Argonne National Laboratory

9700 South Cass Avenue

Argonne, Illinois 60439

REACTOR PHYSICS DIVISION ANNUAL REPORT

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Robert Avery, Division Director

Fred W. Thalgott, Deputy Division Director

Harry H. Hummel, Associate Division Director

W. C. Redman, Associate Division Director

William J. Sturm, Assistant Division Director

Lyman J. Templin, Assistant Division Director

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Foreword

This is the sixth successive issue of the Reactor Physics Division Annual Report and, in format, it simulates the preceding report, ANL-7410. As in previous reports, it emphasizes work performed by members of the Division, rather than integrated programmatic accomplishments. Nevertheless, when reasonable and possible, papers related to a given subject are grouped together within a Section.

In order to assist those who may wish to pursue a given subject more extensively, a list of pertinent references is included in each article. A catalog of open literature and report publications and of abstracts prepared by staff of the Reactor Physics Division during the reporting period are appended in Section VI to further aid those who may be interested in having additional information concerning work done in this Division.

In each of the past issues, there has been a Section on Thermal Reactor Physics. Since the work of the Division was not concerned with this subject during the year reported, no chapter on Thermal Reactor Physics is included in this issue.

In order to include all fast critical facility work at Argonne within one Division, the critical facilities and program at Argonne's Idaho Site were merged into an enlarged Reactor Physics Division in September 1968. This included the ZPPR and the ZPR-3 facilities and related programs. Thus, this issue of the Annual Report includes, for the first time, descriptions of work accomplished by members of the Idaho Site of the Reactor Physics Division.

Furthermore, in September 1968, the Laboratory combined all EBR-II work within one organization—the EBR-II Project—, thus removing from several Divisions EBR-II work for which they had been responsible. Thus, with the exception of two papers describing work completed in the Reactor Physics Division during the year reported, this issue of the Annual Report—unlike those of previous years—includes very little discussion of EBR-II.

At a date later than the period reported in this publication, the name of the Reactor Physics Division was changed to Applied Physics Division. Accordingly, our next annual report will be issued as the Applied Physics Division Annual Report.

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SECTION VI

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Section I

Fission Properties and Cross Section Data

In close support of the reactor program, experimental and theoretical studies of neutron interactions with those nuclei found in the structural and fuel components of reactor systems continue to be carried out. This work is performed principally to provide information necessary for optimal reactor physics design. The major portion of the effort is devoted to studies of fast neutron induced processes, especially those which are important in fast reactors. The studies are directed toward neutron elastic and inelastic scattering, neutron induced reactions including capture, and to characteristics of the fission process.

I-1. Introduction to Fast Neutron Total and Scattering Cross Sections

A. B. SMITH

The neutron scattering program has as its objective the provision of the best possible nuclear data to the reactor development program. The approach is immediate and explicit in the form of measurements in direct response to requests for scattering information. The approach is also implicit in seeking an understanding of neutron scattering requisite to extrapolation and interpolation of measurements, particularly to provide unmeasurable quantities. The processes of interest are so complex as to defy understanding on a nucleon-nucleon basis and resort is made to statistical models such as the optical model, Hauser-Feshbach concepts, and to the statistical properties of nuclear resonances, direct reactions, and channel coupling. The program seeks to provide the broad experimental foundation necessary for such statistical interpretations.

Increased attention is being given to the internal

consistency of the measured partial cross sections with a proper relation to the total cross sections and to such related processes as scattered neutron polarizations. The increased reactor need for precision quantities is recognized and attention given, of necessity, to primary standard cross sections. No result is considered complete until formally reported in such a manner as to be readily available for reactor calculations. (Specific attention is given to the provision of detailed numerical quantities to the National Neutron Cross Section Center and thence to the Evaluated Neutron Data Files.)

The following sub-topics, although not all-inclusive, are illustrative of the results and scope of the activities of the past year. The reactor "user" needing scattering data is encouraged to make specific request to this group since data of interest may be available.

I-2. Fast Neutron Total and Scattering Cross Sections—Bismuth*

A. SMITH, J. WHALEN, E. BARNARD,† J. DE VILLIERS† and D. REITMANN†

The heavy and readily available element bismuth is used in reactor systems as a shielding material. The element has been proposed as a primary target material for the production of intense neutron sources using medium energy positive ion accelerators. The fast neutron properties of bismuth are important in these applications. Despite this fact, the reported total fast-neutron cross sections of bismuth are not particularly consistent in either structure or magnitude. The differential elastic scattering cross sections of bismuth are not well defined and relatively little is known of the inelastic neutron scattering. Fast neutron interactions with bismuth are of basic physical interest in the context of intermediate resonance

structure and of the optical potential in the region of a neutron shell closure.

The above applied need was reasonably met by this experimental study. Total neutron cross sections of bismuth were measured with resolutions of $\gtrsim 1$ keV over the energy range 0.2 to 1.4 MeV. Differential elastic scattering cross sections were determined at intervals of < 50 keV from 0.3 to 1.5 MeV with resolutions of ~ 20 keV. The inelastic neutron excitation of a state at 895 ± 2 keV was observed and the respective differential excitation cross sections determined with incident resolutions of $\gtrsim 10$ keV. Partially resolved resonance structure was evident in all the measured values. The experimental results were assayed for possible intermediate structure and were compared with the results of optical model and statistical calculations. The model calculations were cognizant of the fluctuation and correlation of compound-nucleus resonance widths and of the shell closure at $N = 126$. Representative of the results is

* The numerical data resulting from this work has been transmitted to the National Neutron Cross Section Center, Brookhaven National Laboratory.

† Atomic Energy Board, Pelindaba, Transvaal, Republic South Africa. This work was performed as a joint research effort by staff members of Argonne National Laboratory and Pelindaba Laboratory.

FIG. I-2-1. Comparison of Measured Total (Bars) Cross Sections and Elastic Scattering (Boxes) Cross Sections of Bismuth as Determined in the Present Work. CAL-1 and CAL-2 Curves Were Obtained Using Alternate Choices of an Optical Potential. The Curve Noted as KFK-1000 is from Ref. 6, Noted as BNL-325 from Ref. 5, and that Noted as ANL-5567 from Ref. 7. ANL Neg. No. 113-2373.

the comparison of measured total and elastic scattering cross sections with calculated values using two selected optical potentials (Cal-1 and Cal-2),^{1,2} and comparison of the present measured values with previously reported results shown in Fig. I-2-1.

The energy-averaged total and elastic scattering cross sections were well described by a surface absorption optical potential and statistical theory. Best agreement between experiment and calculation was achieved with a surface absorption smaller than that found to be widely applicable in other mass regions. This reduced absorption was believed due to the shell closure at N-126 and is consistent with similar effects reported elsewhere.³ The optical potential and statistical theory led to calculated inelastic scattering cross sections $\sim 20\%$ larger than observed experimentally. Width fluctuation corrections to the calculated inelastic scattering led to only marginally improved agreement with experiment.⁴ The optical potential derived from the present measurements was extended to a wider energy range and found qualitatively descriptive of reported experimental values. However, the potential was not unique nor was it particularly descriptive of the reported polarization of elastically scattered neutrons. Considerable energy

dependent structure was observed in the measured cross sections. A quantitative assay of the structure observed in the total cross sections led to results consistent with well-known compound nucleus resonance structure and the available experimental resolution.

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I-3. Fast Neutron Total and Scattering Cross Sections—Hafnium, Gadolinium, and Samarium¹

G. L. SHERWOOD, A. B. SMITH and J. F. WHALEN

An extensive study of fast neutron cross sections of hafnium, gadolinium and samarium has been completed. Elastic and inelastic neutron scattering cross sections of elemental hafnium, gadolinium, and samarium were measured at incident neutron energies of 0.3 to 1.5 MeV. The experimental resolution was sufficient to reasonably resolve elastic and inelastic processes and define individual inelastic cross sections for the most appreciably excited states. The total neutron cross sections were determined up to 1.5 MeV with resolutions of $\gtrsim 2.5$ keV. Illustrative

ments the contributions of direct reaction processes were not pronounced.

The ENDF-B evaluated total neutron cross sections are reasonably descriptive of the experimental results.⁹⁻¹¹ The agreement between the ENDF-B evaluated data and the measured elastic and inelastic scattering cross sections is less satisfactory, particularly for gadolinium. The present experimental results should make possible an appreciable improvement in the evaluated elastic and inelastic neutron scattering data sets.

FIG. I-3-1. Total Neutron Cross Sections of Gadolinium. Vertical Bars Indicate Measured Values and Their Uncertainties. The Dashed Line Denotes the Sum of Measured Elastic and Inelastic Scattering Cross Sections. Solid Curve is the "Eye Guide" of BNL-325. ANL Neg. No. 113-1665.

total cross section results obtained for gadolinium are shown in Fig. I-3-1.

The presently measured total cross sections of hafnium, gadolinium, and samarium are in reasonable agreement with some previously reported values but differ from other previous work, particularly in showing no evidence intermediate structure.^{2, 3} The measured elastic and inelastic differential neutron scattering cross sections of the three elements provide new or improved knowledge of the processes up to incident energies of 1.5 MeV. The limitations of experimental resolution and the complexity of these elements preclude extensive interpretations leading to appreciably improved knowledge of nuclear structure.

The experimental results were compared with those obtained from calculations based upon both spherical and deformed optical potentials and statistical theory inclusive of fluctuation corrections.⁴⁻⁸ The calculated results were descriptive of measured total, elastic scattering, and to a lesser extent, inelastic scattering cross sections. At the energies of the present experi-

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I-4. Fast Neutron Total and Scattering Cross Sections—Holmium

J. MEADOWS, J. WHALEN and A. SMITH

Natural holmium is mono-isotopic, odd-A, and of large static deformation. Study of the fast neutron cross sections of this element provides insight into neutron interactions in the rare earth region heavily populated by fission products and, hopefully, to an improved capability for calculating fast neutron cross sections of such products. Particularly, good experimental determinations of fast neutron cross sections of holmium provide a foundation for developing deformed optical potentials and assaying direct interaction processes in odd-A nuclei.^{1,2}

586, 710, 824, 995, 1104, and 1143 keV was observed.*

The physical interpretation of the above experimental results is in progress, initially employing spherical optical model potentials and Hauser-Feshbach concepts and subsequently a deformed optical potential with full channel coupling.¹ Thus far these analyses have proven tedious and non-unique in the selection of a "best" potential. Hopefully, additional polarization measurements now in progress will provide sufficient definition for a reasonably definitive

FIG. I-4.1. Measured Total Neutron Cross Sections of Ho. Results Obtained Using Mono-Energetic and Time-of-Flight Techniques are Indicated. Solid Data Points Result for Present Elastic and Inelastic Scattering Measurements. The Curve Is the "Eye Guide" of Ref. 3. ANL Neg. No. 113-1948.

The experimental portions of a study of holmium total and scattering fast neutron cross sections have been completed. Total cross sections were determined using both time-of-flight and mono-energetic source techniques from 100 to 1500 keV. The total cross section and scattering cross section results are consistent within a few percent, as indicated in Fig. I-4-1. Elastic and inelastic differential scattering cross sections were determined at eight scattering angles between 25 and 155 deg and at incident energy intervals of $\lesssim 50$ keV from 300 to 1500 keV. Inelastic excitation of states at energies of 98, 214, 371, 460, 517,

determination of the nuclear potential in this deformed region.

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* Excitation energies are tentative pending exact energy calibration.

I-5. Fast Neutron Total and Scattering Cross Sections—Titanium

A. SMITH, J. WHALEN, E. BARNARD,* D. REITMANN* and J. DE VILLIERS*

The experimental measurement of elastic and inelastic scattering and total cross sections of titanium has been completed to incident energies of 1.5 MeV.⁽¹⁾ Total cross sections were measured with resolutions of $\gtrsim 1$ keV and elastic scattering cross sections with resolutions of ~ 20 keV. The inelastic excitation of states at 160 and 889 keV was observed. The excitation cross sections of the latter state were measured with resolutions of ~ 5 keV to 1.5 MeV.

All measured cross sections displayed a wealth of

partially resolved compound-nucleus resonance structure. A number of attempts to interpret this structure have not been particularly successful and even a descriptive optical model interpretation of the average behavior has not been achieved. The measured results appear to lead to ambiguous potentials. The interpretation continues in an effort to resolve these uncertainties in the choice of average potential and in an attempt to assay the nature of the observed structure.

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* Atomic Energy Board, Pelindaba, Transvaal, Republic of South Africa. This work was performed as a joint research effort by staff members of Argonne National Laboratory and Pelindaba Laboratory.

I-6. Fast Neutron Total and Scattering Cross Sections—Vanadium

A. B. SMITH, J. F. WHALEN and K. TAKEUCHI

A comprehensive experimental and theoretical study of fast neutron interactions with vanadium has been completed. Total neutron cross sections and elastic and inelastic neutron scattering cross sections were experimentally studied. Total neutron cross sections were determined with good resolution ($\gtrsim 1$ keV) from 0.1 to 1.45 MeV. Differential elastic and inelastic scattering angular distributions were measured at incident neutron energy intervals of $\lesssim 10$ keV from 0.3 to 1.4 MeV with incident neutron resolutions of ~ 20 keV. The inelastic excitation of states at 330 ± 10 keV and 926 ± 10 keV was observed. The experimental results were interpreted in terms of the optical model and statistical concepts inclusive of resonance width fluctuations and correlations. The observed intermediate structure was discussed in the context of strongly overlapping resonances, distributions in resonance widths and spacings, and in terms of an intermediate optical model. Comparison was made with previously reported experimental values and with the pertinent contents of the evaluated data file ENDF-B.

The observed total, elastic, and inelastic scattering cross sections were characterized by both a fine and

intermediate energy-dependent structure. Though of good resolution the results did not provide a definition commensurate with compound nucleus resonance analysis and the interpretation was confined to the intermediate structure and the average energy dependence of the observed quantities.

A phenomenological optical-potential was shown to be descriptive of total and elastic scattering cross sections of vanadium over a wide energy range.² Results of calculations based on this potential and statistical theory were qualitatively in agreement with the observed inelastic scattering but quantitatively deviated from measurement, particularly near the reaction thresholds.³ Consideration of alternative spin-parity assignments and/or the effects of resonance width fluctuations failed to enhance the agreement with the present experiments.⁴

An attempt to employ correlation analysis in a quantitative assay of the observed intermediate structure was inconclusive due to the effects of the finite sample range, the presence of direct reactions, and the uncertainties of the measurements.^{5,6}

The suitability of T. Ericson's concepts⁷ in the context of the present experiments is highly questionable.

However, an interpretation based upon them led to reasonable strength functions, though the derived Γ and D were inconsistent with both experimental evidence and the basic premise ($\Gamma \gg D$).

An experimental interpretation of the observed structure in the total cross section, based upon known compound nucleus distributions in widths and spacings assuming a non-overlapping of partial resonance widths, resulted in compound nucleus level densities similar to those predicted by systematics and by extrapolation from detailed low energy resonance studies. The result was not particularly sensitive to the form of the distribution of either Γ or D or even to the omission of the former.⁶

The results of calculation employing an intermediate optical potential based upon the premise of a few isolated doorway states were in qualitative agreement with the experimental elastic distributions and the phenomenological potential derived therefrom. This intermediate model employed relatively few doorway parameters and comparison with experiment led to parameter values of the magnitude expected from theoretical estimates. A wider application of the method may assist in determining some of the statistical properties of doorway states.⁸

The latter two of the above interpretations of the intermediate structure appear physically applicable and result in qualitative agreement with experiment. The approaches are not unique and other interpretations have led to similar qualitative successes.^{9,10} The various interpretations are not necessarily mutually exclusive and may each appreciably contribute to the physical reality.

The present work indicates that the vanadium data from ENDF-B,* often employed in reactor cal-

culations, is reasonably representative of microscopic measurement and should be suitable for many applied studies. As increasingly fine energy groups are employed in macroscopic calculation, it may become desirable to refine the evaluated data to take more cognizance of the observed fine resonance structure. Further, it may be desirable that the evaluated file be revised to bring it into better agreement with the results of the present and other recent studies of the inelastic neutron scattering cross sections of vanadium. Such revisions will be appreciable near the first inelastic threshold ($Q = -0.32$ MeV).

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I-7. Neutron Scattering Cross Sections of the Isotopes Mo-92, Mo-94, Mo-96, Mo-98 and Mo-100

A. SMITH and J. WHALEN

It was the objective of these measurements to determine the isotopic-spin dependence of the optical potential near the shell closure at $N = 50$.⁽¹⁾ Determinations of elastic and inelastic neutron scattering cross sections were made at $\lesssim 25$ keV intervals from 0.3 to 1.5 MeV. In the isotopes 92 through

100, the inelastic excitation of the $2+$ state was prominent. In addition, the excitation of a number of other states was observed. It was also noted that all of the samples contained oxygen contaminant (4-7% Corrections for this perturbation are now in progress. Complementary total neutron cross section measure-

* Evaluated Nuclear Data File-B, (ENDF-B), National Neutron Cross Section Center, Brookhaven National Laboratory.

FIG. I-7-1. Measured Total, and Elastic Cross Sections of Mo-100. Measured Data Have Not Been Corrected for $\sim 5\%$ Oxygen Contaminant. ANL Neg. No. 113-3100.

ments have been initiated and show a wealth of partially resolved compound-nucleus resonance structure. Typical of the experimental results are the total and elastic neutron cross sections of Mo-100 shown in Fig. I-7-1.

The physical interpretation is now in progress, employing spherical and deformed optical potentials and making provision for the direct excitation of the $2+$ vibrational state. A qualitative inspection indi-

cates a shell dependence of the optical potential similar to that previously noted near $A = 208$.⁽²⁾

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I-8. Polarization in Elastic Neutron Scattering

S. A. COX and E. E. DOWLING WHITING

For the past two years we have conducted a systematic study of polarization produced by the elastic scattering of low energy neutrons for intermediate weight and heavy nuclei. Some of our results for intermediate weight nuclei were reported in Ref. 1. The present report extends the measurements to U-238. Altogether we studied 29 elements from mass numbers 48 to 238. The nuclei were selected to be representative of most of the periodic table^{2,3} and the data are now extensive enough to indicate some generalizations and systematics in relation to different regions of the periodic table.

Data for each element were obtained at 8 angles using a partially polarized incident neutron beam emitted at 51 deg from the $\text{Li}(p,n)$ reaction. The incident neutron energy was always in the energy range from 700 keV to 1 MeV, and for purposes of comparison with the optical model the data were always averaged over an energy span of at least 200 eV within that energy range. The data were corrected for effects due to angular resolution, flux at-

tenuation, multiple scattering, and where necessary, for inelastically scattered neutrons. The polarizing power $P\mathcal{Z}$ was extracted from the measured polarization by dividing by the polarization of the incident neutron beam, which was obtained from data of R. Lane et al.⁴ and that of G. Mongan and R. Walter.⁵ At 20 and 35 deg $P\mathcal{Z}$ was also corrected for effects due to Schwinger scattering, as described in Ref. 1. For most of the elements which we studied the polarization was quite small—rarely greater than 10%.

One of the objectives of the program is to utilize the polarization as an additional constraint on the choice of optical model parameters required to achieve a theoretical fit to the data. By demanding a fit to both the differential scattering cross sections and the polarization, some of the ambiguity in the choice of optical model parameters is removed. It was sometimes found that more than one set of parameters would give a satisfactory fit to the differential cross section but that only one of these sets would at the same time fit the polarization measurements.

FIG. I-8-1. The Real Potential Radius R_1 , Obtained from Optical Model Fits to the Scattering and Polarization Data. ANL Neg No. 118-1989.

As a part of our analysis we made optical model fits, using the ABACUS II program of Auerbach, for all 29 elements. Our procedure for scanning the parameters used in this program has undergone quite a metamorphosis over the past two years. We started out using the parameter set suggested by Moldauer, and we are still using the same form for the three potential wells, namely a Saxon-Woods real potential, a pure surface imaginary potential, and a Thomas spin-orbit potential. On a few occasions we tried using a mixture of surface and volume potentials for the imaginary potential VI and the spin-orbit potential VSO , and we also tried a mixed real and imaginary spin-orbit potential. In every case it was concluded that the original forms were at least as successful. Using the potential forms specified, we varied a number of parameters in order to find better fits to our data. For the first group of elements studied, we found it sufficient to vary the well depth and radius of the real and imaginary potentials and the depth of the spin-orbit well. In doing this we soon found that our original formula for R_1 ,⁽⁷⁾ the real potential radius, did not give a good representation of the data.* We then let R_1 vary as a free parameter for a number of elements. The values of R_1 obtained from the resulting fits are given in Fig. I-8-1. The dashed curve follows our original formula which was obtained from P. Moldauer.⁷ The solid curve is the one finally decided upon. We have since followed a practice of varying R_1 only if a good fit cannot be obtained for the data with $R_1 = 1.26$. This tends to make the resulting choice for VR less ambiguous. More recently, as heavier elements were

studied, we also tried varying A_1 and A_2 , the diffuseness of the real and imaginary potential wells. Increasing the diffuseness parameters is one way of approximating nuclear deformation. For highly deformed nuclei with even Z and even A , the data were also analyzed using the $2^+ - 0^+$ coupling program of C. Dunford⁹ with the appropriate deformation parameter included. Finally, if we were still unable to fit the polarization, we tried varying the radius and/or diffuseness of the spin-orbit potential well independently of R_1 and A_1 , the parameters for the real potential well. It was found in general that the polarization angular distribution is more difficult to fit than the differential cross section and that the curves depend strongly on other parameters besides those of the spin-orbit potential. The objective was to find a single set of parameters that would fit both the differential cross section and the polarization for all elements. When this was possible, the polarization could be used as an additional constraint upon the choice of parameters for a given element. On the other hand, there were a number of elements for which no simultaneous fits could be found. In some cases separate sets of parameters were found that fit either the polarization or the cross section; in a few cases we were unable to fit the polarization no matter what was tried.

Figures I-8-2 through I-8-5 show typical results for 8 of our 29 elements, representing three different mass regions. Our results for titanium and tin are given in Fig. I-8-2. Titanium, with mass number 48, is the lightest-weight element included in this study and presents a good example of the difficulty experienced in fitting the elements with $A < 80$. The solid curves show the best fit to the differential scattering cross section and the corresponding curve for the polarization angular distribution. The dashed curves show the best fit to the polarization and the corresponding curve for the cross section. For most of the elements in this region it was impossible to obtain simultaneous fits to both the cross section and the polarization. On the other hand tin, at $A = 120$, is in a region where it is easy to obtain good fits to both the cross section and polarization. This region includes all elements which were studied between $A \approx 80$ and $A \approx 125$. For $A > 125$, the data again became more difficult to fit, although some elements were worse than others.

Figure I-8-3 gives results for two different calculations for tungsten and gold. In each case the solid curve represents an ABACUS II calculation with a diffuseness parameter of 0.5, an essentially spherical nucleus calculation. It is clear that neither the d

* In all cases the symbols used for the optical model parameters are those of Ref. 7.

FIG. I-8-2. Comparison between Experimental Measurements and Optical Model Calculations for Ti and Sn. *ANL Neg. No. 113-2585*.

FIG. I-8-3. Comparison between Experimental Measurements and Optical Model Calculations for W and Au. *ANL Neg. No. 113-1263*.

ferential cross section nor the polarization data are fitted well. The dashed curves represent the fits obtained using diffuseness parameters of 1.25 for tungsten and 1.00 for gold. This approximates a deformed nucleus. As a result, the agreement with the differential scattering data is considerably improved and the agreement with the polarization is very good.

Calculations using the $2^+ - 0^+$ program of Dunford did not result in any better fit.

Figures I-8-4 gives results for lead and bismuth. Both of these nuclei are essentially spherical and were thus analyzed with a diffuseness parameter of 0.5 f. The fit to the differential cross sections is reasonable and the details of the polarization measurements

FIG. I-8-4. Comparison between Experimental Measurements and Optical Model Calculations with Pb and Bi. *ANL Neg. No. 113-1270.*

FIG. I-8-5. Comparison between Experimental Measurements and Optical Model Calculations for Th and U. *ANL Neg. No. 113-1264.*

are reproduced rather well. The average neutron energies for lead and bismuth were chosen to be 1100 keV because a broad strong resonance in the lead data at 830 keV made comparison with the optical model unreliable for data below 1000 keV.

Figure I-8-5 shows results for thorium and uranium. These are two of the more reluctant elements. After many parameter scans a reasonable fit to the thorium

differential cross section, but a rather poor fit to the polarization data, were obtained. In the case of uranium the contrary was true. The polarization data are fitted rather well but a good fit to the differential cross section data could not be obtained. Since both thorium and uranium are highly deformed tl is perhaps not to be unexpected.

Figure I-8-6 shows the values of VR and VI which

FIG. I-8-6. VR and VI Obtained from Optical Model Fits. The Solid Curve is a Calculation Due to Azziz. ANL Neg. No. 113-2480.

were obtained by fitting our data as best we could through the procedure described. The solid curve is due to a formula by N. Azziz.⁸ His formula depends upon both A and Z for any particular element, so the curve is an average. Our values for VR are consistently above his, but follow the same decreasing trend with mass number. The dotted curves show the values used to make a scan over mass number with the ABACUS II program. For this scan we used a "typical" nucleus, with ground state of $0+$ and one excited state of $2+$ at an energy of 615 keV. We used $r_0 = 1.26$ for all three radii; $A1 = 0.62$ and $A2 = 0.75$; $V_{so} = 5$ MeV. The results of this scan are shown in Figs. I-8-7 and I-8-8.

Figure I-8-7 shows the total elastic cross section and the first two Legendre polynomial coefficients that were obtained for each element plotted against mass number. The curves are the results of the scan described above.

In order to show a systematic trend with mass number for the polarization, the values of $P2$ between 60 and 120 deg were averaged. The three mass regions mentioned previously are seen very clearly in the top part of Fig. I-8-8. The solid points are for elements for which we were able to obtain good optical model fits to both the cross section and the polarization data; the open circles are those for which good fits could not be obtained. For $A < 80$, all the average $P2$ values are positive and all but one were poor fits. For A between 80 and ~ 140 all the average $P2$ values are negative and all but one are good fits. Above $A \sim 180$ we have both good and bad fits, and all the average $P2$ values are posi-

FIG. I-8-7. Experimental Results for the Total Elastic Scattering Cross Sections and the First Two Legendre Polynomial Coefficients Obtained from Fits to the Angular Distribution. ANL Neg. No. 113-2479.

tive. The curve is the result of the ABACUS scan over mass number described above. In the first mass region it shows the positive trend for $P2$, although it does not fit the data exactly. In the center region it fits very nicely. Above $A \approx 140$ there is not

FIG. I-8-8. Experimental Polarization Averaged between 60 and 120 Deg Versus Mass Number Are Given in the Upper Part of the Figure. The Lower Part Gives Representative Values for the S and P-Wave Strength Functions. ANL Neg. No. 113-2482.

much correlation. This may be due to the fact that while VR and VI were varied with mass number, $A1$ was kept constant at a compromise value of 0.75. Below the $P2$ graph (Fig. I-8-8) is a plot of representative experimental values for s - and p -wave strength functions versus mass number. The open points are the s -wave strength functions and the

solid points are the p -wave strength functions. There seems to be a correlation between the strength function peaks and the systematic trends found for the polarization. The first mass region, where we could not obtain good fits to the polarization data, is associated with an s -wave peak. The middle region, where good fits were obtained, is associated with a p -wave peak. The last mass region is one of mixed s - and p -wave strength, and also includes a number of deformed nuclei. Here there are some good fits and some poor fits, and each nucleus seems to present its own special problems.

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I-9. Fast Neutron Total Cross Sections

J. F. WHALEN and J. W. MEADOWS

Additional neutron total cross sections have been determined with the automated on-line computer system reported previously.¹ The measurements were divided into two neutron energy ranges nominally classified as the low energy range (100 to 650 keV) and the high energy range (650 to 1500 keV). The basis for this decision arises from the fact that each of these ranges uses a different detector system and a different executive control program. The detector for

the low energy range is a bank of BF_3 counters whereas the high energy range utilizes a liquid scintillator coupled to a photomultiplier.² The total neutron cross section for the elements gadolinium, holmium, and rhodium were measured over the full low energy range in 1 keV increments. σ_T for samarium was also measured in the low energy range but in 2 keV increments and only in the interval 300 to 650 keV. The elements gadolinium, holmium, hafnium and sa-

FIG. I-9-1. Total Cross Sections. ANL Neg. No. 113-3118.

marium were measured over the full high energy range in 5 keV increments. Calcium was also measured over the full high energy range with 2 keV increments suitable for definition of the fine structure present. The elements titanium and lead were measured with 2 keV increments in the upper two thirds of the high energy range. The results of the above measurements are shown graphically in Fig. I-9-1. Preliminary data for the low energy range measurements for Te-128, Mo-100, Mo-98, Mo-96, Mo-94, and Mo-92 have been accumulated but are not yet ready for distribution.

A series of σ_T measurements were made to determine the effect of sample thickness on the resulting cross sections. Six different samples of Teflon were fabricated of various thicknesses ranging from 1.29×10^{22} to 12.9×10^{22} molecules/cm² in the direction of neutron transmission. Teflon was selected because it was easily machined and contained well-

isolated resonances and relatively flat regions in the total cross section spectrum in the energy range of interest. The total cross section of each sample was determined in the plateau region of near 550 keV and again as the energy was incremented over the sharp resonance at 607 keV. The results of these measurements showed that even with these extreme variations in thickness, any effect on the σ_T due to a wide range of sample thickness determined with this experimental setup was negligible compared with other sources of error.

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I-10. The Total Neutron Cross Section of Carbon from 0.1 to 1.5 MeV

J. W. MEADOWS and J. F. WHALEN

The total neutron cross section of carbon has been measured over the energy region 0.1 to 1.5 MeV using the automated on-line computer system.^{1, 2} Since this cross section is frequently used for detector calibration the measurement was made with care and to a high statistical accuracy. The experimental energy resolution was ~ 2.5 keV and measurements were made at 1.0 keV intervals below 650 keV and at 2.0 keV intervals above that energy. However no structure was observed in the cross section so the data were averaged over 10 keV intervals. The results are shown in Fig. I-10-1.

A least squares analysis of the data gives

$$\sigma_T = 4.8301 - 35580E + 1.5872E^2 - 0.3050E^3 \quad (1)$$

$$s = \left(\frac{\sum d_i^2}{n - m - 1} \right)^{1/2} = 0.035, \quad (2)$$

where σ_T is the total cross section in barns, E is the neutron energy in MeV, s is the standard deviation of the fit, d_i is the difference between the calculated and experimental points, n is the number of data points, and m is the order of the polynomial. The fit is very good since the statistical accuracy

of the data alone requires $s \geq 0.02$. The inclusion of higher order terms gave no significant improvement.

Since σ_T is almost entirely elastic scattering its energy dependence suggests scattering of s-wave neutrons by a bound level. It is probably the one in C-13 at 3.085 MeV excitation,³ this being the only known bound state in that nucleus with $J = +1/2$. Consequently the data were analyzed in terms of scattering by this level plus potential scattering.⁴ It was assumed that the effect of other resonances could be safely absorbed into the potential scattering term since the only nearby ones are the result of scattering from narrow d-wave states.³ The results are shown in the first column of Table I-10-I. The magnitude of s shows that the agreement with the experimental data is fairly good.

In fitting the data the same radius was used for both s and p-wave scattering. However, any reasonable choice of a potential radius predicts $\sim 10\%$ p-wave scattering at 1.5 MeV while the angular distribution measurements of J. Wells et al.⁵ show much less. This suggests that the p-wave radius should be less than the s-wave. No analysis was made treating these as independent variables but a fit was

FIG. I-10-1. The Total Neutron Cross Section of Carbon. Each Point Is the Result of an Average of All Data in a 10 keV Interval. The Curve Was Calculated Assuming s-Wave Scattering from a Potential and a Single Bound Level at $E_\lambda = -2.02$ MeV with $\gamma^2 = 0.580$ MeV and a Radius of 4.8 fm. ANL Neg. No. 113-2688.

ade with the p -wave radius set to zero. The results are shown in Table I-10-I. The size of s shows an improved agreement with the experimental data and Θ^2 is in better agreement with the results of (d,p) reactions.⁶

If a cross section is free of resonances and predominately due to one angular momentum, it may be treated by the effective range analysis. The necessary equations listed below were taken from A. Lane and R. Thomas.⁴ For s -wave interactions

$$k \cot \delta_0 = - \left[\frac{4\pi}{\sigma_T} - k^2 \right] \quad (3)$$

$$= - \frac{1}{\alpha} + \frac{1}{2} r_0 k^2 - P r_0^3 k^4,$$

where δ_0 is the s -wave phase shift, k is the wave number, α is the scattering length, r_0 is the effective range, and P is a shape factor. Furthermore the coefficients of the effective range expansion can be related to a Taylor series expansion of the R function at $E = 0$. The first two terms of the expansion give

$$\gamma^2 = E_\lambda \left(1 - \frac{1}{\beta} \right) \quad (4)$$

$$\Theta^2 = \frac{1}{3} (1 - \beta^2)^2 / \left(1 - \beta + \frac{1}{3} \beta^2 - \frac{r_0}{2R} \right), \quad (5)$$

where

$$\beta = \alpha R \quad (6)$$

and

$$\gamma^2 = \left[\frac{2}{3} \frac{A}{A+1} \frac{mR^2}{\hbar^2} \right]^{-1} \theta^2. \quad (7)$$

In the above equations E_λ is the binding energy, R is the interaction radius, m is in the neutron mass, and A is the mass number of the target nucleus.

Equation (3) was fitted to the data by a least squares procedure. Only the points below 1 MeV were used in order to reduce the effects of any p -wave

TABLE I-10-I. PARAMETERS OF THE FITS TO THE EXPERIMENTAL DATA

Parameter ^a	Single Level Fit		Effective Range
	$\ell \leq 2$	$\ell = 0$	
E_λ , MeV	-2.02	-2.02	-2.02
γ^2 , MeV	1.35	0.540	0.575
Θ^2	0.25	0.185	0.193
R , fm.	3.80	4.80	4.75
$1/\alpha$, fm.			6.18
r_0 , fm.			3.34
s	0.052	0.045	

^a E_λ is the resonance energy, γ^2 is the reduced width, Θ^2 is a dimensionless width defined in Eq. (7), R is the interaction radius, α is the scattering length, r_0 is the effective range, and s is a number relating to the quality of the fit defined by Eq. (2). All energies are in the laboratory system.

scattering. A good fit was obtained with the first two terms; inclusion of the k^4 term gave no improvement. The corresponding values of R , Θ^2 and γ^2 were calculated by Eqs. (4) and (5). The results of the effective range analysis, listed in the last column of Table I-10-I, are in good agreement with the results of the single level fit with only s -wave scattering.

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I-11. The Thermal Absorption Cross Section of Li-6

J. W. MEADOWS and J. F. WHALEN

INTRODUCTION

The $\text{Li}^6(n,T)\text{He}^4$ reaction is often proposed as a secondary standard for neutron flux measurements. Although the characteristics of this reaction make it

very convenient for this purpose, the cross section σ_{nT} is not known to sufficient accuracy even at thermal energies. Most measurements have been made relative to some other reaction and have used natural lithium with-

TABLE I-11-I. THE MASS ANALYSIS AND σ_a OF LI-6
(σ_a was evaluated at 2200 m/sec assuming $1/v$ dependence.)

	Sample A	Sample B	Sample C
Li-6, a/o	98.94	95.88	7.406
σ_a (2200 m/sec), b	936 ± 4	941 ± 5	924 ± 5
σ_a (average)		934 ± 6	

out determining its isotopic content. J. Spaepen¹ has estimated that the error in σ_{nT} at 2200 m/sec is $\pm 3\%$, which is very poor for a standard reaction.

This paper reports the preliminary results of a measurement of the thermal absorption cross section σ_a of Li-6 using lithium of known isotopic content. Although σ_a includes all reactions which absorb neutrons it is virtually identical with σ_{nT} . The only other contributor is the $\text{Li}^6(n, \gamma)\text{Li}^7$ reaction which is approximately 0.005 % of σ_a .

EXPERIMENTAL METHOD AND PROCEDURE

The pulsed neutron technique used in these measurements has been described previously² and is only summarized here. The time dependence of the thermal neutron flux in a source-free water system is compared with that of a very dilute solution. If the experimental conditions for the two measurements are the same, then

$$\frac{R_1(t)}{R_2(t)} \propto \exp(-\bar{v}\sigma_a N_s t),$$

where $R(t)$ is the time dependent count rate of a detector measuring the leakage flux, the subscripts 1 and 2 refer to the pure water and solution respectively, and N_s is the concentration of the solute. The quantity $\bar{v}\sigma_a$ is the result of an average over the neutron velocity distribution. However, if σ_a has a $1/v$ dependence, then

$$\bar{v}\sigma_a = v\sigma_a(v).$$

When more concentrated solutions are used some corrections are required and there are some restrictions on the experimental arrangements. These are discussed in Ref. 2.

Measurements were made on two samples of Li_2CO_3^*

* The Li-6 was obtained from the Isotopes Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee.

and on a sample of natural LiNO_3 . The Li-6 content of all samples was determined by mass analysis. The cross section of each of the enriched samples was measured at four concentrations while the natural lithium was measured at two concentrations. The lithium content of each solution was obtained by chemical analysis.

RESULTS

The results are given in Table I-11-I. They were obtained by assuming that σ_a had a $1/v$ dependence for neutron energies less than 3 eV. No measurements for Li-6 in this energy range have been published, but W. Havens and J. Rainwater³ showed that the total cross section of natural lithium in the energy region 0.021 to 10 eV could be represented by

$$\sigma_t = (11.5/\sqrt{E}) + 1.7,$$

where E is the neutron energy in eV. The elastic scattering cross section is about 1.0 b and when this is subtracted

$$\sigma_a = (11.5/\sqrt{E}) + 0.7$$

which is almost entirely due to the Li-6 content. The size of the constant term indicates that the results in Table I-11-I may be approximately 0.6 % high.

The errors given include the statistical error of the measurements and the random errors in the mass and chemical analysis. The agreement between the lithium samples gives confidence in the lack of systematic errors in the mass analysis. The results are about 1 % less than the 945 b often used⁴ for σ_a at 2200 m/sec but the difference is within the 3 % uncertainty estimated by Spaepen.¹

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I-12. Slow Neutron Scattering by NH_3 Gas and H_2O Vapor

A. K. AGRAWAL

INTRODUCTION

The scattering of slow neutrons by an assembly of molecules is usually inelastic whenever neutron energy is comparable to the system energy levels. Therefore, the calculation of scattering cross section must properly include the effects of translations, rotations and intramolecular vibrations.¹ While a detailed investigation of such calculations is justified by the use of slow neutron scattering experiments as a probe to study molecular dynamics, the scattering cross sections are also needed in the determination of reliable scattering kernels for thermalization studies. The present paper is restricted to the calculation of inelastic scattering cross sections.

Although the formal theory of neutron scattering by molecular systems is by now well developed, practical methods for predicting cross sections are usually based on certain simplifying assumptions.¹ In almost all of the calculations, molecular translations, rotations, and intramolecular vibrations are assumed to be independent of each other. While the effects of translations and vibrations can readily be evaluated for molecular gases, molecular rotations do pose a non-trivial problem. Approximate methods to account for rotational effects are available. The most notable one is perhaps due to T. Krieger and M. Nelkin² in which rotational effects are treated in terms of an effective mass approximation. This approximation, originally given for the calculations of total and angular differential cross sections, has been unsuccessfully applied to calculate the energy and angular differential scattering cross sections. Recently, there has been growing interest in the use of rigorous quantum mechanical treatment of free rotations.³ Such a treatment does give very good agreement with experiments on molecular gases.^{4,5} Unfortunately, the physics of the analysis is lost in the computer programming.

Recently, a semi-classical method based on the so-called Gaussian approximation for the intermediate scattering function has been proposed.⁶ This approach makes use of the classical rotational correlation functions. These authors⁶ give an expression for this correlation function for spherically symmetric, linear, and symmetric molecules. In this calculation of the time correlation function for symmetric rotors, authors had restricted themselves to an approximation applicable to only those symmetric molecules which are almost linear rotors. This limitation has been removed when a

st general expression for the rotational correlation function is derived.⁷ It has also been shown that this

generalized expression can, in special limiting cases, reduce to spherical and linear rotor expressions. In the present paper we apply this generalized result to the analysis of inelastic neutron scattering experiments from the room temperature ammonia gas and water vapor. In the latter case the symmetric rotor correlation function is used after appropriately symmetrizing the asymmetric water molecule. One such method is illustrated which gives rise to a reasonably good agreement with experimental data.

BASIC FORMULAS

It is usually a good approximation to treat the inelastic scattering of neutrons by gases at normal densities in terms of the scattering from each individual molecule. For hydrogenous substances, the double differential scattering cross section per molecule in the incoherent approximation is⁶

$$\sigma(E_0, E, \theta) = n \frac{\sigma_b}{4\pi} \left(\frac{E}{E_0} \right)^{1/2} S(\kappa, \omega), \quad (1)$$

where n is the number of hydrogen atoms in a molecule, σ_b is the bound proton scattering cross section (81 b), E_0 and E , respectively, are the incident and scattered neutron energies, θ is the scattering angle, $\omega = E - E_0$, $|\kappa| = [2m(E + E_0 - 2\sqrt{EE_0} \cos \theta)]^{1/2}$, m is the neutron mass, and $S(\kappa, \omega)$ is the incoherent scattering law for the hydrogen atom in the molecule. If intramolecular vibrations are neglected, $S(\kappa, \omega)$ is determined by molecular translations and intramolecular rotations. Thus, we have

$$S(\kappa, \omega) = e^{-\omega/2T} \exp \left(-\frac{\kappa^2}{8MT} G(T) \right) \cdot \frac{1}{\pi} \int_0^\infty \chi_t(\kappa, t) \chi_r(\kappa, t) \cos \omega t \, dt, \quad (2)$$

where χ_t and χ_r are, respectively, the translational and rotational classical intermediate scattering functions, M is the mass of a molecule, and T is the temperature of the scattering medium. The exponential terms in Eq. (2) are the well-known detailed balance and recoil factors. Function $G(T)$, appearing in the recoil factor, is a correction term arising due to molecular rotation.⁶ For gases, $\chi_t(\kappa, t)$ is well known:

$$\chi_t(\kappa, t) = \exp \left(-\frac{\kappa^2}{2M} T t^2 \right) \quad (3)$$

FIG. I-12-1A. Time-of-Flight Spectra of Neutrons Scattered by Ammonia Gas at $T = 295^\circ\text{K}$. Experimental Points Are Shown by Closed Circles, Our Result with Solid Curve, and Exact Quantum Mechanical Results with Open Circle. Theoretical Spectra Are Computed for Incident Energy of 4.87 MeV, Indicated by the Arrow, and Are Area Normalized. ANL Neg. No. 113-2053.

FIG. I-12-1B. Time-of-Flight Spectra of Neutrons Scattered by Ammonia Gas at $T = 295^\circ\text{K}$. Experimental Points Are Shown by Closed Circles, Our Result with Solid Curve, and Exact Quantum Mechanical Results with Open Circle. Theoretical Spectra Are Computed for Incident Energy of 4.87 MeV, Indicated by the Arrow, and Are Area Normalized. ANL Neg. No. 113-2608.

and $\chi_r(\kappa, t)$ can be written in terms of the classical rotational correlation functions $F_\ell(t)$ as

$$\chi_r(\kappa, t) = \sum_{\ell=0}^{\infty} (2\ell + 1) j_\ell^2(\kappa b) F_\ell(t), \quad (4)$$

where $j_\ell(x)$ is the ℓ th order spherical Bessel function of argument x and b is the equilibrium distance of the

hydrogen atom from the center of mass of the molecule. The inelastic neutron scattering cross section can now be computed by combining Eqs. (1)–(4), provided $F_\ell(t)$ are known.

The above mentioned procedure of computing $\sigma(E_0, E, \theta)$ requires $F_\ell(t)$ for all ℓ from 0 to ∞ . These functions are usually not available except for $\sin$.

uses, as spherically symmetric or linear molecules. It has been shown⁶ that $\chi_r(\kappa, t)$, as given by Eq. (4), can instead be approximated by a Gaussian form in κ . We have

$$\chi_r(\kappa, t) = \exp \left[-\frac{\kappa^2}{2} \frac{2}{3} b^2 (1 - F_1(t)) \right]. \quad (5)$$

This approximation is clearly a very useful one since only the first order rotational correlation function is needed to calculate the rotational contribution to the scattering law. Recently, a generalized expression for $F_1(t)$ for the symmetric rotor has been obtained:⁷

$$F_1(t) = \cos^2 \phi (Y_1 + Y_2) + \sin^2 \phi (Y_3 + Y_4), \quad (6)$$

where

$$Y_1 = \frac{\sqrt{1+c}}{2} \left[-\frac{2}{c\sqrt{1+c}} + \frac{2}{c\sqrt{c}} \cdot \ln(\sqrt{1+c} + \sqrt{c}) \right] \quad \text{if } c \geq 0, \quad (7)$$

$$= \frac{\sqrt{1+c}}{2} \left[-\frac{2}{c\sqrt{1+c}} + \frac{1}{c\sqrt{-c}} \cdot \sin^{-1}(2\sqrt{-c(1+c)}) \right] \quad \text{if } c \leq 0,$$

$$Y_2 = \frac{\sqrt{1+c}}{2} \int_{-1}^1 dx (1-x^2) \frac{1+cx^2-\tau^2}{(1+cx^2)^{5/2}} \cdot \exp \left(-\frac{c^2\tau^2}{2(1+cx^2)} \right), \quad (8)$$

$$Y_3 = \frac{\sqrt{1+c}}{4} \int_{-1}^1 dx (1-x^2) \frac{1+cx^2-c^2x^2\tau^2}{(1+cx^2)^{5/2}} \cdot \exp \left(-\frac{c^2x^2}{2(1+cx^2)} \tau^2 \right), \quad (9)$$

$$Y_4 = \frac{\sqrt{1+c}}{4} \int_{-1}^1 dx (1+x)^2 \frac{1+cx^2-(1+cx)^2\tau^2}{(1+cx^2)^{5/2}} \cdot \exp \left(-\frac{(1+cx)^2}{2(1+cx^2)} \tau^2 \right), \quad (10)$$

$$c = (I - I_z)/I_z,$$

$$\tau = (T/I)^{1/2} t$$

$\phi \equiv$ angle made by the hydrogen atom of a symmetric molecule with its symmetry axis.

It should be noted that $F_1(t)$ given by Eq. (6) is a general result which can be shown to reduce to simpler expressions for spherically symmetric and linear rotors.⁷

RESULTS AND COMPARISON WITH EXPERIMENT

The double differential neutron scattering cross section $\sigma(E_0, E, \theta)$ for ammonia gas and water vapor is computed by combining Eqs. (1)–(3), (5), and (6). In order to compare our results with experimental time-of-flight measurements, the time-of-flight cross section $\sigma(f_0, f, \theta)$ is computed from

$$\sigma(f_0, f, \theta) = 2 \left(\frac{2}{m} \right)^{1/2} E^{3/2} \sigma(E_0, E, \theta), \quad (11)$$

where f_0 and f are the incident and scattered neutron time-of-flights, respectively. In Figs. I-12-1A and I-12-1B we compare, in arbitrary units, our results with Webb's measurement⁸ of scattering of 4.87 MeV neutrons from the room temperature ammonia gas at two different scattering angles. We also compare our results with the exact quantum mechanical values⁵ for $\theta = 20$ deg. Theoretical results include only contributions from hydrogen atoms and they are area-normalized with respect to the experimental points.

We have also applied the present method to analyze the inelastic scattering measurements from water vapor. A water molecule is an asymmetric molecule with its moment of inertia around three principal axes in the ratio of 1:2:3 [$I_x = 1.0243$, $I_y = 1.9207$, and $I_z = 2.9470 \times 10^{-40}$ g cm²], and the angle between H—O—H is $104^\circ 27'$. In the absence of an expression for $F_1(t)$ for an asymmetric molecule, the symmetric rotor expression of $F_1(t)$ [Eq. (6)] is used for an appropriately symmetrized water molecule. The symmetrization procedure used here is to take the arithmetic average of I_x and I_y for I and half the angle between H—O—H for ϕ . In Figs. I-12-2A and I-12-2B we compare our results, in absolute units, with experimental data of W. Gläser⁹ for incident neutrons of 17.3 MeV. In the smaller angle case, we also show the cross sections calculated by using the Krieger-Nelkin² model (with $m_{eff} = 1.81$). The inadequacy of the Krieger-Nelkin model is evident. In view of the good agreement between our results and experiment, a derivation of an expression for $F_1(t)$ for an asymmetric molecule does not appear to be warranted.

The existing discrepancy between our calculations and experiments can perhaps be accounted for by some of the assumptions made here. While all of our calculations were made in the Gaussian approximation, non-Gaussian corrections are expected to be quite small as has been demonstrated⁵ in the case of methane gas. Another assumption used here is the incoherent approximation. For hydrogenous gases this is not serious because the hydrogen cross section is large and pre-

FIG. I-12-2A. Absolute Time-of-Flight Cross Section for Water Vapor at $T = 593^{\circ}\text{K}$ for Incident Neutrons of 17.3 MeV. Experimental Points Are Compared with Our Result (Solid Curve). The Dashed Curve, for Small Scattering Angle Case, Corresponds to Krieger-Nelkin Model. ANL Neg. No. 118-1593.

FIG. I-12-2B. Absolute Time-of-Flight Cross Section for Water Vapor at $T = 593^{\circ}\text{K}$ for Incident Neutrons of 17.3 MeV. Experimental Points Are Compared with Our Result (Solid Curve). ANL Neg. No. 118-2051.

dominantly incoherent. Finally, no efforts were made to include multiple scattering effects.

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I-13. Measurements of the Cross Section Ratios of $U^{235}(n,f)$, $U^{238}(n,\gamma)$ and $Pu^{239}(n,f)$ in the Fast Energy Range

W. P. POENITZ

Accurate data for neutron capture and fission cross sections of U-238, U-235, and Pu-239 are important for fast reactor calculations and design. In some recent calculations inconsistent data for absolute capture and fission cross sections have been used. In general, lower capture cross sections for U-238 and higher fission cross sections for U-235 and Pu-239 seem to be desirable to bring the reactor calculations in agreement with experimental integral data. This suggests a reinvestigation of the ratios $\sigma_\gamma(U-238)/\sigma_f(U-235)$ and $\sigma_f(Pu-239)/\sigma_f(U-235)$ to obtain additional information for selecting consistent absolute cross section sets.

The ratio $\sigma_\gamma(U-238)/\sigma_f(U-235)$ has been measured directly only once before (Ref. 1); however, other values can be derived from cross section measurements based on the same standards (Refs. 2-9). No measurements are known for the ratio $\sigma_\gamma(U-238)/\sigma_f(Pu-239)$. The ratio $\sigma_f(Pu-239)/\sigma_f(U-235)$ has been measured by several experimenters or it may be derived from separate absolute measurements of cross sections (Ref. 8-13). All but one of these fission ratio data have been obtained from dc-measurements.

The present measurements have been carried out in the energy range 130-1400 keV which is accessible with monoenergetic neutrons at 0 degrees using the $Li^7(p,n)$ reaction as a neutron source. The energy of the primary proton beam accelerated by a 3 MeV Van de Graaff accelerator was determined by an electrostatic analyzer calibrated with the threshold energy of the $Li^7(p,n)$ reaction. The neutron energy has been determined from the primary proton energy, target thickness, and average opening angle.

$$\sigma_\gamma(U-238)/\sigma_f(U-235)$$

The method of measuring this ratio was essentially the same as that used by G. Linenberger et al.¹ in 1944.

The relative ratio has been determined as a function of energy by counting the fission fragments above a certain energy threshold and the capture events by the induced Np-239 radioactivity. The unknown efficiencies for the U-235 fission counting and the measurement of the Np-239 activity were eliminated by measuring the same ratio for thermal neutrons and using the ratio at that energy which is known from other independent measurements.

Metallic U-238 foils (0.0125 cm thickness, 1.6 cm diam depleted to 0.4 percent U-235) were sandwiched by two U-235 foils (250 $\mu\text{g}/\text{cm}^2$ thickness each) on molybdenum backing. In this arrangement corrections due to neutrons scattered elastically in the foils and in the walls of the fission counter were eliminated in first order. Only neutrons with different energies (e.g. due to inelastic scattering) require corrections. The sandwich principle was used earlier by Y. Barry.^{4, 14} The fission counter was a low mass ionization chamber designed by J. Meadows.¹⁵ The fission fragment energy spectrum was recorded in a 128 channel analyzer. After completion of the irradiation sufficient time was allowed for the complete decay of U-239 ($T_{1/2} = 23$ min) to its daughter Np-239 ($T_{1/2} = 2.35$ d). The 278 keV γ -ray was counted using a 30 cm^3 Ge (Li) detector. This procedure had been used by H. Menlove and W. Poenitz.⁶ Corrections were considered for neutrons with different energies. Such neutrons may cause erroneous results, as for example, the ratio for thermal neutrons is 0.00473 and 1.02 for the resonance integral, whereas the ratio in the energy region 130-1400 keV is in the range 0.07-0.13. The sufficiently low contribution from thermal or epithermal neutrons could be proved by the $1/r^2$ law using the fission counter. Corrections for neutrons scattered in the target backing and the beam tube support were as high as 5%. A correction up to 6% was caused by

TABLE I-13-I. RESULTS FOR THE CROSS SECTION RATIO $\sigma_\gamma(\text{U-238})/\sigma_f(\text{U-235})$

E_n , keV	R	ΔR
130	0.126	0.006
150	0.126	0.006
250	0.114	0.004
300	0.103	0.003
400	0.104	0.003
500	0.111	0.003
600	0.122	0.004
700	0.133	0.004
900	0.124	0.004
1200	0.097	0.004
1250	0.092	0.004
1400	0.074	0.003

TABLE I-13-II. RESULTS FOR THE CROSS SECTION RATIO $\sigma_\gamma(\text{U-238})/\sigma_f(\text{Pu-239})$

E_n , keV	R	ΔR
250	0.095	0.004
400	0.082	0.003
500	0.082	0.002
700	0.087	0.003
900	0.086	0.003
1400	0.048	0.002

TABLE I-13-III. RESULTS FOR THE RATIO $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$

E_n , keV	R	ΔR
150	0.969	0.024
200	1.084	0.031
250	1.093	0.032
325	1.205	0.030
500	1.335	0.027
700	1.446	0.026
850	1.478	0.041
1000	1.372	0.031
1200	1.437	0.050
1300	1.435	0.031
1400	1.463	0.029

the second neutron group from the reaction $\text{Li}^7(p,n)\text{Be}^7$ which occurs above 2.4 MeV primary energy.

The results are given in Table I-13-I. The uncertainties are typically 3% but somewhat higher at the lower and higher energies. They are caused by the normalization procedure (2%), statistical errors (1-2%), uncertainties of the corrections (0-3%), and energy uncertainties (0-1%).

$$\sigma_\gamma(\text{U-238})/\sigma_f(\text{Pu-239})$$

This ratio was determined by the measurement of the absolute fission rate of a Pu-239 sample and the

absolute Np-239 activity of a U-238 sample, both exposed to the same neutron flux. The uranium foils were outside of the fission counter on the back of the Pu-239 foil. The fission foil was a layer of 25 $\mu\text{g}/\text{cm}^2$ Pu-239 (99.95%) deposited on a steel backing by electroplating. The absolute fission rate was determined by extrapolating the fission fragment energy spectra to zero pulse height and correcting for total absorption. The absolute efficiency of the Ge(Li) detector was determined by an α -calibrated Am-243 sample which was in equilibrium with its daughter Np-239. This Am-243 technique was first used and described by Barry.⁴ The γ -absorption in the uranium

FIG. I-13-1. Comparison of the Present Measurements of $\sigma_\gamma(\text{U-238})/\sigma_f(\text{U-235})$ with Previously Available Data. ANL Neg. No. 113-2604.

FIG. I-13-2. Comparison of the Present Measurements of $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$ with Previously Available Data. ANL Neg. No. 113-2605.

oil was determined in a separate absorption experiment.

Corrections for these measurements were similar to those for the ratio $\sigma_f(\text{U-238})/\sigma_f(\text{U-235})$; however, the backscatter of neutrons from the fission counter caused an additional correction up to 1%.

The results are given in Table I-13-II. The error for the absolute fission and capture rate determination contributes 2.5% to the total error. All other errors are similar to those for the ratio $\sigma_f(\text{U-238})/\sigma_f(\text{U-235})$.

$$\sigma_f(\text{Pu-235})/\sigma_f(\text{U-235})$$

This ratio has been measured for monoenergetic neutrons using the time-of-flight method for low energy background suppression. The proton beam, accelerated by a 3 MeV Van de Graaff accelerator, was pulsed and bunched to 1 nsec pulse width. Measurements were carried out with flight paths between 30 and 80 cm. The fission detector was a double chamber gas scintillation counter. The fission foils were mounted back-to-back in the center of the counter. The walls of this counter were too thin to allow the normal evacuation and filling procedure and therefore a mixture of argon and nitrogen (15%) was continuously passed through the counter. Two photomultipliers detected the scintillations. A coincidence between the two fast anode signals cancelled out pulses produced by fast neutrons in the photomultipliers. The overall time resolution was 3 nsec.

The fission events from the Pu-239 and U-235 samples were recorded in two 2-dimensional matrices (fission fragment energy versus time-of-flight). An integration over all of the time-of-flight range yielded deviations up to 10% compared with the results from the time-of-flight peak only. The extrapolation of the fission fragment energy spectra to zero pulse height required a correction of 1.5% for the Pu-239 sample (about 25 $\mu\text{g}/\text{cm}^2$) and of the same order for the U-235 sample (about 150 $\mu\text{g}/\text{cm}^2$). A very good separation between the fission fragment spectra and the α pile-up was obtained for the Pu-239 sample due to the purity of the material (99.9% Pu-239). Small corrections were applied for scattering in the sample backings and the fission counter. The amount of fissile material has been determined by low geometry α -counting.

The results for the ratio $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$ are given in Table I-13-III. The uncertainties are due to statistics (1-2%), uncertainties in the determination of the amount of fissile materials (1.5%), and uncertainties in corrections and the energy determination (2%). Eye-guide curves through the values given in Tables I-13-I and I-13-III are compared with values measured by other experimenters in Figs. I-

13-1 and I-13-2. The present results for $\sigma_f(\text{U-238})/\sigma_f(\text{U-235})$ confirm the values derived from measurements described in Refs. 4-7. The agreement for $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$ as reported by W. Allen and A. Ferguson,⁸ G. Smirenkin et al.,¹⁰ P. White and Y. Hodgkinson,¹¹ P. White and G. Warner¹² and results of these measurements suggests that an average curve may have an uncertainty as low as 1-2%. The structure in the ratio $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$ around 1 MeV is caused by a step of 10% in the fission cross section of U-235.

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I-14. Isomeric Cross Section Ratio of the Reaction $Rh(n,\gamma)$ in the keV Energy Region

W. P. POENITZ

The occupation probabilities of an isomeric state and the ground state of a nucleus by a γ -ray cascade has been frequently measured in the past. The goals of such experiments have been the investigations of reaction mechanism or the determination of spin sensitive parameters in the nuclear level density formula. It has been shown previously that values of the spin cut-off factor σ obtained from such measurements are strongly dependent on the choice of competing parameters; e.g., the contribution of quadrupole radiation in a γ -ray cascade. The latter applies especially to the determination of the spin cut-off factor from measurements of the isomeric cross section ratio in the (n,γ) process at thermal neutron energies. This would not exclude the possibility of the determination of σ from the shape of the energy dependence of the isomeric cross section ratio.

Measurements of the isomeric cross section ratios of $Rh-103(n,\gamma)$, $Rh-104g$, and $Rh-104m$ have been carried out in the energy range 20–1400 keV. A metallic rhodium disk was irradiated with monoenergetic neutrons using the $Li^7(p,n)Be^7$ reactions as a neutron source and the induced activities were measured with a $4\pi\beta$ counter after completing the irradiation. The time dependence of the count rate has been recorded

with a multichannel analyzer. These measured decay curves have been analyzed assuming a superposition of two exponential functions and a constant background.

The results for the isomeric cross section ratio of $Rh(n,\gamma)$ are shown in Fig. I-14-1. Other values have been obtained from measurements of the activation cross sections by S. A. Cox.² The computer programs CASCADE⁽¹⁾ and ABACUS⁽³⁾ have been used to evaluate the ratios shown as smooth curves in Fig. I-14-1. The higher energy part of this theoretical curve (dashed line) suffers from the lack of knowledge regarding inelastic scattering in this energy region. The upper and lower curves represent theoretical curves for changed parameters of the spin cutoff factor σ (lower curve) and the contribution of quadrupole radiation (higher curve). This shows that the change in the shape of the curves is insufficient for the determination of parameters in the γ -ray cascade model.

Use of the low level occupation probability ratio to determine quantum numbers, e.g., the spin of compound states, was suggested in Refs. 1 and 4. An interesting application would be the investigation of "doorway states." The doorway state concept predicts

FIG. I-14-1. Comparison of Measured Isomeric Cross Section Ratios for $Rh-103(n,\gamma)$ with Theoretical Values. ANL Neg. No. 1... 1405.

FIG. I-14-2. High Resolution Measurements of the Isomeric Cross Section Ratio of Rh-103 (n, γ). ANL Neg. No. 113-1412.

a first step configuration preceding more complex structures in the nucleon capture process. Some intermediate structure was recently found in the total neutron cross section in medium and heavy-weight nuclei by K. Seth⁵ and discussed by H. Feshbach et al. The spin of a doorway state is a good quantum number; therefore one expects that the fine-structure resonances which are fed through a doorway state all have the same spin and that the ratio R should vary between the appropriate values for the different spins in the same way as for the single resonances in the eV region.^{1, 4, 7} This points to the fact that R could very well determine whether the intermediate structure is caused by statistical fluctuations of Γ and D as suggested by P. Singh et al.⁸ or by a doorway state. The latter would cause a fluctuation of R as discussed above; with high probability the former would not cause a fluctuation of R because the probability of finding a "cluster" due to statistical fluctuations in Γ and D , where a predominant amount of the fine structure resonances have the same spin, is much smaller than that of finding a cluster due only to fluctuations in Γ and D .

The result of measurements for Rh(n, γ) in the 100–300 keV region is shown in Fig. I-14-2. In case of a doorway state resonance the ratio would have been expected to occur at one of the dashed lines. The experimental data do not show such structure. However, no strong conclusions can be drawn from these results. New total cross section measurements

by J. Whalen⁹ and by M. Divadeenam¹⁰ show no structure in this cross section at all.

In conclusion, whereas the determination of spins of all kinds of resonances seems to be an interesting possibility for the application of low level occupation probability ratios, the determination of level density parameters is very questionable and should be more favorable when carried out by direct methods.

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I-15. A Compilation of Capture and Fission Cross Sections for Reactor Calculations

W. P. POENITZ

Recently, an effort was made to use in reactor calculations absolute capture and fission cross sections consistent with the cross section standards or the applied experimental techniques employed in the microscopic measurements (Kallfelz and Poenitz¹). The selection of consistent cross section sets had been restricted to $\sigma_f(\text{U-235})$, $\sigma_\gamma(\text{U-238})$ and the ratio of $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$ in the energy range 30 keV to 3 MeV. For all other data the ENDF/B file had been used. Estimated uncertainties given for $\sigma_\gamma(\text{U-238})$ and $\sigma_f(\text{U-235})$ had been used for adjustments of the differential data to obtain a better agreement with the measured integral quantities. The reactor calculations for ZPR-3 Assembly 48 lead to the observation that the ratio $\sigma_f(\text{Pu-239})/\sigma_\gamma(\text{U-238})$ should be higher than that presently included in ENDF/B. This request, however,

supports the use of absolute cross section measurements which are, in the author's opinion, inconsistent.

The present data evaluation has been extended to the energy range 10 keV to 10 MeV. The ratios of $\sigma_\gamma(\text{U-238})/\sigma_f(\text{U-235})$, $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$, $\sigma_\gamma(\text{Au-197})/\sigma_f(\text{U-235})$ and $\sigma_f(\text{U-238})/\sigma_f(\text{U-235})$ have been considered. Absolute cross sections of $\sigma_f(\text{U-235})$, $\sigma_\gamma(\text{U-238})$, and $\sigma_\gamma(\text{Au-197})$ have been compiled. The latter capture cross sections have been converted to $\sigma_f(\text{U-235})$ values using the appropriate ratios.

$$\sigma_\gamma(\text{U-238})/\sigma_f(\text{U-235})$$

This ratio was measured by Linenberger et al.² in 1944 and recently at four energies by the author.³ Other values can be obtained from measurements of $\sigma_\gamma(\text{U-238})/[\sigma_f(\text{U-235}) + \sigma_\gamma(\text{U-235})]$ by Diven et al.⁴ and by

FIG. I-15-1. The Cross Section Ratio $\sigma_\gamma(\text{U-238})/\sigma_f(\text{U-235})$. ANL Neg. No. 118-3080.

FIG. I-15-2. The Cross Section Ratio $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$. ANL Neg. No. 113-3083.

deSaussure et al.⁵ after correction for the neutron capture in U-235. Barry et al.⁶ and Menlove and Poenitz⁷ measured absolute values of U-238 (n,γ). Absolute values of U-235 (n,f) measured by the same techniques were reported by White⁸ and Poenitz⁹, respectively. Combining the appropriate sets of cross section yield ratios of $\sigma_\gamma(\text{U-238})/\sigma_f(\text{U-235})$. These values are shown in Fig. I-15-1. The solid curve is the suggested ratio. It has been established by the good agreement between the values from Refs. 5, 7 and 9 in the lower keV energy range and 3, 6, 7, 8 and 9 in the higher keV energy range. At about 30 keV a curve obtained from the product of $\sigma_\gamma(\text{U-238})/\sigma_\gamma(\text{Au-197})$ and $\sigma_\gamma(\text{Au-197})/\sigma_f(\text{U-235})$ has been used as an additional guide.

$$\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$$

Values measured by Allen and Ferguson,¹⁰ Smith, Henkel and Nobles,¹¹ Dorofeev and Dobrynin,¹² Smirenkin et al.,¹³ White et al.,¹⁴ Perkin et al.,¹⁵ White and Warner,¹⁶ James,¹⁷ and Gilboy and Knoll¹⁸ are shown in Fig. I-15-2. Some of these values have been obtained from measurements of $\sigma_f(\text{Pu-239})$ and $\sigma_f(\text{U-235})$ which had been carried out absolutely or relative to a third standard cross section. In the energy range

above 200 keV the suggested ratio (solid line) has been established by the good agreement between the values from Refs. 10, 11, 13, 14, and 16; however, the measurements by Allen and Ferguson¹⁰ at 2.0, 2.5, and 3.0 MeV have been neglected. In the lower energy range there are still strong discrepancies. Values reported by Allen and Ferguson¹⁰ and White et al.¹⁴ diverge toward lower energies and differ at 40 keV by about 15%. The values measured by James¹⁷ seem to be supported by measurements by Gilboy and Knoll,¹⁸ both supporting the lower values measured by White et al.¹⁴ The structure around 8 keV follows from the measurements of Refs. 17 and 18. The step around 800–1000 keV follows from the measurements by Smirenkin.¹³ This structure is due to a 10% step in the U-235 cross section. The latter also causes the dip around 2.5 MeV, where $\sigma_f(\text{U-235})$ has a relative maximum.

$$\sigma_\gamma(\text{Au})/\sigma_f(\text{U-235})$$

This ratio is needed to utilize absolute measurements of the standard cross section $\sigma_\gamma(\text{Au})$. The available data (Refs. 4, 19–26) are shown in Fig. I-15-3. The solid curve has been established by the agreement among the majority of the measured data.

Fig. I-15-3. The Cross Section Ratio $\sigma_\gamma(\text{Au-197})/\sigma_f(\text{U-235})$. ANL Neg. No. 113-3068.

$$\sigma_f(\text{U-238})/\sigma_f(\text{U-235})$$

Ratios measured recently by Stein et al.²⁷ agree very well with values reported by Kalinin and Ponkratov²⁸ and reevaluated values reported by Hansen et al.²⁹ Recent measurements by White and Warner¹⁶ are about 5 % higher, older values reported by Lamphere³⁰ are about 10 % higher, and values reported by Allen and Ferguson¹⁰ are about 10–15 % lower than the presently recommended curve (see Fig. I-15-4).

$$\sigma_\gamma(\text{Au-197})$$

Absolute values of this cross section have been determined recently by Harris et al.⁴⁰ and Poenitz et al.²⁴ using different experimental methods. These values are supported by measurements by T. Ryves et al.²⁶ at 23 keV using an antimony-beryllium source and values measured by Ferguson and Paul³¹ at 150 keV and 550 keV relative to the scattering cross section of hydrogen. Measurements by Barry²² relative to the scattering cross section of hydrogen agree at 1 MeV with another value measured by Ferguson and Paul,³¹

however, they differ in the overlap range 120–600 keV from the above cited measurements by about 15 %. The cross section suggested here in the energy range 30–600 keV is shown in Fig. I-15-5. Barry's data have been neglected and this will be discussed under $\sigma_f(\text{U-235})$.

$$\sigma_\gamma(\text{U-238})$$

The data available from absolute measurements are shown in Fig. I-15-6. The suggested cross section curve is supported by measurements by Belanova et al.,³² Miller and Poenitz³³ (reevaluation of Belanova's experiment), Tolstikov et al.³⁴ (renormalized with Ref. 33), Hanna and Rose,³⁵ Lyon and Macklin,³⁶ Macklin et al.,³⁷ Miessner and Arai,³⁸ and Menlove and Poenitz.⁷ Data measured by Barry et al.⁶ again have been neglected.

$$\sigma_f(\text{U-235})$$

Both the capture cross section of U-238 and Au-1 have been converted to $\sigma_f(\text{U-235})$ values using the appropriate ratios discussed before (see Fig. I-15-7).

FIG. I-15-4. The Cross Section Ratio $\sigma_f(\text{U-238})/\sigma_f(\text{U-235})$. ANL Neg. No. 113-3067.

The U-235 fission cross section derived from the absolute capture cross sections of U-238 and Au-197 has been confirmed in a preliminary measurement by the author;^{3,42} it also agrees in shape with the measurements by Allen and Ferguson¹⁰ and with the absolute values below 100 keV measured by Perkin et al.¹⁵ and White.⁸ This cross section disagrees, however, in the higher keV energy range from values measured by Perkin et al.,³⁹ Allen and Ferguson,¹⁰ and White.⁸ Two different cross section sets are therefore suggested: Set 1, based on the absolute fission cross section of U-235 as

measured by White;⁸ and Set 2, based mainly on the absolute capture cross section measurements by Harris et al.,⁴⁰ Poenitz et al.,²⁴ Hanna and Rose,³⁵ and Menlove and Poenitz.⁷

The absolute capture cross section measurements by Barry et al.^{6,22} are based on the same normalization as White's measurements. Therefore they have been ignored in recommending values for $\sigma_\gamma(\text{Au-197})$ and $\sigma_\gamma(\text{U-238})$. Also, the agreement between the capture cross section measurements from Refs. 24 and 39 with the preliminary measurements of $\sigma_f(\text{U-235})$ by the author⁹ demonstrate

FIG. I-15-5. The Capture Cross Section of Gold. ANL Neg. No. 113-3081.

only the consistency of these measurements and does not yield additional information. In the lower keV energy range the measurements by Michaudon et al.⁴¹ seem to agree fairly well with the results from Perkin et al.¹⁵ and Knoll and Poenitz,²⁵ although they do not actually overlap. Other σ_f measurements seem to yield higher values.^{17, 42}

In the MeV energy region there is a discrepancy of about 15–20 % between a value measured by White⁸ and results from Kalinin and Ponkratov.²⁸ Measurements by Smith et al.¹¹ agreed very well with the latter; however, they have been recently revised²⁹ and now agree with White's result.

The values for $\sigma_\gamma(\text{U-238})/\sigma_f(\text{U-235})$, $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$, and $\sigma_f(\text{U-238})/\sigma_f(\text{U-235})$ and absolute values of $\sigma_f(\text{U-235})$, suggested here for use in reactor calculations, have uncertainties in the measurements and agreement or disagreement among different measurements. However, it may be more useful not to vary these ratios because the results from such variations are already known. The main success in using the presently suggested ratios in reactor calculations was the discovery of a wrong energy spectrum⁴⁴ indicated by the integral values of $\sigma_\gamma(\text{U-238})/\sigma_f(\text{U-235})$, $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-238})$ and $\sigma_f(\text{U-238})/\sigma_f(\text{U-235})$. This led to the conclusion that $\sigma_{n,n'}(\text{U-238})$ may be too high in the ENDF/B data file.⁴⁵

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FIG. I-15-6. The Capture Cross Section of Uranium 238. ANL Neg. No. 113-3084.

FIG. I-15-7. The Fission Cross Section of Uranium 235. ANL Neg. No. 113-3079.

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I-16. Analysis of the Capture Cross Section of U-238 Between 1 keV and 15 MeV⁽¹⁾

W. G. DAVEY

Since U-238 is an important constituent of fast reactors, it is important to obtain U-238 capture cross sections which may be used for analysis of fast reactors. A major consideration in deriving such cross

Best available values of the U-238 capture cross section in the range of a few keV to 15 MeV were derived by a careful study of twenty individual measurements of the capture cross section. Examina-

Fig. I-16-1. Capture Cross Section—Uranium 238 Curve Fit 5th + 12th Degree. *ANL Neg. No. 103-A11116.*

sections is to obtain values which are derived in a manner consistent with those used for the derivation of other cross sections, and hence, a principal objective of the present work was to obtain values consistent with fission cross sections previously selected^{2,3} by the author.

tion of published data showed that a process of re-normalization, re-evaluation, and selection of data had to be exercised in order to obtain the capture cross section values of U-238. Considerations of internal consistency and a cross-comparison of the results of separate measurements necessitated the

complete rejection of some data, and inadequate and inconsistent reporting of other data caused the rejection of other values. All available experiments were discussed¹ and detailed tabulations of the individual experimental data and selected best cross sections were given.

It was found that the data remaining after careful selection could be grouped into three categories which included a) good absolute measurements, b) good relative measurements, and c) less reliable absolute and relative data. These data were analyzed by weighted fitting of polynomials over two, overlapping, energy ranges. The consistency of the fits to the data indicated that the selected best data had an uncertainty of approximately 5%. The best selected experimental data and the fitted curves are shown in Fig. I-16-1.

Although the examination of resonance parameter data was not a specific objective of this study, a brief examination of the results of calculations of average cross sections up to 45 keV showed that the selected

data are consistent with current resonance data within the rather large range of uncertainties in resonance parameters. This conclusion is tentative since some recent measurements of resonance parameters are not completely in agreement with the earlier data assumed in the calculations.

The selected U-238 capture cross sections when used in conjunction with previously selected fission cross sections should provide a consistent basis for the analysis and understanding of fast reactor integral experiments.

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I-17. Selected Fission Cross Sections for U-236⁽¹⁾

W. G. DAVEY

Fission in the threshold material U-236 constitutes a valuable index of the fraction of the neutron spectrum above the threshold energy, and it is there-

fore important to obtain good nuclear data for this isotope.

New precise U-236/U-235 fission cross section ratios measured in the range of 1 to 5 MeV by W. Stein, R. Smith, and H. Smith² disagreed with previous data and necessitated a re-evaluation of the fission cross sections of these two uranium isotopes in the high energy region. It was concluded that these new measurements were of greater reliability than the previous values and new fission cross section data for U-236 were reevaluated on their basis (Fig. I-17-1). The new U-236 data provide an improved basis for the use of fission in this isotope as a high energy spectrum index in fast reactor studies.

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FIG. I-17-1. Fission Cross Section Data for U-236. ANL Neg. No. 103-11965 Rev. 1.

I-18. Multilevel Parameters for the Pu-239 Fission Cross Sections from 40 to 100 eV

P. LAMBROPOULOS

INTRODUCTION

Below 40 eV, F. T. Adler and D. B. Adler¹ have presented a fairly complete analysis of the fission as well as of the total cross section of Pu-239. More recently, J. Farrell² obtained a set of multi-level R matrix parameters between 14 and 90 eV. These parameters were obtained from the analysis of the Petrel³ fission data for Pu-239, using the Reich-Moore⁴ method. The same data were analyzed by the present author,⁵ between 40 and 150 eV, using the Adler-Adler⁶ method. During that analysis it was realized that the resolution of the Petrel data¹⁰ was rather poor and as a result the parameters obtained were not very reliable. The main advantage of these data is their low background which makes the resonance structure in the low value valleys of the cross section more apparent.

The present analysis of the Saclay⁷ data was undertaken in an effort to determine a more reliable set of fission parameters. The experience gained in the analysis of the Petrel data was useful in deciding on the resonance structure at the low lying valleys. Such structure, in the Saclay data, is masked by the relatively high background. On the other hand, the much superior resolution function of these data reveals more structure above the background level. This analysis seems to reveal a larger number of levels than reported heretofore. The existence of some of them, however, cannot be considered as conclusive until an analysis of the total or capture cross section of Pu-239 is performed and made compatible with the fission cross section. Such analysis was not undertaken in this work due to lack of the necessary experimental data.

Since the Adler-Adler method has been discussed repeatedly⁶ no discussion of it will be presented here. A brief discussion of the method pertaining to the present analysis can be found in Refs. 9 and 10.

RESULTS AND DISCUSSION

The parameters obtained are given in Table I-18-I and the fits in Figs. I-18-1 through I-18-3. In general, the fits are quite good except at some specific areas which will be discussed in detail. By comparing the sent fits to the fits of Ref. 5 at well resolved resonances it was found that the energy scale difference between the two sets of data was very small, if any.

TABLE I-18-I. MULTI-LEVEL FISSION RESONANCE PARAMETERS FOR PU-239 FROM 40 TO 100 EV

μ , eV	ν , eV	$G^{(F)}$, $10^{-4} \text{ (eV)}^{1/2}$	$H^{(F)}$, $10^{-4} \text{ (eV)}^{1/2}$	$\tilde{J}$
40.90	0.400	-0.1877 $\pm$ 0.049	0.1483 $\pm$ 0.055	0
41.50	0.075	2.2242 $\pm$ 0.062	-0.0917 $\pm$ 0.038	1
42.50	0.075	-0.0336 $\pm$ 0.017	-0.0213 $\pm$ 0.017	1
44.46	0.015	1.1200 $\pm$ 0.039	0.0640 $\pm$ 0.019	1
45.70	0.140	0.1049 $\pm$ 0.023	-0.0428 $\pm$ 0.022	1
47.51	0.115	3.0435 $\pm$ 0.076	-0.2208 $\pm$ 0.039	0
49.60	0.550	2.6238 $\pm$ 0.119	-0.8530 $\pm$ 0.107	0
50.05	0.020	1.2309 $\pm$ 0.065	0.5594 $\pm$ 0.051	1
51.60	0.040	0.0174 $\pm$ 0.024	0.0301 $\pm$ 0.023	1
52.50	0.015	2.5341 $\pm$ 0.075	0.0441 $\pm$ 0.033	1
54.40	0.500	-0.0286 $\pm$ 0.068	0.3892 $\pm$ 0.065	0
55.50	0.022	1.1082 $\pm$ 0.052	0.1556 $\pm$ 0.034	1
57.50	0.450	15.1709 $\pm$ 0.263	-2.2762 $\pm$ 0.174	0
59.05	0.030	6.1212 $\pm$ 0.213	-0.2586 $\pm$ 0.127	1
60.00	0.800	1.6524 $\pm$ 0.283	-0.8193 $\pm$ 0.329	0
63.05	0.040	0.7137 $\pm$ 0.078	0.6976 $\pm$ 0.068	1
64.35	0.400	-1.0708 $\pm$ 0.158	1.1020 $\pm$ 0.145	0
65.55	0.036	11.3340 $\pm$ 0.313	1.7846 $\pm$ 0.176	1
66.25	0.080	1.2239 $\pm$ 0.144	0.1023 $\pm$ 0.136	1
67.00	0.100	0.4986 $\pm$ 0.102	0.2157 $\pm$ 0.099	1
68.15	0.500	0.6069 $\pm$ 0.158	0.7854 $\pm$ 0.153	0
71.00	1.000	-0.2732 $\pm$ 0.226	1.4767 $\pm$ 0.209	0
73.35	0.150	-0.4606 $\pm$ 0.105	0.2727 $\pm$ 0.109	1
74.25	0.090	2.5335 $\pm$ 0.257	1.7349 $\pm$ 0.209	1
75.10	0.070	17.5130 $\pm$ 0.665	-0.1221 $\pm$ 0.288	1
76.20	0.350	0.6801 $\pm$ 0.246	-0.4905 $\pm$ 0.220	1
77.50	0.950	-0.1556 $\pm$ 0.339	-0.2204 $\pm$ 0.282	0
81.30	0.995	3.1895 $\pm$ 0.234	-5.3270 $\pm$ 0.185	0
83.30	0.053	1.3791 $\pm$ 0.103	-0.3226 $\pm$ 0.092	1
84.80	0.120	2.4018 $\pm$ 0.208	1.5482 $\pm$ 0.216	1
85.70	0.085	8.0350 $\pm$ 0.364	5.0357 $\pm$ 0.330	1
86.30	0.195	5.3621 $\pm$ 0.308	-0.7772 $\pm$ 0.303	1
88.00	0.750	5.4795 $\pm$ 0.304	3.9537 $\pm$ 0.264	0
89.60	0.180	0.6714 $\pm$ 0.082	0.4432 $\pm$ 0.078	1
90.90	0.030	3.1426 $\pm$ 0.099	0.5829 $\pm$ 0.071	1
92.20	0.210	1.2064 $\pm$ 0.128	0.3705 $\pm$ 0.135	1
93.00	0.180	0.8190 $\pm$ 0.129	-0.1816 $\pm$ 0.012	1
94.50	0.800	2.3920 $\pm$ 0.381	-0.4122 $\pm$ 0.381	0
95.50	0.030	1.7169 $\pm$ 0.133	-0.4193 $\pm$ 0.119	1
96.50	0.100	1.7865 $\pm$ 0.177	-0.5955 $\pm$ 0.168	1
97.60	0.300	2.2886 $\pm$ 0.225	0.8073 $\pm$ 0.196	1
99.00	1.100	1.5486 $\pm$ 0.412	-0.8492 $\pm$ 0.306	0

Thus it appears that the differences which may exist in the positions of the poles between this analysis and that of Ref. 10 are not likely to be due to differences in the energy scale.

The present results seem to reveal 42 levels between 40 and 100 eV, while in ENDF/B File 2 (see Ref. 8)

FIG. I-18-1. Saclay Fission Data for Pu-239 (40-60 eV). ANL Neg. No. 113-2003 T-1.

FIG. I-18-2. Saclay Fission Data for Pu-239 (58-80 eV). ANL Neg. No. 113-2001 T-1.

FIG. I-18-3. Saclay Fission Data for Pu-239 (78–101 eV). *ANL Neg. No. 113-2002 T-1.*

only 23 levels are listed. Compared to Farrell's² results with 28 levels between 40 and 90 eV, the present analysis reveals 34 levels in the same energy range. The angular momentum assignments, given in Table I-18-I, have been made on the basis of level widths and level interference. These assignments, although not conclusive, are in general agreement with those of Farrell² with a few exceptions to be discussed later.

The fit in the valleys of Fig. I-18-1 is not very satisfactory because of the large fluctuations of the cross section there. Thus the parameters of the poles at 40.75, 42.50 and 45.70 eV are rather uncertain as are the positions of the poles themselves. Between 50.05 and 52.50 eV, only one pole at 51.60 eV has been assumed, although the Saclay data seem to indicate the existence of one more pole in that vicinity. However, no such indication is given by the Petrel data. In the next valley, a pole at 54.40 eV, belonging to the $J = 0$ group, has been found necessary. Farrell² does not find it necessary to introduce a level there,

the present analysis gives strong indications in favor of its existence. The poles corresponding to the

peaks of Fig. I-18-1 are rather well established and the fit there is reasonably good.

In Fig. I-18-2, the fit is very good except between 74 and 76 eV. Here, however, the difference between this analysis and that of Farrell is rather substantial. For example, Farrell finds it necessary to introduce a level at 62.70 eV with $J = 0$ while he assigns angular momentum one to a level at 60.65 eV. Here, only one level at 60 eV with $J = 0$ has been introduced. Thus one could say with reasonable certainty that a broad level with $J = 0$ exists between 60 and 62 eV. Its precise position, as well as the existence of a second narrow level with $J = 1$, are not certain. In the vicinity of 74–76 eV, it has not been possible to obtain a better fit. The Saclay data exhibit two rather well resolved peaks there, while in the Petrel data the peak at 74.25 eV appears as a small hump on the wing of the second peak. It is possible that one more hidden level exists in that vicinity. A second possibility is that part of the peak is due to the presence of Pu-241 in the sample. Pu-241 has a resonance around 75 eV.

Perhaps the most interesting aspect of the present

fit is the structure between 80 and 90 eV. In the same energy range, Farrell² finds 4 levels while the present analysis reveals 7 poles. All of these 7 poles seem to be rather well established, with the possible exception of the pole at 86.30 eV. Farrell² introduces a broad level of the group $J = 0$ at 85.60 eV. In this study all four poles between 83 and 87 eV have been assigned to the $J = 1$ group. These results depart from both Farrell's analysis as well as that of Ref. 5. Presumably, this is due to the fact that the structure shown in Fig. I-18-3 is smeared out in the Petrel data.

Above 90 eV, the present results have only been compared to the single-level parameters given in Ref. 8, where 5 levels are listed between 90 and 100 eV. Here 8 poles are found in the same energy range.

Direct comparison of the present parameters to Farrell's parameters is not possible since the latter are R -matrix parameters. Given a set of R -matrix parameters, however, one can construct the inverse level matrix. The complex eigenvalues of this matrix yield μ_k as their real part and ν_k as their imaginary part.⁹ This was done with Farrell's parameters and the eigenvalues of the resulting inverse level matrix were determined numerically. The so-obtained values of μ and ν were then used as guesses in a multi-level analysis of the Saclay data. The resulting fits were not satisfactory. Moreover, about half of the above values of μ and ν differed rather significantly from the values of μ and ν obtained in the present work. The agreement, as expected, was best for well-resolved strong resonances. In this sense, therefore, there is some discrepancy between this work and Farrell's² analysis.

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I-19. Developments Relating to the ENDF/B Project

E. M. PENNINGTON, J. C. GAJNIAK and A. B. COHEN

The ENDF/B system of neutron cross section data and processing codes is used at Argonne to produce library tapes for both the CDC-3600 and IBM 360 ARC system versions of the multigroup cross section code MC².⁽¹⁾ Short descriptions of some of the codes and earlier work on data processing have been given previously.^{2,3} More recent work is described here.

ENDF/B DATA PROCESSING

New versions of the data on ENDF/B tapes 114, 115 and 116 were received. The term "revised cate-

gory 1" is used to refer to these data. Also ENDF/B tape 998 containing preliminary data for eight materials was received. Most of the data were processed through codes DAMMET, ETØE and MERMC2 to yield MC² libraries on both the CDC-3600 and IBM 360. In some cases the data had to be modified using program CRECT to conform to MC² format limitations.

Eight materials of MENDF/B (modified ENDF/B) data were included in the libraries. The data were produced by Atomic Power Development Associates,⁴ and were available in the MC² library

mat for the CDC-3600. In order to convert the data to the IBM 360, two new versions of the code MERMC2⁽²⁾ were written. The first version, which operates on the CDC-3600, reads the 3600 binary tape and produces a binary tape readable by the IBM 360 and which is almost in the MC² library format. The second version is run on the IBM 360 and uses the tape produced by the first version as input. The output tape is exactly in the format of the MC² library. In performing the conversion, two library subroutines written by Mrs. Nancy Clark* were used. A two-step rather than a one-step process was necessary because of the difficulty in converting the A-formats used for the identification of the materials.

Table I-19-I lists all of the data available for the CDC-3600 version of MC². All of the data except for that of deuterium, for which unexplained difficulties occurred in the processing, are also available on the system 360. Table I-19-I gives the ENDF/B identification number MAT, the MC² identification, the presence or absence of elastic scattering Legendre coefficient data, and the status of the data. The four possible entries for the status are revised category 1 ENDF/B, preliminary ENDF/B, MENDF/B, and data modified by Argonne. The materials which were modified by Argonne are described briefly below in the section on Data Revisions.

ENDF/B FORTRAN CODES

Modified versions of the codes CHECKER, CRECT, DAMMET and ETØE were received and made operable on both computers. Brief summaries of the purposes of these codes are given in Ref. 2. Also two new codes, SLAVE3 and PLØTFB, were received. These codes are outlined below.

SLAVE3

Program SLAVE3 gives averages of ENDF/B File 3 tabulated cross section data over various spectra such as a Maxwellian, a fission spectrum, or an arbitrary input spectrum. The code has been made operable on both computers.

PLØTFB

The code PLØTFB lists data from an ENDF/B tape or plots the data using a Calcomp plotter. The version originally received has been made operable on the CDC-3600 only, while the revised version has

* Applied Mathematics Division, Argonne National Laboratory.

TABLE I-19-I. CONTENTS OF 77-MATERIAL MC² LIBRARY TAPE

MAT	MC ² ID	Legendre Data	Status ^a	MAT	MC ² ID	Legendre Data	Status ^a
1005	LI 6	No	R	1007	BE 9	Yes	R
1006	LI 7	No	R	1015	AL 27	Yes	R
1009	B 10	Yes	R	1038	TH232	No	P
1010	C 12	Yes	R	1041	U 233	No	P
1012	N 14	Yes	R	1044	U 2351	No	A
1014	MG	Yes	R	1013	Ø 16	Yes	R
1016	TI	Yes	R	1020	FE	Yes	R
1017	V	Yes	R	1026	XE135	No	R
1019	MN	No	R	1027	SM149	No	R
1025	MØ	Yes	R	1043	U 234	No	R
1030	GD	No	R	1046	U 236	No	R
1035	TA181	No	R	1047	U 238	No	R
1053	PU240	No	R	1051	PU239	No	R
1059	NA 23	Yes	R	1056	AM241	No	R
1060	W 182	No	R	1057	AM243	No	R
1061	W 183	No	R	1018	CR	Yes	R
1062	W 184	No	R	1021	NI	Yes	R
1063	W 186	No	R	1054	PU241	No	R
1042	U3FP1	No	R	1044	U 2352	No	A
1045	U5FP1	No	R	1044	U 2353	No	A
1052	P9FP1	No	R	1087	CU	Yes	P
1066	U3FP2	No	R	1083	RE185	No	P
1067	U3FP3	No	R	1084	RE187	No	P
1068	U5FP2	No	R	1085	CU 63	Yes	P
1069	U5FP3	No	R	1086	CU 65	Yes	P
1070	P9FP2	No	R	1088	HE	No	P
1071	P9FP3	No	R	1022	ZR	Yes	P
1024	NB	No	R	1021	NI	Yes	M
1028	EU151	No	R	1020	FE	Yes	M
1029	EU153	No	R	1044	U235	No	M
1031	DY164	No	R	1047	U238	Yes	M
1032	LU175	No	R	1051	PU239	No	M
1033	LU176	No	R	1059	NAM	Yes	M
1037	AU197	No	R	1025	MØM	No	M
1044	U 235	No	R	1053	PU240M	No	M
1048	NP237	No	R	1054	PU241R	No	A
1050	PU238	No	R	1047	U 238I	No	A
1055	PU242	No	R	1003	D	No	R
1058	CM244	No	R				

* R—Revised Category 1 ENDF/B

P—Preliminary ENDF/B

M—Modified ENDF/B by APDA

A—Modified ENDF/B by Argonne (does not include minor modifications to satisfy MC² formats)

not been compiled. Many changes had to be made in the code for compatibility with CDC-3600 Fortran and the Argonne version of the Calcomp 580 plotting package. A drawback of the code is the fact that the simplicity of the input results in a lack of flexibility in the output. For example, one might wish to plot a certain size graph of one type of File 3 cross section for a material. However, one can plot only all File 3 cross sections for that material, with no choice of graph size or axis scales.

OTHER FORTRAN CODES

UR

Program UR calculates infinite dilution capture, fission, and scattering cross sections from unresolved resonance parameters by integrating over the appropriate statistical distributions. Also an option is available in which neutron and fission widths may be varied to fit input experimental capture and fission cross sections. An early CDC-3600 version of the code including the equations involved has been described previously.³ The code has been converted to the IBM 360 and improvements have been made. Neutron and fission widths for both *s*- and *p*-wave neutrons may now be varied to fit the input data. Originally only *s*-wave widths could be varied. The most significant change is the use of a new subroutine to perform the numerical integration involved in calculating the fluctuation integrals. This new subroutine is about thirty times as fast as the old one, and makes the speed of the program quite acceptable rather than too slow.

MAGIC

MAGIC is the name of the program which is used to make Calcomp plots of data from the MC² library tape. An earlier version was mentioned briefly in Ref. 2.

The MC² library tape consists of six files containing the *W*-table, a table of contents and resonance data, smooth cross sections, inelastic and (*n*,2*n*) secondary energy distributions, fission spectra, and elastic scattering Legendre coefficients. The *W*-table contains the real and imaginary parts of the complex probability function, and is used by MC² for calculation of the ψ and χ Doppler-broadening functions. Program MAGIC does not use the *W*-table file. Almost any desired selection of data from the other five files may be listed and plotted. Cross sections may be calculated and plotted using both the resolved and unresolved resonance parameters of the resonance file. The parameters in the fission spectra file may be used for calculation and plotting of fission spectra. Plotting of smooth data, secondary energy distribution data, and Legendre coefficient data requires no computational processing of the data on the tape.

The input to MAGIC is rather complicated, but a great deal of flexibility in the output is thereby achieved. A wide choice of scales, axis types, graph dimensions and labels is available.

The code is operational on the CDC-3600, and makes use of the Calcomp 580 plotter. It is planned to convert MAGIC to the IBM 360 system in the near future.

DATA REVISIONS

U-235

A version of U-235, referred to in Table I-19-I as U 2351, consists of the ENDF/B U-235 with unresolved resonance parameters added. The derivation of these parameters has been described previously.³ The version referred to as U 2352 is the same as U 2351 except that the smooth cross sections in the unresolved region on the corresponding ENDF/B tape have been changed with the smooth capture and fission set equal to zero. The reduced neutron widths are then altered in processing through ETØE so that the original cross sections, calculated as the sum of smooth and infinite dilution resonance contributions, are produced. This procedure is a result of the fact that energy-dependent reduced neutron widths are not allowed by present ENDF/B formats. The version of U-235 referred to as U 2353 contains unresolved resonance parameters based on data provided by T. Pitterle.⁵

The three versions of U-235 with different unresolved parameters were used in the calculation of Doppler coefficients for an assembly which is the core of ZPR-6 Assembly 4Z with 25% sodium removed. Problems were run using MC², in conjunction with MACH-1,⁽⁶⁾ with temperatures of both 300 and 750°K being used in the resonance calculations. The values of $\Delta k_{eff}/k_{eff}$ for the U-235 temperature change from 300 to 750° were 0.00022, 0.00026, and 0.00037 for U 2351, U 2352 and U 2353, respectively. Nearly all of the Doppler coefficient contribution comes from the unresolved resonance region. The positive Doppler coefficient for U 2352 is larger than that for U 2351 because U 2352 has no smooth capture and fission backgrounds which do not contribute to the Doppler effect. U 2353, which has the largest Doppler coefficient, differs qualitatively from U 2351 or U 2352 in having smaller average fission widths and four degrees of freedom for the chi-squared distribution of fission widths rather than one or two degrees depending on the spin state as in U 2351 or U 2352.

PU-241

Since the ENDF/B Pu-241 has no resonance parameters, a version was prepared in the ENDF/B format including resolved and unresolved resonance parameters. This version is called PU241R in Table I-19-I, and allows MC² calculations to be made in which resonance shielding of Pu-241 occurs.

Resolved resonance parameters are provided the energy range 0.5 to 64 eV, and unresolved parameters cover the range 64 eV to 50 keV. No smooth background cross sections exist in the resonance region.

original ENDF/B data were used for everything except the cross sections in the resonance range.

The resolved resonance data were taken from Table IV of Ref. 7. The unresolved parameters were based on those of Table V of Ref. 7, but differ from them in that $\Gamma_\gamma = 0.030$ eV and $\nu_f = 2$, while $\Gamma_\gamma = 0.040$ eV and $\nu_f = 1$ in Ref. 7. Here ν_f is the number of degrees of freedom for the chi-squared fission width distribution. The Ref. 7 parameters would yield σ_c about 50% higher and σ_f about 20% lower than the present ones.

The fission resonance integral of PU241R differs from that of the ENDF/B Pu-241 by less than one percent for the range 0.5 eV to 46.5 keV. The PU-241R capture resonance integral is about 20% higher than that of ENDF/B. Experimental data on the Pu-241 capture cross section are almost non-existent so that it is difficult to estimate the accuracy of the capture data. Certainly the fission data should be considerably better than the capture data.

U-238

A version of U-238, referred to as U 2381 in Table I-19-I, consists of the ENDF/B version, except that the inelastic scattering cross section and secondary

energy distributions which led to cross section set 224 are used. This new version has a smaller inelastic scattering cross section than that of ENDF/B over much of the energy range.

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I-20. Am-241 Spontaneous Fission Half-Life

R. GOLD, R. J. ARMANI and J. H. ROBERTS*

The Am-241 spontaneous fission decay constant has been determined with solid-state track recorders of pre-etched mica.¹ Measurements have been carried out with electroplated Am-241 sources. Consistent spontaneous fission rate measurements have been obtained for a large range of fission track densities over more than a three-year period. Systematic error

due to source impurities has been established as insignificant relative to statistical error. The present experimental result for the Am-241 spontaneous fission decay constant is: $\lambda_f = (6.04 \pm 0.13) \times 10^{-15} \text{ y}^{-1}$

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* Macalester College, St. Paul, Minnesota.

I-21. The Alpha Half-Life of U-234

J. W. MEADOWS

The amounts of uranium on a series of U-235 fission foils have been determined by alpha counting and chemical analysis. Since the material on the foils contained approximately 1% U-234, this isotope contributed over 95% of the alpha activity and these measurements could be used to determine the U-234 half-life.

Three samples of approximately 93% U-235 having mass analysis of good precision were converted to

obtained by a linear extrapolation to zero deposit thickness plus a 3.0% correction for backscattering. The uncertainties in these corrections can result in errors of a percent or more in the specific activity. However, since the three sets of foils had deposits in the same mass range the relative values should be very good.

The activities of the other uranium isotopes were subtracted from the specific activities and the half

TABLE I-21-I. MASS ANALYSIS OF THE URANIUM SAMPLES AND EXPERIMENTAL HALF LIVES OF U-234

	Sample A	Sample B	Sample C
U-234	0.856 $\pm$ 0.001	1.093 $\pm$ 0.002	0.662 $\pm$ 0.001
U-235	93.250 $\pm$ 0.004	93.157 $\pm$ 0.008	93.227 $\pm$ 0.008
U-236	0.332 $\pm$ 0.001	0.267 $\pm$ 0.001	0.292 $\pm$ 0.001
U-238	5.562 $\pm$ 0.009	5.423 $\pm$ 0.008	5.859 $\pm$ 0.008
$t_{1/2}$ (U-234), 10^6 yrs:			
2π	2.449 $\pm$ 0.006	2.443 $\pm$ 0.031	2.460 $\pm$ 0.031
low geometry	2.428 $\pm$ 0.006	2.433 $\pm$ 0.029	2.543 $\pm$ 0.018

tetrafluoride and deposited on platinum disks by vacuum evaporation. The deposits were 2.54 cm in diameter with thicknesses ranging from ~ 70 to ~ 600 $\mu\text{g}/\text{cm}^2$. The mass analyses are listed in Table I-21-I. Ten foils were prepared from sample A, five from B, and five from C. After alpha-counting the uranium on each foil was dissolved and measured by a colorimetric method. The error as determined from the difference of duplicate measurements was approximately 0.5%.

All foils were counted in a counter with a geometry factor of 1040 ± 2 . The factor was calculated from the counter dimensions and the error is the combined error of these dimensions. The statistical errors were all $< 0.3\%$. There was no indication that the specific activity depended on sample thickness.

The foils were also counted in a 2π proportional counter. The specific activity of each sample was

life of U-234 was calculated. The results are shown in Table I-21-I. The errors were calculated from the scatter of data. The similarity of the errors in the low geometry and 2π results show that most of the scatter is due to the chemical analysis and indicates an error of approximately 0.2% per foil in A and approximately 0.5% per foil in B and C.

The low geometry results for samples A and B are in good agreement but C is high by nearly 5%. There is no obvious explanation for this. Samples B and C were counted and analyzed together, so any malfunction should appear in both. Furthermore, the good agreement of the 2π results seems to rule out any error in the mass or chemical analysis.

A weighted average of the low geometry results gives

$$t_{1/2} = (2.439 \pm 0.014) \times 10^5 \text{ yr.}$$

I-22. Half-Life of Cf-252 from Neutron Counting

A. DeVOLPI, K. G. FORGES and F. OZER

Having already established a capability for absolute neutron source detection (see Paper I-27), it has become both useful and convenient to monitor the decay of Cf-252. This spontaneous fission isotope is the basic standard for the neutron yield in fission (see Paper I-24), as well as having applications in various reactor physics experiments. Previous measurements of its half-life have been made through fission fragment counting. Another technique is to

observe the neutron emission rate resulting directly from spontaneous fission.

For almost six years a sealed sample of Cf-252 has been measured with the manganese bath facility. Two computations of the half-life were made: one based on reference to a standard Ra-Be(α, n) source and the other derived from absolute neutron emission rates. The results agree well, yielding a total half-life of 2.621 ± 0.006 yr, which is about 1% less than that reported for fragment counting.

I-23. Energy Dependence of Delayed Neutron Yield from Neutron-Induced Fission of Th-232, U-235 and U-238

S. A. COX and E. E. DOWLING WHITING

We have made detailed measurements of the delayed neutron yield from neutron induced fission of Th-232, U-235 and U-238. Some preliminary results of our delayed neutron measurements were presented in Ref. 1. In Ref. 1 our data for neutron induced fission of U-235 was discussed but the analysis of the data for neutron induced fission of Th-232 and U-238 was still in progress so that only some qualitative comments could be made. The analysis of the Th-232 and U-238 data have been completed and the results are included in the present report. In the interest of clarity of presentation some sections of Ref. 1 are included in this paper.

Two different aspects of the delayed neutron process were investigated. One series of measurements was made at a number of different incident neutron energies to determine if there was any change in the time dependence of the decay of the delayed neutron activity. A second series of measurements was made to determine if there were any dependence of the total yield of the delayed neutron activity as a function of incident neutron energy. Because the requirements of the two types of measurements were so different, two separate experimental arrangements were necessary. For the time dependence measurements, it was important to have the highest possible yield.

experimental arrangement is shown in Fig. I-23-1. --- cylindrical samples of uranium and thorium were encased in hardened steel and positioned in a

pneumatic tube near the lithium target for irradiation.

After irradiation for a preset time (usually ~ 4 min) the sample was fired a distance of ~ 6 feet into a shielded counting assembly consisting of 12 BF₃ counters moderated by mineral oil. The transit time was ~ 0.2 sec. The data were collected using a 400 channel analyzer with scaling time ranging from 0.075 to 2. sec per channel. The longest scaling times were used for background determination. Because of the high yield requirement which necessitated "poor" geometry the experimental arrangement was not suitable for monitoring of the neutron flux from the lithium target. Thus the yields at different energies could not be directly compared. In fitting the data we made the usual assumption that the delayed neutron decay could be described by 6 components.² Because of the sample transit time of 0.2 sec we did not observe the shortest half-life component. Thus our data were fitted with 5 exponential components. We began the analysis by trying two different methods to extract the amplitudes and half lives of the components from the data. One was the straightforward peeling off procedure. In this method the background was subtracted first, then the longest half-life component was fitted and subtracted, then the next longest half-life component was fitted and subtracted and so on. The other method was to make a simultaneous 5 component fit.³ Neither method was considered

FIG. I-23-1. Experimental Arrangement for "Time Dependence" Measurements. ANL Neg. No. 113-1976 Rev. 1.

FIG. I-23-2. Experimental Arrangement for Measurement of Total Delayed Neutron Yield Versus Neutron Energy. ANL Neg. No. 113-1975.

satisfactory. The peeling off method was considered too subjective and the simultaneous fitting method did not yield unique answers. Hence, we finally decided on a compromise method. The data were fitted in 4 stages starting first with the long time decay portion. In the first stage the data for $t > 60$ sec were fitted with two groups with 4 free parameters. In the second stage the half-life and amplitude of the longest half-life component were constrained and the data for $t > 20$ sec were fitted with 3 components. In the third stage the two longest half-life components were constrained and the data for $t > 5$ sec were fitted with 4 components. In the final stage the three longest half-life components were constrained and the data for $t > 0.5$ sec were fitted with 5 components. In each stage there were 4 free parameters. This method gave the most consistent results and was used for all group determinations.

The experimental arrangement used for the to yield measurements is shown in Fig. I-23-2. The lithium target and sample to be irradiated were placed within a 1.5 ft diam cylindrical cavity. The samples for this phase of the measurements were in the shape of disks approximately 1 in. diam by 0.2 in. thick. For irradiation, a sample was placed 1.25 in. from the lithium target. The sample also served as the active source in a parallel plate fission chamber. Thus the monitored neutron flux was always measured relative to the fission cross section of the irradiated element. The irradiations were for 4 min at the end of which time the proton beam was interrupted and the delayed neutron activity counted by an annular array of 11 BF_3 counters placed just outside the cylindrical cavity in a moderating bath of mineral oil. Measurements were made for U-235 at various energies from 0.25 to 1.5 MeV. For Th-232 and U-238 measurements were made from below the fission threshold to 2.4 MeV. The data for all elements were normalized at 1450 keV to G. R. Keepin's fast neutron determinations. All data were corrected for effects due to the non-isotropic fission fragment angular distribution and isotope impurities in the samples. For all the measurements a 5 channel "leaky current integrator" was used to monitor the degree of saturation of the 5 main delayed neutron activities.

Our results for U-235 are given in Fig. I-23-3 together with Keepin's values for thermal and fast neutron irradiations.² The errors in the individual points are estimated to be $\pm 4\%$ based on both the counting statistics and the reproducibility of the measurements. Within the estimated error there is no significant change in the delayed neutron yield over the energy range from thermal energy to 1.5 MeV. The total yield was normalized to Keepin's fast fission spectrum

FIG. I-23-3. Delayed Neutron Yield Versus Neutron Energy for U-235. ANL Neg. No. 113-1986.

asurements given by the open triangle at ~ 1.4 MeV.⁽¹⁾

Our results for U-238 are given in Fig. I-23-4. The fission cross section of U-238 is included for reference. Here the measurements start at 900 keV, well below the fission threshold and extend well into

FIG. I-23-4. Delayed Neutron Yield Versus Neutron Energy for U-238. The U-238 Fission Cross Sections are Included for Reference. ANL Neg. No. 113-1987.

FIG. I-23-5. Delayed Neutron Yield Versus Neutron Energy for Th-232. The Th-232 Fission Cross Sections are Included for Reference. ANL Neg. No. 113-1988.

the plateau region. The data have been corrected for isotope impurity in the U-238 sample and for effects, due to the changes in the fission fragment angular distribution in the vicinity of the fission threshold. It is clear from the data that there is no significant change in the delayed neutron yield over the energy range from 900 keV–2.4 MeV, although there is some slight indication of a depression in the yield at 1.3 MeV.

Our results for thorium are given in Fig. I-23-5. Here again the fission cross section is given for reference. Again the measurements extend below threshold to well above threshold. The measurements indicated by the open circles were taken specifically to determine whether or not there was any effect on the delayed neutron yield due to the resonance structure in the fission cross section. The yield at the peak of the resonance is slightly higher than the yield off resonance but the difference is not considered to be sufficiently outside the standard deviation of the measurements to be convincing. The overall yield does indicate a slight increase from below threshold to the peak of the first resonance but again the change is not really outside the standard deviation of the measurements.

We conclude that for U-235 the delayed neutron yield is constant to within $\sim 5\%$ up to ~ 1.5 MeV and up to 2.4 MeV for U-238. For thorium there may be some indication of structure due to the resonances in the fission cross section but the effect is no greater than $\sim 10\%$ and is not significantly outside the errors in our data. This is compatible with our time dependence data for which thorium did not show any significant difference in the decay curves on and off resonance.

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I-24. The Absolute Yield of Neutrons per Fission for Cf-252

A. DeVOLPI, K. G. PORGES and F. OZER

Among the prime requirements for breeder reactor development, and in fact frequently foremost among stated needs, is an accurate value of the neutron yield (ν) for Pu-239. Achievement of accuracy in the order of $\frac{1}{2}\%$ has not been possible because of difficulties associated with neutron background in direct measurements. To circumvent this difficulty, a two-step procedure has evolved for determination of ν for Pu-239 and the other fissile materials: (1) precise measurements may be made relative to the neutron yield from the spontaneous fission isotope Cf-252; and (2) the separate measurements of ν (Cf-252) are carried out without background interference.

It has been a goal of the Reactor Physics Laboratory to make such absolute measurements of the yield of neutrons from Cf-252. Up through 1966, current objectives were for an accuracy in the order of 1%, and such a measurement was reported.¹ However, there remained gaps of almost 3% in published values, while requests for even better accuracy from $\frac{1}{4}$ to $\frac{1}{2}\%$ were being made. Accordingly, a thorough reevaluation of all facets of the Argonne measurement was undertaken.

A third fission chamber was added to the series; this chamber provided very high (99%) intrinsic fission fragment efficiency, thus avoiding one of the problem areas connected with the earlier two fission counters. The fission rate calibrations for the first two counters were redone, and the new chamber was calibrated by several techniques, including the neutron-fission coincidence method.

The absolute measurements at Argonne are done in two stages which are approximately concurrent. There are separate evaluations of the absolute neutron rate and of the absolute fission rate; their ratio gives ν directly.

Efforts were made to strengthen the neutron cali-

bration phase. This took the form of improved facilities, extensive supplementary investigations, and cross checking of results. (See Papers I-22 and II-41.) Particular emphasis was placed on standardization of californium fission sources, since some manganese bath correction factors are energy dependent. Remaining unverified correction factors, which are in the order of $\frac{1}{2}\%$, are still being subjected to investigation.

As a result of these additional efforts, a value of ν (Cf-252) = 3.725 ± 0.015 (total yield) can be reported. This measurement is generally independent of cross sections, neutron spectrum assumptions, counting statistics, and fission fragment self-absorption. The error assigned is a combination of statistical and systematic effects at a confidence level of 68%.

This value for ν (Cf-252) is somewhat lower (1.3%) than the previously accepted world average; yet it is only $\frac{1}{2}\%$ below the average of all experiments.² In addition, there are some recently strengthened measurements in England which are quite consistent with this value.³ As a result, it appears that the fissile constants may be generally lower than previously utilized; for example, for U-235, the neutron yield would be about 2.40 neutrons/fission.

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I-25. Determination of the Branching Ratio of Be-7

W. P. POENITZ and A. DeVOLPI

The decay of Be-7 has become an important parameter for various capture and fission cross section measurements of interest to the Liquid Metal Fast Breeder Reactor program. Be-7 is the residual activ-

ity associated with neutron emission from the $\text{Li}^7(p,n)$ reaction, which is frequently used as a neutron source in cross section experiments. Neutrons in the 400-480 keV energy range can be produced in this reaction,

is providing a useful source in the region of fast reactor interest. Moreover, by measuring the Be-7 activity, it is possible to make absolute determinations of the neutron cross sections.¹

The 477 keV gamma-ray emitted in the decay of Be-7 is convenient to detect; however, in order to determine the absolute neutron flux, one must know the frequency with which such gamma emission occurs. The present investigations have been carried out to clarify controversial values reported for the electron capture branching ratio. The 4π Auger-electron γ coincidence technique has been applied by J. Taylor and J. Merritt² to determine the branching ratio of Be-7. Whereas absolute radioactivities can usually be determined with very high precision using the $4\pi\beta$ - γ coincidence method,³ there is a large difficulty in case of Be-7 because of the low energy of the Auger electrons. The ratio $a = 0.1032 \pm 0.0016$ reported by Taylor and Merritt contrasts to the previously reported value of 0.123 ± 0.006 .^(4,5) The latter was obtained by a determination of the absolute neutron flux of the $\text{Li}^7(p,n)\text{Be}^7$ source reaction and the measurement of γ -activity from Be-7. An indirect confirmation of the result reported by Taylor and Merritt was obtained in an experiment to measure the neutron capture cross section of gold at 30 keV, applying both the associated activity method and the neutron flux integration method.^{6,7} Cancelling σ (Au-197) in these experiments yields $a = 0.105 \pm 0.002$. The present measurements have been carried out to resolve the discrepancy between the value reported by Taylor and Merritt² on the one hand and that reported by C. Turner⁴ and J. Dickson and T. Randle⁵ on the other hand. In addition, the branching ratio of Zn-65, which is the associated activity in the $\text{Cu}^{65}(p,n)\text{Zn}^{65}$ reaction, has been measured. For this decay scheme the $4\pi\beta$ - γ -coincidence method is much more appropriate to apply and recent measurements have resulted in fairly good agreement.

In the present experiment the method used by R. Williamson and H. Richards,⁸ and Turner⁴ was applied: The neutron fluxes from the $\text{Li}^7(p,n)\text{Be}^7$ and the $\text{Cu}^{65}(p,n)\text{Zn}^{65}$ reactions have been measured by means of the neutron flux integration technique and the γ -activities have been determined using a calibrated NaI-detector.

The experimental setup is shown in Fig. I-25-1. Protons were accelerated by a 3 MeV Van de Graaff accelerator. The targets consisted of LiF or copper evaporated on a tantalum backing and were deposited in the center of a sphere of radius 30 cm. A thin layer of silver was evaporated on top of the LiF or copper to avoid any loss of associated activity due to thermal evaporation of target material. Both the pro-

ton beam tube and the sphere entrance channel were constructed of 10 mil thick stainless steel in order to reduce the absorption of thermalized neutrons.

The sphere was filled with a solution of VOSO_4 in water. The use of a vanadium bath^{9,10} instead of the usual manganese bath has several advantages due to the shorter half-life of V-52. Besides the reduction of running time, the main advantage for the present experiment is that the neutron counter simultaneously monitors the time-variations of the neutron flux due to fluctuations in the proton beam. The vanadium bath, as applied in this experiment, consisted of a 115-liter sphere filled with vanadyl sulfate dissolved in water to a concentration of about 840 g $\text{VOSO}_4 \cdot 5\text{H}_2\text{O}$ /liter. The activated solution was pumped directly to a sodium-iodide monitor which measured the activity induced by neutron absorption in V-51. With a mixing propeller and with a stream diversion device, efforts were made to homogenize the solution so as to be relatively independent of neutron source energy. A total decay of 25% occurred enroute to the detector.

The saturated activity was calculated with the aid of a computer program which took into account the various time factors. Because of the complex relationship between beam intensity versus V-52 buildup, the total number of counts derived from neutrons was taken as a measure of strength, rather than the saturated activity which is normally derived for steady sources.

It was necessary to make corrections generally of less than 1% for buildup of long-lived contaminants in the supply of vanadium salt, probably arising from residual uranium content. Neutron sources otherwise calibrated independently and cross-checked against international standards¹¹ were used for reference. Both stability and absolute neutron yield were checked with sources of Sb-Be(γ,n), of Cf-252, and Am-Be(α,n). The overall stability of the bath was demonstrated with a root-mean-square value of 0.1%.

After the irradiations, which lasted between 20 and 140 minutes, the associated activities on the targets were counted at least three times using a 3 x 3 in. NaI detector. The photopeaks at 477 and 1114 keV, respectively, were used to obtain a maximum discrimination against background activities. The NaI detector was calibrated using Cr-51, Be-7, and Zn-65 samples, absolutely γ -calibrated by EURATOM at Geel, Belgium, Mn-54 samples γ -calibrated by the National Bureau of Standards, and several gold foils which had been calibrated absolutely using two different $4\pi\beta$ - γ -coincidence apparatus at Argonne National Laboratory.

Four targets of LiF and copper were irradiated.

Fig. I-25-1. Schematic of Experimental Setup. ANL Neg. No. 113-3062.

One of the LiF runs had to be rejected because the neutron production was too high and one of the copper runs was rejected because the associated γ -activity was too low. Corrections have been applied for the leakage of neutrons out of the sphere (1-6%), absorption of thermal neutrons in the construction material (0.0-0.4%) as well as in the calibration sources (0.1-1.0%), and for the neutron background produced in the proton beam tube. The latter was determined in a separate run using an empty tantalum backing. This correction was only important for the

$\text{Cu}^{65}(p,n)\text{Zn}^{65}$ reaction (1-6%). The stability during the experiment was frequently checked by measuring the antimony-beryllium or californium source between the accelerator runs.

The average values for the three runs of each target material are compared with values reported previously in Tables I-25-I and I-25-II. Assuming the value of 0.5065 for the branching ratio of Zn^{65} given by A. Spornol¹⁹ and supported by Taylor Merritt¹⁸ to be exact, allows the evaluation of a Be-7 branching ratio from the present experiment using the

TABLE I-25-I. BRANCHING RATIO OF Be-7

Reference	Year	Branching Ratio
Rumbough et al. ¹²	1938	0.10 ± 0.20 0.07
Williamson and Richard ⁸	1949	0.107 ± 0.020
Turner ⁴	1949	0.118 ± 0.012
Dickson and Randle ⁵ (reevaluation of Turner)	1951	0.123 ± 0.006
Taylor and Merritt ²	1962	0.1032 ± 0.0016
Poenitz ^{6,7} (indirect)	1966	0.105 ± 0.002
This work	1969	0.1051 ± 0.0018

TABLE I-25-II. BRANCHING RATIO OF Zn-65

Reference	Year	Branching Ratio
Good et al. ¹³	1946	0.45
Furberg ¹⁴	1951	0.44 ± 0.03
Sehr ¹⁵	1954	0.475 ± 0.020
Gleason ¹⁶	1959	0.513 ± 0.030
Ricci et al. ¹⁷	1960	0.49
Taylor and Merritt ¹⁸	1963	0.507 ± 0.005
Spornol ¹⁹	1968	0.5065 ± 0.0015
Hammer ²⁰	1968	0.524 ± 0.010
This work	1969	0.485 ± 0.011

relative neutron strength from the two source reactions. The average from both values, 0.106 ± 0.002 , overlaps with the error bars both values from the two (only partly) independent methods.

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I-26. Compilation of Major Fission-Product Gamma Decay Data

D. W. MADDISON

Gamma-ray spectra of mixed fission products are composed of a large number of individual gamma-ray lines. Many of the individual lines of a mixed fission-product sample can be identified quantitatively using the improved resolution obtained from present-day lithium-drifted germanium detectors.

Even with this greatly improved resolution, however, careful analysis of such a spectrum will reveal that many of the gamma-ray lines result from several gamma-rays having similar energies. To understand and properly evaluate these fission-product spectra requires a knowledge of all major gamma rays

emitted from fission product isotopes likely to be in the sample. A compilation of major fission-product gamma decay data is being assembled to provide sufficient information to analyze this type of interference.

Entries in the compilation are limited to isotopes having half-lives between 1 min and 30 yrs. (Longer- and shorter-lived isotopes are not anticipated for routine analysis.) A second limitation is imposed by the product of fission yield for thermal fission of U-235 times transition intensity, gammas per disintegration, for individual gamma rays. [An isotope with a gamma ray intensity of 1.0 (100% of transitions going through this state) and a fission yield of 0.001% would be included in the compilation with a relative yield-intensity factor of 1.0.] Within these limitations, the compilation includes entries for energy and half-life with their uncertainties, transition intensity, and isotope and mass number. Energy precision is quoted to 0.01 keV where possible, and where measured transition intensities are given in the literature,

the absolute intensities are computed with the use of experimental or theoretical conversion coefficients.

Two different half-lives are quoted. The first is the lifetime of the gamma emitting isotope itself, and the second is the equilibrium half-life, defined as the longest lifetime in the preceding decay chain. (This is the half-life that one would observe in a sample of mixed fission products.)

All the data for each gamma ray are contained on a single punched card. This card contains: 1) the gamma-ray energy and its error, 2) the gamma-ray intensity and its error, 3) the equilibrium half-life and its error, 4) the isotopic half-life and its error, 5) the isotope name and mass number, and 6) the yield-intensity factor. Listings of the compilation by gamma-ray energy, by half-life, or by mass number are obtained easily.

At present, the compilation does not have a uniform cutoff date for literature search. Publication is anticipated in the near future, after the data have been updated to about mid-1969.

I-27. Determination of Neutron Source Strengths

A. DeVOLPI, K. G. FORGES and F. OZER

Neutrons are the *sine qua non* of reactor physics. The capability of neutron measurement, especially fast neutrons in the fast breeder program, is an essential ingredient in experimental technique. For some phases of reactor physics, *high accuracy* in fast neutron measurement is another requisite to permit adequate determination of certain nuclear constants. The achievement of adequate accuracy is not an automatic consequence of effort; rather, it requires a deliberate program towards this goal. Once this capability is acquired, numerous applications arise.

This has been the experience at Argonne: with development of ability to determine neutron source strengths to high accuracy, a variety of improved physical constants of use to the nuclear energy program and a number of applications specifically for the Liquid Metal Fast Breeder (LMFBR) Project have occurred.

Initially, Argonne's efforts were directed towards measurement of the neutron yield in fission from U-235. For this purpose a manganese bath facility was chosen as the best avenue for neutron assessment. It was found necessary, however, to improve upon exist-

ing manganese bath technology in order to meet the more stringent objectives which have developed in terms of needed accuracy in ν , for prediction of breeding characteristics of reactors. A report (see Paper I-24) is made separately of the neutron yield measurements; here efforts directed towards the neutron calibrations are summarized.

The more recent advancements in the manganese bath technique have already been reported.¹ With an on-line counter that surveys activity from a continuous flow system, it has been possible to achieve good precision in counting data. Having obtained long-term stability in these measurements, it became feasible to engage in supplementary investigations which were needed to augment the claims of accuracy attached to manganese bath measurements. A prime example of this is the calibration of several neutron sources which have a wide range of emission spectra. Many of the manganese bath correction factors are energy dependent. By varying the concentration of the bath, the dependence upon the neutron energy sensitive term is resolvable. By carrying out this procedure for sources with differing neutron energy spec-

, a better understanding of the correction factors develops.

As a result of these and other supplementary investigations, it has been possible to calibrate independently the international standard Ra-Be(α, n) neutron source, the U.S. standard Ra-Be(γ, n), an Am-Be(α, n) source from the National Physical Laboratory of England, and a number of Cf-252 spontaneous fission emitters. The Argonne data agree quite well with—and in fact advances—the calibrations of the U.S. and the international standards. There remain some discernible discrepancies, but these are now at the half-percent level, whereas previously there were some disagreements of several percent. The remaining differences are still being evaluated.

One of the backup investigations required to assure accuracy in standardization was a cooperative effort in Mn-56 calibration, which is discussed in Paper II-41. Along with other internal calibrations, there is now assurance that radioassay of some active isotopes can be conducted at the ANL Reactor Physics Laboratory with accuracy nearly comparable to that obtainable at the major standardization laboratories of the world.

An example of direct application of this work to the LMFBR program is the use of calibrated Cf-252 fission sources in the reactivity-oscillation method (see Paper II-16) for determination of the capture-to-fission ratio α in fast critical facilities. Both the

source rate and the average number of neutrons per fission are necessary quantities.

Neutron sources have been calibrated for the National Accelerator Laboratory and for use at the National Reactor Testing Station. At the latter facility, the calibrated neutron sources are being applied to β_{eff} measurements at ZPPR, in addition to use as standards for calibration of a manganese bath facility at Idaho Nuclear Corporation.

Another use for the standard neutron sources has been to calibrate the vanadium bath (see Paper IV-28), which is useful in getting at nuclear measurements not otherwise accessible. Such photoneutron sources as antimony-beryllium and sodium-beryllium have also been calibrated for direct production of fission reactions in counters used to measure absolute fission cross sections in the keV region.

Characteristically, as the availability of the neutron reference sources becomes better known, further applications occur. In addition, a viable program of this sort requires that logical improvements in technique and accuracy should continue to be incorporated in anticipation of a future pattern of application.

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I-28. Average Resonance Widths in Single Channel Scattering

P. A. MOLDAUER

A proof has been given in Ref. 1 for the relation

$$T = 1 - \exp [-2\pi\langle\Gamma\rangle/D] \quad (1)$$

for the case of scattering in the presence of only one open channel. Previously this relation had been derived using special models² and numerical calculations³ for one or more open channels. This relation connects the optical model transmission coefficient T to the average

widths $\langle\Gamma\rangle$ and spacings D of the resonance poles of the collision function.

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I-29. Resonance Fine Structure at an Isobaric Analog or Doorway State

P. A. MOLDAUER

General formulas have been derived for the shape of the coarse structure (average cross section) and fine structure (width distribution) at an analog or doorway state for arbitrary degrees of external and internal mixing as well as for directing coupling of fine structure to the continuum. The cases of bound states and of one and two open channels are considered. In the case of a single open channel the results can be expressed in terms of an analog or doorway state energy E_0 , a shift Δ , an internal mixing width W_i , an external mixing width W_e , and escape width $W\uparrow$, and a mixing phase θ . The coarse structure single channel average total cross section is then generally given by

$$\bar{\sigma} = \bar{\sigma}^0 + \frac{2\pi}{k^2} W\uparrow \cdot \frac{(E - E_0 + \Delta) \cos 2\theta + \frac{1}{2}W \sin 2\theta}{(E - E_0 + \Delta)^2 + \frac{1}{4}W^2} \quad (1)$$

where $\bar{\sigma}^0$ is the background cross section far from the analog or doorway state and $W = W_i + W_e + W\uparrow$ is the total analog or doorway width.

The fine structure is characterized by the local transmission coefficient

$$T = T^0 \frac{(E - E_0)^2 + \frac{1}{4}W_i^2 + W_i W\uparrow / T^0}{(E - E_0 + \Delta)^2 + \frac{1}{4}W^2} \quad (2)$$

where T^0 is the background transmission coefficient. In the case of a purely externally mixed analog state, $W_i = 0$, one obtains the familiar Robson formula.¹ In the limit where the internal mixing width is large compared to W_e and Δ , one obtains the symmetric shape of Feshbach, Kerman and Lemmer's doorway states.²

In addition to the average cross sections and transmission coefficient, the distributions of K or R -matrix widths have been calculated, since current fine structure data have been analyzed in terms of these parameters.³

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I-30. Complex Potential Model for Doorway State Intermediate Resonances

K. TAKEUCHI

Intermediate resonances are observed in energy averaged total, elastic, and inelastic scattering cross sections of V-51. It has been attempted to apply the doorway state interpretation to the observed resonances.¹ For a single channel elastic scattering, an intermediate optical model has been proposed by R. Lipperheide,² by H. Feshbach et al.³ and by K. Takeuchi.⁴ This has now been extended to the multi-channel problem with inelastic scattering.

With the doorway states, the exact T -matrix may be separated into three parts: the direct reaction T^{DR} , the doorway state T^d , and the remainder T^a .⁽³⁾ The last part, T^a , is responsible for the fine resonances, while T^{DR} and T^d are for the intermediate resonances. Taking the energy average of the T -matrix over an interval much smaller than the width of the intermediate

resonance but larger than the fine resonance spacings, one gets

$$\langle T \rangle = T^{DR} + T^d + \langle T^a \rangle. \quad (1)$$

An intermediate optical potential is defined such that it yields $\langle T \rangle$. This is a matrix potential with direct coupling between elastic and inelastic channels. Its energy dependence is determined by the doorway state constants. The reaction cross sections are calculated in the usual way except that $T^{DR} + T^d$ corresponds to the direct reaction amplitude and T^a to the compound reaction amplitude.⁵

Since T^d couples several channels, the average cross sections are obtained from a couple channel calculation. A simplification is obtained under the assumption that the coupling strength of the doorway state to the

stic channel is equal to that to the inelastic channel. The coupled channel cross sections can be expressed in terms of the conventional uncoupled optical model cross sections σ_{OM}^{el} , and the elastic and inelastic uncoupled fluctuation cross sections σ_{fi}^{el} and σ_{fi}^{inel} , as follows:

$$\langle \sigma^{el} \rangle = \sigma_{OM}^{el} + \left(1 - \frac{\Gamma_s \uparrow}{\Gamma_s \uparrow + \Gamma_s \downarrow} \right) \sigma_{fi}^{el} \quad (2)$$

and

$$\langle \sigma^{inel} \rangle = \frac{\Gamma_s \downarrow}{\Gamma_s \uparrow + \Gamma_s \downarrow} \cdot \frac{\Gamma_s \uparrow}{\Gamma_s \uparrow + \Gamma_s \downarrow} \sigma_{fi}^{inel}, \quad (3)$$

where $\Gamma_s \uparrow$ and $\Gamma_s \downarrow$ are coupling strengths of the doorway state s to the continuum and to the compound states, respectively.

Calculations were carried out for neutron scattering by V-51. The fluctuation cross sections are estimated

by the Hauser-Feshbach method. A small number of the doorway state parameters yields a good fit of the measured differential elastic scattering and inelastic scattering cross sections. One feature of this model is that it predicts that the average inelastic cross section has a dip where the elastic cross section peaks, and vice versa.

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I-31. Neutron Single Particle Levels in a Wood-Saxon Potential¹

K. TAKEUCHI and P. A. MOLDAUER

The nuclear single particle potential has been studied extensively in the continuum where copious nuclear scattering data have yielded precise determination of the optical model potential. Especially, neutron scattering data are plentiful at low energies near threshold. One study of such data by P. Moldauer² yielded an optical model whose parameters were also found to agree well with fits of high energy neutron scattering data.³

The behavior of neutron single particle bound states in the real part of the above optical model potential has been investigated. One of the purposes of the investigation was to determine the systematic behavior of shell model single particle levels as the nuclear radius changes. Another was to determine the extent to which the predictions of the scattering potential agree with somewhat indirect information on shell model single particle states that can be deduced from single particle transfer experiments.

The calculated single particle levels agree well with the experimental values determined by B. Cohen and his collaborators.⁴ The change in order of the single

particle levels with atomic mass number and the occurrence of abnormally large spin-orbit doublet splittings were noticed by Cohen, who interpreted them as due to the many-body effects of pairing type forces.⁴ According to the results of the present work, these phenomena are explained as a much simpler effect of changing radius of the finite potential. The results constitute justification for the use of the optical model Wood-Saxon potential in shell model calculations and confirm that both good single particle levels and good scattering cross sections are obtainable from the same single particle potential.

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I-32. *R*-Matrix Analysis of Potential Scattering

K. TAKEUCHI and P. A. MOLDAUER

The potential scattering problem is dealt with most simply by numerical integration of the wave equation, leading to a set of phase shifts from which the cross section can be calculated. The purpose in analyzing this problem by the more indirect *R*-matrix method is to clarify the relationship between *R*- and *S*-matrix parameters and to show how the background *R*-matrix can be calculated for reaction problems.

In *R*-matrix theory the energy-dependent scattering phase shift $\delta(E)$ is written as the sum

$$\delta = \delta_H + \delta_R,$$

where δ_H is the hard sphere scattering phase shift appropriate to the *R*-matrix channel radius a and δ_R is the resonance phase shift. While δ_H and δ_R depend strongly on the choice of channel radius, their sum is independent of a . For example, at large energies we have for ℓ -wave neutrons with wave number $k = \sqrt{2ME}/\hbar$,

$$\delta_H \approx (\ell\pi/2) - ka$$

$$\delta_R \approx N\pi - (\ell\pi/2) + ka$$

so that the total phase shift approaches the limiting value of π times the number $-N$ of bound states with the orbital angular momentum ℓ , as required by Levinson's theorem.

We see therefore that at sufficiently large energies there are no scattering resonances—defined as energies at which δ passes through $(n + \frac{1}{2})\pi$ for any integer n . On the other hand δ_R keeps increasing as $\sqrt{E}$ and has an infinite progression of resonance poles wherever δ_R passes through $(n + \frac{1}{2})\pi$. This infinite sequence of poles poses a serious problem to reaction calculations in which the *R*-matrix is calculated as a sum of pole terms. The sum converges very slowly and cannot be adequately represented by a sum over a finite number of terms. On the other hand our analysis shows that this slowly convergent contribution from the high energy *R*-matrix poles has no dynamical content but only serves to cancel the effect of the decreasing hard sphere phase. Moreover, their contribution to the *R*-matrix in an appropriately defined finite energy interval is approximately constant.

We can therefore obtain the potential scattering *R*-matrix in an energy interval by calculating a finite number of *R*-matrix states and obtaining the background *R*-matrix from the difference between the finite pole series and the exact δ_R as calculated from the exact phase shift δ . The significance of this procedure lies in the fact that it can be directly taken over into multi-channel reaction calculations, provided one may assume either that the background *R*-matrix is diagonal, or that one has a physical model for direct reaction processes in the energy interval of interest.

I-33. Shell-Model *R*-Matrix Calculations of Nuclear Reaction Cross Sections

K. TAKEUCHI and P. A. MOLDAUER

A simple and powerful method has been developed for the calculation of resonance cross sections. An *R*-matrix is obtained by a shell model calculation whose basic states are constructed from discrete single particle states in a Wood-Saxon potential and subject to *R*-matrix boundary conditions. This procedure permits the calculation of not only a finite set of *R*-matrix energies and the associated reduced partial widths, but it also yields the important background *R*-matrix associated with that finite set of states.

From this *R*-matrix we calculate the *S*-matrix and total and reaction cross sections, which are independent of the choice of *R*-matrix radii or boundary conditions, and include the effects of wave function antisymmetrization. Sample calculations have been performed to confirm the insensitivity of the results to changes in the channel radii, to study the effects of antisymmetrization, and to study the way in which single particle states and particle-hole excitations are distributed among resonances.

I-34. *R*-Matrix Single-Level Parameters for Pu-239 from 89 to 202 eV

P. LAMBROPOULOS

The fission data used in obtaining the *R*-matrix single-level parameters (Table I-34-I) for Pu-239 from 89 to 202 eV were obtained from the Petrel explosion. A single-level analysis was performed using the program CODILLI of F. T. Adler and D. B. Adler.¹ Note that this program can be used in a single level analysis. Having obtained the Adler-Adler single-level parameters, it was assumed that the capture width was the same for all levels and equal to 40 meV. The same assumption was made by J. Farrell² in his multi-level analysis up to 90 eV. With this assumption and the Adler-Adler single-level parameters, one can determine the fission ($\Gamma_{\lambda f}$) and neutron ($\Gamma_{\lambda n}$) widths by means of a single algebraic procedure. In the single-level approximation, the real parts of the poles of the scattering matrix are identified with the levels of the com-

TABLE I-34-I. SINGLE-LEVEL PARAMETERS FOR Pu-239 FROM 89 TO 202 eV

λ	E_{λ} , eV	$\Gamma_{\lambda/2}$, eV	$\Gamma_{\lambda f}$, meV	$\Gamma_{\lambda n}$, meV	J
1	90.80	0.100	59.04	0.965	1
2	93.00	0.700	659.40	0.597	0
3	94.50	1.600	1555.40	0.460	0
4	95.40	0.200	158.90	1.101	1
5	96.50	1.500	1452.00	8.030	0
6	98.50	2.200	2155.70	4.327	0
7	101.00	0.600	558.01	1.986	0
8	103.10	0.120	79.36	0.636	1
9	104.80	1.000	957.59	2.410	0
10	105.25	0.100	58.66	1.338	1
11	106.80	0.050	8.15	1.850	1
12	108.50	1.600	1559.50	0.498	0
13	110.30	0.060	19.91	0.087	1
14	114.00	1.400	1358.50	1.534	0
15	115.00	0.500	456.11	3.888	1
16	116.10	0.320	277.57	2.434	1
17	118.00	1.000	948.69	11.309	0
18	120.20	0.260	219.58	0.416	1
19	121.00	0.050	7.40	2.599	1
20	123.40	0.120	79.42	0.579	1
21	125.00	1.000	958.27	1.733	0
22	126.30	0.100	59.19	0.806	1
23	127.60	0.254	213.14	0.860	1
24	131.90	2.800	2733.10	26.910	0
25	135.40	0.280	239.37	0.625	1
26	136.80	0.050	9.23	0.771	1
27	139.30	1.000	959.45	0.554	0
28	141.50	0.360	318.43	1.574	1
29	143.20	0.096	52.54	3.451	1
30	146.00	0.090	47.12	2.882	1
31	148.00	0.060	17.98	2.012	1
32	149.40	0.090	48.82	1.174	1

TABLE I-34-I.—Continued.

λ	E_{λ} , eV	$\Gamma_{\lambda/2}$, eV	$\Gamma_{\lambda f}$, meV	$\Gamma_{\lambda n}$, meV	J
33	152.00	0.320	279.89	0.112	1
34	153.20	0.300	259.60	0.399	1
35	155.30	1.500	1444.30	15.734	0
36	157.00	0.180	135.68	4.317	1
37	161.20	0.560	518.96	1.037	0
38	163.20	0.800	754.21	5.789	0
39	164.50	0.100	57.37	2.622	1
40	165.20	0.090	48.11	1.885	1
41	167.10	0.100	57.43	2.564	1
42	170.50	1.000	956.40	3.603	0
43	175.80	0.200	158.90	1.104	1
44	177.10	0.180	139.45	0.552	1
45	178.80	0.100	59.38	0.615	1
46	180.50	0.120	79.48	0.515	1
47	182.00	0.700	655.24	4.762	0
48	184.00	0.500	453.59	6.413	0
49	185.10	1.802	1754.30	7.678	0
50	190.30	0.240	198.73	1.273	1
51	191.50	0.200	157.83	2.174	1
52	194.00	0.200	151.43	8.573	1
53	195.30	0.140	90.71	9.292	1
54	196.40	0.080	25.23	14.771	1
55	199.20	0.300	254.90	5.098	1
56	200.70	0.200	154.11	5.886	1

Average Fission and Neutron Widths

$$\begin{aligned}\langle\Gamma_f\rangle^{J=0} &= 1.142 \text{ eV} \\ \langle\Gamma_f\rangle^{J=1} &= 0.119 \text{ eV} \\ \langle\Gamma_n\rangle^{J=0} &= 0.00567 \text{ eV} \\ \langle\Gamma_n\rangle^{J=1} &= 0.00244 \text{ eV}\end{aligned}$$

pound nucleus; the imaginary parts are identified with the total half-width. The fission and neutron widths are determined by solving an algebraic system of two equations with two unknowns.

The number of levels revealed in this analysis is 56 as compared with 37 listed in ENDF/B in the same energy interval. Of these 56 levels, 19 have been assigned spin 0 and 37 have been assigned spin 1. These assignments have been made on the basis of the widths of the levels alone. If one assumes that most of the $J = 0$ levels have been found, then about 45 % of the $J = 1$ levels have been missed. The above spin assignments should be considered as tentative, as should the parameters themselves. It is worthwhile noticing perhaps that despite the crudeness of the analysis, the average fission width for $J = 0$ comes out to be 1.142 eV as compared with Farrell's value² of 1.033 eV.

The corresponding value listed in ENDF/B is 2.8 eV. For $J = 1$, the present analysis yields 0.119 eV as compared with the value 0.066 eV of Farrell.² The corresponding value of ENDF/B is 0.055 eV. The discrepancy in the values of the average fission width for $J = 1$ is not too surprising in view of the fact that above 90 eV the resolution function of the Petrel data is very broad. In attempting to determine resonance

parameters of narrow levels from such data, one inevitably overestimates the widths.

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I-35. Statistical Behavior of Gamma Ray Cascades

W. P. POENITZ

FIG. I-35-1. Comparison between the (n, γ) Cross Section and the Low Level Occupation Probability Ratio. ANL Neg. No. 113-3058.

The occupation probability of low-lying levels following the (n, γ) process has been intensively investigated in the past by means of isomeric cross section ratios (e.g., see Refs. 1-3). An interesting extension of the experimental technique of measuring the isomeric cross section ratios is the measurement of prompt γ -transitions originating from low-lying levels.⁴ This technique extends the applicability of such investigations to the majority of the existing stable nuclei. A γ -ray cascade model, successful in the description of measured isomeric cross section ratios, had been re-

ported by J. Huizenga and R. Vandenbosch.⁵ A model which avoids most of the simplifying assumptions employed in the Huizenga-Vandenbosch model has been described previously.⁶

Among the many applications for ratios of low-lying level occupation probabilities R discussed in the past,⁷ the determination of spins in the eV energy range is of major interest. R is shown schematically as a function of energy in Fig. I-35-1 in comparison with the (n, γ) cross section. Besides an energy dependence which is negligible as long as the neutron energy is small com-

FIG. I-35-2. The Frequency Distribution of the Low Level Occupation Ratio of Rhodium for Different Resonances. ANL Neg. No. 113-1402.

pared with the binding energy, the above cited γ -ray cascade model predicts the variation of R between two constant values, depending on the spin of the resonance only. This is, however, due to a simplification in the model which does not take into account the fluctuations of the partial gamma widths from resonance to resonance. A fluctuation of R from resonance to resonance around the two constant values shown in Fig. I-35-1 is expected, taking into account such variation of $\Gamma_{\gamma i}$. To evaluate this fluctuation a Porter-Thomas distribution has been assumed for the partial radiation widths. An approximation reported by C. Hastings⁸ was used to generate $\Gamma_{\gamma i}$ by means of the Monte Carlo technique. Results from such evaluations for R in resonances of

the two possible spin values (0 and 1) reached by s -wave neutrons in the $\text{Rh-103 } (n, \gamma)$ process are shown in Fig. I-35-2. The spread of R caused by the fluctuations of the partial radiation widths is very small. This is due mainly to the large number of transitions caused by the high level density which average out the influence of fluctuations of $\Gamma_{\gamma i}$. The fluctuation of R is mainly caused by transitions with a γ -ray multiplicity of 1 or 2. Such transitions contribute only about 1 %⁽⁶⁾ for nuclei with "typical statistical behavior."

The present results show that the expected fluctuations are not contradictory to the application of the low-lying level occupation probability ratios for the determination of spins of resonances.

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Section II

Fast Reactor Physics

The section on Fast Reactor Physics is concerned with the analyses and measurements of liquid metal fast breeder reactor characteristics and parameters. The measurements are generally made on the critical facilities ZPPR (Zero Power Plutonium Reactor), ZPR-3, ZPR-6 and ZPR-9, and are often performed to check calculated results and hence to evaluate the analytical methods and the nuclear constants used in the calculations. Again, the critical assemblies may be constructed to permit measurements on mockups of liquid metal fast breeder power reactors. The results may possibly be extrapolated to aid in the design of the originally conceived reactor. The work reported in this section is of value in understanding the fundamentals of the liquid metal fast breeder and in developing analytic methods for predicting with accuracy the performance of such reactors.

II-1. Startup and Initial Operation of the Zero Power Plutonium Reactor (ZPPR)

P. I. AMUNDSON, R. G. MATLOCK, R. O. VOSBURGH and J. C. YOUNG

The Zero Power Plutonium Reactor is the largest fast critical assembly in the United States. The reactor is designed for physics studies of power breeder reactor systems in support of the AEC's Liquid Metal Fast Breeder Reactor Program, and can accommodate composition mockups characteristic of the proposed 300-500 MWe demonstration plants and 1000 MWe commercial plants. Even though cores involved in these studies may contain up to 3000 kg of plutonium, the power levels required for the experiments are generally less than 100 W.

Uncertainties in currently available basic cross section data and in calculational techniques require that

experiments be performed on mockups of this size to provide checkpoints for theoretical predictions of the performance and safety characteristics of these power reactor systems. ZPPR will provide data on, among other things, enrichment, power distribution, control-rod worth, blanket optimization, and Doppler, temperature, and sodium-void reactivity coefficients for use in evaluation of calculational techniques.

The ZPPR facility is shown in Fig. II-1-1 and consists of two areas: a mound area, where containment is an essential feature, and a support wing, which is a conventional office building. A reactor cell that houses the reactor assembly machine, a vault

FIG. II-1-1. The ZPPR Facility. ANL Neg. No. 103-11840.

FIG. II-1-2. ZPPR—Split-Table Type Critical Assembly. ANL Neg. No. 103-11843.

providing storage of the plutonium inventory while not in use, and a workroom where fuel is inspected and loaded into reactor subassemblies (in the form of drawers) are included in the mound area. Technical staff offices, the ZPPR control room, and the Argonne Fast Source Reactor—a small fast reactor used for instrument and technique development—are located in the support wing.

ZPPR is a split-table type critical assembly as shown in Fig. II-1-2. Located on each table is a reactor matrix consisting of stainless-steel rectangular tubes. Reactor drawers containing simulated reactor materials in the form of small plates are inserted into the matrix tubes to assemble a given reactor design. The core is completed by slowly bringing the two table halves together. Control of the reactor is effected by fuel-bearing control rods which are driven into the core to establish final criticality of each assembly.

Construction of ZPPR began on August 8, 1966,

and was completed in August 1968. Following extensive checkout of the facility, training of operations and supervisory personnel, and receipt of plutonium fuel and other reactor material simulants, the AEC granted operating approval on March 27, 1969. The initial plutonium loading of the reactor immediately commenced and criticality was first achieved on April 18, 1969 at 1525.

The first assembly built in ZPPR was Assembly I of the Fast Test Reactor Resumed Phase-B Critical Experiments Program (FTR-I). This assembly had been built previously on ZPR-3 and was rebuilt on ZPPR as an initial step in proceeding to the next phase (FTR-II) of this program. The FTR-I assembly consisted of a cylindrical plutonium-fueled core surrounded axially and radially by sodium-nickel reflectors. The fuel utilized was a ternary α' clad in 0.015-in. stainless steel and comprised 69.3 w/o depleted uranium, 28.2 w/o plutonium containing 11.5% Pu-240, and 2.5 w/o molybdenum. The

TABLE II-1-I. AS BUILT COMPOSITION OF FTR 1 ON ZPPR
(Loading 17)

Composition by Zone, atoms/b-cm							
Nuclide	Core					Axial Reflector	Radial Reflector
	1-Column Pu Drawer ^a	2-Column Pu Drawer ^b	Cell Average	Movable Fuel Rod ^c	Drawer Adjacent to Poison Safety Rod ^d		
Pu-239	0.0008491	0.0016983	0.0012737	0.0016983	0.0008491		
Pu-240	0.0001125	0.0002251	0.0001688	0.0002251	0.0001125		
Pu-241	0.0000168	0.0000337	0.0000252	0.0000337	0.0000168		
Pu-242	0.0000017	0.0000035	0.0000026	0.0000035	0.0000017		
U-238	0.0055684	0.0063849	0.0059766	0.0063849	0.0055684		
U-235	0.0000122	0.0000139	0.0000131	0.0000139	0.0000122		
O	0.0150075	0.0154338	0.0152206	0.0137617	0.0150075		
Na	0.0107196	0.0063408	0.0085302	0.0063610	0.0062787	0.0133588	0.0065078
C	0.0021888		0.0010944	0.0032025	0.0021888	0.0000284	0.0001547
Fe	0.0087888	0.0162589	0.0125238	0.0180022	0.0097185	0.0079227	0.0068787
Ni	0.0011626	0.0011636	0.0011631	0.0023373	0.0012671	0.0189744	0.0460037
Cr	0.0025337	0.0025358	0.0025348	0.0051353	0.0027964	0.0022823	0.0019747
Mn	0.0002025	0.0002027	0.0002026	0.0004248	0.0002204	0.0002295	0.0002639
Mo	0.0002322	0.0004475	0.0003398	0.0004567	0.0002355	0.0000161	0.0000154

^a Occupies even-numbered matrix columns.^b Occupies odd-numbered matrix columns (center column is odd-numbered).^c Both fuel safety and control rods are of this composition.^d Poison rod withdrawn during operation.

assembly composition is given in Table II-1-I and drawer loadings for the core and reflectors are shown in Figs. II-1-3 and II-1-4. An interface view of the critical reactor loading is illustrated in Fig. II-1-5.

The approach-to-critical for this initial core proceeded as follows: The proposed assembly, including core and reflectors, was completely loaded to form the calculated configuration, with depleted uranium used in place of the plutonium fuel. Starting from the core center, plutonium fuel was then incrementally substituted for the depleted uranium, the amount being determined from assessment of subcritical multiplication data. In addition to the operational nuclear instrumentation channels (two startup, two log-n period, and four linear power level channels), five more neutron counters were positioned about the core (four at the core-reflector interfaces and one on the reactor-cell wall) to monitor the neutron multiplication. The effect of each plutonium fuel loading was assessed by plotting M/CR versus M for each of the five counters, where M is the fissile mass in the assembly and CR is the neutron count rate. The ratio of M/CR was used, rather than the conventional $1/CR$, to help account for the neutron source

ulting from the spontaneous fission of the Pu-240 present in the fuel. Seventeen separate fuel loadings were made to obtain the critical loading of 359.4 kg fissile mass (Pu-239 + Pu-241 + U-235).

The critical mass of FTR-I as achieved on ZPPR is to be compared with 331.9 kg for this core on ZPR-3. The difference is attributed to the larger void fraction of the ZPPR matrix (a 0.10-in. air-cooling channel is provided above each drawer in ZPPR).

The next critical configuration established in ZPPR was the FTR-II core. The core and reflector compositions remained the same as for FTR-I, and a two-drawer-thick simulated boron control ring was introduced at the core/radial reflector interface. The resulting configuration is shown in Fig. II-1-6 and had a total fissile loading of 527.04 kg.

During the course of the FTR-I and FTR-II experiments valuable reactor operating experience was obtained, experimental techniques were examined, and experimental equipment evaluated. In addition, the on-line computer was successfully used for data acquisition and analysis of experimental and operational data.

Although ZPPR has been started in excess of 200 times, less than ten unscheduled reactor scrams have been experienced, all of which have been due to electronic noise in the startup reactor-period channels. No problems have been encountered in reactor operation or personnel radiation exposure during fuel handling and none are anticipated based on these experiences.

(a) 1-COLUMN Pu DRAWER

(b) 2-COLUMN Pu DRAWER

(c) AXIAL REFLECTOR BACK DRAWER (SAME FOR BOTH OF ABOVE CORE DRAWERS)

FIG. II-1-3. Core and Axial Reflector Drawers for FTR-1/2 on ZPPR. ANL Neg. No. 108-A11049.

(a) FRONT DRAWER

(b) BACK DRAWER

FIG. II-1-4. Radial Reflector Drawers for FTR-1/2 on ZPPR. ANL Neg. No. 103-A11051.

ZPPR HALF 1

(HALF 2 IS A MIRROR IMAGE OF HALF 1)

S= SAFETY ROD

C= CONTROL ROD

P= DRAWER ADJACENT TO POISON SAFETY ROD

■= POISON SAFETY ROD (WITHDRAWN DURING OPERATION)

FIG. II-1-5. Critical Loading for ZPPR FTR-1 Reference Core. ANL Neg. No. 103-A11141.

ZPPR HALF I

(HALF 2 IS A MIRROR IMAGE OF HALF I)

S = SAFETY ROD C = CONTROL ROD P = DRAWER ADJACENT TO POISON SAFETY ROD

I = POISON SAFETY ROD (WITHDRAWN DURING OPERATION)

F = U-235 FISSION CHAMBER (FRONT OF CHAMBER AT REACTOR MIDPLANE. NO F's IN HALF 2)

FIG. II-1-6. ZPPR/FTR-II Reference Critical Configuration (Loading 1-70). ANL Neg. No. 103-A11073.

II-2. Analytical Studies of a Basic Physics Series of Plutonium Cores for ZPR-3

V. C. ROGERS, J. M. STEVENSON,* R. G. PALMER and W. G. DAVEY

INTRODUCTION

A series of scoping calculations has been made for possible future assemblies for ZPR-3. Such assemblies with simple compositions could assist in the study of cross-section data, computer codes used in reactor physics calculations, and the persistent discrepancy between calculated and measured central perturbation worths. Safety information required prior to assembly construction and initial estimates of the experimental measurements are given.

The cores in this study have lead or depleted uranium reflectors, 30 cm thick, and are variations of ZPR-3 Assembly 53 (see Paper II-4) in which the single drawer cell contained a mixed plutonium-uranium fuel plate, a plutonium-aluminum fuel plate,

where

$X \equiv \frac{1}{2}$ in. core diluent material consisting of one of the following: Na, Fe, Ni, or U_3O_8

$A \equiv \frac{1}{8}$ in. Fe plate for $X \equiv$ Na, Fe and Ni
 $\equiv \frac{1}{8}$ in. U_3O_8 plate for $X \equiv U_3O_8$

$B \equiv \frac{1}{8}$ in. Pu-Al plate

$C \equiv \frac{1}{4}$ in. Pu-U-Mo plate (ZPPR type).

The nominal atomic concentrations, based upon the existing ZPR fuel inventory, are presented in Table II-2-I.

One of the specific objectives for these cores is the evaluation of existing cross section sets for some of the materials used in fast reactors; that is, sodium, iron, nickel, uranium and oxygen. The large volume fraction of the core diluent in each of the assemblies

TABLE II-2-I. NOMINAL COMPOSITIONS FOR PROPOSED BASIC PHYSICS ASSEMBLIES, atomic density $\times 10^{21}$

Core Materials													
Core Diluent	Pu-239	Pu-240	Pu-241	Pu-242	U-235	U-238	Mo	Al	Fe	Ni	Cr	O	Na
Na	1.944	0.168	0.020	0.0025	0.0055	2.477	0.2276	0.110	17.494	1.442	3.519	0.0	12.084
Fe	1.944	0.168	0.020	0.0025	0.0055	2.477	0.2276	0.110	63.126	0.8316	2.032	0.0	0.0
Ni	1.944	0.168	0.020	0.0025	0.0055	2.477	0.2276	0.110	12.1914	56.5325	2.032	0.0	0.0
U_3O_8	1.944	0.168	0.020	0.0025	0.0279	13.1273	0.2276	0.110	7.632	0.8316	2.032	28.4646	0.0

Reflector Materials						
Reflector	U-235	U-238	Pb	Fe	Ni	Cr
Depleted U	0.0835	39.976	0.0	4.540	0.4942	1.209
Pb	0.0	0.0	27.7836	4.540	0.4942	1.209

and thirteen graphite plates. The first series of variations includes diluents different than graphite—namely sodium, iron, nickel or U_3O_8 , (and a slightly more enriched mixed fuel plate). In the second series, the mixed fuel plate is replaced by a second plutonium-aluminum fuel plate, (and a graphite plate giving fourteen graphite plates in all). Thus there is no U-238 in the core, increasing the sensitivity of experimental results to Pu-239 cross-sections.

ANALYSIS OF CORES CONTAINING PLUTONIUM-URANIUM-MOLYBDENUM FUEL PLATES

The cores in this classification are composed of one-drawer cells with the following configuration:

XABXCX,

* UKAEA, Winfrith, Dorchester, Dorset, England.

increases the dependence of many experimental parameters upon the cross sections of these materials. Hence, a comparison between calculation and experimental data can indicate errors in the cross sections for that particular material, thereby defining future areas for detailed differential cross section measurements.

Homogeneous cross sections, generated with the ENDF/B-MC² system¹ were used in performing the MACH-1⁽²⁾ calculations. The cross sections for lead were obtained from an "adjusted" Bondarenko set³ because of the unavailability of this isotope in the ENDF/B data file. Finite cylinder dimensions were determined using the consistent buckling method. The core height was also restricted to accommodate available fuel-piece lengths and drawer sizes.

TABLE II-2-II. CENTRAL GROUP FLUXES FOR DEPLETED URANIUM REFLECTED ASSEMBLIES

Group	Lower Energy, keV	Core Diluent			
		Na	Fe	Ni	U ₃ O ₈
1	6065.	0.4785	0.2712	0.2602	0.5826
2	3679.	1.931	1.065	0.9834	2.428
3	2231.	4.526	2.765	2.262	6.110
4	1353.	7.364	5.243	4.756	7.068
5	820.8	9.842	8.930	11.081	6.955
6	387.7	20.460	21.993	23.292	15.393
7	183.2	22.064	24.931	20.047	17.120
8	111.1	10.673	10.467	12.890	10.288
9	52.48	10.949	11.608	11.687	12.455
10	19.30	7.800	10.276	6.394	11.310
11	9.119	2.315	1.408	1.509	5.155
12	4.307	0.8607	0.5007	1.852	2.708
13	2.035	0.1958	0.3692	1.255	1.586
14	0.9611	0.4155	0.1296	0.9666	0.6144
15	0.4540	0.1195	0.0373	0.4878	0.1885
16	0.2145	0.0165	0.0120	0.2039	0.0483
17	0.1013	0.00118	0.00270	0.0617	0.00756
18	0.04785	5.892×10^{-4}	4.011×10^{-4}	0.0120	7.418×10^{-4}
19	0.02260	3.22×10^{-6}	2.13×10^{-6}	8.611×10^{-4}	4.590×10^{-5}
20	0.01068	4.53×10^{-7}	1.24×10^{-6}	5.950×10^{-5}	3.34×10^{-6}
21	0.005043	1.06×10^{-7}	7.35×10^{-8}	1.012×10^{-6}	2.64×10^{-7}
22	0.002382	1.04×10^{-7}	6.84×10^{-8}	1.274×10^{-7}	1.28×10^{-7}
23	0.001125	2.96×10^{-8}	3.19×10^{-8}	5.169×10^{-8}	4.42×10^{-8}
24	0.0004140	7.73×10^{-10}	6.61×10^{-10}	1.020×10^{-9}	1.01×10^{-9}

The energy group structure and the real and adjoint central fluxes for the depleted uranium-reflected cores are presented in Tables II-2-II and II-2-III. For the lead reflected cores, the real central flux spectra differ only slightly from the values given in Table II-2-II. The central adjoint flux spectra also have similar shapes as shown for the sodium diluent assembly in Fig. II-2-1. Table II-2-IV contains the parameters relating to critical size, effective beta, and prompt neutron lifetime, and Tables II-2-VA and II-2-VB contain central microscopic reaction ratios and central perturbation worths. The negligible absorption cross section for lead is the main reason for the smaller critical masses of the lead-reflected cores. This reason, coupled with the larger uranium inelastic scattering cross section, also accounts for the longer prompt neutron lifetimes of the lead-reflected cores. Furthermore, the effect of the delayed neutron fraction of the uranium isotopes is observed as an increase in β_{eff} for the U₃O₈ core and for all depleted uranium-reflected cores.

As may be seen in Table II-2-VA, the perturbation worths of the plutonium isotopes for the lead-reflected assemblies are about 1.2 times the perturbation worths in the corresponding depleted uranium reflected assemblies. Because of the similarity of the central real and adjoint flux spectra, these factors

TABLE II-2-III. CENTRAL ADJOINT FLUXES FOR DEPLETED URANIUM REFLECTED ASSEMBLIES

Group	Core Diluent			
	Na	Fe	Ni	U ₃ O ₈
1	4.011	4.190	3.464	3.831
2	3.701	4.011	3.451	3.512
3	3.726	4.063	3.782	3.551
4	3.559	3.899	3.924	3.456
5	3.332	3.666	3.816	3.369
6	3.328	3.682	3.762	3.298
7	3.304	3.556	3.844	3.354
8	3.469	3.536	3.796	3.378
9	3.474	3.373	3.985	3.381
10	3.474	2.984	4.220	3.422
11	3.778	3.851	4.364	3.619
12	3.951	4.002	4.383	3.775
13	4.106	4.238	4.481	3.933
14	4.098	4.283	4.581	4.269
15	4.429	4.913	4.790	4.696
16	4.897	5.262	5.061	5.136
17	5.476	5.633	5.321	5.637
18	6.809	6.735	6.281	6.653
19	3.220	2.805	2.665	3.227
20	5.013	4.656	4.386	5.491
21	4.801	4.626	5.111	5.505
22	5.342	4.644	3.809	5.273
23	4.603	4.030	3.436	4.418
24	4.100	3.361	3.284	3.818

TABLE II-2-IV. CRITICAL SIZE AND MASS FOR PROPOSED BASIC PHYSICS SERIES CORES IN ZPR-3

	Diluent-Reflector							
	Na-U	Na-Pb	Fe-U	Fe-Pb	Ni-U	Ni-Pb	U ₃ O ₈ -U	U ₃ O ₈ -Pb
Radius, cm	37.0	34.0	37.6	34.2	36.7	34.4	40.1	37.3
Height, cm	66.0	66.0	66.0	66.0	66.0	66.0	76.0	71.1
Core vol., ℓ	283.9	240.0	293.1	242.7	279.3	245.4	383.0	311.4
Critical mass (fissile), kg	222.0	187.6	229.1	189.7	218.4	191.9	299.4	243.4
Median energies in the core, keV								
Source	345	363	298	301	277	281	255	257
Absorption	250	257	214	211	211	209	148	140
Flux	280	297	259	265	264	272	208	218
Percent fissions in fertile material	17.5	7.0	12.0	5.4	10.4	4.9	19.8	7.3
β_{eff} , %								
$i = 1$	0.0080	0.0079	0.0077	0.0076	0.0081	0.0081	0.0081	0.0081
$i = 2$	0.0639	0.0612	0.0597	0.0585	0.0624	0.0615	0.0657	0.0631
$i = 3$	0.0562	0.0507	0.0502	0.0475	0.0517	0.0496	0.0585	0.0527
$i = 4$	0.1033	0.0865	0.0875	0.0787	0.0880	0.0814	0.1093	0.0906
$i = 5$	0.0450	0.0338	0.0352	0.0293	0.0342	0.0298	0.0487	0.0359
$i = 6$	0.0150	0.0113	0.0117	0.0098	0.0114	0.0099	0.0162	0.0120
β_{eff} , %	0.2914	0.2514	0.2520	0.2314	0.2558	0.2403	0.3066	0.2624
Prompt neutron lifetime, nsec	173	260	176	251	180	230	195	307
Ih/% reactivity	1066	1124	1149	1181	1102	1125	1034	1090

TABLE II-2-VA. CENTRAL PERTURBATION WORTHS, Ih/kg

Material	Diluent-Reflector							
	Na-U	Na-Pb	Fe-U	Fe-Pb	Ni-U	Ni-Pb	U ₃ O ₈ -U	U ₃ O ₈ -Pb
Pu-239	571	729	842	987	935	1042	593	763
Pu-240	162	210	199	235	196	220	143	186
Pu-241	718	914	1070	1256	1260	1404	813	1045
U-238	-15.6	-18.4	-35.7	-41.5	-56.0	-62.1	-20.1	-23.0
Fe	-11.3	-11.9	-18.4	-20.3	-3.9	-3.9	-10.6	-9.5
Cr	-12.7	-13.7	-21.7	-24.3	-8.0	-9.0	-14.8	-14.1
Ni	-21.8	-26.3	-21.9	-25.7	-16.1	-17.8	-11.6	-21.2
Na	-1.3	4.8						
U ₃ O ₈							-14.8	-15.3

arise from the differences between the perturbation integrals. For the elements in which the scattering component is significant, small changes in the adjoint

spectra greatly affect the values of the total perturbation worths, and these factors do not apply. This is most evident for sodium where there is a change of sign in addition to a change in magnitude of a factor of 3.7. Investigation of the components of the sodium worths reveals that the contribution from scattering increases from 0.8 to 7.4 Ih/kg while the absolute capture contribution increases from -2.1 to -2.6 Ih/kg when changing from the uranium reflector to the lead reflector. The major portion of the change in the scattering worth arises from groups 6 and 7 as the adjoint decreases with energy near 400 keV for the sodium-lead assembly but increases for the sodium-uranium assembly as seen in Fig. II-2-1. This change is so significant because the total scattering worth contains approximately equal positive and negative contributions. Similar changes in

TABLE II-2-VB. CENTRAL REACTION RATIOS

Ratio Numerator ^a	Diluent-Reflector							
	Na-U	Na-Pb	Fe-U	Fe-Pb	Ni-U	Ni-Pb	U ₃ O ₈ -U	U ₃ O ₈ -Pb
Pu-239 fiss	1.08	1.08	1.06	1.06	1.02	1.02	0.984	0.985
Pu-240 fiss	0.355	0.357	0.308	0.308	0.287	0.287	0.297	0.298
Pu-241 fiss	1.30	1.30	1.30	1.30	1.31	1.31	1.30	1.29
U-238 fiss	0.0477	0.0475	0.0311	0.0312	0.0249	0.0249	0.0480	0.04
U-238 cap	0.125	0.125	0.132	0.132	0.141	0.141	0.133	0.13

^a Denominator is U-235 fission.

FIG. II-2-1. Central Adjoint Flux for Sodium Diluent Cores. Note: The Two Spectra Have Been Displaced for Clarity. ANL Neg. No. 103-A11139.

the scattering component of other perturbation samples are also observed; however, in the other cases the capture component is large enough to mask the scattering component changes.

It should also be noted that the significant decrease in the magnitude of the iron and chromium sample worths for the nickel assemblies is due to significant increases in the scattering contribution. This is believed to be caused by the softer flux spectrum in the nickel assemblies and is supported by the larger U-238 sample worth as well as by the smaller U-238 fission to U-235 fission ratio, the increased U-238 capture to U-235 fission ratio, and the lower percentage of fissions in fertile material.

The great similarity of the central flux spectra for assemblies with the same diluent and different reflectors is again demonstrated by the very small differences of their fission ratios as observed in Table II-2-VB.

SAFETY ANALYSIS OF CORES CONTAINING ONLY PLUTONIUM-ALUMINUM FUEL PLATES

These cores consist of one-drawer cells containing two columns of plutonium-aluminum plates and four-

TABLE II-2-VI. COMPOSITION OF PLUTONIUM-GRAPHITE ASSEMBLIES, atomic density $\times 10^{21}$

Isotope	Core	Reflectors	
		Uranium	Lead
Pu-239	2.138		
Pu-240	0.102		
Pu-241	0.0098		
Al	0.222		
C	60.30		
U-235		0.084	
U-238		39.98	
Fe	7.429	4.540	4.540
Cr	1.978	1.209	1.209
Ni	0.809	0.494	0.494
Pb			27.78

teen columns of graphite plates. The nominal compositions are given in Table II-2-VI. The main safety question arises because the cores contain no U-238, with its known large negative Doppler effect. Hence, there is a doubt concerning the sign of the prompt power coefficient. Because of the small expansion characteristics of the plutonium-aluminum plates, it is necessary that the plutonium Doppler coefficient be

TABLE II-2-VII. STATISTICAL PARAMETERS FOR Pu-239

ℓ	0	0	1	1	1
J	0	1	0	1	2
$\langle \Gamma_n^0 \rangle$, eV	0.939×10^{-3}	0.334×10^{-3}	2.195×10^{-3}	1.56×10^{-3}	0.53×10^{-3}
$\langle D \rangle$, eV	8.78	3.12	8.78	3.12	2.12
$\langle \Gamma_f \rangle$, eV	2.8	0.014-0.040	0	1.012	0.672
Γ_γ , eV	0.0387	0.0387	0.0387	0.0387	0.0387
ν_n	1	1	1	2	1
ν_f	2	1	0	1	1

TABLE II-2-VIII. PU-239 DOPPLER COEFFICIENT

Configuration	Doppler Coefficient $10^{-5} dk/kdT$
Plates separate, depleted U reflector	-0.93
Plates separate, Pb reflector	-1.40
Plates together, depleted U reflector	-0.95
Plates together, Pb reflector	-1.46

TABLE II-2-IX. CENTRAL GROUP FLUXES AND ADJOINTS FOR URANIUM REFLECTED PLUTONIUM-GRAPHITE CORES

Group	Lower Energy, keV	Plates Separate		Plates Together	
		ϕ	ϕ^*	ϕ	ϕ^*
1	3680.	2.954	2.940	2.938	2.947
2	2310.	5.467	3.012	5.437	3.020
3	1350.	8.934	3.043	8.885	3.050
4	821.	9.690	3.161	9.636	3.169
5	498.	9.403	3.317	9.350	3.325
6	302.	8.573	3.460	8.525	3.463
7	183.	7.651	3.583	7.608	3.591
8	111.	6.729	3.700	6.690	3.704
9	67.4	6.042	3.781	6.007	3.790
10	40.9	5.442	3.862	5.411	3.872
11	24.8	4.752	3.956	4.724	3.967
12	15.0	4.277	4.026	4.254	4.037
13	9.12	3.805	4.088	3.786	4.101
14	5.53	3.195	4.141	3.182	4.157
15	3.35	2.865	4.196	2.859	4.214
16	2.03	2.485	4.301	2.486	4.322
17	0.961	2.906	4.480	2.937	4.503
18	0.454	2.056	4.852	2.120	4.865
19	0.275	0.923	5.125	0.972	5.119
20	0.0614	1.504	5.316	1.712	5.273
21	0.0137	0.297	5.115	0.400	4.983
22	0.00306	0.0398	5.181	0.0639	5.132
23	6.83×10^{-4}	0.0102	4.975	0.0163	4.995

zero or negative. Initial efforts for these cores were centered upon determining the sign of the Pu-239 Doppler coefficient utilizing recent values of the Pu-239 resonance parameters. The resolved resonance parameters were obtained from Ref. 4. Statistical parameters, listed in Table II-2-VII for the unresolved region above 300 eV were obtained from C.

Durston and S. Katsuragi⁵ with the exception of $\langle \Gamma_f \rangle$ for s-wave neutrons and compound state $J = 1$ which was obtained from E. Fischer.⁶ The symbols ν_f and ν_n refer to the number of degrees of freedom in the fission width and the reduced neutron width chi-squared distributions. These parameters are more reliable as they agree with the presently accepted Pu-239 alpha values.

Two core configurations are used throughout the analysis. In the first, the plutonium-aluminum plates are placed together in the drawer; in the second configuration they are separated by seven columns of graphite so that one drawer is composed of two cells, each consisting of a one-column fuel plate and seven columns of graphite plates. The reflectors are identical to the previous cases; that is, depleted uranium and lead.

Diffusion theory calculations were performed with the MACH-I code using broad group cross sections at 300 and 400°K generated by the ENDF/B-MC² system. The Pu-239 capture and fission cross sec-

TABLE II-2-X. CRITICAL SIZE AND MASS FOR PLUTONIUM-GRAPHITE ASSEMBLIES

	Configuration-Reflector			
	Plates Separate, U Reflector	Plates Separate, Pb Reflector	Plates Together, U Reflector	Plates Together, Pb Reflector
Core radius, cm	27.9	25.0	27.9	25.0
Core half height, cm	25.4	25.4	25.4	25.4
Core volume, liter	124.2	99.4	124.4	99.4
Critical mass (fissile), kg	105.9	84.8	106.1	84.7
β_{eff}^i , %				
$i = 1$	0.00838	0.00844	0.00838	0.00844
$i = 2$	0.06452	0.06241	0.06450	0.06242
$i = 3$	0.05367	0.04808	0.05365	0.04809
$i = 4$	0.09183	0.07321	0.09180	0.07323
$i = 5$	0.03598	0.02305	0.03596	0.0
$i = 6$	0.01202	0.00778	0.01202	0.0
β_{eff} , %	0.2664	0.2230	0.2663	0.2230

3LE II-2-XI. PERCENTAGE CHANGE OF PARAMETER CAUSED BY A 5% REDUCTION IN THE CAPTURE CROSS SECTION OF Pu-239
OVER VARIOUS ENERGY RANGES
(Plates-Separate and U Reflected Assembly)

Parameter	Reference Value	Energy Range				
		Over All Energies	Above 100 keV	Between 10 and 100 keV	Between 1 and 10 keV	Below 1 keV
k_{eff}	1.0000000	0.65	0.04	0.12	0.24	0.24
Median core energy						
Source	73.1 keV	-3.42	-0.14	-0.71	-1.37	-1.37
Absorption	19.4	-0.36	-0.46	-0.72	0.72	0.21
Flux	182.7	-0.82	-0.08	-0.27	-0.33	-0.16
Reaction ratios						
Pu-239 fission/U-235 fission	0.799	-0.47	0.0	-0.05	-0.17	-0.24
Pu-240 fission/U-235 fission	0.200	-1.16	-0.04	-0.17	-0.43	-0.51
Pu-241 fission/U-235 fission	1.363	0.15	0.0	0.0	0.04	0.11
U-238 fission/U-235 fission	0.0318	-1.20	-0.05	-0.19	-0.44	-0.52
U-238 capture/U-235 fission	0.143	-0.13	0.01	0.03	-0.01	-0.15
Perturbation worths						
Pu-239	1114 lh/kg	-0.20	0.09	0.16	0.09	-0.56
Pu-240	21.4	-78.94	-1.42	-7.0	-22.54	-47.48
Pu-241	2093	-1.14	0.01	-0.02	-5.22	-0.97
U-235	721	-4.46	-0.01	-0.20	-1.00	-3.24
U-238	-117.4	3.56	0.08	0.27	1.09	2.10
Fe	6.2	10.70	0.22	1.05	1.87	7.51
Ni	-4.0	-40.40	-0.05	9.02	-15.40	-33.44
Cr	-24.4	0.18	-0.03	-0.09	0.70	-0.40
C	304	3.74	-0.09	-0.01	0.94	2.90
Al	52.8	4.47	-0.04	0.59	1.43	2.47
B-10	-61060	2.94	0.03	0.20	0.80	1.90
β_{eff}	0.266%	0.12	0.0	0.02	0.05	0.04
ℓ prompt	471 nsec	2.55	0.0	0.21	0.64	1.70
lh/% $\Delta k/k$	1065	-0.19	0.0	0.0	-0.09	-0.09
Fissions in fertile material	14.0%	-0.71	0.0	-0.07	-0.21	-0.21

tions in the energy region from 275 eV to 25 keV were calculated using the IDIOT code⁷ which was modified to include the equivalence relation formulation from MC². The RABID code⁸ was used to obtain the Pu-239 fission and capture cross sections from the resolved resonance data below 275 eV. In addition, Pu-239 cross sections were determined for a few small energy intervals in the region from 300 eV to 25 keV by inputting into RABID statistical resonances generated by the RANDOM code.⁹ The resonance spacings are distributed over the Wigner distribution.⁴ In general, these cross sections agree with the IDIOT generated cross sections to within a few percent.

It may be noted from the results listed in Table VIII, the Pu-239 Doppler coefficient is negative for these assemblies. In addition, the relative posi-

tioning of the fuel plates in the two-core configurations has a very small effect on the Doppler coefficient; however, as expected, the Doppler coefficient for the plates-together configuration has a larger magnitude than the Doppler coefficient for the plates-separate configuration. It is also observed that the lead-reflected assemblies have a larger Doppler coefficient owing to their softer spectra.

PARAMETRIC ANALYSIS OF CORES CONTAINING ONLY PLUTONIUM-ALUMINUM FUEL PLATES

Diffusion theory calculations, using the compositions given in Table II-2-VI and the cross sections described in the preceding section indicated that the flux for these assemblies was flatter than that for the cores considered earlier. Comparison of the

TABLE II-2-XII. PERCENTAGE CHANGE OF PARAMETER CAUSED BY A 5% REDUCTION IN THE CAPTURE CROSS SECTION OF PU OVER VARIOUS ENERGY RANGES

(Plates-Separate and Pb Reflected Assembly)

Parameter	Reference Value	Energy Range				
		Over All Energies	Above 100 keV	Between 10 and 100 keV	Between 1 and 10 keV	Below 1 keV
k_{eff}	1.0000000	0.78	0.04	0.14	0.29	0.30
Median core energy						
Source	62.3 keV	-5.04	-0.13	-0.96	-1.96	-2.09
Absorption	14.7	-0.98	-0.41	-1.36	0.48	0.0
Flux	198.0	-0.90	-0.08	-0.25	-0.35	-0.20
Reaction ratios						
Pu-239 fiss/U-235 fiss	0.801	-0.49	-0.01	-0.06	-0.18	-0.26
Pu-240 fiss/U-235 fiss	0.201	-1.24	-0.04	-0.17	-0.46	-0.55
Pu-241 fiss/U-235 fiss	1.363	0.15	0.0	0.0	0.04	0.11
U-238 fiss/U-235 fiss	0.0316	-1.28	-0.05	-0.19	-0.47	-0.56
U-238 cap/U-235 fiss	0.143	-0.12	0.0	0.03	-0.01	-0.15
Perturbation worths						
Pu-239	1382 lh/kg	-0.60	0.08	0.12	-0.06	-0.74
Pu-240	33.0	-66.31	-1.32	-5.96	-19.15	-39.57
Pu-241	2589	-1.44	-0.01	-0.05	-0.28	-1.10
U-235	896	-4.83	-0.03	-0.24	-1.14	-3.40
U-238	-138.1	3.63	0.09	0.27	1.10	2.15
Fe	15.2	5.90	0.05	0.44	0.80	3.59
Ni	0.9	195.2	1.54	-50.52	76.56	167.6
Cr	-20.2	0.26	-0.01	-0.10	1.02	-0.66
C	378	3.25	-0.11	-0.06	0.79	2.65
Al	75.0	3.52	-0.07	0.46	1.12	2.00
B-10	-74920	2.74	0.02	0.18	0.73	1.81
β_{eff}	0.223%	0.28	0.0	0.06	0.10	0.12
ℓ prompt	867 nsec	2.54	0.0	0.11	0.69	1.73
lh/% $\Delta k/k$	1115	-0.27	0.0	-0.09	-0.09	-0.09
Fissions in fertile material	1.1%	-0.55	0.18	0.0	-0.09	-0.09

central fluxes and adjoints for the plutonium-graphite cores, listed in Table II-2-IX, with the fluxes and adjoints listed in Tables II-2-II and II-2-III, supports this conclusion. The greater self-shielding of the resonance cross-sections in the plates-together configuration compared with the plates-apart is shown by the higher fluxes in the low energy groups. In Table II-2-X parameters relating to critical size and effective beta are presented. Finite cylinder dimensions were obtained from the spherical critical radii using shape factors determined by W. Davey.¹⁰

As before, the critical masses for the lead-reflected assemblies are significantly less than for the uranium reflected assemblies. In addition it is observed that the critical mass is insensitive to the relative position of the plutonium-aluminum plates.

Perturbation worths, reaction ratios, and other pa-

rameters for the plates-separate configuration are listed in Tables II-2-XI and II-2-XII. Also given in these tables are the results of sensitivity studies involving a 5% reduction in the radiative capture cross section for Pu-239 over certain energy regions. The first two columns list the parameter and its reference value and the next column represents the percentage change in the parameter where $\sigma(n,\gamma)$ for Pu-239 is reduced over the entire energy range. The remaining four columns give the percentage change in the parameter when $\sigma(n,\gamma)$ for Pu-239 is reduced for energies above 100 keV, between 10 and 100 keV, between 1 and 10 keV, and below 1 keV, respectively. The only parameters in Tables II-2-XI and II-2-XII which show a significant dependence on the value of the Pu-239 capture cross-section are the perturbation worths of Pu and nickel. They are particularly sensitive to the

ure cross-section below 100 keV. It is questionable whether the nickel worth is large enough to enable accurate deductions to be made and the uncertainty in the Pu-240 cross sections will mask the sensitivity of the Pu-240 worth to the Pu-239 parameters.

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II-3. ZPR-3 Assembly 50—Benchmark Core Containing A Graphite Substitution for Sodium

✓ L. MOUNTFORD* and R. O. VOSBURGH

A series of "benchmark" plutonium-fueled, fast critical assemblies have been studied at ZPR-3.⁽¹⁾ This paper describes one assembly of this set, Assembly 50, and presents some of the experimental results. The first assembly of the set was Assembly 48, which had a composition designed to provide flux and adjoint spectra similar to those expected in large fast power reactors. Assembly 50 was related to the earlier assembly in having the same composition except for the substitution of graphite for the cans of sodium. In this way the real flux spectrum was softened with little change in the adjoint spectrum.

DESCRIPTION OF ASSEMBLY 50

Assembly 50 was cylindrical in configuration, with a depleted uranium reflector surrounding the core. The core was 30.66 in. long and had a 14.9 in. radius, giving a length-to-diameter ratio of 1.03. The thickness of the axial reflector was 12 in. and of the radial reflector 14 in. The core outline is shown in Fig. II-3-1.

The core had a one-drawer unit cell whose cross section was 2.183×2.175 in. Each cell contained plates of Pu-Al, Pu-U²³⁸-Mo, depleted uranium, and graphite in the configuration shown in Fig. II-3-2. The safety and control drawers contained more fuel than the normal core drawers to increase their worth. In safety rods the Pu-U-Mo plates were replaced

with Pu-Al; in the control rods they were replaced by one column of Pu-Al and one of depleted uranium.

The average compositions of Assembly 50 are given in Table II-3-I. Two sets of graphite whose masses differed by a few percent were used. The average graphite density is given for both the total core and the central region in which plates from only one of the sets were used. The size of the central region is shown in Fig. II-3-1. The stainless steel composition was 73.4 w/o Fe, 17.0 w/o Cr, 8.4 w/o Ni, 0.75 w/o Mn, and 0.45 w/o Si.

CRITICAL MASS

The Assembly 50 critical mass for a uniform, cylindrically smooth loading was determined by applying corrections to the minimum critical loading. The corrections and the critical parameters of the smoothed assembly are summarized in Table II-3-II. The reactivity effects of the corrections were converted to additional mass of plutonium using the worth of edge drawers relative to blanket material. The critical mass was calculated using the average cell composition and the smoothed core volume. The corrected critical mass was 223.9 kg Pu-239 (240 kg total Pu).

The worths of core edge drawers were measured by a series of core-reflector material substitutions. The worths are shown in Fig. II-3-3. Measurements were made for both the cases where the location in the other matrix directly opposite the substitution

* Atomics International, a Division of North American Rockwell Corporation, Canoga Park, California.

THE CORE INSIDE THE
DOTTED LINE CONT-
AINS ZPR-3 GRAP-
HITE; OUTSIDE, IT
CONTAINS A MIXTURE
OF ZPR-3 AND ZPR-6
GRAPHITE

FIG. II-3-1. Cross Section of Assembly 50. ANL Neg. No. 103-2930 Rev. 1.

FIG. II-3-2. Loading Master, Assembly 50 Core Drawer. ANL Neg. No. 103-2933.

3LE II-3-I. ASSEMBLY 50 COMPOSITION, 10^{22} atoms/cm³

Isotope	Core ^a	Safety-Control Rods		Reflector	
		Safety	Control	Axial	Radial
Pu-239	0.1645	0.3182	0.2121	—	—
Pu-240	0.0106	0.0151	0.0101	—	—
Pu-241	0.0011	0.0014	0.0009	—	—
Pu-242	0.00004	0.000017	0.000013	—	—
U-235	0.0016	0.0009	0.00144	0.0082	0.0083
U-238	0.7404	0.4681	0.7094	3.8377	3.9798
C (Average)	4.594	^b	4.209	—	—
C (Central) ^(c)	4.568	—	—	—	—
Fe	0.7300	1.2187	1.050	0.5622	0.4529
Cr	0.1816	0.3031	0.2613	0.1398	0.1126
Ni	0.0796 ^d	0.1326	0.1143	0.0612	0.0493
Mn	0.0076	0.0126	0.0109	0.0058	0.0047
Si	0.0089	0.0148	0.0128	0.0068	0.0055
Mo	0.0205	—	—	—	—
Al	0.0110	0.0330	0.0220	—	—

^a The core composition does not include the safety and control rod drawers.

^b The graphite in the Half 1 safety rods was one-half ZPR-3 and one-half ZPR-6 graphite, giving a carbon density of 4.209×10^{22} atoms/cm³. In the Half 2 safety rods, all the graphite was from ZPR-3, giving a carbon density of 4.153×10^{22} atoms/cm³.

^c C (Central) is the carbon density in the central region in which all graphite is ZPR-3 graphite (see Fig. II-3-1).

^d The core content of nickel included 0.00013×10^{22} atoms/cm³ as cladding for the Pu-Al fuel.

location contained a core drawer and where it contained reflector material. The linear least squares fit to the two sets of data with the constraint that the slope be the same for both sets showed that a drawer opposite another drawer was worth about 5% more than one opposite the reflector. The sum of the worths of a drawer of each type was used in making the

TABLE II-3-II. SUMMARY OF ASSEMBLY 50 CRITICAL PARAMETERS

(Fuel loading of as-built assembly = 217.05 kg Pu-239; smoothed radius = 14.445 in.)

Corrections to the Loaded Mass	Reactivity, ^a Ih	Pu-239, kg
1. Control and safety rod spiking	+1223 ± 22	
a. replacement of spiked drawers by unspiked drawers	—	-6.15
b. fuel added at boundary to balance removal of spiked drawers	—	+16.24
2. The irregular stepped outline of the core	-108 ± 4	-1.43
3. Core average temperature (reduced from 55.4 to 40.0°C)	-60 ± 3	-0.79
4. Assembly halves interface gap	-40 ± 20	-0.53
5. One control rod partially inserted in reference core	-32 ± 0	-0.42
6. Core subcritical at reference power from spontaneous fission neutron source	+2 ± 0	+0.02

Mass balance, kg Pu-239	223.99
Critical mass of the heterogeneous cylinder, kg Pu-239	223.9 ± 1.1
Core radius (corrected core), cm	37.81
Core height, cm	76.35
Core volume (corrected core), liters	342.9
Radial blanket thickness, cm	38
Axial blanket thickness, cm	30

^a 1% $\Delta k/k = 1002.7$ Ih

critical radius correction. The relation between increments of reactivity and mass at the corrected core edge was

$$\frac{\Delta k}{k} = \frac{1}{6.347} \frac{\Delta m}{m}$$

FIG. II-3-3. Assembly 50 Core Edge Drawer Worths. ANL Neg. No. 103-2940.

CENTRAL REACTIVITY WORTHS

The reactivity worths of several materials were measured at the center of the core inside a 2 x 2 x 2 in. cavity at the front of the fixed half central drawer. The measurements consisted of oscillating a sample and reference between the core center and a location outside the assembly while maintaining a level power with a calibrated autorod. To maintain a constant

power for some of the larger samples, it was necessary also to use a control rod. Two oscillation mechanisms were used for inserting the samples. Small samples (no greater than 0.42 in. diam by 2.0 in. long) were placed in carrier slots in a 1/2 in. diam rod which passed radially through the assembly. Larger samples were placed in a 2 x 2 x 2 in. cavity at the front of a drawer connected to a drive mechanism which

TABLE II-3-III. CENTRAL REACTIVITY WORTHS—ASSEMBLY 50

Material	Sample Dimension and Configuration, in.	Sample Mass, g		$\frac{4VN}{S}^{(a)}$ 10 ²² atoms/cm ²	Material Worth, lh/kg
		Material	Stainless Steel		
Pu ^b	0.020-thick annulus	9.090	6.009	0.687	438.8 ± 7.1 ^c
	0.010-thick annulus	4.673	6.213	0.345	432.2 ± 2.7
	0.006-thick annulus	2.400	6.261	0.173	421.7 ± 5.3
Pu ^d	0.020-thick annulus	9.315	6.498	0.709	523.7 ± 7.9 ^c
	0.010-thick annulus	4.801	6.487	0.349	517.2 ± 2.6
	0.007-thick annulus	2.761	6.597	0.200	514.1 ± 4.9
U	0.39-in. diam × 0.175-in. long	5.128	—	1.80	411.8 ± 3.5
U ^e	0.0106-thick annulus	7.128	1.794	0.441	410.4 ± 1.9
U ^f	0.0052-thick annulus	3.638	1.898	0.226	411.6 ± 3.7
	0.0034-thick annulus	2.327	1.826	0.147	413.7 ± 5.8
U ^g	1.995 × 1.995 × 1.988	2450	—	16.1	-30.93 ± 0.48 ^c
	0.420-diam cylinder	85.76	—	4.61	-32.75 ± 0.18
	0.200-diam cylinder	19.63	—	2.33	-35.53 ± 0.69
	0.100-diam cylinder	4.908	—	1.19	-39.4 ± 2.4
B ^h	0.040-thick annulus	1.399	10.152	2.23	-(1.218 ± 0.013) × 10 ^{4(c)}
	0.040-thick annulus	1.480	10.090	2.36	-(1.187 ± 0.012) × 10 ^{4(c)}
	0.020-thick annulus	0.801	11.464	1.30	-(1.291 ± 0.014) × 10 ^{4(c)}
	0.010-thick annulus	0.448	10.565	0.736	-(1.358 ± 0.018) × 10 ^{4(c)}
	0.200-diam cylinder	1.011	3.096	3.09	-(1.121 ± 0.011) × 10 ^{4(c)}
	0.100-diam cylinder	0.244	1.558	1.54	-(1.291 ± 0.024) × 10 ^{4(c)}
	0.060-diam cylinder	0.088	0.977	0.934	-(1.305 ± 0.014) × 10 ⁴
	0.0153-thick annulus	9.670	1.774	0.831	-290.1 ± 6.0 ^c
Ta	0.0106-thick annulus	6.701	1.836	0.578	-298.4 ± 1.7
	0.0053-thick annulus	3.300	1.815	0.287	-323.3 ± 4.1
	0.200-diam cylinder	17.13	—	2.67	-215.8 ± 3.8 ^c
	0.1005-diam cylinder	4.362	—	1.39	-249.8 ± 3.1
	0.0615-diam cylinder	1.596	—	0.839	-276.4 ± 7.6
	0.42-diam cylinder	35.90	—	—	-15.77 ± 0.38
SS-204	0.42-diam cylinder	35.50	—	—	-12.90 ± 0.48
Fe	0.42-diam cylinder	11.32	6.357	—	-9.46 ± 0.76
Fe ₂ O ₃	0.42-diam cylinder	40.30	—	—	-21.14 ± 0.33
Ni	0.42-diam cylinder	15.70	6.386	—	-37.06 ± 0.50
Mn	0.42-diam cylinder	15.36	6.412	—	-13.06 ± 0.57
Cr	0.42-diam cylinder	4.072	—	—	2232 ± 26 ^c
POLY	0.42-diam cylinder	i	i	—	-11.3 ± 0.9
Na	2 × 2 × 2	i	i	—	—

^a The "atoms" in the unit of 4VN/S refers, for the different materials, to atoms of plutonium, U-235, uranium (for the depleted uranium samples, with all uranium assumed to be U-238), boron and tantalum.

^b 98.78 w/o Pu, 1.09 w/o Al, 0.13 w/o other; 72.24 w/o Pu-239, 22.28 w/o Pu-240, 4.63 w/o Pu-241, 0.79 w/o Pu-242.

^c Rod No. 6 was used for the measurement.

^d 98.62 w/o Pu, 1.22 w/o Al, 0.16 w/o other; 95.05 w/o Pu-239, 4.50 w/o Pu-240, 0.45 w/o Pu-241.

^e 93.20 w/o U-235.

^f 93.10 w/o U-235.

^g 0.21 w/o U-235.

^h 92.8 w/o B, 92.1 w/o B-10.

ⁱ The sodium worth is the average of two measurements with samples of 110.2 and 111.5 g of sodium and 53.47 and 53.99 g of stainless steel, respectively.

FIG. II-3-4. Central Worths of Fissile Materials Versus Sample Size—Assembly 50. ANL Neg. No. 103-2931.

FIG. II-3-5. Boron Worth Versus Sample Size—Assembly 50. ANL Neg. No. 103-2942.

moved it inside the central matrix tube in the stationary half.

The worths of central materials are listed in Table II-3-III. The differences between the worths of the autorod and control rod at their locations corresponding to the two sample positions were corrected where necessary for the effect of oscillating empty carriers, for sample-reference differences, and for the

worth of hydrogen (in the powder samples) to obtain the sample worths. These were divided by the masses to obtain the specific worths given in Table II-3-III. Each uncertainty is the sum of the calculated contribution of reactor noise and the uncertainties in the corrections. For the samples for which a control rod was used the uncertainties include the $\pm 1\%$ uncertainty in the absolute worths of the autorod and

TABLE II-3-IV. SPECIFICATION OF ASSEMBLY 50 CORE MATERIAL SAMPLES

Normal Loading		Pu-U-Mo Removed		Pu-Al Removed	
Material ^a	Mass, g	Material ^a	Mass, g	Material ^a	Mass g
C (3 pieces)	39.39	C (3 pieces)	39.39	3 (3 pieces)	39.39
Depleted U	148.30	Depleted U	148.17	Depleted U	148.301
Pu-Al	# 2U-1454 ^b	Pu-Al	# 2S-1342 ^b	SS can	17.347
Depleted U	148.961	Depleted U	148.17	Depleted U	148.961
C (5 pieces)	65.65	C (5 pieces)	65.65	C (5 pieces)	65.65
Pu-U-Mo	# 2-049 ^b	SS can	17.3	Pu-U-Mo	# 2-049 ^b
Pu-U-Mo	# 2-224 ^b	SS can	17.3	Pu-U-Mo	# 2-224 ^b
C (3 pieces)	39.39	C (3 pieces)	39.39	C (3 pieces)	39.39
2 × 2 in. box	39.438	2 × 2 in. box	39.001	2 × 2 in. box	39.438

^a Each sample consisted of sixteen 2 × 2 × ⅛ in. plates inside a stainless steel box. The materials were loaded in the configuration shown in Fig. II-3-2. They are listed in the order in which they appear in Fig. II-3-2.

^b See Table II-3-V for identification of symbols.

TABLE II-3-V. SPECIFICATIONS OF PLUTONIUM PLATES^a

Pu-1 w/o Al							
Plate Number	Total Mass, g	Core Mass, g	Core Composition		Pu Composition, w/o		
			Al	Pu	Pu-239	Pu-240	Pu-241
2U-1454	86.405	69.582	1.08	98.93	94.97	4.60	0.42
2S-1342	86.295	69.492	1.10	98.89	95.09	4.52	0.39

Pu-U-Mo							
Plate Number	Total Mass, g	Mo	U-238	U-235	Pu-239 + Pu-241 ^b	Pu-240	SS
2-224	94.850	2.00	62.17	0.14	14.31	1.35	14.77
2-049	94.603	2.00	62.09	0.14	14.28	1.34	14.68

^a The values of total mass are from weighings after the measurements. The remaining information is from information supplied by Dow Chemical (Pu-1 w/o Al) and Numec (Pu-U-Mo). The w/o compositions do not total 100 w/o but are as reported by the suppliers.

^b The isotopes 239 and 241 are combined in the ZPR-3 Hot Constants for the Pu-U-Mo alloy. A composition of 90.4 w/o Pu-239 and 0.98 w/o Pu-241 was used in determining the core composition.

control rod. When only the autorod was used this is not included and the uncertainties apply only to comparisons among the worths in which the absolute reactivity normalization factor cancels out.

The worths of the fissile materials and B-10 are plotted as a function of the sample size parameter $4VN/S$ in Figs. II-3-4 and II-3-5, respectively, where V is volume, S is surface area, and N is atom density. Linear least-squares fits were made to the two sets of

plutonium data. The size effect is not linear, in general; however, the fitted lines provide a first-order of the self-shielding effect and extrapolation to infinite dilution. The uranium samples showed no size effect. The weighted average of 411.0 ± 1.3 lh/kg is plotted in Fig. II-3-4. The linear least-squares fit made to the boron data is shown in Fig. II-3-5.

