Radiative Corrections to Expt. 416

Backward π N Charge Exchange

Internal Report # 131

by

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Introduction

The importance of radiative corrections in elastic hadronic reactions has recently been reported by Sogard in the literature (1). In a private communication (2) he has also studied the formalism of radiative corrections to π N charge exchange scattering (CEX) for an experimental apparatus capable of detecting the bremsstrahlung photon. This internal report describes the computational techniques and results of applying the radiative corrections to a backward CEX experiment performed by the Brown High Energy Group in collaboration with others (3).

In the case of elastic π N scattering, Sogard assumes a magnetic spectrometer has been used to detect the outgoing pion. In such an experiment, a cut Δp is generally made on the momentum of the pion, and all events within the cut are assumed to be valid elastic events. Applying the cut excludes from the elastic cross section, true elastic events in which a bremsstrahlung photon from the π or N has reduced the scattered pion momentum to below the cut. Sogard calculates the correction factor for such a cut and finds it to be both appreciable and t -dependent. For example, in $\pi^- p$ elastic scattering at 5 GeV/c the correction turns out to be about 1% in the forward direction and 15% in the dead backward direction. The charge exchange reaction is shown schematically in Figure 1A. An incident π^- (1) collides with a proton (2) to yield a π^0 (3), neutron (4), and a bremsstrahlung photon. The final state particles are assumed to be in a plane in the following calculations. The photon angle is denoted by θ_K and its energy by ω . In the following equations, the subscripts on

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FIGURE 1A

various kinematic quantities refer to the above identified particles.

In our apparatus for backward πN charge exchange (4) we do not measure the pion momentum, but rather are able to detect any brem gamma which exceeds the threshold of a forward shower counter or of an array of gamma spark chambers. Detection of the brem would exclude a true CEX event from the data sample. The measured angular distribution is related to the true angular distribution (radiative free) by the equation

$$\frac{d\sigma_M}{d\Omega} = \frac{d\sigma}{d\Omega} \iiint \frac{d\sigma}{d\Omega_K d\omega d\Omega_3} \quad (1)$$

The calculational problem is to integrate

$$\begin{aligned} \frac{d\sigma}{d\Omega_K d\omega d\Omega_3} &= \frac{\alpha}{(2\pi)^2} \frac{P_3^2}{P_{3e}^2} M\omega \frac{P_{3e} E_4 - E_{3e} \hat{P}_{3e} \cdot (\vec{P}_1 - \vec{P}_{3e})}{E_4 P_3 - E_{3e} \hat{P}_3 \cdot (\vec{P}_1 - \vec{P}_3 - \vec{K})} \\ &\times \left[\frac{2E_1}{\omega(\vec{P}_1 \cdot \vec{K})} - \frac{\nu^2}{(\vec{P}_1 - \vec{K})^2} - \frac{1}{\omega^2} \right] \frac{d\sigma}{d\Omega_3} \quad (2) \end{aligned}$$

over all regions of $\omega - \Omega_K$ for which the brem photon is not detected. A diagram depicting such an area is shown in Figure 1B, where the hatched area represents the range of ω and θ_K for which the brem is observed by the shower counter, and ω_0 is the threshold of the shower counter. The integral naturally divides into sections; Area I may be evaluated exactly and which contains the divergent part from 0 energy photons, and is called the Schwinger term. Area II is numerically integrated.

The remainder of this report will be divided into the following sections:

FIGURE 1 B

2. A study of the curves of constant P_3 in $\omega - \theta_k$ space, and the boundaries of the space.
3. Evaluation of the Schwinger term.
4. Evaluation of the numerical integration term.
5. Evaluation of the region in which the photon is detected and a check of the unitarity condition.
6. Results of the calculation on radiative corrections to BEX.
7. Appendices containing (1) Sogard's works and (2) listings of computer programs written to calculate radiative corrections.

2. In this section we are interested in the brems energy ω as a function of brems angle θ_K for fixed P_3 and fixed incident momentum. The program Omega was written for this purpose. The equation describing this relationship is:

$$\omega = \frac{\mu^2 + M_2 (E_1 - E_3) - E_1 E_3 + \vec{P}_1 \cdot \vec{P}_3}{E_1 + M_2 - E_3 - \vec{k} \cdot (\vec{P}_1 - \vec{P}_3)} \quad (3)$$

where $\mu^2 = \frac{1}{2} (M_1^2 + M_2^2 + M_3^2 - M_4^2)$. Equation 3 is easily derivable from the four vector equations governing the reaction $\pi^- p \rightarrow \pi^0 n \gamma$. Omega operates in the following manner -- the program cycles over various momenta and scattering angles and for each incident π^- momentum and scattering angle θ_3 , the value of P_3 is first calculated assuming an emission of a brems of energy ω at $\theta_K = 0^\circ$ (this brems energy is known as ω_0). The value of P_3 is then held constant as θ_K is incremented, thus tracing out ω as a function of θ_K . As we shall later see, an important quantity in the radiative corrections is Δp , which is defined as the difference between the elastic momentum of the π^0 and the π^0 momentum with a brems in the final state. Figures 2 and 3 show the behavior of ω vs θ_K curves for various pion angles (θ_{π^0}) and various Δp values. The first two figures show the behavior at large ω , which gives the shape of the kinematic limits of the space. The next plot shows the behavior of the curve for an $\omega^0 = 30$ Mev, which is in the range of the shower counter threshold. Each of these plots shows the same characteristic drop off of ω as θ_K increases, given P_3 is held constant.

w vs θ_K

FIGURE 3

One point to note is that in the backward hemisphere, the kinematic demands that a relatively small change of Δp_3 results in a large brems energy. In particular at $\theta_{\pi} = 180^\circ$ scattering, a difference of $\Delta p_3 = 5$ Mev, for incident pion momentum of 6.0 Gev/c, results in an 800 Mev brems in the forward direction ($\theta_K = 0^\circ$). A plot of this is shown in Figure 5. A brems of about 30 Mev at $\theta_K = 0^\circ$ results in a change of only $.164 \times 10^{-3}$ Mev/c in Δp .

A listing of the program is found in the Appendix and run results are tabulated in the computer print-out accompanying this report.

3. Schwinger term

Recall that the radiative correction to BCEX will involve evaluation of equation 1 over all regions in $\omega - \theta_K$ space for which the brem photon will not be detected. The most important contribution to this correction factor comes from the evaluation of the Schwinger term, which includes the divergence from zero energy photons.

Sogard showed in (1) and (2) that an exact evaluation of the Schwinger term is possible, and the resulting radiative correction for this part is

$$\int_I = \frac{d\sigma}{d\Omega_3} e^{-\delta(P_{3e} - P_{30})} = \int_I \frac{d\sigma}{d\Omega_K d\Omega_3} \quad (4)$$

where P_{3e} is the elastic momentum of the π and P_{30} is the momentum of the π given a brem energy of ω° has been emitted at 0° in the lab. δ is defined as

$$\begin{aligned} \delta = & \frac{\alpha}{\pi} \left\{ \left(\frac{1}{2\beta_1} \ln Q_1 - 1 \right) \left(\ln \frac{E_4}{\eta \Delta E} + \frac{\ln E_3}{\eta \Delta E} \right) \right. \\ & + \frac{1}{\beta} \left[\ln \left| \frac{C_{33}}{E_3 a_1} \right| \ln R_1^+ - \ln \left| \frac{C_{34}}{E_3 a_1} \right| \ln R_1^- \right] \\ & \left. - \phi \left(\frac{M(M - E_1^+)(E_3 - E_4)}{C_{33}} \right) + \phi() - \phi() + \phi() \right\} \quad (5) \end{aligned}$$

where the quantities Q_1 , C_{kk} etc. are defined in the Sogard references.

For convenience, we may choose ω° to be the threshold of the shower counter in the evaluation of Δp . This choice maximizes the area in the space for which an exact evaluation of equation 2 exists, thus minimizing the errors introduced in a numerical integration for regions outside the Schwinger term area.

A program "RADCOR" was written to evaluate eqn. (5). RADCOR loops over various incident pion momentum and shower counter thresholds, and for each combination it fixes the π° lab angle, calculates the π° momentum for emission of a brem at angle 0° with energy equal to the shower counter threshold. After calculating some kinematic variables, it calls Subroutine Delta which evaluates δ . By fixing the momentum of the pion, we are able to integrate the area enclosed within the curve of constant P (see figure 1).

As the Schwinger term is the largest part of the radiative corrections to BCEX, and a fundamental parameter in its evaluation is the shower counter threshold ω° , which is not well-known experimentally, a study to see how sensitive the term was to variations in threshold was made. We estimate the threshold of the shower counter to be somewhere between 10-30 Mev, but in a special run we started at 5 Mev, and successively cut the threshold value by a factor of 2 until reaching $\omega^\circ = .3$ Mev. This is diagrammatically shown in figure 6.

Figure 6
 ω vs θ_K

The calculation showed that δ rose from .159 at 5 Mev to .19 at .3 Mev. This change is in the proper direction, since for a lower threshold, the radiative correction would be larger. The interesting point, however, is that for such a large change in ω° , there was a relatively small change in the total correction. This implies that most of the Schwinger term arises from a very small ω° contribution, which is logical since the integrand is proportional to $1/\omega$.

Figures 7 and 8 give the results of the RADCOR term at 6.0 and 2.6 Gev/c incident pion momentum respectively, and show the effect of threshold of the shower counter on the radiative correction term. It is seen that for threshold changes from 5 to 35 Mev, the resulting effect is only a few percent. We are therefore insensitive to the exact value of the threshold, which experimentally is not well-known. However guessing a reasonable value results in very little change in the radiative corrections from this term.

