frac{1}{\pi \Delta A} \int_0^\pi \left(\frac{d_e}{y_e} \right)^{1.25} \eta^{1.75} dA d\phi \right] \\
& = \frac{0.0115}{0.1582} \eta^{0.875} d_e^2 \left(\frac{y_e}{d_e} \right)^{0.875} R_e^{0.125} \frac{d}{d\phi} \left(\frac{y_o}{R + y_o/2} \frac{d\eta}{d\phi} \right) \quad (9-58)
\end{aligned}$$

Examining the derivative in the equation above:

$$\begin{aligned}
& \frac{d}{d\phi} \left(\frac{y_o}{R + y_o/2} \frac{d\eta}{d\phi} \right) = \frac{y_o}{R + y_o/2} \frac{d^2 \eta}{d\phi^2} + \frac{d\eta}{d\phi} \frac{d}{d\phi} \left(\frac{y_o}{R + y_o/2} \right) \quad (9-59) \\
& = \frac{y_o}{R + y_o/2} \frac{d^2 \eta}{d\phi^2} + \frac{R}{(R + y_o/2)^2} \frac{dy_o}{d\phi} \frac{d\eta}{d\phi}
\end{aligned}$$

Therefore, Equation (9-58) can be expanded to:

$$\begin{aligned} & \left[\eta^{1.75} \left(\frac{d_e}{y_e} \right)^{1.25} - \frac{1}{\pi \Delta A} \int_0^\pi \eta^{1.75} \left(\frac{d_e}{y_e} \right)^{1.25} dA d\phi \right] \\ &= \frac{0.0115}{0.1582} \eta^{0.875} d_e^2 \left(\frac{y_e}{d_e} \right)^{0.875} R_e^{0.125} \left[\frac{y_o}{R + y_o/2} \frac{d^2 \eta}{d\phi} + \frac{R}{(R + y_o/2)^2} \frac{dy_o}{d\phi} \frac{d\eta}{d\phi} \right] \end{aligned} \quad (9-60)$$

Rearranging:

$$\begin{aligned} & \frac{y_o}{R + y_o/2} \frac{d^2 \eta}{d\phi^2} + \frac{R}{(R + y_o/2)^2} \frac{dy_o}{d\phi} \frac{d\eta}{d\phi} = \frac{0.1582}{0.0115} \frac{\eta^{-0.875}}{d_e^2} \left(\frac{d_e}{y_e} \right)^{0.875} R_e^{0.125} \frac{dA}{d\phi} \\ & \left[\eta^{1.75} \left(\frac{d_e}{y_e} \right)^{1.25} - \frac{1}{\pi \Delta A} \int_0^\pi \eta^{1.75} \left(\frac{d_e}{y_e} \right)^{1.25} dA d\phi \right] \end{aligned} \quad (9-61)$$

$$\begin{aligned} & \frac{d^2 \eta}{d\phi^2} + \frac{R}{y_o (R + y_o/2)} \frac{dy_o}{d\phi} \frac{d\eta}{d\phi} - \frac{13.7565}{d_e^2} \left(\frac{d_e}{y_e} \right)^{0.875} R_e^{0.125} \frac{dA}{d\phi} \left[\eta^{0.875} \left(\frac{d_e}{y_e} \right)^{1.25} \right. \\ & \left. - \frac{1}{\eta^{0.875}} \cdot \frac{1}{\pi \Delta A} \int_0^\pi \eta^{1.25} \left(\frac{d_e}{y_e} \right)^{1.25} dA d\phi \right] \left(\frac{R + y_o/2}{y_o} \right) = 0 \end{aligned} \quad (9-62)$$

One may simplify the evaluation of Equation (9-62) by defining a ratio of shear stresses Q , such that Q is the ratio of the average shear stress for the nonuniform geometry to that for a uniform geometry:

$$Q = \frac{\tau_{av}}{\tau_u} \quad (9-63)$$

Using Equations (9-34 and 9-50), Q can be expressed by:

$$Q = \frac{1}{\pi \Delta A} \frac{d_e^{1.25}}{u_b^{1.75} \nu^{0.25}} \int_0^\pi \bar{u}^{1.75} \frac{\nu^{0.25}}{y_e^{1.25}} dA d\phi \quad (9-64)$$

or:

$$Q = \frac{1}{\pi \Delta A} \int_0^\pi \eta^{1.75} \left(\frac{d_e}{y_e} \right)^{1.25} dA d\phi \quad (9-65)$$

Equation (9-62) can therefore be expressed by:

$$\frac{d^2 \eta}{d\phi^2} + \frac{R}{y_o (R+y_o/2)} \frac{dy_o}{d\phi} \frac{d\eta}{d\phi} - \frac{13.7565}{d_e^2} \left(\frac{d_e}{y_e}\right)^{0.875} R_e^{0.125} \frac{dA}{d\phi} \left(\frac{R+y_o/2}{y_o}\right) \left[\eta^{0.875} \left(\frac{d_e}{y_e}\right)^{1.25} - \frac{Q}{\eta^{0.875}} \right] = 0 \quad (9-66)$$

The boundary conditions may be transformed directly to:

$$\left. \frac{d\eta}{d\phi} \right|_{\phi=0} = 0 \quad (9-67)$$

$$\left. \frac{d\eta}{d\phi} \right|_{\phi=\pi} = 0 \quad (9-68)$$

The continuity relation yields:

$$\int_0^{\Delta A} \eta \frac{dA}{\Delta A} = 1 \quad (9-69)$$

Solution of Equation (9-66) subject to the boundary conditions, continuity equation and shear stress ratio Q will yield a dimensionless velocity profile. This profile when multiplied by the bulk velocity u_b will give the circumferential distribution of the local mean velocity $\bar{u}$. The use of this distribution will be described in the following section.

The geometrical features such as ΔA , dA , d_e , y_e , y_o , $dy_o/d\phi$, ℓ_o and ϕ^* are derived in Appendix D.

9.4 RADIAL VELOCITY DISTRIBUTION

In Section 9.3, the methods for determining the circumferential distribution of the local mean velocity $\bar{u}$ were discussed. The results of that analysis can be used to determine the radial distribution of velocity and eddy diffusivity for momentum.

The method to be used is to consider each circumferential segment separately. Each circumferential segment is assumed to be adequately approximated by an annulus with mean velocity equal to the local mean velocity $\bar{u}$ for that segment from the circumferential analysis.

For the segments in Regions I and II (Figure 9-3), the annulus is assumed to be defined by an inner radius equal to the fuel pin radius and an outer radius equal to the radial distance from

the pin center to the channel wall at the midpoint of the segment. For the segments in Regions III-V, a fictitious annulus is defined such that the zero shear plane in this annulus is at the same radius as the average radius of the actual outer boundary.

The technique advanced by Levy, ⁽¹⁴⁾ which is a semi-empirical procedure, is then used for each equivalent annulus:

Levy's Annular Model

Consider fully developed turbulent flow in an annulus with outer radius R_o , inner radius R_i , and plane of zero shear at a radius of r_m . If one assumes that Reichardt's expression for the eddy diffusivity of momentum applies to the flow regions on both sides of the plane of zero shear, one can write:

$$\frac{\epsilon_r}{\nu} = \frac{k_o}{6} (R_o - r_m) \frac{\sqrt{\tau_{R_o} g / \rho}}{\nu} \left[1 - \left(\frac{r - r_m}{R_o - r_m} \right)^2 \right] \left[1 + 2 \left(\frac{r - r_m}{R_o - r_m} \right)^2 \right] \quad (9-70)$$

$$\frac{\epsilon_r}{\nu} = \frac{k_i}{6} (r_m - R_i) \frac{\sqrt{\tau_{R_i} g / \rho}}{\nu} \left[1 - \left(\frac{r_m - r}{r_m - R_i} \right)^2 \right] \left[1 + 2 \left(\frac{r_m - r}{r_m - R_i} \right)^2 \right] \quad (9-71)$$

where Equation (9-70) is valid when $r \geq r_m$ and Equation (9-71) is valid when $r \leq r_m$, and where:

- ϵ = Radial component of eddy diffusivity of momentum, ft^2/sec ,
- ν = Kinematic viscosity, ft^2/sec ,
- k = Nondimensional mixing length constant,
- τ_R = Wall shear stress, lb_f/ft^2 ,
- ρ = Fluid density, lb_m/ft^3 ,
- r = Radial position, ft .