The worths of 2 × 2 × 2 in. sections of core materials were measured. The samples, described in Tables II-3-IV and II-3-V, included one having the core composition, two in which the fuel plates were alternately replaced with empty cans, and a single fuel plate. These were compared to an empty cavity. The worths are listed in Table II-3-VI.

An experiment was performed to determine the effect on the neutron flux and adjoint spectra as the result of removing materials to form the central cavity. The reactivity worth and fission rate of U-235 were measured as a function of position in the radial through-tube of the small-sample measurements. The reactivity worth of a 0.39 in. diam by 0.175 in. long stack of U-235 foils was measured by moving it through the central three cells while maintaining a constant power with the autorod. The fission rates were measured with a fission counter having a 2 in. (5 cm) active length.

The fission rate data are shown in Fig. II-3-6. Fig-

TABLE II-3-VI. WORTHS OF 2-IN. CUBE SECTIONS OF CORE MATERIALS—ASSEMBLY 50

Sample	Worth, lh
Normal loading	41.26 ± 0.43
Pu-Al plate replaced with dummy can	1.50 ± 0.31
Pu-U ²³⁸ -Mo plate replaced with dummy can	31.51 ±
Pu-Al plate at center of 2-in. cube cavity	38.17 ±

FIG. II-3-6. Relative U-235 Fission Rates Versus Radius Assembly 50. *ANL Neg. No. 103-2936.*

FIG. II-3-7. Relative U-235 Fission Rates Near the Core Center—Assembly 50. *ANL Neg. No. 103-2937.*

Figure II-3-7 shows the central data on an expanded scale. The data were fitted by a zero-order Bessel function. The extrapolated core radius of 19.8 in. was chosen as giving the best fit to the data in the region with $13 \leq r \leq 14$ in. The fit was normalized to the

measured rate at 13 in. The fission rate was flat across the cavity and at the center, approximately 1% higher than the J_0 curve.

The worth values are shown in Fig. II-3-8. The J_0 curve was squared and normalized to worths using

FIG. II-3-8. Relative U-235 Reactivity Worths Near the Core Center—Assembly 50. ANL Neg. No. 103-2938.

the ratios of J_0^2 and measured worth values outside the cavity. The worth was constant within the cavity but approximately 4% lower at the center than the J_0^2 curve. This result, combined with the higher fission rates, shows a decrease of about 5% in the neutron importance as a result of the removal of core materials to form the cavity.

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II-4. ZPR-3 Assemblies 53 and 54—Continuation of Studies of Basic Physics Cores—Depleted Uranium Blanket and Iron Reflected Versions

R. E. KAISER, J. M. GASIDLO, W. K. LEHTO, L. A. MOUNTFORD,* J. E. POWELL, N. A. HILL,† R. O. VOSBURGH and J. M. STEVENSON‡

INTRODUCTION

In September and October of 1968, Assemblies 53 and 54 were built on ZPR-3. Assembly 53 was a continuation of the benchmark series which began with Assemblies 48, 49 and 50. Assembly 54 was basically the same as Assembly 53 except that the depleted uranium blanket was replaced by an iron reflector. The cores contained graphite as the primary diluent material and had a comparatively low U-238/Pu-239 ratio of 1.59.

* Atomics International, a Division of North American Rockwell Corporation, Canoga Park, California.

† Pacific Northwest Laboratory, Richland, Washington.

‡ UKAEA, Winfrith, Dorchester, Dorset, England.

DESCRIPTION OF ASSEMBLIES

Both assemblies used a single drawer cell with a half height of 12 in. This variation from the 15 in. half-height of Assemblies 48, 49 and 50 was introduced in order to maintain a length-to-diameter (L/D) ratio of slightly less than one. The core drawer configuration used in both assemblies is shown in Fig. II-4-1. Control drawers contained one less column of $\frac{1}{16}$ in. graphite between the drawer and the plutonium-aluminum fuel to allow for the extra thickness of the drawers. Interface diagrams for the two cores are shown in Fig. II-4-2. The cylindrical dimensions of the reference cores are presented in Table II-4-I, neglecting a gap of 0.080 in. at the

FIG. II-4-1. Assembly 53-54 Core Drawer Configuration. ANL Neg. No. 103-2943.

FIG. II-4-2a. Interface Diagram for Assembly 53. ANL Neg. No. 103-2953.

FIG. II-4-2b. Interface Diagram for Assembly 54. ANL Neg. No. 103-2855.

interface which contains 75 v/o stainless steel. In addition, the 0.28 in. spring gap at the back of the 21 in. drawers is not included in the dimensions listed. Atom densities of the core, the depleted blanket, and the iron reflector are included in Table II-4-II.

SPECTRUM MEASUREMENTS

Proton-recoil spectrum measurements were performed at the center of Assembly 53 during the approach-to-critical. Measurements were made with the reactor subcritical by approximately 6.6% Δk , over the range of 308 eV to 1.5 MeV. A special low gamma background zone was built into the core as shown in Fig. II-4-3 using plutonium-aluminum plates which had not been used for several months. The zone was 9-in. deep in the half containing the counter and 6-in. deep in the opposite half. Results of the measurement are shown in Fig. II-4-4, along with a calculated curve based on an MC² ⁽¹⁾ ultrafine group calculation,

smoothed by a Gaussian window to approximate the resolution of the measurement. Cross sections were from the ENDF/B library. A more detailed discussion of the measurement technique may be found in Paper IV-18.

CRITICAL MASS EVALUATION

The loaded masses of fissile material (Pu-239 + Pu-241 + U-235) in Assemblies 53 and 54 were 150.43 and 131.26 kg respectively. The correction for the jagged core outline was found to be negligible, and no control drawers were spiked, so that the only significant correction made to the heterogeneous critical mass was for control rod position. The measured and calculated values of the heterogeneous critical mass, excluding corrections for the interface and spring gaps, are listed in Table II-4-III. All of the calculations were based on homogeneous, one-dimensional, spherical diffusion calculations, corrected

FIG. II-4-3. Interface Diagram for Assembly 53 for Proton-Recoil Spectrometer Measurements. ANL Neg. No. 103-2954.

to cylindrical geometry with use of an appropriate shape factor.² The MACH-1⁽³⁾ program was used for these calculations.

The critical masses calculated by diffusion theory and corrected for heterogeneity are shown in Table II-4-III. The two components of heterogeneity—the resonance self-shielding and the flux advantage factor—were calculated separately and assumed to be additive, although some small degree of duplication was possible in the methods used. The resonance self-shielding effect was calculated from MC²-pre-

ed cross sections for the homogeneous and heterogeneous configurations. The perturbation heterogene-

ity code CALHET⁽⁴⁾ was used for calculating the flux advantage factor caused by the fine spatial flux distribution in the cell. Further details of the heterogeneity studies can be found in Paper II-6.

The kinetics parameters for Assemblies 53 and 54, as calculated with one-dimensional diffusion theory by MACH-1-BAILIFF,⁽³⁾ are presented in Table II-4-IV. The calculations used ENDF/B cross sections with heterogeneous weighting in MC².

FISSION RATIO MEASUREMENTS

Central fission ratios for U-238, U-234, U-233, U-236, Pu-239, and Pu-240 to U-235 have been meas-

FIG. II-4-4. Measured and Calculated Neutron Spectra for Assembly 53. ANL Neg. No. 103-2955.

TABLE II-4-I. CRITICAL DIMENSIONS OF ZPR-3 ASSEMBLIES 53 AND 54

Dimension	Assembly 53		Assembly 54	
	In.	Cm	In.	Cm
Core radius	13.53	34.37	12.65	32.08
Outer radius of blanket	26.97	68.50	25.47	64.69
Radial blanket thickness	13.44	34.14	12.82	32.56
Core height	24.00	60.96	24.00	60.96
Axial blanket thickness	12.00	30.48	12.00	30.48
L/D ratio	0.887		0.949	

TABLE II-4-II. CORE AND REFLECTOR COMPOSITIONS FOR ASSEMBLIES 53 AND 54

Isotope	Composition, 10^{24} atoms/cm ³		
	Core	Assembly 53, Depleted U Blanket	Assembly 54, Iron Reflector
Pu-239	0.001661		
(Pu-Al)	0.001069		
(Sefor)	0.000592		
Pu-240	0.000107		
(Pu-Al)	0.000051		
(Sefor)	0.000056		
Pu-241	0.0000114		
(Pu-Al)	0.0000049		
(Sefor)	0.0000065		
U-235	0.0000061	0.000083	
U-238	0.002610	0.03977	
Al	0.000111		
Mo	0.000208		
Fe	0.007474	0.004847	0.07536
Ni	0.000814	0.000528	0.000528
Cr	0.001859	0.001205	0.001205
Mn	0.0000776	0.000050	0.000555
Si	0.000091	0.000059	0.000059
C	0.055811		0.001575

TABLE II-4-III. COMPARISON OF MEASURED AND CALCULATED VALUES OF CRITICAL MASS IN ASSEMBLIES 53 AND 54

Assembly No.	Cross Section Set	Homogeneous Critical Mass	Advantage Factor Correction	Heterogeneous Calculated Mass	Experimental Mass
53	224	177.04	0.931	164.72	149.15
53	ENDF/B (Het) ^a	150.57	0.931	140.18	149.15
53	ENDF/B (Hom) ^b	163.45	0.931	152.17	149.15
54	224	122.54	0.928	113.72	130.26
54	ENDF/B (Het) ^a	127.44	0.928	118.26	130.26

^a Uses heterogeneously shielded MC² cross sections.^b Uses homogeneously shielded MC² cross sections.

TABLE II-4-IV. CALCULATED KINETICS PARAMETERS FOR ASSEMBLIES 53 AND 54

Parameter	Assembly 53	Assembly 54
β_{eff}	2.893×10^{-3}	2.562×10^{-3}
ℓ_0 , sec	5.735×10^{-7}	8.426×10^{-7}
$l_h/\% \Delta k$	1037.8	1096.9

TABLE II-4-V. FISSION RATIO MEASUREMENTS IN ASSEMBLY 53

Ratio	Measured Value	Calculated Value, ENDF/B
U-233/U-235	1.454 ± 0.014	1.353
U-234/U-235	0.146 ± 0.002	0.141
U-236/U-235	0.053 ± 0.0005	0.054
U-238/U-235	0.0254 ± 0.0002	0.0254
Pu-239/U-235	0.928 ± 0.009	0.825
Pu-240/U-235	0.174 ± 0.002	0.168

TABLE II-4-VI. CENTRAL PERTURBATION MEASUREMENTS IN ASSEMBLIES 50, 53 AND 54

Material	Sample Size, in.	Sample Mass, gm		Material Worth, Ih/kg			Worth Ratio, Assembly 54/ Assembly 53
		Material	Stainless Steel	Assembly 50	Assembly 53	Assembly 54	
Pu ^a	0.010-thick annulus	4.801	6.487	517.2 ± 5.8	655.7 ± 6.4	739.2 ± 8.6	1.127 ± 0.008
	0.007-thick annulus	2.761	6.597	514.1 ± 7.1	650.1 ± 7.7	706.1 ± 11	1.086 ± 0.014
U ^b	0.0034-thick annulus	2.327	1.826	413.7 ± 7.1	497.2 ± 6.4	535.9 ± 11	1.077 ± 0.021
	0.0014-thick annulus	0.983	1.876	—	481.2 ± 10	525 ± 18	1.091 ± 0.042
U ^c	0.200-diam cylinder	19.63	—	-35.53 ± 0.78	-60.67 ± 0.84	-65.4 ± 1.2	1.078 ± 0.019
	0.100-diam cylinder	4.908	—	-39.4 ± 2.4	-75.9 ± 1.9	-72.8 ± 2.8	0.959 ± 0.041
B ^d	0.200-diam cylinder	1.011	3.096	-(1.121 ± 0.026) × 10 ^{4(e)}	-(2.102 ± 0.020) × 10 ^{4(e)}	—	—
	0.100-diam cylinder	0.244	1.558	-(1.291 ± 0.027) × 10 ^{4(e)}	-(2.557 ± 0.002) × 10 ^{4(e)}	-(2.676 ± 0.009) × 10 ^{4(e)}	1.046 ± 0.009
	0.060-diam cylinder	0.088	0.977	-(1.406 ± 0.014) × 10 ^{4(e)}	-(2.911 ± 0.013) × 10 ^{4(e)}	-(3.173 ± 0.022) × 10 ^{4(e)}	1.089 ± 0.009
Ta	0.0053-thick annulus	3.300	1.815	-323.3 ± 5.2	-691.5 ± 7.7	—	—
	0.0022-thick annulus	1.317	1.847	—	-818.9 ± 10	—	—
Mo	0.42-diam cylinder	46.25	—	—	-103.13 ± 1.0	—	—
SS-304	0.42-diam cylinder	35.90	—	-15.77 ± 0.41	-11.07 ± 0.28	—	—
Fe	0.42-diam cylinder	35.50	—	-12.90 ± 0.50	-4.45 ± 0.17	—	—
Cr	0.42-diam cylinder	15.36	6.412	-13.06 ± 0.58	—	—	—
Cr	0.42-diam cylinder	f	f	—	-10.10 ± 0.46	—	—
Ni	0.42-diam cylinder	40.30	—	-21.14 ± 0.39	-20.50 ± 0.29	—	—
C	0.42-diam cylinder	6.910	—	—	+114.2 ± 1.6	—	—
Na	0.42-diam cylinder	3.003	6.665	-11.3 ± 0.91	-57.9 ± 1.8	—	—
Polyethylene							
old	0.416-diam cylinder	4.072	—	2232 ± 58 ^e	3908 ± 39 ^e	—	—
new	0.420-diam cylinder	4.321	—	—	3897 ± 36 ^e	—	—
	0.201-diam cylinder	0.981	—	—	4046 ± 31	—	—
	0.104-diam cylinder	0.258	—	—	4139 ± 37	—	—
Empty Carriers			—	+0.0586 ± 0.0086	-1.3500 ± 0.0029	+0.0972 ± 0.100	—
sample versus reference (Worth in Ih)							

^a 98.62 w/o Pu, 1.22 w/o Al, 0.16 w/o other; 95.05 w/o Pu-239, 4.50 w/o Pu-240, 0.45 w/o Pu-241.

^b 93.10 w/o U-235.

^c 0.21 w/o U-235.

^d 92.8 w/o B; 92.1 w/o B-10.

^e Rod No. 6 was used in measuring this worth.

^f The quoted worth is from the measurements of two samples with 12.0176 and 11.4519 cm of chromium powder, inside 6.7360 and 6.8273 g stainless steel capsules, respectively.

FIG. II-4-5. Normalized Radial Reaction Rate Traverses for Pu-239 in Assembly 53. ANL Neg. No. 103-2945.

FIG. II-4-6. Normalized Radial Reaction Rate Traverses for U-238 in Assembly 53. ANL Neg. No. 103-2950.

ured in ZPR-3 Assembly 53. The counters used were spherical gas flow chambers with the isotope foil to be measured placed back-to-back with a U-235 foil. Table II-4-V lists the results measured in Assembly 53 along with calculations, based on the ENDF/B cross sections, for the U-238, Pu-239 and Pu-240 ratios. The calculations represent ratios calculated for homogeneously weighted cross sections in a spectrum based on heterogeneously weighted cross sections.

CENTRAL REACTIVITY MEASUREMENTS

Central reactivity worth measurements were performed for several materials in Assembly 53 and for Pu-239, U-238, U-235, and B-10 in Assembly 54. Measurements were made by oscillating the samples

and measuring reactivity differences on a low-worth autorod. For high-worth samples such as B-10 and polyethylene, a calibrated control rod was used along with the autorod. Results of the measurements for both assemblies are presented in Table II-4-VI. It can be seen that the worths are slightly higher in the smaller core of Assembly 54, as would be expected.

REACTION RATE TRAVERSES

Reaction rate traverses were made in both assemblies with Pu-239, U-238, U-235, and B-10 as shown in Figs. II-4-5 through II-4-12. The curves have all been normalized to 1.0 at the core center-line. Meas-

FIG. II-4-7. Normalized Radial Reaction Rate Traverses for U-235 in Assembly 53. ANL Neg. No. 103-2946.

FIG. II-4-8. Normalized Radial Reaction Rate Traverses for B-10 in Assembly 53. ANL Neg. No. 103-2947.

FIG. II-4-9. Normalized Radial Reaction Rate Traverses for Pu-239 in Assembly 54. *ANL Neg. No. 103-2948*.

FIG. II-4-10. Normalized Radial Reaction Rate Traverses for U-238 in Assembly 54. *ANL Neg. No. 103-2951*.

urements were made with small gas-filled chambers in a stainless steel guide tube. The data were taken with the same core configuration as was used for the central perturbation measurements, which included a 2 x 2 x 2 in. void at the center of the core. This is believed to be the cause of the slight dip noticeable in some of the curves, particularly that for U-238.

The Assembly 53 data are essentially smooth across the core reflector interface, whereas some of the data from Assembly 54 show a marked inflection at this point. The difference between the two cases is attributed to the softer spectrum in the iron reflector of Assembly 54 as opposed to the depleted uranium blanket of Assembly 53.

GRAPHITE DENSITY VARIATION

The absence of sodium in Assembly 53 precluded any sodium voiding experiments. However, the availability of graphite plates at a lower density allowed analogous experiments to be performed with graphite. Relative to the reference core with a graphite density of 1.16 g/cm³, a central 12.1 liter zone containing graphite at 0.979 g/cm³ gave a reactivity decrease of 183.3 ± 3.5 lh. A similar measurement with a zone at the axial core-reflector interface of 4.9 liters at 0.995 g/cm³ gave a decrease of 46.9 ± 2.4 lh. A more detailed discussion of this experiment along with analytical results is presented in Paper II-7.

FIG. II-4-11. Normalized Radial Reaction Rate Traverses for U-235 in Assembly 54. *ANL Neg. No. 103-2949*.

FIG. II-4-12. Normalized Radial Reaction Rate Traverses for B-10 in Assembly 54. *ANL Neg. No. 103-2952*.

FIG. II-4-13. Reactivity Doppler Results in Assembly 53. ANL Neg. No. 103-2956.

REACTIVITY DOPPLER MEASUREMENTS

The technique of oscillating a heated sample of fissile or fertile material into and out of an unheated core to measure Doppler reactivity effects has been in use at Argonne for several years.⁵ Such measurements were again performed in Assembly 53, with PuO₂ and natural UO₂. A $\frac{3}{8}$ in. graphite buffer zone surrounded the Doppler rod during the experiment.

The plutonium oxide sample contained 85.8 w/o Pu-239, 2.3 w/o Pu-240 and 0.1 w/o Pu-241, and the uranium oxide sample contained 87.6 w/o U-238 and 0.6 w/o U-235. The total measured reactivity effects on heating the samples are shown in Fig. II-4-13, no expansion corrections having been made.

HETEROGENEITY MEASUREMENTS

Three traverses of the Assembly 53-54 unit cell were made: U-235 fission, U-238 fission and U-238 capture. Data from the fission traverses have been reduced, but the U-238 capture data awaits develop-

ment of computer codes. Eight, nominally 0.5-in. diam by 0.005-in. foils were exposed, four at a time, in the central drawer. The locations of the foils are indicated in Fig. II-4-14.

The data from both fission traverses were normalized to the yield of Nb-97, as discussed in Papers IV-5 and I-26. The specific activities of the fission traverse foils and the activation ratios are presented in Table II-4-VII. The errors quoted in the table are based primarily on counting statistics and do not include possible systematic errors. The data were taken with a total exposure of about 10 W-h, giving approximately 10^8 total fissions in each foil.

PULSED-NEUTRON MEASUREMENTS

Pulsed-neutron determinations of β/ℓ were made in each assembly as described in Paper II-5. The results of these measurements are presented in Table II-4-VIII along with calculated values obtained from MACH-1-BAILIFF⁽³⁾ using ENDF/B cross sections with homogeneous and heterogeneous weight-

The remarkable agreement for the Assembly 54 values may be fortuitous, since the cross sections for the iron reflector are possibly incorrect. These cross sections were obtained from a zero-buckling MC^2 run with a small amount of source material and are thus based on an equilibrium spectrum. The degree of error involved would depend on how quickly the real spectrum in the iron blanket reaches an equilibrium value. The substantially larger error in the homogeneous calculation for Assembly 53 is primarily due to differences in the calculated lifetimes, and is consistent with the effects of heterogeneity on other parts of the calculation.

G = GRAPHITE
F = Pu-Al ALLOY FUEL
S = SEFOR FUEL

g. II-4-14. Assemblies 53 and 54 Heterogeneity Foil Placements. ANL Neg. No. 103-2944.

TABLE II-4-VII. SPECIFIC ACTIVITIES AND ACTIVATION RATIOS FOR U-238 AND U-235 FISSION TRAVERSES IN ASSEMBLY 53

Foil Location	U-238		U-235		Activation Ratio 28/25 ^a	
	Asn ^b	% Uncert.	Asn ^b	% Uncert.	RA	% Uncert.
A	3.853	0.74	1.786	1.04	0.0216	1.30
B	5.170	0.39	1.741	0.97	0.0297	1.04
C	4.105	0.30	1.774	0.75	0.0231	0.82
D	3.862	0.36	1.663	1.29	0.0232	1.34
E	4.129	0.93	1.697	2.50	0.0243	2.67
F	4.471	0.58	1.615	1.01	0.0277	1.16
G	4.110	0.41	0.849	1.16	0.0222	1.22
H	4.635	1.86	1.693	1.43	0.0274	2.34

^a Ratio of Nb-97 activities for U-238 foils/U-235 foils at the same location.

^b Relative specific activity, proportional to yield of mass 97 cumulative to Nb, normalized to unit saturated monitor foil activation.

TABLE II-4-VIII. MEASURED AND CALCULATED VALUES OF β/ℓ FOR ASSEMBLIES 53 AND 54

Assembly	Measured Value, 10 ⁸	MACH-1 Diffusion Calculations, 10 ⁸	
		Heterogeneous	Homogeneous
53	4.8 ± 0.1	5.04	5.55
54	3.0 ± 0.1	3.04	

CONCLUSIONS

The object in constructing the iron-reflected Assembly 54 was to minimize the contribution of U-238 fission. This was done in order to ascertain whether significant errors in delayed neutron data were a contributing factor in the large discrepancies almost always encountered in central reactivity measurements and others of a similar nature. Comparisons between calculated and measured critical masses, β/ℓ ratios, and some central reactivities for which preliminary calculations are available, do not indicate a marked decrease in the discrepancies. The slight improvement noted in the calculation of β/ℓ may be due to inaccuracies in the iron reflector cross sections as previously noted. At the present state of the analysis of the two assemblies, it does not appear that calculational discrepancies can be attributed to the uncertainties in the U-238 delayed neutron parameters.

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II-5. Pulsed Neutron Experiments in ZRP-3 Assemblies 53 and 54

W. K. LEHTO

INTRODUCTION

ZPR-3 Assemblies 53 and 54 were the latest of the basic physics cores in the "Benchmark" series.¹ The natural-uranium reflector in Assembly 53 was replaced with iron to make Assembly 54. The main purpose was to minimize the delayed-neutron contribution from the U-238 in Assembly 54 and repeat certain measurements done in Assembly 53, which emphasized the delayed neutrons from the blanket. In addition to reactivity measurements, pulsed-neutron measurements of β_{eff}/ℓ were done in each assembly to obtain a quantitative measure of the contribution of the U-238 reflector to β_{eff} .

EXPERIMENTAL ARRANGEMENT AND PROCEDURE

Neutron bursts were generated with a 150 keV Cockcroft-Walton accelerator; 14 MeV neutrons were produced from the $H^3(d,t)He^4$ reaction. Pulse widths of 1 μ sec and a repetition rate of 10^3 pulses/sec were used. At this pulse rate, the contribution from delayed neutrons appeared as a constant background added to the normal fission-neutron background and the inherent Pu-240 spontaneous-fission source. The tritium target was placed at the core center through the voided central-matrix location in the stationary half. The detectors were $3/8$ by 2 in. BF_3 chambers, positioned approximately one third of the extrapolated dimensions from the core center so that higher harmonics were minimized. The radial distance between the target and detector was 8 in. in Assembly 53 and 6 in. in Assembly 54 (matrix positions 1-P-12 and 1-P-13, respectively). Data were collected with a multi-channel analyzer equipped with a time-of-flight logic unit.

Measurements were made as a function of reactor subcriticality to about 4.5 dollars subcritical. The reactivity was varied by positioning the fuel and safety rods which had been previously calibrated by an inverse-kinetics technique. After correction for analyzer dead time, a function of the form $\phi = Ae^{-\alpha t} + B$ was fitted

to the data by adjusting A , B and α by the least squares technique. The decay constant α is related to the reactivity by

$$\alpha = \frac{k(1 - \beta_{eff}) - 1}{\ell} \quad (1)$$

and in dollar units

$$\alpha = \alpha_c(1 - \$), \quad (2)$$

where k is the neutron multiplication constant, β_{eff} is the delayed neutron fraction, and ℓ is neutron lifetime. The value of $\alpha_c = \beta_{eff}/\ell$ can thus be determined from the \$0 intercept of a plot of α versus dollars subcritical.

RESULTS

The experimentally determined decay constants are shown in Fig. II-5-1 as a function of reactor subcriticality. The intercept α_c was determined by a straight line fit to these data.

FIG. II-5-1. Assembly 53 and 54 α -Values Versus Reactivity. ANL Neg. No. 113-2775.

TABLE II-5-I. β_{eff}/ℓ RESULTS FOR ASSEMBLIES 53 AND 54

Assembly	Experimental, 10 ⁴	Calculated, 10 ⁴	Calc. Exp.
53	0.48 $\pm$ 0.01	0.504	1.05
54	0.30 $\pm$ 0.01	0.304	1.01

The values of β_{eff}/ℓ for the two assemblies are listed in Table II-5-I along with values² from spherical MACH I calculations with input cross sections from MC² using ENDF/B data.

The discrepancy between the calculation and experiment is consistent with the findings in other laboratories;^{3,4} however, the disparity is not as large in this case. It may be only fortuitous that the agreement is better or it may reflect the improved calculational results using the ENDF/B cross sections. This point is open to speculation.

Also it is to be noted that the results of these measurements are at variance with the experimental results of G. Brunson and R. Huber⁵ who found a non-linear relationship between the decay constant and reactivity in certain types of reflected cores. Contrary to their results, these data obey Eq. 2 and extrapolate to $\rho = + \$1$ at $\alpha = 0$ as predicted by linear theory.

These experiments have provided a quantitative

measure of the contribution of the delayed neutrons from the U-238 reflector. Some change in the lifetime is expected between the two cores but the major contributions to the decay constant is in β_{eff} . The linear behavior of the measured decay constants with reactivity indicates that the point reactor theory is applicable for these cores over the range of reactivities studied and that the rod calibration procedures are reliable.

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II-6. Calculation of Heterogeneity Effects in ZPR-3 Assembly 53

J. M. STEVENSON* and A. P. OLSON

INTRODUCTION

A series of computer calculations has been run using the MACH-1 and CALHET⁽¹⁾ codes to estimate the heterogeneity effect for ZPR-3 Assembly 53 (see Paper II-4). This particular assembly had a neutron spectrum which was appreciably softer than usually found in ZPR cores and therefore had a large resonance heterogeneity effect as well as the multiplication effect in the fuel plates.

COMPOSITION OF THE STANDARD DRAWER

The arrangement of the single drawer standard cell of the assembly is shown in Fig. II-6-1. The composition differed slightly from that given in Paper II-4, which included the control rods. All of the plates were 5 cm thick except the Pu-U alloy plate which

was 0.635 cm thick. Both fuel plates had 0.030 cm steel cladding. The thickness of drawer and matrix was 0.233 cm on each side. It will be noted that the fuel in the fuel plates was only 4.514 cm high compared with the 5.08 cm high graphite plates and the 5.526 cm vertical pitch.

MACH-1 CALCULATIONS

Group cross-sections were prepared (in 24 groups) using ENDF/B data and the MC² code with the resonances of the various isotopes in the standard cell shielded according to their densities in the appropriate plates. Thus there were two sets of data for the plutonium isotopes—one corresponding to each type of fuel plate. The average blanket composition was used (see Paper II-4) and the resonances were shielded for the homogeneous composition. The critical radius for a spherical core using the homogeneous composition

* UKAEA, Winfrith, England.

Fig. II-6-1. Schematic Diagram of Standard Cell ZPR-3 Assembly 53. *ANL Neg. No. 113-2821*.

TABLE II-6-I. CENTRAL SPECTRA FROM MACH-1 CALCULATIONS

Group	Lower Energy	"Homogeneous" Flux	"Heterogeneous" Flux	"Heterogeneous" Adjoint Flux
1	3.68 MeV	0.0254	0.0254	4.980
2	2.31	0.0459	0.0458	4.967
3	1.35	0.0734	0.0732	4.863
4	821 keV	0.0827	0.0824	4.826
5	498	0.0875	0.0869	4.981
6	302	0.0855	0.0849	5.137
7	183	0.0797	0.0791	5.264
8	111	0.0719	0.0712	5.376
9	67.4	0.0658	0.0651	5.454
10	40.9	0.0600	0.0593	5.531
11	24.8	0.0530	0.0523	5.638
12	15.0	0.0485	0.0478	5.736
13	9.12	0.0434	0.0428	5.888
14	5.53	0.0367	0.0363	6.040
15	3.35	0.0329	0.0326	6.210
16	2.03	0.0280	0.0279	6.406
17	961 eV	0.0330	0.0335	6.646
18	454	0.0225	0.0238	7.087
19	275	0.0096	0.0106	7.511
20	61.4	0.0131	0.0167	7.860
21	13.7	0.00137	0.00222	8.027
22	3.06	0.000076	0.000156	7.854
23	0.683	0.000013	0.000028	6.778
24	0.00279	6×10^{-9}	14×10^{-9}	9.971

was 37.0 cm, almost identical to the equivalent spherical radius of the as-built cylinder, after a shape factor² was applied. A similar calculation but using the resonances in the core shielded according to the homogeneous composition gave a critical core radius of

37.8 cm. Analysis of the variation of the k -value for various iterations on the radius permitted this difference in radius to be converted to a $\Delta k/k = 1.52\%$. The central neutron fluxes, normalized to unit total, are compared in Table II-6-I where, as expected, the homogeneously shielded resonances remove more of the low energy neutrons than do the heterogeneously shielded resonances. For information, the unnormalized central adjoint fluxes for the heterogeneous case are also included in the table.

The reactivity difference between the two cases was confirmed by taking the shielded cross sections for the heavy isotopes used in both problems and computing the perturbation of each isotope at its atomic density throughout the core with heterogeneous resonance shielding. Thus the difference in reactivity from Pu-239 was obtained by comparing the sum of the two heterogeneously shielded Pu-239 isotopic perturbations at their respective atomic densities with the single homogeneously shielded isotopic perturbation. The only significant $\Delta k/k$ changes relative to the heterogeneous system were

Pu-239	+0.195%
Pu-240	-0.057%
U-238	-1.845%
Total	-1.71%

This total shows reasonable agreement with that of -1.52% above, bearing in mind that no attempt has been made to allow for the spectrum difference between the two systems; i.e., first order perturbation was used.

TABLE II-6-II. GROUP CONTRIBUTIONS TO $\Delta k/k$ FROM CALHET, 10^{-3}

Group	Basic Case	Smeared Cases			1 Group B^2	$DB^2 = 0$	Homogeneously Shielded
		1	2	3			
1	3.563	3.775	4.336	3.520	3.621	3.829	3.579
2	4.694	4.826	5.435	4.438	4.762	4.822	4.739
3	4.037	4.137	4.535	3.695	4.086	4.101	4.101
4	3.161	3.190	3.355	2.742	3.139	3.107	3.217
5	1.880	1.885	1.929	1.579	1.833	1.790	1.914
6	0.857	0.861	0.871	0.712	0.833	0.806	0.877
7	0.270	0.275	0.282	0.228	0.264	0.256	0.286
8	0.000	0.001	0.014	0.007	0.000	0.001	0.012
9	-0.109	-0.109	-0.118	-0.100	-0.106	-0.101	-0.099
10	-0.140	-0.141	-0.131	-0.110	-0.137	-0.131	-0.133
11	-0.143	-0.144	-0.175	-0.147	-0.141	-0.134	-0.136
12	-0.090	-0.092	-0.025	-0.023	-0.088	-0.084	-0.082
13	-0.092	-0.088	-0.105	-0.091	-0.091	-0.086	-0.080
14	-0.051	-0.050	-0.028	-0.027	-0.051	-0.048	-0.032
15	-0.021	-0.017	-0.034	-0.032	-0.021	-0.019	0.011
16	0.026	0.025	0.044	0.034	0.027	0.026	0.076
17	-0.067	-0.077	-0.087	-0.076	-0.066	-0.063	0.066
18	-0.056	-0.060	-0.065	-0.059	-0.055	-0.052	0.230
19	0.034	0.030	0.074	0.058	0.033	0.032	0.200
20	-0.997	-1.075	-1.147	-0.937	-0.990	-0.940	-0.350
21	-0.327	-0.360	-0.375	-0.300	-0.321	-0.305	-0.353
22	-0.045	-0.051	-0.054	-0.041	-0.044	-0.042	-0.047
23	-0.006	-0.007	-0.008	-0.006	-0.006	-0.006	-0.003
24	-0.001	-0.001	-0.001	-0.001	0.000	-0.000	-0.001
Total	16.38	16.73	18.52	15.06	16.48	16.76	17.99

CALHET CALCULATIONS

The CALHET code, developed from the collision probability formulation of Ref. 3 was modified in two ways. First, to significantly reduce the computing time, the method used to calculate collision probabilities was changed from a Simpson's rule integration to Gaussian quadratures. The latter were specifically generated⁴ to evaluate the infinite sums of exponential integral functions which appear in the analytic expressions for collision rates in slab geometry.⁵ Secondly, the treatment of the leakage of neutrons was improved. Equivalent absorption cross sections (DB^2) were not used in calculating the optical thicknesses of the various slabs, but the collision probabilities were reduced by a nonleakage factor in both the homogeneous and heterogeneous arrays.

The cases described here were run with cross sections (representing the plates, clads, and the combined drawer and matrix region), group fluxes, adjoint fluxes and DB^2 values obtained from the heterogeneously shielded MACH-1 run. The basic case considered ten regions (the drawer and matrix on each side of the cell being combined into one region) corresponding to the central section vertically, 4.514 cm high. The group contributions to $\Delta k/k$ are detailed in Table II-6-II,

giving a total of 1.64% $\Delta k/k$. This case is an incorrect representation of the true situation as the steel and carbon above and below the central section are ignored and the infinite slab implies that the plutonium is continuous vertically, whereas there are gaps of ~ 1 cm between each 4.514 cm plutonium piece. These deficiencies were considered in three cases and the corresponding group $\Delta k/k$ values are given in Table II-6-II:

1. The steel and carbon in the top and bottom regions were smeared uniformly into all ten regions, such that the total mass in the cell (with its reduced height of 4.514 cm) was correct. This gave a heterogeneity correction of 1.67% $\Delta k/k$, indicating very little change.
2. The steel and carbon were smeared into the drawer and matrix region and into the carbon regions respectively, retaining the same total mass as above. Because of the extra diluent between fuel plates the heterogeneity increased to 1.85% $\Delta k/k$.
3. The previous case was repeated with all the atomic densities reduced in the ratio 0.817. This corresponded to spreading the cell over the full vertical pitch, and was equivalent to reducing the thickness of all the plates in the previous case.

As expected the heterogeneity decreased to 1.51% $\Delta k/k$. The fault of this case was that the atomic densities of the plutonium pieces were too low.

The best estimate of the heterogeneity would seem to be somewhere between the smeared cases 2 and 3 (say 1.6% for later consideration). The first case ignores the gaps vertically in the plutonium columns and therefore overestimates $\Delta k/k$; the second has the correct average vertical atomic density, but will underestimate $\Delta k/k$ because the individual plates have a higher 4 VN/S (ignoring any interaction between the vertically planar plates) than the diluted slabs.

To study the effect of different assumptions concerning the buckling terms, two further variations on the basic case were run. First, the one-group buckling (produced as part of MC²) was used with the group diffusion coefficients for the group DB^2 values) secondly, the DB^2 were all put equal to zero. Both gave only slight changes from the basic case although some of the group contributions changed by several percent, as seen in Table II-6-II.

DISCUSSION

At first sight the total heterogeneity for ZPR-3 Assembly 53, relative to a completely homogeneous core, would appear to be the sum of the two components:

1. the difference in the k value of the two MACH-1 homogeneous calculations using group cross sections obtained from MC² for homogeneously and heterogeneously shielded resonances (1.5% $\Delta k/k$)
2. the best estimate from CALHET for the effect of group flux variations in the plate arrangement using the group cross sections for the heterogeneously shielded resonances ($\sim 1.6\%$ $\Delta k/k$).

The CALHET contribution is mainly made up of a multiplication effect at high energies ($\sim 1.8\%$ $\Delta k/k$) together with a smaller negative amount ($\sim 0.2\%$ $\Delta k/k$) from the lower energies. This lower energy portion arises from reduced group fluxes in the fuel plates relative to the other plates, reducing the fissions and captures occurring. These reaction rates will dip in the plates because of self-shielding in the fuel plates from the total group average cross sections. In fact the self-shielding from the group capture and fission cross sections (which are energy averaged resonance parameters) has already been taken into account completely and more accurately by using the true resonance parameters provided by MC². Thus some duplication is taking place in the two contributions it is proposed to sum. However, the flux dips introduced by self-shielding from the group capture and fission cross sections will equally affect both reaction rates

in a given plate, and thus the opposite reactivity effects from the changes in the group removal (capture plus fission) and production (fission) rates will tend to cancel. The two types of fuel plate compositions are such that the thickness of the fuel multiplied by the sum of the capture plus fission cross sections for each plate are very similar, in the low energy groups. Thus the (duplicated) flux dips in the two plates will be very similar and the different ratios of captures and fissions taking place in the two types of fuel plates will not be very important.

As a test of the significance of the effect of the duplication, these low energy flux dips were emphasized by using the (higher) fuel-plate group cross sections obtained from the homogeneously shielded resonances in a repeat of the basic CALHET case, i.e., the same fluxes, adjoint, DB^2 , region thickness and atomic densities. The group $\Delta k/k$ values of this case are given in the last column of Table II-6-II. The new total effect is 1.80% compared with 1.64% $\Delta k/k$. Study of the table shows that the sums of the contributions of the first seven groups (the multiplication groups) in the two calculations differ only by about 0.03% in 1.9% $\Delta k/k$. This was expected as the cross sections in these groups were essentially the same for both cases. The difference in the totals arise mainly from groups 15-21. Examination of the fuel plate cross sections shows that these are the groups where the type of resonance shielding used is significant. Because the capture cross sections show a bigger percentage increase from heterogeneous to homogeneous shielding than do those for fission in most of these groups, the net result is a bigger change in the capture rate in going to the plate arrangement and therefore a more positive contribution to $\Delta k/k$. This is observed in Table II-6-II for these groups (11-20). It seems likely that 0.2% $\Delta k/k$ is an overestimate of the effect of the duplication.

This duplication could be overcome by combining the two heterogeneity effects into one calculation, e.g., by CALHET with a sufficient number of groups to deal adequately with all the resonance cross sections. Thus CALHET would correctly take into account self-shielding of the resonances in both the homogeneous and plate cells. The $\Delta k/k$ obtained would be the total heterogeneity effect for the assembly relative to the homogeneous situation. This apparently would require much larger computer storage than is available.

SUMMARY

MACH-1 cases for ZPR-3 Assembly 53, using homogeneously and heterogeneously shielded resonances

cross sections, have shown a difference in reactivity of 1.5% $\Delta k/k$. The MACH-1 cases were used to prepare input data and cards for CALHET calculations, which gave $\Delta k/k$ for the multiplication effect in the plate structure. Several variations of representing the assembly cell were tried in CALHET, and produced small changes in $\Delta k/k$. The significance of the buckling term was shown to be small. The best value from CALHET is about 1.6% $\Delta k/k$.

The total heterogeneity error for this assembly incurred in a calculation assuming a homogenized cell and using homogeneously shielded resonance cross sections is $(3.1 \pm 0.3) \% \Delta k/k$, where the uncertainty arises because of the difficulty in representing the true cell in CALHET and some small duplication in the method. The error in a calculation assuming a homogenized cell and using heterogeneously shielded resonance cross sections is $(1.6 \pm 0.3) \% \Delta k/k$. The quoted errors do not include uncertainties in the data

and the calculational methods used, other than the duplication discussed above.

The best k -value from ENDF/B for the as-built ZPR-3 Assembly 53, applying a shape factor, is 1.015 ± 0.003 .

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II-7. Analysis of Graphite Density Reactivity Measurements in ZPR-3 Assembly 53

S. RAMCHANDRAN and R. G. PALMER

INTRODUCTION

ZPR-3 Assembly 53 was the fourth in a benchmark series of simple-geometry, simple-composition cores designed for basic studies of plutonium systems. Although no sodium was used in Assembly 53, experiments akin to the coolant void coefficient were performed by reducing the density of graphite in various core zones without creating gross void streaming effects. Detailed analyses of the graphite reactivity measurements for the different zones were done with the following objectives in view: to investigate the adequacy of the ENDF/B data file, to check the accuracy of the calculational models, and to provide possible guidelines and assistance to future analysis of the same or similar experiments using modified versions of the existing cross section set.

DESCRIPTION OF ASSEMBLY

The core of ZPR-3 Assembly 53 consists of single drawer unit cells with $1/8$ -in. column of ZPR-3 Pu-Al fuel and $1/4$ in. column of Pu-U-Mo fuel and graphite filling the rest of the drawer (see Fig. II-7-1a). The axial and the radial reflectors are made of depleted uranium blocks.

As compared with the earlier cores of the series (Assemblies 48, 49, and 50), the real neutron spectrum

is appreciably softer (see Table II-7-I) and the adjoint spectrum is sufficiently different by virtue of the reduced ratio of uranium to plutonium (ratio ≈ 1.6 for Assembly 53 as compared with 4.5 for Assembly 48).

EXPERIMENTS ON GRAPHITE REACTIVITY CHANGE

The reference core of Assembly 53 used the higher density ZPR-6 graphite. A central zone of reduced graphite density was built into the core by replacing the ZPR-6 graphite with the lower density ZPR-3 graphite to a depth of 6 in. in each half (in a core of 12 in. half-height) for 13 drawers. Figures II-7-1b, II-7-1c, II-7-2a and II-7-2b give the details of the central and axial drawer configurations and the zone configurations. The smeared graphite density for the center zone was reduced from 1.116 to 0.979 g/cm³, a net reduction of 1.66 kg. The experimental reactivity worth of this change was (-183.9 ± 3.5) Ih. The axial zone for the graphite density change experiment was built at the interface between the core and the axial reflector. The 21 drawers shown in Fig. II-7-2b were modified by replacing the last 3 in. of ZPR-6 graphite with lower density graphite (see Fig. II-7-1c) as was done in the central zone. The graphite density for this zone changed from 1.116 to 0.955 g/cm³, corresponding to a net re-

FIG. II-7-1. Drawer Configurations for the Standard Core and Low-Density Graphite Experiment. ANL Neg. No. 113-3027.

TABLE II-7-I. COMPARISON OF REAL FLUX SPECTRA AT THE CORE CENTER FOR THE BASIC PLUTONIUM SERIES^a

Energy Interval, keV	Assemblies				
	48, %	49, %	50, %	Reference Core 53, %	Perturbed Core 53, %
>25	83.4	84.8	68.8	67.8	69.4
2-25	13.4	13.4	25.6	24.2	23.6
<2	3.2	1.8	5.6	8.0	7.0

^a In all cases the ENDF/B data file was used for the calculations.

duction of 0.59 kg. The experimental reactivity worth of this change was $-(46.9 \pm 2.4)$ lh.

CALCULATION PROCEDURES

CROSS SECTION PREPARATION

Using MC² a 24-group cross section set was prepared from the ENDF/B data file, allowing for the different resonance self-shielding conditions for each of the fuel plates in the cell. The two different cells used in the MC² calculations corresponded to the high and density graphite cases. Heterogeneity calculations were

S = SAFETY ROD C = CONTROL ROD
A = LOW DENSITY GRAPHITE
(a)
CENTER ZONE

S = SAFETY ROD C = CONTROL ROD
A = LOW DENSITY GRAPHITE
(b)
AXIAL ZONE

FIG. II-7-2. Zone Configurations. ANL Neg. No. 113-3072.

performed on the cell to obtain the equivalent homogeneous core cross sections. The one-dimensional transport code TESS with its homogenization routines was used to obtain a real flux weighted, cell-averaged section for each of the high and low density graphases. For easy identification these cross sections, henceforth, will be called COREAL-Hi-C and CO-

REAL-Lo-C for high and low carbon densities, respectively. Homogeneously shielded cross sections were used for the blanket calculations. The perturbation cross sections (i.e., the difference between COREAL-Hi-C and COREAL-Lo-C) showing the δD , $\delta \Sigma_{tr}$, $\delta \Sigma_c$, $\delta \Sigma_f$, and $\delta \Sigma_a$ are as given in Table II-7-II, the symbols having the usual meaning. These group-dependent cross sections will be useful for the interpretation of the results of perturbation analysis.

k-CALCULATIONS

For reference k -calculations, COREAL-Hi-C was used for the core region and for perturbed k -calculations, COREAL-Lo-C was used for the sections of the core region which were perturbed. A 24-group, 2-dimensional (r, z) diffusion k -calculation was done for both the reference and the perturbed cores. The solutions to the real and adjoint flux spectra for the reference core will be known as unperturbed flux spectra (ϕ, ϕ^*) and the same for the perturbed core will be known as the perturbed flux spectra (ϕ', ϕ'^*). Figure II-7-3 shows a flow diagram of the calculational procedures beginning with the cross section data file and ending with the perturbation calculations.

TABLE II-7-II. PERTURBATION CROSS SECTIONS FOR THE CENTRAL AND AXIAL ZONES^a

Group	Lower Energy	δD	$\delta \Sigma_{tr},$ 10^{-2}	$\delta \Sigma_c,$ 10^{-5}	$\delta \Sigma_f,$ 10^{-5}	$\delta \Sigma_a = \delta \Sigma_c$ $+ \delta \Sigma_f,$ 10^{-5}
1	3.68 MeV	0.233	0.76	9.52	8.37	17.89
2	2.31	0.206	1.09	9.18	7.99	17.17
3	1.35	0.198	1.22	8.96	5.42	14.38
4	821 keV	0.189	1.53	0.69	3.25	3.94
5	498	0.159	1.93	-4.64	1.36	-3.28
6	302	0.131	2.29	-3.01	0.36	-2.65
7	183	0.191	2.58	-2.27	-0.04	-2.31
8	111	0.100	2.80	-0.91	-0.15	-1.06
9	67.4	0.094	2.94	-0.71	-0.16	-0.87
10	40.9	0.091	3.01	-0.46	-0.16	-0.62
11	24.8	0.072	3.34	+0.07	-0.26	-0.19
12	15.0	0.089	3.08	-0.09	-0.21	-0.30
13	9.12	0.072	3.12	0.35	-0.35	0.0
14	5.53	0.055	3.17	0.42	-0.54	-0.12
15	3.35	0.065	3.14	0.37	-0.39	-0.02
16	2.03	0.075	3.15	0.76	-0.62	0.14
17	961 eV	0.073	3.21	2.57	0.37	2.94
18	454	0.063	3.21	5.10	5.9	11.00
19	275	0.056	3.22	6.01	4.36	10.37
20	61.4	0.058	3.29	5.88	0.16	6.04
21	13.7	0.050	3.39	-74.68	-2.17	-76.85
22	3.06	0.051	3.41	-0.40	-9.40	-9.80
23	0.683	0.055	3.45	-42.05	4.67	-37.38

^a The perturbation cross section given above is equal to (COREAL-Hi-C)-(COREAL-Lo-C).

FIG. 11-7-3. Flow Diagram of the Calculation Procedures. ANL Neg. No. 113-3026.

PERTURBATION THEORY METHODS

The methods and techniques employed here for the perturbation theory calculations are conventional,¹ and hence very few details are given. The four reactivity components from perturbation analysis are as follows: (a) absorption, which is proportional to $\phi_g'^* \delta \Sigma_{ag} \phi_g$; (b) leakage, which is proportional to $\nabla \phi_g \delta D_g \nabla \phi_g'^*$; (c) fission, which is proportional to $\chi_g \phi_g'^* \sum_g \phi_g \delta (\nu \Sigma_f)_{g'}$; and (d)

moderation, which is proportional to $\phi_g \delta \Sigma_s^{\sigma \rightarrow g'} (\phi_g'^* - \phi_g'^*)$. All the symbols have the usual meaning, except that prime refers to perturbed quantities. The net reactivity is the algebraic sum of these four components and therefore the magnitude of some of the individual components can be larger than the magnitude of the net reactivity itself. The tabulated results for the exact perturbations (*EP*) and first order perturbation (*FOP*) calculations include kk' in the denominator.

PERTURBATION CALCULATIONS

FOP calculations are done using the real and adjoint flux spectra of the unperturbed core. In *FOP* theory in the diffusion theory approximation, the question arises

as to what value of δD one should use in the calculation.^{1,2} If Σ_{trg} and Σ'_{trg} are the unperturbed and perturbed core composition transport cross sections respectively for group g , then the exact expression for δD_g is equal to

$$\frac{1}{3} \Sigma'_{trg} - \frac{1}{3} \Sigma_{trg} = \delta \Sigma_{trg} / 3 \Sigma_{trg} \Sigma'_{trg}.$$

If the perturbation is small, then $\Sigma'_{trg} \approx \Sigma_{trg}$ and

$$\delta D_g \approx \Sigma_{trg} / 3 \Sigma_{trg}^2.$$

Both these expressions were used in the present calculations and the correct use of the expression for δD_g is important while evaluating the leakage contribution to the axial zone.

Perturbation theory can be made to give the exact result for total reactivity change by using the unperturbed flux and perturbed adjoint flux, i.e., (ϕ, ϕ'^*) or vice versa, i.e., (ϕ', ϕ^*) in the calculations. *EP* theory calculations are also used here for sensitivity study of the perturbation components to changes in the real and the adjoint flux spectra. However, the resolution into components no longer has the same physical interpretation as for *FOP* theory and should not be interpreted in terms of exactness relative to the first order component. Only the net reactivity can be compared directly between the two methods. As a check for the

TABLE II-7-III. EXACT PERTURBATION CALCULATION (ϕ, ϕ'^*) FOR THE CENTRAL REGION

Group No.	Leakage, 10^{-6}	Absorption, 10^{-6}	Fission, 10^{-6}	Moderation, 10^{-6}	$\Delta k/kk'$, 10^{-6}
1	-1.34	4.780	-7.36	2.036	-1.844
2	-2.19	8.149	-9.92	8.009	4.048
3	-3.38	10.522	-10.1	6.318	3.360
4	-3.63	3.192	-7.69	-9.784	-17.912
5	-3.31	-2.877	-5.02	-15.972	-27.179
6	-2.72	-2.340	-2.92	-14.021	-22.001
7	-2.32	-1.939	-1.57	-13.471	-19.300
8	-1.77	-0.811	-0.810	-8.998	-12.389
9	-1.50	-0.625	-0.404	-8.918	-11.447
10	-1.30	-0.415	-0.198	-12.429	-14.342
11	-0.917	3.855	-0.097	-10.587	-7.746
12	-1.05	-5.413	-0.047	-15.595	-22.105
13	-0.753	0.001	-0.0229	-14.091	-14.866
14	-0.500	-0.055	-0.0111	-13.517	-14.083
15	-0.540	-0.0075	-0.0054	-14.074	-14.627
16	-0.551	0.052	-0.0026	-14.857	-15.359
17	-0.696	1.375	-0.0017	-21.956	-21.279
18	-0.481	3.977	-0.00576	-14.741	-11.251
19	-0.210	1.791	-0.0016	-8.428	-6.849
20	-0.426	1.906	-0.0013	-0.982	0.497
21	-0.061	-4.090	—	0.168	-3.983
22	-0.006	-4.598	—	0.166	-4.438
23	-0.0012	-0.0325	—	0.0	-0.1
Sum	-32.4	20.950	-46.19	-195.5	-0.02528

FIG. II-7-4. Central Real Flux Per Unit Lethargy as a Function of Lethargy. ANL Neg. No. 103-L5688.

perturbation theory computer programs, the results of *EP* theory calculations should agree with the direct *k*-calculations.

CENTRAL ZONE GRAPHITE DENSITY CHANGE CALCULATIONS

The central real flux per unit lethargy and the adjoint flux as a function of lethargy for both the unperturbed and perturbed cases are as shown in Figs. II-7-4 and II-7-5.

Tables II-7-III and II-7-IV show the group-dependent reactivity components for the two cases of *EP* calculations. Table II-7-V shows the same terms for *OP* calculations. The group dependent moderation term has the sign of the quantity $(\phi_g'^* - \phi_g'^*)$. Essen-

tially, one can look at the moderation term as being proportional to $\phi \frac{\partial \phi^*}{\partial u}$, where ϕ is the flux and $\frac{\partial \phi^*}{\partial u}$ is the lethargy gradient of the adjoint.

The largest contribution to the net reactivity for the central zone is from the moderation term. The negative sign of this net reactivity component is due to various reasons. Because of the low fertile-fissile ratio (≈ 1.6), the high energy importance variations are suppressed, producing an overall decrease of the adjoint flux as energy increases. Also, for this assembly the real flux spectrum is soft ($\approx 1/3$ of the neutrons have energy < 25 keV); therefore, the low energy flux spectra have higher relative weighting as compared to harder spectrum cores like ZPR-3 Assemblies 48 or 49. These fac-

FIG. II-7-5. Central Adjoint Flux as a Function of Lethargy. ANL Neg. No. 103-L5690.

tors contribute to the moderation component in the same direction, thus making this term strongly negative.

Table II-7-VI gives a comparison of the group breakdown of the moderation term from the two *EP* calculations with that of *FOP* calculations. The differences between the *FOP* and *EP* calculations are also given for both the cases of *EP* calculations. The differences are small compared with the net moderation term and

this is due to the perturbed flux and adjoint spectra being not much different from the unperturbed cases.

Table II-7-VII shows a comparison of both perturbation and *k*-calculations with experiment for the central and axial regions. The spread in the results of the different calculations for the central zone is only 4% and the *k*-calculations agree well with that of exact perturbation calculations. The comparison of calculation experiment is poor, the discrepancy being over 40%.

TABLE II-7-IV. EXACT PERTURBATION CALCULATION
(ϕ', ϕ^*) FOR THE CENTRAL REGION

Group No.	Leakage, 10^{-5}	Absorption, 10^{-5}	Fission, 10^{-5}	Moderation, 10^{-5}	$\Delta k/k'$, 10^{-5}
1	-1.44	4.855	-7.44	1.999	-2.026
2	-2.35	8.274	-10.0	7.742	3.666
3	-3.55	10.649	-10.2	5.377	2.276
4	-3.93	3.251	-7.80	-10.486	-18.965
5	-3.84	-2.970	-5.10	-16.448	-28.358
6	-2.92	-2.388	-2.96	-14.404	-22.672
7	-2.48	-1.978	-1.60	-13.772	-19.830
8	-1.83	-0.824	-0.823	-9.574	-13.051
9	-1.52	-0.632	-0.411	-0.511	-12.074
10	-1.28	-0.418	-0.202	-12.709	-14.609
11	-0.857	-0.116	-0.099	-10.886	-11.958
12	-0.941	-0.167	-0.048	-15.473	-16.629
13	-0.662	0.001	-0.023	-13.987	-14.673
14	-0.406	-0.054	-0.011	-13.209	-13.680
15	-0.437	-0.007	-0.006	-13.641	-14.091
16	-0.428	0.051	-0.003	-14.182	-14.562
17	-0.517	1.324	-0.002	-20.503	-19.698
18	-0.311	3.754	-0.0006	-13.375	-9.933
19	-0.105	1.646	-0.0002	-7.266	-5.725
20	-0.180	1.699	-0.00013	-1.269	0.250
21	0.010	-3.256	—	0.197	-3.049
22	0.003	-0.033	—	0.122	0.082
23	0.0006	-0.022	—	0.0	-0.021
Sum	-33.3	22.639	-46.78	-195.3	-0.02528

TABLE II-7-VI. GROUP BREAKDOWN OF THE MODERATION
TERM COMPARING FIRST ORDER AND EXACT PERTURBATION
THEORY AT THE CENTRAL REGION, $10^{-5} \Delta k/k'$

Group No.	First Order	(ϕ, ϕ^*)	Difference	(ϕ', ϕ^*)	Difference
1	1.979	2.036	-0.057	1.999	-0.020
2	7.668	8.009	-0.341	7.742	-0.074
3	3.351	6.318	-2.967	5.377	-2.026
4	-10.400	-9.784	-0.616	-10.486	+0.086
5	-16.120	-15.972	-0.148	-16.448	+0.328
6	-14.280	-14.021	-0.259	-14.404	+0.124
7	-13.660	-13.471	-0.189	-13.772	+0.112
8	-9.543	-8.998	-0.545	-9.574	+0.031
9	-9.515	-8.918	-0.597	-9.511	-0.004
10	-12.770	-12.429	-0.341	-12.709	-0.061
11	-11.020	-10.587	-0.433	-10.886	-0.134
12	-15.710	-15.595	-0.115	-15.473	-0.237
13	-14.290	-14.091	-0.199	-13.987	-0.303
14	-13.630	-13.517	-0.113	-13.209	-0.421
15	-14.120	-14.074	-0.046	-13.641	-0.479
16	-14.780	-14.857	+0.077	-14.182	-0.598
17	-21.560	-21.956	+0.396	-20.503	-1.057
18	-14.320	-14.741	+0.421	-13.375	-0.945
19	-7.972	-8.428	+0.456	-7.266	-0.706
20	-1.431	-0.982	-0.449	-1.269	-0.162
21	0.250	0.168	+0.082	0.197	+0.053
22	0.158	0.166	-0.008	0.112	+0.046
23	0.0	0.0	0.0	0.0	0.0
Total	-201.715	-195.724		-195.268	

TABLE II-7-V. FIRST ORDER PERTURBATION CALCULATION FOR THE CENTRAL REGION, $10^{-5} \Delta k/k^2$

Group No.	Leakage		Absorption	Fission	Moderation	$\Delta k/k^2(c)$	
	a	b				a	b
1	-1.544	-1.436	4.807	-7.403	1.979	-2.161	-2.053
2	-2.553	-2.352	8.196	-9.974	7.668	3.337	3.538
3	-3.990	-3.655	10.6005	-10.192	5.351	1.770	2.097
4	-4.397	-4.006	3.226	-7.767	-10.40	-19.34	-18.952
5	-4.039	-3.670	-2.901	-5.077	-16.12	-28.14	-27.773
6	-3.310	-3.011	-2.367	-2.952	-14.28	-22.91	-22.607
7	-2.830	-2.571	-1.962	-1.592	-13.66	-20.04	-19.790
8	-2.146	-1.958	-0.821	-0.820	-9.543	-13.33	-13.142
9	-1.825	-1.667	-0.633	-0.409	-9.515	-12.38	-12.224
10	-1.590	-1.453	-0.421	-0.201	-12.77	-14.98	-14.847
11	-1.116	-1.025	-0.118	-0.098	-11.02	-12.35	-12.257
12	-1.259	-1.150	-0.170	-0.048	-15.71	-17.19	-17.079
13	-0.917	-0.845	0.001	-0.023	-14.29	-15.23	-15.155
14	-0.605	-0.562	-0.055	-0.011	-13.63	-14.30	-14.258
15	-0.658	-0.609	-0.008	-0.005	-14.12	-14.79	-14.740
16	-0.677	-0.622	0.053	-0.003	-14.78	-15.41	-15.349
17	-0.853	-0.784	1.392	-0.002	-21.56	-21.02	-20.954
18	-0.580	-0.537	4.019	-0.0006	-14.32	-10.88	-10.839
19	-0.249	-0.231	1.806	-0.0002	-7.972	-6.415	-6.397
20	-0.496	-0.460	1.917	-0.0001	-1.431	-0.0101	0.046
21	-0.0743	-0.069	-4.137	—	0.2501	-3.961	-3.956
22	-0.00676	-0.006	-0.046	—	0.1582	0.1054	0.106
23	-0.00143	-0.0001	-0.033	—	0.0	-0.03443	-0.041
Sum	-35.571	-38.500	22.347	-46.577	-200.223	-259.665	-263.641

^a Used $\delta D = \delta \Sigma_{tr}/3\Sigma'_{tr}$.

^b Used $\delta D = \delta \Sigma_{tr}/3\Sigma_{tr}\Sigma'_{tr}$.

^c For the two versions of δD , all components except leakage are the same.

TABLE II-7-VII. COMPARISON BETWEEN CALCULATIONS AND EXPERIMENTAL RESULTS, ZPR-3 ASSEMBLY 53, $10^{-5} \Delta k/k'$

Region	Method	Leakage	Absorption	Fission	Moderation	Total Calculation	C/E ^(a)
Center	1st Order, $\delta D = \frac{\delta \Sigma_{tr}}{3\Sigma_{tr}^2}$	-35.6	22.3	-46.6	-200.2	-259.7	1.48
	1st Order, $\delta D = \frac{\delta \Sigma_{tr}}{3\Sigma_{tr}\Sigma_{tr}'}$	-38.5	22.3	-46.6	-200.2	-263.0	1.50
	Exact, Perturbed Flux	-33.3	22.6	-46.8	-195.3	-252.8	1.44
	Exact, Perturbed Adjoint	-29.6	21.0	-46.2	-195.2	-252.8	1.44
	k-Calculations	—	—	—	—	-253.4	1.44
Axial Edge	1st Order, $\delta D = \frac{\delta \Sigma_{tr}}{3\Sigma_{tr}^2}$	-59.8	2.46	-5.4	-8.4	-71.1	1.58
	1st Order, $\delta D = \frac{3\Sigma_{tr}}{3\Sigma_{tr}\Sigma_{tr}'}$	-64.8	2.46	-5.4	-8.4	-76.1	1.69
	Exact, Perturbed Flux	-56.5	2.35	-5.4	-7.5	-67.0	1.49
	Exact, Perturbed Adjoint	-56.3	2.46	-5.4	-7.8	-67.0	1.49
	k-Calculations	—	—	—	—	-66.8	1.49

^a $\frac{\text{Calculated value}}{\text{Experimental value}}$

TABLE II-7-VIII. EXACT PERTURBATION CALCULATION (ϕ, ϕ'^*) FOR THE AXIAL REGION

Group No.	Leakage, 10^{-5}	Absorption, 10^{-5}	Fission, 10^{-5}	Moderation, 10^{-5}	$\Delta k/k'$, 10^{-5}
1	-1.926	0.628	-0.932	0.879	-1.351
2	-3.582	0.999	-1.210	1.850	-1.943
3	-5.785	1.221	-1.196	2.627	-3.133
4	-6.354	0.367	-0.862	-0.195	-7.044
5	-4.316	-0.357	-0.554	-0.303	-5.530
6	-2.679	-0.307	-0.316	-0.127	-3.429
7	-2.261	-0.260	-0.167	0.090	-2.598
8	-1.972	-0.108	-0.085	0.387	-1.778
9	-1.746	-0.084	-0.041	0.297	-1.574
10	-1.749	-0.055	-0.020	-0.267	-2.091
11	-1.611	-0.015	-0.010	-0.931	-2.567
12	-2.151	-0.021	-0.005	-0.741	-2.918
13	-1.949	0.0001	-0.002	-0.784	-2.735
14	-1.548	-0.0062	-0.001	-0.907	-2.462
15	-1.769	-0.0008	-0.0005	-0.472	-2.242
16	-2.603	0.005	-0.0002	-2.275	-4.873
17	-3.521	0.130	-0.0002	-4.047	-7.438
18	-1.710	0.392	—	-1.301	-2.619
19	-0.707	0.074	—	-0.538	-1.171
20	-2.003	0.167	—	-0.793	-2.629
21	-0.158	-0.413	—	0.024	-0.547
22	-0.016	-0.005	—	0.020	-0.001
23	-0.004	-0.003	—	0.0	-0.007
Total	-56.500	2.350	-5.402	-7.50	-66.80

AXIAL ZONE GRAPHITE DENSITY CHANGE CALCULATIONS

A set of calculations identical to those for the central zone was done for the axial zone and Table II-7-VII

gives a comparison of the different calculations with experiment. For purposes of comparison, the group breakdown of the different reactivity components for only one case of EP calculation is given in Table II-7-VIII. The leakage component is the largest contributor to the net reactivity for the axial zone. For a detailed understanding of the group dependence of the moderation term, reference is made to Fig. II-7-6, where the region averaged adjoint flux as a function of lethargy is given. The different signs associated with the group dependent moderation term is the same as ($\phi_g'^* - \phi_g'$) and this is explained by referring to Fig. II-7-6.

Comparison of *FOP* calculations in Table II-7-VII shows the importance of the correct use of the value for δD in the leakage term for the axial zone. The use of the approximate value for δD overestimates the leakage term by about 9%.

CENTRAL REACTIVITY WORTH CALCULATIONS

Central perturbation worths for the different isotopes were calculated using the DEL perturbation option of the MACH-1 program. For the MACH calculations both the homogeneously shielded cross sections and COREAL-Hi-C were used. For the central perturbation worth calculations of the isotopes, homogeneously shielded cross sections were used. Table II-7-IX gives a comparison of calculation with experiment. In general, the calculation overestimates the central worths by 30 to 40%, and the discrepancy is about same range as found in the central zone graphite reactivity calculations. The COREAL-Hi-C cross sec-

FIG. II-7-6. Axial Adjoint Flux as a Function of Lethargy. ANL Neg. No. 103-L5689.

tions predict central worths larger than the homogeneous cross sections and this is mainly due to the relative softness of the COREAL-Hi-C real flux spectrum.

CONCLUSIONS AND RECOMMENDATIONS

The worth of graphite for the central zone is overestimated by $\approx 40\%$. For the central zone, the largest contributor to the net reactivity is the moderation component. The difficulties in calculating this component can be seen by an examination of the group dependence of this term as given in Tables II-7-III through II-7-V. As the moderation contribution from each group is proportional to small differences in the adjoint flux between groups, the contribution is highly sensitive to cross section uncertainties affecting the adjoint spectrum. The error in each group would also be proportional to errors in the scattering transfer cross section for graphite.

The shape of the adjoint flux spectrum is determined

TABLE II-7-IX. COMPARISON OF CENTRAL PERTURBATION CALCULATIONS WITH CENTRAL REACTIVITY MEASUREMENTS

Isotope	MACH-1 Calculation, Ih/kg		Experiment, Ih/kg	C/E	
	Homogeneous ^a	COREAL-Hi-C ^b		Homogeneous ^a	COREAL-Hi-C
Pu-239	815.3	930.1	672.2 ± 3.0^c	1.21	1.38
U-238	-97.5	-122.3	-91.4 ± 2.3^c	1.07	1.34
U-235	599.7	647.9	512.2 ± 6.2^c	1.17	1.27
C	123.5	162.6	114.2 ± 1.2	1.08	1.42

^a Homogeneously shielded cross sections were used for the core as well as the perturbation worth calculations.

^b Heterogeneously shielded, real flux weighted, and cell averaged cross sections (COREAL-Hi-C) were used for the core calculation. However, homogeneously shielded cross sections were used for the perturbation worth calculations.

^c The experimental values given correspond to zero sample thickness.

predominantly by the energy dependence of the $\eta \left(= \frac{\nu}{1 + \alpha} \right)$ for core composition. Since the uranium-to-plutonium ratio in this assembly is only 1.6, η of Pu-239 dominates. The most important cross section uncertainty in the existing ENDF/B data file is in the Pu-239 alpha value below 25 keV and the recent measurements of R. Gwin et al.³ the analysis by T. A. Pitterle et al.⁴ substantiate this conclusion. Since a third of the real flux is below 25 keV in this assembly, the above cross section uncertainty is all the more important for the present core. Modifications to the existing 24-group cross section set are being made to incorporate R. Gwin's et al. Pu-239 alpha values below 25 keV. Qualitatively, this should result in a reduction of the magnitude of the fission and moderation terms and an increase in the absorption term, all of which are in the direction of better agreement with experiment.

Recent measurements and analyses of the total cross section of graphite by A. E. Profio et al.⁵ and E. A. Straker⁶ indicate that the total cross section for carbon in the existing ENDF/B library is too low by $\approx 10\%$ for the 3.5 MeV resonance. Also, the age calculated for epithermal neutrons in graphite using the ENDF/B graphite cross sections⁷ gave a value of 280 cm² as compared to the measured value⁸ of ~ 308 cm². This difference is attributed to the large value of the epithermal scattering cross section for graphite (≈ 4.85 b) used in the ENDF/B data file. The smaller calculated value of age using the ENDF/B data file indicates that the leak-

age term is overestimated in this particular soft spectrum core. This correction might significantly improve the accuracy of the calculation for the axial zone. Improved agreement of the calculation with experiment for the axial region might also be obtained by doing transport calculations in place of diffusion calculations, and also by changing the Pu-239 alpha values below 25 keV as was done for the central zone.

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II-8. Description of and Some Experimental Data from ZPR-3 Assemblies 55 and 55A

L. LESAGE, W. R. ROBINSON, J. M. GASIDLO and J. E. POWELL

INTRODUCTION

ZPR-3 Assemblies 55 and 55A were constructed in October and November of 1968 in order to measure the spectrum averaged capture-to-fission ratio (i.e., alpha) for Pu-239 by the null reactivity technique. A description of Assemblies 55 and 55A as well as some of the experimental data not reported elsewhere, are contained in this paper. Previous U-235 and Pu-239 null-reactivity alpha measurements were made on ZPR-9 Assembly 24.⁽¹⁾ The alpha measurement results for both assemblies are contained in Ref. 2 and a description of the Assembly 24 measurements is included in Ref. 3.

DESCRIPTION OF ASSEMBLIES

Assembly 55 was cylindrical in geometry and contained a central plutonium-fueled test zone with a k_{∞} of unity, surrounded by concentric buffer, plutonium fueled driver, uranium fueled driver, and reflector regions. The null zone contained ZPR-3 plutonium fuel (with approximately 4.5% Pu-240), depleted uranium, and graphite plates in a one-drawer-cell arrangement. Assembly 55A was identical to Assembly 55 except that a small null zone containing ZPR-3 plutonium plates which had approximately 22% Pu-240 was substituted at the center of Assembly 55 null zone. The buffer region contained

FIG. II-8-1. Half 1 of ZPR-3 Assembly 55. ANL Neg. No. 113-3020.

depleted uranium and graphite plates and the plutonium driver contained ZPR-3, SEFOR and ZPPR plutonium plates and graphite.* Because of the 430 kg limitation on total plutonium in ZPR-3, it was necessary to include some driver drawers containing enriched uranium rather than plutonium along the periphery of the driver. The plutonium driver region had a two-drawer-cell arrangement as discussed below. The loading arrangements for the assemblies, including positions of the control and safety rods, are shown in Figs. II-8-1 and II-8-2, where the drawers which were reloaded to the 22% Pu-240 material for Assembly 55A are indicated by the dashed region at the center of the test zone. The 22% Pu-240 zone had an axial length of 45.7 cm and is also shown by the dashed lines in Fig. II-8-3, which is a plan view of the assembly. The arrangement of the axial and radial reflectors is also shown in Fig. II-8-3. Regional radii and the average homogeneous concentrations are included in Table II-8-I, and a list of

* The ZPR-3 plates are 98.9 w/o Pu and 1.1 w/o Al, and the ZPR-3 Pu is 95.0 w/o Pu-239, 4.55 w/o Pu-240 and 0.44 w/o Pu-241. The SEFOR plates are 19.6 w/o Pu, 77.9 w/o U and 2.5 w/o Mo, and the SEFOR Pu is 91.4 w/o Pu-239 and Pu-241 8.16 w/o Pu-240. The ZPPR plates are 28.0 w/o Pu, 69.5 w/o U and 2.5 w/o Mo, and the ZPPR Pu is 86.8 w/o Pu-239, 11.7 w/o Pu-240 and 1.5 w/o Pu-241. All three types of plates are in stainless steel.

both calculated and measured parameters for the assemblies are contained in Table II-8-II.

The specific drawer loading patterns for each region of both assemblies are as follows, with plate thicknesses given in inches; e.g., 1/8 D implies a 1/8 in. thick depleted uranium plate. (C, E, and SS refer to

FIG. II-8-2. Half 2 of ZPR-3 Assembly 55. ANL Neg. No. 113-3019.

FIG. II-8-3. Elevation View of Reactor—ZPR-3 Assemblies 55 and 55A. ANL Neg. No. 113-3022.

TABLE II-8-I. ATOMIC CONCENTRATIONS AND DIMENSIONS FOR ZPR-3 ASSEMBLIES 55 AND 55A

Region/Assemblies	Atomic Densities, 10 ²¹										Outer Radius, cm	
	Pu-239	Pu-240	Pu-241	U-235	U-238	C	Mo	Al	Fe	Ni		Cr
Test zone/55 ^(a)	1.069	0.051	0.005	0.003	15.380	37.269	—	0.111	6.177	0.839	1.896	25.94
Test zone/55A ^(a)	0.851	0.255	0.053	0.028	12.842	33.313	—	0.119	7.149	0.071	2.194	13.97
Buffer/55 & 55A				0.035	16.927	38.728	—		5.643	0.677	1.398	30.76
Inner driver/55 & 55A	2.002	0.199	0.024	0.012	5.123	48.670	0.434	0.055	7.927	0.952	1.964	47.26
Outer driver/55 & 55A				2.224	0.159	41.775			21.426	2.572	5.307	48.64
Nickel reflector/55 & 55A									5.643	75.245	1.398	63.97
Depleted reflector/55 & 55A				0.081	39.984				4.558	0.547	1.129	78.86

^a Atom densities as loaded after adjustment to null composition.

TABLE II-8-II. LIST OF MEASURED AND CALCULATED PARAMETERS FOR ZPR-3 ASSEMBLIES 55 AND 55A

	Assembly ^a	
	55	55A
Calculated k_{∞} (assembly)	1.0339	1.0333
Calculated k_{∞} (test zone)	0.9836	0.9943
Measured critical mass, kg		
Pu-239	375.9	371.0
Pu-241	3.9	5.0
U-235	35.5	35.4
Total	415.3	411.4
Total volume (excluding reflector), liters	680	680
Volume of central core, liters	193	56
Core height (excluding reflector), cm	91.4	91.4
Axial reflector thickness, cm	15.2 steel 15.2 dep. U	15.2 steel 15.2 dep. U
Calculated reflector savings, cm	16.3 (core)	16.3 (core)
Calculated effective delayed neutron fractions	3.53×10^{-3}	3.53×10^{-3}
Calculated prompt neutron lifetime, sec	6.51×10^{-7}	6.54×10^{-7}
Calculated $\lambda h/\%$ reactivity, $\lambda h/\% \delta k/k$	897.8	896.7
Calculated perturbation denominator ^b	7.67×10^5	8.51×10^5
Calculated Doppler temperature coefficient between 293 and 1100°C, $\delta k/k \cdot ^\circ\text{C}$	-9.20×10^{-6}	
Calculated temperature expansion coefficient between 20 and 545°C, fuel expansion only, $\delta k/k \cdot ^\circ\text{C}$	-7.2×10^{-6}	

^a Cross sections used in the calculations are defined in the footnote of Table II-8-III.

^b The values are normalized such that $\sum_i (\nu \Sigma_{fi})_i \phi_i = 1$ and $\sum_i \chi_i \phi_i^* = 1$ at the center of the core.

graphite, enriched, and stainless steel plates, respectively.)

Test Zone Assembly 55	$\frac{1}{8}$ C, * $\frac{1}{8}$ D, $\frac{1}{8}$ C, $\frac{1}{8}$ D, $\frac{1}{8}$ C, $\frac{1}{8}$ D, $\frac{1}{8}$ C, $\frac{1}{8}$ ZPR-3 Pu, $\frac{1}{8}$ C, $\frac{1}{8}$ D, $\frac{1}{8}$ C, $\frac{1}{16}$ C, ($\frac{1}{16}$ Row: $\frac{3}{8}$ D and $\frac{1}{8}$ C), $\frac{1}{8}$ D, $\frac{1}{8}$ C, $\frac{1}{8}$ D, $\frac{1}{8}$ C.
Test Zone Assembly 55A	$\frac{1}{8}$ C, $\frac{1}{8}$ D, $\frac{1}{8}$ C, $\frac{1}{8}$ D, $\frac{1}{8}$ C, $\frac{1}{8}$ D, $\frac{1}{8}$ C, $\frac{1}{8}$ ZPR-3 Pu (22% 240), $\frac{1}{8}$ C, $\frac{1}{8}$ D, $\frac{1}{8}$ C, $\frac{1}{8}$ D, $\frac{1}{32}$ D, $\frac{1}{32}$ C, $\frac{1}{8}$ C, $\frac{1}{16}$ C, $\frac{1}{4}$ SS frame.
Buffer Assemblies 55 & 55A	$\frac{1}{8}$ C, $\frac{1}{8}$ D, $\frac{1}{8}$ C, $\frac{1}{8}$ D, $\frac{1}{8}$ C, $\frac{1}{8}$ D, $\frac{1}{8}$ C, $\frac{1}{8}$ D, $\frac{1}{4}$ C, $\frac{1}{8}$ D, $\frac{1}{8}$ C, $\frac{1}{8}$ D, $\frac{1}{8}$ C, $\frac{1}{8}$ D, $\frac{1}{8}$ C.
Driver—Type A† Assemblies 55 & 55A	$\frac{1}{8}$ C, $\frac{1}{4}$ SEFOR Pu, $\frac{1}{2}$ C, $\frac{1}{4}$ ZPPR Pu, $\frac{1}{2}$ C, $\frac{1}{4}$ SEFOR Pu, $\frac{1}{8}$ C.
Driver—Type B Assemblies 55 & 55A	$\frac{1}{2}$ C, $\frac{1}{4}$ ZPPR Pu, $\frac{5}{8}$ C, $\frac{1}{8}$ ZPR-3 Pu, $\frac{1}{2}$ C.
U-235 Fuel Drawer Assemblies 55 & 55A	$\frac{1}{8}$ C, $\frac{1}{8}$ SS, $\frac{1}{4}$ C, $\frac{1}{8}$ SS, $\frac{1}{4}$ C, $\frac{1}{16}$ SS, $\frac{1}{8}$ E, $\frac{1}{16}$ SS, $\frac{1}{4}$ C, $\frac{1}{8}$ SS, $\frac{1}{4}$ C, $\frac{1}{8}$ SS, $\frac{1}{8}$ C.

CALCULATIONS

Most of the calculations were made in cylindrical geometry with 25 energy groups using a one-dimensional diffusion code MACH1.⁽⁴⁾ The regional radii were calculated to preserve the cross sectional area of each region. The first order perturbation option in the MACH1 code was used in the calculation of the control rod worths and Doppler coefficient. The axial expansion coefficient was calculated, for each region individually, in one-dimensional slab geometry by the method described in Ref. 5.

Homogeneous cross sections for the central test zone were generated by the MC² code⁶ using ENDF/B

† Type A and B driver drawers were used alternately in vertical columns throughout the driver region with T₁ drawers used in the central column.

TABLE II-8-III. COMPARISON OF CALCULATED REAL AND ADJOINT CENTERLINE FLUXES FOR THE REFERENCE CASES (INFINITE MEDIUM) AND FOR ZPR-3 ASSEMBLIES 55 AND 55A^a

Group	Group Lower Energy Limit, keV	Centerline Real Flux (Reference) Assembly 55	Centerline Real Flux ZPR-3 Assembly 55	Centerline Adjoint Flux (Reference) Assembly 55	Centerline Adjoint Flux ZPR-3 Assembly 55	Centerline Real Flux (Reference) Assembly 55A	Centerline Real Flux ZPR-3 Assembly 55A	Centerline Adjoint Flux (Reference) Assembly 55A	Centerline Adjoint Flux ZPR-3 Assembly 55A
1	3679	1.62	1.58	6.03	6.09	1.57	1.54	6.04	6.09
2	2231	2.65	2.58	5.32	5.40	2.60	2.54	5.38	5.44
3	1353	3.64	3.57	4.66	4.76	3.64	3.56	4.73	4.81
4	821	4.99	4.91	3.94	4.07	5.00	4.92	4.01	4.12
5	500	7.50	7.42	3.74	3.86	7.50	7.43	3.77	3.89
6	302	9.14	9.08	3.54	3.62	9.13	9.08	3.55	3.64
7	183	9.49	9.46	3.37	3.43	9.47	9.45	3.39	3.44
8	111	8.99	8.98	3.24	3.27	8.96	8.97	3.26	3.28
9	67.4	8.60	8.62	3.13	3.13	8.59	8.62	3.15	3.16
10	40.9	8.12	8.16	3.05	3.05	8.11	8.15	3.08	3.07
11	24.8	7.15	7.19	3.04	3.02	7.18	7.23	3.06	3.05
12	15.0	6.26	6.31	3.09	3.06	6.19	6.23	3.10	3.08
13	9.12	5.32	5.37	3.19	3.16	5.32	5.37	3.20	3.18
14	4.31	5.93	5.99	3.35	3.32	5.95	6.01	3.35	3.32
15	2.61	3.07	3.10	3.71	3.67	3.13	3.15	3.69	3.66
16	2.03	1.24	1.26	4.04	3.99	1.27	1.28	3.99	3.96
17	1.23	2.03	2.06	4.22	4.17	2.08	2.09	4.16	4.13
18	0.961	0.776	0.788	4.51	4.45	0.792	0.798	4.41	4.38
19	0.583	1.25	1.27	4.75	4.70	1.27	1.28	4.65	4.61
20	0.275	1.22	1.25	5.22	5.16	1.24	1.26	5.08	5.03
21	0.101	0.735	0.761	5.48	5.42	0.746	0.764	5.27	5.22
22	0.0373	1.231	0.243	5.70	5.63	0.222	0.231	5.42	5.35
23	0.0290	0.0101	0.0107	2.86	2.83	0.00988	0.0104	3.16	3.11
24	0.0226	0.0143	0.0151	3.61	3.57	0.0135	0.0143	3.87	3.81
25	0.0137	0.0116	0.0124	3.20	3.16	0.0110	0.0118	3.23	3.18

^a The calculations were made in cylindrical geometry with the MACH1 one-dimensional diffusion code. Homogeneous cross sections generated by the MC² code using ENDF/B data were corrected for spatial self-shielding in the U-238 resonance region using slab geometry equivalent theory, and individual plate cross sections were re-weighted with fluxes from an S_n unit cell calculation.

data. The resonance-region cross sections for U-238 were then corrected for spatial self-shielding using slab-geometry equivalence theory, and the individual plate cross sections were re-weighted with fluxes from a 25-group S_n unit-cell calculation, giving the final flux and volume-weighted homogenized unit-cell cross sections. The homogeneous MC² cross sections, without equivalence theory or S_n corrections, were used in the driver region in the calculations.

Doppler difference cross sections as a function of σ_p for Pu-239 and U-238 have been generated at Argonne as described in Ref. 7. The Doppler difference cross sections with the appropriate equivalent σ_p values (i.e., the equivalent σ_p for U-238 in the driver was approximately 22 b and that for Pu-239 was approximately 100 b in both the driver and the test zone) were used in the perturbation subroutine to calculate the Doppler coefficients. Only the U-238 gained in the SEFOR and ZPPR plutonium plates was considered in the Doppler calculations; however,

the calculated Doppler coefficient was still negative as indicated in Table II-8-II. The calculated axial temperature expansion coefficient, as listed in Table II-8-II, was also negative so that the combined prompt temperature coefficient for the assembly was strongly negative.

SPECTRUM

The calculated real and adjoint spectra at the center of the test zones for both assemblies are included in Table II-8-III. The spectrum at the center of the Assembly 55 test zone was also measured with the proton recoil spectrometer. The results were collapsed to the same broad group structure as the multigroup calculations, and the calculated and measured curves were normalized to the same total area under the curve (for groups 4 through 20). The results are compared in Fig. II-8-4, and the agreement between the measured and calculated spectra is reasonably good.

FIG. II-8-4. Comparison between Measured and Calculated Spectra. ANL Neg. No. 113-3064.

TABLE II-8-IV. MEASURED SAFETY AND CONTROL ROD WORTHS

Rod Numbers	Rod Function	Type ^a Loading	Mass of Pu-239 and Pu-241, kg	Worth for 10-in. Movement, % $\delta k/k$
1, 9	Safety	1	2.48850	0.1995
2, 8, 10	Safety	1	2.48850	0.1976
3	Safety	2	2.48850	0.2090
4	Safety	3	2.48850	0.2368
7	Safety	4	2.48850	0.2429
5	Control	5	1.81362	0.1059
6	Control	6	1.26489	0.0637

^a Refer to Fig. II-8-5.

CONTROL AND SAFETY RODS

The control and safety rod calculations were made using the first order perturbation subroutine to the MACH1 code by uniformly perturbing the control rod material throughout the driver region. The locations of the individual control and safety rods are shown in Figs. II-8-1 and II-8-2. Rods 5 and 6 were control rods and the other eight were safety rods. In order to achieve the required 1.5% $\delta k/k$ shutdown in the eight safety rods, the plutonium loading of each rod was increased to the limit of 2.5 kg per rod,

and the plutonium loading in the drawers above and below the rods were decreased so that the average plutonium density in the driver remained approximately constant. The rods were loaded entirely with ZPR-3 plutonium metal plates rather than the ZPPR or SEFOR alloy plates used in the driver drawers, and the plutonium plates in some of the rods were zoned either axially or radially. The measured worths of each of the individual rods is given in Table II-8-IV, and the loading of each of the individual rod types is shown in Fig. II-8-5. In the case of both the axially and the radially zoned rods, the side or the end with the heavier plutonium loading was nearer the assembly center. The calculated worth of the uniformly loaded safety rods, numbers 1, 2, 8, 9 and 10, was 0.198% $\delta k/k$ as compared to the average of the measured worths of about 0.20% $\delta k/k$. The results in Table II-8-IV give an indication of the magnitude of the rod worth change achieved by radial and axial zoning of the plutonium.

ACTIVATION RESULTS

The measured Pu-239 activation shapes along the radius at the assembly midplane and along the centerline are shown in Figs. II-8-6 and II-8-7. The one-dimensional diffusion theory calculated radial

FIG. II-8-5. Safety and Control Rod Drawer Loading Types. ANL Neg. No. 113-3070.

curve is also shown in Fig. II-8-6 for comparison. A cosine fit to the axial data is included in Fig. II-8-7, giving an approximate value for the measured reflector savings for the test zone of 16.5 ± 1.0 cm. This is in good agreement with a value of approximately 16.3 cm obtained from a two-dimensional diffusion calculation for this assembly.

REACTIVITY MEASUREMENTS

Table II-8-V contains a list of the measured and calculated reactivity worths for Assemblies 55 and 55A. The measurements were made using an axial oscillator and a calibrated autorod as described in Ref. 2. The worth of a $2 \times 2 \times 2$ in. stainless steel box, loaded with plutonium, depleted uranium and carbon plates in the same arrangement as the test zone, was measured. A small amount of material was added or removed from the plates of a particular material in the box, and its worth was remeasured. The material worth was determined as the reactivity change of the box divided by the mass of the material added or removed from the box. For a strong resonance ab-

FIG. II-8-6. Radial Pu-239 Fission Rate Measurement. ANL Neg. No. 113-3024.

sorber, such as U-238, it is expected that the worths measured in this manner should be less than the cell-averaged worth due to resonance self-shielding in the plates. For the other materials it is expected that the measured worth is approximately equal to the cell-averaged worth, and therefore comparable to the calculated worths, as discussed below. Because of the

precision attainable with the oscillator-autorod system, it was possible to remove only a small fraction of the material from the sample box and thus the spectral perturbation was small. This was not true for the plutonium measurement in Assembly 55A, where a full 1 x 2 x 1/8 in. piece was removed, since this was the only size available. The Pu-239 and U-238 worth values are derived from the plutonium and depleted measurements using the calculated worths of the minor isotopic constituents which are also listed in Table II-8-V. Small samples of Li-6 and aluminum were attached inside empty 2 x 2 x 2 in. sample boxes and oscillated with the axial oscillator to obtain the worths for these materials.

The calculated reactivity worths in Table II-8-V were obtained using first order perturbation theory and fluxes from the one-dimensional diffusion calculation. The equivalence theory and S_n corrected cross sections (see Calculations section) were used for the constituent material, plutonium, depleted uranium, carbon, and stainless steel, so that the calculations should give the cell-average worths. In order for one to have any confidence in the calculated per-

TABLE II-8-V. MEASURED AND CALCULATED REACTIVITY WORTHS IN ASSEMBLIES 55 AND 55A,^(a) 1h/kg

Material	Assembly 55		Assembly 55A	
	Measured	Calculated	Measured	Calculated
Plutonium	150.7 ^b		126.5 ^c	
Pu-239	157.1 ^d	141.3	155.5 ^d	143.1
Depleted U	-7.58		-7.48	
U-238	-7.84 ^e	-9.82	-7.74 ^e	-9.85
Carbon	-18.82	-6.90	-15.69	-7.84
Stainless Steel	-6.23	-4.81	-5.61	-4.82
Aluminum	-7.63 ^f	-8.48		-8.45
Li-6	-148.2 ^f			
Pu-240		4.40		4.88
Pu-241		240.4		
U-235		125.5		128.1

^a The uncertainty in the measurements for all results were approximately 1-2%.

^b Sample contained 95.1% Pu-239, 4.5% Pu-240, and 0.4% Pu-241.

^c Sample contained 72.8% Pu-239, 22.0% Pu-240, 4.6% Pu-241, and 0.6% Pu-242.

^d Derived from plutonium values using calculated worths for Pu-240 and Pu-241.

^e Derived from depleted values using calculated worths of U-235.

^f Aluminum, Li-6 and Li-7 samples were measured in a 2 x 2 x 2 in. cavity at the core center rather than in the normal plate-type sample box.

FIG. II-8-7. Axial Pu-239 Activation Fission Rate, ZPR-3 Assembly 55. ANL Neg. No. 113-3073.

turbation denominator, the calculations should predict the flux distribution in the core. As may be seen from Figs. II-8-6 and II-8-7 the radial flux shape seems to have been calculated reasonably accurately and the axial reflector savings assumed in the calculations agree with the measurements, as discussed in the previous section. It is felt, however, that the perturbation denominator calculation should still be considered rather uncertain due to difficulty in adequately treating the geometric complexity of the zoned geometry.

The measured U-238 worths are smaller than the calculated worths as expected, probably due to resonance self-shielding in the U-238 plates. The results for Pu-239 indicate that the calculated worths are approximately 10% smaller than the measurements. There are several possible causes for the discrepancy, and these will not be discussed; however, the discrepancy is interesting since it has been observed on other assemblies⁸ that the calculated worth of Pu-239 is generally too large by 5 to 30%.

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II-9. Calculational Studies in Support of the Fast Flux Test Facility (FFTF) Critical Experiments on ZPR-3 and ZPPR

A. TRAVELLI, A. J. ULRICH, D. MENEGHETTI and J. C. BEITEL

INTRODUCTION

One effort of the Reactor Physics Division in support of the FFTF Critical Experiments implemented on the ANL facilities consists of planning the experiments according to the reactor design needs of the Pacific Northwest Laboratory (PNL) and in performing computational studies to evaluate the results of the experiments. Long range planning, as well as detailed and operational planning, are carried out by personnel of Argonne's Reactor Physics Division in close cooperation with personnel of the Pacific Northwest Laboratory.

The joint planning effort during this reporting period included preparation of general plans and detailed descriptions of required experimental work. Both the planning and the evaluation of the experiments have concentrated on the PNL concept of a central axial test zone surrounded by an annular core.¹ This concept, in the course of the progressive evolution of the core design as developed at PNL and at Westinghouse, has replaced in emphasis the previous design concept of an axially-split conical core² toward which earlier efforts of the FFTF critical experiments program had been directed.³ To some extent, however, the evaluation effort has included the study of experiments which were completed during previous reporting periods and which concerned the axially-split conical core design concept.

Calculational studies have been conducted in support of experiments performed on ZPR-3 Assembly 51 in a series of criticals denoted as ZPR-3 Assemblies 52a through 52f, on ZPR-3 Assemblies 56A and 56B,

and on the Zero Power Plutonium Reactor (ZPPR) assemblies denoted as ZPPR/FTR-1 and ZPPR/FTR-2. (FTR designates Fast Test Reactor.) The experiments are described in detail in the Reactor Development Program Progress Reports of the Argonne National Laboratory from October 1967 to July 1969.

Assembly 51 was a plutonium-fueled cylindrical core with a nickel-rich reflector. The core composition approximated the driver-core compositions of FFTF if the dilutions due to controls, safety rods, and shims of the reactor design concept are disregarded.

The series of criticals 52a through 52f were scoping experiments with annular or axially-split cores having central or gap sodium regions, respectively, and with the same core composition as Assembly 51.

Assembly 56A was somewhat similar to Assembly 51, because it has a reflected cylindrical core; its core composition, however, approximated the FFTF compositions including the uniform dilution of test loops and control channels which had been disregarded in Assembly 51. The configurations of Assembly 56A and of Assembly 56B differed from each other only in the reflector, which in Assembly 56A was made up in part by steel and in part by the composition of the FFTF reflector, while the latter composition alone was used in the reflector of Assembly 56B. Assembly 56B is also referred to as Assembly ZPR-3/FTR-1 of the resumed Phase B of the critical experiments program.

As the critical program progressed, it came to require larger and larger critical assemblies, comparable in size to FFTF (~1025 liters). The program was then transferred from ZPR-3 to ZPPR, which was at

that time the only facility available for the study of large plutonium-fueled critical assemblies. Assembly ZPPR/FTR-1 was the first assembly built after the transfer, and had the same plate arrangements and geometry as Assembly ZPR-3/FTR-1. The main difference between the two assemblies was due to the different void fractions present in the two facilities, which caused ZPPR/FTR-1 to have a slightly smaller density, larger volume, and heavier critical mass than ZPR-3/FTR-1.

Assembly ZPPR/FTR-2 was designed primarily to study the effect of a peripheral control region on a clean model of FFTF. It was obtained by expanding radially the core of ZPPR/FTR-1 until it became approximately equivalent in volume to the FFTF core, and by compensating the resulting increase in reactivity with a B₄C-rich control annulus which surrounded the core and simulated the FFTF peripheral control system.

All of the criticals described above are part of Phase B of the FFTF critical experiments program. Phase A critical experiments and calculations have been described previously.⁴⁻⁶ Phase A consisted of experiments on a modified ZPR-3 Assembly 48, intended primarily for poison control studies at the center of the reactor and at the core-reflector interface positions; the modified assembly is referred to as ZPR-3 Assembly 48A. Phase B, in addition to the experiments on the assemblies mentioned above, will include experiments performed on an additional assembly, FTR-3, essentially similar to FTR-2 but having a zoned core. Phase B will be followed by Phase C, which will consist of engineering mockup experiments and is to be planned toward finalizing important design parameters.

The calculational studies in support of the critical

TABLE II-9-I. NOMINAL COMPOSITION OF FTR-1 ON ZPPR AS USED IN THE PREPARATION OF CROSS SECTION SET 29004, atoms/b-cm

Nuclide	Core	Axial Reflector	Radial Reflector
O	0.01442066	—	—
Na	0.00860629	0.013358	0.006495
Fe	0.01215677	0.008418	0.007163
Cr	0.00259267	0.002394	0.002048
Ni	0.00129094	0.019264	0.046215
Mn	0.00022017	—	—
Mo	0.00032997	—	—
U-235	0.00001334	—	—
U-238	0.00597782	—	—
Pu-239	0.00128254	—	—
Pu-240	0.00017469	—	—
Pu-241	0.00002376	—	—
C	0.00110393	—	—

experiments have largely consisted of scoping estimates and in pre-analysis of the assemblies to assist in the direction of the critical experiments. Post-experimental analysis was generally limited to some experiments whose evaluation was particularly important in connection with the planning and the interpretation of future experiments.

MULTIGROUP CROSS SECTIONS

A twenty-nine-group cross section set, designated as Set 29004, has been prepared for use in the neutron calculations concerning FTR-1 and FTR-2. As pointed out in the introduction, a critical assembly corresponding to the FTR-1 configuration was built on ZPR-3 and labeled as ZPR-3 Assembly 56B. However, since the FTR-1 configuration was originally conceived for construction on ZPPR, and since FTR-2 was to be built on ZPPR with the same core composition as FTR-1, the nominal core and reflector compositions chosen for the preparation of the cross sections correspond to the FTR-1 compositions for ZPPR as they were anticipated before the construction of FTR-1. These compositions are shown in Table II-9-I.

The cross sections were prepared by means of the MC² code⁷ in conjunction with the basic data of ENDF/B Category I. The neutron spectrum of the P-1 core fundamental mode obtained in a 2100 group calculation was used to collapse the multigroup core cross-sections. The cross sections for the radial and axial reflectors were separately obtained through the calculation of the corresponding 2100 group spectra in *P*₁ calculations and for the compositions that correspond, respectively, to the radial and axial reflectors with a distributed Pu-239 fission source and no leakage.

The heterogeneous option of the MC² code for plane geometry was used to approximate the effects of resonance self-shielding which is present in the thin-plate configuration of the drawers of the critical assemblies. The rather complicated intradrawer structures were approximated as binary cells, because this is the only cell structure for which the current heterogeneous MC² option can be applied directly.

A version of Set 29004, labeled Set 29004.2, was prepared for use in conjunction with the MACH-1⁽⁸⁾ code. This set superseded an earlier similar set, labeled Set 29004.1, in which a code fault had caused the introduction of slight discrepancies in the (*n*,2*n*) cross sections.

The upper energy boundary of Set 29004 is 10 Mev and corresponds to zero lethargy. The lower energy boundary of each of its twenty-nine groups is 1 in Table II-9-II with the corresponding lethargy value.

ZPR-3 ASSEMBLY 51

ZPR-3 Assembly 51⁽⁹⁾ was a cylindrical reactor with a 33.5 cm average radius and an 86.5 cm height. The average compositions of its zones are listed in Tables II-9-III and II-9-IV. The radial configuration of the reference loading of the assembly is shown in Fig. II-9-1, while the axial configuration is shown in Fig. II-9-2. The critical mass, corrected for temperature difference and for incomplete insertion of the control rods amounted to 212.08 kg of fissile elements.

Several calculations concerning ZPR-3 Assembly 51 have been reported in Ref. 3, for comparison with experimental results. They include calculation of: (1) the critical mass of the reactor in various approximations and with various cross-sections sets; (2) the spatial variations of Pu-239 and Pu-238 fission detector responses; (3) spatially dependent fine-energy neutron spectra; (4) safety rod worths; (5) distributed and axial sodium worths; (6) Doppler effects. Other calculations concerning the same assembly and performed during the current reporting period are reported below.

CALCULATION OF THE SPATIAL DEPENDENCE OF THE WORTH OF A SMALL ANNULAR SAMPLE OF B-10

The worth of a small annular sample of B-10 was measured experimentally¹⁰ at various locations along the core centerline and along a radius 1.5 in. from the mid-plane of the core of Assembly 51. The length of the sample was 1.688 in., its outer diameter was 0.391 in., the thickness was 0.010 in., and its weight was 0.4484 g. The sample contained 0.033 g of B-11 and 0.383 g of B-10.

The calculation of the sample worth as a function of radial position was based on first-order perturbation theory. The DEL subroutine of the MACH-1 code was used for this calculation in conjunction with the cross section Set 29001.⁽³⁾

A correction of the results of the perturbation calculations obtained in this way was applied according to two different methods. This correction is meant to take into account, at least partially, the modification that the introduction of the sample into the reactor imposes on the flux distribution.

The first method follows closely the procedure described in Ref. 11. The sample was considered to be an infinitely long thin hollow cylinder and the flux impinging on its surface was assumed to be isotropic and to have the same spectrum and amplitude on every point of the surface. This method assumes a flat distribution of the flux outside the sample and yields group-dependent self-shielding factors that take into account only the depression of the flux inside the

TABLE II-9-II. GROUP BOUNDARIES OF CROSS SECTION SET 29004

Group	Lower Energy Limit	Lethargy
1	6.065 MeV	0.5
2	3.679	1.0
3	2.231	1.5
4	1.353	2.0
5	820.8 keV	2.5
6	497.9	3.0
7	302.0	3.5
8	183.2	4.0
9	111.1	4.5
10	67.38	5.0
11	40.87	5.5
12	24.79	6.0
13	15.03	6.5
14	9.119	7.0
15	5.531	7.5
16	3.355	8.0
17	2.035	8.5
18	1.234	9.0
19	748.5 eV	9.5
20	454.0	10.0
21	275.4	10.5
22	167.0	11.0
23	101.3	11.5
24	61.44	12.0
25	37.27	12.5
26	13.71	13.5
27	5.043	14.5
28	1.855	15.5
29	0.6826	16.5

sample. In general, the flux depression outside the sample is underestimated by this method while the flux depression inside the sample is overestimated because of the assumption that the impinging flux is isotropic. The two approximations tend to balance each other.

The second method that was used to correct the results of the perturbation calculations to take into account the effect of the finite size of the sample is based on the application of the 1D-SNARG⁽¹²⁾ discrete ordinate transport program. The code was used to calculate the depression of the flux due to the presence of the boron sample both within the sample and in the region that surrounds it. A high-order modified¹³ single Gaussian quadrature was used in order to facilitate the calculation of the thin-absorber effects. The sample was approximated by a spherical shell whose radius was chosen in such a way that the spherical shell could have the same thickness, density, and mass as the sample.

Table II-9-V summarizes the results of these calculations. The first column of the table indicates the radial distance from the reactor center line (the radius of the core-reflector interface along the experimental

TABLE II-9-III. CORE COMPOSITIONS FOR ASSEMBLY 51, 10^{24} atoms/cm³

Drawer Type	Pu-239 + 241	Pu-240 + 242	U-235	U-238	Mo	Na	C	O	Al	Fe	Cr	Ni	Mn	Si
Core Type A ^(a)	0.001972	0.000170	0.000012	0.005703	0.000224	0.010262	0.003113	0.011479	0.000110	0.01528	0.003803	0.001664	0.000159	0.000186
Core Type A* ^(a)	0.001498	0.000175	0.000018	0.008306	0.000431	0.008244	0.003113	0.014093	—	0.01600	0.003530	0.001544	0.000147	0.000173
Cell Average	0.001736	0.000172	0.000015	0.007005	0.000328	0.009253	0.003113	0.01279	0.000055	0.01564	0.003666	0.001604	0.000153	0.000180
Control Rod	0.001990	0.000162	0.000015	0.007144	0.000197	0.010123	0.002746	0.008241	0.000124	0.016047	0.003992	0.001747	0.000167	0.000196
Safety Rod	0.002990	0.000329	0.000032	0.014635	0.000796	0.004039	0.000916	0.005362	0.000026	0.017198	0.003829	0.001675	0.000160	0.000188
Drawer above Safety Rod	0.000902	0.000119	0.000012	0.005705	0.000224	0.010340	0.004150	0.017212	—	0.016122	0.003561	0.001558	0.000149	0.000175
Drawer below Safety Rod	0.000597	0.000056	0.000013	0.005829	0.000208	0.010340	0.004150	0.017212	—	0.015803	0.003482	0.001524	0.000145	0.000171

^a A—drawers in even-numbered matrix columns; A*—drawers in odd-numbered matrix columns.

TABLE II-9-IV. REFLECTOR COMPOSITIONS FOR ASSEMBLY 51, 10^{24} atoms/cm³

Location	Na	Ni	Fe	Cr	Mn	Si	C
Axial Reflector	0.010321	0.028926	0.010426	0.002583	0.000182	0.000126	
Radial Reflector I							
Zone A	0.004156	0.056349	0.007637	0.001878	0.000228	0.000092	
Zone B	0.004115	0.056338	0.007539	0.001854	0.000227	0.000091	
Zone C	0.004156	0.056335	0.007511	0.001847	0.000226	0.000091	
Radial Reflector II	—	0.000494	0.074860	0.001153	0.000560	0.000055	0.000561

traverse was 35.9 cm); the second and third column indicate, respectively, the experimental results obtained with the loading of the 2-P row in normal position or rotated by 90 deg as in the 1-P row; the fourth column indicates the results of the direct perturbation calculations; the fifth and sixth column indicate, respectively, the results of the perturbation calculations after correction by the first or by the second method discussed above to take into account the flux depression caused by the sample.

The experimental data have a statistical uncertainty of 60 lh/kg. The effect of the heterogeneous plate structure on the experimental results may be considerably larger than this value, however, and the difference between the data reported in the second and in the third column of Table II-9-V provides an indication of the magnitude of this effect.

The results of the direct perturbation theory calculations overestimate considerably the worth of the boron sample as obtained experimentally. The agreement between calculations and experiments improves when the correction for self-shielding is included with either of the two methods that were considered. It is interesting to note that the two methods give results very close to each other. This is confirmed by the comparison of the group-dependent shielding factors of the two methods, which are listed in Table II-9-VI for the groups of cross section Set 29004 in which the factors are appreciably different from unity.

CALCULATED WORTH OF A BORON CARBIDE BLADE LOCATED AXIALLY

The reactivity change due to insertion of a B₄C blade along the axis of ZPR-3 Assembly 51, as performed in one of the experiments,¹⁴ was calculated in one-dimensional diffusion theory both by perturbation methods and by direct k-calculations.

The blade was assumed to extend through the whole core and through part of the axial reflectors, with size 0.5 x 2 x 36 in. and a weight of 1262.2 g of natural

The composition of the blade material was assumed to be 13.67 w/o B-10, 63.31 w/o B-11, and 23.02 w/o C.

FIG. II-9-1. Reference Core for Assembly 51. ANL Neg. No. 103-2694.

The 29-group cross section Set 29001 was used in the calculations in combination with the MACH-1 program in cylindrical geometry. The cross sections of B-11 were approximated by carbon cross sections. The geometrical shape of the blade was simulated by a cylinder of equal cross-sectional area, and a rather dense mesh distribution was chosen for the region inside the cylinder and for the region immediately surrounding it (~3 mm mesh spacing). The core material was homogenized with the composition reported in Table II-9-III. The transverse leakage was assumed to be constant over all regions and equal to the core transverse leakage, which was evaluated by means of axial calculations in plane geometry ($B_x^2 = 7.17 \times 10^{-4} \text{ cm}^{-2}$).

The reactivity change following replacement of a rod containing only homogenized empty-drawer materials with the B₄C blade was found: (a) by means of perturbation calculations using the unperturbed

FIG. II-9-2. Equivalent Cylindrical Loading for Reference Core of Assembly 51. ANL Neg. No. 103-2725 Rev. 1.

core as the reference state, and (b) by means of direct k -calculations. The results were converted to inhours by using the equivalence $1\% \Delta k/k \approx 1036$ Ih.⁽¹⁵⁾ Table II-9-VII shows the comparison of the final results of these calculations with the experimental results.

ZPR-3 ASSEMBLIES 52a THROUGH 52f

The Assembly 52 series of critical assemblies was conceived and built¹⁶⁻¹⁸ to study various geometrical configurations conceived during the evolution of the FFTF core design. These configurations included axially split cylindrical cores and annular cores with a central test zone. The critical assemblies of the Assembly 52 series are described below, as built on ZPR-3:

52a: Same as Assembly 51 except that the total nominal 30 cm thick radial reflector is nickel-containing, that the depleted drawers next to

the spiked safety rods are normal drawers, and that the control rods are repositioned so as not to fall into the gap-position in subsequent assemblies of this series.

52b: Same as Assembly 52a except that it has a 3 x 3 sodium column region located along the central axis throughout the entire assembly.

52c: Same as Assembly 52b except that it has a 5 x 5 sodium column with the corners of core material retained; i.e., a 5 x 5 column minus the 4 corner columns.

52d: Same as Assembly 52a except that it has a 3 x 11 drawer rectangular sodium gap located axially throughout the entire assembly. The core contains additional safety spiking to insure sufficient rod worths.

52e: Same as Assembly 52d except that the tangular sodium gap is now 3 x 15 drawers; i.e., up to the radial reflector.

TABLE II-9-V. COMPARISON OF CALCULATED AND EXPERIMENTAL RADIAL WORTHS OF B-10 IN ZPR-3 ASSEMBLY 51

Radial Distance from Reactor Center Line, cm	Experimental Worth, Ih/kg		Calculated Worth, Ih/kg		
	2-P Row Normal	2-P Row Rotated 90 deg	Diffusion Theory Perturbation	Self-shielding Included, Method No. 1	Self-shielding Included, Method No. 2
0.	-9647.	-9695.	-10940.	-10381.	-10537.
2.54	-9597.	-9652.	-10859.	-10304.	-10470.
6.35	-9193.	-9274.	-10508.	-9965.	-10117.
12.70	-7939.	-8091.	-9288.	-8782.	-8959.
15.24	-7276.	-7445.	-8718.	-8221.	-8354.
17.78	-6574.	-6748.	-8078.	-7583.	-7701.
20.32	-6024.	-6112.	-7449.	-6944.	-7068.
25.40	-4710.	-4763.	-6475.	-5842.	-5969.
30.48	-3988.	-4037.	-6514.	-5449.	-5614.
33.02	-3643.	-3907.	-7378.	-5826.	-6030.
35.56	-3439.	-3835.	-7773.	-5807.	-5719.
38.10	-2733.	-3380.	-5481.	-4132.	-4075.
40.64	-1901.	-2540.	-3229.	-2495.	-2468.
43.18	-1113.	-1619.	-1902.	-1506.	-1357.
45.72	-573.	-902.	-854.	-701.	-699.
50.80	-112.	-189.	-179.	-158.	-157.
57.15	129.	65.	-31.	-28.	-28.6
63.50	148.	144.	—	—	—
76.20	0	0	—	—	—

TABLE II-9-VII. B₄C BLADE WORTHS, Ih

Perturbation Theory	k-Calculations	Experiment
-1199	-994	-905

52f: Same as Assembly 52c except that it has a 7 x 7 drawer sodium region having the 3 corner drawer-columns at each of the four corners of the sodium region retained as core columns. The core contains additional safety rod spiking (as in 52d and 52e).

Some computational methods were developed and applied to calculate the reactivities and the flux distributions of these rather complicated geometries in simple and yet fairly accurate ways. The methods are described in detail in Papers II-36 and II-37. The main results obtained through these methods and through the application of cross section Set 29601 are summarized in Table II-9-VIII.

ZPR-3 ASSEMBLIES 56A AND 56B

ZPR-3 Assembly 56B^(19,20) corresponds to Assembly I,⁽²¹⁾ which is now labeled as Assembly FTR-1 of the resumed Phase B of the critical program. ZPR-3 Assembly 56A was built immediately before Assembly 56B. The two assemblies were identical, with the exception of a difference in composition in part of the

TABLE II-9-VI. FLUX DEPRESSION AND SELF-SHIELDING FACTORS IN SMALL B-10 REACTIVITY SAMPLE IN ZPR-3 ASSEMBLY 51 USING THE 1-D SNARG PROGRAM

Energy Group	Lower Energy Limit, eV	In Core Composition			In Reflector Composition			Self-Shielding by Formula ^c
		Depression ^a	Self-Shielding ^b	Product	Depression ^a	Self-Shielding ^b	Product	
13	9119.	0.94	1.00	0.94	0.93	1.00	0.93	0.95
14	4307.	1.00	1.00	1.00	0.96	1.01	0.97	0.93
15	2613.	0.99	0.99	0.98	0.95	1.01	0.96	0.92
16	2035.	0.91	0.99	0.90	0.89	1.00	0.89	0.91
17	1234.	0.91	1.00	0.91	0.89	1.02	0.90	0.89
18	961.	0.92	0.99	0.91	0.87	1.01	0.88	0.88
19	583.	0.89	1.00	0.89	0.86	1.01	0.87	0.86
20	275.	0.87	1.00	0.87	0.83	1.01	0.84	0.83
21	101.	0.84	0.99	0.83	0.77	1.00	0.77	0.73
22	29.0	0.84	0.95	0.79	0.68	0.96	0.65	0.70
23	17.6	1.10	0.91	1.01	0.61	0.90	0.55	0.59
24	6.48	0.78	0.84	0.66	0.54	0.83	0.45	0.53
25	3.93	1.02	0.78	0.80	0.49	0.74	0.36	0.40
26	1.44	0.50	0.61	0.31	0.44	0.62	0.28	0.32
27	0.87	0.51	0.48	0.25	0.41	0.48	0.20	0.24
28	0.53	0.52	0.40	0.21	0.40	0.40	0.16	0.19
29	0	0.53	0.28	0.15	0.39	0.28	0.11	0.13

ratio of flux level at sample surface to flat flux level distant from sample.

^a Ratio of average flux level in sample to flux level at outer surface of sample.

^c Self-shielding ratios obtained by application of the formula of Ref. 11.

TABLE II-9-VIII. CALCULATED EFFECTIVE MULTIPLICATION CONSTANTS OF THE ZPR-3 ASSEMBLY 52 SERIES

Assembly No.	Experimental Fissile Critical Mass, kg	Calculated k_{eff}
52a	200.7	1.007 ^a
52b	235.8	1.004 ^a
52c	280.0	1.003 ^a
52d	275.2	1.000 ^b
52e	282.0	0.998, ^b 1.002 ^c
52f	309.2	1.000 ^a

^a Two-dimensional r - z calculation.

^b Two-dimensional x - y calculation with the use of an effective axial buckling (see Paper II-37).

^c One-dimensional slab calculation with the use of a buckling simulation (see Paper II-36).

radial reflector. This difference was due to operational reasons and did not affect in any appreciable way the neutronic properties of the assemblies, so that most of the computations performed in connection with FTR-1 on ZPR-3 apply equally well to both assemblies. The core loading of Assembly 56B, which is practically identical to the loading of Assembly 56A, is shown in Fig. II-9-3. The axial cross section of the core is shown in Fig. II-9-4. The assembly consisted of a 91.59-cm-high cylindrical core

surrounded by a 27.9-cm-thick axial reflector and nominally 15-cm-thick radial reflector regions. The compositions of the core, of the axial reflector, and of the radial reflectors are given in Table II-9-IX. The corresponding plate arrangements are shown in Fig. II-9-5. The outer reflector of Assembly 56B had the same composition as the inner reflector. The outer reflector of Assembly 56A was composed of iron loaded directly into the matrix; the composition of this outer reflector had atomic densities identical to those in the radial reflector II of ZPR-3 Assembly 51, as given in Table II-9-IV.

CALCULATION OF THE CRITICAL MASS

The critical mass of FTR-1 on ZPR-3 was calculated and reported²¹ before the start of the loading of the assembly on the facility. The predicted value was 335 kg for the fissile mass (Pu-239 + Pu-241 + U-235), which correspond to 375 kg of plutonium. The experimental critical mass²⁰ was determined to be 331.94 kg of fissile material, corresponding to 372.4 kg of plutonium.

CALCULATION OF REACTION RATE TRAVERSES

The radial variations of Pu-239 fission, U-238 and B-10 capture detector responses were calculated for

HALF 1

HALF 2

LOADING 56-17

S=SAFETY ROD C=CONTROL ROD N=NEUTRON LEVEL DETECTOR T=SOURCE TUBE DRAWERS

FIG. II-9-3. Critical Loading for Assembly 56B of ZPR-3. ANL Neg. No. 103-2896.

Fig. II-9-4. Equivalent Cylindrical Loading for Assembly 56B of ZPR-3. ANL Neg. No. 103-2902.

TABLE II-9-IX. COMPOSITIONS IN ZPR-3 ASSEMBLY 56B, 10^{-2} atoms/b-cm

Nuclide	Core			Control and Safety Drawers	Reflector			
	A-Drawer	B-Drawer ^a	Cell Average		Axial	Radial		
						Zone 1	Zone 2	Zone 3
Na	1.076	0.6609	0.8685	0.6546	1.346	0.6535	0.6628	0.6523
Fe	1.037	1.710	1.374	1.949	0.8825	0.7613	0.7621	0.7834
Cr	0.2579	0.2406	0.2493	0.3480	0.2188	0.1882	0.1890	0.1943
Ni	0.1129	0.1053	0.1091	0.1523	1.948	4.744	4.769	4.772
Mn	0.01077	0.01005	0.01041	0.01453	0.01810	0.02936	0.02869	0.02891
Si	0.01264	0.01179	0.01222	0.01706	0.01072	0.01127	0.01288	0.01341
Pu-238	0.00004	0.00008	0.00006	0.00007				
Pu-239	0.08857	0.1771	0.1328	0.1708				
Pu-240	0.01177	0.02354	0.01766	0.02268				
Pu-241	0.001672	0.003345	0.002509	0.003058				
Pu-242	0.000172	0.000345	0.0002585	0.000304				
U-235	0.001246	0.001457	0.001352	0.001419				
U-238	0.5762	0.6617	0.6190	0.6426				
Am	0.00008	0.0001639	0.0001229	0.000171				
Mo	0.02283	0.04566	0.03425	0.04403				
O	1.497	1.541	1.519	1.253				
C	0.2075	0	0.1038	0				

^a B-type drawers (two fuel columns per drawer) are located in even-numbered ZPR-3 matrix columns.

U_3O_8
Na_2CO_3
Pu-U-Mo
Na
U_3O_8
Na
Fe_2O_3
Pu-U-Mo
Na
U_3O_8
Fe_2O_3
Pu-U-Mo
Fe_2O_3

Core Two-Drawer Cell

Na	Ni	Na	Ni	Na
----	----	----	----	----

Axial Reflector Drawer Cell

Ni
Na
Ni
Ni
Ni
Na
Ni

Radial Reflector Drawer Cell

FIG. II-9-5. Core and Reflector Drawer Cells for Assembly 56B of ZPR-3. *ANL Neg. No. 103-11964.*

The axial leakage was simulated by a DB² absorber. The transverse buckling ($6.202 \times 10^{-4} \text{ cm}^{-2}$) was obtained through a series of cylindrical and slab calculations converging by iteration to a pair of cal-

Two reaction rate traverses were computed for each of the three detector types from the neutron flux distribution. The calculation of the first reaction rate traverse used the 29-group cross sections obtained from a 2100-group fundamental mode $w_0(\mathbf{r})^2$ flux averaging for the core composition at criticality.

a flux was normalized so that the reaction rate obtained in this calculation was unity at the core center. Calculation of the second reaction rate traverse used the 29-group cross sections obtained from a 2100-group fundamental mode MC² flux-averaging for the reflector composition with zero buckling. The same flux, with the same normalization, was used as in the calculation of the first reaction rate traverse.

The computed radial responses for the three types of detectors are compared in Fig. II-9-6 with the experimental values reported in Ref. 22. The experimental data are indicated by crosses, the reaction rates obtained in the first calculation (core spectrum averaging) are indicated by the dotted curves labeled "1", and the reaction rates obtained in the second calculation (reflector spectrum averaging) are indicated by the dotted curves labeled "2." Thus, type-1 curves are valid close to the core center, and type-2 curves are valid in the regions well inside the reflector. In the region close to the core-reflector interface, and on both sides of it, the spectrum in the reactor changes rapidly and somewhat irregularly from the core spectrum to the reflector spectrum, and the results of more accurate calculations involving space-dependent cross sections would be expected to fall somewhere between curves 1 and curves 2.

The calculated curves shown in Fig. II-9-6 were not corrected for the irregular matrix contour that made the core of Assembly 56B different from a perfect cylinder, nor for heterogeneity effects. These effects are estimated to shift the calculated curves outwards by a distance between 1 and 2 cm in the region close to the core-reflector interface. The results shown in Fig. II-9-6 indicate good agreement between experiments and calculations. The fission rate of Pu-239 is slightly underestimated by the calculations at the edge of the core, and overestimated in the reflector; the largest relative discrepancy is approximately 6%. The calculation of the fission rate traverse in U-238 falls approximately within the experimental accuracy of the measurements. The calculation of the B-10 capture rate traverse shows qualitatively the measured spatial behavior, but overestimates the peak value in the reflector by approximately 12%.

The spread between the calculated curves 1 and 2 shows clearly that the calculations of the fission traverses of Pu-239 are very sensitive to the resonance self-shielding and to the ultrafine spectrum chosen in collapsing the fission cross sections of Pu-239 over the relatively fine energy structure of Set 29004.

The axial variations of the same three detector responses in Assembly 56B have also been calculated in a completely similar manner except that the neutron flux distribution was obtained from a one-di-

FIG. II-9-6. Radial Reaction Rate Traverses in ZPR-3 Assembly 56B. ANL Neg. No. 113-2075.

mensional slab transport theory solution in the S_4 approximation. The transverse buckling ($1.258 \times 10^{-3} \text{ cm}^{-2}$) and value of k (0.998) were consistent with the previous radial calculation. The computed axial responses for the three types of responses are compared in Fig. II-9-7 with the experimental value reported in Ref. 23.

The U-238 experimental axial traverse shows a slight depression at the midplane of the reactor. At this position, which is the mating plane of the two halves, the columns of plates constituting the core loading are interrupted by the stainless steel drawer end plates. A corresponding gap was not introduced into the analytical model.

The calculated curves were based on a solution which assumed simply that the outer edge of the axial reflector was bare, thus causing the calculated detector responses to rapidly approach zero at that

FIG. II-9-7. Axial Reaction Rate Traverses in ZPR-3 Assembly 56B. ANL Neg. No. 113-2431.

boundary. Actually the scattering of neutrons by the stainless steel matrix structure outside the reflector is complicated by axial neutron streaming up the empty matrix tubes. This effect is apparent in the calculated detector responses which are too low near the outer edge of the reflector for Pu-239 fission and B-10 capture, both of which are more sensitive to the neutron spectrum in the reflector than is the U-238 fission.

Otherwise agreement between the calculated and the measured responses was within 7% for the axial variations.

CALCULATED WORTHS OF EDGE DRAWERS

Calculations of the worths of edge drawers in ZPR-3 Assembly 56B were run and their results were compared with later experimental data.

The calculations were performed by means of diffusion program MACH-1 in cylindrical geometry and by using the 29-group cross section Set 29004.2. The reactor sizes and compositions were those anticipated²⁴ before the construction of the assembly and they differ slightly from the actual values used in the experiment.

The transverse buckling of the assembly was chosen to be the same for all groups and for all regions, and was found as the difference between the material buckling of the core region and the transverse buckling needed to achieve criticality in an axial calculation in plane geometry. The calculations yielded $B_g^2 = 6.25 \times 10^{-4} \text{ cm}^{-2}$, and the radial calculations made with this value converged to a critical core radius of 47.27 cm.

The reactivity change following replacement of a ring of reflector material with an equivalent homogeneous ring of A-drawer material and B-drawer material was calculated in perturbation theory. The compositions assumed for the drawers are given in Table II-9-X, and differ slightly from the compositions used in the actual experiment and reported in Table II-9-IX.

The worth of the three materials in consideration (A-drawer material, B-drawer material, and reflector material) at any given radius R was obtained by calculating the real and adjoint fluxes of a reactor with a core radius equal to R , and by averaging the perturbation results of the two mesh intervals enclosing the core-reflector interface. This modified perturbation procedure for evaluating the worths of edge drawers appears to be considerably more accurate than direct perturbation calculations based on the critical configuration, because the shifting of the core-reflector interface causes a large change in both the real and the adjoint fluxes.

The numerical results obtained for various radii

TABLE II-9-X. ASSUMED COMPOSITIONS IN CORE DRAWERS OF ZPR-3 ASSEMBLY 56B, 10^{24} atoms/cm³

Isotope	A-drawer	B-drawer
O	0.0156687	0.0143435
Na	0.0112391	0.0066722
Fe	0.0093696	0.0159309
Cr	0.0026067	0.0026892
Ni	0.0013477	0.0013389
Mo	0.0002289	0.0004578
U-235	0.0000129	0.0000148
U-238	0.0057968	0.0066442
Pu-238	0.0008897	0.0017794
Pu-240	0.0001212	0.0002423
Pu-241	0.0000164	0.0000329
C	0.0022901	0.0000073
Mn	0.0002298	0.0002283

TABLE II-9-XI. EDGE WORTHS OF AN A-DRAWER, Ih/fissile kg

Radius, cm	Direct Pert. Calc.	Modified Pert. Calc.	Experiment
45	38.29	41.85	43.97
46	35.12	38.91	41.31
47	33.28	36.22	38.64

TABLE II-9-XII. EDGE WORTHS OF A B-DRAWER, Ih/fissile kg

Radius, cm	Direct Pert. Calc.	Modified Pert. Calc.	Experiment
45	51.04	52.13	51.17
46	47.10	48.73	48.51
47	44.01	45.60	45.84

in the modified perturbation method are shown in Fig. II-9-8 and in Tables II-9-XI and II-9-XII, where they are compared with the experimental results and with the direct perturbation results based on the critical radius calculated by MACH-1.

CALCULATION OF WORTH TRAVERSES OF B-10

The axial variations of the worth of a small sample of B-10 in ZPR-3 Assembly 56B have been calculated. The flux distribution was obtained using the 29-group cross-section Set 29004.2 in one-dimensional slab diffusion theory. The MACH-1 code was used in the calculations.

The radial leakage was simulated by a DB^2 absorber. The transverse buckling ($1.298 \times 10^{-3} \text{ cm}^{-2}$) was obtained through a series of cylindrical and slab calculations converging by iteration to a pair of calculations corresponding to the same k and to transverse bucklings adding up to the material buckling ($1.939 \times 10^{-3} \text{ cm}^{-2}$). The value of k obtained through this procedure was 0.988.

Two worth traverses were computed from the real and adjoint neutron flux distributions of the unperturbed reactor.

A first calculation of the B-10 worth at each position was obtained by means of first order perturbation theory as applied by the DEL option of the MACH-1 code. The 29-group B-10 cross sections used in this calculation were obtained from a 2100-group fundamental mode MC^2 flux averaging for the core composition at criticality.

A second and improved calculation of the B-10 worth traverse was obtained by applying a correction for self-shielding to the results obtained from perturbation theory. Group-dependent self-shielding factors were derived for the B-10 sample worth in the same way in which they were derived for Assembly

51, which is described above. The sample was considered to be an infinitely long thin hollow cylinder with an isotopic non-space-dependent neutron flux impinging on its surface. For each energy group the self-shielding factor was obtained by use of the method described in Ref. 11.

The computed axial worth traverses are compared with the experimental values²⁵ in the upper half of Fig. II-9-9, where the experimental data for B-10 as a function of axial position are indicated by crosses. The value at the center of the assembly is normalized to unity. The worth computed in the first calculation (first-order perturbation theory) is indicated by the dotted curve labeled "1." The worth computed in the second calculation (first-order perturbation theory modified for self-shielding) is indicated by the dotted curve labeled "2." Both calculated curves are also normalized to unity at the center of the assembly. By comparison of curve 1 with curve 2 it can be seen that the effect of self-shielding on the curve shape is not negligible especially near the reflector where the spectrum is relatively soft. The unnormalized values of experimental and computed worth are compared in Table II-9-XIII.

Radial variations of the worth of a small sample of B-10 in ZPR-3 Assembly 56B have also been calculated in a completely similar manner, except that the real and adjoint neutron flux distributions were obtained from a one-dimensional cylindrical-diffusion

FIG. II-9-8. Worth of Core Edge Drawer in Assembly 56B. ANL Neg. No. 118-3098.

FIG. II-9-9. B-10 Worth Traverses in ZPR-3 Assembly 56B. ANL Neg. No. 113-3122.

calculation. The traverse buckling ($6.406 \times 10^{-4} \text{ cm}^{-2}$) and the value of k (0.988) were consistent with the previous axial calculation.

The normalized experimental and computed radial B-10 worth traverses are compared with each other in the lower half of Fig. II-9-9. The unnormalized values are given in Table II-9-XIV.

COMPUTATIONAL STUDY OF THE EFFECT OF RESIDUAL Na_2CO_3 ON SODIUM-VOIDING EXPERIMENTS

The core compositions of fast critical assemblies considered in connection with the FFTF Critical Experiment Program may contain some sodium carbonate. This material is normally used in critical assemblies in order to achieve sodium and oxygen concentrations very close to the concentrations desired for these elements without disturbing the concentrations of other essential materials.

The use of sodium carbonate plates often provides the experimenter with a considerable advantage for obtaining the proper concentrations in the core. This was the case, for instance, with the core compositions of FTR-1 and FTR-2 of the Resumed Phase B of the FFTF Critical Experiment Program. The advantage is usually an overriding factor in the choice of the compositions, but is accompanied by some disadvantages. The inconvenience of substituting carbon for

oxygen in a core was investigated in the FARET program²⁶ where it was determined that the oxygen to carbon atom ratio for leakage and scattering equivalence is 0.87. Since oxygen is present in the FTR core compositions, an obvious and convenient procedure consists in reducing the oxygen concentration so that the sum of the oxygen concentration and of 87% of the carbon concentration in the core adds up to the desired oxygen concentration.

Another possible disadvantage due to the presence of sodium carbonate in the core is the fact that it is not possible to void entirely the sodium without disturbing seriously the concentration of other materials which contribute substantially to the core reactivity. Therefore, the results of an ideal experiment in which all and only the sodium was removed from a core containing some sodium in the form of elemental sodium and some sodium in the form of sodium carbonate must be inferred by extrapolation from the results obtained by removing all the elemental sodium but not the sodium carbonate. It is generally assumed that the uncertainty introduced by this type of correction is negligible when compared with other uncertainties of the measurements. A series of calcula-

TABLE II-9-XIII. AXIAL B-10 WORTH TRAVERSE

Positions (Inches above Midplane)	Specific Worths, lh/kg		
	Experiment, (± 34)	Perturbation Theory	Perturbation Theory Corrected for Self-Shielding
0	-6692	-7782	-7163
1	-6677	-7749	-7131
2	-6557	-7654	-7044
4	-6213	-7281	-6700
6	-5729	-6689	-6152
8	-5085	-5927	-5447
10	-4365	-5060	-4638
12	-3634	-4175	-3802
14	-2906	-3396	-3034
15	-2604	-3100	-2717
15.5	-2457	-2991	-2583
16	-2335	-2921	-2471
17	-2253	-2877	-2322
18	-2113	-3046	-2313
19	-1980	-3127	-2251
20	-1721	-2825	-1984
22	-1174	-1819	-1252
24	-607	-907	-627
26	-330	-336	-2
28	-130	-55	-36

ns were performed during this reporting period in order to assess the validity of this assumption.

In an experiment in which a large fraction of the core volume is voided of sodium it may be expected that the greatest discrepancy between experiment and calculation of the sodium void effect would be found in the spectral component. This expectation is based on the fact that previous experience within the FFTF Critical Experiments Program and for other assemblies²⁷ indicates that the leakage component of the sodium void coefficient is calculated in good agreement with the experiments. This consideration, in addition to the fact that heterogeneity effects on the leakage component are expected to be small and approximately proportional to the sodium removal, makes it possible to restrict the study of the effect of the residual sodium carbonate on the sodium void coefficient to the study of the effect on the spectral component only.

A series of calculations has been run for the cell of FTR-2 as built on ZPPR. The purpose of the calculations was to ascertain the relative magnitude of the spectral reactivity effects which could be expected by removing progressively the elemental sodium from the core, and to find out whether or not the results could be extrapolated to the spectral reactivity effect of removing all the sodium from the core.

Figure II-9-5 identifies the plate configuration of the two-drawer cell of FTR-1 and FTR-2 that was considered in the calculations.

Three sets of cross sections were produced for this study through the MC² code over a 29-group structure equivalent to the structure of Set 29004. The sets, labeled Sets #1, 2, and 3, provided the cross sections for the various isotopes for the cases, respectively, when no sodium was removed from the cell, when all the elemental sodium was removed, and when all the sodium was removed, including the sodium contained in the cell in the form of sodium carbonate. The main differences between the cross sections of the various sets were due to the different degree of self-shielding of the resonances in the low-energy region for different degrees of sodium voiding. The resonance self-shielding of the resonances is treated by MC² according to the equivalence principle.

A calculation of the material buckling was run with Set #1 for a homogeneous cell equivalent to the FTR-1 cell and for $k = 1$. The result found for the material buckling was $B_m^2 = 1.878 \times 10^{-3} \text{ cm}^{-2}$.

A series of S_{14} calculations with a modified single-Gaussian quadrature²⁸ was then run by means of the dimensional Argonne discrete S_n code SNARG-1D using the three cross-section sets previously derived. All SNARG calculations were run in plane

TABLE II-9-XIV. RADIAL B-10 WORTH TRAVERSE

Positions (Inches from Core Center)	Specific Worths, lh/kg		
	Experiment, (± 34)	Perturbation Theory	Perturbation Theory Corrected for Self-Shielding
0	-6617	-7782	-7163
0.498	-6649	-7770	-7158
0.998	-6622	-7750	-7134
1.498	-6561	-7714	-7070
2.186	-6515	-7632	-7022
2.683	-6514	-7556	-6954
3.183	-6371	-7463	-6869
3.683	-6311	-7357	-6769
4.058	-6155	-7273	-6686
4.368	-6163	-7195	-6614
4.865	-6045	-7056	-6488
5.366	-5919	-6908	-6352
5.864	-5774	-6745	-6202
7.606	-5207	-6111	-5612
8.732	-4766	-5654	-5185
9.161	-4631	-5475	-5018
9.414	-4555	-5368	-4917
9.662	-4489	-5261	-4817
10.039	-4243	-5103	-4668
10.914	-4031	-4733	-4315
13.094	-3231	-3890	-3487
15.278	-2674	-3521	-3085
16.366	-2447	-3312	-2783
17.460	-2228	-3365	-2638
18.550	-2284	-3799	-2738
19.644	-1956	-3699	-2455
20.734	-1453	-2379	-1587
21.826	-924	-1283	-881
24.008	-242	-359	-266
26.190	-51	-90	-73
28.372	-22	-20	-17
30.554	+2	-2	-2
31.864	-17	+0.7	+0.7

geometry and corresponded closely to the cell represented in Fig. II-9-5 with periodic boundary conditions. The stainless steel between the drawers of the cell was considered to be equivalent to additional plates with appropriate thicknesses and compositions. The stainless steel contained in the cans of some of the plates was smeared over the volume of the corresponding plates. All other stainless steel contained in the cell was smeared uniformly over the cell volume, and so was any void present in the cell. The

TABLE II-9-XV. SPECTRAL REACTIVITY EFFECTS FOR VARIOUS SODIUM VOIDING CONFIGURATIONS IN THE FTR-2 CORE CELL

Calculation	% of Na Removed	k_{eff}
Homogeneous	0.0	1.000000
A	0.0	1.006487
B	26.031	1.008901
C	26.031	1.009029
D	54.062	1.011653
E	74.333	1.014657
F	100.0	1.017704

mesh point distribution was chosen so that at least four mesh intervals would be found over every plate region. The transverse buckling was chosen to be equal in all cases to the value of the material buckling obtained in the homogeneous calculation and previously mentioned.

The calculations consisted in the determination of the real fluxes and of k of the cell and for the following cases:

A. The heterogeneous cell as described in Fig. II-9-5, with the cross sections of Set #1.

B. The heterogeneous cell as described in Fig. II-9-5, but with the sodium removed from the 0.5 in. sodium plate in the left drawer of that figure. The cross sections were again taken from Set #1 in all regions, with the exception of the Pu-U-Mo and U_3O_8 plates adjacent to the voided region. The cross sections for these plates were obtained by interpolation between Sets #1 and #2.

C. The heterogeneous cell as described in Fig. II-9-5, but with the sodium removed from the 0.5 in. sodium plate in the right drawer of that figure. The cross sections were again taken from Set #1 in all regions, with the exception of the Pu-U-Mo and U_3O_8 plates adjacent to the voided region. The cross sections for these plates were obtained by interpolation between Sets #1 and #2.

D. The heterogeneous cell as described in Fig. II-9-5, but with the sodium removed from both the 0.5 in. sodium plates present in the two drawers of the cell. The cross sections were taken from Set #1 in all regions, with the exception of the Pu-U-Mo and U_3O_8 plates adjacent to the voided regions. The cross sections for these plates were obtained by interpolation between Sets #1 and #2.

E. The heterogeneous cell as described in Fig. II-9-5, but with the sodium removed from all the plates in the cell which contain elemental sodium. The cross sections of Set #2 were used in all regions.

F. The heterogeneous cell as described in Fig.

II-9-5, but with the sodium removed from all plates in the cell which contain sodium, either in the form of elemental sodium or in the form of sodium carbonate. The cross sections of Set #3 were used in all regions.

The results of these calculations are summarized in Table II-9-XV and in Fig. II-9-10.

The changes in reactivity obtained in these calculations are due to the combination of the effects that sodium removal at various locations of the heterogeneous cell has on the resonance self-shielding and on the broad-group structure of the flux across the cell. The effect on the resonance self-shielding is estimated to account approximately for 13% of the reactivity change following total sodium voiding.

It may be noticed from Fig. II-9-10 that removals of equal amounts of sodium from the cell have different reactivity effects, depending on the location from which the sodium is removed (compare the reactivity difference between points B and A with the reactivity difference between points C and A), on how much sodium has already been removed (compare the reactivity difference between points D and B with the reactivity difference between C and A), and with the thickness of the voided regions (compare the reactivity difference between points E and D with the reactivity difference between D and C). These differences correspond to the higher-order perturbation effects that have prompted this investigation. The high-order reactivity effects, however, appear to be rather small in comparison with the first-order reactivity effects of the sodium removal in the FTR-2 cell. Their magnitude is never larger than 20% of the first-order effects, and it seldom exceeds 10% of those effects. This may be due in part to the judicious way in which the plate arrangement was chosen in the FTR-2 cell, but results obtained both theoretically and experimentally for a very different case²⁹ indicate that this conclusion may have a more general validity.

An important effect of the relatively small ratio of second to first order reactivity effects, and of their being somewhat constant with sodium content, is found in the remarkably regular pattern of the integrated reactivity effects versus fraction of sodium voiding as shown in Fig. II-9-10. It appears that the results of case F, which corresponds to the desired situation of total sodium voiding but which is not experimentally feasible, can be extrapolated with reasonable safety from the results of the experiments corresponding to the other points. The extrapolation is quite satisfactory even when it is based only on points A and E, which correspond physically to the unvoided core and to the core with all the elemental

FIG. II-9-10. Spectral Reactivity Effects for Various Sodium Voiding Configurations in the FTR-2 Core Cell. ANL Neg. No. 113-3124

sodium removed. A relative error smaller than 3% would be found in the evaluation of the spectral component of the reactivity effect for total sodium removal with this procedure and within the applicability of the calculations that have been described.

In conclusion, it appears from this study that the spectral reactivity effect of sodium voiding in cores of FTR-type can be extrapolated with reasonable confidence from the results of experiments in which a small amount of sodium is left in the core in the form of sodium carbonate. The error implicit in the extrapolation may be considered very small in comparison with heterogeneity effects and other phenomena which currently limit the accuracy of sodium-voiding experiments.

CALCULATION OF FISSION REACTION RATES TAKING INTO ACCOUNT HETEROGENEITY EFFECTS

The spatial distributions of the fission rates of U-238, U-235, Pu-239, and Pu-240 have been calculated for the central cell of ZPR-3 Assembly 56B taking into account the effect created by the heterogeneous plate structure of the cell on the broad-group flux distribution across the cell itself.

the calculation of the flux was made by means of the Argonne Transport Code SNARG-1D in the DS_{14} approximation, and by using the modified single-

Gaussian quadrature developed by D. Meneghetti.²⁸ The cross sections were taken from the 29-group cross section Set 29004 which was obtained by means of an MC² calculation using the ENDF/B data file and for a cell configuration approximating closely the core cell of Assembly 56B.

The geometry and composition of every plate of the cell as shown in Fig. II-9-5 was represented very accurately in the calculations. The stainless steel contained in the jackets of some of the plates was considered to be uniformly distributed over the volume occupied by those plates, while the structural steel and the void present in directions other than the direction of calculation was smeared uniformly over the whole cell. A fictitious pure absorber with absorption cross section equal to the fundamental-mode leakage of the homogenized cell divided by the corresponding flux was also smeared over the entire volume of the cell.

The microscopic fission cross sections used in the calculation of the fission traverses were also taken from Set 29004. The results obtained for the spatial distribution of the various fission rates are shown in Fig. II-9-11. The flux used in the calculation of all the fission rates was normalized so that the spatial average of the U-235 fission rate over the cell is unity.

Figure II-9-11 shows clearly some wide oscillations of the U-238 and Pu-240 fission rates across the cell.

FIG. II-9-11. Spatial Distributions of Fission Rates in the Core Cell of ZPR-3 Assembly 56B Relative to the Average U-235 Fission Rate in the Cell. ANL Neg. No. 113-3123.

TABLE II-9-XVI. COMPARISON OF CALCULATED WITH EXPERIMENTAL RESULTS FOR THE FISSION RATIOS IN ASSEMBLY 56B

	Heterogeneous Calculations	Homogeneous Calculations	Experimental Results
$\frac{\phi\sigma_f(\text{U-238})}{\phi\sigma_f(\text{U-235})}$	0.0258	0.0274	0.0308
$\frac{\phi\sigma_f(\text{Pu-239})}{\phi\sigma_f(\text{U-235})}$	0.909	0.922	1.028
$\frac{\phi\sigma_f(\text{Pu-240})}{\phi\sigma_f(\text{U-235})}$	0.204	0.214	0.282

TABLE II-9-XVII. CALCULATED VALUES OF THE RATIOS OF VARIOUS FISSION RATES IN THE FUEL PLATES OF ASSEMBLY 56B

	Heterogeneous Calculations	Homogeneous Calculations
$\frac{\phi\sigma_f(\text{U-238})}{\phi\sigma_f(\text{U-235})}$	12.27	12.11
$\frac{\phi\sigma_f(\text{Pu-239})}{\phi\sigma_f(\text{U-235})}$	89.40	87.65
$\frac{\phi\sigma_f(\text{Pu-240})}{\phi\sigma_f(\text{U-235})}$	2.817	2.735

The oscillations are due to the fact that the broad-group heterogeneity effects affect predominately the behavior of the high-energy groups, where the fission cross sections of these two elements are large. Much smaller heterogeneity effects are found for U-235 and for Pu-239. However, these calculations do not take into account the different degree of resonance shielding which would be found by a thin-walled fission counter used to measure the fission traverses experimentally. The curves depicted in Fig. II-9-11 are valid for a counter whose resonance shielding is con-

stant in space and equal to the one found in the fuel plates of the cell. Thus, the calculations are approximately valid for the interpretation of the measurements of the fission traverses at locations close to the fuel plates. Exploratory calculations indicate that the change in resonance shielding, if taken into account, would cause slight changes in the curves depicted in Fig. II-9-11 at those locations which are far from the fuel plates. The changes, however, were in no case found to be larger than 3% of the unchanged values.

The results obtained through this method for the ratios of the spatial averages of the reaction rates of U-238, Pu-238 and Pu-240 to the corresponding value for U-235 are shown in Table II-9-XVI. In this table, the results calculated for the three fission ratios are compared with the results of homogeneous calculations and with the experimental results.³⁰

The comparison of the calculated values of the fission ratios with the experimental values shows a discrepancy which, even though not much greater than discrepancies found in the past for similar calculations,³¹ may warrant further investigation. The ratios between the fission rates in the fuel plates of the assembly were also calculated by the same method. The results are shown in Table II-9-XVII where they are compared with the results of homogeneous calculations.

FTR-1 ON ZPPR

The FTR-1 configuration, which was built on ZPR-3 as Assembly 56B, was rebuilt³² on ZPPR as a

k of the accuracy with which results obtained from a ZPR-3 assembly could be extrapolated to an equivalent ZPPR assembly, and to study the transition from FTR-1 to FTR-2 on the same facility.

The core loading of the assembly is shown in Fig. II-9-12, and the compositions of the various zones are given in Table II-9-XVIII. Even though the plate arrangements in the drawers of the various

ZPPR HALF 1

(HALF 2 IS A MIRROR IMAGE OF HALF 1)

S= SAFETY ROD

C= CONTROL ROD

P= DRAWER ADJACENT TO POISON SAFETY ROD

☐= POISON SAFETY ROD (WITHDRAWN DURING OPERATION)

FIG. II-9-12. Critical Loading for ZPPR FTR-1 Reference Core. ANL Neg. No. 103-A11048.

TABLE II-9-XVIII. AS-BUILT COMPOSITION—FTR-1 ON ZPPR, LOADING 17, atoms/b-cm

Nuclide	Core					Axial Reflector	Radial Reflector
	1-Column Pu Drawer ^(a)	2-Column Pu Drawer ^(b)	Cell Average	Movable Fuel Rod ^(c)	Drawer Adjacent to Poison Safety Rod ^(d)		
Pu-239	0.0008491	0.0016983	0.0012737	0.0016983	0.0008491		
Pu-240	0.0001125	0.0002251	0.0001688	0.0002251	0.0001125		
Pu-241	0.0000168	0.0000337	0.0000252	0.0000337	0.0000168		
Pu-242	0.0000017	0.0000035	0.0000026	0.0000035	0.0000017		
U-238	0.0055684	0.0063849	0.0059766	0.0063849	0.0055684		
U-235	0.0000122	0.0000139	0.0000131	0.0000139	0.0000122		
O	0.0150075	0.0154338	0.0152206	0.0137617	0.0150075		
Na	0.0107196	0.0063408	0.0085302	0.0063610	0.0062787	0.0133588	0.0065078
C	0.0021888		0.0010944	0.0032025	0.0021888	0.0000284	0.0001547
Fe	0.0087888	0.0162589	0.0125238	0.0180022	0.0097185	0.0079227	0.0068787
Ni	0.0011626	0.0011636	0.0011631	0.0023373	0.0012671	0.0189744	0.0460037
Cr	0.0025337	0.0025358	0.0025348	0.0051353	0.0027964	0.0022823	0.0019747
Mn	0.0002025	0.0002027	0.0002026	0.0004248	0.0002204	0.0002295	0.0002639
Mo	0.0002322	0.0004475	0.0003398	0.0004567	0.0002355	0.0000161	0.0000154

^a Occupies even-numbered matrix columns.

^b Occupies odd-numbered matrix columns (center column is odd-numbered).

^c Both fuel safety and control rods of this composition.

^d Poison rod withdrawn during operation.

TABLE II-9-XIX. AS-BUILT COMPOSITION OF CONTROL RING FOR ZPPR/FTR-2 REFERENCE CONFIGURATION (Loading 1-70)

Nuclide	Composition, atoms/b-cm
Na	0.0082408
C	0.0089729
Fe	0.0094642
Ni	0.0012605
Cr	0.0027303
Mn	0.0002167
Mo	0.0000174
B-10	0.0069348
B-11	0.0281204

zones of the reactor followed closely the corresponding plate arrangements previously described for ZPR-3 Assembly 56B, the compositions of the ZPPR/FTR-1 zones differed slightly from those of the ZPR-3/FTR-1 zones mainly because of the different void fractions that are found in ZPPR and in ZPR-3.

CRITICAL MASS

The calculated critical mass of FTR-1 on ZPPR, as predicted³³ on the basis of the same procedure that was used in the prediction of FTR-1 on ZPR-3, amounted to 367 kg of fissile materials (Pu-239, Pu-241, and U-235). This calculation was based on

nominal compositions that were somewhat different from the compositions found in the as-built system, and did not take into account the results of the experiments performed on ZPR-3/FTR-1. Other calculations of the critical mass were performed after completion of the assembly using the actual compositions as listed in Table II-9-XVIII. These calculations consisted of a series of radial and axial computations for the same value of the multiplication constant that an equivalent procedure had yielded for ZPR-3/FTR-1, and used the cross sections of Sets 29004 and 29004.2. The fissile critical mass was calculated to be 359.66 kg and 360.40 kg when the extrapolation from ZPR-3 to ZPPR was performed, respectively, in diffusion theory through the MACH-1 code and in the S_4 approximation through the SNARG-1D code. The experimental fissile critical mass was 359.41 kg.⁽³⁴⁾

FTR-2 ON ZPPR

The FTR-2 configuration on ZPPR was obtained³⁵ by expanding radially the core of ZPPR/FTR-1 until its volume became approximately equal to 1000 liters, and by introducing between core and radial reflector a nominally two-drawer-thick ring of control material. The composition of the control material is given in Table II-9-XIX. The transverse and axial cross-sectional views of the assembly are shown in Figs. II-9-13 and II-9-14.

ZPPR HALF 1

(HALF 2 IS A MIRROR IMAGE OF HALF 1)

S = SAFETY ROD C = CONTROL ROD P = DRAWER ADJACENT TO POISON SAFETY ROD

I = POISON SAFETY ROD (WITHDRAWN DURING OPERATION)

F = U-235 FISSION CHAMBER (FRONT OF CHAMBER AT REACTOR MIDPLANE. NO F's IN HALF 2)

FIG. II-9-13. ZPPR FTR-2 Reference Critical Configuration (Loading 1-70). ANL Neg. No. 103-11073.

FIG. II-9-14. Equivalent Cylindrical Loading for ZPPR FTR-2 Reference Loading (Loading 1-70). ANL Neg. No. 103-A11074.

CRITICAL MASS

The critical mass of ZPPR/FTR-2 was calculated by assuming that the control region surrounding the core was equivalent to an annular region having the same volume and composition as the FTR-2 control region. The same procedure that was used to extrapolate the FTR-1 critical mass on ZPPR from the FTR-1 critical mass on ZPR-3 was used to extrapolate the critical mass of FTR-2 on ZPPR from the critical mass of FTR-1 on the same facility. The extrapolation procedure yielded a calculated fissile critical mass of 534.23 kg in diffusion theory and of 533.70 kg in transport theory using the S_4 approximation. The mass of ZPPR/FTR-2 was found to be approximately 524 kg from the reported fissile mass³⁵ of the reference loading and the excess reactivity.³⁶

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II-10. The Variable Temperature Rodded Zone (VTRZ) Program

R. A. LEWIS, K. D. DANCE, J. F. MEYER and T. W. JOHNSON

The VTRZ is a facility which is to be built into the ZPR-6 Fast Critical Assembly to allow integral physics measurements to be made in environments which closely simulate the temperature conditions and fuel configuration of fast breeder power reactors. The principle intent of the measurements is to provide confirmation of the adequacy of current analytical techniques used to extrapolate from measurements done in cold plate-type assemblies, typical of critical assemblies like the Argonne ZPR reactors, to hot rodged configurations typical of power reactors.

A conceptual description of the VTRZ Facility together with a description of the experimental program planned for the VTRZ was given in Ref. 1. This report includes a description of the VTRZ preliminary design and a discussion of the progress which has been made on the VTRZ Project during this reporting period.

DESCRIPTION OF THE PRELIMINARY DESIGN

The VTRZ is shown installed in ZPR-6 in Fig. II-10-1. One VTRZ half-zone is shown in an isometric

FIG. II-10-1. VTRZ in ZPR-6. ANL Neg. No. 113-1526.

cut-away assembly view in Fig. II-10-2 and in a section view, normal to its axis, in Fig. II-10-3.

The heatable fuel assemblies consist of 2.036-in. square by 12-in. long calandria cans, made of 18 mil thick 304 SS, containing 16 axial fuel tubes set in a square array as shown in Fig. II-10-4. Rodded fuel in the form of $\frac{3}{8}$ in. diam by 6 in. long 304 SS clad pins, containing various enrichments of UO_2 and mixed $\text{PuO}_2\text{-UO}_2$ fuel pellets, will be inserted into the fuel tubes of the calandria cans. The calandria cans will contain various amounts of sodium depending on their experimental application. Figure II-10-4 shows a typical calandria can and a typical fuel pin compared with a regular ZPR fuel drawer.

The heatable zone consists of a region in the shape of a right cylinder with an octagonal-cross-section, approximately 20 in. inside across the flats of the octagon and 6 ft long, split normal to its long axis into two 3-ft "half-zone" sections, one mounted in the center of each reactor half and mating at the reactor midplane when the halves come together. Each half zone contains a matrix structure which consists of regular ZPR (2.093 in. square inside by 40 mil wall) 304 SS matrix tubes welded

together and mounted horizontally. The matrix structure is supported and enclosed by an octagonal barrel. When operating at room temperature, with either plate or rodded fuel, the inside of the zone may be cooled, if necessary, by circulating air through the zone in exactly the same manner, and using the equipment, described in Ref. 2. In the heatable configuration, the zone is isolated from the reactor cell by gas-tight front and rear covers on each half-zone and a high-purity argon atmosphere, at a slightly negative pressure with respect to the cell, is provided within the zone. These precautions are taken to minimize the consequences if a leak were to occur in a plutonium fuel pin or in a sodium-filled calandria can.

Each heatable half-zone of the VTRZ is supported radially by twelve brackets arranged, six on each side, along the horizontal midplane of the half-zone and attached to the outside of the barrel enclosing the heatable matrix. The brackets are shown in Figs. II-10-2 and II-10-3. The weight of the heatable zone is transmitted through the brackets to a sp welded bridge assembly, composed mainly of reg ZPR matrix tubes, which operates at room tempera-

FIG. II-10-2. VTRZ Half-Zone, Isometric Cutaway. ANL Neg. No. 113-1527.

FIG. II-10-3. VTRZ, Section View Across Axis. ANL Neg. No. 113-1525.

ture and is designed to carry the load of the reactor core above the heatable zone as well as the weight of the zone itself. Expansion of the zone radially and axially is permitted by ball mounts in the support brackets and controlled by radial and axial guides attached to the cover plates and along the bottom of the heatable zone. A 2-in.-thick radial insulation

and cooling-guard-ring annulus separates the heatable zone from the outer bridge assembly.

The inner matrix assembly will be heated by $\frac{3}{8}$ in. diam Inconel-clad electrical cartridge heaters of various lengths up to 36 in. The heaters will reheat fuel pins in selected calandria can fuel tubes. The number of heaters will be selected on the basis of

FIG. II-10-4. Fuel Pins, Calandria Can and Plate-Form Fuel Drawer. ANL Neg. No. 113-94.

the requirements of the particular experiment being performed.

PROGRESS OF VTRZ PROJECT, JULY 1, 1968 TO JUNE 30, 1969

The principal accomplishments of the VTRZ Project during the period July 1, 1969 to June 30, 1969, may be summarized as follows:

1. Establishment of a preliminary design.
2. Construction and testing of prototype models of

one VTRZ half-zone, the inert gas loop, and various pieces of developmental instrumentation.

3. Issuance of the Preliminary Safety Analysis Report and Design Basis Analysis Report.
4. Issuance of technical specifications, evaluation of bidders, and awarding of contracts for the UO_2 and $\text{PuO}_2\text{-UO}_2$ fuel pins.
5. Completion of the design and development of the sodium-filled fuel-mounting calandria cans.

The preliminary design was fixed in July of 1968

... AEC request, a companion document, the Design Basis Analysis Report, was submitted with the PSAR and contained all of the documentation then available on all aspects of the VTRZ Project.

During the summer and fall of 1968 numerous meetings with potential suppliers of the VTRZ fuel pins were held to evaluate their capabilities. This work culminated in the issuance of technical specifications, formal bidding, evaluation of bids, and, finally, in the award of contracts to the Kerr-Magee Corporation for all of the required UO_2 and $\text{PuO}_2\text{-UO}_2$ fuel. The fuel currently on order for the VTRZ is summarized in Table II-10-I. The design and development of the sodium-filled calandria cans was carried out during the winter and spring of 1968-1969. This work included the construction and testing of numerous prototype cans. Thermal cycling tests (20 to 550 to 20°C) and overheating tests (to 1000°C) were performed un-

der conditions simulating the operating environments in the VTRZ. During the period April-June of 1969 the calandria design was fixed, production tooling obtained, and preproduction runs made.

OUTLOOK FOR JULY 1, 1969 TO JUNE 30, 1970

The final VTRZ design should be completed by October of 1969. Assuming availability of construction funds, the heatable zone should be available early in 1970. Fuel pins and calandria cans should be available in the fall of 1969.

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II-11. Analysis of Heterogeneity and Sodium-Void Effects in a 2700-liter Uranium Carbide Fast Core, ZPR-6 Assembly 5

R. A. KARAM, J. E. MARSHALL and K. D. DANCE

INTRODUCTION

All of the measurements conducted on Assembly 5 of ZPR-6, a 2700-liter UC core, were reported previously.¹ In this report analyses of the critical mass, heterogeneity effects, and sodium void effects were performed using ENDF/B data.

SPECIFICATIONS OF ASSEMBLY 5

Assembly 5 of ZPR-6 was a relatively large, dilute uranium carbide core. It was essentially a simple two-region system with cylindrical configuration. Figure II-11-1 shows a schematic diagram of Assembly 5 with all appropriate dimensions. The radial dimension of the core, R_c , and the total radial dimension of the core and reflector, R_t , shown in Fig. II-11-1 are radii of circles whose areas are equivalent to the cross sectional areas of the core and the core and reflector.

The presence of the gap, shown in Fig. II-11-1 in the axial reflector, is due to a safety feature of the ZPR-6 and -9 facilities² which requires that a space be provided at the end of the drawers for the expansion of the fuel plates in case of a minor excursion. This space in Assembly 5 was 1.07 cm. The materials

drawer are held toward the front (mid-section of split table facility) by the action of a steel spring.

The concentrations of the various materials in As-

sembly 5 are given in Table II-11-I. The concentration of U-235 shown in Table II-11-I is based on the use of $\frac{1}{16}$ in. thick plates having an enrichment of 93%. The supply of such plates available was not enough for Assembly 5 and, therefore, double columns of $\frac{1}{32}$ in. and 0.026 in. thick plates were used in some of the drawers near the periphery of the core. The use of such plates resulted in a lower concentration of U-235 by 7%. Figure II-11-2 shows a cross sectional view of Assembly 5 as loaded. Concentration of U-235 in the central region was the same as that given in Table II-11-I and the concentration of U-235 in Region 2 was 1.442×10^{21} atoms/cm³. Concentrations of the other materials are given in Table II-11-I. Regions 1 and 2 of Fig. II-11-2 had equivalent outer radii of 62.73 and 77.86 cm, respectively.

Assembly 5 was loaded with a three-drawer cell pattern which was similar to the loading pattern of Assembly 4Z, a zoned version of Assembly 5.⁽³⁾ Figure II-11-3 shows the pattern of the three-drawer cell of the core regions of Assembly 5. This pattern was slightly different from the pattern of Assembly 4Z, the difference being that in the latter the first and eighth columns of the second drawer of the three-drawer cell were $\frac{1}{16}$ in. thick plates of depleted ura-

FIG. II-11-1. Schematic Diagram for Assembly 5 of ZPR-6. ANL Neg. No. 112-9240 Rev. 1.

TABLE II-11-I. ATOMIC DENSITIES IN ASSEMBLY 5

Material	(atoms/cm ³) $\times 10^{-24}$		
	Region		
	Core	Reflector	Gap for Spring
U-235	0.00154	0.00008	0.00000
U-238	0.01056	0.03993	0.00000
Na	0.00920	0.00000	0.00000
C	0.01293	0.00000	0.00000
Fe	0.00904	0.00432	0.01098
Ni	0.00113	0.00054	0.00091
Cr	0.00239	0.00114	0.00195

nium. These two columns were replaced in Assembly 5 with one $\frac{1}{8}$ in. thick column of the same material. This modification in the loadings of the two assemblies was dictated by the insufficient inventory of $\frac{1}{16}$ in. thick plates. The loading pattern shown in Fig. II-11-3 will henceforth be referred to as "normal loading pattern."

FIG. II-11-2. Cross Sectional View of Assembly 5 of ZPR-6. ANL Neg. No. 112-9161.

FIG. II-11-3. Three Drawer Unit Cell—Normal Loading Pattern. ANL Neg. No. 113-2966.

CRITICAL MASS

MEASUREMENTS

The measured critical mass of Assembly 5 for the loading shown in Fig. II-11-2, corrected for excess reactivity only, was 1585 kg of U-235. The relative reactivity of a drawer in Region 2 (Fig. II-11-2) to that of a drawer in Region 1 was measured as a function of position with a few drawers. The results of these measurements were used to obtain the critical mass of a core that had the same U-235 concentration

in Regions 1 and 2. The critical mass of such a core was 1564 kg of U-235. The extrapolation of the critical mass to one region in the core is a first-order correction; however it is sufficiently accurate because the magnitude of the correction is not large.

CALCULATIONS

The critical mass was calculated with cross section sets obtained from ENDF/B data in the following manner:

(1) A 25-group set was generated for the homogeneous composition of Assembly 5 with the MC² code.⁴ The fundamental-mode, ultra-fine, group fluxes ($\Delta u = 1/120$), used to weight the cross sections in the group collapsing process, were obtained using the ordinary P_1 approximation. This set was designated as ZPR-6-5-ENDF/B-A.

(2) The U-238 capture cross sections in Set ZPR-6-5-ENDF/B-A were corrected for spatial self-shielding in accordance with equivalence theory for plate geometry.⁵

The potential scattering cross section, σ_p per absorber atom in each energy group, was obtained from the relation:

$$\sigma_p = (\Sigma_p + \Sigma_s)/N_0, \quad (1)$$

where Σ_p is the potential scattering cross section for the absorber plate composition and N_0 is the absorber atom density in the plate. The quantity Σ_s is given by the expression⁴

$$\Sigma_s = (S/4V)[a(1 - C)]/[1 + (a - 1)C]. \quad (2)$$

The symbols S and V are respectively the plate surface

and volume, and the constant a is typically 1.09 for plate geometry⁶ and 1.35 for pin geometry.⁷ The Dancoff factor $1 - C$ was obtained using the approximation⁵

$$1 - C = 1 - E_3(\tau_\ell) - E_3(\tau_r), \quad (3)$$

where $E_3(\tau)$ is the well-known exponential integral of order 3 and the subscripts ℓ and r designate the optical thickness τ to the left and to the right of the absorber plate in a sub-cell of the unit cell. The adjusted U-238 capture cross sections obtained by the above method together with the other isotopes in Set ZPR-6-5-ENDF/B-A were designated as Set ZPR-6-5-ENDF/B-B.

(3) Set ZPR-6-5-ENDF/B-B was then used to calculate the fine structure of the fluxes in a unit cell with a 1-D transport code⁸ using $S_{n=16}$ and a modified set of single Gaussian quadratures.⁹ In these calculations the transverse dimensions were chosen in such a way that the full three-dimensional leakage of the overall system was preserved. Figures II-11-4 and II-11-5 show respectively typical distributions of the fluxes in the first and fifteenth energy groups across the unit cell. It is seen from Fig. II-11-4 that the flux peaking in the U-235 plate is about 40% higher than in the rest of the cell. Figure II-11-5 shows that the flux in Group 15 (4.3–2.6 keV) is slightly depressed in and about the U-235 and U-238 plates and significantly depressed in and about the sodium cans due to the large sodium resonance at 2.85 keV.

The adjusted cross sections were finally averaged with the spatial distributions of the fluxes in the unit

FIG. II-11-4. Calculated Spatial Distribution of the First Energy Group (10-3 68 MeV) in Unit Cell of Assembly 5 of ZPR-6. ANL Neg. No. 113-2970.

FIG. II-11-5. Calculated Spatial Distribution of the Fifteenth Energy Group (4.3-2.6 keV) in Unit Cell of Assembly 5 of ZPR-6. ANL Neg. No. 113-2964.

cell. The averaging was carried out according to the simple relation for one-dimensional plate geometry,

$$\bar{\Sigma} = \int_{cell} \Sigma(x)\phi(x) dx / \int_{cell} \phi(x) dx, \quad (4)$$

in any broad energy group.

Equation (4) conserves the reaction rates. For computational convenience, Eq. (4) was rewritten in the form

$$\bar{\sigma} = \sigma \left[\frac{V_t}{\sum_{i=1}^N \phi_i \Delta V_i} \right] \left[\frac{\sum_{i=1}^N \phi_i \rho_i \Delta V_i}{\sum_{i=1}^N \rho_i \Delta V_i} \right], \quad (5)$$

where

$\bar{\sigma}$ $\equiv$ cell averaged cross section

σ $\equiv$ in-plate heterogeneous cell cross section

ϕ_i $\equiv$ flux at point i , $1 \leq i \leq N$

ρ_i $\equiv$ atom density at point i

ΔV_i $\equiv$ width of mesh point i

V_T $\equiv$ width of the unit cell.

The transport cross sections were averaged with the fine structure of the fluxes in accordance with Eq. (4).

The spatially averaged cross section set, obtained using Eq. (5) and designated as ZPR-6-5-ENDF/B-C, was suitable for use in homogeneous calculations with either 1-D or 2-D diffusion or transport codes.

The effect of each of the above three steps on the values of the absorption cross sections of U-235 and U-238 are listed in Tables II-11-II and II-11-III. It is seen from Table II-11-II that the spatial averaging increases the U-235 cross sections in the high energy groups by about 20%. The cross sections in the low energy groups are decreased, due to self-shielding, by about 10-20%. The spatial averaging effect on the U-238 cross sections, as can be seen from Table II-11-III, is not large.

(4) A fourth set designated as ZPR-6-5-ENDF/B-D was generated by spatially averaging the cross sections with the normalized fine structure of the flux obtained from a two-dimensional cell calculation using the 2-D diffusion code.¹⁰ It was expected that diffusion theory would not give adequate convergence for the flux shape. Thus the 2-D fluxes were normalized to the 1-D transport cell calculation using the modified single Gaussian quadratures.⁹ The normalization process was simply multiplying the 2-D fluxes in the unit cell by the ratio of the 1-D flux to the 2-D flux at every space point for each energy group. This rather approximate and somewhat tedious approach was used instead of the direct 2-D transport cell calculation because in the latter method the convergence was extremely slow when 25 groups and an S_n order of 8 or more were used.

(5) In addition to the linear weighting with the fine structure of the real fluxes, another cross section set designated ZPR-6-5-ENDF/B-E was generated with bilinear averaging in space; i.e., weighting with the real and adjoint fluxes. The cell averaged total removal cross section $\bar{\Sigma}_{R_j}$ for any broad group j was calculated according to the relation

$$\bar{\Sigma}_{R_j} = \frac{\int_{cell} \Sigma_{R_j}(x)\phi_j(x)\phi_j^*(x) dx}{\int_{cell} \phi_j(x)\phi_j^*(x) dx}, \quad (6)$$

whereas the scattering matrix terms were calculated by

$$\bar{\Sigma}_{i \rightarrow j} = \frac{\int_{cell} \Sigma_{i \rightarrow j}(x)\phi_i(x)\phi_j^*(x) dx}{\int_{cell} \phi_i(x)\phi_j^*(x) dx}.$$

TABLE II-11-II. COMPARISON OF HOMOGENEOUS AND HETEROGENEOUS ABSORPTION CROSS SECTIONS OF U-235 ASSEMBLY 5

Energy Group		σ_a Homogeneous, ^a b	σ_a Heterogeneous, ^b b
No.	E_L , keV		
1	3670.0	1.370	1.813
2	2230.0	1.338	1.696
3	1350.0	1.346	1.613
4	825.0	1.341	1.486
5	500.0	1.335	1.386
6	300.0	1.450	1.464
7	180.0	1.616	1.616
8	110.0	1.885	1.870
9	67.0	2.213	2.188
10	40.7	2.619	2.582
11	25.0	3.061	3.001
12	15.0	3.550	3.468
13	9.12	4.209	4.072
14	4.31	5.507	5.319
15	2.61	6.823	7.181
16	2.03	8.217	7.125
17	1.23	10.008	9.004
18	0.961	13.224	11.98
19	0.583	15.227	13.69
20	0.275	19.681	17.26
21	0.101	32.934	27.05
22	0.029	35.736	29.35
23	0.0137	49.721	37.12
24	0.00306	57.625	42.96
25	0.00068	23.747	21.65

^a As obtained from the MC² calculation with ENDF/B data for the homogeneous composition of Assembly 5.

^b The homogeneous cross sections were spatially averaged with the fine structure of the fluxes.

The fission source terms were obtained in a similar manner, namely

$$\chi_j(\nu\Sigma_f)_i = \frac{\int_{cell} \chi_j(\nu\Sigma_f)_i \phi_i(x) \phi_j^*(x) dx}{\int_{cell} \phi_i(x) \phi_j^*(x) dx} \quad (8)$$

The bilinearly weighted cross sections do not conserve reaction rates, but for fundamental-mode calculations, it can be shown that bilinear weighting conserves reactivity.

It should be noted that the bilinear weighting was performed with the broad group real and adjoint fluxes over the unit cell in which the assumption of continuity of the flux and current among the plates in both the real and adjoint solutions was used. This is tantamount to bilinear averaging (only spatially) in a homogeneous region on which a heterogeneous perturbation has been superimposed. With such a model the questions raised by A. Henry¹¹ about discontinuities in fluxes and currents across internal boundaries become

inapplicable. Bilinear weighting in energy for the fundamental-mode spectrum may produce significant effects on the cross sections in the resonance region. To do such weighting, however, would require calculating the adjoint with an ultrafine energy group width, as is possible with the MC² code for the real fluxes. There is no code presently, however, that has such a feature.

The results of the calculations of the critical mass are summarized in Table II-11-IV. The first three values of the critical mass illustrate the dependence of the critical mass on the heterogeneity effects. The difference between the first two values, 2330–2157 kg, is due to the change in the resonance self-shielding in U-238 between the truly homogeneous version of Assembly 5 without taking into consideration spatial averaging. (Note that energy self-shielding in all the resonances in set ZPR-6-5-ENDF/B-A is properly accounted for in the MC² code.) Set ZPR-6-5-ENDF/B-B is appropriate to use in the heterogeneous cell calculations provided that equivalence theory as used in this analysis is applicable. The underlying assumptions of equivalence theory, however, are: (1) a simple two-region cell—a fuel region and a non-fuel region; and (2) flat flux existing in both regions (not necessarily

TABLE II-11-III. COMPARISON OF HOMOGENEOUS, σ_p ADJUSTED, AND HETEROGENEOUS ABSORPTION CROSS SECTIONS OF U-238 IN ASSEMBLY 5

Group No.	σ_a Homogeneous, b	σ_a Adjusted According to σ_p Dependence, b	σ_a Heterogeneous, b
1	0.756	Same as homogeneous	0.759
2	0.587	"	0.585
3	0.493	"	0.488
4	0.171	"	0.171
5	0.141	"	0.142
6	0.157	"	0.135
7	0.151	"	0.151
8	0.181	"	0.181
9	0.236	"	0.236
10	0.332	"	0.332
11	0.437	0.433	0.433
12	0.549	0.543	0.541
13	0.657	0.644	0.641
14	0.777	0.745	0.743
15	0.888	0.805	0.838
16	1.033	0.926	0.869
17	0.841	0.787	0.764
18	0.963	0.894	0.881
19	0.961	0.873	0.865
20	0.808	0.710	0.705
21	1.068	0.926	0.918
22	1.268	1.09	1.103
23	2.272	2.010	2.053
24	1.172	1.170	1.169
25	0.990	0.990	0.990

TABLE II-11-IV. CRITICAL MASS CALCULATIONS OF ZPR-6 ASSEMBLY 5

Cross Section Set Used	Code Used for Critical Mass Calc.	Critical Mass, kg U-235	k_{eff} for As-Built ^a System	Remarks
ZPR-6-5-ENDF/B-A	1-D Diffusion	2330	0.971310	Set ZPR-6-5-ENDF/B-A is for the homogeneous composition of Assembly 5.
ZPR-6-5-ENDF/B-B	1-D Diffusion	2157	0.975942	The cross sections in Set ZPR-6-5-ENDF/B-A were adjusted according to their dependence on the σ^{238} of the normal loading pattern (see Fig. II-11-3).
ZPR-6-5-ENDF/B-C	1-D Diffusion	2034	0.980166	Cross sections in Set ZPR-6-5-ENDF/B-B were spatially averaged with the one-dimensional fine structure of the real flux.
ZPR-6-5-ENDF/B-D	1-D Diffusion	2046	—	Cross sections in Set ZPR-6-5-ENDF/B-B were spatially averaged with the two-dimensional fine structure of the real flux.
ZPR-6-5-ENDF/B-E	1-D Diffusion	1930	—	Cross sections in Set ZPR-6-5-ENDF/B-B were bilinearly averaged with the real and adjoint fluxes obtained from 1-D transport calculation.
ZPR-6-5-ENDF/B-C	2-D Diffusion	1881	—	Two-dimensional diffusion calculation using ZPR-6-5-ENDF/B-C.
ZPR-6-5-ENDF/B-C	1-D Transport	1986	0.983229	One-dimensional transport calculation.
ZPR-6-5-ENDF/B-C	1-D Transport	1870	0.988108	One-dimensional transport calculation using the measured extrapolated axial height.

NOTE: THE MEASURED CRITICAL MASS WAS 1585 kg OF U-235.

^a The k_{eff} calculations were done with the measured extrapolated height of 180.0 cm.

equal). Neither of these conditions is met in Assembly 5 and thus it was necessary to introduce approximations. One such approximation was to divide the unit cell into sub-cells in which the plates of different materials between one U-238 plate and the next U-238 plate to the right are homogenized. The materials between the same U-238 plate and the next U-238 plate to the left are separately homogenized. With this approximation the Dancoff correction factor was then calculated according to Eq. (3). While the adequacy of this approximation depends on the internal structure of the unit cell, experimental evidence in Assembly 6 of ZPR-6⁽¹²⁾ showed that the U-238 capture rates at the interface between a U-235 plate and a U-238 plate is about 10% lower than that at the interface between the same U-238 plate and an Fe₂O₃ plate. This indicates that the U-238 capture cross sections in the resonance region would be overestimated. Another approximation in set ZPR-6-5-ENDF/B-B is that the 14 plates of U-238 in the unit cell (Fig. II-11-3) can be represented by a single, average σ_p in each energy group. The effects of this approximation are discussed further below.

Cross section set ZPR-6-5-ENDF/B-B is not suitable for homogeneous-type calculations unless the cross sections in this set are weighted with the spatial distribution of the fluxes in the unit cell. The effects of such weighting on the critical mass is seen from Table II-11-IV to be about 5%.

It should be remarked that the spatial averaging is sensitive to the details of the intrastructure of the fluxes. The 25 broad groups used to calculate the spatial distribution of the fluxes cannot give adequate results (showing the true valleys and peaks) in the resonance region. While the effects of this approximation are hard to assess, it is expected that the U-238 capture cross sections would be overestimated. The 25-group calculation, however, should give reasonable results for the high energy groups where the cross sections are relatively smooth.

Another previously mentioned approximation that was used to obtain the intrastructure of the fluxes was the use of an averaged capture cross section in each group for all the U-238 plates in the unit cell. This approximation was investigated further by using multigroup cross sections, generated for each of the 14 plates of U-238, to obtain the intrastructure of the fluxes. This study showed definite differences in the shape of the fluxes between the two methods; however, when the spatial averaging was performed the two methods gave identical values for the critical mass.

Other approximations that are implicit in the calculations are: (1) continuous, thin slabs in a unit cell when in reality the plates are interrupted with stainless steel in the matrix and drawers (see Fig. II-11-3); (2) spatial self-shielding for scattering events in , nickel, chromium, and sodium are negligible, . . .) isotropic leakages; and (4) interface effects between

ons, i.e., core and reflector, are negligible and fundamental-mode type calculations to generate space independent cross section sets, as is done in MC², are adequate for both the core and reflector.

The effect of the isotropic leakage approximation was investigated by spatially averaging the cross sections with the fluxes along two dimensions in the unit cell. The procedure for this averaging was described earlier. The cross section set obtained in this manner was designated as ZPR-6-5-ENDF/B-D. This set comprised 25 broad energy groups. The value of the critical mass obtained with this set was 2046 kg of U-235. Based on the 25-group, 2-D spatial weighting, it appears that the assumptions of isotropic leakage in the unit cell may not be serious.

The effect of using one cross section set for core and reflector was investigated by generating a 25-group set for the reflector composition with zero buckling in the MC² code. This set was used in conjunction with set ZPR-6-5-ENDF/B-C for the critical mass calculation. The result was that the critical mass value was only 6 kg of U-235 below the value for ZPR-6-5-ENDF/B-C calculation shown in Table II-11-IV.

To evaluate the effects of the other approximations on the critical mass and other parameters is difficult and must await further development of analytical tools.

Other computational effects, such as 1-D versus 2-D diffusion and transport calculation, on the critical mass are listed in Table II-11-IV. It is seen that the critical mass calculated with a 2-D diffusion code is about 7 % lower than that obtained with a 1-D diffusion code even though the same cross section set was used. The reason for this discrepancy is due to the underestimation of the calculated value of reflector saving. The calculated extrapolated height was 171.4 cm compared with a measured value of 180.0 cm. The extrapolated height was obtained by: (1) calculating the radial critical buckling for a two-region slab; (2) calculating the critical height using the critical radius obtained in Step 1. Since the calculated reaction rates¹³ decrease as a function of radius (or height) faster than those measured, Step 1 in the extrapolated height calculation results in a small value for the critical buckling; i.e., large critical radius. Consequently, the one-region calculation of the extrapolated height is underpredicted.

The calculated critical mass with a 1-D transport code, using a calculated extrapolated height of 171.4 cm, was 1986 kg of U-235. The same calculation repeated with the measured extrapolated height of 180.0

resulted in a critical mass of 1870 kg. The critical mass calculated with the bilinearly averaged set, using a diffusion code, was 1930 kg of U-235. This value

should be compared with 2034 kg, the value obtained with set ZPR-6-5-ENDF/B-C.

The k_{eff} values for Assembly 5, calculated with the various sets using the experimental dimensions, are listed in the fourth column of Table II-11-IV.

It is seen from Table II-11-IV that the lowest calculated critical mass value was 1881 kg of U-235. This value would have been reduced further had the bilinearly weighted cross section been used. The reaction rate distribution that corresponds to the 1881 kg value remain lower than the measured values about the core-reflector interface. Furthermore, there is experimental evidence (see Heterogeneity Effects section) that shows that the analytical methods used to calculate the reactivity difference between a homogeneous and a heterogeneous sample, underpredicted this effect by a factor of 4. It is difficult to draw any diagnostic conclusions about ENDF/B data, unless all of these effects are taken into consideration.

HETEROGENEITY EFFECTS

There are two main types of heterogeneity effects associated with fast criticals loaded with plate-type geometries: (1) fast effect, and (2) resonance effects.

The fast effect depends on: (a) the physical shape of the fissile materials, i.e., thickness and length of column; and (b) the physical separation between the fissile and fertile plates. The thickness of the plate of fissile materials contributes positively to the reactivity of the system through an increase in the yield of neutrons per fission due to fission neutrons that never leave the plate before causing another fission, and to neutrons that leave the plate and return to cause fissions before reaching the equilibrium spectrum. The length of the plate affects the reactivity in a similar manner and contributes to the peaking of the fluxes in and around the plate. The fast heterogeneity effect is also affected by the increase or decrease in the total fissions of U-238 in the heterogeneous system relative to the homogeneous system. The nearness of the U-238 atoms to the fissile atoms bears importantly on the total amount of fissions in U-238.

The resonance heterogeneity effect is a self-shielding effect. Resonance neutrons are readily absorbed at the surface of the plate. This process allows fewer such neutrons to reach the interior of the plate and the overall effect is that the average reaction rate per atom in the plate is lowered. The self-shielding effect on the reactivity of the system can be both negative and positive. Self-shielding contributes negatively through the fission process and positively through the capture process. Self-shielding in the scattering process can also have positive and negative effects on the

system; the sign is determined by the relative importance of the neutron before and after scattering.

Both types of heterogeneity effects have been explicitly considered in the calculation of the critical mass. The critical mass parameter, however, does not provide any details about the mechanics of the heterogeneity effects. Thus a series of experiments, designed to yield pertinent data on the heterogeneity effects, was performed on Assembly 5 of ZPR-6.

The following three experiments were conducted:

- (1) measuring the reactivity change associated with two changes of loading patterns and the intrastructure of the fluxes in the unit cell of each loading pattern;
- (2) measuring and comparing the relative reactivity worth at the center of the core of homogeneous, rod-ded-type heterogeneous and pl

type heterogeneous samples;

- (3) measuring the reactivity worth of U-235 and U-238 as a function of sample thickness.

Intercomparison of measurements and calculations are made and certain conclusions are drawn.

LOADING PATTERN EFFECTS

The effect of changing the drawer loading pattern on the reactivity of the system provides convenient means by which methods used to account for heterogeneity effects can be tested. Calculations of σ_p for U-238 show that a considerable change in σ_p^{238} takes place when the normal loading pattern shown in Fig. II-11-3 is rearranged to the bunched loading pattern of Fig. II-11-6. This change in loading pattern was performed in two steps. First the normal loading pattern (Fig. II-11-3) was changed to the "reference" loading pattern shown in Fig. II-11-7. This step was deemed necessary so that the bunching of the U-238, Fig. II-11-6, would not require any shifting in position of either the U-235 plates or the sodium cans. In this manner, the reactivity change associated with the change in loading pattern may be associated with the bunching of U-238. Table II-11-V shows the calculated average σ_p^{238} and the corresponding capture cross sections for the three loading patterns. Table II-11-VI shows the spatially averaged U-238 capture cross sections for the three loading patterns.

The region in which the loading patterns were changed had a square cross section and comprised the central 9 x 9 drawer (Fig. II-11-8) in each half of the reactor. At a radius of about 20 cm from the center of this region there were four control rods per half in which the loading pattern was not changed.

The measured reactivity change associated with the change in loading pattern shown in Fig. II-11-3 to that shown in Fig. II-11-7 in the central square region (154 drawers in both halves) was -13.4 ± 0.4 lh ($-0.028\% \Delta k/k$). The calculated value for such change was -16.4 lh. The calculated value was obtained from the difference in k between the two cases assuming that the central square region could be represented by an equivalent circular region. The cross section set for the reference loading pattern was generated in the manner described earlier.

The change in loading pattern from the reference case (Fig. II-11-7) to the bunched case (Fig. II-11-6) was carried out in steps. The first step was a change in the loading pattern of the 9 central drawers per half (a total of 18 drawers). The second and subsequent steps bunched the U-238 in rings around the central drawers in each half. Table II-11-VII summarizes the experimental results. The total increase

FIG. II-11-6. Bunched U-238 Loading Patterns in Three-Drawer Cell of Assembly 5 of ZPR-6. ANL Neg. No. 112-9278.

activity in the 9×9 drawer central region was $+131.6$ lh ($0.275\% \Delta k/k$). The calculated value for such change was 93.8 lh.

The data shown in Table II-11-VII are compared with a $J_0^2(2.405 \text{ r}/\bar{R})$ distribution in Fig. II-11-9. The symbol $\bar{R}$ is the extrapolated radius of the system. The reactivity worth of a sample as a function of the radial dimension in cylindrical coordinates is proportional to $J_0^2(2.405 \text{ r}/\bar{R})$ distribution in bare reactors. It is seen that the fit is reasonable over the range of measurements. If it is assumed that the J_0^2 distribution is reasonable for the entire core, then the increase in reactivity due to the bunching of U-238 in the entire system is 218 lh. This increase in reactivity corresponds to 5.6% decrease in the critical mass (87.2 kg).

In addition to the measurements of loading pattern effects on the reactivity of the system, an independent series of experiments, measuring the fission rates in U-235 and U-238 as well as the capture rates in U-238, were made as a function of position in the unit cell. Such integral measurements provided tests to verify the adequacy of the theoretical methods for predicting the spatial intrastructure of the fluxes in the unit-cell.

Depleted uranium foils (0.2% enrichment, 5 mil thick by $\frac{5}{8}$ in. diam) and U-235 foils (93% enrichment, 5 mil thick by $\frac{5}{8}$ in. diam) were placed at corresponding locations about the center of the core in the two halves of the ZPR-6 machine. The foils were placed among the adjoining surfaces of all the plates in the unit cell. All foils were irradiated for one hour at a power level of about 50 W. Five hours after irradiation, the activities of the U-238 foils were counted for fission fragments (counting gammas with energies above 0.41 MeV) and for capture by counting the 106 keV gammas from excited Pu-239 in coincidence with x-rays produced by internal k conversion electrons. The five-hour waiting period was necessary to allow sufficient time for the buildup of Np-239 from U-239. The U-235 foils were counted for fission fragments in a manner similar to that used in counting the U-238 foils. A few foils were radiochemically analyzed for the absolute number of fissions in U-235 and the absolute number of fissions and captures in U-238. The radiochemical results were used to obtain efficiency factors to convert the rest of the data to absolute numbers. Irradiations of the U-235 and U-238 foils were performed in each of the loading patterns shown in Figs. II-11-6 and II-11-7.

In another experiment a $2 \times 2 \times \frac{1}{16}$ in. plate of enriched uranium and a $2 \times 2 \times \frac{1}{8}$ in. plate of depleted uranium (0.2% enriched) which were normally in the core, were replaced by packets of foils

FIG. II-11-7. Reference Loading Patterns of Three Drawer Cell for Bunching U-238 in Assembly 5 of ZPR-6. ANL Neg. No. 112-9223 Rev. 1.

equivalent in weight and enrichment to the two plates. This arrangement provided means by which the spatial distribution of the high energy fluxes within the fissile and fertile plates can be verified by measuring the fission rates. The packets were irradiated in the manner described earlier. From each of the $2 \times 2 \times 0.005$ in. foils comprising the packets, a $\frac{5}{8}$ in. diam foil was punched out from the center and counted. This procedure eliminated any requirement for edge effect corrections.

The results are shown in Figs. II-11-10 through II-11-13. Figure II-11-10 shows, across the cell of the reference loading pattern, the measured and calculated fission rates in U-235 and in U-238, the capture rates in U-238, the fission in U-238 to fission in U-235 ratio, and the capture in U-238 to fission in

TABLE II-11-V. DEPENDENCE OF σ_p AND CORRESPONDING σ_c OF U-238 ON LOADING PATTERN (CROSS SECTIONS ARE NOT SPATIALLY WEIGHTED)

Energy Group	E_L , keV	Drawer Heterogeneity							
		Homogeneous		Normal		Reference		Bunched	
		σ_p	σ_c	σ_p	σ_c	σ_p	σ_c	σ_p	σ_c
11	25.0	29.59	0.4367	24.87	0.4328	23.59	0.4318	19.50	0.4278
12	15.0	26.99	0.5487	23.31	0.5426	22.08	0.5405	18.90	0.5341
13	9.12	30.99	0.6575	25.62	0.6442	24.10	0.6402	19.76	0.6271
14	4.31	38.35	0.7770	29.14	0.7447	26.72	0.7348	20.90	0.7079
15	2.61	60.89	0.8882	34.74	0.8046	27.88	0.7732	21.64	0.7386
16	2.03	52.40	1.032	32.83	0.9256	26.44	0.8818	21.24	0.8407
17	1.23	33.95	0.8413	27.14	0.7871	24.78	0.7671	20.20	0.7249
18	0.961	32.20	0.9626	26.19	0.8944	24.71	0.8774	20.11	0.8192
19	0.583	32.46	0.9612	26.28	0.8725	24.91	0.8528	20.21	0.7785
20	0.275	33.67	0.8079	26.83	0.7103	25.45	0.6905	20.47	0.6128
21	0.101	36.23	1.068	27.88	0.9264	26.28	0.8988	20.94	0.7994
22	0.029	37.92	1.268	28.48	1.094	26.73	1.060	21.20	0.9466

TABLE II-11-VI. SPATIALLY AVERAGED U-238 CAPTURE CROSS SECTIONS FOR EACH LOADING PATTERN OF ZPR-6 ASSEMBLY 5

Energy Group		σ_c^{238} As Function of Loading Pattern		
No.	E_L , keV	Normal	Reference	Bunched
1	3670.	0.1242	0.1190	0.1200
2	2230.	0.0303	0.0291	0.0289
3	1350.	0.0733	0.0706	0.0694
4	825.	0.1456	0.1435	0.1439
5	500.	0.1403	0.1399	0.1423
6	300.	0.1351	0.1353	0.1368
7	180.	0.1513	0.1515	0.1522
8	110.	0.1815	0.1817	0.1817
9	67.0	0.2358	0.2367	0.2364
10	40.7	0.3321	0.3336	0.3318
11	25.0	0.4326	0.4340	0.4249
12	15.0	0.5409	0.5385	0.5218
13	9.12	0.6409	0.6391	0.6107
14	4.31	0.7428	0.7398	0.6861
15	2.61	0.8382	0.8746	0.7560
16	2.03	0.8688	0.7592	0.7206
17	1.23	0.7640	0.7174	0.6691
18	0.961	0.8811	0.8612	0.7706
19	0.583	0.8648	0.8545	0.7448
20	0.275	0.7052	0.6997	0.5887
21	0.101	0.9175	0.9182	0.7508
22	0.029	1.105	1.118	0.9545
23	0.0137	2.299	2.447	2.407
24	0.00306	1.162	1.228	1.112
25	0.00068	0.9873	1.007	0.9548

U-235 ratio. The calculated fission rates in U-235 and U-238 were obtained using the cell fluxes from the SNARG-1D code,⁸ with S_n of order 16 and modified Gaussian quadratures,⁹ and cross section set ZPR-6-5-ENDF/B-B with the in-plate capture cross section

FIG. II-11-8. Cross Sectional View of Assembly 5 of ZPR-6 Indicating the Location of the Region Used in Bunching Experiments. ANL Neg. No. 112-9161 Rev. 1.

of U-238; i.e., using 14 sub-sets for the 14 U-238 plates in the unit cell. The capture rates in U-238 were calculated in a similar manner except that the cross sections used outside the U-238 plates were infinite dilution cross sections. The capture cross sections of the U-238 foils in the U-238 plates were the same as those in the respective plates. Calculations reported in Figs. II-11-11 through II-11-13 were obtained in a similar manner.

TABLE II-11-VII. REACTIVITY CHANGE DUE TO BUNCHING U-238 IN ZPR-6 ASSEMBLY

$R_i - R_0$, cm	No. of Drawers	Reactivity Change, Δk
0.0-9.35	18	$+17.5 \pm 0.4$
9.35-15.85	32	$+30.6 \pm 0.4$
15.85-21.95	41	$+33.6 \pm 0.4$
21.95-28.45	63	$+49.9 \pm 0.4$

FIG. II-11-9. Extrapolation of the Worth of U-238 Bunching in Assembly 5 of ZPR-6. ANL Neg. No. 112-9163.

It may be seen from Fig. II-11-10 that the transport calculations underpredict the decrease in the U-235 fissions about the $\frac{1}{16}$ in. U-235 plates. This indicates that spatial self-shielding exists in the U-235 plates and that its magnitude is about 1%. (Note that the relative error in each of the measured points is 0.5%.) The calculated spatial distribution of the U-238 fission rates is in fair agreement with the measurements. The calculated capture rates in U-238 were normalized to the measured point between the $\frac{1}{16}$ and $\frac{1}{8}$ in. U-238 plates in the first drawer of the three-drawer cell. The step-function description of the capture cross sections of the U-238 monitor foils, being equal to those of the respective plates and infinite dilution elsewhere, does not reflect flux depression in the vicinity of the U-238 plates. The calculated ratio of fission in U-238 to fission in U-235 is underestimated by about 13%. The calculated ratio of capture in U-238 to fission in U-235 is overestimated by about 10%.

The accuracies in determining the absolute fission rates in U-235, in U-238, and captures in U-238 are respectively about 2, 5, and 5%.

Figure II-11-11 shows similar reaction rates and ratios across the bunched loading pattern of the unit cell. Again the spatial distributions of the reaction rates are reasonable. The calculated ratio of fission in U-238 to fission in U-235, however, is underestimated by about 26%. (Note that the calculated and meas-

FIG. II-11-10. Measured and Calculated Values of U-235 Fission Rate, U-238 Fission and Capture Rates, and Their Ratios in the Reference Loading Pattern. ANL Neg. No. 113-3061.

ured distributions of the reaction rates are arbitrarily normalized. However, the ratios are not normalized and are directly comparable to measurements.) The calculated ratio of capture in U-238 to fission in U-238 is overestimated by about 7%.

Figure II-11-12 shows the fission rate in the depleted uranium (0.2% enrichment) packet of foils

FIG. II-11-11. Measured and Calculated Values of U-235 Fission Rate, U-238 Fission and Capture Rates, and Their Ratios in the Bunched Loading Pattern. *ANL Neg. No. 115-3060.*

which was placed next to a U-235 plate. The packet simulated a $\frac{1}{8}$ in. depleted uranium plate. The calculated fission rate distribution within the packet is in reasonable agreement with the measured values. Figure II-11-13 shows the fission rate distribution in the enriched uranium (93% enrichment) packet. It

is seen that the distribution within the packet flat.

RELATIVE REACTIVITIES OF HOMOGENEOUS, PELLET-HETEROGENEOUS, AND PLATE-HETEROGENEOUS SAMPLES

The relative reactivity worths of nominally 2 in. cube plate, rodded, and homogeneous samples were measured in Assembly 5 of ZPR-6. The compositions of all the samples were essentially the same (see Table II-11-VIII). The plate-type samples had the loading pattern shown in Fig. II-11-14. Figure II-11-15 shows the pellet-type sample. All samples were encased in 1.90 x 1.90 x 2.00 in. stainless steel (304L) cans with wall thickness of 0.010 in. The homogeneous samples were mixtures of UC (12.7% enrichment) powder and sodium or UC, sodium, and stainless steel powder; the latter to mockup the stainless steel tubes in the rodded samples. The stainless steel and the UC were blended together and sintered; the particle size of the blend was such that only about 55% of the column would be used when packed under vibration. The empty volume fraction was filled with sodium according to the worth of the material in its parameter environment. The errors listed in Table II-11-IX reflect the errors associated with the oscillation method and not the period method.