A listing of the Fortran program is given in Appendix B. Results of the run are given in Table 1.

3. Numerical Integration

Evaluation of the Schwinger term has accounted for Area I in figure 1B. Numerical integration of equation (2) is used to evaluate the remainder of the area in $\omega - \theta_K$ space. The region was divided into several sections as indicated in figure 9 below.

Figure 9

Region IIA covers that area with ω less than ω^0 not including the area already accounted for by the Schwinger term. Regions IIB and IIIB are bounded by the lines $\theta_K = \theta_{SC}$ and $\omega \geq \omega^0$. This area represents the angular coverage of the side gamma chambers, the boundary between them being the threshold of the chambers. Region IIIB covers the sector in which gamma rays are detected by the shower counter. Only IIA and

FIGURE 8
Schwinger Term vs $\cos \theta_{\pi^0}$
for various shower counter thresholds
 π^- Momentum = 6.0 GeV/c

IIB contribute to the radiative correction, but integration of IIIA and B was done as a check on the validity of the numerical integration, since the integral over the entire space, including region I, must sum to 100%.

A program (INTEG) was written to perform this numerical integration, modified slightly for each region's different integration limits. Except at boundary limits, where θ_K and ω are related by eqn. (3), ω and θ_K are considered independent variables, and the numerical integration involves dividing the area into a grid, where, for constant ω , θ_K is stepped, the integrand is evaluated in each box, and the sum calculated.

A study was made to see how many bins were necessary to obtain convergence of the integral at 6 GeV/c. A reasonable balance between cpu time and accuracy was obtained with the values $N_{en} = 200$ where N_{en} is the number of bins between ω° and the kinematic limit of the brems energy at 0° , and $N_{dcosk} = 20$, for the cosk axis. A demonstration of this convergence as a function of the number of bins is shown in Table II, where the integration was performed for various values of n_{en} and n_{dcosk} .

If the bins are infinitely fine, a linear approximation could be made, where the integrand would be evaluated at the center of the box of $\Delta\omega$ and $\Delta dcosk$. However, a more refined method was used. For fixed ω , the integrand behaves very much like

$$I \propto \left[\frac{2E_1}{\omega(\vec{p} \cdot \vec{k})} - \frac{\mu^2}{(p_1 - R)^2} - \frac{1}{\omega^2} \right]$$

which is the last part of equation (2). For constant ω , this part of the

Table II

	nen			
	401	201	101	50
20		9.68 - 8.62	8.99 - 7.93	8.10 - 7.01
40	10.60	10.15 - 8.58	9.46 - 7.89	8.57 - 6.99
80		10.46	9.76 - 7.83	8.87 - 6.94

ndcosk

integrand was averaged over the $\cos k$ interval $\Delta \cos k$. Keeping all other quantities constant, this part is exactly integrable and the average value of the integrand calculated. This was then used in the numerical integration.

Subroutine Calc performs the evaluation of the integrand in a given grid box, and INTEG controls the looping over various limits and sums the partial contributions. Results for various runs of INTEG, for various thresholds and incident momentum were performed and are presented in Table III. Listings of the programs are given in Appendix B.

Figures 10 and 11 show the integrand as a function of ω with $\cos k$ held constant, and with ω held constant. In both plots, it is seen that for small ω , and for $\cos k$ near 1.0 the value of the integrand is large. The shape of the surface is thus a maximum near the origin, falling off quickly as one leaves the origin in either direction.

Figure 12 shows the contribution to the radiative correction in percent, as a function of $\cos \theta_3$, and it is seen to be small, and fairly constant over the entire range of $\cos \theta_3$.

In Figures 13 and 14 we have the contribution from region 3A at two momenta spanning the range of momenta considered, with the effect of threshold of the shower counter plotted on these graphs also. Again, we find that the integrand pion is insensitive in the numerical integration case, as it was with the exact Schwinger term. There is very little structure dependent on $\cos \theta_3$ in these plots. Finally the last two figures, 15 and 16, at 2.6 and 6.0 Gev/c give the dependence of the integration on $\cos \theta_3$. Again the contribution from this region is small and relatively θ_3 dependent except at very forward lab angles, in which we are not particularly interested.

FIGURE 11

FIGURE 12

FIGURE 13

Integration of Area IIIA vs $\cos \theta_{\pi^0} (\text{LAB})$

for various shower counter thresholds

π^- Momentum = 2.6 GeV/c

FIGURE 14

Integration of Area IIIA vs $\cos \theta_{\pi^0} (\text{LAB})$

for various shower counter thresholds

π^- Momentum = 8.0 GeV/c

The sixth column in Table III gives the unitarity condition imposed by summing the percentages from the parts. It is seen that in this approximate numerical integration, the condition is reasonably satisfied. At least, we believe that the Schwinger term gives the exact value in Region I, and the relative magnitudes of the other regions is preserved in the numerical integration.

FIGURE 16

Results as affecting BCX:

The final radiative correction factors are tabulated in Table III. These tables give the RADCOR as a function of specific incident π^- momentum, shower counter threshold and $\mu - 4$ vector transfer. Figures 17 and 18 show the factor as a function of π^0 lab angle for various thresholds. Note that the corrections are on the order of $\sim 10\%$, that they are not strongly influenced by thresholds from 10 - 30 MeV, and that they are not strong functions of $\cos \theta_{\pi^0}$. Figure 19 gives the RCF as a function of momenta and threshold = 20 MeV.

Conclusion being that RADCOR on BCX should give a normalization $\times 10\%$.

RADIATIVE CORRECTION FACTOR

VS

$\cos \theta_{\pi^0}$ (LAB)

FOR VARIOUS SHOWER COUNTER THRESHOLDS

AT BEAM MOMENTUM = 2.6 GEV/C

Figure 17

Figure 18

RADIATIVE CORRECTION FACTOR

VS

$\cos \theta_{\pi^0} \text{ (LAB)}$

SHOWER COUNTER THRESHOLD = 20 MEV

Figure 19

REFERENCES

1. M. Sogard, Phys. Rev. D9, 1486 (1974)
2. Private Communication from D. S. Barton and G. T. Y. Chen (See Appendix #1).
3. C. DeMarzo et al - to be published (Submitted to Physics Letters),
"Differential Cross Sections for Backward $\pi^- p \rightarrow \pi^0 n$ from 2-6 to 8 GeV/c".
4. G. T. Y. Chen Thesis - Internal Report #121, Brown University
(COO-3130TB-165).

APPENDIX I

Private Communication

Sogard's Calculations for BCX

4/25/73
Michael Sogard

Radiation in π^-p charge exchange scattering

Neglect radiation from internal lines:

This has no infrared divergence and/or may only be significant for high energy photons - in any case a model is now required

The matrix element is

$$M = -e\pi \frac{P_1 \cdot \epsilon}{P_1 \cdot k} T - e_p \frac{P_2 \cdot \epsilon}{P_2 \cdot k} T$$

T is the amplitude for CEX scattering - neglect the dependence of T on the photon energy.

Squaring ^{summing} + averaging over spin states we get

$$\frac{1}{2} |M|^2 = \left[-e^2 \frac{P_1^2}{(P_1 \cdot k)^2} - e_p^2 \frac{P_2^2}{(P_2 \cdot k)^2} - 2e\pi e_p \frac{P_1 \cdot P_2}{(P_1 \cdot k)(P_2 \cdot k)} \right] \frac{1}{2} \sum |\Pi|^2$$

$$= e^2 \left[2 \frac{E_1}{\omega(P_1 \cdot k)} - \frac{\mu^2}{(P_1 \cdot k)^2} - \frac{1}{\omega^2} \right] \left(\frac{1}{2} \sum |\Pi|^2 \right)$$

~ let $p_2 = (E_2, \vec{0})$

(2)

$$d\sigma = \frac{(2\pi)^4}{(2\pi)^5} \frac{1}{v} \frac{1}{2E_1} \left(\frac{1}{2}|M|^2\right) \frac{d^3p_3}{2E_3} \frac{M}{E_4} \frac{d^3p_1}{E_4} \frac{d^3k}{2\omega} \delta^4(p_i - p_f)$$

$$\int d^3p_4: \quad d\sigma = \frac{1}{(2\pi)^5} \frac{1}{v} \frac{1}{2E_1} \left(\frac{1}{2}|M|^2\right) \frac{d^3p_3}{2E_3} \frac{M}{E_4} \frac{d^3k}{2\omega} \delta(E_i - E_f)$$

$$\underline{p}_4 = \underline{p}_1 - \underline{p}_3 - \underline{k}$$

$$\int dk: \quad \omega = E_1 + M - E_4 - E_3$$

$$E_i - E_f = E_1 + M - E_4 - \omega - E_3$$

$$= E_1 + M - \sqrt{(\underline{p}_1 - \underline{p}_3 - \underline{k})^2 + m^2} - \omega - E_3$$

$$\therefore \left| \frac{\partial(E_i - E_f)}{\partial k} \right| = \left| -\frac{1}{2} \cdot \frac{2(\underline{p}_1 - \underline{p}_3 - \underline{k}) \cdot (-\hat{k})}{E_4} - 1 \right| = \frac{E_4 + (\underline{k} - \underline{p}_1 + \underline{p}_3) \cdot \hat{k}}{E_4}$$

$$v = \frac{p_1}{E_1}$$

$$d\sigma = \frac{1}{(2\pi)^5} \frac{1}{p_1} \frac{1}{2E_1} \left(\frac{1}{2}|M|^2\right) \frac{d^3p_3}{2E_3} \frac{M}{E_4} \frac{k^2 d\Omega_k}{2\omega} \frac{E_4}{E_4 + (\underline{k} - \underline{p}_1 + \underline{p}_3) \cdot \hat{k}}$$

$$= \frac{1}{8(2\pi)^5} \frac{p_3^2}{p_1 E_3} \cdot \frac{M k}{E_4 + (\underline{k} - \underline{p}_1 + \underline{p}_3) \cdot \hat{k}} \left(\frac{1}{2}|M|^2\right)$$

$$\boxed{\frac{d\sigma}{dp_3 d\Omega_3 d\Omega_k} = \frac{1}{8} \cdot \frac{1}{(2\pi)^5} \cdot \frac{p_3^2}{p_1 E_3} \cdot \frac{M k}{E_4 + \omega - (\underline{p}_1 - \underline{p}_3) \cdot \hat{k}} \left(\frac{1}{2}|M|^2\right)}$$

