Optical Design of a Reaction Chamber for Weakly Absorbed Light.

II. Parallel Mirrors, Multitravel

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J. J. Devaney
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ABSTRACT

This report outlines the possibilities to be found using one or more diffraction-limited high-quality light beams to activate a weakly absorbing gas in a regime where the diffraction spread can be controlled by converging optical devices to within a ratio of $\sqrt{2}$ of the minimum at the beam waist (corresponding lengths between converging elements are within twice the Rayleigh range). Our designs use plane or cylindrical parallel mirrors down which a light beam is repeatedly reflected. In the first design variation, the beam is re-reflected up the parallel mirrors to the entrance aperture where it can be returned repeatedly for a number of multiply reflecting "travels" up and down the parallel mirror reaction chamber. In the second variation, the return of the beam after each multiply reflecting "travel" down the chamber is external to the chamber and is achieved by two mirror reflections. For diffraction control the return mirrors can be made converging.

For multiple laser excitation, any of the external return mirrors can be replaced by a laser.

The advantages of these designs are a high degree of uniformity of chamber illumination with a reasonably high number of passes. Drawbacks of the designs are the large space needed for beam return (many tens of meters for some parameters) and (common to all high optical quality chambers) the figuring and reflectivity demands on the mirrors.

I. INTRODUCTION

This is the second in a series of reports dealing with illumination of a reaction chamber by weakly absorbed light which, as a specific example, we will take here to be 0.4-μm wavelength with a small fractional absorption of $\lambda \equiv 3.675 \times 10^{-6}$ cm$^{-1}$. However, our results are general and not limited to these particular values, nor does this work specifically depend on the first report, designated Vol. I.

We suppose an active chamber size of width, $W$, across which we shall pass a beam of light of height, $h$. The length of the chamber is $l$ (see Fig. 1). We place plane or cylindrical mirrors of size $h \times l$ (or slightly larger) at the ends of the chamber at a separation, $W$. We suppose that the major vertical beam divergence is easily controlled by making one mirror (or if $W$ is very large and/or $h/\lambda$ is not large, making both mirrors) cylindrical; see the pertinent discussion in Vol. I. We further suppose that we have unlimited access to certain regions beyond the active chamber in which can be located as many (converging) mirrors as we please.

Under such circumstances we can highly absorb a sufficiently wide diffraction-limited beam with
a high degree of uniformity of illumination within
the chamber. Moreover these ends are achieved with
the simplest optical elements, namely plane elements,
when beam size is large enough, \( \lambda \) small enough,
and/or the beam path short enough to neglect diffrac-
tion, and by cylindrical elements otherwise. As a
consequence the designs in this paper are appropriate
for large cross-section diffraction-limited beams
where plentiful space is available.

II. INTERNAL BACK REFLECTION DESIGN, SINGLE LASER

General

For the moment we take the beam to be uniform
and of constant width \( a \), height \( h \), and leave dif-
fraction and beam shape for later discussion. The
beam is introduced into the chamber at an angle \( \theta \)
so as to just clear the entrance after one reflec-
tion (Fig. 1). \( \theta \) is given by

\[
\sin \theta = \frac{\lambda}{2W}.
\]

(1)

The number of passes, \( n \), across the chamber is then
two for progression \( a' = a/\cos \theta \) down the chamber,
giving a total

\[
n_1 = 2 \left( \frac{\lambda}{a} \right) + 1 = 2 \left( \frac{\lambda \cos \theta}{a} \right) + 1,
\]

(2)

for a chamber of length, \( \lambda \), which must either be
divisible by \( a' \) or else \( \lambda \) is the least number in
the original length which is divisible by \( a' \). In
case any the position of the mirror \( M \) is the least
divisible number, \( \lambda \). In practice the position and
angle of the incident laser beam and the angle \( \phi \)
of \( M \) are adjusted to fit this requirement. The
additional reflection, the 1 in Eq. (2), is the
last pass traversing \( W \) to the mirror \( M \) in Fig. 1.