It is seen from Table II-11-IX that the reactivity worth of a sample was not particularly sensitive to its environment. The homogeneous and rodded samples had almost the same reactivity worths. The reactivity worth of a plate-type heterogeneous sample, without sodium, was about 13% greater than that of the equivalent homogeneous sample. The reactivity worth of a plate-type heterogeneous sample, with sodium present, was about 20% more than the equivalent homogeneous sample. The reactivity difference between homogeneous Sample No. 6 and plate-heterogeneous Sample No. 2 is 0.464 lh (see Table II-11-IX). If this difference is extrapolated to the entire core volume, assuming that the radial and axial reactivity dependence is respectively $J_0^2(2.405 r/\bar{R})$ and $\cos^2[\pi z/\bar{Z}]$, the difference between the homogeneous and plate-heterogeneous cores would be 3.82% $\Delta k/k$. The calculated reactivity difference between the same two samples was 0.096 lh. If this difference in $\Delta k/k$ per unit volume of sample is extrapolated to the entire core volume, the value would be 0.800% $\Delta k/k$. This indicates that the large extrapolation used to obtain the measured reactivity difference between the homogeneous and heterogeneous cores is better than expected and that the analytical tools used to account for the heterogeneity effects are not sufficiently re-

There were no measurable reactivity differences be-

FIG. II-11-12. Fission Rate in Depleted Uranium Packet of Foils Simulating a $\frac{1}{8}$ in. Plate. *ANL Neg. No. 113-2965.*

FIG. II-11-13. Fission Rate in Enriched Uranium Packet of Foils Simulating a $\frac{1}{8}$ in. Plate in Assembly 5 of ZPR-6. *ANL Neg. No. 113-2968.*

TABLE II-11-VIII. DESCRIPTION OF SAMPLES USED IN HETEROGENEITY EFFECTS STUDY

Sample No.	Physical Description	Composition, g					
		U-235	U-238	C	Na	Stainless Steel	
						In Sample	In Can ^a
1	Heterogeneous ^b	82.95	635.78	25.00	0.00	49.51	23.74
2	Heterogeneous	82.95	635.78	25.00	48.00	49.65	23.74
3	Homogeneous	107.68	739.68	45.05	0.00	0.00	24.45
4	Homogeneous	108.96	751.01	45.74	34.36	0.00	43.37
5	Homogeneous	86.63	597.08	36.36	0.00	60.14	30.08
6	Homogeneous	87.86	605.59	36.88	44.44	61.01	29.08
7	Pellets	86.73	597.83	33.47	0.00	59.96	54.52
8	Pellets	87.38	602.30	33.71	42.29	59.90	54.56

^a 1 samples were canned in stainless steel (304L).
^b These samples were 1.90 x 1.90 x 2.00 in.

FIG. II-11-14. Heterogeneous Sample Loading Pattern. *ANL Neg. No. 112-9244 Rev. 1.*

FIG. II-11-15. Pellet Sample Loading Pattern. ANL Neg. No. 112-9276 Rev. 1.

TABLE II-11-IX. COMPARISON OF MEASURED REACTIVITY WORTHS OF HOMOGENEOUS, RODDED, AND PLATE SAMPLE, AT THE CENTER OF ZPR-6 ASSEMBLY 5

Sample No.	Sample Type	Sample Worth Excluding Can	
		Normal Environment	Bunched Environment
		Worth of UC Only, lh/kg of UC	
3	Homogeneous UC, no Na	3.027 ± 0.007	2.982 ± 0.007
5	Homogeneous UC plus SS, no Na	2.991 ± 0.011	2.932 ± 0.012
7	Rodded UC, SS clad, no Na	2.970 ± 0.009	—
1	Plate U, C, and SS, no Na	3.436 ± 0.007	3.391 ± 0.009
		Worth of UC, SS, Na Mix, lh/kg of Mix	
6	Homogeneous UC plus SS, with Na	2.234 ± 0.006	2.221 ± 0.008
8	Rodded UC, SS clad, with Na	2.227 ± 0.007	—
2	Plate, U, C, and SS, with Na	2.698 ± 0.010	2.644 ± 0.009

tween the rodDED and homogeneous samples. The calculated difference was also nil.

REACTIVITY WORTHS OF U-235 AND U-238 AS A FUNCTION OF THICKNESS

A calibrated fine autorod was used in the measurement of the reactivity worths of U-235 and U-238 as a function of sample thickness. The samples com-

prised 0.005 in. thick foils, with a nominal area sq in., stacked together, and placed in $2 \times 2 \times \frac{1}{8}$ in. stainless steel cans. In some cases only one foil was used. The stainless steel cans had wall thicknesses of 0.015 in. The measurement comprised oscillating the cans containing the foils relative to a similar empty can. The large samples were measured by the period method relative to void without the use of a sample holder.

The reactivity worths of the samples as a function of thickness were calculated by an integral transport method.¹⁴ This method accounts for self-shielding effects inside and flux perturbation outside the samples, as well as for fission, scattering, and edge effect perturbations in each broad energy group. The basic method consists of obtaining the ratio of the average flux in the sample to the unperturbed flux for each energy group and using this ratio to correct the components of $\Delta k/k$; i.e., absorption, fission, scattering, and leakage, obtained from first-order perturbation calculations.

The calculated and measured worths of U-235 and U-238 samples in Assembly 5 are given in Table II-11-X. The resonance-region absorption cross section (groups 11-22) used in the first-order perturbation (FOP) calculation of each U-235 sample are given in Table II-11-XI. These values were generated as a function of sample thickness by the formulation of L. Tirén.¹⁵ The homogeneous cross sections in the core are also listed in Table II-11-XI for comparison. It may be observed that the reason for the core cross sections being smaller than the thinnest foil is due to the presence of a stainless steel enclosure around the sample effecting a decrease in the Dancoff interaction between sample and core. Similar resonance-region cross sections for the U-238 samples are given in Table II-11-XII.

It is seen from Table II-11-X that the calculated and measured values of all U-235 samples are in good agreement. Comparison of the FOP values obtained with the cross sections appropriate for the core composition and those obtained with the proper cross sections generated for each sample indicate that resonance self-shielding effects in U-235 are rather small. The difference between the FOP values and the final results is due to broad group spatial effects. It is seen that these effects produce a net difference of about 3% for the 0.6 cm thick sample. The net difference, however, is the result of a significant high energy peaking inside the sample that is counteracted by a similarly significant flux depression in the low energy groups. For example, the integrated f

TABLE II-11-X. MEASURED AND CALCULATED CENTRAL WORTHS OF U-235 AND U-238 SAMPLES IN ZPR-6 ASSEMBLY 5

Sample Size, cm	Sample Weight, g	Calculated FOP ^a Worth, Ih	Final Calculated Worth, Ih	Measured Worth, Ih	% Difference, (Calculated - Measured) Calculated
U-235 Results					
4.365 × 4.365 × 0.0127	4.200	0.221 (0.221) ^b	0.221	0.209 ± 0.002	+5.8
4.420 × 4.420 × 0.0381	12.935	0.681 (0.681)	0.685	0.659 ± 0.002	+3.8
4.410 × 4.410 × 0.1140	38.582	2.032 (2.032)	2.040	1.938 ± 0.004	+5.0
4.401 × 4.401 × 0.1774	59.755	3.146 (3.147)	3.167	3.079 ± 0.012	+2.8
4.405 × 4.405 × 0.2914	98.472	5.176 (5.186)	5.245	4.978 ± 0.010	+5.1
5.080 × 5.080 × 0.5967	268.146	14.076 (14.118)	14.580	14.691 ± 0.010	-0.8
U-238 Results					
4.530 × 4.530 × 0.0139	5.345	-0.024 (-0.024)	-0.024	-0.026 ± 0.002	-8.3
4.532 × 4.532 × 0.0418	15.917	-0.071 (-0.071)	-0.071	-0.060 ± 0.002	+15.5
4.530 × 4.530 × 0.1670	63.958	-0.277 (-0.284)	-0.277	-0.235 ± 0.002	+15.2
4.865 × 4.865 × 0.3175	140.361	-0.600 (-0.675)	-0.602	-0.506 ± 0.002	+15.6
4.930 × 4.930 × 2.540	1151.27	-4.750 (-5.123)	-4.743	-3.88 ± 0.01	+22.2

^a FOP means first order perturbation; the values were multiplied by 1.099 which is the ratio of the calculated to measured normalization integral. 1% $\Delta k/k = 476$ Ih.

^b Values in parentheses were obtained using cross sections appropriate for the core composition and do not include resonance self-shielding effects in the sample.

TABLE II-11-XI. U-235 ABSORPTION CROSS SECTIONS AS A FUNCTION OF SAMPLE THICKNESSES FOR ZPR-6 ASSEMBLY 5, b

Energy Group		Homogeneous Core Set	Sample Thickness, cm					
No.	E , keV		0.0127	0.0381	0.1140	0.1774	0.2914	0.5967
11	25.0	3.061	3.063	3.062	3.057	3.054	3.050	3.043
12	15.0	3.550	3.553	3.550	3.542	3.538	3.531	3.521
13	9.12	4.209	4.210	4.209	4.208	4.208	4.208	4.211
14	4.31	5.507	5.512	5.509	5.505	5.504	5.504	5.506
15	2.61	6.823	6.838	6.826	6.799	6.784	6.761	6.721
16	2.03	8.217	8.231	8.214	8.175	8.151	8.116	8.050
17	1.23	10.008	10.05	10.02	9.946	9.900	9.833	9.704
18	0.961	13.22	13.31	13.26	13.13	13.05	12.92	12.67
19	0.583	15.23	15.37	15.29	15.09	14.96	14.76	14.35
20	0.275	19.68	20.10	19.87	19.33	18.98	18.52	17.76
21	0.101	32.93	34.18	33.47	32.04	31.17	30.02	28.06
22	0.029	35.74	38.31	36.70	34.17	32.87	31.30	28.90

group 1 (10-3.68 MeV) across the 0.1774 cm thick U-235 sample, is 1.6 times that incident on the sample. The integrated flux in the keV region is about 25% lower than that incident on the sample.

The calculated central reactivity worths of the U-238 samples are (see Table II-11-X) 15-20% greater than the measured values. This trend is not true for the thinnest sample; however, the measured worth of this sample is quite small and could be in error. The results in Table II-11-X also show that resonance self-shielding effects are greater than the group spatial self-shielding effects.

It has been suggested for use in parametric study¹⁷ that the inelastic scattering cross sections of U-238 in the ENDF/B file be reduced by 30% in every energy group. Such a reduction produces a critical mass which is 6% lower than the measured value and about 28% lower than the calculated value without the reduction. Furthermore, the calculated worths of the U-238 sample with the reduced cross sections were in much better agreement with the measured values. Yet such a reduction produced a larger discrepancy between the calculated and measured U-238 fission rate distributions than was obtained

TABLE II-11-XII. U-238 ABSORPTION CROSS SECTIONS AS A FUNCTION OF SAMPLE THICKNESS FOR ZPR-6 ASSEMBLY 5, b

Energy Group		Homogeneous Core Set	Sample Thickness, cm				
No.	E_i keV		0.0139	0.0418	0.1670	0.3175	2.540
11	25.0	0.433	0.434	0.433	0.431	0.429	0.420
12	15.0	0.543	0.544	0.543	0.536	0.529	0.495
13	9.12	0.644	0.649	0.645	0.636	0.628	0.603
14	4.31	0.745	0.761	0.752	0.727	0.710	0.663
15	2.61	0.805	0.823	0.806	0.763	0.735	0.667
16	2.03	0.926	0.947	0.918	0.853	0.817	0.737
17	1.23	0.787	0.825	0.793	0.728	0.694	0.620
18	0.961	0.894	0.968	0.917	0.824	0.779	0.688
19	0.583	0.873	0.981	0.907	0.789	0.739	0.644
20	0.275	0.710	0.850	0.757	0.618	0.562	0.460
21	0.101	0.926	1.234	1.006	0.786	0.703	0.512
22	0.029	1.094	1.526	1.197	0.887	0.758	0.496

before the reduction.¹³ Further evaluation of the U-238 cross sections is needed before the suggested 30% reduction can be accepted.

SODIUM VOID EFFECTS

There are two main effects associated with the removal of sodium from a fast nuclear reactor: (1) spectral effect, and (2) leakage effect. The leakage effect depends on the change in the transport cross sections of the homogenized core materials and on the gradients of the adjoint function and the perturbed real flux. The transport cross sections decrease upon removal of the sodium causing an increase in the mean free paths of neutrons and hence more neutron leakage. Generally, the leakage component of the sodium-void coefficient is negative.

The spectral effect of the sodium-void coefficient depends primarily on: (a) the relative importance of the neutron before and after scattering, and (b) on the number of captures in sodium; and secondarily on the change in averaged cross sections caused by the change in the neutron spectrum due to the removal of sodium. The reactivity effect associated with the scattering of neutrons is very sensitive to the group-dependent distribution of the adjoint function. For example, a neutron scattered from group i to group j results in a reactivity change proportional to the difference in importance between groups j and i . This difference is usually small and can be negative for some energy groups and positive for others. Thus, unless the scattering cross sections of sodium and the energy distribution of the adjoint function are accurately determined, it is extremely difficult to calculate this effect.

The capture of neutrons in sodium results in a positive reactivity effect upon the removal of sodium

from the system. The magnitude of this effect in assembly 5 of ZPR-6 is of the same order as that of the scattering effect.

The change in the averaged cross sections of core materials due to the removal of sodium can be accounted for in a straightforward manner when the voided region comprises the entire core. Conversely this change is negligible for the limiting case when the region size is very small. There are no codes at the present that can treat the situations wherein the size of the regions is neither large nor small. The importance of this effect, however, may be evaluated by studying the limiting cases.

Experimentally a series of measurements designed to study different aspects of the sodium-void effects were performed on Assembly 5 of ZPR-6. The experiments dealt with the following effects:

- (1) effects of sodium removal on the absolute fission rates in U-235 and U-238 and the absolute capture rates in U-238
- (2) mapping of sodium-void coefficient radially and axially
- (3) effects of the loading patterns on the sodium-void coefficient
- (4) effects of U-238 concentrations on the sodium-void coefficient
- (5) the relative sodium-void coefficient in homogeneous, rodged, and plate-type heterogeneous samples.

Intercomparisons of the measured sodium-void coefficients with the calculated values were also made. Cross section averaging in MC² with and without sodium and first-order perturbation versus k calculations in one and two dimensions were also studied.

EFFECTS OF SODIUM REMOVAL ON FISSION AND CAPTURE RATES

The effects of sodium voiding on the fission densities in U-235 and U-238 and the capture density in U-238 were investigated in Assembly 5. Enriched and depleted uranium foils (93 and 0.2% respectively) were irradiated in the central drawer of Assembly 5. Two irradiations were performed: one with the sodium "in" and another with the sodium "out" from a central region of 25 cm in radius. The sodium was removed from the entire axial height of the core. The foils were placed at intervals of two inches along the axial length at approximately the mid-section of the central drawer. The materials adjacent to the foils were a 1/8 in. thick depleted uranium plate on one side and a 1/2 in. thick sodium can in the case of sodium "in", or a 1/2 in. thick empty stainless steel can in the case of sodium "out", on the other side. The depleted foils were placed in one half of the

tor and the enriched foils were placed in the other half. In each irradiation, U-235 monitor foils were placed at the radial core-reflector interface away from the perturbed region. The relative activities of the fission products were determined by scintillation counting. Coincidence counting was used to determine the relative captures in U-238. Absolute determination of the number of fissions in U-238 and U-235 and capture in U-238 was also made on a few foils by the radiochemical techniques described earlier. The results from the radiochemical measurements were used to normalize the results obtained from scintillation counting. The results are shown in Table II-11-XIII. The uncertainty in each of the numbers in Table II-11-XI is 2% for the U-235 foils and 5% for the U-238 foils.

The results show: (1) the fissions per gram of U-238 were not affected by the removal of sodium, (2) the captures per unit weight of U-238 decreased by about 15% when the sodium was voided, and (3) the fissions in U-235 decreased also by about 12% upon removal of the sodium.

The fission and capture rates were calculated for the sodium "in" and sodium "out" cases using a diffusion theory code. Cross section set ZPR-6-5-ENDF/B-C (see Table II-11-IV) was used for the sodium "in" case and a comparable set generated for the sodium "out" case was used. The calculated and measured results are compared in Fig. II-11-16. The calculated spatial distributions of the reaction rates for the sodium "in" case decreases at a faster

TABLE II-11-XIII. THE EFFECTS OF SODIUM VOIDING ON THE ABSOLUTE FISSION DENSITIES IN U-238 AND U-235 AND THE CAPTURE DENSITY IN U-238

Z, cm	Foil No.	Absolute Fissions/g U-238		Absolute Captures/g U-238		Absolute Fissions/g U-235	
		With Na, 10 ⁹	w/o Na, 10 ⁹	With Na, 10 ¹⁰	w/o Na, 10 ¹⁰	With Na, 10 ¹¹	w/o Na, 10 ¹¹
0.00	1	3.66	3.69	1.92	1.63	1.489	1.298
5.08	2	3.64	3.67	1.92	1.61	1.474	1.297
10.16	3	3.64	1.68	1.88	1.59	1.477	1.291
15.24	4	3.57	3.61	1.84	1.57	1.448	1.271
20.32	5	3.46	3.41	1.80	1.51	1.389	1.225
25.40	6	3.37	3.31	1.74	1.45	1.337	1.190
30.48	7	3.09	3.18	1.66	1.40	1.280	1.120
35.56	8	3.08	2.99	1.61	1.31	1.208	1.072
40.64	9	2.72	2.76	1.47	1.22	1.122	0.994
45.72	10	2.79	2.51	1.32	1.14	1.045	0.912
50.80	11	2.22	2.26	1.21	1.03	0.931	0.823
88	12	1.92	1.98	1.07	0.91	0.909	0.733
96	13	1.65	1.62	0.93	0.81	0.710	—
66.04	14	1.29	—	0.76	—	—	—

FIG. II-11-16. Measured and Calculated Values of U-235 and U-238 Fission Rate, U-238 Capture Rate, and Their Ratios for the Sodium "In" and Sodium "Out" Cases in Assembly 5 of ZPR-6. ANL Neg. No. 113-8065.

rate than the measured values. The agreement is better for the sodium "out" case. The calculated U-238 to U-235 fission ratios are about 10% lower than the measured values. The calculated ratios of capture in U-238 to fission in U-235 are about 10% higher than measurements.

SODIUM-VOID COEFFICIENT MAPPING

The sodium-void coefficient was measured as a function of the position along the axial and radial directions in Assembly 5 with the normal loading

TABLE II-11-XIV. MEASURED AND CALCULATED SODIUM-VOID COEFFICIENT AS A FUNCTION OF AXIAL DIRECTION IN NORMAL LOADING OF ZPR-6 ASSEMBLY 5

Sections Voided	ΔZ^a cm	Sodium Weight, kg	Sodium-Void Coefficient			
			Measured ^b		Calculated, lh/kg	
			Ih/Section	Ih/kg Na	1-D Diffusion ^c	2-D Diffusion ^d
A	0.0-5.08	0.886	1.02	1.15	0.591	0.509 ^e (0.797) ^f
B	0.0-10.16	1.772	1.98	1.12	0.582	0.473 (0.779)
C	10.16-20.32	1.955	1.90	0.97	0.420	0.350 (0.628)
D	20.32-30.48	1.955	0.85	0.44	0.116	0.081 (0.333)
E	30.48-40.64	1.955	-0.40	-0.20	-0.287	-0.264 (-0.061)
F	40.64-50.80	1.955	-2.00	-1.02	-0.732	-0.640 (-0.494)
G	50.80	1.955	-2.64	-1.35	-1.161	-0.959 (-0.857)
H	60.96-71.12	1.955	-3.00	-1.54	-1.518	-1.108 (-1.050)
All Sections (measured or calculated)	0.0-71.12	13.502	-3.95	-0.293	-0.361	-0.341 (-0.116)
All Sections (summed)	0.0-71.12	13.502	-3.31	-0.245	-0.252 ^g	

^a ΔZ is the axial length of each section in each half of the core.

^b Typical uncertainty in each measurement was 0.10 lh. 1% $\Delta k/k = 476$ lh.

^c Calculated with the set appropriate to the "sodium-in" configuration with 1-D diffusion code and first order perturbation.

^d Difference in k calculation.

^e Calculated with set appropriate to "sodium-in" configuration.

^f Calculated with two sets: one for the "sodium-in" and one for the "sodium-out" configurations.

^g Difference in k calculation with change in reflector savings due to sodium removal taken into consideration.

pattern of Fig. II-11-3. The measurements in the axial direction consisted of determining the reactivity worth of sodium filled cans in sections 4 in. long in each half of the reactor relative to empty cans which were identical to the sodium containers. The sodium-void effect was measured in 9 central drawers per half (equivalent outer radius of 9.35 cm). The measured and the calculated results are given in Table II-11-XIV. The uncertainty in each of the measured values listed in Table II-11-XIV is about 0.10 lh.

Three sets of calculations are shown in Table II-11-XIV. The first order perturbation (FOP), 1-D diffusion calculations were performed with the MACH-1 code¹⁸ using the set appropriate for the sodium "in" configuration. In these calculations, the unperturbed fluxes, assumed to have a cosine axial distribution, were used. It is seen that the spectral component of the sodium void coefficient (according to FOP the value in section A is 98% spectral and 2% leakage) was incorrectly predicted. The agreement between the measured and calculated values improved as a function of Z , the axial height. The spectral component fraction of the void coefficient decreases as a function of Z and in section H the coefficient is 95% leakage and 5% spectral. Thus the improvement in agreement was primarily due to the leakage component and might be fortuitous since the unperturbed fluxes, assumed to have a cosine distribution, were used.

A check on these assumptions was obtained with a 2-D diffusion, k calculation using the same cross section set. The results show that at the core center, where the leakage component is nil, the unperturbed fluxes increased the coefficient by 15-20%. Comparison of the perturbed fluxes due to voiding the sodium in Section A, with the unperturbed fluxes in group 1 (10-3.68 MeV), group 15 (4.3-2.6 keV) and group 22 (0.10-0.029 keV), is shown in Fig. II-11-17. It is seen that the perturbation in groups 1, 15 and 22 is significant. The 2-D results also show that the distribution of the fluxes as a function of Z does not have a cosine shape.

The last set of 2-D, k calculations (values in parentheses) were obtained using two different cross section sets: one for the sodium-voided region and one for the normal core, Set ZPR-6-5-ENDF/B-C. Although the size of the voided regions may not be large enough to justify the use of cross sections averaged over the fundamental mode spectrum of the sodium-free core, a difference in k of 0.2% between the two sets resulted in the core of the size of Assembly 5 without sodium. This indicates that the difference between the values calculated with 2-D diffusion is due to the spatial self-shielding correction to the U-238 cross sections and the weighting with the fine structure of the fluxes.

The calculated coefficients shown in Table II-11-XIV should be multiplied by 1.10, the ratio of

FIG. II-11-17. Comparison of Perturbed and Unperturbed Fluxes Due to Sodium Voiding in a 2 in. Section at Core Center. ANL Neg. No. 113-3023.

of the calculated-to-measured normalization integral.¹⁶ With this normalization the calculated and measured values are within 20% of each other.

Radial sodium-void coefficient measurements were made by voiding the sodium from the front 8 in. of three drawers in each reactor half at five different radial positions. The centers of the drawers were loaded along a horizontal radius and were 0, 4, 7, 10 and 14 drawer locations from the center of the reactor. The measured and calculated results are given in Table II-11-XV. Some of the values shown were taken on different days, which may be regarded as an indication of reproducibility. It is seen that the

TABLE II-11-XV. MEASURED AND CALCULATED SODIUM-VOID COEFFICIENT AS A FUNCTION OF THE RADIAL DIRECTION IN THE NORMAL LOADING PATTERN OF ZPR-6 ASSEMBLY 5

Section Voided ^a (R_i-R_0), cm	Na Weight, kg	Measured ^b Specific Worth, Ih/kg Na	Calculated ^c Specific Worth, Ih/kg Na
0.0-2.76	1.242	1.10	0.521
0.0-2.76	1.242	1.08	
19.33-24.86	2.585	0.65	0.269
19.33-24.86	2.585	0.64	
35.91-41.43	2.585	-0.15	-0.163
52.48-58.00	2.585	-0.79	-0.599
52.48-58.00	2.585	-0.83	
52.48-58.00	2.585	-0.77	
74.58-80.10	2.585	-0.75	-0.745
74.58-80.10	2.585	-0.77	

^a The voided section extends axially 8 in. into each assembly half.

^b 1% $\Delta k/k = 476$ Ih.

^c 1-D diffusion code—first order perturbation theory—Set ZPR-6-5-ENDF/B-C.

FIG. II-11-18. Position Effects of Bunched U-238 Loading Patterns on Sodium-Void Coefficient. ANL Neg. No. 112-9222.

agreement between the calculated and measured values is similar to that shown in Table II-11-XV.

LOADING PATTERNS EFFECTS

The sodium-void coefficient was also measured in the reference and bunched loading patterns (Figs. II-11-7 and II-11-6, respectively) of the unit cell. The region in which the reference or bunched patterns were made was the central 9 x 9 drawers in each half of the reactor (Fig. II-11-8). The reactivity worth of the sodium was measured in the central 18 drawers, 9 per half. In each core the sodium was removed from the front 4 in., from the front 8 in. and from the entire axial length (28 in.) in each half of the reactor.

The sodium-void coefficient was also measured a slightly modified bunched pattern, Fig. II-11-18. The measurement was made in order to test the effect of the position of the "lump" of U-238 on the reactivity coefficient.

The measured and calculated results are given in Table II-11-XVI. The calculated values were obtained with cross section sets generated from Set ZPR-6-5-ENDF/B-A (see Table II-11-IV) with the appropriate corrections incorporated for spatial self-shielding in each loading pattern. It is seen that the trends of the dependence of the sodium-void coefficient on the loading pattern are predicted, but not the magnitude. It is also seen that by increasing the self-shielding in U-238 the void coefficient decreases significantly and becomes negative.

TABLE II-11-XVI. SODIUM-VOID COEFFICIENT DEPENDENCE ON LOADING PATTERN

Section Voided		Loading Pattern	Measured Specific ^a Worth, 1h/kg Na	Calculated ^b Specific Worth, 1h/kg Na
Length, in.	Weight, kg of Na			
Front: 4	1.772	Normal Loading (Fig. II-11-3) ^c SdCEDCDSDCD CEDSDSDCECD CDSdCDCEDS	1.12 ± 0.11	0.591
	1.955	Ref. Case (Fig. II-11-7) DCSESdDCDCD DCESSDCDECD DCSEEdDCDCD	0.77 ± 0.05	0.456
	1.955	Bunched Case (Fig. II-11-6) CCSESdDDDDC CCESSDDDEDC CCSSEdDDDDC	-0.01 ± 0.15	0.144
	1.955	Position Effect of Bunching (Fig. II-11-18) CCSEScdDDDD CCESSCDEDED CCSSEcdDDDD	0.05 ± 0.15	—
Front: 8	3.727	Normal Loading (Fig. II-11-3)	1.10 ± 0.05 ^d	0.483
	3.91	Ref. Case (Fig. II-11-7)	0.69 ± 0.06	0.397
	3.91	Bunched Case (Fig. II-11-6)	0.03 ± 0.05	0.074
	28	Normal Loading (Fig. II-11-3)	-0.293 ± 0.015	-0.326 -0.252 ^e
	13.62	Ref. Case (Fig. II-11-7)	-0.747 ± 0.015	-0.376 -0.326 ^e
	13.62	Bunched Case (Fig. II-11-6)	-1.248 ± 0.022	-0.630 -0.555 ^e
	13.62	Position Effect of Bunching (Fig. II-11-18)	-1.256 ± 0.040	—

^a 1% $\Delta k/k = 476$ 1h.

^b 1-D diffusion—first order perturbation theory.

^c Key: C = $\frac{1}{8}$ in. thick graphite

S = $\frac{1}{2}$ in. thick sodium can

d = $\frac{1}{16}$ in. thick depleted uranium (0.2% enrichment)

D = $\frac{1}{8}$ in. thick depleted uranium (0.2% enrichment)

E = $\frac{1}{16}$ in. thick enriched uranium (93% enrichment).

^d 1-D diffusion is the sum of two adjacent 4 in. sections.

^e 1-D diffusion— Δk calculation with axial leakage changes considered.

FIG. II-11-19. Cross Sectional View of Assembly 5 of ZPR-6 Indicating the Location of the Modified Region. ANL Neg. No. 112-9161 Rev. 2.

EFFECTS OF U-238 CONCENTRATIONS ON THE SODIUM-VOID COEFFICIENT

The influence of the concentration of U-238 on the sodium-void coefficient was experimentally investigated by altering the material composition in a central region of Assembly 5 (see Fig. II-11-19). The region has an equivalent radius of 30 cm extend-

ing through the axial height of the core (142.74 cm). The U-238 density in this region was reduced by almost a factor of 2, and the sodium density increased by 20%. Furthermore, the U-235 density was decreased by 25% in order to counteract the increase in the reactivity of the system due to the removal of the U-238 plates.

The atomic concentrations of the material, in the "normal" and "modified" compositions of Assembly 5 are shown in Table II-11-XVII. The reflector for both loadings was depleted uranium. The radial and axial thicknesses of the reflector were respectively 27 and 30 cm.

The measurements consisted of determining the reactivity worth of the sodium-filled cans relative to empty cans in the same sections which were described

TABLE II-11-XVII. MATERIAL CONCENTRATIONS OF THE NORMAL AND MODIFIED COMPOSITIONS OF ZPR-6 ASSEMBLY 5

Material in Core	Concentrations, (atoms/cm ³) × 10 ²⁴	
	Normal Assembly 5 Composition	Modified Assembly 5 Composition ^a
U-235	0.00154	0.00115
U-238	0.01056	0.00615
Na	0.00920	0.01120
Fe	0.00904	0.00992
Ni	0.00113	0.00125
Cr	0.002386	0.00264
C	0.01293	0.01293

^a The modified loading comprised a region 30 cm in radius and 142 cm long about the center of the core. The rest of the core had the same composition as the normal loading. This region extended to 30 cm radially.

TABLE II-11-XVIII. MEASURED AND CALCULATED SODIUM-VOID COEFFICIENT AS A FUNCTION OF THE AXIAL DIRECTION IN THE NORMAL AND MODIFIED LOADING PATTERNS OF ZPR-6 ASSEMBLY 5

Sections Voided	ΔZ , ^a cm	Normal Loading Pattern			Modified Loading Pattern		
		Sodium Weight, kg	Measured ^b Specific Worth, lh/kg Na	Calculated ^c Specific Worth, lh/kg Na	Sodium Weight, kg	Measured Specific Worth, lh/kg Na	Calculated ^c Specific Worth, lh/kg Na
B	0.0-10.16	1.772	1.12	0.582	2.157	-0.22	-0.766
C	10.16-20.32	1.955	0.97	0.420	2.380	-0.28	-0.917
D	20.32-30.48	1.955	0.44	0.116	2.380	-0.59	-0.156
E	30.48-40.64	1.955	-0.20	-0.289	2.380	-0.03	-0.549
F	40.64-50.80	1.955	-1.02	-0.732	2.380	-1.25	-1.973
G	50.80-60.96	1.955	-1.35	-1.161	2.380	—	-2.404
H	60.96-71.12	1.955	-1.54	-1.518	2.380	-1.97	-2.801
All Sections Measured	0.0-71.12	13.502	-0.293	-0.361	16.437	-1.12	-1.658
				-0.252 ^d			-1.745 ^d

^a ΔZ is the axial length of each section in each half of the core.

^b $1\% \Delta k/k = 476$ lh.

^c 1-D diffusion code—first order perturbation theory.

^d Difference in k calculation with change in reflector savings due to sodium removal taken into consideration.

earlier. The measured and calculated results are given in Table II-11-XVIII. The void coefficients for the normal loading pattern are also given for comparison. Uncertainties in the measured values of the sodium-void coefficients were typically 0.10 lh.

The calculated values for the modified loading pattern were obtained with a cross section set generated from Set ZPR-6-5-ENDF/B-A (see Table II-11-IV) with the appropriate corrections incorporated for spatial self-shielding.

It is seen that in the modified loading pattern the void coefficient is negative everywhere. Furthermore the calculated values are significantly larger than the measured numbers, indicating the sensitivity of the void coefficient to the adjoint spectrum.

RELATIVE SODIUM-VOID COEFFICIENT IN HOMOGENEOUS, RODDED, AND PLATE-TYPE HETEROGENEOUS SAMPLES

The sodium void coefficient was also measured in homogeneous, rodded, and plate samples at the center

TABLE II-11-XIX. DESCRIPTION OF DEPLETED URANIUM SAMPLES USED IN HETEROGENEITY EFFECTS STUDY

Sample No. ^a	Physical Description	Composition					
		U-235	U-238	C	Na	Stainless Steel	
						In Sample	Can
9	Homogeneous	2.09	680.24	36.25	0.00	0.00	71.58
10	Homogeneous	2.08	679.69	36.22	47.74	0.00	69.40
11	Heterogeneous	1.40	700.02	25.00	48.00	49.51	23.74
12	Heterogeneous	1.40	700.02	25.02	0.000	49.51	23.74

^a These samples were 1.90 x 1.90 x 2.00 in.

of the core. The measurements consisted of oscillat "matched" samples, one with and one without sodium, relative to each other. The matched samples were very nearly identical in composition except for sodium. Some of the samples used in these measurements were the same samples described earlier (Table II-11-VIII). In addition to those samples, the homogeneous and plate heterogeneous samples of depleted uranium listed in Table II-11-XIX were used.

The reactivity worth of the sodium in the three types of samples was measured in the normal and bunched loading patterns (Figs. II-11-3 and II-11-6) environments. The results are given in Table II-11-XX.

The results in Table II-11-XX show that effect of the environment on the sodium-void coefficient in the homogeneous samples was 70%. Such environmental effect was absent with the plate-heterogeneous sample. The sodium-void coefficient in the rodded sample was of the same magnitude as that in the plate sample rather than the homogeneous sample. The sodium-void coefficient in the depleted uranium, plate-heterogeneous sample is a factor of 2 smaller than that in comparable homogeneous samples.

The sodium-void coefficients in the plate-heterogeneous and in the homogeneous samples were calculated with Sets ZPR-6-5-ENDF/B-A and ZPR-6-5-ENDF/B-C, respectively, using the 1-D diffusion, first order perturbation code. It is seen that the heterogeneity effects on the sodium-void coefficient is predicted. The technique for analyzing rodded samples is being developed.

The sodium-void coefficient was also measured in the calandria assembly, without the UC pellets. The void coefficient was 0.653 ± 0.034 lh/kg.

TABLE II-11-XX. HETEROGENEITY EFFECTS ON THE SODIUM-VOID COEFFICIENT

Samples Used in Measurement	Description of Sample	Sodium-Void Coefficient, Ih/kg ^a		
		Loading of Surrounding		
		Normal ^b		Bunched ^c
		Measured	Calculated ^d	Measured
1 & 2	Enriched Plate Heterogeneous	+1.108 ± 0.046	+0.677	+1.148 ± 0.052
3 & 4	Enriched Homogeneous	+1.717 ± 0.070	—	+1.059 ± 0.079
5 & 6	Enriched Homogeneous with SS Powder	+1.697 ± 0.038	+0.695	+1.168 ± 0.059
7 & 8	Pellets in SS Assembly	+1.026 ± 0.043	—	—
9 & 10	Depleted Homogeneous	+2.514 ± 0.073	—	—
11 & 12	Depleted Plate Heterogeneous	+1.283 ± 0.050	—	—

^a 1% $\Delta k/k = 476$ lh.

^b Loading of surrounding is shown in Fig. II-11-3.

^c Loading of surrounding is shown in Fig. II-11-6.

^d 1-D diffusion code—first order perturbation theory.

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II-12. Analysis of Central Reactivity Worths Incorporating Resonance Self-Shielding

K. D. DANCE, R. A. KARAM, J. E. MARSHALL and R. B. POND

An integral transport theory, multiple scattering formulation to treat broad-group spatial-shielding effects in central reactivity calculations has been previously reported.¹ The broad-group spatial-shielding calculations have now been improved by inclusion of exact collision probabilities taken from the work of I. Carlvik,² and by improving the calculation of the outer flux perturbation factor. In addition, resonance self-shielding effects have been incorporated into the calculational technique using the formulation of L. Tirén.³ The self-shielding correction method has been applied to the calculation of central reactivity worths of various samples in a 2700-liter uranium-carbide fast core (ZPR-6 Assembly 5) and a 4000-liter UO₂ fast core (ZPR-6 Assembly 6).

In order to use the formulation of Tirén, which citly treats resonance shielding in the sample as well as resonance interaction between the sample

and the surrounding core, in a routine manner for many different samples, effective cross sections were generated as a function of the potential scattering cross section σ_p by the ERIC-2 code.⁴ These calculations were made with the usual approximations; i.e., narrow isolated resonances with flux proportional to $1/\Sigma_T$, no interference scattering, and small non-resonant absorption. The calculations were made for each resonance sequence separately and the results were then corrected for same-sequence and inter-sequence resonance overlap using the method given by R. Hwang.⁵

ENDF/B parameters were used to calculate fine group ($\Delta u = 0.25$) cross sections from 13 eV to 50 keV for U-238. Within each fine group a constant flux weighting for each resonance integral was used in ERIC-2. The capture and total cross sections calculated in this manner were used in the formulæ of

Hwang to correct the group-averaged cross sections for flux depletion (Hwang's f factor) and for inter-sequence overlap effects. Specifically the group-averaged cross section for process i , $\hat{\sigma}_i$, is given by

$$\hat{\sigma}_i = \frac{\int_{E_L}^{E_L+\Delta E} \phi(E') \sigma_i(E') dE'}{\int_{E_L}^{E_L+\Delta E} \phi(E') dE'} = \sigma_i^{(e)}/f, \quad (1)$$

where $\sigma_i^{(e)}$ and f are given as

$$\sigma_i^{(e)} = \sum_k \sigma_{ik}^{(e)} \left[1 - \sum_{k' \neq k} (\sigma_k^{(e)}/\sigma_p) \right] \quad (2)$$

$$f = 1 - (1/\sigma_p) \sum_k \sigma_k^{(e)} \left[1 - \sum_{k' \neq k} (\sigma_k^{(e)}/\sigma_p) \right];$$

$\sigma_p \equiv$ potential background scattering per absorber atom

$\sum_k \equiv$ summation over all resonance sequences

$\sigma_{ik}^{(e)}, \sigma_k^{(e)} \equiv$ partial and total neutron group cross sections calculated by ERIC-2.

The quantities in the brackets are the inter-sequence overlap correction terms. For the U-238 calculations same-sequence overlap corrections were ignored since these corrections are quite small.

Group cross sections for the 0.25 lethargy intervals were generated for σ_p values between 15 and 5000 b, as well as σ_p equal to infinity. These fine group cross sections were combined, assuming equal Δu weighting, to form cross sections for the broad energy group structures shown in Table II-12-I.

The broad-group cross sections were least squares fit to obtain fitting parameters for use in the resonance self-shielding calculations. The fits were made for the cross sections relative to infinite dilution cross sections as a function of σ_p , and are shown in Figs. II-12-1 and II-12-2. The infinite dilution cross sections are given in Table II-12-II.

Fitting parameters were also obtained for U-235 in basically the same manner used for the U-238 calculations. However, since ENDF/B gives only smooth cross sections above 61 eV for U-235, it was necessary

TABLE II-12-I. GROUP STRUCTURE ENERGY BOUNDS

Upper Energy, eV	Fine Group	Broad Group Structure A	Broad Group Structure B
6.74 × 10 ⁴	1	11	10
5.25	2		
4.09	3	12	11
3.18	4		
2.48	5	13	12
1.90	6		
1.50	7	14	13
1.17	8		
9.12 × 10 ³	9	15	14
7.10	10		
5.53	11	16	15
4.31	12		
3.35	13	17	16
2.61	14		
2.03	15	18	17
1.58	16		
1.23	17	19	18

Upper Energy, eV	Fine Group	Broad Group Structure A	Broad Group Structure B
9.61 × 10 ²	18	19	19
7.49	19	20	
5.83	20	21	20
4.54	21		
3.54	22	22	21
2.75	23		
2.15	24	23	22
1.67	25		
1.30	26	24	22
1.01	27		
7.89 × 10 ¹	28	25	26
6.14	29		
4.79	30	26	27
3.73	31		
2.90	32	27	28
2.26

FIG. II-12-1. Ratio of U-238 Absorption Cross Section to Infinite Dilution Cross Section Versus Potential Scattering Cross Section, σ_p , for Broad Group Energy Structure B. *ANL Neg. No. 113-2759.*

FIG. II-12-2. Ratio of U-238 Absorption Cross Section to Infinite Dilution Cross Section Versus Potential Scattering Cross Section, σ_p , for Broad Group Energy Structure A. *ANL Neg. No. 113-2760 Rev. 1.*

to use the unresolved resonance parameters derived by E. M. Pennington.⁶ The U-235 unresolved resonance parameters used are given in Tables II-12-III and II-12-IV. The constant $\bar{\Gamma}_n^0$ parameters given by Pennington require the addition of smooth capture and fission corrections to match the ENDF/B data. The smooth corrections are given in Table II-12-IV.

Inter-sequence corrections for the U-235 results were made exactly as described for the U-238 results. In addition, approximate same-sequence overlap corrections were made using results calculated by Hwang. The ERIC-2 code, however, is not well suited to calculating broad, low energy resonances such as exist for U-235. Therefore, RABBLE,⁷ a code for the computation of resonance absorption, was used to generate U-235 cross sections in the resolved resonance energy region. For U-238, which has much narrower resonances, RABBLE and ERIC-2 calculations were in good agreement in the entire resolved energy range

TABLE II-12-II. INFINITE DILUTION U-238
ABSORPTION CROSS SECTIONS

Broad Group Structure A		Broad Group Structure B	
Group	σ_a^∞	Group	σ_a^∞
11	0.33255	10	0.33255
12	0.45062	11	0.45062
13	0.58304	12	0.58304
14	0.72740	13	0.72740
15	0.89574	14	0.95838
16	1.11115	15	1.20845
17	1.36102	16	1.51094
18	1.69186	17	1.69186
19	2.74946	18	2.38114
20	3.30838	19	3.32164
21	3.50751	20	3.43244
22	11.7157	21	16.6112
23	21.1783	22	47.0381
24	24.2201		
25	1.53269		
26	91.3845		

TABLE II-12-IV. U-235 RESONANCE PARAMETERS AT
SMOOTH CROSS SECTIONS

Fine Group	Γ_{f02} , eV	Γ_{f01} , eV	σ_γ —Smooth, b	σ —Smooth, b
1	0.570	0.285	0.0005	0.0025
2	0.580	0.290	-0.0111	-0.0561
3	0.525	0.263	0.0046	0.0210
4	0.475	0.238	0.0159	0.0658
5	0.450	0.225	0.0249	0.0975
6	0.440	0.220	0.0350	0.1312
7	0.438	0.219	0.0368	0.1327
8	0.355	0.178	0.0368	0.1327
9	0.323	0.162	0.1231	0.3605
10	0.323	0.162	0.0920	0.2603
11	0.322	0.161	0.1794	0.4921
12	0.325	0.163	0.1449	0.3883
13	0.350	0.175	0.0991	0.2599
14	0.410	0.205	-0.0070	-0.0213
15	0.360	0.180	0.0801	0.2104
16	0.320	0.160	0.3327	0.7777
17	0.270	0.135	0.2946	0.5678
18	0.255	0.128	0.2946	0.5678
19	0.405	0.203	0.6010	1.711
20	0.455	0.228	0.6010	1.711
21	0.380	0.190	0.0367	0.104
22	0.390	0.195	-0.3611	0.735
23	0.280	0.140	1.4227	3.176
24	0.248	0.124	0.1310	0.2420
25	0.242	0.121	-0.347	-0.640
26	0.250	0.125	-0.347	-0.640
27	0.280	0.140	-2.408	-4.885
28	0.295	0.148	-1.328	-8.758

and the ERIC-2 results were used in determining the final U-238 fitting parameters. Curves of the final U-235 cross sections as a function of σ_p are shown in Figs. II-12-3 through II-12-6. The infinite dilution cross sections are shown in Table II-12-V.

The final U-238 and U-235 fitting coefficients have been incorporated into the self-shielding correction code⁴ so that resonance shielding effects for U-238 and U-235 may be treated routinely. Similar fitting coefficients will be determined for the important

TABLE II-12-III. U-235 UNRESOLVED RESONANCE PARAMETERS

Parameter	$\ell = 0$		$\ell = 1$			
	$J = 3$	$J = 4$	$J = 2$	$J = 3$	$J = 4$	$J = 5$
$\bar{D}$, eV	1.00	1.00	1.16	1.00	1.00	1.12
$\bar{\Gamma}_n^0$, eV ^{1/2}	1.0×10^{-4}	1.0×10^{-4}	2.32×10^{-4}	2.00×10^{-4}	2.00×10^{-4}	2.24×10^{-4}
$\bar{\Gamma}_\gamma$, eV	0.035	0.035	0.035	0.035	0.035	0.035
$\bar{\Gamma}_f$, eV	Table V	Table V	0.332	0.127	0.286	0.143
ν_f	2	1	2	1	2	1
μ	1	1	1	2	2	1

FIG. II-12-3. Ratio of U-235 Absorption Cross Section to Infinite Dilution Cross Section Versus Potential Scattering Cross Section, σ_p , for Broad Group Energy Structure B. *ANL Neg. No. 113-2758 Rev. 1.*

FIG. II-12-4. Ratio of U-235 Fission Cross Section to Infinite Dilution Cross Section Versus Potential Scattering Cross Section σ_p , for Broad Group Energy Structure B. *ANL Neg. No. 113-2757.*

FIG. II-12-5. Ratio of U-235 Absorption Cross Section to Infinite Dilution Cross Section Versus Potential Scattering Cross Section, σ_p , for Broad Group Energy Structure A. *ANL Neg. No. 113-2755.*

FIG. II-12-6. Ratio of U-235 Fission Cross Section to Infinite Dilution Cross Section Versus Potential Scattering Cross Section, σ_p , for Broad Group Energy Structure A. ANL Neg. No. 113-2756.

TABLE II-12-V. INFINITE DILUTION U-235 ABSORPTION AND FISSION CROSS SECTIONS

Broad Group Structure A			Broad Group Structure B		
Group	σ_a^∞	σ_f^∞	Group	σ_a^∞	σ_f^∞
11	2.8464	2.1578	10	2.8464	2.1578
12	3.1475	2.3577	11	3.1475	2.3577
13	3.5485	2.6341	12	3.5485	2.6341
14	4.1237	3.0178	13	4.1237	3.0178
15	4.9331	3.5043	14	5.2157	3.7006
16	6.1303	4.3398	15	6.8960	4.9093
17	7.8052	5.6602	16	8.2983	6.0882
18	9.9329	7.0276	17	9.9329	7.0276
19	12.680	8.4903	18	11.883	8.0187
20	16.559	12.149	19	14.482	10.107
21	21.316	15.294	20	20.087	14.544
22	27.357	18.124	21	31.337	20.566
23	35.316	23.008	22	64.157	41.705
24	45.930	30.934			
25	66.882	45.112			
26	79.531	49.668			

plutonium isotopes in order that measurements in future plutonium cores can be calculated.

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II-13. Reaction Rate Calculations in Zero Power Reactor (ZPR) Plate-Type Cells

K. D. DANCE

An analytic method for predicting the space-dependent, effective resonance integral of an absorber foil in a plate-type cell containing the same resonance absorber as the foil has been developed.¹ In an infinitely repeating array of regularly spaced resonance absorber

plates separated by regions of pure scattering diluent of thickness t_1 , the effective resonance integral I of an infinitely thin absorber foil at a distance x from an absorber plate has been shown to be

$$I = C_x I_0^{(2)} + (1 - C_x) I_\infty,$$

are

$$C_x = E_2(\Sigma_{s1}x)/2 + E_2[\Sigma_{s1}(t_1 - x)]/2$$

$I_\infty \equiv$ infinite dilution resonance integral for the absorber material

$I_0^{(2)} \equiv$ resonance integral associated with an absorber plate twice as thick as the actual absorber plates, but separated by the same thickness of diluent, t_1 .

The $E_2(\)$ functions are the usual exponential integrals of order two, and Σ_{s1} is the scattering cross section of the diluent material.

The analytic expression for the space-dependent

resonance integral will be used in the analysis of the detailed reaction rate measurements made in the ZPR cell structures with fissile and fertile foils. Sample calculations have indicated that for regular lattices an accuracy of a few percent or better can be expected when using the simple analytic expression given above. Work is continuing on the derivation of an analytic expression for the resonance integral as a function of position within an absorber plate.

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II-14. An Extended Equivalence Principle Between Homogeneous and Heterogeneous Resonance Integrals in Slab Geometry

J. P. PLUMMER and R. G. PALMER

Currently, nearly all cross section preparation codes utilize a two-region cell equivalence principle in the resonance energy range. The method gives good results for the usual power reactor pin fuel arrangement. In many of the zero-power fast criticals, however, a given resonance absorber can exist in plates at different concentrations and mixed with different diluents. For example, U-238 in the ZPPR materials inventory exists in the forms U_3O_8 , U, Pu-U-Mo, and UO_2 .

In slab geometry, conventional equivalence principles assume that the repeating lattice consists of identical cells with two regions, one containing absorber atoms and perhaps admixed diluent (region A) and the other a diluent region (M). See Fig. II-14-1.

We have extended L. Dresner's treatment¹ of heterogeneous resonance integrals in the narrow resonance approximation to apply to a four-region unit cell, as shown in Fig. II-14-2. The plates A and B contain absorber atom concentrations N_A and N_B and are of thickness d_A and d_B , respectively.

The absorption rate in an A-plate is made up of three contributions:

- I. From neutrons which suffered their last collision in a B-plate:

$$P_{BA}\Sigma_{pB}\phi V_B \frac{\Sigma_{aA}}{\Sigma_{tA}}, \quad (1)$$

where

$\tau_{BA} \equiv$ probability that a neutron which suffered its last collision in a B-plate will collide next in an A-plate.

$\phi \equiv$ asymptotic flux per unit lethargy. This would be the actual flux at the resonance energy if the resonance did not exist.

$V_B \equiv$ volume of a B-plate.

$\Sigma_{aA} \equiv$ macroscopic absorption cross section for an A-plate.

$\Sigma_{tA} \equiv$ macroscopic total cross section for an A-plate.

$\Sigma_{pB} \equiv$ macroscopic non-resonant cross section in a B-plate.

- II. From neutrons which collided last in a diluent region:

$$P_{AM}\phi V_A \Sigma_{aA}, \quad (2)$$

where

$P_{AM} \equiv$ probability that a neutron which suffered its last collision in an A-plate will have its next collision in a diluent region.

$V_A \equiv$ volume of an A-plate.

- III. From neutrons which suffered their last collision

FIG. II-14-1. Two-Region Cell. ANL Neg. No. 113-3029.

FIG. II-14-2. Four-Region Cell. ANL Neg. No. 113-3030.

in an A -plate (including the A -plate in question):

$$P_{AA}\phi\Sigma_{pA}\frac{\Sigma_{aA}}{\Sigma_{tA}}V_A, \quad (3)$$

where

P_{AA} = probability that a neutron which suffered its last collision in an A -plate also has its next collision in an A -plate.

Σ_{pA} = macroscopic non-resonant cross section in an A -plate, $\Sigma_{pA} = \Sigma_{pA}^{(abs)} + \Sigma_A^{(dil)}$. Here, $\Sigma_{pA}^{(abs)} = N_A\sigma_p$ where σ_p is the non-resonant cross section of the absorber atom and $\Sigma_A^{(dil)}$ is the total cross section (assumed to be flat over the resonance of the absorber atom) of the admixed diluents in the A -plate.

Using the reciprocity relation, P_{BA} can be written in terms of P_{AB} :

$$P_{BA} = \frac{d_A}{d_B} \frac{\Sigma_{tA}}{\Sigma_{tB}} P_{AB}. \quad (4)$$

Since the total probability must be normalized, that is,

$$P_{AA} + P_{AB} + P_{AM} = 1, \quad (5)$$

P_{AA} may be replaced by $1 - P_{AB} - P_{AM}$. In addition to a simplification, this replacement insures conservation of total probability no matter what later approximations are made to P_{AB} or P_{AM} .

Collecting terms, the absorption rate per unit lethargy in an A -plate is

$$N_A I' V_A \phi = V_A \frac{\Sigma_{pB}}{\Sigma_{tB}} \Sigma_{aA} \phi P_{AB} + V_A \Sigma_{aA} \phi P_{AM} \\ + V_A \frac{\Sigma_{aA}}{\Sigma_{tA}} \Sigma_{pA} \phi (1 - P_{AB} - P_{AM}). \quad (6)$$

Integrating over the resonance and canceling the common factor $V_A \phi$ gives

$$N_A I = \Sigma_{pA} \int_0^\infty \frac{\Sigma_{aA}}{\Sigma_{tA}} du + \int_0^\infty \Sigma_{aA} \left[\frac{\Sigma_{pB}}{\Sigma_{tB}} \right. \\ \left. - \frac{\Sigma_{pA}}{\Sigma_{tA}} \right] P_{AB} du + \int_0^\infty \Sigma_{aA} \left[1 - \frac{\Sigma_{pA}}{\Sigma_{tA}} \right] P_{AM} du. \quad (7)$$

The integrals over lethargy are extended over the range zero to infinity since the resonance parameters in the cross section cause the integrand to contribute only over the resonance. I is called the effective resonance integral and is the lethargy-integrated absorption cross section which would be required to produce the same amount of absorption in the resonance, assuming the flux has the value it would have if there were no resonance. Notice that the term involving P_{AB} vanishes if there is only a single type of absorber plate ($A \equiv B$)

or when the ratio of admixed diluent-to-absorber is same in both plates A and B . Thus the A - A lattice is a particularly simple special case of the A - B lattice. Since it is very easy to do at this point, and because it will serve to illustrate the method we wish to employ, we shall derive the usual two-region equivalence principle.

First it is necessary to have expressions for the escape probabilities. These are obtained in a straight-forward way, so we shall simply state the results:

$$P_{BA} = \frac{1}{2\bar{\ell}_B \Sigma_{tB}} \left\langle \frac{(1 - \Gamma_A)(1 - \Gamma_B)(\Gamma_1 + \Gamma_2)}{1 - \Gamma_1 \Gamma_2 \Gamma_A \Gamma_B} \right\rangle \quad (8)$$

where $\langle \rangle$ in this paper, denotes an average over the chord length distribution function for a slab, and

$$\Gamma_i = e^{-\Sigma_{ti} r_i}, \quad (9)$$

where r_i is the path length through the slab denoted by i in the direction of a particular chord and Σ_{ti} is the total macroscopic cross section of the material in that slab at the energy in question. The quantity $\bar{\ell}_B$ is the mean chord length in plate B . $\langle \Gamma_i \rangle$ is given exactly, for slab geometry, in terms of exponential integral functions of order 3:

$$\langle \Gamma_i \rangle = 2E_3(\Sigma_{ti} d_i), \quad (10)$$

where d_i is the perpendicular thickness of the slab.

The following identity is also exact:

$$\langle \Gamma_A \Gamma_B \dots \Gamma_q \rangle = 2E_3(\Sigma_{tA} d_A + \Sigma_{tB} d_B + \dots + \Sigma_{tq} d_q). \quad (11)$$

Similarly,

$$P_{AM} = \frac{1}{2\bar{\ell}_A \Sigma_{tA}} \left\langle \frac{1 - \Gamma_A}{1 - \Gamma_1 \Gamma_2 \Gamma_A \Gamma_B} \right. \\ \left. \cdot [(1 - \Gamma_1)(1 + \Gamma_2 \Gamma_B) + (1 - \Gamma_2)(1 + \Gamma_1 \Gamma_B)] \right\rangle. \quad (12)$$

For the two-region cell, $\Gamma_B = \Gamma_A$ and $\Gamma_1 = \Gamma_2 \equiv \Gamma_M$, and thus P_{AM} reduces to

$$P_{AM} = \frac{1}{\bar{\ell}_A \Sigma_{tA}} \left\langle \frac{(1 - \Gamma_A)(1 - \Gamma_M)}{1 - \Gamma_M \Gamma_A} \right\rangle \quad (13)$$

Now we assume that the average of a product equals the product of the averages.* This approximation cannot be justified except by testing the accuracy of the final results.

The Wigner rational approximation must also be invoked. This approximation can be stated as follows:

$$2E_3(\tau) \approx \frac{1}{1 + 2\tau}. \quad (14)$$

This is good for small τ . It is also correct in the heart of the resonance where τ is very large, since both sides

* This is done by K. Dance in Ref. 2.

(14) then approach zero. Thus there is only a region on each shoulder of the resonance where the approximation is not too good.

This approximation is used for energy-dependent transmission probabilities, since energy-independent transmission probabilities, like $\langle \Gamma_M \rangle$, can be treated exactly.

With these approximations, and letting $C \equiv \langle \Gamma_M \rangle$, P_{AM} becomes

$$P_{AM} \approx \frac{1}{1 + \frac{\bar{\ell}_A \Sigma_{tA}}{1 - C}}; \quad (15)$$

substituting this into Eq. (7) yields

$$N_A I = \int_0^\infty \Sigma_a(u) \frac{\Sigma_p^*}{\Sigma_t^*(u)} du, \quad (16)$$

where

$$\Sigma_p^* = \Sigma_p + \frac{1 - C}{\bar{\ell}} \quad (17)$$

and

$$\Sigma_t^* = \Sigma_t + \frac{1 - C}{\bar{\ell}} \quad (18)$$

The cross sections refer to the absorber plate, although we have dropped the indices A and B , since $A \equiv B$ in the two-region lattice.

Equations (16)–(18) represent the two-region equivalence principle in its most rudimentary form. Further embellishments are strictly empirical. An example of a typical modified equivalence relation³ is

$$\Sigma_p^* = \Sigma_p + \frac{a}{\bar{\ell}} \frac{(1 - C)}{1 + (b - 1)C}. \quad (19)$$

For the slab geometry cases examined it was found that the empirical constants a and b were best fitted by the values $a = 1.27$, $b = 1.2$. Hereafter we shall refer to Eq. (19) as the simple equivalence principle (S.E.P.).

A-B Lattice

Again assume that

$$\langle \theta_1 \theta_2 \cdots \theta_n \rangle = \langle \theta_1 \rangle \langle \theta_2 \rangle \cdots \langle \theta_n \rangle \quad (20)$$

and use the rational approximation for $\langle \Gamma_A \rangle$ and $\langle \Gamma_B \rangle$:

$$\langle \Gamma_A \rangle \approx \frac{1}{1 + \bar{\ell}_A \Sigma_{tA}} \quad (21)$$

$$\langle \Gamma_B \rangle \approx \frac{1}{1 + \bar{\ell}_B \Sigma_{tB}} \quad (22)$$

and make the following definitions: $C_1 \equiv \langle \Gamma_1 \rangle$, $C_2 \equiv \langle \Gamma_2 \rangle$, $C_{ave} = \frac{C_1 + C_2}{2}$, and $\bar{C}^2 = C_1 C_2$. With some algebra, but no further approximations, Eq. (12) reduces to

$$P_{AM} = \left\{ 1 + \Sigma_{tA} \bar{\ell}_A \left[\frac{1 + \frac{\Sigma_{tB} \bar{\ell}_B}{\Sigma_{tA} \bar{\ell}_A} C_{ave} + \Sigma_{tB} \bar{\ell}_B}{1 - \bar{C}^2 + (1 - C_{ave}) \Sigma_{tB} \bar{\ell}_B} \right] \right\}^{-1} \quad (23)$$

In comparing this result with Eq. (15) it is evident that if the term in the square brackets of Eq. (23) were not energy dependent, P_{AM} would have the same functional form as in the 2-region case. Then, temporarily neglecting the term in Eq. (7) involving P_{AB} , the resulting equivalence principle would be identical to Eq. (17), except that $\Sigma_p^* - \Sigma_p$ would be some constant other than $(1 - C)/\bar{\ell}$.

In order to get P_{AM} into this desirable form, we write it as follows:

$$P_{AM} \approx \left\{ 1 + \Sigma_{tA} \bar{\ell}_A \left[\frac{1 + (N_B \bar{\ell}_B / N_A \bar{\ell}_A) C_{ave} + k N_B \bar{\ell}_B}{1 - \bar{C}^2 + (1 - C_{ave}) k N_B \bar{\ell}_B} \right] \right\}^{-1}, \quad (24)$$

where k is a constant to be determined empirically. Now, it follows as in the two-region case that the first and third terms on the right hand side of Eq. (7) can be approximated by substituting the following effective potential scattering cross section into the homogeneous resonance integral:

$$\Sigma_{pA}^* = \Sigma_{pA} + \frac{1 - \bar{C}^2 + (1 - C_{ave}) k N_B \bar{\ell}_B}{1 + (N_B \bar{\ell}_B / N_A \bar{\ell}_A) C_{ave} + k N_B \bar{\ell}_B}. \quad (25)$$

This can be modified so that upon reducing to the two-region case, the modified rational approximation of Eq. (19) results. Therefore,

$$\Sigma_{pA}^* = \Sigma_{pA} + \frac{a[1 - \bar{C}^2 + (1 - C_{ave}) k N_B \bar{\ell}_B]}{[1 + (N_B \bar{\ell}_B / N_A \bar{\ell}_A) C_{ave} + k N_B \bar{\ell}_B]} \cdot \frac{1}{1 + (b - 1) C_{ave}}. \quad (26)$$

Unless the admixed diluent in the A -plate is the same type and is present in the same ratio of diluent-to-absorber atom concentration as in the B -plate, the term in Eq. (7) involving P_{AB} cannot be ignored.

Using approximation (20) and the rational approximation again, it is easy to show that

$$P_{AB} \approx \frac{\Sigma_{tB} \bar{\ell}_B C_{ave}}{(1 + \bar{\ell}_A \Sigma_{tA})(1 + \bar{\ell}_B \Sigma_{tB}) - \bar{C}^2}. \quad (27)$$

We wish to approximate P_{AB} with a non-energy-dependent expression and hope that this suffices since the term involving P_{AB} appears to be small compared

to the other terms in Eq. (7). As a very crude approximation, take

$$P_{AB} \approx \frac{N_B \bar{\ell}_B C_{ave}}{N_B \bar{\ell}_B + N_A \bar{\ell}_A}. \quad (28)$$

This expression has the correct limit in the two cases in which $P_{AB} \rightarrow 0$, namely as $N_B \rightarrow 0$ or as $N_A \rightarrow \infty$.

Even with this substitution, it is not possible to incorporate the second term on the right side of Eq. (7) into the resonance integral as a further modification of the effective potential scattering cross section. Instead it must be added at the resonance integral level. Thus, Eq. (7) becomes

$$I = I_h(\sigma_{pA}^*) + \frac{\alpha N_B \bar{\ell}_B C_{ave}}{N_A \bar{\ell}_A + N_B \bar{\ell}_B} [I_h(\sigma_{pB}) - I_h(\sigma_{pA})], \quad (29)$$

where α is an empirical constant and

$\sigma_{pA}^* = \Sigma_{pA}^*/N_A$ and $I_h(\sigma_{pA}^*)$ is the homogeneous resonance integral with a modified potential scattering cross section, σ_{pA}^* , which is exactly what is represented by Eqs. (16)–(18). Thus we have the heterogeneous resonance integral for absorber plate *A*. For plate *B*, simply interchange indices *A* and *B*. We shall call formula (29) the extended equivalence principle (E.E.P.). Note that the E.E.P. reduces exactly to the S.E.P. for a two-region cell.

Extensive numerical comparisons of the accuracy of the two equivalence principles were made. Lattices with four-region unit cells were considered. Since the S.E.P. does not apply directly to such a case, it was necessary to neglect the fact that the *B*-plate was different than

the *A*-plate. Also, if the two diluent regions differed was taken to be C_{ave} for the purposes of S.E.P. The transport theory code RABID (see Paper V-11) was used to compute exact resonance integrals (or effective cross sections in the case of groups of resonances) for a wide range of parameters. The absorber plates were taken to be 0.5 cm thick (no loss in generality, since the optical thickness is the significant parameter) and the absorber atom densities were allowed to vary between 0.0001 and 0.05 atoms/cm³ with the restriction that N_A and N_B never differed by more than a factor of 10 for any given lattice. These criteria evolved from an examination of the ZPR-3 hot constants. The diluent region thicknesses varied from 0.1 to approximately 5 mean-free paths. Admixed diluent in the absorber plates was permitted in widely varying amounts (for example, as much as 1000-to-1 diluent-to-absorber atom ratio for low absorber atom concentrations like 0.0001 cm⁻³). Typical U-238 resonances were examined individually and in groups. Upon optimizing the empirical parameters ($\alpha = 0.5$ and $k = 100$) it was possible to achieve essentially the same accuracy ($\pm 6\%$) with the E.E.P. on four-region cells as one gets with the S.E.P. on two-region cells. However, the S.E.P. varied considerably in accuracy when applied to four-region cells. From the considerable number of results obtained, a brief table (Table II-14-I) was compiled to reflect this situation.

In Table II-14-I, the two diluent regions *M*1 and *M*2 were assumed to be identical, although the accuracy of the E.E.P. is not affected to any significant degree even when X_1 and X_2 differ by as much as a factor of 12 to 1.

TABLE II-14-I. COMPARISON OF CALCULATIONS OF THE EFFECTIVE RESONANCE INTEGRAL (*I*) BY THE SIMPLE EQUIVALENCE PRINCIPLE (S.E.P.) AND THE EXTENDED EQUIVALENCE PRINCIPLE (E.E.P.)

Plate A		Plate B		Diluent Plate Thickness, mfp	% Error in <i>I</i>			
Absorber, atoms/cm ³	Diluent-to-Absorber Ratio	Absorber, atoms/cm ³	Diluent-to-Absorber Ratio		S.E.P.		E.E.P.	
					A-Plate	B-Plate	A-Plate	B-Plate
0.05	4	0.01	0	0.098	1.2	-6.8	0.3	
				0.393	-0.8	4.1	0.4	
				3.14	1.6	0.7	1.6	
0.001	4	0.01	0	0.098	51.0	-18.0	0.5	
				0.393	20.0	-13.0	-2.2	
				3.14	2.2	0.1	1.8	
0.001	5	0.005	1	0.098	28.0	-14.0	-2.5	
				0.393	10.5	-8.6	-2.8	
				3.14	2.0	1.1	1.7	
0.01	0	0.01	5	0.098	-12.5	9.8	-3.5	
				0.393	-4.0	1.6	0.1	
				3.14	0.3	-0.1	0.4	

resonance considered is a U-238 resonance with the following parameters: $E_0 = 3783.7$ eV, $\Gamma_n = 0.2768$ eV, $\Gamma_p = 0.0246$ eV, and $g = 1.0$.

Although, as one can see from the table, it is not difficult to construct a situation where the S.E.P. fails utterly, another interesting fact emerged from this study of heterogeneous resonance integrals. That fact is simply that in all actual cases of ZPR-3 cells examined (e.g., Assemblies 48 and 53) there was no case for which the S.E.P. did not also give quite accurate (within 6%) results. Most plate configurations which eventually are built into critical facility cores are designed with the general idea of being simple to analyze. This usually means that although the unit cell may actually have many regions, it is still not a very great perturbation on a basic two-region cell.

For example, consider the repeating lattice for which the unit cell is depicted in Fig. II-14-3. This was the actual drawer configuration in ZPR-3, Assembly 48. *M* stands for matrix, *D* for drawer, *C* for graphite, etc., and the shaded regions represent cladding. From the point of view of U-238, this is a six-region cell. However, if one homogenizes the two depleted uranium plates together with the Pu-Al plate and the two enclosed cladding plates to form a single region, there are then only two uranium-bearing "plates", and by homogenizing the diluent plates in between, one has an equivalent four-region cell. This can be analyzed using the E.E.P. directly or by applying the S.E.P. in the approximate fashion previously described.

Five different energy ranges were considered, each containing forty U-238 resonances. The lower two energy ranges, 2–2.27 keV and 2.7–3.4 keV are in the resolved range, and the remaining three are in the unresolved range. In addition, eighteen resonances of the structural materials were considered, as well as the 2.85 keV sodium resonance. RABID was again used to

FIG. II-14-3. Drawer Configuration in ZPR-3 Assembly 48. ANL Neg. No. 113-3021.

TABLE II-14-II. COMPARISON OF THE EXTENDED EQUIVALENCE PRINCIPLE (E.E.P.) AND THE SIMPLE EQUIVALENCE PRINCIPLE (S.E.P.) IN CALCULATING THE U-238 EFFECTIVE ABSORPTION CROSS SECTION $\bar{\sigma}_a$ FOR ZPR-3 ASSEMBLY 48

Broad Group Energy Range, keV	% Error in $\bar{\sigma}_a$			
	For U-238 in Homogenized U-Pu Al-U-Plate		For U-238 in Pu-U-Mo Plate	
	S.E.P.	E.E.P.	S.E.P.	E.E.P.
2-2.7	2.6	3.1	0.5	0.0
2.7-3.4	-2.0	2.0	-1.2	0.1
4.6-5.4	-1.9	-1.5	-0.7	-1.0
9.6-10.4	-0.9	-0.6	0.7	0.5
19.6-20.4	0.4	0.5	1.1	1.0

calculate exact effective broad group absorption cross sections for each of the five broad groups. IDIOT⁽⁴⁾ was used to calculate the homogeneous effective cross sections. The results are summarized in Table II-14-II.

As a result of homogenizing the two uranium plates with the plutonium-aluminum plate, either equivalence principle scheme must necessarily predict the same cross section for both depleted uranium plates. The exact transport theory treatment naturally yields cross sections for the two uranium plates which differ to a certain extent. The percent error shown in the table is always the greater error of the two.

In conclusion, the E.E.P. does the job it was intended to do, but for this cell, the S.E.P. does just as well. As mentioned previously, this should be true for most actual critical assembly cores.

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II-15. Null-Reactivity Measurements of Capture-to-Fission Ratio in U-235 and Pu-

C. E. TILL, J. M. GASIDLO, E. F. GROH, L. G. LESAGE, W. R. ROBINSON and G. S. STANFORD

INTRODUCTION

The economics of large fast breeder reactors are dependent in part on the value of the capture-to-fission ratio α^{49} in the main fissile isotope Pu-239. Typical spectra in these reactors are soft enough that a significant fraction of the fissile-element absorption takes place in the low-kilovolt region. It is in just that region, however, that the magnitude of α^{49} has been in some doubt.

Until fairly recently, the values of α^{49} below 25 keV accepted in most cross-section evaluations were based on early measurements made at the Knolls Atomic Power Laboratory¹ (KAPL). Recently, however, programs for measuring $\alpha^{49}(E)$ have been undertaken at several laboratories. The preliminary results of M. Schomberg et al.² were the first to be published. They indicated values of α^{49} in this energy region as much as a factor of two greater than the previously accepted values. The preliminary results of R. Gwin,³ published somewhat later, were higher than the KAPL values but considerably lower than those of Schomberg, particularly for energies above 5 keV.

When publication of the Schomberg work cast doubt on $\alpha^{49}(E)$, the integral measurements reported in this paper were undertaken, using the ANL critical facilities. In this series of experiments, two reference assemblies were constructed, Assembly 24 (ZPR-9) and Assembly 55 (ZPR-3), plus a variant of each containing a small central zone, designated Assemblies 24A and 55A. The four assemblies contained large null-reactivity zones, with spectra designed to emphasize events in the energy region of interest.

The null-reactivity zoned-assembly technique is based on the methods originally developed for the Physical Constant Test Reactor⁴ (PCTR), later applied to fast assemblies by W. Fox et al.⁵ The composition of the test zone is experimentally adjusted to establish a k_{∞} of unity, such that the total fission source can be equated to the total absorption in the test region. In the assemblies described in this paper, the null-zone compositions were made up of fuel (either plutonium or U-235), depleted uranium, graphite, and the structural stainless steel. For this combination of materials, the following equations for the capture-to-fission ratios of U-235 and Pu-239 are obtained by equating the fission sources to the absorptions for the U-235 and plutonium-fueled assemblies, respectively:

$$\alpha^{25} = [\nu^{25} - 1] + (1/F^{25})[(\nu^{28} - 1)F^{28} - C^{28}] + (1/F^{25})[N^h - C^{SS}], \quad (1)$$

$$\alpha^{49} = [\nu^{49} - 1] + (1/F^{49})[(\nu^{28} - 1)F^{28} - C^{28}] + (1/F^{49}) \sum_i [(\nu_i - 1)F_i - C_i] + (1/F^{49})[N^h - C^{SS}]. \quad (2)$$

Here C and F are the gross radiative capture and fission rates in the test region; the superscripts refer to the isotopes, with SS referring to stainless steel; N^h refers to the $(n,2n)$ reaction in the heavy isotopes. The summation over i in the second term of Eq. (2) is for the isotopes 25, 40, 41, and 42. The important terms on the right-hand side of the equations are the first two. The capture and fission rates in the second term of each equation are measured both individually and as ratios of each other. To derive α , values from the literature must be used for ν for the various isotopes, and calculated corrections, generally small, must be made for the third term in Eq. (1) and the third and fourth terms in Eq. (2).

CRITICAL ASSEMBLIES

The two reference assemblies, 24 and 55, were geometrically similar; each had a 25.9 cm radius null-reactivity test zone surrounded by buffer, driver, and reflector zones, each 91.44 cm in length.^{6,7} The essential difference was in the test-zone fuel. Assembly 24 (ZPR-9) was U-235-fueled, and Assembly 55 (ZPR-3) was fueled with plutonium. Assembly 24 was built first, and, to the degree that the U-235 alpha is known, was designed to check the technique. Assembly 24A had a small (1 w/o Pu-240) plutonium null-zone (equivalent spherical radius, 8.4 cm) substituted at the center of the U-235 null-zone of Assembly 24. Its size was a direct result of the 1 kg plutonium limitation then existing on ZPR-9, and it was designed, first, to give immediate information on α^{49} , and second, to investigate whether this technique would allow useful information to be derived from this very limited quantity of fuel. Assembly 55 (ZPR-3) was then built, using 4.6 w/o Pu-240-plutonium fuel, to give definitive information on the α^{49} , and to verify the Assembly 24A results. Assembly 55A had a small, 22 w/o Pu-240, plutonium null-zone (equivalent spherical radius, 18.8 cm) substituted at the center of Assembly 55, to provide some initial information on the higher plutonium isotopes.

The null-zone compositions were designed to emphasize the 0.1 to 25 keV energy range, and approximately one-half the total (calculated) fissile iso-

fissions were in this range. Two-inch cubic samples of the test zone material were used for determination of

test zone reactivity. The ratio of graphite to depleted uranium in the sample was varied until its reactivity, as measured by an oscillation technique, was zero with respect to void. At this point, the total test zone was reloaded to the experimentally determined null composition, and the null was rechecked by further reactivity measurements.

The experimentally established null compositions are given in Table II-15-I. The geometric arrangement of materials in the unit cell can be visualized approximately as follows. The unit cell in each case was a single 2 in. drawer. The fuel plate in each case was approximately at the center of the 2 in. drawer, and alternating rows of carbon and depleted uranium completed the rest of the unit cell. (The width of the rows varied somewhat from assembly to assembly, but were approximately $\frac{1}{8}$ in.)

EXPERIMENTAL METHODS

The individual fission and capture rates in the second term of Eqs. (1) and (2) were measured by a number of independent methods in an effort to assess the reliability of the results.

In the first method, the ratios including *C*-28, *F*-25, and *F*-49 were measured by counting foils irradiated in the test zone and similar foils irradiated simultaneously in a reference thermal spectrum. This technique, called here the thermal calibration technique, has been used for many years for the determination of conversion ratios in thermal reactor criticals (see, for example, Ref. 8). Traverses were made through each plate in the unit cell of the null zone, using stacks of 5 mil foils, and in some cases positioning foil strips so that they integrated across the plate width. Identical foils were irradiated simultaneously on a foil wheel in the thermal column of another reactor.

Secondly, bulk samples of each of the depleted and fuel plates were analyzed radiochemically, to give

absolute determinations of *C*-28, *F*-25, *F*-28, and *F*-49. For the fission rates, the separated Mo-99 activity was counted, and for *C*-28 the separated Np-239 activity was counted.⁹

Finally, the fission rates were also measured using absolute fission chambers, with foils of the relevant materials irradiated back-to-back with the fission chambers, to allow translation of the fission chamber readings to the corresponding fission rates in the relevant plates in the unit cell.

A more detailed discussion of these methods may be found in Ref. 10.

Comparison of the results given by the various techniques serves to illustrate the degree of agreement given by various completely independent absolute techniques as applied to unit-cell measurements in a fast spectrum. A summary of the results (on a per atom basis) for the two reference assemblies is given in Table II-15-II. Each value is a result of several determinations on two or more days. Except for the Pu-239/U-235 fission ratio, the ratios are in-plate unit-cell-averaged values.

TABLE II-15-I. NULL-ZONE COMPOSITIONS, atoms/cm³ × 10²¹

	Assembly			
	24	24A	55	55A
Pu-239	—	0.9161	1.0685	0.8511
Pu-240	—	0.0093	0.0507	0.2554
Pu-241	—	—	0.0049	0.0532
Pu-242	—	—	—	0.0072
U-235	1.172	0.0272	0.0329	0.0275
U-238	12.700	12.719	15.380	12.842
304 SS	9.652	8.119	8.912	10.314
C	44.007	36.471	37.269	33.313
Al	—	0.0993	0.1109	0.1185

TABLE II-15-II. REACTION-RATE RATIOS PER ATOM BY DIFFERENT METHODS

	Assembly 24			Assembly 55		
	c^{28}/f^{25}	f^{28}/f^{25}	f^{49}/f^{25}	c^{28}/f^{49}	f^{28}/f^{49}	f^{49}/f^{25}
Thermal Calibration	0.1325 ± 0.002	—	—	0.1415 ± 0.002	—	0.902 ± 0.02
Radiochemistry	0.1281 ± 0.005	0.01814 ± 0.0006	—	0.1394 ± 0.005	0.01939 ± 0.0006	—
Fission Chambers	—	0.01914 ± 0.0006	0.930 ± 0.02	—	0.02047 ± 0.0006	0.909 ± 0.01
U-238 Radiochemistry U-235 Fission Ch.	0.1310 ± 0.004	0.01831 ± 0.0006	—	—	—	—
Pu-239 Radiochemistry Pu-239 Fission Ch.	—	—	—	0.1393 ± 0.004	0.01953 ± 0.0006	—

RESULTS

CALCULATIONS

The measured results are compared in Table II-15-III with calculations made using ENDF/B data. Homogeneous unit-cell cross sections were first generated using MC².⁽¹⁾ The resonance-region cross sections for U-238 were then corrected for spatial self-shielding using slab-geometry equivalence theory, and the individual plate cross sections were re-weighted with fluxes from a 25-group S_n unit-cell calculation, giving the final flux and volume-weighted homogenized unit-cell cross sections. These cross sections were then used in a zero-buckling fundamental-mode calculation of k_∞ and the various reaction-rate ratios. The numbers given by this procedure should correspond to the actual average reaction rates in the plates making up the unit cell, and thus can be compared directly with the measurements.

An exception is the Pu-239/U-235 fission ratio,

where plates of one or other fuel material was present in each assembly, but not both. Values of this ratio, therefore, were calculated using the cross sections from the homogeneous MC² calculation, before any spatial re-weighting was done, in combination with the cell-averaged fluxes from the final fundamental-mode calculation. Measurements of this fission ratio were made using fission chambers positioned to sense an approximate average of the unit-cell flux, to provide a link between the U-235 and plutonium-fueled assemblies.

Integral values of the Pu-239 in-plate alpha were also generated for two alternative sets of differential $\alpha(E)$ data in the energy region below 25 keV. The first, labeled Calc. b in Table II-15-III, was based on the data of Gwin et al.,³ and the second, labeled Calc. c, on the data of Schomberg et al.² Calc. a used ENDF/B data.

ERRORS

The experimental errors quoted in Table II-15-III are estimated from counting statistics, estimated un-

TABLE II-15-III. NULL-REACTIVITY ASSEMBLY RESULTS

	Assembly			
	24	24A	55	55A
Calculated k_∞ (ENDF/B)	0.979	0.990	0.984	0.994
Average in-plate reaction rate ratios, per atom				
c^{28}/f^{25} Meas.	0.1317 ± 0.002	—	—	—
Calc.	0.1324	—	—	—
c^{28}/f^{49} Meas.	—	0.143 ± 0.004	0.1409 ± 0.002	0.1420 ± 0.002
Calc.	—	0.1489	0.1524	0.1533
f^{28}/f^{25} Meas.	0.0185 ± 0.0004	—	—	—
Calc.	0.01521	—	—	—
f^{28}/f^{49} Meas.	—	0.0196 ± 0.0006	0.0197 ± 0.0005	0.0201 ± 0.0005
Calc.	—	0.01780	0.01857	0.01847
Pu-239/U-235 fission ratio per atom, cell-average flux				
Meas.	0.930 ± 0.02	—	0.908 ± 0.01	0.897 ± 0.01
Calc.	0.8634	0.8158	0.8273	0.8235
Average in-plate capture-to-fission ratio				
α^{25} Meas.	$0.340 \pm 0.03^{(a)}$	—	—	—
Calc. a	0.3291	—	—	—
α^{49} Meas.	—	$0.42 \pm 0.05^{(a)}$	$0.426 \pm 0.03^{(a)}$	$0.490^{(b)} \pm 0.03^{(a)}$
Calc. a	—	0.3193	0.2920	0.02978
Calc. b	—	0.4206	0.3947	0.4048
Calc. c	—	0.5429	0.4940	0.5051

^a Quoted errors are experimental only; Uncertainties in ν and in null composition are not included (see text, "Results" section).

^b Not to be construed as a measurement of α^{49} as such (see text, "Results" section).

ainties of a systematic nature in the calibration of each measurement technique, observed variability among several determinations using the same technique, and finally, the degree of agreement found among the different measurement techniques. The errors quoted on the measured values of α specifically do not include the uncertainty in the values of ν .

The spectrum-averaged values of ν , used in Eqs. (1) and (2) to derive α , were those from the unit-cell fundamental mode calculation described in the above section. The ν values for all isotopes are based on ν_p (Cf-252) = 2.764, giving for example, $\nu^{25} = 2.430$ and $\nu^{49} = 2.870$ at thermal energies. The value of α as determined from Eqs. (1) or (2) is in fact rather sensitive to the value of ν for the main fissile isotope, a change of 1% in ν leading to a change of $\sim 6.5\%$ in α . Thus the values of α inferred from these measurements will change if the standard ν -values change. The slope of $\nu(E)$ does not add significantly to the overall uncertainty.

Another consideration not included in the quoted errors is the uncertainty in the determination of the null composition. The reactivity measurements themselves are very accurate, and the zero worth of the null sample, with respect to void, is established with negligible uncertainty. There are, however, other possible effects, such as non-converged spectrum, heterogeneity changes with sample removal, and anisotropic leakage from the sample, which may cause the measured null composition to be slightly different from the actual null. These effects are assumed to be small. As experimental confirmation of this and of the method in general, we note the agreement between the measured and calculated values of α^{25} for the uranium-fueled Assembly 24 (Table II-15-III).

The corrections for unmeasured, calculated absorptions are relatively minor (the last term on the RHS of Eqs. (1) or (2) is of the order of 0.05), so that very large errors would be required to appreciably affect the results for α .

SPECTRAL DEPENDENCE OF α^{49}

The effect of spectrum uncertainty on the calculated α^{49} was investigated analytically for Assembly 55. The spectrum changes were produced by arbitrarily reducing by 10%, at all energies, the cross sections of three of the constituent materials, one at a time. The results are shown in Table II-15-IV, which also indicates the changes in the group fluxes and fission rates as the spectrum is varied. The analysis was made using 25 energy groups, and the results were collapsed to three groups for presentation in the table. The energy range of Group 1 is above the region of interest. Group 2 covers the region where there is the most serious dis-

agreement between the various differential measurements of α^{49} . We conclude from Table II-15-IV that the error in the calculated value of α^{49} due to incorrect spectrum is probably small in comparison with the experimental error.

The expression for α^{49} can be written as $\sum_i \alpha_i^{49} G_i$, where the summation is over the energy groups, α_i^{49} is the alpha for group i , and G_i is the fraction of Pu-239 fissions in group i . The three-group components of α^{49} , as calculated for Assembly 55 using the Gwin results are given in Table II-15-V. The results indicate the strong dependence of α^{49} on the α_i^{49} values for Groups 2 and 3, even though approximately 72% of the flux and 58% of the fissions occur in Group 1.

ALPHA

The measured α^{25} value of 0.34 ± 0.03 in Assembly 24 agrees well with calculation, lending confidence to both the experimental method and the ENDF/B data for $\alpha^{25}(E)$. The measured α value shown in Table II-15-III for Assembly 55A is the value given by Eq. (2) for α^{49} , but is actually more a test of the fission-capture balance in the higher plutonium isotopes than a check of α^{49} . (See the discussion of higher-isotopic plutonium, below.) The measured α^{49} values 0.426 ± 0.03 , and 0.42 ± 0.05 in Assembly 24A support the ORNL data of Ref. 3, in preference to either ENDF/B or the Schomberg data.²

TABLE II-15-IV. CALCULATED EFFECT OF A 10% REDUCTION IN THE CARBON, U-238, OR STAINLESS STEEL CROSS SECTIONS ON THE SPECTRUM AND α^{49} (ASSEMBLY 55)

	Group i	$\Delta\phi_i/\phi_i$	$\Delta(\sigma_f^{49}\phi_i)/\sigma_f^{49}\phi_i$	$\Delta\alpha^{49}/\alpha^{49}$
Carbon	1	+0.019	+0.048	-0.056
	2	-0.027	-0.008	
	3	-0.125	-0.140	
U-238	1	-0.011	-0.024	+0.029
	2	+0.014	0	
	3	+0.089	+0.076	
Stainless Steel	1	+0.001	+0.002	-0.001
	2	0	0	
	3	-0.002	-0.005	

TABLE II-15-V. CALCULATED SPECTRAL DEPENDENCE OF α^{49} (ASSEMBLY 55)

Group i	Group Lower Energy	$\phi_i/\sum_i \phi_i$	G_i	$\alpha_i^{49}G_i/\alpha^{49}$
1	25 keV	0.725	0.580	0.177
2	2 keV	0.219	0.235	0.391
3	13 eV	0.056	0.185	0.432

NULL-ZONE SIZE

The most reliable value for α^{49} comes from Assembly 55, since the experimental uncertainties are considerably less than for Assembly 24A. However, the results from the very small zone of 24A are largely borne out by the Assembly 55 results, and it can be tentatively concluded that even zones of this size can be made to yield useful information.

REACTION RATES

The ratio c^{28}/f^{25} in Assembly 24 is calculated well. (See Table II-15-III; the lower-case c and f indicate per-atom values.) The calculational technique, therefore, appears to handle the large amount of U-238 resonance-region capture quite satisfactorily. (Approximately 60% of the U-238 capture in this test zone takes place at energies below 25 keV.) The ratio c^{28}/f^{49} in Assembly 55 is overpredicted by about 8%. The explanation probably is in the Pu-239 fission, rather than the U-238 capture, as is shown by the corresponding underprediction of the Pu-239/U-235 fission ratio by about the same amount.

The situation is similar in the other plutonium zones. The ratios f^{28}/f^{25} in Assembly 24, and f^{28}/f^{49} in the plutonium assemblies are underpredicted from 5 to 20%, with a tendency toward smaller discrepancies in the plutonium systems. This is in line with the corresponding underprediction of f^{49}/f^{25} , coupled with the tendency, observed in other studies, toward increasing underprediction of f^{28} with increasing U-238 content per fissile atom.¹²

HIGHER-ISOTOPIC PLUTONIUM

The calculated contributions of the higher plutonium isotopes to the neutron balance in Assembly 55A (the third term in Eq. (2), $i \neq 25$) are 0.312 positive and 0.121 negative, for a net contribution of 0.191. Use of this value in Eq. (2) gives a value for α^{49} of 0.490 ± 0.03 . If the actual value of α^{49} is 0.44 ± 0.03 (0.426 from the Assembly 55 results plus approximately 0.01 for the difference in spectra), a reduction in the higher isotopic contribution of about 0.05 ± 0.04 would be inferred. No definitive conclusions can be drawn. It appears that the contributions of the higher isotopes are calculated reasonably well, possibly within the large experimental uncertainty, with some indication of an overprediction of the net contribution.

SUMMARY

In these spectra, which emphasize the resonance region, the ratios of U-238 capture to U-235 fission and U-235 capture to U-235 fission appear to be calculated adequately, while Pu-239 fission to U-235 fission and U-238 fission to U-235 fission are underestimated by about 10 and 20%, respectively, by ENDF/B data in the calculational techniques used. The α^{49} results support the ORNL data.

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6. Measurement of Pu-239 and U-235 Capture-to-Fission Ratios by the Reactivity-Reaction Rate Method in ZPR-3 Assembly 57

M. M. BRETSCHER, J. M. GASIDLO and W. C. REDMAN

Assembly 57 of ZPR-3 was constructed for the primary purpose of measuring integral alpha values (capture-to-fission ratios) for the fissile isotopes U-233, U-235, and Pu-239 in a relatively soft spectrum. Two independent methods were used to determine the capture-to-fission ratios. In one method high-purity foils were irradiated at high power and subsequently analyzed by mass spectrometric techniques. Results are

calculations show that the spectrum of Assembly 57 was significantly harder than that in Assembly 24 of ZPR-9 in which the earlier alpha measurements were made. The median Pu-239 fission energy in Assembly 57 was calculated to be 135 keV, whereas in Assembly 24 it was only 19 keV.

It was found that delayed photoneutrons from the beryllium in the core made a significant contribution

TABLE II-16-I. ASSEMBLY 57 OF ZPR-3

Region	Outer Radius, cm	Atom Densities, $\times 10^{22}$ atoms/cm ³								
		U-235	U-238	Be	O	Al	Fe	Cr	Ni	Mn
Core	20.48	0.4081	0.6347	3.901	3.901	0.5546	—	—	—	—
Inner reflector	61.92	—	—	—	—	—	7.554	0.1129	0.0494	0.0047
Outer reflector	81.51	0.0083	3.989	—	—	—	0.4540	0.1129	0.0494	0.0047

not yet available for this mass spectrometric determination of alpha. Spectrum-averaged alpha values for U-235 and Pu-239 were also measured by the reactivity-reaction rate technique. This technique was used previously to determine $\bar{\alpha}^{235}$ and $\bar{\alpha}^{239}$ at the center of Assembly 24 of ZPR-9 and the details of the method,¹ as well as the final results of the measurement,² have already been reported.

Assembly 57 was a three-region cylindrical reactor. The central core, containing plates of enriched and depleted uranium and beryllium oxide, was surrounded by an inner reflector of iron and an outer reflector of depleted uranium. Regional radii and average homogeneous atom concentrations are shown in Table II-16-I. A more complete description of Assembly 57 has been reported previously.³

Real and adjoint flux distributions were calculated by a one-dimensional diffusion code, MACH-1,⁽⁴⁾ using cross-section sets generated by MC²⁽⁵⁾ with ENDF/B data. On the basis of these calculations, spectrum-averaged alphas for Pu-239 and U-235 are predicted to be 0.237 and 0.286, respectively. These numbers apply to a small thin sample located at the center of the reactor core. If the recent ORNL-RPI differential cross-section data⁶ for Pu-239 are used, a predicted value of $\bar{\alpha}^{239} = 0.302$ is obtained. These cal-

culations show that the spectrum of Assembly 57 was significantly harder than that in Assembly 24 of ZPR-9 in which the earlier alpha measurements were made. The median Pu-239 fission energy in Assembly 57 was calculated to be 135 keV, whereas in Assembly 24 it was only 19 keV.

It was found that delayed photoneutrons from the beryllium in the core made a significant contribution to the reactor's kinetic behavior. Special "rod-drop" experiments were conducted to determine the relative contributions of fission-product delayed neutrons and delayed photoneutrons. These results are needed in the analysis of reactivity data by inverse kinetics.

Final results for the capture-to-fission ratios in Assembly 57 measured by the reactivity-reaction rate method are not yet available.

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II-17. Capture-to-Fission Ratio Measurements of Pu-239

H. P. ISKENDERIAN

INTRODUCTION

Capsules containing Pu-239 and Pu-241 were attached to the cladding surface of Experimental Boiling Water Reactor fuel rods during the EBWR Plutonium Recycle project,¹ and irradiated to obtain measurements of the capture-to-fission ratios α for these isotopes under actual operating conditions.²⁻⁴ Ten of the capsules had cadmium covers and 14 were without covers. This report describes results for only those 16 capsules which contained Pu-239. The capsules were irradiated four to an element—two capsules with cadmium covers and two without. Eight of the capsules were run in the plutonium zone of the core, and the other eight in the enriched shim zone. Details of the capsule and mounting design have been reported,³ as have the hydrodynamic and heat transfer calculations which showed adequate cooling of the capsules and the fuel rod sections located under the capsules.³

After irradiation, the samples were removed from the reactor, and allowed to decay to reduce fission product activity before the capsules were opened and analyzed. Post-irradiation examination showed the capsules to be in good condition. None of the cadmium covers had melted or fused; in fact all had retained their sharp edges.

Values of α and epicadmium α for Pu-239, α^{49} and α_{epi}^{49} , respectively, were obtained from mass spectrometer measurements of the change in Pu-240 content, and from the production N-137 of Cs-137. The formulae used were

$$\alpha^{49} = \frac{\Delta N^{40}(\tau)}{\Delta N_{fiss}^{49}(\tau)}, \quad (1)$$

where

$$\begin{aligned} \Delta N^{40}(\tau) &\equiv \text{total number of Pu-240 nuclei formed during the irradiation} \\ \tau &\equiv \text{integrated flux during the irradiation} \\ \Delta N_{fiss}^{49}(\tau) &\equiv \text{total number of Pu-239 atoms fissioned during core irradiation;} \end{aligned}$$

$$N^{40}(\tau) = \Delta N^{40}(\tau) - N^{40}(0) \exp(-\sigma_a^{40}\tau), \quad (2)$$

where

$$\begin{aligned} N^{40}(\tau) &\equiv \text{number of Pu-240 atoms as a function of integrated flux} \\ \sigma_a^{40} &\equiv \text{absorption cross section of Pu-240;} \end{aligned}$$

and

$$\Delta N_{fiss}^{49}(\tau) = \frac{N-137}{Y-137}, \quad (3)$$

where Y-137 is the fission product yield of Cs-137.

The value of $N^{40}(\tau)$ was obtained from mass spectrometer measurement of the ratio

$$a = \frac{N^{40}(\tau)}{N^{49}(\tau)}, \quad (4)$$

where

$$N^{49}(\tau) = N^{49}(0) - N_{fiss}^{49}(\tau)(1 + \alpha^{49}). \quad (5)$$

From Eqs. (1)–(5), it follows

$$\alpha^{49} = \frac{a[N^{49}(0) - N_{fiss}^{49}(\tau)] - N^{40}(0) \exp(-\sigma_a^{40}\tau)}{(1 + a)N_{fiss}^{49}(\tau)}. \quad (6)$$

Table II-17-I lists the plutonium content of the Pu-239 samples. Because of the high purity, no correction was necessary for the initial Pu-240 content. Averaged values $\bar{\alpha}$ of α were obtained as the arithmetic averages of the values for each capsule.

RESULTS

Two of the eight samples run without a cadmium cover were not analyzed. One (from the plutonium zone) was lost during handling operations, and the other (from the enriched zone) was contaminated.

Results from the cadmium covered samples are given in Table II-17-II. Similar data from the samples without cadmium are given in Table II-17-III.

Two values of $\bar{\alpha}_{epi}^{49}$ were obtained, one from the plutonium zone and one from the enriched shim zone; they were $0.588 \pm 5\%$ and $0.546 \pm 3\%$ respectively. (Uncertainties quoted are root mean square deviations only.) The two values agree within the uncertainties given. Hence, an average of the two, $\bar{\alpha}_{epi}^{49} = 0.567$ is taken for the spectrum of either zone. For the samples run without cadmium, the $\bar{\alpha}^{49}$ values from the plutonium and enriched shim zones are 0.411 and 0.421, respectively. Analogous to the above, one averaged alpha value is used for either region, namely, $\bar{\alpha}^{49} = 0.416$.

COMPARISON OF MEASURED $\bar{\alpha}^{49}$ WITH CALCULATED α_{epi}^{49}

The value of α_{epi}^{49} in EBWR was calculated by evaluating $\phi\sigma$ from high energies down to 300 eV with the MC² code,⁵ using the ENDF/B⁽⁶⁾ cross section set, and the RABBLE code⁷ using BNL data⁸ for energies () eV down to 0.6 eV, as shown by

$$\alpha_{epi}^{49} = \frac{\sum_{300 \text{ eV}}^{high} \phi \sigma_c(\text{MC}^2 \text{ code}) \times N_c + \sum_{0.6 \text{ eV}}^{300 \text{ eV}} \phi \sigma_c(\text{RABBLE code})}{\sum_{300 \text{ eV}}^{high} \phi \sigma_f(\text{MC}^2 \text{ code}) \times N_f + \sum_{0.6 \text{ eV}}^{300 \text{ eV}} \phi \sigma_f(\text{RABBLE code})}, \quad (7)$$

where ϕ is neutron flux, σ_f is the fission cross section, and the normalizing factors N_c and N_f are obtained by taking the ratios of the RABBLE summations to the MC² summations over the energy interval of 100 to 300 eV. It was found that

$$\alpha_{epi}^{49} = \frac{25.5 \times 1.435 + 252.2}{84 \times 1.568 + 375} = 0.570.$$

The close agreement between calculated and measured values is probably fortuitous.

Of special interest to recent fast reactor physics work

TABLE II-17-I. PLUTONIUM CONTENT OF Pu-239 SAMPLES^a

Isotope	Relative Composition
Pu-239	1.000
Pu-240	0.000538
Pu-241	0.00000

^a The plutonium was in the form of PuO₂, encapsulated in aluminum.³

is an indirect check on the value of α^{49} in the energy interval 0.1 to 20 keV, where nearly 20 % of the epicalcium fissions or captures occur in EBWR; i.e.,

$$\sum_{0.1 \text{ keV}}^{20 \text{ keV}} (\phi \sigma_c) / \sum_{0.6 \text{ eV}}^{high} (\phi \sigma_c) = 18.5\%;$$

$$\sum_{0.1 \text{ keV}}^{20 \text{ keV}} (\phi \sigma_f) / \sum_{0.6 \text{ eV}}^{high} (\phi \sigma_f) = 19.8\%.$$

If we double $\alpha_{epi}^{49} |_{0.1 \text{ keV}}$ by doubling σ_c , leaving $\phi \sigma_f$ unchanged, the calculated α_{epi}^{49} would increase to $\approx 0.570 \times 1.185 = 0.675$. This is much higher than the experimental value of $\bar{\alpha}_{epi}^{49} = 0.57$.

If $\alpha^{49} |_{1.0 \text{ keV}}$ were increased by a factor of 1.5, again leaving $\phi \sigma_f$ unchanged, the calculated value becomes $\alpha_{epi}^{49} = 0.623$.

Thus, the experimental $\bar{\alpha}_{epi}^{49} = 0.57$ is in good agreement with the calculated value. Increasing α^{49} by 50 % in the range 0.1 keV to 20 keV would increase the calculated value to 0.623, which is about 8 % higher than the experimental number.

TABLE II-17-II. RESULTS FROM CADMIUM COVERED SAMPLES IRRADIATED IN PLUTONIUM AND ENRICHED ZONES

Fuel Element	Sample	Pu Weight, μg	$N^{49}(0)$, 10^{15}	$N^{49}(0)$, 10^{15}	$N_{fiss}^{49}(\tau)$, 10^{15}	a	α_{epi}^{49}
Plutonium Zone							
Pu-31	EC-10	243.93	614.36	0.3310	2.551	0.00280	0.5400
Pu-31	EC-11	196.54	495	0.2667	2.351	0.00334	0.5850
Pu-42	EC-16	243.61	613.58	0.3280	2.323	0.002925	0.6267
Pu-42	EC-15	233.88	589.07	0.3160	2.655	0.003262	0.5996
Enriched Zone							
EN-144	EC-14	223.57	563.08	0.3034	2.294	0.002810	0.5533
EN-144	EC-03	212.41	534.97	0.2882	2.068	0.002546	0.5155
EN-292	EC-07	200.34	504.57	0.2718	1.800	0.002487	0.5423
EN-292	EC-08	204.51	515.08	0.2775	2.066	0.002850	0.5718

TABLE II-17-III. RESULTS FROM SAMPLES RUN WITHOUT CADMIUM COVERS IN THE PLUTONIUM AND ENRICHED SHIM ZONES

Fuel Element	Sample	Pu Weight, μg	$N^{49}(0)$, 10^{15}	$N^{49}(0)$, 10^{15}	$N_{fiss}^{49}(\tau)$, 10^{15}	a	α^{49}
Plutonium Zone							
Pu-31	EN-12	212.34	534.80	0.2881	15.85	0.01492	0.4635
Pu-31	EN-11	294.3	741.22	0.3993	28.84	0.01735	0.4077
Pu-42	EN-14	157.79	397.42	0.2141	—	—	—
Pu-42	EN-13	212.42	535.01	0.2882	23.41	0.017504	0.3638
Enriched Zone							
EN-144	EN-10	297.04	748.12	0.4031	20.33	0.014028	0.4637
EN-144	EN-09	269.46	678.66	0.3656	18.38	0.012168	0.4122
EN-292	EN-08	227.44	572.84	0.3070	—	—	—
EN-292	EN-07	146.45	368.85	0.1977	10.20	0.011712	0.388

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II-18. Measurement of Reaction Ratios and Neutron Spectra in a Soft Spectrum Fast Reactor

W. Y. KATO, C. A. PRESKITT,* J. C. YOUNG,* J. M. NEILL,*
C. D. SWANSON, R. J. ARMANI and J. H. ROBERTS†

An experiment to provide an integral check of the U-235, U-238 and Pu-239 fission cross sections and the U-238 capture cross section, has been carried out by measuring the effective fast fission ratios of U-238/U-235, Pu-239/U-235 and the U-238 capture to U-235 fission ratio. This was done in a small soft spectrum fast assembly built on the Gulf General Atomic Subcritical Time-of-Flight Spectrum Facility (STSF) whose spectrum was measured using time-of-flight techniques. An attempt to obtain a measurement of the Pu-239 capture cross section by irradiating a sample of highly purified Pu-239 (~25 ppb Pu-240) in the assembly was not successful because of intensity difficulties.

The STSF facility, previously described by C. A. Preskitt et al.,¹ is a split table facility having a square aluminum matrix structure identical to Argonne's ZPR facilities. Each half of the facility consists of a 25 x 25 array of 2 x 2 x 33 in. ZPR-3 aluminum matrix tubes. The STSF facility is located in such a position that it may be pulsed using the Gulf General Atomic 45 MeV LINAC for time-of-flight neutron spectrum measurements along a flight path having detector stations at 50, 115, and 220 m. Fast reactor assemblies which are subcritical by at least \$10 worth and with loadings up to 600 kg U-235 may currently be studied on STSF.

The STSF was loaded with a small, approximately

54 liter, fast reactor core having BeO, U-238 and U-235 as major constituents. The atom densities for the materials in the core were 3.93, 3.94, 0.57, 0.641 and 0.413×10^{22} atoms/cm³ for Be, O, Al, U-238 and U-235 respectively. The core height was 16 in. and the effective diameter was 8.1 in. In the core 15 in. length drawers were loaded with two 1/8 in. columns of enriched uranium, five 1/16 in. columns of depleted uranium and eleven 1/8 in. columns of BeO plus 7 inches of iron as the axial reflector as shown in Fig. II-18-1. The axial reflector was 17 in. and the radial reflector was 15 in. in thickness. Horizontal and vertical cross sections of the core are shown in Figs. II-18-2 and II-18-3. One dimension cylindrical diffusion calculations using the MACH-1 code² and the ANL Cross Section Sets 224,⁽³⁾ 801, and a specially generated cross section set using MC² and ENDF/B cross sections gave values of 86, 89, and 88 kg U-235 respectively for the critical mass of the system. Multiplication measurements gave an extrapolated value of 89 kg U-235 for the critical mass. The core was estimated to be about \$10 subcritical when loaded with 73 kg of U-235. This core and reflector composition were chosen in order to have a central neutron spectrum in which the number of neutrons in the 10 keV region would be enhanced. This was done since the greatest uncertainties in the Pu-239 capture-to-fission ratio exist in this energy region. An iron reflector was used to minimize the size of the core and thus maximize the available neutron intensity.

* Gulf General Atomic Incorporated, San Diego, California.

† Macalester College, St. Paul, Minnesota.

FIG. II-18-1. Drawer Loading. ANL Neg. No. 113-3099.

FIG. II-18-2. Vertical Cross Section of Core. ANL Neg. No. 113-3075.

FIG. II-18-3. Core Configuration. ANL Neg. No. 113-3063.

Foils of U-235, U-238 and Pu-239 were irradiated close to the center of the STSF assembly. The U-235 and U-238 foils were irradiated for approximately one hour with the LINAC producing about 5×10^{11} neutrons/sec by pulsing an air cooled tungsten-natural uranium target, located near the center of the core, with 30 MeV electrons having a $0.4 \mu\text{sec}$ pulse width and 360 pulses/sec repetition rate. The plutonium foils were irradiated for approximately $2\frac{1}{2}$ hours with about twice the intensity.

Normalization was accomplished using the integrated readings from an ion chamber which monitored the leakage neutron flux. The U-235 and U-238 fissions were determined by separating the fission product Mo-90 radiochemically and by beta counting the activity.

U-238 captures were determined by separating Np-239 and beta counting the decay of Np-239. The

TABLE II-18-I. REACTION RATIOS

	Experimental	Calculated		
		801	224	ENDF/B
$\sigma_f(\text{U-238})/\sigma_f(\text{U-235})$	0.0397	0.0394	0.0391	0.0373
$\sigma_c(\text{U-238})/\sigma_f(\text{U-235})$	0.120	0.122	0.128	0.139
$\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$	1.02	1.01	1.023	0.99

Pu-239 fissions were determined by using solid state track recorders.⁴ The experimentally measured fission ratios are compared to calculated values in Table II-18-I.

Calculations of the $\sigma_f(\text{U-238})/\sigma_f(\text{U-235})$, $\sigma_c(\text{U-238})/\sigma_f(\text{U-235})$, and $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$ ratios

FIG. II-18-4. Neutron Spectrum Near Center of STSF-1A. *ANL Neg. No. 113-3069.*

TABLE II-18-II. COMPARISON OF MEASURED AND CALCULATED SPECTRA

Group	E_L , MeV	Measured	224	801	ENDF/B
1	3.67	0.0220	0.0159	0.0149	0.0168
2	2.23	0.0489	0.0676	0.0643	0.0696
3	1.35	0.0946	0.0994	0.1054	0.1101
4	0.825	0.1250	0.0926	0.1004	0.1036
5	0.500	0.1334	0.1334	0.1334	0.1334
6	0.300	0.1338	0.1132	0.1364	0.1193
7	0.180	0.1290	0.0795	0.1287	0.1305
8	0.110	0.1137	0.1059	0.1116	0.1148
9	0.067	0.0866	0.0950	0.0954	0.1008
10	0.041	0.0702	0.0843	0.0795	0.0890
11	0.025	0.0538	0.0722	0.0686	0.0756
12	0.015	0.0449	0.0686	0.0563	0.0666
13	0.00912	0.0349	0.0537	0.0450	0.0554
14	0.00431	0.0264	0.0437	0.0317	0.0421
15	0.00261	0.0208	0.0301	0.0226	0.0304
16	0.00203	0.0174	0.0230	0.0184	0.0240
17	0.00123	0.0146	0.0161	0.0137	0.0188
18	0.000961	0.0112	0.0150	0.0107	0.0139
19	0.000583	0.0085	0.0113	0.0075	0.0102
20	0.000275	0.0054	0.0048	0.0041	0.0057
21	0.000101	0.0026	0.00195	0.0014	0.0019
22	0.000029	0.0010	0.00026	0.00034	0.00037

were made using the fluxes from the one-dimensional diffusion calculations and the U-238, U-235 and Pu-239 cross sections from the three cross section sets for comparison with the experimental results.

Time-of-flight neutron spectrum measurements were made on this subcritical core using the 220 and 115 m flight paths to confirm the theoretically calculated spectrum. The time-of-flight spectrum measurements from 500 keV to 3 MeV were made using a plastic NE-213 proton recoil scintillation detector at the 220 m station. Measurements in the 50 eV to 1 MeV were made by using a lithium glass NE-908 scintillation detector at the 115 m station. The neutron spectrum measured with the time-of-flight technique is shown in Fig. II-18-4. The neutron beam was taken perpendicular to the center axis approximately 3 in. from the face of the movable half of the facility. A comparison between the measured and calculated spectra is given in Table II-18-II and shown in Fig. II-18-5 where the neutron intensities have been grouped in the same 22 energy groupings as that of the calculations.

FIG. II-18-5. Neutron Spectrum Near Center of STSF-1A. *ANL Neg. No. 113-3101.*

normalization was arbitrarily made in the fifth energy group.

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II-19. Fission Cross Section Ratio Measurements of Pu-239 and U-233 to U-235 from 0.24 to 24 keV

W. K. LEHTO

INTRODUCTION

Considerable effort has been applied to the measurement and subsequent evaluation of the fission cross sections of the fissile isotopes in the energy region of interest in fast-reactor physics. The effort has been motivated primarily by the requirements of the cross section users for accurate cross section data. Recently, measurements in the low keV region have become particularly important to supply the basic data for the design of large fast breeder reactors which possess spectra in this energy region.

A well-known method of cross section measurement which obviates the need for knowing the neutron flux and detector efficiency is to measure the ratio of the desired cross section to that of a standard or reference cross section. For the fissile isotopes, the U-235 cross section has been used extensively as the reference over the neutron energy region covered by the fission spectrum. This method was followed in this work to measure the cross section ratios of Pu-239 and U-233 to U-235. This approach yields a value which is proportional to the absolute cross section ratio. The proportionality constant, which is the ratio of the products of the respective detector efficiencies and numbers of fissile atoms, may be found by detector intercalibration in a flux of known energy where the subject cross sections are well known. Subsequently, accurate measurements^{1,2} of the U-235 fission cross section can be used to obtain the Pu-239 and U-233 cross sections. The complex problems associated with cross section averaging, and with the differing energy resolutions between experiments, place this latter step well beyond the scope of this work.

The fission cross section ratios were measured from 0.24 to 24 keV utilizing a slowing-down lead spectrometer as a neutron source. This energy range is a very difficult one in which to make measurements; its lower limit is unattainable with accelerator neutron sources

and its upper limit is above the range of conventional time-of-flight methods using fission detectors. This energy range is accessible with time-of-flight systems using bomb bursts or booster targets, such as the Harwell system, as sources; however, only limited measurements of these fission ratios have been made.

The slowing-down lead spectrometer is not new, and several such systems have been assembled^{3,4} to provide neutrons with energies from several keV to as low as several eV. After injection of a short burst of fast neutrons into a heavy moderator such as lead, the neutrons slow down by inelastic-scattering collisions to below 0.57 MeV (the energy of the first excited state in lead) and then attain an asymptotic slowing-down distribution with time given by $\bar{E} = C/(t + t_0)^2$, where t is the time after the burst in microseconds and $t_0 = 0.3$ sec.⁽⁴⁾ Nuclear events can be evaluated as a function of neutron energy using conventional time analysis techniques.

The work of A. Bergman et al.³ involved the observation of (n, γ) events in silver, zinc, manganese, cadmium and copper, and compared the energy dependence of (n, α) reactions in B-10 and Li-6. Subsequent work, done at the Lebedev Physical Institute⁵ and at Karlsruhe,^{4,6} extended the techniques to other (n, γ) cross section and resonance-integral measurements and to investigations of Doppler effects in U-238, tantalum, and tungsten.

The spectrometer is a simple, relatively cheap, high intensity, neutron source. Its principal disadvantage is the rather poor energy resolution compared with other sources, such as time-of-flight systems. These inherent resolution problems limit the useful upper energy of the device to several tens of keV. The upper limit was set at 24 keV in this work to provide adequate overlap with other ratio measurements including those obtained with antimony-beryllium sources. The useful lower limit of these measurements is also set by the resolution, which

is too broad to resolve individual resonances and not broad enough to effectively average the measured cross sections over enough resonances to avoid rapidly fluctuating data.

The energy resolution of the system is time dependent and is given by $\Delta E/E = \sqrt{8/3A} + 2\Delta t/t$, where A is the atomic mass, Δt is the pulse width, and t is time after the burst. For a lead moderator, $\sqrt{8/3A} = 0.114$. This is the minimum uncertainty in the neutron energy, and arises from the statistical variation in the time between collisions and the energy lost per collision.

EXPERIMENTAL ARRANGEMENT

Figure II-19-1 is a photograph of the experimental arrangement showing the spectrometer prior to emplacing the thermal neutron shield. A 150-keV Cockroft-Walton accelerator was used to produce 14 MeV neutrons by the (d,T) reaction. Pulses of 1 μ sec duration at a repetition rate of 10^3 pps were injected into the center of a 4 ft lead cube which was surrounded by a 0.040-in.-thick cadmium sheet and ~ 5 in. of borated water to reduce room-return neutrons. Fissions events were collected from parallel-plate, back-to-back fission detectors placed in the detector channel. The chambers, which contained the fission detectors, were positioned one-third of the block dimensions from the center to

reduce contributions from higher harmonics; how, this is not necessary as the measurement is made in the time domain of decaying higher harmonics. The calibration procedure described below precludes any further consideration of harmonic contributions.

The details of the chambers are shown in Fig. II-19-2. Fissile material was electrodeposited on 0.005-in.-thick stainless steel foils, each foil having ~ 1 mg of material deposited over a $1\frac{1}{4}$ -in.-diam circle. The material compositions are shown in Table II-19-I. The foils were placed back-to-back in the chamber. The U-233/U-235 chambers had six foils of each material connected in series, placed with an anode-to-cathode separation of 0.25 in. The Pu-239/U-235 chamber was identical except that alpha-pileup considerations restricted the number of foils to two of each material.

The electronics system is shown in Fig. II-19-3. Data were collected simultaneously from each detector; after amplification the data passed into a routing circuit which provided the proper logic signal to allow the events to be stored in proper time sequence with the accelerator pulse. The data were stored in the appropriate half of the analyzer memory depending on which detector the event originated in. The time analyzer was a TMC 1024 multichannel unit equipped with a Model 211A Time-of-Flight logic unit. Analyzer timing pulses

FIG. II-19-1. Photograph of the Slowing-Down Lead Spectrometer. ANL Neg. No. 103-11861.

FIG. II-19-2. Parallel-Plate Back-to-Back Fission Chamber. ANL Neg. No. 103-A11113.

TABLE II-19-I. ISOTOPIC COMPOSITION OF FISSION FOILS
(Determined by mass spectrometry)

Isotope	U-233, a/o	U-235, a/o	Pu-239, a/o
Pu-242			0.013
Pu-241			0.261
Pu-240			5.254
Pu-239			94.472
U-238	0.06325	5.409	
U-236		0.280	
U-235		93.268	
U-234	0.02070	1.042	
U-233	99.91605		

were generated by a separate circuit which was triggered by the voltage pulse across a resistor from target to ground.

PROCEDURES

The entire system (lead spectrometer and analyzer) was calibrated by recording events from a gamma proportional counter surrounded with a foil of calibration material (copper, zinc, molybdenum) having known

(n, γ) resonances in the energy region of the spectrometer.^{3,4} The time spectrum of gamma events exhibit intensity maxima corresponding to capture of neutrons of resonance energy. The results are summarized in Fig. II-19-4, which shows the slowing-down time to the resonance energy as a function of $E^{-1/2}$. The slope yields a value of $C = 181 \text{ keV} \cdot \mu\text{sec}^2$, or an energy-time relationship $\bar{E} = 181/(t + t_0)^2$, $\bar{E}$ being the neutron energy in keV. The difference in the ion-collection time between the gamma proportional counter and fission chamber would affect the calibration, but in this case the difference is insignificant compared to the uncertainty introduced by the inherent resolution limit of the system.

The number of counts in a time channel is given by

$$C(t) = \epsilon N \sigma_f(\bar{E}) \phi(\bar{E}, t) \Delta t, \quad (1)$$

where

$\sigma_f(\bar{E}) \equiv$ fission cross section at average energy $\bar{E}$
at slowing-down time t

$\phi(\bar{E}, t) \equiv$ neutron flux at energy $\bar{E}$ and t

$\Delta t \equiv$ analyzer channel width

FIG. II-19-3. Schematic of the Data-Collection Equipment. ANL Neg. No. 103-A11056.

FIG. II-19-4. Slowing-Down Time to Mean Energy $\bar{E}$. ANL Neg. No. 103-A11060.

ϵ = detector efficiency (fissile material deposit variations, etc.)

N = number of fissile atoms.

A quantity proportional to the fission-cross section ratio at mean energy $\bar{E}$ is formed from the ratio of the counting rates in two detectors:

$$\frac{\sigma_{f1}(\bar{E}, t)}{\sigma_{f2}(\bar{E}, t)} = \frac{\epsilon_2 N_2 C_1(t)}{\epsilon_1 N_1 C_2(t)}. \quad (2)$$

The constant $K = \epsilon_2 N_2 / \epsilon_1 N_1$ is determined from an intercalibration of the back-to-back detectors in a thermal flux and becomes

$$K = \frac{C'_2 \sigma_{f1}(0.025 \text{ eV}) g_1}{C'_1 \sigma_{f2}(0.025 \text{ eV}) g_2},$$

where C'_1 and C'_2 are the respective count rates in the thermal flux.

The thermal cross sections used are those reported in BNL-325, 2nd Edition, Supp. 2, and the g factors are C. Westcott's 23°C values.⁷ The uncertainty in K is less than 0.5% for all cases and is considerably smaller than that due to the counting statistics of the experiment.

Data were taken with analyzer channel widths of 0.5 and 1 μsec , and the mean energy was chosen to correspond to the center of each channel. Typical running times were 24 hr per experiment.

For these experiments the discriminators were set to eliminate counts due to alpha activity; this procedure resulted in backgrounds of less than 0.1%. A background correction was also made for the contributions from the very small but finite neutron level induced by the target "dark current" ($\sim 0.001\%$) between pulses. Discriminator settings were duplicated for the calibration run by matching bias curves—in this case $\ln(\text{number of counts})$ versus discriminator setting. The systematic errors introduced by the detector-calibration

technique used here are considered to be negligible, because the discriminator settings for each detector were well centered on the plateau of the bias curve and set above the tail of the alpha pileup pulses. Similarly, errors introduced by count-rate differences between the experiment and calibration and by the gamma contribution during the calibration run are negligible.*

RESULTS AND DISCUSSION

The $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$ and $\sigma_f(\text{U-233})/\sigma_f(\text{U-235})$ ratios are shown in Figs. II-19-5 and II-19-6 and are tabulated in Table II-19-II. The uncertainties quoted are those due to the counting statistics only, as the errors due to the detector calibration ($\sim 0.5\%$) are negligible by comparison. The energy spreads corresponding to several of the data points are shown in the figures as the horizontal error bars along the abscissa.

Figure II-19-5 includes the Pu-239/U-235 cross section ratios of several other investigators. Contrary to the case of the U-233/U-235 ratio, no cross section ratio measurements below several keV have been reported. The data of J. Perkin et al.,⁸ W. Allen and A. Ferguson⁹ and W. Gilboy and G. Knoll¹⁰† were obtained by direct measurements of the fission ratio. S. Dubrovina and V. Shigin¹¹ measured only the Pu-239 cross section, and the ratios shown were obtained by W. Davey¹² after dividing by his recommended value of the U-235 fission cross section. The solid line in Fig. 5 is his recommended value of the fission ratio. The ratios attributed to G. James and B. Patrick¹³ were obtained

* The detector intercalibrations were done in the thermal column of the Argonne Fast Source Reactor where the cadmium ratio was 800:1. Since fission gammas were present, they would affect the observed response of the detectors; however, in this case the effect was negligible.

† The data shown here are their corrected values as reported in Ref. 13.

FIG. II-19-5. The $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$ Ratio from 0.24 to 24 keV. ANL Neg. No. 103-A11093.

FIG. II-19-6. The $\sigma_f(\text{U-233})/\sigma_f(\text{U-235})$ Ratio from 0.24 to 24 keV. ANL Neg. No. 103-A11094.

from their deduced cross sections for Pu-239 and U-235 in the energy region below 25 keV.

The results of the present work compare well with those of several other investigators, and in general, verify Davey's selection in the overlap region down to 4 to 5 keV. The agreement with the 24-keV value of Perkin et al. is well within the error of each measurement.

Below several keV these data show an increase in the fission ratio, exhibiting a pronounced peak at about 1.25 keV and another discernible rise at about 0.5 keV. These increases are thought to be due to a series of partially resolved resonances in the Pu-239 cross section while the U-235 cross section is relatively smooth here. The sharp increases are clearly shown in the measurements of the Pu-239 fission cross section by

TABLE II-19-II. $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$ AND $\sigma_f(\text{U-233})/\sigma_f(\text{U-235})$ RATIOS

Mean Neutron Energy, keV	$\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$	$\sigma_f(\text{U-233})/\sigma_f(\text{U-235})$
24.0	0.68 ± 0.02	1.39 ± 0.02
12.5	0.62 ± 0.02	1.39 ± 0.02
7.9	0.58 ± 0.02	1.34 ± 0.02
5.4	0.64 ± 0.03	1.38 ± 0.02
3.9	0.63 ± 0.03	1.40 ± 0.03
3.0	0.58 ± 0.03	1.36 ± 0.03
2.3	0.62 ± 0.03	1.33 ± 0.03
1.9	0.66 ± 0.03	1.37 ± 0.03
1.6	0.68 ± 0.03	1.36 ± 0.03
1.3	0.77 ± 0.03	1.40 ± 0.03
1.1	0.71 ± 0.03	1.33 ± 0.03
0.95	0.66 ± 0.03	1.28 ± 0.03
0.83	0.75 ± 0.03	1.27 ± 0.03
0.73	0.73 ± 0.03	1.18 ± 0.03
0.66	0.81 ± 0.03	1.17 ± 0.03
0.59	0.82 ± 0.03	1.20 ± 0.03
0.53	0.84 ± 0.03	1.23 ± 0.03
0.48	0.81 ± 0.04	1.28 ± 0.03
0.43	0.83 ± 0.04	1.33 ± 0.03
0.39	0.86 ± 0.04	1.35 ± 0.03
0.36	0.85 ± 0.04	1.28 ± 0.03
0.33	0.86 ± 0.04	1.26 ± 0.03
0.30	0.82 ± 0.04	1.27 ± 0.03
0.28	0.92 ± 0.04	1.21 ± 0.04
0.26	0.91 ± 0.04	1.39 ± 0.04
0.24	0.96 ± 0.04	1.35 ± 0.04

E. Shunk et al.,¹⁴ and in the data of James and Endacott and Coté et al., as reported in BNL-325, 2nd Edition, Supp. 2. Similarly, decreases in the capture-to-fission ratios are noted at these energies in the measurements of R. Gwin et al.¹⁵

The $\sigma_f(\text{U-233})/\sigma_f(\text{U-235})$ cross section ratio has not received as much attention as has the Pu-239/U-235 ratio. The measurements of Allen and Ferguson,⁹ R. Lamphere,¹⁶ R. Albert¹⁷ and the monokinetic datum point of Perkin et al.,⁸ are the only direct measurements of the U-233/U-235 fission ratio approaching the low keV region. Some of these data are shown in Fig. II-19-6 along with a ratio attributed to G. James,¹⁸ obtained by dividing his values of the respective cross sections. Shown also is Davey's recommended value based on his evaluation of the U-233/U-235 cross section ratio. The agreement of the results of this work with Albert's data is quite good. Although discrepancies exist between Albert's higher-energy cross sections and those of other investigators,^{19,20} there is no reason to believe that the cross section ratios (from which the questioned data were derived) are in error, especially at lower energies. Discrepancies outside of experimental error are noted in several places, particularly between his values

at 0.85 and 0.95 keV. Both his data and the results of the present work indicate a decrease in the fission ratio below 1.0 keV; they also verify the correctness of a relatively flat fission ratio down to about 1 keV as measured by Lamphere. The agreement is worse with James' values, and the disparity is very marked with the 24 keV value of Perkin et al.

CONCLUSION

The technique of using the slowing-down lead spectrometer with conventional time-analysis techniques has proved to be a simple experimental arrangement for measuring fission-cross section ratios in the low keV region. The limiting factor in these experiments has been the broad energy resolution of the device. Improvements could be made by pulsing the lead block with a much narrower pulse which in turn requires longer running times to attain adequate statistics. In spite of this limitation, the data should be useful when broad-resolution cross sections are desired, and should be of interest to the cross section users.

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II-20. Uranium Doppler Measurements in ZPR-6 and -9 Oxide Cores

J. W. DAUGHTRY and R. A. LEWIS

INTRODUCTION

A series of Doppler experiments has been carried out in three related UO_2 -fueled critical assemblies in order to obtain experimental data on the Doppler effect in a neutron spectrum typical of large dilute sodium cooled and oxide-fueled breeder reactors. In addition, the experiments were designed to test the adequacy of zoned cores for Doppler measurements and to obtain experimental data on expansion and environmental effects in the Doppler samples themselves. The Doppler samples and experimental techniques used were the same as previously reported.¹⁻³ Basically, the technique is a central reactivity measurement in which a small, heatable sample (approximately one kilogram) is oscillated into and out of the center of a critical assembly, and the reactivity change produced by changes in the temperature of the sample is measured with a calibrated control rod.

GENERAL INFORMATION ABOUT ASSEMBLIES

The three critical assemblies in which these measurements were made were ZPR-6 Assembly 6 and ZPR-9 Assemblies 19 and 21. Assembly 6 was one of a series of benchmark assemblies which have been built for physics studies in support of the Liquid Metal Fast Breeder Reactor (LMFBR) program. Its core was about 4000 liters in volume with a unit cell shown in Fig. II-20-1. The oxygen-to-uranium atom ratio was close to two and the core composition

simulated that of a UO_2 -fueled reactor with volume fractions of approximately 32% fuel, 42% sodium, and 22% stainless steel.

The ZPR-9 Assemblies 19 and 21 were zoned cores; i.e., each had the same central core composition and unit cell as Assembly 6 but the central core was surrounded by an enriched driver region designed to achieve criticality at a relatively low critical mass. A buffer region was interposed between the core and

FIG. II-20-1. Central Core Region Loading Pattern. ANL Neg. No. 113-1386A.

TABLE II-20-I. ASSEMBLY PARAMETERS

	ZPR-6 Assembly 6	ZPR-9 Assembly 19	ZPR-9 Assembly 21
Critical mass, kg U-235	1817	480	340
Mass of U-235 in central core, kg		87	26
Total volume (excluding reflector), liters	3953	540	295
Volume of central core, liters		192	58
Core height (excluding reflector), cm	152.4	91.4	91.4
Axial reflector thickness, cm	30 U	15.2 steel 15.2 U	15.2 steel 15.2 U
Radial reflector thickness, cm	30 U	13.5 steel 16.6 U	24.8 steel 16.6 U
Calculated $I_h/\%(\Delta k/k)$	458	435.5	429.5

driver. The buffer region was designed to match core and driver spectra in order to produce real and adjoint centerline fluxes as similar as possible to those of the full-size Assembly 6. The three assemblies are described in detail in Refs. 4-8. Some of the information from these references is given in Table II-20-I for convenience.

EXPERIMENTAL DETAILS

The heated small sample oscillation technique for measuring Doppler effects in fast critical assemblies has been used extensively at Argonne and has been described in previous annual reports.^{1,2} For the experiments reported in this paper the samples were natural and enriched UO_2 , 1 in. in diam by 12 in. long, containing about 1 kg of uranium each. A detailed study⁹ of the effect of the immediate environment around the sample on the measured Doppler effect has led to the adoption of a standard configuration for most Doppler measurements made in ZPR-6 and -9. The purpose of this standard configuration is to reduce the hot-cold resonance interaction effect. The reduction is achieved by introducing a stainless steel filter between the Doppler sample and the reactor core. In the standard environment, a total of $\frac{1}{2}$ in. of stainless steel separates the sample material from the surrounding reactor core materials.

EXPERIMENTAL RESULTS

The results of this series of Doppler measurements are summarized in Table II-20-II. The table gives the reactivity effect caused by heating the sample

TABLE II-20-II. SUMMARY OF DOPPLER MEASUREMENTS IN ZPR-6 ASSEMBLY 6 AND ZPR-9 ASSEMBLIES 19 AND 21

TABLE II-20-11. SUMMARY OF DOPPLER MEASUREMENTS IN LIT-6 REACTOR					
Assembly Designation	Sample Temperature, ^a °K Reactivity Change, ^b 1h/kg-U				Comments
Natural Uranium					
ZPR-6 Assembly 6	505 -0.132	811 -0.266	1081 -0.353		
ZPR-9 Assembly 19	800 -0.497	1055 -0.628			
ZPR-9 Assembly 21	776 -0.594	1041 -0.795			
Enriched Uranium ^c					
ZPP-6 Assembly 6	759 +0.056	764 +0.050	1023 +0.059	1041 +0.055	
ZPR-6 Assembly 6	509 +0.041	768 +0.061	1042 +0.075		¾ in. stainless steel filter
ZPR-6 Assembly 6	406 +0.019	642 +0.041	826 +0.047	1030 +0.058	expansion constrained
ZPR-9 Assembly 21	765 +0.111	1048 +0.114			

^a Data normalized to 0.000 $I_h/\text{kg-U}$ at 293°K and uncorrected for expansion.

^b $\sigma \approx 0.002 I_h/\text{kg-U}$.

^c 97.7% U-235.

FIG. II-20-2. Typical Temperature Profiles in Doppler Sample. ANL Neg. No. 113-2990.

from room temperature, 293°K, to the temperature shown, which represents the average temperature of the sample. The sample temperature is not uniform throughout the sample as shown in Fig. II-20-2; however, the temperature profile is maintained constant while the reactivity measurement is being made. The standard environment was used for all the measurements except for the one case, indicated in Table II-20-II, in which a $\frac{3}{4}$ in.-thick filter was used.

The reactivity effect due to expansion has been found to be negligible for the natural uranium samples. This is not, necessarily, the case for the enriched samples in which the Doppler effect itself is much smaller. In these experiments, one set of data was obtained with a sample having constrained expansion characteristics. The remainder of the data was obtained with freely expanding samples. The constrained expansion was achieved by a special capsule design^{1,3} that permits no sample expansion from room temperature to about 200°C and radial, but not axial, expansion above 200°C. None of the data of Table II-20-II has been adjusted for expansion effects.

The root mean square error σ quoted in Table II-20-II applies to the normalization point of 0.000 lh/kg-U at 293°K as well as to the higher temperature data points.

Figure II-20-3 displays the results obtained with the freely expanding samples in the standard environment. The positive values are the enriched uranium data, and the negative values are the natural uranium data. Note that the positive and negative scales are different in the figure.

It has been pointed out in zoned core studies¹⁰

that the ratio of the worth of an absorbing sample in a zoned core to the worth of the same sample in the full-size reference system is a good measure of the ratio of the perturbation-theory normalization integrals, provided the real fluxes are reasonably well matched. Based on a number of worth measurements of absorbing samples in each of the three assemblies, average values have been determined for this ratio for each of the zoned assemblies relative to Assembly 6. These values are 1.69 ± 0.03 for Assembly 19 and 2.48 ± 0.03 for Assembly 21. In Fig. II-20-4 the Assembly 19 and 21 data have been normalized to

FIG. II-20-3. Reactivity Change Due to Heating Freely Expanding Samples. ANL Neg. No. 113-2994.

FIG. II-20-4. Normalized Data for Freely Expanding Samples. ANL Neg. No. 113-2995.

FIG. II-20-5. Effect of Additional Filter Thickness on Enriched Uranium Doppler Effect in ZPR-6 Assembly 6. ANL Neg. No. 113-3011.

FIG. II-20-6. Comparison of Constrained and Free Expansion Doppler Samples in ZPR-6 Assembly 6 with $\frac{1}{4}$ in. Stainless Steel Filter. ANL Neg. No. 113-2992.

the Assembly 6 results by dividing by these sample worth ratios. The normalized zoned-core results differ from the Assembly 6 results by 5 to 12% for natural uranium and 15 to 20% for enriched uranium. The differences are probably due to uncertainties in the normalization factors and/or some spectral differences in the energy range in which the Doppler effect is important.

Figure II-20-5 shows the effect of an additional $\frac{1}{4}$ in. of stainless steel between an enriched uranium sample and the core materials. In general such a filter has two effects which cause opposite reactivity changes: (1) it reduces the hot-cold resonance interaction effects and, (2) it increases the neutron flux in the energy region of importance to the Doppler effect. From Fig. II-20-5 it is seen that the net result of increasing the filter thickness in this case has been to increase the measured reactivity change.

Figure II-20-6 shows the results obtained in Assembly 6 with two enriched uranium samples having different expansion characteristics. It appears that the expansion effect for enriched uranium in the Assembly 6 spectrum is small.

CONCLUSIONS

The ZPR-6 Assembly 6 Doppler effect data can be used by those interested in temperature-reactivity effects in large dilute oxide-fueled LMFBR's. To do so one should be aware of the expansion and environmental effects discussed earlier and should adjust the results presented in this report as necessary. As already pointed out, the expansion of the natural uranium sample upon heating produces a negligible reactivity change. For the enriched uranium samples the expansion effect, although not necessarily negligible, was found to be small. Thus serious error would not be incurred if the data given were used without making any expansion corrections.

The stainless steel, separating the Doppler samples from the surrounding reactor materials, should be sufficient to eliminate any significant hot-cold resonance interaction effect. On the other hand, one must account for the increase in the measured Doppler effect due to the flux perturbation by the stainless steel. However, this correction is simpler than correcting for the hot-cold resonance interaction. A method for making this correction is given in Ref. 9 and, as a first approximation, the Ref. 9 correction factor could be used directly. This would reduce the experimental results obtained with a $\frac{1}{2}$ in. thick filter by about 15%. With the expansion and environmental effects taken into account the Assembly 6 data are appropriate for use in tests of Doppler effect calculational methods.

The comparison of ZPR-6 and -9 results show that the central Doppler effect can be measured in a zoned core with a relatively small critical mass and this measurement can then be used to infer the results that one would obtain in the full-size reference core. The accuracy with which one can make this extrapolation depends on the accuracy with which one can determine the ratio of the perturbation theory normalization integrals.

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II-21. A Comparison of ENDF/B and Schmidt U-238 Data for Doppler-Effect Calculations

R. B. POND and C. E. TILL

INTRODUCTION

Some recent comparisons of U-238 Doppler effect calculations that have been done in the past using ENDF/B data on the one hand and the compilation of J. Schmidt¹ on the other, showed that there were often considerable differences in the results predicted by the sets of data. The source of these differences was investigated. It was concluded that while detailed differences in the calculational methods used by different individuals at different points in time can be a larger contributor to differences than the variations in the resonance parameters themselves, the major difference between the two sets lies in the assumptions made for *p*-wave capture. The present work is intended (a) to point out differences in the two resonance parameter data sets and (b) to show on a consistent basis how each set of resonance parameters affects the resonance region capture cross sections for Doppler-effect calculations.

BACKGROUND

Doppler-effect measurements are usually made by embedding a sample material in a critical reactor and measuring the reactivity change of the system as the sample is heated. For comparisons with calculation, the reactivity change due to the change in sample temperature is generally considered simply as a small perturbation to the critical system. The major portion of the Doppler-effect calculation is the calculation of appropriately Doppler-broadened cross sections for the sample material.

For purposes of these comparisons, each set of resonance parameters has been used to generate

Doppler-broadened cross sections for a U-238 sample with an effective potential scattering cross section approximately 50 b. This corresponds to the size and density of a U-238 Doppler-sample typically used at Argonne, for which extensive Doppler-effect data are available in a variety of reactor spectra.

REVIEW OF THE U-238 RESONANCE PARAMETERS

RESOLVED PARAMETERS

The number of resolved resonances in the ENDF/B and Schmidt sets are not the same. However, the resonances that are common to both compilations have the same level positions (basically those reported by J. B. Garg et al.² in the 1964 measurements made at Columbia University). There are 209 ENDF/B resolved resonances, 208 which have level positions and partial neutron widths identical to the BNL-325 recommendation. There is one bound level in the ENDF/B compilation which is not included in BNL-325. All partial capture widths in ENDF/B are set equal to 24.6 MeV. Schmidt, on the other hand, includes 239 resonances comprising all 227 resolved resonances of Garg et al. plus 12 additional resonances found by other investigators.¹ Between 1.7977 and 3.9044 keV, the ENDF/B and Schmidt sets are identical except that Schmidt sets Γ_c equal 24.8 MeV for each resonance. There are 117 resonances in this region corresponding exactly to the Columbia University measurements. Schmidt's partial widths below 1.7977 keV are weighted averages of the Columbia University measurements and those of other investigators.¹

(A number of ENDF/B resolved resonances are suspected to be p -wave resonances. However, ENDF/B chooses to treat all resonances as s -wave levels. For consistency in the present calculations all suspected p -wave resonances in the Schmidt compilation have also been treated as s -wave levels.)

UNRESOLVED PARAMETERS

The s - and p -wave strength functions and level spacings in the unresolved resonance region are as follows:

s - and p -wave Strength Functions (S) and Level Spacings (D)

	$\ell = 0$		
	$S_{J=1/2}$	$D_{J=1/2}$	
	9.4×10^{-6} 9.0×10^{-6}	18.5 eV 20.8 eV	
ENDF/B Schmidt			
	$\ell = 1$		
	$S_{J=1/2, 3/2}$	$D_{J=1/2}$	$D_{J=3/2}$
	1.58×10^{-4} 2.50×10^{-4}	18.5 eV 20.8 eV	9.25 eV 11.40 eV
	ENDF/B Schmidt		

The reduced neutron widths in both compilations are assumed to obey a χ^2 distribution with one degree of freedom. The partial capture widths are assumed constant for all resonance sequences and equal to 24.6 MeV in ENDF/B and to 24.8 MeV in Schmidt.

GROUP CROSS SECTION CALCULATIONAL PROCEDURE

The group cross sections were calculated using equivalence theory and the ERIC-2⁽³⁾ resonance

cross section code. The results given by this code have been compared with those given by the MC²⁽⁴⁾ and the RABBLE⁵ codes for both infinitely dilute and shielded cross sections. Fairly close, but not exact, agreement was found for capture cross sections generated by the three codes for identical physical situations. To show precisely how the ENDF/B and Schmidt resonance data were handled in the present calculations it is useful to examine the infinitely dilute cross section results.

INFINITELY DILUTE U-238 CAPTURE CROSS SECTIONS

Columns 2 to 6 of Table II-21-I show the infinitely dilute U-238 capture cross sections calculated by each code using ENDF/B resonance parameters. Columns 7 and 8 of the same table show the same calculation repeated with ERIC-2 but using the Schmidt parameters. (The multi-group energy structure is shown in Table II-21-II.) The fourth and sixth columns are background capture cross sections. Group 15 is the energy interval in which the resolved region stops and the unresolved resonance region begins, and the double entries therein are their respective contributions to the cross section in this group.

The s - and p -wave unresolved resonance region in ENDF/B extends from 3.92 to 50 keV. A smooth background cross section is added to the resolved region that is an extension of the infinitely dilute p -wave cross section in the resolved region. The s - and p -wave unresolved region in the ERIC-2 calculation was assumed to extend from the last resolved resonance (3.9044 keV) to the upper energy bound of Group 10 (67.4 keV), and in place of the smooth background it was assumed that the unresolved p -

TABLE II-21-I. INFINITELY DILUTE U-238 CAPTURE CROSS SECTIONS, b

Group	RABBLE ENDF/B s -Wave	MC ² ENDF/B s -Wave	MC ² ENDF/B Background	ERIC-2 ENDF/B s -Wave	ERIC-2 ENDF/B p -Wave	ERIC-2 Schmidt s -Wave	ERIC-2 Schmidt p -wave
10	—	0.14792	0.18702	0.335872	—	0.328889	—
11	—	0.45427	—	0.455123	—	0.462787	—
12	—	0.58783	—	0.588861	—	0.622019	—
13	—	0.73368	—	0.734660	—	0.799128	—
14	—	0.95789	—	0.958385	—	1.04804	—
15	—	0.21827	—	0.245198	—	0.263762	—
	0.68600	0.69201	0.28669	0.685705	0.277574	0.689321	0.402759
16	1.1992	1.1955	0.3131	1.19697	0.313992	1.20367	0.468333
17	1.4450	1.4285	0.2684	1.43906	0.269705	1.51604	0.411138
18	2.1208	2.1368	0.2255	2.15297	0.228219	2.29712	0.353280
19	3.1074	3.1075	0.1017	3.12973	0.191889	3.17322	0.299636
20	3.3243	3.3157	—	3.43244	—	3.16881	—
21	16.927	14.162	—	16.6112	—	16.1371	—
22	43.561	48.050	—	47.0384	—	49.3953	—
23	78.061	92.927	—	82.6872	—	86.4521	—

ve region extends downward from the last resolved resonance to the lower energy bound of Group 19 (0.583 keV). In the MC² calculation on the other

TABLE II-21-II. MULTI-GROUP ENERGY STRUCTURE

Group	Lower lethargy Bound	Lower Energy Bound
9	5.0	67.38 keV
10	5.5	40.87
11	6.0	24.79
12	6.5	15.03
13	7.0	9.119
14	7.75	4.307
15	8.25	2.613
16	8.5	2.035
17	9.0	1.234
18	9.25	961.1 eV
19	9.75	582.9
20	10.5	275.4
21	11.5	101.3
22	12.75	29.02
23	13.5	13.71

hand, this added background cross section was left in smooth form.

Comparison of the MC² and ERIC-2 cross sections indicated that the difference in the two treatments does not significantly affect the resultant infinitely-dilute cross sections.

SHIELDED U-238 CAPTURE CROSS SECTIONS

Doppler-broadened U-238 capture cross sections at 293 and 1100°K generated for an effective potential scattering cross section of 48.64 b are shown in Table II-21-III. These were calculated with ERIC-2 using ENDF/B and Schmidt resonance parameters in turn, under the same assumptions as described above for the infinitely-dilute cross sections. In addition, however, intersequence resonance interference effects, estimated by the method of R. N. Hwang,⁶ have also been taken into account. The table shows the results as a function of energy group for the s-wave, the p-wave (both sequences), and the total s- plus p-wave

TABLE II-21-III. SHIELDED U-238 CAPTURE CROSS SECTIONS, b

Group	s-Wave	$p_{1/2}$ -Wave	$p_{3/2}$ -Wave	(s + p) Wave	Flux	Group	s-Wave	$p_{1/2}$ -Wave	$p_{3/2}$ -Wave	(s + p) Wave	Flux
A. ENDF/B Cross Sections ($T = 293^\circ\text{K}$)						C. Schmidt Cross Sections ($T = 293^\circ\text{K}$)					
10	0.076626	0.059698	0.195765	0.332091	0.943438	10	0.068993	0.060506	0.193711	0.323212	0.931531
11	0.118532	0.079922	0.245902	0.444356	0.941304	11	0.106661	0.084239	0.256902	0.447803	0.932253
12	0.179613	0.099654	0.284240	0.563508	0.938444	12	0.161454	0.110428	0.317171	0.589053	0.932067
13	0.264854	0.114240	0.299775	0.678869	0.935223	13	0.237726	0.134363	0.359989	0.732078	0.931224
14	0.411246	0.118499	0.281176	0.810921	0.931660	14	0.368312	0.151210	0.369197	0.888719	0.930122
15	0.582157	0.108128	0.236540	0.926825	0.939394	15	0.572401	0.148898	0.333865	1.055163	0.936465
16	0.667991	0.096906	0.205060	0.969957	0.927862	16	0.670983	0.138405	0.298055	1.107443	0.924835
17	0.671589	0.084300	0.174684	0.930573	0.934151	17	0.706460	0.123497	0.258549	1.088506	0.927782
18	0.868523	0.072020	0.147108	1.087651	0.928162	18	0.946594	0.107508	0.220554	1.274656	0.924782
19	1.043012	0.060844	0.123279	1.227136	0.935986	19	1.098111	0.091830	0.186143	1.376085	0.934475
20	0.985240	—	—	0.985240	0.961425	20	0.939962	—	—	0.939962	0.963926
21	1.829007	—	—	1.829007	0.894896	21	1.799605	—	—	1.799605	0.893918
22	2.744724	—	—	2.744724	0.892918	22	2.842967	—	—	2.842967	0.893715
23	3.723837	—	—	3.723837	0.905364	23	3.905160	—	—	3.905160	0.901300
B. ENDF/B Cross Sections ($T = 1100^\circ\text{K}$)						D. Schmidt Cross Sections ($T = 1100^\circ\text{K}$)					
10	0.078443	0.060185	0.197502	0.336131	0.941766	10	0.070693	0.061326	0.197030	0.329050	0.929232
11	0.123075	0.080795	0.248956	0.452826	0.938597	11	0.110885	0.085676	0.262408	0.458970	0.929027
12	0.190212	0.100966	0.288492	0.579670	0.934081	12	0.171247	0.112670	0.325174	0.609091	0.927356
13	0.287970	0.115971	0.304838	0.708778	0.928476	13	0.258932	0.137528	0.370353	0.766813	0.924366
14	0.467035	0.120551	0.286418	0.874004	0.921060	14	0.419002	0.155365	0.381234	0.955601	0.919803
15	0.679615	0.110154	0.241145	1.030915	0.927827	15	0.668404	0.153469	0.345607	1.167480	0.924908
16	0.815630	0.098780	0.209137	1.123547	0.910577	16	0.819350	0.142832	0.308824	1.271007	0.907480
17	0.843565	0.085951	0.178164	1.107680	0.917957	17	0.885396	0.127534	0.267971	1.280902	0.910520
18	1.116129	0.073446	0.150054	1.339629	0.909390	18	1.199098	0.111089	0.228675	1.538862	0.905818
19	1.387354	0.062041	0.125723	1.575119	0.915251	19	1.445683	0.094893	0.192966	1.733543	0.914206
20	1.332166	—	—	1.332166	0.947750	20	1.269966	—	—	1.269966	0.950765
21	2.349719	—	—	2.349719	0.873036	21	2.310388	—	—	2.310388	0.871819
22	3.143779	—	—	3.143779	0.880581	22	3.236633	—	—	3.236633	0.881848
23	4.101563	—	—	4.101563	0.896760	23	4.272143	—	—	4.272143	0.893012

TABLE II-21-IV. U-238 DOPPLER-DIFFERENCE CROSS SECTIONS, b

Group	s-Wave	$p_{1/2}$ -Wave	$p_{3/2}$ -Wave	(s + p) Wave
A. ENDF/B ($\Delta T = 1100 - 293^\circ\text{K}$)				
10	0.001816	0.000486	0.001736	0.004040
11	0.004542	0.000873	0.003054	0.008470
12	0.010599	0.001311	0.004252	0.016162
13	0.023115	0.001730	0.005062	0.029909
14	0.055789	0.002051	0.005241	0.063082
15	0.097457	0.002026	0.004604	0.104089
16	0.147639	0.001874	0.004076	0.153590
17	0.171976	0.001650	0.003480	0.177107
18	0.247606	0.001425	0.002945	0.251977
19	0.344343	0.001196	0.002444	0.347983
20	0.346925	—	—	0.346925
21	0.520712	—	—	0.520712
22	0.399055	—	—	0.399055
23	0.377727	—	—	0.377727
B. Schmidt ($\Delta T = 1100 - 293^\circ\text{K}$)				
10	0.001699	0.000820	0.003319	0.005838
11	0.004223	0.001437	0.005506	0.011166
12	0.009793	0.002241	0.008002	0.020037
13	0.021206	0.003165	0.010364	0.034735
14	0.050689	0.004154	0.012036	0.066881
15	0.096003	0.004571	0.011742	0.112316
16	0.148368	0.004427	0.010768	0.163563
17	0.178936	0.004037	0.009421	0.192396
18	0.252504	0.003581	0.008121	0.264206
19	0.347573	0.003062	0.006823	0.357458
20	0.330004	—	—	0.330004
21	0.510783	—	—	0.510783
22	0.393667	—	—	0.393667
23	0.366983	—	—	0.366983

cross sections, and also the flux integral, $\int (\sigma_p/\sigma_t) dE$.

U-238 DOPPLER-DIFFERENCE CROSS SECTIONS

Table II-21-IV lists the 1100–293°K Doppler-induced changes in the sample capture cross sections given in Table II-21-III for each s- and p-wave component and the sum of all components.

U-238 DOPPLER-EFFECT CALCULATIONS

The Doppler cross sections listed in Table II-21-IV were used in a first-order perturbation diffusion theory calculation of the measured U-238 Doppler effects in the ZPR-9 Assemblies 12 and 17. (Assembly 12^(7,8) was a zoned version of the 2600 liter, uranium-carbide, fast breeder reactor built as Assembly 5 on ZPR-6. Assembly 17^(8,9) was the softest spectrum assembly of a series of polyethylene-degraded fast assemblies built specifically to investigate Doppler

effects as a function of reactor spectra.) END] data were used for the reactor calculation.

The Doppler-effect calculational results are given in Table II-21-V. The table is arranged to emphasize the information on the percentage increase in the U-238 Doppler effect which the p-wave Doppler-effect components add to the total s-wave Doppler effect. The s-wave contributions are listed first. The numbers shown as a function of energy group times the s-wave Doppler effect are the total s-wave plus the cumulative p-wave Doppler effects through any particular energy group.

ASSEMBLY 12

The total p-wave Doppler effect is about 12% of the s-wave Doppler effect for ENDF/B and about 25% for the Schmidt parameters. Overall, the p-wave contribution of Schmidt is about twice that of

TABLE II-21-V. U-238 DOPPLER EFFECT BETWEEN 293 AND 1100°K

Group	$p_{1/2}$	$p_{3/2}$	$p_{1/2}$	$p_{3/2}$
Assembly 12 ^(a) ; 1% $\delta k/k = 448.008$ Ih				
ENDF/B (s-Wave = -0.2716 Ih/kg)			Schmidt (s-Wave = -0.2639 Ih/kg)	
10	1.0309	1.0921	1.0596	1.1937
11	1.0273	1.0790	1.0533	1.1680
12	1.0222	1.0611	1.0446	1.1347
13	1.0160	1.0412	1.0339	1.0963
14	1.0103	1.0243	1.0230	1.0609
15	1.0047	1.0100	1.0114	1.0273
16	1.0034	1.0072	1.0086	1.0199
17	1.0028	1.0058	1.0070	1.0161
18	1.0010	1.0022	1.0027	1.0061
19	1.0005	1.0011	1.0014	1.0032
Assembly 17 ^(b) ; 1% $\delta k/k = 393.257$ Ih				
ENDF/B (s-Wave = -4.8826 Ih/kg)			Schmidt (s-Wave = -4.7739 Ih/kg)	
10	1.0060	1.0159	1.0125	1.0361
11	1.0056	1.0147	1.0120	1.0339
12	1.0051	1.0130	1.0111	1.0306
13	1.0045	1.0107	1.0099	1.0263
14	1.0036	1.0082	1.0083	1.0212
15	1.0023	1.0049	1.0057	1.0135
16	1.0018	1.0038	1.0045	1.0105
17	1.0014	1.0030	1.0036	1.0083
18	1.0007	1.0014	1.0017	1.0039
19	1.0004	1.0008	1.0010	1.0023

^a Experimental Doppler effect = -0.422 Ih/kg with $\sim \delta k/k = 451.2$ Ih.

^b Experimental Doppler effect = -4.045 Ih/kg with 1% $\delta k/k = 391.78$ Ih.

DF/B while the s -wave Doppler effects are only 3% different for the two compilations. The calculated Doppler effect for the s -wave sequences underestimate the measurement by 35.6% and 37.5% for the ENDF/B and Schmidt compilations, respectively. Even with the full p -wave Doppler-effect contribution of Groups 10–19 considered, the calculations still underestimate the measurement by 27.7% for ENDF/B and 21.6% for Schmidt. Use of ENDF/B directly with the constant background between 0.748 and 3.92 keV, and above 50 keV, would give essentially no p -wave Doppler effect in Groups 15–19 and a smaller s - and p -wave Doppler effect in Group 10. This would reduce the total p -wave Doppler effect slightly, to about 10 and 21% of the s -wave Doppler effect, and result in calculational underestimates of the measured Doppler effect of 28.7 and 24.1% for the ENDF/B and Schmidt parameters, respectively.

Additionally, some reduction in the $p_{3/2}$ sequence-Doppler effect should clearly be made above Group 13 (15 keV) because of same-sequence resonance overlap effects. Considering only the s -wave, the $p_{1/2}$ -sequence in Groups 10–14 and the $p_{3/2}$ -sequence in Groups 13–14, the calculations underestimate the measurement by 31.9% using ENDF/B data and by 30.1% using Schmidt data. Under these conditions, both sets of resonance parameters predict about the same U-238 Doppler effect, as the larger contribution from the $p_{3/2}$ sequence given by the Schmidt parameters is effectively suppressed.

A summary is given below. The percentage difference between each calculation and the measurement is given in parentheses.

	ENDF/B	Schmidt
s -wave, Ih/kg	-0.2716 (-35.6%)	-0.2639 (-37.5%)
s -wave + p -wave (Groups 10–19), Ih/kg	-0.3050 (-27.7%)	-0.3307 (-21.6%)
s -wave + p -wave (Groups 10–14), Ih/kg	-0.3010 (-28.7%)	-0.3205 (-24.1%)
s -wave + $p_{1/2}$ -wave (Groups 10–14) + $p_{3/2}$ -wave (Groups 13–14), Ih/kg	-0.2872 (-31.9%)	-0.2948 (-30.1%)
Experimental, Ih/kg	-0.422	-0.422

ASSEMBLY 17

The corresponding calculations for Assembly 17 also show little difference between the two data sets. In this assembly, however, the calculations overestimate the measurement. The p -wave Doppler-effect components add very little to the s -wave Doppler effect. A summary for Assembly 17, is given below:

	ENDF/B	Schmidt
s -wave, Ih/kg	-4.8826 (+20.7%)	-4.7739 (+18.0%)
s -wave + p -wave (Groups 10–19), Ih/kg	-4.9893 (+23.3%)	-5.0062 (+23.8%)
s -wave + p -wave (Groups 10–14), Ih/kg	-4.9534 (+22.5%)	-4.9152 (+21.5%)
s -wave + $p_{1/2}$ -wave (Groups 10–14) + $p_{3/2}$ -wave (Groups 13–14), Ih/kg	-4.9290 (+21.9%)	-4.8674 (+20.3%)
Experimental, Ih/kg	-4.045	-4.045

CONCLUSIONS

The agreement between calculation and measurement in the harder spectrum of Assembly 12 depends to some extent on how much of the p -wave Doppler effect is added to the s -wave Doppler effect. However, even if the total p -wave contribution of Schmidt is included, the agreement between calculation and measurement is still not good. Further, the effect of same-sequence resonance interference effects in reducing the p -wave Doppler effect must be appreciable in the $p_{3/2}$ -sequence. At 50 keV with a temperature of 1100°K, for example, the Doppler-broadened resonance width is about 9 eV and therefore approaches the assumed $p_{3/2}$ -resonance sequence spacing. The assumption in ERIC-2 that the resonances are isolated and non-interfering is better for the s - and $p_{1/2}$ -sequences where the average resonance spacing is still double the Doppler width.

In the softer spectrum of Assembly 17, the s -wave Doppler effect predominates and the total p -wave contribution has very little effect on the net U-238 Doppler effect. The reason for the overestimate of the U-238 Doppler effect in this assembly as opposed to the underestimate in Assembly 12 is still under investigation.

This very simplified calculational procedure indicates the magnitude of the effects on the Doppler-induced cross section changes to be expected when using different assumptions and these two particular sets of resonance parameters. The overall results indicated that about the same U-238 Doppler effect is predicted using either the ENDF/B or the Schmidt resonance compilations. It appears that the reason why Schmidt data has generally predicted U-238 Doppler effects larger than ENDF/B data is due to the substantially larger p -wave contribution given by the Schmidt parameters, coupled with the neglect of the reduction in this effect due to same-sequence resonance interference effects. The effects on the overall flux levels in the assembly of the various assumptions in the two sets of data were not included in this study.

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II-22. Doppler Effect Studies Using Multilevel Formalism

R. N. HWANG

The analytical aspects of the problems in the theory and application of the multilevel formalism to analysis of the Doppler effect have been discussed in Refs. 1 and 2. The most difficult problem is to estimate and interpret the multilevel effect in the unresolved region using a statistical treatment. From the previous discussions of Refs. 1 and 2, it is quite clear that the influence of the multilevel interference effect in the unresolved resonance region depends on many factors and one cannot reach any definite conclusion unless one can isolate each contributing factor. Specifically, it depends not only on the ratio of the average total width to the average spacing, the number of fission channels assumed and the energy region under consideration, but also it depends on the fundamental assumptions used in determining the multilevel and the single level parameters.

Ref. 3 is a sequel to the previous work described in Refs. 1 and 2, and it is intended to provide some quantitative results using a direct numerical approach. The first part of Ref. 3 describes the results in both the resolved and unresolved regions with strong emphasis on the latter. The existing RABBLE code⁴ has been modified to allow for both the single level representation and the multilevel representation in the Adler-Adler form.⁵ In the low energy region, where Adler-Adler parameters are available, the problem becomes quite simple, and the calculations can be made on a routine basis. In the unresolved region, the complex pole parameters and the residues are calculated by the matrix diagonalization code

MATDIAG using the "R-matrix statistical model" for the S-matrix parameters as described by P. A. Moldauer.^{6,7} The original MATDIAG code⁸ has been improved to allow for a maximum of 120 poles and also has been modified to punch cards with the Adler-Adler parameters⁵ in the modified RABBLE format so that meaningful studies can be carried out readily. Calculations have been made for highly enriched UO₂ and PuO₂ rods under various conditions. Detailed discussions are given in Ref. 3 for various cases.

The second part of Ref. 3 describes some further studies on the statistical behavior of the S-matrix parameters. These studies¹⁻³ are believed to be helpful in providing better understanding of the fundamental problems involved. The discussions are centered around the distribution functions of the residues which have not been discussed in Refs. 1 and 2. Since the problems involved are extremely difficult, only the limiting cases pertinent to the fissile isotopes of interest are considered. Again, an illustrative example using two interfering levels is given. Some theoretical predictions and numerical results are also given for the many-level case.

The detailed description of this work will appear in Ref. 8.

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II-23. Measuring Prompt Shutdown Safety Parameters in Plutonium-Fueled Fast Critical Facilities

R. G. MATLOCK, P. I. AMUNDSON and R. J. FORRESTER

The prompt shutdown parameters of primary interest in a fast critical facility are the reactivity removal rates and total reactivity worths of safety related devices. For a split-halves critical facility, the most rapid reactivity removal is obtained from safety rods and the largest reactivity removal from halves separation. Facility operating requirements are such that these parameters must be measured and checked frequently and be maintained at or above specified minimum values.

The above mentioned reactivity parameters can be inferred from time/position correlations with calibrated rods, from source multiplication measurements, or through time-dependent analysis of reactor power level. This last method is now being used on the Zero Power Plutonium Reactor and is described here.

The space-independent reactor kinetics equations can be combined into a single equation¹ and rearranged to yield reactivity as a function of time:²

$$\frac{\Delta k(t)}{k(t)\beta} = \frac{\ell}{k(t)\beta} \frac{dn(t)}{dt} + n(t) - \frac{1}{k(t)} \sum_i \alpha_i [\exp(-\lambda_i t)] \left[k_0 n_0 + \lambda_i \int_0^t (\exp \lambda_i \tau) k(\tau) n(\tau) d\tau \right] - \frac{S\ell}{k(t)\beta} \quad (1)$$

where

$n(t)$ = neutron population

$k(t)$ = effective multiplication factor

$\Delta k(t) = k(t) - 1$

ℓ = prompt neutron lifetime

S = effective neutron source

$\beta, \alpha_i, \lambda_i$ = delayed neutron parameters.

It is essential to include the effective neutron source term in assemblies where the spontaneous fission neutron contributions from such isotopes as Pu-240 is significant. At low power levels, this term can be isolated accurately if the power level history is recorded for two to three minutes after a rod drop.

Time-dependent neutron population data are obtained from high sensitivity ion chambers or counters. This information is multiscaled into an on-line computer, magnetic tape, or multichannel pulse height analyzer. These data along with calculated or otherwise previously determined values for $\ell, \beta, \lambda_i, \alpha_i$, and source are used to determine, from Eq. 1, reactivity as a function of time during the course of a power transient.

Figure II-23-1 shows the reactivity of a plutonium-fueled core (360 kg Pu-239 and Pu-241; 47 kg Pu-240) as a function of time, following the sequential scrambling of four safety rods. The rod scrams were initiated at approximately one second intervals and the reactor power level data taken at a rate of 100 samples per second.

Figure II-23-2 is a plot, on an expanded scale, of reactivity versus time for fuel safety rod No. 6. An accurate determination of the net rod worth, reactivity removal rate, and extent of rod "bounce" or

"rebound" at the end of the scram stroke can be made from this plot. A comparison of sequential data for a single rod gives an accurate indication of rod performance deterioration or changes in rod worth. Figure II-23-3 is a plot of scram data for fuel safety rod No. 6 taken at a later date. Due to core configuration changes the net reactivity worth of this rod decreased and a slight "dip" is apparent in the curve at 70 msec, suggesting a hesitation in rod motion (cleaning and lubricating this safety rod drawer eliminated this "dip").

The reactivity worth of the air gap at the interface between the reactor halves has been measured using the reactor power history method also. Figure II-23-4

FIG. II-23-1. ZPPR Rod Time Data, Week of May 26, 1969. ANL Neg. No. 103-A11152.

FIG. II-23-2. Reactivity Versus Time After Scram, Fuel Safety Rod No. 6. ANL Neg. No. 103-A11153.

FIG. II-23-4. Reactivity Versus Time for Half Separation at 30 in./min. ANL Neg. No. 103-A11155.

shows the system reactivity versus time for separation of the halves, from critical, at 30 in. per minute. The time from $t = 0$ to $t = 0.75$ seconds represents the electrical/electronic time lag between the initiation of table separation and actual table separation. The initial slope of the curve provides an accurate evaluation of the reactivity worth of the gap (table speed is constant).

These data for safety rod motion and table motion are easily obtained at the end of reactor operations which are scheduled for other purposes, i.e., special

FIG. II-23-3. Reactivity Versus Time After Scram, Fuel Safety Rod No. 6. ANL Neg. No. 103-A11154.

s are not required. The data are obtained within minutes after the measurement.

The technique of employing inverse kinetics analysis is used routinely on ZPPR as a rapid, accurate and detailed method of evaluating the performance of reactor safety related devices.

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II-24. Reactivity Measurements and Control Rod Calibrations in High Source Term Fast Critical Assemblies

R. G. MATLOCK, R. E. KAISER, J. M. GASIDLO and P. I. AMUNDSON

In a plutonium fueled fast critical assembly all types of reactivity measurements are affected, to some extent, by the presence of a Pu-240 spontaneous fission source. Since critical facilities are necessarily operated at low power, such a neutron source will influence flux-time evaluations of reactivity. This source contribution must be evaluated for accurate subcriticality determinations.

Historically, reactivity calibrations of control rods and safety devices have been made from asymptotic period measurements. With a source present, however, the time interval required to establish a sufficiently asymptotic period, for reactivity calibration purposes, is impractically long. Inverse kinetics analysis of reactor power level, as a function of time, is now in use at the Zero Power Plutonium Reactor for both reactivity calibrations and source worth determinations and the specific methods are summarized here.

For the determination of the neutron source term, the reactor power is maintained at a constant level for at least ten minutes (this step is necessary for the definition of the initial neutron precursor concentrations). Acquisition of reactor power data is then initiated, from either a counter or current chamber, at a rate of about one sample per second. Following at least ten data samples at constant power, a fixed amount of reactivity is added to establish a period of about 100 sec, i.e., significantly longer than the lifetime of the longest lived decayed neutron precursor. Data is accumulated for approximately five minutes, during which reactivity changes are not made subsequent to the initial reactivity addition, and then the experiment is terminated.

These data are analyzed initially, assuming no neutron source term present in the reactor kinetics equations, for the reactivity-addition versus time in the experiment. A typical calculated result is shown in Fig. II-24-1. After the initial reactivity addition, this

calculation suggests a gradual reactivity removal. This effect results from the assumption that no source was present, since the reactivity worth of the actual source decreases as power increases. Subsequent iterations on these data, with sources included, will produce the correct source term to be used. When the correct source term is used, the correct reactivity-addition versus time curve results as shown in Fig. II-24-2 (i.e., an initial reactivity addition followed by no reactivity change).

The neutron source reactivity worth at steady power is given by

$$\Delta k_s = \frac{S\ell}{n} = \frac{Q}{CR},$$

where

$S \equiv$ neutron source strength

$\ell \equiv$ neutron lifetime

$n \equiv$ neutron population

$Q \equiv$ source term ($S\ell\epsilon$)

$\epsilon \equiv$ detection efficiency (CR/n)

$CR \equiv$ count rate or chamber current,

Once the source term Q has been determined, the degree of subcriticality at any power level is simply obtained from the ratio of Q/CR . A different source term applies to each instrument used, since ϵ varies with each detection channel. For all detectors, at a given power level, the ratios of Q/CR are identical.

For a rod calibration the reactor power is established at a steady, comparatively high level, again for the purpose of neutron precursor concentration stabilization. Power level data are multiplexed at a rate of about one sample per second. Following at least ten data samples at constant power, the rod to be calibrated is moved to remove reactivity. The rod must be moved at a constant speed, or have some other accurate means of position/time correlation, if both total worth and shape calibration are needed. A total worth calibration alone does not require intermediate rod position

FIG. II-24-1. Calculated Result for the Reactivity-Addition Versus Time. ANL Neg. No. 103-A11138.

FIG. II-24-2. Corrected Reactivity-Addition Versus Time Curve. ANL Neg. No. 103-A11137.

information. Data acquisition continues for approximately 60 sec after rod motion ceases.

These data are analyzed, using the reactor kinetics equations, to determine the reactivity change as a function of time (and subsequently position) introduced by the rod motion. The total rod worth is determined from the average values obtained immediately preceding and following rod motion. A typical poison-rod worth versus time curve and a control-rod total worth

and shape calibration are shown in Figs. II-24-3 and II-24-4 respectively.

The reactor temperature coefficient is measured by monitoring and analyzing data in a manner very similar to that for rod calibrations. Reactor power is established at some convenient level and the significant temperatures allowed to stabilize. Power level temperature data are multiplexed at a one sample per second rate. After at least ten initial samples, the

FIG. II-24-3. Typical Poison-Rod Worth Versus Time Curve. *ANL Neg. No. 103-A11055.* FIG. II-24-4. Control Rod Total Worth and Shape Calibration. *ANL Neg. No. 103-A11142.*

reactor cooling air is throttled or shut off completely. Data are accumulated for 15 to 20 min as the reactor temperature rises, and subsequently analyzed for reactivity change versus time. These results, in conjunction with temperature data, provide the temperature coefficient of reactivity.

source constant as well as delayed neutron parameters are necessary in the kinetics equations in order to do these analyses. The source term is provided

as described above and the "best data" delayed neutron parameters are used.

A study of the effect of variations in the delayed neutron parameters used in the calculations of the source term and rod worth indicates a negligible effect on results reported in inhours.

The effect of source uncertainty on rod calibrations was checked by running selected sets of rod calibration data with source constants equal to $Q + \sigma$ and $Q -$

σ , where $\sigma = 8.2\%$. The resulting deviation in rod worth was less than 1%, suggesting little sensitivity to source error. Also, the fact that a fraction of the source neutrons and some delayed neutron precursors are removed from the core by a fuel bearing rod, while it is being calculated, does not appear to have a significant effect on the calibration.

Since subcritical reactivity is directly proportional to the source constant, a precise evaluation of Q is required only for precise subcriticality measurements. Rod calibrations and other measurements using this inverse kinetics method do not place particularly stringent requirements on the precision of the source measurement.

II-25. A Study of the Influence of Reactor Neutron Spectra on Delayed Neutron Parameters

R. E. KAISER

INTRODUCTION

Considerable data are available which demonstrate a dependence of the delayed neutron fractions for various fissile isotopes on the energy of the neutrons causing fission. Although no more than three data points are available for $a_i(E) [= \beta_i/\beta]$ as a function of E for most isotopes, an attempt has been made to estimate the effect of such variations with energy on calculated effective kinetics parameters for plutonium-fueled critical assemblies. The results of the study indicate that a difference of between five and ten percent may result in the errors noted in comparing central reactivity measurements with calculated values.

MATHEMATICAL FORMULATION

The variation of $a_i(E)$ with energy was assumed to be linear between known values which were at thermal and fast neutron energies and at 14 MeV. Values of a_i quoted for fast neutrons represent measurements in spectra whose average energies were in the 1-3 MeV range. The assumption of linearity is based on the lack of sufficient data for any more detailed specification.

The equations used can be developed without difficulty using the subscripts

$i \equiv$ incident neutron energy

$j \equiv$ energy group

$k \equiv$ flux position dependence

$m \equiv$ fissile isotope.

The fission rate as a function of energy group for each fissile isotope is given by

$$FR_m = \sum_k \phi_{jk} \Sigma_{mjk}^f, \quad (1)$$

where ϕ_{jk} is the flux in group j at position k , and Σ_{mjk}^f is the fission cross section for isotope m in energy group j . In the computer program which performs the calculations, Σ^f is treated as varying between regions but not

within a given region. The calculations are then performed in terms of β_i . A particular β_i for isotope m is then given by

$$\langle \beta_i \rangle_m = \frac{\sum_j (\beta_i)_{mj} \bar{v}_{mj} \sum_k \phi_{jk} \Sigma_{mjk}^f}{\sum_j \bar{v}_{mj} \sum_k \phi_{jk} \Sigma_{mjk}^f}, \quad (2)$$

or in terms of Eq. (1),

$$\langle \beta_i \rangle_m = \frac{\sum_j (\beta_i)_{mj} \bar{v}_{mj} FR_m}{\sum_j \bar{v}_{mj} FR_m}. \quad (3)$$

The quantity $(\beta_i)_{mj}$, which represents an average β_i for the j th energy group, is calculated according to the equations

$$(\beta_i)_{mj} = (\beta_i)_m^{thermal} + \left(\frac{d\beta}{dE} \right)_1 (\bar{E}_j - E_{thermal}), \quad E_j < E_B \quad (4)$$

and

$$(\beta_i)_{mj} = (\beta_i)_m^B + \left(\frac{d\beta}{dE} \right)_2 (\bar{E}_j - E_B), \quad E_j > E_B, \quad (5)$$

where E_B is the energy at which the fast a_i values were measured, and where the value of $d\beta/dE$ changes. The energy variation in $\bar{v}$ can be treated by reading $(\bar{v}\Sigma^f)_j$ into the computer program or by reading in Σ_j^f , $\bar{v}_{thermal}$, and $d\bar{v}/dE$, and evaluating $\bar{v}_{mj}$ from

$$\bar{v}_{mj} = \bar{v}_m^{thermal} + \frac{d\bar{v}}{dE} (\bar{E}_j - E_{thermal}). \quad (6)$$

In Eqs. (4)-(6), $\bar{E}_j$ represents the median energy of the j th group.

The computer program, SPECB, which performs these calculations, requires fluxes and adjoints from a MACH-1⁽¹⁾ diffusion calculation in spherical geometry as input. Fission cross sections (with or without values

), and both prompt and delayed neutron emission spectra can be obtained from the cross section library used for the MACH-1 run. The program will handle up to nine fissioning isotopes in a four region reactor with 150 spatial mesh points. Up to 30 energy groups may be used.

In addition to the calculation of spectrally weighted delayed neutron fractions, the program will proceed to calculate effective β_i values, prompt neutron lifetime, and $\text{Ih/percent reactivity}$. The equations used are the same as those used in the MACH-1 BAILIFF⁽¹⁾ calculations, and are listed below for convenience. For the β_i values

$$(\beta_{eff})_{mi} = \frac{\int_v \left[\sum_j (\nu \Sigma^f)_{mj} \phi_j \right] \left[\beta_{im} \sum_j \chi_{mj}^D \phi_j^* \right] dV}{\int_v \left[\sum_j (\nu \Sigma^f)_{mj} \phi_j \right] \left[\beta_{im} \sum_j \chi_{mj}^D \phi_j^* + (1 - \beta_{im}) \sum_j \chi_j \phi_j^* \right] dV}, \quad (7)$$

where χ^D and χ are the delayed and prompt neutron emission spectra, respectively. The prompt neutron lifetime is given by

$$\ell_p = \frac{k \int_v \left[\sum_j \phi_j \phi_j^* / v_j \right] dV}{\Sigma_m \int_v \left[\sum_j (\nu \Sigma^f)_{mj} \phi_j \right] \left[\sum_j \chi_j \phi_j^* \right] dV}, \quad (8)$$

where v_j is the median neutron velocity in group j . The $\text{Ih/percent reactivity}$ is given by

$$\text{Ih}/\%(\Delta k/k) = \frac{0.01}{\rho(3600)},$$

where

$$\rho(3600) = \frac{1}{\ell_p + 3600} \left(\ell_p + 3600 \sum_m \sum_i \frac{(\beta_{eff})_{im}}{1 + 3600 \lambda_{im}} \right).$$

The calculation of effective parameters is based on the

fluxes and adjoints from MACH-1 mentioned previously.

CALCULATED RESULTS

Calculations were performed to obtain two sets of spectrum-weighted delayed neutron parameters, one with 14 MeV values for U-238 and U-235 from C. Masters et al.² and for other isotopes from C. McGarry,³ and the second with data for all isotopes from McGarry.³ Thermal and fast delayed neutron parameters for both sets were from G. Keepin.³ The β_i values at 14 MeV were calculated using a_i values for fast neutrons and total β values from the sources stated above. Both sets of parameters used the same values for χ^D and χ . The two sets will henceforth be referred to as WBS1 (Masters Data) and WBS2. The calculations were performed for ZPR-3 Assemblies 53 and 54, using homogeneously shielded cross sections from MC²(4) based on the ENDF/B library.

The results of the calculations for the two weighted sets and for a standard set obtained directly from MACH-1 BAILIFF are given in Table II-25-I. The change of 5–10% in the reactivity conversion factor

TABLE II-25-I. VARIATION IN EFFECTIVE KINETICS PARAMETERS DUE TO SPECTRUM WEIGHTING IN ZPR-3 ASSEMBLIES 53 AND 54

Parameter	Standard Set	Weighted Sets	
		WBS1	WBS2
Assembly 53			
Ih/% $(\Delta k/k)$	1045.6	1000.3	984.9
ℓ_p , sec	5.16×10^{-7}	5.18×10^{-7}	5.18×10^{-7}
β_{eff}	0.00286	0.00293	0.00300
Assembly 54			
Ih/% $(\Delta k/k)$	1096.9	1047.9	1038.6
ℓ_p , sec	8.43×10^{-7}	8.16×10^{-7}	8.16×10^{-7}
β_{eff}	0.00256	0.00262	0.00266

TABLE II-25-II. COMPARISON BETWEEN MEASURED AND CALCULATED CENTRAL WORTHS IN ZPR-3 SHOWING THE EFFECT OF SPECTRUM-WEIGHTED DELAYED NEUTRON PARAMETERS

Material	Measured Value, Ih/kg	Calculated					
		Standard		WBS1		WBS2	
		Ih/kg	c/m	Ih/kg	c/m	Ih/kg	c/m
Assembly 53							
Pu composite ^a	652.9 ± 7.1	854.6	1.309	817.9	1.253	805.0	1.233
U-235	538.6 ± 8.4	662.3	1.230	633.8	1.177	623.9	1.158
U-238	-61.9 ± 1.5	-100.7	1.627	-96.4	1.557	-94.9	1.533
Assembly 54							
Pu composite ^a	722.6 ± 9.9	1006.5	1.393	962.2	1.332	953.1	1.319
35	580.5 ± 14.9	731.0	1.259	698.8	1.204	692.2	1.192
U-238	-66.8 ± 2.2	-129.6	1.940	-123.9	1.855	-122.7	1.837

^a 98.62 w/o Pu, 1.22 w/o Al, 95.05 w/o Pu-239, 4.5 w/o Pu-240, 0.45 w/o Pu-241.

$\text{Ih}/\% (\Delta k/k)$ is significant in that this factor is usually applied to the results of perturbation calculations to obtain worths in Ih/kg for comparison with experimental values.

The effect of the different β sets on the calibration of the autorod used in measuring central reactivities was found to be negligible. The raw inverse kinetics data for the rod calibration was processed with each of the weighted sets and the standard set and no significant difference in total rod worth was noticed. The result, insofar as central reactivity worths are concerned, is that the calculated results should be adjusted by the 5–10% difference in the reactivity conversion factor.

Table II-25-II presents a comparison between the measured and calculated worths of U-235, U-238, and Pu-239. Measured values are based on the standard set of delayed neutron parameters. The effect of spectrum weighting of the delayed neutron parameters is also illustrated in this table. The corrections obtained do not by any means explain the full discrepancy between measurement and calculation, although in some cases it does account for a significant portion of it.

CONCLUSIONS

The results of this study indicate that in the determination of effective kinetics parameters some signifi-

cant improvement in agreement between reactivity measurements and calculations can be obtained by considering the energy of the neutrons causing fission. This would be particularly true in soft-spectrum cores where the mean neutron energy differs greatly from the conditions under which the values for α , for fast fission were determined. The mean neutron energy in the assemblies studied was about 141 keV. The assumption of linear variation of $\beta(E)$ may not be entirely accurate, but it is felt that this assumption is sufficient to demonstrate the possibility that the type of calculation described in this paper may provide a significant increase in the accuracy of reactivity measurements. It is hoped that future research may provide more extensive data for β as a function of energy on which to base this type of a calculation.

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II-26. Pu-241 and Pu-242 Delayed Neutron Parameters

E. M. BOHN

The plutonium fuel used in the fast critical assemblies contains small amounts of Pu-241 and Pu-242. Some of the delayed neutron data for Pu-241 and all the delayed neutron data for Pu-242 are lacking and it has been the custom to use Pu-239 delayed neutron parameters for Pu-241 and Pu-240 parameters for Pu-242.

TABLE II-26-I. AVERAGE VALUES OF $(Z_A - Z_P)$

Isotope	$(Z_A - Z_P)_{av}$
U-233	3.6 ^a
U-235	4.0 ^a
U-238	4.5 ^a
Pu-239	3.7 ^a
Pu-240	3.8 ^a
Pu-241	3.9 ^b
Pu-242	4.0 ^b

^a From Ref. 1.

^b Extrapolated from Pu-239 and Pu-240 values.

Although valid for cross section comparison, this sort of prescription for missing data is invalid for the case of delayed neutron data. An attempt has been made here to generate the missing data for Pu-241 and Pu-242.

A discussion of the creation of delayed neutron precursors in fission is presented in Refs. 1 and 2, and the features of interests to this work are briefly outlined here.

The properties of delayed neutrons are characteristic of their fission fragment precursors. The probability that bromine, iodine, and many additional fragments become precursors is enhanced as the probability of β -decay to high lying levels in the daughter nuclide is increased. One indication of the probability of β -decay is the fission chain length, $(Z_A - Z_P)$. Z is the atomic number of the most stable nuclide for fission fragment of mass A , and Z_P is the most probable

FIG. II-26-1. Absolute Group Yields Versus $Z_A - Z_P$ for Six Delayed Neutron Groups. Plotted Points are from Reference 3. ANL Neg. No. 113-3071.

TABLE II-26-II. Pu-241 AND Pu-242 DELAYED NEUTRON DATA

Group	Absolute Yield, a_i , Per Fission, 10^{-2}	Relative Yield, a_i/a	$\beta_i = a_i/\nu^{(a)}$, 10^{-2}
Pu-241			
1	0.015 ^b	0.0094	0.0047
2	0.365 ^b	0.2294	0.1163
3	0.275 ^b	0.1728	0.0876
4	0.620 ^b	0.3897	0.1976
5	0.290 ^b	0.1823	0.0924
6	0.026 ^c	0.0163	0.0083
	$\sum_i a_i = a = 1.591 \times 10^{-2}$		$\sum_i \beta_i = \beta = 0.5069$
Pu-242			
1	0.010 ^c	0.0035	0.0031
2	0.550 ^c	0.1952	0.1718
3	0.455 ^c	0.1615	0.1422
4	1.160 ^c	0.4115	0.3625
5	0.615 ^c	0.2183	0.1922
6	0.028 ^c	0.0099	0.0157
	$a = 2.818$		$\beta = 0.8875$

^a Values of 3.14 and 3.2 were used for the ν values for Pu-241 and Pu-242, respectively. These values are characteristic of the neutron spectrum for which experimental values of the ed neutron yields in Fig. II-26-1 were obtained.

^b Values from Ref. 1.

^c Values from Fig. II-26-1.

FIG. II-26-2. Systematics of Total Delayed-Neutron Yields. Yield Versus the Parameter $A-3Z$, where A and Z are the Mass Numbers and Atomic Numbers of the Compound (or Spontaneous) Fissioning Nucleus (from Reference 1). ANL Neg. No. 113-3096.

charge in fission associated with a fragment of mass A . The greater the difference $(Z_A - Z_P)$, the more neutron-rich is a fragment and greater is the possibility of β -decay. The probability of β -decay to high lying levels may be shown to be proportional to $(Q_Z^\beta - B_{N-1})$ where Q_Z^β is the β -decay energy of a nuclide (Z, N) and B_{N-1} is the neutron binding energy of nuclide $(Z + 1, N - 1)$. The fission chain length appears to be directly correlated with $(Q_Z^\beta - B_{N-1})$. Thus, the probability for formation of precursors and subsequent production of delayed neutrons is directly related to the parameter $(Z_A - Z_P)$.

The average value of $(Z_A - Z_P)$ for the fissile nuclides of interest are shown in Table II-26-I. The values for Pu-241 and Pu-242 were selected as a linear extrapolation from the Pu-239 and Pu-240 values. (Reference 3 shows that $(Z_A - Z_P)$ varies linearly with mass for a given Z number of the fissioning nucleus.)

DELAYED NEUTRON YIELDS

The absolute delayed neutron yields per fission for the six delayed groups are plotted as a function of

TABLE II-26-III. DELAYED NEUTRON SPECTRA FOR Pu-241 AND Pu-242

Group	E_L , ev	Pu-241	Pu-242
1	36.7879×10^5	0.00000	0.00000
2	22.3130×10^5	0.00000	0.00000
3	13.5335×10^5	0.296375×10^{-1}	0.315780×10^{-1}
4	82.0850×10^4	0.139152×10^0	0.141191×10^0
5	49.7871×10^4	0.254820×10^0	0.254144×10^0
6	30.1974×10^4	0.234040×10^0	0.230710×10^0
7	18.3156×10^4	0.162040×10^0	0.163333×10^0
8	11.1090×10^4	0.967436×10^{-1}	0.964170×10^{-1}
9	67.3795×10^3	0.561564×10^{-1}	0.553992×10^{-1}
10	40.8677×10^3	0.231655×10^{-1}	0.231640×10^{-1}
11	24.7878×10^3	0.413907×10^{-2}	0.416151×10^{-2}
12	15.0344×10^3	0.00000	0.00000
13	91.1882×10^2		
14	43.0743×10^2		
15	26.1259×10^2		
16	20.3468×10^2		
17	12.3410×10^2		
18	96.1117×10^1		
19	58.2447×10^1		
20	27.5364×10^1		
21	10.1301×10^1		
22	29.0232×10^0		

($Z_A - Z_P$) in Fig. II-26-1. The data for Pu-239, Pu-240 and the first five groups of Pu-241 were obtained from Ref. 1. Smooth curves were drawn through these points and extrapolated to obtain values for Pu-242 and group 6 of Pu-241. The results are tabulated in Table II-26-II. A total yield of 2.82×10^{-2} is predicted for Pu-242 and is compared to the total yields of other fissile isotopes in Fig. II-26-2. The predicted value appears to be consistent with the systematics of Fig. II-26-2. It should be apparent from Fig. II-26-2 that use of Pu-239 and Pu-240 yields for the yields of Pu-241 and Pu-242 underestimates the latter yields by a factor of two to three.

For the present, it is recommended that the group

half lives of Pu-242 be taken as the average for plutonium isotopes.

DELAYED NEUTRON SPECTRA

Twenty-two group delayed neutron spectra for Pu-241 and Pu-242 were generated using the values for the relative yields of delayed neutrons (Table II-26-II) and D. Meneley's program "Delay Spectra."⁴ The program utilizes the experimental delayed neutron spectra of the first four groups for uranium¹ (no other exists) and assumes the spectra of groups 5 and 6 are the same as group 4. The measured uranium spectrum of each delayed group is multiplied by the relative yield of the group. The spectra for all six delayed groups are then combined to give the total delayed neutron spectrum listed in Table II-26-III.

SUMMARY

A simple procedure, based on the systematics of delayed neutron production, was employed to obtain an estimate of delayed neutron parameters for Pu-241 and Pu-242. It was shown that the yields of delayed neutrons from Pu-241 and Pu-242 may be two to three times greater than the yields presently used. Although the concentration of these two plutonium isotopes in the assemblies currently under study may be small, such data are of great importance in the study of high-burnup plutonium fueled cores.

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II-27. β_{eff} Ratio Measurements

S. G. CARPENTER and S. RAMCHANDRAN

INTRODUCTION

One of the most important types of measurements done on zero power critical experiments is that of reactivity of small samples. There have been a number of suggestions that systematic errors exist in the measurements or calculations which would limit the use of such

results to the reactivity ratios only. For example, W. Little and R. Hardie¹ indicate approximately 30% difference in the absolute values for certain cores. Ratio data are valuable but the disagreement is disturbing, and in the case of mockups absolute measurements are often wanted.

FIG. II-27-1. Reactivity Histories with Changing β_{eff} (U-238). ANL Neg. No. 103-A11103.

The units of measurement are commonly cents or inhours while the calculation is in $\Delta k/k$. The conversion factor between the two, β_{eff} , is calculated. The purpose of the present study is to investigate the possible error in the measurement of the reactivity in cents from the most plausible cause, namely, that of an error in the assumed ratio of the effective number of delayed neutrons from the fissile and fertile materials. β_{eff} will be measured in a later experiment.

METHOD

Small sample reactivity measurements are made using Eq. (1) or a simplification of it:

$$\frac{\Delta k_e}{k_e \beta_{eff}} + \frac{l S_e}{k_e \beta_{eff} n} = 1 + \frac{1}{n k_e} \left[\frac{l}{\beta_{eff}} \frac{dn}{dt} - \sum_i a_i e^{-\lambda_i t} \left(n_0 k_0 + \lambda_i \int_0^t k_e n \{ \exp \lambda_i t \} dt \right) \right]. \quad (1)$$

This equation is derived² from the point kinetics model

TABLE II-27-I. ZPR-3 ASSEMBLY 56 CALCULATION OF DELAYED NEUTRON SOURCES

Time	SUM ^a	F ₁ ^b	F ₂	F ₃	F ₄	F ₅	F ₆
Before Drop	12338	59	619	732	1753	1016	337
After Drop	10393	69	724	815	1776	722	169
Later	6181	111	1074	848	1207	491	158
Later	4180	150	1300	670	960	496	162
Later	2620	200	1374	482	936	517	170
50 sec	1842	236	1305	452	974	545	180

^a This value is proportional to the source strength of neutrons in the reactor from the delayed emitters.

^b These values are proportional to the percent of neutrons being emitted at each designated time in the six delayed groups of any one isotope.

which should apply quite well for small sample worth determinations. The right hand side is insensitive to the neutron lifetime (l), the initial neutron multiplication factor (k_0), the effective neutron multiplication factor (k_e), and β_{eff} when measurements are done at a reasonable power. The values of the other constants however, do affect the accuracy of the results.

The terms on the left can be separated if the value of n changes while Δk_e and the effective source (S_e) remain constant. This condition exists after a rod drop. If the values for the a_i 's and λ_i 's are satisfactory, Δk_e and S_e will be constants within statistics, if not at least one of two must vary to satisfy the equality. It is assumed that the values of λ_i and a_i for the individual isotopes are correct as reported by G. Keepin³ (and supported by S. Cox and E. Whiting⁴) but that the relative value of β_{eff} of U-238 and the fissile material can vary to improve the fit to the power history data. This could occur if the isotopic values of β are wrong, the relative fission ratios are calculated incorrectly (this can be checked experimentally but generally the values are not modified from the calculation in analyzing reactivity measurements), or β for each isotope is incorrectly converted into β_{eff} .

The calculated constants were taken from MACH-1 calculations of the respective cores. The values of n were found from an ion chamber connected to a current-to-frequency converter. The reactivity changes were made by moving fuel-bearing safety rods so as to minimize counter efficiency changes during the experiments.

RESULTS

The first core investigated was ZPR-3 Assembly 56. Figure II-27-1 shows the reactivity as a function of time calculated from Eq. (1) with a range of assumed values of β_{eff} for U-238. The value of S_e was taken to make each curve tend to be flat for each choice of β_{eff} for U-238. The minimum deviation from a level line for any S_e occurred at a β_{eff} for U-238 of $8 \pm 2\%$ less than was calculated. The minor contributions calculated for the

other isotopes were left unchanged. It can be seen that for all six cases the measurement of the rod worth (left-most values) was within 1% of 132¢. The rod worth is also insensitive to values of a_i and λ_i at short times after the drop. For the optimum choice of relative β_{eff} , this result persists as it should while the relative precursor densities change significantly. Typical values for the relative number of delayed neutrons F from the six delayed groups of one isotope are given in Table II-27-I. The total of the F 's for all isotopes is a constant. SUM is the total source from delayed neutrons. The reactivity is calculated well for this wide range of relative values and ratio of individual values. As long as measurements are made remaining within the same general bounds, the reactivity errors will not be large.

The second core studied was ZPPR Assembly 1. This assembly was quite similar in composition to the first. Table II-27-II shows the results of the calculations. Again the worth of the rods is insensitive to the β_{eff} ratio assumed. The best fit occurs at $-4 \pm 2\%$ reduction in β_{eff} for U-238 relative to Pu-239 compared with the calculated value.

The third core was ZPR-3 Assembly 57. This core contained primarily U-235, U-238 and BeO. The fission rate in the U-238 was only 6% of the total ($\sim 13\%$ in the other cores). This caused the sensitivity to be lower. The result was $-12 \pm 6\%$ reduction in β_{eff} for U-238 relative to U-235 compared with the calculated value.

CONCLUSIONS

The error in the calculated values of β_{eff} in any one isotope is not large and does not affect the reactivity calibrations to an important extent. In particular, the value of β_{eff} for U-238 is not in serious error relative to Pu-239 or U-235. The small differences found could be due to β -ratio errors, relative fission rate calculations,

TABLE II-27-II. ZPPR ASSEMBLY 1 MEASUREMENT OF β_{eff} (U-238)

β_{eff} Ratio ^a U-238/Pu-239, %	Relative Error ^b	Average Reactivity ^c Subcritical ¢
+30	1.036	187.2
+20	0.811	187.3
+10	0.586	187.4
As Calculated	0.413	187.5
-10	0.425	187.9
-20	0.661	188.1
-30	0.999	188.4

^a It was assumed that the β_{eff} of U-238 was higher or lower than the Mach-1 calculation by the % indicated.

^b Δk_e was picked for each case to minimize the variation of S to satisfy the equation at each data point. The listed values are proportional to the root mean square error at that minimum.

^c This value is the subcriticality of the reactor mentioned in footnote b.

conversion to β_{eff} ratios. The measurement of β_{eff} can therefore probably be done on any core with flux and adjoint flux typical of fast reactors. If there is no correction found for β_{eff} , reactivity measurements in $\Delta k/k$ would seem to be correct as reported. If a scale factor is found due to β_{eff} , it should be quite similar in magnitude for a large number of cores and previous results could be corrected without rebuilding the cores.

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II-28. Fast Neutron Spectra in Uranium Metal

E. F. BENNETT and T. J. YULE

A large (75 x 80 x 90 cm) block of uranium metal is available at the Argonne Thermal Source Reactor (ATSR)¹ for the continuation of exponential experiments, the first of which was reported in Ref. 2.

Experiments in good geometry are in progress and some initial analyses of the results using the ENDF/B Version 1 cross-section material have been made. Most of the measurements at present have been of the neutron spectrum using proton-recoil spectrometers. We will here describe one particular spectrum which is of interest in that a large disagreement with calculation exists. Since the block material is predominantly U-238, the discrepancy is an indication of an inadequate cross-section input to the one-dimensional diffusion code used in the calculation.

The experimental arrangement is sketched in Fig. II-28-1. The uranium block is coupled to leakage neutrons from the ATSR source reactor by a matching graphite pedestal having black boundaries in the transverse directions. Slow neutrons incident upon the uranium metal produce a fission spectrum source which can be measured at various axial distances from the graphite-uranium interface. Fundamental mode neutron flux distributions were maintained through the graphite-uranium column; the large dimensions of the column along the transverse sections held buckling corrections to small values.

The measured spectrum changed in shape and amplitude as measurements were extended axially in the uranium. The harder spectra occur near the graphite-uranium interface and it is the comparison of one measurement (12.7 cm from the interface) with calculation which will be given.

FIG. II-28-1. Experimental Arrangement. ANL Neg. No. 113-2752.

FIG. II-28-2. Comparison of Measured Spectrum and Calculated Energy-Group Histogram. ANL Neg. No. 113-2751.

The experimental method has been described in Ref. 3; numerous improvements in technique have been incorporated since the last reported results.

The measured spectrum (together with a calculated energy-group histogram) are shown in Fig. II-28-2. Corrections to the measured data have been made to include both wall-and-end effect distortion at the higher energies and the effect of nonuniform internal electric fields in the counting tubes.

The cylindrical proportional counters were constructed of thin-walled (16-mil) stainless steel. Even this small amount of steel is sufficient to cause an occasional small flux perturbation at the most prominent scattering resonances in iron.

Group cross sections were generated by the ARC multigroup cross-section code for a depleted uranium block with zero buckling from ENDF/B cross sections. Sixty-eight groups of 1/4 lethargy width plus a thermal group were chosen. The ARC one-dimensional diffusion code was used to determine the spectrum as a function of position in the depleted block. The calculation was an external source type with the source distributed near the front face of the graphite block. The calculated spectrum in the depleted uranium block was found to be quite insensitive to the assumed energy distribution of the external source.

A comparison of the measured spectrum with the

calculated spectrum (arbitrarily normalized) in Fig. II-28-2 indicates significant disagreement, especially at the higher energies. The calculated spectrum is considerably softer than the measured spectrum for energies in excess of about 1 MeV. This is the region where the spectrum depends strongly on the U-238 inelastic scattering cross section. The accuracy of this cross section in the ENDF/B file has been questioned. Fast reactor integral studies have indicated that the cross sections in ENDF/B for U-238 are unsatisfactory. (See Paper II-32.)

The simple geometrical configuration and the fact that an almost-pure U-238 environment is used in the measurement make the comparison of experiment with theory especially useful in understanding U-238 inelastic scattering.

Further measurements in the block are planned. More detailed investigation of the calculational methods, as well as the effect of modifications to the cross sections are under investigation.

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II-29. Fast Neutron Spectrum Measurements in the Materials Test Reactor (MTR) Scandium and Iron Beams

J. E. POWELL and J. W. ROGERS*

INTRODUCTION

Fast neutron spectrum measurements were made with proton-recoil proportional counters in the MTR iron¹ and scandium² beams to check the accuracy of the energy response of the proportional counters. These beams exhibit several sharp lines (see Tables II-29-I and II-29-II) at varying energies due to minima in the iron and scandium cross sections. By measuring the neutron spectrum the counter response can be checked against basic cross section data.