(3)

CEX Scattering (no radiation)

$$d\sigma = \frac{(2\pi)^4}{(2\pi)^6} \frac{1}{v} \frac{1}{2E_1} \left(\frac{1}{2}|T|^2\right) \delta^4(p_i - p_f) \frac{d^3p_3}{2E_3} \frac{M}{E_4} \frac{d^3p_1}{E_4}$$

$$\int d^3p_4 d\sigma = \frac{1}{4} \frac{1}{(2\pi)^2} \frac{1}{v} \frac{1}{E_1} \left(\frac{1}{2}|T|^2\right) \delta(E_i - E_f) \frac{d^3p_3}{E_3} \frac{M}{E_4}$$

$$E_i - E_f = E_1 + M - E_3 - \sqrt{(\underline{p}_1 - \underline{p}_3)^2 + m^2}$$

$$\left| \frac{\partial(E_i - E_f)}{\partial p_3} \right| = \frac{p_3}{E_3} - \frac{\hat{p}_3 \cdot (\underline{p}_1 - \underline{p}_3)}{E_4} = \frac{p_3 E_4 - E_3 \hat{p}_3 \cdot (\underline{p}_1 - \underline{p}_3)}{E_3 E_4}$$

$$v = \frac{p_1}{E_1}$$

$$\int d^3p_3 d\sigma = \frac{1}{4} \frac{1}{(2\pi)^2} \frac{E_1}{p_1} \frac{1}{E_1} \left(\frac{1}{2}|T|^2\right) \frac{E_3 E_4}{p_3 E_4 - E_3 \hat{p}_3 \cdot (\underline{p}_1 - \underline{p}_3)} \frac{M}{E_4} \frac{p_3^2 d\Omega_3}{E_3}$$

$$= \frac{1}{4} \frac{1}{(2\pi)^2} \frac{p_3^2}{p_1 E_3} M \frac{E_3}{p_3 E_4 - E_3 \hat{p}_3 \cdot (\underline{p}_1 - \underline{p}_3)} \cdot d\Omega_3 \frac{1}{2}|T|^2$$

$$\boxed{\frac{d\sigma}{d\Omega_3} = \frac{1}{4} \frac{1}{(2\pi)^2} \frac{p_3^2}{p_1 E_3} M \frac{E_3}{p_3 E_4 - E_3 \hat{p}_3 \cdot (\underline{p}_1 - \underline{p}_3)} \frac{1}{2}|T|^2}$$

(4)

$$\frac{d^3\sigma}{d\Omega_3 d\Omega_k} = \frac{1}{8} \cdot \frac{1}{(2\pi)^5} \frac{P_3^2}{P_1 E_3} \cdot \frac{M k}{E_4 + \omega - (\underline{P}_1 - \underline{P}_3) \cdot \underline{k}} \cdot e^2 \left[\frac{2E_1}{\omega(\underline{P}_1 \cdot \underline{k})} - \frac{\mu^2}{(\underline{P}_1 \cdot \underline{k})^2} - \frac{1}{\omega^2} \right]$$

$\propto \frac{1}{2} \sum_i |T_i|^2$

$$= \frac{1}{8} \frac{1}{(2\pi)^5} \frac{P_3^2}{P_1 E_3} \frac{M k}{E_4 + \omega - (\underline{P}_1 - \underline{P}_3) \cdot \underline{k}} e^2 \left[\right] \cdot \frac{\frac{d\sigma}{d\Omega_3}}{\frac{1}{4} \frac{1}{(2\pi)^3} \frac{P_3^2}{P_1 E_3} \frac{m}{P_3 E_4 - E_3 \hat{P}_3 \cdot (\underline{P}_1 - \underline{P}_3)}}$$

strictly speaking the P_3, E_3 here refer to elastic scattering while the P_3, E_3 over there refer to bremsstrahlung so we can't cancel them. In the above let $P_3 \rightarrow P_e, E_3 \rightarrow E_{3e}$, the elastic value

$$\frac{d^3\sigma}{d\Omega_3 d\Omega_k} = \frac{1}{2} \frac{e^2}{(2\pi)^3} \frac{P_1^2}{P_{3e}^2} \frac{E_{3e}}{E_3} \omega \left(\frac{P_{3e} E_4 - E_{3e} \hat{P}_3 \cdot (\underline{P}_1 - \underline{P}_{3e})}{E_{3e} (E_4 + \omega - (\underline{P}_1 - \underline{P}_3) \cdot \underline{k})} \right) \left[\frac{2E_1}{\omega(\underline{P}_1 \cdot \underline{k})} - \frac{\mu^2}{(\underline{P}_1 \cdot \underline{k})^2} - \frac{1}{\omega^2} \right] \frac{d\sigma}{d\Omega_3}$$

$$= \frac{\alpha}{(2\pi)^2} \frac{P_3^2}{P_{3e}^2} \frac{E_{3e}}{E_3} \omega \left(\frac{P_{3e} E_4 - E_{3e} \hat{P}_3 \cdot (\underline{P}_1 - \underline{P}_{3e})}{E_{3e} (E_4 + \omega - (\underline{P}_1 - \underline{P}_3) \cdot \underline{k})} \right) \left[\frac{2E_1}{\omega(\underline{P}_1 \cdot \underline{k})} - \frac{\mu^2}{(\underline{P}_1 \cdot \underline{k})^2} - \frac{1}{\omega^2} \right] \frac{d\sigma}{d\Omega_3}$$

($\alpha = \frac{e^2}{4\pi}$ - Bjorken-Drell convention)

This is probably not a very convenient form - so calculate $\frac{d^3\sigma}{d\Omega_3 d\Omega_k d\omega}$ instead

(5)

~~the above is wrong~~

$$d\sigma = \frac{1}{(2\pi)^5} \frac{1}{v} \frac{1}{2E_1} \left(\frac{1}{2} \sum_i |M_i|^2 \right) \frac{d^3P_3}{2E_3} \frac{M}{E_4} \frac{d^3k}{2\omega} \delta(E_i - E_f)$$

Integrate over P_3 : $E_3 = E_1 + m - E_4 - k$

$$= E_1 + m - \sqrt{(\underline{P}_1 - \underline{P}_3 - \underline{k})^2 + m^2} - k$$

$$E_i - E_f = E_1 + m - E_3 - E_4 - k$$

$$= E_1 + m - E_3 - \sqrt{(\underline{P}_1 - \underline{P}_3 - \underline{k})^2 + m^2} - k$$

$$\left| \frac{\partial(E_i - E_f)}{\partial P_3} \right| = \frac{P_3}{E_3} + \frac{(\underline{P}_1 - \underline{P}_3 - \underline{k}) \cdot \hat{P}_3}{E_4} = \frac{E_4 P_3 + (\underline{P}_1 - \underline{P}_3 - \underline{k}) \cdot \hat{P}_3 E_3}{E_3 E_4}$$

$$v = \frac{P_1}{E_1}$$

$$d\sigma = \frac{1}{(2\pi)^5} \frac{E_1}{P_1} \frac{1}{2P_1} \left(\frac{1}{2} \sum_i |M_i|^2 \right) \frac{P_3^2 d\Omega_3}{2E_3} \frac{M}{E_4} \frac{\omega d\omega d\Omega_k}{2} \frac{E_3 E_4}{E_4 P_3 + (\underline{P}_1 - \underline{P}_3 - \underline{k}) \cdot \hat{P}_3 E_3}$$

$$= \frac{1}{8} \frac{1}{(2\pi)^5} \frac{P_3^2}{P_1} \frac{M \omega d\omega d\Omega_k d\Omega_3}{E_4 P_3 + (\underline{P}_1 - \underline{P}_3 - \underline{k}) \cdot \hat{P}_3 E_3} \left(\frac{1}{2} \sum_i |M_i|^2 \right)$$