At the end of the chamber at precisely a mul-
tiple of \( a' \), namely \( \lambda \), we place a mirror, \( M \), of
size \( a' \) or greater, at an angle \( \phi \) to the parallel
mirrors. Later, \( M \) will be made concave to compensate
for diffraction, but for simplicity we now consider
a beam of constant width and a plane mirror will
do. At \( \phi = 0 \) the beam is simply lost beyond the
edge, with less and less lost as \( \phi \) increases. For
\( M \) of width greater than \( a' \), one can obtain multiple
reflections with eventual return of the beam down
its incident path. Alternatively one gets split beams, both returning to the entrance side of the
chamber but at different angles. Neither of these
possibilities is advantageous. We begin with
\( \phi = 0 \) which is the case of normal incidence and
was discussed in Vol. I, paragraph 2a. This choice
gives a total of

\[
n_2 \approx 2 \left( \frac{\lambda}{a} \right) + 1 = 2 \left( \frac{2 \lambda \cos \theta}{a} \right) + 1
\]

(3)

passes and is completely uniform for a beam of con-
stant width and low absorption, each position in
the chamber except the ends receiving precisely
four passes, two originating from the incident side
and two from the reflected side, so compensating
somewhat for any absorption. For sufficient absorp-
tion (cross section, density, and width), or suf-
iciently large \( \lambda \), this is the design of choice.
Note that diffraction limits the improvement in
absorption obtained by making \( a \) as small as we
please.

If the combination of chamber size and cross
section does not give enough absorption from the
double travel of Eq. (3), we can increase the
number of travels by increasing \( \phi \) and including
additional mirrors. We define travel, \( t \), to be
the number of progressions down or up chamber in
the dimension \( \lambda \). To better visualize the geometry,
we expand Fig. 1 by "unfolding" the successive
image spaces of the parallel mirrors, much as a
tailor's mirrors repeatedly reflect the buyer.
Figure 2 shows part of the beam for a travel,
\( t = 4 \); Fig. 4 shows six travel.

Four Travel

In Fig. 2 four travels are achieved by choosing
\( \phi \) such that the first reflection from \( M \) exits at
\( A_2 \) and then placing \( M_2 \) sufficiently beyond \( A_2 \)
(which is also aperture \( A_1 \) in real space) so as
not to interfere with the incident beam and the
laser source. Mirror \( M_2 \) is normal to the beam and
therefore returns it through the entire optical
system giving a total of four travels. However,
exiting at \( A_2 \) rather than at \( A_1 \) gives two passes
less across \( W \) per travel. Consequently the total
number of passes is

\[
n_4 \approx 2n_1 + 2(n_1 - 2) = 8 \lambda/a = 8 \lambda \cos \theta_2/a.
\]

(4)

The beam retains its width, \( a \), throughout so
that at the slightly greater angle \( \theta_2 \) that the
beam has exiting at \( A_2 \), the coverage begins to be
In the gaps, the total intensity drops to 50 or 75% of maximum light intensity. The gaps can be somewhat removed by making the mirror \( M \) slightly convex and the mirror \( M_2 \) slightly concave, but we judge the slight improvement gained thereby not worth the additional complexity except for greater angles \( \theta_2 \) or for multiple laser input. Moreover, when diffraction is included the beam will vary in cross section and the mirrors will in fact be concave to control diffraction. From Fig. 2 the entrance angle \( \theta_1 \) at \( A_1 \) is given by

\[
\tan \theta_1 = \frac{a + (a'/2)}{n_1W} = \frac{a'}{2W},
\]

which checks (1) and (2), but the exit angle \( \theta_2 \) at \( A_2 \) is given by

\[
\tan \theta_2 = \frac{\frac{a + (a'/2)}{(n_1 - 2)W} = \frac{a'}{2W} \left( \frac{2a' + a}{2W - a} \right).}
\]

The normal of the mirror, \( M \), bisects the center lines at \( \theta_1 \) and \( \theta_2 \) so

\[
\phi = \frac{1}{2}(\theta_1 + \theta_2)
\]

sets the mirror \( M \) angle.

