Luminosity

for Proton Debunched Beam

Colliding with Gold Bunched Beam

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The luminosity for a beam of $N_1$ particles colliding with a beam of $N_2$ particles is:

$$L = N_1 N_2 B f_{rev} F$$

where:

- $N_1$ is the total number of particles in beam #1, completely unmodulated.
- $N_2$ is the number of particles per bunch in beam #2, assumed modulated.
- $B$, the number of bunches in beam #2.
- $f_{rev}$, revolution frequency common to both beams.

$$F = \frac{1}{2\pi^2 R} \int_{-\frac{y_2}{2}}^{\frac{y_2}{2}} \frac{-\frac{2}{\sigma_{x_2}^2 \sigma_{y}^2}}{2(\sigma_{x_1}^2 + \sigma_{x_2}^2)} \exp \left( \frac{y^2}{2(\sigma_{x_1}^2 + \sigma_{x_2}^2)} \right) \frac{1}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}} \, dy$$
R is the average radius of both beam circumferences

l is the length of the interaction region where the two beams are visible to each other

\( \alpha \) is the total crossing angle

y is the longitudinal coordinate

\( o_x \) and \( o_y \) are the rms beam dimensions in the transverse directions.

The crossing here is assumed to take place in the x-plane, which we assume to be the horizontal one.

In the approximation that the "diamond" region, that is, the region of most immediate interaction is short, we can take \( o_x \) and \( o_y \) for both beams constant.

In which case we have
\[ F^1 = \frac{1}{\pi \sqrt{2\pi} R \alpha \sqrt{\sigma_2^2 + \sigma_2^2}} \]

We consider the case of gold beam colliding with the proton beam.

Gold beam is bunched at 100 GeV/\(N_2 = 1.1 \times 10^9\)
\(B = 5.7\)

Proton beam is unbunched at 250 GeV/\(N_1 = 57 \times 10^{12}\)

Also
\[
\text{Freq} = 78.2 \text{ KHz}
\]
\(\alpha = 2 \text{ mrad}\)
\(R = 640.2 \text{ m}\)
At the crossing point we take

$$\beta^*_{v} = 0.8 \text{ m}$$
$$\beta^*_{\mu} = 0.9 \text{ m} \quad \text{(not required)}$$
$$\eta^* = 0 \text{ m}$$

Normalized em. Hearn (95% L beam)

Production beam = 20 \pi m m rad
Gold beam = 10 \pi m m rad

$$\sigma_{21} = 0.29 \text{ mm}$$
$$\sigma_{22} = 0.32 \text{ mm}$$

This gives a luminosity

$$L = 0.67 \times 10^{28} \text{ cm}^{-2} \text{ s}^{-1}$$