The energy calibration for a proportional counter depends upon an accurate determination of the relationship between the high voltage and the multiplication. For correct counter response, however, the energy loss per ion pair, w , must also be known. Several models for the w of hydrogen have been suggested^{3,4} and they indicate that w varies with energy below 6 keV. However, fast reactor spectrum measurements seem to contradict these models, the measured location of the sodium resonance (3 keV) being the prime indicator.

For these measurements, w for both hydrogen and methane was assumed to be constant for all energies and the calibration techniques suggested by E. Bennett³ were used to obtain a relationship between counter multiplication and high voltage.

EXPERIMENTS

The proton-recoil proportional counting technique that was used for this measurement is discussed in Refs. 3 and 5 and the electronic system is described in Ref. 6. Two proportional counters were used over the energy range of 113 eV to 1 MeV. A predominantly hydrogen-filled counter (5 atm total pressure) 1.5 cm in diameter with a sensitive length of 3.2 cm was used from 113 eV to 130 keV and a methane-filled counter (5 atm total pressure) 2.54 cm in diameter with a sensitive length of 7.6 cm was used from 90 keV to 1 MeV.

The counters were calibrated in the usual manner³ with the $N^{14}(n,p)C^{14}$ reaction. A 5% correction was applied to the hydrogen counter calibration results to account for distortion of the calibration pulses by the short time constants (1.5 μ sec integration, 1.5 μ sec differentiation) of the shaping amplifier in the linear circuit.

* Idaho Nuclear Corporation, Idaho Falls, Idaho.

For both the scandium and the iron beam measurements, the long axis of each counter was placed 1 in. from the end of the collimator, perpendicular to the direction of the neutrons. The collimator is described in Refs. 1 and 2.

RESULTS

The proton spectra (proton flux per unit lethargy) for the iron and scandium beam measurements are shown respectively in Figs. II-29-1 and II-29-2. Twelve voltage settings on the counter (2 methane, 10 hydrogen) were used to obtain these spectra which extend down to 113 eV. The corresponding neutron spectra which were obtained by use of the PSNS Code⁷ are shown in Fig. II-29-3 (iron) and Fig. II-29-4 (scandium). The peak values of the iron beam spectrum are listed in Table II-29-I with values quoted by J. Rainwater⁸ who measured the spectrum of accelerator neutrons transmitted through

TABLE II-29-I. COMPARISON OF PROTON-RECOIL AND TIME-OF-FLIGHT SPECTRUM MEASUREMENTS OF NEUTRONS TRANSMITTED THROUGH IRON

Counter	Proton-Recoil, keV	Time-of-Flight, keV
H ₂	24.8 $\pm$ 0.25	24.74 $\pm$ 0.04
	81.0 $\pm$ 0.8	81.9 $\pm$ 0.08
CH ₄	133 $\pm$ 1.3	136.8 $\pm$ 0.2
	164 $\pm$ 1.6	167.3 $\pm$ 0.2
	207 $\pm$ 2.1	218.3 $\pm$ 0.4
	256 $\pm$ 2.6	242.9 $\pm$ 0.5
	276 $\pm$ 2.8	271.5 $\pm$ 0.5
	314 $\pm$ 3	307.9 $\pm$ 0.6
	356 $\pm$ 3.6	350.1 $\pm$ 0.7
	470 $\pm$ 4.7	465 $\pm$ 1
	560 $\pm$ 5.6	551.9 $\pm$ 1
	651 $\pm$ 6.5	633 $\pm$ 1.2
	959 $\pm$ 9.6	698 $\pm$ 1.4
		946 $\pm$ 2

TABLE II-29-II. COMPARISON OF PROTON-RECOIL AND TIME-OF-FLIGHT SPECTRUM MEASUREMENTS OF NEUTRONS TRANSMITTED THROUGH SCANDIUM

Proton-Recoil, keV	Time-of-Flight, ^a keV
1.95 $\pm$ 0.02	1.95 $\pm$ 0.05
7.94 $\pm$ 0.08	8 $\pm$ 0.2

FIG. II-29-1. Proton-Recoil Flux per Unit Lethargy for the MTR Iron Beam. *ANL Neg. No. 103-A11042.*

FIG. II-29-2. Proton-Recoil Flux per Unit Lethargy for the MTR Scandium Beam. *ANL Neg. No. 103-A11044.*

FIG. II-29-3. Neutron Spectrum for the MTR Iron Beam. *ANL Neg. No. 103-A11089.*

FIG. II-29-4. Neutron Spectrum for the MTR Scandium Beam. ANL Neg. No. 103-A11090.

iron with a 200 m time-of-flight facility. The spectrum of the scandium was measured with the MTR Fast Chopper⁹ up to 10 keV and a comparison of the location of the peaks in that energy range is given in Table II-29-II. An uncertainty of $\pm 1\%$ has been assigned to the proton-recoil calibration which carries through as an uncertainty in peak location.

The resolution of the proportional counter is about 10%, except at the lowest energies where an increase is caused by the multiplication process. At 2 keV the resolution is 45% FWHM (Full Width-Half Maximum).

DISCUSSION

The location of the peaks in the iron and scandium beam spectra, as measured by the proton-recoil spectrometer, are generally in excellent agreement with the time-of-flight results. Differences on the order of 3 to 5% exist between 100 and 300 keV in the iron beam results. This may indicate some structure in w for methane in this region, but this conclusion assumes that the time-of-flight results are correct.

The results of the scandium beam measurements indicate that w for hydrogen is constant down to 2 keV. The nature of the scandium cross section between 185 eV and 1 MeV is also predicted, although the relatively poor resolution of the measurement

limits its usefulness for cross section determination.

In summary, if the time-of-flight results are accurate, the calibration technique described by Bennett³ is probably adequate for reactor spectrum measurements. An exception is the model quoted for the hydrogen w which, instead of varying with energy, appears to be constant from 1 keV to 100 keV.

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II-30. Prescription of Fast Critical Benchmarks for Integral Data Testing at ENDF/B

W. G. DAVEY and A. L. HESS

PARTICIPATION IN THE CROSS-SECTION EVALUATION WORKING GROUP

Argonne is an active agency member of the Cross Section Evaluation Working Group (CSEWG) established by the AEC to provide a basic library of nuclear data for use in reactor analysis and design. The CSEWG duties include the establishment and testing of "best values" for microscopic cross sections (either from measurements or theory), the data comprising the Evaluated Nuclear Data File B (ENDF/B). Testing of ENDF/B in the fast energy range must include the ability to predict the results of integral measurements in the numerous assemblies on the fast critical assemblies. As tests, the experimental results must be examined to establish models for calculations with ENDF/B-based cross sections. The definition of an initial ten fast benchmarks was undertaken in this behalf, and prescriptions for analyses were issued to CSEWG.

PHILOSOPHY OF TESTING

The correlation of integral measurements against calculations utilizing ENDF/B is to indicate in the first place where the data files for particular materials contain gross errors or omissions. Secondly, the efforts could indicate where and how the evaluations require adjustments. The test might also assist in the reevaluation of data, although the "best values" would still be selected principally on the basis of microscopic measurements.

It is recognized that the correlations demonstrate how the data file, together with the computational procedures, predict the reactor properties. Part of the effort is to determine the uncertainties due to the ENDF/B processing and the calculating techniques. The accuracy of the calculational models must also be ascertained. As a test, an experimental value is presumed to be better than the calculated value, either more precise or more appropriate in that the measured parameters are the end points for the calculation. Altogether, a clear understanding of the experimental errors and deficiencies of the calculational techniques is needed.

PRESCRIPTION OF BENCHMARK TESTS

The measurements in fast assemblies of critical size, central reaction ratios, and central worths of materials were selected as the test parameters. For critical sizes, adjustments were applied for the differ-

ence between the as-built assemblies and the simplified calculational models; corrections for heterogeneity, interface gaps, edge irregularities, and the translations from cylindrical to spherical volumes were adopted on the basis of experiments or other studies. The prescriptions thus gave experimentally-derived critical dimensions for homogeneous spheres (and cylinders) with specific assembly compositions. Also given were the probable uncertainties in criticality based on experimental errors and the uncertainties in adjustments.

The central reaction ratios and material worths were given as measured in the as-built environments. Adjustments were applied for flux depression in fission-chamber walls or self-shielding in reactivity samples; however, no attempt was made to evaluate the influence of assembly heterogeneity on these central experiments.

The analytical prescriptions requested transport-theory calculations with multigroup cross sections derived from ENDF/B using MC² or some other averaging code. A half-lethargy broad group structure was specified along with certain options available in MC². Specifications were given for spherical model calculations and the quadrature-order was selected on the basis of curvature. Details for cylindrical calculations were included wherever the assembly was actually cylindrical. The mesh structures were selected to give insignificant error from finite mesh size. Overall, the criteria were intended to minimize the uncertainty from the reactor calculation so that the discrepancies between the calculated and experimental results could be attributed to errors in ENDF/B in conjunction with the methods for broad-group averaging.

Calculations of spectral indices were to use the multigroup cross-sections generated for the particular core (at infinite dilution if not a constituent material) and the central spectrum from the transport theory problem. Likewise, perturbation theory calculations for material worths were to use cross sections averaged in the particular homogeneous core spectrum and the calculated real and adjoint flux distribution. The uncertainty in the validity of correlating the measurements in the heterogeneous cores with values calculated in a homogeneous representation limits the usefulness of the reactivity coefficients as data tests to the identification of large errors, omissions, and misentries in the material data files.

TABLE II-30-I. SPECIFICATIONS OF TEN FAST CRITICAL BENCHMARKS FOR INTEGRAL TESTING OF ENDF/B

	Atomic Densities, 10 ²¹									
	JEZEBEL	VERA 11-A	ZPR-3 Assembly 48	ZEBRA Core 3	GODIVA	VERA 1-B	ZPR-3 Assembly 6F	ZPR-3 Assembly 11	ZPR-3 Assembly 12	ZEBRA Core 2
Core										
Pu-239	37.483	7.213	1.645	3.465	—	—	—	—	—	—
Pu-240	1.768	0.370	0.107	0.183	—	—	—	—	—	—
Pu-241	0.184	0.028	0.011	0.016	—	—	—	—	—	—
U-235	—	—	0.016	0.228	45.115	7.349	6.727	4.567	4.516	2.523
U-238	—	—	7.405	31.560	2.449	0.469	7.576	34.392	16.948	15.550
U-234	—	—	—	—	0.453	0.092	0.069	0.046	0.046	—
H	—	—	—	—	—	0.058	—	—	—	—
C	—	46.200	20.770	—	—	57.540	—	—	26.762	37.950
Na	—	—	6.231	—	—	—	—	—	—	—
Al	—	—	0.233	—	—	—	19.019	—	0.069	—
Fe	—	6.090	10.181	4.559	—	6.283	7.712	5.681	5.704	3.856
Cr	—	1.580	2.531	0.818	—	1.635	1.918	1.486	1.419	1.008
Ni	—	0.660	1.119	0.321	—	0.689	0.839	0.718	0.621	0.397
Mn	—	—	0.106	—	—	—	0.080	0.208	0.059	—
Cu	—	8.050	—	4.794	—	—	—	—	—	—
Mo	—	—	0.206	—	—	—	—	—	—	—
Blanket										
U-235	—	0.250	0.083	0.301	—	0.250	0.089	0.089	0.089	0.301
U-238	—	34.400	39.976	40.990	—	34.400	40.026	40.025	40.026	40.990
Fe	—	6.460	4.925	3.496	—	6.464	4.539	4.925	4.971	3.496
Cr	—	1.680	1.196	0.913	—	1.682	1.129	1.196	1.237	0.913
Ni	—	0.710	0.536	0.360	—	0.708	0.494	0.536	0.541	0.360
Mn	—	—	0.051	—	—	—	0.047	0.111	0.052	—
Al	—	—	0.060	—	—	—	1.359	—	0.060	—
Outer Radii, ^a cm										
Core	6.284	13.985	46.46	23.67	8.714	19.138	22.995	31.61	28.76	45.593
Blanket	None	46.985	76.46	54.17	None	58.588	53.495	61.61	59.26	77.293

^a Two-region spherical model based on as-built critical configurations plus adjustments. Model reactivity $k = 1.000 \pm 0.004$.

Standardization of analytical techniques has been stressed as far as possible. The remaining areas for differences at different CSEWG agencies are principally in the multigroup-averaging codes utilized at the agencies and the corresponding programs for converting the ENDF/B formats for input to the averaging codes. The range of differences in results obtained by the various groups will indicate the precision to be expected in the tests. Additional testing with altered techniques and options has been encouraged to study the possible errors resulting from the basic prescribed criteria.

SELECTION OF TEST CASES

The testing philosophy and ideals have evolved from the initial adoption of a single test assembly

(ZPR-3 Assembly 48) for the general testing of ENDF/B operation with its processing codes and for gross error detection. An expansion to ten test cases considered the need to study systems with diverse spectral characteristics and with simpler compositions. Strong arguments arose about the specific evaluations for the major fast reactor fuel materials, Pu-239, U-235, and U-238, and test cases emphasizing these isotopes were of interest. The ten choices thereby became four plutonium criticals, three high-enrichment uranium criticals, and three low-enrichment uranium criticals. Table II-30-I lists the ten cases along with their compositions and the critical-radii specifications for the prescribed analyses as spheres.

II-31. Review of CSEWG Data Testing of ENDF/B with Analyses of ZPR-3 Assemblies 48 and 11

A. L. HESS

INTRODUCTION

The data-testing subcommittee of the Cross Section Evaluation Working Group (CSEWG) has undertaken the testing of the Evaluated Nuclear Data File-B (ENDF/B) by the analyses of integral experiments which involve the data. As an initial phase of the tests of cross sections pertinent to fast reactors, two "benchmark" cases, ZPR-3 Assembly 48 and ZPR-3 Assembly 11, were prescribed. A memorandum to CSEWG members specified the models for the analyses and requested that standardized procedures be used for the generation of multigroup cross sections from ENDF/B and for the calculation of reactor parameters. Groups at several of the national laboratories and industrial establishments which participate in CSEWG carried out the prescribed analy-

ses and submitted results to the subcommittee chairman. A summary of the results from all the agencies was compiled by the author and this report condenses the important findings.

AGENCY PARTICIPATION AND PROCEDURES USED

The agencies which submitted results according to the prescribed analyses were three national laboratories (Argonne, Los Alamos, and Brookhaven) and three industrial research groups (Atomics International, General Electric Company, and Battelle Northwest Laboratories). The code MC² was used by the national laboratories and Atomics International analysts for the generation of cross sections in a half-lethargy group structure from ENDF/B and its processing code ETOE. Atomics International performed

TABLE II-31-I. SUMMARY OF ASSEMBLY 48 ANALYSES BY CSEWG MEMBERS FOR DATA TESTING OF ENDF/B

Agency Cross-Section Code Treatment		ANL MC ² Ultrafine	LASL MC ² Ultrafine	LASL MC ² Fine	BNL MC ² Fine	AI MC ² Fine	AI GRISM AILMOE	GE ENDRUN FCC-IV	BNW ETOX 1DX
Diffusion Theory Code Calculated <i>k</i>		MACH-1 0.9784	MACH-1 0.975	MACH-1 0.974	CRAM 0.9825	CAESAR 0.9832	CAESAR 0.9758	DEMON 0.9756	1DX 0.9794
Transport Theory Code Calculated <i>k</i>		SNARG 0.9865	DTF-IV 0.9838	DTF-IV 0.9834	ANISN 0.9884	— —	— —	— —	— —
Central Ratio of Fission (<i>f</i>) or Capture (<i>c</i>) to Fission in U-235									
Reaction	Experiment								
<i>f</i> -U-238	0.0307 ± 0.0005	0.0306	0.0305	0.0321	0.0323	0.0338	0.0322	0.0300	0.0306
<i>f</i> -Pu-239	0.976 ± 0.010	0.919	0.920	0.939	0.948	0.941	0.928	0.904	0.926
<i>f</i> -Pu-240	0.243 ± 0.003	0.226	0.226	0.240	0.241	0.244	0.234	0.223	0.227
<i>c</i> -U-238	0.138 ± 0.005	0.139	0.141	0.141	0.140	0.137	0.145	0.141	0.139
Central Reactivity Coefficients, 10 ⁻⁵ Δ <i>k</i> /mole									
Material	Experiment								
Pu-239	106 ± 1	133	131	130	126	130	131	132	133
Pu-240	19.4 ± 5	20.5	20.4	22.6	21.3	22.6	20.1	20.4	20.5
U-235	79 ± 1	101.3	99.6	97.6	92.8	96.4	99.6	103.4	100.9
U-238	-5.9 ± 0.1	-7.65	-7.61	-7.07	-7.33	-7.04	-7.8	-7.6	-7.6
B-10	-89 ± 1	-101	—	—	-90	—	-94.5	-97.1	-99.2
C	-0.05 ± 0.03	-0.036	-0.023	-0.11	-0.037	-0.15	-0.23	-0.20	-0.06
Na	-0.14 ± 0.03	-0.097	-0.07	-0.15	-0.09	-0.22	-0.29	-0.22	-0.05
Fe	-0.69 ± 0.04	-0.87	-0.81	-0.80	-0.83	-0.95	-0.98	-0.88	-0.84
Cr	-0.49 ± 0.04	-1.14	-1.08	-1.05	-1.07	-0.66	-1.11	-1.22	-1
Ni	-1.07 ± 0.04	-1.15	-1.07	-1.18	-1.19	-1.04	-1.32	-1.33	-1
Mo	-4.15 ± 0.05	-9.75	-9.44	-8.42	-8.63	-8.83	-9.37	-9.96	-9.16

tional analyses utilizing their own internally-developed system, with the codes GRISM and AILMOE. The codes used at General Electric (ENDRUN and FCC-IV) and at Battelle (ETOX and 1-DX) involved a Bondarenko formulation for generation of group cross sections in the resonance region. Both the fine and ultrafine group-structure options of MC² were adopted in some of the analyses.

The benchmark prescriptions requested diffusion theory calculations on spherical models, and the various diffusion codes used at the agencies included MACH-1, CRAM, CAESAR, DEMON, and 1DX. Optional analyses were also reported which involved the transport-theory codes SNARG, DTF-IV, and ANISN. Ancillary routines to the diffusion-theory codes calculated core-center worths of materials for comparison with measured reactivity coefficients.

In all of these studies the version of ENDF/B released on July 31, 1968 was used. A new release is anticipated about the end of 1969 and will contain

revisions for several important isotopes. The re-analyses of these benchmarks will be of interest.

COMPARISONS OF RESULTS

The summary paper included complete comparisons of the results obtained by the different agencies at various stages in the analyses; tables listed the real and adjoint spectra output from the MC² fundamental-mode calculations and also spectra at the core-center obtained from the reactor calculations. Such comparisons at different stages are useful for indicating where procedures could be at fault if the results between agencies differed excessively. The summary contains tables of the integral-reactor properties calculated by the agencies in comparison with the measured values; the test parameters were criticality, central reaction ratios, and central material worths. Tables II-31-I and II-31-II below are condensations of the experimental-calculational comparisons for Assembly 48 and Assembly 11, respectively.

TABLE II-31-II. SUMMARY OF ASSEMBLY-11 ANALYSES BY CSEWG MEMBERS FOR TESTING OF ENDF/B

Agency Cross Section Code Treatment	ANL MC ² Ultrafine	LASL MC ² Ultrafine	BNL MC ² Fine	AI MC ² Fine	AI GRISM AILMOE	BNW ETOX 1DX
Diffusion Theory Code Calculated k	MACH-1 0.9669	MACH-1 0.967	CRAM 0.968 ^a	CAESAR 0.9726	CAESAR 0.9792	1DX 0.9664
Transport Theory Code Calculated k	SNARG 0.9740	DTF-IV 0.9739	ANISN 0.975 ^a	— —	— —	— —
Central ratios of fission (f) or capture (c) to U-235 fission						
Reaction	Experiment					
f -U-234	0.31 ± 0.03	0.289	0.288	—	0.291	0.293
f -U-236	0.086 ± 0.004	0.077	0.076	—	0.078	0.080
f -U-238	0.038 ± 0.001	0.033	0.033	—	0.034	0.035
f -Pu-239	1.19 ± 0.02	1.103	1.102	—	—	—
c -U-238	0.112 ± 0.005	0.118	0.119	—	0.117	0.119
Central Reactivity Coefficients, $10^{-5} \Delta k/\text{mole}$						
Material	Experiment					
U-235	123 ± 3	144	139	—	137	138
U-238	-6.6 ± 0.2	-7.98	-7.7	—	-8.26	-7.96
Pu-239	209 ± 5	218	210	—	—	—
B-10	-72 ± 2	-73.1	—	—	—	-72.6
C	-0.85 ± 0.1	-0.43	—	—	—	-0.65
Al	-1.0 ± 0.1	-0.90	—	—	—	-1.11
Fe	-1.7 ± 0.1	-1.69	-1.46	—	-1.90	-2.05
Cr	-1.7 ± 0.1	-1.91	-1.7	—	-0.7	-1.98
Ni	-2.4 ± 0.1	-2.91	-2.6	—	-2.13	-3.14
Mn	-1.8 ± 0.1	-2.15	-1.91	—	-2.4	-2.77
	-5.1 ± 0.2	-5.98	—	—	—	—
	-19.6 ± 0.6	-18.8	—	—	—	-6.02

^a Calculated value less 0.022 correction for use of Pu-239 fission spectrum.

OBSERVATIONS AND CONCLUSIONS

The average k -values given by diffusion theory were 0.978 for Assembly 48 and 0.970 for Assembly 11, and the corresponding transport-theory values were 0.9855 and 0.9743. The transport-diffusion differences are consistent with calculations performed in the past on these assemblies with other cross section sets. Overall, a dispersion of about 1.2% can be seen in the k -values obtained from the different procedures and codes used.

Cross-comparisons between analysts in the preliminary stages of these studies revealed an anomaly in the MACH-1-code results which was subsequently traced to an error in the translation of the MC²-output ($n,2n$) cross sections to MACH-2 input. Correction of the error gave increases in k -values of 0.0053 for Assembly 48 and 0.0097 for Assembly 11. Data in Tables II-31-I and II-31-II include the corrected MACH-1 results. Another discovery was an overestimate of 2.2% k for Assembly 11 if the Pu-239 fission spectrum were inadvertently used in place of the more appropriate U-235 fission spectrum.

For the central fission ratios, the dispersion of calculated results is about on the order of the uncertainty in the experimental values. The calculated material worths, however, have notably greater deviations than the error assigned to the measurements, in addition to exhibiting systematic discrepancies from the measured values. This relatively constant difference in absolute worths between measurements and calculations has been observed before using other data libraries, and is thought somehow to be due to shortcomings of the analytical models.

These analyses prove useful to the data testing effort even though rather limited conclusions are possible about the data quality for any material and in spite of the pending re-evaluations for several of the assembly materials. It appears from the comparisons that the precision of the data testing will be principally limited by the calculational uncertainties rather than the errors in the experiments. The experience of the analysts can be drawn upon for defining the procedures and expected results of the future ENDF/B data testing.

II-32. Modifications to Fissile Element Cross Sections and Their Influence on Calculated Fast Reactor Parameters

J. M. KALLFELZ, B. A. ZOLOTAR and B. R. SEHGAL

INTRODUCTION

The purpose of this study was to test the agreement of calculated and measured integral physics parameters for a number of fast critical assemblies with several differential cross section sets. With ENDF/B cross sections used as the base values, the investigations concerned the influence of changing the capture and fission cross sections of the important fissile elements in a manner consistent with experimental results. The effect of modifications in the inelastic scattering of U-238 and the fission spectrum of U-235 were also studied.

The changes made to the fissile element capture and fission cross sections were consistent in that "correlated" discrepancies between different cross section measurements were considered. "Correlated" discrepancies, discussed in an earlier work by J. Kallfelz and W. Poenitz,¹ result from the fact that many capture and fission cross sections are measured with the same method or standard. Thus a change in one cross section requires associated changes in other cross sections in the same direction. For example, the ratios $\sigma_c(\text{U-238})/$

$\sigma_f(\text{U-235})$ measured by different methods agree much better than the measured absolute values of these two cross sections; in choosing values for these important cross sections, consideration must be given to the correlated discrepancies. The two consistent capture and fission cross section sets used were from an evaluation by Poenitz, whose Paper I-15 gives two different experimental curves for the "standard," $\sigma_f(\text{U-235})$, and a series of experimentally determined ratios of other cross sections to this standard.

The results obtained for these two capture and fission cross section sets showed that many of the discrepancies between calculation and experiment could be explained by errors in the calculated spectrum. This suggested the investigation of U-238 inelastic scattering cross sections and the U-235 fission spectrum, both of which strongly influence the calculated spectrum.

METHODS OF REACTOR CALCULATIONS

Calculations for fifteen fast critical assemblies including a wide variety of composition, size, and spectra,

TABLE II-32-I. SPECIFICATIONS OF SPHERICAL REACTOR MODELS USED IN STUDY

Assembly	Composition, 10 ²² atoms/cm ³																Spherical Model			
	Pu-239	Pu-240	Pu-241	U-235	U-238	U-234	Fe	Cr	Ni	Cu	C	Na	Mn	Mo	O	Al	Core Radius, cm	Reflector Type	Reflector Thickness, cm	S _n Order
Godiva ⁽²⁾				4.5115	0.2449	0.0453											8.714	None	None	16
Jezebel ⁽²⁾	3.7483	0.1768	0.0184														6.284	None	None	16
Vera				0.7349	0.0469	0.0092	0.6283	0.1635	0.0689		5.7598						19.138	A	39.45	8
1B ⁽²⁾							0.6090	0.1580	0.0708	0.8050	4.6200						13.985	A	43.0	16
11A ⁽²⁾	0.7213	0.0370	0.0028																	
Zebra				0.2523	1.5550		0.3856	0.1008	0.0397		3.7950						45.593	B	31.7	4
2 ⁽²⁾				0.0228	3.1560		0.4559	0.0818	0.0321	0.4794							23.67	B	30.5	8
3 ⁽²⁾	0.3465	0.0183	0.0016																	
ZPR-3				0.6727	0.7576	0.0069	0.7712	0.1918	0.0839				0.0080			1.9019	22.995	C	30.50	8
6F ⁽²⁾				0.4567	3.4392	0.0046	0.5681	0.1486	0.0718				0.0208				31.61	D	30.0	4
11 ⁽²⁾				0.4516	1.6948	0.0046	0.5704	0.1419	0.0621		2.6762		0.0059			0.0060	28.76	E	30.5	4
25 ⁽²⁶⁾				0.3421	3.5567	0.0035	0.5773	0.1438	0.0629				0.0060			0.0070	46.484	D	30.0	4
32 ⁽²⁶⁾				0.4451	0.0273	0.0045	5.0213	1.2500	0.5500				0.0522			0.0061	30.7435	D	45.0	8
48 ⁽²⁾	0.1645	0.0107	0.0011	0.0016	0.7405		1.0180	0.2531	0.1119		2.0770	0.6231	0.0106	0.0206		0.0233	46.46	F	30.0	4
ZPR-6																				
5 ⁽²³⁾				0.1541	1.0560		0.9042	0.2386	0.1130		1.2900	0.9200					77.35 ^a	G	30.0	4
6 ⁽²⁴⁾				0.1150	0.5796		1.4100	0.2788	0.1365			0.9142			1.4650		90.87 ^b	G	30.0	4
ZPR-9																				
24 ⁽²⁵⁾				0.1172	1.2700		0.6837	0.1951	0.0864		4.4007						°			16
Reflector																				
A				0.0250	3.4400		0.6464	0.1682	0.0708											
B				0.0301	4.0990		0.3496	0.0913	0.0360											
C				0.0089	4.0026		0.4539	0.1129	0.0494				0.0047			0.1359				
D				0.0089	4.0025		0.4925	0.1196	0.0536							0.0111				
E				0.0089	4.0026		0.4971	0.1237	0.0451				0.0052			0.0060				
F				0.0083	3.9976		0.4925	0.1196	0.0536				0.0051			0.0060				
G				0.0083	4.0080		0.4228	0.1215	0.0567											

^a Two-core-region cylindrical system. Densities given correspond to central region making up 66% of core volume.^b Three-core-region cylindrical system. Densities given correspond to central region making up 72% of core volume.^c Central null-reactivity zone.

FIG. II-32-1. Typical Core Spectra of Assemblies Studied. *ANL Neg. No. 113-2551*.

were considered. For most of the assemblies, calculations were performed for homogeneous spherical models, whose critical radii included adjustments for shape factor, heterogeneity, and geometrical discontinuity effects. Ten of the models used were those prescribed by W. Davey and A. Hess² for the ENDF/B data testing; Table II-32-I gives the details of all the models.

Figure II-32-1 illustrates the range of spectra involved for the testing of the consistent capture and fission cross section sets, which were evaluated only above 9 keV. For this part of the study only the ten assemblies from Ref. 2 were used. The number of neutrons below 9 keV was negligible for the spectra of all these assemblies except for ZPR-3 Assembly 48 and Zebra 2. These latter two soft assemblies are thus less appropriate than the others for the consistent set testing.

The reactor calculations were performed with the transport theory code SNARG-1D⁽³⁾, using the S_n order as given in Table II-32-I for the different assemblies. The k_{eff} from the S_n calculations should differ by no more than about 0.002 from the " S_∞ " eigenvalue. The anisotropy of the scattering was represented by the transport approximation, which introduces an error in k_{eff} . For Godiva, this error is of the order of -0.3% , based on results of G. Bell et al.⁴ and our calculation of $\Delta k/\Delta r$. For large assemblies the error is negligible, but for some assemblies used in this study the magnitude of this error is not clear. Specifically, for ZPR III-6F, a study by G. Joanou and A. Kazi,⁵ indicated that the transport approximation gave quite a large error in k_{eff} ($\approx 3.5\%$). However, a later study by Bell et al.⁴ indicates that the error due to the transport approximation in the k for this assembly is quite small ($\approx 0.2\%$), and that in fact the transport approximation gives better

results than does the use of a full P_1 scattering matrix. To test the influence of the transport approximation, calculations are planned with the transport theory code ANISIN,⁶ which allows higher order anisotropic scattering matrices. It appears, however, that the error in k due to the use of the transport approximation may be small.

Besides k_{eff} , the central reaction rates for U-235, U-238 and Pu-239 fission and for U-238 capture were determined from the transport theory calculations. Based on previous work,¹ the authors felt that these integral values are more useful than central worth calculations for identifying errors in the fissile element differential cross sections, and central worths were not calculated.

It should be noted that while the homogeneous models have been corrected for the influence of heterogeneity on k_{eff} , there is also an effect on the calculated spectrum, which has not been taken into account. A calculation of ZPR-III-11 for the heterogeneous cell showed practically no spectrum change from the homogeneous case. This is in agreement with $\sigma_f(\text{U-238})/\sigma_f(\text{U-235})$ calculations of D. Meneghetti.*⁽⁷⁾ In the same work Meneghetti shows that for several other assemblies inclusion of heterogeneity tends to reduce the cell-averaged $\sigma_f(\text{U-238})/\sigma_f(\text{U-235})$, which, as we will see later, would in general increase the discrepancy between our calculations and experiment. T. Pitterle et al.⁸ performed calculations with ENDF/B for a number of assemblies. They state also that the inclusion of heterogeneity by using cell-homogenized cross sections gives, in general, softer spectra than do homogeneous calcula-

* Meneghetti gives results for ZPR-3 Assembly 22, which is identical to ZPR-3 Assembly 11.

is, worsening the agreement of their calculations with experimental values. While these effects lessen the value of comparing measured reaction ratios with results calculated with homogeneous models, such comparisons are still useful to indicate general trends of cross section errors.

"CONSISTENT" FISSION ELEMENT FISSION AND CAPTURE CROSS SECTION SETS

The selection of the "consistent" capture and fission cross section sets used for the fissile elements are described by Poenitz in Paper I-15. These sets have been improved over those of a previous study¹ by the inclusion of more experimental data and extension to higher energies. The sets, evaluated for energies greater than 9 keV, were consistent in that for a particular set the cross sections are based on the same standard, $\sigma_f(\text{U-235})$. As shown in Fig. II-32-2, two values of this

standard were used, the Set 1 values being as much as 15 % higher than those for Set 2 in the range above 100 keV. Below 2.5 MeV, the present ENDF/B values, also shown in Fig. II-32-2, agree with those of Set 1, as both of these evaluations are based on the measurements of P. White.⁹ Above 2 MeV the values for both consistent sets are up to 15 % lower than those of ENDF/B because of a new measurement and re-evaluation of older results (see Paper I-15). The ratios used to determine the other important cross sections are shown in Figs. II-32-3–II-32-5 along with the ENDF/B values.

In Fig. II-32-3 it can be seen that the evaluation of Poenitz for $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$ agrees with the evaluation of W. Davey,¹⁰ except in the low keV range where the Poenitz evaluation is somewhat lower because of new experimental data, and above 1 MeV where the Poenitz evaluation shows some structure. The ENDF/B values are about 5 % lower than the other two curves

FIG. II-32-2. Fission Cross Sections for U-235 for Two Sets Studied and for ENDF/B. ANL Neg. No. 113-2545.

FIG. II-32-3. Ratio of $\sigma_f(\text{Pu-239})$ to $\sigma_f(\text{U-235})$ for Several Evaluations: ENDF/B Values Not Shown Below 100 keV (Unresolved Resonance Range). ANL Neg. No. 113-2555.

in the 200–500 keV range. This discrepancy is partially caused by a characteristic problem of the original ENDF/B evaluation (which is now being corrected) for which cross sections for different isotopes were determined at different installations; i.e. in the 200–500 keV energy range ENDF/B contains the $\sigma_f(\text{Pu-239})$ of W. Davey,¹¹ which was determined from Davey's $\sigma_f(\text{U-235})$ and $\sigma_f(\text{Pu-239})/\sigma_f(\text{U-235})$, but the ENDF/B $\sigma_f(\text{U-235})$ is from J. Stehn et al.¹²

The ENDF/B ratio $\sigma_c(\text{U-238})/\sigma_f(\text{U-235})$ shown in Fig. II-32-4 is lower than that of Poenitz in the range 40–70 keV. In this range the ENDF/B $\sigma_c(\text{U-238})$ is that of J. Stehn et al.¹² which is influenced by the low values of M. Moxon and C. Chaffey.¹³

DETERMINATION OF GROUP CROSS SECTIONS FOR ENDF/B AND THE CONSISTENT SETS

The ENDF/B group cross sections were determined from the code MC².⁽¹⁴⁾ The calculation for the ZPR-6 and -9 assemblies are described in the references given in Table II-32-I. For all other assemblies, the following procedures were used. ENDF/B cross sections for twenty-six half lethargy width groups (energies from 10 MeV to 22.5 eV) were calculated for each core and blanket composition; core cross sections were determined for a bare critical core, while the blanket cross sections were for an infinite medium. The self-shielding included in the group cross sections was that of the homogeneous region. For the ZPR-6 and -9 assemblies heterogeneous self-shielding and cell weighting corrections were made.

For ten assemblies given in Table II-32-II (those from Ref. 2), group constants for Sets 1 and 2 for $\sigma_f(\text{U-235})$, $\sigma_f(\text{U-238})$, $\sigma_c(\text{U-238})$ and $\sigma_f(\text{Pu-239})$ were determined from the curves of Poenitz see Paper I-15 shown in Figs. II-32-2 through II-32-5, using the ultra-

FIG. II-32-4. Ratio of $\sigma_c(\text{U-238})$ to $\sigma_f(\text{U-235})$ for Poenitz Evaluation and ENDF/B. ANL Neg. No. 113-2754.

FIG. II-32-5. Ratio of $\sigma_f(\text{U-238})$ to $\sigma_f(\text{U-235})$ for Poenitz Evaluation and ENDF/B. ANL Neg. No. 113-2753.

fine group fluxes from the above MC² core and blanket calculations as the weighting spectra. For consistency the group values for some other cross sections which are measured relative to these were also changed. The $\sigma_c(\text{Pu-239})$ and $\sigma_c(\text{U-235})$ were varied the same relative amount as the associated σ_f , while $\sigma_f(\text{Pu-240})$ and $\sigma_c(\text{Pu-240})$ were changed the same relative amount as $\sigma_f(\text{U-235})$ and $\sigma_c(\text{U-238})$ respectively. These changes were all limited to the energy range 9 keV–10 MeV (groups 1–14). All other values used were those of ENDF/B, except for the elastic in-group scattering, which was changed to keep the sum of the partial cross sections equal to the ENDF/B total cross section.

RESULTS FOR ENDF/B AND CONSISTENT SETS

In Table II-32-II the results for k_{eff} and central reaction ratios are given for ten assemblies, arranged in order of core U-238 content. Table II-32-III summarizes the results as discrepancies between calculated and measured values. The following points can be made:

1. Lowering the ENDF/B $\sigma_f(\text{U-235})$ values above 2 MeV (Set 1) provides reasonable k_{eff} values for the hard assemblies.
2. For the assemblies with appreciable U-238 in the core, the calculated central integral c^{28}/f^{25} and f^{49}/f^{25} ratios are about 5 % higher and lower, respectively, than the experimental values.
3. As can be seen in Figs. II-32-6 and II-32-7, calculated f^{28}/f^{25} and k_{eff} values for ENDF/B become progressively too low as the core U-238 content increases.* The k_{eff} dependence is evident for ZPR-3 Assemblies 6F, 12, and 11, which have similar spectra but increasing U-238 content.

* Vera 11-A, one of the exceptions to this rule, has no U-238 in its core. However, k_{eff} for this assembly is actually f sensitive to U-238 cross sections since it has a small core : U-238 blanket.

TABLE II-32-II. EXPERIMENTAL AND CALCULATED REACTOR PARAMETERS

Assembly	k_{eff} , Experimental = 1.0			f^{28}/f^{25} , Central				c^{28}/f^{25} , Central				f^{49}/f^{25} , Central			
	ENDF/B	Set 1	Set 2	Measured ⁽²⁾	ENDF/B	Set 1	Set 2	Measured ⁽²⁾	ENDF/B	Set 1	Set 2	Measured ⁽²⁾	ENDF/B	Set 1	Set 2
Jezebel	1.0073	0.9900	0.9450	0.205 ± 0.008	0.1917	0.1889	0.1944		0.0789	0.0820	0.0812	1.49 ± 0.03	1.3764	1.3924	1.3938
Vera 11-A	0.9801	0.9718	0.9351	0.077 ± 0.002	0.0869	0.0842	0.0839		0.1249	0.1275	0.1299	1.07 ± 0.02	1.1210	1.1307	1.1147
Godiva	1.0245	1.0063	0.963	0.156 ± 0.005	0.1573	0.1531	0.1565	0.0738 ± 0.003	0.0857	0.0892	0.0870	1.42 ± 0.02	1.3499	1.3667	1.3657
Vera 1-B	1.0052	0.9947	0.9703	0.067 ± 0.001	0.0741	0.0714	0.0709	0.131 ± 0.006	0.1305	0.1328	0.1346	1.070 ± 0.026	1.0945	1.1041	1.0871
ZPR-3, 6F	1.0088	0.9942	0.962	0.078 ± 0.002	0.0709	0.069	0.069	0.104 ± 0.003	0.1090	0.112	0.113	1.22 ± 0.03			
ZPR-3, 12	0.9989	0.9832	0.9606	0.047 ± 0.002	0.0427	0.041	0.041	0.123 ± 0.005	0.1292	0.133	0.135	1.12 ± 0.02	1.05		
ZPR-3, 48	0.9865	0.9785	0.9517	0.0307 ± 0.0004	0.0304	0.0291	0.0285	0.138 ± 0.004	0.1403	0.1417	0.1433	0.976 ± 0.010	0.9188	0.9218	0.9059
Zebra 2	0.9745	0.9631	0.9481	0.0320 ± 0.0005	0.0294	0.0282	0.0276	0.136 ± 0.007	0.1404	0.1420	0.1438	0.987 ± 0.010	0.9377	0.9424	0.9267
ZPR-3, 11	0.9740	0.9589	0.9367	0.038 ± 0.001	0.0333	0.0321	0.0319	0.112 ± 0.005	0.1196	0.1214	0.1227	1.19 ± 0.02	1.103	1.129	1.115
Zebra 3	0.9536	0.9466	0.9145	0.0461 ± 0.0008	0.0402	0.0391	0.0392	0.111 ± 0.005	0.1188	0.1205	0.1217	1.190 ± 0.014	1.109	1.135	1.123

TABLE II-32-III. PERCENT DEVIATION OF CALCULATIONS FROM EXPERIMENT

Assembly	k_{eff}			f^{28}/f^{25} , Experimental Accuracy 2-3%			c^{28}/f^{25} , Experimental Accuracy 4-5%			f^{49}/f^{25} , Experimental Accuracy 1-2%		
	ENDF/B	Set 1	Set 2	ENDF/B	Set 1	Set 2	ENDF/B	Set 1	Set 2	ENDF/B	Set 1	Set 2
Jezebel	+0.7	-1.0	-5.5	-6	-8	-5	—	—	—	-8	-7	-6
Vera 11-A	-2.0	-2.8	-6.5	+13	+9	+9	—	—	—	+5	+6	+4
Godiva	+2.5	+0.6	-3.7	+1	-2	0	+16	+21	+18	-5	-4	-4
Vera 1-B	+0.5	-0.5	-3.0	+11	+7	+6	-1	+1	+3	+2	+3	+2
ZPR-3, 6F	+0.9	-0.6	-3.8	-9	-12	-12	+5	+8	+9	—	—	—
ZPR-3, 12	-0.1	-1.7	-3.9	-9	-13	-13	+5	+8	+10	-6	—	—
ZPR-3, 48	-1.3	-2.1	-4.8	-1	-5	-7	+2	+3	+4	-6	-6	-7
Zebra 2	-2.5	-3.7	-5.2	-8	-12	-14	+3	+4	+6	-5	-5	-6
ZPR-3, 11	-2.6	-4.1	-6.3	-12	-16	-16	+7	+8	+10	-7	-5	-6
Zebra 3	-4.6	-5.3	-8.5	-13	-16	-15	+7	+8	+10	-7	-5	-6

FIG. II-32-6. Calculated k_{eff} as a Function of Relative U-238 Concentration in the Core. ANL Neg. No. 113-2436.

4. Set 2 k_{eff} values are on the average about 3% lower than those of Set 1, which are in general already below 1.0. This indicates that, unless large errors exist in many other ENDF/B values, the standard σ_f (U-235) chosen for Set 2 (cf Fig. II-32-2) may be too low.

Several evaluators have suggested using Set 2 σ_c (U-238) with Set 1 σ_f (U-235). While this would help remove the dependence of k_{eff} on U-238 content, and would improve the agreement with central c^{28}/f^{25} measurements, it would be incompatible with the measured differential cross section ratios.

Errors giving calculated spectra which are too soft would help to explain the discrepancies in points 2 and 3 above. This suggested the investigations of the influence of errors in the U-235 fission spectrum and the U-238 inelastic scattering cross sections described below.

FISSION SPECTRUM

For the uranium assemblies studied, use of a U-235 fission spectrum, $\chi^{25}(E)$, which is too soft, could help to

explain the general tendency of the discrepancies between calculations and experiment described in the previous section. The percentage of fissions occurring in U-238 increases with the core U-238 content (although not in the same proportion since an increase in U-238 is partially compensated by a softening of the spectrum) so that the dependence of k_{eff} on U-238 content could be caused by a too soft $\chi^{25}(E)$. Similarly, errors in the U-238 fission spectrum $\chi^{28}(E)$ could cause the observed dependency of the discrepancy between the calculated and measured values of f^{28}/f^{25} on the U-238 content.

Rather conflicting results for $\chi^{25}(E)$ have been published recently, but it is suspected that the ENDF/B $\chi^{25}(E)$ is too soft. $\chi^{25}(E)$ in ENDF/B is represented by a Maxwell distribution with a temperature T of 1.273 MeV and with no dependence on the incident neutron

FIG. II-32-7. Comparison of Calculated and Experimental Central U-238/U-235 Fission Ratios as a Function of Relative U-238 Concentration in the Core. ANL Neg. No. 113-2437.

gy. E. Barnard et al.¹⁵ obtain from their time-of-flight measurements a T of 1.297 ± 0.03 , and determine an average value from eight independent measurements of 1.30 ± 0.01 MeV for thermal incident neutrons. Recently J. A. Grundl¹⁶ has derived $\chi^{25}(E)$ from activation detector measurements which differs considerably in shape from a Maxwellian, and has an average energy, $\bar{E}(U-235)$, about 13 % higher than the results of Barnard et al.¹⁵ Integral results of A. Fabry¹⁷ agree with those of Grundl. Recent measurements by J. Neill et al.¹⁸ agree with the results of Barnard et al., indicating that the ENDF/B value of T for U-235 is only about 3 % low. Furthermore, the ratio $\bar{E}(Pu-239)/\bar{E}(U-235)$ of Grundl and Barnard differ considerably, being 1.039 and 1.085, respectively. The ENDF/B value of T for Pu-239 is 1.41 MeV, while Barnard et al. give 1.407 ± 0.02 and 1.39 ± 0.01 MeV for their measurements and an average of eight independent experiments, respectively.

A few calculations were performed to determine the influence of large discrepancies in $\chi^{25}(E)$, by raising T for U-235 from 1.273 MeV to that of Pu-239, namely 1.41 MeV. The results given in Table II-32-IV, show that for a change of this magnitude considerable increases in k_{eff} are obtained for the high U-238 content cores. For an increase of the magnitude given by Neill and Barnard ($\sim 3\%$) increases in k for the assemblies considered would probably not be more than about 0.5 %.

U-238 σ_n

Since the discrepancies mentioned above could be explained by spectral errors, the U-238 inelastic scattering appeared to be a likely source of error. The data in ENDF/B is the latest recommendation of J. J. Schmidt¹⁹ and is considerably higher (up to 30 %) than the previously recommended values. One of the main reasons for the increase appears to be new level measurements by Barnard, Ferguson, et al.²⁰ Very recent measurements²¹ and theoretical optical model calculations²² at Argonne indicate that these values are probably too high and should be reduced.

To test the influence of lowering the U-238 inelastic

TABLE II-32-IV. Δk OBTAINED FOR CHANGE FROM $\chi(E)$ OF U-235 TO $\chi(E)$ OF Pu-239

Assembly	Central $\frac{f^{28} N^{28}}{f^{25} N^{25}}$	Δk , %	Calculational Code
Godiva	0.0085	0.6	SNARG
3, 6F	0.08	0.8	SNARG
2	0.18	1.6	MACH
ZPR-3, 11	0.25	2.1	MACH

TABLE II-32-V. PERCENT DEVIATION OF CALCULATIONS FROM EXPERIMENT

Assembly	k_{eff}			f^{28}/f^{25} Experimental Accuracy 2-3%		
	ENDF/B	0.7 $\sigma_n(U-238)$	Set 224 $\sigma_n(U-238)$	ENDF/B	0.7 $\sigma_n(U-238)$	Set 224 $\sigma_n(U-238)$
Jezebel	+0.7	+0.7	+0.7	-6	-6	-6
Vera 11-A	-2.0	-0.3		+13	+13	+13
Godiva	+2.5			+1		
ZPR III-32	+2.1		+2.7	-8		-7
Vera 1-B	+0.5			+11		
ZPR-3, 6F	+0.9	+2.5		-9	-2	
ZPR-3, 12	-0.1			-9		
ZPR-3, 48	-1.3	+0.9	-0.3	-1	+11	+4
ZPR-6, 6	-0.7					
Zebra 2	-2.5	-0.1	-1.9	-8	+9	-3
ZPR-6, 5	-0.9	+1.1		-16	-1	
ZPR-3, 11	-2.6	+1.6	-1.4	-12	+10	-4
Zebra 3	-4.6	+0.6	-2.6	-13	+8	-5
ZPR-3, 25	-4.0	+1.2	-1.8	-16	+7	-7
ZPR-9, 24	-2.1	+0.3		-18	-4	

scattering on the calculated results, several new assemblies²³⁻²⁶ (with varying amounts of U-238) were added to the ten assemblies previously studied. ENDF/B results indicated the same discrepancies as seen earlier. The initial parameter change lowered the total $\sigma_n(U-238)$ by 30 % over the entire energy range. As can be seen in Table II-32-V and Figs. II-32-6 and II-32-7 for both k_{eff} and the f^{28}/f^{25} ratio, the U-238 dependence was basically removed but the values were over-corrected in most cases. If additionally the $\sigma_f(U-235)$ values above 2 MeV are lowered (as suggested by Davey)¹⁰ then much better agreement is obtained.

As a test of a more realistic change to the ENDF/B $\sigma_n(U-238)$ several calculations were performed using the older $\sigma_n(U-238)$ level structure of A. Smith²⁷ as used in Argonne Set 224.⁽²⁸⁾ This cross section essentially reduces the contribution of some of the higher inelastic levels and results in a reduction in the total σ_n of about 15 % between 1 and 2 MeV. The new level structure of Smith does not differ appreciably from these older values.²¹ The results of these calculations are also given in Table II-32-V. While this correction did not sufficiently increase the calculated integral values, the agreement is significantly better.

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II-33. Comparison of Criticality Calculations for U-235 and Pu-239 Unmoderated Assemblies

J. T. MADELL, B. SYNDER,* J. SOLYST† and D. GREEN‡

A series of spherical transport calculations were performed with three cross section sets for ten small, unmoderated critical assemblies. The cross section sets used were 224,⁽¹⁾ 238,⁽²⁾ and ASY 11.⁽³⁾ Set 224 is

a 26-group set calculated by MC²,⁽⁴⁾ using the old MC² library tape with weighting spectra characteristic of large oxide and carbide cores. The cross sections for the lowest four energy groups were not used in the calculations. Set 238 is a 22-group set generated with the old MC² library tape and weighting spectra characteristic of the core, reflector and blanket compositions of LBR-

* Student, Northwestern University, Evanston, Illinois.

† Student, University of Wisconsin, Madison, Wisconsin.

‡ Student, University of Missouri, Columbia, Missouri.

TABLE II-33-I. DESCRIPTION OF THE SPHERICAL UNMODERATED CRITICAL ASSEMBLIES

Assembly	Fuel Material	Core			Reflector			Delayed Critical Mass, kg
		Enrichment, w/o	Average Density, g/cm ³	Geometry	Material	Thickness, cm	Average Density, g/cm ³	
6F	uranium	See Ref. 7		sphere	U-238 + Steel	30		131.1
A	uranium	93.9	18.75	thick shells	—	—	—	51.9
B	uranium	93.9	18.75	nesting shells	nat. uranium	10	19.0	19.75
C	uranium	93.18	18.4	nesting shells	aluminum	6.63	2.82	37.3
D	uranium	93.9	18.6	0.375 in. diam $\times$ 0.45 in. high central source cavity	cast iron	5.08	7.16	31.6
E	uranium	93.9	18.4	same as (D)	nickel	5.08	8.35	29.4
F	plutonium	100 ^a	15.44	bare sphere	—	—	—	16.28
G	plutonium	95.1 ^a	15.62	hemispheres	nat. uranium	4.0	18.92	8.39
H	plutonium	95.1 ^a	15.9	hemispheres	aluminum	7.83	2.82	11.15
I	plutonium	—	15.64	2.16 cm central source cavity	iron	10.16	7.87	9.6

^a Pu-239/(Pu-239 + Pu-240) $\times$ 100.

II. Set ASY 11 is a 26-group set generated by MC² with the ENDF/B library tape⁵ and spectra characteristic of the core and blanket of ZPR-3 Assembly 11.

The criticality of ten, small, unmoderated assemblies fueled with either U-235 or Pu-239 was calculated by the SNARG-1D code⁶, using S_8 order. A description of the ten assemblies is given in Table II-33-I. The dimensions of the core and blanket regions as well as their compositions are given in Table II-33-II. Refinements to the input data due to heterogeneity and other effects may be required to represent these small systems accurately for neutron calculation but are not included in this work. The lack of refined data does not detract from the purpose of the work because the calculated values are compared primarily to each other rather than to the experimental results.

The calculated eigenvalues for the ten assemblies are given in Table II-33-III. The measured and calculated values of the critical radii are given in Table II-33-IV along with the percent difference between experimental and calculated critical mass. Calculations were not performed for Assemblies 6F, C, and H with Set ASY 11 because of the lack of cross section data for aluminum.

DISCUSSION OF RESULTS

The eigenvalues calculated for Assembly 6F are surprisingly low. A recent calculation,⁷ using Set 238 and revised compositional data, yielded an eigenvalue for Assembly 6F of 1.0015. The eigenvalues obtained with the three cross section sets are in good agreement with each other for Assemblies A, B and C. However, they are 2 to 4% higher than experimental

values. The addition of the depleted uranium reflector (Assembly B) produces a 1½% increase in the eigenvalue calculated with Set ASY 11, whereas the increase in eigenvalues obtained from the other two sets is zero (Set 238) and 0.5% (Set 224).

The differences in the calculated eigenvalues for Assemblies D and E are primarily attributed to differences in the iron and nickel transport cross sections of the three sets which result, in turn, from the different weighting spectra of the three sets. The iron and nickel cross sections of Set 238 were generated from a weighting spectrum characteristic of 80% steel and 10% sodium mixture. Those of Set 224 were obtained from an ELMOE correction based on a stainless steel composition and then weighted with a spectrum characteristic of a large carbide core. Set ASY 11 cross sections for iron and nickel were obtained from a weighting spectrum characteristic of a depleted uranium blanket. The manner in which the weighting spectrum influences the calculated eigenvalue of a reflected system has been investigated and the conclusions can be applied to the analysis of these systems.⁸

The calculated eigenvalues for a bare plutonium sphere (Assembly F) and a plutonium sphere surrounded with natural uranium (Assembly G) are in good agreement with each other and with the measured values. The calculated eigenvalues for the plutonium assemblies reflected with a light nuclide are in good agreement with each other for Assembly H and fair agreement (0.02Δk) with each other for Assembly I. All calculated values were in discrepancy with experiments. The discrepancy between calculation and experiment for Assembly I is thought to be due,

TABLE II-33-II. COMPOSITIONS AND REGIONAL DIMENSIONS OF THE SPHERICAL UNMODERATED CRITICAL ASSEMBLIES

Assembly	Experimental Radius, cm	Atom Density in Core, $\times 10^{24}$		Atom Density in Reflector, $\times 10^{24}$	
6F	22.83 core	U-235	0.00672	U-235	9.12×10^{-5}
		U-238	0.007632	U-238	0.039984
		SS	0.123 (V%)	SS	0.0731 (V%)
		Al	0.001893	Al	0.00136881
A	8.7	U-235	0.045072		
		U-238	0.002928		
B	6.30 core 16.3 refl.	U-235	0.045072	U-235	0.000336
		U-238	0.002928	U-238	0.04766
C	7.8 core 14.43 refl.	U-235	0.044726	Al	0.0603
		U-238	0.00321		
D	0.58 void	Al	0.000006		
	7.25 core	U-235	0.045072		
	12.48 refl.	U-238	0.002928	Fe	0.0769
E	0.58 void	Al	0.000006		
	7.25 core	U-235	0.045072	Ni	0.0857
	12.33 refl.	U-238	0.002928		
F	6.34 core	Pu-239	0.0389		
G	5.02 core 9.02 refl.	Pu-240 ^a	0.001921	U-235	0.000336
		Pu-239	0.037435	U-238	0.04766
H	5.5 core 13.33 refl.	Pu-240 ^a	0.001955	Al	0.0603
		Pu-239	0.03818		
I	1.08 void	Al	6.0×10^{-6}		
	6.34 core	Pu-240 ^a	0.001921	Fe	0.0769
	16.50 refl.	Pu-239	0.037435		

^a 3.842×10^{-3} atom/cm³ of U-238 substituted for Pu-240 in calculations with cross section Set ASY 11.

in part, to the iron cross sections, as was the case Assembly D. But the magnitude of the discrepancy obtained from all three cross section sets for Assembly I is not understood because the use of the iron cross sections of Sets 224 and 238 in calculating Assembly D gave fairly close agreement with experiment (less than 2% error).

The results in Table II-33-IV show that a small difference in the eigenvalue (or the critical radius) results in a substantial difference in the critical mass of the systems. The 2% difference in the eigenvalue can result in about a 10% difference in the critical mass.

The eigenvalues calculated from Sets 224 and 238 are in good agreement with each other (less than 0.5%) for all assemblies containing primarily heavy isotopes. Differences of 2% or greater in the eigenvalues are seen in assemblies reflected by iron and nickel (Assemblies D, E and I). Sets 224 and 238 differ only in the

TABLE II-33-III. EIGENVALUES FOR TEN CRITICAL ASSEMBLIES CALCULATED WITH THREE CROSS SECTION SETS

Critical Assembly	Cross Section Set		
	224	238	ASY 11
6F	0.9660	0.9705	—
A	1.0267	1.0322	1.0212
B	1.0317	1.0322	1.0394
C	1.0451	1.0431	—
D	1.0197	0.9990	1.0264
E	1.0124	1.0711	1.0561
F	1.0198	1.0174	1.0055
G	1.0175	0.9988	0.9942
H	1.0388	1.0374	—
I	1.2107	1.1916	1.1998

TABLE II-33-IV. COMPARISON OF MEASURED AND CALCULATED RADII AND CRITICAL MASSES FOR TEN CRITICAL ASSEMBLIES

Critical Assembly	Measured Core Radius, cm	Calculated Core Radius, cm			Difference in Critical Mass ($M_e - M_c$)/ $M_e \times 100\%$		
		224	238	ASY 11	224	238	ASY 11
6A	22.83				-17.71	-14.97	—
A	8.70	8.4189	8.4488	8.4752	9.38	8.41	7.55
B	6.30	6.1083	6.1047	6.0500	8.85	9.01	11.44
C	7.8	7.4546	7.4715	—	12.70	12.11	—
D	7.4 ^a	7.2880	6.6996	7.2295	4.47	21.10	6.75
E	7.25 ^a	7.2042	7.4095	6.8292	1.88	-0.38	16.43
F	6.34	6.1937	6.2110	6.2984	6.76	5.98	1.95
G	5.02	4.9658	5.0270	5.0683	3.20	-0.42	-2.91
H	5.50	5.3187	5.3279	—	9.56	9.09	—
I ^b	6.34 ^b	5.1628	5.2514	5.2173	46.23	43.39	44.49

^a Inner core radius = 0.58 cm.

^b Inner core radius = 1.02 cm.

weighting spectra which were used to generate the broad group cross sections. Thus, it is indicated that for the high energy spectrum systems studied here, the broad group cross sections of the heavy nuclides are not sensitive to the weighting spectrum. This, of course, is not the case for materials such as iron or nickel.

Cross section Set ASY 11 used ENDF/B for the basic cross section data. The results obtained from this cross section set are in fairly close agreement with those obtained from Sets 224 and 238 which used old library data. The U-238 cross section of Set ASY 11 appears to be more reactive than those of Sets 224 and 238, as can be seen by comparing the calculated eigenvalues for Assemblies A and B and also for Assemblies F and G. The cross sections for U-235 and Pu-239 of ASY 11 are slightly less reactive than those of Sets 224 and 238. Since the weighting spectrum has such a large effect on the broad group cross sections for the light nuclides (iron and nickel) the influence of the basic cross section data on the calculated eigenvalues is not distinguishable. The influence of the basic cross section data could be determined from a study in which an identical

weighting spectrum was used to generate the broad group cross sections of the light nuclides for the three cross section sets.

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II-34. Approach to Critical Study by Modal Analysis for Plutonium-Fueled Fast Criticals

A. P. OLSON and R. G. PALMER

The purpose of this study was to select a reliable approach-to-critical plotting technique based on an analysis of the harmonic flux distributions as a function of multiplication in plutonium-fueled fast criticals. The availability of such a technique would not only enhance the safety of approach-to-critical operations, but might provide early indication of large errors in design predictions and allow timely changes to the core loading pattern or core length with a minimal perturbation to the program timetable.

The detector response as a function of multiplication in plutonium cores containing significant amounts of Pu-240 is expected to be different from the uranium cores having fixed artificial startup sources. The spontaneous fission source associated with Pu-240 expands with the core loading.

The one-group flux in a cylindrical reactor of height H , core radius a , zero flux boundary b , and flat distributed source in the core, can be expanded in modes as follows:

$$\phi_1(r, z) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \cos(\alpha_m z) J_0(\beta_n r), \quad (1)$$

$$0 \leq r \leq a;$$

$$\phi_2(r, z) = \sum_{m=1}^{\infty} C_m \cos(\alpha_m z)$$

$$\left[K_0(\gamma_m r) - \frac{K_0(\gamma_m b)}{I_0(\gamma_m b)} I_0(\gamma_m r) \right], \quad a \leq r \leq b, \quad (2)$$

where

$$\alpha_m = \frac{m\pi}{H} \quad (m \text{ odd}), \quad (3)$$

$$\gamma_m^2 = \alpha_m^2 + \kappa^2.$$

The expansion coefficients A_{mn} and C_m are

$$A_{mn} = \frac{(-1)^{(m+1)/2}}{(B^2 - \alpha_m^2 - \beta_n^2)} \quad (4)$$

$$\frac{SSJ_1(\beta_n a)}{m\pi a D_1 \beta_n [J_0^2(\beta_n a) + J_1^2(\beta_n a)]},$$

$$C_m = \sum_{n=1}^{\infty} A_{mn} J_0(\beta_n a) / K_0(\beta_n a).$$

The core buckling is B^2 , and κ^2 is the blanket Laplacian. D_1 and D_2 are core and blanket diffusion coefficients. The β_n are roots of

$$\beta_n \frac{D_1 J_1(\beta_n a)}{D_2 J_0(\beta_n a)} - \frac{\gamma m \left[K_1(\gamma_m a) + \frac{K_0(\gamma_m b)}{I_0(\gamma_m b)} I_1(\gamma_m a) \right]}{K_0(\gamma_m a) - \frac{K_0(\gamma_m b)}{I_0(\gamma_m b)} I_0(\gamma_m a)} = 0. \quad (5)$$

k -effective is given by the relation

$$k_{eff} \approx 1 - \frac{(B_g^2 - B^2)L^2}{1 + L^2 B_g^2}, \quad (6)$$

where B_g^2 is the geometrical buckling and L the diffusion length.

Twenty-two-group calculations were performed using ANL cross-section Set 224 in the diffusion and transport theory codes MACH-1 and SNARG to give reference flux shapes and reactivities for a range of core sizes with k_{eff} as small as 0.4, and to give group-collapsed cross sections for one-group modal analyses. Two large (~ 450 liter spherical core) fast-reactor compositions were chosen to represent hard and soft spectra. Geometries studied included cylinders with axial extrapolated lengths of 100, 120, and 140 cm, plus axially infinite systems. Excellent agreement was obtained with reactivities computed by the one-group theory approximation.

Equations (1)–(6) were programmed on the CDC 1604 to yield component and total harmonic fluxes as a function of core radius. A subsequent correlation study of the results showed that $R^{5/2}$ divided by count rate versus core radius R gave good linearity from low multiplications ($k \gtrsim 0.6$).

Figure II-34-1 shows the approach to critical plots for the axially infinite and axially finite calculational

FIG. II-34-1. Correlation of Approach to Critical. ANL Neg. No. 103-K5192 Rev. 1.

models and also the SEFOR mockup (ZPR-3 Assembly 47). Figures II-34-2 and II-34-3 show similar plots for Assemblies 52e and 53 on ZPR-3. The former conventional plots of mass/count rate versus R showed linearity over a smaller range below critical ($k \gtrsim 0.8$).

A parametric study of radial blanket thickness is shown in Fig. II-34-4. It can be seen that the $5/2$ power plot holds well over all blanket thicknesses.

FIG. II-34-2. Approach to Critical—ZPR-III Assembly 52e. ANL Neg. No. 103-2866.

FIG. II-34-3. Approach to Critical—ZPR-III Assembly 53. ANL Neg. No. 103-2867.

FIG. II-34-4. Effect of Blanket Thickness on Approach to Critical. ANL Neg. No. 103-2861.

II-35. Shape Factor Studies on Large, Oxide-Fueled Fast Reactors

A. P. OLSON

Few data exist¹ on experimentally determined fast reactor critical cylindrical core sizes as functions of length to diameter ratio, L/D . The data available are all for small cores with volumes ranging from a few liters to less than 500 liters. Some computer-calculated shape factors—the ratio of spherical critical volume to cylindrical critical volume—are also given in Ref. 1 for 600- and 900-liter uranium metal systems. However, none of this information is for cores at all similar to the large, soft, spectrum cores of interest for base-loaded nuclear power stations, and hence is of doubtful value for Zero Power Plutonium Reactor (ZPPR) core design studies. Consequently a series of five typical, large, oxide-fueled fast reactors ranging from about 300 to 15000 liters have been studied, using a computer to provide representative shape factors.

All cores in the series consisted of the same two-drawer cell, but the plutonium-to-uranium atom ratio in the fuel plate was varied to change the spherical critical size. The same blanket composition and thick-

TABLE II-35-I. PLATE INVENTORIES IN CORE AND BLANKET

Plate Type	Thickness, in.	Number of Plates per Unit Cell	
		Core	Blanket
Fuel	$\frac{1}{4}$	3	0
Na	$\frac{1}{2}$	2	0
Na ₂ CO ₃	$\frac{1}{2}$	1	0
U ₃ O ₈	$\frac{1}{4}$	5	5
Fe ₂ O ₃	$\frac{1}{8}$	2	0
Na	$\frac{1}{4}$	1	3

TABLE II-35-III. SHAPE FACTORS FOR LARGE OXIDE CORES

Series	Radius, cm	Half Height, cm	L/D	Volume, liter	Shape Factor	Normalized Shape Factor
I	39.458	31.545	0.7995	308.59	0.9659	0.9970
	37.503	34.816	0.9284	307.67	0.9688	1.000
	36.637	36.637	1.000	308.99	0.9647	0.9958
	41.439	—	—	298.07	—	—
II	85.301	30.376	0.3561	1388.73	0.6508	0.6864
	80.125	31.705	0.3957	1278.92	0.7066	0.7453
	75.143	33.345	0.4438	1183.01	0.7639	0.8057
	65.026	38.581	0.5933	1025.01	0.8817	0.9300
	55.062	50.040	0.9088	953.24	0.9481	1.000
	59.977	—	—	903.73	—	—
III	66.390	53.050	0.7991	1469.17	0.9362	0.9947
	63.019	58.563	0.9293	1461.32	0.9412	1.000
	61.595	61.595	1.000	1468.31	0.9368	0.9953
	68.990	—	—	1375.45	—	—
IV	166.53	47.286	0.2839	8239.4	0.4573	0.4923
	133.68	52.301	0.3912	5872.5	0.6417	0.6908
	105.61	63.149	0.5979	4425.4	0.8515	0.9167
	93.323	74.639	0.7998	4084.4	0.9226	0.9932
	88.545	82.344	0.9300	4056.4	0.9289	1.000
	86.565	86.565	1.000	4075.7	0.9245	0.9953
	79.056	114.38	1.447	4491.6	0.8389	0.9031
	74.307	162.57	2.187	5640.01	0.6681	0.7192
	72.471	209.65	2.893	6918.4	0.5447	0.5864
	96.53	—	—	3768.2	—	—
	—	—	—	—	—	—
V	150.14	121.48	0.8091	17206	0.9035	0.9929
	142.76	133.41	0.9345	17084	0.9100	1.000
	139.77	139.77	1.000	17156	0.9061	0.9957
	154.82	—	—	15545	—	—

TABLE II-35-II. HOMOGENIZED ATOM DENSITIES, atoms/cm³ × 10²⁴

Material	Blanket	Core Series				
		I	II	III	IV	V
U-235	0.0000176	0.0000153	0.0000163	0.0000168	0.0000172	0.0000175
U-238	0.0079652	0.0072041	0.0077113	0.0079651	0.0081971	0.0083501
Pu-239	—	0.0018525	0.0013443	0.0010900	0.000866	0.000713
Pu-240	—	0.0002398	0.000174	0.0001411	0.0001119	0.000092
Fe	0.007554	0.0106383	0.0106383	0.0106383	0.0106383	0.0106383
Cr	0.0020975	0.0024045	0.0024045	0.0024045	0.0024045	0.0024045
Ni	0.0009292	0.0010653	0.0010653	0.0010653	0.0010653	0.0010653
Mo	—	0.000335	0.000335	0.000335	0.000335	0.000335
C	—	0.0010844	0.0010844	0.0010844	0.0010844	0.0010844
O	0.0207045	0.0164094	0.0164094	0.0164094	0.0164094	0.0164094
Na	0.0060996	0.007596	0.007596	0.007596	0.007596	0.007596

FIG. II-35-1. Peak Shape Factor Correlation. ANL Neg. No. 103-K 5052.

FIG. II-35-2. Shape Factors for Medium and Large Oxide Cores. ANL Neg. No. 103-L 5687.

ness (30 cm) was used in all cases. Plate inventories used are given in Table II-35-I. Homogenized atom densities are given in Table II-35-II.

Spherical critical core sizes were calculated by the MACH-1 diffusion theory code using ANL Set 224 22-group neutron cross sections. Group collapsing to 6 groups produced cross section sets which were used by the two-dimensional diffusion theory code CANDID-2D in (r, z) geometry.

The results, shown in Figs. II-35-1 and II-35-2, are displayed in Table II-35-III. Also shown in the figures is the correlation of experimental data for small cores by W. G. Davey.² It is believed that an analy-

sis by computer of the experimental cores changes only the fissile/fertile atom ratio would yield a distinctive curve of peak shape factor versus spherical core radius for each one. Hence, the difference between W. G. Davey's curve and the computed curve for large oxide cores is caused by different core and blanket compositions, and blanket thicknesses.

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II-36. Use of Group-Space-Dependent Bucklings to Simulate a 3-D Split-Cylindrical Reactor in 2-D⁽¹⁾

A. J. ULRICH and J. C. BEITEL

The r, θ, z coordinate system is the one naturally chosen for solution of neutronic problems for the right circular cylindrical reactor. If a slab-like zone of differing material properties splits the cylinder along a plane containing the axis, as in the split-core concept for the Fast Flux Test Facility (FFTF),⁽²⁾ it is usual to use a two-dimensional $x-y$ computation with an appropriate B_z^2 for z -leakage. Here the z direction is taken to be parallel to the reactor axis and the x direction is perpendicular to the split. It would be highly desirable, however, to use one computational dimension in the direction perpendicular to the split and to reserve a dimension to adequately treat variations in the z -direction, as in the FFTF concept.

In order to achieve this formulation for the case of a split core a way is sought in the next few paragraphs to convert a problem in cylindrical geometry into one in plane geometry.

For the j th group of a multigroup, r, θ, z diffusion problem

$$D_j \left(\frac{1}{\phi_j} \frac{\partial^2 \phi_j}{\partial r^2} + \frac{1}{r \phi_j} \frac{\partial \phi_j}{\partial r} + \frac{1}{r^2 \phi_j} \frac{\partial^2 \phi_j}{\partial \theta^2} + \frac{1}{\phi_j} \frac{\partial^2 \phi_j}{\partial z^2} \right) \phi_j - \Sigma_{r ej} \phi_j + S_j = 0, \quad (1)$$

where $\Sigma_{r ej}$ is the total removal cross section. From a converged solution of this problem the value of the term

$$-B_{vj}^2 = \frac{1}{\phi_j} \frac{\partial^2 \phi_j}{\partial y^2} \equiv \frac{1}{r \phi_j} \frac{\partial \phi_j}{\partial r} + \frac{1}{r^2 \phi_j} \frac{\partial^2 \phi_j}{\partial \theta^2} \quad (2)$$

may be obtained at every mesh interval. This term may be thought of as a group-dependent space-dependent transverse buckling in

$$D_j \left(\frac{1}{\phi_j} \frac{\partial^2 \phi_j}{\partial x^2} - B_{vj}^2 + \frac{1}{\phi_j} \frac{\partial^2 \phi_j}{\partial z^2} \right) \phi_j - \Sigma_{r ej} \phi_j + S_j = 0, \quad (3)$$

This is a diffusion equation in the rectangular coordinates x and z , but its solution, ϕ_j , must be the same as that of the cylindrical equation when x and r are considered to be analogous, because of the definition of B_{vj}^2 . In effect, solution of the rectangular equation is the same as the solution of the cylindrical equation except that the converged value of the second term is used starting with the first iteration.

A geometrical way of looking at this appears in Fig. II-36-1. A pieshaped cylindrical volume element is shown and its conversion to a rectangular volume element is indicated by the dashed lines. The quantities ΔL_{jr} , $\Delta L_{j\theta}$, and ΔL_{jz} are defined as the leakage terms associated with the r, θ and z directions, respectively. In this conversion from cylindrical to rectangular geometry, dx is associated with dr and dy with $r d\theta$. If we define

$$\Delta L_{jz} = -D(\partial^2 \phi_j / \partial z^2) dx dy dz$$

and

$$\Delta L_{ju} = -D(\partial^2 \phi_j / \partial y^2) dx dy dz,$$

we have for the infinitesimal volume element $dz (= r dr d\theta dz)$,

$$\Delta L_{jr} - \Delta L_{jz} = -D dr d\theta dz (\partial \phi_j / \partial r).$$

On the other hand, from Eq. (2)

$$\Delta L_{j\theta} - \Delta L_{ju} = +D dr d\theta dz (\partial \phi_j / \partial r),$$

where

$$\Delta L_{j\theta} \equiv -\frac{D}{r^2} (\partial^2 \phi_j / \partial \theta^2) r dr d\theta dz.$$

Thus the term $-D dr d\theta dz (\partial \phi_j / \partial r)$ is removed from ΔL_{jr} and added to $\Delta L_{j\theta}$ in going from cylindrical to rectangular coordinates. This is exactly what the geometry requires because it can be shown by a separate derivation that this term is simply the net leakage in the r (or x) direction into or out of the volume element through the elementary areas between the solid and dashed lines which face toward the positive and negative r directions. When the problem has cylindrical symmetry, say about $r = 0$, this term gives the dilution of the flux due to expansion of an outward moving diffusion current, or the concentration of the flux due to the contraction of an inward moving diffusion current.

Referring again to Eq. (3) the group-dependent x -dependent transverse buckling is used by a computer program in a rectangular problem, and the computer is caused to generate a solution of the cylindrical problem. The x -dependence of the buckling is formulated in the problem by way of the concept of an assembly made up of slabs. Within each elementary slab the buckling is constant and is handled by the computer in the usual way, but the buckling varies from slab-to-slab, as is necessary to satisfy Eq. (2). As the number of slabs goes to infinity and the thickness

TABLE II-36-I. COMPARISONS IN 1D AND 2D CALCULATIONS

Diffusion Program	Mesh Intervals	Reflected Cylinder, k_{eff}	Slab Simulation, k_{eff}
MACH-1D	30	1.0000	1.0009
MACH-1D	38	1.0000	0.9999
CANDID-2D	14 × 13	1.0098	1.0097
CANDID-2D	19 × 23	1.0084	1.0083

of each slab goes to zero, the solution approaches that of the original cylinder.

In Table II-36-I the results of computations are listed which were performed using the six-group cross-section Set 29601.⁽³⁾ The first two lines of the table compare a 1-D radial solution with a 1-D slab simulation of the solution for different mesh intervals, using the MACH-1 diffusion code.⁴ The first one, having 30 mesh intervals, corresponds to 15 different bucklings per group and the second corresponds to 19 different bucklings per group. This increase in number of bucklings reduces the error from 9 parts per 10,000 to 1 part per 10,000. The last two lines of the table compare an r - z cylinder with an x - z slab simulation, done with the CANDID 2-D diffusion program.⁵ In both cases, the error is 1 part per 10,000. Convergence difficulties, which are known to occur in CANDID 2-D, limit the accuracy to 1 part in 10,000. Agreement between the two problems was actually closer than this. Because of the cylindrical symmetry in these cases the θ -derivative is zero in the definition of B_{vj}^2 in Eq. (2).

Table II-36-II compares fluxes in the fourth group, which is a representative group, for the r - z problem of

FIG. II-36-1. Conversion of Cylindrical Volume Element Into Rectangular Volume Element. ANL Neg. No. 113-2708.

TABLE II-36-II. PERCENT DIFFERENCE OF X-Z FLUX TO R-Z FLUX, GROUP 4 (25 keV-4.3 keV)

		Reactor Center				Radial Reflectors		
Core		-0.00	-0.01	-0.05	-0.15	-0.03	-0.06	-1.19
		-0.00	-0.01	-0.04	-0.14	-0.02	-0.04	-1.22
		-0.00	-0.01	-0.04	-0.15	-0.01	-0.02	-1.24
		0.00	-0.00	-0.05	-0.25	-0.01	-0.01	-1.25
Axial Reflector		0.00	-0.00	-0.05	-0.24	-0.00	0.00	-1.26
		0.00	-0.00	-0.04	-0.23	-0.00	0.01	-1.26
		0.00	-0.00	-0.04	-0.22	0.00	0.01	-1.27
		Axial Reflector Edge						

Table II-36-I with 14×13 mesh intervals. Every other mesh interval is included in the table.

The above calculations confirm the conversion of the cylindrical problem to slab geometry using the derived transverse bucklings. This slab formulation of the problem may now be used as a basis for slab-like perturbations on the slab problem. Consider the two-dimensional problem of the cylindrical reactor with and without the split in $x-y$ geometry with an appropriate B_z^2 to give z -leakage. For each of the two cases, it is possible to use the technique previously described to set up a one-dimensional slab problem in which the flux is equivalent to the flux found in the two-dimensional problem along the x -axis, and k is also the same as in the two-dimensional problem. If it is assumed that the transverse bucklings are the same for each of the two-dimensional problems at any point along the x -axis, then the transverse bucklings to be used in the one-dimensional slab problems will also be identical and the slab problems will differ from each other only in the composition present in the region that corresponds to the split.

Figure II-36-2 shows the application of the method to simulate a split cylindrical core in 2-D $x-y$ geometry by a 1-D slab. All of the problems illustrated in Fig. II-36-2 used cross section Set 29601 and were done using CANDID 1-D or 2-D. The leakage in the z -dimension is handled in the usual way by use of fictitious absorbers with cross section DB_z^2 . The computed reactivities of each of the assemblies shown in Fig. II-36-2 is printed below the assembly. For the purpose of comparison k is determined in $x-y$ geometry for the core with and without fuel in the split. The first step is to set up a 1-D cylinder of the same volume as the $x-y$ cylinder with fuel in the split. The coarse sawtoothed edges of the $x-y$ cylinder compared with the rounded

DIFFUSION THEORY

$$\Delta k_{2D} = 0.1372 \text{ BY } XY$$

$$\Delta k_{1D} = 0.1342 \text{ BY } B_{y1}^2$$

$$\frac{\Delta k_{2D} - \Delta k_{1D}}{\Delta k_{2D}} = 0.022$$

FIG. II-36-2. Worth of Split in Cylindrical Core in XY -Geometry and in One-Dimensional Simulation—By Diffusion Theory. ANL Neg. No. 113-2783 B

edge of the cylindrical problem causes a difference in k of 0.0011. From this 1-D cylinder the transverse bucklings B_{y1}^2 are derived from Eq. (2), where the θ -derivative is zero because of cylindrical symmetry, and the simulation slab problems are solved with and without fuel in the split. These are represented pictorially as having two dimensions in the figure. The Δk due to the split is found to be 0.1372 by $x-y$ geometry and 0.1342 by simulation. They differ by 2.2%.

From the $x-y$ solutions it is possible to compute B_{y1}^2 for the reactor with fuel in the split and without fuel in the split. Table II-36-III gives these bucklings for the first neutron energy group. Columns 1 and 2 compare the derived buckling B_{y1}^2 with the B_{y1}^2 from the $x-y$ problem with fuel in the split. It can be seen that the agreement between the two is quite good near the center of the reactor but as the edge of the core is

oached the effect of the sawtooth edges causes considerable disagreement in the buckling values. Columns 2 and 3 compare B_{y1}^2 from the x - y solutions with and without fuel in the split. Since the buckling B_{yj}^2 is little changed by removal of fuel from the split, conditions for an accurate solution by this perturbation method are good. In Table II-36-IV we have the ratio of fluxes for all six groups in the split, the core, and the reflector. At any position the numerator of the fraction is the flux with sodium in the split divided by the flux with fuel in the split, both obtained by the simulation method; the denominator of the fraction is the flux with sodium in the split divided by flux with fuel in the split, both obtained by a 2D x - y calculation. If the simulation were absolutely correct this ratio should be one. The deviation from unity is a measure of the small error in the flux ratios.

Figure II-36-3 shows the same split cylindrical core problem done in six groups with the SNARG transport code⁶ in the S_2 approximation. Here the Δk due to the

S2 TRANSPORT THEORY

$$\Delta k_{2D} = 0.1364 \text{ BY } XY$$

$$\Delta k_{1D} = 0.1336 \text{ BY } B_{yj}^2$$

$$\frac{\Delta k_{2D} - \Delta k_{1D}}{\Delta k_{2D}} = 0.021$$

FIG. II-36-3. Worth of Split in Cylindrical Core in XY -Geometry and in One-Dimensional Simulation—By Transport Theory. ANL Neg. No. 113-2783 A

TABLE II-36-III. BUCKLINGS FOR THE FIRST ENERGY GROUP

B_{y1}^2 Simulation	B_{y1}^2 From X - Y Geometry	
	Fuel in Split	Na in Split
Center		
0.000610	0.000612	0.000597
0.000619	0.000621	0.000598
0.000645	0.000644	0.000643
0.000700	0.000693	0.000721
0.000831	0.000792	0.000844
0.001284	0.000960	0.001027
Core		
Edge		
0.003437	0.001264	0.001332
0.003009	0.001789	0.001847
Reflector		
Edge		

TABLE II-36-IV. $(\phi_{Na}/\phi_{Fuel})_{sim}/(\phi_{Na}/\phi_{Fuel})_{xy}$

	Group					
	1	2	3	4	5	6
Na	0.981	0.981	0.979	0.977	0.965	0.913
Core						
0.981	0.981	0.983	0.992	0.980	0.969	0.929
0.986	0.986	0.989	0.989	0.988	0.982	0.968
0.994	0.994	0.996	0.997	0.997	0.994	0.991
1.002	1.002	1.004	1.005	1.005	1.004	1.005
1.011	1.011	1.013	1.013	1.013	1.012	1.019
Core						
Edge						
1.018	1.018	1.020	1.019	1.015	1.017	1.022
1.020	1.020	1.026	1.025	1.025	1.025	1.025
Reflector						
Edge						

TABLE II-36-V. k_{eff} USING DIFFUSION THEORY

Assembly	Geom.	k_{eff}	As- sembly	Geom.	k_{eff}	Fissile Critical Mass, kg
52E (fuel in split)	r - z	1.1465	52A	r - z	1.0076	200.7
			52B	r - z	1.0048	235.8
			52C	r - z	1.0038	280.0
52E (Na in split)	x - z	1.0027	52F	r - z	1.0006	282.0
						309.2

split is 0.1364 by x - y calculation and 0.1336 by the simulation method. These two numbers differ by 2.1 %.

Finally an r , θ , z split core problem may be simulated by an x - z slab problem. The Assembly 52E experiment, built on the ZPR-3 facility, was a split core similar to the one shown in Fig. II-36-2. It was part of the series of critical experiments⁷ whose k_{eff} and fissile critical masses are listed in Table II-36-V. Assembly 52A, 52B, 52C, and 52F were all approximately cylindrically symmetric and their k_{eff} were calculated in r - z geometry using the 6-group cross section Set 29601. Assemblies 52B, 52C, and 52F had the same core and reflector compositions as Assembly 52A but each had an axial cylindrical column of canned sodium in the center of the core causing the core to be annular. The diameter of this column of sodium increased in going from Assembly 52B through Assembly 52F. In Assembly 52E the sodium zone was in the form of a slab which split the core, as was previously noted. In the Assembly 52 series the composition of the eight safety rods were spiked with a heavier loading of plutonium than the rest of the core. In the cases of Assemblies 52E and 52F the critical mass was large enough to require an added amount of spiking over that used in Assemblies

FIG. II-36-4. Example of Core Geometries Solvable by the Method Described Here. ANL Neg. No. 113-2709.

52A, 52B and 52C. The k_{eff} of Assembly 52E obtained by the simulation method in an x - z calculation fits into the sequence of the other reactivities given in Table II-36-V in the same order as its critical mass fits into the sequence of critical masses.

The first problem which must be done in the simulation process is to solve the r - z problem of Assembly 52E with fuel in the split. From the fluxes thus obtained the bucklings B_{vj}^2 are derived and the solution of the Assembly 52E problem may proceed. The time required to derive the necessary array of bucklings is small, so that essentially the time required to obtain the Assembly 52E solution is twice that required for an r - z problem with an equal number of mesh intervals, such as Assemblies 52C or 52F.

This simulation method using a derived transverse buckling has been shown to be applicable to the solution

of a split-cylindrical core. Preliminary calculations indicate equal applicability of the same method to solution of the other geometries⁸ shown in summary in Fig. II-36-4. The first is a spherical core with a cylindrical hole through its center. The second is a spherical core with a plane split through its center. The third is a cylinder axially split by a slab and the fourth is a conical core which is axially split by a slab. Each perturbed region may be filled by any material for which diffusion theory applies, provided the derived transverse buckling is not much changed by the perturbation.

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II-37. Use of Effective Axial Bucklings for Non-Core Regions in Criticality Calculations

A. J. ULRICH and D. MENEGHETTI

The computation of k_{eff} of ZPR-3 Assemblies 52A, 52B, 52C and 52F was suited to r - z geometry because of their approximate cylindrical symmetry.^{1,2} Assembly 52A was a cylindrical core reflected radially and axially. Assemblies 52B, 52C and 52F were also cylindrical, reflected, and of the same composition as 52A except that each contained a central cylindrical

region (extending through the axial reflector) filled with canned sodium. These central regions had average radii of 9.23 cm, 14.31 cm and 19.00 cm respectively. Criticality was achieved in each case by varying the outer radius of the core.

The computation of k_{eff} for ZPR-3 Assemblies and 52E,⁽³⁾ however, presented difficulty for two-

nsional analysis because these reflected cylindrical cores contained axial sodium-filled regions which had shapes rectangular in cross section of sizes 16.6×60.8 cm and 16.6×83.0 cm, respectively. This difficulty was circumvented by utilizing an axial buckling characteristic of the combined core and radial reflector regions and an axial buckling characteristic of the non-core materials in the central region in two-dimensional x - y geometry.

The effective axial buckling characteristic of the combined core and radial reflector of the assemblies of the Assembly 52 series was taken to be 6.82×10^{-4} cm⁻². This is the buckling which gave the same k_{eff} for Assembly 52A in a one-dimensional cylindrical calculation using the MACH-1 code⁴ as that given in a two-dimensional r - z calculation using the ANL-CANDID code.⁵ The effective axial buckling for the non-core region was then obtained by adjusting the non-core region axial buckling, while retaining the previous core axial-buckling in the core region, so as to obtain the identical k_{eff} as given by a corresponding two-dimensional analysis. This procedure has been applied to Assemblies 52B, 52C and 52F, as built, and for the same assemblies using a number of differing non-core region compositions. The extent to which axial bucklings for a given composition non-core region is invariant with size was used as a measure of the potential applicability of these bucklings in an analogous system having the axial, centrally-located region of different cross sectional shape or dimension, as in Assemblies 52D or 52E.

Results for various compositions in the central cylindrical region are given in Table II-37-I. Cross section Set 29601⁽⁶⁾ has been employed throughout these calculations. The compositions identified in the first column by volume-percent of the major nuclides

are: 64 v/o Na, 27 v/o SS (the sodium is in stainless steel cans as built on ZPR-3 and extends through the axial reflectors), 16 v/o Na, 62 v/o Ni, 13 v/o SS (the radial reflector composition), graphite, and depleted uranium. The central region radii are those of Assemblies 52B, 52C and 52F.

Listed are the effective axial bucklings of the central regions and the reactivity deviation from criticality which would be obtained if the effective axial buckling derived for the intermediate dimension is used for the other sizes. Along with those computations listed in Table II-37-I another should be considered. The geometry and composition is that of Assembly 52F except that in a cylindrical region 4.00 cm in radius the sodium composition is replaced by axially reflected core material and the core buckling is used in this region. The sodium region in which a buckling search is performed is now an annular region. In this case the effective axial buckling is 4.74×10^{-4} cm⁻² and the reactivity deviation is $\Delta k/k = -0.00004$.

The effective axial bucklings of the central regions do not vary significantly with the central region radial dimension. Also the buckling change is not significant when the region becomes annular as in the last case. These facts together with the corresponding listed reactivity variations indicate that use of the operationally obtained effective buckling for a given central region radius can be applied to other sizes and cross sectional shapes for criticality calculations.

The ZPR-3 Assemblies 52D and 52E were analyzed using regional axial bucklings. In the second column of Table II-37-II the k_{eff} values are listed for the Assemblies 52A, 52B, 52C and 52F as obtained by ANL-CANDID in r - z geometry. Analytical results for Assembly 51 are also recorded for completeness.

TABLE II-37-I. VARIATIONS OF EFFECTIVE AXIAL BUCKLINGS FOR AXIAL-CENTRAL NON-CORE REGIONS

Axial Central Region	Radius of Axial-Central Non-Core Region					
	9.23 cm		14.31 cm		19.00 cm	
	$B_z^{2(a)}$ 10^{-4} cm ⁻²	$\Delta k/k^{(b)}$	$B_z^{2(a)}$ 10^{-4} cm ⁻²	$\Delta k/k^{(b)}$	$B_z^{2(a)}$ 10^{-4} cm ⁻²	$\Delta k/k^{(b)}$
64 Na 27 SS	4.78	-0.00011	4.74	0	4.72	0.00017
64 Na 27 SS refl.*	4.10	-0.00032	3.98	0	3.90	0.00059
16 Na 62 Ni 13 SS	6.41	-0.00017	6.23	0	6.12	0.00025
C	4.68	0.00009	4.77	0	4.83	-0.00018
Depl. U	10.32	-0.00067	9.17	0	9.24	-0.00008

* Effective axial buckling of the axial-central non-core region. Effective axial buckling of core region is in all cases 6.82×10^{-4} (obtained from Assembly 52A).

^(b) Reactivity deviation from criticality where the r - z solution is critical.

* In this case the central region does not penetrate through the axial reflectors. It does in all the other examples given.

TABLE II-37-II. COMPARISON OF CALCULATED k -VALUES FOR ZPR-3 ASSEMBLIES 51, 52a, 52b, 52c, 52e, AND 52f

Assembly No.	CANDID, r - z Geom.	Effective B_z^2 , ^(a) 10^{-4} cm ⁻²		CANDID, x - y Geom.
		Core	Central Region ^b	
51	1.008	6.73	—	1.008
52a	1.007	6.72	—	1.007
52b	1.004	6.72	5.31	1.004
52c	1.003	6.72	4.69	1.003
52d	—	6.72	5.0	1.000
52e	—	6.72	5.0	0.998
52f	1.000	6.72	4.97	1.000

^a Effective values of B_z^2 in core and test regions which force CANDID x - y values to equal r - z CANDID values in Cases 51, 52a, 52b, 52c, and 52f.

^b Approximately 64 v/o sodium and 27 v/o stainless steel.

The effective axial bucklings required for analysis of Assemblies 52D and 52E were to be used in x - y geometry. In order to be consistent with this condition the axial buckling searches on Assemblies 51 and 52A to give the k_{eff} listed in the last column were

also done using x - y geometry. These effective values appear in the third column. Then, as described above, the axial buckling in the central sodium region was varied for Assemblies 52B, 52C and 52F until k_{eff} matched that of the second column. These bucklings appear in the fourth column. The bucklings for the central region of these assemblies were then averaged for use in criticality calculations for Assemblies 52D and 52E. The k_{eff} values thus obtained compare well with the others in the last column.

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II-38. In-Core Gamma-Ray Spectroscopy

R. GOLD

Compton recoil gamma-ray spectroscopy has been applied for in-core gamma spectra measurements. Preliminary accounts of this method of continuous gamma-ray spectroscopy have already been given,^{1,2} and more detailed descriptions of the method will be issued shortly.^{3,4}

Electron recoil spectra observed in the fast neutron environment of the Argonne Thermal Source Reactor depleted uranium Snell-block are displayed in Figs. II-38-1a and II-38-1b. These data were obtained with a lithium-drifted silicon solid-state detector at two different power levels, namely zero power and at low reactor power. The unfolded gamma continuum for these two measurements are shown in Figs. II-38-2a and II-38-2b. Figures II-38-1a and II-38-2a which correspond to operation at zero power, reveal a gamma spectrum almost exclusively due to Pa-234m, a decay product of U-238 which possesses gamma transitions at 0.765 MeV (0.3%) and 1.001 MeV (0.6%).⁽⁵⁾

The low power results, Figs. II-38-1b and II-38-2b

reveal a large and broadly distributed continuum of hard gammas in and above the 2-MeV region as well as an additional peak of annihilation radiation (0.511 MeV). This component of annihilation radiation arises from the high probability of pair production and subsequent positron annihilation in the depleted uranium Snell-block.

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FIG. II-38-1. The Electron-Recoil Spectrum Observed in the Depleted Uranium Snell Block at: (a) Zero Power and (b) Low Reactor Power. Data Obtained with a Channel Width of 0.0167 MeV. *ANL Neg. No. 113-2666*.

FIG. II-38-2. The Unfolded Snell-Block Gamma Continuum for: (a) Zero Power and (b) Low Reactor Power. ANL Neg. No. 113-2664 Rev. 1.

II-39. Investigation of Oxide Fuel Loading for EBR-II

J. T. MADELL and R. E. JARKA

INTRODUCTION

The objective of this study is to investigate some of the neutronic characteristics of a mixed oxide fuel loading for EBR-II. The study is of broad scope and does not look into any details of a design study. In the first part of the study the major static, zero-power, neutronic parameters of a reactor system were calculated, in addition to some other parameters readily obtainable from the computer code. Possible ways of increasing the prompt negative feedback of an oxide core are explored in the second part.

STUDY OF THE STATIC PARAMETERS

The study was carried out using diffusion code MACH-1⁽¹⁾ with two 22-group cross section sets, Sets 224⁽²⁾ and 238.⁽³⁾ In the mixed oxide calculations, EBR-II was represented as a sphere whose enrichment of U-235 was determined to achieve criticality. The radius for a six-row core was based on calculations of the critical radius of a spherical representation of Runs 26 and 27 whose compositions are known. From the critical radius calculations, a radius for a six-ring core was established at 27.5 cm. The radii representing other rows were taken proportional to the enclosed number of subassemblies relative to the 91 subassemblies enclosed in a 27.5 cm sphere.

The 91 subassembly oxide-loaded core presented in these calculations consists of 64 fully loaded fuel subassemblies, 12 control subassemblies, and 2 safety subassemblies which are assumed to have two-thirds the loading of a fuel subassembly. There were assumed to be 13 experimental subassemblies in the core, 9 of which contained fissionable material and 4 of which contained structural material. Their compositions were selected from typical experimental subassemblies present in Run 27. The total number of subassemblies is 91. In the case of a seven-row core of 127 subassemblies, the number of subassemblies of each type is assumed to exist in the same fraction as they do in the 91 subassembly core.

A fully loaded fuel subassembly consists of 36 v/o fuel, 44 v/o sodium, and 20 v/o steel. The mixed oxide fuel is uranium oxide-20 w/o plutonium oxide which is assumed to be at 85% of theoretical density. For this study the plutonium oxide was assumed to consist of 100% Pu²³⁹ O₂.

description of the eight arrangements of the mixed oxide loading of EBR-II is given in Table II-

39-I. Calculations were performed with both cross section Sets 224 and 238 for each of the eight loadings.

A. TABULATION OF RESULTS

The results of the 16-problem study are given in Table II-39-I. The output is normalized, where applicable, to an arbitrary power level of 45 MWt. Because of the approximate nature of the representation used for EBR-II, the results should be considered as relative rather than absolute values. The primary purpose of interpreting the results is to establish the effect on the parameters of the reactor due to changing materials in EBR-II.

B. DISCUSSION OF RESULTS

1. Enrichment

The calculated values of the critical enrichment for the 16 cases range from ~30 to 60 a/o. Since no excess reactivity was specified for these calculations, one should increase the enrichment in order to provide excess reactivity for burnup and power-reactivity decrement. It is seen that the mixed oxide loading allows sufficient flexibility in selection of the enrichment. That is, if an additional reactivity might be needed to compensate an unexpected change in design, it can easily be obtained by increasing the enrichment which presently has a maximum value of 58 a/o. The values of enrichment calculated here are in general agreement with those from previous studies.^{4,5} Comparison of the enrichment and other results obtained from the two cross section sets are presented in a later section.

2. Reflector of Blanket Material

The enrichment calculations indicate nickel is the most effective material followed by steel and depleted uranium. The relative effectiveness is indicated by a difference in the critical enrichment between an all-nickel and all-depleted uranium blanket of 12 a/o. After the calculations were performed, it was discovered that the Pu-239 and stainless steel content in the blanket was too high. Upon correcting these concentrations in a few cases, calculations showed that the depleted uranium blanket was less effective than the steel reflector.

3. Core Fuel Loading

The loading of U-235 for identically sized oxide cores follows the percentage of enrichment. Thus, the

trends in core loading are the same as those in enrichment. It should be noted that a core loading of fissionable material (U-235 and Pu-239) is significantly smaller than the loading calculated for Runs 16, 24, 25, 26, and 27. However, when plutonium is taken to be equivalent to 1.75 times the weight of U-235, the equivalent loading of U-235 in a mixed oxide core is just slightly less than that of the metallic fuel loading.

4. Neutron Flux Level

For calculations based on the same power production, the flux level tends to be higher for lower loadings. The flux required to produce a given power density in an oxide fuel is slightly greater than that for a metal fuel. However, neutronic properties are not among the major considerations for establishing the flux level. The flux level appears to be determined by the allowable power density in a fuel pin rather than the neutronic characteristic of fuel within the pin.

5. Energy Spectrum of the Flux

The results indicate the median energy spectrum of the flux is lower in the oxide loaded core (~ 300 keV)

than in the metallic fuel loading (~ 400 keV). general trend in the oxide cores is that the spectrum is softer for the lower enriched cores. To place these numbers in perspective the median energy of the flux in a very large oxide core is typically 150 keV.

6. Peak-to-Average Power Densities in Core

The peak/average power density ratio is dependent upon the type of material in the inner blanket, and seems relatively insensitive to the type of fuel. The seven-row cores exhibit a larger ratio than the six-row cores.

7. Value of β and ℓ_p

The delayed neutron fraction β is greater in U-235 than in plutonium. Therefore, the value of β is expected to increase with increasing enrichment of an oxide core; the metal cores have a larger β than the oxide core because of the heavier U-235 loading. This is the trend that is seen in the calculations, but the differences in the values of β may not be significant. The trend observed for neutron lifetime is that cores with more effective reflectors/blankets have greater prompt neutron lifetimes. The calculations also show a

TABLE II-39-Ia. DESCRIPTION AND RESULTS OF CALCULATIONS FOR EIGHT CASES OF MIXED OXIDE LOADINGS USING CROSS SECTION SET 224

	Problem Number								
	301 ^a	306	307	308	309	310	311	312	313
Core									
Material	Run 27A	Oxide	Oxide	Oxide	Oxide	Oxide	Oxide	Oxide	Oxide
Outer row	6	6	6	6	6	6	7	7	7
Reflector									
Material	SS	Ni	Ni	Ni	SS	—	Ni	Ni	—
Outer row	8	8	10	15	8	—	10	15	—
Blanket (Depleted U) row	15	15	15	—	15	15	15	—	15
U enrichment, %	~ 52	52.9	48.2	45.0	57.8	57.6	35.6	32.1	42.2
Core loading, kg									
U-235	193.5	98.5	90.7	85.4	106.6	106.6	97.1	88.9	112.5
Pu-239	1.8	45.1	45.1	45.1	45.1	45.1	62.9	62.9	62.9
$\phi(0)$, ^b 10^{16} n/cm ² -sec	0.25	0.31	0.32	0.33	0.30	0.27	0.27	0.29	0.25
Median flux energy, keV	406	330	319	311	345	340	291	283	308
$\nu \Sigma_f \phi$, max/ave	1.44	1.40	1.35	1.31	1.45	1.47	1.42	1.37	1.53
β , 10^{-3}	6.89	5.49	5.42	5.37	5.43	5.58	5.19	5.11	5.35
ℓ_p , 10^{-7}	1.15	1.79	2.69	4.16	1.53	1.19	2.45	4.03	1.39
Perturbed calculations, $-10^{-9} \Delta k/k/\Delta V$									
Na/center	2.57	3.82	3.77	3.66	3.70	3.26	2.68	2.54	2.45
Na/core	4.38	5.43	5.46	5.36	5.16	4.45	4.06	3.98	3.41
Na/reflector	1.18	1.82	1.99	1.95	1.29	0.95	1.19	1.25	0.98
U-238 core ^c	0.59	0.54	0.44	0.36	0.62	0.51	0.21	0.11	0.28
U-235/core ^c	10.08	11.53	11.93	12.23	11.41	11.20	9.98	10.32	9.48
Fe/core	3.12	3.69	3.60	3.43	3.58	2.95	2.56	2.41	2.21

^a Used cross section Set 238.

^b Arbitrarily normalized to 45 MWt.

^c Based on density change of 1×10^{-5} nuclides/cm³.

TABLE II-39-Ib. DESCRIPTIONS AND RESULTS OF CALCULATIONS FOR EIGHT CASES OF MIXED OXIDE LOADINGS USING CROSS SECTION SET 238

	Problem Number								
	302	314	315	316	317	318	319	320	321
Core									
Material	Run 26B	Oxide	Oxide	Oxide	Oxide	Oxide	Oxide	Oxide	Oxide
Outer row	6	6	6	6	6	6	7	7	7
Reflector									
Material	SS	Ni	Ni	Ni	SS	—	Ni	Ni	—
Outer row	8	8	10	15	8	—	10	15	—
Blanket (Depleted U) row	15	15	15	—	15	15	15	—	15
U enrichment, %	~52	49.5	44.2	40.1	57.9	56.9	32.7	28.5	41.6
Core loading, kg									
U-235	186.7	92.9	83.9	78.5	106.9	105.3	90.1	80.4	111.2
Pu-239	2.3	45.1	45.1	45.1	45.1	45.1	62.9	62.9	62.9
$\phi(0)$, ^a 10^{16} n/cm ² -sec	0.26	0.31	0.32	0.34	0.31	0.29	0.28	0.29	0.26
Median flux energy, keV	406	328	316	309	351	343	290	281	313
$\nu\Sigma/\phi$, max/ave	1.44	1.37	1.30	1.26	1.46	1.47	1.37	1.30	1.53
β , 10^{-3}	6.89	5.42	5.33	5.26	5.36	5.53	5.10	4.98	5.30
ℓ_p , 10^{-7}	1.15	2.00	3.34	5.26	1.49	1.17	2.81	5.06	1.35
Perturbed calculations, — 10^{-9} $\Delta k/k/\Delta V$									
Na/center	2.68	2.96	2.84	2.69	2.82	2.46	1.87	1.66	1.74
Na/core	4.62	5.05	4.98	4.81	4.78	4.19	3.66	3.48	3.17
Na/reflector	1.22	1.79	1.91	1.83	1.31	0.96	1.13	1.13	1.00
U-238/core ^b	0.61	0.41	0.27	0.17	0.58	0.49	0.077	0.050	0.26
U-235/core ^b	10.39	11.77	12.21	12.55	11.54	11.38	10.25	10.69	9.64
Fe/core	3.11	3.23	3.03	2.79	3.25	2.69	2.13	1.87	1.97

^a Arbitrarily normalized to 45 MWt.^b Based on a density change of 1×10^{-5} nuclides/cm³.

greater lifetime for the oxide loadings than metallic loadings for the same reflector or blanket.

8. First-Order Perturbation Analysis

The reactivity due to a 1% decrease in the sodium density at the center and throughout the core for the 16 cases were obtained by first-order perturbation calculations and are given in Table II-39-I in terms of $\Delta k/k/\Delta V$. The values of $\Delta k/k/\Delta V$ for sodium depends mostly on the effective size of the core; that is, the physical size plus the reflector (or blanket) saving. This dependence is due to the importance of the leakage component of the sodium reactivity coefficient. For the same size core the sodium coefficient is more negative in the oxide than in the metal fuel loadings. The sodium coefficient in the inner blanket region is larger for the more effective reflector models. As will be discussed in the next section, the magnitude of the calculated sodium coefficient depends on the selection of the cross section set.

The worth of U-238 is dependent upon the energy spectrum; systems of higher enrichment also have a harder spectrum, which results in a greater worth of U-238. The results of the calculations bear out this trend.

In these calculations, the worth of removing a given amount of U-235 is inversely proportional to the amount of U-235 present in any one core. The type of fuel, oxide or metal, does not seem to influence significantly the worth of U-235. The ratio of the worth of Pu-239 to U-235 for identical changes in atomic concentration is 1.62 ± 3 for all 16 cases. The worth of iron in the core and blanket changes significantly with the selection of the cross section set.

9. Further Investigation of Sodium Coefficient in a Mixed Oxide Core

The first order perturbation calculation does not consider any change in the real or adjoint flux due to the change in nuclide concentration. A series of k_{eff} calculations were performed in which the perturbed flux was used to obtain the sodium coefficient. The results of the calculations, presented in Table II-39-II, show that the coefficients in the center of the core are about 15 to 29% more negative when the perturbed fluxes are used. The use of the perturbed fluxes for the sodium coefficients for the entire core changes the results very little. The perturbed fluxes have their greatest influence, therefore, on the scat-

TABLE II-39-II. REACTIVITY DUE TO CHANGE IN SODIUM DENSITY OBTAINED BY SUCCESSIVE k_{eff} CALCULATIONS, $-10^{-6} \Delta k/k$

Change in Na		Problem Number					
Decrease, %	Location	311	312	313	319	320	321
1	Center	1.36	1.29	1.24	0.95	0.84	0.88
1	Core	4.92×10^2	4.83×10^2	4.13×10^2	4.44×10^2	4.22×10^2	3.84×10^2
10	Center	13.6	12.9	12.4	9.5	8.4	8.8
10	Core	4.47×10^3	4.86×10^3	4.14×10^3	4.47×10^3	4.25×10^3	3.85×10^3

TABLE II-39-III. REACTIVITY EFFECT OF TEMPERATURE CHANGE OF U-238 AND PU-239 CROSS SECTIONS FOR FOUR OXIDE LOADINGS OF EBR-II

	Base Problem Number			
	308	311	308	311
Core				
Fuel	U-PuO ₂	U-PuO ₂	U-PuO ₂	U-PuO ₂
Moderator	None	None	6% BeO	6% BeO
Row	6	7	6	7
Reflector				
Material	Ni	Ni	Ni	Ni
Outer row	15	10	15	10
Reactivity, $10^{-4} \Delta k/k$ (300-750°K)				
U-238	-7.81	-9.74	-9.74	-12.22
Pu-239	+1.04	+0.90	+1.78	+2.88
Total	-6.77	-8.84	-7.96	-9.34

tering component of the reactivity, which is most important in the center of the core.

10. Comparison of Cross Section Sets 224 and 238

Sets 224 and 238 were obtained from the same MC² library tape but the cross sections of Set 224 were weighted with a spectrum characteristic of a large oxide or carbide reactor while an EBR-II Run 25 spectrum was the weighting spectrum for Set 238. The deviations of the results obtained from Sets 224 and 238 are small except for the cases with a nickel reflector and the decrease in sodium and iron densities.

The enrichments for the nickel reflected oxide cores using Set 224 (Problems 306, 307, 308, 311 and 312) are from 2.5 to 5% greater than for the same cases using Set 238 (Problems 314, 315, 316, 319 and 320). The values of $\Delta k/k/\Delta V$ for 1% decrease in sodium density at the core center are 30 to 50% larger from Set 224 than from Set 238. A 25-50% difference also exists for a perturbation in the iron density. The light elements (nickel, sodium, iron) appear to be most sensitive to the weighting spectrum. To resolve these discrepancies it may be advisable to generate a cross

section set with a weighting spectrum characteristic of the mixed oxide loading of EBR-II. It should be noted that improvements in MC² were incorporated before Set 238 but after Set 224 were produced.

STUDY OF INCREASING PROMPT NEGATIVE FEEDBACK

Operational experience of the Rhapsodie Reactor⁶ has revealed that the expansion coefficient of the oxide fuel disappears under irradiation. In studying an oxide loading for EBR-II the possible loss of the fuel expansion coefficient which presently contributes a prompt negative feedback was realized, and other sources of a prompt negative feedback were investigated. In this part of the study attention was directed to design considerations which would produce a Doppler effect comparable in magnitude to the fuel expansion effect.

Three approaches to obtaining a Doppler coefficient of the desired magnitude were examined: (1) increase flux in the resonance region by using a nickel reflector, (2) increase U-238 content by using U-233 fuel, and (3) increase resonance flux by using BeO in the core. A few typical loadings were selected from the eight presented in the first part of the report for the purposes of these calculations.

1. NICKEL REFLECTOR

As was seen in the first part of the study, the core spectrum becomes softer with increasing thickness of a nickel reflector and with larger core size. The Doppler coefficient was calculated for a six-row core with a fifteen-row nickel reflector and for a seven-row core with a three-row nickel reflector. The Doppler effects, calculated in terms of $\Delta k/k$ due to a temperature change of U-238 and Pu-239 from 300 to 750°K, are given in Table II-39-III. The spectrum in Problem 311 is about 20 keV softer and the Doppler effect about 30% greater than in Problem 308. Although the calculations are preliminary in stature, the nickel reflector results in a substantial reactivity feed due to the Doppler effect, as compared to that in a reactor with a depleted uranium blanket.

TABLE II-39-IV. NEUTRONIC CHARACTERISTICS OF OXIDE CORES COMBINING U-233 FUEL AND BeO MODERATORS

	Base Problem Number					
	307	308	311	312	308	311
Core						
Material	Oxide	Oxide	Oxide	Oxide	Oxide and BeO	Oxide and BeO
Outer row	6	6	7	7	6	7
Reflector						
Material	Ni	Ni	Ni	Ni	Ni	Ni
Outer row	10	15	10	15	15	10
Blanket (Dep. U) row	15	—	15	—	—	15
U enrichment, %	26 ^a	24 ^a	20 ^a	17.5 ^a	53	42
Core loading, kg						
U-233	44.1	41.0	45.6	40.9		
U-235	9.7	9.7	13.6	13.6	84.5	96.3
Pu-239	45.1	45.1	62.9	62.9	40.4	56.3
$\phi(0)$, 10^{-16} n/cm ² -sec	0.33	0.34	0.28	0.30	0.34	0.28
Median flux energy, keV	305	297	281	273	228	267
$\Sigma_f \phi$, max/ave	1.34	1.30	1.41	1.36	1.31	1.42
β , 10^{-3}	3.51	3.49	3.58	3.54	5.50	5.32
ℓ_p , 10^{-7}	3.15	4.89	2.79	4.63	4.67	2.92

^a Enrichment = atoms of U-233/total U atoms.

2. SUBSTITUTION OF U-233 FOR U-235 FUEL

Since U-233 is more reactive than U-235, it can sustain a larger amount of U-238 in the EBR-II core which, in turn, results in a larger U-238 Doppler effect. The atom enrichment of U-233 for criticality was calculated for the loadings in Problems 307, 308, 311 and 312 and the results presented in Table II-39-IV. The U-238 content in the cores fueled with U-233 is 49% greater than in the cores fueled with U-235. Although the Doppler effect due to U-238 would be increased by ~40% in a U-233 fuel loading, the approach, by itself, is not capable of achieving an increase in an order of magnitude of the desired prompt feedback coefficient.

3. ADDITION OF BeO TO CORE

The core spectrum can be appreciatively degraded by the presence of a light atomic weight, scattering material. BeO qualifies as such a material and, in addition, has a large atomic density and resistance to high temperatures. For the core loadings in Problems 308 and 311, 6% of the oxide fuel was replaced with BeO. Calculation of the neutron parameters for the two loadings given in Table II-39-III indicate that the uranium enrichment must be increased to compensate for a 17% decrease in fuel volume, but that the total loading of fissionable material is slightly less with the BeO present. The BeO also lowers the median flux energy by 20–30 keV. As may be seen in Table II-39-III the reactivity effect of temperature changes in Pu-239 and U-238 for the oxide-BeO load-

ings is slightly more negative (5–15%) than for the loadings without the BeO. Additional amounts of BeO would be required to achieve the desired Doppler coefficient and a seven-row core would probably be necessary to accommodate the increased amount of BeO.

SUMMARY

The limited nature of the study is again stressed, as it was in the initial paragraph of this paper. The study investigates some neutronic properties of the mixed oxide loading of EBR-II, but not all. No design considerations (power production, fuel diameter) were assumed in the calculations; the normalization of the flux to 45 MW was entirely arbitrary.

The uranium enrichment falls within the range of those presently used in EBR-II as fuel or experiment. The flux level is an important measure of merit for an irradiation facility. The calculations indicate that the design considerations (heat transfer, materials, etc.) will have a greater effect on the maximum flux (through the maximum fission density) than the neutronic properties of the fuel. A nickel reflector produces a more attractive value for the peak/average power density than either steel or depleted uranium. The spectrum is another important measure of an irradiation facility. The use of an oxide loading creates a softer spectrum than the metal loadings but not as soft as a large oxide or carbide core.

Some of the reactor characteristics related to safety and control seem to improve while others become

poorer with an oxide loading. It is stressed that no conclusion is drawn about the safety and control of an oxide loading from these calculations. The prompt lifetime is increased by the use of a nickel reflector while the value of β is lower because of the presence of plutonium. The sodium coefficient, appears to be more negative for a 6-row oxide loading and about the same for a 7-row loading as compared to a 6-row metal loading (such as Run 27). The worth of the fuel section of a control rod is proportional to the fraction of fuel loading that its movement represents. The fuel section represents a smaller fraction of the loading in a 7-row oxide core and about the same fraction in a 6-row oxide core compared to a 6-row metal core. The worth of the "follower" section is difficult to estimate from these calculations and the evaluation of control rod systems should await further calculations.

The increase of the Doppler coefficient to compensate for the possible loss of the fuel expansion coefficient looks promising. The most attractive means of increasing the Doppler effect to the desired level is a

combined use of a nickel reflector and a moderator such as BeO, in the core.

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II-40. Dose Rate on Top of the EBR-II Shield Due to Radiation Streaming Through Annuli in the Subassembly Extension

A. E. McARTHUR

To assist in the design of the subassembly extension for EBR-II, calculations were made to estimate the dose rate at the surface of the shield on top of the reactor due to radiation streaming through the void annuli in the subassembly extension and through the assumed void annulus around the extension. The calculations used the standard streaming formulas presented in Ref. 1. These formulas were derived to give conservative values for fluxes streaming through annuli.

The annulus nearest the center of the assembly extension (0.2 in. in width with an inside radius of 0.24 in.) gave a dose rate of 80 mR/hr at the top of the shield. The entering gamma-ray flux of 1.5×10^{10} MeV/cm²-sec was the value at a point 4 ft below the surface of the pool sodium. The calculation determined the dose rate directly above the end of the annulus produced by gamma-rays streaming through the annulus. The entering neutron flux for this annulus was much lower (6×10^4 n/cm²-sec) and produced a negligible dose rate at the top from neutrons.

The smaller annulus (0.063 in. width) inside the assembly extension at a radius of 0.58 in. produced a dose rate, due to streaming, at the exit end on top of the reactor shield of about 20 mR/hr from gamma radiation and a negligible dose rate from neutron radiation.

The annulus at the outside of the subassembly extension was assumed to be $\frac{1}{32}$ in. wide throughout its length (246 cm). The gamma-ray dose rate at the top of the reactor shield due to streaming through the annulus was calculated to be 12 mR/hr. The entering source flux was 4.5×10^9 MeV/cm²-sec.

The above calculations used flux values for the reactor operating at a power of 62.5 MW. The source gamma-ray flux at the top of the sodium is from the equilibrium sodium activity at 62.5 MW and is the value based on measurement. After shutdown this activity will decrease with a half-life of 15 h (a factor of 10 in 50 h).

As already mentioned, the above dose rates calculated to be conservative. If the actual dose rates

as high as the calculated values reported above, additional gamma-ray shielding could be provided for the small area above the annuli openings to reduce the dose rate.

Steel balls around the annuli in the subassembly extension provide adequate attenuation for the radia-

tion from the surface of the sodium—assuming a 40% void fraction for the 40 in. of steel balls.

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II-41. Preliminary Results of the Fission Yield of Mo-99 from Pu-239 in Fast Spectra

R. J. ARMANI, R. GOLD, R. P. LARSEN,* F. J. MARTINO,* R. J. MEYER, R. J. POPEK* and J. H. ROBERTS†

Fission yield measurements for Pu-239 in fast neutron spectra are in progress. Measurements have been planned in three different fast neutron facilities, namely the Gulf General Atomic Assembly, ZPR-3, and the Argonne Thermal Source Reactor (ATSR) Snell block.¹ Irradiations in the Gulf General Atomic Assembly and ZPR-3 have been completed, and these data are currently being analyzed. The ATSR Snell block irradiations are scheduled for the near future.

Preliminary data from the Gulf General Atomic irradiation have been obtained. The Mo-99 activity was determined radiochemically by established techniques,² and the fission rate was measured by the solid-state track recorder technique.³ Using these

methods, the preliminary value for the fission yield of Mo-99 is $(5.81 \pm 0.15)\%$ for Pu-239 in a fast reactor spectrum.

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* Chemical Engineering Division, Argonne National Laboratory.

† Macalester College, St. Paul, Minnesota.

II-42. International Comparison of Mn-56 Activity

A. DEVOLPI, K. G. PORGES, R. J. ARMANI and F. OZER

There are few radioisotopes with a decay scheme convenient for detection with accuracies of a few-tenths of a percent. Mn-56 is one of these, and it happens to be of considerable interest in reactor physics. It is frequently used as an absolute flux monitor in cores. Also, it is the basic isotope produced in the manganese bath system which is applied to the determination of neutron source strengths (see Paper I-27) and to fission neutron yield from Cf-252 (see Paper I-24).

There have been international comparisons of Au-198 and Co-60 activities, but never one of Mn-56.

The special nature of its decay scheme, as well as its great importance, led to an international comparison in which Argonne participated. Despite the short (2.6-h) half-life, the National Physical Laboratory of England was able to distribute active solutions in time for six laboratories to calibrate the samples. Other participants were the International Bureau of Weights and Measures at Sevres, France, the Central Bureau of Nuclear Measurements of EURATOM, the Atomic Energy of Canada, Limited, and the Institute for Nuclear Physics of the Netherlands.

All laboratories calibrated the samples through coin-

cidence techniques, with some variations in specific method. The analysis of results is being undertaken by Argonne. In addition, a specific test was made of an Argonne computer program COINC⁽¹⁾ which has been published for the treatment of coincidence data.

The range of data was 0.6% and the rms spread was 0.1% for all results. There is some indication of possible bias to the extent of 0.3% associated with specific techniques of calibration. The program COINC provides results consistent with calculations conducted by the other laboratories.

In addition to direct verification of Mn-56 counting capability, this comparison aids in supporting

Argonne measurements of other coincidence isotopes on an absolute scale. For example, absolute measurements of reaction rate are a feature of fast critical measurements of a variety of quantities, including alpha by the null-zone reactivity method. The same facility is also used for tritium counting as required by the reactivity-oscillation technique for determination of alpha in low power criticals.

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Section III

Fast Reactor Safety

Analysis of the course of accidents in fast power breeder reactor cores requires a broad range of reactor physics information. It is necessary to know damage and failure thresholds of the fuel; the types of fuel and coolant motion that can result, including acceleration and rate data; reactivity changes brought about by the motions, temperature changes, changes in neutron flux distributions throughout the core, etc.; and, finally, data sufficient to analyze the accident under study to its termination. Such matters are discussed in the papers of this section and, furthermore, the devices and means for acquiring and processing this kind of information are also treated.

III-1. Testing of the Prototype Mark II Integral Sodium Transient Reactor Test (TREAT) Facility Loop

L. E. ROBINSON

The second generation integral sodium TREAT loop, designated Mark II, was developed for TREAT safety experiments with fast-reactor oxide fuels. The Mark II loop was designed to provide an order-of-magnitude higher pressure containment capability, at a higher operating temperature, and with more complete instrumentation than the original Mark I integral sodium TREAT loop, which was used extensively for TREAT experiments with metallic fuel.¹

A line drawing of the Mark II loop is shown in Fig. III-1-1. Briefly, the Mark II integral sodium loop is a direct-flow, recirculating, self-contained facility in which the sodium coolant is driven by a miniaturized, electromagnetic annular linear induction pump (ALIP).² The loop provides a test section having a capacity of 19 EBR-II Mark I fuel pins, or seven oxide fuel pins of 0.290 in. diam, up to 31 in. in length. Sodium pressure-flow-temperature instrumentation sections are located at both inlet and outlet. The primary, or recirculating, portion of the facility is designed for a steady-state pressure of 340 atms (5000 psi) in the Mark IIA version, and of 272 atms (4000 psi) in the Mark IIB, at a temperature of 538°C (1000°F). The primary portion is separated from an auxiliary system of comparable containment capability by a calibrated burst disc/blowout line, and mechanical shut-off valves backed up by freeze valves. The auxiliary system provides a multipurpose high-strength volume which acts as a supply source of sodium, as a dump tank, and as a safety expansion volume.

The in-pile loop facility occupies a secondary enclosure having outer dimensions equivalent to two TREAT fuel elements, and a wall thickness 0.127 cm (0.050 in.). An instrument indication and control console, a power supply module, and a component coolant heat exchanger/circulation supply are located at some distance from the loop when in operation.

The prototype was the first of the Mark IIB integral sodium loops, and contains known deviations from the loop specifications that were considered acceptable in a loop to be used for preliminary tests and development of operational techniques. These include deficiencies that would be most likely to occur during fabrication and result in some compromise of the specifications; detailed testing of the prototype

thus could serve to confirm the design and specifications with a measure of conservatism. Deviations included one weld with an inclusion consisting of extraneous electrode metal, internal undercutting of two welds from incomplete penetration, two welds with filler material of unverified chemistry, and alignment of loop-pump flanges that was outside the specification for degree of parallelism. Despite the deviations from the stringent specifications demanded for the Mark II loops, it was recognized from the outset that the final judgement as to the quality and/or usability of any given loop would rest upon the outcome of the final proof test of the particular unit, at pressure and temperature. Hence, successful proof test of the prototype would qualify the loop for experimental use in the meltdown program, while in no way justifying the relaxation of the quality-control specifications previously enumerated as a means of assuring a conservative design.

During the tests of heater, power, and instrumentation circuitry, unexpected mishaps resulted in minor changes in the design of some components and operational sequences, and supplied considerable information concerning the durability of the loop and its behavior under conditions that had been considered to offer severe potential hazards. Such incidents constitute the essential reason for the construction of a prototype facility, and the behavior of the loop during potentially hazardous occurrences in progressive testing, with the after-effects of the mishaps, become a valid test of the loop design. The incidents with the prototype resulted in the acquisition of data which made unnecessary the performance of further independent tests to determine answers to questions on the loop durability, the consequences of potential hazards, and the malfunction of loop components.

One of the queries concerned the danger to the containment integrity of the loop resulting from a severe electrical malfunction or failure of the electromagnetic sodium pump. Extensive pump-design tests with a "bench-assembled" pump had demonstrated ruggedness in the event of internal electrical breakdown. This pump was used for the prototype loop, although one phase of the 3-phase Annular Linear Induction Pump (ALIP) developed a short to ground. However, the electrical fault only impaired the ef-

FIG. III-1-1. The Mark II Loop. ANL Neg. No. 112-6768.

iciency of the pump and had no effect whatsoever on the integrity of the pump Inconel flow tube or of the loop proper. During the approach to temperature during the first prototype loop test, the ALIP was operated at about half-maximum current to supplement the heat input by the external resistance heaters. After the loop had reached 800°F, the ALIP silicone coolant flow was reduced to near zero. Relatively high temperatures in the ALIP and its coolant circuit resulted. The ALIP stator temperature reached about 550°C, well above the temperature limits of the epoxy potting compound of the pump coils; despite the extensive destruction of the insulation material, as evidenced by the considerable charred epoxy observed in the inspections made afterwards, the prototype ALIP has continued to function well.

A question had been raised concerning the use of temperature-resistant silicone oil as the coolant for ALIP, and the consequences of a coolant spill from line breakage, or seal failure, in direct contact with the loop while at maximum temperature within its enclosure. The Dow-200 low-viscosity silicone fluid was selected for its excellent temperature stability, but it was recognized that the liquid would burn under the proper conditions of temperature and oxygen availability.

This relatively extreme temperature to which ALIP was subjected resulted in the failure of one of the power through-connectors of the pump, which had been mechanically damaged during the outfitting and repaired with an epoxy cement. This provided a leak in the silicone coolant system, at the top of the pump, while the loop was at 950°F. Approximately a half-gallon of the silicone sprayed out into the can, flowed down over the lower half of the loop, and collected in the bottom of the container. Although the silicone saturated the insulation and heaters of the hottest portion of the loop, the only result was the appearance of a large quantity of smoke; there was no ignition of the silicone, nor any perceptible damage to insulation, heaters, or any component of the loop.

In any sodium- or NaK-filled system, the hazards presented by the accidental spillage of the molten liquid metal through vessel rupture or seal leakage are always in question. The proposed use of the Mark II loop in the TREAT reactor enhanced the seriousness of the question as directed to this facility, with its 1.5-liter contents of sodium at high temperatures. This concern was coupled with concern that differential thermal expansion between the two legs of the loop could be a potential hazard. Preliminary verification of the design was obtained by a stress analysis, but because of the complicated geometry, a proof test was required. In the next test, the

The flow-tube temperature was set at 150°F, and the test section temperature was raised to 750°F (i.e., this was a differential of approximately 600°F between the test section and pump legs of the loop). The lower O-ring seal of the ALIP unit developed a small leak and about 500 cc of molten sodium, at 700°F, was dropped down within the secondary can upon the thermal insulation which was still saturated with silicone coolant after the first incident. Again, while some smoke was visible, there was no fire, nor was there detectable damage following the necessary steam-cleaning of the loop.

The use of Inconel X-750 for the ALIP flow tube requires a 25-in. length of a material having a linear thermal expansion coefficient of 8.0×10^{-6} in./in.-°F to be clamped tightly into one leg of a closed, elongated loop composed of Type 316 stainless steel, with a coefficient of 9.6×10^{-6} in./in.-°F. The differential thermal expansion of the opposing legs of the loop, as the device is brought from ambient temperature to 1000°F, could impose severe thermal stresses upon the upper and lower bends. Since there had been some difference of opinion as to the model to use in calculating the resultant stresses in the loop members, the question as to the device's capability to endure such stresses was raised.

Following the 600°F ΔT tests, the ALIP unit was removed from the loop for examination of the flanges and O-rings. While the pump was out of the loop, the pump interflange distance was found to be unchanged (within 0.002 in.), despite the high temperature differential between the Type 316 stainless steel test-section side and the ALIP flow tube of Inconel X-750. This provides an indication that the compound-curved offset bends of the loop, above and below the ALIP, react to the expected axial load of differential thermal expansion as massive but efficient springs. Further, the great strength of the Inconel pump flow tube is not affected by the tensile load applied by the upper and lower loop bends when at temperature, and the massive Inconel pump clamps are too strong to exhibit yield or shift under the maximum loads to be applied within the loop's rated temperature and internal pressure.

The seal leak at the pump flange was found to be the result of using a design in which flange clamps were deliberately slightly sprung open to protect the contact side of the flange. With this type of clamp, the O-ring seal capacity was discovered to be a function of the clamp spring constant as the bolts were tightened; the seal leaked at about 4200 psi, whether the fluid was gas, water, or sodium, and with the clamped flange at room or at elevated temperatures. Replacement of the sprung clamps with

straight clamps, at half the previous bolt torque, provided a seal which was found to easily contain 10,000 psig in tests outside the loop.

The operational tests of the loop indicated a stable output signal from the dc-electromagnetic flowmeters. With an excitation current of 1 A, the signal level is only about 1.2 mV.

The final test performed on the prototype Mark II loop, after completion of the preliminary testing, was a hydrostatic proof-test of the device at temperature. This test consists of filling the loop to the overflow with sodium, under an inert gas blanket of argon, and incrementally pressurizing the loop to 125% of its pressure rating while at rated temperature, as prescribed by the base design code.³ While this type of pressure test is considerably more difficult than the alternative low temperature hydrostatic test, a conservative approach to design and experiment required that meaningful proof of the loop's capabilities could only be obtained by empirical test, while the facility was subject to the actual temperature distribution of experimental operation at the rated temperature. A realistic distribution of temperature required filling the loop with the proper amount of sodium, and circulating the liquid metal through the primary system while pressurizing the 600 cc expansion volume with blanket argon.

In view of the hazard to personnel and equipment presented by hot sodium spill and/or fire, the proof tests on the prototype loop were carried out with the device suspended in a temporary enclosure outside the liquid-metals building. Control, power, and coolant modules were located inside the building and connected to the loop by cables and hoses passed through the building wall. The test pressure source consisted of a gas-pressure-intensifier unit, supplied by an argon gas bottle and connected to the loop by heavy-walled stainless steel tubing designed for ultra-pressure use.

On January 18, 1969, the prototype Mark IIB integral sodium loop was proof tested. Uncertainty as to the capabilities of the much used electrical heaters on the loop prompted the specification of a test temperature of 427°C (800°F), instead of the maximum design temperature of 538°C (1000°F). The required proof pressure at this temperature was calculated by the ASME Code³ formula

$$P_t = \frac{S_{mt}}{S_{mr}} (1.25) P_r,$$

where

S_{mt} = maximum allowable design stress of Type 316 stainless steel of the loop, at the test temperature

TABLE III-1-I. PROOF TEST RECORD

Stage	Time	Time at Pressure	Metered Pressure, psig	Actual Pressure, psig
1	2:20 PM	Start ^a	0	0
2	2:28-2:38	10 min	2500	3000
3	2:38-2:44	6 min	3000	3500
4	2:44-2:50	6 min	3500	4000
5	2:50-2:56	6 min	4000	4500
6	2:56-3:02	6 min	4700	5200
7	3:02-3:06	4 min	5100	5600
8	3:06-3:14	8 min	5700	6200
9	3:14-3:24	10 min	4100 ^b	4600
10	3:24	—	0	0
11	3:40	60 min	1000 ^c	1500

^a Slow pressurization to approximately 50% test pressure.

^b Approximately 75% test pressure.

^c Internal pressure applied while loop cooled to ambient temperature.

S_{mr} = maximum allowable stress at the rated temperature

P_r = rated pressure

P_t = test pressure.

For a loop rated at a pressure of 4000 psi at 538°C (1000°F), the test pressure at the specified temperature was determined to be

$$P_t = \frac{15,600}{14,000} (1.25)(4000) \\ = 5550 \text{ psi.}$$

The rupture of the safety burst disc was avoided by applying the gas pressure simultaneously to the top of the loop and to the auxiliary piping of the prototype. Current to the sodium pump was set at about 15 A to give a flow between 3 and 4 m/sec. After the prototype loop was at the test temperature of 427°C, a breakdown of the gas-pressure-intensifier necessitated the hurried substitution of an alternate

electrically driven hydraulic pressure source for the air-motor driven unit. The electro-hydraulic pressure supply was connected to the intensifier accumulator, from which the pressurized argon was supplied to the loop. Calibration of the pressure gauge revealed a minus 500 psi gauge needle shift in accuracy; the gauge read consistently 500 psig lower than the actual pressure applied to the prototype loop. The proof test data recorded and corrected for calibration of the hydraulic unit pressure gauge are given in Table III-1-I.

The lack of precision in the incremental pressure levels following the initial pressurization (specified $\Delta P = 0.10 P_t$) resulted from the difficulty in controlling the electrically-driven hydraulic unit during the frequent recycling of the intensifier accumulator. Throughout the test, the 4-pole prototype ALIP sodium pump performed satisfactorily, the stator reaching a maximum shell temperature of 77°C (170°F). All control, power, and instrumentation circuits performed properly.

Following the completed test, the loop was removed from its secondary enclosure and carefully examined. No evidence of loop deformation from the proof test was detected.

The prototype proof test demonstrated the capability of the Mark IIB integral sodium loop to safely contain steady state internal pressures in excess of the calculated 415 atms (6100 psi) critical pressure of sodium.

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III-2. Mark II Loop Checkout Experiment on Oxide Meltdown in Flowing Sodium in the Transient Reactor Test (TREAT) Facility

C. E. DICKERMAN, R. PURVIANCE, L. E. ROBINSON, K. SCHMIDT and I. POLLACK*

The successful development of the Mark II integral sodium loop for the Transient Reactor Test (TREAT) Facility has now provided a capability for perform-

* Central Shops Engineering Section, Argonne National Laboratory.

ing meltdown experiments on fast reactor oxide fuel pins inside a flowing sodium environment simulating reactor core flow channel geometry. Following the out-of-pile proof testing (see Paper III-1), preparations were begun for use of the prototype loop in

FIG. III-2-1. X-ray of Fuel Pin and Two Dummy Pins Prior to Loading in the Loop. ANL Neg. No. 113-2773.

the checkout oxide fuel pin meltdown experiment in TREAT. This experiment has two main goals: first, to carry the new loop system through a complete cycle of use as a meltdown facility in-pile in the TREAT reactor; second, to provide early data on the phenomena associated with high specific energy input to an oxide fuel pin in a flowing sodium environment with geometry permitting sodium expulsion and re-entry. The loop design has been described in Ref. 1. The fuel pin consists of a stack of 13% enriched UO_2 pellets of 90% smeared density, approximately 27.9 cm long, inside a type 304 stainless steel jacket with 0.737 cm o.d. by 0.635 cm i.d. A plenum space 11.4 cm long is present inside the cladding above the fuel stack. A spiral spacer wire 0.14 cm diam is wound around the jacket. Figure III-2-1 is an x-ray showing the fuel pin and two dummy pins prior to

loading in the loop. This fueled pin is surrounded by a ring of six dummy pins (hollow jacket tubes) to define the flow channel. Sodium flow is specified at 4 m/sec, which corresponds approximately to the same mass flow rate for the heated pin as the rate for one of many identical pins inside a true subassembly through which the flow would be 8 m/sec. Initial experimental temperature is 400°C. Radiochemical analyses performed on a test pin from a calibration run inside a sister Mark II loop yielded a calibration factor for this experimental arrangement of 3.45 J/g UO_2 per MW-sec of TREAT energy.

A sample input of ~ 2200 J/g was desired in order to provide sufficiently high energy to be of direct use in safety analyses of subsequent loop tests on oxide meltdown in TREAT. On the basis of preliminary safety analyses, it was decided to scale down the

energy input to about 1765 J/g. This conservative lower value makes it possible to use the S-3 stagnant sodium autoclave oxide pin cluster test² (1930 J/g) as a "lead" experiment. The maximum pressure pulse from S-3 was about 36 atm, which is an order of magnitude below the proof test out-of-pile pressure (see Paper III-1). The specified energy input for the checkout experiment is about 35% greater than that required to melt UO₂ under adiabatic conditions.

The out-of-pile proof test of the prototype loop was supplemented by a detailed stress analysis³ showing that stresses generated in the loop under operating conditions are acceptable. These stresses arise from the difference in thermal expansion between the Inconel X pump tube and the remainder of the loop which is of Type 316 stainless steel. Other considerations included: malfunction of the silicone coolant circuit (see Paper III-1); the possible local overheating of the loop arising from the limiting case of dense plugs of solidified fuel forming after sample meltdown; the potential problem of impact loading at the top and bottom of the loop if some of the loop sodium were to be given large increments of kinetic energy by rapidly-vaporizing sodium as a result of heat dump from fragmented and molten fuel to sodium in the flow channel (see Paper III-3); and prompt loop heating due to prompt gamma rays during the reactor power burst. A loop holddown device that braces the top of the loop against the 25-ton top shielding of the reactor has been fabricated. This device is basically a dashpot, loaded with a non-Newtonian silicone grease, which permits slow expansion of the loop during loop heating to operating temperature but "locks" in place under impact loadings. Under conservative assumptions it can be calculated that high conversion of sample energy to kinetic energy of upward-moving sodium inside the loop would result in sufficient upward momentum transfer to the loop to raise it about 38 cm. The holddown is designed to prevent this movement. For the in-between time scale of loop thermal expansion due to prompt gamma heating (time ~ 0.5 sec), short specially-designed springs are inset in the bottom plate of the loop secondary containment can.

One major part of the experimental goals is a check on the degree to which the conversion of thermal to kinetic energy approaches the upper thermodynamic limit. High energy tests (up to 2740 J/g) on oxide pins in water⁴ indicate relatively low conversions of 2.8% or less. The well-known limiting thermodynamic analyses reported by E. Hicks and D. Menzies⁵ can be interpreted as indicating a maximum conversion of

about 11.6% for the checkout. An alternate limiting case calculation was performed in which it was assumed that the total energy input to the sample is 2510 J/g (due to a minor reactor operations error), and this energy is shared isothermally and adiabatically by the 27.9 cm long fueled pin section and by the 16.9 g of sodium that are immediately adjacent to this fueled section. This results in a calculation in which the sodium is heated to its critical point of 410 atm, and then expands at constant pressure while being heated to 2950°C. The constant pressure is then assumed to accelerate 300 g of sodium above the pin for the total expansion distance of 79 cm above the original loop sodium level. The kinetic energy in the 300 g of sodium represents a calculated conversion of approximately 12.7%. Thus conversions much smaller than the order of 12% can be attributed to real effects rather than reflecting large uncertainties in thermodynamic calculations. Such ameliorating real effects could include pin failure into macroscopic fragments rather than idealized particles of the order of 100 μ size, less than "perfect" dispersal of fuel fragments into the sodium, and early separation of the sodium "working fuel" from the hotter fragments due to formation of sodium vapor blankets around the fuel.

Two identical sets of loop control, instrumentation, power supply, and cooling system apparatus were built for use in operations at both TREAT and the Chicago site. The provisional version of the loop operating manual was finished and made available to TREAT personnel for use during the checkout experiment. The prototype loop and its operating apparatus were checked out and operated at TREAT and then turned over to the TREAT staff for the checkout experiment.

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3. Sodium Slug Impact Loading of the Transient Reactor Test (TREAT) Facility Mark II Loop in High Energy Experiments or Accidents

L. W. DEITRICH

INTRODUCTION

Prior to performance of any experiment in TREAT, a safety analysis of the experiment must be prepared by the experimenters and approved by the appropriate reviewing authority. These safety analyses are often based on very conservative limiting-case assumptions regarding the events expected during the experiment. In the course of the safety analyses of the first TREAT oxide fuel meltdown experiment using a Mark II loop, it was assumed that a slug of sodium could be ejected from the loop test section by sodium vapor at its critical pressure, estimated to be 410 atm (6000 psi). The analysis described in this paper was undertaken to provide an estimate of the magnitude of the pressure pulses associated with impact of such a slug on the loop structure, and to determine the possibility of a loop rupture resulting from such an event. The results of the analysis indicate that, while the calculated pressures resulting from impact are in excess of the static burst pressure of the loop, there is not sufficient energy available to do work on the loop wall to cause rupture. However, in view of uncertainties in the analysis and to provide an extra margin of assurance that loop rupture would not occur, orifice plates were placed inside the loop prior to the second, higher energy, oxide fuel meltdown test to dissipate the kinetic energy of the sodium slug without causing excessive pressure. The analysis leading to the placement and sizing of the orifice plates is described herein.

ANALYSIS

Initial work on the analysis of sodium slug impacts employed a simplified description of the assumed sodium slug and its ejection from the test section. Emphasis was placed on analysis of the actual impact of the slug with the loop closure flange to determine the resulting pressure. Later work was concentrated on analysis of the ejection, using a detailed description of the flow path traversed by the slug. The model was used to locate and size orifice plates which dissipate much of the kinetic energy of the slug. These analyses will be described in turn.

ANALYSIS OF SODIUM SLUG IMPACT

Consider the impact on the end of a uniform pipe of a sodium slug traveling at high velocity in the pipe. The end of the pipe is assumed to be flat and perpendicular

to the pipe axis. To a first approximation, this interaction may be considered to be a "water hammer" event. However, the usual assumptions made in water hammer analysis—that the fluid velocity is very much less than the speed of a pressure wave in the fluid and that the speed of the pressure wave is approximately equal to the sonic velocity in the fluid—are not justified for the velocities expected in the present case. It is therefore necessary to solve the complete equations of motion for the fluid. These are¹

$$\frac{\partial H}{\partial x} + v \frac{\partial v}{\partial x} + \frac{\partial v}{\partial t} + \frac{fv|v|}{2D} = 0 \quad (1)$$

$$\frac{\partial H}{\partial t} + a^2 \frac{\partial v}{\partial x} + v \left(\frac{\partial H}{\partial x} - 1 \right) = 0 \quad (2)$$

$$a^2 = \frac{K/\rho}{1 + (K/E)(D/e)c_1} \quad (3)$$

$$c_1 = \frac{2e}{D} (1 + \mu) + \frac{D}{D + e} \left(\frac{5}{4} - \mu \right), \quad (4)$$

where

$H \equiv$ pressure head $= P/\rho$

$P \equiv$ pressure

$\rho \equiv$ fluid density, assumed constant

$v \equiv$ velocity of the fluid

$K \equiv$ fluid bulk modulus of elasticity

$E \equiv$ Young's modulus of the pipe material

$e \equiv$ pipe wall thickness

$D \equiv$ pipe inside diameter

$\mu \equiv$ Poisson's ratio of the pipe material

$a \equiv$ velocity of the pressure wave.

These equations can be solved by the method of characteristics, using essentially the same computer program as that published by V. L. Streeter and J. B. Wylie.¹ This program has been modified to take into account the convective effects, and to calculate strain energies in the fluid and the container walls. The strain energy in the sodium slug is calculated from

$$SE_s = \frac{V}{2\rho c^2} \bar{P}^2, \quad (5)$$

where

$V \equiv$ volume of slug

$c \equiv$ sonic velocity in the slug

$$\bar{P}^2 \equiv \frac{1}{L} \int_0^L P^2(x) dx.$$

The strain energy of the slug per unit length is then

$$SE'_s = \frac{A}{2\rho c^2} P^2, \quad (6)$$

where $A \equiv$ cross sectional area of the sodium slug.

Assuming elastic behavior, the strain energy in the loop wall is given by

$$SE'_w = \frac{5 - 4\mu}{32} \pi L \frac{d^3}{Ee} \bar{P}^2, \quad (7)$$

where $d \equiv$ mean loop wall diameter. The strain energy of the wall per unit length is

$$SE'_w = \frac{5 - 4\mu}{32} \pi \frac{d^3}{Ee} P^2. \quad (8)$$

Note that the strengthening effects of flanges or other reinforcements are not considered.

For purposes of the "sodium hammer" calculation, the following was assumed:

1. The flow is frictionless upflow, at constant density, through a 10 cm² flow area.
2. The pipe is initially filled with argon gas at 1 atm pressure.
3. Impact is uniform against a square end of the pipe.
4. Sodium properties are those given by G. H. Golden and J. V. Tokar² at 1000°F.
5. The loop material, Type 316 stainless steel, has a Young's modulus of 25×10^6 psi and a Poisson's ratio of 0.3.
6. Pipe dimensions are those of the top of the loop adjacent to the closure flange.
7. The sodium slug has a volume of 360 cm³, a mass of 300 grams, and is 36 cm long.
8. The slug is driven upwards by a constant pressure of 410 atm.

The following results were calculated:

1. The slug impacts the end of the pipe at a velocity of about 45,000 cm/sec with a kinetic energy of 30,400 J.
2. A maximum pressure of about 8345 atm results from the impact.
3. The maximum wall strain energy is 5400 J, corresponding to a maximum strain energy per unit length of 172 J/cm. Of the total strain energy of the slug plus the wall, approximately 18% is absorbed by the loop wall, based on the elastic model.
4. The minimum slug kinetic energy is 200 J. At the instant of minimum kinetic energy, the total kinetic and strain energy in the slug is calculated directly to be about 80% of the original slug energy, in good agreement with the wall strain calculation.

The results indicate that, while the pressures generated during the slug impact are very high, the energy

available to do work on the loop wall is not sufficient to cause a loop rupture. The energy per unit length required to rupture the loop near the top is estimated as 1710 J/cm, based on an approximate integration of the stress-strain curve of the material.

It is clear from the results obtained that some of the assumptions made in formulation of the problem are violated. Most important is violation of the assumption of elastic behavior and small deformation of the pipe walls. A rigorous treatment of the problem would require consideration of the compressible hydrodynamics of sodium slug impact coupled with dynamic plastic deformation of the loop wall. The present case does not fit into any of the limiting cases of such problems. The loop wall is not sufficiently flexible that its resistance to shear can be ignored as in the hydrodynamic model; the sodium is not very rigid compared to the wall, so the "highly flexible tubing" model¹ does not apply; the wall is not rigid compared to the sodium so the "water hammer" model discussed above has some weaknesses. In view of the uncertainty in calculating the effects of the slug impact, and the high degree of difficulty of obtaining a rigorous solution to the problem, it was decided to place orifice plates in the flow path to reduce the slug impact velocity. The analysis involved in locating and sizing the orifice plates is considered next.

ANALYSIS OF SODIUM SLUG EJECTION

Assume that a slug of known and constant mass, density, and viscosity is driven through a duct of variable area by a known pressure. The driving pressure is resisted by some known pressure plus the forces due to fluid shear and due to area changes in the duct. These resisting forces are assumed to obey the usual laws of incompressible hydrodynamics. Shock interactions are not considered. The velocity of the slug can be obtained by solution of the momentum equation for transient incompressible flow in a variable area duct,

$$\rho \frac{\partial v}{\partial t} = - \left[\frac{\partial P}{\partial x} + f \frac{\rho v^2}{2D_h} + \rho g + \rho v \frac{\partial v}{\partial x} \right]. \quad (9)$$

The continuity equation reduces to

$$vA = \text{constant}. \quad (10)$$

The flow is assumed to be isothermal and adiabatic, so no solution of the energy equation is necessary. If the velocity of the sodium slug is represented by one value of v_b referred to a "base" area A_b , then

$$\frac{\partial v}{\partial t} = \frac{A_b}{A} \frac{\partial v_b}{\partial t}. \quad (11)$$

Substituting Eq. (11) into Eq. (9), and integrating with respect to x over the length of the slug, one obtains

$$\rho v_b \frac{dv_b}{dt} \int_{x_r}^{x_f} \frac{dx}{A(x)} = P(x_r) - P(x_f) - \frac{\rho}{2} v_b^2 A_b^2 \cdot \int_{x_r}^{x_f} \frac{f dx}{D_h A^2} - \frac{\rho}{2} [v^2(x_f) - v^2(x_r)] - \rho g(x_f - x_r) \quad (12)$$

where

$x_f \equiv$ coordinate of the front of the slug

$x_r \equiv$ coordinate of the rear of the slug.

Now, consider the flow path to be divided into a number of sections, each identified by an index i , and having constant area A_i , friction factor f_i , length L_i , and hydraulic diameter $D_{h,i}$. Equation (12) becomes

$$\frac{dv_b}{dt} = \left\{ \frac{P(x_r) - P(x_f)}{\rho} - \frac{v_b^2}{2} A_b^2 \left[\sum_i \left(\frac{f_i L_i}{D_{h,i}} + K_i \right) \frac{1}{A_i^2} \right] \right\} \frac{1}{A_b \sum_i \frac{L_i}{A_i}} \quad (13)$$

in which the hydrostatic pressure has been ignored and the term K_i has been introduced to account for pressure changes associated with area changes. This coefficient includes provision for the term $\rho[v^2(x_f) - v^2(x_r)]/2$ from Eq. (12). The summations over i are taken to include the length of duct occupied by the slug at any instant of time. We may now write

$$\frac{dv_b}{dt} = F(x, v_b) \quad (14)$$

and

$$v_b = \frac{dx_b}{dt} = \frac{A}{A_b} \frac{dx}{dt}, \quad (15)$$

where $F(x, v_b)$ represents the right-hand side of Eq. (13). Then, dividing Eq. (14) by Eq. (15) and rearranging, one obtains

$$v_b \frac{dv_b}{dx} = \frac{A}{A_b} F(x, v_b) \quad (16)$$

or

$$\frac{dv_b^2}{dx} = 2 \frac{A}{A_b} F(x, v_b). \quad (17)$$

Finally

$$v_b^2 = \frac{2}{A_b} \int_0^x A(x') F(x', v_b) dx'. \quad (18)$$

In finite difference form, Eq. (18) becomes

$$v_{b,n+k}^2 = 2 \sum_{j=n+1}^{n+k} \left\{ \frac{P(x_r) - P(x_f)}{\rho} - \frac{v_{b,j-1}^2}{2} A_b^2 \left[\sum_{i=j-n}^{j-1} \left(\frac{f_i L_i}{D_{h,i}} + K_i \right) \frac{1}{A_i^2} \right] \right\} \cdot \frac{L_j A_j}{A_b^2} \frac{1}{\sum_{i=j-n}^{j-1} \frac{L_i}{A_i}}. \quad (19)$$

In Eq. (19) it is assumed that the duct region is divided into $n + k$ sections, n of which initially contain sodium and k of which are empty. The sections may be defined arbitrarily with the sole restriction being that the volume of the i th section must equal the volume of the $(n + i)$ th section. This is necessary to preserve continuity of the slug, which is not otherwise provided. (This restriction can be a considerable inconvenience in setting up a model for computation.)

Equation (19) has been incorporated into a code for the CDC-160A computer in which Eq. (19) is used exactly as stated, except that $v_{b,j-1}$ is replaced by $\sqrt{1/2 [v_{b,j}^2 + v_{b,j-1}^2]}$ to provide improved accuracy in the computation. This RMS velocity is calculated by iteration at each increment. The friction factor f_i is calculated as a function of the Reynolds number

$$Re_i = \frac{\rho v_{b,j-1} D_{h,i} A_b}{\mu A_i} \quad (20)$$

with values as follows:

$$f_i = 0.04, \quad 0 \leq Re_i \leq 4000 \quad (21a)$$

$$f_i = 0.316/Re_i^{0.25}, \quad 4000 < Re_i \leq 100,000 \quad (21b)$$

$$f_i = 0.184/Re_i^{0.20}, \quad 100,000 < Re_i. \quad (21c)$$

In actual computation $v_{b,j-1}$ in Eq. (20) is replaced with the RMS velocity as discussed above.

The computer code has been used to determine the velocity-displacement relationship for a slug of sodium ejected from the region of the TREAT Mark II loop between the top of the fuel region of the test section and the bottom of the loop overflow part. The reference case assumes that the slug is driven from its initial position into the loop extension by a pressure of 410 atm. The sodium has a density of 0.83 g/cm³ and a viscosity of 0.224 cP. The volume of the slug is 266 cm³. The expansion volume into which the slug is driven is about 510 cm³; the flow area in the upper extension is 7.01 cm². Calculations have been run for various combinations of orifices in the loop extension to determine that combination which gives the greatest dissipation of slug kinetic energy consistent with mechanical and experimental requirements.* It appears that the optimum arrangement of orifices consists of 4 orifice plates, each having a flow area equal to one-half of the free flow area, located in the upper loop extension piece. One orifice plate is located 5 $\frac{5}{16}$ in. (18.57 cm) from the top of the loop, with the others at 3 in. (7.62 cm) intervals below the first mentioned. Each of these orifices has a loss coefficient, defined by

* For example, the orifices must be placed so that there is no impediment to flow before the slug has passed the upper loop flowmeter.

$$K_L = \frac{\Delta P_{unrecoverable}}{\rho v_{ipe}^2 / 2g_c}, \quad (22)$$

equal to 4.07 as calculated from the data given by J. K. Vennard.³

The significant results of the hydrodynamic acceleration calculations for the reference case are as follows:

1. slug velocity at impact with first orifice plate = 22,200 cm/sec.
2. slug velocity at impact with second orifice plate = 17,700 cm/sec.
3. slug velocity at impact with third orifice plate = 12,500 cm/sec.
4. slug velocity at impact with fourth orifice plate = 9200 cm/sec.
5. slug velocity at impact with end of duct = 7800 cm/sec.

The pressure pulse resulting from the slug impact with the top of the loop at 7800 cm/sec is about 1840 atm, or 22% of the value without orifice plates. Since the wall strain energy is proportional to the square of the pressure, the value of the strain energy per unit length of wall is only 8.6 J/cm in the present case—about 5% of the value without the orifice plates and 0.5% of that necessary to rupture the loop wall.

There will also be a pressure pulse resulting from the impact of the sodium slug on the orifice plates. The magnitude of this pulse can be estimated from the results of R. W. Wright.⁴ The maximum pulse, at impact with the lowest plate, is estimated to be 1400 atm,

slightly less than calculated for the impact with top of the loop.

CONCLUSIONS

Analysis of the impact of a sodium slug, ejected from the test section of a Mark II TREAT loop by sodium vapor at its critical pressure, on the top closure flange of the loop indicates that the loop can contain this event. If slug ejection is not impeded in any way, the calculated pressure pulse resulting from impact is about 8345 atm, but the energy available to do work on the loop wall is not sufficient to cause rupture. However, certain assumptions made in calculation of the pressure pulse amplitude are violated. To provide further assurance that the loop will not be ruptured by a sodium slug impact, orifice plates are placed in the loop extension to impede flow of a slug and dissipate its kinetic energy. The location and sizing of the orifice plates was done with the aid of a detailed calculation of the dynamics of the ejection process. With the orifice plates, the pressure pulse resulting from impact is about 1840 atm, and the energy available to do work on the loop wall is only 0.5% of that necessary to rupture the loop.

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III-4. Results of Transient Reactor Test (TREAT) Facility Tests on Motion Detector Instrumentation

A. B. ROTHMAN, C. J. RENKEN,* R. R. STEWART, N. R. TUCKER and C. E. DICKERMAN

INTRODUCTION

Tests were performed in Fiscal Year 1969 to demonstrate the feasibility of an in-pile measurement of axial fuel expansion under transient conditions.² The tests were performed in the TREAT facility on two unirradiated Mark-IA pins which were fabricated from "high-swelling" fuel. A discussion of the results of these tests is presented.

EXPERIMENTAL

Measurements were performed on fuel pins contained in flowing helium in the transparent photographic test

facility used for previous TREAT studies.^{3,4} The facility was modified² to accommodate sample pre-heating and instrumentation. Electromagnetic transducers were positioned, with the sodium bond molten, at the top and bottom of the fuel in order to measure fuel motion in the Mark-IA element during the course of the transient.

The end of the fuel moved through a pulsed induction field generated and detected by transducers that consist of two pickup coils interspersed with three field coils all located coaxially. The coils were fabricated from i-cate-insulated 40-gauge copper wire, with Mu metal magnetic shields, held on zirconium forms. A 2 mil

* Metallurgy Division, Argonne National Laboratory.

less steel heat shield was provided also to protect the coils from radiation heating by the sample.

MOTION DETECTION

The field coils supplied a 1.5 μsec pulse at a pulse repetition rate of 500 Hz. The pulse induced in each pickup was sampled at three different times during its duration, and its amplitude at these sampling points was used to produce 3 voltages at the output of the test system. The two supplementary field coils were wound in series with, but opposite in direction to, the primary coil. In the absence of discontinuities in the test sample, this effected a null for the magnetic flux in the vicinity of each pickup coil. Consequently the induced voltage measured by a given pickup coil depends primarily upon the characteristic interface signal spike caused by the resistivity discontinuity at the moving fuel-sodium interface; and to a lesser extent upon the sodium and fuel resistivities. Due to a difference in the "skin effect" (i.e., effective depth of penetration of the electromagnetic field) between fuel and sodium, time sampling of each pulse in the train of attenuated pulses yields information at various depths in the conductors within the transducer field.

The transducers were positioned with the sodium molten and the fuel at a nominal 200°C prior to the test series. The lower pickup coil of the upper transducer was positioned at the top edge of the fuel. Since the lower pickup coil surrounds the fuel for the majority of the transient, this allows determination of corrections from variations both in fuel resistivity as determined with the lower pickup coil and in known temperature variation in sodium resistivity. The transducer was designed in order to minimize this temperature correction.

The two test transducers were intercalibrated with one another and with a third transducer prior to capsule assembly. The third transducer was used to calibrate the test instrumentation at TREAT before the test series. A half brass, half stainless steel rod, which reproduced approximately the resistivity properties of the sodium and fuel respectively, was moved through the coil in each calibration. In this manner the strip chart recording at each of the three sampling times in the train of attenuated pulses was related to the distance of fuel travel. The final check of the extent of fuel travel was the appearance of the characteristic interface signal spike from the fuel-sodium interface at the top pickup coil of the upper transducer.

TRANSIENTS

EAT test conditions for the clipped transients in this particular test series are given in Table III-4-I. Extreme bowing and separation of the fuel-spacer wire

TABLE III-4-I. CONDITIONS FOR TREAT TESTS ON UNIRRADIATED MARK-1A ELEMENTS

Element	Max. Measured Cladding Temp., °C	Sample Energy Input, J/cm ³	Integrated TREAT Power, MW-sec	Result
SL-33-3	490	1574	22.8	no failure
	631	2430	35.2	no failure
	620	2940	42.6	bent spacer wire
SL-33-6	668	2947	42.7	no failure
	714	2975	43.1	no failure
	918	3838	55.6	no failure

were observed at the termination of the third transient on the first pin, SL-33-3, due to mechanical constraint of the pin by a Lavite insulator at the top of the support frame. This constraint, which caused erratic motion detection measurements for the first fuel element, was corrected in the second test capsule.

TEMPERATURE DETERMINATIONS

Calculations of the transient temperature profile in the fuel element were made with RE-248 ARGUS using methods described in Ref. 2. ARGUS computes the transient temperatures resulting from a transient power input per unit volume of fuel, $P(r, z, t)$, at radius r , axial position z , and time t . The transient power input is given by

$$P(r, z, t) = \theta_0 \mu(r) \eta(z) n(t), \quad (1)$$

where μ , η , and n are arbitrary numerical functions given pointwise for r , z , and t , respectively.

Since no axial power variation could be detected within the precision limits of the radiochemical power calibration, $\eta(z) \equiv 1$. The standardization coefficient θ_0 together with $\mu(r)$ were determined in this calibration from radiochemical analysis of fissions in axial fuel alloy sections and surface foils, assuming a radial power distribution of the form

$$\mu(r) = I_0 r(Kr). \quad (2)$$

From Eqs. (1) and (2), we obtain

$$\frac{\langle P_f \rangle}{P_s} = \frac{2}{Kr_0} \frac{I_1(Kr_0)}{I_0(Kr_0)}, \quad (3)$$

where $\langle P_f \rangle$, P_s , and r_0 are respectively the volume averaged specific fission power in the fuel, the specific fission power at the surface, and the fuel radius for the Mark-1A pin. The attenuation coefficient $7.9 \pm 1.7 \text{ cm}^{-1}$ (uncorrected for burnup) computed from Eq. (3) was used with Eq. (4),

$$\langle P_f \rangle = \frac{2}{Kr_0} \theta_0 I_1(Kr_0), \quad (4)$$

TABLE III-4-II. IN-PILE THERMAL EXPANSION OF UNIRRADIATED FUEL IN A MARK-1A ELEMENT

Time, sec	Transient 1225		Transient 1226		Transient 1227	
	Volume Averaged Fuel Temp., °C	Linear Fuel Thermal Expansion, in./in. × 10 ⁴	Volume Averaged Fuel Temp., °C	Linear Fuel Thermal Expansion, in./in. × 10 ⁴	Volume Averaged Fuel Temp., °C	Linear Fuel Thermal Expansion, in./in. × 10 ⁴
3.50	658	37.0 (121.2)	474	37.0 (45.2)	489	37.0 (48.3)
3.65			586	74.1 (77.0)	627	88.9 (100.3)
3.70			780	118.5 (137.9)		
3.85	797	74.1 (140.6)		792	118.5 (139.9)	
3.90	813	88.9 (142.8)		1014	137.0	
4.20						

to compute a value of 53.96 J/cc-MW-sec, at zero burnup, for the standardization coefficient θ_0 .

Values of the effective heat-transfer coefficient for the surface of the outer cladding were obtained by measuring the rates of temperature drop recorded by the sample cladding thermocouples. The initial axial temperature profile, determined from out-of-pile bench tests, was taken into consideration in computing volume-averaged fuel temperatures during the course of the transients.

RESULTS

The measurement of thermal fuel expansion with an unirradiated fuel element is more difficult than with an irradiated element. In the latter case there is a well defined fuel-sodium interface. In the unirradiated element, however, the fuel is surrounded by a sodium annulus which changes size as the fuel expands. In addition, unirradiated fuel has a negative temperature coefficient of resistivity while that of sodium is positive. Consequently with unirradiated fuel the sensitivity of the fuel motion measurement decreases with temperature. The situation is improved for measurements on irradiated fuel elements. During irradiation, the fuel is converted predominantly to an alpha phase which possesses a positive temperature coefficient of resistivity that will help maintain a useful difference between the resistivities of the fuel and sodium. Nevertheless, measurements of fuel motion as a function of temperature were possible for the course of three transients on element SL-33-6, which had not been irradiated previous to the TREAT test series.

Table III-4-II compares the thermal expansion measurements in the course of each transient with the measurements (and extrapolated measurements above 700°C) of S. Zegler and M. Nevitt,⁵ given in parentheses. The sample was initially at a nominal temperature of 200°C in each transient. Clearly the measurements need not agree. During the TREAT transients the sample was heated and cooled rapidly. Conse-

quently the existence of metastable alloy phases was more possible, especially after TREAT shutdown, than for the slower cycling times used by Zegler and Nevitt.⁵ There is surprisingly excellent agreement, however, between the second and third transients and the expansion calculated from the data of Zegler and Nevitt.

DISCUSSION

The measured thermal expansion of the fuel is considerably lower than the values expected from the data of Zegler and Nevitt for the first transient. This may possibly be explained as the result of an initial alpha-gamma phase transformation on the first transient followed by gamma quenching. The expansion of the gamma stabilized alloy for the two subsequent transients would be greater than that for the expansion of the alpha phase in the first transient. On the other hand the long length of fuel, softened by heating, may have sagged under gravity on the first transient causing a low apparent expansion initially. Such behavior has been observed⁶ for full length fuel samples which were heated to temperatures in excess of 550°C. Imprints from the support pedestal were clearly identified at the bottom of such samples.

While these unirradiated elements were tested in TREAT as metallurgical control specimens for the EBR-II irradiated test series, metallographic examination of alloy phases in the fuel and fuel diameter measurements may also confirm or deny these possibilities. Examination of the unirradiated fuel elements is currently in progress.

It should also be pointed out that there is an inherent error in the initial positioning of the motion detectors. This would represent a larger relative error for smaller values of measured expansion, as in the first transient.

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III-5. Transient Experiments on 1.2 a/o and 2.5 a/o Burnup EBR-II Pins

A. B. ROTHMAN, C. J. RENKEN,* R. R. STEWART, G. G. DEWEY, N. R. TUCKER,
D. R. HUTCHINSON* and C. E. DICKERMAN

INTRODUCTION

Pre-irradiated Mark-IA fuel elements fabricated from "high-swelling" fuel were subjected to a series of TREAT power transients which simulated thermal conditions of over-power operations or loss of EBR-II coolant flow. These experiments were a modification of the program described in Ref. 1. Key points of interest were (1) clad integrity compared with previous tests on pre-irradiated lower swelling fuel,² and (2) thermal expansion characteristics of the highly swollen fuel within the cladding.

EXPERIMENTAL

Two fuel elements, with nominal burnups of 1.2 and 2.5 a/o were tested in TREAT. The test assembly, which consisted of a fuel pin contained in flowing helium in a double-walled capsule suitable for high-speed color photography, was the dry transparent meltdown facility used for previous studies.^{3,4} Instrumentation and pre-transient heating of the test pin were identical to those techniques reported for proof tests of the experimental method in TREAT on unirradiated fuel.⁴ Electromagnetic transducers to study fuel motion were mounted on the sample load frame coaxially to and at the top and bottom of the fuel. Two fast-response chromel-alumel thermocouples were welded to the cladding near the fuel midplane. A beaded thermocouple near the bottom of the fuel pin provided pre-transient temperature control by interruption of power in the heater circuit. When initiating reactor power the heater power was terminated by a latching relay coupled to the ac source.

RESULTS

The test conditions and temperatures for clad failure are given in Table III-5-I. Pre-test inspection of

the sodium bond showed that both irradiated pins had sodium trapped between the fuel and cladding over a considerable length of the fuel pin, caused by expansion of the fuel to the inner diameter of the cladding near the fuel midplane. The high temperatures required for cladding failure and preliminary post-test examination of these pins indicate that clad failure by expansion of trapped sodium is not a serious consideration. Furthermore, the temperatures for clad failure are far in excess of temperatures expected from over-power operations or loss of EBR-II coolant flow.

The pin with 1.2 a/o burnup, element E34, failed at the fuel midplane in a manner similar to previous tests on pins containing pre-irradiated fuel with lower swelling properties. The pin with 2.5 a/o burnup, BF05, in which the fuel had swollen to the restrainer cap, failed mildly on the fourth transient at a somewhat lower temperature. Failure, which was at the top of the pin—evidently from expansion of the pin against the top restrainer—appeared to occur

TABLE III-5-I. TEST CONDITIONS FOR TREAT STUDIES ON "HIGH-SWELLING" EBR-II FUEL

Element	Burnup, a/o	Integrated TREAT Power, MW-sec	Sample Energy Input, J/cm ²	Maximum Clad Temp., °C	Result
E34	1.2	23.1	1390	578	no failure
E34	1.2	34.9	2100	682	no failure
E34	1.2	43.75	2630	801	no failure
E34	1.2	51.6	3100	914	no failure
E34	1.2	58.6	3520	978	failure
BF05	2.5	23.1	1350	531	no failure
BF05	2.5	37.0	2160	677	no failure
BF05	2.5	46.2	2700	766	no failure
BF05	2.5	51.7	3020	866	mild failure
BF05	2.5	58.6	3430	925	second failure

* Metallurgy Division, Argonne National Laboratory.

after the pin had reached peak power. A fifth transient on this pin caused a more severe failure. In both cases molten fuel was expelled from the pin.

The test capsules for elements BF05 and E34 have been disassembled, and post-test examination of the pins and motion detector data are in progress. Preliminary examination of the data from element BF05 indicates fuel thermal expansion data lower by an order of magnitude than data from previous tests on unirradiated fuel elements.⁴ Fuel motion occurred discontinuously, with several arrests in axial expansion, for the course of the transient. Since there was no initial separation gap between the fuel and top restrainer, this behavior appears to be the result of restriction of fuel expansion by the top restrainer.

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4. A. B. Rothman, C. J. Renken, R. R. Stewart and C. E. Dickerman, *In-Pile Measurements of Thermal Expansion of Unirradiated EBR-II Driver Fuel*, Trans. Am. Nucl. Soc. **12**, 349 (1969).

III-6. Interim Examination at 3 a/o Burnup of the First Unencapsulated Mixed Oxide Pins from Experimental Breeder Reactor II

C. E. DICKERMAN

INTRODUCTION

Subassembly X040 was the first EBR-II subassembly to contain unencapsulated experimental mixed oxide fuel pins. The subassembly consisted of 19 ANL pins along with 16 pins from another USAEC contractor (General Electric Co.). The 19 pins were specified for exposure to 5 a/o burnup, to provide a stock of domestic mixed oxide pins irradiated in an unencapsulated environment for use in fast reactor safety transient meltdown experiments in the TREAT reactor. Subassembly X040 was removed from EBR-II at the 3 a/o burnup level for an interim examination in order for the EBR-II project staff to ascertain the following: the condition of the fuel in this first unencapsulated oxide pin subassembly; to verify that the condition of the pins is in accord with expectations based on earlier encapsulated EBR-II oxide pin irradiations; and to confirm that the irradiation could proceed from the standpoint of EBR-II safety and availability. The interim examination showed no conditions that would bar continued irradiation of the subassembly to 5 a/o burnup. One of the 19 pins (pin 012) appeared to have a minor surface defect and it was set aside for detailed destructive examination as a "lead" pin from subassembly X040. The reconstituted subassembly, with pin 012 replaced by a dummy pin, was reloaded into EBR-II as subassembly X040-A.

TABLE III-6-I. PIN DESIGN SPECIFICATIONS

Fuel	
Composition	(U _{0.8} Pu _{0.2})O ₂
Oxygen-to-metal ratio	1.97-2.00
Uranium enrichment	93% U-235
Plutonium enrichment	91% (Pu-239 + Pu-241)
Form	Pellets
Pellet density	82-86% of theoretical
Pellet diameter, in.	0.245-0.247
Length of fuel stack, in.	11
Cladding	
Material	304L
Outer diameter, in.	0.290
Wall, in.	0.020

IRRADIATION SAMPLES

All 19 pins were made to the same design, so that the results from the eventual TREAT testing would reflect to the maximum extent possible only the variations in test conditions. The pin design was based on a conservative application of data available when the design was fixed. In addition, the design was to be similar to those of other EBR-II irradiations then underway, and the design was to be consistent with TREAT experimental apparatus.

Each sample is made in three parts. The central part is the fueled section, which consists of a set of cladding tubing 17.250 in. long, containing fuel stack, one natural UO₂ insulator pellet and

FIG. III-6-1. Pre-Irradiation X-ray of ANL Pins 001, 002, 003, 004, 006, and 007. Bottom of Fuel Section, Showing Bottom Weld. ANL Neg. No. 113-2677.

FIG. III-6-2. Pre-Irradiation X-ray of ANL Pins 001, 002, 003, 004, 006, and 007. Top of Fuel Section, Showing Top Weld. ANL Neg. No. 113-2681.

FIG. III-6-3. Pre-Irradiation X-ray of ANL Pins 011, 012, 014, 016, 018, and 021. Bottom of Fuel Section, Showing Bottom Weld. ANL Neg. No. 113-2676.

FIG. III-6-4. Pre-Irradiation X-ray of ANL Pins 011, 012, 014, 016, 018, and 021. Top of Fuel Section, Showing Top Weld. ANL Neg. No. 113-2680.

FIG. III-6-5. Pre-Irradiation X-ray of ANL Pins 022, 023, 024, 025, 026, 029, and 031. Bottom of Fuel Section, Showing Bottom Weld. ANL Neg. No. 113-2678.

FIG. III-6-6. Pre-Irradiation X-ray of ANL Pins 022, 023, 024, 025, 026, 029, and 031. Top of Fuel Section, Showing Top Weld. ANL Neg. No. 113-2679.

TABLE III-6-II. PIN LOADINGS AND RELATIVE SMEARED-DENSITY ANL PINS IN X040

Pin No.	Mixed Oxide Length, in.	Mixed Oxide Weight, g	Mixed Oxide W/L, g/ft	Maximum Power, ^a kw/ft	Smeared Density, % theoretical	Oxygen-to-Metal Ratio
001	10.65	74.41	8.4	16.0	78.5	1.99
002	11.08	78.66	8.5	16.2	79.5	1.99
003	11.07	79.54	8.6	16.4	80.5	1.99
004	10.79	78.45	8.7	16.6	81.8	1.99
006	10.81	79.57	8.8	16.8	82.5	1.99 ^b
007	10.88	74.04	8.2	15.7	76.5	1.98
011	10.88	75.45	8.3	15.8	78.0	1.98
012	10.89	76.25	8.4	16.0	78.5	1.98
014	10.93	78.72	8.6	16.4	81.0	1.98
016	10.94	76.05	8.3	16.2	78.0	1.98
018	10.91	78.03	8.6	16.4	80.3	2.005
021	10.87	77.56	8.6	16.4	80.2	2.005
022	10.90	78.01	8.6	16.4	80.3	1.98 ^c
023	10.96	79.64	8.7	16.6	81.7	2.005
024	11.00	79.87	8.7	16.6	81.3	2.005
025	10.88	80.02	8.8	16.8	82.6	2.005
026	10.81	80.12	8.9	17.0	82.5	2.005
029	10.81	76.98	8.5	16.2	79.5	1.99
031	11.06	79.23	8.6	16.4	79.8	1.99 ^d

^a Power rating at midplane, calculated using a nominal value including gamma heating of 191 W/g of oxide.

^b Top pellet ratio is 2.005.

^c Pellets 1-6, 8, and bottom pellet ratios are 2.005.

^d Pellets 1-7 ratios are 2.005.

one T_a disc at each end of the stack, and top and bottom end plugs. Steel top and bottom blanket extension sections were made separately and welded to the top and bottom end plugs of the central piece. A spiral spacer wire was wound around each completed test pin at the EBR-II site prior to assembly into X040. Before assembly into a TREAT loop, the steel top and bottom blanket sections are cut off remotely.

The basic design specifications are shown in Table III-6-I. Oxide* composition is typical of that expected for large fast reactor mixed oxide cores. Pelleted fuel was selected to conform to the relative state of development of pelleted versus "powder" fuel, in light of the good irradiation performance of experimental pellet-mixed oxide pins in the Dounreay Fast Reactor and indications from DFR results that it possessed a modest degree in conservatism in operating temperatures and fission gas release over powder fuel. Sample enrichment was set at the practical maximum to ensure adequate power generation

*MET Spec. (U,Pu)O₂-166, January 27, 1966, as revised March 18, 1966.

in variable core regions in EBR-II. Cladding diameter was set by the dimensions of the available irradiation subassembly. The specified cladding wall thickness gives adequate margin—after making allowances for uncertainties in inspection and for conservative criteria of cladding attack during exposure—to meet specified criteria for experimental EBR-II service (see Ref. 1). Fuel oxygen-to-metal ratio was set over as narrow a range as practicable to ensure maximum degree of uniformity: a near-stoichiometric product was consistent with other fuel irradiations at DFR and EBR-II; and the maximum ratio was based on data indicating that fuel-swelling associated with sodium logging is worse for a ratio greater than 2.00 than for a ratio less than 2.00. The fuel pellet density range represents the minimum practical variation and the values correspond to smeared densities for which extensive favorable irradiation data were available. Type 304L stainless steel was specified on the basis of its previous successful service in EBR-II and the extensive experience at ANL in working and welding it. Pellet diameter range was specified to be as small as practical; the initial pellet-cladding gap was the same as that for which extensive favorable irradiation experience existed from DFR.

During the course of fabrication and inspection, the 19 pins were x-rayed for weld and general fuel con-

TABLE III-6-III. MAXIMUM CLADDING SURFACE TEMPERATURES

Pin No.	Maximum Outer Surface, °F	Maximum Inner Surface, °F
001	985	1064
002	956	1036
003	958	1032
004	966	1045
006	966	1048
007	978	1053
011	967	1045
012	947	1026
014	988	1068
016	985	1062
018	977	1057
021	969	1050
022	968	1047
023	972	1054
024	978	1061
025	978	1063
026	966	1048
029	972	1052
031	960	1045

FIG. III-6-7. View of Pin 025 at 3 a/o Burnup, Showing "Brown" Surface Film Spots. *ANL Neg. No. 103-L 5371.*

FIG. III-6-8. View of Pin 012 at 3 a/o Burnup, Showing Spacer Wire. *ANL Neg. No. 103-L 5341.*

FIG. III-6-9. View of Pin 001 at 3 a/o Burnup, Showing "Cleaned" Surfaces at Downstream Edges. *ANL Neg. No. 103-L 5346.*

FIG. III-6-10. View of Pin 012 at 3 a/o Burnup, Showing Possible Surface Anomaly. ANL Neg. No. 103-L 5366.

REACTOR CENTER

FIG. III-6-11. Loading Diagram for Subassembly X040. ANL Neg. No. 113-2705.

ons. Figures III-6-1 through III-6-6 are the x-rays taken at EBR-II of the fueled portions of the pins before assembly into X040. Figures 1, 3, and 5 are

TABLE III-6-IV. DIAMETER DATA FOR PIN 012

Distance from Spade End, ^a in.	Diameter, ^a in.	Distance from Spade End, ^b in.	Diameter, ^b in.
20	0.2911	16	0.2908
21	0.2911	18½	0.2909
23	0.2904	22¼	0.2904
23½	0.2907	24½	0.2908
25½	0.2911	25½	0.2912
26½	0.2914	27	0.2910
28½	0.2909	27½	0.2910
29½	0.2906	28	0.2909
30½	0.2904	30	0.2911
32	0.2912	31	0.2912
32½	0.2906	33	0.2909
34½ ^c	0.2898	34 ^c	0.2906
35½	0.2896	36	0.2906
37½	0.2898	37	0.2906
38¼	0.2896	38¾	0.2888

Pre-irradiation o.d.

maximum 0.2910 in.

minimum 0.2885 in.

average 0.2902 in.

^a 0 deg orientation with respect to the hole in the spade.

^b 90 deg orientation with respect to the hole in the spade.

^c Above fuel.

the bottom parts of the pins, and Figs. 2, 4, and 6 are the top parts. Note that each pair of photos overlap so that the fueled region of each pin is covered completely.

IRRADIATION

Subassembly X040 was irradiated in EBR-II position 5 B2 for runs 30 through 35, except for run 30D. The total number of fissions for the centerline of X040, at the midplane, was reported to be 112×10^{18} fissions/g of oxide. This corresponds to a flux integral of 2.4×10^{22} nvt total, of which approximately 2.0×10^{22} nvt represents neutrons of 0.8 MeV and above. The maximum fluence values (for pins 025 and 031, nearest the core center) are somewhat higher: 2.6×10^{22} and 2.2×10^{22} . Maximum fission power, for pins 025 and 031, is at the midplane and is 190 W/g of oxide. Fission power at the midplane ranges down to about 177 W/g for pin 001. Table III-6-II lists the pins, giving the oxide contents and the nominal maximum pin-power ratings, obtained using a nominal value of 191 W/g for each of the 19 pins.* Maximum power ratings can be

* Pins 025 and 031 have an estimated gamma heating rate in the oxide of 7 W/g additional for a total 197 W/g at the midplane. Gamma heating in the oxide of pin 001 is estimated at 6 W/g, for a total of 183 W/g. In view of experimental uncertainties, a single nominal power rating for all of the 19 pins was adopted for Table III-6-II.

FIG. III-6-12. Neutron Radiograph, Taken at 3 a/o Burnup with Dy Foils of ANL Pins 012 and 024 and GE Pins F3B-9 and F3B-15. ANL Neg. No. 113-2645.

TABLE III-6-V. DIAMETER DATA FOR PIN 024

Distance from Spade End, ^a in.	Diameter, ^a in.	Distance from Spade End, ^b in.	Diameter, ^b in.
20	0.2911	16	0.2911
21	0.2911	18½	0.2911
23	0.2908	22½	0.2905
24	0.2911	24½	0.2910
25½	0.2911	25½	0.2914
26½	0.2914	27	0.2912
28½	0.2916	28	0.2914
29½	0.2917	29¾	0.2912
29¾	0.2915	29⅞	0.2911
31½	0.2909	30	0.2911
31⅝	0.2908	30⅛	0.2913
31½	0.2908	30¼	0.2914
31⅝	0.2908	30⅜	0.2914
31¾	0.2909	30½	0.2913
31⅞	0.2909	30⅝	0.2913
32	0.2911	30¾	0.2912
32⅞	0.2909	30⅞	0.2910
32¼	0.2909	32	0.2911
32⅜	0.2908	31⅞	0.2912
32½	0.2908	32¾	0.2908
34½°	0.2905	33	0.2807
35½	0.2916	34°	0.2899
37	0.2906	36	0.2896
38½	0.2902	37	0.2897
		38¾	0.2889
Pre-irradiation o.d. maximum 0.2910 in. minimum 0.2890 in. average 0.2901 in.			

^a 0 deg orientation with respect to the hole in the spade.

^b 90 deg orientation with respect to the hole in the spade.

^c Above fuel.

converted to average by dividing by the maximum-to-average ratio of 1.12. These values of fluence and fission rates are based on Ref. 1, which indicates a possible uncertainty of 10 to 20%.

Table III-6-III gives the maximum cladding temperatures calculated by F. McGinnis of EBR-II, using the method described in Ref. 2.

3 a/o BURNUP INTERIM EXAMINATION

The non-destructive interim examination at 3 a/o burnup consisted of the following:

1. visual inspection and optical photography
2. neutron radiography
3. diameter measurements

TABLE III-6-VI. DIAMETER SPOT CHECKS FOR PINS 001, 002, 003, 004, 006, AND 007

Pin	Distance from Bottom Weld, ^a in.	Diameter, ^a in.	Pre-Irradiation o.d., in.	Distance from Bottom Weld, ^b in.	Diameter, ^b in.
001	15½°	0.2894	max. 0.2910	15°	0.2895
	10½	0.2859	min. 0.2895	11½	0.2905
	6	0.2913	avg. 0.2901	6½	0.2917
	2	0.2908		2½	0.2906
002	15½°	0.2902	max. 0.2910	14½°	0.2896
	10½	0.2910	min. 0.2881	11½	0.2904
	6½	0.2916	avg. 0.2897	6½	0.2916
	1½	0.2910		1	0.2915
003	15½°	0.2897	max. 0.2910	14½°	0.2900
	10	0.2906	min. 0.2880	11½	0.2905
	6½	0.2912	avg. 0.2899	6½	0.2913
	1½	0.2902		¾	0.2902
004	15½°	0.2899	max. 0.2907	14½°	0.2904
	10	0.2910	min. 0.2889	11½	0.2909
	6½	0.2915	avg. 0.2898	6½	0.2916
	1½	0.2912		¾	0.2909
006	15½°	0.2898	max. 0.2910	14½°	0.2895
	10	0.2908	min. 0.2882	11½	0.2904
	6½	0.2906	avg. 0.2899	6½	0.2913
	1½	0.2904		¾	0.2906
007	15½°	0.2898	max. 0.2910	14½°	0.2902
	10	0.2908	min. 0.2881	11½	0.2909
	6½	0.2913	avg. 0.2900	6½	0.2913
	1½	0.2908		¾	0.2900

^a 0 deg orientation with respect to the hole in the spade

^b 90 deg orientation with respect to the hole in the spade

^c Above fuel.

TABLE III-6-VII. DIAMETER SPOT CHECKS FOR PINS
011, 014, 016, 018, 021, AND 022

Pin	Distance from Bottom Weld, ^a in.	Diam- eter, ^a in.	Pre-Irradiation o.d., in.	Distance from Bottom Weld, ^b in.	Diameter, ^b in.
011	15½°	0.2899	max. 0.2910	14½°	0.2899
	10	0.2910	min. 0.2879	11½	0.2907
	6½	0.2916	avg. 0.2900	6½	0.2917
	1½	0.2909		¾	0.2903
014	15½°	0.2904	max. 0.2910	14½°	0.2906
	10	0.2909	min. 0.2885	11½	0.2906
	6½	0.2906	avg. 0.2900	6½	0.2906
	1½	0.2907		¾	0.2905
016	15½°	0.2908	max. 0.2910	14½°	0.2903
	10	0.2905	min. 0.2887	11½	0.2907
	6½	0.2903	avg. 0.2900	6½	0.2914
	1½	0.2908		¾	0.2905
018	15½°	0.2903	max. 0.2910	14½°	0.2909
	10	0.2910	min. 0.2891	11½	0.2909
	6½	0.2914	avg. 0.2900	6½	0.2916
	1½	0.2908		¾	0.2901
021	15½°	0.2905	max. 0.2910	14½°	0.2904
	10	0.2908	min. 0.2891	11½	0.2907
	6½	0.2910	avg. 0.2900	6½	0.2914
	1½	0.2905		¾	0.2901
022	15½°	0.2904	max. 0.2910	14½°	0.2905
	10½	0.2912	min. 0.2890	11½	0.2907
	6½	0.2917	avg. 0.2902	6½	0.2911
	1½	0.2905		¾	0.2895

^a 0 deg orientation with respect to the hole in the spade.^b 90 deg orientation with respect to the hole in the spade.^c Above fuel.

4. weighting

5. temperature measurements on four selected elements.

After removal of the subassembly from the reactor, it was cleaned of sodium and disassembled following standard Fuel Cycle Facility (FCF) procedures. Sodium remaining on the subassembly surfaces was removed by wet argon gas, followed by a demineralized water flush.

VISUAL INSPECTION

The pins were given a general visual inspection through a FCF air cell window, and then they were viewed and photographed close up using the FCF periscope. The condition of the 19 pins appeared to be good, with one possible exception (pin 012). Some pins were covered with what appeared to be a thin surface film of sodium oxide, but there were no

macroscopic deposits. Figure III-6-7 is a view of some small surface film spots on pin 025. The spiral spacer wires were still apparently "taut" and in good contact with the pin. The spacer wire-pin gap of pin 012 is shown better in Fig. III-6-8. Figure III-6-8 was taken at the location of the largest spacer-pin gap noted: the gap is less than 0.005 in.

The most striking visual feature was local surface regions with a "cleaned" appearance at downstream edges at both the top and bottom fittings of the central (fueled) pin section. One typical example of the "cleaned" regions is shown in Fig. III-6-9 (top fitting of pin 001). Close inspections of these regions failed to produce any visual evidence of actual steel wastage.

Pin 012 had an easily-visible surface scratch of undetermined origin, shown in Fig. III-6-10. For this reason, it was set aside as a "lead" pin for detailed destructive examination.

No bowing measurements were made. From the visual examinations, an upper limit on bowing can be set at about 0.5 in. over the 5 ft length of the entire pin.

TABLE III-6-VIII. DIAMETER SPOT CHECKS FOR PINS
023, 025, 026, 029, AND 031

Pin	Distance from Bottom Weld, ^a in.	Diam- eter, ^a in.	Pre-Irradiation o.d., in.	Distance from Bottom Weld, ^b in.	Diameter, ^b in.
023	15½°	0.2898	max. 0.2910	14½°	0.2906
	10½	0.2907	min. 0.2888	11½	0.2906
	6½	0.2913	avg. 0.2904	6½	0.2915
	1½	0.2910		¾	0.2902
025	15½°	0.2906	max. 0.2910	14½°	0.2899
	10½	0.2908	min. 0.2880	11½	0.2906
	6½	0.2915	avg. 0.2904	6½	0.2914
	1½	0.2908		¾	0.2900
026	15½°	0.2899	max. 0.2910	14½°	0.2906
	10½	0.2905	min. 0.2878	11½	0.2907
	6½	0.2918	avg. 0.2898	6½	0.2904
	1½	0.2911		¾	0.2904
029	15½°	0.2899	max. 0.2910	14½°	0.2903
	10½	0.2908	min. 0.2880	11½	0.2904
	6½	0.2918	avg. 0.2896	6½	0.2906
	1½	0.2906		¾	0.2888
031	15½°	0.2907	max. 0.2910	14½°	0.2902
	10½	0.2907	min. 0.2890	11½	0.2904
	6½	0.2910	avg. 0.2900	6½	0.2903
	1½	0.2906		¾	0.2900

^a 0 deg orientation with respect to the hole in the spade.^b 90 deg orientation with respect to the hole in the spade.^c Above fuel.

LOADING DIAGRAM

The loading diagram for the 19 pins is shown in Fig. III-6-11.

NEUTRON RADIOGRAPHY

Initial neutron radiography was performed on two pins from each of the two groups of pins in X040. Figure III-6-12 is a copy of a neutron radiograph obtained with a dysprosium foil of the fuel section of pins 012 and 024, along with two of the General Electric Co. group of 16 pins for comparison. Additional radiography was performed on the remaining pins. Central voids appeared to be present on the original. The pellet structure visible in Figs. III-6-1 through III-6-6 is no longer prominent.

Measurements were taken of the pin diameter at 0 and 90° with respect to the plane defined by the hole in the spade fitting at the bottom of the lower steel blanket sections. Pins 012 and 024 were measured at approximately 2 and 1 in. intervals respectively. Table III-6-IV gives results for pin 012, and Table III-6-V gives data for pin 024. Also shown in the

TABLE III-6-IX. PRE-IRRADIATION AND POST-IRRADIATION (3 a/o BURNUP INTERIM EXAMINATION) WEIGHTS OF 19 FAST REACTOR SAFETY ANL PINS OF ASSEMBLY X040

Pin No.	Pre-Irradiation, ^a g	Post-Irradiation ^b (3 a/o burnup), g	Difference, g
001	495	513	18
002	499	516.4	17.4
003	500	517.5	17.5
004	499.5	516.3	16.8
006	500	517.3	17.3
007	497	514	17
011	497	513.2	16.2
012	498	514.5	16.5
014	501	517.5	16.5
016	497	513.6	16.6
018	499	515.5	16.5
021	499	516.2	17.2
022	499	515.5	16.5
023	500	517.3	17.3
024	500	517.4	17.4
025	501	518.2	17.2
026	499.5	517.1	17.6
029	496.5	515.0	18.5
031	500	517.2	17.2

^a Pins after welding top and bottom sections to fuel section, but before spiral spacer wires were attached.

^b Pins as assembled, with spiral spacer wires attached.

TABLE III-6-X. TEMPERATURE MEASUREMENTS ON SELECTED ELEMENTS FROM ASSEMBLY X040

A. General Electric Elements			
Element No.	Top of Fuel Region (16 in. from lower weld), °C	Middle of Fuel Region (6 in. from lower weld), °C	Lower End Fitting, °C
F3B-9	40	43	42
F3B-15	51	58	41
B. ANL Elements			
Element No.	Gas Plenum Region (2 in. below top weld), °C	Middle of Fuel Region (6 in. above bottom weld), °C	Lower End Fitting, °C
012	38	62	41
024	39	68	40

tables are the maximum, minimum, and average pre-irradiation outer diameters of the cladding. Spot checks on cladding diameters for the remaining 17 pins are given, along with summarized pre-irradiation data, in Tables III-6-VI through III-6-VIII. Data points for the two orientations, 0 and 90 deg, were taken at the same axial positions when the spacer wire did not interfere.

WEIGHTS

Pre- and post-irradiation weights are given in Table III-6-IX. It should be noted that the pre-irradiation measurements were taken before installation of the spiral spacer wires, while the post-irradiation weights include the wires as welded into place. No anomalies are apparent in the comparison of weights.

TEMPERATURE

After 17 days of cooling temperatures of two pins from each batch were measured by pressing a thermocouple against the cladding. These data are shown in Table III-6-X.

REFERENCES

1. *Guide for Experimental Irradiations in EBR-II, Revision 3, July 1969 with revised pages through September 1969.*
2. R. A. Cushman and F. D. McGinnis, *EBR-II Reactor Heat Transfer: Experience with an Operating LMFBR*, Tenth National Heat Transfer Conference, August 11-14, 1968, Philadelphia, Pa.

III-7. Analysis of Digital Data from Fast Neutron Hodoscope

A. DeVOLPI and J. REGIS

INTRODUCTION

Photographic observation of fuel pin meltdown in transparent capsules has been a successful method of achieving detailed information on the dynamic features of transients at the Transient Reactor Test (TREAT) facility. From pins in an inert gas environment, much data of use to the nuclear safety program have emerged. Present emphasis, however, on sodium loops has eliminated that source of information. Instead, a fast neutron detection system, called a hodoscope, has been developed to provide penetrating data from optically opaque capsules. The hodoscope has recently been upgraded to its full detection capacity and is operational in the sense of being able to acquire data from transients with pins and clusters in sodium loop experiments.

Previous tests have established that not only can the fast neutron hodoscope perceive fuel movement within the capsule, but it is also sensitive to sodium boiling effects. Definitive understanding of the actual quality of data obtainable under very severe shadowing and attenuation conditions is yet to be established, although information from the experiments that have been done is favorable.

Present efforts have been directed towards analysis of the digital data accumulated on 16-mm film.

HODOSCOPE DESIGN

The fast neutron hodoscope consists of two major sections of hardware: a collimator and the combined detection-electronic processing stages. The collimator is made of a stack of steel plates with 334 machined slots. Figure III-7-1 shows its arrangement at the TREAT reactor to view a fuel element located at the center of the core.

At the end of each collimator slot is a fast neutron detector; these detectors provide overlapping coverage of an area 2×20 in. at the fuel plane. Design spatial resolution is 0.15 in. horizontally at a signal/background ratio of seven for fuel pins (6% enrichment) surrounded by a thin capsule. If the fuel pin (6% enrichment) is surrounded by sodium and a steel capsule, then placed in a zircaloy well inside a graphite TREAT dummy element, the resolution deteriorates to 0.25 in. and the signal/background ratio to two (primarily due to the graphite). An example of this situation is provided in Fig. III-7-2.

The electronic features of the system are briefly

as follows: Signals from the fast neutron detectors are carried to a printed circuit card which receives, integrates, and amplifies the linear signal. Another portion of the card discriminates against gamma ray background and then standardizes the amplitude of the accepted pulse. This digital signal is then relayed to the digital processing part of the system.

Digital pulses from each of the 334 detectors are temporarily stored in a like number of integrated circuit binary scalars. On command the contents of these scalars are read out to a neon lamp panel which displays the coded data. In synchronism with the display flash, a high-speed framing camera records the data. The scalars are then reset to zero and the storage sequence re-initiated. Through this procedure of temporary storage and digital photographic recording, it is possible to record the output of all 334 channels with a time resolution of 1.2 msec and a recording duration of 20 sec at that resolving time.

PAST PERFORMANCE

Up to the beginning of this fiscal year, a limited version (50 out of 334 channels) of the hodoscope was applied, collecting data with a temporary analog photographic system. The results from three of these fully analyzed transients are portrayed in Figs. III-7-3 through III-7-5. Figure III-7-3 is the hodoscope record of the destruction of a sodium-bonded

FIG. III-7-1. Fast Neutron Hodoscope Collimator in TREAT. ANL Neg. No. 113-1682.

FIG. III-7-2. Fast Neutron Detector Placement. *ANL Neg. No. 112-4780.* FIG. III-7-3. Transient 917, Meltdown Sequence. *ANL Neg. No. 112-6733.*

FIG. III-7-4. Transient 1007, Meltdown Sequence. ANL Neg. No. 112-6769.

FIG. III-7-5. Transient 1048, Meltdown Sequence. ANL Neg. No. 113-2583.

FIG. III-7-6. Transient 1255, Pretransient and Post-Transient Scan. ANL Neg. No. 113-1898.

pin in inert atmosphere enclosed within a transparent capsule (Transient 917). The detailed pin bending and slumping behavior shown in Fig. III-7-3 was in excellent agreement with the optical photographic record. Note particularly the frame at 4.576 sec, which shows the top portion of the pin falling after failure. Figure III-7-4 is the history of melt-

down of a single pin in a flowing sodium loop, the first evidence of sodium void effects (Transient 1007). The first indication of fuel movement occurs at 3.641 sec, when lowering of count rates suggests fuel displacement in the uppermost sections of the fuel. First indication of failure is at 3.682 sec, in good agreement with timing of a 25 atm pressure spike. Subsequent frames show sodium voiding and fuel dispersal. Collapse of the void (see frames at 3.899 and 3.924 sec) and the subsequent rapid dispersal of fuel indicate collapse of a liquid sodium column on fuel that had been insulated from coolant and a subsequent massive disintegration of fuel. These are in agreement with the timing of a second "impact" pressure spike of ~ 25 atm. Similar results were obtained from Transient 1048 which involved a 7-pin cluster (Fig. III-7-5).