In order to know where the mirrors \( M_2 \) and \( M_3 \) in Fig. 2 must be placed, we must calculate the minimum distance \( b \) along the incident beam from the entrance aperture to where the first reflected beam out of the aperture just clears the entrance beam. Indeed, we will need such calculations in general throughout this paper and will therefore solve the problem in general. Consider Fig. 3: two beams of width \( a \) issue from the same aperture at angles \( \alpha \) and \( \beta \). We wish to calculate the distances \( x \), \( h \), and \( b \) which locate the closest point at which the two beams are just entirely separated.

\[
a = a(\alpha) \cos \alpha \quad (9)
\]

\[
x = h \tan \alpha \quad (10)
\]

\[
x + a(\alpha) = h \tan \beta \quad (11)
\]

\[
h = b \cos \alpha \quad (12)
\]

Eliminating \( x \) in Eqs. (10) and (11)

\[
h = a(\alpha)/(\tan \beta - \tan \alpha), \quad (13)
\]

\[
x = a(\alpha)\tan \alpha/(\tan \beta - \tan \alpha), \quad (14)
\]

In the gaps, the total intensity drops to 50 or 75% of maximum light intensity. The gaps can be somewhat removed by making the mirror \( M \) slightly convex and the mirror \( M_2 \) slightly concave, but we judge the slight improvement gained thereby not worth the additional complexity except for greater angles \( \theta_2 \) or for multiple laser input. Moreover, when diffraction is included the beam will vary in cross section and the mirrors will in fact be concave to control diffraction. From Fig. 2 the entrance angle \( \theta_1 \) at \( A_1 \) is given by

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\[
x = a(\alpha)\tan \alpha/(\tan \beta - \tan \alpha), \quad (14)
\]
Fig. 1. Diagram for calculation of the beam separation distance, b.

\[ b = \frac{[a \tan \beta - a']}{a} \] (15)

Applying Eq. (15) to our present case, setting \( \theta_2 = \beta \) and \( \theta_1 = \alpha \), and using (6), (7) and (9) we have

\[ b = \frac{W}{a} (2 \varepsilon - a') \] (16)

\[ a' = \frac{a}{\cos \theta_1} \]

b is the minimum distance at which the normal mirror, \( M_2 \), can be located from the chamber aperture.

The minimum distance of mirror \( M_2 \) from the chamber, b, can be large. Suppose \( a = 0.5 \) cm, \( \varepsilon = 10 \) cm, and \( W = 200 \) cm for instance, then \( b \approx 80 \) m, to be compared with a total travel of \( n_W = 8 \) m/a = 320 m (160 passes) within the chamber. One must therefore definitely control diffraction as discussed below, and must allow for considerable beam room at the chamber entrance. Indeed a 0.5-cm-wide beam has a Rayleigh range of 49 meters and so is too narrow to be used precisely as the foregoing (without additional converging optical elements). However, the 0.5-cm-wide beam can be used in the External Return Design (see Sec. III) for these parameters.

By giving up both some uniformity and some number of passes in the second and third travel, one may shorten the mirror \( M_2 \) distance, b, to the chamber. Thus if (Fig. 2) the beam exits at some aperture \( A_m \) in place of \( A_2 \) the exit angle is now:

\[ \tan \theta_{1/2} = \tan \theta_{23} = \frac{a'}{W} \left( \frac{2 \varepsilon + a'}{2 \varepsilon - 3a'} \right) \] (21)
The angle $\theta_2$ from mirror $M_3$ is precisely $\phi$:
\[ \theta_2 = \phi. \]  
(22)

The minimum distance $b$ is determined from the greater of the distances $b$ of Eq. (15) for the mirror-mirror
$\theta_1$ and $\theta_2$, namely:
\[ b_{31} = a'^2/[a(tan \theta_1 - tan \theta_2)], \]  
(23)
\[ b_{32} = a''^2/[a(tan \theta_2 - tan \theta_3)]. \]  
(24)

Since by (8) and (22), $\theta_3 = (\theta_1 + \theta_2)/2$, $\tan \theta_3 = -a''/a' \cos^2 \phi$, and $a'' = a/cos \phi$, $a' = a/cos \phi$; the
former expression is proportional to $\cos^3 \theta_1$, the
latter to $\cos^3 \theta_2$ and $\theta_2 > \theta_1$ in the first qua-
drant so that $b_{32} > b_{31}$ and therefore $b_{32} = b$ is
the closest distance of the mirrors, $M_2$ and $M_3$.