Equating the diffusivities at the plane of zero shear ($r = r_m$), one obtains:

$$\frac{k_o}{k_i} = \frac{r_m - R_i}{R_o - r_m} \sqrt{\frac{\tau_{R_i}}{\tau_{R_o}}} \quad (9-72)$$

Assuming the pressure force acts uniformly across the annulus, by writing force balances on the inner and outer regions one can obtain:

$$\frac{\tau_{R_i}}{\tau_{R_o}} = \frac{R_o}{R_i} \left(\frac{r_m^2 - R_i^2}{R_o^2 - r_m^2} \right) \quad (9-73)$$

Substitution into Equation (9-72) then yields:

$$\frac{k_o}{k_i} = \left(\frac{r_m - R_i}{R_o - r_m} \right)^{3/2} \left(\frac{R_o}{R_i} \right)^{1/2} \left(\frac{r_m + R_i}{R_o + r_m} \right)^{1/2} \quad (9-74)$$

Performing a force balance on a slab, the shear stress at a position r can be determined to be

$$\tau_{r_i} = \tau_{R_i} \frac{R_i}{r} \frac{r_m^2 - r^2}{r_m^2 - R_i^2} \quad (r \leq r_m) \quad (9-75)$$

$$\tau_{r_o} = \tau_{R_o} \frac{R_o}{y} \frac{r^2 - r_m^2}{R_o^2 - r_m^2} \quad (r \geq r_m) \quad (9-76)$$

where:

τ_{r_i} = Shear stress in inner region at position r , lb_f/ft^2

τ_{r_o} = Shear stress in outer region at position r , lb_f/ft^2 .

But the shear stress can also be expressed by

$$\tau_r = \rho (\nu + \epsilon_r) \frac{du}{dy} \quad (9-77)$$

where:

u = Local velocity at position r , ft/sec

y = Distance from wall to position r , ft .

Combining Equations (9-70, 9-71, 9-75, 9-76 and 9-77) yields the following:

$$\tau_{R_i} \frac{R_i}{r} \frac{r_m^2 - r^2}{r_m^2 - R_i^2} = \frac{k_i}{6} (r_m - R_i) \frac{\sqrt{\tau_{R_i} g/\rho}}{\nu} \left[1 - \left(\frac{r_m - r}{r_m - R_i} \right)^2 \right] \left[1 + 2 \left(\frac{r_m - r}{r_m - R_i} \right)^2 \right] \frac{du}{dy} \quad (9-78)$$

$$\tau_{R_o} \frac{R_o}{r} \frac{r - r_m^2}{R_o^2 - r_m^2} = \frac{k_o}{6} (R_o - r_m) \frac{\sqrt{\tau_{R_o} g/\rho}}{\nu} \left[1 - \left(\frac{r - r_m}{R_o - r_m} \right)^2 \right] \left[1 + 2 \left(\frac{r - r_m}{R_o - r_m} \right)^2 \right] \frac{du}{dy} \quad (9-79)$$

Defining the following terms,

$$u_i^+ = \frac{u}{\sqrt{\tau_{R_i} g/\rho}} \quad (9-80)$$

$$u_o^+ = \frac{u}{\sqrt{\tau_{R_o} g/\rho}} \quad (9-80a)$$

$$y_i^+ = \frac{(r_m - r) \sqrt{\tau_{R_o} g/\rho}}{\nu} \quad (9-80b)$$

$$y_o^+ = \frac{(r - r_m) \sqrt{\tau_{R_o} g/\rho}}{\nu} \quad (9-80c)$$

$$\beta_i = \frac{r_m - r}{r_m - R_i} \quad (9-80d)$$

$$\beta_o = \frac{r - r_m}{R_o - r_m} \quad (9-80e)$$

$$S_i = \frac{r_m}{R_i} \quad (9-80f)$$

$$S_o = \frac{r_m}{R_o} \quad (9-80g)$$

Equations (9-78, 9-79), can be integrated to obtain the velocity distributions in the inner and outer regions:

$$u_i^+ = \frac{1}{k_i} \ln \left[1.5 y_i^+ \frac{1 + \beta_i}{1 + 2 \beta_i^2} \right] + \frac{2 S_i (1 - S_i)}{k_i (1 + S_i) (2 S_i - 1)} \ln \left(\frac{1 + \beta_i}{2} \right) \\ + \frac{1}{2} \frac{S_i (1 - S_i) (1 - 3 S_i)}{k_i (1 + S_i) \left[S_i^2 + \frac{1}{2} (1 - S_i)^2 \right]} \ln \left(\frac{1 + 2 \beta_i^2}{3} \right) + \frac{6}{k_i (1 + S_i)} \quad (9-81)$$

$$\begin{aligned}
& \frac{\ln \beta_i (1 - S_i) + S_i}{\left[\left(\frac{1 - S_i}{S_i} \right)^2 - 1 \right] \left[1 + 2 \left(\frac{S_i}{1 - S_i} \right)^2 \right]} + \frac{\sqrt{2}}{k_i} \frac{(1 - S_i) S_i}{1 + S_i} \left(\frac{\tan^{-1} \sqrt{2} - \tan^{-1} \beta_i \sqrt{2}}{S_i^2 + \frac{1}{2} (1 - S_i)^2} \right) \\
& + 14.84 - \frac{1}{k_i} \ln 42 \quad . \\
u_o^+ &= \frac{1}{k_o} \ln \left[1.5 y_o^+ + \frac{1 + \beta_o}{1 + 2 \beta_o} \right] + \frac{2 S_o (1 - S_o)}{k_o (1 + S_o) (2 S_o - 1)} \ln \left(\frac{1 + \beta_o}{2} \right) \\
& + \frac{1}{2} \frac{S_o (1 - S_o) (1 - 3 S_o)}{k_o (1 + S_o) \left[S_o^2 + \frac{1}{2} (1 - S_o)^2 \right]} \ln \left(\frac{1 + 2 \beta_o^2}{3} \right) + \frac{6}{k_o (1 + S_o)} \quad (9-82) \\
& \frac{\ln [\beta_o (1 - S_o) + S_o]}{\left[\left(\frac{1 - S_o}{S_o} \right)^2 - 1 \right] \left[1 + 2 \left(\frac{S_o}{1 - S_o} \right)^2 \right]} + \frac{\sqrt{2}}{k_o} \frac{(1 - S_o) S_o}{1 + S_o} \left(\frac{\tan^{-1} \sqrt{2} - \tan^{-1} \beta_o \sqrt{2}}{S_o^2 + \frac{1}{2} (1 - S_o)^2} \right) \\
& + 14.84 - \frac{1}{k_o} \ln 42 \quad .
\end{aligned}$$

The friction factor f can be defined in terms of the axial pressure gradient by:

$$\frac{-dp}{dz} = f \frac{2 \rho \bar{u}^2}{2 R_o \left(1 - \frac{R_i}{R_o} \right)} \quad (9-83)$$

But:

$$\begin{aligned}
\tau_{R_o} g/\rho &= \left(\frac{-dp}{dz} \right) \frac{R_o^2 - r_m^2}{2 R_o} \frac{g}{\rho} \quad (9-84) \\
&= \frac{g}{\rho} \left[\frac{2 R_o \bar{u} (1 - R_i/R_o)}{\nu} \right] \left[\frac{\nu}{2 R_o (1 - R_i/R_o)} \right]^2 \frac{f}{2} \frac{1 - \left(\frac{r_m}{R_o} \right)^2}{1 - \left(\frac{R_i}{R_o} \right)}
\end{aligned}$$

Defining the Reynolds number by:

$$N_{Re} = \frac{2 \bar{u} R_o \left(1 - \frac{R_i}{R_o}\right)}{\nu} \quad (9-85)$$

Then the shear stress at the outer wall is:

$$\frac{\sqrt{\tau_{R_o} g/\rho}}{\nu} = \frac{1}{2} \frac{N_{Re} \sqrt{f/2}}{R_o - r_m} \left\{ \frac{1 - \frac{r_m}{R_o}}{1 - \frac{R_i}{R_o}} \right\}^{3/2} \left(1 + \frac{r_m}{R_o}\right)^{1/2} \quad (9-86)$$