~~the above is wrong~~

⑥

$$\frac{d\sigma}{d\Omega_k d\omega d\Omega_3} = \frac{1}{8} \frac{1}{(2\pi)^5} \frac{P_3^2}{P_1} \frac{M\omega}{E_4 P_3 \hat{p} \cdot (\underline{P}_1 - \underline{P}_2 - \underline{k}) \cdot \hat{P}_3 E_3} e^2 \left[\frac{2E_1}{\omega(\underline{P}_1 \cdot \underline{k})} - \frac{\mu^2}{(\underline{P}_1 \cdot \underline{k})^2} - \frac{1}{\omega^2} \right] \left\{ \frac{1}{2} \frac{E_1}{\omega} \right\}$$

$$= \frac{1}{8} \frac{1}{(2\pi)^5} \frac{P_3^2}{P_1} \frac{M\omega}{E_4 P_3 \hat{p} \cdot (\underline{P}_1 - \underline{P}_2 - \underline{k}) \cdot \hat{P}_3 E_3} e^2 \left[\right] \cdot \frac{\frac{d\sigma}{d\Omega_3}}{\frac{1}{4(2\pi)^2} \frac{P_{3e}^2}{P_1 E_{3e}} \frac{E_{3e}}{P_{3e} E_4 - E_{3e} \hat{P}_3 \cdot (\underline{P}_1 - \underline{P}_2)}} \frac{d\sigma}{d\Omega_3}$$

$$\frac{d\sigma}{d\Omega_k d\omega d\Omega_3} = \frac{d\sigma}{(2\pi)^2} \frac{P_3^2}{P_{3e}^2} \frac{M\omega}{E_4 P_3 - E_3 \hat{P}_3 \cdot (\underline{P}_1 - \underline{P}_2 - \underline{k})} \frac{P_{3e} E_4 - E_{3e} \hat{P}_3 \cdot (\underline{P}_1 - \underline{P}_2)}{E_4 P_3 - E_3 \hat{P}_3 \cdot (\underline{P}_1 - \underline{P}_2 - \underline{k})} \left[\frac{2E_1}{\omega(\underline{P}_1 \cdot \underline{k})} - \frac{\mu^2}{(\underline{P}_1 \cdot \underline{k})^2} - \frac{1}{\omega^2} \right] \frac{d\sigma}{d\Omega_3}$$

$\hat{P}_3 = \hat{P}_{3e}$

Take z along the direction of the incident beam, & the xz plane in the P_1, P_3 scattering plane.

$$\hat{P}_3 \cdot (\underline{P}_1 - \underline{P}_2 - \underline{k}) = P_1 \cos \theta_3 - P_2 - \omega (\cos \theta_3 \cos \theta_k - \sin \theta_3 \sin \theta_k \cos \phi_k)$$

$$\hat{P}_{3e} \cdot (\underline{P}_1 - \underline{P}_{3e}) = P_1 \cos \theta_{3e} - P_{3e}$$

$$\underline{P}_1 \cdot \underline{k} = \omega (E_1 - P_1 \cos \theta_k)$$

⑦

The measured cross section is obtained by integrating over all ω, Ω_k for which the photon is not detected:

$$\frac{d\sigma_m}{d\Omega_3} = \iint d\omega d\Omega_k \frac{d^3\sigma}{d\Omega_3 d\Omega_k d\omega}$$

minimum energy of photon which counts in "anti" chamber

The region of integration includes zero energy photons which make the above expression for $\frac{d^3\sigma}{d\Omega_3 d\Omega_k d\omega}$ blow up, so use the following method:

Get ω as a fun. of θ_k :

$$P_1 + P_2 = P_3 + P_4 + k$$

$$(P_1 + P_2 - P_3 - k)^2 = P_4^2$$

$$2\mu^2 + 2M(E_1 - E_2) - 2P_1 P_3 - 2k \cdot (P_1 + P_2 + P_3) = 0$$

$$\mu^2 + M(E_1 - E_2) - E_1 E_3 + P_1 P_3 - \omega (E_1 + M - E_3) + \omega \hat{k} \cdot (\underline{P}_1 - \underline{P}_2) = 0$$

$$\omega = \frac{\mu^2 + M(E_1 - E_2) - E_1 E_3 + P_1 P_3}{E_1 + M - E_3 - \hat{k} \cdot (\underline{P}_1 - \underline{P}_2)}$$

For E_1, E_3 fixed can show that ω is ⑧
maximum in the forward direction.

We can therefore write

$$\frac{d\sigma_m}{d\Omega} = \iint \frac{d^3\sigma}{d\Omega_3 d\Omega_4 d\omega} = \iint_I + \iint_{II}$$

$$\iint_I = \int \int d\Omega_4 d\omega \frac{d^3\sigma}{d\Omega_3 d\Omega_4 d\omega} = \int_{p_{30}}^{\beta_2} dp_3 \int \frac{d^3\sigma}{d\Omega_3 d\Omega_4 dp_3}$$

$$= \frac{d\sigma}{d\Omega} e^{s(p_{3e} - p_{30})}$$

where $s(p_{3e} - p_{30})$ is the relative correction for
CEX scattering and is given on the next
page - variables are defined as in my paper.

$\iint_{II}$ can be done numerically using the
expression derived above.

Charge exchange ⑧

$$S = + \frac{\alpha}{\pi} \left\{ \left(\frac{1}{2\beta_1} \ln Q_1 - 1 \right) \left(\ln \frac{E_4}{\gamma \Delta E} + \ln \frac{E_3}{\gamma \Delta E'} \right) + \right.$$

$$\left. \frac{1}{\beta_1} \left[\ln \left| \frac{C_{33}}{E_3 a_1} \right| \ln R_1^+ - \ln \left| \frac{C_{34}}{E_3 a_1} \right| \ln R_1^- \right] \right.$$

$$- \phi \left(\frac{m(m - E_1^+)(E_3 - E_4)}{C_{33}} \right) + \phi \left(\frac{m(m - E_1^-)(E_3 - E_4)}{C_{34}} \right)$$

$$- \phi \left(\frac{(mE_1^+ - \mu^2)(E_3 - E_4)}{C_{34}} \right) + \phi \left(\frac{(mE_1^- - \mu^2)(E_3 - E_4)}{C_{33}} \right) \}$$

APPENDIX II

Computer Program Listing

```

FILE: PRINT      SYSIN      P1                      BROWN UNIVERSITY COMPUTING LABORATORY

C      OMEGA..CALCULATES BREM ENERGY AS A FUNCTION OF          OME00010
C      PZ SCATTERING ANGLE AND BREM ANGLE WRT BEAM             OME00020
C      G CHEN      JULY 1973                                     OME00030
C                                                                OME00040
C      COMMON/KINE/E1,P1,M1,E2,P2,M2,E3,P3,M3,E4,P4,M4         OME00050
C      DIMENSION BDCUS(3),DCUS3(3),DCUS4(3),DCMS3(3),DCMS4(3)   OME00060
C      DATA BDCUS/0.,0.,1./                                     OME00070
C      REAL M1,M2,M3,M4                                          OME00080
C      RTOD=360./(2*3.14159)                                     OME00090
C      M1=.1395683                                                OME00100
C      M2=.9382592                                                OME00110
C      M3=.1349645                                                OME00120
C      M4=.9395527                                                OME00130
C      W0=.01                                                      OME00140
C      P1=6.0                                                      OME00150
C      DO 400 I=1,1                                                OME00160
C      E1=SQRT(P1*P1+M1*M1)                                         OME00170
C      P2=0.                                                         OME00180
C      E2=M2                                                         OME00190
C      CTHETA=-1.0                                                  OME00200
C      WRITE (8,102) P1                                             OME00210
102  FORMAT (5X,'PMIM=',F10.5)                                     OME00220
C      WRITE(8,101)                                                 OME00230
101  FORMAT (//,5X,'CTHETA',4X,'THETK',10X,'W',6X,               OME00240
C      -'DCMS3',6X,'DCMS4',6X,'RA',10X,'RADEG')                   OME00250
C      DO 200 I=1,21                                                OME00260
C      STHETA=SQRT(1.-CTHETA*CTHETA)                                OME00270
C      AA=-W0*(E1+M2)+M1*M1+M2*E1+P1*W0                           OME00280
C      BB=-(W0-M2-E1)                                               OME00290
C      CC=W0*CTHETA-P1*CTHETA                                       OME00300
C      A2=AA*AA                                                      OME00310
C      B2=BB*BB                                                      OME00320
C      C2=CC*CC                                                      OME00330
C      TEST=SQRT(4.*A2*B2-4.*(B2-C2)*(A2+C2*M3*M3))              OME00340
C      IF(TEST.LT.0.) GO TO 200                                       OME00350
C      IF(CTHETA.GT.0.) TEST=-TEST                                   OME00360
C      CHANGE MADE 9/5/73                                           OME00370
C      E3P=(2.*AA*BB-TEST)/(2.*(B2-C2))                             OME00380
C      P3P=SQRT(E3P*E3P-M3*M3)                                       OME00390
C      XM=.5*(M1*M1+M2*M2+M3*M3-M4*M4)                             OME00400
C      THETK=0.0                                                     OME00410
C      DO 300 J=1,21                                                OME00420
C      COSK=CUS(THETK/RTOD)                                         OME00430
C      SINK=SQRT(1.-COSK*COSK)                                       OME00440
C      A=M1*M1+M2*(E1-E3P)-E1*E3P+P1*P3P*CTHETA                 OME00450
C      T1=P3P*(COSK*CTHETA-SINK*STHETA)                             OME00460
C      T2=-P1*COSK                                                  OME00470
C      T3=T1+T2                                                      OME00480
C      B=E1+M2-E3P-P1*COSK+P3P*(COSK*CTHETA-SINK*STHETA)         OME00490
C      W=A/B                                                         OME00500
C      E4=E1+E2-E3P-W                                               OME00510
C      P4=SQRT(E4*E4-M4*M4)                                         OME00520
C      CALCULATE DIRECTION COSINES OF PARTICLES 3 AND 4          OME00530
C      THIS IS PLANAR CASE WHERE PHI=0.0*****                   OME00540
C      DCOS4(3)=(P1-P3P *CTHETA-W*COSK)                             OME00550