The precise value $b$ is obtained by substituting
(1) and (21) in (24), a somewhat complicated alge-
braic procedure. We can get a simplified relation
when $a' << a''$, which is usually true, for then the
angles are small enough so we can replace tangents
by their angles. Then
\[ b \approx 2a''^2/[a(\theta_2 - \theta_1)], \]  
(25)
where, to the same accuracy, $a' = a''$ so that:
\[ b \approx \frac{a''}{a}(2\phi - 3\theta_1). \]  
(26)

Compare with the four-travel mirror ($M_4$) distance
given by (16). Again we need plenty of external
beam space for the six travel just as for the four
travel. As in four travel, one may accept some
degradation in uniformity and some loss in total
passes in order to effect a reduction in the dis-
tances of the mirrors $M_2$ and $M_3$ from the chamber.

Higher Travel

In Fig. 4, note that the apertures, $A_m$, are not
equally spaced about a circle whose center is at $M$.
Thus one cannot indefinitely take the apertures $A_m$
to be sufficiently symmetric about the mirror normal
and so continue in the above manner to many travel
designs. For example, the angle $A_6 M A_8$ is obviously
far greater than the angle $A_6 M A_4$. Therefore a
beam through $A_6$ for an $M$ mirror normal at $A_6$
will miss exiting at $A_7$, (indeed also $A_4$), by a large
margin. Consequently higher travel is limited to
$a << t$ and $a << W$. Already that may be sufficient
for some purposes, since, for example, high travel
means a large number of passes and consequent high
transit time, which may not be tolerable for other reasons. However, it is possible to achieve very high travel by the method of external return of the beam which we now study.

II'. EXTERNAL BEAM RETURN DESIGN, SINGLE LASER

In this design the beam is "travelled" down the chamber to a complete exit and is then rerouted externally to reenter the chamber for as many times and thus travels as desired, (within some limitations discussed below).

The advantages of this design are that it yields as many passes as desired, provided the chamber dimensions are sufficiently large, and that the average uniformity of illumination is perfect as in Section II. The limitations of this design as with the internally reflected beam design are the considerable space needed for beam separation and return, and the high-quality beams needed. Figure 5 shows the general plan view with a highly contracted width, \( W \). The upper mirror is of length \( l \), the lower of length \( l + a \). Again the beam is of width \( a \). As before, the angle \( \theta \) is set by the requirement that the beam just clear the entrance upon return after one reflection:

\[
\sin \theta = \frac{a}{2W}.
\]

Again the number of passes is two for every progression \( a' = a \cos \theta \) down the chamber giving a total for the first travel of (Fig. 5)

\[
n'_1 = 2 \left( \frac{a'}{a} + 1 \right) = 2 \left( \frac{\theta \cos \theta}{a} + 1 \right).
\]  

(The prime distinguishes \( n'_1 \) from the passes of the internal-reflection design.) Equation (26) incorporates one additional pass more than (2) because we exit at the top rather than at the bottom. \( \ell \) is required to be divisible by \( a' \), i.e. \( \ell \cos \theta \)/\( a' \) must be an integer. In practice, with a fixed beam width, \( a \), and mirror length, \( \ell \), one adjusts \( \theta \) to make \( \ell/a' \) an integer (see beam \( B_3 \) of Fig. 6).

For the maximum number of passes and travels, one chooses the smallest angle that just permits full clearance of the next beam \( B_2 \) from the first or laser beam \( B_1 \). Let \( m = 1 \) in Fig. 6 and compare \( B_1 \) with \( B_{m+1} \). As before, maximum passes imply the smallest angular difference between beams which implies the greatest distance, \( h_m \), of the mirror \( M_m \), see Figs. 5 and 6. It is clear from Fig. 6 that \( \xi_m \) is determined from

\[
\tan \frac{\xi}{m} = \frac{\ell + a}{n_m W},
\]

where

\[
n'_m = 2 \left( \frac{\ell}{a'} + 2 - m \right),
\]

and

\[
a' = a \cos \frac{\xi}{m},
\]

because for the smallest angle change (\( \theta \)) a beam exiting "cleanly" through the aperture at the bottom of Fig. 6 must leave at the next highest exit which is precisely two passes less. Equations (27), (28) and (29) for \( m = 1 \) give Equation (1).