Using Equation (9-72) and Equation (9-86) yields the following expression for the shear stress at the inner wall.

$$\frac{\sqrt{\tau_{R_i} g/\rho}}{\nu} = \frac{k_o}{k_i} \frac{R_o - r_m}{r_m - R_i} \frac{\sqrt{\tau_{R_o} g/\rho}}{\nu} \quad (9-87)$$

In his paper, ⁽¹⁴⁾ Levy presents an iterative scheme for determining the radius of zero shear r_m . However, this procedure can become unstable if the estimate of r_m is close to, or equal to, $r_m = R_o/2$. An analysis of results using this model have shown that the zero shear radius can be predicted adequately by the expression: ⁽¹⁶⁾

$$r_m = R_i \frac{\left[1 + \left(\frac{R_i}{R_o}\right)^{-0.67}\right]}{\left[1 + \left(\frac{R_i}{R_o}\right)^{0.33}\right]} \quad (9-88)$$

Defining the friction factor by the expression of Osmaschin, ⁽¹⁷⁾ which matches the data of LeTourneau, Grimble and Zerbe: ⁽¹⁸⁾

$$\frac{f}{4} = \frac{1}{\left[1.82 \log_{10} (N_{Re}) - 1.64\right]^2} \quad (9-89)$$

The general method for evaluation of the radial profiles of velocity and eddy diffusivity for momentum (radial) is as follows:

- a. The Reynolds number is evaluated using Equation (9-85) with $\bar{u}$ from the circumferential analysis.
- b. The friction factor (f) and zero shear radius (r_m) are evaluated using Equations (9-88, 9-89).
- c. The inner and outer wall shear stresses are evaluated by Equations (9-74, 9-86, 9-87).
- d. The nondimensional parameters of Equations (9-80b through 9-80g) and the nondimensional velocities u_i^+ and u_o^+ are evaluated at various values of r using Equations (9-81, 9-82).
- e. The nondimensional eddy diffusivities are evaluated using Equations (9-70, 9-71).
- f. The velocities profiles and eddy diffusivities are then obtained by using Equations (9-80, 9-80a) for the velocities and by multiplying the nondimensional diffusivities by the kinematic viscosity.

9.5 CIRCUMFERENTIAL DIFFUSIVITY DISTRIBUTION

The eddy diffusivities of momentum in the radial and circumferential directions are assumed to act according to a mixing length model. Following the procedure of Bender, Switick and Field,⁽¹¹⁾ the following expressions can be written:

$$\epsilon_{mr} = \ell_c^2 \frac{\partial u}{\partial r} \quad (9-90)$$

$$\epsilon_{m\phi} = \ell_c^2 \frac{1}{r} \frac{\partial u}{\partial \phi} \quad (9-91)$$

where:

$$\ell_c = \text{Turbulent characteristic mixing length.}$$

One could attempt to obtain an expression for the mixing length using the method of Hooper.⁽¹⁹⁾ However, the complexities caused by the nonsymmetrical geometry make this infeasible. Since the radial diffusivity components are known, a simpler procedure is available.

Assuming that the mixing lengths in the radial and circumferential directions are equal, one may write:

$$\epsilon_{m\phi} = \frac{\epsilon_{mr}}{r} \frac{\partial u / \partial \phi}{\partial u / \partial r} \quad (9-92)$$

Therefore once the velocity profiles in the radial and circumferential directions and the radial components of diffusivity are known, the circumferential diffusivity components can be determined by Equation (9-92). The derivatives are evaluated by using a first order central difference scheme.

9.6 METHOD FOR CALCULATION OF HEAT TRANSFER INPUT PARAMETERS

9.6.1 Determination of Eddy Diffusivities of Heat

It is assumed that the heat transfer by turbulent mixing is proportional to the momentum transfer by turbulent mixing. Then,

$$\epsilon_H = \bar{\psi} \epsilon_M \quad (9-93)$$

where:

$\bar{\psi}$ = Constant of proportionality

ϵ_H = Eddy diffusivity of heat

ϵ_M = Eddy diffusivity of momentum ..

Using the correlation of Dwyer, (20) the constant of proportionality in the radial direction is:

$$\bar{\psi}_r = 1 - \frac{1.82}{P_R \left(\frac{\epsilon_{mr}}{\nu} \right)_{\max}^{1.4}} \quad (9-94)$$

where:

P_R = Prandtl number..

In a similar analogy, the constant of proportionality for the circumferential heat transfer is:

$$\bar{\psi}_\phi = \left[1 - \frac{1.82}{P_R \left(\frac{\epsilon_{m\phi}}{\nu} \right)_{\max}^{1.4}} \right] \quad (9-95)$$

Therefore the equations for the radial and eddy diffusivities for heat are:

$$\epsilon_{Hr} = \left[1 - \frac{1.82}{P_R \left(\frac{\epsilon_{mr}}{\nu} \right)_{\max}^{1.4}} \right] \epsilon_{mr} \quad (9-96)$$

$$\epsilon_{H\phi} = \left[1 - \frac{1.82}{P_R \left(\frac{\epsilon_{m\phi}}{\nu} \right)_{\max}^{1.4}} \right] \epsilon_{m\phi} \quad (9-97)$$

The maximum values of ϵ_{mr}/ν and $\epsilon_{m\phi}/\nu$ are determined from the calculated values from the velocity distribution analysis.

9.6.2 Effective Sodium Conductivity

An effective conductivity for the sodium coolant can be defined by:

$$K_{eff} = K_{viscous} + K_{turbulent} \quad (9-98)$$

Since the thermal diffusivity (α) is defined by

$$\alpha = \frac{K_{viscous}}{\rho C_p} \quad (9-99)$$

and defining $K_{turbulent}$ by

$$\epsilon_H = \frac{K_{turbulent}}{\rho C_p} \quad (9-100)$$

yields an expression for K_{eff} of:

$$K_{eff} = \rho C_p (\alpha + \epsilon_H) \quad (9-101)$$

Since the assumption of additive diffusivities is commonly used, this expression should be adequate.

9.6.3 Temperature Calculations

The calculations of temperature distributions will be accomplished by use of the generalized nodal heat transfer computer code "THTD." (13)

The input to "THTD" will be the nodal velocity field as well as an effective conductivity for both radial and circumferential directions for each node.

The "THTD" computer code is basically a two-dimensional, transient, and steady state heat transfer code. This code allows each node to have a different conductivity between adjacent nodes. This code does not allow for axial conduction; however, axial variations can be considered as the code allows for heat addition between axial segments.

At this time the "THTD" deck has not yet been set up. However, the basic geometry is as shown in Figure 9-6.

FIGURE 9-6. MODEL FOR TEMPERATURE DISTRIBUTION CALCULATIONS

9.7 NUMERICAL METHODS USED TO CALCULATE VELOCITY DISTRIBUTION

9.7.1 Circumferential Distribution

The details of the numerical methods presently being used to evaluate the circumferential velocity distribution will not be discussed as the code is still being debugged and changes in procedure may be necessary. However, a general view will be presented.

Normally one would take Equation (9-66), substitute a central difference (or comparable method) approximation to the derivatives and step through the profile. However, this procedure does not work well for this case. Since diffusivity is a second order perturbation of the zero diffusivity solution, diffusivity cannot be used to solve for the velocity due to numerical errors. Therefore another method has been adopted.

The zero diffusivity solution is artificially smoothed to assure there are no abrupt changes in the profile. This is then used as a first estimate of the true profile. The first and second derivatives are evaluated based on this profile using a second order central difference scheme (five point formula). The derivatives (numerical value) at each point are then substituted into Equation (9-66) and the equation is solved by iteration until a new profile is obtained. This profile is then used to generate a new series of derivatives. This procedure is repeated until continuity is satisfied by the last profile obtained.