Just before the beginning of the fiscal year 1969, the system was tested with a "flattop" transient of an argon-bonded EBR-II pin in flowing sodium. Although the fuel pin did not disintegrate, it did slump, and the comparison of pre-transient and post-transient scans gave a reliable indication of this action. After this test, the system was returned to Argonne for extension to its full quota of electronic circuits incorporated within an rf-shielded cage.

The first transient accomplished with the hodoscope upon its becoming operational again at TREAT was number 1255. A bare 6% enriched sodium-bonded EBR-II pin inserted in a transparent capsule was selected for this first test of the full hodoscope system with digital photographic recording.

Results are summarized in the next few figures. Comparison of the pin profile before and after the transient indicates severe distortion of the pin (Fig. III-7-6). This is also supported by direct photographic observation through the window of the capsule.

The analysis of the digital photographic data is now reaching the stage where a coherent system for handling results is discernible. Figure III-7-7 indicates the range of brightness that can be presented by the method of rendering intensity data obtained from computer analysis of the coded hodoscope output information. Figures III-7-8 and III-7-9 are two cycles selected from the transient, demonstrating the initial condition of the fuel pin, and the final distorted phase of the pin. There is a mirror reversal of images in these renditions.

In Figs. III-7-8 and III-7-9 the "power level" is a relative scale measured in counts/sec for certain channels at the edges of the viewing area. The occasional bright spots are detector channels which will require either re-calibration or electronic adjustment. Some programming effort is going into filtering anomalous data.

FIG. III-7-7. Transient 1255, Test Pattern. *No ANL Neg. No.*

While it appears at first that there are four parallel displays of the identical data, there are different roles intended for each quadrant. The left-hand image is the direct un-normalized count rate; the next column is for a power-normalized display; the third for count rates less background; and the right-hand pattern for power-normalized background-corrected data. All of these features have not yet been optimized; Fig. III-7-8 is included to show the current

s 3.

Data have also been collected for several other transients, but will not be subjected to analysis until enough of the limitations of the present decoding

program have been lifted. Records were made of the transparent photographic tests of pre-irradiated EBR-II driver fuel elements. Judging from the pre-transient scanning, it is unlikely that adequate information can be acquired from the fast neutron hodoscope, since there was a considerable amount of graphite interceding between the fuel pin and the hodoscope. As a result, signal/background ratio dropped to 0.85 (from 7) and resolution to 0.25 in. (from 0.15 in. design resolution).

Transient 1270 has been a special autoclave-type test with a dummy fuel element in position with the peak power level reaching 2338 MW. Such higher

FIG. III-7-8. Initial Condition of the Fuel Pin. No ANL Neg. No.

levels are typical of those expected for sodium loop tests.

Other tests conducted were Transients 1273-75, 1278, and 1279-81. Transient 1278 was the first done in cooperation with the EBR-II Project testing pin failure in EBR-II. A "flattop" excursion was run with a half-bonded pin in the sodium loop. The series 1279-81 involves an oxide sodium loop.

A power surge at NRTS damaged some of the mechanism of the camera recently, thus causing difficulties in preparing the hodoscope for the latest test with ceramic fuel. The experiment took place

with temporary repairs to the camera (Transient 1083).

DECODING AND ANALYSIS

Steps leading up to the visual translation of the digital data begin with automatic scanning of the photographic film. The data are stored on the film in a 24×30 array of dots (Fig. III-7-10). Each frame of 16-mm film contains the contents of 56 scalers recorded through 12 binary bits for a capacity of 4096 counts. Also recorded on each frame are fiducial marks for scanning purposes, as well as

FIG. III-7-9. Final Distorted Condition of the Fuel Pin. No ANL Neg. No.

coded information revealing which of six phases in a cycle is associated with a given frame. All 334 detectors can be read out in one cycle of six phases. In addition, during the first phase of each cycle, the time is recorded from a clock oscillator.

The roll of film which results from a transient is first edited to locate calibration frames and the main part of the transient. These selected portions are then brought to CHLOE, a flying-spot scanner which produces a magnetic tape containing image density information. This magnetic tape is then transferred to the CDC-3600 computer, where it is decoded,

ultimately resulting in the visual renditions. There are a number of steps involved in this CDC-3600 program labeled DIGLITES.

After reading in a section of the CHLOE-generated tape, the program decodes a block of data corresponding to a single frame. Input information is available in terms of relatively dense images located at certain positions measured in CHLOE coordinates. The first problem is to fit these data to the 720-lamp pattern. The lower-most and upper-most coordinates are found to act as initial fiducials, with further testing on other fiducials for consistency. All image positions

Fig. III-7-10. Hodoscope Detector Array. ANL Neg. No. 113-2609.

are then normalized on the basis of the new reference position.

At this point the subroutine DIGEST is called to establish temporary thresholds of minimum intensity, to determine interlamp distances to set up a 24×30 grid, and to develop procedures for exclusion of spurious images. Generally, only images which fall within a certain band surrounding a grid intersection are accepted as bona-fide. In addition, these images must have a minimum areal density. If these conditions are met, it is still necessary to see if some of the images have merged together; if so, this must be corrected. Once these steps are accomplished, it is possible to quantize the images (0 or 1) at each allotted grid intersection. Finally, an image transformation must be conducted in order to position the coded information into its real-life counterpart.

If the film is not of adequate contrast, it may be necessary to engage in other efforts to adjudge acceptable images. Many of the problems in decoding arise from imperfections in the film and in the scanning process.

The main program now proceeds to a second subroutine, SCALER, which decodes the quantized information. First, the phase number is determined; next, the digital content of each scaler is computed from the binary record; and finally the time of the phase is calculated from the clock information recorded in the first phase. Phase identification is essential to proper allocation of scaler information to its detector correspondence.

If the sixth frame in a cycle has not been completed, the program control returns to the earlier stage of reading in and processing another frame. When the data from all six phases of a cycle have been decoded, then certain calculations are made on the data. First of all, the accumulation time is determined from the time difference information. With this, the count rate of each channel is evaluated, and then deadtime corrections are applied.

Provision at this point is made for selective analysis of the data. For initial examination of the transient, it may be desirable to skip cycles and go through the remaining processing for 1 out of every 5 or 10 frames. Another alternative is to accumulate the data from several cycles and to average them, a feature particularly useful during low power level phases.

Preparations are now made to plot the data. (The existing information may be printed out at this point.) To plot according to the format of Fig. III-7-7, which allows 342 density levels, the count rates are normalized to a maximum of 342. The program then reverts to subroutine SCATPLOT which plots (on the DD80 CRT) a basic grid with labels and then fills in the plot-normalized count rates. The DD80 provides a 35-mm microfilm output which has been used to generate Figs. III-7-7 through III-7-9.

Various other options are being incorporated into the main program DIGLITES. Already mentioned were other normalization procedures and corrections for subtraction of background. About 10 of the channels least likely to be involved in fuel movement—essentially those near the edges—are defined as power level detectors. The count rates from these channels are averaged together to form the “power level”. Testing is conducted to ensure that occasional spurious data (from neutrons, from electronic noise, and from decoding discrepancies) do not distort the power level indication. There are also testing procedures to ensure that the time plotted is not askew. Finally, if certain channels suddenly present a rate anomalously inconsistent with the power pattern (and not related to fuel movement), a filtering process is introduced to replace that rate with an extrapolated value from the previous cycle. When any of these corrective steps are taken, an asterisk is plotted on the display.

Future improvements are being considered, most of which must arise from actual experience in decoding and analysis.

FIG. III-8-3. Effect Upon Radiation Hodoscope Response Produced by Accumulation of Half of the Fuel Into a Single Column Without Loss of Coolant. ANL Neg. No. 113-1678.

FIG. III-8-4. Radiation Hodoscope Response to Blending of Half of Fuel Accompanied by Loss of Sodium Coolant. ANL Neg. No. 113-1676.

sodium-void) detection. In this manner most major movements of fuel or displacement of coolant would be observable. Examples of the response conceived are given in Figs. III-8-1 through III-8-4. Figure III-8-1 indicates the washed out appearance expected from the large quantity of sodium and fuel in a normal subassembly. If the entire fuel content is melted into a single mass column (without loss of coolant), then the profile of Fig. III-8-2 would be anticipated. If only the center half of the fuel accumulates into a single column, then the portrayal given in Fig. III-8-3

is expected. Should the sodium be voided under these conditions, one can calculate that the configuration shown in Fig. III-8-4 would result. In these estimates the sodium "background" condition was considered as currently experienced at TREAT; the radiation hodoscope for the Safety Test Facility can have an accentuated response for sodium that would improve definition of voiding effects.

IN-CORE INSTRUMENTATION

Instrumentation inserted directly into the subassembly would have two possible advantages over the external hodoscope: it could detect much smaller fuel displacements and it would probably be less expensive. On the other hand, it would suffer a severe disadvantage in that the instrumentation is likely to be destroyed just at the time that major events of interest are occurring.

Considering the role of in-core devices as complementary to the hodoscope, an examination was made of some state-of-the-art apparatus which could be employed under the severe radiation and thermal conditions to be encountered. The requirement of rapid transient response primarily to fast neutrons further limits the availability of certain types of detection systems.

Subject to confinement of the sensors to instrumented dummy pins scattered throughout the subassembly, two general classes of devices were examined: detection of externally generated signals which are passed through the tube and analyzed after passage or reflection, and self-generated signals with pulse information directly transmitted to outside receivers. A third category which was also evaluated, represented a combination of these two general classes and called for pre-processing of the self-generated signals to optimize data capacity.

Among the more promising candidates are periodically pulsed gas or liquid fluid systems, which are related to steady state gas stripping techniques already applied to reactors.

Two types of self-generating discrete detectors which may perform satisfactorily are (1) a bundle of self-powered sensors using conversion electrons, and (2) miniature fission fragment counters. In the latter case, a method of local integration accompanied by cyclic interrogation is important to restrain duty cycles to a tolerable level. In this concept, charge integration is allowed until a switch discharges the chamber capacitance into the data transmission line. A multiplexing arrangement is necessary to select one of many such detectors sequentially along the length of the instrumented pin.

III-9. Transient Reactor Test (TREAT) Facility Converter Engineering Reference Design

A. P. GAVIN

An engineering reference design for a modified TREAT reactor core which would include a neutron converter region as proposed by H. Iskenderian in various articles^{1,2} has been developed. The proposed modifications require replacement of twenty-eight of the regular TREAT fuel elements with elements of a new design. These elements will include the fuel requirements² specified by physics calculations and provide the necessary structural integrity and thermal characteristics to withstand the much higher fuel temperatures required by the converter design. The

basic converter element design is illustrated in Fig. III-9-1 and the proposed core arrangement is illustrated in Fig. III-9-2.

In accordance with the proposed arrangement the converter section of the core is divided into two regions. The "inner converter" or inner ring of elements contains oxide fuel which is subdivided radially into two regions of different U-235 enrichments. The "outer converter" elements contain graphite fuel similar to the original TREAT fuel but with higher U-235 content in three radial regions of different en-

FIG. III-9-1. Proposed TREAT Converter Element Design. ANL Neg. No. 113-2706.

Fig.-III-9-2. Center of TREAT Core Showing Proposed Arrangement of Converter Elements. ANL Neg. No. 113-2707.

richment. In order to operate at the proposed maximum fuel temperature of 1800°C without deterioration of the new elements themselves and to protect the regular TREAT elements which will surround the converter region, special insulation is required for all of the converter elements.

As shown in Fig. III-9-1 the insulation system for these elements consists of an argon filled gap with two molybdenum radiation shields. Initially, solid insulation material was considered but preliminary calculations indicated that none of the solid materials which would withstand the conditions contemplated would provide the required insulation in the limited thickness which could be used.

Molybdenum was chosen as the major structural material of the new elements because of its strength at high temperatures and because of its good thermal conductivity. The relatively thick outer can of molybdenum alloy is required to provide a thermal path for heat removal and to provide the structural strength required to withstand buckling due to the reduced pressure required within the element. The

poor oxidation resistance of molybdenum in the temperature range for which the elements are designed requires the addition of a thin stainless steel sheath over the molybdenum can. This sheath also provides the gas tight seal required.

Heat transfer calculations indicate that with the fuel element design illustrated in Fig. III-9-1 and with normal reactor coolant (air) flow the maximum converter element cladding temperature will be well below that which would cause damage to the zircaloy cladding of the adjacent regular TREAT fuel elements or to the central experiment. With such high fuel temperatures and the low heat capacity of the insulation it becomes impossible to assure that no damage to the regular TREAT fuel or to the central experiment will occur in the event of stoppage of the cooling air flow. (The present TREAT core has sufficient heat capacity and is limited to low enough fuel temperatures that it can survive a cooling air flow stoppage without damage to any of the components.) This will require that we provide emergency cooling of the reactor core when the con-

er is used. Since an interruption of even a few minutes' duration would not be expected to result in permanent damage, and since the total power requirement to circulate sufficient cooling air to cool the core to a safe temperature is not great, the proposed modification of the TREAT plant is not expected to be extensive.

REFERENCES

1. H. P. Iskenderian, *Conversion of the Transient Reactor Test (TREAT) Facility for Testing Fast Fuel Subassemblies*, Reactor Physics Division Annual Report, July 1, 1967 to June 30, 1968, ANL-7410, pp. 263-267.
2. H. P. Iskenderian, *Study on Conversion of TREAT Facility for Testing of Fast Reactor Subassemblies*, Trans. Am. Nucl. Soc. 11, 643 (November 1968).

III-10. Physics Calculations for a Fast Neutron Converter for the Transient Reactor Test (TREAT) Facility

H. P. ISKENDERIAN

INTRODUCTION

Two preliminary studies have been made on the physics aspects of replacing part of the core of the Transient Reactor Test (TREAT) Facility by a fast neutron converter section.^{1,2} The converter would be used to provide a neutron spectrum typical of a fast reactor inside meltdown test samples consisting of clusters of fast reactor fuel pins. Since the TREAT reactor serves only to provide nuclear heating of the samples, the index used to indicate whether or not the spectrum is sufficiently fast is merely the radial maximum-to-average sample power ratio, c . Preliminary calculations indicated that a converter section would give significantly better values of c and higher sample powers than a simple Boral filter.

Figure III-10-1 shows the basic converter geometry adopted for the first study.¹ Early calculations were made of the reactor containing the small converter region of Fig. III-10-1 and converter U-235 loadings to provide a value of c of approximately 1.01. The results indicated that sufficient power could be generated in the test section sample to ensure melting fully-enriched mixed oxide pins for a TREAT power burst generating a maximum TREAT core temperature of 400°C. That investigation was extended to determine the maximum sample energy available by relaxing the requirements of c to about 1.1 and increasing the maximum TREAT core temperature to 600°C. With some modification of the basic converter it was found that a total sample energy input of 2.5 times that required to melt the oxide could be obtained.² The modified converter occupied an annular region in the reactor core extending from 8 cm radius to 23 cm radius. It was provided with tungsten and boral linings to simulate refractory thermal insulation and reduce power peaking at interfaces. Substitution of the converter and a Mark II

TREAT sodium loop³ for regular TREAT fuel elements was noted to cause a reactivity loss of 1-2%.

In view of these encouraging results, the final physics design from Ref. 2 was used as the basis for a preliminary engineering design and engineering analysis by Gavin. (See Paper III-9.) This report summarizes the results of physics calculations made on the engineering design. A potential problem—excessive reactivity loss—is identified with the preliminary engineering design, and methods of recovering reactivity are described.

All of the physics calculations, except control rod evaluations and studies of flux peaking at interfaces (between axial reflector and converter or test section), were carried out using the SNARG 1D computer code⁴ in the S_4 approximation with the axial buckling option. The actual slotted TREAT core with the test section pin cluster, steel Mark II loop, and loop instrumentation, was approximated by a cylindrical equivalent. No attempt was made to readjust input (for example, by including a fictitious nuclear poison to account for the large slot) in order to reproduce measured values of k . Our reference core was used to evaluate the positive or negative reactivity changes associated with converter design changes.

The SNARG 2D code⁵ was used to determine flux peaking and to evaluate changes in control rod worth. The Hansen-Roach 16-group set⁶ was used for all calculations. The set was modified for these studies by correcting the thermal group for core temperature as needed.

During the course of these calculations, an experimental "reference" was established for TREAT loaded with a full viewing slot and containing a Mark II loop with a single 13% enriched test pin.* This core loading

* The loading of the first oxide meltdown experiment in the Mark II loop. (See Paper III-2.)

FIG. III-10-1. Basic Converter Design for Original Physics Calculations. *ANL Neg. No. 112-9102 Rev. 1.*

FIG. III-10-2. TREAT Loading for First Meltdown Experiment on an Oxide Pin in the Mark II Loop. *ANL Neg. No. 113-3082.*

is shown in Fig. III-10-2. Excess k for transient initiation in this loading is approximately 2.1%. It is estimated, on the basis of measurements of incremental worth of core elements at the edge of the core, that an

additional 5.5% k would be obtainable if all core positions were filled with fuel.

PRELIMINARY ENGINEERING DESIGN

CALCULATIONS

The preliminary engineering design of the converter by Gavin is described in Paper III-9. To facilitate handling, it is based on converter elements with the same 10 cm square cross section as the standard TREAT fuel elements. Two rows of converter elements were specified. For the case of a 10 × 20 cm cross section loop assembly (as in the case of the Mark II loop³) the cylindrical mockup uses an annular converter extending from a radius of 8 cm to a radius of 30.5 cm. This corresponds to the loop surrounded by a "ring" of 10 "inner converter" elements and an additional ring of 18 "outer converter" elements. Because of the requirements for thermal insulation between the converter fuel and the converter steel cladding, the converter fuel proper occupies a cross section of only 8.9 cm square inside a total thickness of 0.25 cm of molybdenum. Table III-10-I lists the converter U-235 loadings for the problems which cover the range studied for the preliminary engineering design. To reduce flux peaking outer (graphite based) converter is divided into three U-235 loadings, and the inner (oxide based) converter

TABLE III-10-I. CONCENTRATIONS OF U-235 IN THE PRELIMINARY ENGINEERING DESIGN OF THE TREAT CONVERTER REGIONS,^a 10^{24} atoms/cm³

Design	Inner Converter, 10^{-3}		Outer Converter, 10^{-4}			$N^B t$, 10^{-4}	Core Outer Radius, cm
	1	2	1	2	3		
20	1.5	0.75	2.0	1.5	1.0	2.58	95.0
21	1.5	0.75	2.0	1.5	1.0	1.58	95.0
22	1.5	0.75	2.0	1.5	1.0	0	113.5
24	2.25	0.75	2.0	1.5	1.0	0	105
26	3.0	1.0	4.0	3.0	2.0	0	105
28	3.5	2.0	8.0	4.0	1.5	0	105
29	4.0	2.5	8.0	4.0	1.5	0	100
31	4.0	2.5	8.0	4.0	1.5	0	100
33	3.0	2.0	8.0	4.0	1.5	0	95
34	3.0	2.0	8.0	4.0	1.5	0	97
35	2.5	1.8	8.0	4.0	1.5	0	97
40	2.5	1.8	8.0	4.0	1.5	0	97

^a All designs have a 19-pin test sample, except design 40, which has a 7-pin sample.

FIG. III-10-3. Enthalpy Increase Over 50°C Datum for TREAT Core in cal/cc. ANL Neg. No. 118-834.

is divided into two loadings. Also shown in Table III-10-I are the boron concentrations in density times thickness, $N^B t$, for two problems in which a boron lining was placed around the loop region. The test clusters are composed of mixed oxide pins clad with 0.051 cm thick stainless steel tubes having 0.737 cm o.d. The oxide is 0.2 PuO₂ + 0.8 UO₂, with the uranium

enriched. For 7-pin clusters, the loop test section dimensions are those of the current version of the Mark II loop: larger clusters have the loop wall inner diameter scaled up to accommodate the pins, and the

wall thickness increased so that the loop thickness-to-diameter ratio is the same as that for the present Mark II design.

RESULTS

The changes in heat content ΔH in the converter, core, and test section (TS), are conveniently described by the following four parameters:

$$\frac{\Delta H_{TS}^{centerline}}{\Delta H_{core}^{maximum}} = b$$

$$\frac{\Delta H_{converter}^{maximum}}{\Delta H_{TS}^{centerline}} = a_m$$

$$\frac{\Delta H_{converter}^{maximum}}{\Delta H_{core}^{maximum}} = a_m b$$

FIG. III-10-4. Enthalpy Increase Over 50°C Datum for UO₂ in cal/cc. ANL Neg. No. 118-837.

TABLE III-10-II. SUMMARY OF RESULTS FOR THERMALLY-INSULATED PRELIMINARY ENGINEERING DESIGN

Design	Core Radius	Core Temp., °C	k	$-\frac{\Delta k}{\Delta T}, 10^{-4}$	c_1	c_2	b	a_m					$(amb)_{max}$	Results from T_{core}^{max} , °C	$T_{converter}^{max}$, °C	k change, $\frac{\Delta H_{centerline}^{TS}}{\Delta H_{melting}^{UO_2}}$
								Inner converter		Outer converter						
								1	2	1	2	3				
Reference	95.0	22	1.1643	2.32	1.14	4.58	50	—	—	—	—	—	—	375	—	2.24
Core	95.0	390	1.0789	1.92	1.03	1.12	59	—	—	—	—	—	—	440	830	1.60
	20	95.0	22				1.0530	26.3	0.11	0.077	0.072	0.088	0.10			
	95.0	390	0.9822		1.02	1.11	34.0	0.11	0.083	0.076	0.087	0.097	3.8			
21	95.0	390	0.9855		1.02	1.11	34.2	0.11	0.093	0.076	0.087	0.097	3.9			
22	113.5	390	1.0660		1.03	1.12	30.1	0.12	0.100	0.081	0.095	0.11	3.3			
24	105.0	390	1.0448		1.02	1.08	32.5	0.17	0.097	0.078	0.091	0.10	5.4			
26	105.0	390	1.0746		1.01	1.05	35.5	0.18	0.087	0.092	0.11	0.16	6.5			
28	105.0	390	1.1000		1.01	1.02	38.0	0.19	0.14	0.15	0.15	0.12	7.1			
29	100.0	390	1.1005		1.00	1.02	41.0	0.21	0.17	0.15	0.15	0.11	8.4			
31 ^a	100.0	390	1.1130		1.00	1.01	50.3	0.21	0.18	0.13	0.12	0.086	10.4			
33	95.0	390	1.0630		1.01	1.01	48.8	0.17	0.15	0.14	0.13	0.096	8.2	560	2170	3.30
34	97.0	390	1.0695		1.00	1.01	47.7	0.16	0.15	0.14	0.13	0.098	7.8			
35	97.0	22	1.1112	1.46	1.00	1.01	30.7	0.14	0.15	0.14	0.14	0.13	4.5	553	1845	2.9
		390	1.0574				1.00	1.01	44.1	0.14	0.14	0.14	0.13			
40	97.0	22	1.1087	1.49	1.01	—	30.1	0.14	0.15	0.14	0.14	0.13	4.4			
		390	1.0539				1.01	—	44.3	0.14	0.15	0.15	0.14			

^a Designs prior to 31 used UO_2 in inner converter; designs following 31 used $ZrO_2 + CaO + UO_2$.

TABLE III-10-III. SUMMARY OF NUCLEAR EFFECTS OF CONVERTER

Design	$-\frac{\Delta k}{\Delta T}, 10^{-4}$	$\ell_p, 10^{-4}$ sec	$E_{outer\ ring}^{TS}$ MeV	Control Rod Total Worth, %
Unperturbed	2.5	9	—	22
35	1.5	6.7	0.86	17.5

$$\frac{\Delta H_{TS}^{maximum}}{\Delta H_{TS}^{minimum}} = c.$$

Limiting values of converter and core peak temperatures were set at 1800 and 600°C, respectively, with an initial temperature of 50°C. The ΔH values for core (graphite base) and converter (oxide base) were taken from Figs. III-10-3 and III-10-4, respectively.

A summary of the results is given in Table III-10-II. The table includes a reference core problem in which the 19-pin cluster and loop are loaded directly into TREAT without a converter. Two core temperatures, 22 and 390°C, appear in the table; these are the temperatures for which the thermal group constants were evaluated. The first corresponds to the initial conditions (core normally at ambient temperature, with maximum temperature due to heat transfer from the loop less than 50°C), and the second corresponds to a peak core temperature of approximately 550°C. Values of the temperature coefficient of reactivity, $\Delta k/\Delta T$, averaged

over this range, are given for four selected cases. The index c_1 denotes the ratio of maximum power in the 6-pin ring to power at the centerline. Similarly, c_2 denotes the ratio of maximum power in the 12-pin ring to centerline power. Five values of a_m are given, one for each converter sub region. The maximum value of $a_m b$ for each design is designated as $(a_m b)_{max}$. The last three columns of Table III-10-II give the results, for selected designs, of a TREAT excursion producing a total change of 0.05 Δk . The quantity T_{core}^{max} is the maximum core temperature which occurs along the horizontal reactor midplane. Table III-10-II, in conjunction with Table III-10-I, shows the effects of varying the five levels of converter enrichment on sample power and on the relative flux peak levels of the five converter regions. It appears that a maximum ratio $(\Delta H_{TS}/\Delta H_{UO_2}^{melting}) \gtrsim 2.0$ is attainable, even if allowances are made for possible contingencies.

TABLE III-10-IV. EFFECTS OF LARGER CLUSTERS, INCLUDING LARGER LOOP WALLS^a

Cluster Size, No. of Pins	k^b	$\frac{\Delta k}{\Delta T}, 10^{-4}$	b^b	c_1	c_2	c_3
7	1.054	1.49	44.3	1.01	—	—
19	1.057	1.46	44.1	1.00	1.01	—
37	1.063	1.45	43.5	1.00	1.00	1.

^a Based on design 35.

^b Average core temperature of 390°C.

TABLE III-10-V. COMPARISON OF PRELIMINARY ENGINEERING DESIGN CONVERTER WITH BORAL SHIELDED LOOP

Case	b^a	c_1	c_2	c_3	k		$-\frac{\Delta k}{\Delta T},$ 10^{-4}	Inner Core Radius, ^b cm
					22°C	390°C		
7-Pin Cluster								
Reference	58.0	6.55	—	—	1.174	1.081	2.5	7.9
Converter	44.3	1.01	—	—	1.109	1.054	1.5	30.5
Boral	22	1.23	—	—		1.051		8.1
19-Pin Cluster								
Converter	44.1	1.00	1.01	—	1.111	1.057	1.5	30.5
Boral	18.0	1.08	1.31	—	1.133	1.044	2.4	8.1
37-Pin Cluster								
Converter	43.5	1.00	1.00	1.01		1.063	1.5	30.5
Boral	14.8	1.04	1.18	1.41		1.045	—	8.1
Single Pin (Ref. Core) 0.635 cm diam 13% enrichment	75	1.20 ^c	—	—	1.175	1.083	2.5	7.9

^a For 390°C.^b All cores have 97.0 cm outer radius.^c Single test pin maximum-to-average radial power ratio.

Flux peaking at the interface of the converter and axial reflector was virtually eliminated by the use at this interface of a "boral" slab having $N^B t = 1.5 \times 10^{-3}$. The reactivity loss due to the boron is about 0.2–0.3%, depending on design details. There will be no flux peaking for test section pins consisting of a 60 cm long mixed oxide fuel section with natural UO_2 extending through the upper and lower TREAT reflectors.

Effects of the converter on the nuclear characteristics of TREAT are summarized in Table III-10-III, which gives changes in the core temperature coefficient of reactivity, prompt neutron lifetime ℓ_p , average neutron energy producing sample fission in the outer (12 pin) ring of test pins $\bar{E}_{\text{outer ring}}^{TS}$, and total control rod worth for an initial worth of 22%. The unperturbed core used as a comparison standard is the solid, cylindrical core loading without core slots or experimental apparatus.

The changes produced by larger clusters (and the associated larger loop walls necessary for them) are small, as summarized in Table III-10-IV.

COMPARISON OF PRELIMINARY ENGINEERING DESIGN* WITH BORAL SHIELDED LOOP

Additional calculations were run to obtain a more detailed comparison between effects of a converter and sample thermal neutron shielding by a "boral" neutron shield. In the converter concept, the converter absorbs most of the low energy neutrons by fission, which in effect replaces low energy neutrons by high energy neutrons. The comparison was made using a boron concentration lining, the 10×20 cm test hole being

equivalent to that for a 0.635 cm thick boral sheet (see Table III-10-V). The converter used for comparison is that of design 35 (see Table III-10-I), the thermally-insulated, molybdenum-sheathed preliminary engineering design. For convenient reference, Table III-10-V also includes two additional cases: the reference case of Table III-10-I which has a 7 pin test cluster in the Mark II loop, but no converter or neutron shield; and a calculation that mocks up the sample and loop for the first loop oxide meltdown experiment. The inner core radius is the radius, for the cylindrical calculational model, that corresponds to the innermost core boundary (loop secondary container, outer boundary of converter, etc. depending on the case). As before, $\Delta k/\Delta T$ is evaluated as the average over the core temperature range from 22 to 390°C. The converter produces both a higher center pin power (cf. b values) and a better radial power distribution (cf. c values). The temperature coefficient of reactivity for the converter case is still sizeable and negative, although not as large in magnitude as that of the reference and boral filter cases. This indicates that the shutdown of the reactor due to heating will not be compromised by a converter, but that the total change in k during a given transient will be about 40% less than for the reference or boral loadings. This latter point can be important when the total k available in the reactor is limited.

POSSIBLE DESIGN MODIFICATIONS

Table III-10-V indicates a problem with the preliminary engineering design. The single pin case corresponds directly to an actual loading of the reactor. It has a value of k about 2.5% larger than that of the

* See Paper III-9.

converter. Thus, if the converter and a loop were loaded into TREAT, the maximum Δk available for a transient (that is, with TREAT fuel loaded into all available positions) would be about 5%. This is marginal.

The maximum TREAT burst that has been run⁷ is a 2150-MW-sec transient, initiated with 4.9% Δk , and producing a maximum core temperature of 575°C. With the reduced temperature coefficient of the converter loading, this transient could be performed with an initial excess k , $k_{ex}(0)$, of about 3%. The total change in k would be about $2[k_{ex}(0) - \beta] = 4.6\%$. This change would be required for a "flattened" transient in which rods were moved during the burst to maintain near-constant power by compensating for the temperature feedback.

From the calculations reported above, design 35 appears to be satisfactory, except for the marginal value of k . Because of the low neutron absorption in the TREAT core, it was expected that parasitic absorption in the molybdenum was the principal factor in the reactivity loss over the designs reported earlier.^{1,2} This absorption should be particularly severe in the outer converter sections next to the core. The problem of reactivity loss can be approached several ways. First, the converter elements could readily be changed from a 10×10 cm cross section to a 10×20 cm cross section. This change would permit direct elimination

TABLE III-10-VI. CONCENTRATIONS OF U-235 IN THE SURVEY PROBLEMS RUN TO CHECK POSSIBLE MODIFICATIONS TO THE PRELIMINARY ENGINEERING DESIGN,^a 10^{24} atoms/cm³

Design	Inner Converter, 10 ⁻³		Outer Converter, 10 ⁻⁴		
	1	2	1	2	3
41 ^b	2.5	1.8	8.0	4.0	1.5
43	2.5	1.8	4.0	3.0	1.2
44 ^b	2.5	1.8	3.0	2.0	0.8

^a All problems have 97 cm core radius and 19-pin test cluster.

^b The molybdenum is replaced by zirconium in the outer converter (only).

TABLE III-10-VIII. SUMMARY OF THERMAL CALCULATIONS FOR DESIGNS 35, 41, AND 44

Design	T_{max}^{core} , °C	b^a	$T_{max}^{inner\ converter}$, °C	$T_{max}^{outer\ converter}$, °C	$\frac{\Delta H_{centerline}}{\Delta H_{UO_2}} \frac{TS}{\Delta H_{melting}}$	Δk , %
35	400	36.0	1075	1075	1.69	-3.4
	550	37.4	1640	1640	2.73	-4.8
41	400	39.0	1310	1410	1.83	-2.8
	550	40.0	1950	2050	2.92	-3.9
44	400	37.0	1680	1030	1.74	-2.8
	550	38.5	2450	1550	2.79	-3.9

^a Average value used in thermal calculations.

of about 25% of both the molybdenum and steel cladding, and would allow the amount of converter fuel to be increased by about 14%. In addition, the molybdenum could be removed from the interface between the outer converter and the core. Simple replacement by niobium could be done. An alternate approach would be to replace the molybdenum by zirconium, and reduce the outer converter U-235 loading to prevent overheating (taking into account possible iron-zirconium eutectic formation) the zirconium at the converter interface flux peaks. Table III-10-VI lists the converter compositions for three designs investigated to check on the possibility of reducing molybdenum in the converter region. Results are summarized in Table III-10-VII. For reference, results from design 35 are in Table III-10-II. Table III-10-VII indicates that the k of the system consisting of core plus converter plus loop can be significantly increased. Further, the temperature coefficient of reactivity has been reduced somewhat in magnitude but is still adequately large (and requires less loading margin).

It should be noted that no attempts were made to optimize the modified designs given as designs 41, 43, and 44.

Additional detailed thermal calculations were made on designs 41 and 44, for comparison with design 35. Results are shown in Table III-10-VIII. Table III-10-VIII indicates that design 41 somewhat exceeds

TABLE III-10-VII. SUMMARY OF RESULTS FOR SURVEY PROBLEMS ON POSSIBLE CONVERTER DESIGN MODIFICATIONS

Design	Core Temp., °C	k	$-\frac{\Delta k}{\Delta T}, 10^{-4}$	c_1	c_2	b	a_m					a_{mb}		Refractory Metal in Outer Converter
							Inner converter		Outer converter			Inner	Outer	
							1	2	1	2	3			
41	22	1.206	1.2	1.01	1.02	34.5	0.15	0.16	0.17	0.17	0.15	5.6	5.9	Zr
43	22	1.096	—	1.00	1.01	29.0	0.15	0.17	0.10	0.13	0.11	4.9	3.8	Mo
44	22	1.177	1.2	1.01	1.02	32.1	0.15	0.22	0.12	0.14	0.11	7.0	4.4	Zr
	390	1.134		1.00	1.01	44.8	0.15	0.23	0.12	0.13	0.09	10.1	5.7	

gn 35 in total sample energy for the same T_{core}^{max} , but is slightly below design 35 if the design 41 transient is decreased to hold the converter temperatures below the 1800°C maximum. When the outer converter U-235 loading is decreased for design 44—thus allowing more neutrons to reach the inner converter—sample energy input suffers because the converter temperatures are too high.

From Table III-10-VIII, it would appear that if more k is required than in the preliminary engineering design, an optimized modification could be arranged with $\Delta H_{TS}^{centerline} \approx 2 \times \Delta H_{melting}^{OU_2}$.

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III-11. Analysis of Fuel Meltdown Experiment (S-3) with the Transient Reactor Test (TREAT) Facility

A. K. AGRAWAL

INTRODUCTION

For the safe operation of fast breeder reactors it is desirable that, among other things, the fuel pins remain intact at all times during reactor operation. In particular, one is usually concerned with the performance of these pins in an overpower transient and loss of flow (including the beginning and the end of life fuel conditions) situations. For this reason, an accurate analysis of the fuel pin behavior for abnormal reactor power conditions becomes an important task. The problem here is really two-fold: the prediction of pin rupture and then the post-rupture analysis. These problems may depend upon several factors including the behavior of fission gas, fuel expansion during melting, cladding dimensions and properties, and the degree of axial motion of the fuel.

The present report deals with the performance of the fuel pin up to the time of its rupture with special attention towards establishing fuel failure criteria. In this regard extensive use of the digital computer code SAS1A⁽¹⁾ has been made. Since the code does not have any built-in rupture criteria, it has been used to analyze several of the existing experiments on fuel meltdown studies with TREAT. A number

such experiments have been reported and are being planned by General Electric Company² and Argonne National Laboratory^{3,4} which, with a sys-

tematic analysis, could be very valuable in establishing the threshold failure limit of these pins. In here, we have chosen to analyze one such recent experiment performed by the Chemical Engineering Division of Argonne National Laboratory. A brief experimental description of this experiment (S-3) is followed by the SAS1A calculations and their discussions.

EXPERIMENTAL DESCRIPTION

In order to provide a better understanding of the theoretical results to be discussed in this report, the following description of the fuel meltdown experiment^{3,5} in a sodium-filled piston autoclave (series S experiments) is included. This series of experiments involves a cluster of nine rods arranged in a square lattice as shown in Fig. III-11-1. The center rod and its immediate four neighboring rods consist of 93% theoretical density, 10% enriched unirradiated uranium dioxide pellets, while the remaining four rods are hollow dummy rods filled with helium gas at 15 psia. Each fuel rod consists of two separate sections: an upper or holder section 16.2 cm long filled with helium at 15 psia, and a 14.3 cm long fueled section. These two sections are joined by a dowel pin fitting. Each fuel rod is stacked with ten helium bonded pellets in a 304-type stainless steel jacket.

FIG. III-11-1. Geometrical Arrangement of Nine Rod Cluster in the Sodium Autoclave Experiment. *ANL Neg. No. 113-2774.*

These fuel pellets, except for the top one, are 1.27 cm long and 0.640 cm diam. The top pellet was cut to only 0.63 cm in length so that a nominal 0.64 cm axial gap was provided to accommodate the axial thermal expansion of the fuel. The stainless steel jacket was 0.726 cm o.d. and had a wall thickness of 0.038 cm. The dummy rods had the same specifications as the tubing for the fuel rods. The fuel contained in each fuel rod was about 41 g, equivalent to 205 g in the entire array. Considering the clearance between pellet and cladding and axial expansion gap, the smeared density of fuel in the fuel pin is about 82% of theoretical density.

The autoclave, containing a cluster of nine rods in a pool of about 200 g of liquid sodium at 175°C, is placed in the TREAT reactor. The TREAT transient for the experiment under analysis (S-3) is shown in Fig. III-11-2, where time $t = 0$ has been chosen arbitrarily. In this figure, the integrated TREAT power is also shown as a function of time. It is seen that the experiment involved a transient of 669 MW-sec of TREAT power, most of which was deposited in the fuel pin in about 0.4 sec. The fuel pin power is obtained by multiplying the TREAT power with a calibration factor obtained from a test run. The value for this calibration factor used in the present study is 2.880 J/(g- UO_2 -MW-sec of TREAT) for the edge rod and 2.678 J/(g- UO_2 -MW-sec of TREAT) for the center rod. Using these calibration factors, the transient for this experiment is equivalent to fission energies of 1927 J/g- UO_2 for the edge

rod and 1792 J/g- UO_2 for the center rod, assuming that fuel was intact during the transient. These fission energies are well above the fuel melting of uranium dioxide (~ 1300 J/g- UO_2) if adiabatic conditions are assumed.

Experimentally, this transient was found to rupture and destruct all fuel rods. The pressure response showed a group of four pressure pulses (460, 410, 540, and 140 psi) occurring within 20 msec. Indication of the first pressure spike occurred at $t = 0.60$ sec with an energy deposition of about 1860 J/g- UO_2 in the edge rod. A fifth pressure spike of 140 psi was noted at roughly 0.5 sec after the occurrence of the first pressure pulse. The temperature recorded by the thermocouple located at the upper part of the fueled section of the fuel rod is shown in Fig. III-11-3. Notice that a peak sodium temperature of 1100°C

FIG. III-11-2. TREAT Power and the Integrated Power as a Function of Time for S-3 Experiment. The Reactor Peak Power is Normalized to Give the Correct Integrated Power at the Time of Fuel Pin Failure. *ANL Neg. No. 113-2778.*

FIG. III-11-3. Thermocouple Readings of the Sodium Temperature at the Top of Fuel Section as a Function of Time. The Arrows Indicate the Observed Pressure Pulses in Experiment (460, 410, 540, 140 and 140 psi). *ANL Neg. No. 113-2777.*

recorded at about the time of occurrence of the fifth pressure pulse. The residue from this experiment indicated that the five fuel rods were completely destroyed during the transient. Only hollow (0.48 cm i.d.) cylindrical sections of fused UO_2 of about the length of fuel pellets were found. These sections had an outside diameter of 0.673 cm, which is larger than the initial inner diameter of the cladding (0.650 cm).

SAS1A CALCULATIONS

The fuel pin performance up to the time of pin rupture ($t \leq 0.60$ sec) is evaluated by using the fast reactor accident study code SAS1A.⁽¹⁾ Results obtained, therefore, are based on the assumptions and limitations of the code. In particular, the present calculations are made for an equivalent single pin description. It is assumed that the available coolant volume is equally shared by all fuel rods. Furthermore, the effect of bond gas and fission gas pressures on fuel and cladding is not considered. Since the fuel used in this transient was fresh uranium dioxide (zero burnup), the neglect of fission gases is justified.

The heat transfer calculations for both center and edge rods are made by using an equivalent and independent single pin arrangement with the appropriate values of fission energies as noted in the previous section. The most recent measurements of thermal conductivity⁶ and specific heat⁷ values and their explicit temperature dependence of uranium dioxide are used in the calculation of fuel temperature profile. The bond conductance h_b is represented as

$$h_b = \frac{c}{\Delta r}, \quad (1)$$

where c is an input parameter and Δr is the bond gap size. The value of h_b is restricted to a maximum input value of h_{bmax} . In the present calculations, we have taken⁸ $c = 0.0042$ W/cm 2 -°C and $h_{bmax} = 2$ W/cm 2 -°C. The thermal properties of the cladding are taken at an average temperature. Due to the code requirements, the stagnant sodium is replaced by slowly circulating sodium (0.5 m/sec).

The calculated peak fuel temperature and the coolant temperature at the mid-point of the top pellet of the edge rod is shown in Fig. III-11-4 as a function of time of the transient. Also included in this figure is the peak fuel temperature for the center rod. It is seen that the center rod peak fuel temperatures are roughly 7% lower than the temperatures for edge rod because of a difference of 7% in fission energies. No significant change in the coolant temperature from the edge rod to the center rod is

FIG. III-11-4. Calculated Fuel and Coolant Temperature Profiles as a Function of Time for the Edge Rod (Solid Curve). The Dashed Curve (---) is the Calculated Peak Fuel Temperature for the Center Rod and Solid Circles are Coolant Temperature as Sensed by the Thermocouple. ANL Neg. No. 118-2779.

observed due to the rapid transient of the experiment. The thermocouple measurements of coolant temperature at the top of fueled section is included in this figure (these values are taken from Fig. III-11-3) for time up to 0.70 sec. The large difference between code-predicted values and the thermocouple data can be due to a number of reasons including the time constant of the thermocouple, the actual position of the thermocouple, and the replacement of stagnant sodium by circulating coolant. No detailed investigation has been made of this discrepancy.

The thermo-elastic deformation of the fuel and the elastic-plastic deformation of the cladding are evaluated by using the Fuel Deformation module of the code. This module assumes fuel to be uniform and isotropic, i.e., fuel cracking is not allowed. The cladding is considered to behave like a perfectly elastic material until the circumferential stress (hoop stress) becomes equal to the yield point of the cladding. The cladding is treated like perfectly plastic material when the hoop stress exceeds the yield point of the cladding. Although the pressure dependent yield point is not accounted for by the model, the explicit temperature dependence of the cladding yield point is considered.

The chain of events leading to the permanent deformation of the cladding consists of filling the gap between the fuel and the stainless steel due to the thermal expansion of uranium dioxide, the elastic deformation of the cladding primarily due to further thermal expansion of UO_2 until the loading exceeds

the yield point when the cladding undergoes plastic or permanent deformation. From the detailed calculations of the cladding behavior it is found that the melting of fuel causes excessive plastic deformation of the stainless steel cladding. The plastic deformation of the cladding can also be initiated even before any melting of UO_2 depending, of course, upon the gap between the fuel and the cladding. For the experiment under investigation it is found that the loading on the cladding exceeds the yield point value of the cladding before any melting of the fuel takes place.

The cladding rupture is analyzed quantitatively in terms of the percentage plastic deformation which is defined as

percentage plastic deformation

$$= \frac{r_c(t) - r_c(t_0)}{r_c(t_0)} \times 100, \quad (2)$$

where $r_c(t)$ is the outer radius of the fuel pin at time t . Time t_0 is taken as the time when loading on the cladding is equal to the yield point. It should be emphasized that use of the above definition of the plastic deformation does not correspond to a reduction of the outer radius of the pin to the room temperature value before computing the percentage deformation. An alternative way for a quantitative study of the cladding rupture is to calculate the thinning of the cladding after loading exceeds the yield point. The fractional thinning is defined as

$$\text{fractional thinning} = \frac{\delta_c(t_0) - \delta_c(t)}{\delta_c(t_0)}, \quad (3)$$

where $\delta_c(t)$ is the cladding thickness at time t . The percentage plastic deformation and the fractional

FIG. III-11-5. Calculated Plastic Deformation of the Clad as a Function of Time for Both the Edge Rod (Solid Curve) and the Center Rod (---). ANL Neg. No. 113-2776.

FIG. III-11-6. Calculated Plastic Deformation of the Clad and the Fraction of UO_2 Melted as a Function of the Energy Deposition in the Fuel. ANL Neg. No. 113-2780.

thinning are, in general, two different measures of the cladding deformation. Their general behavior is expected to be similar but not identical.

In Fig. III-11-5 we show a graph of the plastic deformation of the cladding as a function of time of the transient for both the edge and center fuel pins. It is seen that both curves indicate the initiation of plastic strain of the cladding at about the same time ($t = 0.28$ sec), which occurs slightly before the peak of the transient. The plastic strain is rather small until fuel begins to melt (~ 0.33 – 0.34 sec, see Fig. III-11-4). This is associated with a 5.8% change in volume due to melting of 93% T.D. uranium dioxide fuel under investigation. It should be noted that in the calculation of change in volume due to melting, it was assumed that molten fuel acquires the density of 100% T.D. uranium dioxide. In Paper III-17, we report the results of sensitivity studies using change in fuel volume due to melting as a parameter.

It is often desirable to plot the plastic deformation of the cladding as a function of energy deposition in the edge fuel pin. Such a plot is shown in Fig. III-11-6 where the energy corresponding to the pin failure (~ 1850 J/g- UO_2) is also shown. Also shown in Fig. III-11-6 is the mass fraction of molten uranium dioxide as a function of the energy deposition in the sample pin. These results remain essentially unchanged for the center rod.

DISCUSSION

The heat transfer calculations⁹ for an idealized single pin configuration indicate a peak fuel temperature of about 4000°C . Present calculations of temperature profile are useful only up to the time or pin rupture (in this case, up to 0.60 sec) since fuel-

ant consequences are not considered in SAS1A. At this time the calculated maximum coolant and the inner surface cladding temperatures are about 760 and 970°C, respectively. There exists a large discrepancy between the code predictions and the measured coolant temperatures; some of the reasons for this discrepancy have been noted in the previous section. It should, however, be noted that the measured coolant temperature at 0.70 sec is very close to the calculated temperature at 0.60 sec.

Understanding the fuel pin rupture mechanism is a major problem. It appears that the pin failure can be due either to excessive loading (hence, excessive plastic deformation) or due to melt-through of the cladding. The former mechanism of fuel failure can be classified as the prompt failure while the latter mechanism may be termed as the delayed action. Such a classification may be useful in understanding fuel pin performance in power or reactivity transients in liquid metal cooled fast breeder reactors.

The occurrence of four pressure pulses within 20 msec at the end of the transient can be due to individual failure of four edge fuel rods in the piston autoclave. At the time of these rod failures, the average temperature of the cladding is calculated to be about 900°C. Since the melting point of stainless steel is about 500°C higher than this temperature, the failure of four edge rods is associated with the excessive plastic deformation rather than the melt-through of the cladding. According to the definition of plastic deformation [Eq. (2)], the code SAS1A predicts a plastic strain of 5.5%.

It is seen from Fig. III-11-3 that the fifth pressure pulse is observed at roughly 0.5 sec after the first pressure pulse is sensed by the pressure transducer. This pressure pulse is perhaps due to central pin failure. The mechanism of the failure in this case, however, is very different from that for edge rods. Since the bulk coolant temperature at $t = 1.1$ sec is recorded to be about 1100°C, the average cladding temperature can be at least equal to the melting temperature of the cladding, which would indicate that the mechanism of central rod failure is the melt through (delayed mechanism) of the cladding rather than excessive strain. This argument is substantiated by the predicted strain of the cladding. From Fig. III-11-5, the predicted strain is 4.6% which is quite a bit smaller than the failure threshold strain value (5.5%). Thus, the S-3 experiment appears to provide a combination of both prompt and delayed mechanisms of pin failure.

The fraction of molten fuel, in a given transient, is often used in establishing the fuel pin failure threshold criteria. While this approach may be quite

useful, particularly for the transient analysis of fast reactor excursions, present investigation indicates a contradictory result. For example, it is seen from Fig. III-11-6 that the fraction of molten uranium dioxide approaches an asymptotic value of 80% with the energy deposition of 1400 J/g- UO_2 . Since the experiment indicates pin failure with an energy deposition of 1850 J/g- UO_2 , which also corresponds to about 80% of molten UO_2 , we feel that the fraction of molten fuel does not seem to be a satisfactory criterion for establishing the threshold of pin rupture.

Finally, the code predicts the physical dimensions of the fuel pin up to the time of pin rupture which can be checked with the experimental values, if they are available. For the S-3 experiment, the code predicts the fuel outer radius at the rupture time to be 0.352 cm for the edge fuel rod and 0.348 cm for the center fuel rod. In order that these numbers can be compared with the room temperature values of the fused uranium dioxide sections the code-predicted values need to be reduced to the room temperature dimensions. Considering an average fuel temperature to be 2000°C, the room temperature fuel outer radius would be 0.340 and 0.336 cm for edge and center fuel rods, respectively. These numbers compare favorably with the experimental value of 0.336 cm.

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III-12. The Problems of Fast Reactor Safety

A. M. JUDD

INTRODUCTION

Work is currently in progress to identify what experimental work is needed in the field of fast reactor safety, and in particular to define the need for new reactor facilities for safety tests. This requires a clear understanding of what is needed to show that a reactor is safe, and a system of priorities to show which of the outstanding problems are most important.

The first need is for a reactor safety standard. Unfortunately no single unequivocal statement of a safety standard is accepted at present. The best we can do is to interpret the current situation in terms of a rough standard. If we do this we conclude that currently it has to be shown for a given reactor that

1. No likely accident can have more than negligible consequences (i.e., it must not breach the primary containment).
2. No unlikely accident can have more than limited consequences (i.e., it must not breach the secondary containment).
3. Accidents with serious consequences are very unlikely (i.e., breach of all containment must be very unlikely).

These three requirements are a gross simplification of the present situation, but in the absence of anything better, we have to use them as a basis for our assessment.

The next step in assessing the need for experimental work in the field of fast reactor safety is to see what technical questions have to be answered to show that a reactor meets a safety standard of this type. In this way we can set a logical basis for a fast reactor safety program.

In this paper we first consider the various accidents which might befall a liquid-metal-cooled fast breeder reactor (LMFBR), and list the things we need to know in order to assess their consequences and probabilities. We then collect these into a list of the outstanding problems, with an indication of their relative importance.

ANALYSIS OF ACCIDENTS

LIKELY ACCIDENTS

Our main concern here is with accident consequences which could give rise to a minor radioactivity release, since a reactor for which a likely accident

could give a major release would be so obviously unsafe it would be altered at the design stage.

Likely accidents (which are usually due to single failures) initially affecting the whole core (such as coolant circulating pump failure or control system malfunction) can be detected easily and damage prevented or minimized by shutting down the reactor.

Coincident failure to shut the reactor down can occur if some event (such as a fire or an earthquake) can cause both one of these whole-core failures and a failure of the shutdown system. Avoidance of such coincidences is normally a design problem, but the development of design principles to minimize the possibility of unforeseen coincidences should be seen as an important safety problem.

Apart from this, however, it is local failures which present the main problem from the point of view of likely accidents. A local failure is relatively hard to detect, but in general the damage done is acceptable if it is confined to one subassembly of a typical LMFBR. Thus, the main problems are the detection of incipient accidents and the prevention of the propagation of damage.

Fuel Pin Failure

As there may be 10^5 fuel pins in a single reactor core, failure of the cladding of individual pins (due to manufacturing defects or hot spots) should, for economic reasons, only very rarely require the reactor to be shut down. Thus, the effects of a single pin failure should be small and propagation of this failure to cause failure of more than a few other pins must be improbable, even if the fuel is near the end of its irradiation life, or has been subject to some previous minor maloperation (such as a minor reactor power or coolant flow transient).

The initial fuel failure may be influenced by such things as manufacturing defects, the mechanics of the fuel and cladding materials, fission-product gas, or hot spots caused by coolant flow irregularities, foreign matter in the coolant, or local power peaks. Propagation could be caused by:

- Fission product gas release (which could cause gas blanketing and overheating of adjacent pins).
- Fuel released from the cladding failure (which might lodge against or stick to adjacent pins causing overheating, or might interact violently

with the sodium thereby damaging adjacent pins by a pressure pulse).

- Distortion of the broken cladding or fuel pin spacer to touch adjacent pins and cause overheating.

It is clear that failure and the possibility of propagation will depend on such things as whether the fuel is vented or not and whether the vent is working properly, and whether the failure occurs while the reactor is at steady power or is caused by a power or coolant flow transient. Without going into detail it is possible to identify the main areas of interest. We need to know:

a. How a fuel pin behaves as it fails. (How big a hole is made in the cladding? Where is it likely to be? How quickly is fuel released? Is it propelled by gas? Does the pin bend? What are the effects of sodium logging, if it can occur?)

b. What happens to gas or fuel released from the pin. (Can the gas cause overheating of adjacent pins? Where does the fuel go? Can it cause a vapor explosion*? If so, is it violent enough to break other pins?)

c. How the answers to a and b depend on burnup, prior maloperation of the fuel, the cause of failure, or the pin design.

Instrumentation to detect failure of a single fuel pin may be important as a guide to a reactor operator.

Flow Blockage in a Single Subassembly

If a partial blockage of a subassembly is big enough, it may lead to coolant boiling and voiding, and fuel melting. This sequence may take place so quickly that, with the difficulty of detecting such a blockage, it is impossible to prevent melting of most of the fuel in the subassembly, even if every subassembly has a flowmeter. This would be acceptable, however, if the probability of such subassembly blockage were low enough, and if the damage due to melting could be confined to the one subassembly. The problem is to be sure that blockage and melting of one subassembly when the reactor is not shut down immediately by a protective system cannot damage any of the rest of the core and initiate a serious power excursion before it is detected (which would be an unacceptable safety risk). It is also desirable to know how big a partial blockage is needed to cause boiling and melting. In addition, the less

likely event of a complete subassembly blockage, which raises similar problems, has to be considered.

Propagation beyond one subassembly could be caused by:

- Pressure pulses from a vapor explosion. (This is a crucial question. Can explosive boiling of the coolant break the subassembly walls and damage the rest of the core? Can it distort the core to increase its reactivity or prevent the insertion of control and safety rods?)
- Fuel movement reactivity effects. (If the fuel can compact at the center, the size of the subassembly must be limited so that the reactivity increase is sufficiently small. Even so it could cause a power transient which could damage the rest of the core.)
- Fuel melting through the subassembly wall. (This seems to take place slowly, so that propagation before the failure is detected is unlikely.)
- Coolant reactivity effects. (This would always be prevented by making the subassemblies small enough so that the maximum coolant void reactivity is much less than a dollar, and a significant power excursion is impossible.)
- Pressure pulses due to collapsing vapor bubbles. (This seems unlikely because the pulses carry little energy.)

Instrumentation to detect boiling or fuel failure in order to shut the reactor down before propagation beyond one subassembly can take place may be important, particularly if a damaging vapor explosion is possible. We need to know:

a. How the coolant moves when boiling occurs. (Once ejected, can it reenter the subassembly and come into contact with hot fuel? Is the reentry accompanied by violent condensation? If cold liquid cannot reenter the subassembly, a vapor explosion is unlikely.)

b. Whether a vapor explosion is possible. (What conditions are needed to cause it? How much damage can it do?)

c. How the disintegrated fuel moves. (Is it ejected from the subassembly? Can it compact at the center? Does it freeze on the subassembly walls? How is its motion affected by the presence of boiling coolant?)

d. Whether analysis predicts the melting of hot fuel through the subassembly walls correctly. (If it can melt through, how quickly does it do so?)

e. What the effects of a partial blockage are. (How big has a blockage to be to cause boiling or fuel damage? What are the effects of fuel support grids or wrapper wires?)

* In the above and in what follows we use the term "vapor explosion" with a special meaning. It refers to the generation of high pressures when a hot liquid comes into intimate contact with a cold liquid, and heat is transferred and vapor formed very rapidly.

f. Whether boiling can be detected quickly by acoustic or other means.

Simultaneous Blockage of Several Subassemblies

This might be regarded as an unlikely event were it not for the Fermi accident. Careful design of the coolant inlets to the subassemblies can reduce the probability of simultaneous blockage so that the fact that the consequences are more severe than those of single subassembly blockage is offset by the lower probability. Analysis of this accident presents no problems other than those discussed under single subassembly blockage above.

Blockages in a Reactor Without Solid Subassembly Walls

Reactors have been proposed in which the fuel subassemblies have either no walls or walls with holes. Both have the advantage that if one subassembly is blocked coolant can enter it from adjacent ones to help prevent serious overheating. A disadvantage is that the strong barrier to the spread of damage presented by the subassembly walls is no longer present.

The problems of this type of design are the effects of blockages between the fuel pins, and fuel pin failure propagation (as failure could possibly spread through the whole core). We need to know:

- a. The effects of a blockage on coolant flow and temperature distributions. (How big a blockage is needed to cause boiling? Or fuel pin failure? How does the boiling coolant behave?)
- b. What happens when the fuel pin fails. (How does the fuel move? How far can the damage spread?)

UNLIKELY ACCIDENTS

These are accidents with a low probability of occurring, and so we are only concerned when the consequences are serious, involving at least breach of the primary coolant system. If the reactor is shut down successfully and the primary containment stays intact indefinitely, the amount of damage done to the core, for example, is of less interest.

In practice this means that our interest is confined to super-prompt-critical excursions, arising, for example, from one of the whole-core accidents mentioned above, but now with coincident failure to shut down. In such a case the problem is to predict:

- Whether prompt critical is reached, and the rate of change of reactivity at prompt critical.
- The amount of energy released (as work and heat) and the amount of damage done by the super-prompt-critical excursion or excursions.

- The behavior of the hot debris after the super-prompt-critical burst, and the possibility of more heat being turned into work to do more damage.
- The amount of radioactivity (fission products, plutonium, radioactive sodium or other materials) released from the primary containment.

Before Prompt Critical

It is important to predict the rate of change of reactivity at prompt critical accurately (i.e., to within about 30%), because the magnitude of the subsequent excursion is strongly dependent on it. Prompt critical may be reached by, for example, control rod movement or the movement of a large gas bubble, but the most difficult cases are when coolant ejection due to boiling, or the motion of fuel as it melts or breaks up, contribute to the reactivity changes.

In the case of an accident initiated by a coolant flow failure with failure to shut down, the coolant will boil and be ejected from at least part of the core before the fuel collapses. For a reactor like one of the present 1000 MW(e) designs, for example, this may cause the reactor to go prompt critical, with the rate of change of reactivity depending on the velocity and void distribution in the transient two-phase coolant flow alone.

If on the other hand, the sodium void reactivity is less positive, or if it is found that coolant boiling causes fuel failure to follow very quickly, or if the accident is initiated by a rapid power transient, the cladding may fail and the fuel melt so that movement of fuel contributes to reaching prompt critical. In such a case the rate of change of reactivity will depend mainly on the motion of the fuel as it disintegrates in the presence of boiling coolant. This motion may be complicated by vapor explosions if hot fuel and liquid coolant come into contact.

In either case, however, there is a lot of inherent randomness: in the location of hot spots and vapor nucleation sites, in the void distribution in two-phase flow, in the behavior of the cladding as it fails and of the fuel as it cracks and melts, and in the fuel movement. This randomness may always prevent an accurate prediction of the rate of change of reactivity at prompt critical.

We need to know:

- a. How to characterize the transient hydrodynamics of coolant ejection from a subassembly by boiling (particularly the velocity and void distribution)
- b. How to predict the motion of broken and melting fuel, either with or without the presence of boiling coolant. (This is a crucial question, about which

: know very little. The first need is for some qualitative information on the sort of motion which may be expected.)

c. The range of uncertainty to be expected in reactivity rate calculations based on a model of this fuel motion.

The Super-Prompt-Critical Excursion

After the reactor goes prompt critical, energy is generated very rapidly until the reactivity transient is ended by the Doppler effect or by material motion. If the Doppler effect is large, the reactor may pass through prompt critical more than once before the excursion is finally terminated.

We have to predict the amount of energy released in the excursion, which appears either as heat or as kinetic energy of the core materials and their immediate surroundings, and the amount of damage this kinetic energy can do to the reactor containment, both as functions of the rate of change of reactivity at prompt critical. Ideally we would like to know two important prompt critical reactivity rates: that which just results in breach of the primary containment, and that which just breaches the secondary containment. If the prompt critical reactivity rate indicated for a certain accident which is unlikely, (but not so unlikely that the probability of its occurrence is negligible) by a meltdown calculation is higher than the latter, the reactor is unsafe; if it is lower than the former, the reactor is reassuringly safe for that accident.

Calculation of the energy generated in the excursion involves coupled neutronics and hydrodynamics, allowing for feedback from the Doppler effect and the disassembly under the influence of very high pressures caused by the rapid nuclear heating. Difficulties arise from the lack of equation of state data for the core materials and in the treatment of the hydrodynamics. The latter is particularly difficult in the case of relatively mild excursions when the strength of the core structure is important.

Damage to the containment and structure by the excursion itself is caused by shock waves generated during the high pressure disassembly of the core, and by the pressure of the expanding vaporized fuel. The effects of these on the structure (including effects such as sodium hammer, etc.) can be studied experimentally using model tests and chemical explosives, provided we understand the equivalence of chemical and nuclear explosives in this respect.

We need to know:

a. Whether our excursion calculations are adequate (i.e., in the treatment of transient neutronics, three-dimensional hydrodynamics, the effects of re-

straints, etc., both for cores which are intact and cores which are damaged at the start of the excursion.)

b. Equation of state and Doppler coefficient data.

c. How much damage is done to the containment. (Is the vessel broken? Is the plug lifted? Are model experiments using chemical explosives adequate?)

After the Excursion

After the super-prompt-critical excursion the core debris is very hot, and we wish to know whether it can do more damage. For example, it might cause violent boiling in the cold coolant which surrounds it, and this might breach the primary containment even if it survived the excursion itself intact, or it might produce missiles which could penetrate the secondary containment. Alternatively, ordinary boiling of the coolant may be enough to pressurize and breach an otherwise intact primary containment.

If the primary containment is breached, either during or after the excursion, we wish to know how much radioactivity escapes. Thus, we need to understand the behavior of fuel or fission products released into the coolant or blanket gas, the behavior of the coolant and gas within the residual core and vessel structure, and especially any dispersal mechanisms such as sodium fires.

Lastly, we wish to know the final disposition of the core materials so that appropriate disposal measures can be prepared, and to assure ourselves that it can be contained and cooled indefinitely.

We need to know:

a. How the core debris behaves. (Can it boil the coolant and pressurize the containment? Can it cause a vapor explosion? Can such an explosion produce damaging missiles? Can coolant or blanket gas be forced out of the primary containment?)

b. How much radioactive material can be released from the fuel debris. (How much fuel is suspended or dispersed in the coolant? How many fission products are retained in the fuel, dissolved in the coolant, or released to the blanket gas?)

c. What the effects of a sodium fire are. (Does it damage the secondary containment? Can the radioactivity it releases be handled? How can it be prevented or minimized?)

d. What happens finally to the debris. (Can it be cooled and contained indefinitely? Is a critical reassembly possible?)

THE MAIN SAFETY PROBLEMS

We can now take all these areas where more information is needed (labeled a, b, c, etc., and introduced by the phrase, "we need to know:" in the previous

section) and summarize them in a list of all the problems about which more knowledge (to be gained experimentally or theoretically, in- or out-of-pile) will materially improve our understanding of LMFBR safety. This is not intended to be a detailed account of all the information needed, but as a survey of the whole field.

MOST IMPORTANT PROBLEMS

These are significant unresolved problems about the safety of a fast reactor which could have a strong influence on the reactor design, and which conceivably could jeopardize our ability to build an adequately safe system.

Fuel Pin Failure and Propagation

We need to understand the behavior of a fuel pin as it fails, and whether the failure can propagate.

This requires an understanding of the "metallurgy" of fuel and cladding, including topics such as fuel swelling, cracking, fission product gas release, and sodium logging; and cladding swelling, ductility, and hardening. This must enable us to predict the effect of burnup or previous maloperation on the mode of failure, and the behavior of unvented fuel or vented fuel in which the vent has become blocked. Typically we need to know how big a hole is made in the cladding, how much fuel and gas are released, whether the pin is bent, and what damage is done to other pins. We must know the mode of pin failure caused either at steady power by manufacturing defects or slowly-developing temperature perturbations, or in power or coolant flow transient situations. These questions must ultimately be answered both for oxide fuel and for more advanced ceramic fuels.

This question must be answered to eliminate the possibility that a very likely event (pin failure) may cause significant local damage (needing replacement of a subassembly), or even damage to the whole core (if failure can spread beyond one subassembly).

This is a persistent question of reactor safety in the sense that fuel failure and failure propagation will always be limits to the safe operation of a reactor. This is true for metal fuels, for example, as well as for possible future fuel configurations such as coated particles or packed beds.

Motion of Molten Fuel

We need to know how the fuel moves as it breaks up or melts in the presence of boiling coolant. We want to know whether it can collect at the core center, and if so how fast, or whether it is inevitably or probably swept out of the core. We are seeking a model of a process of which at present we have no

understanding. If we can make such a model we to know how reliable it is and whether we can be certain of conclusions based on it. We wish to understand the process for reactors either with or without solid subassembly walls.

This problem needs to be solved to enable us to predict whether a super-prompt-critical excursion is possible in the case of, for example, a coolant flow failure with coincident failure to shut down, and if it is, to predict the rate of change of reactivity. This determines the ability of the containment to handle this type of accident. In addition, if whole core accidents with coincident trip failure (which we have called "unlikely" here) come to be thought of as likely in the future, this problem becomes more important because it then determines the possibility of a likely accident leading to a super-prompt-critical excursion.

This is a persistent question in that it will be important for any type of fuel. Once it has been understood for oxide, however, it should be simpler for other ceramic fuels.

Severe Excursions

We need to know whether our super-prompt-critical excursion calculation methods are adequate to predict the amount of energy released, and whether they are over-pessimistic. We wish to be certain that our treatment of the transient neutronics and hydrodynamics is good enough, and particularly in the case of mild excursions whether we can calculate the effects of the strength of the reactor structure well enough.

This problem needs to be solved to give a less pessimistic basis for containment design, and to show whether the risk of complete breach of the containment is acceptable. It is a persistent problem which will be of interest for any fast reactor.

Molten Fuel-Coolant Interaction

We need to understand the violent boiling or "vapor explosion" phenomenon in order to make realistic predictions as to the conditions under which an "explosion" can occur with hot fuel or steel and sodium. (Is it possible on the scale of a single fuel pin, a single subassembly or the whole core?) This understanding should also allow us to make realistic, as opposed to very pessimistic, estimates of the efficiency of conversion of heat to work and thus of the damage capability.

This question needs to be answered to remove the concern that a likely accident (single subassembly blockage) may damage the whole core and possibly even cause a super-prompt-critical excursion (which would make the reactor unacceptable from the safety standpoint). It also has an important bearing on the

bility of fuel pin failure propagation, and on the ability of the containment to withstand a mild super-prompt-critical excursion (if a vapor explosion can be caused by the core debris).

If the question of vapor explosions is answered, it should be possible to eliminate the need (which pessimistic predictions indicate) for certain engineered safety features, such as very rapid boiling detectors, or strong subassembly walls. These would be hard to design (as they would have to function under very stringent conditions), and might have an adverse effect on the economics of the system.

Although this question is of great importance at present, this importance is not permanent, because once an understanding of the basic mechanism or mechanisms has been gained, and reliable predictions of the probability and consequences of vapor explosions can be made, this understanding will be applicable to any reactor design.

Ultimate Containment

We need to understand the behavior of the core debris after a super-prompt-critical excursion and in particular its final disposition and the effects of fission-product-decay heating. This is necessary in order to design some method of cooling and containing the debris indefinitely to prevent an ultimate release of radioactivity even if the containment survives the excursion itself.

This problem has to be solved to prove that unlikely accidents resulting in a super-prompt-critical excursion do not pose an unacceptable risk.

This is unlikely to be a permanent problem, because once it has been shown that an adequate containment and cooling system can be built, the same system should be applicable to any reactor.

Coolant Boiling

We wish to understand superheating in sodium, the hydrodynamics of ejection of coolant by boiling from a subassembly (other than by a vapor explosion), the two-phase flow which ensues, and the reentry of liquid into a voided subassembly.

The ejection phase is of great importance in predicting prompt criticality in a reactor with a large positive coolant void coefficient of reactivity. It is largely understood, at least for single tube geometry. If reentry while the fuel is still hot proves to be impossible, the danger of a vapor explosion in a subassembly blockage accident is reduced.

This question of coolant boiling is of rather less urgency than the first four, because the large amount of work which has been devoted to it has resulted in a good understanding of many aspects. It is included

among the "Important Questions," however, mainly because of the importance of reentry, an aspect which is not yet completely understood. In addition, the dynamics of boiling coolant is closely tied up with other problems, particularly Motion of Molten Fuel and Molten Fuel-Coolant Interaction.

Instrumentation

The development of instrumentation is not usually counted as a safety problem, but it is of great importance to safety. An acoustic or flowmeter boiling detector, for example, would be very useful if it could act fast enough to prevent either pin-to-pin or subassembly-to-subassembly propagation of damage. A fast-acting fission-product detector would also be of importance.

In general, the development and testing of means of detecting abnormalities in the core, especially overheating, loss of integrity, or the presence of foreign material, will always be of importance in safety.

OTHER PROBLEMS

In addition to these main problems there are many others, of which we list the most important here. They are not so important as the first seven, either because they do not have such a strong bearing on reactor design, or because they have been largely solved already.

Doppler Effect

This has to be understood in order to predict behavior in a super-prompt-critical excursion. The present understanding is probably adequate, but some further work is needed.

Sodium Void Reactivity

This has to be calculated accurately to predict behavior in loss-of-coolant accidents. An important problem is the complex geometry the coolant void may adopt.

Explosion Damage

Given the amount of energy generated in a super-prompt-critical excursion we need to be able to predict the amount of damage it can do to the containment. This requires knowledge of the response of the core and blanket structure, the reactor vessel, the plug, and the biological shield, to the shock waves produced.

Post-Burst Behavior of the Core Debris

The behavior of the debris after a super-prompt-critical excursion is not certain. There may be a possibility of a large vapor explosion which could damage

the containment, and there may be other problems. The main need is for a model of what might happen.

Fission-Product Behavior

After a super-prompt-critical excursion fission-products and fuel material may be dispersed from the fuel pins. As they may then be able to escape from the containment, we wish to understand their behavior, and especially the extent to which they may be dissolved and trapped in the coolant.

Equation of State

We need to know the pressure and temperature of the core materials—especially the fuel—as functions of volume and energy content during super-prompt-critical excursions. These data are not known accurately at high energy contents, and so better data may improve the accuracy of the explosion calculations. It may be, however, that the results are not in fact very sensitive to equation of state data.

III-13. Development of Model and Calculational Approach for Multiple Channel Sodium Boiling Calculations

D. R. MACFARLANE

INTRODUCTION

Because of the current limitations in the SAS1A code¹ in treating only the hot channel in the heat transfer and coolant dynamics calculations, considerable effort has been directed toward the development of a multiple core channel model which explicitly calculates the conditions in a number of channels located at different radial positions with lower power densities than the hot channel. This is a considerable improvement over the approximate scheme presently employed in SAS1A wherein an approximate delay time for the initiation of boiling and subsequent coolant voiding is calculated in an attempt to account for the radial incoherence. The multichannel approach gives a much more realistic description of the radial effects across a large core since, even though a relatively few number of channels are calculated, it is an explicit calculation of the detailed temperature distributions, flow rates, and pressures for each of the individual channels.

This improvement in the SAS1A model has application primarily in calculating the so-called whole core transients (i.e., gross core flow reduction and/or reactivity insertions). The code in its present form is adequate to handle the local incidents (i.e., single or few subassembly flow blockage) up to the point where fuel failures and subsequent fuel-coolant interactions may lead to propagation of damage outside of the subassembly. The multiple channel capability has not been incorporated into SAS1A as yet and the results reported here were obtained with a separate and simpler program used for debugging and testing.

COMPUTATIONAL MODEL

The approach taken here is the direct and obvious one in which the calculations for fuel and coolant temperatures, etc., are repeated in complete detail for each of the additional parallel channels considered, in succession. This, of course, means that the computing time and memory requirements increase directly as the number of additional channels to be computed increases. Initial conditions are established by maintaining the power-to-flow ratio selected for the hot channel constant in all the lower power channels. In setting up the initial conditions, the flow rates for the lower power channels are calculated in accordance with their power generation rates to maintain a constant coolant outlet temperature. An orificing coefficient is then calculated for each channel such that this initial flow rate is obtained with the coolant inlet plenum pressure which is available. Thus, the transient analysis is the same as that being used in the single hot channel calculations in that it is assumed that we have parallel channels which respond independently to the imposed plenum-to-plenum pressure drop. There is no cross-flow or other interaction of this type between the separate channels (subassembly cans).

The two-phase flow hydrodynamics model for analysis of coolant voiding is the same as that presently used in SAS1A, and it can handle the coolant dynamics from initiation of boiling through and somewhat beyond flow reversal at the channel inlet. Most of the modifications to the computing algorithm are straightforward; that is, it has been adapted

the single channel calculation so that the series of calculations for additional channels run in parallel, with each channel keeping track of its own real time separately. In order to reduce the running time for the multiple channel calculations, modifications have been made in the algorithm so that a coarser space mesh can be used in the coolant channel than is possible with the present two-phase voiding model employed in the SAS1A code. At the present time in SAS1A the major space node in the coolant channel must be 1 cm. This has been changed in the multichannel calculations so that an arbitrary size for the major space node can be used. The reduction in computing time thus achieved for each individual channel is quite important when running a multichannel problem. A further modification which has been included is the addition of a variable fluid friction factor in the coolant dynamics portion of the calculation. Thus, the user may input the friction factor desired in accordance with the particular channel characteristics of a given subassembly design. This is useful because one may wish to include the pressure drop effect due to spacer wires, separation grids, and other items of this sort which are found in practical fuel subassembly designs. At the present time this friction factor is the same for all channels; however, each channel has its own saturation temperature profile, as determined by the channel pressure gradient just prior to the initiation of coolant boiling. In general, the pressure at a given point in the lower power channels tends to be lower because of the lower flow rate. This results in slightly lower saturation temperatures prior to boiling in these channels.

As stated, the multichannel program treats each channel separately and follows the real time for each channel separately. Therefore, additional work is required to incorporate the model into the SAS1A code. This will involve a cross-correlation between channels so that the feedback contributions from all channels for reactor kinetics would be available at a particular time.

NUMERICAL RESULTS

For purposes of defining the time scales involved in some of the typical reactor power and flow coastdown transients, a series of problems was run for a reactor core which has the parameters given in Table III-13-I. In this particular series of calculations there were two channels, in addition to the hot channel, with power densities 0.8 and 0.6 of the hot channel, respectively. The results summarized in Fig. III-1 give some idea of the time delays for initiating boiling for different power ramp increases. The ab-

TABLE III-13-I. PARAMETERS FOR REACTOR CORE

Core length, cm	100
Blanket thickness, cm	30
Fuel rod diameter, cm	0.635
Coolant volume fraction	0.50
Coolant inlet temperature, °C	500
Coolant outlet temperature, °C	626
Coolant velocity ($t = 0$)	
Channel No. 1, m/sec	8.7
Channel No. 2, m/sec	7.0
Channel No. 3, m/sec	5.2
Peak fuel thermal rating ($t = 0$)	
Channel No. 1, kW/cm	0.40
Channel No. 2, kW/cm	0.32
Channel No. 3, kW/cm	0.24
Channel exit plenum pressure, atm	3.0
Fuel-clad gap thermal conductance, W/cm ² -°C	0.8

scissa is the time required to reach boiling, and hence points at the shorter times represent more rapid power ramps. The ordinate represents the time delays in milliseconds between the hot channel (No. 1) and channels No. 2 and No. 3, respectively. The times to initiation of boiling in the hot channel range between about 0.5 and 3.5 sec. Curves are plotted in Fig. III-13-1 for three values of the fluid friction factor, namely 0.2, 0.4, and 0.6. The 0.2 value is roughly that which is built into the present coolant dynamics module in the SAS1A code, and the higher values represent coolant channels with larger frictional pressure gradients. As shown in Fig. III-

FIG. III-13-1. Boiling Time Delays for Coolant Channels.
ANL Neg. No. 113-3028.

FIG. III-13-2. Flow Reversal Times for Coolant Channels.
ANL Neg. No. 113-3018.

13-1, the time delays between the hot and the two lower power channels are not strong functions of the fluid pressure drop (friction factor) but, rather, are more sensitive to the channel power density (relative to the hot channel) and the rapidity of the transient. Similarly, as shown in Fig. III-13-2, the time for coolant expulsion (start of boiling to flow reversal) is more sensitive to the channel power density and rate of power rise than to the coolant pressure gradient. The results summarized in Figs. III-13-1 and III-13-2 are expected to be typical for a large oxide-fueled fast breeder core and should give a use-

TABLE III-13-II. TIME DELAYS CALCULATED IN FLOW
COASTDOWN TRANSIENT

Fluid Friction Factor	Time to Boil, sec	Time Delay, millisec		Time From Start of Boiling to Flow Reversal, millisec		
	Ch 1	Ch 2	Ch 3	Ch 1	Ch 2	Ch 3 ^a
0.2	4.979	267	680	68	60	—
0.4	5.318	316	783	73	55	—
0.6	5.495	340	837	94	91	—

^a Numerical difficulty was encountered in the low flow channel prior to flow reversal.

ful frame of reference for estimating the time delay for boiling initiation in lower power density channels relative to the hot channel.

Results of the multiple channel calculation for a flow coastdown transient with the same core used in performing the calculations summarized in Figs. III-13-1 and III-13-2 are presented in Table III-13-II. The coastdown rate was such that approximately 50% flow was achieved in 2 sec on an exponential flow decay.

In performing these calculations some difficulty with numerical stability was encountered in the lower power density channels prior to boiling. In order to get around the difficulty it was necessary to limit the time steps to about 15 msec in the cases with the lowest flow and highest pressure gradient (friction factor). This is believed to be related to previously encountered problems with the single channel calculation wherein a stability limitation was encountered when the entrance and exit pressure loss terms were added to the pressure-drop calculation in the program.

CONCLUSIONS

The method used here for estimating the radial dependence of events in a large fast breeder core via explicit calculation of parameters in a few representative channels in different zones of the core gives satisfactory results. It was possible to reduce the running time for a single channel calculation by modifying the extremely fine coolant mesh (1 cm) requirement presently in SAS1A to permit a coarser axial coolant mesh specification. In any event it is impractical, even with these changes, to consider calculations wherein more than perhaps six representative coolant channels are calculated in this manner.

The calculated results indicate that the most important parameters in determining the magnitude of time delays are the channel power density (i.e., peak fuel thermal rating) and the rate of rise of power in the channel. Incorporation of this multichannel analysis into the SAS1A code should lead to a more realistic description of the gross core behavior during reactivity input transients and whole core coolant flow perturbations than is presently possible. This modification, however, will not change the present capability of the code in calculating local or few subassembly incidents such as a local flow blockage.

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-14. Reactivity Transients Leading to Disassembly in Oxide Fueled Fast Reactors

D. R. MACFARLANE, T. J. HEAMES, N. A. MCNEAL, W. T. SHA AND C. K. YOUNGDAHL*

INTRODUCTION

The work described here is a continuation of previously reported calculations^{1,2} on the behavior of large (1000 MWe) sodium cooled fast breeder reactors during sodium boiling accidents. This work is part of Argonne National Laboratory's fast reactor safety program, which has as one of its goals the development of computational models and techniques for the analysis of the response of large fast breeder reactors to coolant flow and reactivity perturbations. Presumably this will lead to a better understanding of the various feedback mechanisms which are likely to be important in such reactor cores. The SASIA computer code, which was used to perform the calculations summarized here, is the result of efforts toward performing a complete calculation of reactor core dynamic behavior (including heat transfer and a fairly detailed description of neutronic feedback mechanisms) in a single integrated computer calculation. Since the physical models and mathematical techniques used in the analysis have already been described in detail elsewhere³ this will not be repeated—except to mention a number of significant additions and changes which have been made since the last annual report.

Basically the model and calculations are aimed toward analyzing transients which are slow enough so that there is time for heat transfer to the coolant before the threshold for fuel subassembly damage is reached. Accordingly, in both power and flow coast-down transients, we are concerned with cases in which the time to reach coolant boiling after the start of a flow or power perturbation from the normal steady state operating conditions, is in the range of 0.5 to 10 sec. Once the threshold for coolant boiling is reached the expulsion of sodium occurs fairly rapidly. Voiding times are typically less than several hundred milliseconds.

ADDITIONS TO CALCULATIONAL SCHEME

In previous annual reports^{1,2} the results of calculations were reported using a computer model in which solutions were obtained to the coupled equations of coolant hydrodynamics (including voiding from sodium boiling), fuel heat conduction, and neutron

kinetics. The results of these calculations showed that relatively slow transients initiated by modest reactivity ramps (\$0.50 to \$2.0/sec) and resulting in sodium boiling could then lead to more rapid reactivity insertions wherein the positive voiding reactivity effects could override the negative Doppler effect and core expansion feedbacks and make the reactor prompt critical. The calculations reported in the last annual report were actually performed with a preliminary version of the SASIA code which consisted of just the fuel and coolant heat transfer and point kinetics (with feedback) portions of the code. In the final version of the SASIA code, which has been used for the calculations reported here, the transient calculations have been extended by adding a weak explosion calculation for core disassembly. This calculational module, which is a fairly extensively modified version of the MARS Code developed by Atomic Power Development Associates, Inc. (APDA),⁴ is called to estimate the disassembly energy yield when certain criteria of fuel temperature or reactor period have been met in the initial portion of the calculation. In addition, the present calculations are much more detailed in the pre-disassembly portion in that a fuel mechanics module has been added which calculates stresses in the fuel and cladding and gives an estimate of when the cladding fails. In summary, some of the salient features of the present SASIA code are:

1. Sodium voiding is calculated explicitly within the code when the coolant temperature exceeds the saturation temperature. A corresponding sodium voiding feedback reactivity is computed using tabular input values of sodium density reactivity as a function of axial core position.

2. The fuel heat transfer model can treat fuel pins which have a central void and, in addition, incorporates a reasonably rigorous treatment of the non-linearities introduced when thermal conductivity varies strongly with temperature.

3. A fuel deformation module computes the stresses and deformations occurring during transients and allows elastic as well as plastic deformation of the cladding.

4. The moment of possible fuel cladding rupture can be badly miscalculated if the possibility of relief of internal pressures (calculated by the fuel deformation module) due to the displacement of

* Reactor Engineering Division, Argonne National Laboratory.

highly molten UO_2 is not taken into account. The fuel dynamics module attempts to do this and since cladding rupture could lead to fuel-coolant interaction and very rapid sodium expulsion and reactivity change, this could supply important input to a future-fuel-coolant interaction module.

5. The reactivity feedbacks due to Doppler effect and axial and radial core expansion are computed. There is also an option for adding a reactivity input due to control rod scram.

6. A two-dimensional (r, z) weak explosion routine is included as an integral part of the code and is called to calculate the energy release in a disassembly when certain criteria (specified as input by the user) are met.

Thus, in developing the code an attempt has been made to include all the important phenomena (with the exception of the fuel-coolant interaction) which would be likely to occur in transients of the types specified above.

NUMERICAL RESULTS

The results presented here are the first which include the MARS weak explosion module as an integral part of the calculation. As part of a program of testing the capability of the code we made a comparison between two typical oxide-fueled LMFBR 1000 MWe designs with respect to their response to

FIG. III-14-1. Sodium Void Worths as a Function of Position for the 50 and 100 cm Cores. ANL Neg. No. 113-2399.

reactivity and flow perturbations. The parameters for these two cores are summarized in Table III-14-I. As shown by the data in the table the cores are basically similar except that one core is 50 cm high and the other is 100 cm high. Both cores produce about 2000 MW of thermal power for the design conditions chosen. The sodium void coefficients for these two cores are given as a function of position in Fig. III-14-1. The net reactivity for voiding the core and axial blankets of the 50 cm core is essentially zero whereas for the 100 cm core it is positive \$2.60. These curves are typical of the behavior found in the large LMFBR designs wherein the coefficient is negative in the blanket and becomes increasingly positive toward the center of the core. Fuel material reactivity worth curves for the weak explosion calculation were obtained by a detailed series of calculations using the DIFF-2D diffusion theory code which is part of the ARC system at Argonne.

A summary of the results of calculations for reactivity ramps of \$1.0/sec, \$2.5/sec, and \$4.0/sec are presented in Table III-14-II. In performing these calculations the switchover to the MARS weak explosion calculation was made at a peak core temperature of 5000°C. As shown by the results, the integrated power for the weak explosion portion of the transient is approximately the same for the two cores for all the ramp inputs. That is, an integrated power in the neighborhood of 1000 MW-sec with a disassembly time of approximately 15 msec was obtained for all cases, except the \$4.0/sec ramp with the 100 cm core. In this latter case the energy yield of 6279 MW-sec is subject to question because of the fact

TABLE III-14-I. PARAMETERS FOR LMFBR CORES

	Flat Core	High Core
Core length, cm	50	100
Axial blanket thickness, cm	30	30
Core diameters, cm		
Zone 1	105	75
Zone 2	150	105
Fuel rod o.d., cm	0.650	0.650
Coolant inlet temp, °C	500	500
Coolant outlet temp, °C	630	630
Coolant velocity at steady state, m/sec	6.0	10.3
Maximum fuel thermal rating, kw/cm	0.46	0.46
Channel exit plenum pressure, atm	3.0	3.0
Core volume fractions		
Sodium	0.47	0.47
Stainless Steel	0.18	0.18
Fuel ($\text{PuO}_2\text{-UO}_2$)	0.30	0.30
Void and Control	0.05	0.05
Doppler coefficient ($T dk/dT$)		
Sodium in	-4.5×10^{-3}	-5.8×10^{-3}
Sodium out	-2.3×10^{-3}	-2.9×10^{-3}
Delayed neutron fraction, β	3.0×10^{-3}	3.0×10^{-3}
Prompt neutron generation time, sec	4.7×10^{-7}	4.7×10^{-7}

TABLE III-14-II. RESULTS OF CALCULATIONS

Core Height, cm	Transient \$/sec	Time to Disassembly, ^a sec	React., \$	Duration of Disassembly, ms	P/P_0	Integrated Power, ^b MW-sec	Peak Temp., °K	Peak Press., atm
50	1.0	1.899	1.90	15.7	256	799	5390	26
	2.5	0.892	2.23	14.6	236	1131	5484	31
	4.0	0.554	2.22	8.6	2335	6279	6404	131
100	1.0	2.011	2.01	15.4	280	1084	5437	28
	2.5	0.986	2.47	15.4	365	1526	5594	37
	4.0	0.701	2.80	15.4	217	1058	5429	28

^a Switch to weak explosion at 5000°C.

^b Used vapor pressure equation of state of the same form as in the original MARS code.⁴

that a spuriously high pressure was obtained in the coolant channel after boiling started. This caused a rapid ejection of sodium thus leading to a more violent transient, as indicated by the results in Table III-14-II. The data in Table III-14-II are primarily directed toward describing the characteristics of the power pulse during the weak explosion disassembly. The column labeled P/P_0 is the peak reactor power during the disassembly normalized to the initial steady state operating power. The peak core temperature and pressure given in the last two columns typically occur in the core geometric center (Zone 1). An interesting feature of the results is that the characteristics of the power pulse during core disassembly is so similar (with the one exception noted) for all the cases presented. Also, the net reactivity inserted by the driving ramp when disassembly starts is roughly constant at about \$2 for all the cases.

Plots of net reactivity and power for the 50 cm core cases for the ramps of \$1.0/sec, \$2.5/sec and \$5.0/sec

(not exactly the \$4.0/sec case presented in Table III-14-II but qualitatively similar) are given in Figs. III-14-2 through III-14-4. In these calculations the temperature, pressure, flow rate, and voiding profiles are similar to those reported earlier^{1,2} and hence have not been included here. The curves in Figs. III-14-2 through III-14-4 are qualitatively similar, with the major difference being that the time scale is correspondingly compressed for the steeper reactivity ramp inputs. Some major features of the curves are:

1. There is an initial reactivity dip when enough fuel melting and corresponding volume increase has taken place to fill the void spaces in the fuel and the gap between the cladding and the fuel, and hence begin to produce fairly significant reactivity feedback due to axial fuel expansion. This initial reduction of reactivity and power becomes successively more apparent and sharper in the more rapid ramp input cases.

2. Shortly thereafter, the start of sodium boiling

FIG. III-14-2. Net Reactivity and Normalized Reactor Power Versus Time, \$1.0/sec Ramp, 50 cm Core. ANL Neg. No. 113-2401.

FIG. III-14-3. Net Reactivity and Normalized Reactor Power Versus Time, \$2.5/sec Ramp, 50 cm Core. *ANL Neg. No. 113-2402*.

FIG. III-14-4. Net Reactivity and Normalized Reactor Power Versus Time, \$5.0/sec Ramp, 50 cm Core. *ANL Neg. No. 113-2398*.

produces the initial sharp negative reactivity change because boiling begins near the upper core-blanket interface where the void reactivity effect is negative.

3. Sodium boiling then progresses into the reactor core and eventually produces the positive reactivity ramp shown. This turns the reactor power upward again sharply. Typically, as shown by the curves, the reactivity approaches prompt critical but is held slightly below by the Doppler effect as the boiling continues to add reactivity.

4. The weak explosion calculation is called when the core maximum temperature exceeds the input value of 5000°C, and the core disassembly produces steady

reduction in reactivity which rapidly reduces the power.

The criteria for switching to the weak explosion calculation in the SAS1A code are either a high core fuel temperature or a short reactor period, whichever occurs first. The calculation is terminated when the core power drops to 95% of the original steady state power. It should be noted that the time scale is broken in Figs. 2-4 and expanded for the weak explosion portion of the transient in order to better show detail in this time region.

For purposes of comparison a problem was run for the 50 cm core and a \$1.0/sec reactivity ramp with all parameters the same except for the sodium void coefficients. In this case the values used were the same as had been used in previously reported calculations,⁵ namely, a net reactivity change for voiding the complete core and axial blankets of +\$3.70. The results of the calculation are presented in Fig. III-14-5. As shown, the net reactivity does go slightly above prompt critical, resulting in a peak reactor normalized power (P/P_0) of 1200 and a disassembly time of 10 msec. The peak reactor temperature and pressure were 6100°K and 85 atm respectively, while the integrated power during the disassembly portion of the transient was 4588 MW-sec. In comparing this result with the first line of Table III-14-II it is seen that the values of the sodium void coefficients input to the problem do have a significant effect on the result. This is as expected since they directly affect the rate of reactivity insertion due to sodium boiling. The rate of sodium boiling reactivity insertion at prompt critical ranges from a few dollars per second up to as high as \$40/sec (although the latter value was for the case of the \$4.0/sec ramp with the 50 cm core and was probably in error on the high side) in this series of power transients.

The corresponding reactivity and power plots for the 100 cm core cases are qualitatively similar to those presented for the 50 cm core. For example, the reactivity and power plots for the \$1.0/sec-100 cm core case presented in Fig. III-14-6 are similar to the plots presented in Fig. III-14-2 for the 50 cm core with the same reactivity ramp.

A pump coastdown problem was run for each of the two cores. In these problems, however, neither core got into the temperature range where the weak explosion calculation would be appropriate. The peak fuel temperature in these calculations (which were run for ~5 sec of real time) increased only 100-200°C and the power remained relatively constant or increased somewhat until boiling began. In the case of the 50 cm core the voiding pattern was such that there was no ramp due to voiding reactivity addition and

FIG. III-14-5. Net Reactivity and Normalized Reactor Power Versus Time, β 1.0/sec Ramp, 50 cm Core with Large Sodium Void Coefficient. ANL Neg. No. 113-2397.

FIG. III-14-6. Net Reactivity and Normalized Reactor Power Versus Time, β 1.0/sec Ramp, 100 cm Core. ANL Neg. No. 113-2396.

the power remained between 70–80% of the initial power level during coolant boilout. With the 100 cm core there were reactivity spikes due to coolant boilout as the various channels triggered but the net reactivity was held below prompt critical by the Doppler effect.

CONCLUSIONS

The core disassemblies predicted by this series of calculations are very mild in the sense that temperatures and pressures achieved are never large. That is, they are relatively slow sub-prompt critical excursions which do not yield very large amounts of ther-

mal energy, and since the pressures generated are only a few tens of atmospheres there is little likelihood of blast damage to surrounding structures. Of course, this conclusion could be altered when the calculations are repeated using a more realistic equation of state for the expanding core. In any event, there is a considerable problem in handling the post-disassembly phase of such an accident wherein we have a large mass of molten and partially vaporized core material which must somehow be cooled down. This part of the problem has not been treated in the present analysis.

As shown by these results, it is feasible to perform

a complete calculation of a reactor transient with an integrated computer code which includes a final reactor disassembly as the terminating event in the excursion. Furthermore the running time for a complete SAS1A reactivity transient of the type described is only 5–10 min on the IBM 360-75. Therefore it is quite feasible to make extensive parametric studies to investigate various safety aspects of proposed LMFBR designs.

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III-15. Coupled Neutronic-Hydrodynamic Power Excursion Analysis

W. T. SHA, T. H. HUGHES and P. M. MARGEL

INTRODUCTION

Evaluation of the safety of large fast breeder reactors has been the primary motivation for extensive theoretical investigation of super-prompt critical power excursions. Although it is a major concern of the fast reactor designer to prevent such an excursion under any adverse circumstances, nevertheless, it is desirable to know what the consequences would be if a super-prompt excursion did occur. The main objective of this report is to discuss a method for estimating the magnitude of the energy released by a reactor undergoing a power excursion. This method is relatively simple and yet reasonably accurate.

In general, there are three approaches to the analysis of reactor power excursions. The first approach uses only the space independent reactor kinetics to estimate the energy release. The second technique is known as Bethe-Tait analysis.¹ It is analytical in nature and has been used to derive scaling laws and check numerical procedures. The following three major assumptions are made in Bethe-Tait analyses: (1) Reactivity changes can be calculated by first order perturbation theory, (2) The duration of the excursion is so short that expansion is negligible, thus permitting the time behavior of the pressure to be calculated by ignoring any change in the density, and (3) The effects of wave propagation can be neglected. The third approach is to solve the system of governing time-space dependent partial differential equations numerically by using a high speed computer.

The first method is the simplest of all. However, it cannot accurately model the appropriate reactivity feedbacks during the excursion. The validity of this approach is very much in doubt. As for the second technique, a number of significant modifications³⁻⁶ have been made since Bethe and Tait's original paper appeared.¹ The modified Bethe-Tait analysis is to date the most widely used method for weak explosion study. However, one must bear in mind that the validity of this method is subject to the assumptions listed above. The third approach² provides a much more accurate and complete model. However, obtaining a solution of the complete time-space dependent coupled neutronic-hydrodynamic equations is no easy task. Experience with multi-group, time-space dependent, two-dimensional neutron diffusion theory calculations indicates that the direct numerical approach is very expensive and perhaps, at present, impractical for the "production-line" type calculation. Recently, very active research has been launched to develop approximations⁷⁻¹³ to the exact time-space neutronic solution with much less computational effort. It is believed that until a very general and efficient approximating method is found, the third approach will not be extensively used in the field of super-prompt critical power excursion analyses. The computational method presented in this report and used in the VENUS computer program¹⁴ is a compromise between the second and third approaches; consists of space independent neutronics and time-space dependent Lagrangian hydrodynamics (*R-Z*

(cylindrical geometry). The feedbacks due to Doppler broadening and reactor material motion (or disassembly) are explicitly taken into account. Thus, the computer time required for a typical power excursion analysis is kept within a practical range (a few minutes on an IBM-360 Model 75) and the reactivity feedback mechanisms are treated with reasonable accuracy during the excursion. In short, it retains simplicity and yet provides essential information with reasonable accuracy.

BASIC ASSUMPTIONS

The following are basic assumptions employed in the VENUS program:

1. The use of space-independent neutron kinetics in the VENUS program implies that the shape function (or spatial power density distribution) is assumed to remain constant throughout the power excursion. It is further assumed that the shape of the material reactivity worth distribution is also independent of time and displacements.

2. The reactivity change due to the motion of the core material is calculated by first order perturbation theory. These values are used throughout the disassembly accident.

3. The finite difference representation of the Lagrangian hydrodynamic equations, i.e., conservation of mass and momentum, will introduce truncation error. As the Lagrangian mesh becomes severely distorted, this truncation error is greatly amplified. Thus in order for the finite difference representation to be valid, the extent of the mesh distortion must not be excessively large.

It is to be noted that assumptions 1 and 2 are also employed in the modified Bethe-Tait calculations.⁵

COMPARISON BETWEEN VENUS AND THE EXISTING METHODS

A. VENUS⁽¹⁴⁾ VERSUS AX-1⁽²⁾

The following are three principal differences between the VENUS and the AX-1 programs:

1. VENUS is a two-dimensional power excursion program and AX-1 is one-dimensional.

2. VENUS accounts for the Doppler reactivity feedback.

3. In neutronics, the one-group space-independent model was employed in VENUS, whereas the time-space-dependent model was used in AX-1.

VENUS⁽¹⁴⁾ VERSUS BETHE-TAIT TYPE CALCULATIONS

As pointed out before, the most widely used method for estimating the total energy generation during a

disassembly accident is the Bethe-Tait type calculation. The method presented in this paper has the following three distinct advantages over the Bethe-Tait and the modified Bethe-Tait analyses.

1. The Bethe-Tait and the modified Bethe-Tait analysis methods employ an equation of state which is most appropriate to the disassembly of a reactor which has been entirely voided of sodium. This is partly because the Bethe-Tait method cannot, because of its basic structure, employ a density- and temperature-dependent equation of state. Pressure, however, can rise extremely rapidly with small changes in density as for "sodium-in" accident conditions. If the reactor accident under analysis should require this type of density-dependent pressure relation, the Bethe-Tait method cannot perform a meaningful disassembly calculation. That VENUS has overcome this limitation represents a major improvement in severe disassembly calculation capability.

VENUS also computes the development of momentum as a function of position. This information can be used to estimate conversion of nuclear energy to reactor damage, including missile damage.

2. Since the density is computed explicitly as a function of time, a temperature (or energy) density-dependent equation of state can readily be employed.

3. Another advantage is that the use of Lagrangian coordinates in the VENUS program provide detailed information on the motion of the reactor material during the excursion. This information is essential to the basic understanding of the mechanics of the accident and is vital to the assessment of possible damage to the surrounding structures.

A comparison of the results of VENUS⁽¹⁴⁾ and of

FIG. III-15-1. Core Characteristics. ANL Neg. No. 113-2416.

MARS⁽⁵⁾ calculations with regard to the total energy release during an excursion indicated that agreement between the two programs is good only for the low density systems.

FIG. III-15-2. Deformed Mesh Configuration at Peak Power. ANL Neg. No. 113-3116.

FIG. III-15-3. Deformed Mesh Configuration at $k_{eff} = 0.997$. ANL Neg. No. 113-3115.

SOME NUMERICAL RESULTS

The following are some of the results from the VENUS program. Fig. III-15-1 presents core dimensions, power density profiles, and mesh set-up used in the VENUS program. Material reactivity-worth profile is similar to power density distribution. Figures III-15-2 and III-15-3 show the detailed motion of core material during the excursion at $t = 1.43$ msec (corresponding to peak power) and $t = 2.43$ msec (corresponding to $k_{eff} = 0.997$), respectively.

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III-16. A Compressible Model for Transient Sodium Boiling

E. SIEGMANN*

An important aspect of the Liquid Metal Fast Breeder Reactor (LMFBR) safety studies is the description of the behavior of sodium coolant during boiling and expulsion. A reduction of sodium density can give either a negative or positive reactivity effect, depending on the vapor void location in the core. In some cases the magnitude of the positive sodium void effect may be large enough to override the negative feedback contributions, producing conditions that could lead to prompt criticality. It is therefore important to know the time-dependent positions of the sodium voids accurately. Earlier sodium void models^{1,2} have used a single vapor region (slug),³ multi-slug region,⁴ or a two-phase region with or without slip.^{5,6} The use of a single vapor region is questionable under certain reactor conditions. If no appreciable superheats exist due to entrained bubbles, sufficient nucleation sites on fuel rod walls, or sufficient impurities, the expulsion could be expected to be a two-phase flow.

The equations⁷ describing two-phase flow are

Continuity:

$$\frac{\partial}{\partial t} [\alpha \rho_g + (1 - \alpha) \rho_L] + \frac{\partial G}{\partial z} = 0; \quad (1)$$

Momentum:

$$\begin{aligned} \frac{\partial G}{\partial t} + \frac{\partial p}{\partial z} + \frac{\partial}{\partial z} \left\{ \frac{G^2}{\rho_L} \left[\frac{\beta x^2}{\alpha} + \frac{(1-x)^2}{(1-\alpha)} \right] \right\} \\ + \frac{\partial p}{\partial z} \Big|_f + g[\alpha \rho_g + (1 - \alpha) \rho_L] = 0; \end{aligned} \quad (2)$$

Energy:

$$\begin{aligned} \frac{\partial}{\partial t} (\alpha \rho_g H_g + (1 - \alpha) \rho_L H_L) \\ + \frac{\partial}{\partial z} [G(x H_g + (1 - x) H_L)] - \gamma \phi = 0, \end{aligned} \quad (3)$$

where

$t \equiv$ time

$z \equiv$ axial position

$p \equiv$ pressure

$G \equiv$ total mass flow rate $= \alpha \rho_g v_g + (1 - \alpha) \rho_L v_L$

$\alpha \equiv$ void fraction = area of vapor/total area of channel

$x \equiv$ quality $= \alpha \rho_g v_g / G$

$v_L, v_g \equiv$ velocity of the liquid, vapor, respectively

$\rho_L, \rho_g \equiv$ density of the liquid, vapor, respectively

$H_L, H_g \equiv$ enthalpy of the liquid, vapor, respectively

$\frac{\partial p}{\partial z} \Big|_f \equiv$ frictional pressure gradient

$g \equiv$ gravitational constant

$\phi \equiv$ heat flux

$\gamma \equiv$ fuel rod circumference/coolant area

$\beta \equiv$ density ratio of liquid to vapor $(= \rho_L / \rho_g)$.

The following assumptions are incorporated into the above equations:

1) The cross sectional area of the coolant channel is constant.

2) The only spacial variable is in the axial direction.

3) Microscopic effects from the statistical motion of turbulent flow is incorporated in the empirically determined surface frictional stress.

4) No effort is made to handle individual bubbles but a slip model is employed to allow for different velocities between vapor and liquid.

5) Heat generation by radiation is neglected.

6) The change in the kinetic energy and the pressure variation have been neglected from the energy equations.

7) Heat transfer is in the radial direction only.

In previous work additional assumptions were introduced to facilitate solutions of these three equations. In the Transfuge I code,⁵ the flow was assumed homogeneous (vapor velocity = liquid velocity) and in thermodynamic equilibrium. The vapor density and the saturation temperature were held constant everywhere in the channel. These assumptions imply that the calculated void rate becomes essentially independent of the momentum equations. This model might underestimate the voiding time.

D. MacFarlane's model⁶ has the following improvements over Transfuge I. The reference pressure or saturation temperature varies along the channel as the steady state profile but is constant with respect to time. Inhomogeneous or slip flow is included with use of the Lockhart-Martinelli⁸ equation. The vapor density is calculated at each point in time and space. To remove numerical instabilities the momentum equation is decoupled from the other two equations by using the momentum integral form. This technique is valid for high-pressure systems, but is questionable for low pressure system (LMFBR's) since large pressurization effects are likely to occur. Thermodynamic equilibrium is again assumed.

The model being described here solves the three equations of mass, momentum, and energy simultaneously in the two-phase region. As in the MacFarlane

* Student, Northwestern University, Evanston, Illinois.

model, slip is included using the Lockhart-Martinelli equation (or specified constant slip) but the saturation temperature (pressure) and the other sodium properties are calculated at every point in time and space. This model also accounts for compressible effects of the coolant.

METHOD OF SOLUTION

The method of characteristics⁹ is used to transform the three partial differential equations to three ordinary differential equations which are only defined on the three characteristic lines (or simply characteristic) in the time-space plane. This transformation is only possible when the three original equations are hyperbolic. The following procedure is employed to make this transformation. Each of the original equations [Eqs. (1), (2), (3)] is expanded and regrouped to form three quasi-linear partial differential equations with the dependent variables being pressure, mass flow rate, and void fraction. The densities and enthalpies are given functions of the saturation pressure only and the quality x is specified as a function of void fraction and pressure. The three equations (labeled L_i) are now of the form

$$L_i = A_{i1} \frac{\partial p}{\partial t} + A_{i2} \frac{\partial G}{\partial t} + A_{i3} \frac{\partial \alpha}{\partial t} + B_{i1} \frac{\partial p}{\partial z} + B_{i2} \frac{\partial G}{\partial z} + B_{i3} \frac{\partial \alpha}{\partial z} + C_i = 0, \quad i = 1, 2, 3, \quad (4)$$

where A_{ij} , B_{ij} , C_i are functions of p , G and α but not of their derivatives. The substitution $\frac{\partial p}{\partial t} = \frac{dp}{dt} - \frac{\partial p}{\partial z} \frac{dz}{dt}$ and similar substitutions for $\frac{\partial G}{\partial t}$ and $\frac{\partial \alpha}{\partial t}$ are made into Eq. (4). The equations are each multiplied by unknown constants λ_1 , λ_2 , λ_3 , then added and rearranged producing

$$\begin{aligned} \lambda_1 L_1 + \lambda_2 L_2 + \lambda_3 L_3 = & (\lambda_1 a_{11} + \lambda_2 a_{21} + \lambda_3 a_{31}) \frac{dp}{dt} \\ & + (\lambda_1 a_{12} + \lambda_2 a_{22} + \lambda_3 a_{32}) \frac{dG}{dt} \\ & + (\lambda_1 a_{13} + \lambda_2 a_{23} + \lambda_3 a_{33}) \frac{d\alpha}{dt} \\ & + (\lambda_1 C_1 + \lambda_2 C_2 + \lambda_3 C_3) \\ & - \left\{ \lambda_1 \left(a_{11} \frac{dz}{dt} - b_{11} \right) + \lambda_2 \left(a_{21} \frac{dz}{dt} - b_{21} \right) \right. \\ & \left. + \lambda_3 \left(a_{31} \frac{dz}{dt} - b_{31} \right) \right\} \frac{\partial p}{\partial z} \end{aligned} \quad (5)$$

$$\begin{aligned} & - \left\{ \lambda_1 \left(a_{12} \frac{dz}{dt} - b_{12} \right) + \lambda_2 \left(a_{22} \frac{dz}{dt} - b_{22} \right) \right. \\ & \left. + \lambda_3 \left(a_{32} \frac{dz}{dt} - b_{32} \right) \right\} \frac{\partial G}{\partial z} \\ & - \left\{ \lambda_1 \left(a_{13} \frac{dz}{dt} - b_{13} \right) + \lambda_2 \left(a_{23} \frac{dz}{dt} - b_{23} \right) \right. \\ & \left. + \lambda_3 \left(a_{33} \frac{dz}{dt} - b_{33} \right) \right\} \frac{\partial \alpha}{\partial z} = 0. \end{aligned}$$

To produce an ordinary differential equation, the coefficients of $\frac{\partial p}{\partial z}$, $\frac{\partial G}{\partial z}$ and $\frac{\partial \alpha}{\partial z}$ in Eq. (5) are put equal to zero. That is,

$$\begin{aligned} \lambda_1 \left(a_{1j} \frac{dz}{dt} - b_{1j} \right) + \lambda_2 \left(a_{2j} \frac{dz}{dt} - b_{2j} \right) \\ + \lambda_3 \left(a_{3j} \frac{dz}{dt} - b_{3j} \right) = 0, \quad j = 1, 2, 3. \end{aligned} \quad (6)$$

To determine the nontrivial solution of Eq. (6), the following determinant is expanded:

$$\begin{vmatrix} a_{1j} \frac{dz}{dt} - b_{1j} \\ a_{2j} \frac{dz}{dt} - b_{2j} \\ a_{3j} \frac{dz}{dt} - b_{3j} \end{vmatrix}_{j=1,2,3} = 0.$$

The expansion of this determinant gives a cubic equation in $\frac{dz}{dt}$. The solution of this cubic equation defines

three specific values for $\frac{dz}{dt}$ (the slope of a characteristic) such that Eq. (5) is an ordinary differential equation on each of these characteristics. The ordinary differential equation can now be defined by substituting the values of $\frac{dz}{dt} \Big|_k$, $k = 1, 2, 3$, into Eq. (5) and any two of the three Eqs. (6) and eliminating λ_1 , λ_2 and λ_3 . The final result will be three equations

$$A_j \frac{dp}{dt} + B_j \frac{dG}{dt} + D_j \frac{d\alpha}{dt} + E_j = 0, \quad j = 1, 2, 3, \quad (7)$$

where A_j , B_j , D_j , and E_j are functions of p , G , α and $\frac{dz}{dt}$ at a point in time and space.

The characteristic slopes $\left(\frac{dz}{dt} \Big|_k, k = 1, 2, 3 \right)$ have important physical meaning. In the case of homogeneous flow, the three characteristics are $U + a$, U , $U - a$, where U is the particle velocity and a is the sonic velocity in the fluid. The two characteristics defined by $\frac{dz}{dt} = U + a$ and $\frac{dz}{dt} = U - a$ represent the path along which a disturbance would propagate in time-space plane. When a slip is introduced, the three

characteristics approximately equal $U + a$, U , $U - a$, where U is some averaged velocity. If thermal equilibrium is assumed the calculated sonic velocities are much smaller than those experimentally measured. It has been shown¹⁰ that if $\left. \frac{\partial x}{\partial p} \right|_H = 0$, where $H = xH_v + (1 - x)H_L$, sonic velocities corresponding to those measured can be calculated. This assumption is used to produce characteristics which agree well with experimentally measured sonic velocities. If the averaged velocity exceeds the sonic velocity, the three characteristics are positive, implying that this is super-sonic flow and disturbance can propagate in one direction only. If with slip the vapor velocity exceeds the characteristic $U + a$, where U is an averaged velocity, the equations become elliptical and the characteristics become imaginary. This is understandable since it is contradictory to have a vapor velocity exceed the disturbance propagation velocity.

The three ordinary differential equations [Eq. (7)] are solved using a fixed rectangular time-space grid. The equations are solved iteratively at each point in time and space until a desired accuracy is obtained. The time step is initially determined by $\Delta t = \Delta z / (U + a)$ where Δz is the spacial mesh and $(U + a)$ is the greatest propagating velocity in the channel at steady state conditions. The time steps are not changed until a

propagation velocity is calculated that exceeds the largest steady state propagation velocity and then Δt is again calculated by the above equation. With the time steps so chosen, convergence is guaranteed but time steps are quite small (i.e., 0.25 msec).

Since the equations derived above are for two-phase fluid, the first problems considered are those for channels containing only two-phase sodium. The boundary conditions are:

1. Exit pressure is held constant.
2. The steady state mass flow rate is given.
3. The entrance void fraction is specified until flow reversal occurs, after which it is calculated.
4. Entrance pressure that is initially consistent with the steady state conditions can vary with time.
5. The heat flux is shaped and can vary with time.

The model has been applied to the calculation of power excursions, pump coastdowns, pressure steps, and pressure pulse problems. To demonstrate the capability of this model, a solution to a pressure pulse problem is shown in Fig. III-16-I. The steady state conditions were:

Entrance void fraction = 0.1

Mass flow rate = 60 g/cm²-sec

Entrance pressure = 3 atm

Heat flux = (1 cal/cm²-sec) $\times$ cos Z ,

where Z is the spacial position along the fuel rod. A

FIG. III-16-1. Pressure Pulse Propagation. ANL Neg. No. 113-2424.

square pressure pulse of 1.0 atm height and 0.84 msec width is introduced at the entrance at $t = 0$. The pulse propagates down the channel and the reflection (an expansion wave) travels back. As shown in Fig. III-16-I, the amplitude of the wave decreases due to dissipative forces and the edge of the pulse flattens. The pulse moves with a sonic velocity comparable to that measured by R. Henry et al.¹⁰ The lower curve (symbol Z, $t = 0.017278$ sec) shows the first reflection starting to propagate up the channel. This expansion wave will reflect at the entrance producing a small pressure pulse. Plots of the mass flow rate and void fraction are also obtained.

In an LMFBR the coolant will be a single phase subcooled liquid under normal operating conditions. Under abnormal conditions leading to boiling, the boiling will most probably start at some point in the coolant channel. This can be described as two incompressible subcooled liquid regions, each upstream and downstream of a compressible two phase region. In the liquid regions, the momentum and energy equations are used to calculate the axial pressure and temperature distribution. At the interface between the single and two phase regions, the mass flow rate, pressure, and temperature are matched and the void fraction is specified as some very small but finite number. Some numerical instabilities have been encountered in calculating interface location, mass flow rate, and saturation temperature; more work is needed in this calculation.

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III-17. Sensitivity Studies for Fuel Deformation Module of SASIA

A. K. AGRAWAL and D. J. DIXON*

INTRODUCTION

The Fuel Deformation module is a part of the integrated fast reactor accident study code SASIA.⁽¹⁾ The purpose of this module is to evaluate the fuel pin behavior during a reactivity or power transient. The analysis, based on a simple, perfectly elastic-plastic model of the cladding, requires the knowledge of thermo-physical and mechanical properties of both fuel and cladding which are rarely known under excursion situations. The objective of the present study is to investigate those properties which are rather important in the evaluation of fuel pin behavior and also to determine to what accuracy they need to be known.

* AUA-ANL Summer Engineering Practice School participant.

The parametric study for the Fuel Deformation module can be conveniently broken down into two sections: one involving fuel properties and the other cladding properties. While fuel thermal properties, as for example thermal conductivity² and specific heat values³, are known or are being made available, molten fuel density for practical fuel elements is not known to the desired degree of accuracy. Since fuel expansion due to melting causes a large pressure on the cladding, the change in volume due to melting is believed to be an important parameter for the present study.

The Deformation module of the code SA treats cladding as a perfectly plastic material when hoop stress exceeds the yield strength. The plastic

in or deformation of the cladding, therefore, depends on both yield strength value and its explicit temperature dependence. For this reason, the calculation of cladding temperature is also an important part of the present sensitivity studies. Assuming that the heat transfer coefficient for the film between clad and coolant to be known, cladding temperature is expected to be strongly dependent on bond (gap) conductance between fuel and cladding. Thus the present sensitivity study would include variation of yield strength and the bond conductance.

BASE CASE

The above mentioned sensitivity studies are performed using a recent experiment (S-3)⁽⁴⁾ on fuel meltdown studies with TREAT as the standard case. This five-pin autoclave experiment is approximated by an equivalent single pin of 10% enrichment and 93% theoretical density (T.D.) UO_2 fuel pellets. Unirradiated pellets of outer radius 0.320 cm are placed in 304 type stainless steel casing of 0.038 cm thickness with a clearance of 0.010 cm between the fuel and the cladding. The fuel pin is assumed to be cooled by an equivalent amount of stagnant liquid sodium. Due to the code requirements, the stagnant sodium is replaced by slowly circulating sodium. The TREAT transient had a peak power of 2600 MW and the full width at half maximum of 0.32 sec. This transient is equivalent to an energy input of 1920 J/g- UO_2 , which is well beyond the full melting of UO_2 under adiabatic conditions. A more detailed description of the experiment and results calculated for the standard case are given in Paper III-11.

RESULTS OF SENSITIVITY STUDIES

We have performed two sets of calculations using two different values for change in volume due to fuel melting. In the calculations performed for the assumed standard case, the molten fuel density of 93% T.D. pellets was taken to be the same as the density of 100% T.D. molten fuel. The resulting change in fuel volume due to phase change, therefore, amounts to 5.8% instead of 13.8% for 100% T.D. fuel. This large difference in volume change due to fuel melting is caused by the neglect of volume occupied by pores and voids in the solid fuel. The calculated plastic deformation of the clad for experiment S-3 is shown by curve 1 in Fig. III-17-1. In this figure, we also show the mass fraction of molten uranium oxide. These calculations have been repeated by giving a larger change in volume of molten uranium dioxide. If only half of the pores and voids which

FIG. III-17-1. Calculated Plastic Deformation of the Clad and the Fraction of UO_2 Melted as a Function of the Energy Produced in the Fuel for S-3 Experiment. Curve 1 Corresponds to the Base Case (5.8% $\Delta V/V$ Due to Fuel Melting) and Curve 2 Corresponds to the Assumed 9.7% $\Delta V/V$. ANL Neg. No. 113-2426 Rev. 1.

existed in the solid fuel are assumed to be filled due to fuel melting, we calculate the change in fuel volume due to melting to be 9.7%. For this case, the calculated plastic strain is shown by curve 2 in Fig. III-17-1. The mass fraction of molten fuel is unchanged since the rate of heat production and loss is not changed. As expected, both curves 1 and 2 are identical until uranium dioxide begins to melt (around 1030 J/g- UO_2). It is evident that a larger change in fuel volume due to melting results in an increased plastic strain of the cladding.

To further illustrate the consequences of volume change due to melting, we find that 5% plastic deformation of the cladding is achieved with an energy release of 1570 J/g- UO_2 for the second case instead of 1740 J/g- UO_2 for the standard case. Alternatively, if it is assumed that molten fuel density is unaffected by the initial porosity of the fuel and that it is equal to the molten fuel density of 100% T.D. uranium dioxide, the results shown in Fig. III-17-1 can be interpreted as due to different initial porosity of the fuel. In other words, curve 1 is the calculated plastic strain for 93% T.D. fuel while curve 2 corresponds to 96.5% T.D. fuel provided all other parameters are unchanged.

Retaining the explicit temperature dependence of cladding yield strength, strength values are increased by a factor of two from the base values. There appears to be no significant change in the plastic deformation except for a small delay in the initiation of

plastic flow. This observation is attributed to the proportionately small role of elastic deformation played in the rapid transient under consideration. It is anticipated that such a variation in yield strength values would be more important for slower transients. No efforts were made to study the sensitivity of clad deformation with respect to change in explicit temperature dependence of the strength values. In all of these calculations the high temperature yield strength values were obtained by a linear extrapolation.

Finally, the effect of varying bond conductance on cladding behavior is studied. In the heat transfer module of the code the bond conductance is expressed as

$$h_b = \frac{c}{\Delta r}, \quad (1)$$

where c is a constant and Δr is the existing clearance between fuel and cladding. The bond conductance h_b as given by Eq. (1) is restricted to a maximum input value of $h_{b \max}$. Theoretically, $h_{b \max}$ is the contact conductance. Neither the constant c nor $h_{b \max}$ is precisely known under the experimental conditions. For this reason we have carried on a sensitivity study in which only the $h_{b \max}$ value is varied. The value of parameter c was taken to be 4.2×10^{-3} W/cm²·°C, while $h_{b \max}$ was varied from 2 W/cm²·°C (corresponding to the standard case) to 10 W/cm²·°C.

FIG. III-17-2. A Plot of Maximum Cladding Temperature and the Plastic Strain of the Clad for Two Different Values of the Contact Conductance. Vertical Lines Indicate the Corresponding Energy Produced in the Pin. Curve 1 is the Base Case ($h_{b \max} = 2$ W/cm²·°C) While Curve 2 Shows Results Obtained with $h_{b \max} = 10$ W/cm²·°C. ANL Neg. No. 113-2596.

°C. This change in bond conductance (or contact conductance) is directly reflected in the cladding temperatures as shown in Fig. III-17-2. This figure is a plot of the maximum cladding temperature as a function of the plastic deformation of the cladding. It should be noted that these values do not necessarily correspond to the same axial position: they are hot spot values. The corresponding energy release is shown by almost vertical lines. Curve 1 in this figure corresponds to the base case while curve 2 is the result obtained with a value five times larger for the maximum bond conductance. It is seen that the cladding temperatures are directly affected by the variation in $h_{b \max}$. Constant energy release lines, which are almost vertical, once again ascertain practically no significant change in the plastic strain due to change in yield strength values which results because of cladding temperature variation. This observation is once again attributed to the model used for plastic strain calculations.

CONCLUSIONS

From the results of sensitivity studies performed using the rapid TREAT transient of fuel meltdown in a sodium filled autoclave experiment (S-3) we arrive at the following conclusions. It is noted that some of the remarks made below may not be directly applicable to less rapid transients which are perhaps typical of fast breeder reactors. In the present analysis, only fresh fuel is considered. Hence, burn-up effects are not considered.

1. The change in fuel volume due to fuel melting is a very important quantity for the purpose of calculation of fuel failure threshold. We find, with reference to TREAT transients, that the onset of fuel melting gives rise to a sharp increase in the plastic deformation of the cladding.

2. If the pores and voids which existed in solid fuel are assumed to be filled due to fuel melting, our calculations confirm the belief that less dense uranium dioxide pellets can tolerate larger energy generated in the fuel pin.

3. Once again, for rapid TREAT transients, the fraction of uranium dioxide which melts during the transient does not appear to be the most meaningful quantity in establishing the threshold of pin failure. This observation is evident from the calculated fraction of UO₂ melted during the transient as shown in Fig. III-17-1. It could still be a useful measure of fuel failure threshold for LMFBR accident situations.

4. The fuel pin failure appears, for these fast transients, to be caused by excessive plastic strain and not due to melt-through of the cladding. Cladding can, however, melt after the power transient is

as the heat energy stored in the fuel flows to the cladding.

5. Both yield strength of the cladding and its temperature dependence need to be known with a reasonably good precision especially at high temperatures. Furthermore, the bond conductance is an important parameter because it determines the cladding temperature and hence plastic strain.

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III-18. Radiation Effects on Superheat in Liquid Sodium

L. W. DEITRICH

INTRODUCTION

It is well known that a radiation particle passing through a superheated liquid can cause nucleation of a vapor bubble and subsequent boiling of the liquid. This phenomenon is exploited in the bubble chamber, and it has been observed in water with the fission fragment as the radiation particle.¹ Since the degree of superheat attained by a liquid before boiling influences the violence of the boiling when it does occur, the ability of reactor radiation to nucleate vapor bubbles in liquid sodium is of interest in the LMFBR safety program. This question has been studied by K. T. Claxton,² using equations based on the "thermal spike" theory of nucleation,³ which was reasonably successful in predicting the observations in water.¹ The present work was undertaken to confirm the results of Claxton² and to determine if further work, analytical or experimental, is required.

ANALYSIS

The basic assumption of the "thermal spike" theory of bubble nucleation by a radiation particle is that the energy required to form a spherical cavity of the "minimum" radius in the liquid and to fill this cavity with vapor must be supplied by the radiation particle as it passes through the liquid. Furthermore, it is necessary that this energy be transferred from the particle to the liquid in a distance which is comparable to the "minimum" radius of the cavity. Based on this as-

sumption, one may establish the following requirements on a particle in order that it be capable of causing bubble nucleation:

1. The particle energy must be at least equal to $E_b(T_s, T)$, where T_s is the superheat, T is the liquid temperature, and E_b is the energy necessary to form a minimum radius cavity and fill it with vapor.
2. The energy E_b must be deposited in a distance which is less than L , the effective deposition length. The value of L is not known a priori but has been estimated to be 2 to ~ 10 times the "minimum" radius r_c .^{1,2,4}

The second requirement is often modified to require that the particle "linear energy transfer" (LET) at the energy E_b be greater than E_b/L , but this modification is not strictly correct, especially when the particle LET is a strong function of its energy.

The equations used for calculating E_b are derived in detail in the literature,¹⁻⁴ and will merely be stated here. The pressure of the vapor in the bubble nucleus is given by

$$P_v = P_{sat} \exp \left[- (P_{sat} - P_l) \frac{M}{\rho_l R T} \right], \quad (1)$$

where

- P_{sat} $\equiv$ saturation pressure at the system temperature
 P_l $\equiv$ liquid pressure
 M $\equiv$ molecular weight
 ρ_l $\equiv$ liquid density

$R \equiv$ universal gas constant

$T \equiv$ system temperature, assumed constant.

The "minimum" radius of the bubble nucleus is calculated from

$$r_c = \frac{2\sigma}{P_v - P_t}, \quad (2)$$

where σ is the liquid surface tension. The rate of bubble growth is calculated from

$$\dot{r} = \frac{4D_t}{r_c}, \quad (3)$$

where D_t is the thermal diffusivity of the liquid. The work necessary to form the liquid-vapor interface is

$$E_1 = 4\pi r_c^2 \sigma. \quad (4)$$

The energy required to fill the bubble nucleus with vapor is, approximately,

$$E_2 = \frac{4}{3} \pi r_c^3 \frac{\Delta H_v}{v_v}, \quad (5)$$

where ΔH_v is the latent heat of vaporization, and v_v is the vapor specific volume. The internal energy of the surface film is

$$E_3 = -4\pi r_c^2 T \frac{d\sigma}{dT}. \quad (6)$$

The kinetic energy imparted to the liquid by the vapor as the nucleus grows is

$$E_4 = 2\pi \rho_l^3 \dot{r}^2. \quad (7)$$

The minimum energy required for nucleation of a bubble in a quasi-static process is then

$$E_b = E_1 + E_2 + E_3 \quad (8)$$

and the minimum energy considering the dynamic character of the process is

$$E_{bd} = E_1 + E_2 + E_3 + E_4. \quad (9)$$

The effective deposition length L is assumed to be given by

$$L = 2\pi r_c \quad (10)$$

in keeping with the suggestion of A. Norman and P. Spiegler.⁴ It is convenient in later work to calculate an effective deposition length in mass thickness units, given by

$$L_m = 2\pi r_c \rho_l. \quad (11)$$

All numerical calculations were performed using a short program for the CDC-160A computer. The necessary thermophysical property values for sodium were those recommended by G. H. Golden and J. V. Tokar.⁵

Consider now the calculation of the energy and LET

available from radiation particles of interest. It is clear to the author that fission fragments are the radiation most likely to be capable of bubble nucleation at temperatures up to about 1300°C and bubble energies in excess of about 0.1 MeV. Fission fragments have the highest energy and LET of any particle normally found in a reactor, and would therefore be most likely to satisfy the nucleation criteria given above.

The most favorable conditions for fission-fragment-induced bubble nucleation will obtain if the fissions are assumed to occur directly in the superheated liquid. In this case, the total fission fragment energy of about 162 MeV is potentially available. If the fissions take place within a fuel element, even if there is a cladding rupture, only one fragment per fission will be available, with a maximum energy of about 100 MeV. Even this energy will be degraded in most cases.

The distance-energy relationship for a fission fragment can be represented by the "square law"

$$E(x) = E_0 \left(1 - \frac{x}{R_a}\right)^2, \quad (12)$$

where

$E(x) \equiv$ fission fragment energy after traveling distance x

$E_0 \equiv$ initial fragment energy in MeV

$R_a \equiv$ fragment range in the stopping material.

The range of a fission fragment in a stopping medium having mass number M_a and atomic number Z_a may be calculated from⁶

$$R_a = [0.0968 + (0.0497 M_a/Z_a^{1/2})] E_0^{1/2} M^{1/2}/Z^{2/3}, \quad (13)$$

where

$M \equiv$ mass number of the fission fragment

$Z \equiv$ atomic number of the fission fragment

$R_a \equiv$ range in mg/cm².

Considering the fission fragment spectrum to be represented by two groups, a light group having $M = 97$, $Z = 38$, and $E_0 = 97$ MeV, and a heavy group having $M = 138$, $Z = 54$, and $E_0 = 65$ MeV, the ranges in sodium calculated from Eq. (13) are 3.80 mg/cm² for the light group and 2.94 mg/cm² for the heavy group. By differentiating Eq. (12),

$$\text{LET} = -\frac{dE}{dx} = \frac{2E_0}{R_a} \left(1 - \frac{x}{R_a}\right). \quad (14)$$

Using the above data, one obtains maximum LET values of 51 MeV-cm²/mg for the light fragments and 44 MeV-cm²/mg for the heavy fragments. Using Eqs. (12) and (14), it may be determined that the light fragment will have a higher LET than the heavy fragment until it has deposited about 24 MeV. This indicates that any bubble nucleated at an energy less than 24 MeV can be a result of the passage of the light

ment alone and the heavy fragment need not contribute. (It will be seen that this includes all the cases of interest.) Therefore, for energy depositions up to 24 MeV, the energy deposited in the effective deposition length for a given bubble may be calculated from

$$E_{dep} = E_0 - E(L) = E_0 \left[1 - \left(1 - \frac{L}{R_a} \right)^2 \right], \quad (15)$$

using the parameters for the light fragment group.

In his paper,² Claxton concludes that knocked-on sodium ions could cause bubble nucleation in liquid sodium. This is a very difficult situation to analyze since there is no satisfactory method of calculating the LET of a sodium ion moving in sodium, especially when the velocity of the ion is as low as is found in the fast neutron knock-on case. The maximum energy of a sodium ion in a reactor is sufficient to nucleate bubbles under some conditions, so a reasonable estimate of the LET of a sodium ion in sodium is highly desirable.

Range-energy data for heavy ions in various stopping materials are available in the literature.^{7,8} While most of these data are for very high energy ions, W. H. Webb et al⁷ give a range-energy curve for nitrogen ions in aluminum which can be extrapolated to sodium ions in sodium by a method suggested for extrapolation of range-energy data by E. L. Hubbard.⁸ In this method, the range of an ion in sodium $R_{i,Na}$ is related to the range of the ion in aluminum $R_{i,Al}$ by

$$R_{i,Na} = \frac{R_{\alpha,Na}}{R_{\alpha,Al}} R_{i,Al}, \quad (16)$$

where

$R_{\alpha,Na}$ $\equiv$ range of an alpha particle in sodium

$R_{\alpha,Al}$ $\equiv$ range of an alpha particle in aluminum.

All particles are assumed to have the same velocity.

Next,

$$\frac{R_{\alpha,Na}}{R_{\alpha,Al}} = \frac{Z_{Na}A_{Na}}{Z_{Al}A_{Al}} = 0.721. \quad (17)$$

In Eq. (17), Z and A represent atomic number and mass number. Also, if the ions are assumed to have the same change,

$$R_{Na,Na} = \frac{A_{Na}}{A_i} R_{i,Na}, \quad (18)$$

where $R_{Na,Na}$ is the range of a sodium ion in sodium. Using the observation that a 5 MeV nitrogen ion has a range of 1.1 mg/cm² in aluminum, we find that this ion would have a range of 0.79 mg/cm² in sodium, and that a sodium ion having the same velocity would have a range of 1.3 mg/cm² in sodium. The energy of a sodium ion having the same velocity as a 5 MeV nitrogen ion is 22 MeV. The resulting average rate of energy deposition, or LET, is 6.31 MeV-cm²/mg. This rough

estimate has been used to assess the possibility of sodium ions nucleating bubbles after knock-on collisions with fast neutrons.

RESULTS

The results of the calculations are presented in Fig. III-18-1. Four sets of lines are plotted in this graph. The sets of lines labeled "energy required-static" and "energy required-dynamic" show the energy required for bubble nucleation according to Eqs. (8) and (9), respectively. The sets of lines labeled "energy available" show the energy which is deposited in the effective deposition length by a light fission fragment and by a sodium ion. Bubble nucleation is possible at any set of conditions where the "energy available" line is above the "energy required" line.

It is seen that, if one considers only the static energy requirement, fission fragments are capable of nucleating bubbles at conditions examined in this study, but that

FIG. III-18-1. Energy Required and Available for Bubble Nucleation in Superheated Liquid Sodium. ANL Neg. No. 113-2761.

if one considers the dynamic energy requirement, temperatures higher by several hundred degrees centigrade will be necessary. Similarly, knocked-on sodium ions may cause nucleation at temperatures as low as 1500°C if the static energy requirement is necessary, but significantly higher temperatures will be required if the dynamic energy is required.

It is interesting to note that, for sodium, the dynamic energy term E_d is very large compared to E_b . This is contrary to the results for water,¹ in which E_d is small compared to E_b . This reflects the fact that sodium has a very high thermal diffusivity compared to that of water, and that a bubble must expand more rapidly in sodium to avoid loss of energy by heat conduction from the thermal spike.

CONCLUSIONS

It may be concluded from this brief study that there is a possibility that fission fragments and sodium ions may be capable of bubble nucleation at temperatures around 1300°C or lower. However, a very large degree of uncertainty exists due to the importance of the kinetic energy imparted to the surrounding liquid by the growing bubble [Eq. (7)]. The real energy deposition required of the incident particle may be very much larger than that predicted on a quasi-static basis. This theory can only provide a fairly rough estimate. An improved analysis of the bubble growth from a "thermal spike" to the nucleus would be needed to identify the correct energy requirement. Similarly, the identification of the correct "effective deposition length" requires improved analysis. None of the available estimates is wholly satisfactory. In fact, it seems probable that no one value is correct for all particles and fluid conditions.

Experimental guidance of any analysis is clearly indicated, due both to the difficulty of the problem and to the lack of any data to indicate the necessity for further consideration of the problem. It is the opinion

of the author that an experiment with fission fragments from fission occurring within the bulk sodium should be pursued first. These particles are most likely to be able to cause nucleation, and can be reasonably well described with energy and LET spectra. In contrast, there is a large uncertainty in the LET value for a knocked-on sodium ion, and Claxton² shows that no other particles seem to have a chance of causing nucleation.

It should be noted that the ability of fission fragments to cause bubble nucleation is not an entirely academic question. While one hopes he does not have fission fragments in the reactor coolant under normal conditions, such interactions may occur in accidents. Fission-fragment-induced nucleation of bubbles in the sodium coolant may play an important role in boiling at the interface between the coolant and molten fuel following fuel element failure.

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III-19. Heat Transfer Problems in UC-PuC Fuels with Heterogeneous Plutonium Distributions and Its Effect on Doppler Feedback

N. A. McNEAL

Over the past few years, interest has developed in the fabrication of fast reactor fuel elements with heterogeneous plutonium distributions. One process of interest is vibratory compaction of high-density spheres.¹ This process consists of introducing uniform sized UC spheres with diameters of several

hundred microns into a fuel tube. PuC spheres about 20 microns in diameter are then infiltrated into the large-sized sphere matrix.

There are several reasons for the interest in type of fuel element. First, the fuel smear density may be varied by changing the size of the particles.

and, the uranium-plutonium ratio may be varied by introducing small UC spheres with the small PuC spheres. Third, under irradiation, the PuC fraction swells, develops interconnecting porosity, and thereby can release its fission gases to the plenum.²

One possible drawback with this type of fuel element lies in its response to accidents. Since nearly all of the negative Doppler effect is in the uranium and nearly all of the heat generation is in the plutonium, there will be a delay in the transfer of heat from the plutonium to the uranium and consequently a delay in the Doppler response.

The question which arises here is how large can the particles in the heterogeneous mixture be made without endangering the safety of the reactor. Several attempts have been made to answer this question for a heterogeneous mixture of UO_2 - PuO_2 . One paper³ suggests that the maximum particle size be $20\ \mu$, another⁴ suggests $40\ \mu$. Others² have said that the acceptable size may be as large as $200\ \mu$. Since the heat transfer properties of the carbide are better than those of the oxide, the allowable particle size in the heterogeneous UC-PuC mixture should be several times larger.³

The above analyses were carried out using a one-dimensional model consisting of a PuO_2 sphere surrounded by a UO_2 spherical shell. When these spherical particles are placed in a cladding jacket, a cylindrical symmetry is imposed on the particles and the heat transfer is no longer one dimensional. The ideal method of analyzing this configuration would be a full three-dimensional heat transfer analysis. However, the complexity of this configuration makes a three-dimensional analysis prohibitive. As a compromise, a two-dimensional analysis was used to obtain the results to be shown below.

To perform the two-dimensional analysis, the general heat transfer code THTB⁽⁵⁾ was used. The complexity of the input to THTB makes it very difficult to change the configuration of the particles under consideration. It, therefore, became necessary to choose one configuration for these preliminary calculations. If only one configuration is to be used, it should incorporate the worst heat transfer characteristics. The worst heat transfer characteristic should be obtained with the largest UC particle size. The fabrication upper limit of $500\ \mu$ was chosen.¹ A regular hexagonal closest packing structure was chosen for the UC spheres to approximate a fresh fuel element. Since a two-dimensional analysis will use a plane perpendicular to the fuel element axis, the plane through the UC spheres centers was chosen since it has the largest UC area and the smallest PuC area.

FIG. III-19-1. Fuel Element Cross Section. ANL Neg. No. 113-2700.

This "worst" configuration is shown in Fig. III-19-1. It is a plane through the centers of $500\ \mu$ diam UC spheres in a hexagonal close pack (hcp) configuration contained in an 0.25 in. diam 15 mil thick cladding. Because of the hexagonal symmetry of the configuration, only the 30 deg wedge shown in Fig. III-19-1 need be considered. The particles in Fig. III-19-1 were subdivided into nodes for THTB input. The PuC particles were lumped into triangular nodes between the UC spheres.

After the configuration of the particles has been established, the thermal properties of the materials involved must be obtained. The thermal properties of PuC and UC were supplied by J. B. Moser.⁶ Those for the cladding were obtained from Ref. 3. The only thermal property left is the contact conductance between the particles. To obtain these data, a metallurgical analysis of a heterogeneous fuel element irradiated in EBR-II was used. The EBR-II element contained highly enriched UC with 80% of the PuC power density. With a power rating of $21.43\ \text{kW/ft}$, the center line temperature was found to be about 1800°C .⁽⁷⁾ Using the power rating of $21.43\ \text{kW/ft}$, several steady state calculations were run with THTB. The contact conductance between particles (assumed to be the same for all particles) was varied until a center line temperature of about 1800°C was obtained. The resulting contact conductance, along with the other thermal properties used, is shown in Table III-19-I.

Using the nodal configuration described above and the thermal properties in Table III-19-I, several transients were run with the THTB code. From the resulting temperature distribution, the volume averaged UC and PuC temperatures were calculated.

Figure III-19-2 shows a plot of the average PuC and UC temperature for an EBR-II type of fuel element undergoing a transient in which the normalized power increases linearly from 1 to 11 in one second. This transient should approximate a 1 dollar/sec ramp insertion rate in a typical large fast re-

TABLE III-19-I. THERMAL PROPERTIES

Material	Thermal Conductivity, W/cm ² ·°C	Specific Heat, J/g·°C	Density, g/cm ³
UC	0.205 (100°C)	0.198 (100°C)	13.63
	0.180 (700°C)	0.256 (700°C)	
	0.180 (1440°C)	0.283 (1500°C)	
PuC	0.096 (200°C)	0.203 (200°C)	13.53
	0.105 (900°C)	0.237 (900°C)	
	0.125 (1300°C)	0.252 (1600°C)	
Stainless Steel	0.210	0.50	7.9
UC-PuC Contact Conductance		2.27 W/cm ² ·°C

FIG. III-19-2. Plot of Average PuC and UC Temperature for an EBR-II Type of Fuel Element Undergoing a Transient of 1 dollar/sec Ramp Insertion Rate. ANL Neg. No. 113-2694.

actor. The PuC and UC temperatures are indistinguishable in Fig. III-19-2. Initially, the UC temperature exceeds the PuC by 5.0°C. After one second, the UC temperature is 3.3°C higher than the PuC. Thus, this type of heterogeneous fuel element in which the uranium power density is 80% that of the plutonium shows practically no temperature lag in the UC particles.

However, most fast reactor fuel elements will not have a high power density in the uranium. Figure III-19-3 shows a plot of UC and PuC average temperatures for a fuel element with only 3% power

density in the uranium. The power ramp is the same as that used to obtain the results in Fig. III-19-2. Initially, the PuC temperature is higher than that of the UC by 26.7°C. After 0.9 sec the PuC temperature exceeds the UC temperature by 282.8°C. At this point, the difference between natural logarithms of the Kelvin temperatures of the UC and of the PuC is 0.076. Assuming a logarithmic Doppler reactivity with a coefficient of say -0.004 , the difference of 0.076 in the temperature logarithms becomes a difference of 0.0003 in the Doppler reactivity. Assuming a fast reactor with a delayed neutron fraction of 0.003, the 0.0003 reactivity difference is about 10 cents. In a homogeneous mixture of UC-PuC the temperature of the uranium should be higher than the uranium temperature and lower than the plutonium temperature in the heterogeneous mixture. Thus, the figure of 10 cents should represent an upper limit on the Doppler reactivity difference between the heterogeneous and homogeneous mixtures.

One additional transient was run. It used the 3% power density in the UC and a mild power rise taken from a power trace calculated by the SAS1A code,⁸ using a 50 cents/sec reactivity insertion ramp. The power trace is shown in Fig. III-19-4. The resulting average temperatures are plotted in Fig. III-19-5. Again, the initial temperature separation was 26.7°C. The final separation was 138.2°C. Using the same assumptions as in the previous case, this temperature

FIG. III-19-3. Plot of UC and PuC Average Temperature for Fuel Element with Only 3% Power Density in the Uranium. Same Power Ramp as Fig. III-19-2. ANL Neg. No. 113-2693.

FIG. III-19-4. Power Trace Using a 50 cents/sec Reactivity Insertion Ramp. ANL Neg. No. 113-2689.

separation leads to a Doppler reactivity difference of 7 cents.

The transients shown in Figs. III-19-2 and III-19-3 were terminated due to excessive fuel temperatures. Beyond these points, other reactivity effects such as fuel movement, coolant boiling, and cladding rupture will become important.

These preliminary calculations indicate that the Doppler reactivity loss for the UC-PuC particle configuration in Fig. III-19-1 will be on the order of 10 cents. Thus, for moderate to mild accidents, the separation of the UC from the PuC in a heterogeneous mixture should not greatly affect the fuel's safety characteristics as far as the Doppler effect is concerned.

There are many avenues left open for further study of the safety characteristics of heterogeneous fuel elements. In the cases presented above, latent heat of melting and variation of particle-to-particle contact conductance with temperature were ignored. This was done because the THTB code will not handle them. Their effects may be significant. Other cross sections besides that in Fig. III-19-1 should be analyzed as well as other temperature node divisions apart from the one used above. Other particle sizes

FIG. III-19-5. Plot of UC and PuC Average Temperatures with a 3% Power Density in the UC and Power Ramp of Fig. III-19-4. ANL Neg. No. 113-2697.

besides the 500 μ size used here should be tried. These variations in configuration will undoubtedly entail considerable work in preparing the input for THTB.

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III-20. The Effect of Melting on UO_2 Thermodynamic Properties

D. MILLER

Many hypothetical accidents in fast reactors fueled with oxides lead to melting of the fuel. To properly analyze these accidents the thermophysical properties of solid and molten fuel are required. Two recent measurements of the enthalpy of $\text{UO}_2^{(1,2)}$ agree well above the

melting point and make it possible to extrapolate thermal properties with reasonable confidence.

Enthalpy measurements on pure, almost stoichiometric, solid UO_2 by L. Liebowitz, L. W. Mishler and M. G. Chasanov² have been fitted to an equation con-

strained to room temperature heat capacity data with the following result:

$$H^S - H_{298} = 15.6820 + 4.73650 \times 10^{-2} T + 9.90824 \times 10^{-5} T^2 - 1.15121 \times 10^4/T, \quad (1)$$

$$298 < T < 3115,$$

where

H = enthalpy, j/g

T = temperature, °K

superscript S designates solid phase.

Using the value of $H_{298} = 45.325$ cal/g-mole recommended by the Vienna Panel,³ this leads to a value of $H^S(T_M) = 1120.95$ j/g at the melting temperature $T_M = 3115^\circ\text{K}$. This value was considered in slightly better agreement with other investigations than the value of 3123°K .⁽³⁾ With the measured heat of melting, $\Delta H_M = 281.93$ j/g, the enthalpy of the liquid at T_M is $H^L(T_M) = 1402.88$ j/g.

Making use of standard thermodynamic relations, the heat capacity of the solid is

$$C_p^S = 4.73650 \times 10^{-2} + 1.98165 \times 10^{-4} T - 1.15121 \times 10^4/T^2, \quad 298 < T < 3115 \quad (2)$$

and the entropy of the solid is

$$S^S - S_{298} = -0.264214 + 4.73650 \times 10^{-2} \ln T + 1.98165 \times 10^{-4} T - 5.75604 \times 10^3/T^2, \quad (3)$$

$$298 < T < 3115,$$

both in units of j/g-°K.

With the recommended $S_{298} = 0.2810$ j/g-°K, these give melting point values for the solid of

$$C_p^S(T_M) = 0.6635 \text{ j/g-°K}$$

$$S^S(T_M) = 1.0187 \text{ j/g-°K}.$$

For the liquid at the melting point,

$$S^L(T_M) = 1.1092.$$

Because the liquid enthalpy is linear with temperature over the brief experimental range, the heat capacity is taken as constant⁴ at

$$C_p^L = 0.5129, \quad T > 3115 \quad (4)$$

$$H^L = 1448.20 + 0.5129 (T - 3115) = 0.5129 T - 149.48 \quad (5)$$

$$S^L = 1.1092 + 0.5129 \ln (T/3115) = -30.166 + 0.5129 \ln T. \quad (6)$$

The vapor pressure above solid UO_2 has been determined by R. Ackermann, et al.,⁵ and supported by

several other investigators. Of the two equations given in Ref. 5, the one used is the "low temperature" equation:

$$P^S = \exp \left(52.5105 - \frac{76250.1}{T} - 4.026 \ln T \right), \quad (7)$$

$$T \leq 3115,$$

where P^S = sublimation pressure, atm. This equation has been extrapolated beyond the temperatures where data were taken and is believed to be the principal error in the following calculations.

By application of the integrated form of the Claperyon equation the effect of melting can be written as

$$\ln P^L = \ln P^S - \frac{\Delta H_M}{R} \left(\frac{1}{T_M} - \frac{1}{T} \right) + \left(\frac{C_p^S - C_p^L}{R} \right) \ln \left(\frac{T}{T_M} \right), \quad (8)$$

where R is the gas constant = 8.314 j/g-mole-°K. Substituting the values for ΔH_M , C_p^S , and C_p^L evaluated at the melting point,

$$\ln P^L = \ln P^S - 36.4060 + 4.8914 \ln T + \frac{9158.75}{T};$$

inserting Eq. (7),

$$\ln P^L = 16.1045 - \frac{85408.9}{T} + 0.8654 \ln T$$

or

$$P^L = \exp \left(16.1045 - \frac{85408.9}{T} + 0.8654 \ln T \right). \quad (9)$$

This equation yields a boiling point of 3680°K which is considerably lower than the value of 3985°K which would result from extrapolating Eq. (7) without regard to melting.

The heat of vaporization ΔH_V derived from Eq. (9) by the Claperyon equation is 2712 j/g at the melting point. Using the estimated critical temperature of 9115°K ,⁽⁶⁾ this can be used in the Watson equation⁷ to extrapolate to higher temperatures:

$$\Delta H_V = 3172 \left(1 - \frac{T}{9115} \right)^{0.375} \text{ j/g}. \quad (10)$$

Volumetric behavior of solid and liquid UO_2 is based on the work of J. Christensen.⁸ The density of the solid ρ^S is given by

$$\rho^S = 10.98 / (1 + 9 \times 10^{-6} t + 6 \times 10^{-9} t^2 + 3 \times 10^{-12} t^3) \text{ g/cm}^3,$$

where t is the temperature in °C. The thermal expansion

coefficient α can be derived from the relation

$$\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P = - \frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_P, \quad (12)$$

and for the solid it is

$$\alpha^s = \frac{9 \times 10^{-6}t + 6 \times 10^{-9}t^2 + 3 \times 10^{-12}t^3}{1 + 9 \times 10^{-6}t + 6 \times 10^{-9}t^2 + 3 \times 10^{-12}t^3} \quad (13)$$

At the melting point the volumes are

$$V^s = 0.1034 \text{ cm}^3/\text{g}$$

$$\Delta V_M = 0.0110 \text{ cm}^3/\text{g}$$

$$V^L = 0.1144 \text{ cm}^3/\text{g},$$

or a volume increase of 10.6% on melting. The liquid at the melting point was found to have a density of

$$\rho^L(T_M) = 8.74 \text{ g/cm}^3$$

and a thermal expansion coefficient $\alpha^L = 1.05 \times 10^{-4}/^\circ\text{K}$. Using these together,

$$\rho^L = 11.5986 - 9.177 \times 10^{-4} T, \quad (14)$$

which should be valid to about 6000°K. The vapor density can be found to this temperature by use of the vapor pressure curves and the ideal gas law without serious error. At higher temperatures, estimates⁶ of vapor and liquid densities should be used.

It is believed that Eqs. (4), (5), (6) and (9) will yield useful estimates for engineering calculations to 5000°K and that Eq. (10) will be useful for extrapolation to 7000°K.

Because the enthalpies of the solid mixed-oxide ($\text{U}_{0.8}\text{Pu}_{0.2}$) O_2 are within 1% of that of pure UO_2 up to 2500°C these equations can be applied until additional enthalpy data are forthcoming.

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III-21. Thermodynamic and Transport Properties of Alkali Metal Vapors

D. MILLER, P. BERTONCINI* and A. C. WAHL*

In principle, calculation of the thermodynamic and transport properties of a dilute gaseous substance requires only the law of force between particles and a satisfactory theory for using the force laws or the derived potential energy laws. The vapors of the alkali metals being of technological as well as theoretical interest, have been studied to understand the forces of interaction between pairs of atoms and the modifications of the theoretical models due to the unsaturated valence electrons.

Previous studies¹ of the estimation of potential energy curves for the forces between atoms and calculation of thermodynamic and transport properties from α^L , indicated that the potential energy ϕ as a function of internuclear spacing r could be extracted from

spectroscopic information if it exists. These studies showed that the theoretical calculation of the properties were in satisfactory to excellent agreement with the limited experimental data and gave the correct asymptotic behavior. The calculated value of coefficients in the virial equation of state and of transport coefficients were not highly sensitive to the exact shape of the potential energy curve, although the disagreement was greater for cesium than for the alkalis of lower atomic weight.

Recent advances² in the *ab initio* calculation of molecular wave functions have made it possible to calculate the interatomic potentials to good accuracy and to test theoretical and empirical relations used in characterizing these potentials and in applying them to the calculation of thermophysical properties.³ The

* Chemistry Division, Argonne National Laboratory.

TABLE III-21-I. CONSTANTS FOR EMPIRICAL POTENTIALS FITTED TO CALCULATED VALUES

System	Exponential, Equation 5		Power Law, Equation 6	
	A	B	A	B
Li ⁷ -Li ⁷	$2.355 \pm 0.216 \times 10^{-4}$	$0.9922 \pm 0.400 \times 10^{-9}$	$20.00 \pm 0.235 \times 10^{-3}$	$4.406 \pm 0.471 \times 10^{-7}$
Na ²³ -Li ⁷	$1.871 \pm 0.708 \times 10^{-4}$	$0.8707 \pm 0.106 \times 10^{-9}$	$108.4 \pm 0.249 \times 10^{-3}$	$5.197 \pm 0.982 \times 10^{-9}$
Na ²³ -Na ²³	$2.137 \pm 0.143 \times 10^{-3}$	$0.8420 \pm 0.459 \times 10^{-9}$	$90.54 \pm 0.697 \times 10^{-3}$	$4.940 \pm 0.656 \times 10^{-8}$

Optimized Valence Configuration (OVC) method² extends the traditional Hartree-Fock model of quantum mechanical calculation to permit proper dissociation and to make the correlation error in the molecule the same as in the atoms.

The OVC method has been applied to the systems Na₂²³, Li₂⁷ and Na²³-Li⁷ in both the attractive singlet ¹Σ ground state and the repulsive triplet ³Σ excited state. The molecular spectra of the first two systems have been studied experimentally and the agreement of calculated properties related to the potential curves is excellent. The sodium-lithium system is unobserved but would be of importance in a mixture of sodium and lithium vapors and serves as a model for mixtures such as would arise in a reactor from release of fission products or from a very severe accident.

This type of calculation has direct application to reactor safety analyses. It makes possible extrapolation of vapor properties to conditions of temperature and pressure that are well beyond the regions for which experimental data exist.

The second virial coefficient $B(T)$ in the virial equation of state

$$\frac{pV}{RT} = 1 + \frac{B(T)}{V} + \frac{C(T)}{V^2}, \quad (1)$$

where

p ≡ pressure

V ≡ volume

T ≡ temperature

R ≡ gas constant,

is given by the relation

$$B(T) = B_{\text{Na-Na}}(T) X_{\text{Na}}^2 + B_{\text{Li-Li}}(T) X_{\text{Li}}^2 + B_{\text{Na-Li}}(T) X_{\text{Na}} X_{\text{Li}}, \quad (2)$$

where B_{ij} represents the i - j interaction. X_i represents the mole fraction of component i , and i and j are lithium or sodium.

Values of B_{ij} are needed for both the singlet⁽¹⁾ and triplet⁽³⁾ states with the appropriate statistical weighting

$$B_{ij} = \frac{1}{4} B_{ij}^{(1)} + \frac{3}{4} B_{ij}^{(3)}, \quad (3)$$

and the virial coefficients are related to potentials $\phi_{ij}^{(1)}$ and $\phi_{ij}^{(3)}$ by, for example,

$$B_{ij}^{(3)} = 2\pi N \int_0^\infty \left\{ 1 - \left[\exp \frac{-\phi_{ij}^{(3)}(r)}{kT} \right] \right\} r^2 dr, \quad (4)$$

where

N ≡ Avogadro's number

k ≡ Boltzmann's constant = R/N .

Virial coefficients have been calculated for the various interactions in the sodium-lithium system and await experimental confirmation.

Traditionally the potential energy curves $\phi(r)$ have been given or fitted by semi-empirical forms or "scaled" or "reduced" potentials such as the Rydberg, Morse, "anti-Morse," power series, etc. Current results indicate that the singlet curves cannot be fit over the range of calculated values by the three-constant Rydberg or Morse curves. The triplet potentials are definitely not reproduced well by an "anti-Morse" curve but are extremely well fitted by an inverse exponential law of the form

$$\phi^{(3)} = A \exp(-Br), \quad (5)$$

and, with larger deviations over the range of available values, by an inverse power

$$\phi^{(3)} = A/r^B. \quad (6)$$

Values of the constants for the three systems and their standard deviations are given in Table III-21-I.

Tests of the combining laws³ for A and B , where

$$A_{ij} = \sqrt{A_i A_j} \quad (7)$$

and

$$B_{ij} = (B_i + B_j)/2, \quad (8)$$

indicate that these laws are crude approximations. More complicated combining laws⁴ may be necessary.

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III-22. Considerations of Leakage Rate Measurements in Containment Using Ice Condenser Pressure Suppression

R. O. BRITTAN

It has been proposed that ice-filled condenser banks be utilized to reduce the high pressure of steam escaping from an accidental breach in the primary system of a pressurized water reactor prior to entry into the secondary containment.¹

At the request of the AEC Advisory Committee on Reactor Safeguards, I examined the problems of conducting leakage rate tests of the secondary containment of such a plant to determine if a valid test could be performed using ordinary precision and care. The examinations are of general interest and so are recorded here.

A leakage rate test is deemed valid if the probable error in measurement during the test can be shown to be less than the allowable leakage rate. However, if a test has not yet been run, validity can only be predicted on the basis that the "possible" error^{2,3} is less than the allowable leakage rate. To do this it must be shown that the planned precision of measurement of the state variables (pressure, temperature and density) will not contribute errors the sum of which is greater than the allowable leakage rate.

Before undertaking this, the nature of the system and the proposed conduction of the tests must be considered.

The proposed system consists of a vaulted region containing the primary system which has access through the ice compartment to the secondary containment volume above. High pressure steam escaping through a break in the primary system passes through trap doors to the ice compartment where heat is removed (and pressure reduced) by the ice heat sink before passage to the upper compartment. The resulting maximum pressure of air and water vapor becomes the test pressure and is considerably lower than if no heat sink were present. The three compartments of the ice-condenser system are linked by gas passages (the trap doors are sealed). Hence with ice present, an automatic orator-condenser system exists which can alter

the vapor content and vapor pressure in all three compartments under normal conditions during the pneumatic leakage rate tests.

If the containment is sealed (to the atmosphere) a quasi-equilibrium condition will be established after some time, in which the water vapor pressure is the same in all three compartments and is that for saturated air in the ice compartment (i.e., wet bulb temperature = dry bulb temperature = dew point temperature = air temperature in ice compartment). If the conditions of the ice compartment remain consistent throughout the test, the vapor pressure will remain constant. The relative humidity in the warmer compartments will be low. For example, with the ice compartment at 470°R and the others at 520°R, the relative humidity would be 100 % in the former and only 12 % in the latter.

However, the first step in the leakage rate test is to add air to the containment (all three compartments) until the test pressure is attained. This will upset the established equilibrium, especially in the ice compartment. During pressurization, the temperature and vapor content of the air-vapor mixture in the ice compartment will increase substantially. Some cooling and condensation will take place there during this time. The true time-temperature-pressure relation will depend on: whether outside air is added directly to each compartment or whether the air will flow into the ice compartment from the upper or lower compartment, the rate at which the added vapor diffuses and mixes with the original atmosphere, the relative state conditions of the original and added air, and the rate at which heat is removed from the air and water vapor in the ice compartment. After pressurization is complete, the atmosphere will begin to approach equilibrium conditions once more, with respect to vapor pressure everywhere, and with respect to temperature in the ice compartment. Time to equilibrium is indeterminate.

In a test conducted on a "dry" containment system

(i.e., one with no water in the solid or liquid phase present) with all temperatures above the dew point temperature, there is no need to determine the water vapor pressure or density, since the latter remains a constant fraction of the air density. This is true of the mixture leaking out of the system as well. However, if water in the solid or liquid phase is present, the water/air density ratio will change with temperature in time, and it is necessary to employ hygrometry during the test to account for these changes.

To evaluate the validity of the ice condenser containment system then, it is necessary to arrive at the effects of "possible" error in determining the water vapor density in the containment air on error in determining the leakage rate.

The change in water vapor content of the air is due to condensation and leakage of water vapor. The latter will always be < 0.02 of the total leakage, and hence an error in determining it will not produce a significant error in determining the leakage rate. Only an error in determining the quantity condensed out between test readings can be significant.

The fractional leakage ℓ determined by measurements of mixture absolute temperature T , mixture absolute pressure P , and mixture water vapor content w_v at two times, Δt apart, is given (if the volume contribution is neglected) as^{2,3}

$$\ell = \frac{\Delta w_t}{w} \approx \frac{\Delta P}{P} - \frac{\Delta T}{T} - \frac{\Delta w_v}{w} \left(\frac{m_a}{m_v} - 1 \right) - \frac{\Delta w_{v_c}}{w},$$

where

- Δw_t $\equiv$ quantity of mixture leaking out in time Δt
- w $\equiv$ quantity of mixture present initially
- ΔP $\equiv$ change in pressure of mixture in time Δt
- P $\equiv$ initial absolute pressure of mixture
- ΔT $\equiv$ change in temperature of mixture in time Δt
- T $\equiv$ initial absolute temperature of the mixture
- Δw_v $\equiv$ change in water vapor in mixture at time Δt
- Δw_{v_c} $\equiv$ quantity of water vapor condensing in time Δt

The corresponding maximum "possible" error in determining fractional leakage $E_{\ell_{max}}$ is given in the same references^{2,3} as

$$E_{\ell_{max}} = \frac{2E_P}{P} + \frac{2E_T}{T} + \frac{2E_{w_v}}{W} \left(\frac{m_a}{m_v} - 1 \right) + \frac{2E_{w_{v_c}}}{W}$$

where

- $E_{\ell_{max}}$ $\equiv$ maximum "possible" error in determining the leakage fraction, $\Delta w_t/w$
- E_P $\equiv$ maximum possible error in measuring absolute pressure
- E_T $\equiv$ maximum possible error in measuring absolute temperature
- E_{w_v} $\equiv$ maximum possible error in measuring water vapor content

$E_{w_{v_c}}$ $\equiv$ maximum possible error in measuring condensed water vapor

m_a/m_v $\equiv$ ratio of molecular weights of air and water vapor = 1.609.

Since $\Delta w_v = \Delta w_{v_t} + \Delta w_{v_c}$, the last two terms of the leakage fraction expression can be rearranged to yield

$$- \frac{\Delta w_{v_t}}{w} \left(\frac{m_a}{m_v} - 1 \right) - \frac{\Delta w_{v_c}}{w} \left(\frac{m_a}{m_v} \right).$$

As noted above, only errors in determining Δw_{v_c} can contribute significantly to the error in determining the leakage rate ($\Delta w_{v_t} < 0.02 \Delta w_t$). The error in determining Δw_{v_c} can be developed as follows:

$$\left(\frac{m_a}{m_v} \right) \frac{\Delta w_{v_c}}{w} \approx - \left(\frac{w_{v_2}}{w} - \frac{w_{v_1}}{w} \right) \left(\frac{m_a}{m_v} \right) = - (s_2 - s_1) \left(\frac{m_a}{m_v} \right),$$

where s is the water vapor/air density ratio ρ_v/ρ_a . Then the maximum "possible" error in determining $\left(\frac{m_a}{m_v} \right)$

$\frac{\Delta w_{v_c}}{w}$ is

$$(E_{s_2} + E_{s_1}) \left(\frac{m_a}{m_v} \right) < 2 \left(\frac{m_a}{m_v} \right)$$

times the error made in determining the largest s , and hence less than the largest "possible" error contribution to the leakage fraction from hygrometric measurements, ΔE_{ℓ_v} . Now

$$s = p_v \left(\frac{m_v}{m_a} \right) / (P - p_v) \approx \left(\frac{p_v}{P} \right) \left(\frac{m_v}{m_a} \right) \left(1 + \frac{p_v}{P} \right), \text{ since } \frac{p_v}{P} \ll 1,$$

where

p_v $\equiv$ water vapor pressure.

Then

$$\frac{ds}{dT_d} = \frac{1}{P} \frac{dp_v}{dT_d} \left(\frac{m_v}{m_a} \right) + \frac{2p_v}{P^2} \frac{dp_v}{dT_d} \left(\frac{m_v}{m_a} \right) \approx \frac{1}{P} \frac{dp_v}{dT_d} \left(\frac{m_v}{m_a} \right),$$

where

$T_d \equiv$ dew point temperature (abs).

Hence

$$E_s = \frac{ds}{dT_d} E_{T_d} = \frac{1}{P} \frac{dp_v}{dT_d} \left(\frac{m_v}{m_a} \right) E_{T_d},$$

where

$E_{T_d} \equiv$ error in determining T_d

and

$$\Delta E_{\ell_v} = 2 \left(\frac{m_a}{m_v} \right) E_s = \left(\frac{2E_{T_d}}{P} \right) \left(\frac{dp_v}{dT_d} \right).$$

Then the total maximum "possible" error in determin-

leakage rates becomes

$$E_{\text{leak}} = \frac{2E_P}{P} + \frac{2E_T}{T} + \frac{2E_{T_d}}{p} \left(\frac{dp_v}{dT_d} \right).$$

$\frac{dp_v}{dT_d}$ varies widely with the dew point temperature T_d . Values from steam tables⁴ are tabulated for the range of interest, along with $\Delta E'_{lv} = \Delta E'_{lv}$ (for one atmosphere overpressure and $E_{T_d} = 1^\circ\text{R}$):

T_d , °R	$\frac{dp_v}{dT_d}$, in. Hg/°R	$\Delta E'_{lv}$, $P = 60$ in. Hg, $E_{T_d} = 1^\circ\text{R}$
460	0.0020	0.000067
480	0.0050	0.00167
500	0.010	0.0033
520	0.019	0.0063

$\Delta E'_{lv}$ can be obtained for other P and E_{T_d} by multiplying the above values of $\Delta E'_{lv}$ by $2E_{T_d}/(1 + a)$ where $a \equiv$ number of atmospheres overpressure in the test.

The validity check can now be applied to the case in hand. If the error in determining T is taken to be $< 0.1^\circ\text{R}$, and the error in determining P is < 0.01 in. Hg, and the error in determining the leakage fraction must be $< \frac{1}{2}\ell$, then the value of $(2E_{T_d}/P)(dp_v/dT_d)$ must be

$$\frac{2E_{T_d}}{P} \left(\frac{dp_v}{dT_d} \right) < \frac{\ell}{2} - \frac{2E_P}{P} - \frac{2E_T}{T}.$$

If $P = 54.4$ in. Hg, $a = 0.813$ atm, $T \approx 480^\circ\text{R}$ and the allowable leakage fraction is $0.0025 = \ell$, then

$$\frac{2E_{T_d}}{P} \left(\frac{dp_v}{dT_d} \right) < \frac{0.0025}{2} - \frac{2 \times 0.01}{54.4} - \frac{2 \times 0.1}{480} = 0.00047.$$

At the ice temperature of 472°R , $dp_v/dT_d = 0.00345$ in. Hg/°R. Thus E_{T_d} would have to be

$$E_{T_d} < \frac{0.00047}{0.00345} \frac{54.4}{2} = 3.7^\circ\text{R}.$$

For other dew point temperatures, E_{T_d} would have to be less than that shown below; that is

$$E_{T_d} < \left(\frac{2}{1 + 0.8133} \right) (0.00047) / \Delta E'_{lv} = 0.00043 / \Delta E'_{lv}$$

T_d	$\Delta E'_{lv}$	E_{T_d} , °R
460	0.000067	< 6.3
480	0.000167	< 2.5
500	0.00033	< 1.3
520	0.00063	< 0.67

Thus, if the dew point temperature can be established to less than $\pm 0.5^\circ\text{R}$, if the error in T is $< \pm 0.1^\circ\text{R}$,

and if P is $< \pm 0.01$ in. Hg, the error in determining the leakage rate will be $< 0.0011/24$ h. If then the leakage rate measured in the test is found to be $< 0.0014/24$ h, there will be no question that the actual leakage rate is less than the allowable leakage rate of $0.0025/24$ h. This validity check is based on maximum "possible" error. With some care in conducting the test, the "possible" error can be shown to be smaller.

If after pressurization, the mixture in all three volumes is allowed enough time to come to quasi-equilibrium with respect to the water vapor content, the water vapor pressure and density will be governed by the ice temperatures and vapor added during pressurization will have condensed out. Since the ice is held at a fairly constant temperature of say $12 \pm 1^\circ\text{F}$ (472°R), the vapor pressure will remain at approximately 0.070 ± 0.003 in. Hg throughout the test and $\Delta w_v/w$ will be approximately ± 0.00003 due to condensation. This uncertainty is small enough to be neglected, and hygrometry would only be needed to roughly establish quasi-equilibrium. Furthermore since p_v is only approximately 0.07 in. Hg, ρ_v/ρ_a is approximately 0.0008 so that the water vapor leaking out is less than 0.1% of the total leakage (i.e., $\Delta w_v/w < 0.001 \ell$). Under these circumstances, once near-equilibrium is established, the effect of $\Delta w_v/w$ on the results could be ignored. The error in establishing the leakage rate would then be $< 0.0008/24$ h.

In contrast, another system using water as the steam condenser such as the pressure suppression containment concept⁵ would be subject to greater error if hygrometry were ignored after quasi-equilibrium is established. In the case where the water sink temperature is $60 \pm 1^\circ\text{F}$ (520°R), the vapor pressure will be substantially higher than for the ice condenser system and would remain at approximately 0.52 ± 0.02 in. Hg and $(\Delta w_v/w)$ will be approximately ± 0.0002 , while ρ_v/ρ_a would be approximately 0.006 and $(\Delta w_v/w)$, the fraction leakage out will be $< 0.006 \ell$. The latter is still negligible, but the error contribution due to ignoring the condensing fraction of the water vapor is approximately 10% of the allowable leakage rate. It can be concluded from these cases that test validity is easier to attain with the ice condenser system than with the pressure suppression system. Valid tests of the latter have been successfully conducted in which the fractional leakage rate is determined to be much smaller than $0.0025/24$ h. Validity of the test of the ice condenser containment system is assured for even smaller allowable leakage rates if the same care is used in making measurements as was used in the pressure suppression containment system test.

On the basis of past tests conducted with reasonable care, the "probable" error will undoubtedly be shown

to be much smaller than the "possible" error and of the order of 0.0003, or approximately 10 % of the allowable leakage rate. Of course this "probable" error cannot be determined until after the test is conducted.

The guide lines to validity developed in conjunction with Refs. 2 and 3 can be useful in planning tests on other future containment systems of the ice condenser or pressure suppression type.

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III-23. Influence of Temperature Sensor Radiation Heating on Containment Leakage Test Validity

R. O. BRITTAN

A wire-resistance-thermometer is recommended as the best method of averaging the gas temperature in a large enclosure such as a containment building.^{1,2} It has been postulated that if the enclosure wall is at a different temperature than the gas, an error is introduced in obtaining the true average temperature of the gas as a result of radiation heating (or cooling) of the wire by the wall. I attempt herein to estimate the possible magnitude of such an error so that the validity of a leakage rate test of the containment can be predicted.

If no radiant heat transfers from wall to the wire, the wire will assume the temperature of the gas with which it is in contact so that the wire temperature T_r equals the gas temperature T_g . This will be the case when the wall temperature T_w equals the gas temperature.

If, on the other hand, a temperature difference exists between the wall and the wire (i.e., $T_w - T_r = \Delta T_w$) then the heat transfer rate per unit area of the wire is given by the Christiansen equation (Ref. 3, p. 230):

$$q_{wr} = \left\{ 1 / \left[1 + \epsilon_r \left(\frac{1}{\epsilon_w} - 1 \right) A_r / A_w \right] \right\} \epsilon_r \sigma (T_w^4 - T_r^4) \\ \approx \epsilon_r \sigma (T_w^4 - T_r^4), \text{ since } A_w \gg A_r, \quad (1)$$

where

ϵ = emissivity

A = area

σ = Stefan-Boltzmann constant = 0.174×10^{-8} Btu/hr-ft²-°R⁴

T = absolute temperature, °R

subscript r refers to wire

subscript w refers to wall.

This heat transfer would cause a temperature change in the wire. However, if T_r should become different from T_g , heat would be transferred between the wire and the gas by conduction-convection at a rate per unit area of the wire given by the Newtonian equation (Ref. 3, p. 3):

$$q_{rg} = h(T_r - T_g) = h\Delta T_g, \quad (2)$$

where

h = convective heat transfer coefficient (wire to air)
subscript g refers to gas.

This results in an opposing change in wire temperature, and the net heating (or cooling) rate is found from Eqs. (1) and (3) to be

$$q_{wrg} = \sigma \epsilon_r (T_w^4 - T_r^4) - h\Delta T_g. \quad (3)$$

This rate becomes zero when equilibrium temperatures are established, and then

$$\sigma \epsilon_r (T_w^4 - T_r^4) = h\Delta T_g. \quad (4)$$

If $T_r \approx T_w$,

$$(T_w^4 - T_r^4) = (T_w - T_r)(T_w + T_r)(T_w^2 + T_r^2) \\ \approx 4T_r^3 \Delta T_w$$

and from Eq. (4),

$$\Delta T_g = \frac{4\sigma \epsilon_r T_r^3}{h} \Delta T_w = T_r - T_g. \quad (5)$$

Here h is the heat transfer coefficient for convection from the wire to the gas, while $4\sigma \epsilon_r T_r^3$ is equivalent radiation heat transfer coefficient. The latter

ily evaluated if the emissivity of the wire is known. Evaluation of h is very complex. It can be accomplished as follows:

For a wire in still air, $h = N_{Nu}k/D_r$, where k is the thermal conductivity of the gas, D_r is the wire diameter, and N_{Nu} is the Nusselt Number. The latter must be obtained from a correlation of experimental data. The correlation is usually made between the Nusselt Number and the product of the Grashof Number (N_{Gr}) and the Prandtl Number (N_{Pr}), where

$$N_{Gr} = \frac{D_r^3 g \beta}{\nu^2} \Delta T_g$$

$$N_{Pr} = \frac{\mu C_p}{k}$$

$$\left. \begin{aligned} g &= \text{gravitational acceleration} = 4.17 \times 10^8 \text{ ft/hr}^2 \\ k &= \text{thermal conductivity of gas} \\ \nu &= \text{kinematic viscosity of gas} \\ \mu &= \text{dynamic viscosity of gas} \\ C_p &= \text{specific heat of gas} \\ \beta &= \text{temperature coefficient of the gas density} \end{aligned} \right\} \begin{array}{l} \text{Ref. 3, Table II-2,} \\ \text{p. 10} \end{array}$$

$\rho = d\rho/\rho dt \approx 1/T_g$ for air.

Such a correlation can be found as a plot of $\log N_{Nu}$ versus $\log (N_{Gr} \cdot N_{Pr})$ on p. 132 of Ref. 3. From this plot I have found that

$$\begin{aligned} \log N_{Nu} &= a + b [\log (N_{Gr} \cdot N_{Pr})] \\ &\quad + c [\log (N_{Gr} \cdot N_{Pr})]^2 \quad (6) \\ &= a + bX + cX^2 \end{aligned}$$

accurately in the range $-5 < X < 4$ for $a = -0.03000$, $b = 0.1244$, $c = 0.01220$, so that

$$h = \frac{kN_{Nu}}{D_r} = 10^{a+bX+cX^2} (k/D_r) \quad (7)$$

and

$$(\Delta T_g / \Delta T_w) = 4\sigma\epsilon_r T_r^3 / [10^{a+bX+cX^2} (k/D_r)]. \quad (8)$$

Unfortunately, ΔT_g is contained in X so that Eq. (8) is transcendental. Since N_{Gr} is a function of ΔT_g , which is sought, it is necessary to evaluate $\Delta T_g / \Delta T_w$ over the range of interest, plot against the assumed ΔT_g for each set of D_r and T_g , and then find the solution for $\Delta T_g / \Delta T_w$ at the intersection of each of these plots with the curve $\Delta T_g = \Delta T_g / \Delta T_w$. [An alternate method (if D_r and T_g are fixed) is to assume an initial value of ΔT_g , solve Eq. (8), use this calculated value of ΔT_g to resolve Eq. (8), and continue iterating to convergence.]

ave undertaken to evaluate ΔT_g over the following range of interest:

$$460 < T_g < 560^\circ\text{R}$$

$$0.001 < D_r < 0.01 \text{ ft}$$

$$0.1 < \Delta T_w < 10^\circ\text{R}$$

$$0.03 < \epsilon_r < 0.30.$$

Fortunately, it was found that $\Delta T_g / \Delta T_w$ is only a weakly varying function of ΔT_w , since N_{Nu} is not strongly dependent on X in the range of interest, and ΔT_g can be approximated by

$$\Delta T_g = C D_r^m T_g^n \Delta T_w^p. \quad (9)$$

ΔT_g was determined for four sets of D_r and T_g with $\Delta T_w = 1$. Using these values of ΔT_g , the powers m and n and the coefficient C were determined. Then ΔT_g was determined for one set of D_r and T_g for several values of ΔT_w . These values of ΔT_g were used to estimate the power p . It was found that

$$\Delta T_g = 2.3 \times 10^{-5} D_r^{0.80} T_g^{2.3} T_w^{0.93} \epsilon_r \quad (10)$$

to within $<5\%$ over the range of interest. Thus it is found that the range of ΔT_g is

$$0.004 < \Delta T_g < 3.0^\circ\text{R}.$$

It is evident then that consideration should be given to the influence of thermal radiation from the wall to the wire when planning and conducting a leakage rate test to assure valid results. In planning the test measurement equipment, D_r and ϵ_r can be selected to reduce the possible magnitude of ΔT_g . It can also be decided to estimate ΔT_w to within a given precision. The range of T_g is such that ΔT_g can be affected at most by a factor of 1.5 by its variation, so deliberate limitation of T_g need not be considered. It is possible to shield the wire in ventilated conduit so that the wire surface area "seen" by the wall is reduced without reducing the area exposed to the gas. The steps which the planner takes must be governed by the relation between the "possible" error fractions introduced in determining pressure, temperature, and (sometimes) hygrometry and the allowable leakage rate fraction.

The "possible" error fraction in determining the leakage rate is (for the "absolute" method)^{4,5}

$$E_L = \frac{2E_P}{P} + \frac{2E_T}{T} + \frac{E_V}{V} + 0.6 \frac{E_{w_v}}{w} + \frac{E_{w_{ve}}}{w} \quad (11)$$

where

E_P $\equiv$ possible error in determining absolute pressure P

E_T $\equiv$ possible error in determining absolute temperature T

E_V $\equiv$ possible error in determining change in volume V

E_{w_v} $\equiv$ possible error in determining change in water vapor content

$E_{w,c}$ = possible error in determining change in condensation

w = weight of air-water vapor mixture in containment.

To assure a valid test, it is necessary that E_L be less than some fraction f_L of the total allowable leakage rate fraction L . Then the error in temperature measurement must be

$$E_T < \frac{T}{2} [f_L L - \sum \text{error contributions} - 2E_T/T]. \quad (12)$$

E_T will be composed of the reading precision and the error introduced in estimating or neglecting ΔT_g .

To illustrate, suppose a pneumatic test is being planned, using the "absolute" method,^{4,5} to determine the leakage rate for a "dry" containment system having an allowable leakage rate fraction L of 0.001/24 h and a test overpressure of 1 atm at $\sim 520^\circ\text{R}$. It is desired to keep the total error less than $\frac{1}{4} L$. It has been decided that the volume change error is negligible and no hygrometry is necessary. The method of measuring the pressure has been selected and is expected to entail a precision of ± 0.005 in. Hg. ΔT_w is estimated to be $< 5^\circ\text{R}$. It is necessary now to plan the temperature measurements to assure that

$$E_T < \left[\frac{1}{4} (0.001) - \frac{2(0.005)}{60} \right] \frac{520}{2} = 0.022^\circ\text{R}.$$

It is planned to use a nickel wire ($\epsilon_r = 0.05$) resistance method to determine the temperature changes. The bridge is chosen to have a precision (following ohm/degree conversion) of $\pm 0.010^\circ\text{R}$. This would mean that the wire diameter must be chosen so that the error entailed in determining ΔT_g (or in neglecting it) must be

$$E_{T_g} < E_T - 0.010 = 0.012^\circ\text{R}.$$

From Eq. (10) it is seen that

$$T_g = 2.3 \times 10^{-5} D_r^{0.8} (520)^{2.3} 0.05 \Delta T_w^{0.93} \\ = 2.02 D_r^{0.8} \Delta T_w^{0.93}.$$

If it is decided to determine ΔT_w to a precision of $\pm 1^\circ\text{R}$, then

$$D_r < \left(\frac{0.012}{2.02} \right)^{1.25} = 0.0017 \text{ ft} \equiv 0.021 \text{ in.},$$

so that a 20 mil wire would yield satisfactory results. However, if it is decided to ignore ΔT_g , since ΔT_w is estimated to be $< 5^\circ\text{R}$,

$$D_r < \left(\frac{0.012}{2.02 \times 5^{0.93}} \right)^{1.25} = 0.00024 \text{ ft} \equiv 0.003 \text{ in.}$$

If it is decided to use a $\frac{1}{2}$ in. diam aluminum conduit shield with $\epsilon = 0.11$ which reduces the area of wire seen by the wall by 95%, the difference between conduit temperature and gas temperature must be obtained by iteration using the general method as outlined earlier, since this diameter lies outside the range for which Eq. (11) is valid. It must also be recognized that the heat transfer surface is doubled, since both inner and outer walls of the conduit are in contact with the air. After convergence of the iterative solution it is found that $h = 0.595$. Then for $4\sigma ET^3 = 0.1076$, $\Delta T_g = \frac{0.1076}{2 \times 0.595} \times 5 = 0.45^\circ\text{R}$. This value of temperature difference now becomes the temperature difference between conduit and wire, so that

$$D_r < \left(\frac{0.012 \times 0.95}{2.02 \times 0.45^{0.93}} \right)^{1.25} = 0.0039 \text{ ft} \equiv 0.047 \text{ in.}$$

Hence, with this shielding, a 50 mil wire could be used and if the radiant heating correction were neglected, E_T would still be less than 0.22°R .

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III-24. Safety Analysis of the Fast Reactor Fuel Assembly Experiment to be Performed in the Engineering Test Reactor (ETR)

J. C. CARTER

This is an aspect of safety analysis which is concerned with the period of time from the initiation of an accident to the failure of one fuel element. An objective of the Fuel Element Failure Propagation (FEFP) Program is that of investigating how the loss of structural integrity of one fuel element affects the other fuel rods in the assembly of rods which constitute the test specimen. Thus this is a program of planned accidents with the possibility of some unplanned ones, all of which must be contained within a test loop located in the J-13 space of ETR.

The time-dependent sodium pressure resulting from both planned and unplanned accidents will be the subject of a later analysis.

The planned accidents are described in Ref. 1; the analysis presented in this paper is concerned with averting damage by possible unplanned accidents, such as a loss of filter or coolant.

The most obvious sources of unplanned accidents in this testing technique stem from loss of filter effectiveness and loss of sodium coolant. The filter effectiveness can be diminished by isotopic changes in filter materials, by melting, and by mechanical damage. The ability of the sodium to remove heat from the test fuel elements can be diminished by pump trouble, by boiling the sodium, and by a break in the piping. Any accident, short of an instantaneous disintegration of the filter and loss of all sodium, will involve a period of time between the instant the filter and/or the coolant effectiveness starts to diminish to the time the UO_2 melts, the sodium boils, and the stainless steel casing of the UO_2 melts or ruptures. The relationship between the period of time from the initiation of trouble to the loss in structural integrity of the UO_2 casing and the period of time it takes to detect and control the trouble is the basis of this preliminary analysis. The detection and control systems may prevent any serious damage to the test loop and reactor if they can function before the casing of the UO_2 fails from other than indirect causes.

The intent is to try to estimate the periods of time involved in loss of filter accidents, loss of coolant accidents, and the highly improbable double accident of filter effectiveness and sodium flow decreasing at the same time.

... filter may be losing its effectiveness while the

coolant remains at full effectiveness, the coolant may be losing its effectiveness while the filter remains at full effectiveness, or both filter and coolant effectiveness may be diminishing simultaneously. In any case, the result is the same, the temperatures and pressures in the test assembly will rise and, unless some provision is made (such as concurrently lowering the power of ETR), eventually the fuel casing will melt or rupture depending upon the relationship between the internal pressure, the metal temperature and ductility. The casing will stretch elastically until the yield stress is exceeded and then it will stretch in accordance with the theory of plasticity until the ultimate stress is exceeded and rupture occurs. The maximum stress and the maximum casing temperature may not occur at the same axial position. The maximum stress is likely to occur near the longitudinal mid-point of the fuel element, where the UO_2 starts melting. The maximum casing temperature is likely to occur near the end of the fuel element, where the sodium starts to boil. The question is where the thermoelastic time relationship exists which will cause the fuel element casing to fail in the shortest period of time after the filter starts to disintegrate and/or the coolant flow starts to decelerate. If there is no appreciable internal pressure, the casing will fail by melting near the end of the fuel element. If there is internal pressure due to expanding UO_2 , it will fail by rupture near the midpoint. If there is internal pressure due to fission gas, it will fail near the hottest point. When the casing loses its structural integrity, the UO_2 and sodium can come into contact and a thermo-chemical reaction may occur with possible rapid increase in sodium pressure.

The physical properties of irradiated stainless steel between 1000 and 1500°C are not precise. The melting point is around 1500°C. Also the effect of molten UO_2 and boiling sodium on irradiated steel and the many variables in the transition from elastic to plastic phenomena cause the performance of the stainless steel casing in this range from 1000 to 1500°C to be uncertain.

It is suggested that the criterion for the structural integrity of the casing be 1000°C. As long as the gap between the UO_2 and its casing is maintained at its present value and the test on new fuel elements are of

FIG. III-24-1. The Ratio of Maximum to Minimum Power in the 19 Fuel Element Assembly. ANL Neg. No. 113-2553.

short duration, low contact and fission gas pressures will exist and thus the probability of exceeding the ultimate stress of the steel is very low.

FILTER

The short duration of individual tests in the program restrict the causes of loss of filter effectiveness to mechanical damage and melting.

When all of the fuel elements in an assembly of 19 are operating at the same power, it is necessary that the center fuel element have a higher enrichment than fuel elements in the ring of 6 and that fuel elements in the ring of 6 have higher enrichment than fuel elements in the ring of 12. The tests are planned so that the total power in each fuel element is of the order of 45 kW. The average power that can be generated in a fuel element without melting UO_2 at its midpoint is calculated to be 60 kW.

The fuel elements are so designed that with a filter in place this value of 60 kW/fuel element is not exceeded if the test assembly is in the J-13 space and the ETR power does not exceed 175 MW. The filter is the equivalent of 35 mils of cadmium. If the filter effectiveness diminishes to zero at a rate for which it is impossible to compensate, the heating rate of the assembly will increase by a factor of 2.62, the power of the fuel elements will increase by a factor of 3.2 in the ring of 12, by a factor of 1.7 in the ring of 6, and by a factor of 1.4 in the center.

As soon as the fuel elements in the ring of 12 melt, their shielding effect upon the elements in the rings of 6 and 1 is removed and the elements in the ring of 6 will melt faster than did those in the ring of 12 since they are of higher enrichment. The center fuel element will melt faster than those of the ring of 6. This is a fuel failure propagation phenomenon unique to this way of testing fast reactor fuel elements or any assembly in which the enrichment varies inversely as the radius of the assembly. The distribution of power in a fuel element of each ring just before melting is shown in Fig. III-24-1. If all of the rods are operating at 45 kW/element, all of them will melt. The rates of melting will increase as successive outer rings are removed.

COOLANT

Since the characteristics of the fluid dynamics of the test loop are not known, the deceleration of sodium velocity (coastdown) is made a parameter. Figure III-24-2 is a plot of velocity versus time. For example, if velocity decreases from 450 to 210 g/cm² sec when the power in the assembly is 60 kW/ele-

FIG. III-24-2. Sodium Flow Rate Versus Time in Coast Down. ANL Neg. No. 113-2552.

the sodium will start to boil in 2.17 sec. Figure III-24-2 was prepared by using the heat transfer module of the SASIA code.²

The following is a concept of an accident which might occur during a typical test. The total power in each rod of the assembly of 19 is 45 kW. The average power density in each rod is 2000 W/cm³ of UO₂. The maximum average power density over the cross section of a fuel element at its longitudinal midpoint is 2600 W/cm³ of UO₂ due to axial buckling. The filter starts to disintegrate and its effectiveness becomes zero in 200 msec. The total power in an outer fuel element increases by a factor of 3.21 in 200 msec. The sodium velocity starts to decelerate following the top curve on Fig. III-24-2 at the instant the filter starts to disintegrate.

The result is that UO₂ starts to melt at the midpoint of the rod in 0.6 sec. The sodium starts to boil in 0.79 sec. The casing temperature is a few degrees above the sodium temperature along the length of the UO₂ fuel element as long as the sodium is in the liquid phase. When sodium starts to boil, the difference between the UO₂ casing temperature and sodium temperature increases due to the lower heat transfer rate of a vapor-liquid mixture and the flow of sodium starts to slow down from its normal rate of 439 g/cm², thus increasing the difference in temperature between casing and sodium. The temperature of the boiling sodium remains constant while the temperature of the casing increases as the percent vapor increases. The maximum casing temperature occurs near the end of the rod. The casing temperature reaches 1000°C in 0.77 sec and its melting point of 1500°C in 1.23 sec.

The stress in the casing is negligible since the gap between the UO₂ and casing is such by design that as the temperature of both increase, their relative expansions do not result in any appreciable contact pressure being exerted on the casing. In case of a smaller gap and/or the presence of fission gas this internal pressure can be appreciable. These test elements have not been exposed to any appreciable radiation.

Since the higher the power at which the rods are operating when an accident occurs the shorter the period of time until the casing loses its structural integrity, the following hypothesized accidents are initiated when the fuel elements are operating at their maximum designed power of 60 kW.

CASE I

The filter is disintegrating from full effectiveness to zero in 0.2 sec and the sodium pump is running at full power. The fuel element is in the ring of 12 in an

assembly of 19 fuel elements and power in the fuel element increases by a factor of 3.21 in 0.2 sec.

Total power in fuel element at time zero	60 kW
Average power density at center of fuel element at time zero	3900 W/cm ³ of UO ₂
Sodium flow at time zero	439 g/cm ² -sec
Sodium inlet temperature	500°C
Maximum UO ₂ temperature	3091°C
Time until UO ₂ starts to melt	0.400 sec
Time until sodium starts to boil	0.629 sec
Time until casing reaches 1000°C	0.525 sec
Time until casing reaches 1500°C	1.016 sec
Stress at the hottest spot of the casing	negligible, dyn/cm ²
Maximum stress in casing (temperature—°C)	negligible, dyn/cm ²
Distance of maximum stress from inlet end of fuel element	negligible, cm

CASE II

The filter is disintegrating from full effectiveness to zero in 0.1 sec and the sodium pump is running at full power. The fuel element is in the ring of 12 in an assembly of 19 fuel elements and power in the fuel element increases by a factor of 3.21 in 0.1 sec.

Total power in fuel element at time zero	60 kW
Average power density at center of fuel element at time zero	3900 W/cm ³ of UO ₂
Sodium flow at time zero	439 g/cm ² -sec
Sodium inlet temperature	500°C
Maximum UO ₂ temperature	3112°C
Time until UO ₂ starts to melt	0.385 sec
Time until sodium starts to boil	0.578 sec
Time until casing reaches 1000°C	0.495 sec
Time until casing reaches 1500°C	0.899 sec
Stress at the hottest spot of the casing	negligible, dyn/cm ²
Maximum stress in casing (temperature—°C)	negligible, dyn/cm ²
Distance of maximum stress from inlet end of fuel element	negligible, cm

CASE III

The filter remains intact and the sodium pump stops. The sodium velocity is shown by the top curve of Fig. III-24-2. The fuel element is in the ring of 12 in an assembly of 19 fuel elements and power in the fuel element remains constant.

Total power in fuel element at time zero	60 kW
Average power density at center of fuel element at time zero	3900 W/cm ³ of UO ₂
Sodium flow at time zero	439 g/cm ² -sec
Sodium inlet temperature	500°C
Maximum UO ₂ temperature	2605°C
Time until UO ₂ starts to melt	3.643 sec
Time until sodium starts to boil	2.168 sec
Time until casing reaches 1000°C	2.030 sec
Time until casing reaches 1500°C	3.648 sec
Stress at the hottest spot of the casing	negligible, dyn/cm ²

Maximum stress in casing (temperature—°C)	negligible, dyn/cm ²
Distance of maximum stress from inlet end of fuel element	negligible, cm

CASE IV

The filter is disintegrating from full effectiveness to zero in 0.2 sec and the sodium pump stops at the instant the filter starts to go. The sodium velocity follows the top curve of Fig. III-24-2. The fuel element is in the ring of 12 in an assembly of 19 fuel elements and power in the fuel element increases by a factor of 3.21 in 0.2 sec.

Total power in fuel element at time zero	60 kW
Average power density at center of fuel element at time zero	3900 W/cm ³ of UO ₂
Sodium flow at time zero	439 g/cm ² -sec
Sodium inlet temperature	500°C
Maximum UO ₂ temperature	3004°C
Time until UO ₂ starts to melt	0.309 sec
Time until sodium starts to boil	0.545 sec
Time until casing reaches 1000°C	0.487 sec
Time until casing reaches 1500°C	0.845 sec
Stress at the hottest spot of the casing	negligible, dyn/cm ²
Maximum stress in casing (temperature—°C)	negligible, dyn/cm ²
Distance of maximum stress from inlet end of fuel element	negligible, cm

CASE V

The filter is disintegrating from full effectiveness to zero in 0.1 sec and the sodium pump stops at the instant the filter starts to go. The sodium velocity follows the top curve of Fig. III-24-2. The fuel element is in the ring of 12 in an assembly of 19 fuel elements and power in the fuel element increases by a factor of 3.21 in 0.1 sec.

Total power in fuel element at time zero	60 kW
Average power density at center of fuel element at time zero	3900 W/cm ³ of UO ₂
Sodium flow at time zero	439 g/cm ²
Sodium inlet temperature	500°C
Maximum UO ₂ temperature	3008°C
Time until UO ₂ starts to melt	0.380 sec
Time until sodium starts to boil	0.504 sec
Time until casing reaches 1000°C	0.465 sec
Time until casing reaches 1500°C	0.725 sec
Stress at the hottest spot of the casing	negligible, dyn/cm ²
Maximum stress in casing (temperature—°C)	negligible, dyn/cm ²
Distance of maximum stress from inlet end of fuel element	negligible, cm

EFFECT OF FILTER LOSS UPON THE
REACTIVITY OF ETR

The experiments planned for the ETR include up to 19 UO₂ fuel elements surrounded by a neutron fil-

ter designed to remove low energy neutrons. The materials to be used for and location of this filter will be designed to reduce the possibility of loss to an acceptably low value. Since the possibility cannot be completely eliminated, the reactivity effect of such a loss on the ETR has been analyzed.

The accident model assumes the filter fails in 200 msec. This represented a conservative value with respect to melt-slumping mechanisms or mechanical loss. The analysis was done using the IREKIN⁽³⁾ code with the following input data: (The results are contained in Ref. 4).

β/ℓ	150 sec ⁻¹
Power trip level for scram	210 MW
Scram delay	20 msec
Scram fuel element insertion time (including 20 msec delay)	270 msec
Total scram reactivity	6.62 dollars
Fuel expansion coefficient	0.00147 dollars/°F
Moderator coefficient	0.02649 dollars/°F
Power absorption	97% fuel 3% moderator
Number of fuel plates	988
Flow area	0.00106 ft ² /half channel
Power level at time zero	175 MW
Reactor inlet pressure	200 psig
Core flow	35,800 gpm
Reactor inlet coolant temperature	0°F (as reference) 110°F (actual)

CONCLUSION

1. The physical properties of UO₂, CdO, boron, and stainless steel at high temperatures and the concurrent phenomena of melting UO₂ and boiling sodium are so encumbered with uncertainties and lack of experimental verification that it is concluded that the casing may lose its structural integrity anywhere during that period of time when thermoelastic conditions are concomitant with a rise in casing temperature from 1000 to 1500°C (the melting point of stainless steel).