The maximum \( m \) occurs for

\[
\frac{\ell}{a'} + 2 = m \quad \text{or} \quad m = \frac{\ell}{a'} + 1
\]

since \( n'_1 = 2 \) (Fig. 5). (We omit passes along the flow having but one reflection at the bottom mirror in Fig. 5 although in principle, these can be made as often as desired so \( \bar{n} \) can be made infinite, but these passes are usually an impractical means of illumination.) Thus the total number of passes is

\[
\bar{n}' = \sum_{m=1}^{\frac{\ell}{a'} + 1} n'_m = 2 \sum_{m=1}^{\frac{\ell}{a'} + 1} \left( \frac{\ell}{a} + 2 - m \right),
\]

\[
\bar{n}' = \sum_{m=1}^{\frac{\ell}{a'} + 1} n'_m = 2 \sum_{m=1}^{\frac{\ell}{a'} + 1} \left( \frac{\ell}{a} + 2 - m \right).
\]
Fig. 5. Expanded Fig. 6 (parallel mirrors). Beams $B_1$, $B_{m+1}$ illustrated. $m$ is travel number.

$$n' = \frac{\ell}{a'} + \frac{\ell}{a} + 2$$

This number for $a' = 0.5$ cm, $\ell = 10$ cm is 462 passes, for example, of which the first travel is

$$n'_1 = 42$$

passes of perfect uniformity for a perfect beam, each succeeding travel having two less passes (and therefore gaps unless broadened). The number of travels is 21, requiring a total of 42 (or 22) mirrors of the type $M$ and $M'$.

These mirrors, $M'_m$, must be located at least a distance $b'_m$ from the entrance aperture so as to selectively reflect but one beam. The same distances apply to the exit mirrors, $M'_m$, except that an alternative design is to combine the mirrors $M'_m$ into a single plane mirror $M'$ located near the exit aperture and rely on the mirrors $M_m$ for selection and redirection of the beam.

IV. CHAMBER EXCITATION BY MULTIPLE LASERS

The foregoing designs are immediately applicable to multiple laser excitation of the reaction cavity by simply replacing the mirrors $M_2$ in Fig. 2, $M'_4$ in Fig. 4, etc., or mirrors $M_m$ in Fig. 6, with lasers directing their beams into the chamber. The location of the lasers is given, just as for the mirrors by Equations (13), (14), and (15) (minimum distances). The angle of inputs $\theta_1$ are given by (1), (6), (7), (17), (21), or (27), depending on just what paths the beams are to follow. The discussion of coverage and gaps is identical, except that for higher angles of insertion $\theta_1$ it is easy to prevent gaps in the first travel by widening the laser beam itself by an amount:

$$a_1 = a' \cos \theta_1 / \cos \theta_1$$

However, as Figs. 2 or 4 show (with a laser in place of $M_2$), the internal return mirror, $M$, can only be precisely set for one laser (in size and angle). Incidentally for larger diameter beams, the input angles $\theta_1$ are slightly larger for a clean, i.e., no beam splitting, exit. That angle is given by Equations (27), (28), and (29) with the larger beam width substituted.

Thus for fully equivalent multi-travel paths in the chamber from multiple lasers, it is necessary to use external return. If the widths are varied as Eq. (32), the first travel of each laser beam will illuminate the chamber uniformly and without gaps (assuming negligible diffraction). Later travels of each beam will have gaps (or diffractive overlaps) unless appropriate optics are inserted in the return paths.
V. THE EFFECTS OF DIFFRACTION, NON-PERFECT BEAMS

The foregoing designs are described using a beam with constant cross-section; actual beams expand because of diffraction and imperfect sources. We discuss (to first order) in this section the effect of real beams on the foregoing designs.