9.8 NOMENCLATURE

ENGLISH SYMBOLSDESCRIPTIONSYMBOL

ΔA	Total flow area (ft ²)
C_p	Specific heat of coolant (Btu/lb _m - °F)
d	Diameter of fuel pin (ft)
dA	Flow area of circumferential segment (ft ²)
d_e	Hydraulic diameter of total section (ft)
dl	Differential length of channel wall
dz	Differential distance in axial direction
$d\phi$	Differential distance in circumferential direction
f	Fanning friction factor
g_c	Gravitational constant (lb _m -ft/lb _f -sec ²)
K_{eff}	Effective coolant conductivity (Btu/h-ft)
k_i	Nondimensional mixing length in inner region of annulus
k_o	Nondimensional mixing length in outer region of annulus
ℓ_c	Turbulent characteristic mixing length (ft)
ℓ_o	Length of channel wall (ft)
N_{Re}	Reynolds number for equivalent annulus
p	Pitch or rod to rod center distance (ft)
P	Pressure (psia)
P_R	Prandtl number of coolant
Q	Ratio of shear stresses - τ_{av}/τ_u
r	Radial position (ft)
r_m	Radius of zero shear (ft)
R	Radius of fuel pin (ft)
Re	Reynolds number of total section
R_i	Inner radius of annulus (ft)
R_o	Outer radius of annulus (ft)
S_i	Ratio of zero shear radius to inner radius of annulus
S_o	Ratio of zero shear radius to outer radius of annulus
u	Coolant velocity in axial direction (ft/sec)
$\bar{u}$	Local mean coolant velocity in circumferential segment (ft/sec)
u_b	Bulk coolant velocity (ft/sec)
u_1^+	Nondimensional coolant velocity in inner region of annulus
u_o^+	Nondimensional coolant velocity in outer region of annulus
u_r	Radial component of coolant velocity (ft/sec)

SYMBOLDESCRIPTION

u_{ϕ}	Circumferential component of coolant velocity (ft/sec)
u_z	Axial component of coolant velocity (ft/sec)
X^*	Distance from rod surface to channel wall at angle of 30° (ft)
y	Distance from rod surface to position r (ft)
y_e	Local equivalent hydraulic diameter (ft)
y_o	Distance from rod surface to outer boundary at position ϕ (ft)
y_i^+	Nondimensional distance from rod surface in annulus
y_o^+	Nondimensional distance from outer wall in annulus
z	Axial position

GREEK SYMBOLSSYMBOLDESCRIPTION

α	Zero when outer boundary is line of symmetry—1.0 when outer boundary is channel wall.
β_i	Defined by $r_m - r/r_m - R_i$
β_o	Defined by $r - r_m/R_o - r_m$
ϵ_{Hr}	Radial component of eddy diffusivity of heat (ft ² /sec)
$\epsilon_{H\phi}$	Circumferential component of eddy diffusivity of heat (ft ² /sec)
ϵ_{mr}	Radial component of eddy diffusivity of momentum (ft ² /sec)
$\epsilon_{m\phi}$	Circumferential component of eddy diffusivity of momentum (ft ² /sec)
η	Nondimensional ratio of local mean velocity to bulk velocity of coolant
ν	Kinematic viscosity of coolant (ft ² /sec)
ρ	Density of coolant (lb _m /ft ³)
τ_{av}	Shear stress averaged over rod surface and channel wall (lb _f /ft ²)
τ_{av1}	Shear stress averaged over rod surface (lb _f /ft ²)
τ_{av2}	Shear stress averaged over channel wall (lb _f /ft ²)
τ_{ri}	Shear stress at position r in inner region of annulus (lb _f /ft ²)
τ_{ro}	Shear stress at position r in outer region of annulus (lb _f /ft ²)
τ_{R_i}	Shear stress on inner surface of annulus (lb _f /ft ²)
τ_{R_o}	Shear stress on inner surface of annulus (lb _f /ft ²)
τ_u	Average shear stress in uniform geometry - annulus (lb _f /ft ²)
$\tau_{\phi 1}$	Local shear stress on rod surface (lb _f /ft ²)

<u>SYMBOL</u>	<u>DESCRIPTION</u>
$\tau_{\phi 2}$	Local shear stress on channel wall (lb_f/ft^2)
$\tau_{\phi\phi}$	Circumferential shear stress due to eddy diffusivity (lb_f/ft^2)
ϕ	Circumferential position
ϕ^*	Circumferential position at which channel wall ends (radians)
$\bar{\psi}_r$	Ratio of eddy diffusivity of heat to eddy diffusivity of momentum - radial component
$\bar{\psi}_\phi$	Ratio of eddy diffusivity of heat to eddy diffusivity of momentum - circumferential component

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APPENDIX A

SINGLE PISTON DECELERATION—ZERO SCRAM FORCENomenclature

ΔP	=	Pressure drop across the hydraulic orifice
C	=	Entrance and exit loss coefficient
f	=	Friction factor
L	=	Characteristic length in which the pressure drop is calculated
D_G	=	Equivalent hydraulic diameter of the orifice
$\dot{m}$	=	Displaced coolant flow rate
ρ	=	Sodium density
A_G	=	Area of orifice
v_f	=	Velocity of piston or control rod
A_p	=	Area of piston of the cushion
w	=	Total weight of control rod and upper extension rod
m	=	Total mass same as above
s	=	Incremental distance traversed by the control rod
v_f	=	Final velocity over incremental distance
v_0	=	Initial velocity of the beginning of increment
a	=	Acceleration of the control rod
A	=	Constant
F_{net}	=	Net force exerted by the pneumatic piston on the upper extension rod
$O.S.$	=	Offset length
W_e	=	Weight of upper extension rod

The pressure drop across the hydraulic snubber can be expressed as

$$\Delta P = \left(c + f \frac{L}{D_G} \right) \frac{\dot{m}^2}{2g_c \rho A_G^2} \quad (A-1)$$

The coolant flow, $\dot{m}$, is defined as the displaced coolant

$$\dot{m} = \rho V_p A_p \quad (A-2)$$

The hydraulic force exerted on the snubber piston is

$$F = \Delta P A_p \quad (A-3)$$

Combining Equations (A-1, A-2, and A-3) the following relationship is obtained

$$F = \left(c + f \frac{L}{D_G} \right) \rho \frac{V_p^2 A_p^3}{2g_c A_G^2} \quad (A-4)$$

which is the hydraulic forces exerted on the piston.

From the equations of linear motion

$$F - W = ma \quad (A-5)$$

$$v_f^2 = v_0^2 + 2as \quad (A-6)$$

Equating Equations (A-5) and (A-6), the resultant force of motion is

$$F = W \left(1 + \frac{v_0^2 - v_f^2}{2sg} \right) \quad (A-7)$$

which is the force required to decelerate the control rod.

The hydraulic force and the force of motion are equated to determine the velocity of the control rod. Equating Equations (A-4) and (A-7), rearranging, and defining

$$k = \left(c + f \frac{L}{D_G} \right)$$

$$V_p = \frac{v_0 + v_f}{2}$$

$$A = \frac{sgk\rho A_p^3}{4wg_c A_G^2}$$

and implicitly expressing in terms of v_f ,

$$v_f^2 (1 + A) + (2A v_0) v_f + \left[v_0^2 (A-1) - 2sg \right] = 0 \quad (A-8)$$

Transient analysis is approximated from Equation (A-8) by calculating the incremental velocity change for an incremental distance change.

APPENDIX B

SINGLE PISTON DECELERATION—SCRAM FORCE $\neq 0$

The hydraulic force exerted on the piston is Equation (A-4). The equation of linear motion is now

$$F - w + F_{\text{net}} = -ma \quad (B-1)$$

Combining Equations (B-1) and (A-6), and solving for the force,

$$F = \frac{v_0^2 - v_f^2}{2s} \left(\frac{w}{g} \right) + w - F_{\text{net}} \quad (B-2)$$

And again defining

$$V_p = \frac{v_0 + v_f}{2}$$

$$k = \left(c + f \frac{L}{D_G} \right)$$

$$A = \frac{ksg \rho A_p^3}{4g_c A_G^2}$$

and solving for v_f implicitly after equating Equations (A-4) and (B-2),

$$(1 + A) v_f^2 + (2Av_0)v_f + \left[v_0^2(A-1) - 2sg + \frac{2sg}{w} F_{\text{net}} \right] = 0 \quad (B-3)$$

Transient analysis is simulated by Equation (B-3) to obtain the velocity profile. The force is calculated by equation (B-2).