```

5/23/74

FILE: PRINT SYSIN P1 BREWN UNIVERSITY COMPUTING LABORATORY

```

      DCOS4(2)=0.0
      DCOS4(1)=- (P3*STHETA+W*SINK)
C     CALCULATE NORMALIZATION FOR DIRECTION COSINES OF PARTICLE 4
C
      XMAG=0.0
      DO 501 K=1,3
501    XMAG=XMAG+DCOS4(K)*DCOS4(K)
      XMAG=SQRT(XMAG)
      DO 502 K=1,3
502    DCOS4(K)=DCOS4(K)/XMAG
      DCOS3(3)=CTHETA
      DCOS3(2)=0.0
      DCOS3(1)=STHETA
      BETACM=P1/(E1+E2)
      CALL LGRTA(BETACM,BDCOS,E3P,P3P,DCOS3,E3CMS,P3PCMS,DCMS3)
      CALL LGRTA(BETACM,BDCOS,E4,P4,DCOS4,E4CMS,P4CMS,DCMS4)
C     CALCULATE RA IN CMS
      RA=0.0
      DO 500 IK=1,3
500    RA=RA-DCMS3(IK)*DCMS4(IK)
      RADEG=RTUD*ARCOS(RA)
C     WRITE (6,100) CTHETA,THETK,W,E3P,P3P,E4,P4,RA
      WRITE(8,100) CTHETA,THETK,W,DCMS3,DCMS4,RA,RADEG
100    FORMAT (5X,11F10.5)
300    THETK=THETK+5.
200    CTHETA=CTHETA+.1
400    P1=P1+1.0
      STOP
      END

```

```

      OME00560
      OME00570
      OME00580
      OME00590
      OME00600
      OME00610
      OME00620
      OME00630
      OME00640
      OME00650
      OME00660
      OME00670
      OME00680
      OME00690
      OME00700
      OME00710
      OME00720
      OME00730
      OME00740
      OME00750
      OME00760
      OME00770
      OME00780
      OME00790
      OME00800
      OME00810
      OME00820
      OME00830
      OME00840

```

FILE: RADCOR FORTRAN P1 BROWN UNIVERSITY COMPUTING LAB

```

C     RADCOR..RADIATIVE CORRECTIONS PROGRAM..MAIN
C     G CHEN
C     JULY 27, 1973
      COMMON/KINE/E1,P1,M1,E2,P2,M2,E3,P3,M3,E4,P4,M4,DP3
      COMMON/TERM/TERMA,TERMB,TERMC
      REAL M1,M2,M3,M4
C     INCIDENT BEAM IS P1 IN LABORATORY
      WOS=.005
      M1=.1395688
      M2=.9382591
      M3=.1349645
      M4=.9395527
      WRITE(8,104)
104    FORMAT (5X,'RADIATIVE CORRECTIONS..SCHWINGER TERM',///)
C     LOOP OVER BREM ENERGY
      DO 400 JJ=1,7
C     LOOP OVER MOMENTUM
      DO 300 II=1,1
C     W0 IS THRESHOLD OF SHOWER COUNTER IN GEV
      IF(II.EQ.1) P1=8.
      IF(II.EQ.2) P1=3.5
      IF(II.EQ.3) P1=4.3
      IF(II.EQ.4) P1=6.0
      E1=SQRT(P1*P1+M1*M1)
      P2=0.0
      E2=M2
C     CTHETA IS THE COSINE OF THE SCATTERING ANGLE PARTICLE 3
      CTHETA=-1.
      S=M1*M1+M2*M2+2.*E1*M2
      WRITE(8,102) P1,WOS,S
102    FORMAT (/,5X,'P1=',F10.5,5X,'W0=',F10.5,5X,'S=',F10.5/)
      WRITE(8,101)
101    FORMAT (1X,'CTHETA',4X,'T',5X,'U',5X,'DEL',6X,'CHNG',
     -      'E4    P4    DP    E3    P3    PCT',/)
C     LOOP OVER CTHETA..ANGLE OF P1 ZERO
      DO 200 I=1,21
      DO 201 IJ=1,2
C     IJ=1..CALCULATES ELASTIC VALUE
C     IJ=2..CALCULATES VALUE OF P3 WITH BREM ENERGY W0
C     ASSUMES BREM ANGLE COSK=1.0 FORWARD
      IF(IJ.EQ.1) W0=0.
      IF(IJ.EQ.2) W0=WOS
      STHETA=SQRT(1.-CTHETA*CTHETA)
      XM=.5*(M1*M1+M2*M2+M3*M3-M4*M4)
      AA=XM*M2*E1+P1*W0-W0*(E1+M2)
      BB=-(W0-M2-E1)
      CC=W0*CTHETA-P1*CTHETA
      A2=AA*AA
      B2=BB*BB
      C2=CC*CC
      TEST=(4.*A2*B2-4.*(B2-C2)*(A2+C2*M3*M3))
      IF(CTHETA.GT.-.001.AND.CTHETA.LT..001) TEST=0.0
C     ABOVE TEST DUE TO ROUND OFF PROB..1/74
      IF(TEST.LT.0.) TEST=0.0
      TEST=SQRT(TEST)

```

5/23/74

FILE: RADCOR FORTRAN P1

BROWN UNIVERSITY COMPUTING LAB

```

      IF(CTHETA.GT.0.0) TEST=-TEST
      IF(CTHETA.EQ.0.0) TEST=0.
      C *****KINEMATIC SINGULARITY AT CTHETA=0.
      E3P=(2.*AA*BB-TEST)/(2.*(B2-C2))
      E3PP=(2.*AA*BB+TEST)/(2.*(B2-C2))
      P3P=SQRT(E3P*E3P-M3*M3)
      P3PP=SQRT(E3PP*E3PP-M3*M3)
      C STORE ELASTIC VALUES
      IF(IJ.EQ.1) E3=E3P
      IF(IJ.EQ.1) P3=P3P
      201 CONTINUE
      DP3=P3-P3P
      T=M1*M1+M3*M3-2.*(E1*E3-P1*P3*CTHETA)
      U=M1*M1+M2*M2+M3*M3+M4*M4-S-1
      E4=E1+E2-E3-W0
      P4=(E4*E4-M4*M4)
      C IF(P4.LT.0.) WRITE(8,103) P4
      103 FORMAT ('>X,'P4 NEG',F10.5)
      IF(P4.LT.0.) P4=0.
      P4=SQRT(P4)
      CALL DELTA(DEL,CTHETA)
      CHNG=EXP(DEL)-1.
      C WRITE(6,1777) CTHETA,TEST,E3,E3P,P3,P3P,DP3
      1777 FORMAT(1X,E12.4,6F8.5)
      PCT=EXP(-DEL)
      202 WRITE(8,100) CTHETA,T,U,DEL,CHNG,E4,P4,DP3,E3P,P3P,PCT
      100 FORMAT(1X,F4.1,6F8.3,E12.5,9F8.3)
      CTHETA=CTHETA+.1
      200 CONTINUE
      300 CONTINUE
      400 WOS=WOS+.005
      STOP
      END

```

just to look at
unphysical solution

FILE: DELTA FORTRAN P1

BROWN UNIVERSITY COMPUTING LAB

```

      SUBROUTINE DELTA(DELTA,CTHETA)
      C CALCULATES DELTA IN RADIATIVE CORRECTION FORMULA
      C G CHEN
      C JUNE 1973
      C
      COMMON/KINE/E1,P1,M1,E2,P2,M2,E3,P3,M3,E4,P4,M4,DP3
      COMMON/TERM/TERMA,TERMB,TERMC
      REAL M1,M2,M3,M4
      ALPHA=1./137.
      PI=3.1416
      H1=P1/E1
      QA=(P1+E1-M1*M1/(M2))/(P1-E1*M1*M1/M2)
      QB=(P1+E1-M2)/(P1-E1*M2)
      Q1=QA*QB
      A=ALOG(Q1)
      A=A/(2.*B1)
      A=A-1.
      ETA=E1/E3
      DE=(1.+E1/M2-(P1*E3*CTHETA)/(M2*P3))
      C$$$$$$$
      P3=P3
      DP3=DP3
      E1=E1
      C$$$$$$$
      DE=DE*P3*DP3/E1
      B=ALOG(ABS(E4/(ETA*DE)))+ALOG(ABS(E3/(ETA*DE)))
      E3P=E3+P3
      E3M=E3-P3
      E1P=E1+P1
      E1M=E1-P1
      C33=M2*M2*E3-M2*E3*E1P+M1*M1*E4-E4*M2*E1M
      C34=M2*M2*E3-M2*E4*E1P+M1*M1*E4-E3*M2*E1M
      A1=M2*M2+M1*M1-2*M2*E1
      R1P=M2*(M2-E1P)/(M2*E1M-M1*M1)
      R1M=M2*(M2-E1M)/(M2*E1P-M1*M1)
      C=(ALOG(ABS(C33/(E3*A1))))*ALOG(R1P)-(ALOG(ABS(C34/(E3*A1))))
      -*ALOG(R1M)
      X=(M2*(M2-E1P)*(E3-E4))/C33
      SP1=SP(X)
      X=(M2*(M2-E1M)*(E3-E4))/C34
      SP2=SP(X)
      X=(M2*E1P-M1*M1)*(E3-E4)/C34
      SP3=SP(X)
      X=(M2*E1M-M1*M1)*(E3-E4)/C33
      SP4=SP(X)
      TERMA=A*B
      TERMB=C/B1
      TERMC=-SP1+SP2-SP3+SP4
      DELTA=(ALPHA/PI)*(A*B+C/B1-SP1+SP2-SP3+SP4)
      RETURN
      END