Vertical diffraction control by the chamber mirrors has been discussed in Vol I. Because of differing diffraction width versus depth it is possible that one may want to invoke the techniques suggested in that report in addition to the vertical diffraction control obtainable through use of vertically concave mirrors \( M, M_{4}, \) and \( M'_{1} \) of Figs. 1-6. In such a case the reader is referred to Vol. I, or to an optics text. We assume here that sufficient control is possible through the smaller mirrors, \( M, M_{4}, M'_{1} \). Moreover, since the theory is identical vertical or horizontal, we do not even discuss vertical control separately but concentrate on horizontal control exclusively with no loss of generality thereby.

The differing diffraction-limited beams ranging from uniform intensity to soft aperture-truncated to a gaussian intensity profile have the same dominant angular diffraction spread, namely the far field angle \( \theta \), given by

\[
\sin \theta \approx \frac{\lambda}{a},
\]

where \( a \) is the (initial) aperture and \( \lambda \) the wavelength. The differing beams have different constants depending primarily on the ease of defining a spread parameter. Thus, for the position of the first minima from the axis in the Fraunhofer (far field) pattern of an initially uniform beam, (33) is precisely the appropriate relation. Such a beam and equation [(33)] were used in Vol. I. For the gaussian beam, the 1/e intensity position spreads in the far field as \( 2\lambda/\pi a \). The smaller spread of the gaussian is apparent rather than real, for first of all 1/e is not much reduction of the beam magnitude, higher reductions having larger angles; and second, Terrell, Suydam, and Goldstein, et al, have shown that truncated gaussians spread to a greater degree than pure gaussian beams do, and are similar to uniform beams of the same truncation. For high power, soft aperture-modulated beams will likely be used; they may be considered (initially) as intermediate between gaussian and uniform. We note that the far-field pattern of all beams, widely truncated or not,\(^3\) is centrally peaked; i.e., gaussian-like. We are therefore perhaps justified in using a somewhat smaller divergence than that of (33) for the spread of the central peak (\% 89% energy in a one-dimensional uniform beam), but complementarily we may not take the beam as uniform. Of course, Eq. (33) gives the angular spread of just the central peak; for a truncated uniform beam the \( k \)th minimum spreads as

\[
\sin \theta = k\lambda/a.
\]

At the moment these considerations are but second order in importance, so we do approximate the beam as roughly uniform and of divergence given by Eq. (33).

We need to know how far and within what limits a beam can be controlled. That requirement is derived from the relation:

\[
2\omega = 2\omega_{0}\sqrt{1 + \left(\frac{\lambda Z}{\pi a_{0}}\right)^{2}} = 2\omega_{0}\sqrt{1 + \left(\frac{Z}{Z_{R}}\right)^{2}}, \quad (34)
\]

where \( 2\omega \) is the width of the beam, \( 2\omega_{0} \) being the width at the waist, and \( Z \) the distance from the waist. Within a variation of \( \sqrt{2} \) times the width at the waist, (going from \( 2\omega_{0} \) to \( 2\omega_{0} \) to \( 2\omega_{0} \)), the total distance, \( D \), of the beam is two Rayleigh ranges, \( Z_{R} \), to wit:

\[
D = 2Z_{R} = \frac{2\omega_{0}}{\lambda}. \quad (35)
\]

\( D \) is therefore the upper practical bound on the distance that a beam can travel without need of refocusing. Table I gives a listing of such distances as a function of waist diameter, \( 2\omega_{0} \), and wavelength, \( \lambda \).

These are the distances within which a (diffraction-limited) beam may be considered to be controlled. Wherever distances between converging optical devices equal or exceed these distances, the designs of this report are not in general appropriate. See rather Vol. I, re: canted mirror designs.