The moment exerted on the offset is determined by

$$M = 0.S. \left[W_e \left(1 - \frac{a}{g} \right) - F_{\text{net}} \right] \quad (B-4)$$

APPENDIX C

DBA CONTAINMENT

To design the containment system, the quantitative methods to establish the following are required:

1. Energy absorption of head hold down mechanism
2. Static strength of head hold down mechanism
3. Thickness of reactor vessel and additional blast cylinders
4. Energy absorption of bottom of vessel and backup structure
5. Post-DBA cooling requirements
6. Radiological release to Equipment and Refueling Cell atmospheres
7. Pressure in Equipment Cell Atmospheres

During the DBA transient, a significant fraction of the fuel in the core may become vaporized. The resulting expansion is the source of mechanical loading. The analytical method used assumes an ideal expansion (isentropic) of the fuel in computing the mechanical work available. The following distribution of energy is assumed:

1. Energy is stored in vaporized fuel prior to its expansion (excluding heat of vaporization and heat required to raise fuel to its vaporization point).
2. Mechanical energy is extracted by isentropic expansion of vaporized fuel.

For mechanical loading it has been the accepted approach to define an equivalent chemical explosion for the energy release. The deficiencies of such an approach are the rate of energy release may be overestimated for the shock wave strength and the incondensibility of the vapors formed may be overestimated for the pressure loadings following the shock wave. However, in lieu of an acceptable alternative, it is assumed that the "TNT Rationale" is a requirement for mechanical loadings during the "explosion."

An ample margin between calculated energy release values and design values is recommended since potential energy extraction processes such as transfer of thermal energy to the sodium that generates a sodium vapor bubble is not included because of the complexity of defining the heat transfer processes which are highly geometry dependent.

Hicks and Menzies, Reference C-1, indicate the secondary processes in which vaporized sodium becoming the working fluid may be explosive in nature and extract about 10% of the residual heat energy as expansion work of sodium vapor, in addition to the work of expanding fuel vapor. The following analysis illustrates an application of the TNT rationale.

C.1 RECOMMENDED DESIGN APPROACH

To provide an ample margin, the energy absorption requirements of the design for the equivalent TNT explosion is selected as 750 pounds, which corresponds to 1500 MW seconds mechanical energy release and defined as the present reference DBA. The discussion of available

test data on explosive tests on similar designs resulted in making ultra-conservative assumptions to establish the head loading leads to very severe energy absorption requirements. By not including design features and configurations which tend to reduce the head loading, it is possible to direct from 15% to 20% of the charge energy into the head. (For a 1500 MW second, a mechanical energy release of 1% would require forty-eight 3-1/2 inch diameter bolts to be 18 inches long. This gives a perspective of magnitude for holddown requirements.) Selecting a vessel wall thickness as small as practical and providing space between the vessel wall and additional external blast cylinders for the vessel to expand into is a recommended design feature. In addition to this the vessel internals should not be designed with more radial rigidity than necessary in the form of support cylinders. The mass of shielding will inevitably tend to direct energy upwards in the expanding bubble process. The above features will make it possible to reduce the design level of energy absorption on the head from about 15% to less than 10% of the charge energy.

Another major consideration is the effect of a gas gap between the sodium surface and the underside of the head. Tests^(C-2) indicate the water hammer loading from the expanding gas bubble are more than the shock wave loading even when a "weak" tank is used. Unfortunately the only test data on a configuration with no gap was taken with a strong tank. It is probable that a combination of weak tank and no gap is the best configuration since (1) the shock-wave loading would be due to a direct reflection which is smaller than shock-wave loading due to the spray-impact loading across a small gap, and (2) the water hammer loading should be small since the bubble pressure should be relieved by vessel deformation. This is a tentative conclusion since applicable test data is not available; it is probable that such an arrangement can limit the head loading to a small fraction (1 to 3%) of the charge energy. The energy transmitted by a direct shock impact is calculated to be approximately 1/2% of the charge energy. The loading during the bubble expansion phase will be controlled by the hoop resistance of the vessel. Three percent (3%) of charge energy is selected as a design value on total kinetic energy imparted to the head. The head must be able to withstand a static pressure loading equal to the pressure required to deform the vessel. The loading on the head during the expansion of the vessel will drop from an initial high value of about 20,000 psi (for TNT) down to about 400 psi. During this loading phase, yielding of the hold down will occur and kinetic energy will be imparted to the head. This yielding decreases as the yield strength of the hold down increases. A preliminary review indicates a numerical method of solution will be required. As a reference, it is recommended the head hold down be designed to absorb 3% of the energy released. A development program will be required to establish the adequacy of this recommendation. An analytical evaluation of the loading mechanisms of an expanding bubble of UO_2 vapor will give an indication of the probable margin of safety in the design against a more realistic model of the accident.

C.2 DISCUSSION OF LOADING MECHANISMS

C.2.1 Shock Wave at Liquid-Solid Interface

For no gap, the head acts essentially as a rigid boundary, and upon impact of the compression shock, a compressive wave is reflected back into the liquid, leaving the liquid essentially at rest. The loading on the head for a normal reflection is the impulse associated with the pressure across the shock front. This can be computed^(C-3):

$$I = 1.46 W^{1/3} \left(\frac{W}{R} \right)^{0.89} \frac{\text{lb-sec}}{\text{sq in}} \text{ for TNT}$$

where:

W = Weight of charge—lb

R = Distance from charge—ft

The velocity of the head associated with this is $V = AI/M$

where:

M = Mass of head

A = Area of head

$$\text{Kinetic energy of head} = 1/2 MV^2 = 1/2 \frac{(AI)^2}{M}$$

C.2.2 Shock Wave—Liquid Gas Interface

For a design in which there is a gas gap between the surface and the head, the shock strength of the refracted wave through the gas is very small due to the gas compressibility. The reflected wave is a tensile wave, and the upward velocity behind the initial shock front is increased further by the reflected tension resulting in a layer of fluid leaving the surface with an upward velocity. A numerical evaluation of this effect has not been performed; it is dependent upon a reasonably precise knowledge of the cavitation pressure which is required to determine the depth of liquid which can be set in motion and transmit kinetic energy by impact on the head. Interpretation of measurements of Reference C-4 indicates this is ~2.5 times the energy transmitted by direct shock impact.

C.2.3 Bubble Expansion

For no gas gap the loading from the expanding bubble will be initially very high. Under such pressure, the head or the vessel or both will deform by plastic flow and create volume to relieve the pressure. If the static strength of the head and hold down is less than the vessel, the head hold-down will take most of the expansion, and some kinetic energy associated with the rate of the expansion will also be delivered from the sodium and head to the hold down.

For the situation in which a free volume exists for the gas to expand into the static pressure, loading is determined by this volume, but the transmittal of kinetic energy to the sodium ahead of the expanding bubble provides a loading mechanism. The amount of energy in the sodium directed upward to impact the head is dependent upon the radial resistance to deformation of the vessel internals and the vessel wall. From the experimental data of Reference C-2, the head loading in a rigid tank was much higher than in a weak tank. Therefore, it appears that an expanding vapor bubble is a stronger loading source than the shock wave. From this discussion it is clear that an analytical approach which adequately predicts head loading must take into account several complex time-dependent interrelated mechanisms.

C.3 PRELIMINARY HEAD LOADING ANALYSES

C.3.1 Static Pressure Loading on Head

The quasi-static pressure loading on the head will be limited by the hoop strength of the vessel.

Hoop strength of vessel,

$$\sigma_h = \frac{pr}{t} \text{ or } p = \frac{\sigma_h t}{r}$$

For 304 stainless steel at 900°F $\sigma_y = 20,000$
 $\sigma_u = 60,000$

$r = 100 \text{ in.}$

$$p = \frac{60,000 t}{100} = 600 t$$

where t is the vessel thickness. The 900°F was chosen to be representative of the minimum upper pool temperature at power and is conservative with respect to the head loadings (as related to vessel deformation).