```

} for debug init

FILE: SPENCE FORTRAN P1

BROWN UNIVERSITY COMPUTING LABOR

```

FUNCTION SP(X)
C
C CALCULATES SPENCE FUNCTION
C THIS SUBROUTINE CALCULATES THE INTEGRAL FROM 0 TO X OF
C (-LOG(ABS(1-Y)) DY)/Y
C
C
C IACC REPRESENTS THE NUMBER OF DIGITS OF ACCURACY IN THE RESULT.
C IF FEWER SIGNIFICANT FIGURES ARE NEEDED, CHANGE THE FOLLOWING CARD
C
IACC=8
IF(ABS(X)-1.) 10,15,20
15 IF(X) 25,30,35
25 SP=-.82246703
RETURN
35 SP=1.6449341
RETURN
20 ARG=1./X
GO TO 50
10 IF(X) 40,30,40
30 SP=0.
RETURN
40 ARG=X
50 TEST=ARG/4.
GOPX=TEST
N=2
60 FN=N
TERM=(ARG**N)/((FN**2)*(FN+1.)**2)
COMP=ABS(TEST/TERM)
IF(COMP-10.**IACC) 70,80,80
70 GOPX=GOPX+TERM
N=N+1
GO TO 60
80 SP=(ARG*(3.+GOPX)+2.*(1.-ARG)*ALOG(1.-ARG))/(1.+ARG)
IF(ABS(ARG).GT.1.E-3) GO TO 85
SP=ARG*(3.+GOPX-2.*(1.-ARG))/(1.+ARG)
85 IF(ABS(X)-1.) 110,15,90
90 TERMS=-0.5*(ALOG(ABS(X))**2-SP
IF(X) 120,30,130
120 SP=-(3.1415927)**2/6.+TERMS
RETURN
130 SP=(3.1415927)**2/3.+TERMS
110 RETURN
END

```

MS
MS
MS

INTEG2

FILE: INTEG2 FORTRAN P1

BROWN UNIVERSITY COMPUTING LABORATORY

```

C INTEG2..DOES NUMERICAL INTEGRATION FOR RADIATIVE
C CORRECTIONS IN REGION 2
C G CHEN OCT. 17, 1973
C RUN COMMENT
C PUR WO AND COSK IN WEIGHTED VALUE SEE CALC
C
COMMON/KINE/E1,P1,M1,E2,P2,M2,E3P,P3P,M3,E4,P4,M4
COMMON/SAVE/SINK,CUSK,E3E,P3E,CTHETA,STHETA,DCOSK
REAL M1,M2,M3,M4
PTOD=360./(2*3.14159)
M1=.1395688
M2=.9382592
M3=.1349645
M4=.9395527
XM=.5*(M1*M1+M2*M2+M3*M3+M4*M4)
NEN=11
DCOSK=.05
THRESH=.010
DO 500 ITH=1,4
C LOOP OVER THE THRESHOLD
DW=THRESH/10.
C ONCE FOR ELASTIC VALUES, ONE FOR REGION 2
PI=2.0
C LIMIT OF SHOWER COUNTER IS 35 DEGREES
XLIM=180./PTOD
C LOOP OVER MOMENTUM
DO 400 II=1,5
E1=SQRT(PI*PI+M1*M1)
P2=0.
E2=M2
WRITE(6,102) P1,THRESH,NEN
102 FORMAT (//,2X,'PMUM=',F10.5,3X,'THRESH=',F8.5,5X,'NDCOSK=',I5)
WRITE(6,101)
101 FORMAT (/,1X,'CTHETA E3E P3E E30 P30
-E4 P4 SUM NDCOSK')
C LOOP OVER CTHETA
CTHETA=-1.0
DO 200 I=1,21
SUM=0.0
STHETA=SQRT(1.-CTHETA*CTHETA)
C CALCULATE P30 FOR FORWARD BREM
WO=THRESH
AA=-WO*(E1+M2)+XM+M2*E1+P1*WO
BB=-(WO-M2-E1)
CC=WO*CTHETA-P1*CTHETA
A2=AA*AA
B2=BB*BB
C2=CC*CC
TEST=(4.*A2*B2-4.*(B2-C2)*(A2+C2*M3*M3))
IF(TEST.GE.0.) TEST=SQRT(TEST)
IF(TEST.LT.0.) GO TO 302
IF(CTHETA.GT.0.) TEST=-TEST
E30=(2.*AA*BB-TEST)/(2.*(B2-C2))
C CHECK FOR PHYSICAL SOLUTION
P30=SQRT(E30*E30-M3*M3)

```

INT0001
INT0002
INT0003
INT0004
INT0005
INT0006
INT0007
INT0008
INT0009
INT0010
INT0011
INT0012
INT0013
INT0014
INT0015
INT0016
INT0017
INT0018
INT0019
INT0020
INT0021
INT0022
INT0023
INT0024
INT0025
INT0026
INT0027
INT0028
INT0029
INT0030
INT0031
INT0032
INT0033
INT0034
INT0035
INT0036
INT0037
INT0038
INT0039
INT0040
INT0041
INT0042
INT0043
INT0044
INT0045
INT0046
INT0047
INT0048
INT0049
INT0050
INT0051
INT0052
INT0053
INT0054
INT0055
INT0056
INT0057
INT0058
INT0059
INT0060

INTEB2

BROWN UNIVERSITY COMPUTING LABORATORY

[illegible]

```

IF(1A.EQ.1) E3E=E3P
IF(1A.EQ.1) P3E=P3P
IF(1A.EQ.1) GO TO 302
E4=E1+E2-E3P-W0
TESTA=E4*E4-M4*M4
IF(TESTA.LT.0.) WRITE(6,203)
203 FORMAT(5X,'P4 NEG',/)
IF(TESTA.LT.0.) GO TO 302
P4=SQRT(E4*E4-M4*M4)
CALL CALC(VAL,W0)
SUM=SUM+VAL*DCOSK*DW
DSUM=VAL*DCOSK*DW
C WRITE(6,100) THETA,CUSK,W0,E3P,P3P,E4,P4,VAL,SUM,DSUM,IA
100 FORMAT(2X,2F10.5,8E12.3,15)
300 COSK=COSK-DCUSK
C WRITE(6,100) THETA,CUSK,W0,E3P,P3P,E4,P4,VAL,SUM,DSUM,IA
302 W0=W0+DW
SUM=SUM*2.*314.159
C 314. IS PI TIMES 100..SUM IS IN PERCENT
WRITE(6,105) CTHETA,E3E,P3E,E30,P30,E4,P4,SUM,NDCOSK
104 FORMAT(1X)
105 FORMAT(1X,F0.3,7F10.5,15)
301 CONTINUE
200 CTHETA=CTHETA+.1
400 PI=PI+1.0
500 THRESH=THRESH+.01
GO TO 3000
202 FORMAT(5X,'NEG P3',F10.5,2I5)
3000 WRITE(6,103)
103 FORMAT(/,5X,'END OF RUN')
STOP
END

```

INTEG

FILE: INTEG FURTRAN P1 BROWN UNIVERSITY COMPUTING LABORATORY

```

C    INTEG..DUES NUMERICAL INTEGRATION FOR RADIATIVE    INT0001
C    CORRECTIONS    INT0002
C    G CHEN    UCT. 11, 1973    INT0003
C    RUN COMMENT    INT0004
C    PUT W0 AND COSK IN MIDDLE OF INTEGRATING BINS    INT0005
C    REVISED 11/26/73 TO BE DCOSK FLEXIBLE    INT0006
C    REVISED TO USE WAVG INSTEAD OF MIDDLE OF BIN    INT0007
C    COMMON/KINE/E1,P1,M1,E2,P2,M2,E3P,P3P,M3,E4,P4,M4    INT0008
C    COMMON/SAVE/SINK,COSK,E3E,P3E,CTHETA,STHETA,DCOSK    INT0009
C    COMMON/SAVE2/WU,WL    INT0010
C    REAL M1,M2,M3,M4    INT0011
C    TWOP1=2.0*3.14159    INT0012
C    RTOD=360./TWOP1    INT0013
C    M1=.1395688    INT0014
C    M2=.9382592    INT0015
C    M3=.1349645    INT0016
C    M4=.9395527    INT0017
C    NEN..NUMBER OF ENERGY BINS    INT0018
C    NEN=201    INT0019
C    DCOSK=.0250    INT0020
C    WRITE(6,996)    INT0021
996    FORMAT(1H1,5X,'RADCUK PERCENT REGION 3B AND 2B ONLY')    INT0022
C    READ(5,998) NEN    INT0023
C    READ(5,997) DCOSK    INT0024
998    FORMAT(13)    INT0025
997    FORMAT(F10.5)    INT0026
C    NDCOSK=(COS(35./RTOD)+1.)/DCOSK    INT0027
C    THRESH=.005    INT0028
C    DO 500 IT=1,5    INT0029
C    P1=2.0    INT0030
C    LIMIT OF SHOWER COUNTER IS 35 DEGREES    INT0031
C    XLIM=180./RTOD    INT0032
C    LOOP OVER MOMENTUM    INT0033
C    DO 400 II=1,1    INT0034
C    E1=SQRT(P1*P1+M1*M1)    INT0035
C    P2=0.    INT0036
C    E2=M2    INT0037
C    XM=.5*(M1*M1+M2*M2+M3*M3-M4*M4)    INT0038
C    WRITE(6,102) P1,THRESH,NEN,NDCOSK    INT0039
102    FORMAT (///,5X,'PMUM=',F10.5,5X,'THRESHOLD=',F6.5,5X,'NEN,NDCOSK',    INT0040
-215)    INT0041
C    WRITE (6,101)    INT0042
101    FORMAT (/,2X,'CTHETA    E3E    P3E    DW    DCOSK    WMAINT0043
-X    SUM')    INT0044
C    LOOP OVER CTHETA    INT0045
C    CTHETA=-1.    INT0046
C    DO 200 I=1,21    INT0047
C    SUM=0.0    INT0048
C    STHETA=SQRT(1.-CTHETA*CTHETA)    INT0049
C    W0=0.    INT0050
C       INT0051
C       INT0052
C       INT0053
C       INT0054
C       INT0055

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INTEG

FILE: INTEG FURTRAN P1 BROWN UNIVERSITY COMPUTING LABORATORY