Between converging mirrors we want the beam restored as much as possible to the original waist, \( 2\omega_{0} \), which acts now as the distance \( Z \) of Figs. 1, 2, 4, 5, and 6. Consequently we require that the radius of curvature of the concave mirrors, \( M, M_{4}, M'_{1} \) of these figures be given by

\[
R(Z) = Z[1 + \left(\frac{\omega_{0}^{2}}{Z}\right)^{2}] = Z[1 + \left(\frac{Z}{Z_{R}}\right)^{2}] \quad (36)
\]
TABLE 1

CONTROLLED (TO WITHIN \( \frac{\lambda}{2} \) OF WAIST) BEAM SPREAD DISTANCE, \( D \) (m)

<table>
<thead>
<tr>
<th>Wavelength, ( \lambda ) (( \mu )m)</th>
<th>Waist Diameter, ( 2\omega_0 ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>24.5 98.2 393. 1570. 6283.</td>
</tr>
<tr>
<td>0.6</td>
<td>16.4 64.4 262. 1047. 4188.</td>
</tr>
<tr>
<td>0.8</td>
<td>12.3 49.1 196. 785. 3142.</td>
</tr>
<tr>
<td>1.0</td>
<td>9.82 39.3 157. 628. 2513.</td>
</tr>
<tr>
<td>2.0</td>
<td>4.91 19.6 78.5 314. 1257.</td>
</tr>
<tr>
<td>4.0</td>
<td>2.45 9.82 39.3 157. 628.</td>
</tr>
<tr>
<td>8.0</td>
<td>1.23 4.91 19.6 78.5 314.</td>
</tr>
<tr>
<td>10.0</td>
<td>0.98 3.93 15.7 62.8 251.</td>
</tr>
<tr>
<td>16.0</td>
<td>0.62 2.45 9.82 39.3 157.</td>
</tr>
</tbody>
</table>

Where, for best control, \( Z \) is half the beam distance from one converging element to the next.

Table 1 gives twice the Rayleigh range, \( 2Z_R \). Thus (Fig. 4) for travel, \( t \approx 6 \), we would choose radii for \( M \) and say \( M'_2 \) from (36), substituting

\[
Z = \frac{1}{2} (b + (n_1 - 2)R);
\]

\( M'_2 \) would then be left planar. Also, where possible the mirrors should considerably exceed our nominal beam width for diffraction control.

At the maximum recommended distance between focusing elements (\( 2Z_R \)), the radius of curvature of a concave element is \( Z_R \) and the beam diameter is \( 2\sqrt{Z} \omega_0 \) leading to a depth of concavity of \( 1/\lambda \). For smaller or larger distances, \( Z \), the radius is larger, Eq. (36). Larger distances than \( 2Z_R \) are unacceptable against diffraction; much smaller do not need converging elements for the diffractive loss is usually negligible. One must also allow for a scattering loss (mirrors, condensate, density variations, etc.) from the beam as well as absorptive and diffractive loss. It is not the purpose of this early study to include all effects to all possible accuracy; rather we present these designs as exploratory for the purposes of comparison with other designs, with the intent to include parameter ranges from pessimistic to optimistic.

We conclude our discussion with an example. Suppose our illumination chamber is 300-cm wide (\( W \) of Figs. 1 or 5), 10-cm long (\( L \)), and 1-cm deep with a beam waist, \( 2\omega_0 \) (\( \lambda/a \)), of 1 cm; \( \theta_1 \), the angle of the beam to the mirror normals is given by (1) to be 0.00167 radians. Our laser must be at least 5 m from the entrance aperture, using Eq. (10), and furnishes a diffraction-limited beam. We expect 22 passes, \( n' \), from Eq. (26) for a total in-chamber distance of 6.6 m plus a return distance to mirror \( M_1 \) of Fig. 5 of greater than 57 m (with or without separate mirrors \( M' \)). The total distance, laser to \( M_1 \), is about 125 m which, for 0.4-\( \mu \)m light, gives an excursion of 1.19 to 1 to 1.19 cm from laser to mirror \( M_1 \). So the beam is within \( \pm 10 \% \) of constant cross section. The total possible number of passes is given by (31) and is 132 for these parameters.

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REFERENCES


6. Concave mirror surfaces can be figured to as little as 50 \( \AA \) and then uniform coatings can be applied; D. Morelli, OCLI, private communication (November 1974).