For:

$t = 3/4 \text{ in.}$	$p = 450 \text{ psi}$
$t = 1 \text{ in.}$	$p = 600 \text{ psi}$
$t = 2 \text{ in.}$	$p = 1200 \text{ psi}$
$t = 3 \text{ in.}$	$p = 1800 \text{ psi}$

C.3.2 Shock Wave Loading on Head

The mass of head was assumed to be 150,000 pounds for this analysis. Actually, the head and its appurtenances (lead shielding) are considerably heavier than this value. This is a conservative assumption with respect to the head-bolt specifications.

Free impulse at head 16 ft from charge.

$$\begin{aligned} I_{sp} &= 1.46 W^{1/3} \left(\frac{W^{1/3}}{R} \right)^{0.89} \\ &= 1.46 \times 750^{1/3} \left(\frac{750^{1/3}}{20} \right)^{0.89} \\ &\cong 7.0 \text{ lb sec/in}^2 \end{aligned}$$

where:

W = Weight of TNT—lb

R = Distance from charge—ft

Kinetic energy of head after impact,

$$U_K = \frac{1}{2} \frac{W}{2} v^2 = \frac{1}{2} \frac{W}{g} \left(\frac{IA}{W/g} \right)^2 = \frac{1}{2} \left(\frac{IA}{W} \right)^2 g$$

$$A = \text{Area of head} = \pi \times 100^2 = 3.14 \times 10^4 \text{ in}^2$$

$$U_K = \frac{1}{2} \frac{(7.0 \times 3.14)^2 \times 386 \times 10^8}{150,000} = \underline{\underline{62.2 \times 10^6 \text{ in-lb}}}$$

Percent of charge energy (α_s)

$$\alpha_s = \frac{62.2 \times 10^6}{1500 \times 8.85} \times 10^6 = \underline{\underline{0.47\%}}$$

$$\text{Say} = \underline{\underline{1/2\%}}$$

C.4 HOLD DOWN BOLT REQUIREMENTS

C.4.1 Static Pressure

For:

Vessel thickness t = 0.75 in.

Static pressure p_s = 450 psi

Design stress in bolt = σ yield = 30,000 psi

$$\begin{aligned} \text{Bolt area required} &= \frac{pA}{\sigma} = \frac{450 \times 3.14 \times 10^4}{30 \times 10^3} \\ &= \underline{\underline{465 \text{ sq in.}}} \end{aligned}$$

Assume 48 bolts

$$A_b = \frac{465}{48} = 9.65 \text{ sq in.}$$

$$\text{Bolt diameter} = \sqrt{\frac{4}{\pi} \times 9.65} = \sqrt{12.3}$$

$$= 3.5 \text{ in. diam}$$

C.4.2 Length Required

Energy absorption per inch with 20% elongation

Average stress = 50,000 psi Strain = 0.2

$$U_s = \sigma \epsilon A = 50,000 \times 0.2 \times 9.65 = 96,500 \text{ in.-lb./ft bolt}$$

$$= 48 \times 96,500 = 4.65 \times 10^6 \text{ in.-lb./ft}$$

$$\text{Energy delivered to head } 3\% \times 1500 \times 8.85 \times 10^6 = 396 \times 10^6$$

$$\text{Length} = \frac{396 \times 10^6}{4.65 \times 10^6} = 85 \text{ in.} \quad \underline{\text{Say, 7 ft}}$$

C.5 DISCUSSION OF AVAILABLE TEST DATA

Reference C-2 reports experimental information taken on model tests in a 24-inch diameter tank 24 inches high with a central roof plug 7-3/4 inches in diameter and a weight of 50 pounds. Tests were performed using a bare charge and also a charge surrounded by a simulated core structure. From velocity measurements of the plug, the kinetic energy imparted to it is established.

	<u>KE of Plug as % of Charge Energy</u>
Strong Tank Bare Charge	0.2%
Strong Tank Enclosed Charge	1.5%
Weak Tank Bare Charge	0.2%
Weak Tank Enclosed Charge	0.6%

The percentage of charge energy imparted to the plug from direct shock wave impact can be computed using Reference C-3 as follows:

Energy release quoted is 203,000 ft lb

Corresponding weight of TNT W = 0.138 lb

Distance from charge to undersurface of head

$$R = 16.6 \text{ in.} = 1.38 \text{ ft}$$

$$\text{Specific impulse } I_{sp} = 1.46 W^{1/3} \left(\frac{W^{1/3}}{R} \right)^{0.89}$$

$$= 0.32 \text{ lb sec/sq in.}$$

$$\text{Head area } A = 47 \text{ in.}^2$$

$$\text{Impulse on head} = I_{sp} \times A = 15 \text{ lb-sec}$$

$$\begin{aligned}\text{Kinetic energy of head} &= \frac{1}{2} \frac{W}{g} v^2 = \frac{1}{2} \frac{I^2}{W} g \\ &= \frac{1}{2} \times \frac{15^2}{50} \times 32 = 72 \text{ ft-lb}\end{aligned}$$

$$\text{Percent charge energy imparted to head by direct shock wave impulse} = \frac{72}{203,000} = \underline{\underline{0.035\%}}.$$

Reference C-4 reports on similar tests in which the impulse on the head due to shock wave effects was separated out from the total loading. For tests using three different charge weights, the impulse on the head in which a gap existed between the surface and the bottom of the head was from two to three times the impulse obtained from the direct impact as calculated above using Reference C-3. So, for the tests of Reference C-2, the percentage of charge energy delivered to the plug by the shock wave is approximately $3 \times 0.035\% \cong 0.1\%$.

The major part of the energy transmitted to the roof is associated with the expanding vapor bubble of explosion products and is strongly dependent upon the radial restraint by the rigidity of the vessel internals and vessel wall.

The area of the model plug is approximately 10% of the roof area. For the strong tank, the impulse on the roof was approximately $1.25 \times$ the impulse on the plug. The kinetic energy imparted to the roof is approximately $10 \times 1.25 \times$ energy imparted to the small central plug.

Percent of Energy Given to Head

<u>Tank-Charge</u>	<u>Total</u>	<u>Shock Wave</u>	<u>Gas Bubble</u>
Strong-bare	25%	1%	24%
Strong-enclosed	17%	1%	16%
Weak-bare	2.5%	1%	1.5%
Weak-enclosed	7.5%	1%	6.5%

Since 50% of the energy release is in the gas bubble 3 to 50% of this can be transmitted to the head depending upon the system geometry. For the FFTF the percentage of charge energy absorbed by the head and its hold-down structure will also depend upon the radial constraints to bubble growth. A weak tank with the radial stiffness of the core internals minimized is desired to minimize energy absorption requirements in the head.

In the above experiments, the internal structure included a mockup of a relatively massive graphite shield. It is not anticipated that the FFTF design would offer as much restraint. The height to diameter ratio of the reactor vessel is greater than in the tests quoted. The effect of this is not clear; however, it is most likely that if the vessel is designed to be relatively weak, the gun barrel effect will be offset by deformation of the vessel, creating a volume for the gases to expand into.

C.6 HEAD LOADING FROM SHOCK WAVE WITH A GAP BETWEEN SURFACE AND HEAD

Reference C-4 reports the following initial impulse data associated with shock wave with a gap:

<u>W, grams of Hexogene</u>	<u>Impulse (psi-msec)</u>
15	570
30	715
60	900

The charge energies were reported as 2×10^4 , 4×10^4 and 8×10^4 calories. Since TNT releases 1050 cal/gm, the equivalent weights of TNT are 19, 38, and 76 grams or 0.042, 0.084 and 0.168 pounds. From Reference C-3 the impulse is:

$$I = 1.46 W^{1/3} \left(\frac{W^{1/3}}{R} \right)^{0.89} \text{ lb sec/in}^2$$

R is estimated to be 1.1. Therefore,

$$\begin{array}{ll}
 I = 1.35 W^{0.63} & \\
 = 184 \text{ psi-msec for 15 gm Hexogene} & 570/184 = 3.1 \\
 = 284 \text{ psi-msec for 30 gm Hexogene} & 715/284 = 2.5 \\
 = 439 \text{ psi-msec for 60 gm Hexogene} & 900/439 = 2.1
 \end{array}
 \left. \vphantom{\begin{array}{l} I = 1.35 W^{0.63} \\ = 184 \text{ psi-msec for 15 gm Hexogene} \\ = 284 \text{ psi-msec for 30 gm Hexogene} \\ = 439 \text{ psi-msec for 60 gm Hexogene} \end{array}} \right\} \begin{array}{l} \text{ratio of measured im-} \\ \text{pulse with a gap to} \\ \text{calculated impulse based} \\ \text{on no gap.} \end{array}$$