On the basis of this criterion the period of time is between 0.465 sec and 0.892 sec for the case of the filter completely disintegrating in 0.1 sec and the cooling system failing at the same instant that the filter starts to disintegrate. This double accident gives the shortest time in which to detect and control. Since the filter material has a higher melting point than 1500°C and since the structural integrity of the filter is enhanced by its stainless steel casing, the probability of losing a filter is very low. In the event of this highly improbable double accident (Cases IV and V) the time required for the casing to reach 1000 s 1.72 times as long as the period of time required to detect and scram ETR. The period of time for the

casing to reach 1500°C is 2.68 times as long as the period of time required to detect and scram ETR.

2. The time required for a melting filter to be removed from the test section is estimated to be 0.2 sec. If this should actually be 0.1 sec, it will reduce the period of time for the casing temperature to reach 1000°C by 0.03 sec. This change is largely due to the low thermal conductivity of UO_2 .

3. The size of the gap between the UO_2 and the inside surface of the stainless steel casing determines what stress will exist provided the fuel elements are not in the reactor long enough to build up fission gas pressure. In these experiments the gap is wide enough

not to cause any appreciable stress and the fuel elements are all new and the tests are of such duration that no fission gas pressure occurs.

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III-25. An Investigation of Fast Reactor Fuel Testing in the Engineering Test Reactor (ETR)

J. C. CARTER

INTRODUCTION

At present no facilities exist for testing assemblies of fast reactor fuel elements in a fast reactor environment. An alternative is the simulation of a fast reactor environment in a thermal reactor. This alternative method of testing requires a physical arrangement wherein a small volume of fast reactor fuel is placed in a large thermal core. Whether or not the fast reactor fuel is surrounded by a neutron filter depends upon the degree of simulation required by the test.

If only the same average power density over the cross section of each fuel element in an assembly is desired, no filter is required. The power density can be obtained by varying the enrichment in each ring of fuel elements inversely as the distance from the center of the assembly. However, the fission gradient on the diameter of each element is very steep. If it is also desired to keep the fission gradient on the diameter of each element within 10%, a simple filter of uranium, boron, or cadmium is adequate.

If it is desired to simulate the fission spectrum of a particular fast reactor, it is theoretically possible to construct a filter which will produce any desired spectrum by combining materials which have cross sections with the appropriate energy dependence. Such a filter is expensive, difficult to construct, and requires experimental research.

The specific test program described in this paper is based on assemblies of 7-, 19- and 37-fuel-elements

in the J-13 space of the ETR core.¹ It is desired that the fission rate over cross sections of the assembly and over the cross sections of individual fuel elements not vary over 10%. Therefore, a neutron filter is required. The intent of the program is to show how the principle parameters in the design of this type of test affect the phenomena occurring within the assembly and to present theoretical designs of 7-, 19-, and 37-fuel-element assemblies. The principal parameters which can be varied are the number of fuel elements in an assembly, the enrichment of UO_2 , the material of the filter, the thickness of the filter, and the diameter and pitch of the fuel elements in the assembly.

Preliminary calculations have been made to show the effects of (1) the number of fuel elements in a test assembly, (2) the enrichment of the nuclear fuel, and (3) the material of the neutron filter. An estimate was made of the gamma heating in the fuel elements, in the neutron filter, and in the containment vessel. Preliminary designs of a 7- and 19-fuel-element assembly have been made and indicate that the desired power densities can be obtained in the test fuel elements when they are in ETR.

The Advanced Reactivity Measurements Facility² (ARMF-II), will be used to measure nuclear parameters and to verify calculations. Measurements are being made in the Argonne Thermal Source Reactor (ATSR) to check filter materials and fission gradients in the proposed 7- and 19-fuel-element assemblies, and in individual fuel elements.

BASIS OF CALCULATIONS

The exploratory calculations were based on the following conditions:

1. The ETR is operating at 175 MW.

2. The J-13 core position is a square hole, 15.2 cm on a side and 91.4 cm long. Since this volume is larger than the volume of the test assembly, the excess area is assumed to be ETR core material. The assembly of fuel elements consists of a center fuel element surrounded by one or more rings of fuel elements. For calculational purposes, the rings of fuel elements are converted to equivalent concentric cylinders. The equivalent surfaces and volumes are presented in Table III-25-I. The four cylinders of UO_2 are equivalent to the center fuel element and to the rings of 6,

TABLE III-25-I. EQUIVALENT VOLUMES AND SURFACES OF CONSTITUENTS OF FUEL ELEMENT ASSEMBLIES

7-Fuel-Element Assembly	
Region 1	
Lateral surface area of 1 fuel element =	157.42 cm ²
Volume of 1 fuel element =	21.56 cm ³
Region 3	
Lateral surface area of fuel in 6 elements =	944.52 cm ²
Lateral surface area of Region 3 =	909.49 cm ²
Volume of fuel in 6 elements =	129.36 cm ³
Volume of Region 3 =	129.95 cm ³
Region 5	
Thickness of cadmium filter =	0.241 cm
Regions 2 and 4 contain sodium and stainless steel	
Regions 6, 7, 8, 9, and 10 are ETR core	
19-Fuel-Element Assembly	
Regions 1, 2, 3, and 4 are the same as for the 7-fuel-element assembly	
Region 5 now becomes a cylinder of UO_2	
Lateral surface area of fuel in 12 fuel elements =	1889.04 cm ²
Lateral surface area of Region 5 =	1861.80 cm ²
Volume of fuel in 12 fuel elements =	258.72 cm ³
Volume of Region 5 =	253.88 cm ³
Region 7	
Thickness of cadmium filter =	0.2465 cm
Region 6 contains sodium and stainless steel	
Regions 8, 9, and 10 are ETR core	
37-Fuel-Element Assembly	
Regions 1, 2, 3, 4, 5, and 6 are the same as for the 19-fuel-element assembly	
Region 7 is a UO_2 cylinder	
Lateral surface area of fuel in 18 fuel elements =	2833.56 cm ²
Lateral surface area of Region 7 =	2833.56 cm ²
Volume of fuel in 18 fuel elements =	388.08 cm ³
Volume of Region 7 =	387.71 cm ³
Region 9	
Thickness of cadmium filter =	0.254 cm
Region 8 contains sodium and stainless steel	
Region 10 is ETR core	

TABLE III-25-II. DIMENSIONS AND VOLUMES OF THE UO_2 FUEL ELEMENT

(All dimensions at room temperature)

Length, cm	91.44
O.D. cladding, cm	0.635
Cladding thickness, cm	0.039
O.D. UO_2 , cm	0.548
Lateral surface of UO_2 , cm ²	157.42
Vol. of fuel element and spacers, cm ³	30.10
Vol. of cladding, cm ³	6.67
Vol. of spacers, cm ³	1.15
Vol. void, cm ³	0.72
Vol. of UO_2 , cm ³	21.56
Vol. of U-235 and U-238, cm ³	10.57
Vol. % UO_2	71.63
Vol. % SS	25.98
Vol. % void	2.39

12, and 18 fuel elements. The UO_2 may be in regions 1, 3, 5, and 7. Filter material may be in regions 5, 7, or 9 depending upon whether a 7-, 19-, or 37-fuel-element assembly is to be tested. Sodium and stainless steel may be distributed in regions 2, 4, 6, and 8. The amount of stainless steel in a region is that contained in the cladding, spacers, and structural parts, with sodium occupying the remaining volume. The lateral surfaces and volumes of the UO_2 cylinders are approximately equivalent to the lateral surfaces and volumes of the fuel in the fuel elements they represent.

3. The ETR core materials and geometry are based upon information obtained from drawings and reports.³ The dimensions of a fuel element are given in Table III-25-II.

4. In order to perform rapid parametric studies, the test assembly is assumed to be at the center of ETR rather than in the J-13 core position. The effect of moving the test assembly from the center to the off-center J-13 position was determined from x-y calculations with the SNARG-2D code.

5. The codes used are SNARG-1D and SNARG-2D⁽⁴⁾ with cross section set 201 and DIF-2D⁽⁵⁾ of the ARC system with cross section set 233. All codes give consistent results.

THE NUMBER OF RODS IN AN ASSEMBLY

The number of fuel elements that can be in an assembly and the length of these elements depends upon the reactor available, the characteristics of the fuel elements, and the power density required for the test.

ETR has the highest flux and longest core of any reactor available. The fuel elements are similar to those anticipated for use in large thermal breeders. The maximum average power density specified for the

TABLE III-25-III. POWER DENSITY IN ASSEMBLIES CONTAINING 7, 19, AND 37 UO_2 FUEL ELEMENTS^a

Center Fuel Element		Ring of 6 Fuel Elements		Ring of 12 Fuel Elements		Ring of 18 Fuel Elements	
Enrich., %	W cc UO_2	Enrich., %	W cc UO_2	Enrich., %	W cc UO_2	Enrich., %	W cc UO_2
93	4117	93	4886				
93	3175	93	3520	93	4495		
93	2641	93	2817	93	3256	93	4172
70	3099	70	3678				
70	2390	70	2649	70	3383		
70	1988	70	2120	70	2451	70	3140
50	2213	50	2626				
50	1707	50	1892	50	2417		
50	1420	50	1515	50	1751	50	2243
30	1328	30	1576				
30	1024	30	1135	30	1450		
30	852	30	909	30	1050	30	1346

^a Power density was computed for the midpoint of 91.44-cm-long fuel elements with the assemblies so positioned in the reactor that the center line, or the center fuel element, is coincident with the longitudinal center line of the ETR core. The thickness of the cadmium filter is 35 mils. The filter is in Region 5 for the 7-fuel-element assembly, in Region 7 for the 19-fuel-element assembly, and in Region 9 for the 37-fuel-element assembly.

test is 2000 W/cm³ of UO_2 . The power density at the midpoint is 2600 W/cm³ of UO_2 .

The J-13 space in the ETR core limits the test assembly to 37 UO_2 fuel elements. Each UO_2 pellet is 0.548 cm diam and the length of UO_2 is 90 cm. The enrichment of the center element is 93%. The enrichment of the ring of 6 is 85%, the ring of 12 is 74%, and the ring of 18 is 58%.

The effect of increasing the number of rings of fuel elements in a test assembly is that of successively increasing the shielding of the inner rings of fuel elements. This decreases both the magnitude of the fission rate in the fuel elements of each successive ring and the gradient of the fission rate along that diameter of a fuel element which is coincident with the radius of the assembly. No attempt has been made to calculate the angular distribution of fission rate in a fuel element except for the center one. Experiments are underway to measure the angular and radial variation over cross sections of fuel elements in the rings of 6- and 12-fuel-elements.

Table III-25-III shows the effect of adding successive rings of 12- and 18-fuel-elements to an assembly of 7-fuel-elements. All fuel elements are of the same enrichment. The filter material and thickness are maintained constant, but the filter diameter increases as rings of fuel elements are added. The difference in power density between rings becomes

less and the gradient on the radius of the assembly becomes less as rings of fuel elements are added, indicating that low energy neutrons are being captured in the outer rings of fuel elements. Since the outer fuel elements capture the largest percentage of low energy neutrons, they have the steepest fission gradient on that diameter which is coincident with the radius of the assembly.

FUEL ENRICHMENT

The ability to vary the enrichment of the test fuel elements provides the primary means of obtaining the desired average fission rate period throughout the assembly. A fairly uniform average fission rate for all

FIG. III-25-1. Heat Generation and Enrichment. ANL Neg. No. 113-2684.

FIG. III-25-2. Effect of Filter Thickness Upon Power Density. ANL Neg. No. 113-2685.

NEUTRON FILTER

Filter materials with a high neutron absorption cross section for low energy neutrons are desired.

Theoretically, rare earths such as europium, samarium, dysprosium, and erbium appear attractive as filter materials.⁶ They have large resonance integrals, with the resonances closely spaced so that large average cross sections well into the keV range are produced. There is essentially no burnup problem with these materials, since neutron absorption produces another rare-earth isotope, usually with a large cross section.

Materials such as tantalum and rhenium are also potential filter materials because of high melting points and good absorption cross sections at low energies. Their cross sections are not as high as those of rare earths, cadmium, and boron; therefore, filters of these materials must be thicker than filters of cadmium, boron, and the rare earths, thus creating heat-transfer and space problems.

A uranium filter, when it is feasible to remove its fission heat, has the unique advantage of being able to provide a heat generating boundary for the flowing sodium, thus enhancing the chances of attaining dynamic similarity between a small test assembly and a large fast reactor.

FIG. III-25-3. Flux Distribution in Test Assembly. ANL Neg. No. 113-2695.

aterials with low melting points are a potential source of trouble because the sodium temperature in the test assembly is not less than 500°C. The actual choice of a filter is a compromise of performance, economics, and safety. At present, cadmium oxide and boron steel appear to be the best filter materials.

Cadmium absorbs almost all neutrons below 0.41 eV and allows nearly all of the higher energy neutrons to pass through. Cadmium offers little flexibility in altering the epithermal flux. The effect of the thickness of cadmium on the power density in 93% enriched UO_2 fuel elements in a 7-fuel-element assembly is shown in Fig. III-25-2. The optimum thickness of pure cadmium appears to be 35 mils. Additional thickness above 35 mils appears to have little effect on the power density. A 16-group flux plot and a plot of $\Sigma_f / \Sigma \sigma \Sigma_f$ are shown on Figs. III-25-3 and III-25-4, respectively, for the case of the cadmium filter.

The gamma heating of cadmium is estimated to be 15 W/g of cadmium. The gamma heating would be approximately 80 kW for a 35-mil cadmium filter. The cadmium filter must be formed of cadmium oxide because pure cadmium melts at 321°C. The

cadmium oxide filter absorbs most all of the thermal neutrons but has no flexibility in controlling the high energy neutrons. Its melting point is 1426°C. The thickness equivalent to 35 mils of pure cadmium is 50 mils. The filter would probably be a thin cylinder of cadmium oxide clad with stainless steel.

Boron is a $1/v$ absorber that is stable at high temperatures. Therefore, boron offers more flexibility than cadmium in controlling the neutron spectrum. The effect of the thickness of the boron on power density in a 93%-enriched UO_2 , 7-fuel-element assembly is shown in Fig. III-25-2.

The irradiation of boron produces helium. The $\text{B}^{10}(n, \alpha)$ reaction is undesirable, but must be taken into consideration along with the effect of irradiation upon the structural integrity. For the anticipated test time in ETR, the irradiation effects do not seem to present insurmountable problems. The heat to be removed from the boron is estimated at 20 W/g. This is slightly higher than for cadmium because of the energy of the (n, α) reaction in addition to the gamma heating.

A boron filter may be boron between plates of

FIG. III-25-4. Fission Spectrum in a 7 Fuel Element Assembly. ANL Neg. No. 113-2699.

stainless steel or simply an alloy such as boron steel, zirconium-boron, or titanium-boron. A boron steel filter is recommended as practical and safest for a testing program in which many tests of duration of less than a month are to be conducted. Such a filter is a cylinder of boron steel clad with stainless steel on both surfaces. The thickness of 2 w/o boron steel equivalent to 15 mils of solid boron is 240 mils. The melting point of 1400°C and mechanical strength of this simple steel cylinder reduces the possibility of filter trouble. It is inexpensive and easy to fabricate in quantity.

TABLE III-25-IV. THE EFFECT UPON $\sigma \Sigma_{f,j} / \sum \sigma \Sigma_{f,j}$ BORON, AND URANIUM FILTERS^a

Group	E	Δu	Center Fuel Element			A Fuel Element in the Ring of 6		
			Cd	B	U	Cd	B	U
1	∞ -3 MeV		2.88	3.12	4.11	2.28	2.38	3.48
2	3-1.4 MeV	0.762	6.07	6.45	8.22	5.02	5.22	7.03
3	1.4-0.9 MeV	0.442	2.26	2.42	3.51	1.93	2.13	3.00
4	0.9-0.4 MeV	0.811	2.47	3.83	5.02	2.94	2.97	4.20
5	0.4-0.1 MeV	1.386	4.32	4.33	5.92	3.65	3.90	4.49
6	100-17 keV	1.772	4.94	5.04	6.02	3.75	4.20	5.02
7	17-3 keV	1.735	6.07	6.14	6.62	5.37	5.52	5.92
8	3-0.55 keV	1.696	9.78	9.97	10.03	8.62	8.30	8.49
9	550-100 eV	1.705	16.93	16.61	13.04	16.12	14.54	11.53
10	100-30 eV	1.204	12.77	11.08	4.11	13.59	12.46	4.10
11	30-10 eV	1.099	10.92	9.16	3.01	13.89	12.00	3.50
12	10-3 eV	1.204	5.15	4.13	0.80	8.31	6.48	0.89
13	3-1 eV	1.099	12.25	7.15	10.53	11.46	6.52	9.92
14	1-0.4 eV	0.916	2.93	6.55	11.53	2.63	6.23	11.90
15	0.4-0.1 eV	1.386	0.26	3.52	7.02	0.30	4.47	11.02
16	Thermal		0.00	0.50	0.50	0.00	2.50	5.00

^a The power for the 7-fuel-elements with the cadmium filter is 302 kW. The power of ETR is 175 MW. The enrichment of the center fuel element is 54% and that of the element in the ring of 6 is 46%. The fuel element is that described in Table III-25-II.

TABLE III-25-V. THE EFFECT OF THE FILTER UPON THE MAXIMUM-TO-AVERAGE FISSION RATE IN A FUEL ELEMENT^a

Center Fuel Element			A Fuel Element in the Ring of 6		
Cd	B	U	Cd	B	U
1.048	1.055	1.056	1.11	1.12	1.14

^a The gradient is taken on that diameter of a fuel element which is coincident with the radius of the assembly 7 fuel elements.

The reason that the ratio is greater for boron than for cadmium is that the thickness of filters is chosen so that the total power in the 7-fuel-element is the same for all filters. Under this condition, more fissions occur in the lower energy groups when the boron filter is used than when the cadmium filter is used (see Table III-25-IV).

If the filter consists of both boron and cadmium nearly all neutrons of thermal energy would be filtered out by the cadmium and as many of the epithermal neutrons as desired could be filtered out by varying the amount of boron. A filter composed of plates of boron and cadmium oxide combines the ability to absorb most of the thermal neutrons and the epithermal neutrons as a function of their energy and the thickness of the boron plate.

A comparison of the effect of filter material on the average value of $\Sigma_{f,j} / \sum \sigma \Sigma_{f,j}$ in each rod for each energy group j is shown in Table III-25-IV. The effect of the filters on the gradient of fission rate on the diameter of each fuel element is shown in Table III-25-V.

This investigation indicates that a variety of materials may be used as filters by varying their thicknesses if a uniform heat-generating rate is the only requirement. If a particular spectrum is desired, the filter must be composed of materials which have cross sections of the appropriate energy dependence.

The choice of a filter is a compromise between performance and economics.

GAMMA HEATING

The gamma heating rate in a 19-fuel-element test assembly irradiated in the center of the ETR core has been estimated (see Paper III-26). The gamma fluxes at the midplane of the five-region assembly were calculated using formulas for cylindrical geometry and a uniformly distributed source.⁷ Gamma flux values were found for points at the center, half-way through the five-region assembly, outside the five-region assembly, and at the outer edge of the test assembly. Gamma radiation from the core region outside the test assembly was determined. This surrounding radiation was attenuated by the intervening material of the assembly before being absorbed at the designated points and contributing to the heating rates.

During irradiation, heating of the steel pressure vessel is calculated to be 15 W/g and heating of the cadmium filter surrounding the test assembly to be 15 W/g. The calculated gamma heating in fuel elements in a test assembly is: central fuel elements, 25 W/g of UO_2 ; ring of 12 fuel elements, 23 W/g of UO_2 .

During irradiation, a major portion of the heating is produced by radiation from the surrounding core. Calculations show that immediately after the test assembly is removed from the ETR core, the equilibrium fission products in the test assembly will produce gamma heating in the five central regions equal to 6 W/g of UO_2 , and heating in the steel pressure vessel is equal to 2 W/g.

FIG. III-25-5. Power Gradient on the Diameter of Fuel Elements. ANL Neg. No. 113-2687.

PRELIMINARY DESIGN OF 7- AND 19-FUEL-ELEMENT TEST ASSEMBLIES

The total power required per fuel element is assumed to be 45 kW. The average power density at the mid-point of each fuel element is then 2600 W/cm³ of UO₂. The neutron filter is equivalent to 35 mils of cadmium. The ETR is assumed to be operating at 175 MW.

A 7-FUEL-ELEMENT TEST ASSEMBLY

In order for the average power density at the mid-point of each fuel element to be 2000 W/cm³, the calculated enrichment of the center fuel element must be 54% and that of the surrounding ring of 6 fuel elements must be 46%. The gradient of the fission rates in each fuel element is shown in Fig. III-25-5. The percentage of the fissions in the fuel elements attributable to each of the sixteen groups into which the energy spectrum is divided⁸ is shown in Fig. III-25-6. The percentage is the average over the cross section of a fuel element.

A 19-FUEL-ELEMENT TEST ASSEMBLY

In order for the average power density at the mid-point of each fuel element to be 2600 W/cm³, the

FIG. III-25-6. Fission Distribution Among Energy Groups. ANL Neg. No. 113-2686.

calculated enrichment of the center fuel element 72%, the enrichment of the fuel elements in the ring of 6 is 65%, and the enrichment of the fuel elements in the ring of 12 is 55%. The gradients of power density for fuel elements in each ring are shown in Fig. III-25-5. The percentages of the fissions averaged over the cross section of the fuel elements attributable to each of the 16 groups into which the energy spectrum is shown in Fig. III-25-7.

Each fuel element whether in the 7- or 19-fuel-element assembly will be producing an average of 2000 W/cm³ of UO₂. Steady state and transient temperatures have been computed for a fuel element and the surrounding sodium.

The steady state temperatures have been computed for 11 radial nodes at each of 13 axial nodes. The casing temperature was computed for the midpoint and each surface at 13 axial nodes. The average

FIG. III-25-7. Fission Distribution Among Energy Groups. ANL Neg. No. 113-2696.

FIG. III-25-8. The Steady State Axial Temperatures in a Fuel Element. ANL Neg. No. 113-2682.

FIG. III-25-9. The Steady State Radial Temperatures in a Fuel Element. ANL Neg. No. 113-2683.

FIG. III-25-10. The Transient Temperature Distribution. ANL Neg. No. 113-2691.

TABLE III-25-VI. PHYSICAL PROPERTIES

UO ₂		
Melting temp., °C		2800
Vaporization temp., °C		3200
Molecular weight		268
Coeff. of linear expansion in solid		1.33601×10^{-5}
Coeff. of linear expansion in liquid		4.58861×10^{-5}
Fractional linear expansion due to melting		1.12006×10^{-2}
Poisson ratio		0.30
Youngs modulus, dynes/cm ²		2×10^8
Thermal conductivity (K), $\frac{W}{g \cdot ^\circ C} \times 10^2$		see tabulation
Specific heat (S.H.), $\frac{J}{g \cdot ^\circ C}$		see tabulation
<u>°C</u>	<u>K × 10²</u>	<u>S.H.</u>
100	5.12	0.3036
500	3.81	0.3127
1000	3.06	0.3245
1500	3.14	0.3367
2000	3.88	0.3494
2500	3.15	0.3626
2800	5.84	0.3764
3000	5.84	0.4378
4000	5.84	0.5080
5000	5.84	0.5967

SS Casing	
Stainless steel melting point, °C	1500
Coefficient of linear expansion	2.16×10^{-5}
S.H. × density, $\frac{J}{cm^3 \cdot ^\circ C}$	4.6
Thermal conductivity, $\frac{W}{cm \cdot ^\circ C}$	0.225
Poisson ratio	0.25

Sodium at 1000°C	
S.H., $\frac{J}{g \cdot ^\circ C}$	1.2907
Density, g/cm ³	0.704
Viscosity, g/cm · °C	0.00148
Thermal conductivity, $\frac{W}{cm \cdot ^\circ C}$	0.0461

Filter Materials										
Material	B	Cd	Hf	Re	Ta	Dy	Eu	Gd	CdO	BN
Mol. wt.	10.82	112.41	178.6	186.3	180.88	162.46	152	156	128.41	24.83
$\sigma_a(lh) \left\{ \begin{array}{l} \text{b/atom} \\ \text{cm}^2/\text{g} \end{array} \right.$	750	2400	115	98	21	950	4300	46000		
g/cm ³	42	12.9	0.39	0.23	0.07	3.5	17	176		
Melt. pt., °C	2.34	8.65	13.36	20.53	16.6	8.56	5.24	7.94	6.95	2.30
Coeff. of expansion, 10 ⁻⁶	2000	3210	2130	3440	3027				1426	3000
S.H., $\frac{J}{g \cdot ^\circ C}$	8.3	31.8	5.9	12.45	6.5					7.51
Thermal conductivity, $\frac{W}{cm \cdot ^\circ C}$				0.137	27.26					
	0.92	0.48			0.54					6.20

ium temperature was computed for each of 91 axial nodes.

The temperatures at these 143 nodes for the UO_2 and casing and at 91 nodes for sodium have been computed and are available for powers of 45 and 60 kW/fuel element. The sodium entering temperature and flow rate for each case are 500°C and $450\text{ g/cm}^2\text{-sec}$, respectively.

Figure III-25-8 is a plot of the steady state center-line UO_2 temperature, the temperature near the outside edge of the UO_2 , and the sodium temperature which is a few degrees (5 to 10) below the outside surface temperature of the casing. Figure III-25-9 is a plot of the steady state radial temperature at the mid-section of the fuel element.

The transient temperatures at the same nodes have been computed for 45 and 60 kW/fuel element. In addition, the percent of UO_2 melted and the time at which the sodium starts to boil were computed. Figure III-25-10 shows these for a power ramp of 3.21 in 0.2 sec. The entering sodium temperature and flow rate are 500°C and $450\text{ g/cm}^2\text{-sec}$, respectively.

These three figures are intended to show what types of information can be calculated as the design of the test loop evolves. It takes about 10 to 15 min of computing time to produce the data on each of the

figures. The calculations were made using the heat transfer module of SASIA.⁽⁹⁾

The physical properties of the materials are given in Table III-25-VI.

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III-26. Calculated Gamma-Ray Heating in an Assembly of Fast Reactor Fuel Elements Irradiated in the Center of the Engineering Test Reactor (ETR) Core

A. E. MCARTHUR

As a contribution to an investigation concerned with placing a small region of fast reactor fuel in a large thermal core, calculations were made to estimate the gamma-ray heating rate in an assembly of fast reactor fuel elements when irradiated in the center of the ETR core. The radius of the cylindrical fast reactor fuel assembly with a total of seven regions was 4.06 cm and included two inner regions (2 and 4) of sodium coolant and two outside regions of sodium, cadmium, and steel. The length of the assembly was 91.4 cm. Regions 1, 3, and 5 were highly enriched UO_2 fuel. Figure 1 shows the location of assembly regions and surrounding ETR core.

Gamma-ray source values were obtained using a fission density of 5.3×10^{18} fissions/cm³-sec for the

five-region central section. For conservatism this density was determined by increasing the average fission density for the five-region central section volume by 33%. Gamma-ray fission source strengths were calculated using information (MeV/fission) given in Ref. 1 and include 1.15 MeV/fission of energy due to capture gamma-rays. Fission densities for the ETR core and the assembly fuel regions are those given in Paper III-25.

Gamma-ray fluxes were calculated at the horizontal midplane of the five-region central section using formulas² for cylindrical geometry and a uniformly distributed source. Calculations were made for four photon energies (1, 2, 4, and 6 MeV) with appropriate attenuation coefficients. Flux values were calculated

FIG. III-26-1. Fast Reactor Fuel Assembly Irradiated in ETR. ANL Neg. No. 113-2698.

for points at the center of the assembly, 0.86 cm from the center, outside the five-region central section, and at the outside surface of the assembly.

The ETR core around the assembly was considered to be a thick uniform gamma-ray source produced by a fission density of 1.53×10^{13} fissions/cm³-sec. The

flux at the surface of this source was determined used as radiation incident on the surrounded fuel assembly. This radiation was attenuated by the intervening assembly material before being absorbed at the specified points to contribute to the heating rates reported.

Results of the calculations show that the heating rate in UO₂ at the center of the assembly and at 0.86 cm from the center are approximately equal, with a value of 25 W/g. The heating rate at the outside of the five-region central section (radius = 1.72 cm) is 23 W/g of UO₂.

During irradiation the heating in the steel at the outside of the assembly is calculated to be 15 W/g of steel. The heating of the cadmium in the outside of the assembly will be approximately the same as the heating in the steel, 15 W/g.

During irradiation a major part of the heating is produced by radiation from the surrounding ETR core. Calculations show that right after the assembly is removed from the ETR core the equilibrium fission products in the assembly will produce gamma-ray heating in the five central regions equal to 6 W/g of UO₂ and heating in the steel at the outside of the assembly equal to 2 W/g of steel.

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III-27. Studies of the Liquid Metal Fast Breeder Reactor (LMFBR) Safety Test Facilities Project

C. N. KELBER, G. CINELLI,* R. HOLTZ,* A. JUDD, W. LIPINSKI,* D. MACFARLANE, D. MOHR,*
R. NICHOLSON and R. SINGER*

Argonne National Laboratory has undertaken for the AEC an activity titled: "Studies of LMFBR Safety Test Facilities." This activity is synonymous with Task 10-5.1, Analysis of Facility Requirements, described in the LMFBR Program Plan (Element 10, Safety).

The studies were initiated in the first quarter of FY 1969 and are to be completed at the end of the first quarter of FY 1971. Their purpose is to obtain in-

* Reactor Engineering Division, Argonne National Laboratory.

depth definition of program and design concept, and justification for those key facilities which may be required in the LMFBR safety program. The specific objectives are: (1) to determine the type and number of in-reactor (and supporting ex-reactor) experiments needed for the LMFBR safety program; (2) determine the test conditions and types of measurements appropriate to the experiments; (3) prescribe the proper priorities for the experiments; (4) determine the facility capabilities required to conduct the experiments; (5) determine the practicability of

forming the experiments in existing (or firmly planned) facilities, including reasonable modifications; (6) develop an optimum scheduling of the experiments; (7) make recommendations for the utilization and modification of existing facilities; (8) make recommendations as to whether any new facility is required, and if so, recommend the general nature and needed capabilities of such a facility in sufficient detail to enable the Division of Reactor Development and Technology (RDT) and the Program office (PRO) to prepare a work scope for a conceptual design; and (9) justify and document adequately all of the above.

The work plan for conducting the Studies provides for three Details. Detail A relates solely to the establishment of the mechanics of conducting the Studies and needs no description here. Details B and C relate to determination of the needed program of experiments and to the needed test facilities, respectively, and are described in what follows.

DETAIL B: PROGRAMMATIC DEFINITION AND JUSTIFICATION

The objectives of this Detail are to: (1) determine the type and number of in-reactor experiments which should be performed in the LMFBR safety program; (2) determine appropriate priorities for the required experiments; (3) determine the optimum time schedule for conducting the experiments; and, (4) document adequately all of the above.

Included is the development of adequate justification for the recommended (program of) experiments, their priorities, and their schedule. To accomplish this, delineation will be made of the objectives for the selected tests and in-depth evaluations of the proposed tests will be effected. The alternate approaches studied (that is, the options of approach considered but discarded) will be described briefly and the rationale for rejecting them will be noted. Test conditions and required measurements for the various test series will be defined in sufficient detail to enable determination (not in this Detail, but in Detail C) of key test facility capability requirements.

Detail B consists of three Phases which are to be implemented consecutively.

Phase 1 is a preliminary phase, and is now completed. In this phase a tentative, but complete, listing has been made of the type and number of in-pile experiments applicable to the LMFBR safety program. Evaluations of these are provided and tentative justifications developed. The general test conditions and red measurements associated with the various experiments have been determined. A preliminary (Phase 1) draft report incorporates this information.

(Note that priorities of experiments, scheduling of experiments, test facility capabilities required or available, etc. are not meant to be part of this report; these areas will be addressed in subsequent reports of this Detail, or in Detail C, as indicated below.) This draft is being sent to RDT, PRO, the Division of Reactor Licensing (DRL), and industry for review and comment.

In Phase 2, the study will be extended to include determination of the most appropriate priority for each (class of) experiment. The priority indication system used will be that described in Volume 1, Overall Plan, LMFBR Program Plan. Both an importance indicator and a schedule indicator will be ascribed to each (class of) experiment. An "ideal" time schedule for the entire experiment program will be developed as a first step in arriving at an optimum schedule. This ideal schedule necessarily will be formulated without benefit of adequate information as to which facilities the experiments would best be performed in, or the cost or availability of these facilities. Thus, the priorities and time scheduling determined in this phase will be tentative. Input from industry, PRO, and related Argonne National Laboratory (ANL) safety groups will be utilized in pursuit of this work.

An intermediate (Phase 2) draft report will be prepared. This will consist of: a revision of the Phase 1 report to reflect appropriate comments received as a result of the outside reviews of that document; and, extension to include all of the above material plus feedback available from simultaneous Detail C work. This draft will be reviewed by the ANL safety groups. It will then be sent simultaneously to RDT, PRO, the Advisory Committee on Reactor Safety (ACRS), and industry for review and comment.

Phase 3 is the final phase of Detail B. Here, the Phase 2 study results will be further refined. The previously determined tentative list of type and number of experiments will be additionally examined and evaluated, as will the earlier priority assignments and the ideal time schedule. This reexamination will be done in cognizance of, and will benefit from, comments received as a result of the outside reviews of the Phase 2 report and the studies conducted under Detail C, particularly study of potential utilization of existing (or firmly planned) facilities including modifications and the possible need for new facilities. This effort will result in an improved recommended program of experiments, firm priority assignments, and an optimum (and practical, as opposed to the earlier "ideal") schedule.

A final (Phase 3) draft will be prepared describing

the above. The draft will not be issued as a separate report, but will be incorporated directly into the overall report "Studies of LMFBR Safety Test Facilities" to be prepared as part of Detail C. It will be reviewed internally by ANL when the latter document is reviewed. Similarly, it will be reviewed by RDT, PRO, DRL, ACRS, and industry when the complete report is reviewed by those groups.

DETAIL C: FACILITY DEFINITION AND UTILIZATION

The objectives of this Detail are to: (1) enumerate, define and substantiate the facility capabilities needed to carry out the recommended program; (2) determine the capabilities of existing or firmly planned facilities, such as the Southwest Experimental Fast Oxide Reactor (SEFOR), Experimental Breeder Reactor-II (EBR-II), and Power Burst Facility (PBF), including reasonable modifications; (3) provide recommendations on the use of existing facilities (including modifications); (4) provide recommendations on the need for any new facilities; (5) develop the general nature and capability requirements of such facilities in sufficient detail to enable RDT and PRO to prepare a detailed work scope for the conceptual design; and, (6) document adequately all of the above.

Included is the development of adequate justification for the capabilities found necessary, the recommended facility modifications and facility usage, and recommendations concerning possible need for new facilities. The work reported under this Detail will demonstrate: (1) the technical feasibility of various options for carrying out the program; (2) the technical advantages and disadvantages of the various options; (3) the comparative approximate cost and achievable time schedules of the different options; and, (4) the rationale for rejection of the options not recommended.

In establishing needed facility capabilities the following factors will be considered plus whatever special factors are pertinent for each experiment: (1) specimen size and type; (2) specimen power density ranges and rates of change; also, power density distribution; (3) time duration of power runs; (4) coolant flow and temperature ranges and rates of changes; (5) energy containment capability; (6) instrumentation—including the research and development; (7) pre-irradiation of specimens; and, (8) hot cell and specimen examination.

This Detail consists of two Phases which are to be implemented consecutively.

Phase 1 work has only recently been started. Initially, this work will consist of: (1) determining the

major facility capabilities required to perform experiments defined in Detail B; and, making a preliminary survey of the potential utilization of existing facilities to perform these experiments, including reasonable modifications of the facilities.

For each (class of) experiment outlined in Detail B, a detailed description of required facility capabilities will be prepared. Justification for the capabilities will be developed which will be commensurate with the desired test conditions and measurements associated with the various experiments as delineated in Detail B. The need for specific capabilities also will be reviewed with respect to the various options available for implementing the recommended program. This will involve technical trade-offs and (at a later stage in conjunction with study of facility utilization) cost trade-offs and effects of various options on the program time schedule.

The preliminary survey of the potential utilization of existing facilities such as SEFOR, EBR-II, PBF, Transient Reactor Test (TREAT) Facility, and Fast Flux Test Facility (FFTF) will be made with the possibility kept in mind of making reasonable modifications to these facilities. This preliminary study will be based on the determination of key facility capabilities needed to perform the experimental program, and will form the basis for the preparation of work scopes for subcontracts to develop information about specific facility utilization in detail.

As these efforts are completed, subcontracts will be let with the operators of those existing facilities which appear capable of most effective use. The purpose of the subcontracts will be to effect detailed study of the feasibility and practicality of conducting (classes of) experiments in those reactors, including questions on possible modifications, scheduling conflicts, and approximate costs.

Concurrently with the subcontract work, further analysis by ANL of overall facility utilization will be continued. Important feedback to this work is expected to flow from the Detail B, Phase 2 report. After the results of the subcontracts have become available and factored in, specific recommendations on the use of existing facilities, including modifications, will be formulated. During the latter portion of this work, a recommendation also will be developed on whether any new facilities are required and, if the recommendation is affirmative, work will be started on determination of the general nature and required capabilities of such facilities. All of this effort will provide feedback of importance to Task B, Phase 1 work, particularly in aiding in firming up the

- a. Develop preliminary program (type, number of experiments; test conditions; justification).
- b. Prepare draft of preliminary, Phase 1, report. (Consists of work of a., above.)
- c. Send Phase 1 report draft to RDT, DRL, PRO, and industry for review and comment.
- d. Receive comments on Phase 1 report draft from RDT, DRL, PRO, and industry.
- e. Develop tentative experiment priorities and "ideal" time schedule.
- f. Prepare letter report on preliminary judgement of problem need for "whole-subassembly" and "whole-core" tests.
- g. Prepare draft of intermediate, Phase 2, report. (Consists of revised Phase 1 report plus work of e., above.)
- h. Send Phase 2 report draft to RDT, PRO, ACRS, and industry for review and comment.
- i. Receive comments on Phase 2 report draft from RDT, PRO, ACRS, and industry.
- j. Develop final program, priority assignments, and optimum time schedule of experiments.
- k. Prepare final, Phase 3 draft. (Consists of revised Phase 2 report plus work of j., above.)
- l. Incorporate Phase 3 draft into draft of overall report "Studies of LMFBR Safety Test Facilities".

FIG. III-27-1. Schedule and Milestones; Detail B. *ANL Neg. No. 113-3097*.

list of experiments, their priorities, and the optimum time schedule.

A draft of a preliminary, Phase 1, report will be prepared. This will document the work above and will set forth the major facility capabilities required, provide recommendations on use of existing facilities including modifications, provide a recom-

and (if appropriate) indicate the general nature and

required capabilities of any new facilities. This draft is not expected to be complete in detail; in particular, the nature and capabilities of any new facilities will not be fully developed. This draft will be incorporated into an integrated report titled "Studies of LMFBR Safety Test Facilities" consisting of the Task C, Phase 1 draft and the Task B, Phase 3 draft. The integrated report then will be distributed to RDT, PRO, DRL, and industry for review and comment.

- a. Determine major facility capabilities required to perform experiments recommended in Detail B.
- b. Conduct preliminary study of potential utilization of existing facilities, including reasonable modifications.
- c. Prepare scope of subcontracts to answer questions of technical feasibility, cost, and schedule with respect to the utilization of specific facilities.
- d. Negotiate subcontracts.
- e. Start of subcontracts.
- f. Execution of subcontracts.
- g. End of subcontracts.
- h. Formulate recommendations on the use of existing facilities including modifications.
- i. Develop recommendation on whether any new facilities are required, and, if recommendation is affirmative, determine the general nature and required capabilities of such facilities.
- j. Prepare draft of preliminary, Phase 1, report. (Consists of work of a.-i., above. This draft will not be complete--some information on the general nature and capabilities of any new facilities may be missing because, if needed, this material will still be under development.)
- k. Send preliminary draft of integrated report "Studies of LMFBR Safety Test Facilities" (consisting of Phase 1 draft, above, and Detail B, Phase 3, draft) to RDT, PRO, DRL and ACRS, and industry for review and comment.
- l. Receive comments on integrated report preliminary draft from RDT, PRO, DRL, ACRS, and industry.
- m. Prepare final, Phase 2, draft. (Consists of revised integrated report preliminary draft, plus completion with respect to the general nature and requirements of new facilities, if any.)
- n. Send final draft of integrated report to RDT.

FIG. III-27-2. Schedule and Milestones; Detail C. ANL Neg. No. 113-3095.

In Phase 2, a final draft report will be prepared. This will consist of the Phase 1 report refined and revised to reflect the comments resulting from the out-

side reviews of the integrated report, plus completion of the report with respect to the general nature requirements of any new facility(ies). The latter ma-

al will be in sufficient detail to enable preparation by RDT and PRO of a detailed work scope for the conceptual design of any such facility(ies). At this time, refinement of the Task B, Phase 3 portion of the integrated report also will be effected to reflect any comments that may have been received on it from the outside reviews. The resulting final draft of

the integrated report then will be reviewed within ANL and issued.

SCHEDULE

The currently anticipated time schedule for the Studies is shown by the following bar charts, one for each of Details B (Fig. III-27-1) and C (Fig. III-27-2).

Section IV

Experimental Techniques and Facilities

Measured integral reactor physics parameters are necessary for either direct introduction into reactor design calculations or for comparison with values obtained analytically using basic microscopic cross section data. In the latter case the objective is to verify either the cross section data or the calculational techniques. Reactor integral parameters are obtained using special measurement techniques in exponential or critical facilities. The papers presented in this section deal largely with the development of such special techniques, facilities, and radiation detection devices. Included also are papers which describe specially developed equipment useful in obtaining microscopic nuclear data.

IV-1. On-Line Computer System for the Zero Power Plutonium Reactor (ZPPR)

R. W. GOIN, C. L. BECK, J. E. HUTTON and A. L. HESS

An on-line computer system was acquired for the ZPPR facility during the 1968-69 fiscal year. This system was designed for on-line automatic recording of data from the varied experiments performed on ZPPR, with the potential for concurrent on-line data reduction and evaluation of experimental data. The ZPPR system acquisition was made in conjunction with that for the experimental data processing system of the ZPR-6 and -9 facility. Compatibility of the systems at the two facilities was a primary consideration in the computer selection.

The ZPPR computer system is composed of a Systems Engineering Laboratories (SEL) 840 MP computer and SEL-supplied standard peripheral equipment. The salient features of the computer are: 24-bit word length, 16 K word memory, 3 index registers, hardware floating point and double precision, 31 interrupt levels, and memory protect and instruction trap. The peripheral equipment includes: movable head disc, 2 magnetic tape drives, high-speed paper tape reader/punch, card reader, line printer, X-Y drum plotter, CRT display with light pen (located in the ZPPR control room), remote teletype (located in ZPPR control room), and digital I/O.

At the present time, one digital I/O channel is connected to the data collection system in the control room through a digital multiplexor (see Paper IV-2). During a reactor run, data such as power level and control rod positions can be recorded at rates varying from 100 samples/sec to 1 sample/min. An improved interface for experimental data is in the final stages of construction, and is described in Paper IV-3.

Programs have been written which allow an experi-

menter in the control room to initiate and stop the recording of data and to specify which of the 27 digital multiplexor channels are to be recorded. The recorder data is stored on magnetic tape in records containing 100 multiplexor scans. At the completion of a run, the data is merged onto a permanent data tape.

During off-line operation the SEL 840 MP is used for reduction of ZPPR, ZPR-3, and AFSR data. An inverse kinetics program has been written which reads the data from the merged data tape, computes reactivities for each time step, and plots reactivity versus time (or control rod position) on the X-Y plotter or the CRT. Coefficients of a fourth-order polynomial fit to the reactivity versus position data are calculated.

The proton-recoil data unfolding program, PSNS, has been converted for use on the 840 MP. This program reads the paper tape output from the Data 620 computer, so the time-consuming process of paper tape to card conversion is eliminated.

In addition, service functions are provided such as listing tapes, reformatting or copying tapes, card listings, curve fitting and plotting, copying paper tapes, reformatting paper tapes, copying cards to paper tape or data to card image magnetic tape for punching cards on the IBM 1620, and compacting card decks to paper tape for input to the SEL 810A. Other routines presently in the conversion stage include activation analysis data reduction programs and fuel inventory. In addition, routines are compiled and executed for the ZPPR staff including Monte Carlo studies and fuel loading programs.

IV-2. The Zero Power Plutonium Reactor (ZPPR) Data Collection System

J. E. HUTTON

INTRODUCTION

The ZPPR Data Collection System collects control autorod position data, reactor power data, and related digital data for display in the control room and processing by the ZPPR on-line computer (Paper

IV-1). The system also digitizes and stores thermocouple and other analog signals in IBM-compatible format on magnetic tape.

At present the system is being expanded to provide a more versatile and efficient input to the ZPPR com-

FIG. IV-2-1. ZPPR Data Collection System Block Diagram. ANL Neg. No. 103-A11157.

puter through the control room remote I/O terminal (Paper IV-3). The expanded system will also allow the operator to select digital data displayed in the control room for storage on the magnetic tape with the analog data.

TECHNICAL DESCRIPTION

The system can best be described by dividing it functionally into three sections:

1. The encoder subsystem which includes the posi-

tion encoders, and their control, multiplexing, and readout circuits.

2. The controller and digital data collection subsystem which controls the analog recording subsystem and transmits all digital data to the computer. The counters used with the voltage-to-frequency converters and the system timer are included in this section.

3. The analog data collection subsystem comprised of the DCS signal cabinet and the Data Logger and its incremental tape recorder.

Each of these sections is described in detail below. Figure IV-2-1 depicts the system in block form.

ENCODER READOUT SUBSYSTEM

This subsystem provides digital position information from control rods, the auto rod, traverse systems and other experimental equipment in the reactor cell. It is composed of:

1. Shaft encoders and two encoder multiplexers in the reactor cell.
2. An encoder control and display (EC&D) unit and the slave display in the control room.

The major components of these units and their interconnections are shown in Fig. IV-2-2.

The EC&D counter address is transmitted to the encoder multiplexers where it is used to select an encoder and a decade from that encoder. The outputs of the lead and lag brushes from the encoder selected are decoded by the lead-lag logic and stored in the output shift register. After the five decades of an encoder have been sampled, the EC&D control logic generates a "display encoder" (DEX) pulse, where X is the number of the encoder just sampled.

The display selectors in the EC&D and the encoder slave display select the proper display encoder pulse, as set on the selector switches, to load the encoder position into their respective displays. The encoder position data and timing pulses are also available to the data collection subsystem for transmission to the computer.

CONTROL AND DATA COLLECTION SUBSYSTEM

This section contains the timer, experimental counters, manual data entry, digital multiplexer, and channel selector and display (CS&D) unit. Each of these units is described below.

Timer

The timer unit outputs sample rate pulses and time in hours, minutes, and seconds to the CS&D and digital multiplexer units, respectively. Digital data can be sampled at 0.01, 0.1, 1, 10, and 60 sec intervals while the analog data sampling rates are 0.1 through 60 sec intervals. The controls for selecting the sample rate and starting and stopping the recording of analog data are also on this unit.

Experimental Counter

Pulse rate and other frequency information is collected with the experimental counters. These units are read by the computer and reset once each sampling

period with a resultant dead time of 300 μ sec. When required, the two 24 bit counters can be separated into four smaller counters.

Manual Data Entry Unit

This unit is used by the operator to set the timer and to enter five-digit identification numbers. Numbers are entered with the most significant digit first. They are stored in a shift register in the CS&D unit for display and transmission to the computer.

Digital Multiplexer

All the information to be transmitted to the digital computer or displayed on the CS&D is sequentially gated to the output data buss in the digital multiplexer. To obtain maximum multiplexing speed without providing additional storage for the encoder outputs, the EC&D counter is used to control the multiplexer; the timer output and other digital data are sampled between the "display encoder pulses."

Channel Selection and Display (CS&D) Unit

In addition to displaying all data to be transmitted to the computer, this unit houses the analog block selector and control logic which performs the following functions:

1. Synchronizes data to the computer and the magnetic tape with the sample rate pulse by resetting the EC&D counter.
2. Prevents the outputs of the experimental counters from changing during readout to the computer and resets them after reading.
3. Generates interrupt pulses to the computer when data is on the output lines.
4. Controls analog data recording by gating sample pulses to the data logger when the record switch on the timer is set. This gate is held open until the next "end of record" after the switch has been reset. This feature insures that only complete records are recorded on the tape.
5. Stores data from the manual data entry unit.

ANALOG DATA COLLECTION SUBSYSTEM

This portion of the system was designed to collect temperature data on the reactor core.

Sixty thermocouple outputs enter the system through the DCS signal cabinet. The cabinet contains seven 36-pole relays controlled by the Analog Block Selector. The relays connect the thermocouples in groups of fifteen, or the reference voltage, or the reference ground, or the outputs of the DCS patch panel

FIG. IV-2-2. Functional Diagram ZPPR Encoder Read Out Sub System. ANL Neg. No. 103-A11158.

the data logger. In the data logger the fifteen inputs selected with the Analog Block Selector are multiplexed. The information is then digitized as 17 bits of binary coded decimal data which are recorded with channel number and gain information.

The data logger also provides the following controls for its magnetic tape unit:

1. record length control, which may be set from 1 to 10,000 scans per record
2. beginning of tape control
3. end of file control.

These controls enable the system to produce IBM compatible magnetic tapes for use on the ZPPR computer.

IV-3. A Remote Input/Output System for the Zero Power Plutonium Reactor (ZPPR) Computer

J. E. HUTTON

INTRODUCTION

Collection and on-line analysis of experimental data at the ZPPR Facility will be accomplished through the remote I/O system described in this report. The system collects and transmits experimental data from various locations in the ZPPR facility to the central 840MP computer (see Paper IV-1) where it is recorded and analyzed. Where required, results of the analysis can be returned to the appropriate remote terminal for display and control.

Although the design criteria for this system have their foundations in the justification and design study for the ZPPR computer, the actual design could not proceed until the 840MP computer had been chosen and details of installation had been settled. At present end-to-end checks between a single remote terminal and the computer have been successfully completed, and fabrication is continuing on the remaining I/O terminals and signal conditioning circuits.

DESIGN CRITERIA

This system will be used by an organization which is responsible for the collection and analysis of data from the ZPPR reactor. This work includes both the development of new experimental techniques and the repeated use of these techniques after development for measurements on new reactor cores. Therefore, the remote I/O terminal must be a flexible and easily used experimental tool for development of new techniques while at the same time it is important that configuration control can be maintained when it is desired to set up and run previously developed collection systems.

provide flexibility:

Modular construction and standard interfaces are used throughout the system, thus insuring

interchangeability of experiments and equipment between stations.

2. Computer channels, control lines, and interrupts are routed to remote stations as needed.
3. Control logic at the data link selector and each remote station is tailored to each specific job by removable patch panels.

The following features were included to assist experimental personnel in the setup and checkout of new techniques:

1. Data are transmitted on data busses to minimize the number of individual connections required for setup.
2. Standard interface and timing logic is prewired to eliminate race and other control logic problems.
3. Manual controls are synchronized to the system clock and sync circuits are provided for external signals to avoid async-to-sync problems.
4. Local displays and controls operate independently from the computer for checkout of new systems and problem isolation.

To assist in maintaining configuration control, all control logic for the experimental equipment is connected through a removable patch panel which, for an established task, will be permanently wired or provided with a cover.

The data buss structure and computer channels will not normally change between jobs. Therefore, many routine jobs will only require changing of the experimental inputs, patch panel, and computer programs.

HARDWARE DESCRIPTION

The system can be separated into three types of subsystems: the SEL digital I/O unit, data link selector unit, and the remote I/O terminal. The first

FIG. IV-3-1. Data Link Selector Block Diagram. ANL Neg. No. 103-A11118.

two units are located in the computer room while one unit of the third type is located at each remote location. Figures IV-3-1 and IV-3-2 detail these subsystems and their interconnectors. The remainder of this report explains the functions of the components shown in the figures.

SEL DIGITAL I/O UNIT

This unit provides an interface between the 840MP I/O buss and the data link selector, the necessary synchronizing signals for I/O operations, and the signal conditioning for the interrupt lines. Four 24-bit output channels, six 24-bit input channels, twenty-four control lines, two 16-bit status channels, and ten interrupt lines are available to the experimenter.

DATA LINK SELECTOR (DLS)

This unit routes the I/O data and signals to the remote I/O terminal. Figure IV-3-1 shows its major components which are described below.

I/O Busses

Busses to the remote I/O terminals are divided functionally into three categories:

1. twenty-four lines of data from the computer
2. twenty-four lines of data to the computer and one line acknowledging computer acceptance of the data
3. twenty-four lines of control information. 16 of these lines provide connection between the DLS and remote I/O patch panels; the remaining lines carry the system clock, address for the address counters, and synchronizing signals.

All signals are transmitted through identical 25-pair connectors. Therefore, the configuration of the busses can be adjusted to meet the requirements of particular experiments by moving connectors on the DLS connector panel.

System Clock and Divider

The system clock is provided by the data link selector. This 100 kc clock is divided to provide synchronized timing pulses on the control patch panel at 10 kc, 1 kc, and 100 cps. The BCD outputs of these dividers are available to the computer so that event times may be recorded to the nearest 10 μ sec without interrupting the computer at these higher rates.

FIG. IV-3-2. Remote I/O Terminal Block Diagram. ANL Neg. No. 108-A11119.

Sync Logic

Synchronization pulses for each input channel are shaped and delayed until the computer has accepted the input. The signals then go directly to the input connector associated with that channel. Other sync signals which are used only for specific applications are routed to the control patch panel.

Address Logic

These circuits decode computer bits 16 and 17 of channel 73 and send an "Address to Unit" (ATU) pulse to the control patch panel after bits 18-23 of Channel 73 have been output to all the remote I/O terminals. The ATU pulse is patched to the appropriate I/O terminal where it is used in conjunction with bits 18-23 to address experimental equipment connected to that I/O terminal.

Patch Panel

This panel is used to route interrupts, status signals, control bits, timing pulses, sync signals, and ATU signals between the remote I/O terminals, the computer, and the DLS timing, sync, and address

logic. Standard logic circuits are also provided on the panel for setup of non-routine I/O control problems.

REMOTE I/O TERMINAL

One remote I/O terminal located at each experimental station provides operator controls and displays in addition to data collection and timing equipment. Each terminal is modular in construction containing three types of units: display, control, and experimental equipment housing. Each of these units is described below and Fig. IV-3-2 shows their functional components.

Control Unit

The control unit provides an interface for all data and control signals. It is composed of the following functional blocks:

Signal Conditioning Modules

These modules contain line drives and receivers and balun isolation units. There is one module for each I/O buss. Each control unit holds one control module and up to three driver or receiver modules.

Manual Controls and Indicators

Eight alternate action and three monitory contact switches are wired to the patch panel for operator control. The switch outputs are buffered and synchronized with the system clock. Other switches are provided for simulation of computer control bits and selection of signals for display. Eight indicators from the patch panel are also provided.

Patch Panel

This panel provides the decoded outputs of the address counters, eight outputs from the DLS patch panel, outputs from the sense switches, inputs for the indicator lights and eight inputs to the DLS patch panel. Connections to the experimental control logic and connection to the six 18-pin connectors on the control module for control of experimental equipment are also provided.

Address Setting Logic

These circuits set and control the address counters during the transmission of data to and from the computer. Pulses are also provided for resetting experimental equipment after data has been read by the computer.

Experimental Equipment Control Logic

The inputs and outputs of several DTL gates, clocked flip flops, and synchronizing latch circuits are provided on the patch panel to facilitate the control of experimental equipment through the connectors on the control module.

Display Unit

This unit provides a display of computer generated data and a display independent of the computer output of the experimental equipment at the station. Since this display is taken from the I/O buss it is useful in setup and checkout. This unit contains the following functional blocks.

Display

This section contains six rear projection displays, with $\frac{7}{8}$ in. figures and their associated drivers and registers. The register is normally loaded from the I/O buss that sends data to the computer. The load command is synchronized with the desired data by the display selector.

Display Selector

This circuit accepts the decoded outputs from the cycling address counter. The output selected by the display selector switch is then transmitted to the display register as a load command.

Address Counters

There are two address counters: One for transmitting data to the computer, the other for receiving data from the computer. Their outputs, which appear on the patch panel, are used to gate data from experimental equipment to the I/O buss and to gate computer data to the display.

The first of these is a cycling counter. It is normally counting at a 100 kc rate. This provides read pulses which are used for local display of data on the computer input buss. When the computer is reading data, the address logic sets and holds the cycling counter at the proper address. The second counter is used to direct data from the computer to experimental equipment or the display. It is set by the address logic and does not normally cycle.

Experimental Equipment Housing

Standard nuclear instrument module (NIM) bins are used for the experimental equipment housing. In addition to experimental equipment built for the system these bins will accept many commercial counters, A/D converters, discriminators, amplifiers, and high voltage power supplies.

IV-4. Automated Fuel Inventory Procedures for the Zero Power Plutonium Reactor (ZPPR) and ZPR-3

R. W. GOIN and E. MISCHKE

During the period before ZPPR startup became a reality, a project to automate fuel inventory procedures for ZPPR and ZPR-3 was initiated. The project resulted from cognizance of the large number of plutonium fuel plates necessary to construct antic-

ipated cores. Ideally, a completely automated system as proposed by H. Lawroski et al.¹ would involve a mechanical or optical device which could read an identification code on each fuel plate, examine the fuel plate for out-of-tolerance conditions, and transmit this

mation along with the physical location of the plate, to some recording device. Much research needs to be done before the above-described system can be achieved.

The first logical step in automating the fuel inventory was simply to punch on cards the information that is regularly recorded. As standard practice, Special Materials personnel record specific information about each plate when fuel is checked in and out of the vault; and similar information is recorded when reactor fuel drawers are loaded and unloaded. To facilitate transferring the needed information to punched cards, the recording forms were modified so they could be used as keypunch forms. The complete plutonium fuel inventory was assimilated and stored on magnetic tape. Recorded information for each fuel plate includes a complete isotopic analysis, date of fabrication, weight of the stainless steel can, name of the manufacturer, a compact identification, and the current physical location of the plate.

The recording procedure is as follows: When a fuel piece is taken from the vault to the checking station, its identification symbol, location in the vault, and any out-of-tolerance conditions are recorded. The plate is then taken to the loading station, where its identification symbol and intended location in the

reactor are recorded. This procedure is followed in reverse when fuel is transferred from the reactor to the vault. This information from both stations is key-punched on cards which are then used as input for the fuel inventory program.

The fuel inventory program includes cross checks to verify the input. The location from which a fuel piece came is checked against the entry on the magnetic tape; and identification numbers from the loading station entries are correlated with those from the checking station. Any discrepancies cause a message to be printed, indicating the type of error. Discrepancies can be caused by an identification having been misread at one of the stations, or keypunch errors. The magnetic tape is updated according to the newest correct location information.

In addition to the obvious advantages of this limited system to the Special Materials personnel who are responsible for fuel inventory, this procedure opens new possibilities for determining exact atomic densities at any given location in any given reactor loading.

REFERENCE

1. H. Lawroski, J. E. Hutton and R. N. Curran, *A Magnetic Signaled Automatic Fuel Surveillance System*, Trans. Am. Nucl. Soc. **10**, 639 (1967).

IV-5. Characteristics and Capabilities of Automated Counting Facilities for the Zero Power Plutonium Reactor (ZPPR)

D. W. MADDISON and L. S. BELLER*

The recently-completed ZPPR Counting Facility was designed for support of all phases of the experimental program with a minimum of special setup or equipment required for a wide variety of radiation measurements. The major requirement was for a facility to accommodate a greatly expanded program of activation measurements of heterogeneity effects and of detailed intracell measurements of reaction rates and spectral indices in plate-type lattices. The design requirement was for a method of obtaining precise (1% or better internal precision) activation measurements of up to several hundred foils of different types at a time. In addition, there was a general requirement for the capability for doing, on a less frequent basis, individual measurements involving trace-

element analysis, low-level gamma-ray analysis, coincidence measurements, and alpha activity determination.

The requirements were satisfied by a system based primarily on spectroscopy. The major detector is the lithium-drifted germanium [Ge(Li)] gamma-ray detector. The principle advantages over competing detector types are very fine resolution, excellent stability, and a well understood response function (necessary for proper intercomparison of reaction rates in different materials). The major disadvantage is a lower detection efficiency (typically three to five percent of that of NaI(Tl) detectors for medium sized crystals). The fine resolution of Ge(Li) detectors, however, makes the lower limit for quantitative detection much higher than these figures would indicate, especially in mixed spectra. The present configuration consists of three such detectors of nominal

* Atomics International, a Division of North American Rockwell Corporation, Canoga Park, California.

30 cm³ active volume, in a permanent installation with automatic sample changers.

The permanent installation is intended to handle the large numbers of foils required in many experimental programs. The configuration of two of the detector systems includes 2 × 2 in. NaI(Tl) detectors mounted so that they can be used either independently or in coincidence with the Ge(Li) detectors. The third detector system includes a lithium-drifted silicon [Si(Li)] charged-particle detector which may be used in the same coincidence/independent modes. The resolution and efficiency of the Si(Li) detector is sufficient for either conversion electron spectroscopy or alpha spectroscopy of the fertile and fissile isotopes.

A medium-sized (3 × 3 in.) NaI(Tl) detector is available in a standard-geometry lead cave. This is used for very low level counting and for standard-source comparisons.

One small (11 cm³) Ge(Li) detector is available in a non-permanent installation for miscellaneous measurements or special setups. This can be used in the standard lead cave when low background and high resolution must be combined.

The heart of the entire installation is a small general-purpose computer. This is a Systems Engineering Laboratories Model 810A, with a memory consisting of 4096 words of 16 bits each. Internally, the machine has an instruction cycle of 1.75 microseconds, hardware multiply/divide, and a very complete and flexible instruction repertoire for real-time data acquisition and some forms of analysis. Externally, the machine is easy to interface to experimental equipment. This last property is used to give the machine the capability for reading a considerable amount of external data and performing control functions not normally available to multichannel analyzer systems. The computer can read and control up to four analog-to-digital converters (ADC), three automatic sample changers (and identify each sample uniquely in any order, checking to be sure the samples are in position before beginning the count) and the functions of external controller/timers for the detector systems. It can read and record the time of day, most of the important switch and selector settings in the nuclear electronics systems, and other auxiliary data. Many additional input and output functions can be implemented easily.

Output can be obtained optionally on an incremental magnetic tape recorder or a standard teletype keyboard or tape punch. Program input is through either keyboard, perforated paper tape, or magnetic tape. In addition, the computer has complete command over a large storage display oscilloscope.

The experimental configuration of the entire counting facility is controlled by the computer's program. For most operations, this program specifies that the facility is a multichannel analyzer system. The computer then becomes from one to three independent multichannel analyzers, or a single multiparameter analyzer with up to four parameters. The full power of a general purpose computer is available for real-time or preliminary data reduction. It could, for example, determine the length of each count on the basis of accumulated statistics rather than fixed time.

The small 4k memory size is not as severely limiting as might be expected. Most spectra contain large regions with no important information; even in mixed fission-product spectra, for example, only 15 to 30% of the channels contain essential information, and programs are normally written to ignore unwanted regions of the spectrum.

The computer program can be changed readily to handle any particular job requirement. Programs are presently available to operate all three sample changers, Ge(Li) detectors, and ADC's simultaneously or singly. The single-analyzer program allows 3350 channels of data storage taken from designated regions of the 8192 channel ADC spectrum. The three-analyzer program allows for about 1000 channels of storage from each of the three 8192 channel ADC spectra, again from designated regions specified at the beginning of the counting interval. This allows the maximum resolution of the system to be utilized and only the portion of the spectrum with significant information to be collected.

An extensive set of data reduction and analysis programs had been written to handle the data produced by the counting facility. The output can be varied to suit a large number of experimental conditions. All programs start with a peak locator routine. From this point, a number of types of output are possible. Results equivalent to any desired number of standard differential or integral single-channel analyzers (SCA), with or without background subtraction, can be obtained. The parameters of the SCA units are derived from the observed spectrum or peak shape so that any drifts are self-cancelling. (The original differential data are available for other types of analysis if desired.) More elaborate programs have been written (see Paper IV-20) to analyze spectra, identify individual gamma rays and provide exact energy, half-life, transition intensity, and isotope name, go through a thorough statistical analysis, and write output in terms of absolute activities of the parents of all gamma rays in the spectrum.

The following list provides significant details of the major components of the system.

Detectors

1. Lithium drifted germanium; gamma-ray analysis.
 - a. Volume and type: 30 cm³, 5 sided trapezoidal
Resolution, FWHM: 3.3 keV at 1332 keV (Co-60), 2.0 keV at 81 keV (Ba-133)
Efficiency, relative to NaI: 4.1 % at 1332 keV
Lower useful energy limit: 60 keV
 - b. Volume and type: 30 cm³, 5 sided trapezoidal
Resolution: 3.5 keV at 1332 keV, 1.8 keV at 81 keV
Efficiency: 3.6 % at 1332 keV
Lower useful energy limit: 60 keV
 - c. Volume and type: 30 cm³, coaxial
Resolution: 3.2 keV at 1332 keV, 2.1 keV at 81 keV
Efficiency: 3.7 % at 1332 keV
Lower useful energy limit: 20 keV
 - d. Volume and type: 11 cm³, coaxial
Resolution: 4.2 keV at 1332 keV
Efficiency: 1.2 % at 1332 keV
2. Lithium drifted silicon; alpha, beta, and x-ray analysis.
Size: 3 cm² × 2000 microns depth
Resolution: Am-241 alphas—35 keV at 20°C,
Cs-137 conversion electrons—15 keV at 20°C
3. Sodium Iodide; gamma ray analysis.
 - a. One 3 × 3 in. NaI, 8 % resolution
 - b. Two 2 × 2 in. NaI, 8½ % resolution
 - c. Two 1½ × ¼ in. NaI for low energy gamma- and x-rays

B. Shields and Sample Changers

1. One 4-in. lead shielded cave with graded cadmium and copper liner, inside dimensions—32 × 32 × 30 in. Background with 11 cm³ Ge(Li) detector—10 counts per min.
2. Three sample changers with lead shielding.
Sample capacity—120
Sample size—2 in. diam × ¼ in. thick, maximum
Special foil holder for 5 mil thick by ½ to 1 in. diam foils
Background for 30 cm³ Ge(Li) detectors—6 counts per sec
Sample positioning reproducibility: ±0.001 in. vertically, 0.010 in. horizontally

C. Electronics

1. Preamplifiers: integral connection with each detector.
2. Amplifiers: pole-zero corrected, active filter, Gaussian output. Prompt and delayed outputs, unipolar or bipolar pulse shaping.
3. Linear ratemeter: 0–100,000 counts per sec, 8 scales.
4. Single channel analyzer: leading edge or cross-over timing, base line restorer, strobe and gate, variable delayed output.
5. Biased amplifier and pulse stretcher.
6. High voltage supplies:
 - a. 0–3000 V, 10 ppm stability, 0–6 mA
 - b. two 0–3000 V, 100 ppm stability, 0–6 mA
 - c. two 0–1000 V, 50 ppm stability, 30 µA
7. Analog to digital converters—three.
Conversion rate—100 MHz
Full-scale conversion—8192 channels
Digital and analog offsets
Linearity: 0.075 % integral, 0.75 % differential
Base-line restorer: active or passive
8. Precision controller/timer—three.
Modes: real time, live time, fixed dead time per event
Presets: real and live time—0–999 minutes, fixed dead time—0–999 microseconds per event
Accuracy: real time—10 ppm, live time—0.1 % (estimate), fixed dead time—0.01 % (estimate)

D. Data Handling and Control

1. SEL-810A computer, 4k memory, standard teletype and digital input/output unit, couples to sample changers, ADC's, controller/timers, time of day clock, storage scope, and magnetic tape unit.
2. Storage oscilloscope, 8 × 10 in. screen, separate storage and non-storage modes, for live and static data display, computer command of all functions.
3. Magnetic tape unit, read and write modes, 200, 556, or 800 bpi density, incremental, slew, or high speed write or read.

IV-6. Measuring Reaction Rates in Heterogeneous Fast-Reactor Cells

G. S. STANFORD, C. E. TILL and W. R. ROBINSON

Measurements of the reaction rates in the cell constituents are among the more readily interpretable of the measurements made on fast critical assemblies. The information is directly related to the neutron balance, and is invaluable for diagnosing areas of uncertainty in cross sections and calculational methods. Although in reactor analysis the reaction rates generally enter as ratios, to determine those ratios the measurements of the individual reaction rates must usually be absolute. Since absolute measurements tend to be plagued by unsuspected systematic errors, agreement between independent methods is important.

To examine the status of various methods for measuring reaction rates in fast critical assemblies, several such methods were used in recent measurements on ZPR-3 and ZPR-9. The available methods fall into four general categories which, in order of decreasing spatial resolution, are foils, solid-state track recorders (SSTR), radiochemistry, and fission counters. In this paper, in-cell methods for measuring fission rates in fissionable isotopes and capture rates in U-238 will be discussed.

The primary data from activated foils consist usually of relative counting rates, which must be reduced to an absolute basis by means of some form of calibration. Several independent calibration methods are available. Except for threshold reactions, there is the standard thermal calibration technique, involving the irradiation in a thermal-neutron flux of foils similar to those irradiated in the fast assembly. Accuracies approaching $\pm 1\%$ can be obtained.

For the fission reactions, one can calibrate the foils by positioning some of them against the face of an

absolute fission counter located somewhere in the fast assembly during the irradiation.

Relative capture rates in U-238 foils can be determined by counting the 100-keV x-ray-gamma-ray coincidences resulting from the decay of Np-239.^(1, 2) Calibration can be accomplished both by the thermal calibration technique and by counting, along with the foils, a calibrated Am-243 source. At present, however, full application of the latter technique to thick foils has been prevented by the lack of an accurate correction for self-absorption of the 100-keV radiation by the foil.

The SSTR technique³ is potentially useful for making absolute fission-rate measurements in heterogeneous reactor cells; its usefulness will increase when automatic track counting becomes available.⁴

Radiochemical analysis⁵ can be used to determine fission rates in general, and capture in U-238 in particular, in rather large pieces of material, yielding volume-average results. Reproducibility is approximately $\pm 3\%$.

Because of their size and construction, fission counters—either solid-state or gas-ionization—are unsuitable for detecting intra-cell fine structure. However, they can provide absolute fission rates and can indicate cell-average values if the geometry is suitable. Also, as mentioned above, they can be used for absolute calibration of fission foils.

Except for the SSTR and Am-243 techniques, which need some further development, agreement between the various methods has in general been satisfactory, when suitable precautions are taken. This is indicated in Table IV-6-I, which gives results obtained in two heterogeneous assemblies.

TABLE IV-6-I. REACTION-RATE RATIOS (PER ATOM) BY DIFFERENT METHODS

	Assembly 24 (Uranium Fuel)		Assembly 55 (Plutonium Fuel)		
	c^{28}/f^{25}	f^{28}/f^{25}	c^{28}/f^{49}	f^{28}/f^{49}	f^{49}/f^{25}
Thermal calibration	0.1325 ± 0.002	—	0.1415 ± 0.002	—	0.902 ± 0.02
Radiochemistry	0.1281 ± 0.005	0.01814 ± 0.0006	0.1394 ± 0.005	0.01939 ± 0.0006	—
Fission chambers	—	0.01914 ± 0.0006	—	0.02047 ± 0.0006	0.909 ± 0.01
U-238 Radiochemistry	0.1310 ± 0.004	0.01831 ± 0.0006	—	—	—
U-235 Fission Chamber	—	—	—	—	—
U-238 Radiochemistry	—	—	0.1393 ± 0.004	0.01953 ± 0.0006	—
Pu-239 Fission Chamber	—	—	—	—	—

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IV-7. Thickness Corrections for Neutron-Activated Gold Foils

G. S. STANFORD and J. H. SECKINGER

Curves for determining thickness-correction factors for treating counting data for sets of neutron-activated unmatched gold foils are presented in Ref. 1. The curves were determined experimentally by making use of the first derivative of the curve of activation versus thickness. This was done for both epithermal and thermal activation, using gold foils located in a void in a graphite thermal column and irradiated in a near-isotropic flux. The epithermal activation was found to agree well with a published theoretical calculation. An apparent disagreement with theory for the thermal case (less important for thickness-correction purposes) when a low counting bias was used seems to have been due to an inaccurate counting rate correction for gamma-ray scattering and self-absorption.

A computer code was written to perform a least-squares fitting process in order to obtain the coefficients

C_i for the equation

$$A(t) = C_1 + C_2 f(t) + C_3 f^2(t) + C_4 f^3(t),$$

where $A(t)$ is the specific activation of a foil of thickness t , and $f(t)$ is a differentiable function with a theoretical basis for being at least an approximate fit to the data. This results in the function $A(t)$ being differentiable. For the epithermal data, the calculated self-shielding function for a single-resonance approximation was used for $f(t)$. For the thermal case, $f(t)$ was taken to be the standard thermal self-shielding function. One advantage of this approach, over fitting to an arbitrary function such as a polynomial or a sum of exponentials, is that extrapolation to zero thickness can be made with greater assurance.

REFERENCE

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IV-8. Mass Determination of Thin Foils

R. J. ARMANI, E. R. EBERSOLE* and J. M. STEVENSON†

Fission rate measurements are often obtained in fast critical assemblies by use of fission chambers, as thoroughly described by F. Kirn.¹ Thin fission foils are used in these chambers to reduce loss of fission fragments. Typically, foil thicknesses vary from 15 to 100 Mg/cm².

In order for the measurement to be absolute, the mass of the fissionable material must be accurately determined (within approximately 1%). Three methods can be used to determine the mass of the thinly deposited sources, two of which are nondestructive and one of which is destructive. The last of these is undesirable since the foils are customarily used for more than one experiment; hence that method is not considered here.

*Idaho Division, Argonne National Laboratory.
 †United Kingdom Atomic Energy Authority, Winfrith, England.

TABLE IV-8-I. RESULTS OF DIFFERENCE METHOD AND ABSOLUTE ALPHA COUNTING

Sample No.	Fissionable Material	Mass in μg by Difference Method	Mass in μg by α Counting
101	Natural U	95.7	101.3
21	Enriched U	109.1	108.5
69	Pu-239	106.3	107.2
71	Pu-240	94.6	97.9

The two methods used are (1) the difference method and (2) absolute alpha counting. The difference method is a combination of mass spectroscopy, uranium fluorimetry, and alpha assay.¹ A known quantity of fissionable material is placed in an electroplating cell, and the plating process is allowed to proceed. At the end of the process, the electrolyte is analyzed for the mass of material remaining in solution. A material balance provides the mass which has been electroplated. The absolute alpha counting method consists of determining the alpha activity by 2π or low-geometry counting.² The specific alpha activity is calculated by use of the isotopic analysis and proper decay constants, and the mass is obtained by dividing the measured alpha activity by the specific alpha activity.

Relative measurements are often made by exposing unknown foils together with previously calibrated similar foils in in-core irradiations. The details of

such comparisons are described by P. Amundson et al.³

Each method has its own advantages, and any given experimenter may feel more confident using one of these methods in preference to the other or a relative measurement. Of course, it is most desirable to use both methods and a relative measurement and then to compare the results. Generally, ANL-Chicago uses the absolute counting method while ANL-Idaho uses the difference method plus relative measurements. In order to compare results, the masses of four fission foils, which were prepared by electrodeposition, are to be determined by relative measurements and the two methods described.

Preliminary data from the difference method and absolute alpha counting are available. Table IV-8-I shows these results. Data from relative measurements with in-core exposures compared to known foils will be available shortly.

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3. P. I. Amundson, A. M. Broomfield, W. G. Davey and J. M. Stevenson, *An International Comparison of Fission Detector Standards*, Proc. International Conference on Fast Critical Experiments and Their Analysis, Argonne National Laboratory, 1966, ANL-7320, pp. 679-687.

IV-9. An Evaluation of Methods to Measure the Effective Delayed Neutron Fraction

E. M. BOHN, R. A. KARAM and A. B. LONG

In general, reactivity calibration is accomplished through use of the point reactor kinetics equation¹ which uses values of the effective delayed neutron fractions given by²

$$[\gamma_i \beta_i]^q = \frac{\int_V \int_{E'} \beta_i^q \chi_i^q(E') \phi^+(r, E') dE' \int_E \nu^q \Sigma_f^q \phi(r, E) dE dV}{\sum_q \int_V \int_E [(1 - \beta^q) \chi^q(E') + \sum_i \beta_i^q \chi_i^q(E')] \phi^+(r, E') dE' \int_E \nu^q \Sigma_f^q \phi(r, E) dE dV}, \quad (1)$$

where superscript q denotes values for isotope q , subscript i denotes the i th delayed neutron group, β_i is the fraction of delayed neutrons in the i th group, γ_i

is the importance of delayed neutrons in the i th group, χ_i^q is the delayed neutron spectrum for isotope q , χ^q is the prompt neutron spectrum for isotope q ; and V is the reactor volume. The total effective delayed

neutron fraction $\overline{\gamma\beta}$ is given by the sum,

$$\overline{\gamma\beta} = \sum_q \sum_i (\gamma_i \beta_i)^q.$$

$[\gamma_i \beta_i]^q$ are usually computed by multigroup calculations for which values of β_i^q and χ_i^q must be supplied. There exists experimental data for the β_i^q for nearly all isotopes of interest except Pu-241 and Pu-242. These data have typical uncertainties of 5–10%⁽³⁾. The isotopic abundance of Pu-241 and Pu-242 in plutonium fuel is small and reasonably estimated values of β_i^q for these isotopes may be used (see Paper II-26). However, the only data which exist for the χ_i^q are the data for the first four delayed groups of U-235. In the computation of $[\gamma_i \beta_i]^q$ (and for reactor calculations in general), a composite delayed neutron spectrum for each isotope, χ_b^q , is used rather than the individual group values, χ_i^q . This composite spectrum is an average of the χ_i^q weighted by the relative delayed neutron group yield a_i^q , where the χ_i^q are taken as those for U-235. In addition, the fifth and sixth delayed neutron group spectra are assumed to correspond to the fourth group spectrum. Thus,

$$\chi_b^q(E') = \sum_i a_i^q \chi_i^{235}(E') \quad (3)$$

is substituted into Eq. (1) for the χ_i^q .

This is equivalent to assuming that the effectiveness of all delayed neutron groups for isotope q is the same (i.e., $\gamma_i^q = \bar{\gamma}^q$). From Eq. (2) the total effective delayed neutron fraction is then written as

$$\bar{\gamma}\beta = \sum_q \bar{\gamma}^q \beta^q. \quad (4)$$

Thus, the calculation of $\bar{\gamma}\beta$ entails certain assumptions and approximations which introduce uncertainty in reactivity calibration (through use of the period-reactivity relation), especially for fast reflected assemblies containing plutonium fuels. Therefore, because of the inherent uncertainties in the calculation of $\bar{\gamma}\beta$,

$$\sum_{\Delta V} \left. \frac{\delta k}{k} \right|_{\Delta V} = 1 - \frac{\int_V \int_{E'} \int_E \phi^+(r, E') [(\Sigma_a^f - \Sigma_a^s) - (\Sigma_s^f - \Sigma_s^s)] \phi(r, E) dE dE' dV}{\int_V \int_{E'} \int_E \phi^+(r, E') \nu \Sigma_f^f \phi(r, E) dE dE' dV}, \quad (8)$$

a measurement of $\bar{\gamma}\beta$ with a precision of 10% or better would be useful. An evaluation of methods to measure $\bar{\gamma}\beta$ for typical ZPR Assemblies is presented here.

SUBSTITUTION EXPERIMENT

In an ideal substitution experiment, a non-fissionable substance having the same scattering and absorption cross sections as the fissile fuel is substituted for the fuel in a region ΔV of the reactor. The reactivity of the substance is

$$\left. \frac{\delta k}{k} \right|_{\Delta V} = \frac{-\int_{\Delta V} \int_{E'} \chi(E') \phi^+(r, E') dE' \int_E \nu \Sigma_f \phi(r, E) dE dV}{\int_V \int_{E'} \chi(E') \phi^+(r, E') dE' \int_E \nu \Sigma_f \phi(r, E) dE dV}, \quad (5)$$

where the small contribution from delayed neutrons has been neglected. If the reactivity $\rho_{\Delta V}$ due to this substitution is measured in terms of dollars,

$$\bar{\gamma}\beta \rho_{\Delta V}(\$) = \left. \frac{\delta k}{k} \right|_{\Delta V}. \quad (6)$$

The calibration of control rods in terms of dollars for the $\rho_{\Delta V}$ measurements can be accomplished without a knowledge of $\bar{\gamma}\beta$ by pulsed neutron, prompt jump, or stable period measurements.*

If fuel is replaced over the entire core, the total change in k is 1.0 and

$$\bar{\gamma}\beta = \frac{1}{\sum_{\Delta V} \rho_{\Delta V}(\$)}. \quad (7)$$

The substitution experiment was used a number of times to measure $\bar{\gamma}\beta$ for thermal reactors to an accuracy of about $\pm 4\%$.⁽⁴⁻⁶⁾ The actual experiment deviated from the ideal experiment discussed above due to a mismatch in absorption and scattering cross sections between the fuel and the substitution substance. The substance usually substituted for fuel was boron. Only absorption in the thermal group had to be considered since absorption in higher energy groups and scattering were relatively small. The mismatch in cross sections between the substitution substance (s) and the fuel (f) was treated as a correction factor in Eq. (5). Summing $\left. \frac{\delta k}{k} \right|_{\Delta V}$ over the entire core yields

where the second term on the right-hand-side is the correction term. If the correction term is small compared to 1.0, an inexact estimate of the term would not greatly affect the precision in $\bar{\gamma}\beta$. For example, R. Perez et al., computed a correction factor of 0.55 ± 0.017 which gave an uncertainty of $\pm 2.5\%$ in $\bar{\gamma}\beta$.⁽⁶⁾

Preliminary calculations have been performed with

* If prompt jump or stable period measurements are used, the assumption of $\gamma_i \approx \bar{\gamma}$ must be good for the system under study.

TABLE IV-9-I. PERTURBATION CALCULATION FOR Th-232 SUBSTITUTION EXPERIMENT IN ZPR-6, ASSEMBLY 6

	$\delta\nu\Sigma_f$	$\delta\Sigma_a$	δ (Leakage)	$\delta\Sigma_s$	Total
Removal of U-235 and U-238	-1.0000	+0.7406	-0.0571	+0.0373	-0.2792
Addition of Th-232 ^a	+0.0530	-0.7264	+0.0853	-0.0705	-0.6734
Net $\delta k/k$	-0.9470	+0.0142	+0.0282	-0.0332	-0.9526

^a Th-232 cross sections from set 224 were used for these calculations.

ZPR-6 Assembly 6,⁽⁷⁾ to determine the feasibility of a substitution experiment in a typical fast core. Th-232 was selected as the substitution substance since it closely matches the absorption and scattering cross-sections of U-235 and U-238 and has a high threshold for fission. Perturbation calculations using MACH-1⁽⁸⁾ simulated the replacement of all U-235 and U-238 with Th-232. The results are presented in Table IV-9-I. Since thorium does fission, a fission term must be included in the numerator of the correction factor in Eq. (8). Also included in the MACH-1 calculations, is the correction component due to leakage. The absorption, leakage, and scattering components can be matched well by thorium and the total net change in $\delta k/k$ ($= -0.9526$) is due mostly to a 5% contribution by thorium to the fission change. If one assigns a 10% random uncertainty to the calculation of each component of $\delta k/k$ in Table IV-9-I, an uncertainty in the total net $\delta k/k$ of ± 0.104 or 11% is found.

The uncertainty in each reactivity measurement due to uncertainties in control rod calibration and reproducibility can be estimated to give a 2% uncertainty in the measured $\bar{\gamma}\beta$.⁽⁶⁾ Thus, the total uncertainty in $\bar{\gamma}\beta$ would be about 12% considering all sources of error above, with the largest uncertainty due to the calculation of the correction factor to $\delta k/k$. This analysis has not considered heterogeneity effects in the substitution

sample which may add an additional one or two percent to the uncertainty.

VARIANCE-TO-MEAN EXPERIMENT

The variance-to-mean experiment is a variation of the Rossi- α type measurement and is based on the correlation in time of fission chain events in the reactor. E. Bennett⁹ has derived the following expression for the ratio of the variance σ^2 to the mean number of neutrons $\bar{c}$ detected during a counting time T :

$$\frac{\sigma^2}{\bar{c}} = 1 + 2\epsilon \frac{(\bar{\nu}^2 - \bar{\nu})}{\bar{\nu}^2} \sum_{i=1}^7 \frac{A_i B_i}{\omega_i} \left(1 - \frac{1 - e^{-\omega_i T}}{\omega_i T} \right), \quad (9)$$

where ϵ is the detector efficiency (detection rate/total fission rate), ω_i are the roots of the zero power reactor transfer function,

$$T(s) = \sum_{i=1}^7 \frac{A_i}{s + \omega_i}, \quad (10)$$

and

$$B_i = \sum_{j=1}^7 \frac{A_j}{\omega_j + \omega_i}. \quad (11)$$

Equation (9) may be written

$$\frac{\sigma^2}{\bar{c}} = 1 + 2\epsilon \frac{(\bar{\nu}^2 - \bar{\nu})}{\bar{\nu}^2} Y. \quad (12)$$

The correlated part of the variance-to-mean Y was evaluated in a one-delayed group model using parameters characteristic of ZPR-6 Assembly 6. The results are displayed in Fig. IV-9-1 as a function of the counting time interval and k . For short time intervals, only prompt neutrons contribute to Y which reduces to

$$Y = \frac{1}{2} \frac{(1 - \bar{\gamma}\beta)^2}{(\bar{\gamma}\beta - \rho)^2} \left[1 - \frac{1 - e^{-\alpha T}}{\alpha T} \right], \quad (13)$$

where α is the Rossi- α . If the counting interval is chosen short enough to ignore delayed neutron effects and long enough that $(1 - e^{-\alpha T})/\alpha T$ is small, Eq. (13) reduces to

$$Y = \frac{1}{2} \frac{(1 - \bar{\gamma}\beta)^2}{(\bar{\gamma}\beta - \rho)^2}. \quad (14)$$

From Fig. IV-9-1, it is noted that times of the order

FIG. IV-9-1. Correlated Part of the Variance-to-Mean Count Rate in ZPR-6 Assembly 6. ANL Neg. No. 118-3025.

... few milliseconds would be sufficient for the application of Eq. (14) to a typical subcritical fast assembly. As criticality is approached, the prompt neutron correlation term of Eq. (14) reduces to

$$Y \approx \frac{1}{2} \frac{1}{(\gamma\beta)^2}. \quad (15)$$

However, from Fig. IV-9-1, it is seen that the Y including delayed neutrons diverges for very short time intervals. Thus, the prompt neutron approximation of the variance-to-mean experiment in Assembly 6 is only good for slightly subcritical assemblies.

One problem encountered in the application of Eq. (14) is that of determining the reactivity ρ which depends on $\gamma\beta$. A method which has been suggested¹⁰ assumes a value for ρ , computes $\gamma\beta$, and then calculates a ρ based on the $\gamma\beta$ obtained. Iteration continues until the same $\gamma\beta$ is obtained from two successive calculations. Another method which could be used requires a reactivity calibration in terms of dollars. (This can be accomplished independently of a knowledge of $\gamma\beta$.) Equation (14) then becomes

$$Y = \frac{1}{2} \frac{(1 - \gamma\beta)^2}{(\gamma\beta)^2 [1 - \rho(\$)]^2}. \quad (16)$$

The variance-to-mean experiment has been applied to the measurement of $\gamma\beta$ in fast reactors,¹⁰⁻¹² using essentially the formulation obtained by substituting Eq. (14) into Eq. (12). The most restrictive limitation in these measurements has been the relatively low efficiency of the detectors used. From Eq. (12), it is obvious that one must have an efficiency at least as large as Y^{-1} to measure a reasonable σ^2/ϵ . Detectors which have been used include BF_3 counters,¹⁰ fission chambers,^{10,11} and gas scintillators¹² and in each case the efficiency was no greater than 10^{-4} . It has been necessary to cope with high count rates and only recently has the problem of dead time correction in the variance-to-mean measurement been fully realized.¹³⁻¹⁵

In order to estimate the uncertainty in $\gamma\beta$ for a variance-to-mean experiment, a parameter Y' is defined such that Eq. (12) may be written,

$$Y' = 2 DY, \quad (17)$$

with $D = \frac{\bar{\nu}^2 - \bar{\nu}}{\bar{\nu}^2}$. For small reactivities,

$$\gamma\beta \approx \left[\frac{\epsilon D}{Y'} \right]^{1/2}. \quad (18)$$

The relative error in $\gamma\beta$ is then given by

$$\frac{\Delta\gamma\beta}{\gamma\beta} = \frac{1}{2} \left[\left(\frac{\Delta\epsilon}{\epsilon} \right)^2 + \left(\frac{\Delta D}{D} \right)^2 + \left(\frac{\Delta Y'}{Y'} \right)^2 \right]^{1/2}. \quad (19)$$

N. Pacilio¹⁶ has calculated $[\Delta Y'/Y']^2$ to be

$$\left[\frac{\Delta Y'}{Y'} \right]^2 = \frac{1}{N} \left[\left(4 + \frac{3}{\langle M \rangle} \right) \frac{1}{Y'} + \left(2 + \frac{3}{\langle M \rangle} \right) + \frac{Y'}{\langle M \rangle} \right], \quad (20)$$

where N is the number of time intervals sampled and $\langle M \rangle$ is the average number of counts per interval. The relative uncertainties in ϵ and D can be estimated at 3% and 2%, respectively.¹² The uncertainty in Y' was calculated from parameters given by A. Bergstrom et al.¹² and found to be 7% for 3000 time intervals. The total uncertainty in $\gamma\beta$ is 4% with the uncertainty in counting statistics ($\Delta Y'/Y'$) contributing 3.5% alone. It would appear that the uncertainty in $\gamma\beta$ could be reduced to about 1% if the N in Eq. (20) is allowed to be arbitrarily large. However, long data accumulation times (of the order of a minute) subject the experiment to fluctuations not due to fission chain statistics, introducing unwanted correlations. An uncertainty of 4% is considered a minimum for the variance-to-mean experiment.

Although attractive for small cores, the variance-to-mean technique cannot be applied to large fast criticals due to efficiency limitations. Using MACH-1 to compute the fission density at the center of ZPR-6, an efficiency of about 1.0×10^{-6} was calculated for a fission chamber containing one gram of U-235. From the discussion above, this is a factor of 10^2 too small.

OTHER TECHNIQUES

The experiments discussed above are the only methods which have been used in the past to measure $\gamma\beta$. Other techniques which were investigated include a method based on pulsing the reactor. The decay constant α is measured as a function of subcritical reactivity ρ , where ρ is determined from a control rod calibration using an assumed value for $\gamma\beta$. If the resulting relationship is linear, as was observed for ZPR-6 Assembly 5,⁽¹⁷⁾ one has,

$$\alpha = -\frac{\alpha_c}{\gamma\beta} \rho + \alpha_c, \quad (21)$$

where α_c is the decay constant measured at delayed critical $\gamma\beta/\ell$. The straight line of Eq. (21), when extrapolated to the abscissa ($\alpha = 0$), should intersect the axis at the point $\rho = \gamma\beta$ if the value of $\gamma\beta$ assumed in the rod calibration is correct. $\gamma\beta$ is varied until a value is found which satisfies the extrapolation.

This method was tested through a series of calculations where the "measured" values of α were generated

TABLE IV-9-II. COMPARISON OF EXTRAPOLATED AND ASSUMED VALUES OF $\overline{\gamma\beta}$ IN THE PULSING EXPERIMENT

Assumed $\overline{\gamma\beta}$, 10 ⁻²	Extrapolated $\overline{\gamma\beta}$, 10 ⁻²	% Difference
0.674	0.674	0.0 ^a
0.700	0.704	0.6
0.983	1.007	2.4

^a This is the "correct" value of $\overline{\gamma\beta}$ as described in the text.

by computing

$$\alpha = \frac{1 - k(1 - \overline{\gamma\beta})}{\ell}, \quad (22)$$

using as "correct" values for $\overline{\gamma\beta}$ and ℓ , the values calculated by the BAILIFF routine of MACH-1 for ZPR-6 Assembly 6. The control rods of ZPR-6 Assembly 6 were calibrated for various values of $\overline{\gamma\beta}$, including the "correct" $\overline{\gamma\beta}$. The "measured" α values were plotted as a function of ρ for the case of each $\overline{\gamma\beta}$ in order to obtain an extrapolated $\overline{\gamma\beta}$. The extrapolated $\overline{\gamma\beta}$ and assumed $\overline{\gamma\beta}$ are compared in Table IV-9-II. There is a difference of only 2.4% between the assumed and extrapolated $\overline{\gamma\beta}$ for an assumed $\overline{\gamma\beta}$ nearly 50% greater than the correct value. In order to measure $\overline{\gamma\beta}$ within 10%, one would have to be able to distinguish a difference of about 0.5% between the assumed and extrapolated $\overline{\gamma\beta}$ values. For such an experiment the minimum difference discernible is about 5%.

Another method very similar to the pulsing experiment involves the comparison of $\rho/\overline{\gamma\beta}$ measured from a prompt jump and $\rho/\overline{\gamma\beta}$ measured from the stable period following the jump. The correct $\overline{\gamma\beta}$ is determined when a value of $\overline{\gamma\beta}$ used in the period measurement gives a $\rho/\overline{\gamma\beta}$ equal to that of the prompt jump. The sensitivity of such an experiment to assumed values of $\overline{\gamma\beta}$, however, is the same as that found above for the pulsing method.

Thus, although techniques similar to the methods described here have been used in the past to indicate that the calculated values of $\overline{\gamma\beta}$ are correct,¹⁷ it must be concluded that this agreement results from the insensitivity of these experiments to the value of $\overline{\gamma\beta}$.

SUMMARY

Methods which have been used to measure the effective delayed neutron fraction $\overline{\gamma\beta}$ in thermal reactors (substitution experiment) and in fast reactors (variance-to-mean experiment) have been considered for use with large fast critical assemblies. The substitution experiment would yield a result with an uncertainty of about 12% for the case of Th-232 substituted in ZPR-6 Assembly 6. The variance-to-mean measurement

was shown to have a maximum precision of about 1%; however, the experiment is limited to small assemblies. Methods based on pulsed neutron and prompt jump (and power history techniques) were found to be insensitive to $\overline{\gamma\beta}$.

The measurement of the effective delayed neutron fraction is important, especially in the case of fast reflected assemblies and for plutonium fueled assemblies in particular. Measurements yielding a precision of 10% in $\overline{\gamma\beta}$ would be useful. Thus, it is suggested that a small plutonium core be built for which $\overline{\gamma\beta}$ may be determined by the variance-to-mean method.

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IV-10. Precision of Reactivity Measurements by Servo-Control Techniques in a Reactor with a Source

R. E. KAISER and J. M. GASIDLO

In plutonium fueled critical assemblies, small reactivity measurements are complicated by the presence of a strong source consisting of the spontaneous fission of Pu-240 in the fuel. One technique commonly employed in making such measurements involves determination of the reactivity worths as indicated by differences in servo-controlled autorod positions. Using this technique, fluctuations in autorod position due to reactor noise can become a significant source of uncertainty in the results.

This study is an extension of previous work by E. Bennett and R. Long¹ on U-235 fueled reactors. The assumption is that the autorod fluctuations due to reactor noise, power drift, and external reactivity influences are independent of one another, which is reasonable if the reactivity changes are small. The frequency spectrum of the noise caused by autorod motion may then be determined from the transfer function of the automatic control system.

The derivation of the autorod noise spectrum is based on the configuration shown in Fig. IV-10-1, assuming that there is no external reactivity modulation or reactor drift, whence the external reactivity $\rho_e(t) = 0$. The spectrum of the rod-motion noise can then be derived from the closed loop transfer function. The resulting expression for the autorod noise spectrum is

$$W(\omega) = \frac{A_s^2(\omega) \frac{2}{F_0} \left[\epsilon^{-1} + A_T^2(\omega) \left(\frac{\nu(\nu-1)}{\bar{p}^2} \right) \right]}{1 + 2A_s(\omega)A_T(\omega) \cos[\phi_s(\omega) + \phi_T(\omega)] + [A_T(\omega) + A_s(\omega)]^2} = \frac{F(\omega)}{\epsilon} + G(\omega), \quad (1)$$

where application of noise analysis techniques^{2,3} gives, for zero frequency,

$$W(0) = \frac{2}{F_0} \left[\frac{(1-k)^2}{\epsilon} + \frac{\nu(\nu-1)}{\bar{p}^2} \right], \quad (2)$$

in terms of the reactor fission rate F_0 , the detector efficiency ϵ , the amplitudes of open loop transfer function A_s , and the reactor transfer function A_T , and their respective phase angles. This function describes the frequency spectrum of the fluctuation in autorod position at a constant power level.

In a typical experiment, the difference in reactivity corresponding to two positions of the autorod is sought. The average reactivity ρ_k at a single position k is defined as

$$\rho_k = \frac{1}{\tau} \int_{(k-1)\tau}^{k\tau} \rho(t) dt. \quad (3)$$

The differences D_k are calculated from the average positions according to

$$D_k = (-1)^{k+1} [\rho_k - \frac{1}{2}(\rho_{k+1} + \rho_{k-1})], \quad (4)$$

which cancels drifts of first and second order in time. The average D_k is then the value sought.

Due to the random nature of the fluctuations, the D_k 's may be considered to be random variables. However, there is some correlation between successive D_k 's because of the manner in which the differences are calculated. It will be assumed that the correlations vanish between D_j and D_k , where $j > k + 2$. The degree of correlation of adjacent values is related to the ensemble average of $\langle D_j D_k \rangle$ and can be expressed in terms of the autorod noise spectrum, the integration time τ , and the values of j and k . This expression may be written in integral form, as

$$\begin{aligned} \langle D_j D_k \rangle &= (-1)^{j+k} (8\pi\tau^2)^{-1} \left(\frac{1}{\epsilon} \int_0^\infty \frac{F(\omega)}{\omega^2} \right. \\ &\quad \cdot \sum_i A_i \cos \ell\omega\tau d\omega + \int_0^\infty \frac{G(\omega)}{\omega^2} \sum_i A_i \cos \ell\omega\tau d\omega \Big); \\ \langle D_j D_k \rangle &= 0 \quad \text{if } j > k + 2. \end{aligned} \quad (5)$$

$F(\omega)$ and $G(\omega)$ are the components of the rod noise spectrum. The ensemble averages can be evaluated by

performing the indicated integration. In his earlier paper, E. Bennett¹ assumed that $F(\omega)/\epsilon$ was small compared with $G(\omega)$ so that the detector efficiency term was not significant and that $W = W(0)$ could be used in evaluating the integral, which he then evaluated by contour techniques. However, it can be seen from Eq. (2) that the assumption that the efficiency term can be neglected is not necessarily valid in a reactor which is subcritical due to a spontaneous fission source.

The function $G(\omega)$ is constant from $\omega = 0$ to 100 and then drops off sharply. The integral over this function can be evaluated by contour methods with the assumption $G(\omega) = G(0)$. The break at high frequency has little effect. The function $F(\omega)$ increases sharply from its initial value, however, and its integral must be evaluated numerically. The magnitude of this increase from $F(0)$ to F_{max} is on the order of 3 decades

FIG. IV-10-1. Schematic Diagram of the Autorod Control System. ANL Neg. No. 103-11819 Rev. 1.

and the plateau where $F(\omega) = F_{max}$ ranges from about $\omega = 0.1$ to $\omega = 10^2$ to 10^3 for the cases studied.

Using the integrals thus determined, the ensemble averages $\langle D_j D_k \rangle$ can be calculated. The square of the measurement error E was then determined from

$$E^2 = \frac{1}{N} \langle D_k^2 \rangle + \frac{2}{N} \langle D_j D_k \rangle \quad (6)$$

and the square of the measured deviation σ was predicted from

$$\sigma^2 = \frac{1}{N} \langle D_k^2 \rangle, \quad (7)$$

where N is the number of differences D_k . Bennett's

TABLE IV-10-I. COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL EVALUATIONS OF AUTOROD ERROR
(Power = 20 W)

ϵ	Integrated $W(\omega)$, 10^{-2}	Experimental, 10^{-2}	$W(0)$ Approx., 10^{-2}
0.76×10^{-4}	1.27	1.2 ± 0.1	1.24
0.25×10^{-4}	1.36	1.2 ± 0.1	1.24
0.36×10^{-5}	1.87	1.5 ± 0.2	1.25
0.49×10^{-6}	4.00	2.6 ± 0.2	1.26
0.45×10^{-7}	12.6	8.8 ± 0.9	1.31

analysis, being analytic throughout, yielded the further result that $E/\sigma = \sqrt{8/3N}$. This ratio can be calculated by the method described here but cannot be stated explicitly.

An experiment was performed on Argonne's ZPR-3 reactor in which five detectors, placed to give widely varying efficiencies, were each used to feed a signal into the servo-amplifier which controlled the autorod. Forty integrations of autorod position were then performed with each detector and the appropriate differences calculated. The standard deviations and errors of the measurements were determined and compared with the predicted errors. The results of this comparison are shown in Table IV-10-I. The efficiencies quoted must be considered known to possibly 20% as they are based on the power calibration of the ZPR-3 reactor and the detector response.

The calculated errors and measured errors correspond quite well at higher detector efficiencies, indicating that both methods of calculation are reliable under some conditions. At lower detector efficiencies, however, the zero frequency approximation does not indicate the sharp rise indicated by the measurements and the numerically integrated calculations. This approximate knowledge of detector efficiency can enable one to plan an experiment, in particular to

se a power level, so that the sharp increase in error at low detector efficiencies can be avoided. An estimate of the efficiency can be obtained from the source constant, $Q = S\ell\epsilon$, which is determined by inverse kinetics (see Paper II-24), for every assembly. Using the measured Q , a calculated lifetime ℓ , and S from

$$S = \lambda^{240} N^{240},$$

a value of ϵ of sufficient accuracy can be obtained, where the neutron source is due to spontaneous fission of Pu-240. The results shown in Table IV-10-I indicate

that the expression

$$E/\sigma = \sqrt{8/3N}$$

is valid as long as one maintains certain minimum requirements on power level and detector efficiency.

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IV-11. Developments in Computer Circuitry

C. E. COHN

INTRODUCTION

Three generally useful developments in computer circuitry have arisen from work with the various small digital computers used in the Reactor Physics Division for experimental assistance.

UNIVERSAL PULSE INPUT FOR RTL LOGIC¹

The circuit shown in Fig. IV-11-1 will accept either positive or negative pulses of 1 V or larger and shape them into a form suitable for use with RTL logic. The output from the circuit will have sharp transitions even if the input pulses have very long rise and fall times.

In the quiescent state, the transistor will be cut off and the inverter following it will be full on. When a pulse appears, the transistor will receive base drive and begin to conduct. The transistor will receive additional base drive when the inverter comes out of saturation. The resulting regenerative action will

FIG. IV-11-1. Universal Pulse Input for RTL Logic. ANL Neg. No. 113-2546.

FIG. IV-11-2. Power Switch Incorporating Initialization Circuit. ANL Neg. No. 113-2556.

rapidly turn the transistor full on and the inverter full off. When the pulse terminates, the inverse action will occur. The regenerative loop has enough hysteresis to give sharp, bounce-free transitions. This circuit has been incorporated into the design of binary data scalars for the DATA-622/i and SEL-840MP computers.

SIMPLE TURN-ON INITIALIZATION FOR DIGITAL SYSTEMS²

When power is applied to a digital system, measures must be taken to insure that the flip-flops in the system will be initialized to the correct state. Also, destruction of memory contents by the turn-on transient must be prevented. Currently, these functions are performed by various "system normalizers," "initial-condition drivers", and other more or less elaborate arrangements.

The initialization can be done more simply and economically with the power switch arrangement shown in Fig. IV-11-2. Here the power switch is a two-pole, three-position rotary with *shorting* contacts. The extreme positions of this switch are the

FIG. IV-11-3. Transformation of J-K Flip-Flops. ANL Neg. No. 113-2554.

OFF and ON positions, while the middle position is traversed during every switching operation. One of the poles switches the power, while the other switches the initialization circuit.

In the OFF position, the power circuit is open, but the initialization circuit is closed. On switching to the middle position, the power comes on but the initialization is held without interruption by the shorting contacts. Switching to the ON position opens the initialization circuit with no interruption to the power.

On turnoff, an analogous sequence takes place. Switching from the ON position to the intermediate position actuates the initialization with no interruption in the power. Switching to the OFF position brings down the power while holding the initialization during the turn-off transient.

The above description applies to an initialization circuit that closes to initialize. A circuit that opens to initialize would be connected differently, as shown. Additional initialization circuits could be connected to additional poles.

Ordinarily, solid-state power supplies come to equilibrium after turn-on in just a few cycles of the line voltage. Therefore, normal operation of the switch would give enough dwell time in the middle position to wait out the turnon transient, with no conscious pause or hesitation required.

The existing power switch on the DATA-622/i computer was replaced by this circuit to allow a straightforward power switching procedure. Originally, the procedure was quite cumbersome and tedious. It was necessary to press the system reset button after turnon and to open the console door and throw the memory disable switch before and after every power switching. Now the power switch performs all these functions automatically. A three-pole switch was used, with separate poles allocated to the system reset and memory disable circuits.

TRANSFORMATION OF J-K FLIP-FLOPS³

The J-K flip-flop is a common element in integrated-circuit logic. Its application is complicated by the existence of two types, which differ in the condition for complementing. The first, which we shall call Type 1, is typified by DTL logic and complements when clocked with the J and K inputs both true. The second, called here Type 2, is typified by RTL logic and complements with the J and K inputs both false.

This note presents a simple recipe for transforming a circuit for one type to use with the other type. The transformation is done by replacement of each flip-flop with the block shown in Fig. IV-11-3a where the J and K inputs are inverted and transposed. The result is then simplified by the usual methods.

As an example of the use of this recipe, consider the modulo-3 counter⁴ shown in Fig. IV-11-3b, which was designed for Type 1 flip-flops. Figure IV-11-3c shows its output waveforms. Transformation by this recipe yields Fig. IV-11-3d, which simplifies by inspection to the final version of Fig. IV-11-3e, for Type 2 flip-flops.

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IV-12. Computer-Aided Calibration of a Fine Autorod¹

C. E. COHN, R. A. KARAM, J. E. MARSHALL and J. M. VAN DOORNINCK*

The fine autorod on the fast critical ZPR-6 Assembly 6,⁽²⁾ a servo-controlled rod of very small worth, was accurately calibrated with the aid of the DDP-24 computer³ in Argonne's Reactor Physics Laboratory. Use of the computer gave the needed accuracy in much less running time than use of non-computer methods.

The autorod was a right triangle of $\frac{1}{4}$ in. Boral sheet, $86\frac{3}{4}$ in. long and 2 in. wide at the base, extending axially through the entire assembly. The total travel of the rod was $11\frac{1}{8}$ in. The rod was driven by a 60-Hz two-phase servomotor and chopper servoamplifier with velocity feedback; the system was activated by flux error. The total worth of the rod was 1.67 lh, with deviations from linearity attributed to nonuniform distribution of boron in the Boral and of stainless steel in the structure. The sensitive reactivity measurements done with this rod required that the slope and the deviations from linearity be known to a high degree of accuracy.

Calibration was done by the inverse-kinetics technique⁴ in regular use at Argonne. The rod was set up to run repeatedly back and forth through its full travel. The position was digitized by a voltage-to-frequency converter connected to the position-readout potentiometer. The flux was digitized by another voltage-to-frequency converter connected to a current amplifier and ionization chamber.

The accumulated counts from the voltage-to-frequency converters were read every second by the computer and the reactivity was calculated. Each reactivity datum had a standard deviation of about 0.03 lh. The run proceeded for one hour, giving 73 in-and-out cycles of the rod. The reactivities were fitted to the sum of a series of Legendre polynomials in rod position and a series of Gram polynomials⁵ in time, the latter to allow for reactor drift. The signifi-

cance of each additional term in these series was checked by the F test.⁶ The limits of significance came at the eighth order in position and first order in time.

The fit was compared with results of other measurements observing the response of the autorod to the insertion and removal of a sample consisting of an aliquot of a U-235 foil of accurately-known reactivity worth. The differential worths were derived in a manner⁷ which eliminated the influence of reactor drift. These "other" measurements required a total of eight hours.

The small-sample measurements agree with the computer fit to within their own errors and inconsistencies. These measurements were unable to yield information near the ends of the rod travel. It is estimated that one week of small-sample measurements would have been needed to obtain results comparable in quality to the computer fit.

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* Visiting Scientist, Technological University, Delft, The Netherlands.

IV-13. Automatic Testing of Analog-Digital Converters

C. E. COHN

In computerized instrumentation having analog to digital converters (ADC), it is convenient to use the computer to test the ADC's and verify satisfactory performance. The most difficult part of such an opera-

FIG. IV-13-1. Block Diagram of Typical Voltage-to-Frequency Converter. ANL Neg. No. 113-2418.

FIG. IV-13-2. Block Diagram of Test Voltage Generator. ANL Neg. No. 113-2422.

FIG. IV-13-3. Typical Plot of Test Results. ANL Neg. No. 113-2967.

is to produce accurate test voltages under computer control. Previous systems¹⁻³ have used digital-analog converters with resistive ladder networks. These have to be built of high quality, expensive components for otherwise they show serious step nonlinearities where the higher-order bits change state.

It is possible to build an accurate test voltage generator without step nonlinearities by adapting an ordinary voltage-to-frequency converter. The voltage-to-frequency converter is a device which produces a pulse train whose repetition rate is proportional to an input voltage. Its principle of operation is shown in Fig. IV-13-1. It contains an analog integrator which integrates the input signal. When the integrator output reaches a certain level, a discriminator is triggered. This causes a standardized pulse generator to produce a pulse whose total area is highly stabilized. This pulse enters the integrator input in such a polarity as to reduce its charge. The pulse is also transmitted to the output. Thus, each pulse neutralizes a given amount of charge inserted by the input signal.

It is evident that we have here all that is needed to produce an accurate test voltage generator. In principle, all that is needed is to remove the input voltage and to break the connection between the integrator output and the discriminator input. The standardized pulse generator is now excited by a command pulse from the computer applied to the discriminator input. Each such command produces a step in the integrator output. A sequence of such commands produces a staircase waveform with highly uniform steps—just what is needed for ADC testing.

A test voltage generator of this type was made by adapting a commercial voltage-to-frequency converter. The generator is shown in block diagram form in Fig. IV-13-2. The step command pulses from the computer enter the discriminator and cause the production of standardized pulses to step the integrator. Unfortunately, the operational amplifier that came with the voltage-to-frequency converter was unsuitable for this application. It was apparently intended for operation at fairly low-voltage swings and it was unable to cover the voltage range desired. Therefore,

FIG. IV-13-4. Typical Plot of Deviations from Linearity. ANL Neg. No. 113-2969.

a precision solid-state operational amplifier was substituted. Thus, all that remains of the voltage-to-frequency converter is the discriminator and standardized pulse generator with their power supplies.

The unit was set up to cover the zero to 10-V range in 4,096 steps. For this purpose, the 0.01- μ F integrating capacitor was replaced by a 2- μ F capacitor with a polystyrene dielectric. A relay was installed to short the integrating capacitor⁴ and thus to initialize the integrator to zero on command from the computer.

The integrator feeds a unity-gain inverter/follower which allows outputs of either polarity to be obtained. The trimming potentiometers allow the instrument span to be precisely adjusted.

The computer program for ADC testing begins with the transmission of a command to reset the integrator. A test cycle then begins with the transmission of a step command to the generator. After a few microseconds to allow settling, another command pulse is transmitted to strobe the ADC. The computer then waits for a conversion-complete signal from the ADC. If such a signal is not forthcoming within a preset time, it is concluded that the ADC does not respond to that voltage level. Such nonresponse may occur at very low levels due to threshold effects or at very high levels due to saturation. The computer notes it and proceeds to the next cycle.

If the ADC responds within the preset time, the ADC reading is recorded. After the entire range has been covered the responses are fitted to a straight line. The slope of this line indicates the gain characteristic of the ADC, while the intercept indicates its zero setting. The intercept is helpful in obtaining a

correct setting of the ADC's bias potentiometer or compensating for an existing setting.

Figure IV-13-3 shows the results of a typical test as plotted by the computer. This test was on a commercial 4,096-channel ADC. The observed ADC response is plotted as a solid line which is interrupted at voltages where the ADC did not respond. The fitted line is shown dotted. The ordinate is the voltage out of the generator while the abscissa is the ADC response. The run took less than half a minute exclusive of graph plotting.

The computer also plots the residuals from the least-squares fit, as shown in Fig. IV-13-4. The jagged appearance of the curve results from the fact that the generator and the ADC both cover 4,096 steps and step boundaries are not necessarily in phase. Some channel-to-channel jitter is also apparent.

The method described here will yield important information about the behavior of an ADC which is impractical or impossible to obtain by other means.

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IV-14. A Computer-Controlled Microscope for Scanning Fission Track Plates¹

C. E. COHN, R. GOLD and T. W. PIENIAS*

The solid-state track recorder technique² has come into wide use for precision radiation measurements. A major inconvenience of this technique has been a necessity for manual counting of the tracks, and thus there is considerable interest in the automation of this task. A number of approaches are under development by various groups. The method followed here involves an optical microscope coupled to a digital computer where the specimen is mechanically scanned

under computer control. The scanning is done by a specimen stage which is moveable in two dimensions by stepping motors coupled to micrometer screws. This approach has three major advantages over the approaches being pursued by other groups. First, the inclusion of a digital computer allows more elaborate handling of the data than noncomputerized methods. Secondly, the specimen is scanned directly with intermediate photographic recording required. Thirdly, the entire useful area of the specimen may be scanned as a unit.

* Reactor Engineering Division, Argonne National Laboratory.

Figure IV-14-1 shows the microscope. The specimen stage moves on linear ball bearings. It is moved in each of two axes (called x and y) by a stepping motor of 800 steps per revolution coupled to an ordinary micrometer screw of 40 threads per in. This combination gives a step size of 0.8μ . The stage is spring loaded to eliminate backlash. The coupling between one of the screws and the stage is via an antifriction roller on the screw and a tungsten-carbide pad on the stage. This allows for relative motion between the screw and the stage when the latter is moved in the direction called y . Most of the motion normally occurs in the direction called x .

For focusing, the microscope tube is moved up or down relative to the stage (called the z -direction) in 3.2-micron steps by a stepping motor of 200 steps per revolution driving a third micrometer screw. The light detection equipment is mounted above the microscope tube.

Figure IV-14-2 shows the optical signal path. The light is produced by a conventional incandescent microscope lamp. (This is being replaced by a quartz-iodine lamp to increase light intensity by about a factor of three and so improve light collection statistics.) The light passes through the normal substage mirror and condenser, through the specimen, and into a 40-

FIG. IV-14-1. Computer-Controlled Scanning Microscope. ANL Neg. No. 113-2186.

FIG. IV-14-2. Optical Signal Path. ANL Neg. No. 113-2133.

FIG. IV-14-3. Motor Control Circuits. ANL Neg. No. 113-2134.

power objective of 0.65 numerical aperture. A 10-power eyepiece projects an image to a 16-mil aperture located 8 in. above the eyepiece.

This aperture defines one picture element. It is viewed by a $\frac{3}{4}$ -in. photomultiplier. The photomultiplier output is in the 0-10 μ A range. A current amplifier converts this to a voltage signal in the 0-10 V range. This signal goes to the input of a linear gate which drives an analog-digital converter (ADC). When the linear gate is strobed by the motor circuits, the ADC digitizes the light values existing at that instant. The ADC sends channel advance pulses to a scaler. On completion of conversion, the scaler contents are dumped into a buffer register in the computer.

Figure IV-14-3 shows the motor control circuits. A step in any direction is initiated by a control pulse from the computer which is developed by an "output

control pulse" instruction with the appropriate address. This control pulse performs three functions: First, it commands the motor drive circuit to cause a step. Second, it strobes the ADC, causing the light signal existing at the beginning of the step to be digitized. Third it triggers a delay multivibrator to produce a "motor busy" signal. The computer is programmed to sense this signal and to wait for its termination before initiating another step. In that way the stepping rate is kept to within the speed capabilities of the motors, which are at present 1000 steps per second for the stage motors and 200 steps per second for the focus motor.

Figure IV-14-4 shows the sequence in which things happen during scanning. The computer initiates a step and then reads the contents of the buffer register. The light value read is that existing at the time, not of the step just initiated but rather of the step immediately preceding. The computer processes the light value and then waits out "motor busy" before initiating the next step.

The computer is programmed to follow a raster-like scan pattern of the type shown in Fig. IV-14-5. Before the program is started, the microscope is manually set up so that the photomultiplier views the specimen where no track exists. With the light source obscured, the computer first takes the basal or zero light reading of the ADC. With the light restored, the computer performs a one-step back-and-forth motion which causes 100 light readings to be taken. The mean and standard deviation of these readings are

FIG. IV-14-4. Timing Diagram. ANL Neg. No. 113-2421.

FIG. IV-14-5. Scan Pattern. ANL Neg. No. 113-2417.

FIG. IV-14-6. Focus Check. ANL Neg. No. 113-2420.

FIG. IV-14-7. Correlation of Successive Scans. ANL Neg. No. 113-2423.

FIG. IV-14-8. Convergence and Divergence of Spots. ANL Neg. No. 113-2419.

calculated and the light level below which a spot will be considered to exist is set at the mean minus three standard deviations.

With these preliminaries complete, the scan begins. After the full distance is traversed in the x -direction, one step in the y -direction is taken and the x -axis is then traversed again in the opposite direction. This continues until the total desired area has been covered.

The depth of field of the microscope objective is quite small—just a few microns. Neither the glass slide nor the specimen itself is flat to this tolerance over the scanned area. Therefore, focus checking during the scan is mandatory. Whenever the scan traverses a spot it is stopped at the far edge of the spot. The microscope tube is then raised one step and the spot is back scanned to its near edge, as shown in Fig. IV-14-6. The tube is then lowered two steps and the scan returns to the far edge. The minimum light values observed during each of the three traverses of the spot are compared. The microscope tube is left in that position which gave the smallest minimum and the scan resumes. In that way the focus is continuously optimized on a maximum-opacity basis. Limitation of focus adjustment to one step each time minimizes the probability that the system will lock onto an extraneous feature far outside the plane of interest. However, the system can lock onto dust particles on the specimen's surface. These must therefore be removed before scanning.

Figure IV-14-7 shows how successive scans are correlated. If a given spot is traversed on two successive scans, the two traverses are recognized as belonging to the same spot if they overlap to at least the degree shown. That is, at least their edges must coincide. The total length of all traverses belonging to the spot is recorded as the spot area. (The detailed shape of the spot is not treated by the present version of the program.) A traverse on the current scan that does not overlap one on the previous scan is considered as a new spot and a new spot entry is opened in memory. If a traverse on the previous scan is not overlapped by one on the current scan the associated spot is considered to have ended and the spot entry is closed. After a run is complete, all spot entries remaining open are automatically closed.

Figure IV-14-8 shows the provisions that are made for the convergence or divergence of spots. If a traverse on the current scan overlaps two previously separated spots, the latter are said to have converged and their entries are combined into one. On the other hand, if one traverse on the previous scan overlaps two or more traverses on the present scan, the spot has diverged. The boundaries for recognition of overlaps on the next scan are expanded to encompass all the diverging traverses.

The optical properties of a track, particularly in the mica specimens studied to date, are such that a track does not appear as a single simply connected dark region, but rather as a number of dark regions with light areas interspersed. Thus, simply adding up

the individually recognized spots would give a track count that was much too high. Therefore, the individual spots are examined for the possibility of consolidation. If the centroids of any two spots lie within a prescribed distance of each other, the two spots are consolidated into one. This distance was arbitrarily set at 15 steps or 12 μ .

The results of this program do not yet agree within statistics with the results of a manual scanning. Therefore, additional refinements are under exploration, concerned mainly with discrimination against spots that are not tracks.

The stepping motors that drive the stage are the fastest that were available at the time this instrument was designed. With their speed capability and step size, it would take approximately 43 h to scan a specimen area of 1 sq. cm. Newer motors have since become available which are capable of at least 2000 steps per second at 160 steps per revolution. It is planned to incorporate one of these motors for driving the stage in the x -direction. It appears that the resulting five-fold increase in step size will be satisfactory for fission track measurements since 1 sq. cm could be scanned in less than 1 h which is a much more reasonable time.

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IV-15. Electric Field Effects in Proportional Counters and Their Influence on Spectroscopy

E. F. BENNETT

INTRODUCTION

Proportional counters in which fast charged particles are released uniformly over the gas volume produce an induction effect per event that can be used to estimate particle energy. Workable counters are, of necessity, limited both as to size and pressure of contained gas and this limitation degrades the quality of results in spectroscopy applications.

Two distinct types of degradation have been identified. The first is usually described as a "wall-and-end" effect in which the track length of the fast particle is sufficiently long to permit entrance into or exit

from the counting volume before stopping. Information relating to the wall-and-end effect problem can be found in Refs. 1 and 4.

The second effect has its origin in the finite length of detectors and in the manner in which an effective length, over which counting is done, is defined. This "end effect" difficulty has also been the object of investigation. One of the earliest relevant studies is the familiar text of B. Rossi and H. Staub.² More recent work can be found in Refs. 3 and 4.

The simplest method of defining an end is by inserting a section of tubing (several times the diame-

ter of the anode) over the anode. The tube extends well inside the counter from the electrical insulation at the extreme end. Tubing and anode are in electrical contact and the tube also serves as a support for the anode (which may be extremely fragile). This simple method is the most widely used and counters of small size are as readily assembled in this manner as are larger ones.

The difficulty with counters built in this way occurs in the transition region near the tip where field lines are not directed radially over their entire length and where field strengths at the anode surface are

FIG. IV-15-1. Proportional Counter Internal Construction Detail. ANL Neg. No. 113-2297.

weakening (ultimately to a value too low to cause multiplication). If the transition were rapid, multiplication would be reduced over an axial distance too short to be of significance. In practice, and even with a minimal increase of diameter in the end region, multiplication decreases gradually. This produces a low energy tail on the distribution of mono-ionizing events and degrades the instrument for spectroscopy.

A schematic of the "simple" end consisting of a concentric field tube (of several anode radii) and maintained at anode potential is shown in Fig. IV-15-1. Some electric field lines have also been included. The drawing is not to scale; in practice the cathode-to-anode ratio may easily be 1000 while the field-tube-to-anode ratio may be 10 or so. The field tube radius must be sufficiently greater than the anode radius to reduce gas multiplication to a negligible value over its entire length.

The origin of systematic errors lies in the distorted path of field lines near the tip discontinuity. The field in the immediate vicinity of the anode (where gas multiplication occurs) shifts continuously from values too low to cause multiplication to an asymptotic value achieved at a distance of about 1.5 cathode radii from the discontinuity. The asymptotic value of electric field strength is that appropriate to infinite concentric cylinders, $V/(r \log c/a)$, where V is the potential difference. In addition to a gradual weakening of the anode field, field lines will not lie precisely along the radial direction; and since ionization

FIG. IV-15-2. Dependence of Electric Field Upon Distance from Tip Discontinuity. ANL Neg. No. 113-2298.

ected will follow field lines, the electrical volume of the detector may be different from the mechanical volume.

All of these effects are described qualitatively in Ref. 2, for example. A calculation of the field was not possible, however, prior to the availability of fast computers. The electrode structure of Fig. IV-15-1 does not, apparently, allow a simple solution in tabulated functions.

FIELD CALCULATION

In order to study in detail the variation of the electric field near tips and the resulting effect upon pulse-height spectra, Laplace's equation for the potential with appropriate boundary conditions was solved by iteration. A standard finite difference approximation was used, and, in the most elementary way, the potential at any interior mesh point was expressed as a suitable "four point" average over adjacent mesh points. The radial dimension r was first mapped into a variable u according to $u = \log(r/a)$, where a is the radius of the anode. A fixed mesh interval was chosen in u space. The axial dimension was broken into mesh intervals which increased (in geometric progression) as distance from the discontinuity increased both along the anode region and along the end. This initial mapping of both radial and axial distances permitted a much more efficient (mesh point number and iteration time) solution to the problem.

The boundary conditions maintained anode (including field definition tube) and cathode at a fixed potential difference. At the mid-plane of the counter, potential gradients along the z axis were set to zero for all radial mesh points. At the extreme counter end boundary an arbitrary set of radial potentials could be input in order to ascertain any possible effect of end construction upon anode field. The true field pattern at the extreme end is generally complicated since lead-in seals and anode supporting structures will distort field patterns. The calculation indicated an insensitivity of anode field to end potentials where field tube insertion exceeded 1.5 cathode radii.

After a sufficient number of iterations of the finite difference expression, potentials converged to values that were essentially independent of details of dimensioning and mesh spacing over a broad range of these variables. The electric field at the anode was taken to be the value of potential at the radial mesh point adjacent to the anode; absolute electric fields were relevant to the problem.

In practice, the field tube inside diameter will be several times the diameter of the anode; and, apart

from misalignment in construction, the actual region near the tip will be as shown in Fig. IV-15-1. The calculation was set up to solve this case but no significant effect upon anode field was observed when a solid field tube structure was used, as would occur for instance if solder filled the anode-field-tube space during construction of the counter.

The anode field variation appropriate to a counter used in a neutron spectroscopy application is shown in Fig. IV-15-2. The counter had an anode radius of 0.0127 mm ($1/2$ mil). The ratio of field tube radius to anode radius was 10.0 and the cathode-to-anode ratio was 1000. The total length of anode was 6400 anode radii and at the half-anode distance (3200 radii) axial potential gradients were set to zero. At a distance of 1000 radii from the discontinuity measured along the field definition tube, the potentials were maintained at infinite cylinder values. The initial rapid increase in anode field is seen in Fig. IV-15-2 followed by a rather slow approach to asymptotic. Gas multiplication is extremely sensitive to field strength and the results of Fig. IV-15-2 clearly indicate that end effects will persist for some considerable distance along the anode region.

In order to understand the influence upon "electrical" volume of the distorted pattern of field lines, lines originating at each mesh point along the anode were followed to their termination at the cathode. Even at the first mesh boundary, which is only an anode diameter from the discontinuity, the warping of field lines is small; this line terminates at the cathode after an axial displacement of less than 1μ . The conclusion can be safely drawn that, for the counter described, electrical and mechanical volumes are not different.

COUNTER RESPONSE FUNCTIONS AND THE EFFECT UPON SPECTRA

The observation that only a very slight "bowing" of field lines occurs in an ordinary coaxial cylinder proportional counter design permits a relatively direct method of converting the observed axial field variation at the anode to a distribution in pulse amplitude where the initial ionization is of point extension and is uniformly distributed.

By an initial calibration, made with a source of known ionization, one can relate gas gain A to voltage V .

Various parametric representations of gas multiplication versus voltage have been offered.^{5, 6} None is entirely adequate except over a limited range of variables. A prescription relating A to V , which is at least as good as others suggested, is

$$(\log A)/V^n = CV + D. \quad (1)$$

The power factor α is determined empirically by the requirement that it minimize residuals after linear fitting to a measured sequence of A and V values.

Gas multiplication occurs only in the immediate

vicinity of the anode in proportional counters. If electric field variation along the anode is known and if a calibration relating gas multiplication to voltage is also available, it is an easy matter to drive a re-

FIG. IV-15-3. Pulse Amplitude Distribution from Ar-37 Beta Particle. ANL Neg. No. 113-2299.

FIG. IV-15-4. Effect of Response Corrections Upon a Neutron Spectrum. ANL Neg. No. 113-2414.

use function for the counter at any particular voltage. To do this, replace V in Eq. (1) with the calculated anode field (a function of axial distance z) normalized such that for large z the function equals V . The resulting expression then relates gas multiplication to axial distance.

The density of events (per cm of anode) will be uniform across the whole anode region where field bowing is insignificant. In order to drive an amplitude distribution of events, one only has to transform from the (constant) density per unit anode to the corresponding density per unit multiplication. The actual prescription for expressing A versus V is plainly irrelevant to the transformation from events per unit anode length to events per unit amplitude. Any relationship superior to Eq. (1) will improve, correspondingly, the end result. Equation (1) fails badly for very low values of multiplication (in the transition region between ion collection and proportional modes of counter behavior).

A comparison of predicted response, with that actually observed, is given in Fig. IV-15-3. The discrete points are from a pulse height distribution of Ar-37 in the counter described. Two distinct de-excitation modes occur; one, involving betas at about 3 keV, is the dominant peak in the figure. Another mode with energy about 200 eV also occurs, however, and it is this line which causes the sharp increase in the spectrum below channel 20. A discriminator blocks all events below about channel 7. Statistical effects introduce substantial broadening into the measured result. The pulse height distribution, determined using the calculated electric field (and the observed gain-voltage relationship), was area-normalized to the experimental distribution of Fig. IV-15-3 in the region above channel 20. The mean value of multiplication was determined from the calculated distribution (it was 0.87 of the asymptotic multiplication reached at a large distance from the tip). This mean value was set equal to the pulse-height channel corresponding to the peak of the observed spectrum and the calculated low energy tail

(the solid line in Fig. IV-15-3) was then drawn. The calculated distribution agrees well with the measured result within the limited region between contamination from the soft beta decay at low channels and statistical broadening effects at high channels.

The presence of nonideal internal field patterns has been observed, under some circumstances, to produce extremely large systematic errors in neutron spectra measured by the proton-recoil method. The worst situation arises when measurements at low energies, where the flux is relatively low, are made in the presence of high energy fluxes which are much more intense. The degradation due to field effects will produce a systematically higher neutron flux than actually exists in the low energy measurement for the same reason that the "tail" in Fig. IV-15-3 at low energies leads to an over-prediction of the spectrum of low energy Ar-37 betas.

An example of the effects of fields upon a neutron spectrum measured in depleted uranium is given in Fig. IV-15-4. The spectrum was measured in the interior of a depleted uranium slab; a detailed account of the measurement will be published. The curve labeled "Corrected for Response Function" includes wall-and-end effects as well as field effects but field corrections predominate below about 500 keV. It is quite apparent that large systematic errors in the low energy spectrum are introduced by internal electric field patterns.

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IV-16. Predictions of Geometrical Distortions in Proton-Recoil Counters and Application to Neutron Spectroscopy

T. J. YULE

INTRODUCTION

The measurement of fast-neutron spectra with proton-recoil proportional counters is well established.¹⁻³ If neutron spectra are to be accurately determined by this method, it is necessary to correct the measured proton-recoil energy distribution for distortions introduced by the finite size of the detector. Distortions arise from the truncation of proton-recoil tracks by the counter walls or by the extension of tracks into an end region where the multiplication is very low. The above distortions are termed geometrical. Distortions are also introduced by the behavior of the electric field near the ends of the counter, where the anode is desensitized. The electric field undergoes a transition, which leads to nonuniform gas multiplication in a region near the ends. We are mainly concerned with corrections for geometrical distortions. Corrections for field distortions are considered in Paper IV-15.

In general, to be able to correct for geometrical distortions in 4π proton-recoil proportional counters, it is

necessary to determine the energy distribution from monoenergetic protons of energy E_0 which were generated uniformly and isotropically throughout a given counter. The variation of the energy distribution as a function of E_0 constitutes the response function for the counter. Monte Carlo methods have been used to obtain response functions for spherical counters.^{3,4} A method has been developed to empirically determine response functions for cylindrical counters.⁵ Recently, analytical solutions to the geometry problem have been found.⁶ We have compared analytically determined distributions with measured distributions and investigated the application of the analytic solutions to corrections in neutron spectroscopy.

COMPARISON OF MEASURED DISTRIBUTIONS AND CALCULATED DISTRIBUTIONS

We may express the equation for the energy distribution from monoenergetic protons in terms of geometrical functions:

FIG. IV-16-1. Measured Pulse-Height Spectrum from the $^{14}\text{N}(n,p)^{14}\text{C}$ Reaction with Thermal Neutrons. The Range, R_0 , of 585-keV Protons Was 1.53 cm. See Text for Explanation of Distribution. ANL Neg. No. 119-2309.

FIG. IV-16-2. Calculated Distorted Part of Response Function for a Counter with a Radius of 1.27 cm and a Length of 8.25 cm Filled with Methane. The Range, R_0 , for Protons of Energy, E_0 , Equal to 585 keV Is Indicated. The Undistorted Part of the Response Function Is Not Shown. ANL Neg. No. 113-2304.

$$R(E, E_0) = B \left\{ F(R_0) \delta(E - E_0) + \left. \frac{dR}{dE} \right|_{E_0-E} N[R_0 - R(E_0 - E)] \right\}, \quad (1)$$

where B is a constant, $\delta(E - E_0)$ is a delta function, E_0 is the energy of the proton at the beginning of the track, $R(E)$ is the range of a proton of energy E , and $R_0 = R(E_0)$. The geometrical function $F(R_0)$ is the probability that a path length is greater than R_0 and the function $N[R_0 - R(E_0 - E)] dR$ is the probability that a path length is between $R_0 - R(E_0 - E)$ and $R_0 - R(E_0 - E) + dR$. Tracks starting in the end regions have been neglected, as well as any resolution effects. $R(E, E_0)$ is the idealized geometrical response function of the counter. The first term indicates the fraction of events whose paths are not truncated or do not extend into an end region and is referred to as the undistorted part of the response function. The second term yields the energy distribution of truncated tracks or tracks which pass into the end regions and is referred

to as the distorted part of the response function. $\left. \frac{dR}{dE} \right|_{E_0-E}$ is proportional to the inverse of the stopping cross section of the counter gas.

Experimental energy distributions were determined by filling a counter with methane and nitrogen, placing the counter in a thermal flux, and recording the pulse height distribution from the 585-keV protons of the $N^{14}(n,p)C^{14}$ reaction. Measurements were made for various pressures. Figure IV-16-1 shows a measured distribution for which the range of the protons was 1.53 cm. The rapid increase in the number of counts per channel at lower energies results from the detection of Compton electrons from the interaction of gamma rays with the walls of the counter. At low energies carbon recoils are also detected.

The presence of the tail from gamma-ray induced events, distortion introduced by field effects, and the finite resolution of the detector prevent a straightforward comparison. However, an examination of calculated distorted distributions indicate how a comparison

FIG. IV-16-3. Measured Fraction of the Distribution that is Undistorted Versus the Range, R_0 , of the 585 keV Protons. The Solid (Dashed) Curve is the Calculated Fraction and Takes into Account Geometrical (Geometrical and Field) Effects. *ANL Neg. No. 113-2307*.

FIG. IV-16-4. Measured Slope of the Straight Line Fits to the Distorted Distributions Versus the Range, R_0 , of the 585 keV Protons. The Solid (Dashed) Curve is the Calculated Slope and Takes into Account Geometrical (Geometrical and Field) Effects. *ANL Neg. No. 113-2308*.

be made. Figure IV-16-2 shows two calculated distributions—one for which R_0 is small and one for which R_0 is large. In both cases the calculations indicate that from zero energy to almost the peak energy a straightline fit to the curve is possible. For small values of R_0 there is a rapid change in the distribution near the peak energy. However, the rapid variation occurs so near the peak energy that it is impossible to distinguish these events from undistorted events if the resolution of the counter is taken into account.

It was decided to compare two parameters of the measured and calculated distributions—the fraction of events which are not distorted and the slope of the straightline fits to the distributions. Figure IV-16-3 shows the measured fraction of undistorted events versus R_0 . The solid curve is calculated and takes into account only geometrical sources of distortion. The solid curve provides a poor approximation to the data. This is not surprising since measurements from events of negligible track length indicate that the electric field transition region is a significant source of distortion.⁵ In order to account for the influence of the transition region, we assign to the region a length which is independent of voltage and range and assume that any track which starts in this region is distorted. The dashed line takes into account the transition region.

Figure IV-16-4 shows the slopes of the straightline fits versus R_0 . Once again, the solid curve, which takes into account only geometrical distortions, provides a poor approximation to the data. The dashed line takes into account the influence of the transition region by assuming that the energy distribution of events which start in the transition region can be approximated by a straightline whose slope is independent of voltage and range.

It is possible to become much more sophisticated in the treatment of field distortion. (See Paper IV-15.) However, the above simple assumptions lead to fair agreement between measured and predicted distributions and indicates that, to good approximation, it is possible to separate the two causes of distortion.

APPLICATION TO SPECTROSCOPY

We assume that it is possible to independently correct measured proton-recoil distributions for geometrical distortions and electric field distortions. For application to spectroscopy we are interested, for a fixed pressure of counter gas, in the behavior of the response function as a function of energy. For a given energy we approximated the response function with the first four Legendre polynomials. Figure IV-16-5 shows the behavior of the coefficient of the first Legendre polynomial as energy. It was found that the coefficients of the Legendre polynomials vary smoothly with energy. The energy dependence of each coefficient was approxi-

FIG. IV-16-5. The Energy Dependence of the Coefficient, C_1 , of the First Legendre Polynomial for a Counter with a Radius of 1.27 cm and a Length of 8.25 cm Filled with 95.3 psi of Methane at 25°C. The Curve is a Sixth-Order Least-Squares Polynomial Approximation. ANL Neg. No. 113-2915.

imated with sixth-order least-square polynomial fits. The complete representation of the response function is

$$R(E, E_0) = [1 - C_1(E_0)]\delta(E - E_0) + \sum_{i=1}^4 C_i(E_0)P_i(E/E_0), \quad (2)$$

with

$$C_i(E_0) = \sum_{j=1}^6 a_{ij}(E_0)E_0^{j-1}.$$

The a_{ij} are the coefficients of the least-square polynomial approximation to the Legendre polynomial coefficients, C_i , and the P_i are the Legendre polynomials. The above response function can be used to correct a measured proton-recoil distribution for geometrical distortions using either iterative or integral methods.⁵

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IV-17. Comparison of Proton-Recoil Proportional Counter and Time-of-Flight Neutron Spectrum Measurements in a U-235-Fueled Fast Reactor

J. E. POWELL

INTRODUCTION

A facsimile of Assembly 1A of the Subcritical Time-of-Flight Spectrum Facility¹ (STSF-1A) at Gulf General Atomic (GGA) was constructed in ZPR-3 and proton-recoil spectrum measurements were made for a comparison with time-of-flight results. Details, such as the amount of subcriticality (10 dollars), core construction, core volume fractions, and location of the re-entrant hole and source tube, were reproduced as closely as possible to those of STSF-1A so a meaningful comparison could be made. A MACH-1 calculation² using the ANL-723 cross section set (from MC² using ENDF/B parameters) was made of the central spectrum for both STSF-1A and the ZPR-3 version and results showed that the two reactors were essentially identical. The arrangement of the assembly, which was part of the ZPR-3 Assembly 57 experiment,³ is shown in Fig. IV-17-1. Table IV-17-I is a comparison of ZPR-3 and STSF-1A core volume fractions.

EXPERIMENT

The proton-recoil measurement utilized techniques that have been discussed in detail previously.⁴⁻⁶ The proportional counters are described in Paper II-29 and the electronics system in Ref. 7.

The counters were positioned in the matrix with the center lines of their sensitive volumes 9.85 cm out axially from the center of the core, the approximate distance of the GGA re-entrant hole from the center of the core. Data were taken over the energy range of 400 eV to 2.3 MeV at maximum counting rates of 8000 cps. Two subcritical measurements were made:

TABLE IV-17-I. COMPARISON OF ZPR-3 AND STSF-1A CORE VOLUME FRACTIONS

STSF-1A		Assembly 57	
Atom Density, cm ⁻³ /b		Atom Density, cm ⁻³ /b	
Nuclide	Core	Nuclide	Core (Zone 1)
U-235	0.004130	U-235	0.004081
U-238	0.006408	U-238	0.006347
Be	0.039345	Be	0.03901
O	0.039409	O	0.03901
Al	0.0057	Al	0.005546

B - MOCKUP OF BEAM TUBE
C - CONTROL RODS
P - PROTON RECOIL COUNTER
S - SAFETY RODS
T - STARTUP SOURCE TUBE

FIG. IV-17-1. Arrangement of Facsimile of STSF-1A. ANL Neg. No. 103-2927 Rev. 1.

FIG. IV-17-2. Proton-Recoil Spectrum Measurement in ZPR-3 (STSF-1A). ANL Neg. No. 103-2923.

FIG. IV-17-3. Comparison of Results of Proton-Recoil and Time-of-Flight Spectrum Measurements and Calculations in STSF-1A. ANL Neg. No. 103-2922.

one with the startup sources (Po-Be) at 20 in. and one with the sources at 24 in. from the center of the core, which represented the full range of source positioning for practical counting rates. These measurements served as a rough check on the dependency of the neutron spectrum upon source position.

Results of the measurement are shown in Fig. IV-17-2. Resolution is about 10% except at the lowest energies where it increased to about 45% to 2 keV. The spectra with the sources at the different positions were identical within the experimental error of the measurement. Structure in the spectrum is primarily due to resonances in oxygen and aluminum (aluminum matrix tubes and drawers were used in the core).

COMPARISONS WITH TIME-OF-FLIGHT CALCULATIONS

A comparison with the time-of-flight measurements⁸ and a 25-group MACH-1 calculation² is shown in Fig. IV-17-3. For each of the three results, the integral flux from 1 keV to 2.3 MeV has been set equal to one. There is general agreement between the proton-recoil results and the calculation from 1 keV to 2.3 MeV. The data were not corrected for the electric field effect which probably explains the large discrepancy below 1 keV.

Differences on the order of 30% exist between the time-of-flight and proton-recoil results over most of the energy range of the measurement and these dif-

ferences are, as yet, unexplained. The results of the two measurements should be comparable, however, since the measurements were made near the center of the core.

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IV-18. Electronic System Developments for Proton Recoil Spectra Measurements in Plutonium Critical Facilities

J. M. LARSON and J. E. POWELL

INTRODUCTION

Proton-recoil proportional counting has proven to be a very useful method for measuring fast neutron spectra between 1 keV and 2 or 3 MeV in zero power reactors.¹⁻⁴

One of the problems encountered in making spectrum measurements in a fast critical assembly is distortion of the measured proton-recoil spectrum by amplifier overloads. This distortion occurs when measurements are made in the energy region of 1 to 10 keV where the counter multiplication is high. Protons with energies in the MeV range create overload pulses in the ionization amplifiers that may result in severe amplifier paralysis. Consequently count rate restrictions must be placed upon the spectrometer since pulses that occur during the time the amplifiers are paralyzed are either lost or distorted.

IMPROVED AMPLIFIER SYSTEM

An amplifier system having improved overload capabilities has been developed for use with proton-

recoil spectrometers and is described in this report. The count rate capability of this system, when measuring in spectra which generate severe overloads, is several times greater than that which has been obtained with conventional double-differentiated RC shaping systems.

A block diagram of the amplifier system is shown in Fig. IV-18-1. A low-noise charge-sensitive preamplifier, a pole-zero compensated linear amplifier, and an active base line restorer are used in this design.

The preamplifier is direct coupled and has a single differentiating time constant τ of 250 μsec ($\tau = R_f C_f$). The long differentiating time constant of the preamplifier is reduced to 1.5 μsec by the pole-zero compensation network at the input of the linear amplifier. The pulses out of the pole-zero compensation network are amplified and integrated (single RC integration time constant of 1.5 μsec) by the three operational amplifier gain stages of the linear amplifier. All of the gain stages within the linear amplifier are coupled by very large capacitors (coupling time con-

FIG. IV-18-1. Block Diagram for Compensated Electronic System. ANL Neg. No. 103-11929.

FIG. IV-8-2. Pulse Profiles. ANL Neg. No. 103-11930.

stants between stages are greater than 0.1 sec) so that base line undershoot, introduced by the coupling networks, is reduced to negligible levels.

The output of the linear amplifier drives an active base line restorer. This restorer operates on the amplified diode principle⁵ and is used to remove the slow component of the collected charge from the proportional counter that appears at the output of the shaping amplifiers when large overloads occur. The need for the restoring circuit can be better explained through the use of Fig. IV-18-2.

Figure IV-18-2A illustrates a typical profile of the charge collected from the proportional counter after an ionizing event. The collected charge from the proportional counter is made up of a slow rising component β superimposed on the fast rising step α . The fast rising step α is shaped by the shaping networks of the linear amplifier resulting in the unipolar pulse

shown in Fig. IV-18-2B. The slow rising component β is greatly attenuated by the shaping networks of the linear amplifier and thus contributes negligibly to the pulse height and profile of the primary pulse of Fig. IV-18-2B.

Under conditions of overload, the primary pulse saturates; however, the slow component grows as the overload increases, thus giving rise to a pedestal, following the overload, as shown in Fig. IV-18-2C. (The dotted portion of the profile of Figs. IV-18-2C and IV-18-2D represents the pulse shape that would be obtained on an overload if there were no slow component in the collected charge.) The pedestal lasts for the duration of the collection time of the slow component and may have typical durations of 0.2 to 1 msec, depending upon the characteristics of the proportional counter used. The pedestal may be a source

FIG. IV-18-3. Specific Ionization Spectra for Compensated Amplifier System at 4000, 10,000, and 15,000 counts/sec. ANL Neg. No. 103-2844.

FIG. IV-18-4. Specific Ionization Spectra for Uncompensated Amplifier System at 4000, 10,000 and 15,000 counts/sec. ANL Neg. No. 103-2845.

of severe distortion when measuring in spectra that generate numerous overloads since those pulses occurring in coincidence with the pedestal will be distorted.

The pedestal is removed by adjusting the pole-zero compensation for a negative undershoot that just exceeds the amplitude of the pedestal. The adjustment results in the pulse of Fig. IV-18-2D. This pulse may be restored by the base line restorer and after restoration appears as shown in Fig. IV-18-2E. The restorer must have a fast restoring rate to minimize the duration of the negative undershoot of the restored pulse. In this application, the fast restoring rate was obtained by using unbalanced currents in the restoring diodes as shown in Fig. IV-18-1.

In order to take full advantage of the overload

characteristics of the pole-zero compensated linear amplifier it is necessary to use a pre-amplifier having a large output signal capability. This capability is required to prevent the preamplifier from saturating on large overload pulses, as saturation results in increased dead time in the linear amplifiers.⁶ The pre-amplifier design described by J. Larson⁶ has a 20 V output swing, in addition to low noise and fast response, and for that reason was selected for use in this application.

TEST IN ZPR-3

The performance of the compensated amplifier was tested and compared with the performance of the conventional double-differentiated system by measuring ionization and specific ionization spectra in the core of ZPR-3 for count rates varying from 4000 to 15,000 counts/sec.

Specific ionization spectra taken from the two systems over the proton energy range of 0.5 to 3.0 keV for counting rates of 4000, 10,000 and 15,000 counts/sec are shown in Figs. IV-18-3 and IV-18-4. With the compensated system the gamma-ray background can be easily subtracted out of the proton data. However, the uncompensated system showed noticeable distortion at 5000 counts/sec, and at 10,000 and 15,000 counts/sec there was no longer discrimination between proton and gamma-ray events.

To study the effects of overload distortion on the proton-recoil spectrum two-parameter data were taken at 4000, 10,000 and 15,000 counts/sec with both amplifier systems.⁷ A comparison of the 4000 counts/sec data is shown in Fig. IV-18-5 for the proton energy range of 0.66 to 3.0 keV, counting time for the two sets of data being the same. In the energy region between 0.66 and 1.5 keV spectra taken with the two systems differ by several percent, indicating that some distortion is present in the uncompensated amplifier data.

Comparisons of proton-recoil spectra could not be made at 10,000 or 15,000 counts/sec because the specific ionization spectra, taken with the uncompensated amplifiers, were smeared and the gamma ray background could not be subtracted out of the two-parameter data (see Fig. IV-18-4).

In order to study the performance of the pole-zero compensated amplifiers at high count rates, the proton-recoil data taken at 4000 counts/sec were compared with the data taken at 10,000 and 15,000 counts/sec. The 10,000 counts/sec data compared within statistics but at 15,000 counts/sec the two differed by a few percent. The effect is shown in IV-18-6 where normalization is over the energy range

FIG. IV-18-5. Comparison of Proton-Recoil Spectra Taken With Compensated and Uncompensated Amplifier Systems. *ANL Neg. No. 103-2846*.

FIG. IV-18-6. Comparison of Proton-Recoil Spectra Taken With the Compensated Amplifier System at 4000 and 15,000 counts/sec. *ANL Neg. No. 103-2843*.

of the measurement. These differences could possibly be explained by a change in the reactor spectrum in going from 0.6% (10,000 counts/sec) to 0.2% (15,000 counts/sec) subcritical. However, it is more probable that these differences are due to pulse pile-up since no change in the proton spectrum was observed in going from 2 to 0.6% subcritical.

CONCLUSION

significant improvement of the overload capabilities of the ionization amplifiers has resulted from

a design that utilizes single RC differentiation and pole-zero compensation in conjunction with active base line restoring. This technique improves the overload capabilities of the ionization amplifiers by eliminating lengthy amplifier paralysis due to base line undershoot and pulse pedestal, following overloads, due to the slow component of collected charge from the proportional counter.

At present the spectrometer will produce undistorted spectra up to 10,000 counts/sec at the 1 keV energy range when the compensated electronics are

used. About 10,000 counts/sec pile-up effects begin to be noticeable in the spectrum; however, work to date indicates that the spectrometer may be operated at rates as high as 20,000 counts/sec when pile-up rejection electronics are added to the system.

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IV-19. A Preamplifier for Proton Recoil Proportional Counting

J. M. LARSON

The shaping amplifiers of proton-recoil proportional counting systems are subjected to severe overloads when spectrum measurements are made at neutron energies between 1 and 10 keV. Many of the larger overloads that occur when measuring in this energy range are several thousand times greater than the signals of interest and as a consequence preamplifier saturation is likely to occur during these events. Preamplifier saturation is undesirable as the resulting amplifier paralysis or dead time is generally much longer than that which would have occurred had the preamplifier not saturated.¹

A new preamplifier having a large output dynamic range has been developed for proton-recoil proportional counting and is described in this paper. This preamplifier features a 20 volt peak output swing that significantly reduces the probability of preamplifier saturation during large overload events.

CIRCUIT

The preamplifier circuit is shown schematically in Fig. IV-19-1. The circuit is charge-sensitive and uses a field effect transistor (FET) in the first stage. The output of the input FET (Q_1) is coupled to a current source load, formed by the collector of Q_4 , via the common base-connected transistors Q_2 and Q_3 . Transistors Q_5 , Q_6 , and Q_7 are emitter followers, having near unity gain, and comprise the output driving stage of the preamplifier.

The preamplifier is powered from +24, +40 and -24 volt supplies. The plus and minus 24 volt levels are obtained from a standard NIM power source and a separate supply provides the +40 volt level. The +20 volt peak output capability is obtained by bias-

ing the base of Q_3 to +24 volts and by supplying bias currents to Q_1 , Q_2 and Q_3 from the +40 volt source.

The circuit has a nominal open loop gain of 10,000 so that excellent closed loop gain stabilities are obtained. Direct coupling is used, so that the output pulse decays with a single time constant, determined by the feedback elements C_f and R_f , which makes the preamplifier compatible with pole-zero compensated shaping amplifiers.

PREAMPLIFIER CHARACTERISTICS

The equivalent input noise charge generated by the preamplifier is low and has a typical magnitude of 33×10^{-18} C (for external shaping time constants of 2 μ sec and with 10 pF of external input capacity). The preamplifier also has a typical 10 to 90% rise time of 15 nsec with 10 pF of external input capacity.

The circuit is built on a $1\frac{3}{8} \times 4\frac{3}{4}$ in. board, as shown in Fig. IV-19-2.

CONNECTION TO THE PROPORTIONAL COUNTER

When the preamplifier is used with proportional counters, as in proton-recoil systems, it has been found that the most satisfactory connection is with the anode of the counter direct coupled to the preamplifier as shown in Fig. IV-19-1. This method of connection increases the signal-to-noise ratio of the preamplifier-counter combination by decreasing stray input capacity at the input of the preamplifier, and by eliminating the thermal noise that would be generated by a detector isolation resistance.

An additional feature of the direct coupled connection is the isolation provided by the counter between the high voltage supply and the gate of the

FIG. IV-19-2. Preamplifier. ANL Neg. No. 103-11862.

preamplifiers input FET. This isolation has eliminated FET failures induced by transients occurring on the detectors high voltage supply.

OTHER APPLICATIONS

Because of its small size, the preamplifier has been used in other applications where it has been desirable to mount the preamplifier in close proximity to a detector, as for example, near the base of a photomultiplier tube. These applications in general do not require a 20 volt peak output swing and the +40 volt supply is eliminated with only minor modifications to the circuit.

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IV-20. Automated Spectrum Analysis Methods for Ge(Li) Detectors

L. S. BELLER* and D. W. MADDISON

Activation measurements of reaction rates and heterogeneity effects in ZPPR assemblies rely heavily on the precision and resolution obtainable from Ge(Li) gamma-ray detectors. This process generates large quantities of data which can be handled routinely only by automated methods when full advantage of the properties of the detectors is to be obtained.

A computer code system has been written to satisfy this need. The codes provide completely automatic analysis from raw data through peak finding and analysis; identification of the peak energy, half-life, and isotope; statistical analysis; and, as output, count rates and either absolute or relative concentrations of most gamma-emitting isotopes. The codes are an outgrowth of development work that has been going on over the past year and a half. The codes are intended to handle the most general case—complex spectra from mixed fission and capture products—but are applicable to many other forms of analysis.

The data are obtained with three Ge(Li) detector chains equipped with automatic sample changers, then fed to fast analog-to-digital converters and a small general-purpose computer. Depending on the program, the data are stored as (up to three) independent multichannel analyzer systems with selective block acquisition or as a single multiparameter prob-

lem of up to four parameters. The computer records the counting and other pertinent data from each detector system, times and controls the system operation, and writes the results on magnetic tape. In normal operation, all foils are counted repeatedly to establish a rough measure of half-lives.

The code system begins by sorting spectrum data from each foil into order by ascending time-of-count. The next section processes these individual spectra.

A peak finder locates each peak and its boundaries in terms of three points of inflection. The routine works by a moving-average method, and defines prospective peaks by the statistical probability that they differ from nearby reference areas. Additional criteria are based on energy, apparent peak width, and system parameters. The routine finds peaks reliably over a wide range of parameters and is remarkably insensitive to statistical precision, or the lack of it, in the data. It may be used for spectra from NaI-(Tl) as well as Ge(Li) detectors.

Backgrounds, which are usually Compton continua from higher energy gammas, are obtained from those nearby areas not occupied by peaks. In the present version, net counts in each peak are obtained simply as the peak integral less the interpolated background. There is an option, in the case of peaks that are not fully resolved, for an analytic continuation based on an assumed Gaussian shape.

Energy calibration can be inserted as an input

* Atomic International, a Division of North American Rockwell Corporation, Canoga Park, California.

option or obtained from the spectrum itself, if the code is given the exact energy and approximate channel number of any two reasonably isolated peaks known to be in the spectrum. In this mode, drifts in calibration between spectra have no effect on subsequent analysis.

When all spectra have been processed, net counts for each peak are correlated as a function of time (succeeding spectra) to obtain an approximate decay constant. The peak energy and approximate half-life are used to identify the peak by reference to an extensive library of activation and fission product gamma rays (see Paper I-26). In general, energy alone is sufficient for identification. The decay constant and relative amplitudes remove almost all ambiguities.

The library also contains "best" values of half-life, and where available, the absolute transition intensity. This information, together with the previously deter-

mined efficiency function for each detector, is used to compute saturated absolute or relative activities at each point. A statistical analysis routine compares these with confidence-interval criteria, rejecting any in apparent disagreement with the overall mean. A final weighted mean activity, proportional to the relative or absolute concentration of the parent isotope is printed out for each gamma ray.

Precisions of 1% or better, as measured by internal agreement and reproducibility, are obtained routinely for many isotopes in mixed fission products.¹ The precision seems to be limited primarily by the adequacy of the background interpolation algorithms in the presence of densely crowded peaks. This problem is being investigated.

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1. L. S. Beller and D. W. Maddison, *Relative U-238 Fission Yields by Gamma-Ray Analysis*, Trans. Am. Nucl. Soc. **11**, 607 (1968).

IV-21. Techniques for Determination of Selected Fission-Product Yields in Fast Reactor Spectra by Gamma-Ray Analysis

L. S. BELLER* and D. W. MADDISON

The resolution and efficiency of medium-sized Ge(Li) gamma-ray detectors (30 cm³ nominal volume) have been found sufficient to allow quantitative and non-destructive determination of individual gamma-ray intensities from mixed fission products contained in foils irradiated in fast reactor critical facilities. This information is being used to derive absolute yields and yield ratios for several fissile isotopes in a range of neutron spectra typical of large, dilute fast reactors.

Certain yields, ratios of yields in several fissile species, and any changes in these with neutron spectrum in the range of spectra represented in fast critical assemblies must be known and understood for proper normalization of present and projected studies of heterogeneous reaction rates.

The most intense and easily analyzed gamma rays observable with Ge(Li) detectors in the first few days after irradiation come from isotopes of yttrium, strontium, niobium, iodine, barium, and lanthanum. A number of others are also present with enough intensity to be useful, but these are the isotopes that

contribute most to statistical precision in determination of relative fission rates.

The method being used determines the absolute yield directly, but requires the knowledge of absolute detector efficiency as a function of energy and of the absolute transition intensity (gamma/disintegration) for the individual gamma rays.

The absolute detection efficiencies were determined for several counters relative to six standard sources of varying energies. These are individually traceable to the Bureau of Standards by two routes, and were internormalized independently using standard NaI(Tl) detectors and calculated efficiencies.¹ The efficiency function for each detector was then derived by a least-squares fitting of the standard-source data to a second-order polynomial in the logarithms of intensity and energy, with small correction terms for attenuation by cryostat, detector, dead layer, and samples.

Good data on transition intensities for many isotopes are available in the recent literature. This is supplemented where needed with corrections based on experimental or theoretical internal conversion coefficients. For many other important isotopes, notably I-135, the required transition intensities are relatively unknown, but can be obtained by calibrating the sys-

* Atomics International, a Division of North American Rockwell Corporation, Canoga Park, California.

tem against well-known relative yields in thermal fission of U-235.^(2, 3)

Foil samples are nominally 0.500 in. diam by 0.005 in. thick of metallic Th-232, U-235, U-238, and Pu-239. A total of about 10^8 fissions in each sample are sufficient to give good statistical precision on more than 20 gamma-rays in 12 mass chains. The overall studies have been restricted to gamma-rays having half-lives between about 20 minutes and several months and energies between 200 and 1700 keV.

Total fissions in each sample foil are determined by comparison in back-to-back geometry to the fission rate of a standard fission counter of known effective mass. Each foil is counted repeatedly for up to three months using standard multichannel analyzer techniques. Data reduction, which includes peak location, energy calibration, identification, and determination of net peak counts, is done by computer techniques. Precision of the measurements for favorable gamma-rays is better than 1%, as measured by internal statistics and external reproducibility.

Several techniques of increasing sensitivity are used to assure that the results indeed represent the desired activity of individual isotopes. The apparent half-life of each gamma-ray is determined individually over as long a time as possible, and compared to that expected from the parent activity. This test is not as sensitive as might be desired, but serves to eliminate gamma rays with obvious interferences.

Using the known half-life of each isotope, the individual observations of each gamma ray are corrected to an equivalent saturated or time-zero activity, intercompared statistically for evidences of disagreement, and correlated as a function of time to reveal any evidence for interfering activity. These tests appear to be very sensitive.

Next, there are two or more usable gamma rays in secular equilibrium in all the mass chains in the preferred list. The ratios of observed intensities of these lines are followed for as long a time as possible. The ratios are independent of energy and efficiency calibrations and must stay constant unless there are interferences. Very small interferences can be detected by these techniques. Data surviving all these tests are combined into a final weighted mean for each gamma ray and corrected for transition intensity and

detection efficiency. These values are proportional to the absolute yields, with the proportionality constant being derived from the auxiliary fission counter measurement.

Preliminary values for yields in U-238 have been given,⁴ and values for all fissile isotopes being studied are subject to further analysis. Qualitatively, the absolute yields appear to be intermediate between those given in various compilations for "fast" fission and thermal fission. Values for yields ratios, such as those for mass 97/mass 91, show the same pattern. As a tentative conclusion, it appears that radiochemical analyses which use "fast" spectrum yields to determine absolute fission rates may be in considerable error.

In the range of neutron spectra studied in ZPR-3 assemblies, there is good evidence for changes in some yield ratios caused by differences in neutron spectra. In one set of measurements in ZPR-3 Assembly 53, there was a statistically valid linear correlation between the yield ratio I-132/Nb-97, for fission of U-238, and the position in the cell at which it was measured. The ratio was about 4% higher at positions near fuel plates (where the majority of fissions are attributed to the primary fission spectrum) than in foils three centimeters away in graphite (where the spectrum is considerably degraded). Foils at intermediate locations had intermediate changes in yield ratios. The change is interpreted as being due to an increase in the yield of mass 132 (or a change in the distribution of independent yields) as the mean energy of the spectrum causing U-238 fission becomes softer. The corresponding change in fission rates was 25%. There is as yet no evidence for a corresponding change in yield ratios for thermally fissile species.

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IV-22. Statistical Performance of Surveillance Count Channels

K. G. PORGES

When some steady process which releases nuclear particles or photons is monitored by detectors delivering a count for each detected particle, intervals between successive counts are usually nearly Poisson-distributed. This fact allows the display or presentation of the count rate (presumably proportional to the process rate), by a number of relatively simple devices. The indication inevitably fluctuates, reflecting the Poisson distribution of the input as well as the design of the processor. The latter operates with a basic time constant, which amounts to an integrating time. Increasing this time constant has two effects: output fluctuations decrease and the response of the output to a sudden change in input rate becomes slower. Thus, the time when the output reaches a trip level (or announcement delay) increases.

A safety surveillance channel based on particle detection—whether individual pulses are amplified and fed to a count rate meter, or are already integrated at the amplifier input—is thus subject to a considerable range of performance characteristics, depending on the choice of instrument and adjustment of instrument parameters. Yet this performance is often neglected in the specification and employment of such channels. The reason for this neglect may well be due, in part, to the lack of a reliable theoretical framework on which performance prediction can be based. To some extent, instrumental fluctuations caused by instabilities (rather than by the input statistics), which are difficult to prevent in the long run where analog circuitry is employed, may lead to a general policy of best-guess instrument adjustments.

These circumstances point to a need for instrument improvement, accompanied by a systematic investigation of performance statistics and their dependence on adjustable parameters. A number of digital instruments which are practically free from nonstatistical fluctuations are discussed in Paper IV-23, and one particular instrument is described in some detail in Paper IV-24 as well as in Ref. 1. Here we wish to discuss briefly the difficulties encountered with performance predictions based on statistical theory, and to describe a modest effort aimed at a semi-empirical formulation which has been under way for the past year.

The performance of a warning channel with trip level, considered in Paper IV-23 for three specific circuits, is best expressed in terms of three related

output parameters: *the false trip frequency*, due to random excursions at a steady background rate, the *announcement delay* and the *detection probability* for a given increase in this background. The false trip frequency is sometimes specified without taking the other two parameters into account; moreover, false trip frequencies may be required to be as low as once per year. At the present state of the art, this kind of performance can be neither predicted with any reasonable reliability by existing theory, nor developed on the basis of tests carried out under realistic conditions.

The exact point where theory fails cannot be established; however, this failure can be connected with the progressive deviation of higher moments of the distribution of the channel output from the theoretical model on which predictions are based.² The actual distribution can be equivalently given as a distribution in time, or as a distribution of short samples of the output, which can be readily obtained by repeatedly opening a linear gate connected to an analog-to-digital converter system and digital store. The theoretical model, on the other hand, requires the specific assumption that (a) the distribution does not differ strongly from normal; and (b) general statistical theorems such as Carson's theorem or Campbell's theorem are applicable. Both assumptions, however become strictly valid only in the limit where the product of the input rate and the instrument integrating time or inspecting time becomes very large. Even for quite large values, deviations between actual and model distributions appear when the higher moments of these distributions are compared. The most noticeable effect of these higher moments occurs at some distance from the distribution peak or mean output. Where extremely small false alarm frequencies are demanded, the alarm level must be set at just such a distance from the mean level, or else the higher moments of the distribution made small by increasing the input rate or inspecting time. The input rate, however, is often as high as one can reasonably make it by deploying a number of detectors of the highest available sensitivity. The inspecting time cannot be stretched beyond certain limits without making the whole channel useless for warning purposes.

Tests under realistic conditions may be feasible for certain warning systems, but may still require an inordinate amount of running time. For systems

such as fuel failure detectors of nonintegrating type destined for some of the 1000-MWe LMFBR reactors now under development, realistic conditions would be not only time-consuming but also rather costly.

A partial solution of this difficulty has been sought by a series of tests under somewhat artificial conditions, with a simulated Poisson distribution, considerably scaled up in mean rate, at the input of a test channel designed for very high rates. The channel output is sampled and samples are stored in a 1024-channel memory, retrieved on paper tape, and processed by computing the moments up to the ninth moment, which terminates the Edgeworth correction series² with the third term. The moments are then further converted into semi-invariants and compared with the semi-invariants computed by Campbell's theorem. For this, the filter response of the count rate meter must be known with high precision, and must, moreover, remain very stable. Considerable effort was required to achieve this stability, and reproducible data did not result until the whole unit was set

up in an insulated enclosure at regulated constant temperature.

The first part of the program is directed at the development of a semi-empirical formula for the distribution obtained with Poisson input at a certain rate and certain processing time constants. The false trip rate for these parameter values will then be computed on the basis of such a formulation and compared with measurements at a representative number of alarm settings with respect to the mean. A computer program will then finally be sought for the calculation of false trip rates over a useful range of parameter values, and a similar program developed in which false trip rate and input rate may be specified to obtain a set of alarm levels and time constants.

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IV-23. Statistical Performance of Digital Count-Rate Meters

K. G. PORGES

The fact that digital circuits, based on certain logical connections of elementary "go/no-go" units, offer almost absolute long-term stability has been recognized for some time. Only recently, however, with the advent of integrated (single-chip) digital units, has the cost of constructing an extensive logical network (required to provide the memory feature characteristic of count-rate meters) become low enough to make digital count-rate meters a practical possibility. These developments have similarly influenced the cost and availability of computers. The following discussion of the statistics of digital count-rate metering may thus apply equally to a fully integrated instrument, or to a computer program through which similar basic circuit elements, incorporated in a computer, perform exactly the same function.

Practical application of a count-rate meter demands not only long-term stability, but also adequate statistical performance for the specific task the instrument is to perform. Thus, a count-rate meter employed in a trip channel, say in a safety surveillance system, cannot be allowed excessive fluctuations leading to false trips; at the same time, rapid response to real events may be required, and hence fluctuations cannot be removed by

heavy integration (corresponding to a low-pass filter network).

The statistical performance of digital count-rate meters must therefore be carefully considered before this type of instrument (or computer program) is employed in a trip channel system. Moreover, a number of different digital schemes are possible in which increasing complexity presumably secures a commensurate improvement in statistical performance. Again, a careful evaluation of this performance can best determine what level of complexity is practically worthwhile.

In this brief exposition, we consider specifically three types of digital count-rate presentation: (1) an elementary single cyclic scaler; (2) a "queuing" instrument based on temporary storage in a shift register, described in Paper IV-24; and (3) a digital instrument which simulates the exponential memory of an analog count-rate meter.^{1,2}

The quality of performance of such instrument channels is readily described in terms of two related parameters: the *announcement delay* and the *mean false trip rate*. The announcement delay must evidently be specified in terms of a specific excursion, e.g. a step function in input rate of a specific magnitude. It is expected to make this magnitude just equal to the trip level,

that there is a 50% chance of detection at the specified announcement delay. The announcement delay, thus specified, may be readily seen to fall between one and two cycle times T_c for the cyclic scaling CRM (count-rate meter), one shift register delay T_s for the "queuing" CRM, and rather longer—perhaps four to five integrating periods $T_i = RC$ —for an analog CRM (or a CRM which simulates analog performance). For the cyclic scaler, the actual announcement delay depends on the phase defined by the onset of a sudden input rate increase with respect to the cycle. If one now considers a large count-rate increase, and requires that the surveillance system must respond to the event giving rise to this increase as fast as possible in order to forestall some possibly serious consequences, the added phase delay characteristic of the cyclic scaling system may become intolerable. The announcement delay of the "queuing" CRM, in contrast, may be readily seen to always equal the minimum delay possible with a cyclic scaler, while the analog CRM trip is delayed only very slightly beyond this minimum. In fact, the performance of a "queuing" CRM, incorporating an M -bit shift register as described in Paper IV-24, amounts to the same performance as that of M parallel cyclic scaling channels arranged with phases uniformly spaced over the count interval. Thus, the added complexity of the processing system secures a shorter and more reliable announcement delay; at the same time, one must expect a certain increase in the false trip rate.

Any discussion of the false trip rate must inevitably be based on a number of more or less unrealistic presuppositions: (a) the background input rate n is assumed to be constant and perfectly Poisson-distributed; (b) no spurious counts or excursions in parameter values are supposed to occur over very long periods.

For these assumptions, the probability that a random sample count in the cyclic count channel exceeds some specified number A is given by

$$P_A = \sum_{K=A}^{\infty} [\exp(-nT_c)](nT_c)^K / K!, \quad (1)$$

where $T_c \equiv$ count interval. Since the device has no memory, each such sample is entirely independent of past performance. The mean time between trips is thus given by

$$t = P_A T_c + 2T_c(1 - P_A)P_A + 3T_c(1 - P_A)^2 P_A + \dots = T_c / P_A. \quad (2)$$

The reciprocal of the mean false trip interval is defined the false alarm frequency, or false trip rate,

$$f_A = P_A / T_c. \quad (3)$$

For sufficiently large numbers, Eq. (1) can be expressed in more convenient form through the parameter

$$\beta = (K - nT_c) / \sqrt{nT_c}, \quad (4)$$

and Stirling's theorem. Expansion, rearrangement, and collection of terms in decreasing order of magnitude yields

$$[\exp(-nT_c)](nT_c)^K / K! \approx (2\pi nT_c)^{-1/2} e^{-\beta^2/2} [1 + (\beta^3 - 3\beta) / 6\sqrt{nT_c} + \dots]. \quad (5)$$

The series, known as the Edgeworth series, evidently diverges for large values of β , which implies that estimates of parameter settings for particularly small specified false trip rates must be based on an evaluation of Eq. (1) rather than on the Gaussian approximation [Eq. (5)]. (Similar restrictions apply to other types of count-rate meters, as further discussed below.)

Inserting Eq. (5) into Eq. (1) and passing from sum to integral, one finds that the correction series terms can be dropped provided

$$A - nT_c \ll nT_c. \quad (6)$$

Put

$$B = (A - nT_c) / \sqrt{nT_c}; \quad (7)$$

then, whenever inequality (6) holds, one may express the false trip rate in terms of the error integral,

$$f_A \approx (2T_c)^{-1} [1 - \Phi(B/\sqrt{2})]. \quad (8)$$

In turn, this expression can be expanded if $B^2 \gg 1$ (which amounts to specifying large values for nT_c if, at the same time, the inequality (6) must be satisfied). Thus, finally,

$$f \approx (2\pi)^{-1/2} (BT_c)^{-1} e^{-B^2/2} \left\{ 1 - \frac{1}{B^2} \dots \right\}, \quad (9)$$

for a single cyclic scaler CRM.

We turn now to the false-alarm frequency of an assembly of M such count-and-dump channels, arranged to cycle at evenly spaced intervals, which also applies to the digital count-rate meter described in Paper IV-24, or to an analog count-rate meter that develops square pulses. This false-alarm frequency is evidently smaller than M times the rate obtained from Eq. (9), since a false alarm, when it occurs, will probably appear in several of the M channels, yet such an alarm should be counted only once. We shall use the digital count-rate meter as a basic model for developing a false-alarm-frequency formula, and note that the number of pulses stored in the shift register at any time must always be exactly equal to the number appearing in the add-subtract scaler. The situation we are considering has a strong resemblance to certain aspects of queuing theory, a subject covered by "an incredibly voluminous literature"

to quote a remark by W. Feller.³ Nevertheless, a search of this literature for the solution of the exact problem posed here turned out to be unavailing. The development given below is therefore not based on previously published work—admitting the possibility that a more exhaustive search may discover a treatment of a closely similar problem in traffic control, design of a servicing facility, or telephone communications.

We shall assume that the number of available shift register bits M is considerably larger than the mean number, $N = nT_d$, of occupied bits. The system thus has a negligible digital dropout; that is, the chance that two inputs arrive within the shifting period T_d/M is very small. (For a practical unit, such a second pulse can be temporarily stored; hence the digital dropout is a manageable problem even at high input rates, as discussed in Paper IV-24.) For every shift, the system plays a coin-tossing game, with an a priori probability (N/M) of "success" and adjoint probability $1 - (N/M)$ of "failure." Hence, the probability of finding exactly K pulses stored in the shift register (and therefore also in the scaler) is given by the binomial rather than the Poisson distribution law:

$$P(K) = \{M!/[M - K]!K!\} (N/M)^K [1 - (N/M)]^{M-K}. \quad (8)$$

A false alarm occurs whenever the store, upon the completion of a game, happens to contain exactly $A - 1$ events and the next game results in adding another event. This further implies that the game played T_d earlier had a negative result, such that the shift register does not lose one event at its delivery end while it acquires a new one at its receiving end. Finally, the inspection interval for this situation has been shortened to T_d/M , whence the false-alarm frequency becomes

$$\begin{aligned} f &= (M/T_d)[1 - (N/M)](N/M)P_{M-1}(A - 1) \\ &= (A/T_d)[1 - (N/M)]P_M(A). \end{aligned} \quad (9)$$

As before, this can be readily processed by Stirling's formula and the Taylor-McLaurin expansion. We further introduce the alarm-level parameter

$$A = N + B\sigma, \quad (10)$$

where

$$\sigma = \sqrt{[1 - (N/M)]N} \quad (11)$$

is the variance of the binomial distribution given by Eq. (8). After some manipulation, we obtain the Edgeworth representation,

$$\begin{aligned} f &= [1 - (N/M)](N + B\sigma) \\ &\cdot \{e^{-B^2/2}/T_d(2\pi\sigma)^{1/2}\} \{1 + (B^3 - 3B) \\ &\cdot [1 - (2N/M)]/6\sigma^2 + \dots\}. \end{aligned} \quad (12)$$

Dropping small quantities, we obtain

$$f \approx \sqrt{n/2\pi T_d} e^{-B^2/2}. \quad (13)$$

Equation (13) can be derived also on the basis of the square-pulse count-rate meter model if we express the false alarm frequency in terms of a level distribution $P(a) da$ and slope distribution $P(s) ds$:⁴

$$f = P(a) \int_0^\infty sP(s) ds, \quad (14)$$

where

$$P(a) = e^{-B^2/2}/\sqrt{2\pi\sigma^2} \quad (15)$$

is the level distribution probability. The slope distribution $P(s)$ is given here by a sum of delta functions. Realistically supposing that the square pulses actually have trapezoidal shape with constant rise and fall slope s_0 , we may put

$$P(s) = b[\delta(s - s_0) + \delta(s + s_0)] + (1 - 2b)\delta(s). \quad (16)$$

The weighting constant b is the probability that a random sampling of the trace will catch the trace while rising or falling:

$$b = n/s_0. \quad (17)$$

The second moment of the level distribution may be calculated by means of Campbell's theorem:

$$\sigma^2 = nT_d. \quad (18)$$

After inserting Eqs. (15) through (18) into Eq. (14) and integrating over all positive slopes, one finds a false-alarm frequency given exactly by Eq. (13).

The false-trip frequency for the analog (or quasi-analog) count-rate meter is readily found by statistical considerations, embodied in Eqs. (14) and (15) or in their equivalent formulation

$$f = \left\{ \int_0^\infty (\dot{F})^2 dt / \int_0^\infty F^2 dt \right\}^{1/2} (e^{-B^2/2}/\pi), \quad (19)$$

where

$$F(t) = \frac{T_b}{T_b - T_i} [\exp(-t/T_b) - \exp(-t/T_i)] \quad (20)$$

is the channel response to a single count for the case of a conventional analog count-rate meter circuit with an RC time constant T_b , followed by an integration stage of time constant T_i .^{*} Insertion of the response and its time derivative into Eq. (19) yields the analog CRM false trip rate

$$f_A \approx \frac{1}{2\pi\sqrt{T_b T_i}} e^{-B^2/2}. \quad (21)$$

* In practice, integration is always present to some extent usually through the readout instrument (chart recorder) and/or trip level reading circuit.

1 comparing the false-trip rates predicted for the "queuing" and the analog simulation types of digital CRM, given by Eqs. (13) and (21), respectively, it becomes apparent that the analog instrument does effect stronger smoothing of fluctuations, resulting in a correspondingly lower false-trip rate for the same time $T_c = T_b$ and same alarm level A .

As mentioned above, the announcement delay for large excursions of the input rate is, for these comparative parameter values, only slightly longer for the analog instrument. At the same time, the announcement delay for a barely detected increase is considerably longer for the analog system.

If, on the other hand, there is no premium on fast reaction, the cyclic scaler is not only the simplest but also the most reliable means of presenting count rates, with the lowest false trip rate. In many cases, a few such scalars, staggered over the count interval, should provide a sufficiently frequent sampling (hence, sufficiently small announcement delay) at the lowest cost. The latter consideration evidently must be strongly weighed when instrumenting a large plant, such as some of the power reactors now under study, where a large number of surveillance channels will be required.

In conclusion, a brief remark may be added concerning the limitations of statistical concepts in predicting instrument performance. A somewhat more thorough discussion is presented in Paper IV-22. In many prac-

tical situations the occurrence of false alarms or trips is not only costly, but imposes an intolerable psychological burden on operating personnel. For that reason, specifications may call for false trip rates as low as once per year or less. Such specifications are meaningful only in the sense of encouraging instrument designers to do their best; in a more literal sense, they may be unverifiable, inasmuch as no adequate practical test of such instrument performance is feasible, while statistical theory becomes highly unreliable. Moreover, the kind of major plant failure for which the instrument channel is designed may (or at least may be hoped to) occur very rarely; thus, the specified announcement delay and detection efficiency of the channel may turn out to be similarly unverifiable. The intercomparison of differing digital CRM systems presented here is thus practically limited to applications where specifications allow some confidence in statistical theory or performance tests can be made.

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IV-24. Continuous Digital Ratemeter¹

K. G. PORGES, S. J. RUDNICK* and P. L. MICHAUD*

Ratemeters are widely used in reactor instrumentation, alarm systems, and other nuclear applications. Currently, a large number of very stable analog ratemeter designs is available. But the input to a ratemeter is essentially digital and an output which is also digital is often required; for example, when the ratemeter is used in conjunction with an alarm level discriminator. Furthermore, digital computers are increasingly deployed to monitor, diagnose, and correct reactor power levels, processing plant feed rates, etc. It is somewhat awkward to develop an analog function from the digital output of a count channel, as one may have the tendency to do using readily available existing ratemeters, only to digitize this again for alarm system or computer input. The continuously reading digital linear ratemeter proposed here is intended to short-circuit this analog step.

Two digital linear ratemeters with exponential-decay memory characteristics were previously described.^{2,3} In comparison the instrument described here has several advantages:

1. relative logical simplicity
2. simple and easily analyzed "square" memory characteristic with a well-defined finite maximum memory time
3. provision of a delayed replica of the input pulse train.

A number of digital ratemeters of the cyclic count and dump type (otherwise known as scaler/timers) have been described in the literature⁴⁻⁷ and some are commercially available. These count and dump circuits produce outputs only at the end of each count interval and respond to step inputs in a manner determined not only by the choice of counting period but also by the

* Electronics Division, Argonne National Laboratory.

FIG. IV-24-1. Simplified Diagram of the Basic Continuous Digital Ratemeter. ANL Neg. No. 143-2818.

particular phase of the count cycle at the instant the step occurs. On the other hand, a continuous ratemeter, such as the one described here, has an output which is always present and its response to a step change in input begins immediately and proceeds at a rate determined only by the choice of integrating period.

The instrument outlined in Fig. IV-24-1 functions by entering derandomized incoming pulses into an *add-subtract scaler* in the add mode, and also feeding the same pulses to a *shift register*. The logic is arranged so that each and every pulse which is counted in the scaler is ensured entry into the shift register and vice versa. Pulses leaving the shift register at the other end are put into the add-subtract scaler in the subtract mode. As a result, the number of counts in the scaler is always exactly equal to the number of pulses in the shift register. This number equals the number of input pulses in the preceding time interval m/f and has an average value mn/f , where $m \equiv$ number of bits in the shift register, $f \equiv$ clock (shifting) frequency determined by a stable crystal-controlled oscillator and $n \equiv$ mean input rate. The ratio m/f is evidently equal to the delay through the register and corresponds to the length of "square" pulses entering an equivalent hypothetical analog circuit where such pulses are allowed to pile up. The number appearing in the scaler is then equivalent to the voltage appearing across the corresponding analog circuit output. Hence, the unit is said to have a "square" memory characteristic. The dynamic range of the basic instrument extends from f/m to f . Frequency scaling as detailed below serves to shift the lower limit downward. The maximum slewing rate is the full scale rate divided by the delay through the register: $f/(m/f) = f^2/m$.

When the derandomizer used is only of first order, that is, if it stores pulses merely to ensure that only those which enter the shift register are counted in the scaler, the instrument has a finite resolving time of $1/f$. Thus, it tends to lose some counts and at most one

input pulse is entered into the scaler and the shift register during each $1/f$ period. The probability of j inputs ($j = 0, 1, 2 \dots$) in time $1/f$ is

$$P_j = (N^j/j!)e^{-N}, \quad (1)$$

where $N = n/f$; thus, the total loss in time m/f becomes

$$L = m \sum_{j=1}^{\infty} (j-1)P_j = m \sum_{j=1}^{\infty} jP_j - (1 - e^{-N}) = m(N - 1 + e^{-N}). \quad (2)$$

Observing that the quantity $(N - 1 + e^{-N})$ must be the loss per interval $1/f$ and the total count per interval is $n = n/f$, it follows that for a first-order derandomizer the observed count rate is

$$c_1 = f(1 - e^{-N}) = n[1 - n/(2f) + n^2/(6f^2) - n^3/(24f^3) + \dots]. \quad (3)$$

Alternatively, one can improve the loss situation by using a higher-order derandomizer.⁸ A second-order derandomizer is shown at the left of Fig. IV-24-2. The auxiliary buffer functions as a second storage channel and is activated whenever a first pulse is registered in the primary buffer. The count is stored in this second channel until the next clock pulse, whence it is transferred to the primary channel. For the second-order case illustrated in Fig. IV-24-2, the observed count rate is

$$c_2 = f(2 - 2e^{-N} - Ne^{-N}) = n[1 - n^2/(6f^2) + n^3/(12f^3) - n^4/(40f^4) + \dots]. \quad (4)$$

Here a numerical example might be useful to illustrate the value of the second-order derandomizer. Assuming $n/f = 0.25$, we obtain

$$c_1 = 0.885n$$

$$c_2 = 0.991n.$$

FIG. IV-24-2. Design Details for a Practical Continuous Digital Ratemeter. The Synchronizer is Essentially an Additional Bit of the Shift Register. ANL Neg. No. 148-2816.

Obviously this improvement is well worth the second buffer. Neglecting the effect on the dynamic range and the slew rate of the instrument, one can observe that as the order of the derandomizer increases, the required number of shift register bits decreases for a specified precision; the delay remains constant if the frequency f is changed in proportion to m . The order of the derandomizer may be increased indefinitely by adding identical additional buffer stages. A general expression for observed count rate c_x as a function of the derandomizer's order x is

$$c_x = n[1 - N^x/(x+1)! + xN^{x+1}/(x+2)! - (\frac{1}{2})x(x+1)N^{x+2}/(x+3)! + \dots] \quad (5)$$

In a practical instrument, the shift register capacity m must be large if fluctuations are to be reasonably small. This circumstance has made the construction of such an instrument impractical in the past; however, the rapidly developing technology of MOS single-chip circuit fabrication has resulted in a very substantial decrease in the cost of large shift registers. Currently available static shift registers cost as little as \$0.15 per bit; dynamic shift registers are available at \$0.12 per bit; and further price reductions are likely. An instrument suitable for reactor control purposes may thus incorporate a shift register with as many as 20K bits at an acceptable cost.

For any specific shift register length, fluctuations can be minimized by choosing as low a driving frequency f as is consistent with the allowable resolving time loss and with the required response time. The driving frequency f may be adjusted by running an oscillator of frequency f_0 through a scaling ladder, resulting in

$$f = f_0/2^k \quad (9)$$

where

$k \equiv$ number of binary stages.

From this the first-order fractional loss (with second-order derandomizer) from Eq. (4) becomes $(2^k n^2)/(6f_0^2)$ and the first-order fractional fluctuation from Eq. (6) becomes $(2^k nm/f_0)^{-1/2}$. Frequency scaling is thus the equivalent of time constant switching in conventional analog instruments. The mean content of the scaler is, however, also shifted to

$$C(k) = (2^k/f_0)mn = 2^k C(0); \quad (10)$$

hence, some means to compensate for this shift is required. This can be done by switching the inputs from

the first to the K th stage of the scaler; alternative may be preferable to insert a "translator" switch between the scaler and the output or display. In a practical instrument the frequency scaling switch should be ganged with the scaler input or "translator" switches.

An instrument incorporating a second-order derandomizer, a static shift register with 20K capacity, and a 10^6 Hz crystal oscillator, scaled down through up to 13 binaries ($0 < k < 13$), can deliver a digital output at a statistical fluctuation of 2% and a precision of 1% throughout a range of four orders of magnitude. In principle it should operate without calibration or maintenance, delivering continuous and digital information. To maintain the precision and the small relative fluctuation, it is necessary to adjust the frequency to keep $4n \leq f \leq 8n$. This may be accomplished either manually or automatically using a signal taken from the scaler or display register. After each change in frequency, however, it is necessary to reset the scaler and clear the shift register. The add-subtract scaler can be of standard design and the display and other outputs depend on the application. Display in binary form is obviously the easiest and least expensive but decimal output can also be obtained by means of a suitable decoder.⁹⁻¹¹

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IV-25. Reduction of Processing Losses in On-Line or Off-Line Acquisition of Random Counts at High Rates

K. G. PORGES, C. J. RUSH* and G. E. CAYA*

In a number of experimental situations, count rates from nuclear detector channels must be processed to extract various kinds of information such as time constants, correlation coefficients, etc. The conventional means of acquiring this information is a series of sequential scaling operations, readily performed by a pulse-height analyzer in "multiscaling" mode or else by a suitably programmed on-line computer.

When the number of storage channels required by the experiment exceeds the capacity of the core memories with which these processors are equipped, an on-line magnetic tape recorder can be substituted. The operation of the acquisition system then requires (in addition to the scaler and control circuitry) a translator and a series of tape driving circuits through which the number found in the scaler at the end of a given count interval is encoded and inscribed on the tape. This amount of circuitry may be difficult to accommodate in some experiments for which the bulk and weight of equipment must be kept to a minimum. The tape recorder may then be used in a quite different mode, without intervention of the scaler, command logic, and translator: events are simply inscribed as they occur and time is kept track of by means of clock pulses inscribed in a parallel channel.

Direct tape writing generally makes less efficient use of the storage medium. Moreover, serious processing losses can develop at high rates, in view of the relatively long deadtime (about 5 to 100 μsec) characteristic of digital tape units. In a like vein, the problem of processing losses may well arise in connection with very high count rates and slow on-line scalers or computer interfaces. Nuclear detector channels with intrinsic deadtimes of the order of 10 nsec are readily available; on the other hand, economic reasons suggest the choice of inexpensive but slow circuit elements (such as RTL single chip circuits with response times of 300–500 μsec) for computer interfacing.

The reduction of deadtime losses, insofar as these result entirely from the acquisition process, is therefore of some interest in either of the two rather different experimental situations described. Some of the means available for loss reduction are discussed here.

For the case of very high input rates (say, in excess of $10^6/\text{sec}$), the most expedient means of reducing deadtime loss resulting from an input scaler with several hundred nsec deadtime is a prescaler. For the deadtime

problem arising in connection with real-time input, the use of a cyclic distributor suggests itself. Examples of a prescaler and a distributor are shown schematically in Figs. IV-25-1a and IV-25-1b. The deadtime, or recovery time, of the binaries is shown explicitly in these figures as a veto gate imposed across the entrance after each input pulse (either set or reset).

The purpose of prescaling is to make the content of the main (slow) scaler at the end of a given count interval agree within a small error with the input divided by the scaling factor. When the main scaler content is very much larger than the prescaler content, it is not necessary to read out the prescaler, and the latter can then be simply added to the existing equipment with-

FIG. IV-25-1. (a) Block Diagram of a Prescaler; and (b) a Two-Track Distributor. Deadtimes are Shown Explicitly as Veto Gate Generators. ANL Neg. No. 118-2324.

* Electronics Division, Argonne National Laboratory.

TABLE IV-25-I. SERIES COEFFICIENTS FOR EQUATIONS (2) AND (3)

k	a_k	b_k
0	1	1
1	1/3	1/5
2	1/6	1/30
3	1/30	1/210
4	1/120	1/840
5	1/840	1/7560
$\vdots$	$\vdots$	$\vdots$

out further connections than a signal, a count/stop, and a reset command line. Even these latter two can be omitted without unduly increasing the statistical error where the prescaling factor is modest.

A distribution system, as shown in Fig. IV-25-1b, is similarly meant to store a total number of counts (in several channels) which agrees as closely as possible with the number of events which have appeared at the input of the unit. As mentioned above, this stratagem is particularly suited for direct tape input, where four or more separate channels are usually available together with additional channels which are required for the clock.

The effectiveness of prescaling or distribution can be evaluated by computing the fractional loss after k stages (or with k -fold distribution, respectively). Calculations of this sort can be made on the basis of stochastic theory¹ or with a more direct, heuristic approach.

Either method can be readily shown to yield a fractional scaling loss

$$[(n/2) - c_M]/c_M = e^{-n\tau}(nT^2) \sum_k a_k (nT)^k \quad (1)$$

for a single binary prescaler of negligible deadtime, where $n \equiv$ input rate, $t \equiv$ main scaler deadtime, and $c_M \equiv$ main scaler count rate. Some of the coefficients a_k are given in Table IV-25-I.

For a prescaler consisting of two binaries with negligible input paralysis, one finds

$$[(n/4) - c_M]/c_M = e^{-n\tau}[(nT)^4/6] \sum_k b_k (nT)^k. \quad (2)$$

The coefficients b_k are also listed in Table IV-25-I.

For the two-track distribution system shown in Fig. IV-25-1b, we obtain

$$\begin{aligned} [n - (c_A + c_B)]/(c_A + c_B) \\ = e^{-n\tau}[(nT)^2/2] \sum_k a_k (nT)^k, \end{aligned} \quad (3)$$

whence one concludes that the fractional loss in a k -fold distribution setup is just $1/k$ times the loss in a

corresponding scaledown system. We may thus immediately write the four-track fractional loss:

$$\begin{aligned} [n - (c_A + c_B + c_C + c_D)]/(c_A + c_B + c_C + c_D) \\ = e^{-n\tau}[(nT)^4/24] \sum_k b_k (nT)^k. \end{aligned} \quad (4)$$

These equations show that the counting loss can become so small after either prescaling or distribution that the input scaler paralysis τ may no longer be negligible by comparison, assuming that the ratio T/τ amounts to at most two orders of magnitude. The slow scaler input after one fast binary in fact becomes

$$(n/2)(1 + n\tau)^{-1},$$

rather than $n/2$ as assumed above. Events which occur within τ of each other cannot reach the main scaler and must be excluded. The probability of finding additional inputs capable of reaching the second stage within $(T - \tau)$ is then obtained by integration over the parts of the paralysis T "available" to these inputs. By this method, one readily finds the overall loss for a factor of two scaledown system

$$\begin{aligned} [(n/2) - c_M]/c_M = n\tau + \\ (1 + n\tau)e^{-n\tau} \sum_k a_k \{n[T - (k+2)\tau]\}^{k+2}. \end{aligned} \quad (1')$$

Similarly, a fourfold scaledown system results in fractional losses

$$\begin{aligned} [(n/4) - c_M]/c_M = n\tau + (1 + n\tau)(e^{-n\tau}/6) \\ \cdot \sum_k b_k \{n[T - (k+4)\tau]\}^{k+4}. \end{aligned} \quad (2')$$

For a two-track distribution system,

$$\begin{aligned} [n - (c_A + c_B)]/(c_A + c_B) = n\tau + (1 + n\tau) \\ \cdot (e^{-n\tau}/2) \sum_k a_k \{n[T - (k+2)\tau]\}^{k+2}, \end{aligned} \quad (3')$$

and, finally, for a four-track distribution setup,

$$\begin{aligned} [n - (c_A + c_B + c_C + c_D)]/(c_A + c_B + c_C + c_D) \\ = n\tau + (e^{-n\tau}/24) \sum_k b_k \{n[T - (k+4)\tau]\}^{k+4}. \end{aligned} \quad (4')$$

Equations (2), (1'), and (2') have been plotted in Fig. IV-25-2a; Eqs. (2), (3'), and (4') are similarly plotted in Fig. IV-25-2b. The assumption $\tau/T = 0.01$ has been made for this illustration. The figures show that a scaledown by 2 or a two-track distribution scheme still results in appreciable corrections when nT is above 0.5, corresponding to an input rate of 10^6 and main scaler deadtime of 500 nsec (or similarly, input rate of 10^4 and magnetic tape deadtime of 50 μ sec). In contrast, a fourfold system keeps corrections well below for values of nT which are in excess of unity. For case of a fast nuclear channel whose output is to be

FIG. IV-25-2. Fractional Losses for Two and Four Fast Stages, Ahead of a Slow Stage, at Different Values of the Input Rate—Slow Stage Deadtime Product nT . A Fast Stage Deadtime of 1% of the Slow Stage is Assumed. (a) Prescaling System; and (b) Distribution System. ANL Neg. No. 113-2322.