```

C    LOOP OVER BREM ENERGY    INT0056
C    DO 302 IA=1,NEN    INT0057
C    COSK=COS(35./RTOD)-DCOSK/2.    INT0058
C    LOOP OVER COSK    INT0059
C    DO 300 J=1,NDCOSK    INT0060
C    IF(ARCOS(COSK).GT.XLIM) WRITE (6,106)    INT0061
106    FORMAT (5X,'XLIM EXCEEDED')    INT0062
C    IF(ARCOS(COSK).GT.XLIM) GO TO 302    INT0063
C    SINK=SQRT(1.-COSK*COSK)    INT0064
C    AA=-W0*(E1+M2)+XM+M2*E1+P1*W0*COSK    INT0065
C    CALCULATE VALUES OF P AND E GIVEN W0,COSK,CTHETA    INT0066
C    BB=-(W0-M2-E1)    INT0067
C    CC=W0*(CTHETA*COSK+SINK*STHETA)-P1*CTHETA    INT0068
C    A2=AA*AA    INT0069
C    B2=BB*BB    INT0070
C    C2=CC*CC    INT0071
C    TEST=(4.*A2*B2-4.*(B2-C2)*(A2+C2*M3*M3))    INT0072
C    IF(TEST.GE.0.) TEST=SQRT(TEST)    INT0073
C    IF(TEST.LT.0.) WRITE(6,202) SUM,IA,J    INT0074
C    IF(TEST.LT.0.) GO TO 302    INT0075
C    IF(CTHETA.GT.0.) TEST=-TEST    INT0076
C    E3P=(2.*AA*BB-TEST)/(2.*(B2-C2))    INT0077
C    IF(E3P.LT.M3) WRITE (6,107) IA,J    INT0078
107    FORMAT (5X,'E3P.LT.M3',215)    INT0079
C    IF(E3P.LT.M3) GO TO 302    INT0080
C    CHECK FOR PHYSICAL SOLUTION    INT0081
C    P3P=SQRT(E3P*E3P-M3*M3)    INT0082
C    STORE ELASTIC VALUES    INT0083
C    IF(1A.NE.1) GO TO 109    INT0084
C    E3E=E3P    INT0085
C    P3E=P3P    INT0086
C    WMAX=(XM+M2*(E1-M3)-E1*M3)/(E1+M2-M3-P1)    INT0087
C    DW=(WMAX-THRESH)/FLOAT(NEN-1)    INT0088
C    W0=W0+THRESH-DW/2.    INT0089
C    GO TO 302    INT0090
109    E4=E1+E2-E3P-W0    INT0091
C    IF(E4.LT.0.) WRITE(6,108) E4    INT0092
108    FORMAT(5X,'E4 LT 0.',F10.5)    INT0093
C    IF(E4.LT.0.) GO TO 302    INT0094
C    TESTA=E4*E4-M4*M4    INT0095
C    IF(TESTA.LT.0.) WRITE(6,203)    INT0096
203    FORMAT(5X,'P4 NEG',/)    INT0097
C    IF(TESTA.LT.0.) GO TO 302    INT0098
C    P4=SQRT(E4*E4-M4*M4)    INT0099
C    WL=W0-DW/2.    INT0100
C    WU=W0+DW/2.    INT0101
C    WAVG=W0    INT0102
C    CALL CALC(VAL,WAVG)    INT0103
C    SUM=SUM+VAL*DCOSK*DW    INT0104
C    DSUM=VAL*DCOSK*DW    INT0105
C    WRITE(6,100)CTHETA,COSK,W0,E3P,P3P,E4,P4,VAL,SUM,DSUM,IA    INT0106
100    FORMAT(2X,2F10.5,8E12.3,15)    INT0107
C       INT0108
C       INT0109
C       INT0110
C       INT0111

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INTEG

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FILE: INTEG      FORTRAN P1      BROWN UNIVERSITY COMPUTING LABORATORY

300 COSK=COSK-DCOSK
C WRITE(6,100) THETA,COSK,W0,E3P,P3P,E4,P4,VAL,SUM,DSUM,IA
302 W0=W0+DW
SUM=SUM*TWOPI*100.
WRITE(6,105) CTHETA,E3E,P3E,DW,DCOSK,WMAX,SUM
104 FORMAT(1X)
105 FORMAT (F8.3,6F10.5)
200 CTHETA=CTHETA+.1
400 P1=P1+1.0
500 THRESH=THRESH+.005
GO TO 3000
202 FORMAT(5X,'NEG P3',F10.5,2I5)
3000 WRITE(6,103)
103 FORMAT(//,5X,'EOJ')
STOP
END

```

INT0111
INT0112
INT0113
INT0114
INT0115
INT0116
INT0117
INT0118
INT0119
INT0120
INT0121
INT0122
INT0123
INT0124
INT0125
INT0126

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FILE: INTEG3A  FORTRAN P1      BROWN UNIVERSITY COMPUTING LABORATORY

C INTEG..DOES NUMERICAL INTEGRATION FOR RADIATIVE
C CORRECTIONS
C G CHEN OCT. 11, 1973
C RUN COMMENT
C PUT W0 AND COSK IN MIDDLE OF INTEGRATING BINS
C REVISED 11/26/73 TO BE DCOSK FLEXIBLE
C REVISED TO USE WAVE INSTEAD OF MIDDLE OF BIN
COMMON/KINE/E1,P1,M1,E2,P2,M2,E3P,P3P,M3,E4,P4,M4
COMMON/SAVE/SINK,COSK,E3E,P3E,CTHETA,STHETA,DCOSK
COMMON/SAVE2/WU,WL
REAL M1,M2,M3,M4
TWOPI=2.0*3.14159
RTOD=360./TWOPI
M1=.1395688
M2=.9382592
M3=.1349645
M4=.9395527
XM=.5*(M1*M1+M2*M2+M3*M3+M4*M4)
C NEN..NUMBER OF ENERGY BINS
NEN=201
DCOSK=.0250
WRITE(6,996)
996 FORMAT(5X,'RADCOR PERCENT SHOWER COUNTER ONLY')
C READ(5,998) NEN
C READ(5,997) DCOSK
998 FORMAT(I3)
997 FORMAT(F10.5)
NDCOSK=2./DCOSK
THRESH=.03
DO 500 IT=1,1
C LIMIT OF SHOWER COUNTER IS 35 DEGREES
XLIM=35./RTOD
C
C LOOP OVER MOMENTUM
C
DO 400 II=1,1
IF(II.EQ.1) P1=2.0
IF(II.EQ.2) P1=3.5
IF(II.EQ.3) P1=4.3
IF(II.EQ.4) P1=8.0
E1=SQRT(P1*P1+M1*M1)
P2=0.
E2=M2
WRITE(6,102) P1,THRESH,NEN,NDCOSK
102 FORMAT (1H1,5X,'PMOM=',P10.5,5X,'THRESHOLD=',P6.5,5X,'NEN,NDCOSK',
-2I5)
WRITE (6,101)
101 FORMAT(/,2X,'CTHETA E3E P3E DW DCOSK WMAINT00480
-X SUM')
C
C LOOP OVER CTHETA
C
CTHETA=-.01
DO 200 I=1,3
SUM=0.0

```

INT00010
INT00020
INT00030
INT00040
INT00050
INT00060
INT00070
INT00080
INT00090
INT00100
INT00110
INT00120
INT00130
INT00140
INT00150
INT00160
INT00170
INT00180
INT00190
INT00200
INT00210
INT00220
INT00230
INT00240
INT00250
INT00260
INT00270
INT00280
INT00290
INT00300
INT00310
INT00320
INT00330
INT00340
INT00350
INT00360
INT00370
INT00380
INT00390
INT00400
INT00410
INT00420
INT00430
INT00440
INT00450
INT00460
INT00470
INT00480
INT00490
INT00500
INT00510
INT00520
INT00530
INT00540
INT00550