C.7 RADIAL BLAST SHIELD REQUIREMENTS

For TNT the thickness of a single cylinder required to contain a charge without rupture (or to a prescribed maximum deformation) can be calculated using the method of Reference C-5.

$$W = \left[\frac{1.407A(\sigma_t)\epsilon}{w^{-0.85} 10^5} \right]^{0.811}$$

$$\sigma_t = \sigma_y + \frac{\epsilon}{\epsilon_u} [\sigma_u (1 + \epsilon_u) - \sigma_y]$$

where the geometry factor A is,

$$A = \frac{(3.41 + 0.117 R_i/h_o) (R_e^2 - R_i^2)^{1.85}}{(1.47 + 0.0373 R_i/h_o)^{0.15} R_i^{0.15}}$$

- W = Weight of TNT—lb
 σ_y = Yield stress psi
 σ_u = Ultimate stress psi
 ϵ = Hoop strain, ϵ_u = Ultimate elongation
 R = Cylinder radius, i = inside, e = outside, ft
 h = Cylinder thickness, ft
 w = Material density, lb/ft³

For $R_i = 8$, $R_e = 8 + h$.

SS 304 at 900°F $\sigma_y = 16,000$; $\sigma_u = 60,000$; $\epsilon_u = 0.44$

$$A = \frac{(3.41 + 0.936/h) 124.8 h^{1.85}}{(1.47 + 0.298/h)^{0.15}}$$

$$\frac{\sigma_t}{10^5} = (0.16 + 1.6 \epsilon)$$

$$w^{+0.85} = (500)^{0.85} = 197$$

Maximum radial deflection = $U_r = R\epsilon = 100 \epsilon$, in.

$$\frac{1}{w^{0.811}} = \frac{1}{750^{0.811}} = 3600$$

$$\epsilon \frac{\sigma_t}{10^5} = (0.16 + 0.016 U_r) \frac{U_r}{100}$$

For $h = 1, 2, 4$ in.; $A = 14.4, 34.8$, and 89.4 , respectively.

$$(0.16 + 0.016 U_r) \frac{U_r}{100} = \frac{3600}{1.407 (197A)}$$

$$(0.16 + 0.016 U_r) U_r = \frac{1300}{A}$$

Thus for $h = 1, 2$, and 4 in., the radial expansion would be $73, 44$, and 25.6 inches, respectively.

Figure C-1 shows this expansion as a function of vessel thickness.

It is clear that additional radial support will be required for the vessel to reduce the expansion.

C.8 DOWN BLAST REQUIREMENTS

Down loads from the shock wave and blast pressure may be absorbed by elongation of the vessel; however, the difficulty from theoretical calculations is showing that the stress levels at the knuckle region between the bottom head and the cylinders will not cause fracture. Tests will establish this; however, it is recommended that the assumption be made until test data is available to indicate to the contrary that the knuckle does fail. Crush structure beneath the vessel is recommended as a means of absorbing the kinetic energy imparted to the lower head. Preliminary

FIGURE C-1. DEFLECTION OF VESSEL AND BLAST CYLINDERS

calculations using the direct impulse of a shock wave on the grid plate of 20 lb-sec/sq in. (based upon 6 ft from 750 lb charge) show that the energy absorption requirements of the crush structure = 1.4×10^9 in.-lb.

C.9 ENERGY TRANSFER TO EQUIPMENT CELL ATMOSPHERES

From analogy to the SEFOR - DBA it has been assumed that the analyses will indicate an energy stored in vaporized fuel above its boiling point of about 1000 MW-sec. The fuel vapor leaks to the equipment and refueling cells following the "explosive" expansion and has 60% of the initial stored energy, i.e., 600 MW sec stored in it as heat. This is transferred to the equipment cell atmospheres and raises the pressure and temperature.

Assuming the equipment cells and refueling cell contain 80,000 cu ft N_2 and 120,000 cu ft Ar, then Figure C-2 shows the pressure rise in 200,000 cu ft for 1000 MW-sec is 10 psi for N_2 and 21 psi for Ar.

$$\text{Balanced pressure} = \frac{80}{200} \times 10 + \frac{120}{200} \times 21 = \underline{\underline{17 \text{ psi/1000 MW sec}}}$$

$$\text{For 600 MW sec p} = \frac{600}{1000} \times 17 \text{ psi} = \underline{\underline{10 \text{ psi}}}$$

Using a factor of safety of 1.5 leads to a recommended design pressure based upon the assumed volumes of 15 psig and a corresponding temperature of approximately 500° F.

These values are subject to change with further analysis, but serve as a reference for preliminary containment design.

C.10 POST DBA COOLING

The agglomerate of material which accumulates in the bottom of the vessel following the DBA generate heat due to decay of fission products. The heat level is 20 MW initially, decaying to 4 MW after three hours, and about 1 MW after one day. The temperature rise of the sodium in the reactor is approximately 350° F in the first three hours assuming no cooling. A decay heat removal system capable of removing 5 MW would appear to be adequate. A 10 MW system has been suggested to permit redundancy to be incorporated.

C.11 REFERENCES

- C-1 Hicks, E. P. and Menzies, D.C., "Theoretical Studies of Fast Reactor Maximum Accident," (ANL-7120).
- C-2 Rees, N.J. M., "A Model Investigation of Explosion Containment in Single Tank Fast Reactors," (ANL-7120).
- C-3 Cole, R., "Underwater Explosions," Dover Pub., p. 241.

FIGURE C-2. PRESSURE RISE IN ATMOSPHERES PER 1000 MW-sec ENERGY DEPOSITED

- C-4 Drevon, G.A.V., et al., "Comparison of Pressure Loading Produced by Containment Explosions in Water and Sodium," (ANL-7120).
- C-5 Wise, W.R. and Proctor, J.F., "Explosion Containment Laws for Nuclear Reactor Vessels," (Equation 9-1), (NOLTR-63-140).

APPENDIX DGEOMETRICAL FEATURES OF VELOCITY DISTRIBUTION MODEL

D.1 CIRCUMFERENTIAL DISTRIBUTION

In deriving the geometrical features, one may consider two separate parts. First, the features such as total flow area, total hydraulic diameter, etc. are constants. Secondly, the local properties such as local flow area, local hydraulic diameter, etc., will vary with circumferential location, and formulas must be obtained for the five regions shown in Figure 9-3.

D.1.1 Properties of Total Flow Section

Referring to Figure D-1, the length of channel wall, ℓ_o , is given by

$$\ell_o = (L_1 + P) \sin 30^\circ \quad (D-1)$$

where:

P = Rod to rod spacing

The length L_1 is:

$$L_1 = \frac{X^* + R}{\cos 30^\circ} \quad (D-2)$$

where:

X^* = Tangential distance from rod surface to channel wall

R = Rod radius

Therefore,

$$\ell_o = \left[\frac{X^* + R}{\cos 30^\circ} + P \right] \sin 30^\circ \quad (D-3)$$

which can be reduced to:

$$\ell_o = 0.5P + 0.57735 (X^* + R) \quad (D-4)$$

The angle at which the channel wall ends, ϕ^* , can be found from the law of cosines to be

$$\phi^* = 2 \arctan \sqrt{\frac{(S - L_2)(S - L_1)}{S(S - \ell_o)}} \quad (D-5)$$

FIGURE D-1. MODEL FOR GEOMETRY CALCULATIONS – TOTAL AREA

where:

$$S = \frac{\ell_o + L_1 + L_2}{2} \quad (D-6)$$

$$L_2 = \sqrt{L_1^2 + \ell_o^2 - \ell_o L_1} \quad (D-7)$$

The total flow area is given by:

$$\Delta A = \frac{1}{2} \ell_o L_3 - \frac{1}{2} L_4 L_5 - \frac{\pi R^2}{2} \quad (D-8)$$