FIG. IV-25-3. 100 MHz Scale of Four with Direct Parallel Output Having the Same Paralysis as the Scaled-Down Output. ANL Neg. No. 143-2850 Rev. 1.

essed on-line by a slow computer interface, it thus
ars that the most practical solution is a scaledown
by a factor of four. Figure IV-25-2a shows that the frac-
tional loss just reaches 8% for $nT = 1$ after a fourfold

scaledown. For a practical example, let $\tau = 5$ nsec and
 $T = 0.5$ μ sec; then this loss occurs for $n = 2 \times 10^6$ /sec.
If the correction formula or procedure which allows for
this relatively modest loss is some 10% in error (due to

its dependence on the "effective" deadtime, which may be imprecisely known), the final result still is known to better than 1 %.

A fourfold distribution system for real-time tape recording can operate without serious loss up to $nT = 2$. This makes such a system particularly useful in connection with an "excursion" monitor or similar equipment subjected to unpredictably high temporary input rates, occurring at unpredictable times. Some other considerations which are believed to be important in the design of such equipment have been described elsewhere.²

A prescaler of four intended for on-line computer processing of reactor experiments at input rates varying between a maximum in excess of 10^6 and a minimum of 100 per second, is shown in Fig. IV-25-3. The unit is

used free-running, i.e. without commands, in connection with a dual scaler (computer interface) which has a deadtime between 0.3 and 0.5 μ sec. The availability of the other main scaler allows one to parallel the scaled input with a direct input, from which better statistical precision is available at low rates. A simple program selects the proper channel and applies necessary dead-time corrections to either main scaler content.

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IV-26. Liquid Organic Fast Neutron Detector Assembly

G. G. SIMONS and J. M. LARSON

A high efficiency NE213 fast-neutron detector assembly complete with a pulse shape discrimination (PSD) circuit was constructed. The detector assembly is an integral part of a steady state fast-neutron spectrometer system being developed to measure spectra associated with the Zero Power Plutonium Reactor (ZPPR), Zero Power Reactor-3 (ZPR-3) and Argonne Fast Source Reactor (AFSR) reactors operated by Argonne (see Paper IV-27). It was desired that this detector assembly have sufficient sensitivity and pulse shape discrimination capability to be operational over a neutron energy range from about 0.5 to 14 MeV and possess adequately high gamma ray discrimination to allow routine operation in the combined neutron and gamma ray field associated with the above reactors.

Based on the acceptable performance exhibited by the NE-213 liquid organic scintillation solution in other fast-neutron detector assemblies,^{1,2} this solution was selected to be used as the recoil proton medium in the present detector assembly. The scintillator offers high efficiency over the neutron energy range from 0.5 to 14 MeV, does not exhibit the nonisotropic behavior characteristic of stilbene, and is amenable to pulse shape discrimination. Also since the scintillator is a liquid, detectors containing NE-213 can be fabricated in various sizes and shapes to optimize performance under various operating environments. Accurate knowledge is required of the response and/or efficiency for each type of detector configuration. This tends to

discourage either usage of nonstandard detectors or the assemblage of a large number of unique detectors. The NE-213 detector described below is similar to an NE-213 detector calibrated using the Los Alamos 8 MeV Vertical Van de Graaff.³

NEUTRON AND GAMMA RAY INTERACTIONS IN THE NE-213 DETECTOR

A high probability exists that neutrons or gamma rays entering the NE-213 detector will undergo an interaction within the active detector volume. Neutron events consist of neutron-proton single or multiple elastic scatterings and either neutron-carbon elastic or inelastic scattering. Gamma ray interactions are predominantly Compton scattering. Each of the above events will cause fluorescence photons which will be detected by the photomultiplier tube and subsequently analyzed unless the undesirable gamma ray induced events are removed.

Gamma ray induced pulses are removed by electronically observing NE-213 scintillation pulse shapes following either neutron or gamma ray excitation. The pulse shape following excitation is the result of a fast and slow decay component in the scintillator. The fast component is due to the decay of the initial solvent excitation; the decay time is less than 10 nsec long. The second type of decay consists of complex, little understood ion recombination reactions with a decay time of about 1 μ sec.⁽⁴⁾ Fast and slow pulse components have different reaction mechanisms; these

mechanisms appear to be dependent upon the nature of the ionizing particle. Observed specific ionization for protons in NE-213 solution is greater than that for electrons, and further the proton pulse has a greater ratio of slow-to-fast pulse decay component.

DETECTOR ASSEMBLY COMPLETE WITH PULSE SHAPE DISCRIMINATOR

The NE-213 detector assembly consists of the mounted detector and tube base. The mounted detector includes an NE-213 detector containing 103 cc

of deoxygenated solution in a standard size 2×2 in. cylindrical clear glass cell. A 10^6 centistokes silicone compound was used to optically couple the flat base of the glass cell to a $\frac{1}{8}$ in. thick plexiglass light pipe which was optically coupled to the photocathode of a photomultiplier tube. A light tight assembly was achieved by wrapping the detector, aluminum reflector, and photomultiplier tube with black plastic electrician's tape. This assembly was placed in a metal static shield.

The detector assembly, shown in Fig. IV-26-1, in-

FIG. IV-26-1. NE-213 Detector Assembly. ANL Neg. No. 103-11160.

The preamplifier that is connected to dynode 11 (shown schematically in Fig. IV-26-4) is a modification of the charge sensitive preamplifier described in Paper IV-19. This circuit isolates the 11th dynode of the photomultiplier tube from external loading and provides a stable charge gain. The gain of the preamplifier is determined by the magnitude of the preamplifier feedback capacitor C_f , which has a value of 120 pfd in this application.

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IV-27. Liquid Organic Fast-Neutron Spectrometer

G. G. SIMONS

INTRODUCTION

Fast-neutron flux and spectra measuring instruments have steadily been developed using the proton recoil phenomenon. These devices are capable of performing accurate measurements since they depend primarily upon the smoothly varying $H^1(n,n)$ cross section. Three main types of proton recoil counters include the telescope¹ using a thin solid polyethylene radiator, the gaseous proportional counter² containing either hydrogen or methane, and the large and very efficient counters composed of either solid stilbene³ or liquid NE-213.^(4,5) The proportional counter can be used to measure neutron spectra inside zero power fast reactors over a neutron energy range of about 1 keV to 2 MeV.⁽²⁾

In order to expand the measurable energy range above 2 MeV a beam can be extracted from a reactor and either a time-of-flight system⁶ or a steady state fast-neutron spectrometer system (described in this paper) can be used to measure spectra up to several MeV. The time-of-flight technique requires a pulsed source and considerable reactor operating time and hence cannot be applied to steady state reactors such as the Zero Power Plutonium Reactor (ZPPR), the Zero Power Reactor-3 (ZPR-3) or the Argonne Fast Source Reactor (AFSR).

The NE-213 spectrometer was constructed such that it is a highly efficient spectrometer meeting the requirements for obtaining meaningful data in the environment of a zero power critical reactor. The operation of the neutron spectrometer is described and a plutonium-beryllium packaged neutron source pulse height and unfolded energy spectra are presented.

PURPOSE

An important parameter which must be measured during the mockup of either basic fast reactor physics cores or specific fast reactor configurations, using the ANL zero power critical reactors, is the neutron energy spectrum. Precise neutron spectral measurements made at locations of particular interest to the reactor designer or where large uncertainties exist in the calculated fluxes—for example, reactor core centerline, core-reflector interface, reflector-shield interface, and through the shield—allow needed comparisons to be performed between experimentally measured and theoretically calculated spectra.

NE-213 SPECTROMETER ELECTRONICS

The electronics used for the fast-neutron spectrometer system are shown as a block diagram in Fig. IV-27-1. They consist of a proton recoil detector and tube base gamma ray discrimination circuit and associated electronics. The detector and operating principle of the gamma ray discrimination circuit are discussed in Paper IV-26 while the remaining system and associated topics are described below.

As shown in Fig. IV-27-1, pulses from the eleventh dynode (containing both neutron and gamma ray induced pulses) are fed directly to a low noise preamplifier which is mounted in the tube base. The preamplifier pulses are sent through a long 50 Ω cable to the instrument rack housing the associated electronics. It is split and each portion is sent to different pulse shaping amplifiers. One of the amplifiers sh and amplifies the pulses so that they are compatible with the input requirements of the analog-to-digital

FIG. IV-27-1. Block Diagram of NE-213 Spectrometer Electronics. *ANL Neg. No. 103-11120.*

converter (ADC) and they are then delayed and sent to a linear gate. The other linear pulse from the eleventh dynode is shaped in such a manner, in the second amplifier, to diminish walk of the timed logic signal and is sent to the timing single channel analyzer (SCA) where it is timed at the leading edge of the positive pulse. The resulting logic signal can be delayed in the SCA and the width varied from 0.5 to 11 μ sec using the gate generator. This logic signal following proper shaping and delaying is finally sent to the coincidence unit. Likewise the pulses from the pulse shape discrimination circuit (PSD) are amplified, timed, delayed, stretched and then sent to the coincidence unit. Proper timing of the logic signals from the linear (11th dynode) and the PSD signals requires that the positive PSD neutron induced logic signals are in time with the linear neutron and gamma ray induced logic signals and that the large positive overshoot pulses from gamma ray induced events are not in time with the linear logic signal. Thus, ideally, only neutron induced events in the NE-213 detector will result in an output pulse from the coincidence unit. The coincidence output pulse is delayed and stretched in the two gate generators operating in series until it correctly opens the linear gate for passing the linear signal. Finally, ideally, only neutron induced events in the NE-213 detector will result in a linear pulse being sent to the ADC for pulse height analysis.

The neutron energy range over which the neutron spectrometer will routinely operate is from 0.5 to 14 MeV. Below about 0.5 MeV, the fast component of the neutron induced pulse is approximately the same as that induced by gamma ray events and cannot be separated from gamma ray events.

Complete elimination of gamma ray events is impossible. However, they can be reduced to a negligible level for most applications. A Na-22 gamma ray rejection ratio of 1000:1 is realizable at a count rate (without PSD circuit in operation) of up to 5000 counts/sec. Maximum performance of this system has yet to be determined.

PLUTONIUM-BERYLLIUM SPECTRUM MEASUREMENT

A raw data pulse height spectrum due to plutonium-beryllium neutron interactions in the NE-213 detector is shown in Fig. IV-27-2. These data were obtained from a 10 g plutonium-beryllium neutron source positioned at a nominal 12 in. from the side of the detector along the detector centerline. The Na-22 gamma ray rejection ratio was greater than 1000:1 at a total gamma ray induced count rate of 1000 counts/sec. Thus, since the gamma ray-to-neutron ratio from a plutonium-beryllium source is approximately 0.7, the distortion in the raw pulse height spectrum resulting from gamma ray induced pulses leaking through the discrimination circuitry would be negligible.

FIG. IV-27-2. PuBe Pulse Height Spectrum Measured with the NE-213 Spectrometer. ANL Neg. No. 103-A11122.

M. Anderson has shown³ that the spectra from plutonium-beryllium sources are dependent upon the source plutonium content. Also the spectra depend upon source fabrication. These two factors, as well as resolution variations between different neutron spectrometers, make it difficult to compare plutonium-beryllium spectra measurements completed at various laboratories. However, since the same general neutron spectrum exists in most plutonium-beryllium sources,

FIG. IV-27-3. PuBe Neutron Spectrum from a 10 g Source Measured with the NE-213 Spectrometer. ANL Neg. No. 103-A11121.

a rough comparison between spectrometer systems spectra measurements is possible using different plutonium-beryllium sources. This is shown in Fig. IV-27-3 where two spectra measured by Anderson³ (his two sources contained 2 and 116 grams of plutonium) with a stilbene spectrometer are normalized at 4.8 MeV and exhibited along with the normalized spectrum obtained by unfolding the raw NE-213 data shown in Fig. IV-27-2 with the FERDoR code.⁷

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IV-28. The Vanadium Bath: A New Neutron Detection System

A. DeVolpi and F. OZER

For neutron experiments involving high accuracy, relatively large detection systems are needed to avoid dependence on angular correlations and inadequately understood corrections for fast neutron escape. If the experiment involves irradiation in a high gamma background environment, one of the best detectors is the manganese bath¹ (see Paper I-27). However, this device has certain inconveniences when applied to integration of neutron outputs produced by fluctuating sources, such as reactor or accelerator beams. The vanadium bath has been devised as a relatively fast response system to circumvent the limitations of the manganese bath.

Manganese-55 has a half-life of 2.6 h; thus, the time to reach activity saturation is about one day, to be followed by another day for decay. Vanadium-52, on the other hand, has a half-life of 3.8 min, which reduces the full irradiation and decay cycle to just over an hour. Following a study² of the various alternatives to manganese as an activation element, vanadium was found to meet the general requisites of efficiency, availability, stability, and suitability. With the continuous circulation on-line counter developed for the manganese bath,¹ it is practical to count vanadium activity at a relative efficiency of about 50% of that obtainable with the manganese system.

Three applications of the vanadium bath system have already arisen. Two experiments involving thermal neutron beams from the Argonne CP-5 reactor were expeditiously conducted with the vanadium detector. These were an absolute measurement of the neutron yield (ν) in fission from U-235 and precision measurements of the number of neutrons emitted per neutron absorbed (η) in Pu-239 and U-235.

At the Reactor Physics Division Van de Graaff accelerator, it has been possible to use the bath as the neutron detector in a measurement of the Be-7 and Zn-65 decay schemes involving the associated activity method.

Further applications of the vanadium bath are likely for rapidly decaying neutron sources. In all cases, calibration of the vanadium bath is carried out with standard neutron sources, previously measured by the manganese bath.

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IV-29. Gas Cherenkov Reactor Power Monitor

R. GOLD, K. G. PORGES and W. C. CORWIN

INTRODUCTION

It has been recognized for some time that the next generation of nuclear power plants of fast breeder type will develop temperature and radiation levels which preclude the use of conventional sensors such as fission chambers or proportional counters in or near the reactor core.

Such sensors measure the current developed by ionizing nuclear reactions involving neutrons, and thus respond to the power level by reading the neutron flux. In order to collect this current, a collecting field must be applied between two electrodes in the detector. This, in turn, requires that at least one of the electrodes be insulated from the grounded casing;

similarly, the connecting path or transmission line which links the collecting electrode to the preamplifier must maintain adequate insulation.

Intense radiation causes various types of damage to insulation, resulting in leakage currents and noise. Particularly severe problems arise in unavoidable interconnection points. Other deleterious effects of combined heat and radiation are the gradual deterioration of the counter gas and structural deformation of critical spacings, which strongly affect the performance of the detector. Finally, depletion of fissionable coatings (or other media which interact with neutrons to produce ionization) occurs at a rapid rate in the extreme flux levels encountered in such a plant.

In the past, most of these effects could be overcome to a satisfactory extent by improving the radiation hardness and heat tolerance of detectors, cables, connectors, and other components. As such efforts inevitably reach the end of the line, it becomes necessary to locate detectors well away from the core, which introduces geometric effects in their response. Moreover, a program of frequent inspection and replacement of components (and recalibration of detectors whose fissionable coatings have been depleted) must be countenanced.

Faced with the evident cost, complexity, and inconvenience of this situation, one is led to search for ways of sensing reactor power which do not require devices subject to heat and radiation deterioration. In particular, one would like to dispense with applied voltages, insulation, cables, connectors, and depletable coatings.

Any useful sensor must indicate the power level with adequate sensitivity and speed of response. Sensors which respond to neutrons satisfy this requirement, since the neutron flux is strongly correlated with the fission rate, hence reactor power. As regards the gamma radiation produced by the reactor, only that fraction of the gamma flux which originates in *prompt* processes (capture and fission) varies with the power level; copious gamma radiation is also emitted by *decay* processes originating from induced activities and fission products. Since conventional gamma detectors cannot discriminate to a practically useful extent against this delayed gamma component, neutron detectors have become standard; moreover, considerable efforts have been made to reduce the gamma sensitivity of these sensing channels.

It may be pointed out, however, that the gamma spectrum associated with decay rapidly falls off above 1 MeV, whereas the spectrum associated with prompt events has a relatively strong component at considerably higher energies. If, therefore, an energy dis-

crimination principle which has a response threshold somewhere between, say, 2 and 5 MeV can be incorporated into a gamma detector, the resulting instrument should meet the requirement of rapid response to power level.

A detection channel which has such a threshold feature and also appears to satisfy radiation and heat resistance specifications to a much larger degree than other known systems has been recently proposed.¹ It is based on the detection of the *Cherenkov* light generated by relativistic Compton recoil electrons which originate in scattering processes involving only the high-energy component of the gamma flux. The Cherenkov effect is characterized by a sharp threshold, depending on the index of refraction of the medium. The index of refraction in a gaseous medium is a unique function of the gas density and can therefore be adjusted over a considerable range. This allows one to choose the Cherenkov threshold anywhere above about 2 MeV at technically feasible pressures, for a large variety of gases and gas mixtures.

The detector simply consists of a long metal tube or channel, one end of which is located in or near the reactor core while the opposite end is equipped with an optical system. The optical system is designed for the transmission of light out of the reactor environment, via a suitable system of mirrors and filters, to an optical window viewed by a photomultiplier tube. The metal channel is filled with a given gas to a suitable pressure.

The gas-filled channel thus serves both as the detecting medium and as the signal transmission line, circumventing the radiation damage problems described above. Moreover, absence of depletable coatings minimizes the need for frequent recalibration. Gas density can be readily adjusted to local temperature, and the gas filling is easily renewed in situ.

EXPERIMENTAL MEASUREMENTS

A series of preliminary proof-of-principle tests of the gas Cherenkov detection system concept has been carried out using the LINAC and the Van de Graaff-electron accelerators. The relative intensity of scintillation and Cherenkov light was measured as a function of gas density in the range from zero to well above the Cherenkov threshold.

To measure the relative Cherenkov yield due to pulses of monoenergetic electrons, a detector was designed which provided a short passage of the accelerator beam between two thin beam windows, machined into a stainless steel body which could withstand pressures up to 3000 psi. Approximately halfway between

e beam windows, a 45-deg mirror, consisting of a thin and highly polished stainless steel foil, deflected any light generated in the front part of the detector chamber into a lateral tube of about 1-m length, whose end was closed with an optical window. A photomultiplier was coupled to this window, and the whole region around the photomultiplier and window was heavily shielded. The detector is drawn schematically in Fig. IV-29-1. After the necessary accelerator tuning and focussing, the detector head was carefully aligned with the electron beam; in some measurements, a collimator was inserted between beam port and detector. The detector was then filled with gas, and the accelerator pulsed at 30 pps while the photomultiplier anode current was measured pulse by pulse, by means of electronic instrumentation which sorted anode current pulses according to their pulse height and stored them in a multichannel analyzer. Such pulse-height spectra were accumulated for about 3000 machine bursts, whereupon the gas pressure was varied and the measurement cycle was repeated. The spectra were eventually printed out and the mean pulse height determined and plotted against gas pressure.

The most complete set of data was obtained for pure methane with 3-MeV Van de Graaff electrons. These results are displayed in Fig. IV-29-2, where the smooth curve has been calculated using the CHERY program (CHERENKOV YIELD) and fitted to the measured points with an arbitrary intensity scale. As demonstrated by Fig. IV-29-2, agreement between the

FIG. IV-29-1. Gas Cherenkov Detector System Used in the Electron Accelerator Experiments. ANL Neg. No. 113-2388.

FIG. IV-29-2. Resume of the Light Yield Data for Methane from the Van de Graaff Experiments. The Smooth Curve is the Calculated Cherenkov Light Yield Obtained from the Computer Code CHERY. ANL Neg. No. 113-2386.

FIG. IV-29-3. Resume of the Light Yield Data for Ethane from the Van de Graaff Experiments. The Smooth Curve is the Calculated Cherenkov Light Yield Obtained from the Computer Code CHERY. ANL Neg. No. 113-2387.

measured points and theoretical calculations is generally satisfactory.

Similar results for ethane are presented in Fig. IV-29-3. The subthreshold region of this plot does not differ significantly from Fig. IV-29-2; however, a strong saturation effect appears in the region beyond the Cherenkov threshold. This effect can be attributed to the fact that ethane is near its dew point at room temperature in this pressure region. Consequently, the density change with pressure is largely uncertain, and possible condensation on the walls severely reduces light collection efficiency. This interpretation is indicated also by the fact that data reproducibility in this pressure region was very poor. At the higher temperatures prevailing in a reactor environment, these effects would probably not occur; hence, ethane for a reactor power monitor appears favorable, in view of the weak subthreshold light.

The CHERY program from which the expected light yield was calculated is based on an integration of the Frank and Tamm formula, which gives the number of Cherenkov photons per unit frequency per centimeter traversed by the electron in a medium of index of refraction n . This integration must take into account the continuous slowing down of electrons due to Coulomb interactions, which can very reasonably be approximated by constant energy loss per centimeter in the energy region considered here. The result is still not absolute since light transmission efficiency and photocathode efficiency can only be estimated. At the high pressures prevailing in these measurements, the dependence on gas pressure must be calculated on the basis of a fairly exact equation of state, such as Van der Waal's with coefficients computed from tabulated critical constants. In terms of the index of refraction n , input energy γ_i , and final energy γ_f , the yield integral becomes

$$Y \propto \frac{(\gamma_i - \gamma_f)(n^2 - 1)}{n^2} + \left(\frac{1}{2n^2}\right) \ln \frac{(\gamma_i + 1)(\gamma_f - 1)}{(\gamma_i - 1)(\gamma_f + 1)} \quad (1)$$

The relativistic energy parameters γ used in this equation have their usual meaning; the final energy γ_f is that energy at which the electron ceases to emit Cherenkov light. A slightly different formula is obtained in the region well above threshold, where electrons impinge on the mirror before they have slowed down to final energy; the curves shown in Figs. IV-29-1 and IV-29-2 have been corrected for that effect. No account was taken of the variation of light collection efficiency with energy, due to the increasingly steep angle at which Cherenkov light is emitted. The noticeable discrepancy at high pressures between prediction and measurement for the methane data may be attributed to this loss, which, however, is not very serious for practical purposes.

The observed ratio of the Cherenkov "plateau" to the below-threshold scintillation background is approximately two orders of magnitude for both methane and ethane. Actually, these ratio values can only be regarded as lower bounds in view of various geometry factors which reduce light collection efficiency and create the deviation between theory and experiment in the region well beyond threshold. In view of the need for discrimination between high-energy prompt gamma radiation and low energy fission product and other non-prompt decay gamma radiation, either methane or ethane appear to be promising candidates for a gas Cherenkov reactor power monitor.

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IV-30. Fast Neutron Generator Facility

A. B. SMITH

The experimental determination of many neutron cross sections vital to the fast reactor program are among the most difficult of nuclear physics measurements; for example, the inelastic scattering cross sections of fissile materials. Yet the accuracy required is continually increasing. Many of the neutronic properties and the long term performance characteristics of fast reactor systems are sensitive to relatively high

energy neutron processes (neutron spectra and irradiation damage, for example). Precision in difficult measurements at relatively high energies is demanding of technology, and thus requires special purpose neutron generators of unique intensity. The Fast Neutron Generator, now being installed (Fig. IV-30-1) is such a device and is the key to a successful program in support of the fast reactor development effort.

FIG. IV-30-1. View of Fast Neutron Generator. In the Foreground is the Negative-Ion Injector with the Main Generator Vessel in the Background. ANL Neg. No. 201-9145.

The Fast Neutron Generator will provide an advanced capability for fast neutron cross section measurements. The Generator operates in two modes, tandem and single ended. Particularly intense dc positive ion beam currents of up to approximately 1 mA will be available at energies which will provide both monoenergetic and white neutron sources with energies to approximately 20 MeV. The Generator will also operate in nano-second pulsed modes with peak pulsed currents of ≥ 10 mA. The Generator is located in a building designed to provide proper experimental areas, control and work areas, and office space. Experimental equipment already available includes: a) four on-

line digital computers, b) an advanced multi-angle time-of-flight apparatus, c) an automated total neutron cross section facility, d) a large liquid scintillator with extensive vacuum paths for time-of-flight measurements, e) a spin-flip solenoid for studying polarization properties, f) a charged particle scattering chamber, and g) several general purpose target positions.

The facility is nearing operation with all major Generator components in position and partially operational. The experimental program should begin within the coming months.

IV-31. Energy Calibration Studies for the Reactor Physics Van de Graaff Accelerator

J. F. WHALEN and J. W. MEADOWS

Recent monoenergetic total cross section measurements of titanium and lead in the vicinity of 1500 keV neutron energy, made with the Reactor Physics Division Van de Graaff Accelerator, located prominent

resonance structure 5 to 15 keV higher than the same structure measured by other laboratories. This systematic inconsistency warranted a calibration effort the results of which are described below.

Before the calibration measurements were started, the power supply and the associated control of the electrostatic analyzer were examined, cleaned, and tuned up according to the recommendations of the design engineer. Several minor modifications were made, due to an abnormal sensitivity to zero adjust. It is this electrostatic analyzer which controls the accelerating energy of the Van de Graaff Accelerator by means of the $^2\text{H}^+$ ion content of the original beam.

A literature search provided a convenient secondary check at the $\text{B}^{11}(p,n)\text{C}^{11}$ threshold. This was chosen because the targets are easily produced and remain stable in air. Although an accurate measurement of this threshold was reported by only one group, E. Bechner et al.,¹ this same group, using identical means, also measured the well known $\text{C}^{13}(p,n)\text{N}^{13}$ and $\text{Li}^7(p,n)\text{Be}^7$ thresholds² and obtained very good agreement with the accepted values. Since two of these threshold values measured by Bechner et al.¹ [$\text{B}^{11}(p,n)\text{C}^{11}$ at 3016.4 ± 1.5 keV and $\text{C}^{13}(p,n)\text{N}^{13}$ at 3235.3 ± 0.7 keV] are relatively close together and the latter agrees well with the accepted value of 3235.7 ± 1.0 keV, it was felt that the $\text{B}^{11}(p,n)\text{C}^{11}$ threshold could be relied upon.

Over a five day period, a number of measurements of the $\text{B}^{11}(p,n)\text{C}^{11}$ and $\text{Li}^7(p,n)\text{Be}^7$ thresholds were made on the Reactor Physics Van de Graaff Accelerator. Assuming the $\text{Li}^7(p,n)\text{Be}^7$ threshold as 1880.6 keV,⁽²⁾ the resulting values of $\text{B}^{11}(p,n)\text{C}^{11}$ averaged out to 3021.0 ± 1.0 keV, indicating a systematic error in the σ_T measurements of $+5.0 \pm 1.0$ keV at this energy.

In assigning the source of this systematic error it was noted that the digital-to-analog converter which provides the reference voltage for the electrostatic analyzer power supply during the σ_T measurements under computer control, agreed consistently with the

potentiometric bridge which provides the reference voltage during manual energy control. This would seem to relieve any suspicion directed toward these two instruments. The other two possible sources of error are the power supply for the electrostatic analyzer and the actual analyzer plates themselves. Fortunately it was possible to greatly diminish the possibility of assigning any responsibility to the former of these by resurrecting some old silicon data which were obtained using an earlier model power supply. The location of the same resonance structure was determined with the present system and these values agreed within 2 keV of the former values. This would seem to indicate that the source of error must be in a slight non-linearity in the deflection coefficient of the electrostatic analyzer plates.

It has been decided that for the σ_T measurements a linear correction from 0 to -5 keV should be applied to the proton energy between 1881 keV and 3016 keV with appropriate extrapolation to higher energies. This method of correction is supported by the fact that the sulfur resonance at 585 keV which has been located on the energy scale with great accuracy by R. Paterson et al.,³ falls 2 keV below the energy found by the Reactor Physics Division. The above linear correction when applied to our data would bring these two energies into coincidence within experimental error.

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IV-32. Construction of Improved On-Line Computer Interface

J. F. WHALEN, R. J. PECINA*, R. M. KASH* and R. N. LARSEN*

A replacement interface for the two CDC 160-A on-line computers servicing the Van de Graaff experimental program has been constructed. In its present configuration it will provide four major channels of communication between the experimental domain and the computer. Two of these are input channels: one for the analog-to-digital converters and one for a manual keyboard. The other two channels are output

channels: one for the display system and one for an external relay control system. The modular form of these channels is designed so that additional units can be added as the need arises.

The new system operates logically in the same manner as the original interface but with improved capabilities. The actual hardware utilizes current in graded circuitry and standard NIM packaging which facilitates unit exchange and power supply replace-

* Electronics Division, Argonne National Laboratory.

it in case of failure. Two new analog-to-digital converters were included (with the option of a third in the future) having an increased precision which extends to 8192 channels, although a maximum of 4096 is permitted with this data channel. The keyboard input has been substituted for the present contour level switches for ease of manual operation and the addition of interrupt options. This keyboard will be used mainly for operator-computer interaction through the display system medium. It has a twelve bit holding register with an octal read out. The display system itself has the same options as the present system, with improved digital-to-analog converters permitting the display of 1024 channels individually distinguishable and 4096 channels with continuous line display for

shape discrimination. The external relay control system provides six computer selectable bistable units. A single twelve bit word transmitted to this channel will set, reset, or pass any combination of these bistable elements whose outputs control 500 mA relays. These will be used for controlling beam gates, sample changers, or any other relatively slow mechanical devices in the target area.

The circuitry for channel selection by the computer in this interface has been designed so that selection codes can be adjusted to accommodate other peripheral devices with fixed selection codes. In addition, either computer can be selected for interface connection with a front panel switch.

IV-33. Grey Neutron Detector

W. P. POENITZ

The grey neutron detector is a semi-prompt neutron detector^{1,2} applicable in the energy range from thermal to several MeV. The operation of the detector is based on the slowing down and subsequent capture of the neutrons in a moderating medium and the measurement of the capture γ -rays leaking through the surface of the moderator. This detector was recently employed to measure the energy dependence of some neutron cross sections important for fast reactor calculations.³⁻⁵ In principle the efficiency of the detector is constant over a large energy range; however, due to the absorption of the capture γ -rays in the moderator corrections amounted to as much as 15% in the energy range from 0-3 MeV. Therefore, it was felt that an experimental proof of the calculated energy dependence of the efficiency² would be desirable.

Two experimental methods were used to measure the efficiency of the grey neutron detector:

a) The associated activity technique⁶

The $\text{Li}^7(p,n)\text{Be}^7$ and the $\text{V}^{51}(p,n)\text{Cr}^{51}$ reactions were used as neutron sources. The neutron energy range 30-1000 keV was covered using a 3 MeV Van de Graaff accelerator. The targets were placed in the center of the grey neutron detector which was 76 cm on a side.⁵ The space angle subtended by the entering proton beam tube was only 5×10^{-4} rad. After irradiation of the targets and measurement of the integrated flux with the grey neutron detector, the activities of the targets were counted. The ratios of the count rates of the photopeaks measured with a NaI-detector at 0.477 MeV (Be-7) or 0.320 MeV

(Cr-51) to the count rate of the photopeak at 2.2 MeV gave directly the energy dependence of the grey neutron detector efficiency. The ratio between the measured and calculated efficiency is shown in Fig. IV-33-1. These values were normalized to unity at 30 keV.

b) The flux integration technique⁷

In this experiment a collimated neutron beam was used. This beam was produced using the $\text{Li}^7(p,n)$ reaction and a cylindrical collimator. The neutron source was shielded by at least 60 cm of boron- and lithium-loaded polyethylene. The grey neutron detector tank was filled with a VOSO_4 -water solution. Two stirrers provided a fast homogeneous distribution of the activated vanadium over the whole tank. The prompt 2.22 MeV γ -rays from the capture in hydrogen and the delayed 1.43 MeV γ -rays from the decay of the activated vanadium nuclei were measured simultaneously using a 3×3 in. NaI(Tl) detector. The ratios of the count rates in the photopeaks again yield the energy dependence of the grey neutron detector. This experiment is essentially equivalent to the application of the grey neutron detector as a flux monitor in a neutron cross section experiment. The deviation from the calculated efficiency curve is also shown in Fig. IV-33-1.

The error bars on the values shown in Fig. IV-33-1 are partly due to counting statistics and uncertainties in fitting the photopeaks (0.6-1.0)% and partly due to the uncertainties in the choice of parameters for the calculation of the efficiency curve (0.0-1.6)%.

The agreement of the results obtained by the two

FIG. IV-33-1. The Ratio of the Experimental Efficiency to the Theoretical Calculated Efficiency of the Grey Neutron Detector as a Function of Neutron Energy. *ANL Neg. No. 118-1465*.

different techniques excludes the possibility of a significant energy-dependent influence of the neutron entrance channel on the efficiency curve. This can also indirectly be concluded from the small size of the opening space angle due to the channel (about 4×10^{-3} rad). A measurement of the effect of the neutron entrance channel on the efficiency was performed at 30 keV by comparing the count rates of the grey neutron detector for the original neutron entrance channel and for one decreased in diameter by a factor of about 10. The difference of the count rates was $(1 \pm 1)\%$.

An additional check of the efficiency of the grey neutron detector was obtained from a measurement of the $\text{Li}^7(p,n)\text{Be}^7$ cross section using this detector. Thin lithium-metal targets were placed in the center of the grey neutron detector and bombarded with protons in the energy range (1.9–3.1) MeV. The proton current on the target was monitored with a charge integrator. The curve was normalized at 2.25 MeV to the absolute values measured by R. Macklin and T. Gibbons^{8,9}. An absolute value obtained using the associated Be-7 activity was in agreement with this normalization within 3%. In Fig. IV-33-2 the results are compared with measurements performed by Macklin and Gibbons^{8,9} using a graphite sphere detector and values obtained by D. Lister¹⁰ who measured the angular distribution of the $\text{Li}^7(p,n)$ reaction with a recoil proton counter and normalized to the

FIG. IV-33-2. The Total $\text{Li}^7(p,n)\text{Be}^7$ Cross Section. Comparison with Results from Macklin and Gibbons^{8,9} and Lister.¹⁰ *ANL Neg. No. 118-1468 Rev. 1*.

absolute values obtained by Macklin and Gibbons.^{8,9} The difference above 2.4 MeV is due to the second neutron group.

The many current applications of the grey neutron detector demonstrates its capability as a smooth-efficiency neutron monitor for the intermediate energy range.

Lister collaborated in carrying out and evaluating the $\text{Li}^7(p,n)$ cross section measurements.

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IV-34. A Large Double-Chamber Gas Scintillation Counter for Fast Neutron Fission Cross Section Measurements

W. P. POENITZ

For some time gas scintillation counters¹ have been employed as detectors in neutron fission cross section experiments. The fast time-response of such counters² makes them particularly valuable in time-of-flight measurements in the fast neutron energy range. However, the large α -activity of most fissionable nuclei poses an appreciable problem. Further, for time-of-flight experiments in the fast energy range large quantities of fissile material are needed for cross section measurements and this increases the α pileup and causes difficulties in measuring reasonable fission particle pulse-height spectra. These difficulties were overcome with a double-chamber gas scintillation counter designed for cross section measurements in beam-type experiments. The counter was applied to absolute measurements of the U-235 and Pu-239 fission cross sections at 30 keV.

The detector consists of two identical chambers (Fig. IV-34-1). Each of these chambers consists of a brass cylinder 10.5 cm high and 10.5 cm diam. and has a 0.02 cm thick bronze window at one end. The chambers are silver-plated to obtain a high light reflection from the walls. Two photo-multipliers view the scintillation light in each chamber through quartz windows. Employing an indium wire seal, the two chambers are sealed together vacuum tight.

After flushing the counter several times with argon gas and evacuating it to about 10^{-5} mm Hg the counter was filled with an argon-nitrogen mixture using about 15% nitrogen. This gas mixture, sug-

gested by C. Eggler and C. Huddleston³ is inexpensive and the pulse height obtained is sufficiently high to discriminate against the tube noise. A more expensive gas (e.g., xenon) would complicate the detector usage involving frequent sample changes.

The fission foils used in the present experiments were electro-deposits of uranium acetate and of plutonium acetate on Vyns-backings. The diameter of the deposits was 5 cm. The foil thicknesses were 100, 300, and 500 $\mu\text{g}/\text{cm}^2$. They were positioned between the two chambers. These foils were produced by the Central Bureau of Nuclear Measurements, EURATOM.

Figure IV-34-2 shows a schematic block diagram of circuitry used with the present measurements. Measurements were carried out, gated or ungated, by a coincidence between the two chambers. In both cases the fast anode output of all four photo-multipliers was added. Fast electronics were used until the last step, at which point the time-to-pulse height converter and a pulse stretcher following the linear gate produced the input pulses for a double analogue-to-digital converter. Time-of-flight versus energy spectra were stored in a CDC-160A on-line computer system.⁴

Figure IV-34-3 shows the spectra obtained for a Pu-239 sample with a deposit of 100 $\mu\text{g}/\text{cm}^2$ (96.26 wt/o Pu-239, 3.57 wt/o Pu-240 and 0.17 wt/o Pu-241).

The dashed line of Fig. IV-34-3 is the sum spectrum adding the four photo-multiplier outputs; the smooth line is the same spectrum gated with coincidence between the two chambers. The essential advantage of such coincidence is that counts caused by fast neutrons in the multipliers were canceled out. The

FIG. IV-34-1. Schematic of the Detector. ANL Neg. No. 113-1464.

FIG. IV-34-2. Block Diagram of the Electronics. ANL Neg. No. 113-1466.

FIG. IV-34-3. Pulse Height Spectra Gated and Ungated for a $100 \mu\text{g}/\text{cm}^2$ Pu-239 Foil. ANL Neg. No. 113-1438.

FIG. IV-34-4. Pulse Height Spectra for U-235 and Pu-239 ($500 \mu\text{g}/\text{cm}^2$ Foil). ANL Neg. No. 113-1439.

α -pileup spectrum is only partly suppressed because the foils were translucent. However, the separation between α -pulses and fission pulses was found to be sufficient due to the summing of the energies of both fission fragments. Figure IV-34-4 shows that the extrapolation to zero pulse height for the plutonium sample is not a severe problem. After normalization to the same spectrum-peak height and the same count rate (above channel 50), the shape of the U-235 fission sum spectrum and of the Pu-239 fission sum spectrum were found to be so similar that the measured U-235 spectrum can be used for the necessary

FIG. IV-34-5. Pulse Height Spectra for U-235 Foils of Different Thickness. ANL Neg. No. 113-1440.

extrapolation to zero pulse height with a high degree of accuracy. Figure IV-34-5 shows the spectra obtained for two different U-235 sample thicknesses, 100 and $500 \mu\text{g}/\text{cm}^2$. The number of fission counts to be determined by extrapolation to zero pulse height is as low as 1.5% for the $100 \mu\text{g}/\text{cm}^2$ foil. In a recent cross section measurement⁵ this number was about 8% and its uncertainty contributed significantly to the uncertainty of the cross section. A counter similar to that shown in Fig. IV-34-1 has been built to be used in open geometry experiments. For this detector the walls were only 0.017 cm thick and the multipliers view the scintillator light directly. A time resolution of 2 nsec has been obtained with this detector.

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IV-35. Shielding Design and Analysis for Modifications of JANUS

A. E. McARTHUR and D. H. SHAFTMAN

INTRODUCTION

JANUS is a biological irradiation facility which has been constructed at Argonne National Laboratory for research on the effects of neutron radiation on animals.¹ Two irradiation rooms are included in JANUS. One is a small High Flux Room, for acute-exposure biological experiments. The other is a much larger Low Flux Room, for chronic-exposure experiments at neutron dose rates of the order of 1 rad/h.

The JANUS facility includes a 200-kW(t) water-moderated reactor with graphite reflectors. Movable converter plates containing highly enriched uranium are located beyond the radial graphite reflectors. The converter plates for the two rooms are at different distances from the reactor core, to provide the different levels of exposures. These plates absorb neutrons thermalized in the graphite and provide fission-spectrum sources of neutrons close to the two irradiation rooms. When personnel access is required to one of the irradiation rooms, that converter plate is lowered to a shielded region and shutters are inserted to attenuate neutron and gamma-ray radiation from the core and reflectors. This design was intended to permit access to either irradiation room without interruption to the experiments in the other room, and, in particular, without changing the power level of the reactor core.

Certain design problems have limited the usefulness of JANUS.²⁻⁵ These problems pertain to the types and energy spectra of the radiations to which animals are exposed, and to the exposure of personnel to undesirably high radiation dose rates during operation of the facility and during preparations for experiments in the High Flux Room. (For more details, see Ref. 2.) Experimental and theoretical studies of these problems led to formation of a task force whose charter was to determine what should be done to modify the JANUS facility so as to enhance its usefulness for these basic research studies and to reduce the radiation exposure of personnel. Reference 2 summarizes the radiation problems and the principal recommendations made by the task force. The task force proposed to modify the High Flux Room and the associated shutters and converter plate, and to defer modification of the Low Flux Room. Reference 6 summarizes analyses which led to the task force recommendations for improved shutters for the High Flux Room.

At the beginning of FY-1969, in preparation for detailed design and performance of JANUS modifications, the shielding and neutronics aspects of the task force report were reviewed in detail by D. H. Shaftman.^{7,8} This review provided basic confirmation that the modifications proposed would fulfill the desired objectives. Certain areas of modification were noted which required special additional consideration and which would be resolved best by performance of specific experiments during the pre-operational work with the modified facility.

Following this review, work on the detailed design of the modifications proceeded, with participation by the authors in two working groups which include personnel from Reactor Operations, Biological and Medical Research, Plant Engineering, Central Shops, and Reactor Physics. The JANUS facility is now in the process of structural modification. It is expected that the facility will be ready for experiments within the second quarter of FY-1970.

For the convenience of the reader, this report summarizes the principal tasks of the particular JANUS modifications which relate to personnel dose rates, as proposed by the task force and studied during the more recent review. The balance of this paper reviews some of the interesting detailed-design problems of radiation shielding, and particularly the reduction of radiation dose rates to personnel working in the High Flux Room during periods between animal exposures.

TERMINOLOGY

Unless otherwise indicated, the terms "room", "converter plate", and allied terms, will be used to abbreviate the corresponding terms "High Flux Room", "converter plate for the High Flux Room", etcetera.

SUMMARY OF PRINCIPAL MODIFICATIONS TO REDUCE PERSONNEL EXPOSURE

When lowered into the position of full insertion, the former shutters for the High Flux Room permitted excessive transmission of radiation (especially neutrons) into the High Flux Room. The shutters were not long enough to prevent scattering of neutrons into that room from regions at elevations above that of the top of the shutters, thereby bypassing the shutters. Nor were the shutters thick enough to provide the desired bulk shielding against neutron and gamma radiation from the reactor. In the former de-

sign, the converter plate was located between the shutters and the reactor core, as shown in Fig. IV-35-1, which is a reproduction of Fig. 11 of the JANUS design manual.¹ (Note the differences in terminology, e.g., "High Dosage Area" is now "High Flux Room".) In the modified JANUS, the new converter plate will be located on the room side of the new shutters. This change was made primarily to provide space for much thicker shutters (28 versus 20 in.). The final compositions and detailed thicknesses of the new, thicker shutters will be discussed in the next section of this report, on the bulk radial biological shield for the High Flux Room.

The use of longer shutters will help also to reduce the radiation dose rates to operating personnel in the Reactor Work Room and Reactor Control Room areas, on the level above. The reduction of personnel dose rates in these areas is one of the principal purposes of the modifications. One of the modifications is to locate shutter actuators on the level above, and to fill the resulting holes in the shields with new plugs of heavy concrete. The combined effect of these changes is expected to eliminate the

potential problems of high dose rates to personnel in these areas of the main floor of JANUS. In the past operation of JANUS, shielding was provided locally to reduce personnel exposure on the main floor.

The remaining principal modifications to reduce dose rates to personnel are the changes of the High Flux Room to reduce decay-gamma-ray dose rates to people working in the room between periods of irradiation experiments. Heretofore personnel have not been permitted to enter the High Flux Room very soon after the termination of an animal-exposure period, and they have been severely limited in their work time, in part because of the high residual gamma-ray radiation from the "hot" walls, floor, and ceiling of the room, and from the exposed activants such as light fixtures and ventilation ductwork. The most important improvement of the room is to line the inside of the room with layers of materials which will reduce the levels of neutron activation and provide an effective shield against residual gamma-rays. A design criterion for the room modifications is that high-activity materials will be replaced by materials

FIG. IV-35-1. Vertical Section Through the Mid-Plane of the Converter Plates (Unmodified JANUS). ANL Neg. No. 144-260A.

h such desirable characteristics as low cross section for neutron activation and/or short half-life. Where possible, activating materials will be relocated to shielded locations. The goal is to permit reasonable entry to the High Flux Room after a "cooling" time of 5 min or less following termination of an animal exposure.

A false ceiling will be installed in the room. This change will serve several purposes. The new ceiling will constitute a protective lining of the old concrete ceiling to reduce residual gamma-ray dose rates to personnel in the room. It will reduce the degree of asymmetry in the room, with an expected improvement toward a more nearly uniform spatial distribution of neutron dose rate in the animal population. And it will permit relocation of actuators for the converter plate and for attenuator plates to a crawl space between the two ceilings. The attenuator plates will provide additional flexibility to the experimenters in shaping the spatial distribution of exposure to the animals.

The total biological dose rates to personnel in the modified High Flux Room will be sums of dose rates from many sources. The theoretical analyses indicate that the major contributor will be high-energy neutrons that will penetrate the new bulk radial biological shield. The total gamma-ray dose rate will be a sum of dose rates from such sources as: penetration of prompt and decay gamma-rays through the bulk radial shield, from sources in the reactor core and the reflectors; incompletely-shielded activity in the original concrete walls, floor, and ceiling; activation of the lamp bulbs and the lamp fixtures in the new ceiling; activation of materials (steel, aluminum, impurities in the lead, etcetera) in the new ceiling and in the crawl space above it; activation of materials in the floor liners, in the wall liners, and supporting bolts and screws; residual gamma-rays from the converter plate in its "out" position; and activation of the animals and their housing. Therefore, in the detailed designs of the modifications, a special effort was made to minimize the dose-rate contributions from each of these sources, within the constraints of sound engineering design, needs of the experimenters, dollar cost, availability of materials, and the efficient timing of the work.

THE BULK RADIAL BIOLOGICAL SHIELD FOR THE HIGH FLUX ROOM

In this application, the term "bulk radial biological shield" designates the radiation-shielding zones between the reactor core and the inside of the High Flux Room. In this sense, the graphite reflector is part of the bulk shield. Figure IV-35-2 shows a vertical

section of that portion of the modified bulk shield which is beyond the graphite reflector.

In Reference 6, C. N. Kelber and A. E. McCarthy have summarized results of some analyses relating to possible replacement of part of the graphite reflector by a zone of lead. The hope was that the lead would improve the neutron-shielding effectiveness of the bulk shield by degrading the spectrum of high-energy neutrons by inelastic scattering. As shown in Table VII-5-II of Ref. 6, this hope was well founded. Unfortunately, the calculated level of the thermal neutron flux at the converter plate was reduced significantly. In the more recent analyses by D. H. Shaftman⁷ this problem was reexamined. It was observed that the level of the thermal neutron flux at the former converter plate was basically determined by the diffusion of thermal and near-thermal neutrons entering the external graphite reflector, and not by slowing-down of more energetic neutrons in that portion of the reflector. Briefly, this is due to the large thermal migration area of graphite by comparison with the very much smaller "age" for high-energy neutrons. Substitution of high-purity natural lead for some of the graphite in the external reflector would interpose a relatively highly capturing medium, or, rather, a medium with a much smaller thermal migration area; this would result in a large net loss in the intensity of the thermal neutron flux at the converter plates. There appears to be little or no net advantage to such a modification of the graphite region rather than the recommended provision of a zone of lead radially beyond the new shutters. Therefore, the graphite reflector will not be modified.

In the modified JANUS there will be a new, more heavily loaded converter plate for the High Flux Room. It will be located radially beyond the shutters rather than in front of them. (See Figs. IV-35-1 and IV-35-2.) In the reference redesign, between the position of the new converter plate and the position of the shutters there will be a wall of lead made up of abutting one-inch-thick sheets. The reference thickness is 6 in. Radially beyond the new plate there will be an additional lead wall, with a reference thickness of 2 in. As shown in Fig. IV-35-2, provision is being made for thicknesses of 6 or 7 in., and 1 to 3 in., for these two lead zones. It is intended that any deviations from the reference thicknesses will be guided by experimental data obtained in the modified facility. The best present estimate is that a total thickness of 8 in. of lead in these two zones will be needed for shielding against gamma-rays and neutrons traversing the shutters, when personnel entry into the room is required. The 2-in.-thick lead wall provides additional shielding against residual gamma-

FIG. IV-35-2. Vertical Section Through the Mid-Plane of the Converter Plate for the High Flux Room (Modified JANUS). ANL-Neg. No. 113-3117.

rays from the converter plate when it is in its "out" position, that is, lowered into a shield pit. If a thicker wall is needed for shielding of the converter plate, this wall can be increased in thickness to 3 in. For experiment purposes, however, it would be desirable to *reduce* the wall thickness to 1 in. in order to minimize energy degradation of the fission neutrons from the converter plate in their traversal of that lead wall. Special additional provisions of shielding of the converter plate, when it is in the pit, include the use of a 10-in.-thick ledge of lead on the room side of the plate, and an 8-in.-thick floor liner of lead for a distance of 1 ft from the ledge.

The 6-in.-thick lead wall preceding the converter plate, in its "in" position, is calculated to reduce the level of the thermal neutron flux at the converter plate to one-third. Nevertheless, this lead is an essential feature of the bulk shield, and especially as a gamma-ray shield. In part, this reduction in thermal neutron flux is compensated by the increased blackness of the new plate. In the unmodified JANUS, a 6-in.-thick lead wall was included, on the room side of the converter plate.

The calculations summarized in Ref. 6 utilized the

shielding-analysis code MAC.⁹ For the more recent calculations, one-dimensional multigroup S_n calculations were made, using an Argonne National Laboratory version¹⁰ of the Bondarenko group constants,¹¹ to determine the relative fission rates with and without lead in the external graphite reflector.

The new radiation shutters are longer and thicker than the former shutters. The study described in Ref. 6 (Tables VII-5-III and VII-5-IV) indicated that a shutter fill of 2 in. of lead followed by 25 in. of a borated polyethylene would be more effective in stopping neutrons than a fill of heavy concrete. Moreover, the weight of each of the three shutter sections would double if a heavy-concrete fill were used. A fill of lead and borated polyethylene will be used. The above-mentioned lead walls of total thickness 8 in. are needed in addition to the 2 in. of lead in the shutter fill, because otherwise the total optical thickness of the bulk shield would be smaller than the previous thickness with respect to the attenuation of gamma-rays from the core and reflector. The 8-in.-thickness of lead is important also in attenuating high-energy neutrons which penetrate the shutters.

The borated-polyethylene fill is inserted in the form

offset "bricks" of typical dimensions $1 \times 4 \times 8$ in. The total radial thickness of the polyethylene fill is 25 in. The total radial thickness of the new shutters is $28\frac{1}{4}$ in. In contrast, the previous shutters had a total radial thickness of 20.0 in., including the steel shutter casings with wall thicknesses of 1.0 in. The bulk fill of the previous shutters was a ferrophosphorous concrete with a density of ~ 290 lb/ft³—almost five times the density of the polyethylene.

Additional details of the new shutters are given in the following section.

THE NEW SHUTTERS AND SHUTTER PEDESTALS: PHYSICAL COMPOSITION AND SHIELDING EFFECTIVENESS

The reference "shutter shielding configuration" noted in Ref. 6 (Table VII-5-III) has been retained for the shutter for the High Flux Room. At the vertical level of the midplane of the reactor core, and starting at the reactor-core side of the shutter: a $\frac{1}{4}$ -in.-thick sheet of Boral is attached to the outside of the steel shutter casing, to reduce activation of the casing; the wall thickness of the new casing is $\frac{1}{2}$ in. The first 2 in. of the shutter is filled with lead. This is followed by 25 in. of borated polyethylene, with a specified minimum density of 0.95 g/cm³ and specified minimum average densities of 7.9×10^{22} atoms H/cm³ and 1.0×10^{21} atoms B/cm³. The outer wall of the steel casing is also $\frac{1}{2}$ in. thick.

The shutters and shutter pedestals are curved at front and rear, and special shaping is required for some of the polyethylene bricks. To match other dimensions, pieces of bricks are used to minimize gaps. The commercially made polyethylene bricks are supplied with nominal dimensions $1 \times 4 \times 8$ in., and the special shaping and cutting will be accomplished by Argonne National Laboratory. Specifications of the bricks include tolerances on: spatial dimensions; density and voids; impurities; minimum hydrogen content; and, for boron: minimum atom density, uniformity of spatial distribution, and maximum (B_4C) particle size. The polyethylene bricks are installed in step patterns, to minimize radiation-streaming paths. Spatial gaps are included in the shutters and shutter pedestals, to allow for thermal expansion of the polyethylene. These gaps are thin enough that there is no concern about radiation streaming. As a final precaution, the design makes provision for later addition of a liquid organic

to the polyethylene-filled shutters and pedestals, this is judged to be desirable for an additional reduction of neutron dose rate to personnel in the room.

As shown in Fig. IV-35-2, the shutters are stepped to reduce streaming of radiation. There are mating steps in the fixed pedestals for the shutters, with the higher step on the reactor-core side of the pedestal. The basic radial configuration of the upper portion of the pedestal is the same as that for the shutter, namely: Boral; steel casing; 2 in. lead; borated polyethylene; and the outer wall of the steel casing. Starting 32 in. below the highest elevation of the shutter pedestal, a high-density hardboard (Benelex 401) is used as the entire fill for the bottom 2 ft of the pedestal. Even though the Benelex 401 is not borated, it will provide adequate shielding at this distance below the bottom of the reactor core. It is less expensive than the polyethylene, and it is superior to the concrete as a neutron attenuator, which is the purpose of this Benelex-401 fill.

The bottom faces of the shutters and the top faces of the shutter pedestals are not lined with lead, because of the concern of introducing a thick zone which is more easily penetrated by neutrons in the energy range of hundreds of keV. Approximate modeling of the problem as a one-dimensional array permitted application of the computer program MAC. The MAC calculation indicated that use of lead as the entire shutter fill (27 in.) would increase the total neutron dose rate by more than 2 decades, by comparison with the bulk-shield effectiveness of the reference shutter. The facings considered included a total thickness of 4 in. of lead, representing the sum of the two 2-in.-thick facings, plus a total thickness of 1 in. of steel, representing the two $\frac{1}{2}$ -in.-thick steel casings. It is considered unlikely that a feasible stepping of shutters and pedestals would eliminate this streaming problem. Therefore, the concept of somewhat hardening the high-energy neutron exposure spectrum in the room, by using such lead facings of shutters and pedestals, was discarded.

The evaluation of the shielding effectiveness of the new bulk radial biological shield was based largely on a comparison with the calculated shielding effectiveness of the former shield plus measured data of dose rates to personnel in the room. Calculations of two types were performed to determine neutron fluxes: one-dimensional multigroup S_n ($n = 4$ or 8), with P_1 scattering by hydrogen; and MAC. As noted above, an Argonne National Laboratory version of the 26-group Bondarenko cross sections has been generated. This set was applied. Knowing that these calculations might be severely limited by the inadequacy of the P_1 scattering approximation, check-calculations were made of shielding experiments performed with the Bulk Shielding Reactor (BSR) with thick water shields.¹² The predicted neutron fluxes

were much lower than the measured fluxes. Thus these multigroup S_n calculations were non-conservative. The MAC calculation, on the other hand, yielded somewhat conservative results when applied to the BSR experiments. For subsequent analyses of the neutron-shielding effectiveness of the high-attenuation bulk shields, MAC was used.

In Ref. 6 it was reported that calculations with MAC implied a neutron dose rate of 27 mrem/h and a gamma-ray dose rate of 29 mrem/h to a human receptor at the room-side of the outer lead wall of the modified bulk radial shield. In those analyses, scattering of neutrons by the lined wall opposite this surface was not included. In the course of the review (by D. H. Shaftman), additional MAC calculations provided a measure of the augmentation of the neutron dose rate by such scattering, but these one-dimensional calculations cannot properly account for the actual geometrical effects. With the reactor operating at full power [200 kW(t)], the shutters inserted, and without animals in the room, the calculated neutron dose rate was 80 mrem/h at the position corresponding to the room. This result is in essential agreement with the task force report,¹³ where it was stated that backscattering of neutrons from the walls of the room was estimated to double the calculated neutron dose rate.

By comparison with similar calculations for the previous bulk radial shield, the modified bulk shield provides an additional attenuation factor of ~ 50 for the neutron dose rate. Of course, to this factor of increased effectiveness a second factor (~ 10) must be applied to account for the elimination of paths of neutron bypass of the previous shutters, because of the increased height of the new shutters. The net factor of increased shielding effectiveness for neutrons therefore is estimated to be 500.

In these one-dimensional calculations with MAC, the reactor core and bulk-shield zones were approximated as infinitely long cylinders. The MAC code requires an input of multigroup neutron flux at the surface of the reactor core; this was obtained by a spherical-geometry calculation, and it is conservative. The effect of these two approximations of geometry is to overestimate the neutron dose rate at the radial distance of the High Flux Room from the core. One important measure of non-conservatism in these one-dimensional analyses is the failure to account fully for the effects of multiple reflections of neutrons from the walls, floor, and ceiling of the room. The room wall opposite the converter plate is included in the calculation, and, as noted above, this inclusion more than doubled the calculated neutron dose rate—an albedo effect. But scatterings by the end

walls, the floor, and the ceiling are not included. Since the lead liners offer a higher albedo for neutrons than does the concrete, the relative neutron dose rate in the room might be higher than the value indicated by the comparison of MAC calculations for the modified and unmodified bulk radial shields. The best present estimate is that the maximum neutron dose rate to personnel in the room will be smaller than 0.2 mrem/h. Probably it will be smaller than the calculated value of 80 mrem/h, at full reactor power and with the shutters inserted and the converter plate in its "out" position.

In calculating (with MAC) the gamma-ray dose rates due to penetration of the bulk radial shield, the core-averaged fission rate of the actual reactor was used to provide the source of prompt and fission-product gamma-rays. MAC calculates the capture-gamma-ray sources in the reflector on the basis of the neutron fluxes calculated by MAC. However, subsequently the gamma-ray dose rates are calculated in slab geometry. The net effect of these geometrical approximations is conservative in calculating the gamma-ray dose rate in the room due to gamma-rays penetrating the bulk shield. Reference 6 reports a MAC-calculation value of 29 mrem/h for this contribution to the gamma-ray dose rate to personnel in the room. More recent calculations, also with MAC, indicate that the preponderant contribution is from gamma-rays produced in the reactor core and the graphite reflectors. In contradiction to the earlier calculation, the contribution from the steel casing of the shutters is minimized by the Boral attached to the room-side of the shutters and by the boron in the polyethylene fill. This type of inconsistency in computer output from the Argonne version of the MAC code has occurred on other occasions, and usually it has been difficult to discern that the result is erroneous.

Neither the neutron dose rate nor the gamma-ray dose rate appears to be very sensitive to the boron content of the polyethylene fill. The principal gamma-ray contribution is from the reactor core and the graphite reflectors. The neutron dose rate is due largely to penetration of the bulk radial shield by high-energy neutrons, as noted in Ref. 6, and this is little affected by the boron content.

PRINCIPAL MODIFICATIONS OF THE HIGH FLUX ROOM: PHYSICAL DESCRIPTIONS; MOTIVATIONS

The principal modifications of the High Flux Room are:

1. lining of the walls and floor, first with layers of material containing boron and hydrogen, and then

an outer layer of lead. The composite purpose of these liners is to reduce the level of the activating (thermal) neutron flux in the original walls and floor, and to shield against the remaining gamma-ray activity. (See Fig. IV-35-3.)

2. installation of a false ceiling, with a zone of a special concrete containing more hydrogen than the usual shielding concrete; this concrete ("boro-bauxite" concrete) also includes boron, for capture of thermal

neutrons. The false ceiling includes an 8-in.-thick zone of boro-bauxite concrete above a 4-in.-thick layer of lead. The purposes of the new ceiling are: to reduce the residual gamma-ray dose rate to people in the room; to improve the spatial uniformity of the neutron dose rate to the animal population; and to provide a crawl space, between the two ceilings, for actuator mechanisms to move the converter plate and the attenuator plates.

FIG. IV-35-3. Vertical Section—High Flux Room JANUS Reactor. No ANL Neg. No.

3. minimization of use of exposed materials which would provide long-lived sources of residual gamma-rays or would degrade the high-energy-neutron spectrum in the High Flux Room. The latter criterion is essential to the research needs of the experimenters.

In the following paragraphs are summarized some of the more interesting and unusual problems of redesign of the room to minimize the dose-rate contributions from residual gamma-rays. Many other detailed problems of this nature were resolved which are not included in this report.

The task force had recommended use of a special, boro-bauxite shielding concrete for the wall lining, but with the proviso that the expected high hydrogen content of this concrete indeed was attainable.² During the early stages of detailed redesign, measurements were made of the hydrogen content of an experimental slab of boro-bauxite concrete as a function of the total curing time. For this investigation, an accelerated curing was achieved in a vacuum chamber from which water vapor was removed continuously. Extrapolation of the data of weight loss to infinite curing time indicated that the hydrogen content of the cured boro-bauxite concrete might be as low as one-third of the content (8×10^{22} atoms H/cm³) originally sought.^{14,15} Perhaps a development effort could resolve this problem of hydrogen loss, e.g., by use of a special coating of the concrete. In the interest of time, and because this particular resolution is acceptable for the liner of most of the wall area, and for the floor liner, it was decided to use a borated hardboard instead of the boro-bauxite concrete. Use of a high-density (1.4 g/cm³) hardboard would have been preferable because of the higher hydrogen content, but it was indicated that addition of the desired load of boron was not feasible. The density of the borated hardboard is 1.0 g/cm³. The Argonne National Laboratory specifications for this material require low sodium content, a net hydrogen content of at least 3.9×10^{22} atoms H/cm³, and a uniformly-distributed net boron content of at least 1.4×10^{21} atoms B/cm³.

The typical thickness of the liner of borated hardboard is 4 in. The typical thickness of the lead liner also is 4 in. In certain areas it was not feasible to retain this ratio of 4 in. hardboard to 4 in. lead, e.g., in the vicinity of ventilation openings. The choice was to increase the thickness of lead at the expense of the hardboard, because the extra gamma-ray attenuation provided by the additional lead more than compensated for the increase in activation level resulting from the removal of moderating material.

The actuators for the converter plate and for the attenuator plates will be located in the crawl space

between the original ceiling and the new ceiling. Access to this space requires removal of a shield plug. Specifications have been established for all materials used in these devices, in electrical wiring, etcetera, to meet the criterion of minimizing use of materials with high (n, γ) cross section and long half-life. For this application, a half-life of the order of hours is acceptably short if this activity is not high enough to contribute significantly to the dose rate to personnel in the High Flux Room. An example is the use of steels of low manganese content and low cobalt content since these materials are important activants and are long-lived.

Bolts which anchor the lead liners to the ceiling support structure are inset into the lead and are covered with lead, e.g., by lead plugs. The steel I-beams, which support the weight of the lead liner and the boro-bauxite concrete of the new ceiling, will be painted with red lead, and the lower portions of these beams will have an additional coating of a special neutron-absorbing paint to reduce the activation of the manganese and cobalt in the steel. A gadolinium-based paint has been chosen because of the high capture cross section for thermal neutrons and because typically neutron capture results in production of another stable gadolinium isotope. High-purity gadolinium is impracticably expensive, but the small associated admixture of other rare earths is tolerable because most of the neutron captures would be by gadolinium. Gadolinium-based paints have been used at other installations, with reported success.¹⁵

The ceiling-light fixtures, and especially the light bulbs, have received considerable attention because they are potential sources of high gamma-ray activity. There are two principal problems. With the new ceiling installed to improve the experiment environment and to provide the crawl space, the ceiling is too low for the typical installation of light fixtures. Therefore the light fixture and light bulb must be inset into the ceiling, but it is highly undesirable to remove considerable areas of shielding-lead in the process. The second principal problem relates to neutron activation of the materials in the light bulb and in the fixture, including the light reflector used.

The first problem has been resolved by special shaping of the opening in the lead so as to accommodate the receptacle and the partially inset bulb. The second problem has been resolved by selecting a relatively lightweight floodlamp bulb which has a very thin layer of a low-activating reflecting material inside the bulb. The glass of the bulb is a boro-silicate glass with low sodium content. In the following portion of this report the importance of having low sodium content is discussed in some detail. An ordi-

light bulb of the wattage needed contains enough sodium to become a significant contributor of residual gamma-rays to personnel in the room. The half-life of the activated sodium (Na-24) is 15 h, and this is intolerably long since it is important for personnel to be able to enter the room within minutes after termination of a given exposure.

SHIELDING ANALYSES FOR MODIFICATIONS OF THE HIGH FLUX ROOM

APPROXIMATIONS FOR CALCULATIONS OF ACTIVATION (NEUTRON) FLUXES

The calculations of levels of activation in the modified room, the liners, and the original walls, floor, and ceiling of the room have been normalized to measurements of neutron flux in the original JANUS.¹⁶ All calculations reported here are for one-dimensional, slab-array approximations to the actual geometrical configurations. Multigroup S_n theory was used; the neutron cross sections are those of the aforementioned Argonne National Laboratory version (set No. 233) of the Bondarenko 26-group set. The typical calculation determined the spatial and energy distribution of the neutron flux which would result from a specified source of fission-spectrum neutrons in the converter plate. These problems do not involve deep penetrations by neutrons, in contrast with calculations of bulk-shield effectiveness, and calculations of this type should be reasonably accurate within the geometrical model adopted.

In actuality, there are large gaps between the converter plates and the room, and between the new converter plate and the graphite reflector. Simply moving a converter plate away from the graphite reflector tends to reduce the fission rate in the plate, but then this plate is correspondingly closer to the High Flux Room. These two counter-balancing geometric effects are ignored in the slab-geometry model.

Another difference is that in the original JANUS there was a "tunnel" formed between the top faces of the shutter pedestals and the bottom faces of the raised shutters. The old converter plate was located at the far end of this tunnel. In the modified JANUS the new converter plate will be very much closer to the room. (See Figs. IV-35-1 and IV-35-2.) The tunnel, however, was more than 3 ft high, and the influence of the tunnel on the thermal neutron flux in the unmodified room is believed to have been small.

The slab-geometry model does not account fully for the albedo effects of the room, and there is a predicted reduction in height of the modified room. The neutron albedo of the lead is greater than that of concrete. Of course, the more recent one-dimensional

calculations do include certain neutron albedo effects but the height of the modified room is roughly commensurate with the distance from the new converter plate to the opposite wall of the room and probably the net albedo of the room is increased. In this respect, the model of one-dimensional geometry is nonconservative in the comparison of neutron flux levels in the modified JANUS with flux levels in the unmodified JANUS, when personnel are working in the room with the reactor at power.

In the calculations of neutron-activation levels in materials near the room surface of the lead liner, and inside the room, a conservative approach was to assume a maximum animal-content of the room, for this moderating matter greatly increased the calculated level of the thermal neutron flux in the room. The calculated relative levels of thermal neutron flux are 56 (1000 mice), and 28 (500 mice), where the reference level, normalized to unity, corresponds to the case where there is no moderating material inside the room. These are the relative values of the thermal neutron flux at the center of the zone simulating the cages of animals.

To determine activation levels in the original concrete walls of the room, comparison calculations were made without moderator in the room, for then the thermal neutron flux in the concrete was highest.

These source-to-flux calculations indicated that, for the same *effective* source rates in the converter plates (including the effects of gaps, which the slab-geometry calculations ignore), the thermal neutron flux in the modified High Flux Room, with a maximum anticipated animal population, was very nearly equal to twice the calculated thermal neutron flux in the unmodified room with no animals present. This comparison does not include the net effects of the higher albedo of the lead liners of the modified room, which is believed to increase the relative flux. This result emphasizes the importance of minimizing the use of unshielded highly activating materials within the High Flux Room when the reactor is operating at high power and the radiation-research experiment requires a large animal population. The calculated amplification of the thermal neutron flux in the room when animals are present is related principally to the high effectiveness of the wall liners in reducing the thermal neutron flux in the room in the experimentally perhaps less interesting case where very few animals are being irradiated.

THE RELATIVE FISSION RATES IN THE TWO CONVERTER PLATES

The original converter plate included an alloy of highly enriched uranium (U-235) with aluminum.

A total of 30 elemental strips of the alloy were used to make up the plate; each strip was $\frac{1}{2}$ -in.-thick and contained approximately 320 g of uranium. The new converter plate contains a total of 33.7 kg of highly enriched uranium in the form of 0.021-in.-thick foils which are individually jacketed in stainless steel envelopes.

The fission rate per unit lateral surface area of the new converter plate is calculated to be appreciably lower, at given reactor power, than the fission rate in the old converter plate in the previous location. In part this reduction is due to the positioning of the new plate at a greater distance from the graphite reflector, and, with respect to the irradiation neutron dose rate to animals in the room, it is counterbalanced by now having the converter plate correspondingly closer to the experiment zone. These counteracting effects of geometry were discussed earlier in this report.

There are two other principal influences on the net fission rate in the new foil, the one partially counterbalancing the other. These two influences are: the interposition of a 6-in.-thick (or, possibly, 7-in.-thick) lead wall between the new plate and the graphite reflector; and the increased grayness of the new plate to thermal and near-thermal neutrons. It is calculated that the lead reduces the fission rate by two-thirds, by capture of thermal neutrons entering the lead from the external graphite reflector. The optical thickness of the new plate for thermal neutrons, $\bar{\Sigma}_{a_{th}} \times \text{thickness}$, is three times as large as the value for the previous converter plate. It is calculated that this increase in grayness contributes a factor of increased effectiveness of the foil of between 50 and 100%.

Therefore, in the calculations of relative thermal neutron flux in the unmodified and modified JANUS High Flux Room, it would appear to be conservative to assume that the effective source rates in the two converter plates are equal. Except for additional (3-D) effects of a higher albedo for neutron scattering from the lined walls of the modified room, the best present estimate is that, for one-dimensional calculations, the effective source rate in the new plate is between one-half and two-thirds of the rate in the former plate. The actual maximum fission rate per unit lateral surface area in the new converter plate is calculated to be less than one-third of the rate in the old plate.

ACTIVATION LEVELS IN THE BOUNDARIES OF THE HIGH FLUX ROOM

In the above, the basis was laid for the calculation of activation levels in the boundaries of the modified

High Flux Room. In summary, at full reactor power the effective source rate in the new converter plate is at least half that of the old plate. An additional factor must be applied to account for the difference in neutron albedos in the modified and unmodified High Flux Rooms. It has not been feasible to perform these calculations of multiple reflection and slowing down of neutrons in view of the very large geometric dimensions involved. The best present estimate is that the additional effects of multiple scattering from the ceiling, the floor, and the far walls of the room are to multiply the relative thermal neutron flux in the modified room by a factor greater than unity but less than two. Equivalently, the assumption is made that, for the one-dimensional slab-geometry comparison calculations, the effective fission-neutron source rates in the two converter plates are equal.

The normalization to absolute levels of activating flux in the modified room was based on measured spatial and energy distributions of activity of standard 1-mil-thick gold foils in the unmodified room. At each of three locations in that room, two gold foils were activated: one foil with a thin aluminum cover; and the second foil with a 20-mil-thick cadmium cover. At each location the activity of the cadmium-covered foil was slightly less than half of the total activity (cadmium ratio ~ 1.8). The measurements were made without animals or other masses of moderating matter in the room. From these measurements it is inferred that in the earlier JANUS configuration the maximum level of thermal neutron flux at the room surface of the wall opposite the converter plate was approximately 1×10^8 n-cm/cm³-sec. This value was attained also at the center of the room; at the center of the ceiling the thermal neutron flux was half as great. The normalizations of calculated thermal neutron flux were to a value of 1×10^8 n-cm/cm³-sec at the room walls of the earlier JANUS, at full reactor power and without moderator inside the room.

As noted earlier, in calculating the relative activation of the original concrete walls behind the liners of lead and borated hardboard, it was assumed that there was no moderator within the room. This tends to overestimate the equilibrium activity in the concrete. Accounting for the many factors of differences between the original JANUS and the modified JANUS, the best present estimate is that, for given reactor power, the peak thermal neutron flux in the original concrete walls is reduced to less than one-third of its value by the modifications. In this estimate the differences in neutron albedos are included.

The principal contributors of gamma-ray activity in the original walls and structures include alumi-

, sodium, and manganese. The decay gamma-rays from the aluminum and manganese have energies less than 2 MeV; 2.8-MeV gamma-rays are emitted by Na-24. The composite effective gamma-ray attenuation factor of the 4-in.-thick lead liner is calculated to be at least 200 for the decay gamma-rays from the concrete. Therefore, in the modified JANUS and at given reactor power, the absolute contribution of gamma-ray activity in the concrete walls to the personnel dose rate in the High Flux Room is calculated to be less than 1/500th of the contribution in the original JANUS.

To determine equilibrium gamma-ray activities of materials exposed at or near room surfaces of the lead liners, and materials exposed within the room, a thermal neutron flux level of 2×10^8 n-cm/cm²-sec is assumed. This value is obtained as twice the thermal neutron flux in the original JANUS room, at full reactor power. The juxtaposition of factors which have led to this estimate has been described above. The model is that there is a maximum animal loading in the room of 1000 mice in containers. The estimated total masses of hydrogen, carbon, and oxygen in this loading are: 6.4 kg H; 26.0 kg C; and 19.5 kg O.⁽¹⁷⁾ It is assumed that these animals are distributed over a volume with a lateral area of 4 m² and a thickness of 0.2 m.⁽¹⁷⁾ In the one-dimensional calculations, these atom densities of hydrogen, carbon, and oxygen in the animal zone were assumed to apply throughout the room zone.

SPECIAL PROBLEMS OF ACTIVATIONS OF MATERIALS WITHIN THE HIGH FLUX ROOM OR AT ITS BOUNDARY SURFACES

Moderation of neutrons by the animals under irradiation might be a significant amplifier of gamma-ray activity due to impurities in the lead wall-liners. Reference 8 suggests that there might be such impurities. The lead used for liners of the room originate from lead bricks accumulated from various sources at Argonne National Laboratory. Activation measurements on some lead bricks, indicate distributions of impurity levels.¹⁸ For the liners of walls and ceilings, bricks were melted and lead blocks of the desired shapes and sizes were cast. The bulk of the lead floor liner is made up of lead bricks, and thin sheets of lead will be laid on top of them to form a smoother surface. The general level of activity of the lead is expected to be low enough that this should not be an important absolute contributor to the total biological dose rate to personnel working in the room. It is intended that if there should be some annoyingly "hot" local regions of lead, this lead

TABLE IV-35-I. ESTIMATED DOSE RATES FROM EQUILIBRIUM ACTIVITIES OF ELEMENTS IN PROPOSED LIGHTS FOR THE MODIFIED JANUS HIGH FLUX ROOM^a

Element	Active Isotope	Half-Life	Dose Rate, mrem/h/g ^b		
			Distance from source, in. (assuming point-source geometry)		
			2	6	12
Sodium	Na-24	15 h	72.0	8.0	2.0
Aluminum	Al-28	2.3 min	7.2	0.8	0.2
Argon	A-41	1.8 h	14.	1.6	0.4
Copper	Cu-64	12.8 h	6.2	0.7	0.2
	Cu-66	5.2 min	0.6	6×10^{-2}	2×10^{-2}
Molybdenum	Mo-99	67 h	0.1	1×10^{-2}	2×10^{-3}
	Mo-101	14.6 min	0.16	2×10^{-2}	4×10^{-3}
Silver	Ag-108	2.3 min	1.0	0.1	2×10^{-2}
Tin	Sn-123	40 min	8×10^{-3}	8×10^{-4}	2×10^{-4}
	Sn-125	9.5 min	1×10^{-3}	1×10^{-4}	2×10^{-5}
Tungsten	W-185	1.7 min	8.0	0.8	0.2
	W-187	24 h	2.0	0.2	6×10^{-2}

^a In these calculations, the equilibrium gamma-ray dose rate is calculated per gram of the element. The source is considered to be an equivalent point source. Since the light source is above the exposed person, the net gamma-ray dose rate would be obtained as a weighted integral over space.

^b A thermal neutron flux of 2×10^8 n-cm/cm²-sec is assumed.

would be replaced or coated with a thermal-neutron absorber, e.g., gadolinium.

The light bulbs and light fixtures of the modified room were calculated to be significant potential sources of equilibrium gamma-ray activity for people in the room. Approximating each lighting fixture as a point source of radiation, Table IV-35-I summarizes the gamma-ray dose rates, as a function of effective distance to the human receptor, for each material that might be in the bulb or fixture. By far the most important contributor is sodium, from the glass of the light bulb, if an ordinary light bulb is used. As noted earlier, lightweight, borosilicate-glass flood-lamp bulbs are available with only a few grams of sodium per bulb. These bulbs will be used. Moreover, there is a useful reduction in the average activating flux in the glass because of the boron present. A lesser problem which remains to be resolved is the receptacle material. Porcelain receptacles typically contain significant masses of aluminum, from the constituent clay and feldspar. It is believed that a commercially available plastic receptacle would become unacceptably hot (thermally) because of the restricted circulation of air in the region of the inset receptacle.

A comparison has been made of the relative activation of structures of magnesium and of aluminum,

e.g., positioning-structures for animals, in the High Flux Room. The purpose of this comparison was to investigate the possible advantages of using magnesium for such structures. Two important disadvantages of magnesium are hazards of combustion and its corrosion behavior under handling and under sterilization procedures. These might be overriding considerations against the use of magnesium. The activation advantage of magnesium is very great, however. For thermal-neutron activation, 100% of aluminum (Al-27) but only 11% of magnesium (Mg-26) have significant cross sections. For equal volumes of material, there are ~ 13 times as many thermally activating atoms of aluminum as of magnesium. Taking the microscopic activation cross sections and the gamma-ray energies into account, in the same thermal neutron flux field the gamma-ray dose rates from equal volumes of structural materials would be ~ 170 to 200 times as high from aluminum as from magnesium.

TABLE IV-35-II. MULTIGROUP NEUTRON FLUX SPECTRA IN THE MIDDLE OF THE ANIMAL ZONE IN THE MODIFIED HIGH FLUX ROOM^a

Neutron Group Number	Neutron Energy Range	Calculated Relative Neutron Flux		
		No Animals	500 Mice	1000 Mice
1	6.5-10.5 MeV	0.0063	0.0057	0.0053
2	4.0-6.5 MeV	0.042	0.037	0.033
3	2.5-4.0 MeV	0.167	0.138	0.117
4	1.4-2.5 MeV	0.558	0.418	0.332
5	0.8-1.4 MeV	0.757	0.506	0.378
6	0.4-0.8 MeV	1.196	0.676	0.476
7	0.2-0.4 MeV	0.882	0.495	0.349
8	0.1-0.2 MeV	0.517	0.345	0.252
9	46.5-100. keV	0.376	0.291	0.217
10	21.5-46.5 keV	0.226	0.234	0.179
11	10.0-21.5 keV	0.149	0.201	0.157
12	4.7-10.0 keV	0.112	0.184	0.148
13	2.2-4.7 keV	0.096	0.173	0.142
14	1.0-2.2 keV	0.085	0.164	0.136
15	0.47-1.0 keV	0.076	0.155	0.132
16	0.22-0.47 keV	0.067	0.147	0.128
17	0.10-0.22 keV	0.061	0.139	0.125
18	47.-100. eV	0.054	0.131	0.120
19	22.-47. eV	0.048	0.123	0.116
20	10.-22. eV	0.043	0.116	0.112
21	4.7-10. eV	0.041	0.109	0.108
22	2.2-4.7 eV	0.040	0.104	0.105
23	1.0-2.2 eV	0.034	0.097	0.101
24	0.47-1.0 eV	0.025	0.089	0.096
25	0.22-0.47 eV	0.015	0.080	0.091
26 (Thermal)	0.025 eV (average)	0.021	0.580	1.157

^a These calculated fluxes are normalized to a fission-spectrum-neutron source of one neutron per second per cm² of lateral surface area (one surface) of the converter plate.

At high neutron energies (above 1 MeV), aluminum activates under (n,γ) , (n,α) , and (n,p) reactions, and magnesium activates under (n,γ) and (n,p) reactions. It has been calculated that even for epi-1 MeV neutrons aluminum is a much worse activator than magnesium. A crucial point is that the $\text{Mg}^{24}(n,p)\text{Na}^{24}$ reaction has a nominal threshold energy of 5 MeV. There are very few neutrons above 5 MeV in the room. By contrast, the nominal threshold energy for the $\text{Al}^{27}(n,p)\text{Mg}^{27}$ reaction is only 1.9 MeV.

Table IV-35-II lists calculated relative multigroup neutron fluxes in the middle of the animal zone in the High Flux Room of the modified JANUS. These are results of one-dimensional slab-geometry calculations of the neutron flux due to a unit fission-spectrum-neutron production rate (one neutron per second per cm² of lateral surface area) in the converter plate. The source neutrons are assumed to be emitted isotropically. The additional neutron-albedo effects of the end walls, ceiling, and floor are not included; it is not known how the detailed relative and absolute values of the multigroup fluxes will be affected by these reflections.

SPECIAL PROBLEMS OF ACTIVATIONS OF MATERIALS WITHIN THE CRAWL SPACE OF THE HIGH FLUX ROOM

Boro-bauxite concrete will be used as the moderator liner of the ceiling, above a 4-in.-thick liner of lead. These two liners will constitute the principal materials of the new ceiling, and a crawl space will be left between the two ceilings. Since the boro-bauxite concrete will have a smaller hydrogen atom density ($\sim 2.8 \times 10^{22}$ atoms H/cm³), and since there is enough space for a thicker moderator zone of the new ceiling, a thickness of 8 in. of the boro-bauxite concrete will be poured.

In the vicinity of each lighting fixture the new ceiling structure must be modified. The local structure has been designed to include extra lead so as to avoid paths of gamma-ray shine from the original ceiling into the High Flux Room. Also, above each fixture area there will be provision for an additional moderator block of 4 in. of borated hardboard, to avoid additional hot spots of activation of materials in the crawl space and in the original concrete ceiling.

The motivation for these special design features is that there will be sizeable masses of activating structures and devices in the crawl space, including such materials as steel and aluminum. For the radiation-protection of personnel in the High Flux Room and for reasonable access to the crawl space, it is important to keep the activation levels down.

Modelling the ceiling as receiving the same incident neutron currents from the converter plate as the wall opposite the converter plate, one-dimensional slab-geometry calculations indicate that the peak thermal neutron flux in the crawl space is less than 1/30th of the peak thermal neutron flux in the concrete walls of the unmodified JANUS, at given reactor power. Even applying a factor of two to account for additional neutron-albedo effects within the modified room, the thermal neutron flux in the crawl space is calculated to be less than 1×10^7 . Additionally, there will be flux reductions there because of neutron self-shielding and over-all flux depression effects of the materials in the crawl space. The contribution of the gamma-ray activity in the crawl space and in the original ceiling to the biological dose rate to personnel in the High Flux Room is considered to be small enough that no additional measures need be taken for their protection from this activity. It might prove to be advantageous to apply a neutron-absorbing coating to some of the materials in the crawl space, to reduce gamma-ray dose rates to personnel entering the crawl space. Such a coating could be applied later rather easily, and it is not planned to do this as part of the reference modification.

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IV-36. Reactor Physics Analyses for Modifications of JANUS

D. H. SHAFTMAN

INTRODUCTION

In Paper IV-35, shielding design and analysis for the modifications of the JANUS facility are reviewed. The facility modifications are in progress. The reader is referred to that paper for descriptions of the modifications as given in the text and in Figs. IV-35-1 and IV-35-2. The present report summarizes some of the reactor physics studies in support of the modifications.

Included here are: reactivity effects of the new converter plate; neutron-albedo effects of the lead walls and the lead liners of the original concrete walls; a char-

acterization of spectrum hardness at higher neutron energies; and the relative levels of thermal neutron flux in the modified JANUS.

Most of the computer calculations were performed by L. K. Volodka.

CALCULATIONAL MODELS

All calculations were performed with a discrete-ordinates transport-theory code (SNARG 1-D)⁽¹⁾ for one-dimensional geometries. Typically the S_4 approximation was used, but there were S_6 confirmatory computations.

With the exception of calculations of reactivity effects, all of the computations are of the source-to-flux type. A unit fission-spectrum-neutron source is supplied, corresponding to fissions in the converter plate, and the resulting unmultiplied neutron flux is determined as a function of neutron energy and of spatial location.

An Argonne National Laboratory version (Set No. 233) of the Bondarenko multigroup cross sections was used, with 26 groups of neutron energies covering the range from 10.5 MeV down to thermal energies.^{2,3} The approximation of linearly anisotropic scattering of neutrons by hydrogen atoms was opted; for all other nuclides, isotropic scattering is postulated. Table IV-36-I lists the energy-group and lethargy-interval structure of the Bondarenko set.² Group No. 26 is referred to as the thermal energy group.

The SNARG-1D computer program permits calculations for three regular one-dimensional geometries: slab; sphere; and cylinder. The one-dimensional models were chosen for two reasons: (1) for most of these comparison-type calculations, a 1-D model was judged to be acceptably accurate; and (2) because of the very large physical distances involved (e.g., dimensions of the High Flux Room), the requirements of the numbers of mesh points for meaningful two-dimensional computa-

tions were beyond the machine capability for tested S_n -type computer programs then available at Argonne National Laboratory.

To determine the relative fission rate in the new converter plate, and to investigate reactivity effects of this plate, the model of sphere geometry was chosen. In the neighborhood of the new converter plate, 1-D modelling by slab geometry seemed to be most appropriate for the source-to-flux calculations. One inadequacy of this slab-geometry model is that spatial gaps have no influence on the neutron flux in other regions. In actuality, there are effects of geometrical curvature. Also, unless special measures are taken, there actually is a large spatial variation of fission rate across the lateral dimensions of the converter plate. The reason is that the reactor core is approximately a right-circular cylinder consisting of 19 fuel assemblies. The active core height is 66 cm and the core radius is 23 cm. By contrast, the new converter plate has little curvature, and the lateral dimensions of the plate are much larger: height of the active plate, ~ 100 cm; width, ~ 350 cm. The design of the modified JANUS, however, provides for special neutron-attenuation plates to permit various degrees of spatial flattening of the fission rate across the converter plate. Therefore the implicit assumption of a spatially uniform fission rate, in the 1-D model, is not inconsistent with the actual design.

Most of the calculations have been performed to supply comparative values of neutron fluxes in the modified and unmodified facilities. Such comparisons inherently compensate for some of the inadequacies of the geometric idealizations, but not for all of them. One example is the inability of the 1-D comparison calculations to account fully for the relative effectiveness of the modified High Flux Room in multiple reflections of neutrons within the room.

Figure IV-36-1 gives schematic representations of the geometric models of the source-to-flux calculations. In these analyses, only a portion of the graphite reflector is included; the graphite at greater distances from the room has no significant effect on fluxes in the room, in this model.

REACTIVITY EFFECTS

In the "in" position, the new converter plate is calculated to contribute less than 1¢ to the net reactivity of the system. Omitting the reactivity contribution of fissions in the reactor core, $k_{eff} = 0.25$ was calculated for the restricted system consisting of the converter plate and its neighboring regions when in the "in" position. These analyses were needed to delineate certain aspects of safety of the system.

The new converter plate is made up of 34 steel-jacketed foils of fully enriched (93%) uranium metal

TABLE IV-36-I. THE PARTITIONING OF THE NEUTRON FLUX SPECTRUM IN THE BONDARENKO MODEL

Number of Neutron Energy Group	Energy Range	Size of Lethargy Interval	Lethargy Range
1	6.5-10.5 MeV	0.48	0-0.48
2	4.0-6.5 MeV	0.48	0.48-0.96
3	2.5-4.0 MeV	0.48	0.96-1.44
4	1.4-2.5 MeV	0.57	1.44-2.01
5	0.8-1.4 MeV	0.57	2.01-2.58
6	0.4-0.8 MeV	0.69	2.58-3.27
7	0.2-0.4 MeV	0.69	3.27-3.96
8	0.1-0.2 MeV	0.69	3.96-4.65
9	46.5-100. keV	0.77	4.65-5.42
10	21.5-46.5 keV	0.77	5.42-6.19
11	10.0-21.5 keV	0.77	6.19-6.96
12	4.7-10.0 keV	0.77	6.96-7.73
13	2.2-4.7 keV	0.77	7.73-8.50
14	1.0-2.2 keV	0.77	8.50-9.27
15	0.47-1.0 keV	0.77	9.27-10.04
16	0.22-0.47 keV	0.77	10.04-10.81
17	0.10-0.22 keV	0.77	10.81-11.58
18	47.-100. eV	0.77	11.58-12.35
19	22.-47. eV	0.77	12.35-13.12
20	10.-22. eV	0.77	13.12-13.89
21	4.7-10. eV	0.77	13.89-14.66
22	2.2-4.7 eV	0.77	14.66-15.43
23	1.0-2.2 eV	0.77	15.43-16.20
24	0.47-1.0 eV	0.77	16.20-16.97
25	0.22-0.47 eV	0.77	16.97-17.74
26	(Thermal) 0.025 eV	—	—

which are placed side-by-side within a strong framework. Each uranium foil is 0.053 cm (0.021 in.) thick and contains ~ 1 kg uranium. There are published experimental data which indicate that 1/2-in.-thick slabs of fully enriched uranium metal are grossly subcritical even when reflected by hydrogenous matter.⁴ Therefore, when it is in its shield pit, the effective multiplication factor of the converter plate is very small. The shield pit has steel-lined zones of concrete at the lateral surfaces.

NEUTRONICS EFFECTS OF LEAD WALLS ON BOTH SIDES OF THE CONVERTER PLATE

In the previous JANUS there was a 6-in.-thick wall of lead bricks between the High Flux Room and the converter plate. This lead wall served as a radiation shield, against neutrons and gamma-rays. In the modified facility, with the shutters raised for exposure of animals in the room, there will be a total thickness of 8 in. of lead between the room and the graphite reflector, but there will be a lead-wall thickness of 2 in. on the room side of the new converter plate. This thickness of 8 in. is the reference value; the design provides for a variation between 7 and 10 in., with a possible thickness in the range of 1 to 3 in. for the reference 2-in.-thick wall. These walls are formed by abutting 1-in.-thick sheets of lead.

The increased thickness of lead, and the use of an additional 2 in. of lead in the shutters and shutter pedestals, is an essential feature of the new bulk radial biological shield, principally for shielding against gamma-rays. The re-allocation of lead, from the room side of the plate to the reactor-core side, is intended to minimize the degradation of the fission-neutron energies in their traversal of that lead, with the purpose of improving somewhat the usefulness of the facility for the planned biological research. As discussed later in this report, there are calculational indications that this change might result in a degree of hardening of the flux spectrum of high-energy neutrons within the High Flux Room.

The positioning of a 6-in.-thick lead wall on the reactor-core side of the new converter plate is calculated to reduce the fission rate to one-third of the rate in the absence of that lead, assuming that this is pure natural lead. The reduction is due largely to capture by the lead of thermal neutrons entering the lead region from the external graphite reflector. On the other hand, from other analyses it has been inferred that this lead wall acts to enhance the neutron flux in the room in Bondar-ko groups Nos. 1 to 10 (above 21.5 keV) by reflecting neutrons toward the room. The estimated enhancement is an average of 50% in that energy range, and the calculated neutron flux spectrum is essentially

A UNMODIFIED JANUS

B MODIFIED JANUS

FIG. IV-36-1. Schematic One-Dimensional Representations of the Multi-Region Geometric Configurations of JANUS Near the High Flux Room. *ANL Neg. No. 113-3066*.

unchanged in shape. Thinning the lead wall on the room side of the converter plate enhances the neutron flux spectrum at high energies. And, finally, the new converter plate is optically thicker to thermal and near-thermal neutrons entering it, and this tends to compensate for the reduction in incident flux by the 6-in.-thick lead wall. (See Paper IV-35 for additional details.)

REMARKS ON CHARACTERIZATION OF HARDNESS OF THE NEUTRON FLUX SPECTRUM IN THE HIGH FLUX ROOM

The primary purpose of the JANUS modifications, including the lining of walls, floor, and ceiling with layers of moderator and of lead, is to reduce the biological dose rates to the personnel engaged in reactor operation and in performance of the experiments. In the earlier JANUS, both neutron radiation and gamma-ray radiation contributed importantly. (For details, see Paper IV-35.) An important auxiliary benefit was sought, namely hardening of the neutron flux spectrum to increase the utility of JANUS for the planned research.

Characterization of relative hardness of the neutron flux spectrum by a single parameter appears to be highly approximate and controversial. There are a number of single-parameter characterizations that one might choose for special purposes. Perhaps the value of the flux-weighted energy would be of interest to some readers. Others might prefer a biological-dose averag-

TABLE IV-36-II. APPROXIMATE VALUES OF KERMA PER UNIT FLUENCE⁶

Number of Neutron Energy Group	Range of Neutron Energies	Group-Averaged Kerma per Unit Fluence in Units of ergs/gram/unit fluence
1	6.5-10.5 MeV	5.0×10^{-7}
2	4.0-6.5 MeV	4.3×10^{-7}
3	2.5-4.0 MeV	3.9×10^{-7}
4	1.4-2.5 MeV	3.2×10^{-7}
5	0.8-1.4 MeV	2.6×10^{-7}
6	0.4-0.8 MeV	1.7×10^{-7}
7	0.2-0.4 MeV	1.1×10^{-7}
8	0.1-0.2 MeV	$8. \times 10^{-8}$
9	46.5-100. keV	$4. \times 10^{-8}$
10	21.5-46.5 keV	$2. \times 10^{-8}$

ing of neutron energy. For such characterizations it must also be decided whether to average over a restricted energy range or over the entire neutron energy range, down to and perhaps including thermal energies.

A simple solution to this controversy would be to avoid it and present the neutron flux spectra for the various cases of interest so that the reader might perform his own preferred evaluation. In the following sections of this report, there are such detailed tabulations of spectra. But, in addition, a particular characterization of relative hardness has been selected, and numerical values are presented for some of the more interesting cases studied. The selected characterization parameter is an "equivalent mean neutron energy" which is defined as the energy for which the kerma⁵ per unit fluence is equal to the neutron-flux averaged kerma per unit fluence of the given spectrum. Using the group-averaged values of kerma per unit fluence given in Table IV-36-II,⁽⁶⁾ and interpolating with a monotonically increasing function of neutron energy, this definition is mathematically acceptable because there is then a one-to-one bicontinuous correspondence between neutron energy and kerma per unit fluence over the restricted energy range chosen. Because, for all cases studied, the fractional contribution to the total kerma from neutron energies of the low-keV range and below is small, it has been decided to restrict the averaging to Bondarenko groups Nos. 1 to 10, covering the neutron energy range from 10.5 MeV down to 21.5 keV (see Table IV-36-I).

Values of an auxiliary characterizing parameter are supplied as well, namely, the fraction of the total neutron flux which is in groups Nos. 1 to 10, i.e., above 21.5 keV.

Given a neutron radiation field, the "kerma" for a given material exposed to this field for a given period of time is defined to be the ratio $\Delta E_k / \Delta m$, "where ΔE_k is

the sum of the initial kinetic energies of all the charged particles liberated by indirectly ionizing particles in a volume element of the specified material, Δm is the mass of the matter in that volume element."⁵ This mass, Δm , is "so small that its introduction does not appreciably disturb the radiation field."⁵ If the neutrons are monoenergetic, the kerma is determined for neutrons of that energy. It is convenient here to define also the "kerma per unit fluence of neutrons of energy-group j " as the total kerma resulting from exposure of the material to an idealized neutron radiation field which is of zero intensity outside the energy range of group j , and which has the spectrum of group- j in the energy range of group j , for unit total fluence (nvt).

Let $\bar{K}(E)$ be the kerma per unit fluence for neutrons of energy E , and let $\bar{K}_j$ be the kerma per unit fluence for group- j neutrons. Then the equivalent mean neutron energy $\hat{E}$ is defined here to be that value of energy for which

$$\bar{K}(\hat{E}) = \frac{\sum_{j=1}^{10} \phi_j \bar{K}_j}{\sum_{j=1}^{10} \phi_j}.$$

Table IV-36-II lists group-averaged values of kerma per unit fluence, $\bar{K}_j$.⁽⁶⁾ These approximate values were used in evaluating $\hat{E}$ for the cases reported.

ANALYSIS OF MULTIPLE REFLECTIONS WITHIN THE ROOM, IN THE APPROXIMATION OF SLAB GEOMETRY

Three cases have been calculated to investigate the spectrum effects of multiple reflections of neutrons within the room. As noted earlier, these 1-D computations do not account for additional reflections of neutrons from the far walls, the ceiling, and the floor of the room. Nor do they account for variation in flux across the room. Nevertheless, some hints of spectral effects may be discerned from a study of results of these simplified calculations.

The three cases are:

Case A: the reference configuration of the modified JANUS facility, including a liner of a 4-in. thickness of a borated hardboard and an outer liner of 4 in. of lead. (See Paper IV-35 for descriptive details. Also see Fig. IV-36-1 of the present paper.)

Case B: same as Case A, but without the wall liners.

Case C: same as Case A, but with no reflection from the opposite wall, i.e., with a vacuum-boundary condition at the outside surface of the 2-in.-thick lead wall on the room-side of the new converter plate.

Values of the neutron fluxes per unit lethargy "

Cases A and B are included in Table IV-36-III.

Case C, let it suffice to state that $\hat{E} \approx 0.9$ MeV, and that the total kerma for groups Nos. 1 to 10 is calcu-

TABLE IV-36-III. NEUTRON FLUX PER UNIT LETHARGY IN THE HIGH FLUX ROOM

Number of Neutron Energy Group	Upper Bound of Energy	Upper Bound of Lethargy in the Group	Flux Per Unit Lethargy				
			Case A	Case B	Case D	Case E	Case F
			Modified JANUS; No Animals	Modified JANUS; No Animals; No Wall Liners	Modified JANUS; 1000 Mice in the Room	Unmodified JANUS; No Animals	Unmodified JANUS; 1000 Mice
1	10.5 MeV	0.48	0.013	0.012	0.011	0.0018	0.0016
2	6.5 MeV	0.96	0.087	0.079	0.069	0.013	0.011
3	4.0 MeV	1.44	0.348	0.279	0.244	0.059	0.045
4	2.5 MeV	2.01	0.979	0.688	0.582	0.224	0.151
5	1.4 MeV	2.58	1.328	0.788	0.663	0.355	0.213
6	0.8 MeV	3.27	1.733	0.971	0.690	0.537	0.271
7	0.4 MeV	3.96	1.278	0.696	0.506	0.413	0.210
8	0.2 MeV	4.65	0.749	0.510	0.365	0.306	0.159
9	100. keV	5.42	0.488	0.355	0.282	0.229	0.127
10	46.5 keV	6.19	0.294	0.273	0.233	0.185	0.109
11	21.5 keV	6.96	0.194	0.230	0.204	0.175	0.102
12	10.0 keV	7.73	0.145	0.205	0.192	0.156	0.098
13	4.7 keV	8.50	0.125	0.187	0.184	0.148	0.097
14	2.2 keV	9.27	0.110	0.183	0.177	0.144	0.095
15	1.0 keV	10.04	0.099	0.171	0.171	0.134	0.094
16	0.47 keV	10.81	0.087	0.162	0.166	0.126	0.093
17	0.22 keV	11.58	0.079	0.153	0.162	0.119	0.091
18	100. eV	12.35	0.070	0.144	0.156	0.111	0.089
19	47. eV	13.12	0.062	0.136	0.151	0.103	0.087
20	22. eV	13.89	0.056	0.130	0.145	0.096	0.084
21	10. eV	14.66	0.053	0.125	0.140	0.090	0.082
22	4.7 eV	15.43	0.052	0.122	0.136	0.084	0.079
23	2.2 eV	16.20	0.044	0.116	0.131	0.078	0.077
24	1.0 eV	16.97	0.032	0.107	0.125	0.073	0.074
25	0.47 eV	17.74	0.019	0.096	0.118	0.068	0.071

Note: (a) The relative values of the total thermal neutron flux (group No. 26) are (in units of n-cm/cm²-sec)

Case A: 0.021; Case B: 0.796; Case D: 1.157; Case E: 0.581; Case F: 1.027.

(b) The lower energy limit of group No. 25 is 0.22 eV.

(c) The fluxes all are normalized to unit source of fission neutrons per unit lateral surface area of the converter plate.

lated to be 44 % of the corresponding kerma for Case A. Below group No. 10, the neutron flux drops off very rapidly; the flux in group No. 10 is only $\frac{1}{5}$ th of the group-10 flux for Case A. Referring to Table IV-36-III, in the absence of wall liners (Case B), grossly speaking the neutron flux spectrum is softened in the sense that a much larger fraction of the total neutron flux is in groups Nos. 11 to 26 (46 % versus 17 % for Case A.) However, $\bar{E}$ (Case A) $\approx$ 0.6 MeV $\approx$ $\bar{E}$ (Case B).

The values of the total kerma, for the range of groups Nos. 1 to 10, supply additional data. When wall liners are present (Case A), the calculated total kerma is $\sim$ 60 % higher than when the original concrete walls are unlined (Case B).

PRELIMINARY EVALUATION OF RELATIVE SPECTRUM HARDNESS IN THE MODIFIED HIGH FLUX ROOM

Table IV-36-III includes also tabulations of the neutron fluxes per unit lethargy for three other cases:

Case D: modified JANUS, with the maximum anticipated population of animals (1000 mice) in the room. (See Paper IV-35 for additional details.)

Case E: unmodified JANUS; no animals in the room.

Case F: unmodified JANUS; 1000 mice in the room.

Comparing the neutron flux spectra in the modified room (Case A) and the unmodified room (Case E), when there are no animals in the room:

1. $\bar{E}$ (Case A) $\approx$ 0.6 MeV $>$ $\bar{E}$ (Case E) $\approx$ 0.5 MeV.

2. For Case A, 83 % of the total neutron flux is calculated to be in groups Nos. 1 to 10; the corresponding value for Case E is 45 %.

Comparing the modified room (Case D) and the unmodified room (Case F), when there are 1000 mice in the room:

1. $\bar{E}$ (Case D) $\approx$ 0.6 MeV $>$ $\bar{E}$ (Case F) $\approx$ 0.5 MeV.

2. For Case D, 44 % of the total neutron flux is in groups Nos. 1 to 10; the value for Case F is 30 %.

On the preliminary basis of these idealized, one-

dimensional calculations, it is estimated that there will be some hardening of the neutron flux spectrum in the High Flux Room as a result of the modifications of JANUS. This hardening is a composite of the effects of: reducing the thickness of the lead wall on the room-side of the converter plate; and lining the room with borated-moderator and lead.

THE THERMAL NEUTRON FLUX IN THE CONCRETE AND IN THE HIGH FLUX ROOM

It has been indicated above that the modifications to JANUS are calculated to result in a substantial increase in the high-energy neutron flux in the room at given fission rate per unit area in the converter plates. In part this is due to the higher neutron albedo of the lead liners of the room walls. And yet it is calculated also that the peak thermal neutron flux in the concrete wall will be only one-sixth of the calculated level for the unmodified facility, at given fission rate in the converter plates. The moderator-liner, borated hardboard, is effective in reducing the level of the thermal neutron flux, both by slowing down source neutrons and capturing them and by preventing an enhancement due to multiple scatterings within the room.

When there are essentially no animals or other moderating matter in the room, the calculated thermal neutron flux in the modified room is only one-thirtieth of the level calculated for the unmodified JANUS. The

introduction of significant numbers of animals is calculated to raise the thermal neutron flux in the room. In the middle of the maximum anticipated population of animals (1000 mice and their containers), the calculated thermal neutron flux is more than 50 times as high as it is in the complete absence of moderator there. Halving the animal population, by halving the thickness of the animal zone, results in the halving of the thermal neutron flux in the room. Bear in mind, however, that these are all idealized 1-D calculations, and that the results all are normalized to the same (unit) spatially uniform source of fission neutrons.

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Section V

Reactor Computation Methods and Theory

To a large degree, the quality of reactor design and performance depends upon the quality of the conceptual models and their portrayal in accurate mathematical representation, and upon the quality and efficiency of computational methods. A priori, the continuous development and refinement of theory and computational methods leads to the design of more dependable, safer, and better performing reactors. It is with this intent that the studies described in this section were undertaken.

V-1. Status and Developments in the Argonne Reactor Computation (ARC) System

B. J. TOPPEL and L. JUST*

The ARC system is oriented toward two types of users. The first makes use of the system to effectively provide conventional stand alone code capability for production calculations. To this user, the modular aspects of the system, except for the resulting benefits of standardized user input formats, are largely irrelevant. The second type of user, the methods developer, takes full advantage of the flexibility afforded by the modular approach and utilizes earlier modules as building blocks to facilitate development of a new computational capability.

The computational modules, standard path modules, and system routines of the ARC system have progressed to the point that substantial production capability now exists in the system. In addition, the run-time linking capability of the system together with the standardized module interfaces and existing computational modules are providing an environment which is encouraging and which is facilitating new algorithm developments.

As more capability has become available in the system, the corresponding user involvement has resulted in expected and desirable user-programmer interactions and in substantial improvements and revisions in various modules and standard paths in the system.

Owing to the modular aspect of the ARC system together with the various internal options available within individual modules, the number of calculational options available to the user is very large.

A substantial testing program for the ARC modules and standard paths has consisted of option verification on at least one problem of significance to the LMFBR program. This verification consists of comparison with results obtained from older codes or by means of hand computational checking. In spite of this initial verification, subsequent user involvement will inevitably point out further coding errors. No code is ever completed in the ordinary sense of the word. In this regard it is interesting to note that the testing of ARC modules against "established" codes of long standing has revealed two heretofore unknown errors in the earlier coding.

Table V-1-I indicates for various of the established $\square$ modules the percentage of all realistic options

*Applied Mathematics Division, Argonne National Laboratory.

which have been implemented and the percentage which have been tested, in the above sense. The modules and their names given here differ from earlier listings.^{1,2} The current list reflects the influence of experience on our earlier planning. For example, a number of the Input and Output Edit modules originally planned for are now missing, the function of these modules having been absorbed in the corresponding Computational modules. Another notable change is the absorption of the criticality search algorithm within the various neutronics modules. This change was dictated by execution time efficiency, and is typical of the sort of dynamic change that is expected in the algorithm repertoire of the ARC system as operational experience accrues.

In addition to the modules listed in Table V-1-I, extensive coding has been carried on in the area of accident analysis and safety evaluation which will be directly applicable to ARC system modules. Incorporation of this coding into the ARC system will be initiated during the summer of 1969.

The production use of the ARC system is achieved through use of various standard path modules which invoke other modules (or standard paths) within the system. Table V-1-II lists some of the currently fully operational standard paths available to the user and shows the modules invoked. The listing illustrates graphically the elimination of programming duplication in a modular approach. One should note also that the system provides the enormously powerful capability of a standard path or a module invoking another standard path, as well as individual computational modules. For example, although not listed in Table V-1-II, a standard path has been written to generate trial functions for the synthesis algorithm using 1D-diffusion theory calculations. This standard path invokes STP001 and hence effectively invokes in one call all of the modules pertinent to the 1D-diffusion capability. The ARC system routines provide this capability with the same ease with which one may call a Fortran subroutine.

The current version of the ARC system operates under the PCP (Primary Control Program) mode on the IBM/360. This corresponds to a single job, batch operation. The system has been modified to accommodate the forthcoming MVT (Multiprogramming with

TABLE V-1-I. STATUS OF ARC SYSTEM MODULES

Module	Percent of Options	
	Imple- mented	Tested
Epithermal Cross-Section Specifications CSI001	100	90
Resonance Cross-Sections CSC001	100	90
Nonresonant Cross-Sections and Fundamental Mode Spectrum CSC002	100	70
Broad-Group Cross Sections and Fundamental Mode Spectrum CSC003	100	70
Cross-Section Homogenization Specifications NUI001	90	75
Code Independent Specifications NUI002	90	90
1D-Transport Specifications NUI005	100	90
Run-Time Microscopic Cross-Section Modifications NUI006	100	100
Cross-Section Homogenization NUC001	90	75
1D-Diffusion External Source and <i>K</i> Calculation NUC002	95	95
1D-Transport NUC003	100	85
1D-Diffusion Search NUC004	95	95
2D-Diffusion External Source and <i>K</i> Calculation NUC005	95	50
2D-Diffusion Search NUC006	95	50
1D-Neutronics Output Manipulation NUE001	100	80
1D-Neutron Inventory AJC001	100	95
2D-Neutron Inventory AJC002	85	85
1D-Diffusion Perturbation AJC003	95	90
2D-Diffusion Perturbation AJC004	95	90
Spatial Synthesis AJC005	100	95
Geometry Dataset Writer AJC006	100	100
Microscopic Cross-Section Group Collapse AJC007	100	90
Macroscopic Cross-Section Group Collapse AJC008	100	90
Fuel Cycle Specifications FCI001	20	0
Fuel Cycle FCC001	95	65

a Variable number of Tasks) mode of operation. Although most ARC system production jobs require so much core that the computer cannot be shared, nevertheless the MVT environment will provide several benefits to users of the system. Under MVT, only a single Input/Output module need be resident for the job. Under PCP, each module which was linked to, required its own edition of the I/O routines with consequent loss of core space. Also, with the advent of MVT, IBM software support of the the two million byte Large Core Storage (LCS) area will become available. Utilization of the LCS as an extension of fast core will make possible less use of input-output with consequent improved run time efficiencies.

From the production users' point of view, MVT provides a very attractive capability through the use of symbolic parameters in catalogued procedures. With

the current system, every production run requires a mission of a large number of job control language (JCL) cards which constitute the standard path procedure. The use of catalogued procedures will permit reduction in the number of run time cards required by an order of magnitude and provide a much more user-oriented language for interacting with the IBM operating system than does the current JCL.

In a system of such complexity, organization of the source cards and of the programs on disks is very important. Until recently two system libraries have been kept, one containing each module in load module form and the other containing object modules of any segments the programmer wishes to save. The source cards were kept by the individual programmers. Recent attempts to get listings of the whole system showed the need for more system libraries to enable requests from those wanting to use the ARC system to be fulfilled.

The current system organization includes four libraries.

1. A source library (SOURCE) made up of card images of source segments of reasonable size (of the order of 500 cards).
2. A partially linked library (SEGLIB) of the same source segments after they have gone through the linkage editor. This library is probably not useful to a requestor.
3. A library (OVERLAY) where organization information for the modules is kept in card image form.

TABLE V-1-II. MODULES INVOKED BY VARIOUS STANDARD PATHS

Standard Path Module	Modules Invoked
1D-Diffusion STP001	NUI001, NUI002, NUI006, NUC001, NUC002, NUC004, NUE001, AJC001, AJC003
1D-Transport STP002	NUI001, NUI002, NUI005, NUI006, NUC001, NUC003, NUE001, AJC001
2D-Diffusion STP003	NUI001, NUI002, NUI006, NUC001, NUC005, NUC006, AJC002, AJC004
Burnup Diffusion STP004	NUI001, NUI002, NUC001, NUC005, NUC006, AJC007, FCI001, FCC001
Multigroup Cross-Sections STP005	CSI001, CSC001, CSC002, CSC003
1D-Diffusion Perturbation STP006	NUI001, NUI002, NUI006, NUC001, NUC002, NUC004, AJC003, AJC006
2D-Diffusion Perturbation STP007	NUI001, NUI002, NUI006, NUC001, NUC005, NUC006, AJC006

4. A load module library (MODLIB) containing the executable load modules made up from the material in SEGLIB and OVERLAY.

A listing of the SOURCE library and the OVERLAY library and of the directory of the MODLIB library gives a complete picture of what the ARC system contains. Copies of the SOURCE library, the OVERLAY library and the MODLIB library with the appropriate

data should enable another installation to run ARC jobs with the possibility of making changes.

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V-2. Capabilities of MC² in the Argonne Reactor Computation (ARC) System

C. G. STENBERG and A. L. RAGO*

The MC² capability in the ARC system on the IBM System/360 consists of one input module, CSI001 (INPUT), and three computational modules, CSC001 (RESXEC), CSC002 (SCSFM), and CSC003 (BCSFM). These four modules are available for production use and are being extensively tested for reliability.

The current testing of these modules has consisted of comparing the results of the MC² capability in the ARC system on the IBM System/360 to those of the operational MC² cross-section code¹ on the CDC-3600 over the full range of available user input options. This testing has revealed a minor coding error in subroutine ALRAGØ⁽¹⁾ and in subroutine AVER1⁽¹⁾ of module CSC003. The errors have been corrected. To date, 75% of the testing is complete. These four ARC modules provide the same calculational capability as the version of MC² on the CDC-3600. A discussion of improvements and additions to the MC² capability follows.

The standard path capability for multigroup cross-sections, STP005, that presently exists in the ARC system with regard to MC² variants is given in Table V-2-I and the main flow diagram for the standard path is given in Fig. V-2-1. STP005 has been tested on a production basis over all user input options. The MC² variants are invoked through information supplied by the user input data and also by the proper JØB CØN-TRØL LANGUAGE (JCL) cards supplied to the IBM operating system.

Modifications of the modules CSI001 and CSC003, and of the user input data module A.XSSPC⁽²⁾ now permit the user to add isotopes to a new or existing mi-

croscopic group cross section data module XS.ISØ.⁽²⁾ The versatility of this modification allows one to include additional isotopes at a later time. When isotopes are added to an existing data set, a consistency check is made in module CSI001 to insure that the number of broad groups is compatible with the existing data set. The broad group energy boundaries of the existing data set and the isotopes to be added to this set must agree to within 0.01%.

A recent concept being explored is that of recompiling all source subroutines to produce new object modules of each of the ARC modules which provide the MC² capability. This was done using a new FØRTRAN(H) ØPT = 2 (optimization) compiler. Test problems involving a variety of input options are now being run using the new compiler and the results are compared with those of previous reliable runs. Currently the various subroutines which represent the MC² capability have been compiled using the presently available FØRTRAN(H) ØPT = 0, 1, 2 and FØRTRAN(G) compilers. It has been necessary to invoke all these options for the various subroutines in order to get correctly compiled modules. The significance of successfully recompiling all the subroutines of the associated modules of MC² with a single compiler and optimization will allow a reorganization of the subroutines within each module and provide the user, who may wish to examine the source program, with a clearer representation of the structure for each module.

The maximum code dimensions available for the calculations of an MC² problem in ARC are dictated by the available space in the main core of the computer. At present, the maximum dimensions of pertinent quantities are 20 materials, 4 isotopes for each material, 70 broad energy groups, 70 fine energy groups,

* Applied Mathematics Division, Argonne National Laboratory.

FIG. V-2-1. Main Flow Diagram for Standard Path STP005. ANL Neg. No. 113-2692.

TABLE V-2-I. CAPABILITIES OF STANDARD PATH STP005

	Modules Invoked	Capabilities
Multigroup Cross Sections	CSI001, CSC001, CSC002, and CSC003	Complete MC ² problem
Resonance Cross Sections	CSI001 and CSC001	Calculates resolved and unresolved resonance cross sections only
Spectrum Average and Group Collapsing	CSI001, CSC002, and CSC003	Generates smooth cross sections and a fundamental mode spectrum, and collapses to a broad group set. Uses previously generated resonance cross sections
Group Collapse	CSI001 and CSC003	Generates broad group set using previously generated data and spectrum
Multigroup Cross Section Sets	CSI001, CSC001, CSC002, and CSC003	Invokes complete MC ² problem followed by repeated invokings of spectrum averaging and group collapsing. This uses one set of resonance data for several spectrum and broad group generations

2100 ultrafine energy groups, and 2000 resolved resonances. It is anticipated that the next effort in the refinement of MC² in ARC will be to variably dimension the arrays in modules CSI001, CSC001, CSC002, and CSC003 by the use of BPØINTR.⁽³⁾ With this refinement the user can utilize bulk memory to expand the code dimensions. Another option is to expand some of the code dimensions at the expense of other code di-

mensions by utilizing only the main core available in the computer.

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V-3. Solution Techniques Used in the Fast Reactor Fuel Cycle Code REBUS

J. HOOVER, D. A. MENELEY and P. M. WALKER*

This work describes the solution techniques used in the Reactor Burnup System (REBUS). The problem formulation and the code capabilities are presented in Ref. 1. REBUS is addressed to the problem of fast reactor fuel cycle calculations. The approach taken was to specifically include models of all physical procedures which are expected in Liquid Metal Fast Breeder Reactor (LMFBR) operation within defined limits. These are: one reactor (as opposed to an interacting power system); fuel contained in discrete lumps; fully defined external conditions—that is, complete specification of bred fuel disposal and refabrication make-up; and fixed fuel management definition (no cycle optimization).

Fuel cycle calculations may be divided into two

* Applied Mathematics Division, Argonne National Laboratory.

classes, conceptual design analysis and final design analysis. Conceptual design analysis involves a large number of survey and parametric studies in which the reactor is represented simply, the purpose being to obtain the general performance and economic characteristics of the system. The external cycle (reprocessing and refabrication) must also be modeled. Final design analysis requires a much more detailed spatial model of the reactor for determining local power and burnup distributions and accurate mass balances. From the point of view of programming, the difference between these two types of problems is mainly in the input data logic. For survey work the input data must be simple and easily varied from case to case. Input for detailed analysis cannot be simple, so emphasis is placed on arranging the data in a clear and logical manner.

The present version of REBUS reflects major empha-

sis on the first type of problem defined above, that is, the survey problem. The main omission from REBUS is the time dictionary necessary for running a non-equilibrium problem. This dictionary would permit real-time calculation of a series of burn-shuffle-refuel steps, with changing reactor operating conditions, control rod banking, burned fuel disposal, and fresh fuel specifications. These calculations fall into the category of detailed design analysis. REBUS has been designed so that the model can properly treat these problems when the extra control modules are written.

The REBUS system solves for the equilibrium (infinite time) operating conditions of a recycle system under fixed conditions. Iterations are performed on the burn step time such that a required burnup is achieved at fuel discharge; iterations are performed on the fresh fuel enrichment such that the uncontrolled reactor is just critical at the specified fraction α of the burn step time; in addition iterations are performed on the control poison density so that the reactor is critical at each time node specified within the burn interval. Other iterations are required to achieve equilibrium conditions as enrichment, burn time, and control density are varied to satisfy the given constraints. The user has control of some of the iterations through convergence criteria so that different sorts of problems can be solved. Flux calculations are performed at each of the time nodes. The fluxes are normalized to a specified total power. Region- and time-averaged group fluxes are used to form burn matrix elements in each sub-interval between time nodes. (A region may be as small as one spatial interval of the neutronics solution.)

The isotope chain matrix may contain β^- , β^+ , and α -decay terms as well as (n,r) , (n,p) , (n,α) , $(n,2n)$, and (n,f) reactions. Fractional yields of fission products may be fissioning-isotope dependent. Isomeric states may also be considered. Isotope decay chains are computed both in the reactor and throughout the external cycle.

Within the limitation of solid fuel elements, all physically possible fuel management schemes are permitted. Typical movement patterns such as out-in shuffling and fractional batch are included as special cases.

An indexing system was developed which permits location of each discrete bundle of fuel in space throughout the whole period during which it is in the reactor. A fuel bundle might be loaded into the reactor at a particular position and irradiated for some time in a flux which is determined by the interaction of all the fuel in the reactor as well as by control motions. After this initial "burn", the bundle may or may not be repositioned in space; in any case if any of the fuel in the reactor is moved, charged, or discharged, the

irradiation rate will undergo a discontinuous change the so-called "shuffling step". After several burn-shuffle sequences the bundle is discharged from the system.

The spatial movement of the fuel bundle is constrained by requirements of volume preservation. Whenever it is shuffled (repositioned) in space, another lump of equal volume must be removed from its position and inserted in another location, or discharged, and a third must be moved to occupy the space vacated by the original fuel lump. This linked series of element motions is the basis of the indexing system. The indices which are used to identify and locate the fuel bundle are "path", "stage", and "region".

The first index is given the name "path". A path consists of all those fuel bundles in the reactor at a particular time which are linked by a single fuel management sequence. The individual fuel bundles, the "stages", are identified by the second index. This second index is 1 for fuel which has just been charged into the reactor, 2 for a bundle which has been shuffled once, and so on up to the maximum stage index, after which the fuel bundle is discharged. The third index, the "region", is used to locate each stage in space. A region is the volume over which the average fluxes are computed in obtaining the fuel burnup; in current neutronics models the region boundaries are restricted to rather simple surfaces. A region in a cylindrical model may be an annular ring containing a number of fuel bundles of different composition and burnup history. Thus we must allow several different stages to be assigned to a single region, with the constraint that the sum of the volumes of all stages in a region must equal the volume of that region.

Following reactor discharge, fractions of a discharged batch may be assigned to one or more reprocessing plants. Provisions are included for sale both before and after reprocessing. Each of the reprocessing plants has associated with it a reprocessing time, and a recovery factor for each isotope, which determines the amount of waste and losses. The output of each reprocessing plant is divided into two categories, which simplifies the refabrication specifications. One category is class 1 (high reactivity output) and the other is class 2 (low reactivity output).

EQUATION SET

FUEL DISCHARGE AND REPROCESSING

Figure V-3-1 is a schematic of material movements in the external cycle. Consider L discharges (originally L charges or "paths") which are to be cooled, then sent directly, sent to J reprocessing plants, or held for recharging. Define E , a block diagonal matrix of order

FIG. V-3-1. Schematic Diagram of External Cycle. ANL Neg. No. 112-7982 Rev. 2.

ML , where M is the number of active isotopes (i.e., those undergoing transmutation reactions) in the problem. All elements in the l th M -dimensioned diagonal block are equal to the fraction of charge l which is to be sold without reprocessing. Next define the destination matrix D , with MJ rows and ML columns. Each M -dimensioned submatrix D_{jl} is diagonal with constant elements equal to the fraction of the unsold remainder of charge l which is to be sent to reprocessing plant j . Waste and reprocessing losses are accounted for by the recovery matrix R , block diagonal of order MJ . Each M -dimensional block is diagonal with elements equal to the fraction of each isotope which is recovered by reprocessing plant j . The reprocessing plant output vector nV_0 is then given by

$$nV_0 = RD(I - E)V\hat{n} \quad (1)$$

where

$I \equiv$ identity matrix, order ML .

$V \equiv$ volume matrix, block diagonal of order ML . Each element of a diagonal $M \times M$ block is equal to the volume of the discharge l .

$\hat{n} \equiv$ atom density vector (ML) for the discharge at the end of the burn step.

FABRICATION

Refabrication is done by reference to reactor charge specifications which are defined by the user through a

priority scheme. The priority scheme is an absolute order of preference for use of each reprocessing plant output and external feed in makeup of each charge. The object of the priority scheme is to arrive at two delivery matrices, designated Q and Q_F , which represent the fraction of the atoms of each isotope in each reprocessing plant output and each external feed which are to be used in fabrication of each fuel charge, respectively. The elements are computed from the given enrichment, the required volume, the given full atom densities of each charge batch, and the contents of the make-up storage.

FUEL CHARGE ATOM DENSITIES

Grouping the various parts of the above computations, we can define the atom density vector n_i of the fuel charge in terms of the discharge and external feed vectors. First define S and P as

$$\begin{aligned} S &\equiv V^{-1}QRD(I - E)V \\ P &\equiv V^{-1}Q_FV_F. \end{aligned} \quad (2)$$

Then

$$n_i = S\hat{n} + Pn_F. \quad (3)$$

The vector $\hat{n}$ is related to the previous fuel charge vector n_{i-1} through the burn matrix B , representing all transmutations within the core (see Reactor Burnup Equations). Then

$$n_i = SBn_{i-1} + Pn_P. \quad (4)$$

The equilibrium solution is obtained when

$$n_i = n_{i-1} = n.$$

SOLUTION METHODS

In this section, we describe the solution techniques developed to obtain the solution of the equilibrium problem. Acceleration techniques, used to reduce the computation time, are presented in a later section.

The purpose of an equilibrium calculation is to find the density for each charge and an operating time for any burn step such that the charges produced in the external cycle are the same for every burn step. The solution is subject to the following constraints:

(1) Let L denote the total number of fuel charges under consideration and e_ℓ , $1 \leq \ell \leq L$, the enrichment associated with the charge density for each fuel charge ℓ . The solution is sought by varying the enrichment vector $(e_1, e_2, \dots, e_L)$ along a specified straight line.

(2) The burnup of the system at the end of each burn step assumes a specified value.

(3) The unpoisoned k -effective of the reactor at a given fraction of the burn step time takes on a specified value.

REACTOR BURNUP EQUATIONS

Consider any region r of the reactor and let n_r denote an atom density vector associated with the region r . The dimension of n_r is M , where M is the number of isotopes being considered. The following set of equations is then taken to govern the reactor as a function of time:

$$\frac{dn_r(t)}{dt} = A_r[\phi(t)]n_r(t), \quad (5)$$

where r ranges over the regions of the reactor, and n_r over the atom density vectors in the reactor. The column vector $n_r(t)$ is

$$n_r(t) = [n_{r,1}(t), n_{r,2}(t), \dots, n_{r,M}(t)],$$

where M is the number of active isotopes (those undergoing transmutations). The matrices $A_r[\phi(t)]$ have the form

$$A_r[\phi(t)] = \Lambda_r + \hat{A}_r[\phi(t)], \quad (6)$$

where Λ_r is a constant matrix arising from radioactive decay, and the elements $a_{ij}(t)$ of $\hat{A}$ have the form

$$a_{ij}(t) = \sum_{g=1}^G \sigma_{ij}^g \phi_{gr}(t). \quad (7)$$

Here ϕ_{gr} is the average flux in group g over the region r and σ_{ij}^g denotes the microscopic cross-section for each isotope and type of reaction. In fact, the element

$(A_r)_{i,j}$ is the formation rate of isotope j from isotope i . The diagonal terms $(A_r)_{i,i}$ are the total loss rate of isotope i due to all reactions. The coupling between the isotope densities and the flux is governed by a fixed power, static neutronics calculation with or without criticality being maintained, at the option of the user. The neutronics calculation will be noted symbolically by

$$F[\phi(t); n(t); t] = 0. \quad (8)$$

An approximate solution to the coupled system of Eqs. (5) and (8) over the burn step time interval $[0, T]$ is achieved in the following manner. The interval is divided into P subintervals (where P is an input quantity) of equal length by the points $t = ph$, $0 \leq p \leq P$, with $h = T/P$. Using the slowly varying character of the matrix $A_r[\phi(t)]$, the following approximation is made over each subinterval $[t_{p-1}, t_p]$:

$$A_r[\phi(u)] \approx \frac{1}{2} \{A_r[\phi(t_{p-1})] + A_r[\phi(t_p)]\}, \quad (9)$$

$$t_{p-1} \leq u \leq t_p.$$

With this approximation the coupled system takes the form

$$\frac{dn_r}{dt} = A_{rp} n_r, \quad t_{p-1} \leq t \leq t_p \quad (10)$$

$$F[\phi(t); n(t); t] = 0,$$

where r ranges over all regions of the reactor and $p = 1, 2, \dots, P$. Here A_{rp} denotes the right hand side of Eq. (9).

This system is solved iteratively in the following manner. Assuming that $n_r(t_{p-1})$ has been found for each r , we define the sequence $n_r^{(q)}(t)$ in the following manner:

$$B_r^{(q)}[t_p, n_r^{(q)}(t_p)] = \exp[hA_{rp}^{(q)}] \quad (11)$$

$$n_r^{(q)}(t_p) = B_r^{(q)}[t_p, n_r^{(q)}(t_p)]n_r(t_{p-1}) \quad (12)$$

$$F[\phi^{(q+1)}(t_p); n_r^{(q)}(t_p); t_p] = 0 \quad (13)$$

and

$$A_{rp}^{(q+1)} = \frac{1}{2} \{A_r[\phi(t_{p-1})] + A_r[\phi^{(q+1)}(t_p)]\}, \quad (14)$$

for

$$q = 0, 1, 2, 3, \dots,$$

For $p = 1$, $A_{r1}^{(0)}$ is taken to be $A_{r0}^{(0)}$. However, for $p > 1$, $A_{rp}^{(0)}$ is the result of the following extrapolation from the preceding intervals

$$A_{rp}^{(0)} = 2A_{r(p-1)} - A_{r(p-2)}.$$

The convergence of the above sequence (the inner iterations of the density vectors) is based on convergence of the column vector $n_r(t)$. If $q > 0$, convergence of the column vector $n_r(t)$ is tested after the

lulation in Eq. (12). The convergence criterion for $n_r(t)$ is

$$e_r = \sum_{i=1}^M |n_{r,i}^{(q)} - n_{r,i}^{(q-1)}| / n_{r,i}^{(q)}$$

$$e_r \leq \text{epsn},$$

where *epsn* is an input quantity. The inner iterations on the density vectors are converged if the above criterion is satisfied for every region r .

If the convergence criterion is not satisfied, the calculation proceeds as indicated above; however, an extrapolation of the column vector $n_r(t)$ is done every third iteration ($q = 2, 5, 8, \dots$) before the neutronics calculation indicated in Eq. (13). This extrapolation is based on the rate of convergence of the diagonal terms of the $B_r^{(q)}$ matrix in Eq. (11). The elements of the vector $n_r^{(q)}(t)$ upon extrapolation become

$$n_{r,i}^{(q)} = (n_{r,i}^{(q-1)} - n_{r,i}^{(q-2)} R_{r,i}) / (1 - R_{r,i}),$$

where

$$R_{r,i} = (B_{r,i}^{(q)} - B_{r,i}^{(q-1)}) / (B_{r,i}^{(q-1)} - B_{r,i}^{(q-2)}),$$

and $B_{r,i}^{(q)}$ is the i th diagonal element of the array $B_r^{(q)}$ in Eq. (11). The extrapolation is done only every third iteration to insure extrapolating to $n_r^{(q)}$ with B_r matrices associated with $n_r^{(q-1)}$ and $n_r^{(q-2)}$.

The evaluation of the exponential of the A matrix is performed using a technique proposed by B. Duane.² The method is based on the following observations. The rate of convergence of an expansion for a matrix $f(C) = a_0 I + a_1 C + a_2 C^2 + \dots$ is determined by the spectral radius of C . If C is renormalized so that it has a smaller spectral radius, the above series will converge faster. If $f(C)$ can be obtained in some simple manner from the approximation to $f(C')$, (C' is the renormalized matrix), then the problem of slow convergence is solved. In the present case $C = tA$ and we choose an integer N such that $W = 2^{-N} tA$ has all its eigenvalues less than one in modulus. This can be achieved by choosing N as the least integer such that

$$2^{-N} t \text{Max}_{1 \leq j \leq m} \sum_{i=1}^m |a_{ij}| < 1, \text{ i.e., } 2^{-N} t \|A\|_{\infty} < 1.$$

Now for any matrix C we define

$$f(C) = \sum_{\ell=0}^{\infty} \frac{C^{\ell}}{(\ell+1)!}.$$

Thus $f(C) = (e^C - 1) C^{-1}$ if C^{-1} exists. In addition, we define

$$g(C) = \frac{1}{2}(e^C + 1) = \frac{1}{2}[2 + Cf(C)].$$

ius

$$e^{tA} = I + tAf(tA), \quad (15)$$

and $f(2C) = f(C)g(C)$ which will provide the means for recovering $f(tA)$ from $f(W)$. Now, having defined W , we approximate $f(W)$ by

$$f(W) \approx \sum_{\ell=0}^L \frac{W^{\ell}}{(\ell+1)!},$$

where L is determined by

$$\frac{\|W\|_{\infty}^{\ell}}{(\ell+1)!} < \epsilon,$$

and ϵ is a convergence criterion, which in REBUS is 10^{-8} . Then using the functional relation for f and g , we have

$$\begin{aligned} f(2W) &= f(W)g(W); g(2W) \\ &= \frac{1}{2}[2I + 2Wf(2W)] \\ &\vdots \\ f(2^{k+1}W) &= f(2^k W)g(2^k W); g(2^{k+1}W) \\ &= \frac{1}{2}[2I + 2^{k+1}Wf(2^{k+1}W)]. \end{aligned} \quad (16)$$

Hence for $k = N - 1$, we have an approximation for $f(2^N W) = f(tA)$, which in turn yields an approximation for e^{tA} .

If a distribution of atom densities $n_{ik}(0)$ is given at $t = 0$ for each stage k of path ℓ , then the above numerical procedure yields an approximation to the atom density distribution at any later time. The resulting density $n_{ik}(t)$ can be represented in the form

$$n_{ik}(t) = B_r[t, n(0)]n_{ik}(0), \quad (17)$$

where $r = r(\ell, k)$ is the region in which stage k of path ℓ resided. The column matrix n_{ik} is $n_{ik} = (n_{ik,1}, n_{ik,2}, \dots, n_{ik,M})$ where $n_{ik,i}$ is the density of isotope i in stage k of path ℓ . Note that through the coupling of the region-averaged flux, the matrix B_r depends on the total initial density distribution in the entire reactor. This dependence is indicated by the symbol $n(0)$.

CYCLIC MODE

Consider the repetitive loading, burning, and discharging of the reactor governed by the equations described in the preceding section. Let the operating time for each burn step be fixed, and let the charge density for each path which is loaded in at the beginning of each burn step be fixed. Consider any path ℓ , and suppose that during its K_{ℓ} stages the path resides in regions $r_1, r_2, \dots, r_H$ such that it is in r_1 for the first k_1 stages, r_2 for the next k_2 stages, and so on. Thus, $k_1 + k_2 + \dots + k_H = K_{\ell}$; moreover when the path reaches the K_{ℓ} th stage in region r_H it is discharged. Let $n_{\ell,1}$ denote the charge density for stage 1 of path ℓ which is loaded in at the beginning of every burn step. Starting

at the beginning of some burn step, we follow this charge through the K_ℓ burn steps until it is discharged. Note that the number of stages coincide with the number of burn steps for which the charge resides in the reactor. After k_1 burn steps in region r_1 , the density is given by

$$B_{r_1}^{(k_1)} \dots B_{r_1}^{(2)} B_{r_1}^{(1)} n_{\ell,1}.$$

Note that since the B matrices depend on the total density distribution in the reactor at the beginning of each burn step, a different B matrix will be generated at each burn step. This dependence is indicated by the superscript index. Having completed k_1 stages in region r_1 , the charge then moves to region r_2 for the next k_2 burn steps and then on to the next region until at the end of the K_ℓ th burn step after it was introduced, it is discharged. This discharge density is given by

$$B_{r_H}^{(K_\ell)} \dots B_{r_2}^{(k_1+k_2)} \dots B_{r_2}^{(k_1+1)} B_{r_1}^{(k_1)} \dots B_{r_1}^{(1)} n_{\ell,1}.$$

Now if we had followed the same charge density $n_{\ell,1}$ which was introduced one time step later than the preceding one, then its value upon discharge would be given by

$$B_{r_H}^{(K_\ell+1)} \dots B_{r_2}^{(k_1+k_2+1)} \dots B_{r_2}^{(k_1+2)} \dots B_{r_1}^{(2)} n_{\ell,1}$$

Thus we see that since the B matrices differ from one time step to the next, the discharge densities will vary with each time step. In the limit when the conditions in the reactor are the same at the beginning of every burn step, the B matrices would not depend on the time steps. Thus there would exist matrices

$$B_{r_1}, B_{r_2}, \dots, B_{r_H}$$

such that the discharge density for path ℓ after every time step would be given by

$$B_{r_H}^{k_H} \dots B_{r_2}^{k_2} B_{r_1}^{k_1} n_{\ell,1}, \quad (18)$$

where superscript k_i specifies the k_i th power of B_{r_i} . This limiting state will be referred to as the cyclic mode.

The cyclic mode is achieved iteratively by repeatedly loading a fixed charge for each path and maintaining a fixed burn step time. This procedure is accelerated in two ways. First the iterative procedure is modified by ignoring the dependence of the B matrices on the flux. For each path ℓ , let a charge density $n_{\ell,1}$ be loaded, and assume that the densities of the remaining stages are arbitrary. Then it is clear that it would take $K = \max(K_\ell)$ time steps just to clear the reactor of the arbitrary densities. By ignoring the flux dependence we can accomplish this in one time step. Thus if

$$B_{r_1}^{(1)}, B_{r_2}^{(1)}, \dots, B_{r_H}^{(1)}$$

were B matrices for path ℓ resulting from the first loading when all but the first stage densities are arbitrary,

then for the next burn step time we define the densities for each stage of path ℓ in the following manner:

$$n_{\ell,1}, B_{r_1}^{(1)} n_{\ell,1}, \dots, [B_{r_1}^{(1)}]^{k_1} n_{\ell,1}, B_{r_2}^{(1)} [B_{r_1}^{(1)}]^{k_1} n_{\ell,1}, \dots, [B_{r_H}^{(1)}]^{k_H-1} \dots [B_{r_1}^{(1)}]^{k_1} n_{\ell,1}. \quad (19)$$

With this distribution for each path, we burn for one burn step using the procedure in Eqs. (11)–(14), thereby generating a second set of matrices. This second set is then used in place of the first set, and the process (cyclic mode iteration) repeated until a converged set is obtained. The cyclic mode convergence is based on the stage densities at the end of cycle conditions. After q cyclic mode iterations ($q = 1, 2, 3, \dots$), the densities for each stage of path ℓ at the end of cycle are

$$\begin{aligned} & B_{r_1}^{(q)} n_{\ell,1}, [B_{r_1}^{(q)}] [B_{r_1}^{(q-1)}] n_{\ell,1}, \\ & \dots, [B_{r_1}^{(q)}] [B_{r_1}^{(q-1)}]^{k_1-1} n_{\ell,1}, [B_{r_2}^{(q)}] [B_{r_1}^{(q-1)}]^{k_1} n_{\ell,1}, \\ & \dots, [B_{r_2}^{(q)}] [B_{r_2}^{(q-1)}]^{k_2-1} [B_{r_1}^{(q-1)}]^{k_1} n_{\ell,1}, \\ & \dots, [B_{r_H}^{(q)}] [B_{r_H}^{(q-1)}]^{k_H-1} \dots [B_{r_1}^{(q-1)}]^{k_1} n_{\ell,1}. \end{aligned} \quad (20)$$

Using this definition of the stage densities, $n_{\ell k}(T)$, the error for $n_{\ell k}(T)$ is defined by

$$e_{\ell k} = \sum_{i=1}^M |n_{\ell k,i}^{(q)} - n_{\ell k,i}^{(q-1)}| / n_{\ell k,i}^{(q)},$$

where $n_{\ell k,i}^{(q)}$, the isotope densities in each stage, are the elements of column matrix $n_{\ell k}^{(q)}$:

$$n_{\ell k}^{(q)} = (n_{\ell k,1}^{(q)}, n_{\ell k,2}^{(q)}, \dots, n_{\ell k,M}^{(q)}).$$

For convergence, $e_{\ell k}$ must satisfy

$$e_{\ell k} \leq \text{epsc}$$

for each pair (ℓ, k) . epsc is an input quantity.

If the convergence criterion is not satisfied, the calculation proceeds as indicated above; however, an extrapolation of the stage densities, $n_{\ell k}(0)$, at the beginning of the cycle is done after every third cyclic mode iteration. This extrapolation is based on the rate of convergence of the diagonal terms of the B_r matrix. The extrapolated elements of the vector $n_{\ell k}^{(q)}(0)$, which is in region r , are

$$n_{\ell k,i}^{(q)} = (n_{\ell k,i}^{(q-1)} - n_{\ell k,i}^{(q-2)} R_{r,i}) / (1 - R_{r,i}),$$

where

$$R_{r,i} = (B_{r,i}^{(q-1)} - B_{r,i}^{(q-2)}) / (B_{r,i}^{(q-2)} - B_{r,i}^{(q-3)}),$$

and $B_{r,i}^{(q)}$ is the i th diagonal element of the array $B_r^{(q)}$. The column matrices $n_{\ell k}(0)$ are extrapolated before iterations ($q = 4, 7, 10, \dots$), and the extrapolated are used to obtain the cyclic mode instead of the stage density definition in Eq. (19).

NUP CONVERGENCE

After the reactor is in its cyclic mode, the burnup is calculated and a burn time search to obtain the desired burnup is initiated. In the burnup search, the reactor burnup equations are solved and the cyclic mode is recalculated until the burnup converges, with the exception that the neutronics solution in Eq. (13) is not done. Instead the previously calculated fluxes are used in the A matrices with each new time estimate to calculate the B matrices.

The isotopes and the paths in which the burnup is monitored are specified in the input. These are typically the heavy isotopes in the core. The burnup is the fractional loss of the specified isotopes due to fission in the interval charge-discharge. Using previously defined quantities, the burnup is

$$b(T) = \sum_{\ell} \sum_k \sum_i \sum_j (B_r)_{ij} n_{ik}(0) V_{\ell k} / \left(\sum_{\ell} \sum_k n_{ik}(0) V_{\ell k} \right), \quad (21)$$

where

j is summed over the fission products

i is summed over the specified isotopes

k is summed over the stages of path ℓ

ℓ is summed over the specified paths

$r \equiv$ region where stage k of path ℓ resides

$V_{\ell k} \equiv$ volume of stage k of path ℓ .

The burnup is converged when

$$|b(T) - b_d| \leq \text{epsg},$$

where b_d and epsg are input quantities.

The second burn time estimate is based on the following linear extrapolation of the burnup using the rate of fission destruction of the appropriate isotopes at time T . The definition of b_d is

$$b_d \equiv b(T) + \sum_{\ell} K R_{\ell k}(T) \Delta T / \left(\sum_{\ell} \sum_k n_{ik}(0) V_{\ell k} \right),$$

where

$$R_{\ell k}(T) = \sum_i \sum_j (A_r)_{ij} n_{\ell k, i}(T) V_{\ell k};$$

K is the last stage of path ℓ and the sums over i and j are defined above. Solving for the second time, we obtain

$$T = T + [b_d - b(T)] \left(\sum_{\ell} \sum_k n_{ik}(0) V_{\ell k} / \left(\sum_{\ell} K R_{\ell k}(T) \right) \right). \quad (22)$$

The successive burn time estimates are made by linear interpolation of the two previous burn times and the corresponding burnups. The linear interpolation is

$$T = T - \frac{(T - T')}{[b(T) - b(T')]} [b(T) - b_d]. \quad (23)$$

EXTERNAL CYCLE

We now consider the constraints on the problem as well as the interaction between the external and internal paths. First of all, the enrichments are constrained by the relation $e_{\ell}(x) = e_{\ell}(0) [1 - \delta_{\ell}(1 - x)]$ for $1 \leq \ell \leq L$ and $0 \leq \delta_{\ell} \leq 1$. Thus it is the enrichment parameter x which will be adjusted to achieve a solution. Consider next a charge distribution

$$n^{(0)}(x) = [n_1^{(0)}(x), \dots, n_L^{(0)}(x)] \quad (24)$$

having an enrichment parameter x associated with it. Let $t^{(0)}(x)$ denote the burn step time of the reactor such that when the core is in its cyclic mode, the burnup $b[t^{(0)}(x); x; n^{(0)}(x)]$ achieves its desired value b_d . For each path ℓ , set

$$B_{\ell}^{(0)} = [B_{rH}^{(0)}]^{k_H} \dots [B_{r1}^{(0)}]^{k_1}$$

and

$$B^{(0)} = \text{diag} [B_1^{(0)}, \dots, B_L^{(0)}]. \quad (25)$$

Note that the B matrices are those associated with the core in its cyclic mode and

$$B^{(0)} = B[t^{(0)}(x), x, n^{(0)}(x)].$$

Let the discharge density distribution $B^{(0)} n^{(0)}$ be passed through the external cycle, thereby generating a new charge distribution $n^{(1)}$ which is defined by

$$n^{(1)} = S[B^{(0)} n^{(0)}, x] B^{(0)} n^{(0)} + P[B^{(0)} n^{(0)}, x] n_F. \quad (26)$$

The matrices S and P were defined above, and as indicated depend on the discharge density and the enrichment. Let the charge distribution $n^{(1)}$ be used in place of $n^{(0)}$ thereby generating $t^{(1)}(x)$, $B^{(1)}$, and $n^{(2)}$. Repeating this process until the limiting state is reached, we would expect to obtain a charge distribution $n(x)$, a burn step time $t(x)$, and a B -matrix $B(x)$ such that

$$n(x) = S[B(x)n(x), x] B(x)n(x) + P[B(x)n(x), x] n_F. \quad (27)$$

When the system has achieved this state, it will be said to be in its equilibrium mode relative to a given enrichment. If the enrichment x is then adjusted so that the unpoisoned k_{eff} achieves its desired value k_d at a specified fraction α of the burn step time $t(x)$ when the system is in its equilibrium mode, then we have the solution to the equilibrium problem.

This is the basic approach used in solving the equilibrium problem. However, there are non-linearities present at every level of the problem; thus to adhere

too closely to the above procedure would involve a prodigious amount of calculation. For this reason the equilibrium mode is only approximated. Thus starting with a charge distribution $n^{(0)}$ and an associated enrichment x , the burn step time $t^{(0)}$ and the matrix $B^{(0)}$ are determined as before. The discharge density also passes through the external path generating $n^{(1)}$ as in Eq. (26). Now, however, we ignore the dependence of $t^{(0)}$ and $B^{(0)}$ on the initial charge distribution. Thus we pass $n^{(1)}$ through the core using $B^{(0)}$, thereby obtaining a discharge distribution $B^{(0)}n^{(1)}$. This is passed through the external path and the process repeated

$$x^{(q+1)} = x^{(q)} + \frac{F^{(q)} F^{(q-2)} \left[\frac{x^{(q)} - x^{(q-1)}}{F^{(q)} - F^{(q-1)}} \right] + F^{(q-2)} \left[\frac{x^{(q)} - x^{(q-2)}}{F^{(q)} - F^{(q-2)}} \right] [F^{(q-2)} - F^{(q-1)} - F^{(q)}]}{[F^{(q-1)} - F^{(q-2)}]}.$$

until a converged distribution $\hat{n}^{(0)}$ is obtained. Having produced $B^{(0)}$, $t^{(0)}$, and $\hat{n}^{(0)}$ we shall say that the system is in its approximate equilibrium mode relative to this enrichment. The convergence of these external cycle iterations is determined by the convergence of the charge densities $n_{\ell 1}(0)$. The error for $n_{\ell 1}(0)$ is

$$e_{\ell} = \sum_{i=1}^M |n_{\ell 1, i}^{(q)} - n_{\ell 1, i}^{(q-1)}| / n_{\ell 1, i}^{(q)}.$$

For convergence, the error for each path ℓ must satisfy

$$e_{\ell} \leq \text{epse},$$

where *epse* is an input number.

EQUILIBRIUM SOLUTION

With these concepts in mind, the solution to the equilibrium problem is obtained in the following manner. Using $B^{(0)}$, $t^{(0)}$, and $\hat{n}^{(0)}$ the unpoisoned value of k_{eff} , at the specified fraction α of the step time $t^{(0)}$, is calculated. Recall that for calculational purposes, the time interval $[0, t^{(0)}]$ was divided into P equal parts ($P \geq 1$). We restrict the possible specified values for α by insisting that $\alpha = p'/P$ for some p' , $1 \leq p' \leq P$. Since $B_r^{(0)} = B_r(t_p) \cdots B_r(t_1)$ for each region r , we can form

$$\bar{B}_r^{(0)} = B_r(t_{p'}) \cdots B_r(t_1).$$

These matrices are then used to approximate the isotope density distribution in the reactor at a time $\alpha t^{(0)}$ with the initial loading given by

$$\hat{n}_{\ell}^{(0)}, B_{r_1}^{(0)} \hat{n}_{\ell}^{(0)}, \dots, [B_{r_H}^{(0)}]^{k_H-1} \cdots [B_{r_1}^{(0)}]^{k_1} \hat{n}_{\ell}^{(0)}.$$

A neutronics calculation is then performed to obtain the value of the unpoisoned k_{eff} . The convergence of the unpoisoned k_{eff} is achieved if

$$|k_{eff} - k_d| < \text{epsf},$$

where k_d and *epsf* are input numbers.

Two enrichment parameters are input by the user as guesses. If the unpoisoned k_{eff} is not converged

with the second enrichment parameter, a linear enrichment interpolation, based on enrichment versus k_{eff} , is used to obtain a new enrichment parameter

$$x^{(2)} = x^{(1)} - \frac{[x^{(1)} - x^{(0)}]}{[F^{(1)} - F^{(0)}]} F^{(1)},$$

where

$$F^{(1)} = k_{eff}^{(1)} - k_d.$$

Successive enrichment estimates are based on a parabolic fit

If the enrichment obtained does not fulfill the criteria

$$\begin{aligned} x^{(q+1)} &> 0, \\ x^{(q)} - 10 |x^{(q)} - x^{(q-1)}| &< x^{(q+1)} \\ &< x^{(q)} + 10 |x^{(q)} - x^{(q-1)}|, \end{aligned}$$

the preceding linear interpolation is used.

At this point, a linear burn time interpolation, based on burn time versus enrichment, is done:

$$T = T^{(q)} + \frac{[T^{(q)} - T^{(q-1)}]}{[x^{(q)} - x^{(q-1)}]} [x^{(q+1)} - x^{(q)}]. \quad (28)$$

Using this enrichment parameter and the discharge density $\hat{n}^{(1)} = B^{(0)} \hat{n}^{(0)}$, we pass through the external cycle, generating a charge $n^{(1)}$ with enrichment $x^{(1)}$. Using $B^{(0)}$ and $n^{(1)}$, the beginning of cycle atom densities are approximated by employing the following distribution of stages for each path ℓ :

$$n_{\ell}^{(1)}, B_{r_1}^{(0)} n_{\ell}^{(1)}, \dots, [B_{r_H}^{(0)}]^{k_H-1} \cdots [B_{r_1}^{(0)}]^{k_1} n_{\ell}^{(1)}. \quad (29)$$

With this distribution, the above procedures are repeated giving rise to an unpoisoned k_{eff} corresponding to $x^{(1)}$. The process is repeated until the unpoisoned k_{eff} obtains the desired value.

ACCELERATION TECHNIQUES

The method presented in the preceding sections involves a prodigious amount of computation to solve the equilibrium problem. Various techniques have been included in REBUS to reduce the computation effort without decreasing the accuracy, that is, to accelerate the convergence of the equilibrium calculation. Thus in REBUS three levels of search procedures are sequentially invoked to solve the equilibrium problem. The first and second procedures are special cases of the solution techniques described in the preceding section.

In the preliminary search procedure, the burn time interval is divided into one subinterval. The iterations of the region densities in Eqs. (11)–(14) are

done and the cyclic mode is obtained with an enrichment parameter and burn step time specified in the input. The cyclic mode is then iterated until the change in k_{eff} over the burn step converges. The discharge density distribution is passed through the external cycle, thereby generating a new charge distribution with the second enrichment parameter specified in the input. Using this charge and the previously calculated B matrix the cyclic mode is again obtained. A linear search of the enrichment parameter is done until the desired multiplication constant is achieved. Control searches are not done in the neutronics calculation.

Specifically, the preliminary search is done in the following manner. For each path ℓ , a charge density of $n_{\ell,1}$ is loaded based on the user defined priority scheme. The arbitrary densities in the remaining stages of path ℓ are assumed to be equal to $n_{\ell,1}$. With this distribution for the density vector in region r , we do a neutronics calculation to obtain a set of B_r matrices:

$$\begin{aligned} F[\phi(0); n_r(0); 0] &= 0, \\ A_r^{(0)} &= A_r[\phi(0)], \\ B_r^{(0)} &= \exp[TA_r^{(0)}]. \end{aligned} \quad (30)$$

If

$$B_{r_1}^{(0)}, B_{r_2}^{(0)}, \dots, B_{r_H}^{(0)}$$

are the B matrices for path ℓ resulting from this first loading, then we define the densities for each stage of path ℓ by

$$\begin{aligned} n_{\ell,1}, B_{r_1}^{(0)} n_{\ell,1}, \dots, [B_{r_1}^{(0)}]^{k_1} n_{\ell,1}, B_{r_2}^{(0)} [B_{r_1}^{(0)}]^{k_1} n_{\ell,1}, \\ \dots, [B_{r_H}^{(0)}]^{k_H-1} \dots [B_{r_1}^{(0)}]^{k_1} n_{\ell,1}. \end{aligned} \quad (31)$$

With this distribution for each ℓ , we form $n_r(0)$. One burn step subinterval is now used to obtain the cyclic mode. We define the sequence $n_r(t_p)$ by

$$\begin{aligned} B_r[t_p, n(t_p)] &= \exp[hA_{rp}], \\ n_r(t_p) &= B_r[t_p; n(t_p)] n_r(t_p), \end{aligned} \quad (32)$$

$$F[\phi(t_p); n_r(t_p); t_p] = 0,$$

and

$$A_{rp} = \frac{1}{2} \{A_r[\phi(t_{p-1})] + A_r[\phi(t_p)]\},$$

where $0 \leq p \leq 1$.

This generates a second set of matrices

$$B_{r_1}^{(1)}, B_{r_2}^{(1)}, \dots, B_{r_H}^{(1)}.$$

This second set is used in place of the first set and the process (cyclic mode iteration) is repeated until convergence is achieved. Convergence is defined by

$$\begin{aligned} ||k^{(q)}(0) - k^{(q)}(T)| - |k^{(q-1)}(0) \\ - k^{(q-1)}(T)|| < epsf, \end{aligned}$$

where

$k^{(q)}(0) \equiv k_{eff}$ at the beginning of the cycle, t_0 , for the q th cyclic mode iteration

$k^{(q)}(T) \equiv k_{eff}$ at the end of cycle, t_1 , for the q th cyclic mode iteration

$epsf \equiv$ an input number.

The desired k_{eff} at the beginning of cycle is now estimated by assuming that k_{eff} varies linearly during the at-power phase of the reactor. If k_d is the desired k_{eff} at the specified fraction α of the burn step time, then the desired k_{eff} at the beginning of the cycle is

$$k_d(0) = k_d(\alpha T) - \frac{k(T) - k(0)}{T} \alpha T.$$

By linear interpolation the new enrichment parameter is

$$x = x - \frac{x - x'}{k(0) - k'(0)} [k(0) - k_d(0)].$$

The interaction with the external cycle is now considered. The discharge density distribution $B^{(0)} n^{(0)}$, defined by Eqs. (24) and (25), is passed through the external cycle, generating a new charge $n^{(1)}$:

$$n^{(1)} = S[B^{(0)} n^{(0)}, x] B^{(0)} n^{(0)} + P[B^{(0)} n^{(0)}, x] n_F. \quad (33)$$

The matrices S and P were previously defined, and as indicated depend on the discharge density and the enrichment parameter. Using this charge and the B_r matrices previously obtained, the steps in Eqs. (31)–(33) are repeated until the desired k_{eff} is obtained. The preliminary search procedure is converged if

$$|k(0) - k_d(0)| < epsf.$$

When convergence has been achieved in the preliminary search, the intermediate search is initiated with further refinements to obtain a better enrichment and burn time. This procedure is identical to the method described in the preceding sections with the following restrictions. Only one inner iteration of the density vectors (see REACTOR BURNUP EQUATIONS) and one cyclic mode iteration (see CYCLIC MODE) are done. Control searches are not done in the neutronics calculation. An enrichment giving the desired k_{eff} at the specified fraction α of the burn step time and a burn step time giving the desired burnup are obtained.

With this enrichment, burn step time, and the B matrices obtained the complete method previously presented is used to obtain the solution to the equilibrium problem.

NEUTRONICS CALCULATION

The fuel cycle code, REBUS, is a module in the Argonne Reactor Computation (ARC) System. At

present REBUS can be used in conjunction with the two dimensional diffusion theory module DIF2D⁽³⁾ or the one dimensional diffusion theory module DIF1D.⁽³⁾ The region-averaged group fluxes and k_{eff} are the only interfaces to the burnup module of REBUS from the neutronics calculation specified in Eq. (13).

The region-averaged atom densities and the control search specifications are the only interfaces to the neutronics module from the burnup module of REBUS. There is no geometric information in the burnup module itself, so that any neutronics module can be used (zero to three dimensional, diffusion or transport theory, direct or synthesis solution).

This section is concerned with acceleration techniques used to reduce the neutronics computation time for control searches. The burnup module in REBUS alters the control search specifications which interface with the neutronics module.

The general search expression³ used is

$$P(x) = P(0) + xM,$$

where P is the quantity being varied, x is the search parameter, and M is the quantity modifier. In composition searches, $P(x)$, $P(0)$, and M are macroscopic cross sections which are defined by the user. In addition to a value of x input, the user can also specify the derivative of k_{eff} with respect to x . The object of the control search is to find the value of x for which the reactor is just critical.

In REBUS the control search is done at each of the time nodes during the final search procedure (see Acceleration Techniques). The following procedure is used in the burnup module to estimate the x giving criticality for each of the P time points and to calculate dk/dx at each time point.

At the first control search at t_1 , two input values specified by the user are used. With these values, a linear search of x as a function of k is done until criticality of the reactor is achieved. From the neutronics module, $x(t_1)$ and $dk(t_1)/dx$ are obtained.

Assuming the x varies linearly over the burn step time, we obtain as an estimate for $x(t_2)$

$$x(t_2) = x(t_1)(1 - 1/\alpha).$$

The estimate for $dk(t_2)/dx$ used is the value obtained for t_1 . These values are used in the neutronics module, with a linear search of x versus k to obtain criticality. Successive estimates, in the burnup module, of the parameter and the derivative are obtained by linear extrapolation from two preceding values. For time point p ,

$$\begin{aligned} x(t_p) &= 2x(t_{p-1}) - x(t_{p-2}), \\ \frac{dk(t_p)}{dx} &= 2 \frac{dk(t_{p-1})}{dx} - \frac{dk(t_{p-2})}{dx}. \end{aligned} \quad (34)$$

During the inner iterations on the atom den vector at time t_p , which iterations are presented in Eqs. (11)–(14), the burnup module does extrapolations only on the parameter x . For $q = 2$, the estimate for $x^{(q)}(t_p)$ is

$$x^{(q)}(t_p) = 2x^{(q-1)} - x^{(q-2)}.$$

For successive iterations, an extrapolation based on assuming linear variation of the difference in the $x^{(q)}$ values is used:

$$\frac{x^{(q)} - x^{(q-1)}}{x^{(q-1)} - x^{(q-2)}} = \frac{x^{(q-1)} - x^{(q-2)}}{x^{(q-2)} - x^{(q-3)}}.$$

For cyclic mode iterations the following scheme is used to estimate x . For the first cyclic mode iteration ($q = 1$), the procedure previously presented is used. If $q = 2$, the parameter $x^{(1)}(t_1)$ and the derivative $dk^{(1)}(t_1)/dx$ obtained for $q = 1$ are used as values at t_1 . Successive values are estimated by linear extrapolation from the two preceding cyclic mode iterates

$$\begin{aligned} x^{(q)}(t_1) &= 2x^{(q-1)}(t_1) - x^{(q-2)}(t_1), \\ \frac{dk^{(q)}(t_1)}{dx} &= 2 \frac{dk^{(q-1)}(t_1)}{dx} - \frac{dk^{(q-2)}(t_1)}{dx}. \end{aligned} \quad (35)$$

At time t_2 , the parameter $x^{(q)}(t_2)$ is determined by the change in the value of x at t_1 . For $q \geq 2$, we use

$$x^{(q)}(t_2) = x^{(q-1)}(t_2) - [x^{(q-1)}(t_1) - x^{(q)}(t_1)],$$

and

$$\frac{dk^{(q)}(t_2)}{dx} = \frac{dk^{(q)}(t_1)}{dx}.$$

For any q , at successive time points t_p the parameter and the derivative are calculated as shown in Eq. (34).

For iterations on the enrichment parameter, the above procedure for cyclic mode iterations is used except for Eq. (35). For any q and time t_1 the parameter and the derivative obtained for the previous enrichment are used. That is, information is stored only for the preceding enrichment, and the extrapolation in Eq. (35) is not done.

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V-4. Argonne Reactor Computation (ARC) System One-Dimensional Neutronics Capabilities

H. HENRYSON

FIG. V-4-1. One-dimensional Neutronics Standard Path.
ANL Neg. No. 113-3074

A complete one-dimensional neutronics capability is available to users of ARC system standard paths or to users of the computational and adjunct calculational modules in a non-standard configuration. The former capability is equivalent to the stand alone diffusion and transport codes of old, but the modular framework provides a number of advantages over that approach. (1) There has been a minimum of programming dupli-

cation. Six of the eight input, computational and adjunct calculational modules invoked by the transport and diffusion theory standard paths are identical. The two modules which differ are concerned with the specific computational algorithms in question. In addition both standard paths use the same ARC system routines. One consequence of this method to the user has been a standardization of output formats. (2) Extended capabilities may easily be incorporated into a number of programs. For example, the cross section modification module was written on the request of a user and was easily incorporated into both of the one-dimensional neutronics standard paths as well as the perturbation and two-dimensional diffusion theory standard paths. (3) Input has been standardized. There are three types of alphanumeric input data which must be supplied to execute a neutronics calculation: path dependent data, general neutronics input, and module dependent input. The first two data types are identical to users of the two standard paths. The module dependent input for modules common to the two paths, e.g., inventory and output manipulation, are, of course, identical. Consequently only the data sets which describe information pertinent to the discrete ordinates and diffusion theory algorithms differ, e.g., angular quadrature order.

The flow through the diffusion theory standard path is illustrated in Fig. V-4-1. The transport path differs from this only in the substitution of NUI005 and NUC003 in the box representing the neutronics calculation. It should be noted that the neutronics calculation modules include complete criticality search and inhomogeneous source calculations as well as standard real and adjoint eigenvalue calculations. Through the use of catalogued procedures the execution of a problem invoking either of these standard paths requires a minimum knowledge of OS/360 job control language and very few input cards. However, extreme flexibility is available to the user who wishes to make full use of the ARC system and OS/360 capabilities. Documentation has been prepared and distributed to provide users with the information necessary to run problems using the neutronics standard paths in several orders of complexity.

V-5. Testing of the Diffusion Perturbation Modules in the Argonne Reactor Computation (ARC) System

P. H. KIER

TABLE V-5-I. UNPERTURBED, FINITE CYLINDRICAL CONFIGURATION FOR 2-D PERTURBATION CALCULATIONS

Region	Composition	Material	Volume Fraction	Outer Radius, cm
1	C1	M1	1	56.80
		M6	1	
2	C2	M2	1	137.60
		M7	1	
3	C3	M5	1	181.80
		M8	1	
4	C4	M9	1	227.60

Testing of the one-dimensional perturbation theory module and its associated driver module, STP006, and of the two-dimensional perturbation theory module and its associated driver module, STP007, has begun.¹ The purposes of these series of tests are to reveal programming mistakes by testing all possible options and to determine the accuracy of the perturbation computations.

These modules provide full capabilities for 1-D and 2-D perturbation theory calculations. The driver paths link the perturbation module with diffusion theory, criticality search, and k -calculation modules so that the real and adjoint fluxes used by the perturbation module may either be saved from an earlier calculation or generated in the current calculation. There are two levels of homogenization in ARC: "isotopes" may be combined to form "materials" and "isotopes" or "materials" are combined to form "compositions." The reactor configuration is specified by assigning "compositions" to the "regions" of the reactors. The types of perturbation that are permitted are: changes in the concentration of isotopes, changes in the volume fraction of materials, changes in the assignment of compositions to regions, and changes in the transverse buckling.

2-D PERTURBATION MODULE

The 2-D perturbation module has been tested for a finite cylindrical configuration with respect to changes in the assignment of compositions to regions, changes in the concentration of isotopes, and changes in the volume fractions of materials. Because of the unavailability of another perturbation code, the ARC

perturbation module computations were compared with ARC diffusion module computations. To reduce discrepancies arising from the limitations of perturbation theory, only small perturbations resulting in $\Delta k/k^2$ of the order 10^{-4} were considered. The diffusion calculations were run with very tight convergence criteria so that these small changes in reactivity were calculated with accuracy in at least three significant figures.

The reactor configuration, which is described in Table V-5-I, is that of a fictitious, finite cylindrical fast reactor with an extrapolated half-height of 52.220 cm. The materials are made up of isotopes from a

TABLE V-5-II. ISOTOPIC CONCENTRATIONS IN THE MATERIALS

Material	Isotope No.	Isotope Name	Concentration, $10^{24}/\text{cm}^3$
M1	53	U-235	0.00003156
	68	U-238	0.01035688
M2	8	Pu-241	0.00007676
	53	U-235	0.00002201
	55	Pu-239	0.0010974
	64	Pu-240	0.00039041
	68	U-238	0.00722140
M5	53	U-235	0.00003892
	68	U-238	0.01277050
M6	34	Na	0.007696
	35	O	0.0207769
	37	Fe	0.0101123
	38	Ni	0.0010399
	39	Cr	0.0026407
M7	34	Na	0.0078528
	35	O	0.0176160
	37	Fe	0.012836
	38	Ni	0.00132
	39	Cr	0.003352
M8	34	Na	0.0045745
	35	O	0.0256184
	37	Fe	0.0124689
	38	Ni	0.0012822
	39	Cr	0.00325613
M9	34	Na	0.010689
	37	Fe	0.030564
	38	Ni	0.003143
	39	Cr	0.007983

ely used 22-group cross section set, ARC.ID1, with concentrations as shown in Table V-5-II.

The perturbation module permits the perturbation in the diffusion coefficient, δD , to which the contribution of leakage to $\Delta k/k^2$ is proportional, to be defined in two ways. In the first definition, δD is simply the difference in the diffusion coefficient between the perturbed and the unperturbed systems; that is

$$\delta D_1 = D' - D = -(\Sigma'_{tr} - \Sigma_{tr})/3\Sigma'_{tr}\Sigma_{tr}, \quad (1)$$

where Σ'_{tr} and Σ_{tr} are the transport cross sections for the perturbed and the unperturbed systems respectively. In the second definition, δD is proportional to the change in the transport cross section.

$$\delta D_2 = \frac{D}{D'} (D' - D) = -(\Sigma'_{tr} - \Sigma_{tr})/3\Sigma_{tr}\Sigma_{tr}. \quad (2)$$

When $\Sigma'_{tr} < \Sigma_{tr}$, the use of δD_1 will yield larger leakage effects than will the use of δD_2 . The perturbation calculations were run with both definitions to determine if one yields consistently better agreement with diffusion theory.

COMPOSITION-REGION ASSIGNMENT PERTURBATIONS

Four problems were run in which the composition of a part of a region was changed. These sub-regions are described in Table V-5-III and the results of the perturbation and diffusion computations are given in Table V-5-IV. As seen from the table, there is significantly better agreement between the perturbation and the diffusion computations when δD is defined

TABLE V-5-III. SUB-REGIONS IN WHICH COMPOSITIONS ARE CHANGED

Region	Composition		Radius, cm		Height, cm	
	Unperturbed	Perturbed	Inner	Outer	Lower	Upper
5	C1	C4	51.12	56.80	0	4.5125
6	C2	C3	56.80	60.84	31.5875	36.10
7	C3	C4	137.60	142.02	31.5875	36.10
8	C3	C2	164.12	181.80	0	4.5125
9	C4	C2	181.80	190.96	0	4.5125

TABLE V-5-IV. COMPARISON OF 2-D PERTURBATION AND DIFFUSION COMPUTATIONS FOR COMPOSITION-REGION ASSIGNMENT CHANGES

Problem	Regions Perturbed	$\Delta k/k^2, 10^{-4}$		
		Diffusion	Perturbation δD_1	Perturbation δD_2
1	5	3.624	2.667	3.629
2	6	-5.210	-5.385	-5.189
3	7	-0.312	-1.165	-0.0819
4	8, 9	1.154	1.119	1.134

by Eq. (2). With this definition, the discrepancy is less than 1% for perturbations 1 and 2 and is less than 2% for perturbation 4. However for perturbation 3 there is serious disagreement between the diffusion and perturbation calculations. In this case there were large flux gradients in the test region so that leakage effects were very important and also the net reactivity effect was more than an order of magnitude smaller than the individual terms. Thus the errors introduced by the approximations to δD are apparent.

ISOTOPE CONCENTRATION PERTURBATIONS

Problems have also been run to test the perturbation module with respect to changes in the concentration of isotopes. The concentration of one isotope in a different composition was changed in each of four problems. The results of these calculations along with a comparison with diffusion theory are given in Table V-5-V. For the first two problems, in which the concentration of fissile nuclides are changed, the agreement between diffusion and perturbation calculation is good. Here leakage gives a relatively small contribution to the net reactivity so both approximations to δD give essentially the same result. In the third problem in which a non-fissile isotope is removed from region 3, the agreement is somewhat poorer and the use of δD_2 is again preferable. In the last problem where an unrealistically large amount of oxygen is added to the outermost region, the change in leakage is the dominant effect. Here there is serious dis-

TABLE V-5-V. COMPARISON OF 2-D PERTURBATION AND DIFFUSION COMPUTATIONS FOR ISOTOPE CONCENTRATION CHANGES

Prob.	Comp.	Matl.	Isotope	Concentration, $10^{24}/\text{cm}^3$		$\Delta k/k^2, 10^{-4}$		
				Initial	Perturbed	Diffusion	Pert. δD_1	Pert. δD_2
1	C1	M1	53(U ²³⁵)	0.00003156	0.00003748	1.154	1.1708	1.1708
2	C2	M2	55(Pu ²³⁹)	0.0010974	0.0010970	-1.718	-1.6988	-1.6988
3	C3	M8	38(Ni)	0.0012822	0.0	-1.775	-1.8850	-1.8269
4	C4	M9	35(O)	0.0	2.0	0.237	-3.3591	0.79499

TABLE V-5-VI. COMPARISON OF 2-D DIFFUSION AND 2-D PERTURBATION CALCULATIONS FOR MATERIAL VOLUME FRACTION PERTURBATIONS

Prob.	Comp.	Matl.	Volume Fractions		$\Delta k/k^2, 10^{-4}$		
			Unperturbed	Perturbed	Diffusion	Pert. δD_1	Pert. δD_2
1	C1	M1	1.0	0.95	1.190	1.1742	1.2283
2	C2	M2	1.0	1.0003	1.324	1.3421	1.3421
3	C3	M8	1.0	0.98	-1.467	-1.4394	-1.4079
4	C4	M2	0	1.0	0.618	0.9381	0.9527

TABLE V-5-VII. BUCKLING CHANGES FOR PARALLELEPIPED REACTOR

Prob.	Δh_1	Reg.	Δh_1	Reg.	Δh_2	Reg.	Δh_2	Reg.
1	+2	1-4						
2					-2	1-4		
3	+2	1-4			-2	1-4		
4			+2	1-4				
5							-2	1-4
6	+2	1-4	+2	1-4	-2	1-4	-2	1-4
7	+4	1						
8			+2	2				
9					-4	3		
10							-10	4
11	+4	1	+2	2	-4	3	-10	4
12	+2	1, 3	-3	2, 4	+4	1, 4	-1	2, 3

agreement between diffusion theory and the perturbation computations but again the use of δD_2 yields better results than the use of δD_1 .

MATERIAL VOLUME FRACTION CHANGE

Four problems were run to test the 2-D perturbation module with respect to changes in the volume fraction of a material in a composition. The description of the perturbations and the comparison with diffusion theory are given in Table V-5-VI. In the first two problems the volume fractions of fissile materials are altered, in the third problem the volume fraction of a non-fissile material is altered, and in the last problem a fissile material is added to an originally non-fissile composition. As may be seen from the tables, the agreement between the results of diffusion and perturbation calculations is good for the first problems since the discrepancies lie between 1 and 2%. In contrast to the results for other types of perturbations, use of δD_1 in this instance yields slightly better agreement with diffusion theory use of δD_2 .

Only in the last problem in which a large amount of fissile material is added to the large outer region does the perturbation calculation significantly overestimate the reactivity change. Although the net re-

activity change is small, the addition of material to the outer region does not represent a small change in the system. There is a sufficient change in the flux distribution to make the assumptions of perturbation theory tenuous. Therefore, for this case, the discrepancy between diffusion and perturbation calculations most likely arose from the limitations of perturbation theory rather than from an error in the perturbation module.

FUTURE TESTS

The testing of the 2-D perturbation module and its driver module STP007 is far from complete. It will be tested for other geometric configurations such as a parallelepiped reactor or a cylindrical with $r-\theta$ dependence. It will be tested for buckling changes in which the actual half-height or transverse extrapolation distances will be changed regionally.

These tests have been made with an isotopic cross section set in which all the fissile isotopes have the same chi fission vector. Tests will be run in which the fissile isotopes have chi fission matrices to determine if the perturbation module correctly computes the reactivity effect of perturbation in the chi fission matrix.

1-D PERTURBATION MODULE

Testing of the one-dimensional perturbation module began before the two-dimensional module became available. The early calculations tested the 1-D perturbation module by comparing the results of the perturbation calculations with diffusion calculations. These early comparisons revealed errors in the module; they were subsequently corrected and will not be discussed further in this paper.

Because of the availability of a 1-D perturbation code, the DEL-MACH-1⁽²⁾ code, it was decided to compare the ARC perturbation module calculations with DEL-MACH-1 calculations to eliminate uncertainties arising from the limitations of perturbation theory. The results of these comparisons will now be presented.

BUCKLING CHANGES FOR PARALLELEPIPED

Buckling perturbations in the 1-D perturbation module are specified by changes in the transverse half-height and extrapolation distance. Changes in the transverse dimensions may be region-dependent. It was decided to test the module with regard to buckling changes for the complicated case of a parallelepiped reactor which has two transverse directions. The fictitious reactor had four regions with the same compositions previously used. The outer dimensions of the regions as measured from the midplane were

ken as 9.2085, 54.0035, 94.3083 and 148.4063 cm. These dimensions were chosen such that a parallelepiped reactor with a bare height of 200 cm would have the same regional volumes as the cylindrical reactor previously considered.

With an extrapolation distance of 16 cm, the critical half-height in each of the transverse directions was found to be 101.96 cm using a buckling search. These transverse dimensions were used in a k -calculation to generate the real and adjoint fluxes used by the perturbation module. The perturbations are described in Table V-5-VII. Δh_i and Δl_i refer to changes in the half-height and extrapolation distance in the i th transverse direction, respectively, and Reg. refers to the region or regions for which the perturbation applies.

The results of the calculations are given in Table V-5-VIII. In the first six problems the perturbations apply to all regions; in the last six problems the perturbations are region-dependent. From this table it is easily seen that there is excellent agreement between the ARC module and the DEL-MACH-1 computations. In all the problems, the discrepancy is less than 1%.

COMPOSITION CHANGES

The 1D perturbation module was tested with respect to changes in the volume fraction of materials in a composition. However, in MACH-1 there is no material level, since isotopes are combined directly into compositions whose volume fraction in a region can be changed. Hence we will refer to these perturbations as composition perturbations and when we change the volume fraction of a composition in the ARC perturbation module we are changing the volume fractions of all materials in the composition by the same amount. The definition of the perturbation in the diffusion coefficient given by Eq. (1), δD_1 , was used in both codes. The same cylindrical reactor used in the testing of the 2-D perturbation module is considered here.

The description of the perturbations and the results of the calculations are given in Table V-5-IX. In the first four problems, the volume fraction of the composition originally assigned to a region is changed; in the last four problems the composition originally assigned to a region is augmented by a fraction of another composition. From the table we see that there are fairly large discrepancies ranging from 2.7 to 6% between the calculations of the two codes. It is difficult to explain the existence of these discrepancies, which occur in all terms contributing

TABLE V-5-VIII. COMPARISON OF STP006 AND DEL COMPUTATIONS FOR PARALLELEPIPED REACTOR

Prob.	$\Delta k/k^2$	
	ARC	DEL-MACH-1
1	0.0010696	0.0010611
2	-0.0011254	-0.0011165
3	-0.00005580	-0.00005540
4	0.0010696	0.0010611
5	-0.0011254	-0.0011165
6	-0.00022352	-0.00022111
7	0.00023717	0.00023618
8	0.00090738	0.00089950
9	-0.00008770	-0.00008752
10	-0.000000518	-0.000000522
11	0.0010563	0.0010476
12	-0.0019238	-0.0019073

TABLE V-5-IX. COMPARISON OF ARC AND MACH-DEL COMPUTATIONS FOR COMPOSITION PERTURBATIONS

Region	Composition	Change in Vol. Fraction	$\Delta k/k$ DEL-MACH-1	$\Delta k/k^2$ ARC	% Difference
1	C1	0.01	-8.0031×10^{-6}	-8.4839×10^{-6}	6.0
2	C2	-0.001	-3.8165×10^{-4}	-4.0025×10^{-4}	4.9
3	C3	0.01	5.0425×10^{-6}	5.2614×10^{-6}	4.3
4	C4	1.0	-6.9550×10^{-7}	-7.1421×10^{-7}	2.7
4	C6 ^a	0.0001	9.6307×10^{-6}	10.0054×10^{-6}	3.9
2	C6 ^a	0.0001	3.1265×10^{-2}	3.2799×10^{-2}	4.9
3	C2	0.001	2.5962×10^{-5}	2.7206×10^{-5}	4.8
1	C5 ^b	0.0001	-9.1330×10^{-4}	-9.4908×10^{-4}	3.9

^a Composition C6 is U-235 with an atom density of $1 \times 10^{24}/\text{cm}^3$.

^b Composition C5 is B-10 with an atom density of $1 \times 10^{24}/\text{cm}^3$.

to the net reactivity change. Study of the output from the two codes showed that the identical perturbations in macroscopic cross sections were used so it might be thought that the discrepancies result from significant differences in the flux distributions used by the two codes. However, when several real and adjoint fluxes were plotted, no noticeable differences between those from the ARC module and DEL-MACH-1 were detected. Investigation of the source of these discrepancies as well as testing of all options of the 1-D perturbation module will be continued.

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V-6. Plans for an Argonne Reactor Computation (ARC) System ZPR Heterogeneity Package

B. A. ZOLOTAR

A series of ARC¹ modules are under development in an attempt to provide a convenient and reliable method for obtaining group cross sections for use in the analysis of ZPR lattices. The generation of these cross sections will include both spectral and spatial cell weighting with approximations to the heterogeneous effects included in the calculation. For the first version of this heterogeneity package the following procedure has been adopted:

1. A modified version of the MC²(²) code will be used to generate broad-group cross sections for each lattice plate. The assumption is made that except in the resonance range these plate cross sections are identical to those for a homogeneous cell of the same average composition. In the resolved and unresolved resonance region the capture and fission cross sections are shielded using equivalence theory on an ultra-fine group basis. The two-sided E_3 formulation of D. Meneghetti³ is used to find the Dancoff corrected potential cross section for each individual absorber plate. Problems relating to the proper treatment of interacting resonances in different plates are still under study.

2. In order to determine cell-averaged cross sections, the plate cross sections will be used with the ARC 1-D transport¹ modules to obtain fluxes within the unit cell. These calculations will be made with a high-order modified single Gaussian angular quadrature.⁴ If desired, a group-dependent leakage term will

be included to simulate the overall leakage from the core.

3. The broad-group flux shapes in the cell will be used to obtain homogenized cross sections appropriate for overall reactor studies.

Upon completion, this heterogeneity package will be extensively used as part of a study to test many of the assumptions inherent in such a calculation. Alternatives to the standard formulations will be examined and comparisons will be made with results from integral transport codes such as RABBLE⁽⁵⁾ and RABID.⁽⁶⁾

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V-7. Modification of the Multigroup Resonance Absorption Code RABBLE

P. H. KIER

Users of the RABBLE⁽¹⁾ program have found that for problems in which the resonance escape probability is small, negative fluxes or negative resonance escape probabilities are sometimes computed. The difficulty arises because of the buildup of errors in the calculation of regional slowing down sources as thousands of fine groups are covered. Various methods of correcting these errors have been devised and tested. Although none of these methods has completely eliminated the difficulties,

the domain in which they occur has been considerably reduced and work is continuing to eliminate them completely.

Similar difficulties have arisen in the use of RABID, (see Paper V-II); a resonance absorption code using periodic boundary conditions, which is suitable for the analysis of fast critical assemblies. Our effort to correct RABBLE is correlated with the effort to correct RABID.

In RABBLE, the lethargy range of interest is divided into many very narrow intervals, called fine groups. These fine groups are sufficiently narrow that it is reasonable to assume that a neutron can suffer only one collision within a fine group and that the collision density is independent of lethargy within a fine group. With these assumptions, the rate with which neutrons are scattered into the k th fine group, which extends from u_k to $u_k + \Delta u$ is given by $S_k \Delta u$ where

$$S_k = \int_{u_k - \epsilon}^{u_k} du' \frac{F_s(u')}{1 - \alpha} e^{-(u-u')} \quad (1)$$

$$= \int_{u_k - \epsilon}^{u_k} du' F_s(u') P(u' \rightarrow u).$$

$F_s(u')$ is the scattering rate and $P(u' \rightarrow u)$ is the probability per unit lethargy of a neutron being scattered from u' to u . For simplicity of notation, we have assumed only one scattering nuclide in Eq. 1. When the range of integration is broken up into fine groups it can be shown that Eq. (1) can be reduced to

$$S_k = e^{-\Delta u} S_{k-1} + P_1 F_{s_{k-1}} - e^{-\Delta u} P_L F_{s_{k-L-1}}, \quad (2)$$

where P_n is the probability that a neutron is scattered down n fine groups and L is the maximum number of groups through which a neutron can be scattered. If more than one scattering nuclide is present, the last two terms in Eq. (2) are summations over the number of scatterers. P_n is given by

$$P_n \Delta u = \frac{1}{1 - \alpha} (1 - e^{-\Delta u})^2 e^{-(n-1)\Delta u}$$

$$\approx \frac{\Delta u}{1 - \alpha} (1 - e^{-\Delta u}) (1 - \Delta u/2) e^{-(n-1)\Delta u} \quad (3)$$

$$\approx \frac{\Delta u}{1 - \alpha} (1 - e^{-\Delta u}) e^{-(n-1)\Delta u}.$$

In Eq. (3), the factor $(1 - \Delta u/2)$ was dropped so that the sum of P_n over all possible n is unity, as required by neutron conservation. The appearance of this factor is due to the approximation that neutrons are not scattered from one lethargy to another lethargy in the same fine group. Because of this approximation, the lethargy range from which neutrons are scattered into a fine group is displaced slightly such that the P_n are slightly too small. By dropping the factor $(1 - \Delta u/2)$ the approximation of no in-group scattering is compensated for somewhat.

An exact evaluation of Eq. (2) can be made quickly enough but requires great storage since the last term requires scattering rates as a function of nuclide, region and fine group. In the original version of RABBLE, the limited memory of the CDC-3600 precluded exact evaluation of the last term, which was approximated

by the average intermediate group scattering rate for a composition multiplied by the ratio of the intermediate group flux in the region to the intermediate group flux in the composition. In RABBLE terminology, a composition is subdivided into regions and an intermediate group is equivalent to an ultrafine group in MC².⁽²⁾ A. Olson (see Paper V-II) has found a significant improvement in the last term of Eq. (2) when the average regional intermediate group scattering rate is used instead of the original approximation.

Several expressions for the slowing down source S_k have been derived and tested. The best of these relates S_k to the slowing down density q_k . The slowing down density, or rate with which neutrons are scattered past lethargy u_k in some region is given by

$$q_k = \int_{u_k - \epsilon}^{u_k} du' \frac{F_s(u')}{1 - \alpha} \{e^{-(u-u')} - \alpha\}. \quad (4)$$

Upon comparing Eqs. (1) and (4), we see that

$$q_k = S_k - \frac{\alpha}{1 - \alpha} \int_{u_k - \epsilon}^{u_k} F_s(u') du' = S_k - h_k, \quad (5)$$

where it is easily shown that h_k satisfies the recursion relationship

$$h_k = h_{k-1} + \frac{\alpha}{1 - \alpha} [F_{s_{k-1}} - F_{s_{k-L-1}}]. \quad (6)$$

From Eqs. (5) and (6), it is easily seen that S_k can be written as

$$S_k = S_{k-1} + q_k - q_{k-1} + h_k - h_{k-1}$$

$$= S_{k-1} + q_k - q_{k-1} \quad (7)$$

$$+ \frac{\alpha}{1 - \alpha} [F_{s_{k-1}} - F_{s_{k-L-1}}] \Delta u.$$

$q_k - q_{k-1}$ is found by differentiating Eq. (4) and then integrating from u_{k-1} to u_k :

$$q_k - q_{k-1} = (F_{s_{k-1}} - S_{k-1}) \Delta u. \quad (8)$$

Equation (7) has been found to yield better slowing down sources than Eq. (2), but it is still possible for problems to give erroneous fluxes and resonance escape probabilities. It is felt that the errors arise from the approximation of using intermediate group averages in evaluating the scattering rate from one scattering interval distance, $F_{s_{k-L-1}}$. The approximation of representing the scattering rate by a three term polynomial is currently under investigation.

For resonant nuclides the expression for the scattering rate at $u_k - \epsilon$ is

$$F_{s_{k-L-1}} = a + bU + cU^2, \quad (9)$$

where $U = (u_k - \epsilon) - u_j$ and u_j is the lethargy at which the j th intermediate group begins. $u_k - \epsilon$ lies in

the j th intermediate group. The three coefficients in Eq. (9), which are dependent upon nuclide, region and intermediate group, are obtained from the first three lethargy moments of the scattering rate about u_j .

For non-resonant nuclides, the expression for the scattering rate at $u_k - \epsilon$ is

$$F_{s_{k-L-1}} = \Sigma_p(a' + b'U + c'U^2), \quad (10)$$

where Σ_p is the constant potential scattering cross section of the nuclide and the quantity within the parentheses approximates the neutron flux at $u_k - \epsilon$. The three coefficients in Eq. (10), which are dependent upon region and intermediate group, are obtained from the first three moments of the neutron flux about u_j .

The refinement RABBLE offers is its use of space and lethargy-dependent slowing down sources, in which true deviations from the asymptotic form are accumulated, in computing resonance absorption. Unfortunately, it appears that errors in the source, arising from either numerical round off or from approximations in the model are also accumulated. These errors first appear in negative regional slowing-down densities. If a negative slowing down density can be modified when it first appears, negative sources and fluxes will be precluded.

Several ad hoc methods of modifying negative regional slowing-down densities subject to the restraint of neutron conservation have been investigated. From Eq. (8), it is seen that the change in the slowing down density depends on the difference between the scattering rate and the source. Several methods of increasing the scattering rate have been tried and have failed. A scheme currently under investigation is to reduce the source in a region having a negative slowing-down den-

sity. The source neutrons that are removed from the region are distributed among the other regions according to the flux in those regions. The net result is to redistribute the slowing-down source in such a manner as to reduce the chance of absorption. This procedure has the disadvantage that a change in the source distribution causes a change in the collision density distribution. Thus it is necessary to iterate until all regional slowing-down densities resulting from a source distribution are positive, which might appreciably increase the running time of a problem.

If the current efforts to eliminate the troubles in RABBLE fail, it is possible to evaluate the scattering rate from one scattering interval distance, $F_{s_{k-L-1}}$, on a fine group basis, for heavy resonant nuclides at least, because of the large memory of the IBM-360. To do so would require some limitations on the minimum width of a fine group but these would not be seriously restrictive. However for light resonant nuclides a neutron could be scattered through so many fine groups that it does not appear to be feasible to store the fine group scattering rates for these nuclides and therefore some more highly approximate method must be used. Fortunately, the most important light-nuclide resonance, that of sodium, is so wide that the calculation for $F_{s_{k-L-1}}$ on an intermediate group basis might be adequate.

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V-8. Cross Section Averaging Schemes for Group Collapsing and Cell Homogenization in Neutron Transport Calculations for Critical Assemblies

R. B. NICHOLSON

INTRODUCTION

Fast critical assemblies of the type under operation at Argonne, e.g., ZPPR and ZPR-3, -6 and -9, are constructed by stacking plates of the several reactor materials in many drawers which are slid into a close-packed array of rectangular matrix tubes. Except for small interruptions by matrix and drawer walls and by voids due to cooling channels and clearances, the drawer loadings are usually made such that the plates in different drawers line up and form plane sheets of the individual materials. The core of the assembly can then be well approximated by a stack of thin slabs bounded by the core blanket interface which, although jagged, is usually made to approximate a cylinder sometimes a sphere. The slab structure is, if possible, a repeating cell structure with the cell thickness equal to one or a small number of drawers.

The technique under development for analysis of assemblies of this type involves the generation of cell-averaged cross sections for use in one- or two-dimensional transport theory calculations for the homogenized system. The fluxes and adjoints required for the homogenization routines are obtained from one-dimensional transport theory cell calculations with periodic or reflective boundary conditions. The purpose of this paper is to develop homogenization expressions which utilize the angular cell fluxes and adjoints to best advantage in the calculation of criticality and other quantities measured in the critical experiments.

One can develop flux weighting and bilinear weighting expressions quite independently of one another and it is difficult to choose one over the other from purely theoretical considerations. In order to test the homogenization errors, to compare them with ordinary unweighted homogenization, and to compare flux weighting and bilinear weighting, we develop expressions for one-dimensional slab geometry which can be calculated "exactly" (without homogenization). This is also the geometry in which the homogenization formulas can be developed with the firmest theoretical foundation. Some weaker arguments are required when there are flux gradients in directions that are not perpendicular to the slab faces, as is always the case in the real critical experiments. Since we rely mainly on the one-dimensional "Gedanken Experiments" for evaluation of the methods, it is necessary to point out that there remains some further uncertainty in application to two or three dimensional situations. However, one particular two-dimensional system can be coped with in a reasonable fashion, and it suggests how to proceed in more complex situations. We believe that the formulas developed represent about the best that can be done with the cell homogenization approach for the calculation of criticality and that the accuracy is good enough for most purposes. The situation with regard to critical assembly measurements of a more localized nature, such as foil activations and perturbation sample measurements, requires further investigation.

While the primary interest is in cell homogenization, it does not complicate matters greatly to develop simultaneously the capability for group collapsing. It may under some conditions be desirable and good enough to carry out the flux and adjoint calculations for the cell in many groups and then collapse to few groups for the subsequent homogeneous calculation which might be done in two- or even three-dimensional transport theory.

We present the development for bilinear weighting obtained from the variational principle. Using some of the same arguments, a flux weighting scheme can be developed relatively easily from the neutron transport equation. As far as the formulas are concerned, the same result is obtained by setting the adjoints equal to unity in the bilinear expressions. The bilinear formulation therefore encompasses both, and we do not present the separate development for flux weighting. However, it is important that the flux weighting stands on equally good theoretical ground and is not to be regarded as an approximation to more accurate bilinear expressions. Any choice between them is to be made on the basis of numerical "Gedanken Experiments".

A FUNCTIONAL FOR THE TRANSPORT EQUATION

The first step in the development is to define a functional for use in the variational principle. The requirements on it are that when we set its first variation equal to zero, the resulting set of equations are the multigroup transport equations for neutron flux and adjoint and the proper interface conditions for the neutron transport problem. Here we proceed in a manner similar to that of A. F. Henry¹ and A. J. Buslik,² who have shown that one has to be careful about interface conditions when making approximations in the functional.

The functional is to yield the equations for the complete heterogeneous problem with all energy groups. Once it is found, then we make some approximations in the functional. The flux in the g th group is approximated by

$$N_g(x, \Omega) = N_g^c(x, \Omega) N_g^h(x, \Omega), \quad (1)$$

$$N_g^*(x, \Omega) = N_g^{c*}(x, \Omega) N_g^{h*}(x, \Omega), \quad (2)$$

where the $N_g^c(x, \Omega)$ and $N_g^{c*}(x, \Omega)$ are assumed to be known, namely the solution of the cell problem with periodic boundary conditions. The functions $N_g^h(x, \Omega)$ and $N_g^{h*}(x, \Omega)$ are the unknown functions. The variables x and Ω are vector position and neutron direction. These functions are smooth through the plate structure, the variations due to structure being contained in N_g^c . The superscript h simply indicates that it is the function we will solve for after carrying out the cross section homogenization and group collapsing approximations. The approximation in Eqs. (1) and (2) is that the homogeneous functions are the same for each fine multigroup g contained in the coarser multi-group G .

The normalization of the cell flux is important if one wishes the homogeneous functions to give the coarse energy spectrum. Normally the solutions of the cell problem as output of a computer code would contain both the fine

structure (depending on number of groups) of the energy spectrum and an approximate coarse structure. The coarse structure is removed by a renormalization as follows:

$$\sum_{g \text{ in } G} \iint N_g^c(x, \Omega) dV d\Omega = 1, \quad (3)$$

for each large group G . If this were not done, the downscattering and fission matrix group constants would be modified in such a way that N_g^h would be almost energy independent, providing only the correction for the fact that the cell calculation did not get the leakage quite right. This normalization does not affect the eigenvalue. It is easy to see from the formulas developed below for homogenized cross sections that a similar normalization of the adjoint would not influence the cross sections and is unnecessary.

The procedure is then to substitute Eqs. (1) and (2) into the first functional, make some approximations, and thereby obtain what is called the reduced functional. The approximations are made in such a way that the reduced functional F_R will have the same form as the original one and will therefore also yield the standard transport multi-group equations, but this time equations for N_g^h and its adjoint; and also the usual boundary conditions. The assumption is that if we require F_R to be stationary, then the solutions of the resulting equations for N_g^h when inserted into Eq. (1) will give a good approximation to $N_g(x, \Omega)$ and that the eigenvalue of the equations will closely approximate the eigenvalue of the original equations. If sample problems bear this out we can conclude that we have done something worthwhile. We have little other justification except our intuition and the success of others in similar problems.

Once one has set down the above-ground rules and aims, the procedures and approximations required are fairly straightforward. We define the functional

$$\begin{aligned} F[N_g(x, \Omega), N_g^*(x, \Omega), \alpha_g(x, \Omega), \alpha_g^*(x, \Omega)] = & \sum_g \iint d\Omega dV \left[\frac{1}{2} N_g^*(x, \Omega) \Omega \cdot \nabla N_g(x, \Omega) - \frac{1}{2} N_g(x, \Omega) \Omega \cdot \nabla N_g^*(x, \Omega) \right. \\ & + N_g^*(x, \Omega) \Sigma_g(x) N_g(x, \Omega) - N_g^*(x, \Omega) \sum_j \int d\Omega' \Sigma_{jg}(x, \Omega, \Omega') N_j(x, \Omega') - \frac{N_g^*(x, \Omega) \chi_g}{\lambda} \sum_j \int d\Omega' \nu_j \Sigma_{fj}(x) N_j(x, \Omega') \Big] \\ & + \frac{1}{2} \sum_g \iint_{i_s \cdot \Omega > 0} d\Omega dS_g N_g^*(x, \Omega) \alpha_g(x, \Omega) - \frac{1}{2} \sum_g \iint_{i_s \cdot \Omega < 0} d\Omega dS_g N_g(x, \Omega) \alpha_g^*(x, \Omega) \\ & - \frac{1}{2} \sum_g \iint d\Omega dS_i \{ \alpha_g^*(x, \Omega) [N_g^+(x, \Omega) - N_g^-(x, \Omega)] - \alpha_g(x, \Omega) [N_g^{*+}(x, \Omega) - N_g^{*-}(x, \Omega)] \}. \end{aligned} \quad (4)$$

In this equation, g , Ω , V , S_g and S_i are groups, angles, volumes, external vacuum surfaces and internal surfaces, respectively. The cross sections Σ_g , Σ_{jg} , and Σ_{fg} are total, energy and angle transfer, and fission, respectively. The parameters ν and χ are neutrons per fission and fission spectrum. The unknowns are N_g , N_g^* , α , and α^* . The volume integrals go over all volumes interior to the external vacuum boundaries. Note also in the external surface terms that in the first one the angular integration is only over Ω values pointing out of the surface and in the second one only over Ω values pointing into the surface. The vector i_s is the outward directed unit normal. In the internal surface terms, the $+$ and $-$ superscripts represent the inside and outside of the surfaces, respectively. Thus we have allowed that in general N and N^* can be discontinuous across a boundary. We will show shortly, however, that the N and N^* which make F stationary are the continuous ones.