FILE: INTEG3A FORTRAN P1

BROWN UNIVERSITY COMPUTING LABORATORY

```

      STHETA=SQRT(1.-CTHETA*CTHETA)
      W0=0.
C
C      LOOP OVER BREM ENERGY
C
      DO 302 IA=1,NEN
      COSK=1.0-DCOSK/2.
C
C      LOOP OVER COSK
C
      DO 300 J=1,NDCOSK
      IF(ARCOS(COSK).GT.XLIM) WRITE(6,106)
106  FORMAT(5X,'XLIM EXCEEDED')
      IF(ARCOS(COSK).GT.XLIM) GO TO 302
      SINK=SQRT(1.-COSK*COSK)
      AA=-W0*(E1+M2)+XM+M2*E1+P1*W0*COSK
C      CALCULATE VALUES OF P AND E GIVEN W0,COSK,CTHETA
      BB=-(W0-M2-E1)
      CC=W0*(CTHETA*COSK+SINK*STHETA)-P1*CTHETA
      A2=AA*AA
      B2=BB*BB
      C2=CC*CC
      TEST=(4.*A2*B2-4.*(B2-C2)*(A2+C2*M3*M3))
      IF(CTHETA.GT.-.001.AND.CTHETA.LT..001) TEST=0.
      IF(TEST.GE.0.) TEST=SQRT(TEST)
C      IF(TEST.LT.0.) WRITE(6,202) SUM,IA,J
      IF(TEST.LT.0.) GO TO 302
      IF(CTHETA.GT.0.) TEST=-TEST
      E3P=(2.*AA*BB-TEST)/(2.*(B2-C2))
C      IF(E3P.LT.M3) WRITE(6,107) IA,J
107  FORMAT(5X,'E3P.LT.M3',2I5)
      IF(E3P.LT.M3) GO TO 302
C      CHECK FOR PHYSICAL SOLUTION
      P3P=SQRT(E3P*E3P-M3*M3)
C      STORE ELASTIC VALUES
      IF(IA.NE.1) GO TO 109
      E3E=E3P
      P3E=P3P
      WMAX=(XM+M2*(E1-M3)-E1*M3)/(E1+M2-M3-P1)
      DW=(WMAX-THRESH)/FLOAT(NEN-1)
      W0=W0+THRESH-DW/2.
      GO TO 302
C
109  E4=E1+E2-E3P-W0
      IF(E4.LT.0.) WRITE(6,108) E4
108  FORMAT(5X,'E4 LT 0.',F10.5)
      IF(E4.LT.0.) GO TO 302
      TESTA=E4+E4-M4*M4
C      IF(TESTA.LT.0.) WRITE(6,203)
203  FORMAT(5X,'P4 NEG',/)
      IF(TESTA.LT.0.) GO TO 302
      P4=SQRT(E4+E4-M4*M4)
      WL=W0-DW/2.
      WU=W0+DW/2.
      WAVG=W0
      CALL CALC(VAL,WAVG)

```

```

      INT00560
      INT00570
      INT00580
      INT00590
      INT00600
      INT00610
      INT00620
      INT00630
      INT00640
      INT00650
      INT00660
      INT00670
      INT00680
      INT00690
      INT00700
      INT00710
      INT00720
      INT00730
      INT00740
      INT00750
      INT00760
      INT00770
      INT00780
      INT00790
      INT00800
      INT00810
      INT00820
      INT00830
      INT00840
      INT00850
      INT00860
      INT00870
      INT00880
      INT00890
      INT00900
      INT00910
      INT00920
      INT00930
      INT00940
      INT00950
      INT00960
      INT00970
      INT00980
      INT00990
      INT01000
      INT01010
      INT01020
      INT01030
      INT01040
      INT01050
      INT01060
      INT01070
      INT01080
      INT01090
      INT01100

```

FILE: INTEG3A FORTRAN P1

BROWN UNIVERSITY COMPUTING LABORATORY

```

      SUM=SUM+VAL*DCOSK*DW
      DSUM=VAL*DCOSK*DW
C      WRITE(6,100) CTHETA,COSK,W0,E3P,P3P,E4,P4,VAL,SUM,DSUM,IA
100  FORMAT(2X,2F10.5,8E12.3,I5)
300  COSK=COSK-DCOSK
C      WRITE(6,100) THETA,COSK,W0,E3P,P3P,E4,P4,VAL,SUM,DSUM,IA
302  W0=W0+DW
      SUM=SUM*TWOPI*100.
      WRITE(6,105) CTHETA,E3E,P3E,DW,DCOSK,WMAX,SUM
104  FORMAT(1X)
105  FORMAT(F8.3,6F10.5)
200  CTHETA=CTHETA+.01
400  CONTINUE
500  THRESH=THRESH+.010
      GO TO 3000
202  FORMAT(5X,'NEG P3',F10.5,2I5)
3000  WRITE(6,103)
103  FORMAT(/,5X,'EOJ')
      STOP
      END
      SUBROUTINE CALC(VAL,WAVG)
C      G CHEN SPT 1973
C      CALCULATES VALUE OF INTEGRAND FOR THE RADIATIVE
C      CORRECTIONS IN CEX..CORRESPONDS TO EVALUATION
C      OF INTEGRAL TWO FOR SPECIFIED INPUT VALUES
C      REVISED 11/26/73 TO BE DCOSK DEPENDENT
C      REVISED TO USE WAVG
C      REVISED TO FIND INTEGRAND AVERAGE WRT WAVG
      COMMON /KINE/E1,P1,M1,E2,P2,M2,E3P,P3P,M3,E4,P4,M4
      COMMON/SAVE/SINK,COSK,E3E,P3E,CTHETA,STHETA,DCOSK
      COMMON/SAVE2/WU,WL
      DATA ALPHA/.007297/,P22/39.478/
      REAL M1,M2,M3,M4
      T1=(ALPHA*P3P*P3P*M2*WAVG)/(P22*P3E*P3E)
      T21=P3E*E4-E3E*(P1*CTHETA-P3E)
      T22=E4*P3P-E3P*(P1*CTHETA-P3P-WAVG*(CTHETA*COSK+STHETA*SINK))
C      MODIFICATION ON NUMERICAL EVAL OF T3..AVERAGE VALUE
      COSU=COSK+DCOSK/2.
      COSL=COSK-DCOSK/2.
      T31=2*E1/(DCOSK*WAVG*WAVG*P1)
      T31=T31*ABS(ALOG(E1-P1*COSU)-ALOG(E1-P1*COSL))
      T32=M3*M3/(P1*WAVG*WAVG)
      T32=T32*ABS(1./(E1-P1*COSU)-1./(E1-P1*COSL))
      T33=1./(WAVG*WAVG)
      T3=T31-T32-T33
      VAL=T1*T21*T3/T22
      RETURN
      END

```

```

      INT01110
      INT01120
      INT01130
      INT01140
      INT01150
      INT01160
      INT01170
      INT01180
      INT01190
      INT01200
      INT01210
      INT01220
      INT01230
      INT01240
      INT01250
      INT01260
      INT01270
      INT01280
      INT01290
      INT01300
      INT01310
      INT01320
      INT01330
      INT01340
      INT01350
      INT01360
      INT01370
      INT01380
      INT01390
      INT01400
      INT01410
      INT01420
      INT01430
      INT01440
      INT01450
      INT01460
      INT01470
      INT01480
      INT01490
      INT01500
      INT01510
      INT01520
      INT01530
      INT01540
      INT01550
      INT01560
      INT01570
      INT01580

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FILE: CALC FORTRAN P1

BROWN UNIVERSITY COMPUTING LABORATORY

C	SUBROUTINE CALC (VAL,WAVG)	CAL00010
C	G CHEN SPT 1973	CAL00020
C	CALCULATES VALUE OF INTEGRAND FOR THE RADIATIVE	CAL00030
C	CORRECTIONS IN CEX..CORRESPONDS TO EVALUATEON	CAL00040
C	OF INTEGRAL TWO FOR SPECIFIED INPUT VALUES	CAL00050
C	REVISED 11/26/73 TO BE DCOSK DEPENDENT	CAL00060
C	REVISED TO USE WAVG	CAL00070
C	REVISED TO FIND INTEGRAND AVERAGE WRT WAVG	CAL00080
	COMMON /KINE/E1,P1,M1,E2,P2,M2,E3P,P3P,M3,E4,P4,M4	CAL00090
	COMMON/SAVE/SINK,COSK,E3E,P3E,CTHETA,STHETA,DCOSK	CAL00100
	COMMON/SAVE2/WU,WL	CAL00110
	DATA ALPHA/.007297/,P22/39.478/	CAL00120
	REAL M1,M2,M3,M4	CAL00130
	T1=(ALPHA*P3P*P3P*M2*WAVG)/(P22*P3E*P3E)	CAL00140
	T21=P3E*E4-E3E*(P1*CTHETA-P3E)	CAL00150
	T22=E4*P3P-E3P*(P1*CTHETA-P3P-WAVG*(CTHETA*COSK+STHETA*SINK))	CAL00160
C	MODIFICATION ON NUMERICAL EVAL OF T3..AVERAGE VALUE	CAL00170
C	IF COSU GT 1.0 SET TO 1.0	CAL00180
	IF (COSK.GT..9875) COSK=.9875	CAL00190
	COSL=COSK-DCOSK/2.	CAL00200
	COSU=COSK+DCOSK/2.	CAL00210
	T31=2*E1/(DCOSK*WAVG*WAVG*P1)	CAL00220
	T31=T31*ABS (ALOG (E1-P1*COSU) -ALOG (E1-P1*COSL))	CAL00230
	T32=M3*M3/(P1*WAVG*WAVG)	CAL00240
	T32=T32*ABS (1./(E1-P1*COSU) - 1./(E1-P1*COSL))	CAL00250
	T33=1./(WAVG*WAVG)	CAL00260
	T3=T31-T32-T33	CAL00270
	VAL=T1*T21*T3/T22	CAL00280
C	WRITE (6,100) CTHETA,COSK,VAL	CAL00290
100	FORMAT (5X,3P10.5)	CAL00300
	RETURN	CAL00310
	DEBUG INIT	CAL00320
	END	CAL00330

APPENDIX III

Computer Program Output from G. T. Y. Chen's runs,
see separate computer binder labelled "Radiative
Corrections".