But from the geometry:

$$L_3 = \frac{\ell_o}{\tan 30^\circ} \quad (D-9)$$

$$L_4 = \frac{P}{2} \quad (D-10)$$

$$L_5 = L_4 \tan 30^\circ \quad (D-11)$$

Therefore, the area formulation becomes:

$$\Delta A = \frac{1}{2} \left(\frac{\ell_o \cdot \ell_o}{\tan 30^\circ} \right) - \frac{1}{2} \left(\frac{P}{2} \cdot \frac{P}{2} \tan 30^\circ \right) - \frac{\pi R^2}{2} \quad (D-12)$$

$$= \frac{\ell_o^2}{2 \tan 30^\circ} - \frac{P^2}{8} \tan 30^\circ - \frac{\pi R^2}{2}$$

$$= \frac{\ell_o^2}{2} \sqrt{3} - \frac{P^2}{8} \frac{\sqrt{3}}{3} - \frac{\pi R^2}{2}$$

$$\Delta A = \frac{1}{2} \left[\sqrt{3} \ell_o^2 - \frac{\sqrt{3}}{12} P^2 - \pi R^2 \right] \quad (D-13)$$

The total hydraulic diameter is then obtained from:

$$d_e = \frac{4 \Delta A}{\pi R + \ell_o} \quad (D-14)$$

D.1.2 Properties of Local Flow Sections

The local geometrical properties will be derived for Region I in Figure 9-3. The formulas which apply in the remaining regions would have similar derivations and will therefore be presented without derivation.

The distance from the rod surface to the outer boundary measured radially, y_o , at an arbitrary position ϕ is given by (Figure D-2):

$$\cos(\pi/6 - \phi) = \frac{X^* + R}{y_o + R} \quad (D-15)$$

$$y_o = \frac{X^* + R}{\cos(\pi/6 - \phi)} - R \quad \left\{ \begin{array}{l} \text{Region I} \\ 0 \leq \phi \leq \pi/6 \end{array} \right\} \quad (D-16)$$

The argument of the cosine will vary with the region of interest and in regions III-V, the distance $X^* + R$ can be replaced by $P/2$. The formulas are:

$$y_o = \frac{X^* + R}{\cos(\phi - \pi/6)} - R \quad \left\{ \begin{array}{l} \text{Region II} \\ \pi/6 \leq \phi \leq \phi^* \end{array} \right\} \quad (D-17)$$

$$y_o = \frac{P/2}{\cos(2\pi/3 - \phi)} - R \quad \left\{ \begin{array}{l} \text{Region III} \\ \phi^* \leq \phi \leq 2\pi/3 \end{array} \right\} \quad (D-18)$$

$$y_o = \frac{P/2}{\cos(\phi - 2\pi/3)} - R \quad \left\{ \begin{array}{l} \text{Region IV} \\ 2\pi/3 \leq \phi \leq 5\pi/6 \end{array} \right\} \quad (D-19)$$

$$y_o = \frac{P/2}{\cos(\pi - \phi)} - R \quad \left\{ \begin{array}{l} \text{Region V} \\ 5\pi/6 \leq \phi \leq \pi \end{array} \right\} \quad (D-20)$$

The rate of change of y_o with ϕ , $dy_o/d\phi$, can be obtained by differentiating Equations (D-16) to D-20) directly:

$$\frac{dy_o}{d\phi} = -(X^* + R) \frac{\sin(\pi/6 - \phi)}{\cos^2(\pi/6 - \phi)} \quad \left\{ \begin{array}{l} \text{Region I} \\ 0 \leq \phi \leq \pi/6 \end{array} \right\} \quad (D-21)$$

$$\frac{dy_o}{d\phi} = (X^* + R) \frac{\sin(\phi - \pi/6)}{\cos^2(\phi - \pi/6)} \quad \left\{ \begin{array}{l} \text{Region II} \\ \pi/6 \leq \phi \leq \phi^* \end{array} \right\} \quad (D-22)$$

FIGURE D-2. MODEL FOR GEOMETRY CALCULATIONS - REGION I

$$\frac{dy_o}{d\phi} = - (P/2) \frac{\sin(2\pi/3 - \phi)}{\cos^2(2\pi/3 - \phi)} \quad \left\{ \begin{array}{l} \text{Region III} \\ \phi^* \leq \phi \leq 2\pi/3 \end{array} \right\} \quad (D-23)$$

$$\frac{dy_o}{d\phi} = (P/2) \frac{\sin(\phi - 2\pi/3)}{\cos^2(\phi - 2\pi/3)} \quad \left\{ \begin{array}{l} \text{Region IV} \\ 2\pi/3 \leq \phi \leq 5\pi/6 \end{array} \right\} \quad (D-24)$$

$$\frac{dy_o}{d\phi} = - (P/2) \frac{\sin(\pi - \phi)}{\cos^2(\pi - \phi)} \quad \left\{ \begin{array}{l} \text{Region V} \\ 5\pi/6 \leq \phi \leq \pi \end{array} \right\} \quad (D-25)$$

The local flow area, dA , can be expressed by

$$dA = \frac{1}{2} (X^* + R) (X^* + R) \tan \left[\pi/6 - (\phi - d\phi)/2 \right] - \frac{1}{2} (X^* + R) (X^* + R) \tan \left[\pi/6 - (\phi + d\phi)/2 \right] - \pi R^2 \left(\frac{d\phi}{2\pi} \right) \quad (D-26)$$

which can be reduced to:

$$dA = \frac{1}{2} (X^* + R)^2 \left[\tan(\pi/6 - \phi + d\phi/2) - \tan(\pi/6 - \phi - d\phi/2) \right] - \frac{R^2 d\phi}{2} \quad (D-27)$$

Region I $0 \leq \phi \leq \pi/6$

In the remaining regions dA has the form:

$$dA = \frac{1}{2} (X^* + R)^2 \left[\tan(\phi + d\phi/2 - \pi/6) - \tan(\phi - d\phi/2 - \pi/6) \right] - \frac{R^2 d\phi}{2} \quad (D-28)$$

Region II $\pi/6 \leq \phi \leq \phi^*$

$$dA = \frac{1}{2} (P/2)^2 \left[\tan(2\pi/3 - \phi + d\phi/2) - \tan(2\pi/3 - \phi - d\phi/2) \right] - \frac{R^2 d\phi}{2} \quad (D-29)$$

Region III $\phi^* \leq \phi \leq 2\pi/3$

$$dA = \frac{1}{2} (P/2)^2 \left[\tan(\phi + d\phi/2 - 2\pi/3) - \tan(\phi - d\phi/2 - 2\pi/3) \right] - \frac{R^2 d\phi}{2} \quad (D-30)$$

Region IV $2\pi/3 \leq \phi \leq 5\pi/6$

$$dA = \frac{1}{2} (P/2)^2 \left[\tan(\pi - \phi + d\phi/2) - \tan(\pi - \phi - d\phi/2) \right] - \frac{R^2 d\phi}{2} \quad (D-31)$$

Region V $5\pi/6 < \phi < \pi$

The local hydraulic diameter (y_e) can be determined easily from:

$$y_e = \frac{4dA}{P_w} \quad (D-32)$$

where:

P_w = Wetted perimeter

From Figure A-2:

$$P_w = Rd\phi + (X^* + R) \left[\tan(\pi/6 - \phi + d\phi/2) - \tan(\pi/6 - \phi - d\phi/2) \right] \quad (D-33)$$

Then y_e becomes:

$$y_e = \frac{4dA}{Rd\phi + (X^* + R) \left[\tan(\pi/6 - \phi + d\phi/2) - \tan(\pi/6 - \phi - d\phi/2) \right]} \quad (D-34)$$

Region I $0 \leq \phi \leq \pi/6$

In the remaining regions:

$$y_e = \frac{4dA}{Rd\phi + (X^* + R) \left[\tan(\phi + d\phi/2 - \pi/6) - \tan(\phi - d\phi/2 - \pi/6) \right]} \quad (D-35)$$

Region II $\pi/6 \leq \phi \leq \phi^*$

$$y_e = \frac{4dA}{Rd\phi} \quad (D-36)$$

Region III-V $\phi^* < \phi < \pi$

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