First consider the variation of F with respect to N_g^* . To keep the notation condensed we suppress the arguments x and Ω . The gradient terms can be manipulated as follows:

$$\frac{1}{2} N_g^* \Omega \cdot \nabla N_g - \frac{1}{2} N_g \Omega \cdot \nabla N_g^* = N_g^* \Omega \cdot \nabla N_g - \frac{1}{2} \nabla \cdot (\Omega N_g N_g^*). \quad (5)$$

The volume integral of the gradient terms can then be written as follows by making use of the divergence theorem:

$$\begin{aligned} \sum_g \iint d\Omega dV \left(\frac{1}{2} N_g^* \Omega \cdot \nabla N_g - \frac{1}{2} N_g \Omega \cdot \nabla N_g^* \right) = & \sum_g \iint d\Omega dV N_g^* \Omega \cdot \nabla N_g \\ & - \frac{1}{2} \sum_g \iint d\Omega dS_i (N_g^+ N_g^{*+} - N_g^- N_g^{*-}) i_s \cdot \Omega \\ & - \frac{1}{2} \sum_g \iint_{i_s \cdot \Omega > 0} d\Omega dS_g (N_g N_g^*) i_s \cdot \Omega - \frac{1}{2} \sum_g \iint_{i_s \cdot \Omega < 0} d\Omega dS_g (N_g N_g^*) i_s \cdot \Omega, \end{aligned} \quad (6)$$

where merely for convenience we have divided up the surface terms first into interior and exterior surfaces and then secondly the exterior surface terms into outgoing and incoming directions.

None of the other terms require any manipulation and we can now write the expression for

$$\begin{aligned} \delta F(N_\theta, \delta N_\theta^*, \alpha, \alpha^*) = & \sum_\theta \int \int d\Omega dV \delta N_\theta^* \\ & \left[\Omega \cdot \nabla N_\theta + \Sigma_\theta N_\theta - \sum_j \int d\Omega' \Sigma_{j\theta} N_j - \frac{\chi_\theta}{\lambda} \sum_j \int d\Omega' \nu_j \Sigma_{fj} N_j \right] \\ & + \frac{1}{2} \sum_\theta \int \int_{i_\theta \cdot \Omega > 0} d\Omega dS_\theta (\alpha_\theta - i_\theta \cdot \Omega N_\theta) \delta N_\theta^* - \frac{1}{2} \sum_\theta \int \int_{i_\theta \cdot \Omega < 0} d\Omega dS_\theta i_\theta \cdot \Omega N_\theta \delta N_\theta^* \\ & + \frac{1}{2} \sum_\theta \int \int d\Omega dS_\theta (\alpha_\theta - i_\theta \cdot \Omega N_\theta^+) \delta N_\theta^+ - \frac{1}{2} \sum_\theta \int \int d\Omega dS_\theta (\alpha_\theta - i_\theta \cdot \Omega N_\theta^-) \delta N_\theta^-. \end{aligned} \quad (7)$$

Now the basis of the variational principle is that if δF is to be zero for completely arbitrary δN_θ^* , then the integrands of all the above integrals must separately vanish. Thus the conditions for $\delta F = 0$ are

$$\Omega \cdot \nabla N_\theta + \Sigma_\theta N_\theta - \sum_j \int d\Omega' \Sigma_{j\theta} N_j - \frac{\chi_\theta}{\lambda} \sum_j \int d\Omega' \nu_j \Sigma_{fj} N_j = 0, \quad (8)$$

$$\alpha_\theta = i_\theta \cdot \Omega N_\theta \quad \text{for } i_\theta \cdot \Omega > 0 \quad \text{on exterior surfaces,} \quad (9)$$

$$N_\theta = 0 \quad \text{for } i_\theta \cdot \Omega < 0 \quad \text{on exterior surfaces,} \quad (10)$$

$$\alpha_\theta = i_\theta \cdot \Omega N_\theta^+ = i_\theta \cdot \Omega N_\theta^- \quad \text{or} \quad N_\theta^+ = N_\theta^- \quad \text{on interior surfaces.} \quad (11)$$

Equation (8) is the transport equation. Equation (9) states that on an external surface α_θ is the partial current across the surface. Equation (10) states that no neutrons are entering from the vacuum. Equation (11) states that on interior surfaces α_θ is the current and N_θ is continuous across interior surfaces. Thus, we see that the requirement that $\delta F = 0$ for arbitrary variations in N^* leads to the result that N is the solution of the usual multigroup transport equation with normal boundary conditions.

We will not go through the manipulations for variations in N . The development is the same except that it is necessary to interchange orders of integration over Ω and Ω' and summations over g and j . The result is the adjoint transport equation for N_θ^* and the normal boundary conditions:

$$-\Omega \cdot \nabla N_\theta^* + \Sigma_\theta N_\theta^* - \sum_j \int d\Omega' \Sigma_{\theta j} N_j^* - \frac{\nu_\theta \Sigma_{f\theta}}{\lambda} \sum_j \int d\Omega' \chi_j N_j^* = 0, \quad (12)$$

$$\alpha_\theta^* = i_\theta \cdot \Omega N_\theta^* \quad \text{for } i_\theta \cdot \Omega < 0 \quad \text{on exterior surfaces,} \quad (13)$$

$$N_\theta^* = 0 \quad \text{for } i_\theta \cdot \Omega > 0 \quad \text{on exterior surfaces,} \quad (14)$$

$$\alpha_\theta^* = i_\theta \cdot \Omega N_\theta^{*+} = i_\theta \cdot \Omega N_\theta^{*-} \quad \text{or} \quad N_\theta^{*+} = N_\theta^{*-} \quad \text{on interior surfaces.} \quad (15)$$

Finally, it is necessary to calculate δF due to variations in α and α^* . This involves only the surface terms and one can see immediately that requiring δF to vanish in this case leads to

$$N_\theta^* = 0 \quad \text{for } i_\theta \cdot \Omega > 0 \quad \text{on exterior surfaces,} \quad (16)$$

$$N_\theta = 0 \quad \text{for } i_\theta \cdot \Omega < 0 \quad \text{on exterior surfaces,} \quad (17)$$

$$N_\theta^+ = N_\theta^- \quad \text{on interior surfaces,} \quad (18)$$

$$N_\theta^{*+} = N_\theta^{*-} \quad \text{on interior surfaces.} \quad (19)$$

Thus the α and α^* variations only lead to conditions already obtained from the N and N^* variations. It is important that no inconsistencies are obtained.

THE REDUCED FUNCTIONAL

Following the program outlined above, the next step is to introduce the approximations of Eqs. (1) and (2) into the functional (4). Consider first the total, fission, and scattering cross section terms in the volume integrals as the treatment of these terms is independent of reactor geometry.

In the term containing the total cross section, we replace $N_\theta^* N_\theta^+ \Sigma_\theta$ by its value averaged over the cell and summed g groups g contained in G :

$$\sum_g \int \int d\Omega dV N_\theta^* \Sigma_\theta N_\theta^+ = \sum_g \int \int d\Omega dV N_\theta^{h*} \overline{N^* \Sigma N^h}^g N_\theta^h, \quad (20)$$

where

$$\overline{N^* \Sigma N}^g \equiv \frac{1}{4\pi V} \sum_{g \text{ in } g} \iint d\Omega dV N_g^{c*} \Sigma N_g^c. \quad (21)$$

This approximation is equivalent to replacing the average of a product by the product of the averages. The justification or rationale for it is that fluctuations in the cell functions are closely correlated with the cross section and not with the smooth homogeneous functions. The approximation is expected to be good if the cell is a small part of the reactor so that the homogeneous function does not change much across a cell. Note that $\overline{N^* \Sigma N}^g$ is not a function of Ω and is spatially constant within a region of the reactor consisting of a given cell type but changes from one region to another if the cell type changes, or if one includes a different DB^2 leakage in the different regions in the cell calculations.

In the fission term we make the approximation

$$\sum_{g \text{ in } g} \sum_{j \text{ in } J} \iiint d\Omega d\Omega' dV N_g^*(x, \Omega) \chi_{g\nu_j} \Sigma_{fj}(x) N_j(x, \Omega') \approx \iiint d\Omega d\Omega' dV N_g^{h*}(x, \Omega) \langle N^* \chi \nu \Sigma_f N \rangle_{Jg} N_j^h(x, \Omega'), \quad (22)$$

where the angular brackets mean

$$\langle N^* \chi \nu \Sigma_f N \rangle_{Jg} \equiv \frac{1}{16\pi^2 V} \sum_{j \text{ in } J} \sum_{g \text{ in } g} \iiint d\Omega d\Omega' dV N_g^{c*}(x, \Omega) \chi_{g\nu_j} \Sigma_{fj}(x) N_j^c(x, \Omega). \quad (23)$$

The quantity $\langle N^* \chi \nu \Sigma_f N \rangle_{Jg}$ is of course now a matrix multiplier rather than a simple constant and one cannot (without additional approximation) define separate fission spectra independent of incoming neutron energy, and fission cross sections independent of outgoing neutron energy. The cross section and spectrum are linked together in the matrix. Just as with the total cross section, these matrix elements are independent of Ω and depend upon space only stepwise from region to region.

Equation (23) can be simplified in form by noting that the integrals over Ω and Ω' merely integrate N and N^* to give the total flux and adjoint and therefore

$$\langle N^* \chi \nu \Sigma_f N \rangle_{Jg} = \frac{1}{16\pi^2 V} \sum_{g \text{ in } g} \chi_{g\nu_j} \sum_{j \text{ in } J} \int dV \phi_g^*(x) \Sigma_{fj}(x) \phi_j(x), \quad (24)$$

where

$$\phi_j(x) \equiv \int d\Omega N_j^c(x, \Omega) \quad (25)$$

$$\phi_j^*(x) \equiv \int d\Omega N_j^{c*}(x, \Omega). \quad (26)$$

It is convenient to divide the scattering source term in the functional into two parts, the isotropic and anisotropic components. Thus we write the scattering cross section as

$$\Sigma_{jg}(x, \Omega, \Omega') = \Sigma_{jg}^{is}(x) + \Sigma_{jg}^{an}(x, \Omega, \Omega'), \quad (27)$$

where

$$\Sigma_{jg}^{is}(x) = \frac{1}{16\pi^2} \iint \Sigma_{jg}(x, \Omega, \Omega') d\Omega d\Omega'. \quad (28)$$

Because we are normally concerned with an isotropic medium (within plates) in which the differential scattering cross section is a function only of $|\Omega - \Omega'|$, Eq. (28) can normally be replaced by

$$\Sigma_{jg}^{is}(x) = \frac{1}{4\pi} \int \Sigma_{jg}(x, \Omega, \Omega') d\Omega. \quad (29)$$

In consequence of the definition of $\Sigma_{jg}^{is}(x)$, the quantity $\Sigma_{jg}^{an}(x, \Omega, \Omega')$ provides the angular dependence but has a z integrated value. The isotropic part is treated exactly as the fission matrix.

We considered three different ways of approximating the anisotropic term and chose the following as the best for

with existing computer programs with a minimum of modification required. The anisotropic cross section is expanded in the usual fashion in Legendre polynomials:

$$\Sigma_{j\theta}^{an}(x, \Omega, \Omega') = \sum_{\ell=1}^{\infty} \Sigma_{j\theta}^{(\ell)}(x) P_{\ell}(\mu_0) \left(\frac{2\ell+1}{4\pi} \right) \quad (30)$$

where

$$\mu_0 = \cos \theta_0,$$

and θ_0 is the angle between Ω and Ω' . The Ω and Ω' dependence is through $P_{\ell}(\mu_0)$. Since this is a strong function of Ω and Ω' we cannot replace $N_{\theta}^{c*} \Sigma_{j\theta}^{(\ell)} N_j^c$ by its cell average without some type of Ω and Ω' weighting. Consider the integral

$$I(x) = \iint d\Omega d\Omega' N_{\theta}^{h*} N_{\theta}^{c*} P_{\ell}(\mu_0) N_j^c N_j^h. \quad (31)$$

This is to be approximated by

$$I(x) \approx A_{j\theta}^{\ell}(x) \iint d\Omega d\Omega' N_{\theta}^{h*} P_{\ell}(\mu_0) N_j^h, \quad (32)$$

and then $A_{j\theta}^{\ell}(x)$ is to be the cell weighting function for $\Sigma_{j\theta}^{(\ell)}(x)$. We seek an $A_{j\theta}^{\ell}(x)$ as a suitable $P_{\ell}(\mu_0)$ weighted average of $N_{\theta}^{c*} N_j^c$. One cannot simply carry out the normal weighted average over all Ω and Ω' because the weighting function integrates to zero over the complete range. In the cell coordinate system in which μ and μ' are the cosine of the neutron angles relative to the normal to the plates, $P_{\ell}(\mu_0)$ can be expanded by the spherical harmonics addition theorem:

$$P_{\ell}(\mu_0) = P_{\ell}(\mu) P_{\ell}(\mu') + 2 \sum_{m=1}^{\ell} \frac{(\ell-m)!}{(\ell+m)!} P_{\ell}^m(\mu') P_{\ell}^m(\mu) \cos m(\phi - \phi'), \quad (33)$$

where

$$\mu = \Omega \cdot \mathbf{i}_x \quad \text{and} \quad \mu' = \Omega' \cdot \mathbf{i}_x, \quad (34)$$

and the P_{ℓ}^m are the associated Legendre functions; ϕ and ϕ' are the azimuthal angles. Now we temporarily divide the ranges of μ and μ' into the subranges of positive and negative values for $P_{\ell}(\mu)$ and $P_{\ell}(\mu')$ and note that any integrals over ϕ which contain $\cos m(\phi - \phi')$ will vanish because the cell functions are independent of azimuthal angle in this coordinate system. Therefore, terms containing the associated functions vanish, and our weighting function is just $P_{\ell}(\mu) P_{\ell}(\mu')$, which is now separable in μ and μ' allowing $A_{j\theta}(x)$ to be separated in j and g :

$$A_{j\theta}^{\ell}(x) = A_j^{\ell}(x) A_{\theta}^{\ell}(x); \quad (35)$$

but we have $A_j^{\ell+}$, $A_j^{\ell-}$, $A_{\theta}^{\ell+}$ and $A_{\theta}^{\ell-}$ because of the division into subranges of μ and μ' :

$$A_j^{\ell+}(x) \equiv \frac{\int N_j^c(x, \Omega) P_{\ell}(\mu) d\mu}{\int P_{\ell}(\mu) d\mu}; \quad P_{\ell}(\mu) > 0, \quad (36)$$

$$A_j^{\ell-}(x) \equiv \frac{\int N_j^c(x, \Omega) P_{\ell}(\mu) d\mu}{\int P_{\ell}(\mu) d\mu}; \quad P_{\ell}(\mu) < 0, \quad (37)$$

and similarly for $A_{\theta}^{\ell+}$ and $A_{\theta}^{\ell-}$ with N_j^c replaced by N_{θ}^{c*} . Since P_{ℓ} integrates to zero over the complete range in μ the denominators in these expressions are equal and opposite in sign. The $A_j^{\ell+}(x)$ and $A_j^{\ell-}(x)$ are both positive but not in general equal. Similarly for $A_{\theta}^{\ell+}(x)$ and $A_{\theta}^{\ell-}(x)$. As it is difficult to carry the subranges we now average the A^+ and A^- values to obtain a single $A_{j\theta}^{\ell}(x)$. It is given simply by

$$A_{j\theta}^{\ell}(x) \equiv \frac{\int N_j^c(x, \Omega) |P_{\ell}(\mu)| d\Omega \int N_{\theta}^{c*}(x, \Omega) |P_{\ell}(\mu)| d\Omega}{\left(\int |P_{\ell}(\mu)| d\Omega \right)^2} \quad (38)$$

This will be a good approximation if the individual A^+ and A^- values are not too different. This has not been verified.

Now performing the cell average and sums over j in J and g in G we have as an approximation for the anisotropic term in the functional

$$\sum_{j,g} \iiint dV d\Omega d\Omega' N_g^*(x, \Omega) \Sigma_{jg}^{an}(x, \Omega, \Omega') N_j(x, \Omega') \quad (39)$$

$$\approx \sum_{j,g} \sum_{\ell} \langle N^* \Sigma^{an} N \rangle_{jg}^{(\ell)} \iiint dV d\Omega d\Omega' N_g^{h*}(x, \Omega) P_{\ell}(\mu_0) N_j^h(x, \Omega'),$$

where

$$\langle N^* \Sigma^{an} N \rangle_{jg}^{(\ell)} = \frac{\frac{2\ell+1}{4\pi V} \sum_{j \text{ in } J} \sum_{g \text{ in } G} \int dV \Sigma_{jg}^{(\ell)}(x) \int d\Omega |P_{\ell}(\mu)| N_g^{c*}(x, \Omega) \int d\Omega' |P_{\ell}(\mu')| N_j^c(x, \Omega')}{\left(\int |P_{\ell}(\mu)| d\Omega \right)^2} \quad (40)$$

One is not likely in the near future to carry the expansion beyond P_3 . Up to P_3 the normalization integrals are given in Table V-8-I.

If the homogeneous problem is one dimensional spherical or slab geometry the $P_{\ell}(\mu_0)$ in Eq. (39) can again be expanded in spherical harmonics and the associated functions drop out due to azimuthal symmetry of N^h and N^{h*} leaving just $P_{\ell}(\mu)P_{\ell}(\mu')$.

Notice that the procedure developed here for the anisotropic cross section has an evident deficiency. The cross section weighting is independent of the orientation of the plates relative to the geometry of the homogeneous problem. We would expect that the homogenized anisotropic cross section should depend on the orientation but have not found an appropriate way of accounting for this in a simple manner.

GEOMETRY OF THE HOMOGENEOUS PROBLEM

The treatment of the gradient and surface terms is dependent upon the geometry of the homogeneous problem and orientation of the plates. We consider slab and spherical geometry and in the slab geometry we consider the situations of plates parallel to the slab surfaces and perpendicular to the slab surfaces. The most simple case requiring the least approximation is the slab geometry with plates parallel to slab surfaces.

Consider the gradient terms

$$\sum_g \iiint d\Omega dV \left(\frac{1}{2} N_g^* \Omega \cdot \nabla N_g - \frac{1}{2} N_g \Omega \cdot \nabla N_g^* \right) \quad (41)$$

$$= \frac{1}{2} \sum_g \iiint d\Omega dV [N_g^{c*} N_g^{h*} \Omega \cdot (N_g^c \nabla N_g^h + N_g^h \nabla N_g^c) - N_g^c N_g^h \Omega \cdot (N_g^{c*} \nabla N_g^{h*} + N_g^{h*} \nabla N_g^{c*})].$$

In Eq. (41) we make the same type of approximations we have used previously. The cell functions are replaced by their cell averaged and group collapsed values. Thus

$$\sum_g \iiint d\Omega dV \left(\frac{1}{2} N_g^* \Omega \cdot \nabla N_g - \frac{1}{2} N_g \Omega \cdot \nabla N_g^* \right) \approx \sum_g \iiint d\Omega dV \overline{N N^*}^g \left(\frac{1}{2} N_g^{h*} \Omega \cdot \nabla N_g^h - \frac{1}{2} N_g^h \Omega \cdot \nabla N_g^{h*} \right) \quad (42)$$

$$+ \sum_g \iiint d\Omega dV N_g^h N_g^{h*} \overline{\left(\frac{1}{2} N^* \Omega \cdot \nabla N - \frac{1}{2} N \Omega \cdot \nabla N^* \right)}^g$$

TABLE V-8-I. NORMALIZATION INTEGRALS FOR
| P_{ℓ} | AVERAGING

ℓ	$\int P_{\ell} d\Omega$
0	4π
1	2π
2	$8\pi/3\sqrt{3}$
3	$13\pi/10$

The second term in this approximation, the one involving the cell average of the gradient terms, is small compared to the first. However, the heterogeneity effect under investigation is a small effect and numerical calculations have demonstrated that this term can not be dropped. Since the cell averaged quantity multiplies $N_a^h N_a^{h*}$ in the reduced functional it has the effect of producing a shift in the total cross section. It is positive in most cases in fast criticals and therefore corresponds to an absorption effect. In pseudo-physical terms it is the result of the fact that the cell integrated flow of neutron importance does not vanish as does the cell integrated flow of neutrons. Because of cell boundary conditions the two cell averaged gradient terms are numerically equal and can be replaced by $N^* \mu \frac{dN}{dx}$ where we have also replaced $\Omega \cdot \nabla$ by $\mu \frac{d}{dx}$ in the cell coordinate system.

In the first term on the right in Eq. (42) the cell averaged quantity $\overline{NN^*}$ should be averaged with some form of Ω weighting because of the $\Omega \cdot \nabla$ terms in the integrand. The direction of the gradient of the homogeneous functions depends upon the geometry and therefore the nature of the Ω weighting required depends on the geometry. We consider some special cases.

SLAB GEOMETRY WITH PLATES PARALLEL TO SLAB SURFACES

In this most simple case the coordinate systems are the same for the cell and homogeneous problems and the gradient is normal to slab and plate surfaces, in the direction i_x . The proper weighting function is $\Omega \cdot i_x$, which is just μ . We divide the Ω integration temporarily into two subranges, angles for which μ is positive and angles for which it is negative and define the averages:

$$\overline{N_a^c N_a^{c*+}} \equiv \eta_a^+ \equiv \frac{\frac{1}{V} \sum_{\theta \text{ in } a} \iint_{\mu > 0} dV d\Omega \mu N_a^c(x, \Omega) N_a^{c*}(x, \Omega)}{\int_{\mu > 0} \mu d\Omega}, \quad (43)$$

$$\overline{N_a^c N_a^{c*-}} \equiv \eta_a^- \equiv \frac{\frac{1}{V} \sum_{\theta \text{ in } a} \iint_{\mu < 0} dV d\Omega \mu N_a^c(x, \Omega) N_a^{c*}(x, \Omega)}{\int_{\mu < 0} \mu d\Omega}. \quad (44)$$

Now we can either proceed with separate consideration for the angles $+$ and $-$, or if η_a^+ and η_a^- are nearly equal we can merge the problem again and define

$$\begin{aligned} \eta_a &= \frac{\eta_a^+ + \eta_a^-}{2} = \frac{\frac{1}{V} \sum_{\theta \text{ in } a} \iint dV d\Omega |\mu| N_a^c(x, \Omega) N_a^{c*}(x, \Omega)}{\int |\mu| d\Omega} \\ &= \frac{1}{2\pi V} \sum_{\theta \text{ in } a} \iint dV d\Omega |\mu| N_a^c(x, \Omega) N_a^{c*}(x, \Omega). \end{aligned} \quad (45)$$

We take this latter approach and expect it to be good enough. The reason for originally splitting the angular integration was of course that the weighting function μ is an odd function which integrates to zero over the complete range of μ . Thus we have as our approximation for the gradient terms

$$\begin{aligned} \frac{1}{2} \sum_{\theta} \iint d\Omega dV (N_a^* \Omega \cdot \nabla N_a - N_a \Omega \cdot \nabla N_a^*) &\approx \frac{1}{2} \sum_{\theta} \iint d\Omega dV \eta_a (N_a^{h*} \Omega \cdot \nabla N_a^h - N_a^h \Omega \cdot \nabla N_a^{h*}) \\ &\quad + \sum_{\theta} \iint d\Omega dV N_a^h N_a^{h*} \overline{N^* \mu \frac{dN}{dx}}. \end{aligned} \quad (46)$$

In the terms in the functional involving surface integrals we make the approximation analogous to Eqs. (1) and (2):

$$\alpha_a = A_a \alpha_a^h(x, \Omega) \alpha_a^c(x, \Omega); \quad \alpha_a^* = A_a \alpha_a^{h*}(x, \Omega) \alpha_a^{c*}(x, \Omega), \quad (47)$$

where α_g^c and α_g^{c*} are specified functions,

$$\alpha_g^c = \mathbf{i}_s \cdot \boldsymbol{\Omega} N_g^c(x, \Omega), \quad \alpha_g^{c*} = \mathbf{i}_s \cdot \boldsymbol{\Omega} N_g^{c*}(x, \Omega), \quad (48)$$

and α_g^h and α_g^{h*} are unknowns to be determined from the variational principle. We maintain these functions as independent of the N_g^h and N_g^{h*} functions in spite of the fact that α and N are related for the exact solution by Eqs. (11) and (15). The variational principle applied to the reduced functional then establishes the relation between the approximate solutions α_g^h , α_g^{h*} and N_g^h , N_g^{h*} .

The constants A_g are merely conveniences. We are free to specify them at will since the α_g^h functions are anyway regarded as unknowns. The ultimate relation between the α_g^h and N_g^h will depend upon the choice for A_g .

Equations (47) and (1) are substituted into the surface integrals and the quantity

$$\sum_{g \text{ in } g} N_g^{c*}(x, \Omega) N_g^c(x, \Omega) \quad (49)$$

is extracted from the surface integrals by approximating it by its $\boldsymbol{\Omega} \cdot \mathbf{i}_s$ weighted average value on the surface. Now the convenient choice for the A_g is

$$A_g \equiv \eta_g / \overline{N_g^{c*} N_g^c}, \quad (50)$$

the bar denoting the same as in Eq. (42) except that the average is over the surface instead of the volume. With this choice a typical surface integral approximation is

$$-\frac{1}{2} \sum_g \iint d\Omega dS_i \{ \alpha_g^{h*}(x, \Omega) [N_g^+(x, \Omega) - N_g^-(x, \Omega)] \} \approx -\frac{1}{2} \sum_g \eta_g \iint d\Omega dS_i \{ \mathbf{i}_s \cdot \boldsymbol{\Omega} \alpha_g^{h*} [N_g^{h+}(x, \Omega) - N_g^{h-}(x, \Omega)] \} \quad (51)$$

and the reason for the choice of A_g is now evident. We want the surface terms to be multiplied by the same constant (η_g) as the gradient terms. The result is that $\mathbf{i}_s \cdot \boldsymbol{\Omega} \alpha_g^h$ and $\mathbf{i}_s \cdot \boldsymbol{\Omega} \alpha_g^{h*}$ will then be found to bear the same relation to N_g^h and N_g^{h*} as did α_g and α_g^* to N_g and N_g^* .

A further evident result is that there will be a slight contradiction between α_g as calculated from Eqs. (9) or (11) by recombining N_g^h and N_g^c through Eq. (1); and α_g as calculated from A_g , α_g^c , and α_g^h inserted into Eq. (47). The contradiction is from the fact that the $|\mu|$ weighted average of $N_g^c N_g^{c*}$ over the volume is not identical to the average over the surface so that A_g is not quite identically unity. Notice that there is never any need to calculate A_g or $\overline{N_g^{c*} N_g^c}$ unless one wishes to check the magnitude of this contradiction.

It might appear that one could avoid this conflict by not leaving α_g^h and α_g^{h*} as additional undetermined functions in the approximate functional but instead to specify them as identical to N_g^h and N_g^{h*} . One then finds that the development proceeds similarly except that it leads to discontinuous boundary conditions on N_g^h and N_g^{h*} at internal boundaries, and external boundary conditions that say that both N_g^h and N_g^{h*} vanish for all angles. The internal discontinuities could be handled, but the external conditions can be satisfied only for the trivial solution $N_g^h = N_g^{h*} = 0$ everywhere.

On the other hand, the contradiction on the value of α is not serious provided we can demonstrate numerically that the procedure leads to a good estimate of the flux and more importantly to an accurate eigenvalue. We therefore proceed with the approach of keeping α_g^h as a function to be determined by the variation principle.

Further comment is in order on the meaning of the surface integrals in the approximate functional. In our original functional representing a critical assembly system the pertinent surfaces were the interfaces between plates of fuel, sodium, steel, etc. Since N_g^c and N_g^{c*} were designed to account for the fine structure, we require that N_g^h and N_g^{h*} are a priori continuous across such surfaces. In the approximate functional the coefficients are constants within larger regions of the reactor, i.e., core and blanket, or perhaps the core might be divided into zones for power flattening. We allow that N_g^h and N_g^{h*} could conceivably be discontinuous across such surfaces between zones. The variation of the functional being set equal to zero will be seen to lead to continuity of these functions across such boundaries also. However, we cannot assume so in advance. Therefore, the surface integrals are over macroscopic region surfaces of this type and there will be a value of the constants like η_g , and other cell-averaged quantities developed above, for each such region.

We have now to gather together the several approximations outlined above, put them into the original functional, and thereby obtain the approximate reduced functional. The result for slab geometry is

$$\begin{aligned}
F_R[N_\sigma^h(x, \Omega), N_\sigma^{h*}(x, \Omega), \alpha_\sigma^h(x, \Omega), \alpha_\sigma^{h*}(x, \Omega)] = & \sum_\sigma \iint dV d\Omega \left[\frac{1}{2} \eta_\sigma N_\sigma^{h*}(x, \Omega) \mathbf{\Omega} \cdot \nabla N_\sigma^h - \frac{1}{2} \eta_\sigma N_\sigma^h(x, \Omega) \mathbf{\Omega} \cdot \nabla N_\sigma^{h*}(x, \Omega) \right. \\
& + \overline{N^* \mu} \frac{dN^a}{dx} N_\sigma^{h*}(x, \Omega) N_\sigma^h(x, \Omega) + \overline{N^* \Sigma N}^a N_\sigma^{h*}(x, \Omega) N_\sigma^h(x, \Omega) - N_\sigma^{h*}(x, \Omega) \sum_j \langle N^* \Sigma^{is} N \rangle_{j\sigma} \int d\Omega' N_j^h(x, \Omega') \\
& - \frac{1}{\lambda} N_\sigma^{h*}(x, \Omega) \sum_j \langle N^* \chi \nu \Sigma_j N \rangle_{j\sigma} \int d\Omega' N_j^h(x, \Omega') - N_\sigma^{h*}(x, \Omega) \sum_{\ell, j} \langle N^* \Sigma^{an} N \rangle_{j\sigma}^{(\ell)} P_\ell(\mu) \int d\Omega' N_j^h(x, \Omega') P_\ell(\mu') \Big] \quad (52) \\
& + \frac{1}{2} \sum_\sigma \eta_\sigma \iint_{i_s \cdot \mathbf{\Omega} > 0} d\Omega dS_\sigma N_\sigma^{h*}(x, \Omega) \alpha_\sigma^h(x, \Omega) - \frac{1}{2} \sum_\sigma \eta_\sigma \iint_{i_s \cdot \mathbf{\Omega} < 0} d\Omega dS_\sigma N_\sigma^h(x, \Omega) \alpha_\sigma^{h*}(x, \Omega) \\
& - \frac{1}{2} \sum_\sigma \eta_\sigma \iint d\Omega dS_i \{ \alpha_\sigma^{h*}(x, \Omega) [N_\sigma^{h+}(x, \Omega) - N_\sigma^{h-}(x, \Omega)] - \alpha_\sigma^h(x, \Omega) [N_\sigma^{h*+}(x, \Omega) - N_\sigma^{h*-}(x, \Omega)] \}.
\end{aligned}$$

For convenience, we consolidate the definitions of the various cell averaged quantities here:

$$\eta_\sigma \equiv \frac{1}{2\pi V} \sum_{\text{in } \sigma} \iint d\Omega dV N_\sigma^{c*}(x, \Omega) | \mu | N_\sigma^c(x, \Omega), \quad (53)$$

$$\overline{N^* \Sigma N}^a \equiv \frac{1}{4\pi V} \sum_{\text{in } \sigma} \iint d\Omega dV N_\sigma^{c*}(x, \Omega) \Sigma_\sigma(x) N_\sigma^c(x, \Omega), \quad (54)$$

$$\langle N^* \Sigma^{is} N \rangle_{j\sigma} \equiv \frac{1}{16\pi^2 V} \sum_{\text{in } \sigma} \sum_{j \text{ in } j} \iiint dV d\Omega d\Omega' N_\sigma^{c*}(x, \Omega) \Sigma_{j\sigma}^{is} N_j^c(x, \Omega'), \quad (55)$$

$$\langle N^* \chi \nu \Sigma_j N \rangle_{j\sigma} \equiv \frac{1}{16\pi^2 V} \sum_{\text{in } \sigma} \sum_{j \text{ in } j} \iiint dV d\Omega d\Omega' N_\sigma^{c*}(x, \Omega) \chi_{\sigma\nu j} \Sigma_{j\sigma}(x) N_j^c(x, \Omega'), \quad (56)$$

$$\langle N^* \Sigma^{an} N \rangle_{j\sigma}^{(\ell)} \equiv \frac{\frac{2\ell+1}{4\pi V} \sum_{\text{in } \sigma} \sum_{j \text{ in } j} \int dV \Sigma_{j\sigma}^{(\ell)}(x) \int d\Omega | P_\ell(\mu) | N_\sigma^{c*}(x, \Omega) \int d\Omega' | P_\ell(\mu') | N_\sigma^c(x, \Omega')}{\left[\int | P_\ell(\mu) | d\Omega \right]^2} \quad (57)$$

$$\overline{N^* \mu} \frac{dN^a}{dx} \equiv \frac{1}{4\pi V} \sum_{\text{in } \sigma} \iint d\Omega dV N_\sigma^{c*}(x, \Omega) \mu \frac{dN_\sigma^c(x, \Omega)}{dx}. \quad (58)$$

Comparing Eqs. (52) and (4) it can be seen that the reduced functional has the same form as the original functional¹ except that one of the group constants [Eqs. (53)–(58)] is contained in each of the terms. We can therefore write down directly the equations and boundary conditions that will result from setting the variation of F_R equal to zero. In these equations we divide through the group constant η_σ in order that the equations will be in the standard form. The result is

$$\mathbf{\Omega} \cdot \nabla N_\sigma^h + \Sigma_\sigma N_\sigma^h - \frac{1}{\lambda} \sum_j \langle \chi \nu \Sigma_j \rangle_{j\sigma} \int N_j^h d\Omega - \sum_j \Sigma_{j\sigma}^{is} \int d\Omega N_j^h - \sum_{\ell, j} \Sigma_{j\sigma}^{(\ell)} P_\ell(\mu) \int d\Omega N_j^h P_\ell(\mu) = 0 \quad (59)$$

and

$$-\mathbf{\Omega} \cdot \nabla N_\sigma^{h*} + \Sigma_\sigma N_\sigma^{h*} - \frac{1}{\lambda} \sum_j \langle \chi \nu \Sigma_j \rangle_{j\sigma} \int N_j^{h*} d\Omega - \sum_j \Sigma_{j\sigma}^{is} \int d\Omega N_j^{h*} - \sum_{\ell, j} \Sigma_{j\sigma}^{(\ell)} P_\ell(\mu) \int d\Omega N_j^{h*} P_\ell(\mu) = 0; \quad (60)$$

with boundary conditions

$$N_\sigma^h \quad \text{and} \quad N_\sigma^{h*} \quad \text{are continuous functions} \quad (61)$$

$$N_\sigma^h = 0; \quad \mathbf{\Omega} \cdot \mathbf{i}_s < 0 \quad \text{on external boundary} \quad (62)$$

$$N_\sigma^{h*} = 0; \quad \mathbf{\Omega} \cdot \mathbf{i}_s > 0 \quad \text{on external boundary.} \quad (63)$$

ie unknowns α_σ^h and α_σ^{h*} are determined to be

$$\alpha_\sigma^h = N_\sigma^h, \quad \alpha_\sigma^{h*} = N_\sigma^{h*}. \quad (64)$$

In Eqs. (59) and (60) we have defined the group cross sections:

$$\Sigma_g \equiv \frac{\overline{N^* \Sigma N}^g}{\eta_g} + \frac{\overline{N^* \mu \frac{dN}{dx}}^g}{\eta_g} \quad (65)$$

$$\langle \chi \nu \Sigma_f \rangle_{Jg} \equiv \langle N^* \chi \nu \Sigma_f N \rangle_{Jg} / \eta_g, \quad (66)$$

$$\Sigma_{Jg}^{is} \equiv \langle N^* \Sigma^{is} N \rangle_{Jg} / \eta_g, \quad (67)$$

$$\Sigma_{Jg}^{(l)} \equiv \langle N^* \Sigma^{an} N \rangle_{Jg}^{(l)} / \eta_g, \quad (68)$$

where the quantities on the right hand side of the equations are defined in Eqs. (53)–(58).

SLAB GEOMETRY WITH PLATES PERPENDICULAR TO SLAB SURFACES

In this case the gradient of the homogeneous functions is parallel to plate surfaces. In the cell coordinate system $\Omega \cdot \mathbf{i}_x$ is not μ but rather $\sqrt{1 - \mu^2} \cos \phi$ where ϕ is the azimuthal angle. The cell functions are independent of ϕ so we need not consider any ϕ weighting in η , but we do have the $\sqrt{1 - \mu^2}$ weighting:

$$\begin{aligned} \eta_g &= \frac{\frac{1}{V} \sum_{g \text{ in } g} \iint N_g^c N_g^{c*} \sqrt{1 - \mu^2} dV d\Omega}{\int \sqrt{1 - \mu^2} d\Omega} \\ &= \frac{1}{\pi^2 V} \iint N_g^c N_g^{c*} \sqrt{1 - \mu^2} dV d\Omega. \end{aligned} \quad (69)$$

The other expressions remain unchanged.

SPHERICAL GEOMETRY

In spherical geometry the gradient is in the radial direction which is not a single direction but depends upon the angular position of the point in question. Since we do not want position dependent group constants in the homogeneous model it is necessary to average the $\Omega \cdot \nabla$ weighting function over angular position. This averaging certainly weakens the μ dependence of the weighting and while we have not provided a well founded basis for it, our recommendation is to use no μ weighting in spherical problems:

$$\eta_g = \frac{1}{4\pi V} \sum_{g \text{ in } g} \iint N_g^c N_g^{c*} dV d\Omega. \quad (70)$$

Again, the other expressions are the same as for slab problems.

CYLINDRICAL GEOMETRY

The situation in cylindrical geometry is complex and we have not developed a general expression for η_g . The recommendation is to judge whether the cylinder is more like a sphere or a slab and choose one of the above definitions for η . It is fortunate that a few preliminary results indicate that the calculated multiplication constant is not strongly dependent upon the recipe used for η_g . In one particular "Gedanken Experiment", which was a slab problem with plates parallel to slab surfaces, the homogenization procedure with flux weighting only, and with μ weighting in η gave a k differing from the "exact" calculation by 0.02 % compared to 0.11 % with no μ weighting. The total heterogeneity effect was 0.63 %. Thus, the weighting was not insignificant for this case, but it is evident that a good estimate can be made if one just uses the weighting for the geometry which most nearly approximates the problem under investigation.

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V-9. A Fast Accurate Technique for Calculation of the Resonance J -Function

R. B. NICHOLSON and G. GRASSESCHI

The well-known J -Function occurring in calculations of resonance capture of neutrons is

$$J(\theta, \beta) = \int_0^\infty \frac{\Psi(\theta, x) dx}{\Psi(\theta, x) + \beta}, \quad (1)$$

where Ψ is the Doppler broadened shape function.¹ The Ψ function can be calculated quite accurately by the ANL QUICKW code.² However, in multigroup cross section preparation codes such as MC² the J function must be calculated many times. To avoid excessive computer time it is important that the J function be calculated accurately with as few ordinates as possible, thus limiting the number of entries into QUICKW. We have found that a simple manipulation of the integral results in accurate calculations by low order Gauss-Legendre quadrature.

Define $I(\theta, \beta, x)$ to be the integrand in Eq. (1):

$$I(\theta, \beta, x) \equiv \frac{\Psi(\theta, x)}{\Psi(\theta, x) + \beta}. \quad (2)$$

The range of integration is broken into two subranges $0 \rightarrow x_1$ and $x_1 \rightarrow \infty$ and in the latter range the change of variable $x = 1/y$ is made. The result is:

$$J(\theta, \beta) = \int_0^{x_1} I(\theta, \beta, x) dx + \int_0^{1/x_1} \frac{I(\theta, \beta, 1/y) dy}{y^2}. \quad (3)$$

The qualitative behavior of the Ψ function is such that the two integrands in Eq. (3) are both finite and slowly varying over their entire ranges, providing a good choice is made for x_1 . The result is that the integrals can be done to high accuracy with surprisingly low order quadratures. For this to be true for all values of θ and β , the break point x_1 must be a function of θ and β . By

considering the asymptotic behavior of the Ψ function for large and small θ and by a little numerical experimentation we have been able to find an approximate minimum for the errors in the quadrature formula. Define

$$x_a = [(\beta + 1)/\beta]^{1/2}; \quad (4)$$

$$x_b = \frac{2}{\theta} [\ln(6.6 + 4.4 \theta/\beta)]^{1/2}. \quad (5)$$

The break point is taken to be the greater of x_a and x_b :

$$x_1 = x_a \quad \text{if} \quad x_a \geq x_b; \quad (6)$$

$$x_1 = x_b \quad \text{if} \quad x_a < x_b. \quad (7)$$

If the two integrals in Eq. (3) are evaluated by four-point Gauss-Legendre quadratures the error in J is always less than 0.1% for the ranges $\theta = 10^{-3}$ to 1.0 and $\beta = 10^{-3}$ to ∞ . Outside these ranges the errors sometimes but not often approach 0.2%. The errors decrease very rapidly as the order of the quadrature is increased, but it would not often be important to attain higher accuracy than that given by the four-point quadrature. In fact, for some purposes the two-point quadratures would be sufficiently accurate, the errors being about 1%. This could be quite useful in calculations with a small computer because a two-point quadrature is nearly equivalent to an analytic result.

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V-10. Summing Exponential Integral Functions with Application to the Calculation of Collision Probabilities

A. P. OLSON

Integral transport theory was used in a companion paper (see Paper V-11) to derive plate-to-plate collision probabilities in infinite slab geometry as used in the RABID code. The "exact" analytic expressions derived

involve many infinite sums of the exponential integral functions

$$E_n(z) = \int_1^\infty e^{-zt} dt/t^n, \quad n \geq 0. \quad (1)$$

A related function¹ which is needed is

$$\alpha_n(z) = \int_1^\infty t^n e^{-zt} dt, \quad n \geq 0. \quad (2)$$

The sum we wish to evaluate numerically for a unit cell of optical thickness h is

$$S_n(z, h) = \int_1^\infty \frac{e^{-zt} dt}{(1 - e^{-ht})t^n} \quad (3)$$

$$= \sum_{k=0}^\infty E_n(z + kh), \quad n \geq 0.$$

A very fast, accurate method for calculating $S_n(z, h)$ has been derived based on special Gaussian quadrature formulas.² That is, let

$$\int_1^\infty e^{-xz} f(x) dx/x^n = \sum_{i=1}^m w_{i,n} f(t_{i,n}) + R_m(z) \quad (4)$$

$$\int_1^\infty g(x) dx/x^n \quad (5)$$

$$= \sum_{i=1}^m [w_{i,n} \exp(t_{i,n})] g(t_{i,n}) + R_m(z)$$

where $R_m(z)$ is the remainder (or error in the m th order quadrature), $f(x)$ is an arbitrary function $g(x) = e^{-xz}f(x)$, and $w_{i,n}$ and $t_{i,n}$ are quadrature weights and nodes. The quadratures are exact if $f(x)$ is a polynomial of degree less than or equal to $2m - 1$. One obtains

$$S_n(z, h) \approx \sum_{i=1}^m \frac{w_{i,n} \exp(t_{i,n})}{1 - \exp(-ht_{i,n})} \exp(-zt_{i,n}). \quad (6)$$

These Gaussian quadratures are least accurate for either z or h small (say < 0.1). Also, the error increases as z or h approaches zero and as m decreases. A complementary numerical technique has been derived which rapidly becomes more accurate as h decreases. The Euler-MacLaurin summation formula³ leads to

$$S_n(z, h) = E_{n+1}(z)/h + E_n(z)/2 + hE_{n-1}(z)/12$$

$$- h^3 E_{n-3}(z)/720 + h^5 E_{n-5}(z)/30,240$$

$$- h^7 E_{n-7}(z)/1,209,600 \quad (7)$$

$$+ h^9 E_{n-9}(z)/47,900,160 - \dots,$$

where $E_{n-\ell}(z)$ is replaced by $\alpha_{\ell-n}(z)$ if $n - \ell < 0$. The truncation error of the series in Eq. (7) is less than the absolute value of the first neglected term for h sufficiently small.

Much simpler expressions can be obtained using the recursion relation

$$E_n(z) = [e^{-z} - zE_{n-1}(z)]/(n-1) \quad (8)$$

and the definition

$$E_0(z) = e^{-z}/z. \quad (9)$$

For example,

$$S_2(z, h) \approx \frac{1}{2} \left\{ \exp(-z) \left[\frac{1}{h} + \frac{h}{6z} - \frac{h^3(1+z)}{360z^2} \right] \right.$$

$$\left. + E_2(z) \left(1 - \frac{z}{h} - \frac{h}{6z} \right) \right\} + \frac{h^5}{5040z^4}, \quad (10)$$

$$|\text{error}| < h^7 \alpha_5(z)/1,209,600 < h^7/(10,080z^6);$$

$$S_3(z, h) \approx \frac{1}{12} \left\{ \exp(-z) \right.$$

$$\left. \left[\frac{4-2z}{h} + 3 - \frac{h^3([1+z+z^2]/2)}{1260z^3} \right] \right.$$

$$\left. + E_2(z) \left(h - 3z + \frac{2z^2}{h} \right) \right\}, \quad (11)$$

$$|\text{error}| < h^7 \alpha_4(z)/1,209,600 < h^7/(50,400z^5);$$

$$S_4(z, h) \approx \frac{1}{12} \left\{ \exp(-z) \right.$$

$$\left. \left[\frac{3-z}{h} + 2 + \frac{h^3(1-z)}{60z^2} \right] \right.$$

$$\left. + E_3(z) \left(\frac{z^2}{h} + h - 2z - \frac{h^3}{30z^2} \right) \right\}, \quad (12)$$

$$|\text{error}| < h^5 \alpha_1(z)/30,240 < h^5/(30,240z^2).$$

Equations (10)–(12) require $z \geq h$ for accuracy. Equation (10) in particular benefits by the restriction $z \geq 2h$. For small arguments, $E_2(z)$ and $E_3(z)$ are quickly obtained from series expansions.

Another numerical difficulty arises for optically thin plates, such as thin foils. The collision rate in the foil involves taking the difference $S_{n+1}(z, h) - S_{n+1}(z + \Delta, h)$ where Δ is very small. In this case, all numerical accuracy is eventually lost for Δ sufficiently small. A

Taylor series expansion about $z + \frac{\Delta}{2}$ yields

$$E_{n+1} \left(z - \frac{\Delta}{2} \right) - E_{n+1} \left(z + \frac{\Delta}{2} \right) \quad (13)$$

$$= \Delta \left[E_n(z) + \frac{\Delta^2 E_{n-2}(z)}{24} + \frac{\Delta^4 E_{n-4}(z)}{1920} + \dots \right],$$

where again $\alpha_{\ell-n}(z)$ is substituted for $E_{n-\ell}(z)$ if $n - \ell < 0$.

Equation (13) is a rapidly convergent expansion for $\Delta < 0.01$ which improves in accuracy as $\Delta \rightarrow 0$. It also leads to easy numerical evaluation via the Gaussian quadrature of Eq. (6):

$$S_{n+1}(z, h) - S_{n+1}(z + \Delta, h)$$

$$\approx \Delta \sum_{i=1}^m \left[\frac{w_{i,n} \exp(t_{i,n})}{1 - \exp(ht_{i,n})} \right] \exp \left[- \left(z + \frac{\Delta}{2} \right) t_{i,n} \right] \quad (14)$$

$$\cdot \left(1 + \frac{\Delta^2 t_{i,n}^2}{24} + \frac{\Delta^4 t_{i,n}^4}{1920} \right), \quad \Delta < 0.01.$$

The given numerical techniques are all in use in several codes such as RABID, and versions of the perturbation theory heterogeneity effect codes CALHET⁽⁴⁾ and CALHET-2.⁽⁵⁾

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V-11. Developments in Integral Transport Methods for Resonance Region Studies in Plate-Type Lattices

A. P. OLSON

INTRODUCTION

A tractable approach to the computation of resonance absorption, average neutron fluxes, and group cross sections for multiplate fast reactor cells of the ZPR type has been developed by P. Kier and A. Robba¹ (see Paper V-7). Their technique, herein described as the "RABBLE Method", assumes that the neutron currents at plate interfaces can be resolved into two components: one with a cosine angular distribution, the other with a (cosine)² angular distribution. This assumption enables the definition of transmission probabilities for these current components which, together with escape probabilities for spatially flat iso-

tropic sources, leads to a simple solution of the spatial part of the integral transport equation.

A more accurate treatment of the spatial dependence of neutron flux is possible if one makes no assumptions at all about the angular distribution of the interface currents. Instead, the integral transport equation is solved "exactly" by the RABID code under the assumption that the slowing-down source is spatially linear, rather than flat as in RABBLE.

THEORY

INTEGRAL TRANSPORT EQUATION

The integral form of the Boltzmann transport equation in a source-free medium is

$$\phi(\mathbf{r}, u) = \iiint d\mathbf{r}' \int_0^u \frac{du' F_s(u', \mathbf{r}') P(u' \rightarrow u) \exp[-\Sigma_t(u, \mathbf{r}') |\mathbf{r} - \mathbf{r}'|]}{4\pi |\mathbf{r} - \mathbf{r}'|^2}, \quad (1)$$

where the scattering rate at lethargy u' and space point $\mathbf{r}'$ is

$$F_s(u', \mathbf{r}') = \Sigma_s(u') \phi(u', \mathbf{r}'). \quad (2)$$

The probability per unit lethargy that a neutron is scattered from u' to u is

$$P(u' \rightarrow u) = \begin{cases} \frac{1}{1-\alpha} e^{-(u-u')}, & u \geq u' \geq u - \epsilon \\ 0, & u' < u - \epsilon \end{cases} \quad (3)$$

where

$$\alpha = \left(\frac{A-1}{A+1} \right)^2 \quad \text{and} \quad \epsilon = 2 \ln \left(\frac{A+1}{A-1} \right),$$

and A is the ratio of scattering material mass to neutron mass. Equation (1) can be reduced to 1-dimensional form for infinite slab geometry by integrating over the other two space coordinates as follows:

$$\begin{aligned} \iiint \frac{dx' F_s(u', \mathbf{r}') \exp[-\Sigma_t(u, \mathbf{r}') |\mathbf{r} - \mathbf{r}'|]}{4\pi |\mathbf{r} - \mathbf{r}'|^2} &= \int dx' F_s(u', x') \int_0^\infty \frac{2\pi r dr \exp[-\Sigma_t(u, \mathbf{r}') (r^2 + |x - x'|^2)^{1/2}]}{4\pi (r^2 + |x - x'|^2)} \\ &= \int dx' F_s(u', x') \int_1^\infty \exp[-\Sigma_t(u) |x - x'| t] dt/t = \frac{1}{2} \int dx' F_s(u', x') E_1[\Sigma_t(u) |x - x'|], \end{aligned} \quad (4)$$

where the scattering source now depends only on x' and u' , and the exponential integral functions are

$$E_n(Z) \equiv \int_1^\infty \exp(-Zt) dt/t^n. \quad (5)$$

Equation (1) now becomes

$$\phi(x, u) = \frac{1}{2} \int_0^u du' F_s(u', x') P(u' \rightarrow u) E_1[\Sigma_t(u) |x - x'|]. \quad (6)$$

A similar equation for the neutron current can be obtained:

$$\mathbf{J}(x, u) = \frac{1}{2} \int dx' \int_0^u du' F_s(u', x') P(u' \rightarrow u) E_2[\Sigma_t(u) |x - x'|]. \quad (7)$$

The lethargy dependence must now be converted from a continuous function to a multigroup formalism in which the group lethargy width is very narrow ($\Delta u \lesssim 10^{-3}$). Now define

$$\phi_\ell(x) \Delta u = \int_{\text{group } \ell} \phi(x, u') du' \quad (8)$$

$$f_{s\ell}(x') \Delta u = \int_{\text{group } \ell} du' F_s(u', x'), \quad (9)$$

$$\begin{aligned} P_\ell \Delta u &= \frac{1}{1 - \alpha} \int_{u_0}^{u_0 + \Delta u} du \int_{u_0 - \ell \Delta u}^{u_0 - (\ell-1) \Delta u} du' e^{-(u-u')} \\ &= \frac{(1 - e^{-\Delta u})^2}{1 - \alpha} e^{-(\ell-1) \Delta u}, \quad \ell = 1, 2, \dots, L; \end{aligned} \quad (10)$$

$$\begin{aligned} P_s \Delta u &= \frac{1}{1 - \alpha} \int_{u_0}^{u_0 + \Delta u} du \int_{u_0}^u du' e^{-(u-u')} \\ &= \frac{1}{1 - \alpha} (\Delta u - 1 + e^{-\Delta u}). \end{aligned} \quad (11)$$

Let $L = \epsilon/\Delta u$ be an integral number of groups (by a minor adjustment to the mass ratio A , if necessary). Then the slowing down source in group k from all other groups is

$$S_{0k}(x') = \sum_{\ell=1}^L f_{s\ell}(x') P_\ell \Delta u - \alpha \Sigma_{sk-L} \phi_{k-L} P_s \Delta u, \quad (12)$$

and the in-group (self-scatter) source is

$$S_{sk}(x') = \Sigma_{sk} \phi_k P_s \Delta u,$$

where $P_s \Delta u$ is the probability for self-scatter. Some of the neutrons scattered in group $k - L$ cannot reach group k , leading to the correction term

$$\alpha \Sigma_{sk-L} \phi_{k-L} P_s \Delta u.$$

Now Eqs. (6) and (7) become

$$\phi_k(x) = \frac{1}{2} \int dx' \left[\sum_{l=1}^L f_{sk-l}(x') P_l \Delta u + \Sigma_{sk} \phi_k P_s \Delta u \right] E_1(\Sigma_{lk} | x - x' |), \quad (13)$$

$$J_k(x) = \frac{1}{2} \int dx' \left[\sum_{l=1}^L f_{sk-l}(x') P_l \Delta u + \Sigma_{sk} \phi_k P_s \Delta u \right] E_2(\Sigma_{lk} | x - x' |). \quad (14)$$

COLLISION RATES

The collision rate within a plate is the difference between the uncollided current in and the uncollided current out. For example, the current at τ mean free paths beyond a plate of optical thickness τ_1 is

$$J(\tau, \tau_1) = \frac{1}{2} \int_0^{\tau_1} dx' [S_0(x') + S_s(x')] E_2(\tau + \Sigma_{11} x'). \quad (15)$$

Since the self-scatter term is never more than 1 or 2% of the total source, it is a good approximation to assume that both source components have the same x' -dependence:

$$S(x') = S_0(x') + S_s(x') = \bar{S} + \left(x' - \frac{t_1}{2} \right) \frac{\Delta S}{t_1}. \quad (16)$$

Then

$$J(\tau, \tau_1) = \frac{1}{2} \int_0^{\tau_1} dx' S(x') E_2(\tau + \Sigma_{11} x') = \frac{\bar{S}}{2\tau_1} [E_3(\tau) - E_3(\tau + \tau_1)] \\ + \frac{\Delta S}{2\tau_1} \left\{ \frac{1}{2} [E_3(\tau) + E_3(\tau_1 + \tau)] - \frac{1}{\tau_1} [E_4(\tau) - E_4(\tau_1 + \tau)] \right\}. \quad (17)$$

To obtain the collision rate in plate 2 due to the source in plate 1, in which τ mean free paths separate the plates, one evaluates Eq. (17) at both sides of plate 2:

$$\mathbf{CR}(1 \rightarrow 2) = J(\tau, \tau_1) - J(\tau + \tau_2, \tau_1). \quad (18)$$

Assuming an array of unit cells which repeat to infinity in both directions (periodic boundary conditions), the contribution by all plates of type 1 in a given direction is

$$\mathbf{CR}_\infty(1 \rightarrow 2) = \sum_{m=0}^{\infty} [J(\tau + mh, \tau_1) - J(\tau + \tau_2 + mh, \tau_1)]. \quad (19)$$

The average collided flux is

$$\phi = \mathbf{CR} / \Sigma_t. \quad (20)$$

If the source strength is unity ($\bar{S}t = 1$), then Eq. (19) evaluated in forward and backward directions yields collision probabilities in an N -region unit cell of the form

$$\underline{P} = \begin{pmatrix} P_{1 \rightarrow 1} & P_{2 \rightarrow 1} & \cdots & P_{N \rightarrow 1} \\ P_{1 \rightarrow 2} & & & \\ \vdots & & & \\ P_{1 \rightarrow N} & \cdots & & P_{N \rightarrow N} \end{pmatrix}.$$

Writing sources and collision rates as vectors, the transport equation becomes

$$\mathbf{CR} = \underline{P}[\mathbf{S}_0 + \underline{R}\mathbf{CR}], \quad (21)$$

where $\underline{R}$ is a diagonal matrix:

$$\underline{R} = \begin{pmatrix} \left(\frac{\Sigma_s P_s}{\Sigma_t} \right)_1 & & \circ \\ & \ddots & \\ \circ & & \left(\frac{\Sigma_s P_s}{\Sigma_t} \right)_N \end{pmatrix}.$$

Solving Eq. (21),

$$\mathbf{CR} = [\underline{P}^{-1} - \underline{R}]^{-1} \mathbf{S}_0. \quad (22)$$

Equation (22) could be solved, but in general it is not necessary to go to this amount of trouble to account for in-group scattering if the fine group size $\Delta u < 10^{-3}$. It should be noted, however, that considerable computation time could be saved if Δu could be increased (at least between resonances). It is not known at this time whether the extra time involved in solving Eq. (22) using a coarse Δu would be less than the time used to calculate an approximation to Eq. (22) using a fine Δu .

A simple but reasonable approximation is to solve

Eq. (21) using an estimate of $\mathbf{CR}$ on the right-hand side. In general, the variation from fine group $k - 1$ to fine group k of collision rates is a few percent or less. Also, the self-scatter source contribution is very small. Equation (21) is approximated as follows:

$$\mathbf{CR}_k \approx \mathbf{CR}_{k-1}, \quad \text{in right-hand member;} \quad (23)$$

$$\mathbf{CR}'_k = \underline{P}[\tilde{S}_0 + \underline{R}\mathbf{CR}_{k-1}] = \underline{P}\mathbf{S}'. \quad (24)$$

Now correct the source using the best estimate available for $\mathbf{CR}_k$:

$$\mathbf{S} - \mathbf{S}' \approx \underline{R}(\mathbf{CR}'_k - \mathbf{CR}_{k-1}) \quad (25)$$

accounts for single in-group scattering collisions. Multiple in-group scattering collisions lead to

$$\mathbf{S} - \mathbf{S}' \approx \underline{R}(\underline{I} - \underline{R})^{-1}(\mathbf{CR}'_k - \mathbf{CR}_{k-1}). \quad (26)$$

The increment to the collision rate vector is $\mathbf{S} - \mathbf{S}'$, as is also true if one ignores the spatial transfer of this very small correction. The collision rate vector becomes

$$\mathbf{CR}_k \approx \mathbf{CR}'_k + \mathbf{S} - \mathbf{S}'. \quad (27)$$

In the limiting case of one region (homogeneous) problems, the exact solution of Eq. (22) is

$$\mathbf{CR}_k = \mathbf{S}_0 / (1 - \mathbf{R}), \quad (28)$$

which is also obtained by the approximate method.

SLOWING DOWN SOURCE

The slowing down source would be very time consuming to calculate directly because hundreds or even thousands of fine-group contributions are involved. Using the property that $P_i = e^{-\Delta u} P_{i-1}$, a recursion relation can be obtained:

$$\begin{aligned} S_k &= e^{-\Delta u} S_{k-1} + (P_1 - e^{-\Delta u} P_s)[(\Sigma_s \phi)_{k-1} \\ &\quad - \alpha(\Sigma_s \phi)_{k-L-1}] + P_s[(\Sigma_s \phi)_k - \alpha(\Sigma_s \phi)_{k-L}]. \end{aligned} \quad (29)$$

Equation (29) is written for a single isotope in a given region. Generalization to many isotopes is not a problem. The only unknown is ϕ_k , as ϕ_{k-1} was last calculated and $(\Sigma_s \phi)_{k-L-1}$ and $(\Sigma_s \phi)_{k-L}$ are obtained by parabolic interpolation from a table of scattering rates. Tabular entries are usually one per fine group (no lethargy averaging). A procedure has been devised which auto-

matically averages over the minimum number of fine groups when necessary to ensure coverage of the lethargy range between groups k and $k - L - 1$.

It is not necessary to use the same Δu throughout a problem to maintain a prescribed level of numerical accuracy. The Doppler width $w = \left(\frac{4EkT}{A}\right)^{1/2}$, where k is the Boltzmann constant, determines Δu by the empirical rule, $\Delta u \leq w/4E$. At lower energies, Δu can be increased. Then an interpolation scheme is necessary to extract $(\Sigma_s \phi)_{k-L}$ and $(\Sigma_s \phi)_{k-L-1}$ from the table since the group boundaries extrapolated back from group k will not in general match up with the group boundaries previously used when the scattering rates were stored.

FOILS BETWEEN REGIONS

One of the standard experiments on ZPR-type fast reactor mockups is that of determining relative reaction rate traverses within a unit cell by "small" (in both thickness and diameter) foils placed between the plates. It is reasonable to assume that the foils have essentially no effect on the neutron flux distribution in a typical unit cell. Using this assumption, one can obtain the collision rate in the foil by evaluating Eq. (4) at both sides of the foil (τ_f mean free paths thick):

$$\mathbf{CR}(1 \rightarrow \text{foil}) = \mathbf{J}(\tau, \tau_1) - \mathbf{J}(\tau + \tau_f, \tau_1). \quad (30)$$

Again, assuming an array of unit cells which repeat to infinity in both directions, the contribution by all plates of type 1 in a given direction is

$$\begin{aligned} \mathbf{CR}_\infty(1 \rightarrow \text{foil}) &= \sum_{m=0}^{\infty} [\mathbf{J}(\tau + mh, \tau_1) \\ &\quad - \mathbf{J}(\tau + \tau_f + mh, \tau_1)]. \end{aligned} \quad (31)$$

The assumption that the foil has no effect on the cell means that the optical thickness of the unit cell h does not include τ_f , and that there is no slowing down source in the foil. Also the optical thickness τ between plate 1 and the foil does not include contributions by foils at interfaces.

REFERENCE

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V-12. A Note on the Use of Prompt Neutron Spectra in MACH-1 Calculations

E. M. BOHN and C. D. SWANSON

The one-dimensional diffusion-theory code, MACH-1,⁽¹⁾ allows the use of one prompt neutron spectrum, independent of incident neutron energy in each region of an assembly. This restriction and its effect on central worth and criticality calculations was investigated by comparing MACH-1 calculations using standard prompt neutron spectra (as obtained from MC²)⁽²⁾ and calculations using effective prompt neutron spectra. An effective prompt neutron spectrum for each region of an assembly weighted by the isotopic concentrations of the fissile isotopes and effective cross sections in the region was considered. The primary interest in this study developed during the design calculations for ZPR-6 Assembly 6C, a plutonium zone core being considered for further study. The fissile isotope concentrations in each region of the assembly are listed in Table V-12-I.

EFFECTIVE PROMPT NEUTRON SPECTRA

TEMPERATURE OF THE PROMPT NEUTRON SPECTRA

For each fissile isotope I the average energy $\bar{E}_R^I$ of neutrons inducing fission in region R is

$$E_R^I = \frac{\sum_j \bar{E}_j (\nu \sigma_f)_i^I \phi_{R,j}}{\sum_j (\nu \sigma_f)_i^I \phi_{R,j}} \quad (1)$$

The fluxes used in Eq. (1) are regional integral fluxes obtained from MACH-1 calculations using standard prompt neutron spectra. The average energy of a fission neutron from isotope I in region R is then given by³

$$\bar{\epsilon}_R^I = 0.74 + 0.65(\bar{\nu}_R^I + 1)^{1/2}, \quad (2)$$

where the average number of neutrons $\bar{\nu}_R^I$ was computed with the linear relation

$$\bar{\nu}_R^I = \nu_{TH} + \left| \frac{d\nu}{dE} \right| \bar{E}_R^I. \quad (3)$$

The prompt neutron spectrum $\chi_{R,j}^I$ for isotope I in region R was computed by numerical integration of a Maxwellian distribution of temperature T_R^I , where

$$T_R^I = (\frac{2}{3}) \bar{\epsilon}_R^I.$$

ie regional dependence of T_R^I in the case of U-235 and U-238 was found to be negligible due to the low

TABLE V-12-I. FISSILE ISOTOPE CONCENTRATIONS IN ZPR-6 ASSEMBLY 6C, $\times 10^{24}$ atoms/cm³

Assembly 6C	Region		
	1	2	3
Outer radius ^a , cm	46.96	95.49	125.49
U-235	0.000013	0.001150	0.000083
U-238	0.005814	0.005796	0.040080
Pu-239	0.000884		
Pu-240	0.000114		
Pu-241	0.000022		
Pu-242	0.000005		

^a Critical dimensions as predicted with MC² generated cross section sets using ENDF/B data.

TABLE V-12-II. PROMPT NEUTRON SPECTRA TEMPERATURES IN ZPR-6 ASSEMBLY 6C

Isotope	ν_{TH}	$d\nu/dE$	Temperature, MeV	
			This Work	MC ² -ENDF/B
U-235	2.432 ^a	0.066 ^a	1.30	1.273 ^b
U-238	2.304 ^a	0.616 ^a	1.34	1.35 ^c
Pu-239	2.867 ^a	0.148 ^a	1.36	1.41 ^d
Pu-240	3.08 ^e	0.15 ^f	1.40	1.37 ^g
Pu-241	2.94 ^h	0.15 ^f	1.35	1.35 ^h
Pu-242	3.08 ^e	0.15 ^f	1.40	1.34 ^d

^a See Ref. 3.

^b Source not given.

^c See Refs. 4 and 5.

^d See Ref. 5.

^e Extrapolated from data in ANL-5800.

^f Value predicted by evaporation theory.

^g See Ref. 6.

^h See Ref. 7.

value of $d\nu/dE$ for U-235 and the high fission threshold of U-238. Values of T^I and the constants used in Eq. (3) are presented in Table V-12-II along with the temperatures of the standard prompt neutron spectra. The difference in the temperatures computed for U-235 and Pu-239 and those given by ENDF/B is significant and is due primarily to a difference in evaluation of basic fission parameters. The spectrum parameters used in this work are based on data compiled by G. Keepin³ and represents a more extensive evaluation than the ENDF/B study.

TABLE V-12-III. EFFECTIVE AND MC²-ENDF/B PROMPT NEUTRON SPECTRA IN THE FIRST TEN GROUPS FOR ZPR-6 ASSEMBLY 6C

Group	E_L , MeV	Region 1		Region 2		Region 3	
		Effective	MC ² - ENDF/B ^(b)	Effective	MC ² - ENDF/B ^(c)	Effective	MC ² - ENDF/B ^(d)
1	3.670	1.426×10^{-1}	1.551×10^{-1}	1.298×10^{-1}	1.244×10^{-1}	1.366×10^{-1}	1.408×10^{-1}
2	2.230	2.058	2.104	2.004	1.970	2.034	2.051
3	1.350	2.255	2.237	2.270	2.277	2.262	2.257
4	0.825	1.749	1.704	1.798	1.826	1.772	1.756
5	0.500	1.155	1.112	1.202	1.230	1.176	1.161
6	0.300	6.702×10^{-2}	6.406×10^{-2}	7.029×10^{-2}	7.232×10^{-2}	6.852×10^{-2}	6.747×10^{-2}
7	0.180	3.507	3.337	3.696	3.814	3.593	3.532
8	0.110	1.707	1.620	1.804	1.865	1.751	1.720
9	0.067	8.540×10^{-3}	8.093×10^{-3}	9.358×10^{-3}	8.770×10^{-3}	8.770×10^{-3}	8.609×10^{-3}
10	0.041	4.139	3.918	4.387	4.542	4.252	4.172

^a All numbers in this table were generated by numerical integration using a 50 point Simpson's Rule formulation.

^b Pu-239 spectrum with a temperature of 1.41 MeV.

^c U-235 spectrum with a temperature of 1.273 MeV.

^d U-238 spectrum with a temperature of 1.35 MeV.

TABLE V-12-IV. ZPR-6 ASSEMBLY 6C CENTRAL WORTH RESULTS USING EFFECTIVE AND MC²-ENDF/B PROMPT NEUTRON SPECTRA IN MACH-1, 1h/kg

Materials	Effective Spectra	MC ² -ENDF/B Spectra
U-235	91.41	95.61
U-238	-7.92	-8.12
Pu-239	111.05	115.94
Na	-2.14	-2.25
Fe	-2.80	-2.96
k	1.0000	1.0008

EFFECTIVE PROMPT NEUTRON SPECTRA

The effective prompt neutron spectrum for each region of the assembly was calculated with the relation

$$\bar{\chi}_{R,k} = \frac{\sum_i \chi_k^i \sum_j (\nu \Sigma_f)_{R,j}^i \phi_{R,j}}{\sum_i \sum_j (\nu \Sigma_f)_{R,j}^i \phi_{R,j}} \quad (4)$$

The fluxes initially used in Eq. (4) were regional integral fluxes from MACH-1 calculations using standard prompt neutron spectra. The $\bar{\chi}_{R,k}$ thus obtained were then used to generate new $\phi_{R,j}$. The subsequent $\bar{\chi}_{R,k}$ agreed with the first $\bar{\chi}_{R,k}$ for each group to within one percent. The first 10 groups of the effective prompt neutron spectrum in each region of Assembly 6C is compared in Table V-12-III with the MC²-ENDF/B spectrum which is normally used in MACH-1 calculations for these assemblies.

RESULTS

Criticality and central worth calculations for Assembly 6C were performed with both prompt neutron

spectra of Table V-12-III. The results are presented in Table V-12-IV. The k value computed with the MC²-ENDF/B spectra assumed the critical dimensions obtained with the effective spectra. The difference in k is small. The central worths computed with the effective spectrum for Assembly 6C are all about 5 % lower than the worths computed with the MC²-ENDF/B prompt neutron spectra. The difference is accounted for by a slight softening of the flux spectrum with use of the effective prompt neutron spectrum. It is noted that this decrease in calculated central worths would offer an improvement in central worth MACH-1 calculations based on past experience with plutonium fueled assemblies (i.e.; ZPR-3 Assembly 48).⁽⁸⁾

Thus, the careful evaluation of fission neutron parameters and use of effective prompt neutron spectra results in central worth predictions for Assembly 6C which differ a few percent from those obtained using standard fission neutron spectra in MACH-1 calculations.

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V-13. A Comparison of Fast Reactor Cross Section Codes

E. M. PENNINGTON

INTRODUCTION

Two different approaches are being used to generate effective multigroup-cross sections for fast reactor calculations. In the first approach, used by the Argonne MC² code¹ and the Gulf General Atomics GAFGAR code,² many narrow energy groups (~ 1000 to 5000) are used, especially in the resonance range. Broad-group cross sections for each isotope are calculated using weighting spectra which depend on the energy-dependent cross sections of all other isotopes in the reactor mix.

In the second approach, described in detail in Ref. 3, few-group (~ 25 – 75) infinite-dilution cross sections and shielding factors for each isotope are first generated for a number of discrete temperatures and σ_0 's. Here σ_0 is the total cross section of all other isotopes in the mix, averaged over the group, per atom of the isotope under consideration. These computations are performed by the Battelle Northwest Laboratory ETOX⁴ and the General Electric Company ENDRUN⁵ codes. Effective multigroup cross sections for a specified composition and temperature are then computed by interpolation to the desired σ_0 and temperature in a two-dimensional interpolation table. Appropriate in-

TABLE V-13-I. ISOTOPE DENSITIES FOR TEST ASSEMBLIES

Isotope	Density, atoms/b-cm		
	ZEBRA Assembly 6A	ZPR-3 Assembly 50	ZPR-3 Assembly 52
Pu-239	0.001894	0.001655	0.001799
Pu-240	0.000144	0.000106	0.000178
U-235	0.000046	0.000016	0.000015
U-238	0.006353	0.007406	0.007208
Ni	0.000443	0.000796	0.001606
Fe	0.004525	0.007300	0.015639
Cr	0.001270	0.001816	0.003672
Al	0.002504	0.000110	0.000057
Na	0.004474	0.0	0.009122
C	0.029590	0.045940	0.003029
O	0.0	0.0	0.012416
Mo	0.0	0.000206	0.000338
B-10	0.0	0.0	0.0

TABLE V-13-II. COMPARISONS OF k_{eff} AND δk_{eff} FOR THREE ASSEMBLIES USING FOUR CODES

	MC ²	GAFGAR	ETOX (1DX)	ENDRUN (FCC-IV)
ZPR-3 Assembly 50				
k_{eff} (Base Case)	0.9859	0.984	0.987	0.981
δk_{eff} [T (U-238) 1100°K $\rightarrow$ 300°K]	0.0172	0.0163	0.0164	0.0169
δk_{eff} [δN (Na) = 0.001 atoms/b-cm]	0.0073	0.0057	0.0058	0.0057
ZEBRA Assembly 6A				
k_{eff} (Base Case)	0.9828	0.985	0.984	0.978
δk_{eff} [T (U-238) 1100°K $\rightarrow$ 300°K]	0.0084	0.0080	0.0075	0.0076
δk_{eff} [δN (Na) = 0.001 atoms/b-cm]	0.0069	0.0066	0.0069	0.0067
ZPR-3 Assembly 52				
k_{eff} (Base Case)	0.9923	0.990	0.995	0.997
δk_{eff} [T (U-238) 1100°K $\rightarrow$ 300°K]	0.0047	0.0049	0.0043	0.0046
δk_{eff} [δN (Na) = 0.001 atoms/b-cm]	0.0043	0.0042	0.0044	0.0039

terpolation routines have been incorporated in the 1DX⁽⁶⁾ and FCC-IV⁽⁷⁾ codes.

W. Little of Battelle Northwest Laboratory suggested a comparison of results from the four codes. He proposed homogeneous models for three typical fast assemblies, ZEBRA Assembly 6A, ZPR-3 Assembly 50, and ZPR-3 Assembly 52. The isotope densities for these assemblies are given in Table V-13-I. The following specifications give the general rules and the calculations to be performed.

A. General Rules for Base Case

1. $T = 300^\circ\text{K}$ [except $T(\text{U-238}) = 1100^\circ\text{K}$]
2. Pu-239 fission source spectrum
3. Homogeneous cross sections

4. ENDF/B⁽⁸⁾ revised category 1 data
5. Use the following bucklings:

Assembly	B^2 , cm ⁻²
ZEBRA Assembly 6A	0.0033
ZPR-3 Assembly 50	0.0030
ZPR-3 Assembly 52	0.0029

B. Calculations

For each system, compute the following parameters using a fundamental mode model:

1. k_{eff} (Base Case)
2. k_{eff} [$T(\text{U-238}) = 300^\circ\text{K}$]
3. k_{eff} [$\delta N(\text{Na}) = 0.001$ atoms/b-cm]
4. Base case reaction rate ratios (see Table V-13-III).

NUMERICAL RESULTS

The MC² calculations were carried out using the Argonne CDC-3600 computer. Both the GAFGAR and ETOX (1DX) computations were done by Battelle Northwest Laboratory, while the ENDRUN (FCC-IV) computations were done by General Electric Company. Some of the results have been reported previously.⁹ Table V-13-II compares results of the codes for the Base Case k_{eff} and the U-238 temperature and sodium void changes in k_{eff} . Activation ratio comparisons are presented in Table V-13-III.

TABLE V-13-III. ACTIVATION RATIOS FOR THREE ASSEMBLIES USING FOUR CODES

	MC ²	GAFGAR	ETOX (1DX)	ENDRUN (FCC-IV)
ZPR-3 Assembly 50				
$\sigma_f(\text{U-235})/\sigma_f(\text{Pu-239})$	1.17	1.15	1.17	1.22
$\sigma_f(\text{U-238})/\sigma_f(\text{Pu-239})$	0.0309	0.0289	0.0306	0.0304
$\sigma_c(\text{U-238})/\sigma_f(\text{Pu-239})$	0.178	0.179	0.176	0.179
$\sigma_c(\text{Pu-239})/\sigma_f(\text{Pu-239})$	0.276	0.278	0.277	0.283
$\sigma_a(\text{B-10})/\sigma_f(\text{Pu-239})$	1.76	1.70	1.76	1.83
ZEBRA Assembly 6A				
$\sigma_f(\text{U-235})/\sigma_f(\text{Pu-239})$	1.09	1.07	1.08	1.12
$\sigma_f(\text{U-238})/\sigma_f(\text{Pu-239})$	0.0399	0.0371	0.0397	0.0393
$\sigma_c(\text{U-238})/\sigma_f(\text{Pu-239})$	0.159	0.160	0.156	0.159
$\sigma_c(\text{Pu-239})/\sigma_f(\text{Pu-239})$	0.213	0.215	0.212	0.217
$\sigma_a(\text{B-10})/\sigma_f(\text{Pu-239})$	1.44	1.45	1.42	1.48
ZPR-3 Assembly 52				
$\sigma_f(\text{U-235})/\sigma_f(\text{Pu-239})$	1.04	1.03	1.03	1.06
$\sigma_f(\text{U-238})/\sigma_f(\text{Pu-239})$	0.0329	0.0308	0.0324	0.0320
$\sigma_c(\text{U-238})/\sigma_f(\text{Pu-239})$	0.149	0.151	0.147	0.150
$\sigma_c(\text{Pu-239})/\sigma_f(\text{Pu-239})$	0.176	0.178	0.176	0.183
$\sigma_a(\text{B-10})/\sigma_f(\text{Pu-239})$	1.23	1.23	1.23	1.28

FIG. V-13-1. Fluxes for Half-Lethargy Groups for Three Assemblies as Calculated by MC². ANL Neg. No. 113-2710.

DISCUSSION

Agreement among the codes on the Base Case k_{eff} is rather good. The reactors represent a range of fast spectra with ZPR-3 Assembly 52 having the hardest and ZPR-3 Assembly 50 the softest spectrum of the three. Figure V-13-1 shows the fluxes for half-lethargy groups for the three assemblies as calculated by MC². These group fluxes are normalized to sum to unity.

The four codes are in reasonable agreement in the Doppler effect for U-238. In the case of the sodium void coefficient, the codes agree well for ZPR-3 Assembly 52 and ZEBRA Assembly 6A, but MC² gives a considerably larger δk_{eff} than the other codes for ZPR-3 Assembly 50. The reason for the discrepancy is not known. However, the spectra for the two sodium atom-densities differ to a greater extent for ZPR-3 Assembly 50 than for the other two reactors, since the Base Case for ZPR-3 Assembly 50 contains no sodium. For the other two cores, the change in sodium density is only around 10–20%.

Generally the activation ratios are in good agreement with no significant trends among the codes being noted.

The few-group shielding factor codes (ETOX and ENDRUN) have computer running times smaller than the ultra-fine-group codes (MC² and GAFGAR) by factors of the order of 10 or 20. The agreement among the codes demonstrated above shows that the few-group codes may be useful for reactor design calculations. In practice, in the case of multizone cores, an MC² calculation may be done only for the core because of the cost and difficulty in data handling. The result-

ing cross sections are then used in all regions of the reactor. Since the shielding factor codes are cheaper, faster, and easier to make automatic, it is more feasible to run problems for all regions of a complex reactor. Thus the final calculation of the reactor could be more accurate than the MC² method.

The ultra-fine group codes can be used for detailed analyses, such as those of critical assemblies, and for spot-checking the results of the few-group codes.

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V-14. Inelastic Moderation of Neutrons

M. SEGEV*

INTRODUCTION

The problem of analytic representation of fast reactor spectra has recently been the subject of a number of studies.¹⁻⁵ T. Murley¹ and K. Inoue² follow the degradation of the fission spectrum by repeated inelastic and elastic scatterings into a final energy spectrum. R. Malakoff and B. Mataviya³ and M. Driscoll and I. Kaplan⁴ use the Greuling-Goertzel approximation of the asymptotic moderating ratio to describe the energy spectrum. F. Dunn and M. Becker⁵ use the Greuling-

Goertzel expression for the moderating ratio, but redefine the parameters ξ and γ of this expression in order to obtain satisfactory results in the range of inelastic scattering.

In three of these studies³⁻⁵ the analytic form of the spectrum is compared, or adjusted to compare,⁴ with numerical solutions for the gross energy structure of the collision density in a number of fast mixtures. Dunn and Becker⁵ allow also fine structure in the collision density due to fine structure in ξ ; by so doing they succeed in describing the flux under the sodium resonance, eliminating a previous error by Driscoll.

The application of any of the analytic formulas from

* Visiting Scientist, Israel Atomic Energy Commission, Soreq Research Center, Yavne, Israel.

the aforementioned studies always presents an uncertainty: one is not sure whether a given mixture or a given energy range are within the validity-scope of the proposed formula. Clarification of such problems can stem only from basic, non-presumptive studies of the slowing-down process. An example of this approach is a recent paper⁶ on elastic slowing down: it affirms that the structured ξ ought to be used for a resonance as wide as the sodium resonance in a fast mixture; it also establishes, however, that the nonstructured ξ should be used for a narrow or a weak resonance in a fast mixture.

The present study starts with the basic equations of asymptotic slowing down by inelastic and elastic scattering in an infinite, homogenous, and weakly absorbing mixture of elements. The proposed solution to these equations is a composite expression which reduces to the approximate solutions in the limit of inelastic scattering by infinitely heavy scatterers and in the limit of purely elastic scattering. The composite expression is an adequate approximation also in intermediate cases, namely where neither the slowing down by target recoil nor the slowing down by level excitation is negligible, provided the absorption is small and that the mixture contains a substantial amount of medium-weight or light moderators.

Practically, the present analytic approach is found suitable in providing the detailed structure of the reactor flux, needed for the evaluation of group cross sections.

STATEMENT OF THE PROBLEM: THE ASYMPTOTIC SLOWING DOWN EQUATIONS

The problem is asymptotic slowing down of neutrons in an infinite, homogenous mixture of elements. As the neutrons collide in the mixture their energy is reduced by recoil and level-excitation of the target nuclei. A level is to be identified by element and by threshold, but will be counted by a single index for the sake of simple notation.

Consider a neutron of energy E' , colliding in the mixture. The probability that the neutron excites the λ th level is $s_\lambda(E')$. s_λ is determined by Σ_λ and Σ_t , the macroscopic cross sections, respectively, for the excitation of the λ th level and the total cross section of the mixture:

$$s_\lambda(E') = \Sigma_\lambda(E')/\Sigma_t(E'). \quad (1)$$

The λ th level being excited, the probability that the neutron emerges with energy E is $P_\lambda(E', E)$; it is related by the mechanics of inelastic scattering to $P_\lambda(E', \mu)$, the probability that the flight direction of the neutron changes by the center-of-mass angle $\cos^{-1} \mu$. The relation is:

$$P_\lambda(E', E) = \begin{cases} \frac{2}{\alpha_\lambda E'} \frac{1}{\sqrt{1 - E_\lambda/E'}} P_\lambda(E', \mu) & \text{for } E_{L,\lambda} \leq E \leq E_{H,\lambda} \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

α_λ is $4A_\lambda/(1 + A_\lambda)^2$, where A_λ is the mass number of the element containing the λ th level.

E_λ is the λ th threshold; it is related to the center-of-mass excitation energy Q_λ by Eq. (3) (Q_λ will be assigned the value zero for "elastic levels"):

$$E_\lambda = \left(\frac{1 + A_\lambda}{A_\lambda} \right)^2 Q_\lambda. \quad (3)$$

$E_{H,\lambda}(E', A_\lambda, E_\lambda)$ and $E_{L,\lambda}(E', A_\lambda, E_\lambda)$ are given by

$$\left. \begin{matrix} E_{H,\lambda} \\ E_{L,\lambda} \end{matrix} \right\} = \left(\frac{1 \pm A_\lambda \sqrt{1 - E_\lambda/E'}}{1 + A_\lambda} \right)^2 E'. \quad (4)$$

The asymptotic slowing down equations are

$$\frac{dq}{dE} = aF \quad (5)$$

$$F(E) = \sum_{\lambda=\text{levels}} \int_{E_{L,\lambda}}^{E_{H,\lambda}} dE' s_\lambda(E') P_\lambda(E', E) F(E'), \quad (6)$$

where

$q \equiv$ slowing down density

$F \equiv$ collision density

$a \equiv$ absorption ratio Σ_a/Σ_t .

$E'_{H,\lambda}(E, A_\lambda, E_\lambda)$ and $E'_{L,\lambda}(E, A_\lambda, E_\lambda)$ are, respectively, the high and low limits of a range of energies E' , outside which $P_\lambda(E', E)$ is zero; they can be derived from the mechanics of inelastic scattering and are given by

$$\left. \begin{matrix} E'_{H,\lambda} \\ E'_{L,\lambda} \end{matrix} \right\} = \left(\frac{1 + \frac{1}{A_\lambda} + \left(1 - \frac{1}{A_\lambda + 1}\right) \frac{E_\lambda}{E}}{\frac{1}{A_\lambda} \pm \sqrt{1 + \left(1 - \frac{2}{A_\lambda + 1}\right) \frac{E_\lambda}{E}}} \right). \quad (7)$$

If q is given for a certain energy E_0 , then $q(E)$, $E < E_0$, can be expressed as $q(E_0)$ times the non-absorption probability in the range $[E, E_0]$. Define

$$M \equiv q \frac{dq}{dE} \quad (8)$$

and integrate Eq. (8) from E to E_0 , giving

$$q(E) = q(E_0) \exp \left[- \int_E^{E_0} \frac{dE'}{M(E')} \right]; \quad (9)$$

the collision density is given by Eqs. (5) and (8):

$$F = \frac{q}{aM} \quad (10)$$

d the problem now consists in obtaining an approximation for M .

SLOWING DOWN IN THE LIMIT OF PURELY ELASTIC SCATTERING

If all the levels in the mixture had zero thresholds, i.e., there were no inelastic scattering, then Eq. (6) would read

$$F(E) = \sum_{\lambda=\text{isotopes}} \int_E^{E/(1-\alpha_\lambda)} dE' s_\lambda(E') \cdot P_\lambda(E', E) F(E') \quad (11)$$

Two changes of variable are now made in Eq. (11). The first change is from E' to μ ; the second change is from E to u , the lethargy variable. Assuming that $P(E', \mu)$ is constant in E' , the result is

$$F(u) = \sum_{\lambda=\text{isotopes}} \int_{-1}^{+1} d\mu s_\lambda(u + \ln \Lambda_\lambda) \cdot F(u + \ln \Lambda_\lambda) P_\lambda(\mu), \quad (12)$$

where

$$\Lambda_\lambda(\mu) \equiv 1 - \frac{\alpha_\lambda}{2} + \frac{\alpha_\lambda}{2} \mu. \quad (13)$$

The product $s_\lambda F$ is developed in a Taylor series about μ :

$$-aF = \sum_{n=1}^{\infty} (-)^{n-1} \frac{d^n}{du^n} [s\xi^{(n)}F], \quad (14)$$

where

$$s(u)\xi^{(n)}(u) \equiv \sum_{\lambda=\text{isotopes}} \frac{1}{n!} s_\lambda(u) \xi_\lambda^{(n)} \quad (15)$$

$$\xi_\lambda^{(n)} \equiv \left| \int_{-1}^{+1} d\mu (\ln \Lambda_\lambda)^n P_\lambda(\mu) \right|. \quad (16)$$

Eq. (5) is substituted for $-aF$ in Eq. (14) and Eq. (10) is substituted into the right hand side of Eq. (14). After integration the result is

$$q = \sum_{n=1}^{\infty} (-)^{n-1} \frac{d^{n-1}}{du^{n-1}} \left[\frac{s\xi^{(n)}}{aM} \right] q. \quad (17)$$

If the following conditions are satisfied

$$\left| \frac{d}{du} \left(\frac{\xi^{(2)}}{\xi^{(1)}} \right) \right| \ll 1, \quad a \ll 1, \quad (18)$$

then Eq. (17) is a rapidly converging series,⁶ and M can be approximated by successive steps. The first-step approximation is obtained by truncating the series after the first term; the second-step approximation is obtained by truncating the series after the second term, then substituting the first-step approximation into the second term. Neglecting, due to Eq. (18), the derivative of $\xi^{(2)}/\xi^{(1)}$, the result is

$$M(E) = \frac{s}{a} \xi^{(1)} E \quad (\text{first-step app.}) \quad (19a)$$

$$M(E) = \left(\frac{s}{a} \xi^{(1)} + \frac{\xi^{(2)}}{\xi^{(1)}} \right) E \quad (\text{second-step app.}) \quad (19b)$$

In the case of center-of-mass isotropic scattering Eq. (19b) reduces to the Greuling-Goertzel approximation for M . The difference between Eq. (19a) and Eq. (19b) can be interpreted from the case of constant cross sections. In this case Eq. (17) is solved by $M(u) = \text{constant}$. In order to determine M , the relation $dq/du = -q/M$ is substituted into Eq. (17):

$$\frac{a}{s} = \sum_{n=1}^{\infty} \xi^{(n)} \left(\frac{1}{M} \right)^n. \quad (20)$$

An inversion of an equation similar to Eq. (20) was suggested by G. Placzek.⁷ Inverting Eq. (20) yields

$$M = \frac{s}{a} \xi^{(1)} E + O \left[\left(\frac{a}{s} \right)^2 \right], \quad (\text{constant cross sections}) \quad (21a)$$

$$M = \left(\frac{s}{a} \xi^{(1)} + \frac{\xi^{(2)}}{\xi^{(1)}} \right) E + O \left[\left(\frac{a}{s} \right)^3 \right], \quad (\text{constant cross sections}). \quad (21b)$$

Extrapolating now from Eq. (21), the difference between Eq. (19a) and Eq. (19b) will be interpreted as a difference in accounting for the effect of absorption. L. Dresner³ interpreted in the same way the difference between the Wigner and the Greuling-Goertzel approximation.

SLOWING DOWN IN THE LIMIT OF INFINITELY HEAVY SCATTERERS

If all the elements in the mixture were infinitely heavy, then a scattering collision with a neutron of energy E' which excites the λ th level would degrade the neutron energy to $E' - Q_\lambda$, where Q_λ is the excitation energy in center of mass coordinates

$$P_\lambda(E', E) = \delta[E' - (E + Q_\lambda)] \quad (22)$$

Substituting Eq. (22) into Eq. (6) and developing $s_\lambda(E + Q_\lambda)F(E + Q_\lambda)$ in a Taylor series about E yields

$$aF = \sum_{n=1}^{\infty} \frac{d^n}{dE^n} [sQ^{(n)}F], \quad (23)$$

where

$$s(E)Q^{(n)}(E) \equiv \sum_{\lambda=\text{levels}} \frac{1}{n!} s_\lambda(E) Q_\lambda^n. \quad (24)$$

Substituting Eq. (5) into the left hand side and Eq. (10) into the right hand side of Eq. (24), then integrating, yields

$$q = \sum_{n=1} \frac{d^{n-1}}{dE^{n-1}} \left[\frac{sQ^{(n)}}{aM} \right] q. \quad (25)$$

If the following assumptions are made

$$\left| \frac{d}{dE} \left(\frac{Q^{(2)}}{Q^{(1)}} \right) \right| \ll 1, \quad a \ll 1, \quad (26)$$

then approximations for $M(E)$ can be derived from Eq. (25) in much the same way the approximations for $M(u)$ were derived from Eq. (17) in the previous section:

$$M(E) = \frac{s}{a} Q^{(1)}, \text{ (first step app.)} \quad (27a)$$

$$M(E) = \frac{s}{a} Q^{(1)} + \frac{Q^{(2)}}{Q^{(1)}}, \text{ (second-step app.)} \quad (27b)$$

In the previous section a reference was made to a study showing that Eq. (18) is a sufficient condition for a rapid convergence of the right hand side of Eq. (17) beyond the second term. Since the equations of this section are analogous to those of the previous section, it will be conjectured that Eq. (26) is a sufficient condition for the rapid convergence of Eq. (29) beyond the second term.

In the case of constant cross sections, Eq. (25) is solved by $M(E) = \text{constant}$. Substituting $dq/dE = q/M$ into Eq. (25) and then inverting the series, yields

$$M = \frac{s}{a} Q^{(1)} + O \left[\left(\frac{a}{s} \right)^2 \right], \quad (28a)$$

(constant cross sections)

$$M = \frac{s}{a} Q^{(1)} + \frac{Q^{(2)}}{Q^{(1)}} + O \left[\left(\frac{a}{s} \right)^3 \right], \quad (28b)$$

(constant cross sections).

THE COMPOSITE MODERATING RATIO

The general approximation of $M(E)$ should reduce to Eq. (19) in the limit of purely elastic scattering and to Eq. (27) in the limit of infinitely heavy scatterers. A composite expression that exhibits these tendencies is suggested by the following considerations.

Let $\Delta_\lambda(E, \mu)$ denote the energy loss associated with the scattering of a neutron of energy E by the λ th level, through an angle of $\cos^{-1} \mu$. The mechanics of inelastic scattering yields

$$\Delta_\lambda(E, \mu) = \frac{\alpha_\lambda}{2} [1 - \mu \sqrt{1 - E_\lambda/E}] E + Q_\lambda. \quad (29)$$

Inelastic scattering will be assumed to be isotropic, i.e., $P_\lambda(E, \mu) = 1/2$; the anisotropy (if any) of the elastic scattering will be assumed to be independent of E , i.e., $P_\lambda(E, \mu) = P_\lambda(\mu)$. The average energy loss, $\langle \Delta(E) \rangle_\lambda$, therefore is

$$\langle \Delta(E) \rangle_\lambda = \frac{\alpha_\lambda}{2} (1 - \langle \mu \rangle_\lambda) E + Q_\lambda, \quad (30)$$

where $\langle \mu \rangle_\lambda$ is the average of μ over $P_\lambda(\mu)$.

If the mixture contains only heavy elements, then, by Eq. (16),

$$\xi_\lambda \approx \frac{\alpha_\lambda}{2} (1 - \langle \mu \rangle_\lambda) \quad (31)$$

(the superscript (1) will be omitted from $\xi_\lambda^{(1)}$, $\xi^{(1)}$ and $Q^{(1)}$ whenever ambiguity cannot occur) and Eq. (30) reads

$$\langle \Delta(E) \rangle_\lambda \approx \xi_\lambda E + Q_\lambda. \quad (32)$$

In the limit of infinitely heavy scatterers, $\langle \Delta \rangle_\lambda$ is Q ; in this case M is s/a times the average energy loss per scattering in the mixture [see Eq. (27a)]. In the limit of purely elastic scattering $\langle \Delta \rangle_\lambda$ is $\xi_\lambda E$ and, again, M is s/a times the average loss per scattering in the mixture [see Eq. (19a)]. The fact that M has the same meaning in both extreme cases suggests that M be the average energy loss in any mixture of heavy elements, namely

$$M \approx \frac{s}{a} (\xi E + Q), \quad (33a)$$

where $s\xi$ is given by Eqs. (15) and (31).

If $s\xi$ is given by Eqs. (15) and (16), then the scope of Eq. (33a) as a composite moderating ratio will be extended to include mixtures with lighter elements, namely any non-hydrogenous mixture. The form of Eq. (33a) further suggests that a second order approximation for M can be constructed from Eqs. (19b) and (27b):

$$M \approx \left(\frac{s}{a} + \frac{\xi^{(2)}}{\xi^{(1)}} \right) E + \left(\frac{s}{a} Q + \frac{Q^{(2)}}{Q^{(1)}} \right). \quad (33b)$$

Equations (33) exhibit the expected tendencies in the limits:

$$\begin{array}{ccc} \text{Eq. (33a)} & \xrightarrow{Q/\xi E \rightarrow 0} & \text{Eq. (19a)} \\ \text{Eq. (33b)} & & \text{Eq. (19b)} \end{array} \quad (34)$$

$$\begin{array}{ccc} \text{Eq. (33a)} & \xrightarrow{\xi E/Q \rightarrow 0} & \text{Eq. (27a)} \\ \text{Eq. (33b)} & & \text{Eq. (27b)} \end{array} \quad (35)$$

THE COMPOSITE MODERATING RATIO AS A SOLUTION OF THE SLOWING DOWN EQUATIONS

It needs to be shown that Eqs. (33) are non-trivial approximations, namely that they are approximate solutions of the slowing down equations also in cases intermediate between the limit of Eq. (34) and the limit of Eq. (35). The demonstration of this fact consists in a complementary use of analytic and numerical substitutions of Eqs. (33) into Eq. (6). In general, the composite moderating ratio is found to be a good approximation

mixtures with weak absorption and a considerable amount of light and medium-mass scatterers.

Most of the mathematical steps involved in the above demonstration are rather lengthy and will be omitted here. Following, however, is an assessment of the applicability of the composite moderating ratio in the "evaporation" region.

Consider, for example, a simplified case of a heavy element with constant total inelastic and elastic scattering cross sections, in mixture with a lighter element having a constant, purely elastic, scattering cross section.

Equations (18) and (26), respectively the criteria for the approximations of Eqs. (19a) and (27a), may be combined in a single criterion for the composite moderating ratio. Equation (18) will carry the weight 1, because there is energy loss by target recoil in every scattering collision; Eq. (26) will carry the weight s_{in}/s , the frequency of inelastic level excitations among scattering collisions:

$$\left| \frac{d}{du} \left(\frac{\xi^{(2)}}{\xi^{(1)}} \right) \right| + \frac{s_{in}}{s} \left| \frac{d}{dE} \left(\frac{Q^{(2)}}{Q^{(1)}} \right) \right| \ll 1. \quad (36)$$

The first term in Eq. (36) is zero. In order to evaluate the second term, the definition of $Q^{(n)}(E)$ [Eq. (24)] is transformed to an integral form

$$\frac{1}{n!} \sum_{\lambda} s_{\lambda}(E) Q_{\lambda}^n \rightarrow \frac{s_{in}(E)}{n!} \int_0^E dE^* (E - E^*)^n P_{in}(E, E^*), \quad (37)$$

where $P_{in}(E, E^*)$ is the probability that inelastic scattering degrades the neutrons from E to E^* . A frequently used approximation⁹ for $P_{in}(E, E^*)$ is

$$P_{in}(E, E^*) = \frac{E^*}{\theta^2(E)} \exp \left[-\frac{E^*}{\theta(E)} \right], \quad (38)$$

where $\theta(E)$ is the "nuclear temperature."

Equation (38) has been normalized so that its integral over dE^* from zero to infinity is 1. (Actually, the integral is from zero to E , but there is a negligible numerical difference between this integral and the integral from zero to infinity.) $Q^{(1)}(E)$ and $Q^{(2)}(E)$ may now be evaluated by substituting Eq. (38) into Eq. (37) and replacing E , the integration upper limit, by infinity. If $\theta(E)$ is approximated as $CE^{1/2}$, where $C \approx 0.2$ MeV^{-1/2} for heavy elements, then, by Eqs. (33a) and (36),

$$aM \approx (s\xi + s_{in})E - 2s_{in}\theta(E) \quad (39)$$

provided

$$\frac{1}{2} \frac{s_{in}}{s} \left| 1 - \theta^2(1) \left[\frac{1}{\theta(E)} + \frac{1}{\theta(1)} \right] \right| \ll 1, \quad (40)$$

(E in MeV).

Equation (40) asserts that, as E and s_{in} decrease, Eq. (39) becomes a better approximation of M . A numerical examination of this assertion was carried out. The solution $F = q/aM$, with M given by Eq. (39), was substituted in the slowing down equation for the problem above, and the function $R_{evp}(E)$ was defined as the ratio between the right hand side and the left hand side of the equation. A number of calculations were run, the results affirming the assertion above. Figure V-14-1 is an example of these runs. Note also that the values of R_{evp} ($a \neq 0$) are not as close to 1 as the values of R_{evp} ($a = 0$), but that a second-order version of Eq. (39), composed from Eqs. (19b) and (27b), reduces the values of R_{evp} to a great extent. Finally, the agreement between the criterion, Eq. (40), and the numerical results supports a previous conjecture, namely that Eq. (26) was a sufficient condition for Eqs. (27). [The expression inside the absolute value brackets in Eq. (40) is $(d/dE) (Q^{(2)}/Q^{(1)})$.]

ON THE APPLICATION OF THE THEORY

The application of the first-order composite moderating ratio, Eq. (33a), requires the evaluation of three mixture parameters: s , $s\xi$, and sQ . s is the total scattering ratio of the mixture Σ_s/Σ_t ; sQ is the sum $\sum_{\lambda} s_{\lambda} Q_{\lambda}$, where Q_{λ} is the center-of-mass excitation energy for the λ th level; $s\xi$ is the sum $\sum_{\lambda} s_{\lambda} \xi_{\lambda}$, where ξ_{λ} equals the average lethargy gain per scattering from the isotope containing the λ th level. Note that the latter sum includes the inelastic levels as well as the elastic levels. This is a reflection of the fact that degradation by target

FIG. V-14-1. R Values in the Evaporation Region. ANL Neg. No. 113-2302.

recoil occurs in inelastic as well as in elastic scattering (see, e.g., Eq. (32) for the average energy loss in a scattering collision).

The fact that the parameters s , $s\xi$ and sQ are simply related to the measurable properties of the mixture, namely cross sections, threshold energies, angular distribution, and energy distributions, renders the approximation simple in content as well as in form. This simplicity is not without some price. The previous section carries the implication that the composite moderating ratio be applied only if the mixture contains a substantial amount of medium-weight or light moderators. Another requirement involved is small absorption. This seems to have been a common restriction in all of the simple analytic solutions of slowing down problems. An approximation which better accounts for the effects of absorption is given by the second-order formula for M , Eq. (33b). This formula remains simple in content and form, involving $sQ^{(2)}$ and $s\xi^{(2)}$ in addition to s , $sQ^{(1)}$ and $s\xi^{(1)}$. Higher-order composite formulae for M will probably further enhance the accuracy of the solution.

If the neutron source in a medium consists of more than a point-source, then the non-virgin energy spectrum is the sum of slowing-down spectra. If the neutron source is distributed smoothly over a wide range of energies, then the collision density at any energy, either below or within the range of the source, is made up mainly from asymptotic spectra. In other words, if non-asymptotic features are neglected in all the slowing down spectra that make up the flux at a certain energy, then a certain error is introduced; this error disappears as the source becomes infinitely wide (for $q(E)$ and $F(E)$ may be interpreted as slowing down and collision density per source neutron at E and Eqs. (5) and (6) may be interpreted as the exact non-asymptotic slowing down equations for a constant source).

If $\chi(E)$, the fission source, is assumed to be suffi-

FIG. V-14-2. Fine-Group Flux of Mixture 11 (Ref. 10). ANL Neg. No. 113-2991.

TABLE V-14-I. THE RATIO $\frac{k_{eff} [Eq. (41)]}{k_{eff} (MC^2)}$

System	1	2	3	11	12	13
$\frac{k_{eff} [Eq. (41)]}{k_{eff} (MC^2)}$	1.009	1.010	1.011	1.012	1.012	1.012

ciently wide, such that non-asymptotic effects can be neglected, then the flux F/Σ is given by

$$\phi(E) = \frac{1}{[\Sigma(E)][aM(E)]} \int_E^\infty dE_0 \chi(E_0) s(E_0) \cdot \exp \left[- \int_E^{E_0} \frac{dE'}{M(E')} \right] + \frac{\chi(E)}{\Sigma(E)}, \quad (41)$$

where $M(E)$ is given by Eqs. (33).

NUMERICAL COMPARISONS

An important application of Eq. (41) is in replacing numerical solutions for the fine-structure of the reactor flux, rendering the weighting of broad-group cross sections an easier task. In order to assess the usefulness of Eq. (41) in group computations, a series of comparisons with MC² outputs has been started. The infinite media studied consist of the core-mixtures in the systems chosen for intercomparison calculations at the Argonne Conference of 1965 (see Table I of Ref. 10). So far only all fine group calculations were carried out with MC², dividing the range of 10 MeV to 1 keV into 66 fine groups.

Figure V-14-2 compares the fine-group flux, calculated numerically by MC², with the analytic flux, given by Eq. (41), for system 11 of Ref. 10. The overall analytic spectrum is somewhat harder than the numerical spectrum, but more important, the fine details of the flux are reproduced quite well by the analytic formula, indicating that Eq. (41) is useful in providing weighting spectra for broad-group calculations.

The fine-group cross sections were then weighted with the fine-group flux to produce broad-group cross sections and fluxes and values of k_{eff} . The k_{eff} values obtained through the use of Eq. (41) were very close to the values obtained through the use of the MC² fine-flux. Table V-14-I summarizes the ratios of $k_{eff} [Eq. (41)]/k_{eff} (MC^2)$ for six of the mixtures of Ref. 10.

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V-15. Functionals for Discontinuous Trial Function Neutron Flux Synthesis¹

V. LUCO and P. LAMBROPOULOS

It has often been stated² that the functionals originally used as a basis for the formulation of variational methods of neutron flux synthesis will not remain finite when the class of admissible functions includes sectionally continuous functions. New functionals having first order Euler-Lagrange equations have been proposed^{3,4} which, it has been said, instead of showing unmanageable divergencies remain finite but to some extent undetermined.

It is our purpose to show that the divergencies and indeterminacies mentioned in the literature are easily removed if the admissible functions are consistently interpreted as sectionally continuous functions with sectionally continuous first derivatives.

We will also show that functionals of both the originally proposed form and of the first-order kind can be used for discontinuous trial function synthesis, if modified to give physically meaningful interface conditions.

EVALUATION OF THE FUNCTIONALS

The functionals originally used to formulate the variational flux synthesis methods with continuous trial functions are of the following type:²

$$J[\phi, \phi^*] = \int_a^b \left(\phi^* \frac{d\phi}{dx} D \frac{d\phi}{dx} + \phi^* \Lambda \phi \right) dx. \quad (1)$$

The type proposed to be used with discontinuous trial functions is³

$$F[\phi, \phi^*, j, j^*] = \int_a^b \left(\phi^* \frac{dj}{dx} - j^* \frac{d\phi}{dx} + \phi^* \Lambda \phi - j^* D^{-1} j \right) dx. \quad (2)$$

For simplicity a one-dimensional system from $x = a$ to $x = b$ has been assumed, and all the non-diffusion terms in the group diffusion operator have been lumped together into the Λ operator. For the same reason the time dimension has been excluded and zero flux and adjoint boundary conditions will be imposed throughout. Any or all of these limitations could be removed without affecting the argument.

The classes of admissible functions to be used in the evaluation of the functionals are: for $J[\phi, \phi^*]$ the class of functions $\phi(x)$ and $\phi^*(x)$ which are sectionally continuous⁵ with sectionally continuous first derivatives in $[a, b]$, assuming zero values at $x = a$ and $x = b$; for $F[\phi, \phi^*, j, j^*]$ the same class of functions $\phi(x)$ and $\phi^*(x)$, plus the class of functions $j(x)$ and $j^*(x)$ which are sectionally continuous with sectionally continuous first derivatives in $[a, b]$.

The term "sectionally continuous" is used here in the conventional sense, and it implies that the function in question is continuous almost everywhere in $[a, b]$; that is, at all points except for a set of points of measure zero. For the purposes of this paper, such a set will be assumed to consist of a finite number of points. At the points of discontinuity, the value of the function is assumed to have finite (but different) limits from the left and the right. This set of discontinuity will be referred to as jump discontinuity.[†]

[†] These conditions could be relaxed to admit discontinuous functions with unbounded limits from either side provided such functions were square integrable.

Again for the sake of simplicity, it will be assumed that there is only one point ($x = x_0$) inside $[a, b]$ where functions $\phi, \phi^*, d\phi/dx, d\phi^*/dx, j$ and j^* can have a jump discontinuity. At that point x_0 , the derivatives $d\phi/dx, d\phi^*/dx, dj/dx$ and dj^*/dx do not exist. The functions ϕ and ϕ^* or may may not be defined at $x = x_0$.

The functionals $J[\phi, \phi^*]$ and $F[\phi, \phi^*, j, j^*]$, being the integrals of sectionally continuous functions are now well defined, finite, and are given by⁶

$$\begin{aligned} J[\phi, \phi^*] &= \int_a^b \left[\frac{d\phi^*}{dx} D \frac{d\phi}{dx} + \phi^* \Lambda \phi \right] dx \\ &= \int_a^{x_0^-} \left[\frac{d\phi^*}{dx} D \frac{d\phi}{dx} + \phi^* \Lambda \phi \right] dx + \int_{x_0^+}^b \left[\frac{d\phi^*}{dx} D \frac{d\phi}{dx} + \phi^* \Lambda \phi \right] dx, \end{aligned} \quad (3)$$

and

$$\begin{aligned} F[\phi, \phi^*, j, j^*] &= \int_a^b \left[\phi^* \frac{dj}{dx} - j^* \frac{d\phi}{dx} + \phi^* \Lambda \phi - j^* D^{-1} j \right] dx \\ &= \int_a^{x_0^-} \left[\phi^* \frac{dj}{dx} - j^* \frac{d\phi}{dx} + \phi^* \Lambda \phi - j^* D^{-1} j \right] dx \\ &\quad + \int_{x_0^+}^b \left[\phi^* \frac{dj}{dx} - j^* \frac{d\phi}{dx} + \phi^* \Lambda \phi - j^* D^{-1} j \right] dx. \end{aligned} \quad (4)$$

The integral

$$\int_a^{x_0^-} f(x) dx + \int_{x_0^+}^b f(x) dx$$

is a notation for

$$\lim_{\epsilon \rightarrow +0} \left(\int_a^{x_0 - \epsilon} f(x) dx + \int_{x_0 + \epsilon}^b f(x) dx \right),$$

where the limit is taken after the integration is performed. It is in this sense that all integrals over $[a, b]$ should be understood in this paper. It should be noted here that contrary to what has been asserted elsewhere,^{3,4} the integration across the point of discontinuity x_0 contributes nothing to the value of the functionals. In other words, in both cases the integral

$$\int_{x_0^-}^{x_0^+} [\] dx$$

which has been the source of the mathematical difficulties mentioned in the introduction is unequivocally found to have the value

$$\int_{x_0^-}^{x_0^+} [\] dx = 0$$

For a more thorough discussion of this point the reader is referred to Ref. 1.

For the class of functions considered here, integrals like those appearing in Eqs. (3) and (4) will always be finite and independent of whatever value one might assign to the function at x_0 , for this is a set of measure zero. Moreover, since the functions in question are continuous in each of the intervals $[a, x_0 - \epsilon]$ and $[x_0 + \epsilon, b]$, one can integrate by parts and then take the limit for $\epsilon \rightarrow +0$. Using this procedure of integration by parts in Eq. (3), one finds that the functional $J[\phi, \phi^*]$ can be written in two other equivalent forms:

$$J[\phi, \phi^*] = \phi_-^* D_- \phi_- - \phi_+^* D_+ \phi_+ + \int_a^b \phi^* \left[-\frac{d}{dx} \left(D \frac{d\phi}{dx} \right) + \Lambda \phi \right] dx \quad (5)$$

or

$$J[\phi, \phi^*] = \phi_-'^* D_- \phi_- - \phi_+'^* D_+ \phi_+ + \int_a^b \left[-\frac{d}{dx} \left(\frac{d\phi^*}{dx} D \right) + \phi^* \Lambda \right] \phi dx,$$

where

$$\lim_{\epsilon \rightarrow 0} D(x_0 \pm \epsilon) = D_{\pm}, \quad \lim_{\epsilon \rightarrow 0} \phi(x_0 \pm \epsilon) = \phi_{\pm}, \quad \lim_{\epsilon \rightarrow 0} \left. \frac{d\phi}{dx} \right|_{x_0 \pm \epsilon} = \phi'_{\pm},$$

and similarly for ϕ^* and its derivatives. Note that if a function has a jump discontinuity, say at x_0 , its derivative may approach infinity from the left and/or the right. One can readily show, however, that this will be an integrable singularity. Therefore since, for example, $d\phi(x)/dx$ is assumed to have only jump discontinuities the integral

$$\int_a^b \phi^* \frac{d}{dx} \left(D \frac{d\phi}{dx} \right) dx$$

will be finite (provided of course that $D(x)$ is bounded and differentiable, which is assumed to be the case). By means of the same transformation F can be written in three other equivalent forms.¹

The stationarity conditions for $J[\phi, \phi^*]$ can be obtained from any of its equivalent forms. Using Eq. (1),

$$\delta J = \int_a^b \left[\frac{d\delta\phi^*}{dx} D \frac{d\phi}{dx} + \frac{d\phi^*}{dx} D \frac{d\delta\phi}{dx} + \delta\phi^* \Lambda\phi + \phi^* \Lambda\delta\phi \right] dx \quad (7)$$

and integrating by parts

$$\begin{aligned} \delta J = \int_a^b \left\{ \delta\phi^* \left[-\frac{d}{dx} \left(D \frac{d\phi}{dx} \right) + \Lambda\phi \right] + \left[-\frac{d}{dx} \left(\frac{d\phi^*}{dx} D \right) + \phi^* \Lambda \right] \delta\phi \right\} dx \\ + \delta\phi^* D_- \phi'_- - \delta\phi^* D_+ \phi'_+ + \phi'^*_- D_- \delta\phi_- - \phi'^*_+ D_+ \delta\phi_+. \end{aligned} \quad (8)$$

As $\delta\phi^*_-$, $\delta\phi^*_+$, $\delta\phi_-$, $\delta\phi_+$ are independent arbitrary variations, $\delta J = 0$ implies the following conditions at x_0 :

$$\phi'_- = \phi'_+ = \phi'^*_- = \phi'^*_+ = 0. \quad (9)$$

On physical grounds, the desired conditions at x_0 are

$$\left. \begin{aligned} \phi_+ &= \phi_-, & \phi^*_+ &= \phi^*_- \\ D_+ \phi'_+ &= D_- \phi'_-, & D_+ \phi'^*_+ &= D_- \phi'^*_- \end{aligned} \right\}. \quad (10)$$

Clearly the functional $J[\phi, \phi^*]$ will be stationary for functions ϕ, ϕ^* which do not satisfy the appropriate interface conditions at $x = x_0$ and are therefore unacceptable. In a similar manner it is easily shown that the functional F has its stationary value for inappropriate functions ϕ, ϕ^*, j, j^* . In order to obtain the proper interface conditions it is necessary to modify the functionals J and F by adding terms defined at the interface.

The following modified functionals have their stationary value for functions ϕ, ϕ^*, j, j^* which satisfy conditions (10) or equivalent, at $x = x_0$:

$$J_1[\phi, \phi^*] = (\phi^*_+ - \phi^*_-)\alpha + \beta(\phi_+ - \phi_-) + \int_a^b \left[\frac{d\phi^*}{dx} D \frac{d\phi}{dx} + \phi^* \Lambda\phi \right] dx, \quad (11)$$

$$J_2[\phi, \phi^*] = \alpha(D_+ \phi'_+ - D_- \phi'_-) + \beta(\phi_+ - \phi_-) + \int_a^b \phi^* \left[\frac{d}{dx} \left(D \frac{d\phi}{dx} \right) - \Lambda\phi \right] dx, \quad (12)$$

$$J_3[\phi, \phi^*] = (\phi^*_+ - \phi^*_-)\alpha + (\phi'^*_+ D_+ - \phi'^*_- D_-)\beta + \int_a^b \left[\frac{d}{dx} \left(\frac{d\phi^*}{dx} D \right) - \phi^* \Lambda \right] \phi dx, \quad (13)$$

$$\begin{aligned} J_4[\phi, \phi^*, \gamma] &= [\phi^*_+ - \phi^*_-][\gamma D_+ \phi'_+ + (1 - \gamma) D_- \phi'_-] + [\gamma \phi'^*_+ D_+ + (1 - \gamma) \phi'^*_- D_-][\phi_+ - \phi_-] \\ &\quad + \int_a^b \left[\frac{d\phi^*}{dx} D \frac{d\phi}{dx} + \phi^* \Lambda\phi \right] dx, \end{aligned} \quad (14)$$

$$F_1[\phi, \phi^*, j, j^*] = \alpha(\phi_+ - \phi_-) + \beta(j_+ - j_-) + \int_a^b \left(\phi^* \frac{dj}{dx} - j^* \frac{d\phi}{dx} + \phi^* \Lambda\phi - j^* D^{-1} j \right) dx, \quad (15)$$

$$F_2[\phi, \phi^*, j, j^*] = \alpha(\phi_+ - \phi_-) + (\phi^*_+ - \phi^*_-)\beta + \int_a^b \left(-\frac{d\phi^*}{dx} j - j^* \frac{d\phi}{dx} + \phi^* \Lambda\phi - j^* D^{-1} j \right) dx, \quad (16)$$

$$F_3[\phi, \phi^*, j, j^*] = (j^*_+ - j^*_-)\alpha + \beta(j_+ - j_-) + \int_a^b \left(\phi^* \frac{dj}{dx} + \frac{dj^*}{dx} \phi + \phi^* \Lambda\phi - j^* D^{-1} j \right) dx, \quad (17)$$

$$F_4[\phi, \phi^*, j, j^*] = (j_+^* - j_-^*)\alpha + (\phi_+^* - \phi_-^*)\beta + \int_a^b \left(\frac{dj^*}{dx} \phi - \frac{d\phi^*}{dx} j + \phi^* \Lambda \phi - j^* D^{-1} j \right) dx, \quad (18)$$

$$F_5[\phi, \phi^*, j, j^*, \gamma] = [\gamma \phi_-^* + (1 - \gamma) \phi_+^*] [j_+ - j_-] - [\gamma j_+^* + (1 - \gamma) j_-^*] [\phi_+ - \phi_-] + \int_a^b \left(\phi^* \frac{dj}{dx} - j^* \frac{d\phi}{dx} + \phi^* \Lambda \phi - j^* D^{-1} j \right) dx, \quad (19)$$

where α and β are undetermined multipliers defined at the interface and γ is a numerical parameter.[†] A. J. Buslik⁷ first proposed functionals $J_1[\phi, \phi^*]$ and $F_2[\phi, \phi^*, j, j^*]$. The functionals J_2, J_3, F_1, F_3 and F_4 are of the same type. Functional $J_4[\phi, \phi^*, \gamma]$ is of the type proposed by G. Pomraning⁴ for the case of the self adjoint Sturm-Liouville equation. F_5 for $\gamma = 1/2$ coincides with the functional used by Wachspress-Becker,³ and a similar form was proposed by Pomraning⁴ for time-dependent problems.

It will now be shown that the stationarity conditions for $J_1[\phi, \phi^*]$, and $J_4[\phi, \phi^*, \gamma]$, are the appropriate ones for the physical problem. The demonstration for $J_2, J_3, F_1, F_2, F_3, F_4, F_5$ would essentially be the same and will thus not be carried out. The first variations for J_1 and J_4 , are

$$\delta J_1 = \int_a^b \left\{ \delta \phi^* \left[-\frac{d}{dx} \left(D \frac{d\phi}{dx} \right) + \Lambda \phi \right] + \left[-\frac{d}{dx} \left(\frac{d\phi^*}{dx} D \right) + \phi^* \Lambda \right] \delta \phi \right\} dx + (\phi_+^* - \phi_-^*) \delta \alpha + \delta \beta (\phi_+ - \phi_-) + \delta \phi_-^* (D_- \phi_- - \alpha) + \delta \phi_+^* (\alpha - D_+ \phi_+) + (\phi_+^{*'} D_- - \beta) \delta \phi_- + (\beta - \phi_-^{*'} D_+) \delta \phi_+. \quad (20)$$

$$\begin{aligned} \delta J_4(\gamma) = & \int_a^b \left\{ \delta \phi^* \left[-\frac{d}{dx} \left(D \frac{d\phi}{dx} \right) + \Lambda \phi \right] + \left[-\frac{d}{dx} \left(\frac{d\phi^*}{dx} D \right) + \phi^* \Lambda \right] \delta \phi \right\} dx \\ & + [\phi_+^* - \phi_-^*] [\gamma D_+ \delta \phi_+ + (1 - \gamma) D_- \delta \phi_-] + [\gamma \delta \phi_+^{*'} D_+ + (1 - \gamma) \delta \phi_-^{*'} D_-] [\phi_+ - \phi_-] \\ & - [\gamma \delta \phi_-^* + (1 - \gamma) \delta \phi_+^*] [D_+ \phi_+ - D_- \phi_-] - [\phi_+^{*'} D_+ - \phi_-^{*'} D_-] [\gamma \delta \phi_- + (1 - \gamma) \delta \phi_+]. \end{aligned} \quad (21)$$

The stationarity conditions, as usual, follow from requiring $\delta J_1 = \delta J_4 = 0$. In these cases they are

$$\left. \begin{aligned} -\frac{d}{dx} \left(D \frac{d\phi}{dx} \right) + \Lambda \phi &= 0 \\ -\frac{d}{dx} \left(\frac{d\phi^*}{dx} D \right) + \phi^* \Lambda &= 0 \end{aligned} \right\} \text{ for } J_1 \text{ and } J_4 \quad (22)$$

in the reactor volume, plus the interface conditions

$$\phi_+ = \phi_-, \quad \phi_+^* = \phi_-^*, \quad D_+ \phi_+ = D_- \phi_- = \alpha, \quad \phi_+^{*'} D_+ = \phi_-^{*'} D_- = \beta, \quad \text{for } J_1, \quad (23)$$

$$\phi_+ = \phi_-, \quad \phi_+^* = \phi_-^*, \quad D_+ \phi_+ = D_- \phi_-, \quad \phi_+^{*'} D_+ = \phi_-^{*'} D_-, \quad \text{for } J_4. \quad (24)$$

These are the required differential equations and interface conditions. Consequently the functionals J_1 and J_4 , can be used for the formulation of a variational flux synthesis approximation using discontinuous trial functions. The same conclusion applies to $J_2, J_3, F_1, F_2, F_3, F_4$ and F_5 .

Finally, as shown before for J and F , the functionals $J_1, J_2, J_3, J_4, F_1, F_2, F_3, F_4$ and F_5 can be written in several equivalent forms.

For the sake of brevity the different equivalent forms will be shown only for J_1 and J_4 :

$$\begin{aligned} J_1 &= \phi_-^* (D_- \phi_- - \beta) + \phi_+^* (\beta - D_+ \phi_+) + \alpha (\phi_+ - \phi_-) + \int_a^b \left(\phi^* \left[-\frac{d}{dx} \left(D \frac{d\phi}{dx} \right) + \Lambda \phi \right] \right) dx \\ &= (\phi_-^{*'} D_- - \alpha) \phi_- + (\alpha - \phi_+^{*'} D_+) \phi_+ + (\phi_+^* - \phi_-^*) \beta + \int_a^b \left[-\frac{d}{dx} \left(\frac{d\phi^*}{dx} D \right) + \phi^* \Lambda \right] \phi dx, \end{aligned} \quad (25)$$

$$\begin{aligned} J_4(\gamma) &= [\gamma \phi_+^{*'} D_+ + (1 - \gamma) \phi_-^{*'} D_-] [\phi_+ - \phi_-] - [\gamma \phi_-^* + (1 - \gamma) \phi_+^*] [D_+ \phi_+ - D_- \phi_-] \\ &+ \int_a^b \phi^* \left[-\frac{d}{dx} \left(D \frac{d\phi}{dx} \right) + \Lambda \phi \right] dx = [\phi_+^* - \phi_-^*] [\gamma D_+ \phi_+ + (1 - \gamma) D_- \phi_-] \\ &- [\phi_+^{*'} D_+ - \phi_-^{*'} D_-] [\gamma \phi_- + (1 - \gamma) \phi_+] + \int_a^b \left[-\frac{d}{dx} \left(\frac{d\phi^*}{dx} D \right) + \phi^* \Lambda \right] \phi dx. \end{aligned} \quad (26)$$

[†] Values of γ other than 0 or 1 generate an excess of interface conditions in some cases. For a discussion of this point see Paper V-17.

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V-16. Boundary Conditions in Variational Flux Synthesis

V. LUCO

INTRODUCTION

The functional proposed in Ref. 1 to be used as the basis for a variational flux synthesis procedure, is such that at all of the outside boundaries of the reactor the neutron flux and its adjoint are required to satisfy the following conditions:

$$\frac{\phi_g}{2} = -D_g \frac{\partial \phi_g}{\partial n}$$

$$\frac{\phi_g^*}{2} = -D_g \frac{\partial \phi_g^*}{\partial n},$$

where ϕ_g and ϕ_g^* are group fluxes and adjoints, D_g is the diffusion coefficient and $\frac{\partial}{\partial n}$ indicates derivative with respect to the positive (outgoing) normal direction at the outside reactor surfaces.

A modified functional has been formulated and is presented here, which gives the user of the method ample freedom to choose the conditions to be satisfied by the neutron flux and its adjoint at the reactor boundaries.

THE FUNCTIONAL

The functional proposed for the case of a cylindrical reactor described in cylindrical coordinates is

$$\begin{aligned}
 J[U_g^k(r,z), U_g^{k*}(r,z)] \equiv & \sum_{k=1}^K \left\langle 2\pi \int_{V_k} \sum_{g=1}^G \left\{ \nabla U_g^{k*} \cdot D_g \nabla U_g^k + U_g^{k*} \left[\Sigma_{r,g} U_g^k - \sum_{g'=1}^{G-1} \Sigma_{g',g} U_{g'}^k - \lambda \chi_g \sum_{g'=1}^G (\nu \Sigma_f)_{g'} U_{g'}^k \right] \right\} r dr dz \right\rangle \\
 & - \sum_{k=1}^{K-1} \left\langle 2\pi R_k \int_0^h \sum_{g=1}^G \left\{ \left(\frac{\partial U_g^{k*}}{\partial r} \right)_{r=R_k} [U_g^{(k+1)}(R_k, z) - U_g^k(R_k, z)] + [U_g^{(k+1)*}(R_k, z) - U_g^{k*}(R_k, z)] \left(\frac{\partial U_g^k}{\partial r} \right)_{r=R_k} \right\} dz \right\rangle \\
 & + \sum_{g=1}^G \left[\sum_{k=1}^K \left\langle 2\pi \int_{R_{(k-1)}}^{R_k} \left\{ \alpha a_g^1 U_g^{k*}(r, h) U_g^k(r, h) - (1 - \alpha) \left[U_g^{k*}(r, h) D_g \left(\frac{\partial U_g^k}{\partial z} \right)_{z=h} + \left(\frac{\partial U_g^{k*}}{\partial z} \right)_{z=h} D_g U_g^k(r, h) \right] \right\} r dr \right. \right. \\
 & \left. \left. + \eta \left\{ \beta a_g^2 U_g^{k*}(r, 0) U_g^k(r, 0) + (1 - \beta) \left[U_g^{k*}(r, 0) D_g \left(\frac{\partial U_g^k}{\partial z} \right)_{z=0} + \left(\frac{\partial U_g^{k*}}{\partial z} \right)_{z=0} D_g U_g^k(r, 0) \right] \right\} r dr \right\rangle \right] \\
 & + 2\pi R \int_0^h \left\{ \gamma a_g^3 U_g^{K*}(R, z) U_g^K(R, z) - (1 - \gamma) \left[U_g^{K*}(R, z) D_g \left(\frac{\partial U_g^K}{\partial r} \right)_{r=R} + \left(\frac{\partial U_g^{K*}}{\partial r} \right)_{r=R} D_g U_g^K(R, z) \right] \right\} dz \\
 & + 2\pi R \int_0^h \left(\frac{\partial U_g^{1*}}{\partial r} \right)_{r=0} \cdot \left(\frac{\partial U_g^1}{\partial r} \right)_{r=0} dz \Bigg], \quad (1)
 \end{aligned}$$

FIG. V-16-1. Schematic Representation of the Reactor. ANL
Neg. No. 113-666 T-1.

where V_k is the volume between $R_{k-1} \leq r \leq R_k$ and $0 \leq z \leq h$; $U_\theta^k(r, z)$ and $U_\theta^{k*}(r, z)$ are independent-variable functions defined in region k and whose stationary values will be identified later as $\sigma_\theta^k(r, z)$ and $\sigma_\theta^{k*}(r, z)$ respectively; the constants D_θ , $\Sigma_{r\theta}$, $\Sigma_{\theta'\theta}$, χ_θ , $\Sigma_{f\theta}$, and ν_θ are the usual multi-group parameters; the constants a_θ^1 , a_θ^2 , a_θ^3 are parameters to be selected by the user when specifying a logarithmic derivative boundary condition; the parameters ζ , α , η , β , θ , and γ can take the values 0 or 1 and should be specified by the user, as will be explained shortly, to obtain the desired combination of boundary conditions.

There are K synthesis regions (see Fig. V-16-1), and G energy groups, $R_0 = 0$, and $R_k = R$. The interface conditions in this functional are different from those proposed in Ref. 1. Here the formulation proposed in Paper IV-17 is used. The computer code for two-dimensional spatial synthesis, SYN1D, developed on the basis of these formulations allows the user to choose one or the other synthesis interface specification.

THE STATIONARITY CONDITIONS

The stationarity conditions are obtained as usual by varying the functional with respect to U_θ^k and U_θ^{k*} and then requiring this variation to be zero. The first variation of the functional has volume terms which are of no interest here as they do not affect the boundary conditions. For simplicity also only those surface terms in the first variation resulting in boundary conditions for the flux will be considered. The argument leading to adjoint boundary conditions is exactly parallel.

The pertinent surface terms in the first variation are then

$$\begin{aligned} & \sum_{g=1}^G \left\{ \sum_{k=1}^K \left[2\pi \int_{R_{k-1}}^{R_k} \left\langle \zeta \left\{ \alpha a_\theta^1 \delta U_\theta^{k*}(r, h) U_\theta^k(r, h) - (1 - \alpha) \left[\delta U_\theta^{k*}(r, h) D_\theta \left(\frac{\partial U_\theta^k}{\partial z} \Big|_{z=h} \right) + \delta \left(\frac{\partial U_\theta^{k*}}{\partial z} \Big|_{z=h} \right) D_\theta U_\theta^k(r, h) \right] \right\} \right. \right. \\ & + \delta U_\theta^{k*}(r, h) D_\theta \left(\frac{\partial U_\theta^k}{\partial z} \Big|_{z=h} \right) + \eta \left\{ \beta a_\theta^2 \delta U_\theta^{k*}(r, 0) U_\theta^k(r, 0) + (1 - \beta) \left[\delta U_\theta^{k*}(r, 0) D_\theta \left(\frac{\partial U_\theta^k}{\partial z} \Big|_{z=0} \right) + \delta \left(\frac{\partial U_\theta^{k*}}{\partial z} \Big|_{z=0} \right) \right. \right. \\ & \cdot D_\theta U_\theta^k(r, 0) \left. \left. \right\} \delta U_\theta^{k*}(r, 0) D_\theta \left(\frac{\partial U_\theta^k}{\partial z} \Big|_{z=0} \right) \right\rangle r dr \right] + 2\pi R \int_0^h \left\langle \Theta \left\{ \gamma a_\theta^3 \delta U_\theta^{K*}(R, z) U_\theta^K(R, z) - (1 - \gamma) \left[\delta U_\theta^{K*}(R, z) D_\theta \right. \right. \right. \\ & \cdot \left(\frac{\partial U_\theta^K}{\partial r} \Big|_{r=R} \right) + \delta \left(\frac{\partial U_\theta^{K*}}{\partial r} \Big|_{r=R} \right) D_\theta U_\theta^K(R, z) + \delta U_\theta^{K*}(R, z) D_\theta \left(\frac{\partial U_\theta^K}{\partial r} \Big|_{r=R} \right) + \left(\frac{\partial U_\theta^{1*}}{\partial r} \Big|_{r=0} \right) \left(\frac{\partial U_\theta^1}{\partial r} \Big|_{r=0} \right) \left. \left. \right\} dz \right\rangle = 0, \end{aligned} \quad (2)$$

implying the following stationarity conditions:

$$\zeta \left[\alpha a_g^1 \phi_g^k(r, h) = (1 - \alpha) D_g \left(\frac{\partial \phi_g^k}{\partial z} \Big|_{z=h} \right) \right] + D_g \left(\frac{\partial \phi_g^k}{\partial z} \Big|_{z=h} \right) = 0 \quad (3)$$

$$\zeta (1 - \alpha) D_g \phi_g^k(r, h) = 0 \quad (4)$$

$$\eta \left[\beta a_g^2 \phi_g^k(r, 0) + (1 - \beta) D_g \left(\frac{\partial \phi_g^k}{\partial z} \Big|_{z=0} \right) \right] - D_g \left(\frac{\partial \phi_g^k}{\partial z} \Big|_{z=0} \right) = 0 \quad (5)$$

$$\eta (1 - \beta) \phi_g^k(r, 0) = 0 \quad (6)$$

$$\theta \left[\gamma a_g^3 \phi_g^K(R, z) - (1 - \gamma) D_g \left(\frac{\partial \phi_g^K}{\partial r} \Big|_{r=R} \right) \right] + D_g \left(\frac{\partial \phi_g^K}{\partial r} \Big|_{r=R} \right) = 0 \quad (7)$$

$$\theta (1 - \gamma) \phi_g^K(R, z) = 0 \quad (8)$$

$$\left(\frac{\partial \phi_g^1}{\partial r} \Big|_{r=0} \right) = 0, \quad (9)$$

with $g = 1, \dots, G$ in all of them.

By a proper choice of the parameters ζ , α , η , β , θ , and γ it is possible to select zero flux, or zero derivative, or a logarithmic derivative boundary condition at any of the bounding reactor surfaces. For example, at the top of the reactor, that is for $z = h$ and $0 \leq r \leq R$, the boundary conditions

$$a_g^1 \phi_g^k(r, h) + D_g \frac{\partial \phi_g^k}{\partial z} \Big|_{z=h} = 0, \quad \begin{cases} h = 1, \dots, G \\ k = 1, \dots, K \end{cases}$$

are obtained by selecting $\zeta = 1$ and $\alpha = 1$.

The choice $\zeta = 1$ and $\alpha = 0$ gives

$$\phi_g^k(r, h) = 0, \quad \begin{cases} g = 1, \dots, G \\ k = 1, \dots, K. \end{cases} \quad (10)$$

Finally, $\zeta = 0$ and $\alpha = 0$ implies

$$\frac{\mu \phi_g^k}{\partial z} \Big|_{z=h} = 0, \quad \begin{cases} g = 1, \dots, G \\ k = 1, \dots, K. \end{cases}$$

As the choices of parameters ξ , α , η , β , θ , and γ are completely free, it is possible to impose any desired combination of these boundary conditions at the different portions of the outside reactor surface, i.e., the top plane $z = h$, $0 \leq r \leq R$, the bottom plane $z = 0$, $0 \leq r \leq R$ and the cylindrical surface $r = R$, $0 \leq z \leq h$ in the present example.

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V-17. Solutions to the Problem of Interface Overdetermination in Spatial Flux Synthesis

V. LUCO and W. L. WOODRUFF*

INTRODUCTION

Most of the formulations of variational flux synthesis methods using discontinuous trial functions have been used on functionals of the type proposed by E. Wachspress and M. Becker.¹ A. Buslik² pointed out that this type of functional generates an excess of interface conditions when both fluxes and currents are discontinuous simultaneously. He proposed a functional that eliminates this difficulty. In his functional the interface conditions of continuity of κ and adjoint flux are incorporated into the functional by means of undetermined Lagrange multipliers. The solu-

* Student, Texas A&M University, College Station, Texas.

tions of the stationarity conditions for this functional are shown to also satisfy the continuity conditions for current and adjoint current at the discontinuity interfaces.

It will be shown here that functionals of a type similar to those first proposed by G. Pomraning³ can be used to formulate variational flux synthesis procedures for stationary reactor problems without the overdetermination of the Wachspress-Becker functional. Moreover it is not necessary, as in Buslik's formulation, to deal separately with synthesis expansions for currents at the discontinuity interfaces.

THE FUNCTIONALS

The functionals proposed are

$$J[\phi, \phi^*, \gamma] = \sum_{k=1}^K \int_{V_k} dV \left[\nabla \phi^* \cdot D \nabla \phi + \phi^* \left(A - \frac{M}{\lambda} \right) \phi \right] \\ + \sum_{\ell=1}^L \int_{S_\ell} dS n_\ell \cdot \{ [\phi_+^* - \phi_-^*] [\gamma (D \nabla \phi)_+ + (1 - \gamma) (D \nabla \phi)_-] + [\gamma (\nabla \phi^* D)_+ + (1 - \gamma) (\nabla \phi^* D)_-] [\phi_+ - \phi_-] \} \quad (1)$$

and

$$F[\phi, \phi^*, j, j^*; \gamma] = \sum_{k=1}^K \int_{V_k} dV \left[-\nabla \phi^* \cdot j - j^* \cdot \nabla \phi + \phi^* \left(A - \frac{M}{\lambda} \right) \phi - j^* \cdot D^{-1} j \right] \\ - \sum_{\ell=1}^L \int_{S_\ell} dS n_\ell \cdot \{ [\gamma j_+^* + (1 - \gamma) j_-^*] [\phi_+ - \phi_-] + [\phi_+^* - \phi_-^*] [\gamma j_+ + (1 - \gamma) j_-] \}. \quad (2)$$

Here the reactor volume has been divided into K non-overlapping partial volumes V_k with interfaces S_ℓ . The class of admissible functions is the class of sectionally continuous vector functions ϕ, ϕ^*, j, j^* (column and row vectors in energy-space for direct and adjoint quantities respectively) with sectionally continuous first derivatives in the reactor volume, having their jump discontinuities at the surfaces S_ℓ . For simplicity it will be assumed that ϕ and ϕ^* go to zero at the reactor boundary. λ is the eigenvalue; D , A and M are the diffusion, absorption-scattering and fission matrices; n_ℓ is the unit normal at the surface S_ℓ ; γ is a numerical parameter that can take the values 0 or 1.

The nomenclature $\phi_+, \phi_-, \phi_+^*, \phi_-^*, (D \nabla \phi)_+, (D \nabla \phi)_-, (\nabla \phi^* D)_+, (\nabla \phi^* D)_-, j_+, j_-, j_+^*, j_-^*$ is used to indicate the limits attained on both sides of a discontinuity interface S_ℓ by the corresponding quantities. The stationarity conditions for these functions are obtained by varying $\phi, \phi^*, j, j^*, \phi_+, \phi_-, (\nabla \phi)_+, (\nabla \phi)_-, \phi_+^*, \phi_-^*, (\nabla \phi^*)_+, (\nabla \phi^*)_-, j_+, j_-, j_+^*$ and j_-^* arbitrarily in Eqs. (1) and (2) and applying the following transformations:^{4,5}

$$\sum_{k=1}^K \int_{V_k} dV [\nabla \delta \phi^* \cdot D \nabla \phi] = \sum_{\ell=1}^L \int_{S_\ell} dS n_\ell \cdot \{ \delta \phi_-^* (D \nabla \phi)_- - \delta \phi_+^* (D \nabla \phi)_+ \} + \sum_{k=1}^K \int_{V_k} dV \delta \phi^* [-\nabla \cdot (D \nabla \phi)] \\ \sum_{k=1}^K \int_{V_k} dV [\nabla \phi^* \cdot D \nabla \delta \phi] = \sum_{\ell=1}^L \int_{S_\ell} dS n_\ell \cdot \{ (\nabla \phi^* D)_- \delta \phi_- - (\nabla \phi^* D)_+ \delta \phi_+ \} + \sum_{k=1}^K \int_{V_k} dV [-\nabla \cdot (\nabla \phi^* D)] \delta \phi \\ \sum_{k=1}^K \int_{V_k} dV [-\nabla \delta \phi^* \cdot j] = \sum_{\ell=1}^L \int_{S_\ell} dS n_\ell \cdot [\delta \phi_+^* j_+ - \delta \phi_-^* j_-] + \sum_{k=1}^K \int_{V_k} dV [\delta \phi^* \nabla \cdot j] \\ \sum_{k=1}^K \int_{V_k} dV [-j^* \cdot \nabla \delta \phi] = \sum_{\ell=1}^L \int_{S_\ell} dS n_\ell \cdot [j_+^* \delta \phi_+ - j_-^* \delta \phi_-] + \sum_{k=1}^K \int_{V_k} dV [\nabla \cdot j^* \delta \phi]$$

The resulting stationarity conditions for J are

$$\left. \begin{aligned} & -\nabla \cdot (D \nabla \phi) + \left(A - \frac{M}{\lambda} \right) \phi = 0 \\ & -\nabla \cdot (\nabla \phi^* D) + \phi^* \left(A - \frac{M}{\lambda} \right) = 0 \\ & \phi_+ = \phi_- \\ & \phi_+^* = \phi_-^* \\ & n_\ell \cdot [(D \nabla \phi)_+ - (D \nabla \phi)_-] = 0 \\ & n_\ell \cdot [(\nabla \phi^* D)_+ - (\nabla \phi^* D)_-] = 0 \end{aligned} \right\} \begin{array}{l} \text{in } V \\ \\ \text{at } S_\ell, \\ \ell = 1, \dots, L \end{array}; \quad (3)$$

adjoint for F :

$$\left. \begin{aligned} \nabla \phi + D^{-1} \mathbf{j} &= 0 \\ \nabla \cdot \mathbf{j} + \left(A - \frac{M}{\lambda} \right) \phi &= 0 \\ \nabla \phi^* + \mathbf{j}^* D^{-1} &= 0 \\ \nabla \cdot \mathbf{j}^* + \phi^* \left(A - \frac{M}{\lambda} \right) &= 0 \end{aligned} \right\} \text{in } V, \quad (4)$$

$$\left. \begin{aligned} \phi_+ &= \phi_- \\ \phi_+^* &= \phi_-^* \\ \mathbf{n}_\ell \cdot [\mathbf{j}_+ - \mathbf{j}_-] &= 0 \\ \mathbf{n}_\ell \cdot [\mathbf{j}_+^* - \mathbf{j}_-^*] &= 0 \end{aligned} \right\} \text{at } S_\ell, \quad \ell = 1, \dots, L$$

The equivalence of the variational problems posed and the solution of the neutron diffusion problem and its adjoint is thus established.

FLUX SYNTHESIS

It will be shown now that no problem of overdetermination arises when these functionals are used to generate synthesis equations with trial functions for direct and adjoint quantities having simultaneous discontinuities at the surfaces S_ℓ . For the sake of brevity the demonstration will be carried out only for the functional J .

A two-dimensional problem described in x - y coordinates will be considered in order to simplify the presentation. The reactor is divided into two synthesis regions by a line parallel to the y -axis at the $x = x_0$ point. Synthesis region number one and the $(-)$ side of the interface are located to the left of the internal boundary. For this geometry the functional J takes the following form:

$$\begin{aligned} J[\phi, \phi^*; \gamma] &= \iint_S dx dy \left[\frac{\partial \phi^*}{\partial x} D \frac{\partial \phi}{\partial x} + \frac{\partial \phi^*}{\partial y} D \frac{\partial \phi}{\partial y} + \phi^* \left(A - \frac{M}{\lambda} \right) \phi \right] \\ &+ \int_L dy \{ [\phi_+^* - \phi_-^*] [\gamma D_+ \phi_+' + (1 - \gamma) D_- \phi_-'] + [\gamma \phi_+' D_+ + (1 - \gamma) \phi_-'^* D_-] [\phi_+ - \phi_-] \}, \end{aligned} \quad (5)$$

where

S is the area defined by $a \leq x \leq b, c \leq y \leq d$

L is the line segment $c \leq y \leq d$, at the $x = x_0$ point

$$\begin{aligned} \phi_+' &= \lim_{\epsilon \rightarrow 0} \frac{\partial \phi}{\partial x} \Big|_{x_0+\epsilon} & \phi_+^{*'} &= \lim_{\epsilon \rightarrow 0} \frac{\partial \phi^*}{\partial x} \Big|_{x_0+\epsilon} \\ \phi_- ' &= \lim_{\epsilon \rightarrow 0} \frac{\partial \phi}{\partial x} \Big|_{x_0-\epsilon} & \phi_-^{*'} &= \lim_{\epsilon \rightarrow 0} \frac{\partial \phi^*}{\partial x} \Big|_{x_0-\epsilon} \end{aligned}$$

with similar definitions for $\phi_+, \phi_+^*, D_+, \phi_-, \phi_-^*$, and D_- .

The following synthesis expansions are then introduced into Eq. (5):

$$\phi_k(x, y) = \sum_{i=1}^{n_k} H_{ik}(y) \psi_{ik}(x) \quad (6)$$

$$\phi_k^*(x, y) = \sum_{i=1}^{n_k} \psi_{ik}^*(x) H_{ik}^*(y), \quad (7)$$

where

$k = 1, 2$ indicates synthesis regions—number 1 to the left and number 2 to the right of the synthesis boundary.

$H_{ik}(y)$ and $H_{ik}^*(y)$ are the known trial functions used as the basis for the expansions.

$\psi_{ik}(x)$ and $\psi_{ik}^*(x)$ are the mixing functions to be determined variationally.

n_k is the number of trial functions used in region k .

After these substitutions have been made, it is possible to integrate over y in Eq. (5), obtaining the following reduced functional:

$$J'[\psi_k, \psi_k^*; \gamma] = \sum_{k=1,2} \int dx \left(\frac{d\psi_k^*}{dx} [D_k] \frac{d\psi_k}{dx} + \psi_k^* \left\{ [A_k(x)] - [L_k(x)] - \frac{[M_k(x)]}{\lambda} \right\} \psi_k \right) \\ + \{ \psi_2^*(+) [\alpha^{21}(-)] - \psi_1^*(-) [\alpha^{11}(-)] \} (1 - \gamma) \psi_1'(-) + \{ \psi_2^*(+) [\alpha^{22}(+)] - \psi_1^*(-) [\alpha^{12}(+)] \} \gamma \psi_2'(+), \quad (8) \\ + (1 - \gamma) \psi_1^{*'}(-) \{ [\alpha^{12}(-)] \psi_2(+) - [\alpha^{11}(-)] \psi_1(-) \} + \gamma \psi_2^{*'}(+) \{ [\alpha^{22}(+)] \psi_2(+) - [\alpha^{21}(+)] \psi_1(-) \},$$

where

$\psi_k(x)$ is a column vector with components $\psi_{ik}(x)$.

$\psi_k^*(x)$ is a row vector with components $\psi_{ik}^*(x)$.

$\psi_1(-)$, $\psi_1^*(-)$, $\psi_1'(-)$, and $\psi_1^{*'}(-)$ are the limits of $\psi_1(x)$, $\psi_1^*(x)$, $d\psi_1/dx$, and $d\psi_1^*/dx$ for $x \rightarrow x_0$ in region 1.

$\psi_2(+)$, $\psi_2^*(+)$, $\psi_2'(+)$, and $\psi_2^{*'}(+)$ are the limits of $\psi_2(x)$, $\psi_2^*(x)$, $d\psi_2/dx$, and $d\psi_2^*/dx$ for $x \rightarrow x_0$ in region 2.

$[D_k(x)]$, $[A_k(x)]$, $[L_k(x)]$ and $[M_k(x)]$ are $n_k \times n_k$ matrices defined by their elements

$$D_{ij}^k(x) = \int_c^d dy H_{ik}^*(y) D_k(x, y) H_{jk}(y)$$

$$A_{ij}^k(x) = \int_c^d dy H_{ik}^*(y) A_k(x, y) H_{jk}(y)$$

$$L_{ij}^k(x) = \int_c^d dy \frac{dH_{ik}^*}{dy} D_k(x, y) \frac{dH_{jk}}{dy}$$

$$M_{ij}^k(x) = \int_c^d dy H_{ik}^*(y) M_k(x, y) H_{jk}(y).$$

$[\alpha^{kk'}(+)]$ and $[\alpha^{kk'}(-)]$, with $k, k' = 1, 2$, are $n_k \times n_{k'}$ matrices defined by their elements

$$\alpha_{ij}^{kk'}(-) = \int_c^d dy H_{ik}^*(y) D_{k'}(x_0, y) H_{jk'}(y),$$

$$\alpha_{ij}^{kk'}(+) = \int_c^d dy H_{ik}^*(y) D_k(x_0, y) H_{jk'}(y).$$

The first variation for J' is obtained by arbitrarily varying ψ_k , ψ_k^* , $\psi_1(-)$, $\psi_2^*(+)$, $\psi_1'(-)$, $\psi_2'(+)$, $\psi_1^*(-)$, $\psi_2^*(-)$, $\psi_1^{*'}(-)$, $\psi_2^{*'}(+)$, then integrating by parts the terms in $d\delta\psi_k/dx$, $d\delta\psi_k^*/dx$, noting that

$$[D_1] \equiv [\alpha^{11}(-)], \quad [D_2] \equiv [\alpha^{22}(+)],$$

and finally imposing the boundary conditions

$$\psi_1(a) = \psi_1^*(a) = \psi_2(b) = \psi_2^*(b) = 0. \quad (9)$$

The resulting first variation is:

$$\delta J' = \sum_{k=1,2} \int dx \left\langle \delta\psi_k^* \left\{ - \frac{d}{dx} \left([D_k] \frac{d\psi_k}{dx} \right) + \left([A_k(x)] - [L_k(x)] - \frac{[M_k(x)]}{\lambda} \right) \psi_k \right\} + \left\{ - \frac{d}{dx} \left(\frac{d\psi_k^*}{dx} [D_k] \right) \right. \right. \\ + \psi_k^* \left([A_k(x)] - [L_k(x)] - \frac{[M_k(x)]}{\lambda} \right) \Bigg\rangle + \{ \psi_2^*(+) [\alpha^{21}(-)] - \psi_1^*(-) [\alpha^{11}(-)] \} (1 - \gamma) \delta\psi_1'(-) \\ + \{ \psi_2^*(+) [\alpha^{22}(+)] - \psi_1^*(-) [\alpha^{12}(+)] \} \gamma \delta\psi_2'(+), \quad (10) \\ + (1 - \gamma) \delta\psi_1^{*'}(-) \{ [\alpha^{12}(-)] \psi_2(+) - [\alpha^{11}(-)] \psi_1(-) \} \\ + \gamma \delta\psi_2^{*'}(+) \{ [\alpha^{22}(+)] \psi_2(+) - [\alpha^{21}(+)] \psi_1(-) \} - \gamma \delta\psi_1^*(-) \{ [\alpha^{12}(+)] \psi_2'(+)- [\alpha^{11}(-)] \psi_1'(-) \} \\ - (1 - \gamma) \delta\psi_2^*(+) \{ [\alpha^{22}(+)] \psi_2'(+)- [\alpha^{21}(-)] \psi_1'(-) \} - \gamma \delta\psi_1(-) \{ \psi_2^{*'}(+) [\alpha^{21}(+)] - \psi_1^{*'}(-) [\alpha^{11}(-)] \} \\ - (1 - \gamma) \delta\psi_2(+) \{ \psi_2^{*'}(+) [\alpha^{22}(+)] - \psi_1^{*'}(-) [\alpha^{12}(-)] \}$$

If γ is set equal to zero in $\delta J'$ the following set of stationarity conditions are obtained:

$$- \frac{d}{dx} \left\{ [D_k] \frac{d\psi_k}{dx} \right\} + \left\{ [A_k(x)] - [L_k(x)] - \frac{[M_k(x)]}{\lambda} \right\} \psi_k = 0 \quad (11)$$

$$- \frac{d}{dx} \left\{ \frac{d\psi_k^*}{dx} [D_k] \right\} + \psi_k^* \left\{ [A_k(x)] - [L_k(x)] - \frac{[M_k(x)]}{\lambda} \right\} = 0 \quad (12)$$

$$\left. \begin{aligned} [\alpha^{12}(-)]\psi_2(+) - [\alpha^{11}(-)]\psi_1(-) &= 0 \\ \psi_2^*(+)[\alpha^{21}(-)] - \psi_1^*(-)[\alpha^{11}(-)] &= 0 \\ [\alpha^{22}(+)]\psi_2'(+) - [\alpha^{21}(-)]\psi_1'(-) &= 0 \\ \psi_2^{*'}(+)[\alpha^{22}(+)] - \psi_1^{*'}(-)[\alpha^{12}(-)] &= 0 \end{aligned} \right\} \text{ at } x = x_0. \quad (13)$$

$$\psi_2^*(+)[\alpha^{21}(-)] - \psi_1^*(-)[\alpha^{11}(-)] = 0 \quad (14)$$

$$[\alpha^{22}(+)]\psi_2'(+) - [\alpha^{21}(-)]\psi_1'(-) = 0 \quad (15)$$

$$\psi_2^{*'}(+)[\alpha^{22}(+)] - \psi_1^{*'}(-)[\alpha^{12}(-)] = 0 \quad (16)$$

If instead the value $\gamma = 1$ is chosen Eqs. (11) and (12) are unchanged but the interface conditions are now:

$$\left. \begin{aligned} [\alpha^{22}(+)]\psi_2(+) - [\alpha^{21}(+)]\psi_1(-) &= 0 \\ \psi_2^*(+)[\alpha^{22}(+)] - \psi_1^*(-)[\alpha^{12}(+)] &= 0 \\ [\alpha^{12}(+)]\psi_2'(+) - [\alpha^{11}(-)]\psi_1'(-) &= 0 \\ \psi_2^{*'}(+)[\alpha^{21}(+)] - \psi_1^{*'}(-)[\alpha^{11}(-)] &= 0 \end{aligned} \right\} \text{ at } x = x_0. \quad (17)$$

$$\psi_2^*(+)[\alpha^{22}(+)] - \psi_1^*(-)[\alpha^{12}(+)] = 0 \quad (18)$$

$$[\alpha^{12}(+)]\psi_2'(+) - [\alpha^{11}(-)]\psi_1'(-) = 0 \quad (19)$$

$$\psi_2^{*'}(+)[\alpha^{21}(+)] - \psi_1^{*'}(-)[\alpha^{11}(-)] = 0 \quad (20)$$

To determine the $n_1 + n_2$ unknown functions $\psi_{i1}(x)$, $\psi_{i2}(x)$ defined by the set of second order differential equations it is necessary to impose $2(n_1 + n_2)$ conditions. If γ is equal to zero there are n_1 interface conditions, Eq. (13), n_2 interface conditions, Eq. (15), and $n_1 + n_2$ boundary conditions, Eq. (9). When γ is equal to unity there are n_2 interface conditions, Eq. (17), n_1 interface conditions, Eq. (19), and again the same $n_1 + n_2$ boundary conditions, Eq. (9). Thus for both cases the problem is well posed. In a similar way it can be shown that the adjoint problem is also properly defined.

In this same manner the functional of Eq. (2) with appropriate expansions for the currents can also be shown to yield a well defined problem for the values of $\gamma = 0, 1$. To complete the argument, it suffices to say that for both functionals all other choices for γ (other than $\gamma = 0, 1$) will lead to the same overdeterminations discussed by Buslik.²

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V-18. Comparison of Weighting Function Choices for Spatial Reactor Kinetics

E. L. FULLER

Consider a two-mode expansion for the neutron flux in an infinite slab reactor:

$$\bar{\phi}(x,t) = \psi_1(x) N_1(t) + \psi_2(x) N_2(t). \quad (1)$$

The functions $\psi_1(x)$ and $\psi_2(x)$ are trial (shape) functions; $N_1(t)$ and $N_2(t)$ are undetermined amplitude functions. Several approximate methods,¹⁻³ all essentially weighted residual methods⁴ differing only in their method of choosing trial functions, have been developed to determine the amplitude functions. For these, a proper choice of trial functions is crucial for a good approximate solution. The effect of weighting

function choice on accuracy has not been investigated, however. We here attempt to rectify this situation.

Only one reactor is analyzed in this study. It is a 240 cm slab reactor which, for convenience, is treated in a symmetric manner. That is, the only transients studied result from perturbations symmetric about the center of the reactor so that only half of the reactor is considered. Hence, a zero current boundary condition is imposed at the origin. The parameters for criticality are taken from Ref. 5 and are listed in Table V-18-I. The initial flux shape is shown in Fig. V-18-1.

Two transients are analyzed. Both are initiated by a ramp change in the fission cross section in Region 1 (0-15 cm from the origin) for 0.1 sec which is then held constant for the remainder of the transient (up to $t = 1$ sec). The final values of the fission cross section

TABLE V-18-I. PARAMETERS FOR CRITICALITY

Parameter	Region 1, 0-15 cm	Region 2, 15-60 cm	Region 3, 60-120 cm
D , cm	1.69531	1.69531	1.69531
$\nu\Sigma_f$, cm^{-1}	0.0194962	0.0194962	0.0194962
Σ_a , cm^{-1}	0.0194962	0.0194962	0.0183343
ν , cm/sec	10^6	10^6	10^6

FIG. V-18-1. Shape Functions Used in Analysis of Transients. ANL Neg. No. 118-1894.

in Region 1 are given in Table V-18-II, and the final shapes appear in Fig. V-18-1. One group of delayed neutrons is used, with $\beta = 0.0064$, and $\lambda = 0.08 \text{ sec}^{-1}$. When the transients were analyzed with the "exact" finite difference computer code WIGLE,⁽⁶⁾ it was found that Transient 1 was sub-prompt critical and Transient 2 super-prompt critical. The WIGLE solutions also yielded the final shapes shown in Fig. V-18-1. Also shown in Fig. V-18-1 is the final shape of a transient initiated by simultaneously increasing the fission cross section in the 15-30 cm region and the absorption cross section in the 0-15 cm region in ramps until $t = 0.1$ sec, and then terminating the transient at $t = 1$ sec. (This shape is called "poor".)

In order to study the effect of trial- and weighting-function choice on accuracy, three sets of functions are formed by coupling the initial shape with the final shapes shown in Fig. V-18-1. These sets are in various combinations as trial and weighting functions; the results are shown in Tables V-18-III and V-18-IV, and in Figs. V-18-2 and V-18-3. Note that $N_1(t)$ is the mode 1 amplitude, and $N_2(t)$ is the mode 2 amplitude. (Mode 1, i.e., $\psi_1(x)$ is the initial shape, and mode 2 is the final shape.) The amplitude function is defined by

TABLE V-18-II. REGION 1 FISSION CROSS SECTIONS AT END OF RAMP

	Transient 1	Transient 2
$\nu\Sigma_f$, cm^{-1}	0.0201461	0.0202962

TABLE V-18-III. TRANSIENT 1, RESULTS AT $t = 1$ sec

Approximation	Reciprocal Period, sec^{-1}	Amplitude Function	Mode 1 Amplitude	Mode 2 Amplitude
Exact (WIGLE)	1.91	102.90		
Point Kinetics	0.49	10.87	10.87	
Transient 1 Shapes				
Galerkin wt.	1.91	102.91	0.01	102.77
Transient 2 wt.	1.91	102.83	0.00	102.83
"Poor" wt.	1.91	101.92	0.06	101.86
Subdomain 1 wt. ^a	1.90	102.58	0.03	102.55
Subdomain 2 wt. ^b	1.92	103.82	-0.25	104.07
Transient 2 Shapes				
Transient 1 wt.	1.93	105.36	20.71	84.64
Galerkin wt.	1.92	104.80	20.69	84.11
"Poor" wt.	2.07	120.64	20.75	99.88
Subdomain 1 wt. ^a	1.88	100.20	20.03	80.17
Subdomain 2 wt. ^b	1.96	108.13	20.88	87.26
"Poor" Shapes				
Transient 1 wt.	1.89	100.34	-216.44	316.78
Transient 2 wt.	1.80	93.52	-195.86	289.37
Galerkin wt.	0.52	11.60	9.51	2.08

^a Subdomains are non-overlapping: 0-15 cm and 15-120 cm from origin.^b Subdomains are non-overlapping: 0-60 cm and 60-120 cm from origin.

TABLE V-18-IV. TRANSIENT 2 RESULTS AT $t = 1$ sec

Approximation	Reciprocal Period, sec^{-1}	Amplitude Function	Mode 1 Amplitude	Mode 2 Amplitude
Exact (WIGLE)	29.51	3.94×10^{12}		
Point Kinetics	8.84	4.87×10^4	4.87×10^4	
Transient 2 Shapes				
Transient 1 wt.	29.51	3.94×10^{12}	1.14×10^8	3.94×10^{12}
Galerkin wt.	29.51	3.93×10^{12}	3.86×10^8	3.93×10^{12}
"Poor" wt.	29.52	3.92×10^{12}	-7.69×10^8	3.93×10^{12}
Subdomain 1 wt. ^a	29.71	4.72×10^{12}	-3.08×10^{10}	4.75×10^{12}
Subdomain 2 wt. ^b	29.44	3.70×10^{12}	5.56×10^9	3.69×10^{12}
Transient 1 Shapes				
Galerkin wt.	29.48	3.82×10^{12}	-9.56×10^{11}	4.78×10^{12}
Transient 2 wt.	29.49	3.85×10^{12}	-9.64×10^{11}	4.82×10^{12}
"Poor" wt.	28.99	2.45×10^{12}	-5.45×10^{11}	3.00×10^{12}
Subdomain 1 wt. ^a	29.82	5.24×10^{12}	-1.35×10^{12}	6.59×10^{12}
Subdomain 2 wt. ^b	29.36	3.43×10^{12}	-8.24×10^{11}	4.25×10^{12}
"Poor" Shapes				
Transient 1 wt.	28.75	1.99×10^{12}	-5.67×10^{12}	7.66×10^{12}
Transient 2 wt.	29.34	3.34×10^{12}	-9.81×10^{12}	1.31×10^{13}
Galerkin wt.	9.92	1.24×10^5	9.68×10^4	2.73×10^4

^a Subdomains are non-overlapping: 0-15 cm and 15-120 cm from origin.^b Subdomains are non-overlapping: 0-60 cm and 60-120 cm from origin. FIG. V-18-2. Transient 2 Flux Distribution at $t = 1$ sec Using Transient 1 Shapes as Trial Functions. ANL Neg. No. 113-1892.

$$P(t) = \frac{\int \psi_1(x) \bar{\phi}(x, t) dx}{\int \psi_1^2(x) dx} \quad (2)$$

and the shape function, at any time t , is then determined by

$$\psi(x, t) = \bar{\phi}(x, t) / P(t). \quad (3)$$

The reciprocal period is asymptotic at $t = 1$ sec for all of the transients studied. Therefore, it is calculated by

FIG. V-18-3. Transient 1 Shape Functions at $t = 1$ sec Using "Poor" Trial Functions. ANL Neg. No. 113-1893.

$$\alpha_{as} = \frac{1}{t_2 - t_1} \ln \frac{P(t_2)}{P(t_1)}, \quad t_2 > t_1, \quad (4)$$

where t_1 and t_2 are times after which the period has become asymptotic. If this reciprocal period is the same as for the "exact" case, the error in the amplitude function represents the total accumulated error for the transient.

When the "bracketing" shapes (initial and final shapes of the transient being analyzed) are used as trial functions, the approximations are quite good; the effect of weighting-function choice is minimal.

Mode 2 dominates in these cases, indicating that the proper shape has been selected at $t = 1$ sec. The proper shape is also selected at intermediate times, but the results are not tabulated here. When the trial functions are "fair", such as when the Transient 1 shapes for Transient 2 are used, a good choice of weighting functions becomes more important in obtaining an accurate amplitude function, although each case gives about the same shape, as is illustrated in Fig. V-18-2 for Transient 2. When "poor" trial functions are used, the shape functions are incorrect, as can be seen by observing Fig. V-18-3. The amplitude function is little better than point kinetics for the choice of Galerkin weighting.⁴ Using weighting functions more characteristic of the "bracketing" shapes of the transient results in fairly good estimates of both the amplitude function and the reciprocal period. Such choices are fortuitous, however; one would most likely use Galerkin weighting.

One can conclude that, given a reasonably good choice of trial functions, Galerkin weighting is sound strategy. One does not have to decide how to arrange subdomains, nor does one have to somehow find a

different set of good weighting functions. Although Galerkin weighting does not help when a poor set of trial functions is chosen, it does not appear to hinder a good choice, at least for the one-energy-group model, for which the Galerkin method is equivalent to using adjoint weighting.

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V-19. Integration of the Multimode Kinetics Equations by the Method of Undetermined Parameters

E. L. FULLER

INTRODUCTION

In solving the time-dependent neutron diffusion equations, the neutron flux distribution can be approximated by expansions of the form

$$\bar{\phi}(x,t) = \sum_{i=1}^I \psi_i(x,t) N_i(t) = \psi(x,t) N(t), \quad (1)$$

where ϕ is a G -dimensional column vector in a G -energy group formulation. The trial function matrices ψ_i are also G -dimensional column vectors, and the $N_i(t)$ are scalar functions, so that ψ is the $G \times I$ matrix

$$\psi(x,t) = [\psi_1(x,t) \cdots \psi_I(x,t)], \quad (2)$$

and N is the I -dimensional column vector of the N_i :

$$N(t) = \text{col } [N_1(t) \cdots N_I(t)]. \quad (3)$$

It can be shown¹ that all of the approximate methods²⁻⁹ that have been proposed to date to solve the space-time problem are based on making an expansion in the form of Eq. (1), and then utilizing a weighted

residual procedure¹⁰ to perform the spatial integration. The only differences that exist among the approximate methods are the ways in which their trial functions and weighting functions are chosen for use in the weighted residual method.¹ Once the spatial weighted residual method is used, systems of first order, ordinary differential equations in time results in all cases. With proper definitions of the parameters, these systems can all be put into a matrix shorthand form analogous to the point reactor model. This form, known as the multimode kinetics equations,² can be written as

$$\Lambda(t) \frac{dN}{dt} = [\rho(t) - \beta(t)]N(t) + \sum_{j=1}^J \lambda_j C_j(t), \quad (4)$$

$$\frac{dC_j}{dt} = \beta_j(t)N(t) - \lambda_j C_j(t), \quad (5)$$

$$\beta(t) = \sum_{j=1}^J \beta_j(t). \quad (6)$$

The matrices $\Lambda(t)$, $\rho(t)$, $\beta(t)$, and $\beta_j(t)$ are $I \times I$

atrices, called, respectively, the generation time matrix, the reactivity matrix, and the delayed neutron fraction matrices.

SOLUTION METHODS

Once the multimode kinetics equations are integrated, the space-time solution is complete. To carry out the integration, however, some care must be taken, because conventional numerical integration methods, such as finite difference or Runge-Kutta methods, have proved to be poor choices for integrating the point reactor model. Because the time derivative is multiplied by Λ (a very small parameter), very small time steps must be taken to ensure a stable solution. Consequently, approximate methods based on integral equation formulations have been developed¹¹⁻¹⁴ to successfully overcome the finite-difference time step limitation. Most of these methods formally integrate the point reactor model and approximate the slowly varying portions of the integrands with trial solutions. The kinetics equations are put into a quasilinear form, in which an integrating factor is chosen in some manner. We choose an alternate approach here: we multiply by a set of weighting functions and use the method of undetermined parameters, generalizing the earlier work of R. Brittan¹² and J. Kaganove¹³ to integrate the multimode kinetics equations.

If Eq. (5) is substituted into Eq. (4), then

$$\Lambda \frac{dN}{dt} = \rho(t)N(t) - \sum_{j=1}^J \frac{dC_j}{dt}. \quad (7)$$

Also, Eq (5) can be integrated formally over the time interval $t_i \leq t \leq t_{i+1}$ as follows:

$$C_j(t) = C_j(t_i)e^{-\lambda_j(t-t_i)} + \int_{t_i}^t \beta_j(t') N(t') e^{-\lambda_j(t-t')} dt'. \quad (8)$$

Then, expressions for dC_j/dt can be obtained by differentiating Eq. (8):

$$\frac{dC_j}{dt} = -\lambda_j C_j(t_i)e^{-\lambda_j(t-t_i)} + \beta_j(t)N(t) - \lambda_j \int_{t_i}^t \beta_j(t') N(t') e^{-\lambda_j(t-t')} dt' \quad (9)$$

The system represented by Eqs. (7) and (9) can be solved in the interval $t_i \leq t \leq t_{i+1}$ by first expanding the amplitude functions in the trial functions $T_k(t)$ as follows:

$$\bar{N}(t) = \sum_{k=0}^K A_k T_k(t). \quad (10)$$

Each $T_k(t)$ is a scalar function of time, so that each A_k is an I-dimensional column vector. The factor A_0 is the

value of the amplitude function vector $\bar{N}(t_i)$, so that $T_0(t)$ is unity. Each element of each remaining A_k must be determined to complete the solution.

If Eq. (9) is substituted into Eq. (7), the following equation results:

$$\begin{aligned} \Lambda \frac{dN}{dt} &= [\rho(t) - \beta(t)]N(t) \\ &= \sum_{j=1}^J \lambda_j \left[C_j(t_i) e^{-\lambda_j(t-t_i)} \right. \\ &\quad \left. + \int_{t_i}^t \beta_j(t') N(t') e^{-\lambda_j(t-t')} dt' \right]. \end{aligned} \quad (11)$$

The approximate solution, Eq. (10), can now be substituted into Eq. (11) to obtain the residual

$$\begin{aligned} R_N(t) &= \Lambda \frac{d\bar{N}}{dt} - [\rho(t) - \beta(t)]\bar{N}(t) \\ &= \sum_{j=1}^J \lambda_j \left[C_j(t_i) e^{-\lambda_j(t-t_i)} \right. \\ &\quad \left. + \int_{t_i}^t \beta_j(t') \bar{N}(t') e^{-\lambda_j(t-t')} dt' \right]. \end{aligned} \quad (12)$$

The weighting functions $V_k(t)$, $k = 1, \dots, K$, can be used in the following weighted residual method to determine the unknown constants in the vectors A_k :

$$\int_{t_i}^{t_{i+1}} V_r(t) R_N(t) dt = 0, \quad r = 1, \dots, K. \quad (13)$$

Equation (13) is an example of the method of undetermined parameters. Once the parameters in the A_k are determined, the calculation can be repeated over the next time step, with $\bar{N}(t_{i+1})$ now serving as the vector A_0 . Let us now consider the use of piecewise polynomial functions for the $T_k(t)$ to form a trial solution.

The approximate solution to the multimode kinetics equations is a piecewise polynomial if, for each time step considered, the trial solution is expressed as a K th degree polynomial.¹⁵ The coefficients of the polynomials are constant over a time step, but differ from one time step to the next, hence the name "piecewise polynomials."

The best known examples of piecewise polynomials are spline functions¹⁵ of degree M , which are defined as M th degree polynomials over each interval, such that the function and its first $M-1$ derivatives are continuous at each point where the polynomial coefficients change. These points are known as joints. The most commonly used spline functions are cubic splines, i.e., $M = 3$.

Spline functions are not used in the ensuing development. Instead, piecewise polynomial functions are used for which the only continuity requirements demanded are for the amplitude functions, not for any of their derivatives. Such a choice has been very successful in

the past for point kinetics.^{12,13} It is now extended to multimode kinetics. In the time interval $t_i \leq t \leq t_{i+1}$, let the trial solution, Eq. (10), be given by the polynomial

$$\bar{N}(t) = \sum_{k=0}^K A_k(t - t_i)^k. \quad (14)$$

Then,

$$\frac{d\bar{N}}{dt} = \sum_{k=1}^K k A_k(t - t_i)^{k-1}. \quad (15)$$

If the system under analysis is nonlinear, the matrices $\rho(t)$ and $\beta_j(t)$, can be approximated over the time interval by

$$\rho(t) = \rho(t_i) + (t - t_i)(\rho_1 + 2\rho_2 t_i) + \rho_2(t - t_i)^2 \quad (16)$$

$$\beta_j(t) = \beta_j(t_i) + (t - t_i)(\beta_{j1} + 2\beta_{j2} t_i) + \beta_{j2}(t - t_i)^2. \quad (17)$$

The $I \times I$ matrices ρ_1 , ρ_2 , β_{j1} , and β_{j2} are evaluated by fitting results generated from the feedback model to the forms given by Eqs. (16) and (17). The feedback occurs through the matrix operators D_k , A_k , and F_k^T which, for nonlinear systems, are functions of the neutron flux. Therefore, the matrix elements of $\rho(t)$ and $\beta_j(t)$ become functionals of the neutron flux. Equations (14)–(17) can now be substituted into Eq. (13). The result is

$$\begin{aligned} 0 = & \int_{t_i}^{t_{i+1}} \sum_{k=1}^K V_r(t) \{ [k\Lambda(t - t_i)^{k-1} \\ & - \rho(t_i)(t - t_i)^k - (\rho_1 + 2\rho_2 t_i)(t - t_i)^{k+1} \\ & - \rho_2(t - t_i)^{k+2}] A_k \} dt \\ & - \int_{t_i}^{t_{i+1}} V_r(t) \left\{ [\rho(t_i) + (\rho_1 + 2\rho_2 t_i)(t - t_i) \right. \\ & \left. + \rho_2(t - t_i)^2] A_0 - \sum_{j=1}^J \frac{dC_j}{dt} \right\} dt. \end{aligned} \quad (18)$$

Equation (18) yields IK simultaneous algebraic equations, from which the elements of each of the A_k , $k = 1, \dots, K$, can be found. The weighting functions, $V_r(t)$, must be specified before the solution can be obtained. We consider the following three possibilities:

COLLOCATION WEIGHTING

The residual $R_N(t)$ is allowed to vanish at the K points t_r , $r = 1, \dots, K$, all lying within the time interval. The weighting functions are the Dirac delta functions

$$V_r(t) = \delta(t - t_r), \quad r = 1, \dots, K. \quad (19)$$

SUBDOMAIN WEIGHTING

The subdomain method was first used for point kinetics by R. Brittan¹² and J. Kaganove.¹³ The weighting functions are the unit step functions

$$V_r(t) = U(t) - U(t - t_r), \quad r = 1, \dots, K. \quad (20)$$

A good choice for the subdomains is given by

$$t_r = t_i + \frac{(t_{i+1} - t_i)}{2^{r-1}}, \quad r = 1, \dots, K. \quad (21)$$

GALERKIN WEIGHTING

For the Galerkin method, the weighting functions are the trial functions

$$V_r(t) = (t - t_i)^r, \quad r = 1, \dots, K. \quad (22)$$

Consequently, each weighting function is continuous over the entire time interval. The results of substituting the weighting functions shown above into Eq. (18) are given in Ref. 1. The integrals are evaluated in a straightforward manner, but, since the results are rather cumbersome, they are not reproduced here.

SELECTION OF TIME STEP SIZE

When the method of undetermined parameters is used to solve the multimode kinetics equations, methods that regulate the size of the time intervals can be incorporated. Such methods allow large time steps to be taken when the transient is proceeding slowly, and automatically shorten the time step when more rapid changes occur. They also ensure a stable solution for every time step taken. One such method is the time-step halving technique¹³ that is now described.

Consider the time step

$$\Delta t_i = t_{i+1} - t_i, \quad (23)$$

where the solution to the multimode kinetics equations is known at t_i , and desired at t_{i+1} . Two independent solutions to the multimode kinetics equations are now found. The first of these integrates over the whole time step, yielding a solution at $t = t_{i+1}$. The second solution is for only half the step, yielding a solution at

$$t = t_i + \frac{\Delta t_i}{2}. \quad (24)$$

Each solution is found by determining a different set of parameters. The half-step solution is expressed as a different polynomial for each mode. Suppose each of these polynomials is extrapolated to obtain values at $t = t_{i+1}$. These values will not be the same as the found by integration over the whole time step, but they should be close. The degree of closeness of these values determines the next step.

To compare the two sets of values, form the following error norm:

$$\epsilon = \left[\frac{\sum_{k=1}^K [N_{k,ext} - N_k(t_{i+1})]^2}{\sum_{k=1}^K N_{k,ext}^2} \right]^{1/2}, \quad (25)$$

where K is the number of modes, $N_{k,ext}$ is the value of the k th amplitude function calculated for the half-step and extrapolated to the end of the step, and $N_k(t_{i+1})$ is the value of the k th amplitude function from the full-step calculation. The error norm is now compared with a preselected tolerable error ϵ_1 . If $\epsilon < \epsilon_1$, then the amplitude functions $N_k(t_{i+1})$, $k = 1, \dots, K$, are acceptable as the solutions, and the multimode kinetics equations can be integrated over the next time step. If, in addition, $\epsilon < C\epsilon_1$, where C is a preselected parameter less than unity, the size of the next time step is doubled. A good value for C is 0.1.

If, on the other hand, $\epsilon > \epsilon_1$, the values $N_k(t_{i+1})$ are not acceptable, and the time step is halved. Two independent solutions to the multimode kinetics equations are again found, this time for the times $t = t_i + (\Delta t_i/2)$, and $t = t_i + (\Delta t_i/4)$. Note that the first of these has already been made for the original comparison, so that it does not have to be repeated. The solutions are again compared as described above. If acceptable values for

TABLE V-19-I. PARAMETERS FOR CRITICALITY

Parameter	Region 1 (0-15 cm)	Region 2 (15-60 cm)	Region 3 (60-120 cm)
D , cm	1.69531	1.69531	1.69531
$\nu\Sigma_f$, cm^{-1}	0.0194962	0.0194962	0.0194962
Σ_a , cm^{-1}	0.0194962	0.0194962	0.0183343
v , cm/sec	10^6	10^6	10^6

FIG. V-19-1. Shape Functions Used in Analysis of Transient. ANL Neg. No. 113-2690.

TABLE V-19-II. RESULTS AT $t = 1$ sec OF VARIOUS APPROXIMATIONS FOR A 0.1 sec RAMP INSERTION

	Run Time, sec	No. Time Steps	Amplitude Function
Exact (WIGLE)	440	2000	3.94×10^{12}
Point kinetics	4.75	25	4.87×10^4
Two continuous modes (good trial functions)	11.32	90	3.93×10^{12}
Two nodes	7.52	43	1.71×10^6
Five nodes	23.27	44	1.23×10^8
Eight nodes	59.92	45	3.19×10^9
Ten nodes	162.64	90	3.17×10^{11}

$N_k[t_i + (\Delta t_i/2)]$ are found, the solution at $t = t_{i+1}$ will again be attempted in the same manner. This time, however, $t = t_i + (\Delta t_i/2)$ is the lower end of the time step, so that independent solutions are formed for $t = t_i + 3\Delta t_i/4$, and $t = t_{i+1}$.

If the values $N_k[t_i + (\Delta t_i/2)]$ are not acceptable, the time step is halved again and the calculation is repeated for the quarter step. The method continues until an acceptable solution has been found for $t = t_{i+1}$. The procedure then begins anew to determine the $N_k(t_{i+2})$, etc., until the end of the transient is reached.

NUMERICAL RESULTS

The results presented here are primarily intended to show how the speed and accuracy of the undetermined parameter method vary as the number of modes increases. We also wish to demonstrate the capability of the method to obtain an essentially exact solution when a truly good approximate method is used for the spatial integration.

Only one reactor is analyzed in this study. It is a 240 cm infinite slab reactor which, for convenience, is treated in a symmetric manner. That is, a zero current boundary condition is imposed at the origin. One group diffusion theory is used: the parameters for criticality are listed in Table V-19-I and the initial flux shape is shown in Fig. V-19-I. A super-prompt critical transient is analyzed. It is initiated by a ramp change in the fission cross section in Region 1 (0-15 cm from the origin) for 0.1 sec which is then held constant for the remainder of the transient (up to $t = 1$ sec). The final value of $\nu\Sigma_f$ in Region 1 is 0.0202962, corresponding to a total reactivity insertion of \$1.23. One group of delayed neutrons is used, with $\beta = 0.0064$, and $\lambda = 0.08 \text{ sec}^{-1}$. The WIGLE code¹⁶ yielded the final shape, shown in Fig. V-19-1.

Table V-19-II contains the results of analyzing the transient by various space-time approximate methods. In all cases, subdomain weighting is used with quadratic trial functions ($K = 2$) and $\epsilon = 0.01$. The amplitude

functions are characteristic of the reactor as a whole, and are obtained from the approximate solutions by

$$P(t) = \frac{\int_x \psi(x,0) \bar{\phi}(x,t) dx}{\int_x \psi^2(x,0) dx}, \quad (26)$$

where $\psi(x,0)$ is the initial shape. The two-continuous-mode case uses the initial and final shape functions as trial functions and weighting functions in the spatial integration. Since these bracket the true solution, the approximation gives essentially the exact result throughout the transient. The nodal methods use the initial shape in the various regions as the trial and weighting functions. The nodes are coupled through the diffusion theory interface conditions, so that, in effect, each node becomes a mode. The coupling coefficients are proportional to the partial currents at the region interfaces.¹

The run time depends strongly on the number of modes used, because $IK \times IK$ matrices must be inverted at each time step. The matrix inversion routine used is such that the inversion time increases cubically as the order of the matrix increases. It is felt that the situation could be improved from two standpoints. First, a more efficient inversion routine could be used. Second, orthogonal polynomials could be used as the $T_k(t)$ and $V_k(t)$, thus decoupling the calculations for the A_k . The decoupling feature reduces the size of the matrices from $IK \times IK$ to $I \times I$, an especially obvious advantage when many spatial modes are required.

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V-20. The Simulation of Large Mathematical Models of Nuclear Reactors

N. F. MOREHOUSE* and J. C. CARTER

INTRODUCTION

The expense both in time and computing effort rises rapidly when the mathematical model of a reactor consists of more than point kinetics and a single core region. When more than one energy group, more than one region, and the reactor auxiliaries are

included in the model it is necessary to consider the computer on which the model is to be simulated. The types of computers currently available are the digital, the analog, and the hybrid. The hybrid is new and has the potential of accommodating larger dynamic systems.

Two trends appear to be developing: one is toward larger and larger digital computers and the other is

* Applied Mathematics Division, Argonne National Laboratory.

ward hybrid computers wherein computer hardware is traded for numerical analysis, knowledge of computers, and programming skill. Exploration in the field of hybrid computers is motivated by the desire to simulate more comprehensive mathematical models of nuclear reactors in the most effective manner.

A brief description of the hybrid computer at Argonne National Laboratory is followed by a description of the model, the technique of using the hybrid computer, and some results of the simulation of a simple model of a reactor.

DESCRIPTION OF THE HYBRID COMPUTER

Argonne's Hybrid Computer System consists of the pre-existing analog computer and an IBM 1130 digital computer which communicate by means of an interface. The analog computer consists of two Electronic Associates 131-R consoles and two Reeves 550 consoles. The entire system has about 300 amplifiers, although much of the equipment is out of date. The IBM 1130 is equipped with disk bulk storage, a card reader punch, and a line printer. The entire system is either Fortran or Assembly Language programmable at the programmer's option.

Although this system is far from representing the current state of the art, particularly in terms of convenience for the programmer and operator, it does permit a number of hybrid applications to be developed and tested. From the amount of investigating that has been done it appears that many reactor systems can be done effectively on a computer which is composed of both analog and digital components wherein each component is used for the type of equations for which it is best suited.

THE MODEL

The model of a reactor consists of the following categories of phenomena: the neutronic, the thermodynamic, and the mechanical. The equations which define the phenomena of the respective categories have widely varying frequency responses and complex interactions between those variables which are common to each category. These make the attainment of continuity and compatibility in energy, space, and time difficult to achieve in a simultaneous solution of all the equations comprising the mathematical model of the reactor system.

The differencing schemes, particularly with respect to time, present a formidable task in numerical analysis and programming in order to couple the equations in one category to those in another. This

can be reduced when the hybrid system is used because equations with high frequency response can be solved on the analog components.

In the case of a cylindrical reactor core the equations in any section concentric with and perpendicular to the longitudinal axis are similar at any position Z along the longitudinal axis. This suggests dividing the core volume into annular increments $\Delta r \times \Delta z$ and multiplexing. Multiplexing and iteration enables a system of $M \times N$ equations to be solved N times, using initial conditions determined in the preceding Δz for the M equations in each Δz , rather than solving an $M \times N$ matrix.

The characteristics of the equations comprising a model must be analyzed and decisions made as to which parts of the model will be programmed for the digital components and which for the analog components. It is a matter of trading mathematical analysis and knowledge of computers for computer hardware.

A very simple mathematical model of a 1000 MW fast breeder core is presented in an effort to indicate the potentialities of the hybrid computer. This model consists of the two group, space and time dependent neutron diffusion equations, the two dimensional heat transfer equations in three core regions each of thickness Δz , and the feedback equations relating physical changes in the core to the cross sections in the neutron diffusion equations.

With the simulation at steady state a disturbance is made to equilibrium by arbitrarily changing any variable as a function of time. The variations are usually in the form of a jump, a ramp, or a sine wave. A disturbance in any one category—neutronic, thermodynamic, or mechanical—is reflected in the other two categories and creates feedbacks. The neutrons try to redistribute in energy, space, and time in such a manner as to restore equilibrium if the reactor is designed to be a damped system. If it is not a damped system, the reactor will start toward prompt critical.

Figure V-20-1 shows a schematic representation of the basic model. The radial regions are coupled by neutron flux and the axial regions are coupled by flowing sodium.

There is no limitation on increasing the number of energy groups, the number of space increments, or adding reactor auxiliaries to the system, except time and cost.

TECHNIQUE

The system shown on Fig. V-20-1 is in a steady state and the inlet conditions to each radial increment on the plane Z_0 are given. An arbitrary change in any time-dependent variable in any category can be made. The variable in this simulation is the fission cross sections in core region 1.

The iteration scheme used is the same as that used in Ref. 1. In brief, one feedback used on iteration n

FIG. V-20-1. A Schematic Diagram of the Mathematical Model. ANL Neg. No. 113-2635 Rev. 1.

FIG. V-20-2. Response of the Model to a Step in Σf . $W = 0.5$. ANL Neg. No. 113-2639.

is calculated from the temperature distribution computed in run $n - 1$. Initially, the feedback is set equal to zero in most cases, but for large changes in a variable it is set equal to a large enough negative value to prevent the flux, during the first iteration, from increasing excessively.

Since the iteration scheme introduces a fictitious delay equal to the problem solution time for the feedback, there is a possibility that instabilities may be introduced into the reactor system due entirely to the method of computation. Such was found to be the case and a damping method is introduced and described along with the result.

The differential equations representing the model

(Fig. V-20-1 were all programmed on the analog computer, the digital computer being used for storing temperatures on iteration n and computing the resulting feedback to be entered into the analog computer on run $n + 1$.

The particular division of the equations in this model between the analog and digital computer was chosen firstly because it was easy to program and secondly because it was desired to extend the work begun in a previous paper.¹ Actually, it is better to program the temperature equations on the digital computer, leaving only the neutronics on the analog. This would permit faster solution times and permit a more detailed model to be simulated.

RESULTS

This is the first problem to be solved using the present Argonne Hybrid System.

Since the feedback for iteration n was computed from the temperatures in iteration $n-1$, one would expect some instability due to over-correction if the time span of the calculation was long enough. Such an instability was indeed found, as is shown in Fig. V-20-2. One way of overcoming this problem is to put

Feedback run $(n + 1)$

$$= W \times \text{Feedback run } (n - 1)$$

$$+ (1 - W) \text{ Feedback run } (n),$$

where W is a weighting factor. This does not affect the convergence but does damp the functional instability, as is shown in Fig. V-20-3.

FIG. V-20-3. Response of the Model to a Step in Σf . $W = 0.85$. ANL Neg. No. 113-2637.

FIG. V-20-4. Response of the Model to a Ramp in Σf . ANL Neg. No. 113-2636 Rev. 1.

Solutions for a three axial region temperature model are shown in Figs. V-20-4 and V-20-5.

It should be noted that rather long run-times were chosen since one purpose of the calculation was to

FIG. V-20-5. Response of the Model to a Sine Wave in Σf . ANL Neg. No. 113-2638 Rev. 1.

determine if a model which is capable of giving good short-time accuracy in dynamic calculations could also be made stable over rather long time intervals.

CONCLUSIONS

The success of hybrid computers in the simulation of large dynamic systems occurring in the NASA space programs suggests that hybrid computers may provide an equal degree of success in the simulation of comprehensive dynamic systems of nuclear reactors. This first simulation is a very crude one but was done with considerably less hardware than if it were made on an all-digital computer.

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V-21. SIGBAR—A Program for Computing Average Fission Cross Sections in Fast Neutron Spectra

R. GOLD and I. K. OLSON

A computer code, SIGBAR, has been developed for the CDC 3600 to compute average fission cross sections in fast reactor spectra. This code was developed principally to predict fission rates using experimentally measured fast neutron spectra (proton-recoil measurements). It can be used to test fission cross sections in different cross section libraries (e.g.,

ENDF/B) by comparison with independently measured fission rates. In addition, average fission cross sections can also be calculated with MC² generated spectra to provide alternate comparisons between theory and experiment.

Average fission cross sections $\bar{\sigma}_f$, are computed from the customary definition

$$\bar{\sigma}_f = \frac{\int \sigma_f(E) \phi(E) dE}{\int \phi(E) dE}, \quad (1)$$

where $\phi(E)$ is the flux per unit energy at energy E and $\sigma_f(E)$ is the microscopic fission cross-section at energy E . For computational convenience, the lethargy variable μ can be introduced into Eq. (1), yielding

$$\bar{\sigma}_f = \frac{\int \sigma_f(\mu) \phi(\mu) d\mu}{\int \phi(\mu) d\mu}. \quad (2)$$

Four different forms of fast neutron spectra (flux per unity lethargy) have been used:

1. A proton-recoil spectrum obtained from the PSNS code¹ in the energy range of approximately 1 keV to 2 MeV
2. A theoretical spectrum obtained with the MC² code² for the reactor environment representative of the experimental measurement
3. MC² spectrum smoothed by a Gaussian response function
4. A combined experimental and theoretical spectrum, where the Gaussian smoothed theoretical spectrum is used below and above the limits of the experimental measurement, with appropriate area normalization.

For all of the available spectral types described above, the neutron spectrum $\phi(E)$ possesses a histogram form with a constant lethargy interval $d\mu = \Delta$. Consequently Eq. (2) reduces to

$$\bar{\sigma}_f = \frac{\sum_i \sigma_f(\mu_i) \phi(\mu_i) \cdot \Delta}{\sum_i \phi(\mu_i) \cdot \Delta} \quad (3)$$

The cross section value at the energy mesh point E_i is obtained by interpolation. Different interpolation formulas are used depending upon the specific

isotope involved as well as the energy region covered. The fission cross sections and corresponding energy range used are as follows:

Th-232	63 eV to 15 MeV
U-235	63 eV to 27 MeV
U-238	63 eV to 15 MeV
Pu-239	129 eV to 14 MeV

Theoretical and experimental spectra were found in the energy range from 1 keV to 1-2 MeV. Least squares polynomial extrapolations were used to obtain flux estimates over the extended energy range of the fission cross sections.

The Pu-239 data contained negative values below 1 keV. To offset this lack of resonance contribution, an infinite dilution problem was run with the MC² program.² Concentrations of 10^{-20} for Pu-239 and 10^{-2} for carbon were used. The addition of carbon was needed to offset the self-shielding properties of Pu-239. A number of fine group problems were run to obtain a set of 389 cross section values between 0.129 keV and 14 MeV. The Pu-239 data is input from higher to lower energy and is reordered within the program.

Provision has also been made for calculating average fission cross sections from fission track counting data. In this case, the average fission cross section is determined from experimental measurements with Solid State Track Recorders.³ For this calculation, PSNS is used to obtain the experimental value of the integral fast neutron flux.

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-22. NUMQUAD—A Program for Numerical Integration of Experimental Data Using Gauss-Legendre Quadrature

I. K. OLSON and R. GOLD

A computer code, NUMQUAD (in FORTRAN-IV), has been developed for the DDP-24 computer to numerically integrate a set of experimental data points. Gauss-Legendre quadrature is used. An approximation of the integral

$$I = \int_A^B f(x) dx \quad (1)$$

is computed, where the limits A and B are input quantities and $f(x)$ is a representation of the set of experimental data points.

The order N approximation of I , by Gauss-Legendre Quadrature, is given by:¹

$$I = \left(\frac{B-A}{2} \right) \sum_{i=1}^N w_i f(x_i), \quad (2)$$

where the arguments x_i of $f(x)$ are given by

$$x_i = [(B-A)z_i + (A+B)]/2, \quad i = 1, 2, \dots, N, \quad (3)$$

with z_i the zeros of the order N Legendre polynomial and w_i the associated weights. There are N real zeros and weights of the order N Legendre polynomial. Since these zeros and weights are symmetric about the origin, less storage is required. The zeros are stored for the interval $(-1, 1)$ in the form:²

$$\pm ZEWT(I, 1), \quad I = 1, 2, \dots, N/2 \text{ for } N \text{ even,}$$

$$\pm ZEWT(I, 1), \quad I = 1, 2, \dots$$

$$(N-1)/2$$

and

$$ZEWT[(N+1)/2, 1] =$$

$$-ZEWT[(N+1)/2, 1]$$

for N odd.

The error E_N of the N th order approximation is calculated from the formula³

$$E_N = \frac{1}{(2N+1)} \left\{ \frac{(B-A)}{2} [f(B) + f(A)] - \bar{I} - \left(\frac{B-A}{2} \right)^2 \sum_{i=1}^N w_i z_i f'(x_i) \right\}, \quad (4)$$

where

$\bar{I}$ is the integration approximation,
 x_i is as defined in Eq. (3),

TABLE V-22-I. TEST PROBLEM RESULTS

Integral	No. of Data Points Used	No. of Zeros Used	Values
Eq. (5a)	16	12	0.99998 $\pm$ 0.00001
Eq. (5b)	21	10	3.7874 $\pm$ 0.0001
Eq. (5c)	27	15	0.4938 $\pm$ 0.0000

$f'(x_i)$ is the derivative of the interpolation polynomial.

For example, for polynomial interpolation:

$$f'(x_i) = b + 2cx_i + 3dx_i^2 + 4ex_i^3;$$

for logarithmic interpolation:

$$f'(x_i) = y(b + 2cx_i + 3dx_i^2 + 4ex_i^3),$$

where y is the interpolated function value.

In the interpolation of the function f at a given Legendre zero, x_i , the least squares polynomial fit is calculated using those data points closest to the zero. The number of coefficients used may be ≥ 2 , ≤ 5 , or interpolation may be performed between data points.

TEST PROBLEMS

The following three known integrals were tested with this program and the results obtained using NUMQUAD are tabulated in Table V-22-I:

$$\int_0^{\pi/2} \cos(x) dx = 1, \quad (5a)$$

$$\int_0^3 e^{x/2} \cos(x/2) dx = 3.7875, \quad (5b)$$

$$\frac{1}{\sqrt{2\pi}} \int_0^{2.5} e^{-y^2/2} dy = 0.4938 \quad (5c)$$

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Section VI

Publications

July 1, 1968 to June 30, 1969

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